Computer algebra independent integration tests

6-Hyperbolic-functions/6.1-Hyperbolic-sine/6.1.1-c+d-x-^m-a+b-sinh-^n

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3.246 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 985
3.247 $\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ . . . 989
3.248 $\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 991
3.249 $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 997
3.250 $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1007
3.251 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1014
3.252 $\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ . . . 1018
3.253 $\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1021
3.254 $\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1025
3.255 $\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1029
3.256 $\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1032
3.257 $\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$ . . . 1035
3.258 $\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$ . . . 1037
3.259 $\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1039

3.260 $\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1043

3.261 $\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1047

3.262 $\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1050

3.263 $\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$ . . . 1053

3.264 $\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$ . . . 1056

3.265 $\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1059

3.266 $\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1064

3.267 $\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1068

3.268 $\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1071

3.269 $\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$ . . . 1074

3.270 $\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$ . . . 1078

3.271 $\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1082

3.272 $\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1088

3.273 $\int \frac{(e+fx)\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1093

3.274 $\int \frac{\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1097

3.275 $\int \frac{\text{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$ . . . 1100

3.276 $\int \frac{\text{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$ . . . 1103

3.277 $\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1106

3.278 $\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1112

3.279 $\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1117

3.280 $\int \frac{\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1121

3.281 $\int \frac{\text{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$ . . . 1124

3.282 $\int \frac{\text{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$ . . . 1127

3.283 $\int \frac{(e+fx)^3\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1130

3.284 $\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1138

3.285 $\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1144

3.286 $\int \frac{\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$ . . . 1149

3.287 $\int \frac{\text{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$ . . . 1152

3.288 $\int \frac{\text{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$ . . . 1156

3.289 $\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1160

3.290 $\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1164

3.291 $\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1168

3.292 $\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1171

3.293 $\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ . . . 1174

3.294 $\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1176

3.295 $\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1182

3.296 $\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1187

3.297 $\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1191
3.298 $\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 1195
3.299 $\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1197
3.300 $\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1204
3.301 $\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1211
3.302 $\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1216
3.303 $\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 1219
3.304 $\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1221
3.305 $\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1226
3.306 $\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1231
3.307 $\int \frac{\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1236
3.308 $\int \frac{\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 1239
3.309 $\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1241
3.310 $\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1249
3.311 $\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1256
3.312 $\int \frac{\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1261
3.313 $\int \frac{\text{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 1265
3.314 $\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1267
3.315 $\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1278
3.316 $\int \frac{\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1286
3.317 $\int \frac{\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 1290
3.318 $\int \frac{x^m\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1293
3.319 $\int \frac{x^m\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1295
3.320 $\int \frac{x^m\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1297
3.321 $\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$ 1299
3.322 $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$ 1302
3.323 $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$ 1306
3.324 $\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$ 1311
3.325 $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$ 1314
3.326 $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$ 1318
3.327 $\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$ 1323
3.328 $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$ 1327
3.329 $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$ 1334
3.330 $\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$ 1343
3.331 $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$ 1347
3.332 $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$ 1354
3.333 $\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1363

3.334 $\int \frac{(e+fx)^2 \cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1368
3.335 $\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1373
3.336 $\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1377
3.337 $\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 1380
3.338 $\int \frac{(e+fx)^3 \cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1383
3.339 $\int \frac{(e+fx)^2 \cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1391
3.340 $\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1398
3.341 $\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1404
3.342 $\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 1408
3.343 $\int \frac{(e+fx)^3 \cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1411
3.344 $\int \frac{(e+fx)^2 \cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1420
3.345 $\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1429
3.346 $\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1435
3.347 $\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 1439
3.348 $\int \frac{(e+fx)^3 \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1442
3.349 $\int \frac{(e+fx)^2 \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1449
3.350 $\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1455
3.351 $\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1460
3.352 $\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 1463
3.353 $\int \frac{(e+fx)^3 \operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1465
3.354 $\int \frac{(e+fx)^2 \operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1473
3.355 $\int \frac{(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1480
3.356 $\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1486
3.357 $\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 1490
3.358 $\int \frac{(e+fx)^2 \operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1493
3.359 $\int \frac{(e+fx)\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1505
3.360 $\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1513
3.361 $\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 1517
3.362 $\int \frac{(e+fx)^3 \cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1520
3.363 $\int \frac{(e+fx)^2 \cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1528
3.364 $\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1534
3.365 $\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1539
3.366 $\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 1542
3.367 $\int \frac{(e+fx)^3 \cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1545
3.368 $\int \frac{(e+fx)^2 \cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1554
3.369 $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1561
3.370 $\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1567

3.371 $\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ . . . 1572
3.372 $\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1575
3.373 $\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1586
3.374 $\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1594
3.375 $\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1601
3.376 $\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ . . . 1605
3.377 $\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1608
3.378 $\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1616
3.379 $\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1623
3.380 $\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1629
3.381 $\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ . . . 1632
3.382 $\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1635
3.383 $\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1644
3.384 $\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1651
3.385 $\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1657
3.386 $\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ . . . 1661
3.387 $\int \frac{(e+fx)^2\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1663
3.388 $\int \frac{(e+fx)\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1675
3.389 $\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1683
3.390 $\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ . . . 1687
3.391 $\int \frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1690
3.392 $\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1698
3.393 $\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1706
3.394 $\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1712
3.395 $\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ . . . 1715
3.396 $\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1718
3.397 $\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1728
3.398 $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1736
3.399 $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1744
3.400 $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ . . . 1749
3.401 $\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1752
3.402 $\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1759
3.403 $\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1770
3.404 $\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1779
3.405 $\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ . . . 1783
3.406 $\int \frac{(e+fx)^3\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 1786

3.407 $\int \frac{(e+fx)^2 \sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1796
3.408 $\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1804
3.409 $\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1812
3.410 $\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 1816
3.411 $\int \frac{(e+fx)^3 \sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1819
3.412 $\int \frac{(e+fx)^2 \sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1829
3.413 $\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1838
3.414 $\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1845
3.415 $\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 1849
3.416 $\int \frac{(e+fx)^2 \tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1852
3.417 $\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1864
3.418 $\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1872
3.419 $\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 1876
3.420 $\int \frac{(e+fx)^3 \coth(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1879
3.421 $\int \frac{(e+fx)^2 \coth(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1885
3.422 $\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1890
3.423 $\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1894
3.424 $\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 1897
3.425 $\int \frac{(e+fx)^3 \cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1899
3.426 $\int \frac{(e+fx)^2 \cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1905
3.427 $\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1910
3.428 $\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1915
3.429 $\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 1919
3.430 $\int \frac{(e+fx)^3 \cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1922
3.431 $\int \frac{(e+fx)^2 \cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1930
3.432 $\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1936
3.433 $\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1941
3.434 $\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 1944
3.435 $\int \frac{(e+fx)^3 \text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1947
3.436 $\int \frac{(e+fx)^2 \text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1955
3.437 $\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1963
3.438 $\int \frac{\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1969
3.439 $\int \frac{\text{csch}(c+dx)\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 1973
3.440 $\int \frac{(e+fx)^3 \text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1975
3.441 $\int \frac{(e+fx)^2 \text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1986
3.442 $\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 1995
3.443 $\int \frac{\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 2002

3.444 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ . . . 2007
3.445 $\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2010
3.446 $\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2019
3.447 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2029
3.448 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ . . . 2034
3.449 $\int \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2037
3.450 $\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2044
3.451 $\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2050
3.452 $\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2055
3.453 $\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ . . . 2058
3.454 $\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2061
3.455 $\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2069
3.456 $\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2076
3.457 $\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2082
3.458 $\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ . . . 2086
3.459 $\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2088
3.460 $\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2096
3.461 $\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2103
3.462 $\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2109
3.463 $\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ . . . 2112
3.464 $\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2115
3.465 $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2126
3.466 $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2136
3.467 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2143
3.468 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ . . . 2147
3.469 $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2150
3.470 $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2162
3.471 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2170
3.472 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ . . . 2175
3.473 $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2178
3.474 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2187
3.475 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ . . . 2192
3.476 $\int \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2195
3.477 $\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2205
3.478 $\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2214
3.479 $\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ . . . 2220

3.480 $\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 2223
3.481 $\int \frac{(e+fx)^3\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$ 2226
3.482 $\int \frac{(e+fx)^2\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$ 2238
3.483 $\int \frac{(e+fx)\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$ 2248
3.484 $\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$ 2255
3.485 $\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 2260
3.486 $\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ 2263
3.487 $\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ 2270
3.488 $\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ 2280
3.489 $\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ 2287
3.490 $\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 2290
3.491 $\int \frac{(e+fx)^3\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$ 2293
3.492 $\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$ 2301
3.493 $\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$ 2315
3.494 $\int \frac{\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$ 2325
3.495 $\int \frac{\text{csch}^3(c+dx)\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 2329
3.496 $\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 2332
3.497 $\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 2342
3.498 $\int \frac{\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$ 2354
3.499 $\int \frac{\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 2360
3.500 $\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ 2363
3.501 $\int \frac{\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$ 2373
3.502 $\int \frac{\text{csch}^3(c+dx)\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$ 2378

4 Listing of Grading functions **2381**

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [502]. This is test number [160].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (502)	% 0. (0)
Mathematica	% 92.63 (465)	% 7.37 (37)
Maple	% 67.93 (341)	% 32.07 (161)
Maxima	% 51.39 (258)	% 48.61 (244)
Fricas	% 90.44 (454)	% 9.56 (48)
Sympy	% 18.73 (94)	% 81.27 (408)
Giac	% 39.04 (196)	% 60.96 (306)

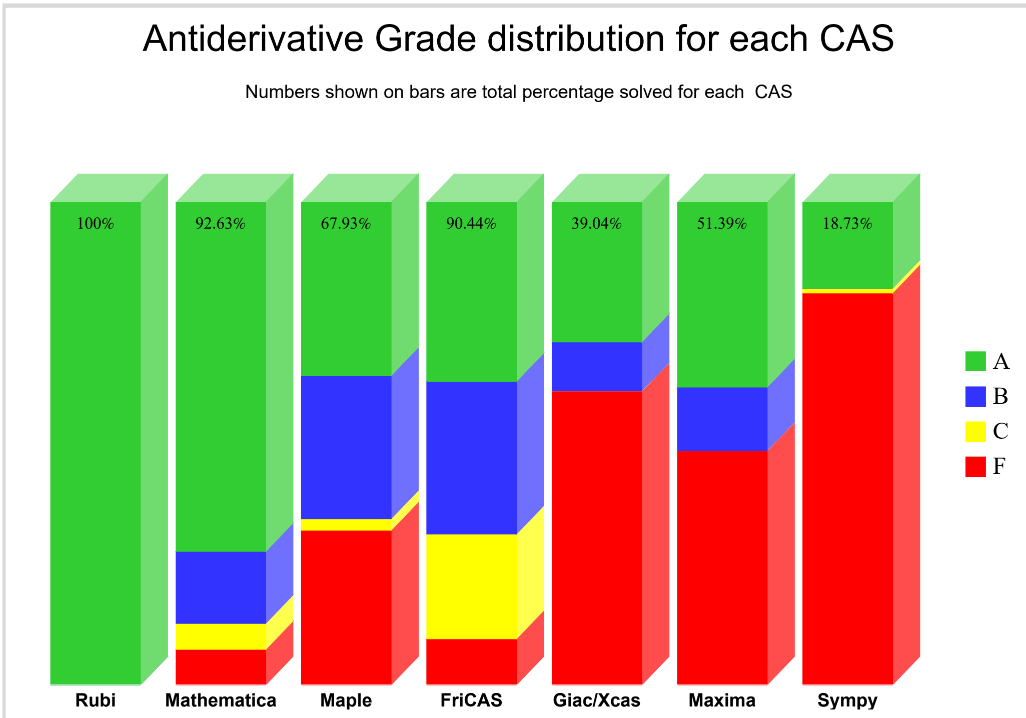
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

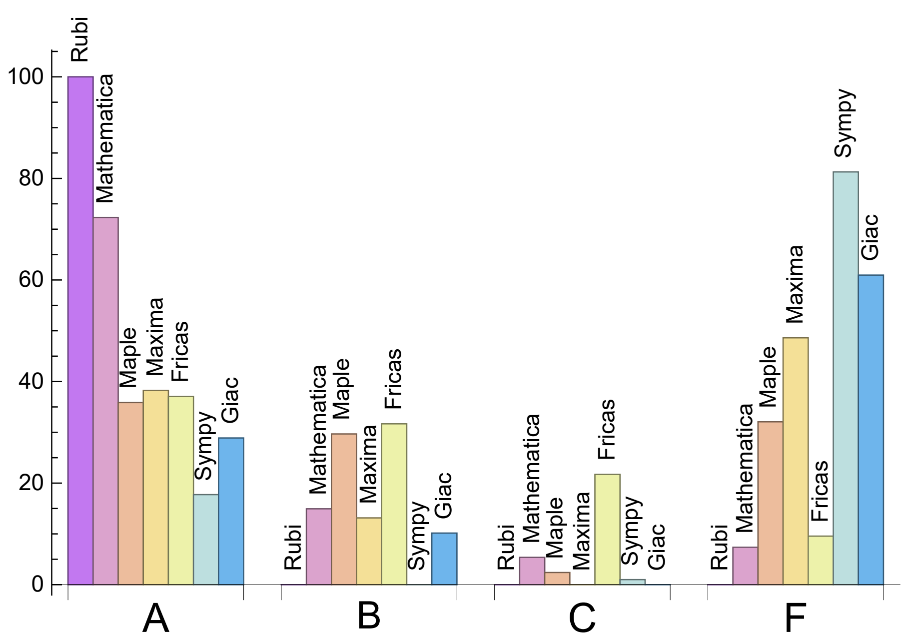
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	72.31	14.94	5.38	7.37
Maple	35.86	29.68	2.39	32.07
Maxima	38.25	13.15	0.	48.61
Fricas	37.05	31.67	21.71	9.56
Sympy	17.73	0.	1.	81.27
Giac	28.88	10.16	0.	60.96

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.46	267.92	0.79	139.	1.
Mathematica	12.22	787.06	1.65	173.	0.93
Maple	0.26	387.95	1.7	150.	1.61
Maxima	0.89	183.91	1.31	105.	1.21
Fricas	2.28	3413.25	8.39	881.5	5.6
Sympy	6.65	156.88	1.46	32.5	0.9
Giac	0.86	187.32	1.69	128.	1.82

1.4 list of integrals that has no closed form antiderivative

{26, 27, 31, 32, 36, 37, 65, 66, 67, 72, 76, 77, 111, 112, 116, 117, 139, 140, 144, 145, 149, 150, 151, 155, 156, 172, 173, 176, 177, 179, 180, 181, 185, 186, 191, 192, 197, 198, 203, 204, 209, 210, 215, 216, 221, 222, 227, 232, 237, 242, 247, 252, 257, 258, 275, 276, 281, 282, 287, 288, 293, 298, 303, 308, 313, 317, 318, 319, 320, 337, 342, 347, 352, 357, 361, 366, 371, 376, 381, 386, 390, 395, 400, 405, 410, 415, 419, 424, 429, 434, 439, 444, 448, 453, 458, 463, 468, 472, 475, 480, 485, 490, 495, 499, 502}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {23, 29, 45, 205, 217, 238, 248, 271, 277, 283, 284, 300, 306, 310, 314, 328, 329, 332, 338, 339, 340, 343, 344, 350, 354, 358, 362, 363, 372, 373, 379, 383, 387, 391, 392, 396, 397, 398, 401, 402, 407, 408, 412, 416, 425, 430, 435, 436, 440, 441, 445, 449, 450, 459, 460, 464, 465, 466, 469, 470, 476, 477, 481, 486, 487, 491, 492, 493, 496, 497, 500}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
```

```
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima abs_integrate was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

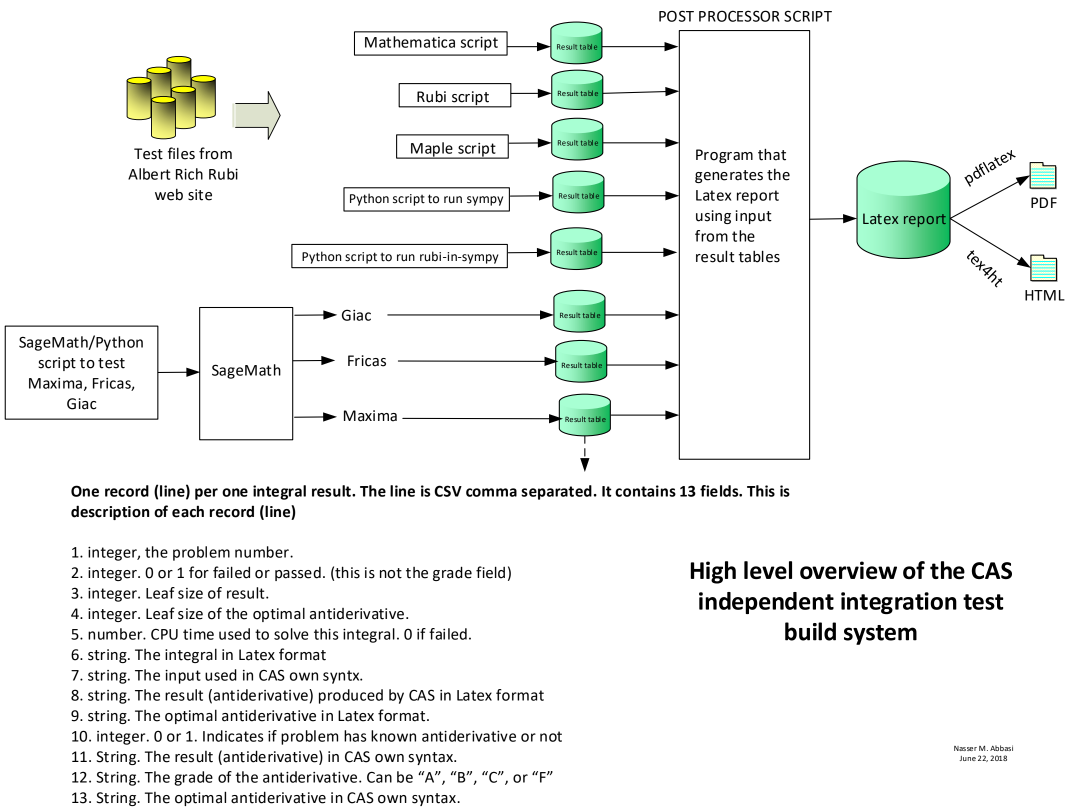
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 52, 53, 54, 55, 56, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 194, 195, 196, 199, 201, 202, 205, 206, 208, 209, 210, 214, 215, 220, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 274, 275, 278, 279, 280, 281, 286, 287, 289, 290, 291, 292, 293, 294, 295, 296, 298, 301, 302, 303, 306, 307, 308, 309, 310, 311, 312, 313, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 330, 335, 336, 337, 341, 345, 346, 350, 351, 352, 353, 354, 355, 356, 357, 359, 360, 361, 364, 365, 367, 368, 369, 370, 374, 375, 379, 382, 383, 384, 385, 388, 393, 394, 399, 403, 404, 406, 408, 411, 412, 413, 414, 417, 422, 423, 424, 425, 426, 427, 428, 429, 432, 433, 439, 440, 441, 442, 443, 446, 447, 448, 451, 452, 454, 455, 456, 457, 461, 462, 466, 467, 468, 471, 472, 473, 478, 479, 484, 488, 489, 493, 494, 495, 498 }

B grade: { 34, 49, 51, 57, 59, 110, 130, 135, 189, 193, 200, 207, 211, 212, 213, 217, 218, 219, 248, 249, 262, 273, 277, 283, 284, 285, 299, 300, 304, 305, 314, 328, 329, 331, 332, 333, 334, 343, 344, 348, 349, 358, 362, 363, 372, 373, 377, 378, 387, 391, 392, 401, 402, 407, 416, 420, 421, 430, 431, 435, 436, 437, 445, 450, 460, 464, 465, 469, 477, 482, 487, 491, 492, 496, 500 }

C grade: { 25, 29, 35, 71, 250, 297, 338, 339, 340, 380, 389, 396, 397, 398, 409, 418, 438, 449, 459, 470, 474, 476, 481, 483, 486, 497, 501 }

F grade: { 197, 198, 203, 204, 216, 221, 222, 237, 252, 276, 282, 288, 342, 347, 366, 371, 376, 381, 386, 390, 395, 400, 405, 410, 415, 419, 434, 444, 453, 458, 463, 475, 480, 485, 490, 499, 502}

2.1.3 Maple

A grade: { 4, 5, 6, 12, 13, 19, 20, 21, 25, 26, 27, 30, 31, 32, 36, 37, 65, 66, 67, 72, 76, 77, 99, 100, 104, 105, 106, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 139, 140, 144, 145, 149, 150, 151, 155, 156, 160, 161, 165, 166, 167, 172, 173, 176, 177, 179, 180, 181, 185, 186, 189, 190, 191, 192, 195, 197, 198, 200, 201, 203, 204, 207, 208, 209, 210, 214, 215, 216, 220, 221, 222, 226, 227, 231, 232, 235, 237, 241, 242, 246, 247, 251, 252, 256, 257, 258, 261, 263, 264, 266, 267, 268, 269, 270, 273, 275, 276, 278, 279, 280, 281, 282, 287, 288, 292, 293, 298, 303, 307, 308, 312, 313, 317, 318, 319, 320, 336, 337, 342, 347, 351, 352, 356, 357, 361, 365, 366, 371, 376, 381, 385, 386, 390, 394, 395, 400, 405, 410, 414, 415, 419, 423, 424, 429, 434, 438, 439, 443, 444, 448, 452, 453, 458, 463, 467, 468, 471, 472, 475, 479, 480, 484, 485, 490, 494, 495, 498, 499, 502 }

B grade: { 1, 2, 3, 7, 8, 9, 10, 11, 14, 15, 16, 17, 18, 22, 23, 24, 28, 29, 33, 34, 35, 96, 97, 98, 101, 102, 103, 107, 108, 109, 113, 157, 158, 159, 162, 163, 164, 168, 171, 175, 178, 187, 188, 193, 194, 196, 199, 202, 205, 206, 211, 212, 213, 217, 218, 219, 225, 230, 236, 240, 245, 250, 253, 254, 255, 259, 260, 262, 265, 271, 272, 274, 277, 283, 284, 285, 286, 291, 296, 297, 301, 302, 306, 311, 315, 316, 321, 322, 324, 325, 327, 328, 330, 331, 335, 340, 341, 345, 346, 350, 355, 359, 360, 364, 369, 370, 374, 375, 379, 380, 384, 388, 389, 393, 398, 399, 403, 404, 408, 409, 413, 417, 418, 422, 427, 428, 432, 433, 437, 442, 446, 447, 451, 456, 457, 461, 462, 466, 470, 473, 474, 478, 483, 488, 489, 493, 497, 500, 501 }

C grade: { 60, 61, 62, 63, 64, 78, 79, 80, 81, 82, 83, 84 }

F grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 68, 69, 70, 71, 73, 74, 75, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 152, 153, 154, 169, 170, 174, 182, 183, 184, 223, 224, 228, 229, 233, 234, 238, 239, 243, 244, 248, 249, 289, 290, 294, 295, 299, 300, 304, 305, 309, 310, 314, 323, 326, 329, 332, 333, 334, 338, 339, 343, 344, 348, 349, 353, 354, 358, 362, 363, 367, 368, 372, 373, 377, 378, 382, 383, 387, 391, 392, 396, 397, 401, 402, 406, 407, 411, 412, 416, 420, 421, 425, 426, 430, 431, 435, 436, 440, 441, 445, 449, 450, 454, 455, 459, 460, 464, 465, 469, 476, 477, 481, 482, 486, 487, 491, 492, 496 }

2.1.4 Maxima

A grade: { 5, 6, 7, 10, 11, 12, 13, 14, 15, 20, 21, 22, 26, 27, 31, 32, 36, 37, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 63, 64, 65, 66, 67, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 98, 99, 100, 101, 104, 105, 106, 107, 110, 111, 112, 116, 117, 139, 140, 144, 145, 149, 150, 151, 155, 156, 159, 160, 161, 162, 164, 165, 166, 167, 168, 172, 173, 176, 177, 179, 180, 181, 185, 186, 189, 190, 191, 192, 196, 197, 198, 202, 206, 208, 209, 210, 215, 216, 220, 221, 222, 227, 232, 237, 242, 247, 252, 254, 256, 257, 258, 263, 264, 268, 271, 272, 275, 276, 277, 279, 281, 282, 287, 288, 292, 293, 298, 303, 307, 308, 313, 316, 317, 318, 319, 320, 337, 342, 347, 351, 352, 357, 360, 361, 366, 371, 376, 380, 381, 386, 389, 390, 395, 400, 405, 409, 410, 415, 418, 419, 424, 429, 434, 438, 439, 444, 447, 448, 453, 458, 463, 467, 468, 472, 474, 475, 480, 485, 490, 494, 495, 499, 502

B grade: { 1, 2, 3, 4, 8, 9, 16, 17, 18, 19, 23, 24, 28, 30, 33, 34, 38, 39, 40, 41, 60, 61, 62, 96, 97, 102, 103, 108, 113, 115, 157, 158, 163, 187, 193, 195, 205, 211, 212, 214, 217, 218, 253, 259, 260, 261, 262, 274, 280, 283, 284, 286, 302, 336, 346, 365, 375, 394, 404, 423, 433, 452, 462, 479, 489, 501 }

C grade: { }

F grade: { 25, 29, 35, 68, 69, 70, 71, 74, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 109, 114, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 152, 153, 154, 169, 170, 171, 174, 175, 178, 182, 183, 184, 188, 194, 199, 200, 201, 203, 204, 207, 213, 219, 223, 224, 225, 226, 228, 229, 230, 231, 233, 234, 235, 236, 238, 239, 240, 241, 243, 244, 245, 246, 248, 249, 250, 251, 255, 265, 266, 267, 269, 270, 273, 278, 285, 289, 290, 291, 294, 295, 296, 297, 299, 300, 301, 304, 305, 306, 309, 310, 311, 312, 314, 315, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 338, 339, 340, 341, 343, 344, 345, 348, 349, 350, 353, 354, 355, 356, 358, 359, 362, 363, 364, 367, 368, 369, 370, 372, 373, 374, 377, 378, 379, 382, 383, 384, 385, 387, 388, 391, 392, 393, 396, 397, 398, 399, 401, 402, 403, 406, 407, 408, 411, 412, 413, 414, 416, 417, 420, 421, 422, 425, 426, 427, 428, 430, 431, 432, 435, 436, 437, 440, 441, 442, 443, 445, 446, 449, 450, 451, 454, 455, 456, 457, 459, 460, 461, 464, 465, 466, 469, 470, 471, 473, 476, 477, 478, 481, 482, 483, 484, 486, 487, 488, 491, 492, 493, 496, 497, 498, 500 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 9, 10, 11, 12, 18, 19, 20, 26, 27, 31, 32, 36, 37, 41, 48, 56, 62, 65, 66, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 99, 100, 101, 104, 105, 106, 110, 111, 112, 115, 116, 117, 139, 140, 144, 145, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 172, 173, 176, 177, 179, 180, 181, 182, 183, 184, 185, 186, 189, 190, 191, 192, 195, 196, 197, 198, 201, 202, 203, 204, 208, 209, 210, 215, 216, 221, 222, 227, 232, 237, 242, 247, 252, 255, 256, 257, 258, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 275, 276, 279, 280, 281, 282, 287, 288, 293, 298, 303, 306, 307, 308, 313, 317, 318, 319, 320, 337, 342, 347, 350, 351, 352, 357, 361, 366, 371, 376, 379, 380, 381, 386, 390, 395, 400, 405, 410, 415, 419, 423, 424, 429, 434, 438, 439, 444, 453, 458, 463, 468, 472, 480, 485, 490, 499 }

B grade: { 6, 7, 8, 13, 14, 15, 16, 17, 21, 22, 25, 29, 30, 35, 38, 39, 40, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 63, 64, 69, 96, 97, 102, 103, 107, 109, 114, 162, 168, 171, 175, 178, 188, 194, 200, 207, 213, 214, 219, 220, 225, 226, 230, 231, 235, 236, 240, 241, 245, 246, 250, 251, 259, 260, 273, 274, 278, 285, 286, 291, 292, 296, 297, 301, 302, 311, 312, 315, 316, 321, 322, 324, 325, 327, 328, 330, 331, 335, 336, 340, 341, 345, 346, 355, 356, 359, 360, 364, 365, 369, 370, 374, 375, 384, 385, 388, 389, 393, 394, 398, 399, 403, 404, 408, 409, 413, 414, 417, 418, 422, 427, 428, 432, 433, 437, 442, 443, 446, 447, 451, 452, 456, 457, 461, 462, 466, 467, 470, 471, 474, 478, 479, 483, 484, 488, 489, 493, 494, 497, 498, 501 }

C grade: { 23, 24, 28, 33, 34, 108, 113, 169, 170, 174, 187, 193, 199, 205, 206, 211, 212, 217, 218, 223, 224, 228, 229, 233, 234, 238, 239, 243, 244, 249, 253, 254, 271, 272, 277, 283, 284, 289, 290, 294, 295, 299, 300, 304, 305, 309, 310, 314, 323, 326, 329, 332, 333, 334, 338, 339, 343, 344, 348, 349, 353, 354, 358, 362, 363, 367, 368, 372, 373, 377, 378, 382, 383, 387, 391, 392, 396, 397, 402, 406, 407, 411, 412, 416, 420, 421, 425, 426, 430, 431, 435, 436, 440, 441, 449, 450, 454, 455, 459, 460, 464, 465, 469, 476, 477, 481, 482, 487, 492 }

F grade: { 67, 68, 70, 71, 92, 93, 94, 95, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 150, 248, 401, 445, 448, 473, 475, 486, 491, 495, 496, 500, 502 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 8, 9, 10, 11, 16, 17, 18, 19, 26, 27, 31, 32, 36, 37, 65, 66, 67, 72, 76, 77, 96, 97, 98, 102, 103, 104, 110, 115, 139, 140, 144, 145, 150, 157, 158, 159, 163, 164, 165, 185, 186, 190, 195, 196, 202, 226, 242, 247, 256, 259, 260, 261, 262, 265, 266, 267, 268, 275, 281, 292, 308, 313, 317, 318, 319, 320, 336, 352, 357, 361, 365, 381, 386, 390, 394, 410, 415, 419, 424, 429, 434, 453, 458, 463, 490 }

B grade: { }

C grade: { 60, 61, 62, 63, 64 }

F grade: { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 28, 29, 30, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 68, 69, 70, 71, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 105, 106, 107, 108, 109, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 187, 188, 189, 191, 192, 193, 194, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 263, 264, 269, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 309, 310, 311, 312, 314, 315, 316, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 355, 356, 358, 359, 360, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 382, 383, 384, 385, 387, 388, 389, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 411, 412, 413, 414, 416, 417, 418, 420, 421, 422, 423, 425, 426, 427, 428, 430, 431, 432, 433, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 454, 455, 456, 457, 459, 460, 461, 462, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502 }

2.1.7 Giac

A grade: { 4, 5, 10, 11, 12, 19, 20, 26, 27, 31, 32, 36, 37, 38, 39, 40, 41, 60, 61, 62, 65, 66, 67, 72, 76, 77, 98, 99, 100, 104, 105, 110, 111, 112, 115, 116, 117, 139, 140, 144, 145, 149, 150, 151, 155, 156, 159, 160, 161, 165, 166, 167, 172, 173, 176, 177, 179, 180, 181, 185, 186, 190, 191, 192, 196, 197, 198, 202, 203, 204, 208, 209, 214, 220, 226, 227, 231, 232, 236, 237, 241, 246, 251, 256, 257, 258, 262, 263, 268, 269, 275, 276, 279, 280, 281, 292, 293, 297, 298, 302, 303, 307, 308, 312, 318, 319, 320, 336, 337, 341, 342, 346, 347, 351, 356, 360, 365, 366, 370, 371, 376, 380, 385, 389, 394, 395, 399, 400, 405, 409, 414, 418, 423, 424, 433, 434, 438, 439, 443, 452, 462, 471, 479, 490, 498 }

B grade: { 1, 2, 3, 6, 7, 8, 9, 14, 15, 16, 17, 18, 21, 22, 30, 96, 97, 101, 102, 103, 106, 107, 157, 158, 162, 163, 164, 168, 189, 195, 201, 259, 260, 261, 265, 266, 267, 274, 286, 316, 375, 404, 428, 447, 457, 467, 474, 484, 489, 494, 501 }

C grade: { }

F grade: { 13, 23, 24, 25, 28, 29, 33, 34, 35, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 63, 64, 68, 69, 70, 71, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 108, 109, 113, 114, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 152, 153, 154, 169, 170, 171, 174, 175, 178, 182, 183, 184, 187, 188, 193, 194, 199, 200, 205, 206, 207, 210, 211, 212, 213, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 228, 229, 230, 233, 234, 235, 238, 239, 240, 242, 243, 244, 245, 247, 248, 249, 250, 252, 253, 254, 255, 264, 270, 271, 272, 273, 277, 278, 282, 283, 284, 285, 287, 288, 289, 290, 291, 294, 295, 296, 299, 300, 301, 304, 305, 306, 309, 310, 311, 313, 314, 315, 317, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 338, 339, 340, 343, 344, 345, 348, 349, 350, 352, 353, 354, 355, 357, 358, 359, 361, 362, 363, 364, 367, 368, 369, 372, 373, 374, 377, 378, 379, 381, 382, 383, 384, 386, 387, 388, 390, 391, 392, 393, 396, 397, 398, 401, 402, 403, 406, 407, 408, 410, 411, 412, 413, 415, 416, 417, 419, 420, 421, 422, 425, 426, 427, 429, 430, 431, 432, 435, 436, 437, 440, 441, 442, 444, 445, 446, 448, 449, 450, 451, 453, 454, 455, 456, 458, 459, 460, 461, 463, 464, 465, 466, 468, 469, 470, 472, 473, 475, 476, 477, 478, 480, 481, 482, 483, 485, 486, 487, 488, 491, 492, 493, 495, 496, 497, 499, 500, 502 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	76	547	440	350	311	437
normalized size	1	1.	0.84	6.01	4.84	3.85	3.42	4.8
time (sec)	N/A	0.12	0.347	0.033	1.123	2.599	3.237	1.168

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	61	308	300	231	202	275
normalized size	1	1.	0.87	4.4	4.29	3.3	2.89	3.93
time (sec)	N/A	0.079	0.22	0.007	1.059	2.537	1.502	1.2

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	44	147	181	139	112	151
normalized size	1	1.	0.9	3.	3.69	2.84	2.29	3.08
time (sec)	N/A	0.05	0.157	0.006	1.118	2.561	0.722	1.167

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	53	92	72	46	62
normalized size	1	1.	0.96	1.89	3.29	2.57	1.64	2.21
time (sec)	N/A	0.02	0.063	0.004	1.088	2.529	0.265	1.148

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	49	82	77	193	0	77
normalized size	1	1.	0.96	1.61	1.51	3.78	0.	1.51
time (sec)	N/A	0.109	0.085	0.059	1.289	2.58	0.	1.157

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	65	133	108	315	0	197
normalized size	1	1.	0.92	1.87	1.52	4.44	0.	2.77
time (sec)	N/A	0.127	0.238	0.033	1.386	2.678	0.	1.275

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	88	277	127	518	0	406
normalized size	1	1.	0.85	2.66	1.22	4.98	0.	3.9
time (sec)	N/A	0.166	0.558	0.039	1.384	2.61	0.	1.23

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	132	910	516	663	660	505
normalized size	1	1.	0.81	5.62	3.19	4.09	4.07	3.12
time (sec)	N/A	0.105	0.681	0.016	1.257	2.683	7.344	1.24

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	104	523	355	459	456	328
normalized size	1	1.	0.78	3.9	2.65	3.43	3.4	2.45
time (sec)	N/A	0.074	0.403	0.007	1.146	2.702	3.8	1.258

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	75	262	223	288	264	184
normalized size	1	1.	0.79	2.76	2.35	3.03	2.78	1.94
time (sec)	N/A	0.054	0.311	0.007	1.14	2.693	1.771	1.18

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	52	103	119	165	126	85
normalized size	1	1.	0.95	1.87	2.16	3.	2.29	1.55
time (sec)	N/A	0.026	0.162	0.006	1.218	2.626	0.8	1.219

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	66	97	97	232	0	92
normalized size	1	1.	0.85	1.24	1.24	2.97	0.	1.18
time (sec)	N/A	0.165	0.122	0.074	1.384	2.64	0.	1.2

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	75	152	119	365	0	0
normalized size	1	1.	0.93	1.88	1.47	4.51	0.	0.
time (sec)	N/A	0.155	0.444	0.072	1.389	2.721	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	102	299	134	597	0	446
normalized size	1	1.	0.91	2.67	1.2	5.33	0.	3.98
time (sec)	N/A	0.195	0.916	0.076	1.477	2.714	0.	1.158

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	123	555	149	857	0	725
normalized size	1	1.	0.76	3.43	0.92	5.29	0.	4.48
time (sec)	N/A	0.187	0.888	0.118	1.515	2.739	0.	1.156

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	150	1217	863	1141	772	883
normalized size	1	1.	0.67	5.41	3.84	5.07	3.43	3.92
time (sec)	N/A	0.36	1.019	0.156	1.354	2.822	12.508	1.208

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	127	676	587	759	495	559
normalized size	1	1.	0.73	3.86	3.35	4.34	2.83	3.19
time (sec)	N/A	0.226	0.992	0.007	1.377	2.578	6.796	1.23

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	86	320	363	466	284	311
normalized size	1	1.	0.7	2.6	2.95	3.79	2.31	2.53
time (sec)	N/A	0.131	0.404	0.007	1.357	2.625	3.839	1.183

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	59	115	190	255	126	132
normalized size	1	1.	0.79	1.53	2.53	3.4	1.68	1.76
time (sec)	N/A	0.058	0.183	0.007	1.227	2.625	1.82	1.186

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	102	166	158	400	0	153
normalized size	1	1.	0.84	1.37	1.31	3.31	0.	1.26
time (sec)	N/A	0.282	0.239	0.083	1.393	2.531	0.	1.201

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	160	271	196	662	0	400
normalized size	1	1.	1.1	1.87	1.35	4.57	0.	2.76
time (sec)	N/A	0.262	1.341	0.091	1.387	2.696	0.	1.246

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	220	562	196	1123	0	811
normalized size	1	1.	1.2	3.05	1.07	6.1	0.	4.41
time (sec)	N/A	0.425	0.843	0.089	1.6	2.665	0.	1.186

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	191	541	450	986	0	0
normalized size	1	1.	1.28	3.63	3.02	6.62	0.	0.
time (sec)	N/A	0.136	2.94	0.072	1.507	2.788	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	118	306	263	635	0	0
normalized size	1	1.	1.19	3.09	2.66	6.41	0.	0.
time (sec)	N/A	0.09	2.299	0.03	1.549	2.644	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	174	60	0	340	0	0
normalized size	1	1.	3.48	1.2	0.	6.8	0.	0.
time (sec)	N/A	0.046	0.079	0.004	0.	2.635	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	11.29	0.051	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	11.125	0.034	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	185	473	432	2689	0	0
normalized size	1	1.	1.8	4.59	4.19	26.11	0.	0.
time (sec)	N/A	0.227	2.298	0.059	2.	2.853	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	74	198	240	0	1521	0	0
normalized size	1	1.	2.68	3.24	0.	20.55	0.	0.
time (sec)	N/A	0.148	5.399	0.03	0.	2.79	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	52	56	123	431	0	108
normalized size	1	1.	1.79	1.93	4.24	14.86	0.	3.72
time (sec)	N/A	0.03	0.099	0.02	1.143	2.577	0.	1.317

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	17.713	0.061	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	18.673	0.072	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	440	876	817	8979	0	0
normalized size	1	1.	1.72	3.42	3.19	35.07	0.	0.
time (sec)	N/A	0.274	10.083	0.06	1.974	3.475	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	420	444	531	5252	0	0
normalized size	1	1.	2.73	2.88	3.45	34.1	0.	0.
time (sec)	N/A	0.166	10.747	0.044	1.843	3.169	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	313	197	0	2677	0	0
normalized size	1	1.	3.4	2.14	0.	29.1	0.	0.
time (sec)	N/A	0.081	2.674	0.033	0.	2.838	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	74.269	0.484	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	79.6	0.81	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	108	0	416	1179	0	313
normalized size	1	1.	0.63	0.	2.43	6.89	0.	1.83
time (sec)	N/A	0.37	0.059	0.033	1.226	2.752	0.	1.372

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	106	0	362	894	0	273
normalized size	1	1.	0.73	0.	2.48	6.12	0.	1.87
time (sec)	N/A	0.248	0.095	0.026	1.193	2.643	0.	1.335

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	104	0	311	718	0	227
normalized size	1	1.	0.85	0.	2.53	5.84	0.	1.85
time (sec)	N/A	0.187	0.078	0.023	1.156	2.734	0.	1.275

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	244	273	0	124
normalized size	1	1.	1.	0.	2.35	2.62	0.	1.19
time (sec)	N/A	0.135	0.038	0.03	1.151	2.629	0.	1.217

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	120	0	139	814	0	0
normalized size	1	1.	1.02	0.	1.18	6.9	0.	0.
time (sec)	N/A	0.2	0.149	0.023	1.111	2.768	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	161	0	154	1226	0	0
normalized size	1	1.	1.08	0.	1.03	8.23	0.	0.
time (sec)	N/A	0.251	0.754	0.025	1.336	2.787	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	168	0	154	1820	0	0
normalized size	1	1.	0.97	0.	0.89	10.46	0.	0.
time (sec)	N/A	0.308	0.572	0.026	1.345	2.955	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	239	239	190	0	379	2319	0	0
normalized size	1	1.	0.79	0.	1.59	9.7	0.	0.
time (sec)	N/A	0.449	6.591	0.062	1.749	3.198	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	163	0	323	1789	0	0
normalized size	1	1.	0.77	0.	1.53	8.48	0.	0.
time (sec)	N/A	0.321	2.297	0.058	1.723	3.315	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	129	0	255	1438	0	0
normalized size	1	1.	0.78	0.	1.54	8.66	0.	0.
time (sec)	N/A	0.296	0.579	0.055	1.629	2.912	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	142	0	144	369	0	0
normalized size	1	1.	1.02	0.	1.04	2.65	0.	0.
time (sec)	N/A	0.241	0.13	0.062	1.681	2.824	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	570	0	157	1442	0	0
normalized size	1	1.	4.01	0.	1.11	10.15	0.	0.
time (sec)	N/A	0.253	4.858	0.066	1.3	2.856	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	156	0	159	2053	0	0
normalized size	1	1.	0.9	0.	0.91	11.8	0.	0.
time (sec)	N/A	0.321	1.215	0.066	1.309	3.016	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	825	0	159	2974	0	0
normalized size	1	1.	3.75	0.	0.72	13.52	0.	0.
time (sec)	N/A	0.322	9.187	0.067	1.316	3.084	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	222	0	159	3951	0	0
normalized size	1	1.	0.88	0.	0.63	15.74	0.	0.
time (sec)	N/A	0.395	0.563	0.069	1.294	3.576	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	243	0	693	4817	0	0
normalized size	1	1.	0.64	0.	1.82	12.64	0.	0.
time (sec)	N/A	1.066	9.446	0.082	1.826	3.426	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	243	0	581	3622	0	0
normalized size	1	1.	0.75	0.	1.79	11.14	0.	0.
time (sec)	N/A	0.799	3.891	0.078	1.793	3.301	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	209	0	450	2966	0	0
normalized size	1	1.	0.76	0.	1.64	10.79	0.	0.
time (sec)	N/A	0.522	0.291	0.078	1.886	2.998	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	191	0	240	595	0	0
normalized size	1	1.	0.84	0.	1.05	2.61	0.	0.
time (sec)	N/A	0.401	0.194	0.095	1.701	2.725	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	2058	0	266	3312	0	0
normalized size	1	1.	8.37	0.	1.08	13.46	0.	0.
time (sec)	N/A	0.454	10.265	0.088	1.473	3.014	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	253	0	265	4826	0	0
normalized size	1	1.	0.91	0.	0.96	17.42	0.	0.
time (sec)	N/A	0.673	3.061	0.084	1.428	3.153	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	3211	0	266	7120	0	0
normalized size	1	1.	9.7	0.	0.8	21.51	0.	0.
time (sec)	N/A	0.775	18.188	0.089	1.425	3.811	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	50	132	236	467	133	194
normalized size	1	1.	0.45	1.19	2.13	4.21	1.2	1.75
time (sec)	N/A	0.159	0.013	0.052	1.151	2.752	152.835	1.36

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	49	120	201	356	99	136
normalized size	1	1.	0.53	1.3	2.18	3.87	1.08	1.48
time (sec)	N/A	0.11	0.016	0.024	1.225	2.735	3.05	1.404

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	47	71	157	138	70	82
normalized size	1	1.	0.61	0.92	2.04	1.79	0.91	1.06
time (sec)	N/A	0.076	0.009	0.022	1.218	2.636	1.477	1.314

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	49	120	100	355	94	0
normalized size	1	1.	0.56	1.38	1.15	4.08	1.08	0.
time (sec)	N/A	0.112	0.02	0.022	1.136	2.699	7.424	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	84	132	77	454	129	0
normalized size	1	1.	0.74	1.16	0.68	3.98	1.13	0.
time (sec)	N/A	0.149	0.083	0.024	1.187	2.648	175.807	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	23.482	0.042	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	21.786	0.043	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	6.015	0.033	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	17	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.113	0.075	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	0	0	375	0	0
normalized size	1	1.	0.92	0.	0.	15.62	0.	0.
time (sec)	N/A	0.06	0.071	0.069	0.	2.621	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	33	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.117	0.092	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	68	0	0	0	0	0
normalized size	1	1.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	1.242	0.073	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	2.994	0.063	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	206	0	217	799	0	0
normalized size	1	1.	0.87	0.	0.92	3.37	0.	0.
time (sec)	N/A	0.318	0.198	0.089	1.546	2.921	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	131	0	0	597	0	0
normalized size	1	1.	0.91	0.	0.	4.15	0.	0.
time (sec)	N/A	0.197	0.168	0.074	0.	2.692	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	101	0	107	377	0	0
normalized size	1	1.	0.92	0.	0.97	3.43	0.	0.
time (sec)	N/A	0.093	0.055	0.043	1.282	2.779	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	6.427	0.029	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	3.779	0.033	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	74	258	0	0
normalized size	1	1.	0.92	1.24	1.25	4.37	0.	0.
time (sec)	N/A	0.078	0.026	0.042	1.231	2.79	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	53	73	74	258	0	0
normalized size	1	1.	0.9	1.24	1.25	4.37	0.	0.
time (sec)	N/A	0.073	0.019	0.048	1.34	2.733	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	74	258	0	0
normalized size	1	1.	0.92	1.24	1.25	4.37	0.	0.
time (sec)	N/A	0.074	0.022	0.04	1.299	2.744	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	53	73	74	225	0	0
normalized size	1	1.	0.9	1.24	1.25	3.81	0.	0.
time (sec)	N/A	0.07	0.017	0.034	1.319	2.705	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	67	58	236	0	0
normalized size	1	1.	1.	1.37	1.18	4.82	0.	0.
time (sec)	N/A	0.069	0.022	0.041	1.217	2.693	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	51	67	74	258	0	0
normalized size	1	1.	0.93	1.22	1.35	4.69	0.	0.
time (sec)	N/A	0.069	0.018	0.046	1.285	2.688	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	71	74	258	0	0
normalized size	1	1.	0.92	1.2	1.25	4.37	0.	0.
time (sec)	N/A	0.071	0.022	0.03	1.325	2.698	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	79	0	0	427	0	0
normalized size	1	1.	0.92	0.	0.	4.97	0.	0.
time (sec)	N/A	0.159	0.127	0.046	0.	2.748	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	78	0	0	427	0	0
normalized size	1	1.	0.92	0.	0.	5.02	0.	0.
time (sec)	N/A	0.136	0.123	0.044	0.	2.759	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	79	0	0	427	0	0
normalized size	1	1.	0.92	0.	0.	4.97	0.	0.
time (sec)	N/A	0.14	0.13	0.076	0.	2.773	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	76	0	0	375	0	0
normalized size	1	1.	0.89	0.	0.	4.41	0.	0.
time (sec)	N/A	0.127	0.105	0.053	0.	2.704	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	63	0	0	365	0	0
normalized size	1	1.	0.88	0.	0.	5.07	0.	0.
time (sec)	N/A	0.127	0.076	0.069	0.	2.754	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	72	0	0	424	0	0
normalized size	1	1.	0.87	0.	0.	5.11	0.	0.
time (sec)	N/A	0.137	0.1	0.046	0.	2.796	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	427	0	0
normalized size	1	1.	0.92	0.	0.	5.08	0.	0.
time (sec)	N/A	0.143	0.112	0.049	0.	2.781	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	0.094	0.077	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	0.124	0.073	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	45	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.13	0.11	0.081	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	63	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.207	0.144	0.071	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	128	494	317	656	573	356
normalized size	1	1.	1.31	5.04	3.23	6.69	5.85	3.63
time (sec)	N/A	0.14	0.832	0.01	1.099	2.787	2.23	1.297

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	88	249	190	402	350	205
normalized size	1	1.	1.19	3.36	2.57	5.43	4.73	2.77
time (sec)	N/A	0.097	0.472	0.01	1.065	2.648	1.532	1.263

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	48	96	89	192	177	96
normalized size	1	1.	0.96	1.92	1.78	3.84	3.54	1.92
time (sec)	N/A	0.053	0.084	0.009	1.043	2.632	0.944	1.297

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	60	96	96	162	0	96
normalized size	1	1.	0.86	1.37	1.37	2.31	0.	1.37
time (sec)	N/A	0.161	0.295	0.045	1.208	2.773	0.	1.291

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	83	153	119	290	0	225
normalized size	1	1.	0.87	1.61	1.25	3.05	0.	2.37
time (sec)	N/A	0.189	0.488	0.053	1.221	2.788	0.	1.284

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	109	303	134	471	0	451
normalized size	1	1.	0.83	2.31	1.02	3.6	0.	3.44
time (sec)	N/A	0.232	0.669	0.06	1.22	2.73	0.	1.348

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	220	1082	709	1283	1153	788
normalized size	1	1.	0.9	4.42	2.89	5.24	4.71	3.22
time (sec)	N/A	0.287	1.49	0.018	1.152	3.273	4.628	1.346

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	189	550	440	768	706	455
normalized size	1	1.	1.09	3.16	2.53	4.41	4.06	2.61
time (sec)	N/A	0.197	0.713	0.014	1.269	3.045	3.206	1.26

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	86	215	225	389	364	215
normalized size	1	1.	0.7	1.76	1.84	3.19	2.98	1.76
time (sec)	N/A	0.105	1.209	0.015	1.186	2.956	2.064	1.24

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	117	193	203	309	0	188
normalized size	1	1.	0.79	1.3	1.36	2.07	0.	1.26
time (sec)	N/A	0.352	0.395	0.119	1.388	2.811	0.	1.334

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	214	313	247	581	0	485
normalized size	1	1.	1.26	1.84	1.45	3.42	0.	2.85
time (sec)	N/A	0.34	0.661	0.141	1.395	2.708	0.	2.11

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	198	625	277	959	0	953
normalized size	1	1.	0.84	2.65	1.17	4.06	0.	4.04
time (sec)	N/A	0.53	2.355	0.154	1.397	2.905	0.	1.401

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	206	435	320	863	0	0
normalized size	1	1.	1.56	3.3	2.42	6.54	0.	0.
time (sec)	N/A	0.302	2.894	0.112	1.683	2.727	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	150	227	0	483	0	0
normalized size	1	1.	1.49	2.25	0.	4.78	0.	0.
time (sec)	N/A	0.219	2.219	0.056	0.	2.697	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	185	66	101	151	56	97
normalized size	1	1.	2.94	1.05	1.6	2.4	0.89	1.54
time (sec)	N/A	0.074	0.451	0.056	1.068	3.194	0.69	1.217

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	20.624	0.098	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	21.087	0.164	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	443	723	856	2147	0	0
normalized size	1	1.	1.45	2.37	2.81	7.04	0.	0.
time (sec)	N/A	0.398	6.247	0.141	1.999	2.604	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	269	374	0	1162	0	0
normalized size	1	1.	1.12	1.55	0.	4.82	0.	0.
time (sec)	N/A	0.278	3.557	0.113	0.	2.504	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	241	113	346	398	173	285
normalized size	1	1.	1.53	0.72	2.19	2.52	1.09	1.8
time (sec)	N/A	0.109	1.081	0.103	1.196	2.539	8.021	1.325

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	36.176	0.823	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	37.907	1.103	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	141	174	0	0	0	0
normalized size	1	1.	0.78	0.96	0.	0.	0.	0.
time (sec)	N/A	0.212	0.236	0.087	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	125	151	0	0	0	0
normalized size	1	1.	0.92	1.11	0.	0.	0.	0.
time (sec)	N/A	0.172	0.3	0.062	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	105	128	0	0	0	0
normalized size	1	1.	0.95	1.15	0.	0.	0.	0.
time (sec)	N/A	0.141	0.224	0.06	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	87	105	0	0	0	0
normalized size	1	1.	1.32	1.59	0.	0.	0.	0.
time (sec)	N/A	0.075	0.164	0.057	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	96	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	0.159	0.305	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	133	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.175	0.254	0.062	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	170	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.198	0.34	0.059	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	269	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.351	1.403	0.046	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	173	0	0	0	0	0
normalized size	1	1.	0.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.251	1.125	0.046	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	138	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	0.776	0.043	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	146	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.273	0.785	0.045	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	243	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.29	0.974	0.046	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	638	638	2918	0	0	0	0	0
normalized size	1	1.	4.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.64	7.439	0.063	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	506	506	300	0	0	0	0	0
normalized size	1	1.	0.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.39	1.807	0.046	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	218	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.208	1.387	0.046	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	242	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.418	1.497	0.046	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	444	444	347	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.437	2.434	0.046	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	536	536	4751	0	0	0	0	0
normalized size	1	1.	8.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.638	7.686	0.044	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	493	493	331	0	0	0	0	0
normalized size	1	1.	0.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.261	1.27	0.201	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	276	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.202	0.974	0.069	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	221	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.505	0.066	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	3.843	0.052	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	3.952	0.053	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	807	807	546	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.438	3.239	0.047	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	506	506	384	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.311	1.764	0.047	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	332	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.169	0.853	0.046	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	21.625	0.046	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	24.086	0.044	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1016	1016	1200	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.677	4.532	0.046	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	689	689	482	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.46	2.284	0.048	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	411	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.243	1.577	0.045	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	35.33	0.046	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	3.972	0.073	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	3.916	0.048	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	339	0	0	857	0	0
normalized size	1	1.	0.83	0.	0.	2.09	0.	0.
time (sec)	N/A	0.602	1.585	0.12	0.	2.832	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	229	0	0	581	0	0
normalized size	1	1.	0.85	0.	0.	2.17	0.	0.
time (sec)	N/A	0.361	0.799	0.101	0.	2.617	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	207	0	0	301	0	0
normalized size	1	1.	1.53	0.	0.	2.23	0.	0.
time (sec)	N/A	0.154	0.56	0.056	0.	2.542	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	4.278	0.034	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	15.745	0.058	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	123	482	316	365	264	351
normalized size	1	1.	1.38	5.42	3.55	4.1	2.97	3.94
time (sec)	N/A	0.143	0.451	0.01	1.312	2.424	1.933	1.267

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	83	240	188	231	151	200
normalized size	1	1.	1.24	3.58	2.81	3.45	2.25	2.99
time (sec)	N/A	0.096	0.309	0.007	1.186	2.456	0.88	1.328

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	91	88	128	68	89
normalized size	1	1.	0.96	2.02	1.96	2.84	1.51	1.98
time (sec)	N/A	0.048	0.111	0.007	1.302	2.46	0.365	1.281

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	57	94	96	230	0	95
normalized size	1	1.	0.89	1.47	1.5	3.59	0.	1.48
time (sec)	N/A	0.124	0.144	0.023	1.425	2.494	0.	1.249

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	71	149	119	351	0	225
normalized size	1	1.	0.82	1.71	1.37	4.03	0.	2.59
time (sec)	N/A	0.169	0.366	0.022	1.455	2.471	0.	1.307

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	95	296	134	572	0	441
normalized size	1	1.	0.77	2.41	1.09	4.65	0.	3.59
time (sec)	N/A	0.205	0.639	0.024	1.466	2.458	0.	1.247

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	235	1061	702	880	779	813
normalized size	1	1.	0.94	4.24	2.81	3.52	3.12	3.25
time (sec)	N/A	0.29	1.383	0.014	1.334	2.495	5.313	1.263

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	249	535	435	541	456	470
normalized size	1	1.	1.37	2.94	2.39	2.97	2.51	2.58
time (sec)	N/A	0.202	0.753	0.016	1.271	2.479	2.357	1.296

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	98	208	221	294	219	220
normalized size	1	1.	0.84	1.79	1.91	2.53	1.89	1.9
time (sec)	N/A	0.106	0.745	0.014	1.255	2.3	0.935	1.237

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	134	201	200	483	0	200
normalized size	1	1.	0.86	1.29	1.28	3.1	0.	1.28
time (sec)	N/A	0.325	0.298	0.108	1.593	2.507	0.	1.294

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	232	319	244	778	0	487
normalized size	1	1.	1.27	1.74	1.33	4.25	0.	2.66
time (sec)	N/A	0.35	0.642	0.128	1.502	2.564	0.	2.087

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	395	626	274	1237	0	948
normalized size	1	1.	1.63	2.59	1.13	5.11	0.	3.92
time (sec)	N/A	0.447	0.96	0.135	1.587	2.47	0.	1.31

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	404	318	0	0	2412	0	0
normalized size	1	1.	0.79	0.	0.	5.97	0.	0.
time (sec)	N/A	0.826	0.261	0.168	0.	2.721	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	233	0	0	1733	0	0
normalized size	1	1.	0.79	0.	0.	5.85	0.	0.
time (sec)	N/A	0.669	0.156	0.12	0.	2.52	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	142	393	0	1127	0	0
normalized size	1	1.	0.76	2.1	0.	6.03	0.	0.
time (sec)	N/A	0.369	0.037	0.054	0.	2.545	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.944	0.036	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.939	0.036	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	549	549	428	0	0	8910	0	0
normalized size	1	1.	0.78	0.	0.	16.23	0.	0.
time (sec)	N/A	1.037	1.935	0.29	0.	3.528	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	194	519	0	4073	0	0
normalized size	1	1.	0.76	2.04	0.	16.04	0.	0.
time (sec)	N/A	0.442	1.075	0.103	0.	3.115	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	53.355	0.283	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	55.645	0.47	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	544	544	836	1232	0	13728	0	0
normalized size	1	1.	1.54	2.26	0.	25.24	0.	0.
time (sec)	N/A	2.081	9.854	0.192	0.	3.711	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	104.981	0.61	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	98.068	0.79	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	4.448	0.042	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	543	448	0	0	1885	0	0
normalized size	1	1.	0.83	0.	0.	3.47	0.	0.
time (sec)	N/A	0.807	1.747	0.108	0.	2.967	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	254	0	0	1191	0	0
normalized size	1	1.	0.9	0.	0.	4.24	0.	0.
time (sec)	N/A	0.375	0.764	0.089	0.	2.493	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	118	0	0	585	0	0
normalized size	1	1.	0.9	0.	0.	4.47	0.	0.
time (sec)	N/A	0.148	0.192	0.046	0.	2.731	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	1.194	0.03	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	5.302	0.068	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	232	501	428	1096	0	0
normalized size	1	1.	1.42	3.07	2.63	6.72	0.	0.
time (sec)	N/A	0.356	3.276	0.103	1.731	2.483	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	188	269	0	649	0	0
normalized size	1	1.	1.45	2.07	0.	4.99	0.	0.
time (sec)	N/A	0.272	2.455	0.069	0.	2.464	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	239	86	146	235	0	180
normalized size	1	1.	2.66	0.96	1.62	2.61	0.	2.
time (sec)	N/A	0.112	0.637	0.079	1.267	2.538	0.	1.215

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	61	67	49	78	24	46
normalized size	1	1.	1.74	1.91	1.4	2.23	0.69	1.31
time (sec)	N/A	0.044	0.207	0.026	1.152	2.322	0.353	1.235

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	48.143	0.091	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	41.715	0.102	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	857	688	906	1904	0	0
normalized size	1	1.	3.56	2.85	3.76	7.9	0.	0.
time (sec)	N/A	0.526	6.595	0.183	1.983	2.628	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	260	374	0	1123	0	0
normalized size	1	1.	1.41	2.03	0.	6.1	0.	0.
time (sec)	N/A	0.391	3.46	0.13	0.	2.723	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	238	134	325	416	449	367
normalized size	1	1.	2.	1.13	2.73	3.5	3.77	3.08
time (sec)	N/A	0.177	1.089	0.106	1.341	2.666	2.36	1.528

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	59	107	100	178	102	89
normalized size	1	1.	1.13	2.06	1.92	3.42	1.96	1.71
time (sec)	N/A	0.089	0.221	0.036	1.009	2.463	0.748	1.411

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	180.001	0.128	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	180.002	0.218	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	376	928	0	2449	0	0
normalized size	1	1.	0.96	2.36	0.	6.23	0.	0.
time (sec)	N/A	0.702	7.311	0.155	0.	2.782	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	1661	508	0	1419	0	0
normalized size	1	1.	5.79	1.77	0.	4.94	0.	0.
time (sec)	N/A	0.547	5.136	0.128	0.	2.678	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	325	197	0	586	0	479
normalized size	1	1.	1.86	1.13	0.	3.35	0.	2.74
time (sec)	N/A	0.263	1.802	0.109	0.	2.556	0.	1.437

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	109	196	132	243	180	127
normalized size	1	1.	1.31	2.36	1.59	2.93	2.17	1.53
time (sec)	N/A	0.08	0.164	0.041	1.029	2.6	1.391	1.47

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	180.003	0.137	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	180.002	0.157	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	313	313	363	1034	783	2367	0	0
normalized size	1	1.	1.16	3.3	2.5	7.56	0.	0.
time (sec)	N/A	0.482	5.599	0.268	1.969	2.824	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	275	573	468	1368	0	0
normalized size	1	1.	1.23	2.56	2.09	6.11	0.	0.
time (sec)	N/A	0.346	4.153	0.131	1.856	2.71	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	345	211	0	548	0	0
normalized size	1	1.	2.74	1.67	0.	4.35	0.	0.
time (sec)	N/A	0.147	1.252	0.132	0.	2.625	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	52	42	84	154	0	72
normalized size	1	1.	1.27	1.02	2.05	3.76	0.	1.76
time (sec)	N/A	0.06	0.064	0.037	1.086	2.496	0.	1.243

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	46.109	0.742	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	57.749	1.383	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	419	419	1042	1535	1268	5860	0	0
normalized size	1	1.	2.49	3.66	3.03	13.99	0.	0.
time (sec)	N/A	0.805	17.279	0.266	2.224	3.037	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	715	847	814	3237	0	0
normalized size	1	1.	2.42	2.86	2.75	10.94	0.	0.
time (sec)	N/A	0.583	13.087	0.151	2.081	2.763	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	454	316	0	1272	0	0
normalized size	1	1.	2.79	1.94	0.	7.8	0.	0.
time (sec)	N/A	0.228	5.436	0.151	0.	2.592	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	61	79	149	378	0	128
normalized size	1	1.	1.07	1.39	2.61	6.63	0.	2.25
time (sec)	N/A	0.086	0.205	0.04	1.072	2.566	0.	1.228

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	131.951	0.571	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	180.001	1.329	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	546	546	2479	2058	1777	10116	0	0
normalized size	1	1.	4.54	3.77	3.25	18.53	0.	0.
time (sec)	N/A	1.212	68.67	0.256	2.749	3.725	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	1378	1107	1165	5378	0	0
normalized size	1	1.	3.74	3.01	3.17	14.61	0.	0.
time (sec)	N/A	0.856	17.044	0.192	2.444	2.959	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	541	423	0	2163	0	0
normalized size	1	1.	2.53	1.98	0.	10.11	0.	0.
time (sec)	N/A	0.367	2.887	0.184	0.	2.772	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	90	119	213	640	0	142
normalized size	1	1.	1.03	1.37	2.45	7.36	0.	1.63
time (sec)	N/A	0.123	0.419	0.049	1.152	2.603	0.	1.215

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	180.023	1.332	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	180.047	2.257	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	453	453	607	0	0	2677	0	0
normalized size	1	1.	1.34	0.	0.	5.91	0.	0.
time (sec)	N/A	0.794	2.312	0.241	0.	3.122	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	366	0	0	1913	0	0
normalized size	1	1.	1.09	0.	0.	5.68	0.	0.
time (sec)	N/A	0.709	1.691	0.179	0.	2.742	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	163	440	0	1243	0	0
normalized size	1	1.	0.74	2.	0.	5.65	0.	0.
time (sec)	N/A	0.413	0.699	0.069	0.	2.53	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	64	87	0	473	371	115
normalized size	1	1.	1.19	1.61	0.	8.76	6.87	2.13
time (sec)	N/A	0.075	0.106	0.003	0.	2.473	169.228	1.296

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	9.225	0.065	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	551	551	979	0	0	5847	0	0
normalized size	1	1.	1.78	0.	0.	10.61	0.	0.
time (sec)	N/A	1.027	2.97	0.105	0.	3.192	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	453	0	0	3897	0	0
normalized size	1	1.	1.11	0.	0.	9.57	0.	0.
time (sec)	N/A	0.853	3.032	0.088	0.	2.811	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	299	510	0	2272	0	0
normalized size	1	1.	1.13	1.93	0.	8.61	0.	0.
time (sec)	N/A	0.487	2.186	0.071	0.	2.661	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	74	132	0	822	0	161
normalized size	1	1.	1.04	1.86	0.	11.58	0.	2.27
time (sec)	N/A	0.128	0.275	0.03	0.	2.521	0.	1.168

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	136.201	0.088	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	712	712	1407	0	0	11113	0	0
normalized size	1	1.	1.98	0.	0.	15.61	0.	0.
time (sec)	N/A	1.235	4.584	0.099	0.	4.027	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	522	522	740	0	0	7156	0	0
normalized size	1	1.	1.42	0.	0.	13.71	0.	0.
time (sec)	N/A	1.044	4.36	0.086	0.	3.252	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	307	589	0	3992	0	0
normalized size	1	1.	0.92	1.76	0.	11.92	0.	0.
time (sec)	N/A	0.593	2.687	0.067	0.	2.91	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	101	262	0	1434	0	217
normalized size	1	1.	0.94	2.45	0.	13.4	0.	2.03
time (sec)	N/A	0.221	0.305	0.03	0.	2.428	0.	1.203

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	180.002	0.091	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	605	605	757	0	0	3903	0	0
normalized size	1	1.	1.25	0.	0.	6.45	0.	0.
time (sec)	N/A	0.965	3.12	0.438	0.	3.052	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	433	454	0	0	2674	0	0
normalized size	1	1.	1.05	0.	0.	6.18	0.	0.
time (sec)	N/A	0.814	2.12	0.334	0.	2.682	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	306	532	0	1621	0	0
normalized size	1	1.	1.17	2.04	0.	6.21	0.	0.
time (sec)	N/A	0.46	1.95	0.112	0.	2.547	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	69	65	0	591	0	144
normalized size	1	1.	1.08	1.02	0.	9.23	0.	2.25
time (sec)	N/A	0.088	0.082	0.003	0.	2.662	0.	1.587

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	5.925	0.065	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	745	745	1353	0	0	13974	0	0
normalized size	1	1.	1.82	0.	0.	18.76	0.	0.
time (sec)	N/A	1.311	18.596	0.692	0.	4.129	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	535	535	795	0	0	8581	0	0
normalized size	1	1.	1.49	0.	0.	16.04	0.	0.
time (sec)	N/A	1.08	16.147	0.743	0.	3.497	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	405	626	0	4370	0	0
normalized size	1	1.	1.32	2.05	0.	14.28	0.	0.
time (sec)	N/A	0.565	5.601	0.105	0.	2.892	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	100	105	0	1200	0	177
normalized size	1	1.	1.25	1.31	0.	15.	0.	2.21
time (sec)	N/A	0.147	0.734	0.003	0.	2.869	0.	1.63

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	117.797	0.879	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1053	1053	2800	0	0	0	0	0
normalized size	1	1.	2.66	0.	0.	0.	0.	0.
time (sec)	N/A	1.752	45.331	0.806	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	725	725	1531	0	0	22066	0	0
normalized size	1	1.	2.11	0.	0.	30.44	0.	0.
time (sec)	N/A	1.372	25.65	0.655	0.	4.142	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	420	420	736	861	0	10647	0	0
normalized size	1	1.	1.75	2.05	0.	25.35	0.	0.
time (sec)	N/A	0.726	7.984	0.162	0.	2.998	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	145	164	0	2844	0	248
normalized size	1	1.	1.28	1.45	0.	25.17	0.	2.19
time (sec)	N/A	0.404	1.944	0.003	0.	3.025	0.	1.231

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	180.001	1.941	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	118	635	356	752	0	0
normalized size	1	1.	0.85	4.57	2.56	5.41	0.	0.
time (sec)	N/A	0.213	0.078	0.118	1.641	2.173	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	94	393	221	482	0	0
normalized size	1	1.	0.89	3.71	2.08	4.55	0.	0.
time (sec)	N/A	0.184	0.05	0.073	1.562	2.229	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	66	188	0	252	0	0
normalized size	1	1.	0.9	2.58	0.	3.45	0.	0.
time (sec)	N/A	0.111	0.026	0.073	0.	2.188	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	23	27	57	22	45
normalized size	1	1.	1.	1.	1.17	2.48	0.96	1.96
time (sec)	N/A	0.027	0.015	0.013	1.124	2.184	4.537	1.236

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	27.507	0.088	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	32.427	0.09	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	106	448	504	605	610	1084
normalized size	1	1.	0.98	4.15	4.67	5.6	5.65	10.04
time (sec)	N/A	0.164	0.779	0.045	2.022	2.218	2.666	1.346

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	78	223	366	373	376	648
normalized size	1	1.	0.95	2.72	4.46	4.55	4.59	7.9
time (sec)	N/A	0.126	0.484	0.042	1.598	2.165	1.947	1.261

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	57	84	254	178	194	312
normalized size	1	1.	1.02	1.5	4.54	3.18	3.46	5.57
time (sec)	N/A	0.072	0.638	0.04	1.509	2.151	1.102	1.163

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	139	85	59	92	82	47
normalized size	1	1.	6.32	3.86	2.68	4.18	3.73	2.14
time (sec)	N/A	0.043	0.145	0.044	1.062	2.113	0.434	1.224

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	62	103	103	159	0	109
normalized size	1	1.	0.82	1.36	1.36	2.09	0.	1.43
time (sec)	N/A	0.204	0.321	0.09	1.515	2.19	0.	1.189

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	85	164	124	278	0	0
normalized size	1	1.	0.83	1.59	1.2	2.7	0.	0.
time (sec)	N/A	0.221	0.528	0.095	1.657	2.166	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	134	429	0	906	1061	1357
normalized size	1	1.	0.58	1.86	0.	3.92	4.59	5.87
time (sec)	N/A	0.261	1.3	0.16	0.	2.22	4.774	1.287

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	99	241	0	520	644	764
normalized size	1	1.	0.58	1.41	0.	3.04	3.77	4.47
time (sec)	N/A	0.186	0.911	0.138	0.	2.13	3.36	1.222

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	60	113	0	242	330	332
normalized size	1	1.	0.61	1.15	0.	2.47	3.37	3.39
time (sec)	N/A	0.1	1.133	0.121	0.	2.145	2.076	1.195

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	29	81	120	136	65
normalized size	1	1.	0.82	0.85	2.38	3.53	4.	1.91
time (sec)	N/A	0.048	0.049	0.015	1.163	2.089	0.766	1.159

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	112	180	0	258	0	208
normalized size	1	1.	0.85	1.37	0.	1.97	0.	1.59
time (sec)	N/A	0.324	0.419	0.114	0.	2.122	0.	1.187

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	212	299	0	510	0	0
normalized size	1	1.	1.18	1.66	0.	2.83	0.	0.
time (sec)	N/A	0.392	0.713	0.125	0.	2.213	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	463	463	767	1152	923	3418	0	0
normalized size	1	1.	1.66	2.49	1.99	7.38	0.	0.
time (sec)	N/A	0.483	11.38	0.244	2.029	2.454	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	501	613	522	1958	0	0
normalized size	1	1.	1.87	2.29	1.95	7.31	0.	0.
time (sec)	N/A	0.264	11.362	0.132	1.995	2.388	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	707	268	0	902	0	0
normalized size	1	1.	4.39	1.66	0.	5.6	0.	0.
time (sec)	N/A	0.142	2.691	0.151	0.	2.289	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	30	91	117	267	0	147
normalized size	1	1.	0.71	2.17	2.79	6.36	0.	3.5
time (sec)	N/A	0.057	0.048	0.046	1.168	2.12	0.	1.153

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	65.483	0.714	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	180.007	1.079	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	450	450	1049	1001	984	3393	0	0
normalized size	1	1.	2.33	2.22	2.19	7.54	0.	0.
time (sec)	N/A	0.589	12.276	0.18	2.502	2.413	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	564	509	0	1760	0	0
normalized size	1	1.	1.74	1.57	0.	5.42	0.	0.
time (sec)	N/A	0.387	8.262	0.154	0.	2.246	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	194	143	339	527	0	352
normalized size	1	1.	1.23	0.91	2.15	3.34	0.	2.23
time (sec)	N/A	0.158	1.134	0.164	1.339	2.298	0.	1.219

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	75	140	144	0	81
normalized size	1	1.	1.	1.6	2.98	3.06	0.	1.72
time (sec)	N/A	0.055	0.046	0.049	1.182	2.071	0.	1.159

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	122.852	0.248	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	180.02	1.753	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	667	667	1804	2026	1800	9095	0	0
normalized size	1	1.	2.7	3.04	2.7	13.64	0.	0.
time (sec)	N/A	0.711	13.394	0.259	6.253	2.896	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	423	423	1284	1044	1081	5145	0	0
normalized size	1	1.	3.04	2.47	2.56	12.16	0.	0.
time (sec)	N/A	0.398	12.986	0.208	3.734	2.77	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	1290	445	0	2423	0	0
normalized size	1	1.	5.54	1.91	0.	10.4	0.	0.
time (sec)	N/A	0.19	6.668	0.213	0.	2.445	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	101	180	243	763	0	250
normalized size	1	1.	1.11	1.98	2.67	8.38	0.	2.75
time (sec)	N/A	0.081	0.105	0.056	1.149	2.104	0.	1.191

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	115.857	1.718	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	180.034	2.361	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	329	0	0	2128	0	0
normalized size	1	1.	0.92	0.	0.	5.98	0.	0.
time (sec)	N/A	0.483	0.149	0.259	0.	2.475	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	244	0	0	1504	0	0
normalized size	1	1.	0.92	0.	0.	5.7	0.	0.
time (sec)	N/A	0.411	0.146	0.204	0.	2.311	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	157	412	0	964	0	0
normalized size	1	1.	0.92	2.42	0.	5.67	0.	0.
time (sec)	N/A	0.234	0.035	0.077	0.	2.162	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	104	41	45
normalized size	1	1.	1.	1.06	1.33	5.78	2.28	2.5
time (sec)	N/A	0.027	0.007	0.002	1.157	2.043	1.097	1.161

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	12.865	0.197	0.	0.	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	527	527	933	0	0	4691	0	0
normalized size	1	1.	1.77	0.	0.	8.9	0.	0.
time (sec)	N/A	0.908	3.284	0.269	0.	3.064	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	447	0	0	3163	0	0
normalized size	1	1.	1.15	0.	0.	8.13	0.	0.
time (sec)	N/A	0.785	2.919	0.23	0.	2.75	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	258	901	0	1872	0	0
normalized size	1	1.	1.02	3.58	0.	7.43	0.	0.
time (sec)	N/A	0.443	1.958	0.089	0.	2.325	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	492	174	0	684	0	157
normalized size	1	1.	7.24	2.56	0.	10.06	0.	2.31
time (sec)	N/A	0.117	2.086	0.039	0.	2.166	0.	1.188

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	25.094	0.111	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	642	642	10263	0	0	9852	0	0
normalized size	1	1.	15.99	0.	0.	15.35	0.	0.
time (sec)	N/A	0.785	23.902	0.28	0.	3.065	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	477	477	3021	0	0	6356	0	0
normalized size	1	1.	6.33	0.	0.	13.32	0.	0.
time (sec)	N/A	0.642	17.629	0.234	0.	2.647	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	251	975	0	3507	0	0
normalized size	1	1.	0.84	3.27	0.	11.77	0.	0.
time (sec)	N/A	0.358	1.366	0.11	0.	2.369	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	53	291	171	848	0	128
normalized size	1	1.	0.9	4.93	2.9	14.37	0.	2.17
time (sec)	N/A	0.069	0.051	0.043	1.16	2.271	0.	1.163

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	81.998	0.234	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	786	786	10644	0	0	4177	0	0
normalized size	1	1.	13.54	0.	0.	5.31	0.	0.
time (sec)	N/A	1.457	28.679	0.351	0.	3.035	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	558	558	1639	0	0	2745	0	0
normalized size	1	1.	2.94	0.	0.	4.92	0.	0.
time (sec)	N/A	1.034	21.549	0.267	0.	2.617	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	334	334	439	954	0	1547	0	0
normalized size	1	1.	1.31	2.86	0.	4.63	0.	0.
time (sec)	N/A	0.596	2.733	0.128	0.	2.419	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	114	100	128	247	0	171
normalized size	1	1.	1.65	1.45	1.86	3.58	0.	2.48
time (sec)	N/A	0.07	0.096	0.004	1.67	2.156	0.	1.149

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	18.473	0.074	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	780	780	1143	0	0	14519	0	0
normalized size	1	1.	1.47	0.	0.	18.61	0.	0.
time (sec)	N/A	1.688	13.884	0.705	0.	4.216	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	548	548	905	0	0	8370	0	0
normalized size	1	1.	1.65	0.	0.	15.27	0.	0.
time (sec)	N/A	1.295	8.36	0.539	0.	3.29	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	284	1928	0	3164	0	0
normalized size	1	1.	0.96	6.54	0.	10.73	0.	0.
time (sec)	N/A	0.717	3.162	0.169	0.	2.48	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	104	90	0	884	0	165
normalized size	1	1.	1.35	1.17	0.	11.48	0.	2.14
time (sec)	N/A	0.1	0.173	0.003	0.	2.158	0.	1.234

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	68.394	0.567	0.	0.	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	928	928	3368	0	0	24035	0	0
normalized size	1	1.	3.63	0.	0.	25.9	0.	0.
time (sec)	N/A	1.772	31.523	0.477	0.	5.246	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	560	560	588	2051	0	11385	0	0
normalized size	1	1.	1.05	3.66	0.	20.33	0.	0.
time (sec)	N/A	0.938	7.076	0.176	0.	3.515	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	104	468	292	2215	0	400
normalized size	1	1.	0.87	3.93	2.45	18.61	0.	3.36
time (sec)	N/A	0.144	0.188	0.003	1.963	2.437	0.	1.367

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	133.342	1.058	0.	0.	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	11.348	0.072	0.	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	8.58	0.115	0.	0.	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	5.523	0.068	0.	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	78	164	0	1015	0	0
normalized size	1	1.	1.05	2.22	0.	13.72	0.	0.
time (sec)	N/A	0.073	0.503	0.214	0.	2.118	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	175	491	0	3195	0	0
normalized size	1	1.	0.75	2.1	0.	13.65	0.	0.
time (sec)	N/A	0.44	1.378	0.19	0.	2.401	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	368	0	0	5462	0	0
normalized size	1	1.	1.06	0.	0.	15.7	0.	0.
time (sec)	N/A	0.74	2.567	0.32	0.	2.787	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	78	164	0	1015	0	0
normalized size	1	1.	1.05	2.22	0.	13.72	0.	0.
time (sec)	N/A	0.07	0.396	0.	0.	2.609	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	175	491	0	3195	0	0
normalized size	1	1.	0.75	2.1	0.	13.65	0.	0.
time (sec)	N/A	0.44	0.551	0.001	0.	2.932	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	368	0	0	5462	0	0
normalized size	1	1.	1.06	0.	0.	15.7	0.	0.
time (sec)	N/A	0.731	0.388	0.	0.	2.899	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	112	308	0	2811	0	0
normalized size	1	1.	1.	2.75	0.	25.1	0.	0.
time (sec)	N/A	0.098	1.162	0.319	0.	2.314	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	306	306	623	805	0	11416	0	0
normalized size	1	1.	2.04	2.63	0.	37.31	0.	0.
time (sec)	N/A	0.52	16.412	0.3	0.	3.171	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	631	631	5753	0	0	24868	0	0
normalized size	1	1.	9.12	0.	0.	39.41	0.	0.
time (sec)	N/A	1.095	24.68	0.43	0.	4.171	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	112	308	0	2811	0	0
normalized size	1	1.	1.	2.75	0.	25.1	0.	0.
time (sec)	N/A	0.095	0.374	0.	0.	2.219	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	623	805	0	11416	0	0
normalized size	1	1.	2.04	2.63	0.	37.31	0.	0.
time (sec)	N/A	0.521	7.106	0.	0.	3.044	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	631	631	5753	0	0	24868	0	0
normalized size	1	1.	9.12	0.	0.	39.41	0.	0.
time (sec)	N/A	1.087	7.562	0.	0.	4.705	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	448	10378	0	0	4582	0	0
normalized size	1	1.	23.17	0.	0.	10.23	0.	0.
time (sec)	N/A	0.647	20.959	0.198	0.	2.771	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	1301	0	0	3051	0	0
normalized size	1	1.	3.94	0.	0.	9.25	0.	0.
time (sec)	N/A	0.55	14.217	0.126	0.	2.475	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	206	483	0	1760	0	0
normalized size	1	1.	0.97	2.28	0.	8.3	0.	0.
time (sec)	N/A	0.312	1.057	0.064	0.	2.279	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	33	35	112	354	65	84
normalized size	1	1.	0.97	1.03	3.29	10.41	1.91	2.47
time (sec)	N/A	0.057	0.039	0.013	1.075	2.123	1.476	1.149

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	73.846	0.118	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	696	696	2961	0	0	8699	0	0
normalized size	1	1.	4.25	0.	0.	12.5	0.	0.
time (sec)	N/A	1.13	14.694	0.191	0.	3.104	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	510	510	2170	0	0	5658	0	0
normalized size	1	1.	4.25	0.	0.	11.09	0.	0.
time (sec)	N/A	0.962	10.612	0.145	0.	2.716	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	327	327	1549	1012	0	3200	0	0
normalized size	1	1.	4.74	3.09	0.	9.79	0.	0.
time (sec)	N/A	0.55	4.619	0.082	0.	2.382	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	109	260	0	1150	0	223
normalized size	1	1.	1.15	2.74	0.	12.11	0.	2.35
time (sec)	N/A	0.181	0.428	0.035	0.	2.279	0.	1.152

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	180.001	0.109	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	864	864	7460	0	0	17256	0	0
normalized size	1	1.	8.63	0.	0.	19.97	0.	0.
time (sec)	N/A	1.117	51.077	0.24	0.	3.589	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	636	636	3509	0	0	10886	0	0
normalized size	1	1.	5.52	0.	0.	17.12	0.	0.
time (sec)	N/A	0.875	16.722	0.191	0.	3.049	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	400	551	1102	0	5828	0	0
normalized size	1	1.	1.38	2.76	0.	14.57	0.	0.
time (sec)	N/A	0.485	3.435	0.099	0.	2.569	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	75	428	247	1613	0	215
normalized size	1	1.	0.88	5.04	2.91	18.98	0.	2.53
time (sec)	N/A	0.122	0.164	0.039	1.067	2.207	0.	1.152

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	180.001	0.155	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1021	1021	2333	0	0	4177	0	0
normalized size	1	1.	2.29	0.	0.	4.09	0.	0.
time (sec)	N/A	1.33	25.527	0.353	0.	3.12	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	716	716	1640	0	0	2743	0	0
normalized size	1	1.	2.29	0.	0.	3.83	0.	0.
time (sec)	N/A	1.067	19.14	0.285	0.	2.701	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	421	421	438	1287	0	1544	0	0
normalized size	1	1.	1.04	3.06	0.	3.67	0.	0.
time (sec)	N/A	0.597	2.631	0.154	0.	2.467	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	51	113	128	247	0	115
normalized size	1	1.	0.74	1.64	1.86	3.58	0.	1.67
time (sec)	N/A	0.077	0.077	0.002	1.605	2.139	0.	1.446

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	16.827	0.244	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	917	917	1143	0	0	14924	0	0
normalized size	1	1.	1.25	0.	0.	16.27	0.	0.
time (sec)	N/A	1.712	13.295	0.728	0.	4.408	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	648	648	906	0	0	8627	0	0
normalized size	1	1.	1.4	0.	0.	13.31	0.	0.
time (sec)	N/A	1.321	8.188	0.931	0.	3.445	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	285	1858	0	3272	0	0
normalized size	1	1.	0.85	5.55	0.	9.77	0.	0.
time (sec)	N/A	0.663	3.106	0.168	0.	2.509	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	104	100	0	884	0	158
normalized size	1	1.	1.33	1.28	0.	11.33	0.	2.03
time (sec)	N/A	0.112	0.193	0.001	0.	2.157	0.	1.277

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	60.499	0.559	0.	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1176	1176	3390	0	0	24520	0	0
normalized size	1	1.	2.88	0.	0.	20.85	0.	0.
time (sec)	N/A	1.696	31.477	0.454	0.	5.548	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	711	711	587	2074	0	11628	0	0
normalized size	1	1.	0.83	2.92	0.	16.35	0.	0.
time (sec)	N/A	0.993	8.545	0.19	0.	3.697	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	105	474	294	2240	0	304
normalized size	1	1.	0.86	3.89	2.41	18.36	0.	2.49
time (sec)	N/A	0.199	0.208	0.003	1.588	2.469	0.	1.359

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	142.055	1.159	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	606	606	2872	0	0	8556	0	0
normalized size	1	1.	4.74	0.	0.	14.12	0.	0.
time (sec)	N/A	0.879	28.481	0.216	0.	2.986	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	449	449	1496	0	0	5513	0	0
normalized size	1	1.	3.33	0.	0.	12.28	0.	0.
time (sec)	N/A	0.717	11.766	0.302	0.	2.646	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	423	565	0	3054	0	0
normalized size	1	1.	1.52	2.03	0.	10.99	0.	0.
time (sec)	N/A	0.418	1.087	0.116	0.	2.387	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	49	54	161	784	87	123
normalized size	1	1.	0.89	0.98	2.93	14.25	1.58	2.24
time (sec)	N/A	0.081	0.071	0.017	1.051	2.211	3.597	1.299

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	180.002	0.26	0.	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	897	897	1667	0	0	15351	0	0
normalized size	1	1.	1.86	0.	0.	17.11	0.	0.
time (sec)	N/A	1.473	7.795	0.217	0.	3.591	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	649	649	966	0	0	9708	0	0
normalized size	1	1.	1.49	0.	0.	14.96	0.	0.
time (sec)	N/A	1.202	4.989	0.228	0.	3.493	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	676	1128	0	5268	0	0
normalized size	1	1.	1.68	2.8	0.	13.07	0.	0.
time (sec)	N/A	0.69	3.058	0.106	0.	2.809	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	123	398	0	1882	0	309
normalized size	1	1.	0.87	2.82	0.	13.35	0.	2.19
time (sec)	N/A	0.498	0.392	0.037	0.	2.138	0.	1.169

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	180.001	0.135	0.	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1123	1123	8734	0	0	26953	0	0
normalized size	1	1.	7.78	0.	0.	24.	0.	0.
time (sec)	N/A	1.524	42.185	0.273	0.	4.358	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	819	819	5198	0	0	16764	0	0
normalized size	1	1.	6.35	0.	0.	20.47	0.	0.
time (sec)	N/A	1.171	19.325	0.243	0.	3.447	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	499	499	853	1217	0	8818	0	0
normalized size	1	1.	1.71	2.44	0.	17.67	0.	0.
time (sec)	N/A	0.675	3.21	0.118	0.	2.758	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	98	614	316	2631	0	298
normalized size	1	1.	0.87	5.43	2.8	23.28	0.	2.64
time (sec)	N/A	0.163	0.302	0.046	1.05	2.238	0.	1.202

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	180.001	0.194	0.	0.	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1218	1218	3387	0	0	4721	0	0
normalized size	1	1.	2.78	0.	0.	3.88	0.	0.
time (sec)	N/A	1.694	26.838	0.652	0.	3.275	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	861	861	1759	0	0	3077	0	0
normalized size	1	1.	2.04	0.	0.	3.57	0.	0.
time (sec)	N/A	1.366	19.743	0.49	0.	2.841	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	516	516	438	3882	0	1767	0	0
normalized size	1	1.	0.85	7.52	0.	3.42	0.	0.
time (sec)	N/A	0.776	2.875	0.225	0.	2.503	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	78	153	149	284	0	128
normalized size	1	1.	1.05	2.07	2.01	3.84	0.	1.73
time (sec)	N/A	0.16	0.086	0.001	1.628	2.309	0.	1.524

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	180.001	0.421	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1118	1118	1147	0	0	14864	0	0
normalized size	1	1.	1.03	0.	0.	13.3	0.	0.
time (sec)	N/A	1.98	13.669	0.732	0.	4.223	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	772	772	910	0	0	8585	0	0
normalized size	1	1.	1.18	0.	0.	11.12	0.	0.
time (sec)	N/A	1.533	8.489	0.576	0.	3.41	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	284	1928	0	3274	0	0
normalized size	1	1.	0.74	5.01	0.	8.5	0.	0.
time (sec)	N/A	0.758	3.052	0.162	0.	2.585	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	106	103	0	883	0	163
normalized size	1	1.	1.18	1.14	0.	9.81	0.	1.81
time (sec)	N/A	0.106	0.196	0.001	0.	2.124	0.	1.378

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	180.009	0.487	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1256	1256	3390	0	0	24496	0	0
normalized size	1	1.	2.7	0.	0.	19.5	0.	0.
time (sec)	N/A	1.968	31.353	0.448	0.	5.43	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	760	760	588	2068	0	11621	0	0
normalized size	1	1.	0.77	2.72	0.	15.29	0.	0.
time (sec)	N/A	1.148	9.5	0.174	0.	3.666	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	130	475	296	2241	0	305
normalized size	1	1.	1.07	3.93	2.45	18.52	0.	2.52
time (sec)	N/A	0.229	0.327	0.003	1.771	2.417	0.	1.261

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	180.001	1.098	0.	0.	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	792	792	7460	0	0	14880	0	0
normalized size	1	1.	9.42	0.	0.	18.79	0.	0.
time (sec)	N/A	1.197	29.333	0.268	0.	3.389	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	578	578	3510	0	0	9353	0	0
normalized size	1	1.	6.07	0.	0.	16.18	0.	0.
time (sec)	N/A	0.928	14.951	0.208	0.	2.858	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	447	671	0	5007	0	0
normalized size	1	1.	1.28	1.93	0.	14.39	0.	0.
time (sec)	N/A	0.526	1.73	0.1	0.	2.532	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	66	73	231	1484	105	171
normalized size	1	1.	0.87	0.96	3.04	19.53	1.38	2.25
time (sec)	N/A	0.097	0.154	0.016	1.122	2.497	2.837	1.257

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	180.003	0.162	0.	0.	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1038	1038	7520	0	0	22876	0	0
normalized size	1	1.	7.24	0.	0.	22.04	0.	0.
time (sec)	N/A	1.827	29.101	0.238	0.	4.206	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	755	755	5115	0	0	14272	0	0
normalized size	1	1.	6.77	0.	0.	18.9	0.	0.
time (sec)	N/A	1.514	19.522	0.198	0.	3.298	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	474	474	2915	1213	0	7611	0	0
normalized size	1	1.	6.15	2.56	0.	16.06	0.	0.
time (sec)	N/A	0.865	11.889	0.106	0.	2.798	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	153	624	0	2803	0	379
normalized size	1	1.	0.83	3.39	0.	15.23	0.	2.06
time (sec)	N/A	0.793	1.989	0.049	0.	2.295	0.	1.203

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	180.001	0.123	0.	0.	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1443	1443	5157	0	0	0	0	0
normalized size	1	1.	3.57	0.	0.	0.	0.	0.
time (sec)	N/A	2.187	21.873	0.23	0.	0.	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1049	1049	3179	0	0	24563	0	0
normalized size	1	1.	3.03	0.	0.	23.42	0.	0.
time (sec)	N/A	1.623	14.96	0.211	0.	4.609	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	641	641	958	1363	0	12709	0	0
normalized size	1	1.	1.49	2.13	0.	19.83	0.	0.
time (sec)	N/A	0.944	4.34	0.115	0.	4.196	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	123	804	405	4018	0	383
normalized size	1	1.	0.87	5.7	2.87	28.5	0.	2.72
time (sec)	N/A	0.223	0.388	0.052	1.318	2.587	0.	1.198

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	180.002	0.223	0.	0.	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1519	1519	2861	0	0	10368	0	0
normalized size	1	1.	1.88	0.	0.	6.83	0.	0.
time (sec)	N/A	2.154	26.213	1.115	0.	4.307	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1067	1067	3418	0	0	6611	0	0
normalized size	1	1.	3.2	0.	0.	6.2	0.	0.
time (sec)	N/A	1.691	19.677	0.853	0.	3.659	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	631	631	481	4066	0	3537	0	0
normalized size	1	1.	0.76	6.44	0.	5.61	0.	0.
time (sec)	N/A	0.957	4.787	0.309	0.	3.115	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	91	196	198	733	0	169
normalized size	1	1.	1.02	2.2	2.22	8.24	0.	1.9
time (sec)	N/A	0.196	0.176	0.06	1.566	2.723	0.	1.414

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	180.002	0.934	0.	0.	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1294	1294	1111	0	0	16386	0	0
normalized size	1	1.	0.86	0.	0.	12.66	0.	0.
time (sec)	N/A	2.455	13.038	0.785	0.	5.391	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	904	904	937	0	0	9671	0	0
normalized size	1	1.	1.04	0.	0.	10.7	0.	0.
time (sec)	N/A	1.821	8.541	0.651	0.	4.138	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	454	317	1897	0	3753	0	0
normalized size	1	1.	0.7	4.18	0.	8.27	0.	0.
time (sec)	N/A	0.892	4.543	0.214	0.	3.014	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	96	158	0	1126	0	182
normalized size	1	1.	0.79	1.31	0.	9.31	0.	1.5
time (sec)	N/A	0.209	0.502	0.003	0.	2.539	0.	1.432

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	180.003	0.753	0.	0.	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1479	1479	3368	0	0	24035	0	0
normalized size	1	1.	2.28	0.	0.	16.25	0.	0.
time (sec)	N/A	2.453	30.882	0.447	0.	5.787	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	894	894	588	2284	0	11386	0	0
normalized size	1	1.	0.66	2.55	0.	12.74	0.	0.
time (sec)	N/A	1.415	7.864	0.202	0.	4.002	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	152	472	293	2217	0	301
normalized size	1	1.	1.27	3.93	2.44	18.48	0.	2.51
time (sec)	N/A	0.201	0.402	0.002	1.675	2.906	0.	1.449

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	180.001	0.905	0.	0.	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	451	451	1924	0	0	2996	0	0
normalized size	1	1.	4.27	0.	0.	6.64	0.	0.
time (sec)	N/A	0.77	19.046	0.495	0.	2.858	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	1296	0	0	2053	0	0
normalized size	1	1.	3.99	0.	0.	6.32	0.	0.
time (sec)	N/A	0.654	14.156	0.39	0.	2.617	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	236	451	0	1243	0	0
normalized size	1	1.	1.15	2.2	0.	6.06	0.	0.
time (sec)	N/A	0.38	0.946	0.134	0.	2.486	0.	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	35	101	170	0	85
normalized size	1	1.	0.82	1.03	2.97	5.	0.	2.5
time (sec)	N/A	0.047	0.021	0.001	1.083	2.392	0.	1.324

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	29.08	0.379	0.	0.	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	638	638	802	0	0	3588	0	0
normalized size	1	1.	1.26	0.	0.	5.62	0.	0.
time (sec)	N/A	1.275	2.476	0.734	0.	3.418	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	462	462	489	0	0	2483	0	0
normalized size	1	1.	1.06	0.	0.	5.37	0.	0.
time (sec)	N/A	1.04	1.871	0.594	0.	2.734	0.	0.

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	339	970	0	1544	0	0
normalized size	1	1.	1.19	3.39	0.	5.4	0.	0.
time (sec)	N/A	0.579	1.835	0.187	0.	2.725	0.	0.

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	80	150	0	559	0	188
normalized size	1	1.	1.13	2.11	0.	7.87	0.	2.65
time (sec)	N/A	0.213	0.146	0.002	0.	2.581	0.	2.397

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	57.994	0.826	0.	0.	0.	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	656	656	2574	0	0	7715	0	0
normalized size	1	1.	3.92	0.	0.	11.76	0.	0.
time (sec)	N/A	1.23	18.452	1.22	0.	3.658	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	486	486	1521	0	0	5030	0	0
normalized size	1	1.	3.13	0.	0.	10.35	0.	0.
time (sec)	N/A	1.025	14.301	1.063	0.	2.862	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	296	932	0	2799	0	0
normalized size	1	1.	0.92	2.89	0.	8.69	0.	0.
time (sec)	N/A	0.592	1.925	0.255	0.	2.998	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	48	178	176	533	0	146
normalized size	1	1.	0.84	3.12	3.09	9.35	0.	2.56
time (sec)	N/A	0.128	0.082	0.078	1.088	3.2	0.	1.548

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	180.002	1.079	0.	0.	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1049	1049	3000	0	0	5839	0	0
normalized size	1	1.	2.86	0.	0.	5.57	0.	0.
time (sec)	N/A	1.341	32.847	0.756	0.	3.698	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	734	734	3268	0	0	3794	0	0
normalized size	1	1.	4.45	0.	0.	5.17	0.	0.
time (sec)	N/A	1.097	33.66	0.58	0.	3.056	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	439	439	1540	1065	0	2091	0	0
normalized size	1	1.	3.51	2.43	0.	4.76	0.	0.
time (sec)	N/A	0.637	2.69	0.171	0.	2.522	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	92	123	186	350	0	211
normalized size	1	1.	1.02	1.37	2.07	3.89	0.	2.34
time (sec)	N/A	0.17	0.134	0.003	1.726	2.695	0.	1.272

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	27.58	0.098	0.	0.	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1164	1164	1467	0	0	21639	0	0
normalized size	1	1.	1.26	0.	0.	18.59	0.	0.
time (sec)	N/A	2.199	16.941	1.674	0.	5.446	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	795	795	1245	0	0	12713	0	0
normalized size	1	1.	1.57	0.	0.	15.99	0.	0.
time (sec)	N/A	1.614	11.537	2.283	0.	3.992	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	442	442	459	1815	0	5233	0	0
normalized size	1	1.	1.04	4.11	0.	11.84	0.	0.
time (sec)	N/A	0.806	7.246	0.279	0.	2.639	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	171	136	0	1455	0	209
normalized size	1	1.	1.51	1.2	0.	12.88	0.	1.85
time (sec)	N/A	0.26	0.265	0.001	0.	3.117	0.	1.219

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	180.001	1.864	0.	0.	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1185	1185	4072	0	0	0	0	0
normalized size	1	1.	3.44	0.	0.	0.	0.	0.
time (sec)	N/A	2.225	36.451	1.372	0.	0.	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	746	746	886	2580	0	17816	0	0
normalized size	1	1.	1.19	3.46	0.	23.88	0.	0.
time (sec)	N/A	1.059	10.841	0.244	0.	4.229	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	196	530	358	3066	0	485
normalized size	1	1.	1.22	3.31	2.24	19.16	0.	3.03
time (sec)	N/A	0.271	0.793	0.003	1.814	4.251	0.	1.766

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	174.592	3.421	0.	0.	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	601	601	5638	0	0	9829	0	0
normalized size	1	1.	9.38	0.	0.	16.35	0.	0.
time (sec)	N/A	1.003	48.702	0.909	0.	3.623	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	419	419	1595	0	0	6053	0	0
normalized size	1	1.	3.81	0.	0.	14.45	0.	0.
time (sec)	N/A	0.818	20.236	0.668	0.	2.939	0.	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	416	528	0	3152	0	0
normalized size	1	1.	1.71	2.17	0.	12.97	0.	0.
time (sec)	N/A	0.464	1.894	0.138	0.	2.409	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	35	149	554	0	132
normalized size	1	1.	1.	0.7	2.98	11.08	0.	2.64
time (sec)	N/A	0.075	0.041	0.001	1.179	2.146	0.	1.337

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	180.001	0.916	0.	0.	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	721	721	1350	0	0	10496	0	0
normalized size	1	1.	1.87	0.	0.	14.56	0.	0.
time (sec)	N/A	1.636	8.687	0.827	0.	3.238	0.	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	517	517	792	0	0	6512	0	0
normalized size	1	1.	1.53	0.	0.	12.6	0.	0.
time (sec)	N/A	1.296	8.037	0.713	0.	2.817	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	364	1017	0	3430	0	0
normalized size	1	1.	1.24	3.46	0.	11.67	0.	0.
time (sec)	N/A	0.688	3.669	0.213	0.	2.356	0.	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	98	147	0	975	0	201
normalized size	1	1.	1.27	1.91	0.	12.66	0.	2.61
time (sec)	N/A	0.266	0.441	0.003	0.	2.255	0.	1.84

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	180.	0.999	0.	0.	0.	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	718	718	10534	0	0	13009	0	0
normalized size	1	1.	14.67	0.	0.	18.12	0.	0.
time (sec)	N/A	1.645	13.512	0.986	0.	3.424	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	518	518	1720	0	0	8101	0	0
normalized size	1	1.	3.32	0.	0.	15.64	0.	0.
time (sec)	N/A	1.287	14.832	0.81	0.	2.945	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	313	938	0	4255	0	0
normalized size	1	1.	0.97	2.9	0.	13.13	0.	0.
time (sec)	N/A	0.743	2.585	0.23	0.	2.478	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	52	172	177	751	0	153
normalized size	1	1.	0.88	2.92	3.	12.73	0.	2.59
time (sec)	N/A	0.123	0.086	0.001	1.226	2.292	0.	1.388

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	180.002	0.944	0.	0.	0.	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1428	1428	7234	0	0	21464	0	0
normalized size	1	1.	5.07	0.	0.	15.03	0.	0.
time (sec)	N/A	2.284	14.016	1.48	0.	5.387	0.	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	982	982	2107	0	0	12925	0	0
normalized size	1	1.	2.15	0.	0.	13.16	0.	0.
time (sec)	N/A	1.668	13.796	1.095	0.	4.04	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	591	591	535	1529	0	6273	0	0
normalized size	1	1.	0.91	2.59	0.	10.61	0.	0.
time (sec)	N/A	0.9	6.528	0.315	0.	3.039	0.	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	160	159	234	1098	0	284
normalized size	1	1.	1.54	1.53	2.25	10.56	0.	2.73
time (sec)	N/A	0.17	0.594	0.003	1.743	2.819	0.	1.319

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	76.395	1.513	0.	0.	0.	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	914	914	2677	0	0	23058	0	0
normalized size	1	1.	2.93	0.	0.	25.23	0.	0.
time (sec)	N/A	2.037	28.63	1.658	0.	5.03	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	499	499	1862	1771	0	9453	0	0
normalized size	1	1.	3.73	3.55	0.	18.94	0.	0.
time (sec)	N/A	0.978	8.95	0.268	0.	3.129	0.	0.

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	135	174	0	2483	0	262
normalized size	1	1.	0.94	1.21	0.	17.24	0.	1.82
time (sec)	N/A	0.311	2.645	0.003	0.	3.115	0.	1.724

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	154.862	1.777	0.	0.	0.	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	978	978	1337	3280	0	0	0	0
normalized size	1	1.	1.37	3.35	0.	0.	0.	0.
time (sec)	N/A	1.411	10.954	0.273	0.	0.	0.	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	227	478	473	5906	0	637
normalized size	1	1.	1.26	2.66	2.63	32.81	0.	3.54
time (sec)	N/A	0.259	0.93	0.004	1.885	4.706	0.	1.367

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	F(-1)	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.135	180.002	3.345	0.	0.	0.	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	752	752	6441	0	0	24935	0	0
normalized size	1	1.	8.57	0.	0.	33.16	0.	0.
time (sec)	N/A	1.353	65.606	0.936	0.	4.926	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	502	502	1816	0	0	14549	0	0
normalized size	1	1.	3.62	0.	0.	28.98	0.	0.
time (sec)	N/A	1.004	30.406	0.615	0.	3.814	0.	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	376	649	0	7004	0	0
normalized size	1	1.	1.26	2.18	0.	23.5	0.	0.
time (sec)	N/A	0.57	6.872	0.143	0.	2.604	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	60	73	217	1370	0	181
normalized size	1	1.	0.83	1.01	3.01	19.03	0.	2.51
time (sec)	N/A	0.111	0.101	0.001	1.269	2.193	0.	1.436

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	180.001	1.971	0.	0.	0.	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1038	1038	2384	0	0	29336	0	0
normalized size	1	1.	2.3	0.	0.	28.26	0.	0.
time (sec)	N/A	2.241	43.631	0.889	0.	4.754	0.	0.

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	714	714	1529	0	0	17416	0	0
normalized size	1	1.	2.14	0.	0.	24.39	0.	0.
time (sec)	N/A	1.728	26.417	0.684	0.	3.626	0.	0.

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	413	413	734	1284	0	8606	0	0
normalized size	1	1.	1.78	3.11	0.	20.84	0.	0.
time (sec)	N/A	0.929	8.171	0.233	0.	2.719	0.	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	145	162	0	2272	0	298
normalized size	1	1.	1.31	1.46	0.	20.47	0.	2.68
time (sec)	N/A	0.569	1.281	0.003	0.	2.463	0.	1.993

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	180.008	2.181	0.	0.	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	972	972	11848	0	0	0	0	0
normalized size	1	1.	12.19	0.	0.	0.	0.	0.
time (sec)	N/A	2.203	73.768	0.872	0.	0.	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	689	689	2277	0	0	18048	0	0
normalized size	1	1.	3.3	0.	0.	26.19	0.	0.
time (sec)	N/A	1.708	42.209	0.671	0.	2.996	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	435	455	1098	0	8753	0	0
normalized size	1	1.	1.05	2.52	0.	20.12	0.	0.
time (sec)	N/A	0.976	4.727	0.171	0.	2.389	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	64	194	234	1554	0	224
normalized size	1	1.	0.8	2.42	2.92	19.42	0.	2.8
time (sec)	N/A	0.105	0.118	0.001	1.294	1.848	0.	1.417

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	180.001	0.499	0.	0.	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1795	1795	9045	0	0	0	0	0
normalized size	1	1.	5.04	0.	0.	0.	0.	0.
time (sec)	N/A	3.272	87.99	2.194	0.	0.	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1219	1219	2784	0	0	29429	0	0
normalized size	1	1.	2.28	0.	0.	24.14	0.	0.
time (sec)	N/A	2.222	42.449	1.398	0.	6.108	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	762	762	913	1478	0	13535	0	0
normalized size	1	1.	1.2	1.94	0.	17.76	0.	0.
time (sec)	N/A	1.138	7.784	0.204	0.	3.736	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	164	219	319	2503	0	370
normalized size	1	1.	1.26	1.68	2.45	19.25	0.	2.85
time (sec)	N/A	0.235	0.364	0.003	1.691	3.375	0.	1.346

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	158.407	3.241	0.	0.	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1245	1245	2574	0	0	0	0	0
normalized size	1	1.	2.07	0.	0.	0.	0.	0.
time (sec)	N/A	3.437	27.558	1.557	0.	0.	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	699	699	863	2767	0	24195	0	0
normalized size	1	1.	1.23	3.96	0.	34.61	0.	0.
time (sec)	N/A	1.356	8.417	0.276	0.	4.714	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	185	233	0	5921	0	317
normalized size	1	1.	0.9	1.13	0.	28.74	0.	1.54
time (sec)	N/A	0.42	2.471	0.002	0.	4.718	0.	1.77

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.134	180.001	3.536	0.	0.	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1122	1122	2868	3563	0	0	0	0
normalized size	1	1.	2.56	3.18	0.	0.	0.	0.
time (sec)	N/A	1.807	10.2	0.289	0.	0.	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	237	539	564	7170	0	652
normalized size	1	1.	1.12	2.55	2.67	33.98	0.	3.09
time (sec)	N/A	0.367	0.808	0.003	1.764	7.085	0.	1.775

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	F(-1)	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.134	180.001	3.319	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [496] had the largest ratio of [0.9167]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	2	1.	14	0.143
2	A	4	2	1.	14	0.143
3	A	3	2	1.	14	0.143
4	A	2	2	1.	12	0.167
5	A	3	3	1.	14	0.214
6	A	4	4	1.	14	0.286
7	A	5	4	1.	14	0.286
8	A	6	4	1.	16	0.25
9	A	4	3	1.	16	0.188
10	A	4	4	1.	16	0.25
11	A	2	1	1.	14	0.071
12	A	5	4	1.	16	0.25
13	A	5	5	1.	16	0.312
14	A	7	6	1.	16	0.375
15	A	7	7	1.	16	0.438
16	A	12	4	1.	16	0.25
17	A	8	4	1.	16	0.25
18	A	6	4	1.	16	0.25
19	A	3	3	1.	14	0.214
20	A	8	4	1.	16	0.25
21	A	8	4	1.	16	0.25
22	A	12	5	1.	16	0.312
23	A	9	5	1.	14	0.357
24	A	7	4	1.	14	0.286
25	A	5	3	1.	12	0.25
26	A	0	0	0.	0	0.
27	A	0	0	0.	0	0.
28	A	6	6	1.	16	0.375
29	A	5	5	1.	16	0.312
30	A	2	2	1.	14	0.143
31	A	0	0	0.	0	0.
32	A	0	0	0.	0	0.
33	A	15	8	1.	16	0.5
34	A	9	6	1.	16	0.375
35	A	6	4	1.	14	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
36	A	0	0	0.	0	0.
37	A	0	0	0.	0	0.
38	A	8	5	1.	16	0.312
39	A	7	5	1.	16	0.312
40	A	6	5	1.	16	0.312
41	A	5	4	1.	16	0.25
42	A	6	5	1.	16	0.312
43	A	7	5	1.	16	0.312
44	A	8	5	1.	16	0.312
45	A	10	8	1.	18	0.444
46	A	9	7	1.	18	0.389
47	A	8	6	1.	18	0.333
48	A	7	5	1.	18	0.278
49	A	7	6	1.	18	0.333
50	A	9	7	1.	18	0.389
51	A	9	8	1.	18	0.444
52	A	11	7	1.	18	0.389
53	A	23	7	1.	18	0.389
54	A	20	7	1.	18	0.389
55	A	14	6	1.	18	0.333
56	A	12	5	1.	18	0.278
57	A	12	5	1.	18	0.278
58	A	18	6	1.	18	0.333
59	A	19	7	1.	18	0.389
60	A	7	5	1.	12	0.417
61	A	6	5	1.	12	0.417
62	A	5	4	1.	12	0.333
63	A	6	5	1.	12	0.417
64	A	7	5	1.	12	0.417
65	A	0	0	0.	0	0.
66	A	0	0	0.	0	0.
67	A	0	0	0.	0	0.
68	A	2	1	1.	18	0.056
69	A	2	1	1.	20	0.05
70	A	3	1	1.	20	0.05
71	A	4	3	1.	22	0.136
72	A	0	0	0.	0	0.
73	A	8	3	1.	16	0.188
74	A	5	3	1.	16	0.188
75	A	3	2	1.	14	0.143
76	A	0	0	0.	0	0.
77	A	0	0	0.	0	0.
78	A	3	2	1.	12	0.167
79	A	3	2	1.	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	3	2	1.	12	0.167
81	A	3	2	1.	10	0.2
82	A	3	2	1.	12	0.167
83	A	3	2	1.	12	0.167
84	A	3	2	1.	12	0.167
85	A	5	3	1.	14	0.214
86	A	5	3	1.	14	0.214
87	A	5	3	1.	14	0.214
88	A	5	3	1.	12	0.25
89	A	5	3	1.	14	0.214
90	A	5	3	1.	14	0.214
91	A	5	3	1.	14	0.214
92	A	4	2	1.	20	0.1
93	A	4	2	1.	20	0.1
94	A	5	2	1.	20	0.1
95	A	7	5	1.	24	0.208
96	A	6	3	1.	21	0.143
97	A	5	3	1.	21	0.143
98	A	4	3	1.	19	0.158
99	A	5	4	1.	21	0.19
100	A	6	5	1.	21	0.238
101	A	7	5	1.	21	0.238
102	A	10	6	1.	23	0.261
103	A	9	7	1.	23	0.304
104	A	6	4	1.	21	0.19
105	A	9	5	1.	23	0.217
106	A	9	5	1.	23	0.217
107	A	15	6	1.	23	0.261
108	A	7	7	1.	23	0.304
109	A	6	6	1.	23	0.261
110	A	3	3	1.	21	0.143
111	A	0	0	0.	0	0.
112	A	0	0	0.	0	0.
113	A	10	9	1.	23	0.391
114	A	9	9	1.	23	0.391
115	A	4	4	1.	21	0.19
116	A	0	0	0.	0	0.
117	A	0	0	0.	0	0.
118	A	6	3	1.	21	0.143
119	A	5	3	1.	21	0.143
120	A	4	3	1.	21	0.143
121	A	3	3	1.	19	0.158
122	A	4	4	1.	21	0.19
123	A	5	5	1.	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
124	A	6	5	1.	21	0.238
125	A	9	5	1.	21	0.238
126	A	7	5	1.	21	0.238
127	A	4	4	1.	19	0.21
128	A	9	5	1.	21	0.238
129	A	9	5	1.	21	0.238
130	A	14	5	1.	21	0.238
131	A	10	5	1.	21	0.238
132	A	5	4	1.	19	0.21
133	A	12	5	1.	21	0.238
134	A	12	5	1.	21	0.238
135	A	21	6	1.	21	0.286
136	A	10	6	1.	21	0.286
137	A	8	5	1.	21	0.238
138	A	6	4	1.	19	0.21
139	A	0	0	0.	0	0.
140	A	0	0	0.	0	0.
141	A	16	9	1.	21	0.429
142	A	10	7	1.	21	0.333
143	A	7	5	1.	19	0.263
144	A	0	0	0.	0	0.
145	A	0	0	0.	0	0.
146	A	23	10	1.	21	0.476
147	A	13	8	1.	21	0.381
148	A	8	5	1.	19	0.263
149	A	0	0	0.	0	0.
150	A	0	0	0.	0	0.
151	A	0	0	0.	0	0.
152	A	12	5	1.	23	0.217
153	A	9	5	1.	23	0.217
154	A	5	3	1.	21	0.143
155	A	0	0	0.	0	0.
156	A	0	0	0.	0	0.
157	A	6	3	1.	18	0.167
158	A	5	3	1.	18	0.167
159	A	4	3	1.	16	0.188
160	A	5	4	1.	18	0.222
161	A	6	5	1.	18	0.278
162	A	7	5	1.	18	0.278
163	A	10	6	1.	20	0.3
164	A	9	7	1.	20	0.35
165	A	6	4	1.	18	0.222
166	A	10	5	1.	20	0.25
167	A	11	7	1.	20	0.35

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
168	A	14	8	1.	20	0.4
169	A	12	7	1.	20	0.35
170	A	10	6	1.	20	0.3
171	A	8	5	1.	18	0.278
172	A	0	0	0.	0	0.
173	A	0	0	0.	0	0.
174	A	18	10	1.	20	0.5
175	A	11	8	1.	18	0.444
176	A	0	0	0.	0	0.
177	A	0	0	0.	0	0.
178	A	35	11	1.	18	0.611
179	A	0	0	0.	0	0.
180	A	0	0	0.	0	0.
181	A	0	0	0.	0	0.
182	A	18	5	1.	20	0.25
183	A	10	5	1.	20	0.25
184	A	5	3	1.	18	0.167
185	A	0	0	0.	0	0.
186	A	0	0	0.	0	0.
187	A	9	9	1.	29	0.31
188	A	8	8	1.	29	0.276
189	A	5	4	1.	27	0.148
190	A	2	2	1.	22	0.091
191	A	0	0	0.	0	0.
192	A	0	0	0.	0	0.
193	A	14	11	1.	31	0.355
194	A	12	10	1.	31	0.323
195	A	8	6	1.	29	0.207
196	A	4	4	1.	24	0.167
197	A	0	0	0.	0	0.
198	A	0	0	0.	0	0.
199	A	19	13	1.	31	0.419
200	A	17	13	1.	31	0.419
201	A	11	7	1.	29	0.241
202	A	2	2	1.	24	0.083
203	A	0	0	0.	0	0.
204	A	0	0	0.	0	0.
205	A	17	10	1.	29	0.345
206	A	14	11	1.	29	0.379
207	A	9	7	1.	27	0.259
208	A	3	3	1.	22	0.136
209	A	0	0	0.	0	0.
210	A	0	0	0.	0	0.
211	A	24	10	1.	31	0.323

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
212	A	20	11	1.	31	0.355
213	A	12	7	1.	29	0.241
214	A	5	5	1.	24	0.208
215	A	0	0	0.	0	0.
216	A	0	0	0.	0	0.
217	A	40	13	1.	31	0.419
218	A	30	13	1.	31	0.419
219	A	19	8	1.	29	0.276
220	A	6	6	1.	24	0.25
221	A	0	0	0.	0	0.
222	A	0	0	0.	0	0.
223	A	14	9	1.	26	0.346
224	A	12	8	1.	26	0.308
225	A	10	6	1.	24	0.25
226	A	4	4	1.	19	0.21
227	A	0	0	0.	0	0.
228	A	19	11	1.	28	0.393
229	A	16	10	1.	28	0.357
230	A	13	8	1.	26	0.308
231	A	6	6	1.	21	0.286
232	A	0	0	0.	0	0.
233	A	24	13	1.	28	0.464
234	A	21	13	1.	28	0.464
235	A	16	9	1.	26	0.346
236	A	6	6	1.	21	0.286
237	A	0	0	0.	0	0.
238	A	22	9	1.	26	0.346
239	A	18	8	1.	26	0.308
240	A	14	7	1.	24	0.292
241	A	5	5	1.	19	0.263
242	A	0	0	0.	0	0.
243	A	29	11	1.	28	0.393
244	A	24	12	1.	28	0.429
245	A	17	9	1.	26	0.346
246	A	7	7	1.	21	0.333
247	A	0	0	0.	0	0.
248	A	45	14	1.	28	0.5
249	A	34	14	1.	28	0.5
250	A	24	10	1.	26	0.385
251	A	7	7	1.	21	0.333
252	A	0	0	0.	0	0.
253	A	6	6	1.	29	0.207
254	A	5	5	1.	29	0.172
255	A	4	4	1.	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
256	A	2	2	1.	22	0.091
257	A	0	0	0.	0	0.
258	A	0	0	0.	0	0.
259	A	6	4	1.	31	0.129
260	A	5	4	1.	31	0.129
261	A	4	3	1.	29	0.103
262	A	2	2	1.	24	0.083
263	A	5	5	1.	31	0.161
264	A	6	6	1.	31	0.194
265	A	10	8	1.	31	0.258
266	A	7	5	1.	31	0.161
267	A	6	6	1.	29	0.207
268	A	2	1	1.	24	0.042
269	A	9	6	1.	31	0.194
270	A	11	7	1.	31	0.226
271	A	22	13	1.	29	0.448
272	A	13	10	1.	29	0.345
273	A	10	8	1.	27	0.296
274	A	4	3	1.	22	0.136
275	A	0	0	0.	0	0.
276	A	0	0	0.	0	0.
277	A	20	12	1.	31	0.387
278	A	16	12	1.	31	0.387
279	A	7	7	1.	29	0.241
280	A	3	3	1.	24	0.125
281	A	0	0	0.	0	0.
282	A	0	0	0.	0	0.
283	A	32	16	1.	31	0.516
284	A	17	12	1.	31	0.387
285	A	11	7	1.	29	0.241
286	A	4	3	1.	24	0.125
287	A	0	0	0.	0	0.
288	A	0	0	0.	0	0.
289	A	11	6	1.	26	0.231
290	A	9	5	1.	26	0.192
291	A	7	4	1.	24	0.167
292	A	2	2	1.	19	0.105
293	A	0	0	0.	0	0.
294	A	18	11	1.	28	0.393
295	A	15	10	1.	28	0.357
296	A	12	8	1.	26	0.308
297	A	5	5	1.	21	0.238
298	A	0	0	0.	0	0.
299	A	21	14	1.	28	0.5

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
300	A	16	10	1.	28	0.357
301	A	13	10	1.	26	0.385
302	A	3	2	1.	21	0.095
303	A	0	0	0.	0	0.
304	A	29	10	1.	26	0.385
305	A	24	9	1.	26	0.346
306	A	19	8	1.	24	0.333
307	A	6	6	1.	19	0.316
308	A	0	0	0.	0	0.
309	A	29	13	1.	28	0.464
310	A	24	14	1.	28	0.5
311	A	15	11	1.	26	0.423
312	A	5	5	1.	21	0.238
313	A	0	0	0.	0	0.
314	A	39	14	1.	28	0.5
315	A	31	12	1.	26	0.462
316	A	7	6	1.	21	0.286
317	A	0	0	0.	0	0.
318	A	0	0	0.	0	0.
319	A	0	0	0.	0	0.
320	A	0	0	0.	0	0.
321	A	4	4	1.	24	0.167
322	A	9	6	1.	26	0.231
323	A	11	7	1.	26	0.269
324	A	4	4	1.	24	0.167
325	A	9	6	1.	26	0.231
326	A	11	7	1.	26	0.269
327	A	6	6	1.	24	0.25
328	A	12	9	1.	26	0.346
329	A	19	11	1.	26	0.423
330	A	6	6	1.	24	0.25
331	A	12	9	1.	26	0.346
332	A	19	11	1.	26	0.423
333	A	16	9	1.	32	0.281
334	A	13	8	1.	32	0.25
335	A	10	7	1.	30	0.233
336	A	4	3	1.	25	0.12
337	A	0	0	0.	0	0.
338	A	23	14	1.	34	0.412
339	A	20	14	1.	34	0.412
340	A	15	10	1.	32	0.312
341	A	5	5	1.	27	0.185
342	A	0	0	0.	0	0.
343	A	30	16	1.	34	0.471

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
344	A	23	13	1.	34	0.382
345	A	17	12	1.	32	0.375
346	A	4	3	1.	27	0.111
347	A	0	0	0.	0	0.
348	A	39	11	1.	26	0.423
349	A	32	10	1.	26	0.385
350	A	25	9	1.	24	0.375
351	A	6	5	1.	19	0.263
352	A	0	0	0.	0	0.
353	A	36	14	1.	32	0.438
354	A	30	15	1.	32	0.469
355	A	18	12	1.	30	0.4
356	A	5	5	1.	25	0.2
357	A	0	0	0.	0	0.
358	A	49	15	1.	34	0.441
359	A	38	13	1.	32	0.406
360	A	8	7	1.	27	0.259
361	A	0	0	0.	0	0.
362	A	22	14	1.	34	0.412
363	A	17	10	1.	34	0.294
364	A	14	10	1.	32	0.312
365	A	4	3	1.	27	0.111
366	A	0	0	0.	0	0.
367	A	31	16	1.	36	0.444
368	A	25	16	1.	36	0.444
369	A	19	12	1.	34	0.353
370	A	8	8	1.	29	0.276
371	A	0	0	0.	0	0.
372	A	40	17	1.	36	0.472
373	A	28	14	1.	36	0.389
374	A	22	13	1.	34	0.382
375	A	4	3	1.	29	0.103
376	A	0	0	0.	0	0.
377	A	46	12	1.	32	0.375
378	A	38	11	1.	32	0.344
379	A	30	10	1.	30	0.333
380	A	7	6	1.	25	0.24
381	A	0	0	0.	0	0.
382	A	45	15	1.	28	0.536
383	A	37	16	1.	28	0.571
384	A	21	13	1.	26	0.5
385	A	8	7	1.	21	0.333
386	A	0	0	0.	0	0.
387	A	53	15	1.	34	0.441

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
388	A	42	13	1.	32	0.406
389	A	8	7	1.	27	0.259
390	A	0	0	0.	0	0.
391	A	30	15	1.	34	0.441
392	A	22	10	1.	34	0.294
393	A	18	11	1.	32	0.344
394	A	4	3	1.	27	0.111
395	A	0	0	0.	0	0.
396	A	38	18	1.	36	0.5
397	A	31	18	1.	36	0.5
398	A	24	14	1.	34	0.412
399	A	9	8	1.	29	0.276
400	A	0	0	0.	0	0.
401	A	55	18	1.	36	0.5
402	A	40	15	1.	36	0.417
403	A	31	14	1.	34	0.412
404	A	4	3	1.	29	0.103
405	A	0	0	0.	0	0.
406	A	61	15	1.	34	0.441
407	A	50	14	1.	34	0.412
408	A	39	13	1.	32	0.406
409	A	7	6	1.	27	0.222
410	A	0	0	0.	0	0.
411	A	53	18	1.	34	0.529
412	A	44	19	1.	34	0.559
413	A	25	15	1.	32	0.469
414	A	9	8	1.	27	0.296
415	A	0	0	0.	0	0.
416	A	71	17	1.	28	0.607
417	A	55	15	1.	26	0.577
418	A	7	6	1.	21	0.286
419	A	0	0	0.	0	0.
420	A	18	8	1.	26	0.308
421	A	15	7	1.	26	0.269
422	A	12	6	1.	24	0.25
423	A	4	4	1.	19	0.21
424	A	0	0	0.	0	0.
425	A	33	14	1.	32	0.438
426	A	27	13	1.	32	0.406
427	A	21	11	1.	30	0.367
428	A	6	6	1.	25	0.24
429	A	0	0	0.	0	0.
430	A	34	17	1.	34	0.5
431	A	26	13	1.	34	0.382

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
432	A	22	13	1.	32	0.406
433	A	4	3	1.	27	0.111
434	A	0	0	0.	0	0.
435	A	40	13	1.	32	0.406
436	A	33	12	1.	32	0.375
437	A	26	11	1.	30	0.367
438	A	7	6	1.	25	0.24
439	A	0	0	0.	0	0.
440	A	53	22	1.	34	0.647
441	A	44	23	1.	34	0.676
442	A	26	19	1.	32	0.594
443	A	10	9	1.	27	0.333
444	A	0	0	0.	0	0.
445	A	57	23	1.	34	0.676
446	A	43	20	1.	32	0.625
447	A	9	7	1.	27	0.259
448	A	0	0	0.	0	0.
449	A	27	11	1.	32	0.344
450	A	22	12	1.	32	0.375
451	A	15	9	1.	30	0.3
452	A	4	3	1.	25	0.12
453	A	0	0	0.	0	0.
454	A	41	17	1.	28	0.607
455	A	34	18	1.	28	0.643
456	A	25	14	1.	26	0.538
457	A	7	7	1.	21	0.333
458	A	0	0	0.	0	0.
459	A	48	19	1.	34	0.559
460	A	37	17	1.	34	0.5
461	A	28	15	1.	32	0.469
462	A	4	3	1.	27	0.111
463	A	0	0	0.	0	0.
464	A	64	20	1.	34	0.588
465	A	53	21	1.	34	0.618
466	A	37	18	1.	32	0.562
467	A	7	6	1.	27	0.222
468	A	0	0	0.	0	0.
469	A	51	25	1.	36	0.694
470	A	30	20	1.	34	0.588
471	A	13	11	1.	29	0.379
472	A	0	0	0.	0	0.
473	A	57	27	1.	34	0.794
474	A	9	7	1.	29	0.241
475	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
476	A	34	14	1.	34	0.412
477	A	26	14	1.	34	0.412
478	A	19	11	1.	32	0.344
479	A	4	3	1.	27	0.111
480	A	0	0	0.	0	0.
481	A	67	22	1.	34	0.647
482	A	52	22	1.	34	0.647
483	A	38	17	1.	32	0.531
484	A	8	8	1.	27	0.296
485	A	0	0	0.	0	0.
486	A	62	23	1.	28	0.821
487	A	47	20	1.	28	0.714
488	A	36	18	1.	26	0.692
489	A	3	2	1.	21	0.095
490	A	0	0	0.	0	0.
491	A	87	28	1.	34	0.824
492	A	71	26	1.	34	0.765
493	A	49	23	1.	32	0.719
494	A	7	6	1.	27	0.222
495	A	0	0	0.	0	0.
496	A	88	33	1.	36	0.917
497	A	44	22	1.	34	0.647
498	A	17	12	1.	29	0.414
499	A	0	0	0.	0	0.
500	A	65	28	1.	34	0.824
501	A	9	7	1.	29	0.241
502	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int (c+dx)^4 \sinh(a+bx)\,dx$

Optimal. Leaf size=91

$$-\frac{24d^3(c+dx)\sinh(a+bx)}{b^4}+\frac{12d^2(c+dx)^2\cosh(a+bx)}{b^3}-\frac{4d(c+dx)^3\sinh(a+bx)}{b^2}+\frac{24d^4\cosh(a+bx)}{b^5}+\frac{(c+dx)}{}$$

```
[Out] (24*d^4*Cosh[a + b*x])/b^5 + (12*d^2*(c + d*x)^2*Cosh[a + b*x])/b^3 + ((c +
 d*x)^4*Cosh[a + b*x])/b - (24*d^3*(c + d*x)*Sinh[a + b*x])/b^4 - (4*d*(c +
 d*x)^3*Sinh[a + b*x])/b^2
```

Rubi [A] time = 0.120453, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3296, 2638}

$$-\frac{24d^3(c+dx)\sinh(a+bx)}{b^4}+\frac{12d^2(c+dx)^2\cosh(a+bx)}{b^3}-\frac{4d(c+dx)^3\sinh(a+bx)}{b^2}+\frac{24d^4\cosh(a+bx)}{b^5}+\frac{(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sinh[a + b*x],x]
```

```
[Out] (24*d^4*Cosh[a + b*x])/b^5 + (12*d^2*(c + d*x)^2*Cosh[a + b*x])/b^3 + ((c +
 d*x)^4*Cosh[a + b*x])/b - (24*d^3*(c + d*x)*Sinh[a + b*x])/b^4 - (4*d*(c +
 d*x)^3*Sinh[a + b*x])/b^2
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^4\sinh(a+bx)\,dx = \frac{(c+dx)^4\cosh(a+bx)}{b} - \frac{(4d)\int (c+dx)^3\cosh(a+bx)\,dx}{b}$$
$$= \frac{(c+dx)^4\cosh(a+bx)}{b} - \frac{4d(c+dx)^3\sinh(a+bx)}{b^2} + \frac{\left(12d^2\right)\int (c+dx)^2\sinh(a+bx)\,dx}{b^2}$$
$$= \frac{12d^2(c+dx)^2\cosh(a+bx)}{b^3} + \frac{(c+dx)^4\cosh(a+bx)}{b} - \frac{4d(c+dx)^3\sinh(a+bx)}{b^2} - \frac{\left(24d^3\right)}{}$$
$$= \frac{12d^2(c+dx)^2\cosh(a+bx)}{b^3} + \frac{(c+dx)^4\cosh(a+bx)}{b} - \frac{24d^3(c+dx)\sinh(a+bx)}{b^4} - \frac{4d(c}{}$$
$$= \frac{24d^4\cosh(a+bx)}{b^5} + \frac{12d^2(c+dx)^2\cosh(a+bx)}{b^3} + \frac{(c+dx)^4\cosh(a+bx)}{b} - \frac{24d^3(c+dx)}{b}$$

Mathematica [A] time = 0.347484, size = 76, normalized size = 0.84

$$\frac{\cosh(a+bx)\left(12b^2d^2(c+dx)^2+b^4(c+dx)^4+24d^4\right)-4bd(c+dx)\sinh(a+bx)\left(b^2(c+dx)^2+6d^2\right)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Sinh[a + b*x],x]
```

```
[Out] ((24*d^4 + 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cosh[a + b*x] - 4*b*d*
(c + d*x)*(6*d^2 + b^2*(c + d*x)^2)*Sinh[a + b*x])/b^5
```

Maple [B] time = 0.033, size = 547, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sinh(b*x+a),x)
```

```
[Out] 1/b*(c^4*cosh(b*x+a)+1/b^4*d^4*((b*x+a)^4*cosh(b*x+a)-4*(b*x+a)^3*sinh(b*x+
a)+12*(b*x+a)^2*cosh(b*x+a)-24*(b*x+a)*sinh(b*x+a)+24*cosh(b*x+a))+1/b^4*d^
4*a^4*cosh(b*x+a)-4/b^4*d^4*a*((b*x+a)^3*cosh(b*x+a)-3*(b*x+a)^2*sinh(b*x+a
)+6*(b*x+a)*cosh(b*x+a)-6*sinh(b*x+a))+4/b^3*d^3*c*((b*x+a)^3*cosh(b*x+a)-3
*(b*x+a)^2*sinh(b*x+a)+6*(b*x+a)*cosh(b*x+a)-6*sinh(b*x+a))+6/b^4*d^4*a^2*(
(b*x+a)^2*cosh(b*x+a)-2*(b*x+a)*sinh(b*x+a)+2*cosh(b*x+a))+6/b^2*d^2*c^2*((
b*x+a)^2*cosh(b*x+a)-2*(b*x+a)*sinh(b*x+a)+2*cosh(b*x+a))-4/b^4*d^4*a^3*((b
*x+a)*cosh(b*x+a)-sinh(b*x+a))+4/b*d*c^3*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))-
4/b^3*d^3*a^3*c*cosh(b*x+a)+6/b^2*d^2*a^2*c^2*cosh(b*x+a)-4/b*d*a*c^3*cosh(
b*x+a)-12/b^3*d^3*a*c*((b*x+a)^2*cosh(b*x+a)-2*(b*x+a)*sinh(b*x+a)+2*cosh(b
*x+a))+12/b^3*d^3*a^2*c*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))-12/b^2*d^2*a*c^2*
((b*x+a)*cosh(b*x+a)-sinh(b*x+a)))
```

Maxima [B] time = 1.1229, size = 440, normalized size = 4.84

$$\frac{c^4e^{(bx+a)}}{2\,b} + \frac{2\,(bxe^a - e^a)c^3de^{(bx)}}{b^2} + \frac{c^4e^{(-bx-a)}}{2\,b} + \frac{2\,(bx+1)c^3de^{(-bx-a)}}{b^2} + \frac{3\left(b^2x^2e^a - 2\,bxe^a + 2\,e^a\right)c^2d^2e^{(bx)}}{b^3} + \frac{3\left(b^2x^2 + 2\,bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*c^4*e^(b*x + a)/b + 2*(b*x*e^a - e^a)*c^3*d*e^(b*x)/b^2 + 1/2*c^4*e^(-b
*x - a)/b + 2*(b*x + 1)*c^3*d*e^(-b*x - a)/b^2 + 3*(b^2*x^2*e^a - 2*b*x*e^a
 + 2*e^a)*c^2*d^2*e^(b*x)/b^3 + 3*(b^2*x^2 + 2*b*x + 2)*c^2*d^2*e^(-b*x - a
)/b^3 + 2*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*c*d^3*e^(b*x)/b
^4 + 2*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*c*d^3*e^(-b*x - a)/b^4 + 1/2*(b^4*
x^4*e^a - 4*b^3*x^3*e^a + 12*b^2*x^2*e^a - 24*b*x*e^a + 24*e^a)*d^4*e^(b*x)
/b^5 + 1/2*(b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*d^4*e^(-b*x - a
)/b^5
```

Fricas [A] time = 2.59911, size = 350, normalized size = 3.85

$$\frac{\left(b^4d^4x^4 + 4\,b^4cd^3x^3 + b^4c^4 + 12\,b^2c^2d^2 + 24\,d^4 + 6\left(b^4c^2d^2 + 2\,b^2d^4\right)x^2 + 4\left(b^4c^3d + 6\,b^2cd^3\right)x\right)\cosh\left(bx + a\right) - 4\left(b^3d\right.}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 + 12*b^2*c^2*d^2 + 24*d^4 + 6*(b^
4*c^2*d^2 + 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d + 6*b^2*c*d^3)*x)*cosh(b*x + a) -
 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d + 6*b*c*d^3 + 3*(b^3*c^2*d^2
+ 2*b*d^4)*x)*sinh(b*x + a))/b^5
```

Sympy [A] time = 3.23735, size = 311, normalized size = 3.42

$$\begin{cases} \frac{c^4\cosh\left(a+bx\right)}{b} + \frac{4c^3dx\cosh\left(a+bx\right)}{b} + \frac{6c^2d^2x^2\cosh\left(a+bx\right)}{b} + \frac{4cd^3x^3\cosh\left(a+bx\right)}{b} + \frac{d^4x^4\cosh\left(a+bx\right)}{b} - \frac{4c^3d\sinh\left(a+bx\right)}{b^2} - \frac{12c^2d^2x\sinh\left(a+bx\right)}{b^2} \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\sinh\left(a\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sinh(b*x+a),x)
```

```
[Out] Piecewise((c**4*cosh(a + b*x)/b + 4*c**3*d*x*cosh(a + b*x)/b + 6*c**2*d**2*
x**2*cosh(a + b*x)/b + 4*c*d**3*x**3*cosh(a + b*x)/b + d**4*x**4*cosh(a + b
*x)/b - 4*c**3*d*sinh(a + b*x)/b**2 - 12*c**2*d**2*x*sinh(a + b*x)/b**2 - 1
2*c*d**3*x**2*sinh(a + b*x)/b**2 - 4*d**4*x**3*sinh(a + b*x)/b**2 + 12*c**2
*d**2*cosh(a + b*x)/b**3 + 24*c*d**3*x*cosh(a + b*x)/b**3 + 12*d**4*x**2*co
sh(a + b*x)/b**3 - 24*c*d**3*sinh(a + b*x)/b**4 - 24*d**4*x*sinh(a + b*x)/b
**4 + 24*d**4*cosh(a + b*x)/b**5, Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c
**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sinh(a), True))
```

Giac [B] time = 1.16781, size = 437, normalized size = 4.8

$$\frac{\left(b^4d^4x^4 + 4\,b^4cd^3x^3 + 6\,b^4c^2d^2x^2 - 4\,b^3d^4x^3 + 4\,b^4c^3dx - 12\,b^3cd^3x^2 + b^4c^4 - 12\,b^3c^2d^2x + 12\,b^2d^4x^2 - 4\,b^3c^3d + 24\,b^2\right.}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 4*
b^4*c^3*d*x - 12*b^3*c*d^3*x^2 + b^4*c^4 - 12*b^3*c^2*d^2*x + 12*b^2*d^4*x^
2 - 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*d^2 - 24*b*d^4*x - 24*b*c*d^3
 + 24*d^4)*e^(b*x + a)/b^5 + 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2
*d^2*x^2 + 4*b^3*d^4*x^3 + 4*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + b^4*c^4 + 12*
b^3*c^2*d^2*x + 12*b^2*d^4*x^2 + 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*
d^2 + 24*b*d^4*x + 24*b*c*d^3 + 24*d^4)*e^(-b*x - a)/b^5
```

3.2 $\int (c+dx)^3 \sinh(a+bx) \, dx$

Optimal. Leaf size=70

$$\frac{6d^2(c+dx)\cosh(a+bx)}{b^3}-\frac{3d(c+dx)^2\sinh(a+bx)}{b^2}-\frac{6d^3\sinh(a+bx)}{b^4}+\frac{(c+dx)^3\cosh(a+bx)}{b}$$

```
[Out] (6*d^2*(c + d*x)*Cosh[a + b*x])/b^3 + ((c + d*x)^3*Cosh[a + b*x])/b - (6*d^
3*Sinh[a + b*x])/b^4 - (3*d*(c + d*x)^2*Sinh[a + b*x])/b^2
```

Rubi [A] time = 0.0790137, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3296, 2637}

$$\frac{6d^2(c+dx)\cosh(a+bx)}{b^3}-\frac{3d(c+dx)^2\sinh(a+bx)}{b^2}-\frac{6d^3\sinh(a+bx)}{b^4}+\frac{(c+dx)^3\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sinh[a + b*x],x]
```

```
[Out] (6*d^2*(c + d*x)*Cosh[a + b*x])/b^3 + ((c + d*x)^3*Cosh[a + b*x])/b - (6*d^
3*Sinh[a + b*x])/b^4 - (3*d*(c + d*x)^2*Sinh[a + b*x])/b^2
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\sinh(a+bx)\,dx &= \frac{(c+dx)^3\cosh(a+bx)}{b}-\frac{(3d)\int (c+dx)^2\cosh(a+bx)\,dx}{b}\\
&= \frac{(c+dx)^3\cosh(a+bx)}{b}-\frac{3d(c+dx)^2\sinh(a+bx)}{b^2}+\frac{\left(6d^2\right)\int (c+dx)\sinh(a+bx)\,dx}{b^2}\\
&= \frac{6d^2(c+dx)\cosh(a+bx)}{b^3}+\frac{(c+dx)^3\cosh(a+bx)}{b}-\frac{3d(c+dx)^2\sinh(a+bx)}{b^2}-\frac{\left(6d^3\right)}{}\\
&= \frac{6d^2(c+dx)\cosh(a+bx)}{b^3}+\frac{(c+dx)^3\cosh(a+bx)}{b}-\frac{6d^3\sinh(a+bx)}{b^4}-\frac{3d(c+dx)^2\text{si}}{b^2}\end{aligned}$$

Mathematica [A] time = 0.21966, size = 61, normalized size = 0.87

$$\frac{b(c+dx)\cosh(a+bx)\left(b^2(c+dx)^2+6d^2\right)-3d\sinh(a+bx)\left(b^2(c+dx)^2+2d^2\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sinh[a + b*x],x]
```

```
[Out] (b*(c + d*x)*(6*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] - 3*d*(2*d^2 + b^2*(c
+ d*x)^2)*Sinh[a + b*x])/b^4
```

Maple [B] time = 0.007, size = 308, normalized size = 4.4

$$\frac{1}{b}\left(\frac{d^3\left((bx+a)^3\cosh(bx+a)-3\,(bx+a)^2\sinh(bx+a)+6\,(bx+a)\cosh(bx+a)-6\,\sinh(bx+a)\right)}{b^3}-3\,\frac{d^3a\left((bx+a)^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sinh(b*x+a),x)
```

```
[Out] 1/b*(1/b^3*d^3*((b*x+a)^3*cosh(b*x+a)-3*(b*x+a)^2*sinh(b*x+a)+6*(b*x+a)*cos
h(b*x+a)-6*sinh(b*x+a))-3/b^3*d^3*a*((b*x+a)^2*cosh(b*x+a)-2*(b*x+a)*sinh(b
*x+a)+2*cosh(b*x+a))+3/b^2*d^2*c*((b*x+a)^2*cosh(b*x+a)-2*(b*x+a)*sinh(b*x+
a)+2*cosh(b*x+a))+3/b^3*d^3*a^2*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))-6/b^2*d^2
*a*c*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))+3/b*d*c^2*((b*x+a)*cosh(b*x+a)-sinh(
b*x+a))-1/b^3*d^3*a^3*cosh(b*x+a)+3/b^2*d^2*a^2*c*cosh(b*x+a)-3/b*d*a*c^2*c
osh(b*x+a)+c^3*cosh(b*x+a))
```

Maxima [B] time = 1.0586, size = 300, normalized size = 4.29

$$\frac{c^3e^{(bx+a)}}{2\,b}+\frac{3\,(bxe^a-e^a)c^2de^{(bx)}}{2\,b^2}+\frac{c^3e^{(-bx-a)}}{2\,b}+\frac{3\,(bx+1)c^2de^{(-bx-a)}}{2\,b^2}+\frac{3\left(b^2x^2e^a-2\,bxe^a+2\,e^a\right)cd^2e^{(bx)}}{2\,b^3}+\frac{3\left(b^2x^2+2\,bx\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*e^(b*x + a)/b + 3/2*(b*x*e^a - e^a)*c^2*d*e^(b*x)/b^2 + 1/2*c^3*e^(
-b*x - a)/b + 3/2*(b*x + 1)*c^2*d*e^(-b*x - a)/b^2 + 3/2*(b^2*x^2*e^a - 2*b
*x*e^a + 2*e^a)*c*d^2*e^(b*x)/b^3 + 3/2*(b^2*x^2 + 2*b*x + 2)*c*d^2*e^(-b*x
 - a)/b^3 + 1/2*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*d^3*e^(b*
x)/b^4 + 1/2*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*d^3*e^(-b*x - a)/b^4
```

Fricas [A] time = 2.53654, size = 231, normalized size = 3.3

$$\frac{\left(b^3d^3x^3+3\,b^3cd^2x^2+b^3c^3+6\,bcd^2+3\left(b^3c^2d+2\,bd^3\right)x\right)\cosh(bx+a)-3\left(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d+2\,d^3\right)\sinh(bx+}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 6*b*c*d^2 + 3*(b^3*c^2*d + 2*b*
d^3)*x)*cosh(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d + 2*d^3)
*sinh(b*x + a))/b^4
```

Sympy [A] time = 1.50237, size = 202, normalized size = 2.89

$$\begin{cases} \frac{c^3\cosh(a+bx)}{b} + \frac{3c^2dx\cosh(a+bx)}{b} + \frac{3cd^2x^2\cosh(a+bx)}{b} + \frac{d^3x^3\cosh(a+bx)}{b} - \frac{3c^2d\sinh(a+bx)}{b^2} - \frac{6cd^2x\sinh(a+bx)}{b^2} - \frac{3d^3x^2\sinh(a+bx)}{b^2} + 6c \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sinh(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sinh(b*x+a),x)
```

```
[Out] Piecewise((c**3*cosh(a + b*x)/b + 3*c**2*d*x*cosh(a + b*x)/b + 3*c*d**2*x**
2*cosh(a + b*x)/b + d**3*x**3*cosh(a + b*x)/b - 3*c**2*d*sinh(a + b*x)/b**2
 - 6*c*d**2*x*sinh(a + b*x)/b**2 - 3*d**3*x**2*sinh(a + b*x)/b**2 + 6*c*d**
2*cosh(a + b*x)/b**3 + 6*d**3*x*cosh(a + b*x)/b**3 - 6*d**3*sinh(a + b*x)/b
**4, Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin
h(a), True))
```

Giac [B] time = 1.19993, size = 275, normalized size = 3.93

$$\frac{\left(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx - 3\,b^2d^3x^2 + b^3c^3 - 6\,b^2cd^2x - 3\,b^2c^2d + 6\,bd^3x + 6\,bcd^2 - 6\,d^3\right)e^{(bx+a)}}{2\,b^4} + \frac{\left(b^3d^3x^3 + 3\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x - 3*b^2*d^3*x^2 + b^3*c^
3 - 6*b^2*c*d^2*x - 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 6*d^3)*e^(b*x + a
)/b^4 + 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*b^2*d^3*x^2
+ b^3*c^3 + 6*b^2*c*d^2*x + 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 + 6*d^3)*e^
(-b*x - a)/b^4
```

3.3 $\int (c+dx)^2 \sinh(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{2d(c+dx)\sinh(a+bx)}{b^2}+\frac{2d^2\cosh(a+bx)}{b^3}+\frac{(c+dx)^2\cosh(a+bx)}{b}$$

```
[Out] (2*d^2*Cosh[a + b*x])/b^3 + ((c + d*x)^2*Cosh[a + b*x])/b - (2*d*(c + d*x)*
Sinh[a + b*x])/b^2
```

Rubi [A] time = 0.0496901, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3296, 2638}

$$-\frac{2d(c+dx)\sinh(a+bx)}{b^2}+\frac{2d^2\cosh(a+bx)}{b^3}+\frac{(c+dx)^2\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sinh[a + b*x],x]
```

```
[Out] (2*d^2*Cosh[a + b*x])/b^3 + ((c + d*x)^2*Cosh[a + b*x])/b - (2*d*(c + d*x)*
Sinh[a + b*x])/b^2
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\sinh(a+bx)\,dx &= \frac{(c+dx)^2\cosh(a+bx)}{b}-\frac{(2d)\int (c+dx)\cosh(a+bx)\,dx}{b}\\ &= \frac{(c+dx)^2\cosh(a+bx)}{b}-\frac{2d(c+dx)\sinh(a+bx)}{b^2}+\frac{\left(2d^2\right)\int \sinh(a+bx)\,dx}{b^2}\\ &= \frac{2d^2\cosh(a+bx)}{b^3}+\frac{(c+dx)^2\cosh(a+bx)}{b}-\frac{2d(c+dx)\sinh(a+bx)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.157426, size = 44, normalized size = 0.9

$$\frac{\cosh(a+bx)\left(b^2(c+dx)^2+2d^2\right)-2bd(c+dx)\sinh(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sinh[a + b*x],x]
```

```
[Out] ((2*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] - 2*b*d*(c + d*x)*Sinh[a + b*x])/b
^3
```

Maple [B] time = 0.006, size = 147, normalized size = 3.

$$\frac{1}{b}\left(\frac{d^2\left((bx+a)^2\cosh(bx+a)-2(bx+a)\sinh(bx+a)+2\cosh(bx+a)\right)}{b^2}-2\frac{d^2a((bx+a)\cosh(bx+a)-\sinh(bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sinh(b*x+a),x)
```

```
[Out] 1/b*(1/b^2*d^2*((b*x+a)^2*cosh(b*x+a)-2*(b*x+a)*sinh(b*x+a)+2*cosh(b*x+a))-
2/b^2*d^2*a*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))+2/b*d*c*((b*x+a)*cosh(b*x+a)-
sinh(b*x+a))+1/b^2*d^2*a^2*cosh(b*x+a)-2/b*d*a*c*cosh(b*x+a)+c^2*cosh(b*x+a
))
```

Maxima [B] time = 1.1176, size = 181, normalized size = 3.69

$$\frac{c^2e^{(bx+a)}}{2b}+\frac{(bxe^a-e^a)cde^{(bx)}}{b^2}+\frac{c^2e^{(-bx-a)}}{2b}+\frac{(bx+1)cde^{(-bx-a)}}{b^2}+\frac{\left(b^2x^2e^a-2bxe^a+2e^a\right)d^2e^{(bx)}}{2b^3}+\frac{\left(b^2x^2+2bx+2\right)d^2}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*e^(b*x + a)/b + (b*x*e^a - e^a)*c*d*e^(b*x)/b^2 + 1/2*c^2*e^(-b*x -
 a)/b + (b*x + 1)*c*d*e^(-b*x - a)/b^2 + 1/2*(b^2*x^2*e^a - 2*b*x*e^a + 2*e
^a)*d^2*e^(b*x)/b^3 + 1/2*(b^2*x^2 + 2*b*x + 2)*d^2*e^(-b*x - a)/b^3
```

Fricas [A] time = 2.56058, size = 139, normalized size = 2.84

$$\frac{\left(b^2d^2x^2+2b^2cdx+b^2c^2+2d^2\right)\cosh(bx+a)-2\left(bd^2x+bcd\right)\sinh(bx+a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*d^2)*cosh(b*x + a) - 2*(b*d^2*x +
 b*c*d)*sinh(b*x + a))/b^3
```

Sympy [A] time = 0.721562, size = 112, normalized size = 2.29

$$\begin{cases} \frac{c^2\cosh(a+bx)}{b}+\frac{2cdx\cosh(a+bx)}{b}+\frac{d^2x^2\cosh(a+bx)}{b}-\frac{2cd\sinh(a+bx)}{b^2}-\frac{2d^2x\sinh(a+bx)}{b^2}+\frac{2d^2\cosh(a+bx)}{b^3} & \text{for } b\neq 0 \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sinh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sinh(b*x+a),x)
```

```
[Out] Piecewise((c**2*cosh(a + b*x)/b + 2*c*d*x*cosh(a + b*x)/b + d**2*x**2*cosh(
a + b*x)/b - 2*c*d*sinh(a + b*x)/b**2 - 2*d**2*x*sinh(a + b*x)/b**2 + 2*d**
2*cosh(a + b*x)/b**3, Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sinh(a)
, True))
```

Giac [B] time = 1.16676, size = 151, normalized size = 3.08

$$\frac{\left(b^{2} d^{2} x^{2} + 2 \, b^{2} c d x + b^{2} c^{2} - 2 \, b d^{2} x - 2 \, b c d + 2 \, d^{2}\right) e^{\left(b x + a\right)}}{2 \, b^{3}} + \frac{\left(b^{2} d^{2} x^{2} + 2 \, b^{2} c d x + b^{2} c^{2} + 2 \, b d^{2} x + 2 \, b c d + 2 \, d^{2}\right) e^{\left(-b x - a\right)}}{2 \, b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*b*d^2*x - 2*b*c*d + 2*d^2)*e^(
b*x + a)/b^3 + 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*b*d^2*x + 2*b*c
*d + 2*d^2)*e^(-b*x - a)/b^3
```

3.4 $\int (c+dx)\sinh(a+bx)\,dx$

Optimal. Leaf size=28

$$\frac{(c+dx)\cosh(a+bx)}{b}-\frac{d\sinh(a+bx)}{b^2}$$

```
[Out] ((c + d*x)*Cosh[a + b*x])/b - (d*Sinh[a + b*x])/b^2
```

Rubi [A] time = 0.0199479, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3296, 2637}

$$\frac{(c+dx)\cosh(a+bx)}{b}-\frac{d\sinh(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sinh[a + b*x],x]
```

```
[Out] ((c + d*x)*Cosh[a + b*x])/b - (d*Sinh[a + b*x])/b^2
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\sinh(a+bx)\,dx &= \frac{(c+dx)\cosh(a+bx)}{b}-\frac{d\int \cosh(a+bx)\,dx}{b}\\ &= \frac{(c+dx)\cosh(a+bx)}{b}-\frac{d\sinh(a+bx)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.0625134, size = 27, normalized size = 0.96

$$\frac{b(c+dx)\cosh(a+bx)-d\sinh(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sinh[a + b*x],x]
```

```
[Out] (b*(c + d*x)*Cosh[a + b*x] - d*Sinh[a + b*x])/b^2
```

Maple [A] time = 0.004, size = 53, normalized size = 1.9

$$\frac{1}{b}\left(\frac{d\left((bx+a)\cosh\left(bx+a\right)-\sinh\left(bx+a\right)\right)}{b}-\frac{da\cosh\left(bx+a\right)}{b}+c\cosh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sinh(b*x+a),x)
```

```
[Out] 1/b*(1/b*d*((b*x+a)*cosh(b*x+a)-sinh(b*x+a))-1/b*d*a*cosh(b*x+a)+c*cosh(b*x
+a))
```

Maxima [B] time = 1.08766, size = 92, normalized size = 3.29

$$\frac{ce^{(bx+a)}}{2\,b}+\frac{(bxe^{a}-e^{a})de^{(bx)}}{2\,b^{2}}+\frac{ce^{(-bx-a)}}{2\,b}+\frac{(bx+1)de^{(-bx-a)}}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*c*e^(b*x + a)/b + 1/2*(b*x*e^a - e^a)*d*e^(b*x)/b^2 + 1/2*c*e^(-b*x - a
)/b + 1/2*(b*x + 1)*d*e^(-b*x - a)/b^2
```

Fricas [A] time = 2.52931, size = 72, normalized size = 2.57

$$\frac{(bdx+bc)\cosh\left(bx+a\right)-d\sinh\left(bx+a\right)}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*d*x + b*c)*cosh(b*x + a) - d*sinh(b*x + a))/b^2
```

Sympy [A] time = 0.265362, size = 46, normalized size = 1.64

$$\begin{cases}\frac{c\cosh\left(a+bx\right)}{b}+\frac{dx\cosh\left(a+bx\right)}{b}-\frac{d\sinh\left(a+bx\right)}{b^{2}} & \text{for}\;b\neq 0\\ \left(cx+\frac{dx^{2}}{2}\right)\sinh\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a),x)
```

```
[Out] Piecewise((c*cosh(a + b*x)/b + d*x*cosh(a + b*x)/b - d*sinh(a + b*x)/b**2,
Ne(b, 0)), ((c*x + d*x**2/2)*sinh(a), True))
```

Giac [A] time = 1.14771, size = 62, normalized size = 2.21

$$\frac{(bdx+bc-d)e^{(bx+a)}}{2\,b^{2}}+\frac{(bdx+bc+d)e^{(-bx-a)}}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x + b*c - d)*e^(b*x + a)/b^2 + 1/2*(b*d*x + b*c + d)*e^(-b*x - a)/
b^2
```

3.5 $\int \frac{\sinh(a+bx)}{c+dx} dx$

Optimal. Leaf size=51

$$\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] (CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*c)/d])/d + (Cosh[a - (b*c)/d]*Sinh
Integral[(b*c)/d + b*x])/d
```

Rubi [A] time = 0.109284, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3303, 3298, 3301}

$$\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]/(c + d*x),x]
```

```
[Out] (CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*c)/d])/d + (Cosh[a - (b*c)/d]*Sinh
Integral[(b*c)/d + b*x])/d
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(a+bx)}{c+dx}\,dx &= \cosh\left(a-\frac{bc}{d}\right)\int \frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\sinh\left(a-\frac{bc}{d}\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\\ &= \frac{\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{d}+\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.0848559, size = 49, normalized size = 0.96

$$\frac{\sinh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{bc}{d}+bx\right)+\cosh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{bc}{d}+bx\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]/(c + d*x),x]
```

```
[Out] (CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*c)/d] + Cosh[a - (b*c)/d]*SinhInte
gral[(b*c)/d + b*x])/d
```

Maple [A] time = 0.059, size = 82, normalized size = 1.6

$$\frac{1}{2d}\mathrm{e}^{-\frac{da-cb}{d}}\mathrm{Ei}\left(1,bx+a-\frac{da-cb}{d}\right)-\frac{1}{2d}\mathrm{e}^{\frac{da-cb}{d}}\mathrm{Ei}\left(1,-bx-a-\frac{-da+cb}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)/(d*x+c),x)
```

```
[Out] 1/2/d*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-1/2/d*exp((a*d-b*c)/d)*Ei(1
,-b*x-a-(-a*d+b*c)/d)
```

Maxima [A] time = 1.2888, size = 77, normalized size = 1.51

$$\frac{e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{2d}-\frac{e^{\left(a-\frac{bc}{d}\right)}E_1\left(-\frac{(dx+c)b}{d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*e^(-a + b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d - 1/2*e^(a - b*c/d)*e
xp_integral_e(1, -(d*x + c)*b/d)/d
```

Fricas [A] time = 2.58023, size = 193, normalized size = 3.78

$$\frac{\left(\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)-\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)+\left(\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)+\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\sinh\left(-\frac{bc-ad}{d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*((Ei((b*d*x + b*c)/d) - Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) + (E
i((b*d*x + b*c)/d) + Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(sinh(a + b*x)/(c + d*x), x)
```

Giac [A] time = 1.15718, size = 77, normalized size = 1.51

$$\frac{\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)} - \mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(Ei((b*d*x + b*c)/d)*e^(a - b*c/d) - Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d
))/d
```

3.6 $\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=71

$$\frac{b\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{b\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{\sinh(a+bx)}{d(c+dx)}$$

```
[Out] (b*Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x])/d^2 - Sinh[a + b*x]/(d*(c
 + d*x)) + (b*Sinh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/d^2
```

Rubi [A] time = 0.126858, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3297, 3303, 3298, 3301}

$$\frac{b\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{b\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{\sinh(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]/(c + d*x)^2,x]
```

```
[Out] (b*Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x])/d^2 - Sinh[a + b*x]/(d*(c
 + d*x)) + (b*Sinh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/d^2
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx = -\frac{\sinh(a+bx)}{d(c+dx)} + \frac{b\int \frac{\cosh(a+bx)}{c+dx}\,dx}{d}$$

$$= -\frac{\sinh(a+bx)}{d(c+dx)} + \frac{\left(b\cosh\left(a-\frac{bc}{d}\right)\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d} + \frac{\left(b\sinh\left(a-\frac{bc}{d}\right)\right)\int \frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}$$

$$= \frac{b\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d^2} - \frac{\sinh(a+bx)}{d(c+dx)} + \frac{b\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d^2}$$

Mathematica [A] time = 0.238479, size = 65, normalized size = 0.92

$$\frac{b\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(b\left(\frac{c}{d}+x\right)\right) + b\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(b\left(\frac{c}{d}+x\right)\right) - \frac{d\sinh(a+bx)}{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]/(c + d*x)^2,x]
```

```
[Out] (b*Cosh[a - (b*c)/d]*CoshIntegral[b*(c/d + x)] - (d*Sinh[a + b*x])/(c + d*x
) + b*Sinh[a - (b*c)/d]*SinhIntegral[b*(c/d + x)])/d^2
```

Maple [A] time = 0.033, size = 133, normalized size = 1.9

$$\frac{b\mathrm{e}^{-bx-a}}{2\,d\,(bdx+cb)} - \frac{b}{2\,d^2}\mathrm{e}^{-\frac{da-cb}{d}}\,\text{Ei}\left(1, bx+a-\frac{da-cb}{d}\right) - \frac{b\mathrm{e}^{bx+a}}{2\,d^2}\left(\frac{cb}{d}+bx\right)^{-1} - \frac{b}{2\,d^2}\mathrm{e}^{\frac{da-cb}{d}}\,\text{Ei}\left(1, -bx-a-\frac{-da+cb}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)/(d*x+c)^2,x)
```

```
[Out] 1/2*b*exp(-b*x-a)/d/(b*d*x+b*c)-1/2*b/d^2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d
-b*c)/d)-1/2*b/d^2*exp(b*x+a)/(b*c/d+b*x)-1/2*b/d^2*exp((a*d-b*c)/d)*Ei(1,-
b*x-a-(-a*d+b*c)/d)
```

Maxima [A] time = 1.38578, size = 108, normalized size = 1.52

$$-\frac{b\left(\frac{e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{d} + \frac{e^{\left(a-\frac{bc}{d}\right)}E_1\left(-\frac{(dx+c)b}{d}\right)}{d}\right)}{2\,d} - \frac{\sinh\left(bx+a\right)}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*(e^(-a + b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d + e^(a - b*c/d)*e
xp_integral_e(1, -(d*x + c)*b/d)/d)/d - sinh(b*x + a)/((d*x + c)*d)
```

Fricas [B] time = 2.67794, size = 315, normalized size = 4.44

$$\frac{\left((bdx+bc)\text{Ei}\left(\frac{bdx+bc}{d}\right)+(bdx+bc)\text{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)-2\,d\sinh\left(bx+a\right)+\left((bdx+bc)\text{Ei}\left(\frac{bdx+bc}{d}\right)-(bdx+\right.}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(((b*d*x + b*c)*Ei((b*d*x + b*c)/d) + (b*d*x + b*c)*Ei(-(b*d*x + b*c)/d
))*cosh(-(b*c - a*d)/d) - 2*d*sinh(b*x + a) + ((b*d*x + b*c)*Ei((b*d*x + b*
c)/d) - (b*d*x + b*c)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/(d^3*x +
c*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27503, size = 197, normalized size = 2.77

$$\frac{bdx\text{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}+bdx\text{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}+bc\text{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}+bc\text{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}-de^{(bx+a)}+de^{(-bx-a}}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) + b*d*x*Ei(-(b*d*x + b*c)/d)*e
^(-a + b*c/d) + b*c*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) + b*c*Ei(-(b*d*x + b*
c)/d)*e^(-a + b*c/d) - d*e^(b*x + a) + d*e^(-b*x - a))/(d^3*x + c*d^2)
```

3.7 $\int \frac{\sinh(a+bx)}{(c+dx)^3} dx$

Optimal. Leaf size=104

$$\frac{b^2\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{2d^3}+\frac{b^2\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{2d^3}-\frac{b\cosh(a+bx)}{2d^2(c+dx)}-\frac{\sinh(a+bx)}{2d(c+dx)^2}$$

```
[Out] -(b*Cosh[a + b*x])/(2*d^2*(c + d*x)) + (b^2*CoshIntegral[(b*c)/d + b*x]*Sin
h[a - (b*c)/d])/(2*d^3) - Sinh[a + b*x]/(2*d*(c + d*x)^2) + (b^2*Cosh[a - (
b*c)/d]*SinhIntegral[(b*c)/d + b*x])/(2*d^3)
```

Rubi [A] time = 0.166022, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3297, 3303, 3298, 3301}

$$\frac{b^2\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{2d^3}+\frac{b^2\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{2d^3}-\frac{b\cosh(a+bx)}{2d^2(c+dx)}-\frac{\sinh(a+bx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]/(c + d*x)^3,x]
```

```
[Out] -(b*Cosh[a + b*x])/(2*d^2*(c + d*x)) + (b^2*CoshIntegral[(b*c)/d + b*x]*Sin
h[a - (b*c)/d])/(2*d^3) - Sinh[a + b*x]/(2*d*(c + d*x)^2) + (b^2*Cosh[a - (
b*c)/d]*SinhIntegral[(b*c)/d + b*x])/(2*d^3)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sinh(a+bx)}{(c+dx)^3}\,dx = -\frac{\sinh(a+bx)}{2d(c+dx)^2} + \frac{b\int \frac{\cosh(a+bx)}{(c+dx)^2}\,dx}{2d}$$
$$= -\frac{b\cosh(a+bx)}{2d^2(c+dx)} - \frac{\sinh(a+bx)}{2d(c+dx)^2} + \frac{b^2\int \frac{\sinh(a+bx)}{c+dx}\,dx}{2d^2}$$
$$= -\frac{b\cosh(a+bx)}{2d^2(c+dx)} - \frac{\sinh(a+bx)}{2d(c+dx)^2} + \frac{\left(b^2\cosh\left(a-\frac{bc}{d}\right)\right)\int \frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{2d^2} + \frac{\left(b^2\sinh\left(a-\frac{bc}{d}\right)\right)\int \frac{\text{co}}{}}{2d^2}$$
$$= -\frac{b\cosh(a+bx)}{2d^2(c+dx)} + \frac{b^2\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{2d^3} - \frac{\sinh(a+bx)}{2d(c+dx)^2} + \frac{b^2\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+b\right.}{2d^3}$$

Mathematica [A] time = 0.558116, size = 88, normalized size = 0.85

$$\frac{b^2\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(b\left(\frac{c}{d}+x\right)\right)+b^2\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(b\left(\frac{c}{d}+x\right)\right)-\frac{d(b(c+dx)\cosh(a+bx)+d\sinh(a+bx))}{(c+dx)^2}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]/(c + d*x)^3,x]
```

```
[Out] (b^2*CoshIntegral[b*(c/d + x)]*Sinh[a - (b*c)/d] - (d*(b*(c + d*x)*Cosh[a +
 b*x] + d*Sinh[a + b*x]))/(c + d*x)^2 + b^2*Cosh[a - (b*c)/d]*SinhIntegral[
b*(c/d + x)])/(2*d^3)
```

Maple [B] time = 0.039, size = 277, normalized size = 2.7

$$-\frac{b^3e^{-bx-a}x}{4d\left(b^2d^2x^2+2b^2cdx+c^2b^2\right)} - \frac{b^3e^{-bx-a}c}{4d^2\left(b^2d^2x^2+2b^2cdx+c^2b^2\right)} + \frac{b^2e^{-bx-a}}{4d\left(b^2d^2x^2+2b^2cdx+c^2b^2\right)} + \frac{b^2}{4d^3}e^{-\frac{da-cb}{d}}\text{Ei}\left(1,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)/(d*x+c)^3,x)
```

```
[Out] -1/4*b^3*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*x-1/4*b^3*exp(-b*x
-a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c+1/4*b^2*exp(-b*x-a)/d/(b^2*d^2*
x^2+2*b^2*c*d*x+b^2*c^2)+1/4*b^2/d^3*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)
/d)-1/4*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)^2-1/4*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)
-1/4*b^2/d^3*exp((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*c)/d)
```

Maxima [A] time = 1.38369, size = 127, normalized size = 1.22

$$-\frac{b\left(\frac{e^{\left(-a+\frac{bc}{d}\right)}E_2\left(\frac{(dx+c)b}{d}\right)}{(dx+c)d}+\frac{e^{\left(a-\frac{bc}{d}\right)}E_2\left(-\frac{(dx+c)b}{d}\right)}{(dx+c)d}\right)}{4d} - \frac{\sinh(bx+a)}{2(dx+c)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*(e^(-a + b*c/d)*exp_integral_e(2, (d*x + c)*b/d)/((d*x + c)*d) + e^(
a - b*c/d)*exp_integral_e(2, -(d*x + c)*b/d)/((d*x + c)*d))/d - 1/2*sinh(b*
x + a)/((d*x + c)^2*d)
```

Fricas [B] time = 2.60996, size = 518, normalized size = 4.98

$$-\frac{2\,d^2\sinh\left(bx+a\right)+2\left(bd^2x+bcd\right)\cosh\left(bx+a\right)-\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)-\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{E}\cdots\right.}{4\left(d^5x^2+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d^2*sinh(b*x + a) + 2*(b*d^2*x + b*c*d)*cosh(b*x + a) - ((b^2*d^2*x
^2 + 2*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) - ((b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c
^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23032, size = 406, normalized size = 3.9

$$\frac{b^2d^2x^2\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}-b^2d^2x^2\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}+2\,b^2cdx\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}-2\,b^2cdx\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}+b^2c^2\cdots}{4\left(d^5x^2+2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^2*d^2*x^2*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) - b^2*d^2*x^2*Ei(-(b*d*x
 + b*c)/d)*e^(-a + b*c/d) + 2*b^2*c*d*x*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) -
 2*b^2*c*d*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + b^2*c^2*Ei((b*d*x + b*c)
/d)*e^(a - b*c/d) - b^2*c^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - b*d^2*x*e
^(b*x + a) - b*d^2*x*e^(-b*x - a) - b*c*d*e^(b*x + a) - b*c*d*e^(-b*x - a)
- d^2*e^(b*x + a) + d^2*e^(-b*x - a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

3.8 $\int (c+dx)^4 \sinh^2(a+bx)\,dx$

Optimal. Leaf size=162

$$-\frac{3d^3(c+dx)\sinh^2(a+bx)}{2b^4}+\frac{3d^2(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b^3}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{3d^4\sinh(a+bx)}{4b^5}$$

```
[Out] (-3*d^4*x)/(4*b^4) - (d*(c + d*x)^3)/(2*b^2) - (c + d*x)^5/(10*d) + (3*d^4*
Cosh[a + b*x]*Sinh[a + b*x])/(4*b^5) + (3*d^2*(c + d*x)^2*Cosh[a + b*x]*Sin
h[a + b*x])/(2*b^3) + ((c + d*x)^4*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (3*
d^3*(c + d*x)*Sinh[a + b*x]^2)/(2*b^4) - (d*(c + d*x)^3*Sinh[a + b*x]^2)/b^
2
```

Rubi [A] time = 0.104895, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 32, 2635, 8}

$$-\frac{3d^3(c+dx)\sinh^2(a+bx)}{2b^4}+\frac{3d^2(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b^3}-\frac{d(c+dx)^3\sinh^2(a+bx)}{b^2}+\frac{3d^4\sinh(a+bx)}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sinh[a + b*x]^2,x]
```

```
[Out] (-3*d^4*x)/(4*b^4) - (d*(c + d*x)^3)/(2*b^2) - (c + d*x)^5/(10*d) + (3*d^4*
Cosh[a + b*x]*Sinh[a + b*x])/(4*b^5) + (3*d^2*(c + d*x)^2*Cosh[a + b*x]*Sin
h[a + b*x])/(2*b^3) + ((c + d*x)^4*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (3*
d^3*(c + d*x)*Sinh[a + b*x]^2)/(2*b^4) - (d*(c + d*x)^3*Sinh[a + b*x]^2)/b^
2
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (c+dx)^4\sinh^2(a+bx)\,dx = \frac{(c+dx)^4\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{d(c+dx)^3\sinh^2(a+bx)}{b^2} - \frac{1}{2}\int (c+dx)^4\,dx + \frac{(3d\cdots}{}$$
$$= -\frac{(c+dx)^5}{10d} + \frac{3d^2(c+dx)^2\cosh(a+bx)\sinh(a+bx)}{2b^3} + \frac{(c+dx)^4\cosh(a+bx)\sinh(a+b\cdots}{2b}$$
$$= -\frac{d(c+dx)^3}{2b^2} - \frac{(c+dx)^5}{10d} + \frac{3d^4\cosh(a+bx)\sinh(a+bx)}{4b^5} + \frac{3d^2(c+dx)^2\cosh(a+bx)\sin\cdots}{2b^3}$$
$$= -\frac{3d^4x}{4b^4} - \frac{d(c+dx)^3}{2b^2} - \frac{(c+dx)^5}{10d} + \frac{3d^4\cosh(a+bx)\sinh(a+bx)}{4b^5} + \frac{3d^2(c+dx)^2\cosh(a\cdots}{2b^3}$$

Mathematica [A] time = 0.681253, size = 132, normalized size = 0.81

$$\frac{10\sinh(2(a+bx))\left(6b^2d^2(c+dx)^2+2b^4(c+dx)^4+3d^4\right)-20bd(c+dx)\cosh(2(a+bx))\left(2b^2(c+dx)^2+3d^2\right)-8b^5x\left(10\cdots\right.}{80b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Sinh[a + b*x]^2,x]
```

```
[Out] (-8*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) - 2
0*b*d*(c + d*x)*(3*d^2 + 2*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)] + 10*(3*d^4 +
 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sinh[2*(a + b*x)])/(80*b^5)
```

Maple [B] time = 0.016, size = 910, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(12/b^3*d^3*c*a^2*(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1/4*(b*x+a)^2-1/
4*cosh(b*x+a)^2)-12/b^3*d^3*c*a*(1/2*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)-1/6*
(b*x+a)^3-1/2*(b*x+a)*cosh(b*x+a)^2+1/4*cosh(b*x+a)*sinh(b*x+a)+1/4*b*x+1/4
*a)+6/b^4*d^4*a^2*(1/2*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)-1/6*(b*x+a)^3-1/2*
(b*x+a)*cosh(b*x+a)^2+1/4*cosh(b*x+a)*sinh(b*x+a)+1/4*b*x+1/4*a)-4/b^4*d^4*
a^3*(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)+4
/b^3*d^3*c*(1/2*(b*x+a)^3*cosh(b*x+a)*sinh(b*x+a)-1/8*(b*x+a)^4-3/4*(b*x+a)
^2*cosh(b*x+a)^2+3/4*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+3/8*(b*x+a)^2-3/8*cosh
(b*x+a)^2)+6/b^2*d^2*c^2*(1/2*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)-1/6*(b*x+a)
^3-1/2*(b*x+a)*cosh(b*x+a)^2+1/4*cosh(b*x+a)*sinh(b*x+a)+1/4*b*x+1/4*a)+4/b
*d*c^3*(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2
)-4/b^3*d^3*a^3*c*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*a)+6/b^2*d^2*a^2
*c^2*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*a)-4/b*d*a*c^3*(1/2*cosh(b*x+
a)*sinh(b*x+a)-1/2*b*x-1/2*a)-4/b^4*d^4*a*(1/2*(b*x+a)^3*cosh(b*x+a)*sinh(b
*x+a)-1/8*(b*x+a)^4-3/4*(b*x+a)^2*cosh(b*x+a)^2+3/4*(b*x+a)*cosh(b*x+a)*sin
h(b*x+a)+3/8*(b*x+a)^2-3/8*cosh(b*x+a)^2)+1/b^4*d^4*(1/2*(b*x+a)^4*cosh(b*x
+a)*sinh(b*x+a)-1/10*(b*x+a)^5-(b*x+a)^3*cosh(b*x+a)^2+3/2*(b*x+a)^2*cosh(b
*x+a)*sinh(b*x+a)+1/2*(b*x+a)^3-3/2*(b*x+a)*cosh(b*x+a)^2+3/4*cosh(b*x+a)*s
inh(b*x+a)+3/4*b*x+3/4*a)+1/b^4*d^4*a^4*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*
x-1/2*a)-12/b^2*d^2*c^2*a*(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1/4*(b*x+a)^
2-1/4*cosh(b*x+a)^2)+c^4*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*a))
```

Maxima [B] time = 1.2571, size = 516, normalized size = 3.19

$$-\frac{1}{4}\left(4x^2 - \frac{\left(2bxe^{(2a)} - e^{(2a)}\right)e^{(2bx)}}{b^2} + \frac{(2bx+1)e^{(-2bx-2a)}}{b^2}\right)c^3d - \frac{1}{8}\left(8x^3 - \frac{3\left(2b^2x^2e^{(2a)} - 2bxe^{(2a)} + e^{(2a)}\right)e^{(2bx)}}{b^3} + \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*x^2 - (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 + (2*b*x + 1)*e^(-2*b
*x - 2*a)/b^2)*c^3*d - 1/8*(8*x^3 - 3*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) +
e^(2*a))*e^(2*b*x)/b^3 + 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3)*c^
2*d^2 - 1/8*(4*x^4 - (4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2*a)
 - 3*e^(2*a))*e^(2*b*x)/b^4 + (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x
 - 2*a)/b^4)*c*d^3 - 1/80*(8*x^5 - 5*(2*b^4*x^4*e^(2*a) - 4*b^3*x^3*e^(2*a)
 + 6*b^2*x^2*e^(2*a) - 6*b*x*e^(2*a) + 3*e^(2*a))*e^(2*b*x)/b^5 + 5*(2*b^4*
x^4 + 4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^5)*d^4 - 1/8*c^
4*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)
```

Fricas [B] time = 2.68257, size = 663, normalized size = 4.09

$$-\frac{2b^5d^4x^5 + 10b^5cd^3x^4 + 20b^5c^2d^2x^3 + 20b^5c^3dx^2 + 10b^5c^4x + 5\left(2b^3d^4x^3 + 6b^3cd^3x^2 + 2b^3c^3d + 3bcd^3 + 3\left(2b^3c^2\ldots\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/20*(2*b^5*d^4*x^5 + 10*b^5*c*d^3*x^4 + 20*b^5*c^2*d^2*x^3 + 20*b^5*c^3*d
*x^2 + 10*b^5*c^4*x + 5*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d + 3*
b*c*d^3 + 3*(2*b^3*c^2*d^2 + b*d^4)*x)*cosh(b*x + a)^2 - 5*(2*b^4*d^4*x^4 +
 8*b^4*c*d^3*x^3 + 2*b^4*c^4 + 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 + b
^2*d^4)*x^2 + 4*(2*b^4*c^3*d + 3*b^2*c*d^3)*x)*cosh(b*x + a)*sinh(b*x + a)
+ 5*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d + 3*b*c*d^3 + 3*(2*b^3*c
^2*d^2 + b*d^4)*x)*sinh(b*x + a)^2)/b^5
```

Sympy [A] time = 7.34425, size = 660, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((c**4*x*sinh(a + b*x)**2/2 - c**4*x*cosh(a + b*x)**2/2 + c**3*d*x
**2*sinh(a + b*x)**2 - c**3*d*x**2*cosh(a + b*x)**2 + c**2*d**2*x**3*sinh(a
 + b*x)**2 - c**2*d**2*x**3*cosh(a + b*x)**2 + c*d**3*x**4*sinh(a + b*x)**2
/2 - c*d**3*x**4*cosh(a + b*x)**2/2 + d**4*x**5*sinh(a + b*x)**2/10 - d**4*
x**5*cosh(a + b*x)**2/10 + c**4*sinh(a + b*x)*cosh(a + b*x)/(2*b) + 2*c**3*
d*x*sinh(a + b*x)*cosh(a + b*x)/b + 3*c**2*d**2*x**2*sinh(a + b*x)*cosh(a +
 b*x)/b + 2*c*d**3*x**3*sinh(a + b*x)*cosh(a + b*x)/b + d**4*x**4*sinh(a +
b*x)*cosh(a + b*x)/(2*b) - c**3*d*sinh(a + b*x)**2/b**2 - 3*c**2*d**2*x*sin
```

```
h(a + b*x)**2/(2*b**2) - 3*c**2*d**2*x*cosh(a + b*x)**2/(2*b**2) - 3*c*d**3
*x**2*sinh(a + b*x)**2/(2*b**2) - 3*c*d**3*x**2*cosh(a + b*x)**2/(2*b**2) -
 d**4*x**3*sinh(a + b*x)**2/(2*b**2) - d**4*x**3*cosh(a + b*x)**2/(2*b**2)
+ 3*c**2*d**2*sinh(a + b*x)*cosh(a + b*x)/(2*b**3) + 3*c*d**3*x*sinh(a + b*
x)*cosh(a + b*x)/b**3 + 3*d**4*x**2*sinh(a + b*x)*cosh(a + b*x)/(2*b**3) -
3*c*d**3*sinh(a + b*x)**2/(2*b**4) - 3*d**4*x*sinh(a + b*x)**2/(4*b**4) - 3
*d**4*x*cosh(a + b*x)**2/(4*b**4) + 3*d**4*sinh(a + b*x)*cosh(a + b*x)/(4*b
**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4
+ d**4*x**5/5)*sinh(a)**2, True))
```

Giac [B] time = 1.24048, size = 505, normalized size = 3.12

$$-\frac{1}{10}\,d^4x^5 - \frac{1}{2}\,cd^3x^4 - c^2d^2x^3 - c^3dx^2 - \frac{1}{2}\,c^4x + \frac{\left(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 12\,b^4c^2d^2x^2 - 4\,b^3d^4x^3 + 8\,b^4c^3dx - 12\,b^3cd^3x^2 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/10*d^4*x^5 - 1/2*c*d^3*x^4 - c^2*d^2*x^3 - c^3*d*x^2 - 1/2*c^4*x + 1/16*
(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 8*b
^4*c^3*d*x - 12*b^3*c*d^3*x^2 + 2*b^4*c^4 - 12*b^3*c^2*d^2*x + 6*b^2*d^4*x^
2 - 4*b^3*c^3*d + 12*b^2*c*d^3*x + 6*b^2*c^2*d^2 - 6*b*d^4*x - 6*b*c*d^3 +
3*d^4)*e^(2*b*x + 2*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4
*c^2*d^2*x^2 + 4*b^3*d^4*x^3 + 8*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + 2*b^4*c^4
 + 12*b^3*c^2*d^2*x + 6*b^2*d^4*x^2 + 4*b^3*c^3*d + 12*b^2*c*d^3*x + 6*b^2*
c^2*d^2 + 6*b*d^4*x + 6*b*c*d^3 + 3*d^4)*e^(-2*b*x - 2*a)/b^5
```

3.9 $\int (c+dx)^3 \sinh^2(a+bx)\,dx$

Optimal. Leaf size=134

$$\frac{3d^2(c+dx)\sinh(a+bx)\cosh(a+bx)}{4b^3}-\frac{3d(c+dx)^2\sinh^2(a+bx)}{4b^2}-\frac{3d^3\sinh^2(a+bx)}{8b^4}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}$$

```
[Out] (-3*c*d^2*x)/(4*b^2) - (3*d^3*x^2)/(8*b^2) - (c + d*x)^4/(8*d) + (3*d^2*(c
+ d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^3) + ((c + d*x)^3*Cosh[a + b*x]*Si
nh[a + b*x])/(2*b) - (3*d^3*Sinh[a + b*x]^2)/(8*b^4) - (3*d*(c + d*x)^2*Sin
h[a + b*x]^2)/(4*b^2)
```

Rubi [A] time = 0.0742048, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3311, 32, 3310}

$$\frac{3d^2(c+dx)\sinh(a+bx)\cosh(a+bx)}{4b^3}-\frac{3d(c+dx)^2\sinh^2(a+bx)}{4b^2}-\frac{3d^3\sinh^2(a+bx)}{8b^4}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sinh[a + b*x]^2,x]
```

```
[Out] (-3*c*d^2*x)/(4*b^2) - (3*d^3*x^2)/(8*b^2) - (c + d*x)^4/(8*d) + (3*d^2*(c
+ d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^3) + ((c + d*x)^3*Cosh[a + b*x]*Si
nh[a + b*x])/(2*b) - (3*d^3*Sinh[a + b*x]^2)/(8*b^4) - (3*d*(c + d*x)^2*Sin
h[a + b*x]^2)/(4*b^2)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)^3 \sinh^2(a+bx)\,dx = \frac{(c+dx)^3\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{3d(c+dx)^2\sinh^2(a+bx)}{4b^2} - \frac{1}{2}\int (c+dx)^3\,dx + \frac{(3\cdots}{}$$
$$= -\frac{(c+dx)^4}{8d} + \frac{3d^2(c+dx)\cosh(a+bx)\sinh(a+bx)}{4b^3} + \frac{(c+dx)^3\cosh(a+bx)\sinh(a+b\cdots}{2b}$$
$$= -\frac{3cd^2x}{4b^2} - \frac{3d^3x^2}{8b^2} - \frac{(c+dx)^4}{8d} + \frac{3d^2(c+dx)\cosh(a+bx)\sinh(a+bx)}{4b^3} + \frac{(c+dx)^3\cosh(a\cdots}{}$$

Mathematica [A] time = 0.403, size = 104, normalized size = 0.78

$$\frac{2b(c+dx)\sinh(2(a+bx))\left(2b^2(c+dx)^2+3d^2\right) - 3d\cosh(2(a+bx))\left(2b^2(c+dx)^2+d^2\right) - 2b^4x\left(6c^2dx+4c^3+4cd^2x^2+\cdots\right.}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sinh[a + b*x]^2,x]
```

```
[Out] (-2*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 3*d*(d^2 + 2*b^2*(c
 + d*x)^2)*Cosh[2*(a + b*x)] + 2*b*(c + d*x)*(3*d^2 + 2*b^2*(c + d*x)^2)*Si
nh[2*(a + b*x)])/(16*b^4)
```

Maple [B] time = 0.007, size = 523, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^3*d^3*(1/2*(b*x+a)^3*cosh(b*x+a)*sinh(b*x+a)-1/8*(b*x+a)^4-3/4*(b*
x+a)^2*cosh(b*x+a)^2+3/4*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+3/8*(b*x+a)^2-3/8*
cosh(b*x+a)^2)-3/b^3*d^3*a*(1/2*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)-1/6*(b*x+
a)^3-1/2*(b*x+a)*cosh(b*x+a)^2+1/4*cosh(b*x+a)*sinh(b*x+a)+1/4*b*x+1/4*a)+3
/b^3*d^3*a^2*(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1/4*(b*x+a)^2-1/4*cosh(b*
x+a)^2)-1/b^3*d^3*a^3*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*a)+3/b^2*c*d
^2*(1/2*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)-1/6*(b*x+a)^3-1/2*(b*x+a)*cosh(b*
x+a)^2+1/4*cosh(b*x+a)*sinh(b*x+a)+1/4*b*x+1/4*a)-6/b^2*c*d^2*a*(1/2*(b*x+a
)*cosh(b*x+a)*sinh(b*x+a)-1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)+3/b^2*c*d^2*a^2*
(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*a)+3/b*c^2*d*(1/2*(b*x+a)*cosh(b*x
+a)*sinh(b*x+a)-1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)-3/b*c^2*d*a*(1/2*cosh(b*x+
a)*sinh(b*x+a)-1/2*b*x-1/2*a)+c^3*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*
a))
```

Maxima [B] time = 1.14597, size = 355, normalized size = 2.65

$$-\frac{3}{16}\left(4x^2 - \frac{\left(2bxe^{(2a)} - e^{(2a)}\right)e^{(2bx)}}{b^2} + \frac{(2bx+1)e^{(-2bx-2a)}}{b^2}\right)c^2d - \frac{1}{16}\left(8x^3 - \frac{3\left(2b^2x^2e^{(2a)} - 2bxe^{(2a)} + e^{(2a)}\right)e^{(2bx)}}{b^3} + \frac{3(2\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -3/16*(4*x^2 - (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 + (2*b*x + 1)*e^(-2*
b*x - 2*a)/b^2)*c^2*d - 1/16*(8*x^3 - 3*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a)
+ e^(2*a))*e^(2*b*x)/b^3 + 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3)*
c*d^2 - 1/32*(4*x^4 - (4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2*a
) - 3*e^(2*a))*e^(2*b*x)/b^4 + (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*
x - 2*a)/b^4)*d^3 - 1/8*c^3*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)
```

Fricas [A] time = 2.70187, size = 459, normalized size = 3.43

$$-\frac{2\,b^4d^3x^4+8\,b^4cd^2x^3+12\,b^4c^2dx^2+8\,b^4c^3x+3\left(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d+d^3\right)\cosh\left(bx+a\right)^2-4\left(2\,b^3d^3x^3+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(2*b^4*d^3*x^4 + 8*b^4*c*d^2*x^3 + 12*b^4*c^2*d*x^2 + 8*b^4*c^3*x + 3
*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*cosh(b*x + a)^2 - 4*(2
*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 + 3*b*c*d^2 + 3*(2*b^3*c^2*d + b
*d^3)*x)*cosh(b*x + a)*sinh(b*x + a) + 3*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2
*b^2*c^2*d + d^3)*sinh(b*x + a)^2)/b^4
```

Sympy [A] time = 3.79951, size = 456, normalized size = 3.4

$$\begin{cases} \frac{c^3x\sinh^2(a+bx)}{2}-\frac{c^3x\cosh^2(a+bx)}{2}+\frac{3c^2dx^2\sinh^2(a+bx)}{4}-\frac{3c^2dx^2\cosh^2(a+bx)}{4}+\frac{cd^2x^3\sinh^2(a+bx)}{2}-\frac{cd^2x^3\cosh^2(a+bx)}{2}+\frac{d^3x^4\sinh^2(a+b\cdots}{8} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\sinh^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((c**3*x*sinh(a + b*x)**2/2 - c**3*x*cosh(a + b*x)**2/2 + 3*c**2*d
*x**2*sinh(a + b*x)**2/4 - 3*c**2*d*x**2*cosh(a + b*x)**2/4 + c*d**2*x**3*s
inh(a + b*x)**2/2 - c*d**2*x**3*cosh(a + b*x)**2/2 + d**3*x**4*sinh(a + b*x
)**2/8 - d**3*x**4*cosh(a + b*x)**2/8 + c**3*sinh(a + b*x)*cosh(a + b*x)/(2
*b) + 3*c**2*d*x*sinh(a + b*x)*cosh(a + b*x)/(2*b) + 3*c*d**2*x**2*sinh(a +
 b*x)*cosh(a + b*x)/(2*b) + d**3*x**3*sinh(a + b*x)*cosh(a + b*x)/(2*b) - 3
*c**2*d*sinh(a + b*x)**2/(4*b**2) - 3*c*d**2*x*sinh(a + b*x)**2/(4*b**2) -
3*c*d**2*x*cosh(a + b*x)**2/(4*b**2) - 3*d**3*x**2*sinh(a + b*x)**2/(8*b**2
) - 3*d**3*x**2*cosh(a + b*x)**2/(8*b**2) + 3*c*d**2*sinh(a + b*x)*cosh(a +
 b*x)/(4*b**3) + 3*d**3*x*sinh(a + b*x)*cosh(a + b*x)/(4*b**3) - 3*d**3*sin
h(a + b*x)**2/(8*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3
 + d**3*x**4/4)*sinh(a)**2, True))
```

Giac [B] time = 1.2579, size = 328, normalized size = 2.45

$$-\frac{1}{8}\,d^3x^4-\frac{1}{2}\,cd^2x^3-\frac{3}{4}\,c^2dx^2-\frac{1}{2}\,c^3x+\frac{\left(4\,b^3d^3x^3+12\,b^3cd^2x^2+12\,b^3c^2dx-6\,b^2d^3x^2+4\,b^3c^3-12\,b^2cd^2x-6\,b^2c^2d+\cdots\right.}{32\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*d^3*x^4 - 1/2*c*d^2*x^3 - 3/4*c^2*d*x^2 - 1/2*c^3*x + 1/32*(4*b^3*d^3*
x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x - 6*b^2*d^3*x^2 + 4*b^3*c^3 - 12*b^
2*c*d^2*x - 6*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 3*d^3)*e^(2*b*x + 2*a)/b^
4 - 1/32*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x + 6*b^2*d^3*x^2
 + 4*b^3*c^3 + 12*b^2*c*d^2*x + 6*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 + 3*d^3
)*e^(-2*b*x - 2*a)/b^4
```

3.10 $\int (c+dx)^2 \sinh^2(a+bx)\,dx$

Optimal. Leaf size=95

$$-\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}+\frac{d^2\sinh(a+bx)\cosh(a+bx)}{4b^3}+\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{d^2x}{4b^2}-\frac{(c+dx)^3}{6d}$$

```
[Out] -(d^2*x)/(4*b^2) - (c + d*x)^3/(6*d) + (d^2*Cosh[a + b*x]*Sinh[a + b*x])/(4
*b^3) + ((c + d*x)^2*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (d*(c + d*x)*Sinh
[a + b*x]^2)/(2*b^2)
```

Rubi [A] time = 0.0543412, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 32, 2635, 8}

$$-\frac{d(c+dx)\sinh^2(a+bx)}{2b^2}+\frac{d^2\sinh(a+bx)\cosh(a+bx)}{4b^3}+\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{d^2x}{4b^2}-\frac{(c+dx)^3}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sinh[a + b*x]^2,x]
```

```
[Out] -(d^2*x)/(4*b^2) - (c + d*x)^3/(6*d) + (d^2*Cosh[a + b*x]*Sinh[a + b*x])/(4
*b^3) + ((c + d*x)^2*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (d*(c + d*x)*Sinh
[a + b*x]^2)/(2*b^2)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (c+dx)^2 \sinh^2(a+bx)\,dx = \frac{(c+dx)^2\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{d(c+dx)\sinh^2(a+bx)}{2b^2} - \frac{1}{2}\int (c+dx)^2\,dx + \frac{d^2\int}{}$$
$$= -\frac{(c+dx)^3}{6d} + \frac{d^2\cosh(a+bx)\sinh(a+bx)}{4b^3} + \frac{(c+dx)^2\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{d(c+}{}$$
$$= -\frac{d^2x}{4b^2} - \frac{(c+dx)^3}{6d} + \frac{d^2\cosh(a+bx)\sinh(a+bx)}{4b^3} + \frac{(c+dx)^2\cosh(a+bx)\sinh(a+bx)}{2b}$$

Mathematica [A] time = 0.310909, size = 75, normalized size = 0.79

$$\frac{3\sinh(2(a+bx))\left(2b^2(c+dx)^2+d^2\right) - 6bd(c+dx)\cosh(2(a+bx)) - 4b^3x\left(3c^2+3cdx+d^2x^2\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sinh[a + b*x]^2,x]
```

```
[Out] (-4*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) - 6*b*d*(c + d*x)*Cosh[2*(a + b*x)] +
 3*(d^2 + 2*b^2*(c + d*x)^2)*Sinh[2*(a + b*x)])/(24*b^3)
```

Maple [B] time = 0.007, size = 262, normalized size = 2.8

$$\frac{1}{b}\left(\frac{d^2}{b^2}\left(\frac{(bx+a)^2\cosh(bx+a)\sinh(bx+a)}{2} - \frac{(bx+a)^3}{6} - \frac{(bx+a)(\cosh(bx+a))^2}{2} + \frac{\cosh(bx+a)\sinh(bx+a)}{4} + \frac{bx}{4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^2*d^2*(1/2*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)-1/6*(b*x+a)^3-1/2*(b*
x+a)*cosh(b*x+a)^2+1/4*cosh(b*x+a)*sinh(b*x+a)+1/4*b*x+1/4*a)-2/b^2*d^2*a*(
1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)+1/b^2*
d^2*a^2*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*a)+2/b*c*d*(1/2*(b*x+a)*co
sh(b*x+a)*sinh(b*x+a)-1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)-2/b*c*d*a*(1/2*cosh(
b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*a)+c^2*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-
1/2*a))
```

Maxima [A] time = 1.14048, size = 223, normalized size = 2.35

$$-\frac{1}{8}\left(4x^2 - \frac{\left(2bxe^{(2a)} - e^{(2a)}\right)e^{(2bx)}}{b^2} + \frac{(2bx+1)e^{(-2bx-2a)}}{b^2}\right)cd - \frac{1}{48}\left(8x^3 - \frac{3\left(2b^2x^2e^{(2a)} - 2bxe^{(2a)} + e^{(2a)}\right)e^{(2bx)}}{b^3} + \frac{3\left(2b^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(4*x^2 - (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 + (2*b*x + 1)*e^(-2*b
*x - 2*a)/b^2)*c*d - 1/48*(8*x^3 - 3*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e
^(2*a))*e^(2*b*x)/b^3 + 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3)*d^2
 - 1/8*c^2*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)
```

Fricas [A] time = 2.69329, size = 288, normalized size = 3.03

$$-\frac{2\,b^3d^2x^3+6\,b^3cdx^2+6\,b^3c^2x+3\left(bd^2x+bcd\right)\cosh\left(bx+a\right)^2-3\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2+d^2\right)\cosh\left(bx+a\right)\mathrm{si}}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/12*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 + 6*b^3*c^2*x + 3*(b*d^2*x + b*c*d)*co
sh(b*x + a)^2 - 3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*cosh(b*x
+ a)*sinh(b*x + a) + 3*(b*d^2*x + b*c*d)*sinh(b*x + a)^2)/b^3
```

Sympy [A] time = 1.77054, size = 264, normalized size = 2.78

$$\begin{cases}\frac{c^2x\sinh^2(a+bx)}{2}-\frac{c^2x\cosh^2(a+bx)}{2}+\frac{cdx^2\sinh^2(a+bx)}{2}-\frac{cdx^2\cosh^2(a+bx)}{2}+\frac{d^2x^3\sinh^2(a+bx)}{6}-\frac{d^2x^3\cosh^2(a+bx)}{6}+\frac{c^2\sinh(a+bx)\cosh(a}{2b}\\\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sinh^2(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((c**2*x*sinh(a + b*x)**2/2 - c**2*x*cosh(a + b*x)**2/2 + c*d*x**2
*sinh(a + b*x)**2/2 - c*d*x**2*cosh(a + b*x)**2/2 + d**2*x**3*sinh(a + b*x)
**2/6 - d**2*x**3*cosh(a + b*x)**2/6 + c**2*sinh(a + b*x)*cosh(a + b*x)/(2*
b) + c*d*x*sinh(a + b*x)*cosh(a + b*x)/b + d**2*x**2*sinh(a + b*x)*cosh(a +
 b*x)/(2*b) - c*d*sinh(a + b*x)**2/(2*b**2) - d**2*x*sinh(a + b*x)**2/(4*b*
*2) - d**2*x*cosh(a + b*x)**2/(4*b**2) + d**2*sinh(a + b*x)*cosh(a + b*x)/(
4*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sinh(a)**2, True))
```

Giac [A] time = 1.17973, size = 184, normalized size = 1.94

$$-\frac{1}{6}\,d^2x^3-\frac{1}{2}\,cdx^2-\frac{1}{2}\,c^2x+\frac{\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-2\,bd^2x-2\,bcd+d^2\right)e^{(2\,bx+2\,a)}}{16\,b^3}-\frac{\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/6*d^2*x^3 - 1/2*c*d*x^2 - 1/2*c^2*x + 1/16*(2*b^2*d^2*x^2 + 4*b^2*c*d*x
+ 2*b^2*c^2 - 2*b*d^2*x - 2*b*c*d + d^2)*e^(2*b*x + 2*a)/b^3 - 1/16*(2*b^2*
d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 2*b*d^2*x + 2*b*c*d + d^2)*e^(-2*b*x -
2*a)/b^3
```

3.11 $\int (c+dx)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=55

$$-\frac{d\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{cx}{2}-\frac{dx^2}{4}$$

```
[Out] -(c*x)/2 - (d*x^2)/4 + ((c + d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (d*S
inh[a + b*x]^2)/(4*b^2)
```

Rubi [A] time = 0.0262664, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3310}

$$-\frac{d\sinh^2(a+bx)}{4b^2}+\frac{(c+dx)\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{cx}{2}-\frac{dx^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sinh[a + b*x]^2,x]
```

```
[Out] -(c*x)/2 - (d*x^2)/4 + ((c + d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (d*S
inh[a + b*x]^2)/(4*b^2)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\sinh^2(a+bx)\,dx &= \frac{(c+dx)\cosh(a+bx)\sinh(a+bx)}{2b}-\frac{d\sinh^2(a+bx)}{4b^2}-\frac{1}{2}\int (c+dx)\,dx\\ &= -\frac{cx}{2}-\frac{dx^2}{4}+\frac{(c+dx)\cosh(a+bx)\sinh(a+bx)}{2b}-\frac{d\sinh^2(a+bx)}{4b^2}\end{aligned}$$

Mathematica [A] time = 0.161799, size = 52, normalized size = 0.95

$$\frac{2b((c+dx)\sinh(2(a+bx))-2ac-bx(2c+dx))-d\cosh(2(a+bx))}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sinh[a + b*x]^2,x]
```

```
[Out] (-(d*Cosh[2*(a + b*x)]) + 2*b*(-2*a*c - b*x*(2*c + d*x) + (c + d*x)*Sinh[2*
(a + b*x)]))/(8*b^2)
```

Maple [B] time = 0.006, size = 103, normalized size = 1.9

$$\frac{1}{b}\left(\frac{d}{b}\left(\frac{(bx+a)\cosh(bx+a)\sinh(bx+a)}{2}-\frac{(bx+a)^2}{4}-\frac{(\cosh(bx+a))^2}{4}\right)-\frac{da}{b}\left(\frac{\cosh(bx+a)\sinh(bx+a)}{2}-\frac{bx}{2}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b*d*(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)-1/4*(b*x+a)^2-1/4*cosh(b*x+
a)^2)-1/b*d*a*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*a)+c*(1/2*cosh(b*x+a
)*sinh(b*x+a)-1/2*b*x-1/2*a))
```

Maxima [A] time = 1.21832, size = 119, normalized size = 2.16

$$-\frac{1}{16}\left(4x^2-\frac{\left(2bxe^{(2a)}-e^{(2a)}\right)e^{(2bx)}}{b^2}+\frac{(2bx+1)e^{(-2bx-2a)}}{b^2}\right)d-\frac{1}{8}c\left(4x-\frac{e^{(2bx+2a)}}{b}+\frac{e^{(-2bx-2a)}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(4*x^2 - (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 + (2*b*x + 1)*e^(-2*
b*x - 2*a)/b^2)*d - 1/8*c*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)
```

Fricas [A] time = 2.62637, size = 165, normalized size = 3.

$$-\frac{2b^2dx^2+4b^2cx+d\cosh(bx+a)^2-4(bdx+bc)\cosh(bx+a)\sinh(bx+a)+d\sinh(bx+a)^2}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*b^2*d*x^2 + 4*b^2*c*x + d*cosh(b*x + a)^2 - 4*(b*d*x + b*c)*cosh(b*
x + a)*sinh(b*x + a) + d*sinh(b*x + a)^2)/b^2
```

Sympy [A] time = 0.800126, size = 126, normalized size = 2.29

$$\begin{cases}\frac{cx\sinh^2(a+bx)}{2}-\frac{cx\cosh^2(a+bx)}{2}+\frac{dx^2\sinh^2(a+bx)}{4}-\frac{dx^2\cosh^2(a+bx)}{4}+\frac{c\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{dx\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{d\sinh^2(a}{4b^2}\\ \left(cx+\frac{dx^2}{2}\right)\sinh^2(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((c*x*sinh(a + b*x)**2/2 - c*x*cosh(a + b*x)**2/2 + d*x**2*sinh(a
+ b*x)**2/4 - d*x**2*cosh(a + b*x)**2/4 + c*sinh(a + b*x)*cosh(a + b*x)/(2*
b) + d*x*sinh(a + b*x)*cosh(a + b*x)/(2*b) - d*sinh(a + b*x)**2/(4*b**2), N
```

```
e(b, 0)), ((c*x + d*x**2/2)*sinh(a)**2, True))
```

Giac [A] time = 1.21919, size = 85, normalized size = 1.55

$$-\frac{1}{4}dx^2 - \frac{1}{2}cx + \frac{(2bdx + 2bc - d)e^{(2bx+2a)}}{16b^2} - \frac{(2bdx + 2bc + d)e^{(-2bx-2a)}}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*d*x^2 - 1/2*c*x + 1/16*(2*b*d*x + 2*b*c - d)*e^(2*b*x + 2*a)/b^2 - 1/1
6*(2*b*d*x + 2*b*c + d)*e^(-2*b*x - 2*a)/b^2
```

3.12 $\int \frac{\sinh^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=78

$$\frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{2d}-\frac{\log(c+dx)}{2d}$$

```
[Out] (Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*c)/d + 2*b*x])/(2*d) - Log[c + d*x
]/(2*d) + (Sinh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Rubi [A] time = 0.16549, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3312, 3303, 3298, 3301}

$$\frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{2d}-\frac{\log(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2/(c + d*x),x]
```

```
[Out] (Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*c)/d + 2*b*x])/(2*d) - Log[c + d*x
]/(2*d) + (Sinh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sinh^2(a+bx)}{c+dx}\,dx = -\int \left(\frac{1}{2(c+dx)} - \frac{\cosh(2a+2bx)}{2(c+dx)}\right)dx$$
$$= -\frac{\log(c+dx)}{2d} + \frac{1}{2}\int \frac{\cosh(2a+2bx)}{c+dx}\,dx$$
$$= -\frac{\log(c+dx)}{2d} + \frac{1}{2}\cosh\left(2a-\frac{2bc}{d}\right)\int \frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx + \frac{1}{2}\sinh\left(2a-\frac{2bc}{d}\right)\int \frac{\sinh\left(\frac{2bc}{d}+2b\right.}{c+dx}$$
$$= \frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{2d} - \frac{\log(c+dx)}{2d} + \frac{\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

Mathematica [A] time = 0.121673, size = 66, normalized size = 0.85

$$\frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2b(c+dx)}{d}\right)+\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2b(c+dx)}{d}\right)-\log(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2/(c + d*x),x]
```

```
[Out] (Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*(c + d*x))/d] - Log[c + d*x] + Sin
h[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])/(2*d)
```

Maple [A] time = 0.074, size = 97, normalized size = 1.2

$$-\frac{\ln(dx+c)}{2d} - \frac{1}{4d}e^{-2\frac{da-cb}{d}}\text{Ei}\left(1, 2bx+2a-2\frac{da-cb}{d}\right) - \frac{1}{4d}e^{2\frac{da-cb}{d}}\text{Ei}\left(1, -2bx-2a-2\frac{-da+cb}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2/(d*x+c),x)
```

```
[Out] -1/2*ln(d*x+c)/d-1/4/d*exp(-2*(a*d-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/
4/d*exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)
```

Maxima [A] time = 1.38425, size = 97, normalized size = 1.24

$$-\frac{e^{\left(-2a+\frac{2bc}{d}\right)}E_1\left(\frac{2(dx+c)b}{d}\right)}{4d} - \frac{e^{\left(2a-\frac{2bc}{d}\right)}E_1\left(-\frac{2(dx+c)b}{d}\right)}{4d} - \frac{\log(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*e^(-2*a + 2*b*c/d)*exp_integral_e(1, 2*(d*x + c)*b/d)/d - 1/4*e^(2*a -
 2*b*c/d)*exp_integral_e(1, -2*(d*x + c)*b/d)/d - 1/2*log(d*x + c)/d
```

Fricas [A] time = 2.63968, size = 232, normalized size = 2.97

$$\frac{\left(\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)+\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{2\,(bc-ad)}{d}\right)+\left(\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)-\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sinh\left(-\frac{2\,(bc-ad)}{d}\right)-2\,\log\left(dx+\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*((Ei(2*(b*d*x + b*c)/d) + Ei(-2*(b*d*x + b*c)/d))*cosh(-2*(b*c - a*d)/d
) + (Ei(2*(b*d*x + b*c)/d) - Ei(-2*(b*d*x + b*c)/d))*sinh(-2*(b*c - a*d)/d)
 - 2*log(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sinh(a + b*x)**2/(c + d*x), x)
```

Giac [A] time = 1.19975, size = 92, normalized size = 1.18

$$\frac{\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}+\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}-2\,\log\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) + Ei(-2*(b*d*x + b*c)/d)*e^(-2
*a + 2*b*c/d) - 2*log(d*x + c))/d
```

3.13 $\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=81

$$\frac{b\sinh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{b\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\sinh^2(a+bx)}{d(c+dx)}$$

```
[Out] (b*CoshIntegral[(2*b*c)/d + 2*b*x]*Sinh[2*a - (2*b*c)/d])/d^2 - Sinh[a + b*
x]^2/(d*(c + d*x)) + (b*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*
x])/d^2
```

Rubi [A] time = 0.154983, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3313, 12, 3303, 3298, 3301}

$$\frac{b\sinh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{b\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\sinh^2(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2/(c + d*x)^2,x]
```


```
[Out] (b*CoshIntegral[(2*b*c)/d + 2*b*x]*Sinh[2*a - (2*b*c)/d])/d^2 - Sinh[a + b*
x]^2/(d*(c + d*x)) + (b*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*
x])/d^2
```


Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx &= -\frac{\sinh^2(a+bx)}{d(c+dx)} - \frac{(2ib)\int \frac{i\sinh(2a+2bx)}{2(c+dx)}\,dx}{d}\\ &= -\frac{\sinh^2(a+bx)}{d(c+dx)} + \frac{b\int \frac{\sinh(2a+2bx)}{c+dx}\,dx}{d}\\ &= -\frac{\sinh^2(a+bx)}{d(c+dx)} + \frac{\left(b\cosh\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sinh\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d} + \frac{\left(b\sinh\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,d}{d}\\ &= \frac{b\operatorname{Chi}\left(\frac{2bc}{d}+2bx\right)\sinh\left(2a-\frac{2bc}{d}\right)}{d^2} - \frac{\sinh^2(a+bx)}{d(c+dx)} + \frac{b\cosh\left(2a-\frac{2bc}{d}\right)\operatorname{Shi}\left(\frac{2bc}{d}+2bx\right)}{d^2}\end{aligned}$$

Mathematica [A] time = 0.44441, size = 75, normalized size = 0.93

$$\frac{b\sinh\left(2a-\frac{2bc}{d}\right)\operatorname{Chi}\left(\frac{2b(c+dx)}{d}\right)+b\cosh\left(2a-\frac{2bc}{d}\right)\operatorname{Shi}\left(\frac{2b(c+dx)}{d}\right)-\frac{d\sinh^2(a+bx)}{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] (b*CoshIntegral[(2*b*(c + d*x))/d]*Sinh[2*a - (2*b*c)/d] - (d*Sinh[a + b*x]
^2)/(c + d*x) + b*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])/d^
2
```

Maple [A] time = 0.072, size = 152, normalized size = 1.9

$$\frac{1}{2\,d\,(dx+c)} - \frac{b{\rm e}^{-2\,bx-2\,a}}{(4\,bdx+4\,cb)\,d} + \frac{b}{2\,d^2}{\rm e}^{-2\,\frac{da-cb}{d}}{\rm Ei}\left(1,2\,bx+2\,a-2\,\frac{da-cb}{d}\right) - \frac{b{\rm e}^{2\,bx+2\,a}}{4\,d^2}\left(\frac{cb}{d}+bx\right)^{-1} - \frac{b}{2\,d^2}{\rm e}^{2\,\frac{da-cb}{d}}{\rm Ei}\left(1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] 1/2/d/(d*x+c)-1/4*b*exp(-2*b*x-2*a)/(b*d*x+b*c)/d+1/2*b/d^2*exp(-2*(a*d-b*c
)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/4*b/d^2*exp(2*b*x+2*a)/(b*c/d+b*x)-1/2
*b/d^2*exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)
```

Maxima [A] time = 1.38888, size = 119, normalized size = 1.47

$$-\frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_2\left(\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)d} - \frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_2\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)d} + \frac{1}{2\,\left(d^2x+cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*e^(-2*a + 2*b*c/d)*exp_integral_e(2, 2*(d*x + c)*b/d)/((d*x + c)*d) -
1/4*e^(2*a - 2*b*c/d)*exp_integral_e(2, -2*(d*x + c)*b/d)/((d*x + c)*d) + 1
/2/(d^2*x + c*d)
```

Fricas [B] time = 2.72063, size = 365, normalized size = 4.51

$$-\frac{d\cosh\left(bx+a\right)^{2}+d\sinh\left(bx+a\right)^{2}-\left(\left(bdx+bc\right)\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)-\left(bdx+bc\right)\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{2\,(bc-ad)}{d}\right)-\left((bdx+b\right.}{2\left(d^{3}x+cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(d*cosh(b*x + a)^2 + d*sinh(b*x + a)^2 - ((b*d*x + b*c)*Ei(2*(b*d*x +
b*c)/d) - (b*d*x + b*c)*Ei(-2*(b*d*x + b*c)/d))*cosh(-2*(b*c - a*d)/d) - ((
b*d*x + b*c)*Ei(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*Ei(-2*(b*d*x + b*c)/d))*
sinh(-2*(b*c - a*d)/d) - d)/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^{2}\left(a+bx\right)}{\left(c+dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(sinh(a + b*x)**2/(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^2/(d*x + c)^2, x)
```

3.14 $\int \frac{\sinh^2(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=112

$$\frac{b^2\cosh\left(2a-\frac{2bc}{d}\right)\mathrm{Chi}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{b^2\sinh\left(2a-\frac{2bc}{d}\right)\mathrm{Shi}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)}-\frac{\sinh^2(a+b}{2d(c+dx)}$$

```
[Out] (b^2*Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*c)/d + 2*b*x])/d^3 - (b*Cosh[a
 + b*x]*Sinh[a + b*x])/(d^2*(c + d*x)) - Sinh[a + b*x]^2/(2*d*(c + d*x)^2)
+ (b^2*Sinh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/d^3
```

Rubi [A] time = 0.195081, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3314, 31, 3312, 3303, 3298, 3301}

$$\frac{b^2\cosh\left(2a-\frac{2bc}{d}\right)\mathrm{Chi}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{b^2\sinh\left(2a-\frac{2bc}{d}\right)\mathrm{Shi}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)}-\frac{\sinh^2(a+b}{2d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2/(c + d*x)^3,x]
```

```
[Out] (b^2*Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*c)/d + 2*b*x])/d^3 - (b*Cosh[a
 + b*x]*Sinh[a + b*x])/(d^2*(c + d*x)) - Sinh[a + b*x]^2/(2*d*(c + d*x)^2)
+ (b^2*Sinh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/d^3
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^3}dx = -\frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2} + \frac{b^2\int\frac{1}{c+dx}dx}{d^2} + \frac{(2b^2)\int\frac{\sinh^2(a+bx)}{c+dx}dx}{d^2}$$

$$= \frac{b^2\log(c+dx)}{d^3} - \frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2} - \frac{(2b^2)\int\left(\frac{1}{2(c+dx)} - \frac{\cosh(2a+2bx)}{2(c+dx)}\right)d}{d^2}$$

$$= -\frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2} + \frac{b^2\int\frac{\cosh(2a+2bx)}{c+dx}dx}{d^2}$$

$$= -\frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2} + \frac{\left(b^2\cosh\left(2a-\frac{2bc}{d}\right)\right)\int\frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx}{d^2} + \frac{(b^2\sin}{}$$

$$= \frac{b^2\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{d^3} - \frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2} + \frac{b^2\sinh\left(2a\right.}{}$$

Mathematica [A] time = 0.916274, size = 102, normalized size = 0.91

$$\frac{2b^2\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2b(c+dx)}{d}\right) + 2b^2\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2b(c+dx)}{d}\right) - \frac{d\left(b(c+dx)\sinh(2(a+bx))+d\sinh^2(a+bx)\right)}{(c+dx)^2}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2/(c + d*x)^3,x]
```

```
[Out] (2*b^2*Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*(c + d*x))/d] - (d*(d*Sinh[a
 + b*x]^2 + b*(c + d*x)*Sinh[2*(a + b*x)]))/(c + d*x)^2 + 2*b^2*Sinh[2*a -
(2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])/(2*d^3)
```

Maple [B] time = 0.076, size = 299, normalized size = 2.7

$$\frac{1}{4d(dx+c)^2} + \frac{b^3e^{-2bx-2a}x}{4d(b^2d^2x^2+2b^2cdx+c^2b^2)} + \frac{b^3e^{-2bx-2a}c}{4d^2(b^2d^2x^2+2b^2cdx+c^2b^2)} - \frac{b^2e^{-2bx-2a}}{8d(b^2d^2x^2+2b^2cdx+c^2b^2)} - \frac{b^2}{2d^3}e^{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2/(d*x+c)^3,x)
```

```
[Out] 1/4/d/(d*x+c)^2+1/4*b^3*exp(-2*b*x-2*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)
*x+1/4*b^3*exp(-2*b*x-2*a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c-1/8*b^2*
```

```
exp(-2*b*x-2*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)-1/2*b^2/d^3*exp(-2*(a*d
-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/8*b^2/d^3*exp(2*b*x+2*a)/(b*c/d+b*
x)^2-1/4*b^2/d^3*exp(2*b*x+2*a)/(b*c/d+b*x)-1/2*b^2/d^3*exp(2*(a*d-b*c)/d)*
Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)
```

Maxima [A] time = 1.47718, size = 134, normalized size = 1.2

$$\frac{1}{4\left(d^3x^2+2\,cd^2x+c^2d\right)}-\frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_3\left(\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^2d}-\frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_3\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4/(d^3*x^2 + 2*c*d^2*x + c^2*d) - 1/4*e^(-2*a + 2*b*c/d)*exp_integral_e(3
, 2*(d*x + c)*b/d)/((d*x + c)^2*d) - 1/4*e^(2*a - 2*b*c/d)*exp_integral_e(3
, -2*(d*x + c)*b/d)/((d*x + c)^2*d)
```

Fricas [B] time = 2.71413, size = 597, normalized size = 5.33

$$-\frac{d^2\cosh(bx+a)^2+d^2\sinh(bx+a)^2+4\left(bd^2x+bcd\right)\cosh(bx+a)\sinh(bx+a)-d^2-2\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(d^2*cosh(b*x + a)^2 + d^2*sinh(b*x + a)^2 + 4*(b*d^2*x + b*c*d)*cosh(
b*x + a)*sinh(b*x + a) - d^2 - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(
2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-2*(b*d*x + b
*c)/d))*cosh(-2*(b*c - a*d)/d) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*E
i(2*(b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-2*(b*d*x +
 b*c)/d))*sinh(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh^2(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(sinh(a + b*x)**2/(c + d*x)**3, x)
```

Giac [B] time = 1.15751, size = 446, normalized size = 3.98

$$\frac{4\,b^2d^2x^2\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}+4\,b^2d^2x^2\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}+8\,b^2cdx\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}+8\,b^2cdx\mathrm{Ei}\left(-\frac{2\,(\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^2*d^2*x^2*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) + 4*b^2*d^2*x^2*
Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) + 8*b^2*c*d*x*Ei(2*(b*d*x + b*c)/
d)*e^(2*a - 2*b*c/d) + 8*b^2*c*d*x*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d
) + 4*b^2*c^2*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) + 4*b^2*c^2*Ei(-2*(b*
d*x + b*c)/d)*e^(-2*a + 2*b*c/d) - 2*b*d^2*x*e^(2*b*x + 2*a) + 2*b*d^2*x*e^
(-2*b*x - 2*a) - 2*b*c*d*e^(2*b*x + 2*a) + 2*b*c*d*e^(-2*b*x - 2*a) - d^2*e
^(2*b*x + 2*a) - d^2*e^(-2*b*x - 2*a) + 2*d^2)/(d^5*x^2 + 2*c*d^4*x + c^2*d
^3)
```

3.15 $\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=162

$$\frac{2b^3\sinh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{2b^3\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{2b^2\sinh^2(a+bx)}{3d^3(c+dx)}-\frac{b\sinh(a+bx)\cosh}{3d^2(c+dx)}$$

```
[Out] -b^2/(3*d^3*(c + d*x)) + (2*b^3*CoshIntegral[(2*b*c)/d + 2*b*x]*Sinh[2*a -
(2*b*c)/d])/(3*d^4) - (b*Cosh[a + b*x]*Sinh[a + b*x])/(3*d^2*(c + d*x)^2) -
 Sinh[a + b*x]^2/(3*d*(c + d*x)^3) - (2*b^2*Sinh[a + b*x]^2)/(3*d^3*(c + d*
x)) + (2*b^3*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/(3*d^4)
```

Rubi [A] time = 0.18726, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3314, 32, 3313, 12, 3303, 3298, 3301}

$$\frac{2b^3\sinh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{2b^3\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{2b^2\sinh^2(a+bx)}{3d^3(c+dx)}-\frac{b\sinh(a+bx)\cosh}{3d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2/(c + d*x)^4,x]
```

```
[Out] -b^2/(3*d^3*(c + d*x)) + (2*b^3*CoshIntegral[(2*b*c)/d + 2*b*x]*Sinh[2*a -
(2*b*c)/d])/(3*d^4) - (b*Cosh[a + b*x]*Sinh[a + b*x])/(3*d^2*(c + d*x)^2) -
 Sinh[a + b*x]^2/(3*d*(c + d*x)^3) - (2*b^2*Sinh[a + b*x]^2)/(3*d^3*(c + d*
x)) + (2*b^3*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/(3*d^4)
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx = -\frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2} - \frac{\sinh^2(a+bx)}{3d(c+dx)^3} + \frac{b^2\int\frac{1}{(c+dx)^2}\,dx}{3d^2} + \frac{\left(2b^2\right)\int\frac{\sinh^2(a+bx)}{(c+dx)^2}\,dx}{3d^2}$$

$$= -\frac{b^2}{3d^3(c+dx)} - \frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2} - \frac{\sinh^2(a+bx)}{3d(c+dx)^3} - \frac{2b^2\sinh^2(a+bx)}{3d^3(c+dx)} - \frac{\left(4ib^3\right)\int\frac{i\sinh}{2(}}{3d^3}$$

$$= -\frac{b^2}{3d^3(c+dx)} - \frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2} - \frac{\sinh^2(a+bx)}{3d(c+dx)^3} - \frac{2b^2\sinh^2(a+bx)}{3d^3(c+dx)} + \frac{\left(2b^3\right)\int\frac{\sinh(2}{c-}}{3d^3}$$

$$= -\frac{b^2}{3d^3(c+dx)} - \frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2} - \frac{\sinh^2(a+bx)}{3d(c+dx)^3} - \frac{2b^2\sinh^2(a+bx)}{3d^3(c+dx)} + \frac{\left(2b^3\cosh\left(2\right.\right.}{}$$

$$= -\frac{b^2}{3d^3(c+dx)} + \frac{2b^3\text{Chi}\left(\frac{2bc}{d}+2bx\right)\sinh\left(2a-\frac{2bc}{d}\right)}{3d^4} - \frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2} - \frac{\sinh^2(a+b}{3d(c+dx)^3}$$

Mathematica [A] time = 0.888442, size = 123, normalized size = 0.76

$$\frac{4b^3\sinh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2b(c+dx)}{d}\right) - \frac{d\left(\cosh(2(a+bx))\left(2b^2(c+dx)^2+d^2\right)+d(b(c+dx)\sinh(2(a+bx))-d)\right)}{(c+dx)^3} + 4b^3\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2b(c+dx)}{d}\right)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2/(c + d*x)^4,x]
```

```
[Out] (4*b^3*CoshIntegral[(2*b*(c + d*x))/d]*Sinh[2*a - (2*b*c)/d] - (d*((d^2 + 2
*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)] + d*(-d + b*(c + d*x)*Sinh[2*(a + b*x)]
)))/(c + d*x)^3 + 4*b^3*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/
d])/(6*d^4)
```

Maple [B] time = 0.118, size = 555, normalized size = 3.4

$$\frac{1}{6d(dx+c)^3} - \frac{b^5e^{-2bx-2a}x^2}{6d\left(b^3d^3x^3+3b^3cd^2x^2+3b^3c^2dx+c^3b^3\right)} - \frac{b^5e^{-2bx-2a}cx}{3d^2\left(b^3d^3x^3+3b^3cd^2x^2+3b^3c^2dx+c^3b^3\right)} - \frac{}{6d^3\left(b^3d^3x^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2/(d*x+c)^4,x)
```

```
[Out] 1/6/d/(d*x+c)^3-1/6*b^5*exp(-2*b*x-2*a)/d/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^
3*c^2*d*x+b^3*c^3)*x^2-1/3*b^5*exp(-2*b*x-2*a)/d^2/(b^3*d^3*x^3+3*b^3*c*d^2
*x^2+3*b^3*c^2*d*x+b^3*c^3)*c*x-1/6*b^5*exp(-2*b*x-2*a)/d^3/(b^3*d^3*x^3+3*
b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c^2+1/12*b^4*exp(-2*b*x-2*a)/d/(b^3*d^
3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*x+1/12*b^4*exp(-2*b*x-2*a)/d^2
/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c-1/12*b^3*exp(-2*b*x-
2*a)/d/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)+1/3*b^3/d^4*exp(
-2*(a*d-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/12*b^3/d^4*exp(2*b*x+2*a)/(
b*c/d+b*x)^3-1/12*b^3/d^4*exp(2*b*x+2*a)/(b*c/d+b*x)^2-1/6*b^3/d^4*exp(2*b*
x+2*a)/(b*c/d+b*x)-1/3*b^3/d^4*exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+b
*c)/d)
```

Maxima [A] time = 1.51533, size = 149, normalized size = 0.92

$$\frac{1}{6\left(d^4x^3+3\,cd^3x^2+3\,c^2d^2x+c^3d\right)}-\frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_4\left(\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^3d}-\frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_4\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6/(d^4*x^3 + 3*c*d^3*x^2 + 3*c^2*d^2*x + c^3*d) - 1/4*e^(-2*a + 2*b*c/d)*
exp_integral_e(4, 2*(d*x + c)*b/d)/((d*x + c)^3*d) - 1/4*e^(2*a - 2*b*c/d)*
exp_integral_e(4, -2*(d*x + c)*b/d)/((d*x + c)^3*d)
```

Fricas [B] time = 2.73919, size = 857, normalized size = 5.29

$$d^3-\left(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d+d^3\right)\cosh\left(bx+a\right)^2-2\left(bd^3x+bcd^2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)-\left(2\,b^2d^3x^2+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(d^3 - (2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*cosh(b*x + a
)^2 - 2*(b*d^3*x + b*c*d^2)*cosh(b*x + a)*sinh(b*x + a) - (2*b^2*d^3*x^2 +
4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*sinh(b*x + a)^2 + 2*((b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*Ei(2*(b*d*x + b*c)/d) - (b^3*d^3*x^3
 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*Ei(-2*(b*d*x + b*c)/d))*cosh(
-2*(b*c - a*d)/d) + 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3
*c^3)*Ei(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + b^3*c^3)*Ei(-2*(b*d*x + b*c)/d))*sinh(-2*(b*c - a*d)/d))/(d^7*x^3 + 3*c
*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2/(d*x+c)**4,x)
```

```
[Out] Integral(sinh(a + b*x)**2/(c + d*x)**4, x)
```

Giac [B] time = 1.15627, size = 725, normalized size = 4.48

$$\frac{4\,b^3d^3x^3\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)} - 4\,b^3d^3x^3\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)} + 12\,b^3cd^2x^2\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)} - 12\,b^3cd^2x^2\mathrm{Ei}\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/12*(4*b^3*d^3*x^3*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) - 4*b^3*d^3*x^3
*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) + 12*b^3*c*d^2*x^2*Ei(2*(b*d*x +
 b*c)/d)*e^(2*a - 2*b*c/d) - 12*b^3*c*d^2*x^2*Ei(-2*(b*d*x + b*c)/d)*e^(-2*
a + 2*b*c/d) + 12*b^3*c^2*d*x*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) - 12*
b^3*c^2*d*x*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) - 2*b^2*d^3*x^2*e^(2*
b*x + 2*a) - 2*b^2*d^3*x^2*e^(-2*b*x - 2*a) + 4*b^3*c^3*Ei(2*(b*d*x + b*c)/
d)*e^(2*a - 2*b*c/d) - 4*b^3*c^3*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d)
- 4*b^2*c*d^2*x*e^(2*b*x + 2*a) - 4*b^2*c*d^2*x*e^(-2*b*x - 2*a) - 2*b^2*c^
2*d*e^(2*b*x + 2*a) - b*d^3*x*e^(2*b*x + 2*a) - 2*b^2*c^2*d*e^(-2*b*x - 2*a
) + b*d^3*x*e^(-2*b*x - 2*a) - b*c*d^2*e^(2*b*x + 2*a) + b*c*d^2*e^(-2*b*x
- 2*a) - d^3*e^(2*b*x + 2*a) - d^3*e^(-2*b*x - 2*a) + 2*d^3)/(d^7*x^3 + 3*c
*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

3.16 $\int (c+dx)^4 \sinh^3(a+bx)\,dx$

Optimal. Leaf size=225

$$-\frac{8d^3(c+dx)\sinh^3(a+bx)}{27b^4}+\frac{160d^3(c+dx)\sinh(a+bx)}{9b^4}-\frac{80d^2(c+dx)^2\cosh(a+bx)}{9b^3}+\frac{4d^2(c+dx)^2\sinh^2(a+bx)}{9b^3}$$

```
[Out] (-488*d^4*Cosh[a + b*x])/(27*b^5) - (80*d^2*(c + d*x)^2*Cosh[a + b*x])/(9*b
^3) - (2*(c + d*x)^4*Cosh[a + b*x])/(3*b) + (8*d^4*Cosh[a + b*x]^3)/(81*b^5
) + (160*d^3*(c + d*x)*Sinh[a + b*x])/(9*b^4) + (8*d*(c + d*x)^3*Sinh[a + b
*x])/(3*b^2) + (4*d^2*(c + d*x)^2*Cosh[a + b*x]*Sinh[a + b*x]^2)/(9*b^3) +
((c + d*x)^4*Cosh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (8*d^3*(c + d*x)*Sinh[a
 + b*x]^3)/(27*b^4) - (4*d*(c + d*x)^3*Sinh[a + b*x]^3)/(9*b^2)
```

Rubi [A] time = 0.359891, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 3296, 2638, 2633}

$$-\frac{8d^3(c+dx)\sinh^3(a+bx)}{27b^4}+\frac{160d^3(c+dx)\sinh(a+bx)}{9b^4}-\frac{80d^2(c+dx)^2\cosh(a+bx)}{9b^3}+\frac{4d^2(c+dx)^2\sinh^2(a+bx)}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sinh[a + b*x]^3,x]
```

```
[Out] (-488*d^4*Cosh[a + b*x])/(27*b^5) - (80*d^2*(c + d*x)^2*Cosh[a + b*x])/(9*b
^3) - (2*(c + d*x)^4*Cosh[a + b*x])/(3*b) + (8*d^4*Cosh[a + b*x]^3)/(81*b^5
) + (160*d^3*(c + d*x)*Sinh[a + b*x])/(9*b^4) + (8*d*(c + d*x)^3*Sinh[a + b
*x])/(3*b^2) + (4*d^2*(c + d*x)^2*Cosh[a + b*x]*Sinh[a + b*x]^2)/(9*b^3) +
((c + d*x)^4*Cosh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (8*d^3*(c + d*x)*Sinh[a
 + b*x]^3)/(27*b^4) - (4*d*(c + d*x)^3*Sinh[a + b*x]^3)/(9*b^2)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (c+dx)^4 \sinh^3(a+bx)\,dx = \frac{(c+dx)^4\cosh(a+bx)\sinh^2(a+bx)}{3b} - \frac{4d(c+dx)^3\sinh^3(a+bx)}{9b^2} - \frac{2}{3}\int (c+dx)^4 \sinh(a$$

$$= -\frac{2(c+dx)^4\cosh(a+bx)}{3b} + \frac{4d^2(c+dx)^2\cosh(a+bx)\sinh^2(a+bx)}{9b^3} + \frac{(c+dx)^4\cosh(a$$

$$= -\frac{8d^2(c+dx)^2\cosh(a+bx)}{9b^3} - \frac{2(c+dx)^4\cosh(a+bx)}{3b} + \frac{8d(c+dx)^3\sinh(a+bx)}{3b^2} + 4d^2$$

$$= -\frac{8d^4\cosh(a+bx)}{27b^5} - \frac{80d^2(c+dx)^2\cosh(a+bx)}{9b^3} - \frac{2(c+dx)^4\cosh(a+bx)}{3b} + \frac{8d^4\cosh^3}{81b}$$

$$= -\frac{56d^4\cosh(a+bx)}{27b^5} - \frac{80d^2(c+dx)^2\cosh(a+bx)}{9b^3} - \frac{2(c+dx)^4\cosh(a+bx)}{3b} + \frac{8d^4\cosh}{81}$$

$$= -\frac{488d^4\cosh(a+bx)}{27b^5} - \frac{80d^2(c+dx)^2\cosh(a+bx)}{9b^3} - \frac{2(c+dx)^4\cosh(a+bx)}{3b} + \frac{8d^4\cosh}{8}$$

Mathematica [A] time = 1.01925, size = 150, normalized size = 0.67

$$\frac{-243\cosh(a+bx)\left(12b^2d^2(c+dx)^2+b^4(c+dx)^4+24d^4\right)+\cosh(3(a+bx))\left(36b^2d^2(c+dx)^2+27b^4(c+dx)^4+8d^4\right)-}{324b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Sinh[a + b*x]^3,x]
```

```
[Out] (-243*(24*d^4 + 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cosh[a + b*x] + (
8*d^4 + 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cosh[3*(a + b*x)] - 24
*b*d*(c + d*x)*(-242*d^2 - 39*b^2*(c + d*x)^2 + (2*d^2 + 3*b^2*(c + d*x)^2)
*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(324*b^5)
```

Maple [B] time = 0.156, size = 1217, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sinh(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^4*d^4*(-2/3*(b*x+a)^4*cosh(b*x+a)+1/3*(b*x+a)^4*cosh(b*x+a)*sinh(b
*x+a)^2+28/9*(b*x+a)^3*sinh(b*x+a)-80/9*(b*x+a)^2*cosh(b*x+a)+488/27*(b*x+a
)*sinh(b*x+a)-1456/81*cosh(b*x+a)-4/9*(b*x+a)^3*sinh(b*x+a)*cosh(b*x+a)^2+4
/9*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)-8/27*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)
^2+8/81*sinh(b*x+a)^2*cosh(b*x+a))-4/b^4*d^4*a*(-2/3*(b*x+a)^3*cosh(b*x+a)+
1/3*(b*x+a)^3*cosh(b*x+a)*sinh(b*x+a)^2+7/3*(b*x+a)^2*sinh(b*x+a)-40/9*(b*x
+a)*cosh(b*x+a)+122/27*sinh(b*x+a)-1/3*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^2+
2/9*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)-2/27*sinh(b*x+a)*cosh(b*x+a)^2)+6/b^4
*d^4*a^2*(1/3*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)-2/3*(b*x+a)^2*cosh(b*x+a)
-2/9*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2+14/9*(b*x+a)*sinh(b*x+a)+2/27*sinh(b
*x+a)^2*cosh(b*x+a)-40/27*cosh(b*x+a))-4/b^4*d^4*a^3*(-2/3*(b*x+a)*cosh(b*x
+a)+1/3*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)+7/9*sinh(b*x+a)-1/9*sinh(b*x+a)*c
osh(b*x+a)^2)+1/b^4*d^4*a^4*(-2/3+1/3*sinh(b*x+a)^2)*cosh(b*x+a)+4/b^3*c*d^
```

```
3*(-2/3*(b*x+a)^3*cosh(b*x+a)+1/3*(b*x+a)^3*cosh(b*x+a)*sinh(b*x+a)^2+7/3*(
b*x+a)^2*sinh(b*x+a)-40/9*(b*x+a)*cosh(b*x+a)+122/27*sinh(b*x+a)-1/3*(b*x+a
)^2*sinh(b*x+a)*cosh(b*x+a)^2+2/9*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)-2/27*si
nh(b*x+a)*cosh(b*x+a)^2)-12/b^3*c*d^3*a*(1/3*(b*x+a)^2*sinh(b*x+a)^2*cosh(b
*x+a)-2/3*(b*x+a)^2*cosh(b*x+a)-2/9*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2+14/9*
(b*x+a)*sinh(b*x+a)+2/27*sinh(b*x+a)^2*cosh(b*x+a)-40/27*cosh(b*x+a))+12/b^
3*c*d^3*a^2*(-2/3*(b*x+a)*cosh(b*x+a)+1/3*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)
+7/9*sinh(b*x+a)-1/9*sinh(b*x+a)*cosh(b*x+a)^2)-4/b^3*c*d^3*a^3*(-2/3+1/3*s
inh(b*x+a)^2)*cosh(b*x+a)+6/b^2*c^2*d^2*(1/3*(b*x+a)^2*sinh(b*x+a)^2*cosh(b
*x+a)-2/3*(b*x+a)^2*cosh(b*x+a)-2/9*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2+14/9*
(b*x+a)*sinh(b*x+a)+2/27*sinh(b*x+a)^2*cosh(b*x+a)-40/27*cosh(b*x+a))-12/b^
2*c^2*d^2*a*(-2/3*(b*x+a)*cosh(b*x+a)+1/3*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)
+7/9*sinh(b*x+a)-1/9*sinh(b*x+a)*cosh(b*x+a)^2)+6/b^2*c^2*d^2*a^2*(-2/3+1/3
*sinh(b*x+a)^2)*cosh(b*x+a)+4/b*c^3*d*(-2/3*(b*x+a)*cosh(b*x+a)+1/3*(b*x+a)
*sinh(b*x+a)^2*cosh(b*x+a)+7/9*sinh(b*x+a)-1/9*sinh(b*x+a)*cosh(b*x+a)^2)-4
/b*c^3*d*a*(-2/3+1/3*sinh(b*x+a)^2)*cosh(b*x+a)+c^4*(-2/3+1/3*sinh(b*x+a)^2
)*cosh(b*x+a))
```

Maxima [B] time = 1.35361, size = 863, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*c^3*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 - 27*(b*x*e^a - e^a)*e^
(b*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 + (3*b*x + 1)*e^(-3*b*x - 3*a)/b^
2) + 1/24*c^4*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b + e^(
-3*b*x - 3*a)/b) + 1/36*c^2*d^2*((9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(
3*a))*e^(3*b*x)/b^3 - 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 - 81
*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 + (9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x
 - 3*a)/b^3) + 1/54*c*d^3*((9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e
^(3*a) - 2*e^(3*a))*e^(3*b*x)/b^4 - 81*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x
*e^a - 6*e^a)*e^(b*x)/b^4 - 81*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x -
a)/b^4 + (9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4) + 1/648*
d^4*((27*b^4*x^4*e^(3*a) - 36*b^3*x^3*e^(3*a) + 36*b^2*x^2*e^(3*a) - 24*b*x
*e^(3*a) + 8*e^(3*a))*e^(3*b*x)/b^5 - 243*(b^4*x^4*e^a - 4*b^3*x^3*e^a + 12
*b^2*x^2*e^a - 24*b*x*e^a + 24*e^a)*e^(b*x)/b^5 - 243*(b^4*x^4 + 4*b^3*x^3
+ 12*b^2*x^2 + 24*b*x + 24)*e^(-b*x - a)/b^5 + (27*b^4*x^4 + 36*b^3*x^3 + 3
6*b^2*x^2 + 24*b*x + 8)*e^(-3*b*x - 3*a)/b^5)
```

Fricas [B] time = 2.82189, size = 1141, normalized size = 5.07

$$\frac{\left(27\,b^4d^4x^4+108\,b^4cd^3x^3+27\,b^4c^4+36\,b^2c^2d^2+8\,d^4+18\left(9\,b^4c^2d^2+2\,b^2d^4\right)x^2+36\left(3\,b^4c^3d+2\,b^2cd^3\right)x\right)\cosh\left(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/324*((27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 + 36*b^2*c^2*d^2 +
8*d^4 + 18*(9*b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3*d + 2*b^2*c*d^3)
*x)*cosh(b*x + a)^3 + 3*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 +
36*b^2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3
```

```
*d + 2*b^2*c*d^3)*x)*cosh(b*x + a)*sinh(b*x + a)^2 - 12*(3*b^3*d^4*x^3 + 9*
b^3*c*d^3*x^2 + 3*b^3*c^3*d + 2*b*c*d^3 + (9*b^3*c^2*d^2 + 2*b*d^4)*x)*sinh
(b*x + a)^3 - 243*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 + 12*b^2*c^2*d^2
 + 24*d^4 + 6*(b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d + 6*b^2*c*d^3)*x
)*cosh(b*x + a) + 36*(27*b^3*d^4*x^3 + 81*b^3*c*d^3*x^2 + 27*b^3*c^3*d + 16
2*b*c*d^3 - (3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d + 2*b*c*d^3 + (9
*b^3*c^2*d^2 + 2*b*d^4)*x)*cosh(b*x + a)^2 + 81*(b^3*c^2*d^2 + 2*b*d^4)*x)*
sinh(b*x + a))/b^5
```

Sympy [A] time = 12.5076, size = 772, normalized size = 3.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((c**4*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c**4*cosh(a + b*x)**3/
(3*b) + 4*c**3*d*x*sinh(a + b*x)**2*cosh(a + b*x)/b - 8*c**3*d*x*cosh(a + b
*x)**3/(3*b) + 6*c**2*d**2*x**2*sinh(a + b*x)**2*cosh(a + b*x)/b - 4*c**2*d
**2*x**2*cosh(a + b*x)**3/b + 4*c*d**3*x**3*sinh(a + b*x)**2*cosh(a + b*x)/
b - 8*c*d**3*x**3*cosh(a + b*x)**3/(3*b) + d**4*x**4*sinh(a + b*x)**2*cosh(
a + b*x)/b - 2*d**4*x**4*cosh(a + b*x)**3/(3*b) - 28*c**3*d*sinh(a + b*x)**
3/(9*b**2) + 8*c**3*d*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**2) - 28*c**2*d**
2*x*sinh(a + b*x)**3/(3*b**2) + 8*c**2*d**2*x*sinh(a + b*x)*cosh(a + b*x)**
2/b**2 - 28*c*d**3*x**2*sinh(a + b*x)**3/(3*b**2) + 8*c*d**3*x**2*sinh(a +
b*x)*cosh(a + b*x)**2/b**2 - 28*d**4*x**3*sinh(a + b*x)**3/(9*b**2) + 8*d**
4*x**3*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**2) + 28*c**2*d**2*sinh(a + b*x)
**2*cosh(a + b*x)/(3*b**3) - 80*c**2*d**2*cosh(a + b*x)**3/(9*b**3) + 56*c*
d**3*x*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**3) - 160*c*d**3*x*cosh(a + b*x)
**3/(9*b**3) + 28*d**4*x**2*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**3) - 80*d*
*4*x**2*cosh(a + b*x)**3/(9*b**3) - 488*c*d**3*sinh(a + b*x)**3/(27*b**4) +
 160*c*d**3*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**4) - 488*d**4*x*sinh(a + b
*x)**3/(27*b**4) + 160*d**4*x*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**4) + 488
*d**4*sinh(a + b*x)**2*cosh(a + b*x)/(27*b**5) - 1456*d**4*cosh(a + b*x)**3
/(81*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3
*x**4 + d**4*x**5/5)*sinh(a)**3, True))
```

Giac [B] time = 1.2083, size = 883, normalized size = 3.92

$$\frac{\left(27\,b^4d^4x^4 + 108\,b^4cd^3x^3 + 162\,b^4c^2d^2x^2 - 36\,b^3d^4x^3 + 108\,b^4c^3dx - 108\,b^3cd^3x^2 + 27\,b^4c^4 - 108\,b^3c^2d^2x + 36\,b^2d^4x^2 - 3\right.}{648\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/648*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 - 36*b^3*d^
4*x^3 + 108*b^4*c^3*d*x - 108*b^3*c*d^3*x^2 + 27*b^4*c^4 - 108*b^3*c^2*d^2*
x + 36*b^2*d^4*x^2 - 36*b^3*c^3*d + 72*b^2*c*d^3*x + 36*b^2*c^2*d^2 - 24*b*
d^4*x - 24*b*c*d^3 + 8*d^4)*e^(3*b*x + 3*a)/b^5 - 3/8*(b^4*d^4*x^4 + 4*b^4*
c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 4*b^4*c^3*d*x - 12*b^3*c*d^
3*x^2 + b^4*c^4 - 12*b^3*c^2*d^2*x + 12*b^2*d^4*x^2 - 4*b^3*c^3*d + 24*b^2*
c*d^3*x + 12*b^2*c^2*d^2 - 24*b*d^4*x - 24*b*c*d^3 + 24*d^4)*e^(b*x + a)/b^
```

```
5 - 3/8*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^3*d^4*x^3
+ 4*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + b^4*c^4 + 12*b^3*c^2*d^2*x + 12*b^2*d^
4*x^2 + 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*d^2 + 24*b*d^4*x + 24*b*c
*d^3 + 24*d^4)*e^(-b*x - a)/b^5 + 1/648*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3
 + 162*b^4*c^2*d^2*x^2 + 36*b^3*d^4*x^3 + 108*b^4*c^3*d*x + 108*b^3*c*d^3*x
^2 + 27*b^4*c^4 + 108*b^3*c^2*d^2*x + 36*b^2*d^4*x^2 + 36*b^3*c^3*d + 72*b^
2*c*d^3*x + 36*b^2*c^2*d^2 + 24*b*d^4*x + 24*b*c*d^3 + 8*d^4)*e^(-3*b*x - 3
*a)/b^5
```

3.17 $\int (c+dx)^3 \sinh^3(a+bx)\,dx$

Optimal. Leaf size=175

$$-\frac{40d^2(c+dx)\cosh(a+bx)}{9b^3}+\frac{2d^2(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{9b^3}-\frac{d(c+dx)^2\sinh^3(a+bx)}{3b^2}+\frac{2d(c+dx)^2\sinh(a+$$

```
[Out] (-40*d^2*(c + d*x)*Cosh[a + b*x])/(9*b^3) - (2*(c + d*x)^3*Cosh[a + b*x])/(
3*b) + (40*d^3*Sinh[a + b*x])/(9*b^4) + (2*d*(c + d*x)^2*Sinh[a + b*x])/b^2
 + (2*d^2*(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^2)/(9*b^3) + ((c + d*x)^3*C
osh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (2*d^3*Sinh[a + b*x]^3)/(27*b^4) - (d
*(c + d*x)^2*Sinh[a + b*x]^3)/(3*b^2)
```

Rubi [A] time = 0.226485, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 3296, 2637, 3310}

$$-\frac{40d^2(c+dx)\cosh(a+bx)}{9b^3}+\frac{2d^2(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{9b^3}-\frac{d(c+dx)^2\sinh^3(a+bx)}{3b^2}+\frac{2d(c+dx)^2\sinh(a+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sinh[a + b*x]^3,x]
```


```
[Out] (-40*d^2*(c + d*x)*Cosh[a + b*x])/(9*b^3) - (2*(c + d*x)^3*Cosh[a + b*x])/(
3*b) + (40*d^3*Sinh[a + b*x])/(9*b^4) + (2*d*(c + d*x)^2*Sinh[a + b*x])/b^2
 + (2*d^2*(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^2)/(9*b^3) + ((c + d*x)^3*C
osh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (2*d^3*Sinh[a + b*x]^3)/(27*b^4) - (d
*(c + d*x)^2*Sinh[a + b*x]^3)/(3*b^2)
```


Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
```

]

Rubi steps

$$\int (c+dx)^3 \sinh^3(a+bx)\,dx = \frac{(c+dx)^3\cosh(a+bx)\sinh^2(a+bx)}{3b} - \frac{d(c+dx)^2\sinh^3(a+bx)}{3b^2} - \frac{2}{3}\int (c+dx)^3 \sinh$$

$$= -\frac{2(c+dx)^3\cosh(a+bx)}{3b} + \frac{2d^2(c+dx)\cosh(a+bx)\sinh^2(a+bx)}{9b^3} + \frac{(c+dx)^3\cosh(}{}$$

$$= -\frac{4d^2(c+dx)\cosh(a+bx)}{9b^3} - \frac{2(c+dx)^3\cosh(a+bx)}{3b} + \frac{2d(c+dx)^2\sinh(a+bx)}{b^2} + \frac{2}{}$$

$$= -\frac{40d^2(c+dx)\cosh(a+bx)}{9b^3} - \frac{2(c+dx)^3\cosh(a+bx)}{3b} + \frac{4d^3\sinh(a+bx)}{9b^4} + \frac{2d(c+d}{}$$

$$= -\frac{40d^2(c+dx)\cosh(a+bx)}{9b^3} - \frac{2(c+dx)^3\cosh(a+bx)}{3b} + \frac{40d^3\sinh(a+bx)}{9b^4} + \frac{2d(c+}{}$$

Mathematica [A] time = 0.992243, size = 127, normalized size = 0.73

$$\frac{-162b(c+dx)\cosh(a+bx)\left(b^2(c+dx)^2+6d^2\right)+6b(c+dx)\cosh(3(a+bx))\left(3b^2(c+dx)^2+2d^2\right)-4d\sinh(a+bx)\left(\right.}{216b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sinh[a + b*x]^3,x]
```

```
[Out] (-162*b*(c + d*x)*(6*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] + 6*b*(c + d*x)*(
2*d^2 + 3*b^2*(c + d*x)^2)*Cosh[3*(a + b*x)] - 4*d*(-242*d^2 - 117*b^2*(c +
 d*x)^2 + (2*d^2 + 9*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(21
6*b^4)
```

Maple [B] time = 0.007, size = 676, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sinh(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^3*d^3*(-2/3*(b*x+a)^3*cosh(b*x+a)+1/3*(b*x+a)^3*cosh(b*x+a)*sinh(b
*x+a)^2+7/3*(b*x+a)^2*sinh(b*x+a)-40/9*(b*x+a)*cosh(b*x+a)+122/27*sinh(b*x+
a)-1/3*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^2+2/9*(b*x+a)*sinh(b*x+a)^2*cosh(b
*x+a)-2/27*sinh(b*x+a)*cosh(b*x+a)^2)-3/b^3*d^3*a*(1/3*(b*x+a)^2*sinh(b*x+a
)^2*cosh(b*x+a)-2/3*(b*x+a)^2*cosh(b*x+a)-2/9*(b*x+a)*sinh(b*x+a)*cosh(b*x+
a)^2+14/9*(b*x+a)*sinh(b*x+a)+2/27*sinh(b*x+a)^2*cosh(b*x+a)-40/27*cosh(b*x
+a))+3/b^3*d^3*a^2*(-2/3*(b*x+a)*cosh(b*x+a)+1/3*(b*x+a)*sinh(b*x+a)^2*cosh
(b*x+a)+7/9*sinh(b*x+a)-1/9*sinh(b*x+a)*cosh(b*x+a)^2)-1/b^3*d^3*a^3*(-2/3+
1/3*sinh(b*x+a)^2)*cosh(b*x+a)+3/b^2*c*d^2*(1/3*(b*x+a)^2*sinh(b*x+a)^2*cos
h(b*x+a)-2/3*(b*x+a)^2*cosh(b*x+a)-2/9*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2+14
/9*(b*x+a)*sinh(b*x+a)+2/27*sinh(b*x+a)^2*cosh(b*x+a)-40/27*cosh(b*x+a))-6/
b^2*c*d^2*a*(-2/3*(b*x+a)*cosh(b*x+a)+1/3*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)
+7/9*sinh(b*x+a)-1/9*sinh(b*x+a)*cosh(b*x+a)^2)+3/b^2*c*d^2*a^2*(-2/3+1/3*s
inh(b*x+a)^2)*cosh(b*x+a)+3/b*c^2*d*(-2/3*(b*x+a)*cosh(b*x+a)+1/3*(b*x+a)*s
inh(b*x+a)^2*cosh(b*x+a)+7/9*sinh(b*x+a)-1/9*sinh(b*x+a)*cosh(b*x+a)^2)-3/b
```

```
*c^2*d*a*(-2/3+1/3*sinh(b*x+a)^2)*cosh(b*x+a)+c^3*(-2/3+1/3*sinh(b*x+a)^2)*
cosh(b*x+a))
```

Maxima [B] time = 1.37749, size = 587, normalized size = 3.35

$$\frac{1}{24}c^2d\left(\frac{\left(3\,bxe^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx)}}{b^2}-\frac{27\,(bxe^a-e^a)e^{(bx)}}{b^2}-\frac{27\,(bx+1)e^{(-bx-a)}}{b^2}+\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{b^2}\right)+\frac{1}{24}c^3\left(\frac{e^{(3\,bx+3\,a)}}{b}-\frac{9\,e^{(}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*c^2*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 - 27*(b*x*e^a - e^a)*e^
(b*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 + (3*b*x + 1)*e^(-3*b*x - 3*a)/b^
2) + 1/24*c^3*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b + e^(
-3*b*x - 3*a)/b) + 1/72*c*d^2*((9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*
a))*e^(3*b*x)/b^3 - 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 - 81*(
b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 + (9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x -
 3*a)/b^3) + 1/216*d^3*((9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e^(3
*a) - 2*e^(3*a))*e^(3*b*x)/b^4 - 81*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^
a - 6*e^a)*e^(b*x)/b^4 - 81*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x - a)/
b^4 + (9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4)
```

Fricas [B] time = 2.57785, size = 759, normalized size = 4.34

$$\frac{3\left(3\,b^3d^3x^3+9\,b^3cd^2x^2+3\,b^3c^3+2\,bcd^2+\left(9\,b^3c^2d+2\,bd^3\right)x\right)\cosh\left(bx+a\right)^3+9\left(3\,b^3d^3x^3+9\,b^3cd^2x^2+3\,b^3c^3+2\,bcd^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 + 2*b*c*d^2 + (9*b^3*
c^2*d + 2*b*d^3)*x)*cosh(b*x + a)^3 + 9*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 +
3*b^3*c^3 + 2*b*c*d^2 + (9*b^3*c^2*d + 2*b*d^3)*x)*cosh(b*x + a)*sinh(b*x +
 a)^2 - (9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d + 2*d^3)*sinh(b*x + a
)^3 - 81*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 6*b*c*d^2 + 3*(b^3*c^2*
d + 2*b*d^3)*x)*cosh(b*x + a) + 3*(81*b^2*d^3*x^2 + 162*b^2*c*d^2*x + 81*b^
2*c^2*d + 162*d^3 - (9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d + 2*d^3)*
cosh(b*x + a)^2)*sinh(b*x + a))/b^4
```

Sympy [A] time = 6.79592, size = 495, normalized size = 2.83

$$\begin{cases}\frac{c^3\sinh^2(a+bx)\cosh(a+bx)}{b}-\frac{2c^3\cosh^3(a+bx)}{3b}+\frac{3c^2dx\sinh^2(a+bx)\cosh(a+bx)}{b}-\frac{2c^2dx\cosh^3(a+bx)}{b}+\frac{3cd^2x^2\sinh^2(a+bx)\cosh(a+bx)}{b}-\frac{2cd^2x^2\,c}{} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\sinh^3(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((c**3*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c**3*cosh(a + b*x)**3/
(3*b) + 3*c**2*d*x*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c**2*d*x*cosh(a + b
*x)**3/b + 3*c*d**2*x**2*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c*d**2*x**2*c
osh(a + b*x)**3/b + d**3*x**3*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*d**3*x**
3*cosh(a + b*x)**3/(3*b) - 7*c**2*d*sinh(a + b*x)**3/(3*b**2) + 2*c**2*d*si
nh(a + b*x)*cosh(a + b*x)**2/b**2 - 14*c*d**2*x*sinh(a + b*x)**3/(3*b**2) +
 4*c*d**2*x*sinh(a + b*x)*cosh(a + b*x)**2/b**2 - 7*d**3*x**2*sinh(a + b*x)
**3/(3*b**2) + 2*d**3*x**2*sinh(a + b*x)*cosh(a + b*x)**2/b**2 + 14*c*d**2*
sinh(a + b*x)**2*cosh(a + b*x)/(3*b**3) - 40*c*d**2*cosh(a + b*x)**3/(9*b**
3) + 14*d**3*x*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**3) - 40*d**3*x*cosh(a +
 b*x)**3/(9*b**3) - 122*d**3*sinh(a + b*x)**3/(27*b**4) + 40*d**3*sinh(a +
b*x)*cosh(a + b*x)**2/(9*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d
**2*x**3 + d**3*x**4/4)*sinh(a)**3, True))
```

Giac [B] time = 1.23016, size = 559, normalized size = 3.19

$$\frac{\left(9 b^3 d^3 x^3 + 27 b^3 c d^2 x^2 + 27 b^3 c^2 d x - 9 b^2 d^3 x^2 + 9 b^3 c^3 - 18 b^2 c d^2 x - 9 b^2 c^2 d + 6 b d^3 x + 6 b c d^2 - 2 d^3\right) e^{(3 bx+3 a)}}{216 b^4} - \frac{3 \left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x^2 + 27*b^3*c^2*d*x - 9*b^2*d^3*x^2 +
9*b^3*c^3 - 18*b^2*c*d^2*x - 9*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 2*d^3)*e
^(3*b*x + 3*a)/b^4 - 3/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x - 3
*b^2*d^3*x^2 + b^3*c^3 - 6*b^2*c*d^2*x - 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^
2 - 6*d^3)*e^(b*x + a)/b^4 - 3/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2
*d*x + 3*b^2*d^3*x^2 + b^3*c^3 + 6*b^2*c*d^2*x + 3*b^2*c^2*d + 6*b*d^3*x +
6*b*c*d^2 + 6*d^3)*e^(-b*x - a)/b^4 + 1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x
^2 + 27*b^3*c^2*d*x + 9*b^2*d^3*x^2 + 9*b^3*c^3 + 18*b^2*c*d^2*x + 9*b^2*c^
2*d + 6*b*d^3*x + 6*b*c*d^2 + 2*d^3)*e^(-3*b*x - 3*a)/b^4
```

3.18 $\int (c+dx)^2 \sinh^3(a+bx)\,dx$

Optimal. Leaf size=123

$$-\frac{2d(c+dx)\sinh^3(a+bx)}{9b^2}+\frac{4d(c+dx)\sinh(a+bx)}{3b^2}+\frac{2d^2\cosh^3(a+bx)}{27b^3}-\frac{14d^2\cosh(a+bx)}{9b^3}-\frac{2(c+dx)^2\cosh(a+b}{3b}$$

```
[Out] (-14*d^2*Cosh[a + b*x])/(9*b^3) - (2*(c + d*x)^2*Cosh[a + b*x])/(3*b) + (2*
d^2*Cosh[a + b*x]^3)/(27*b^3) + (4*d*(c + d*x)*Sinh[a + b*x])/(3*b^2) + ((c
 + d*x)^2*Cosh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (2*d*(c + d*x)*Sinh[a + b*
x]^3)/(9*b^2)
```

Rubi [A] time = 0.131249, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3311, 3296, 2638, 2633}

$$-\frac{2d(c+dx)\sinh^3(a+bx)}{9b^2}+\frac{4d(c+dx)\sinh(a+bx)}{3b^2}+\frac{2d^2\cosh^3(a+bx)}{27b^3}-\frac{14d^2\cosh(a+bx)}{9b^3}-\frac{2(c+dx)^2\cosh(a+b}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sinh[a + b*x]^3,x]
```

```
[Out] (-14*d^2*Cosh[a + b*x])/(9*b^3) - (2*(c + d*x)^2*Cosh[a + b*x])/(3*b) + (2*
d^2*Cosh[a + b*x]^3)/(27*b^3) + (4*d*(c + d*x)*Sinh[a + b*x])/(3*b^2) + ((c
 + d*x)^2*Cosh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (2*d*(c + d*x)*Sinh[a + b*
x]^3)/(9*b^2)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (c+dx)^2 \sinh^3(a+bx)\,dx = \frac{(c+dx)^2\cosh(a+bx)\sinh^2(a+bx)}{3b} - \frac{2d(c+dx)\sinh^3(a+bx)}{9b^2} - \frac{2}{3}\int (c+dx)^2 \sinh$$
$$= -\frac{2(c+dx)^2\cosh(a+bx)}{3b} + \frac{(c+dx)^2\cosh(a+bx)\sinh^2(a+bx)}{3b} - \frac{2d(c+dx)\sinh^3(}{9b^2}$$
$$= -\frac{2d^2\cosh(a+bx)}{9b^3} - \frac{2(c+dx)^2\cosh(a+bx)}{3b} + \frac{2d^2\cosh^3(a+bx)}{27b^3} + \frac{4d(c+dx)\sinh(}{3b^2}$$
$$= -\frac{14d^2\cosh(a+bx)}{9b^3} - \frac{2(c+dx)^2\cosh(a+bx)}{3b} + \frac{2d^2\cosh^3(a+bx)}{27b^3} + \frac{4d(c+dx)\sinh}{3b^2}$$

Mathematica [A] time = 0.403865, size = 86, normalized size = 0.7

$$\frac{-81\cosh(a+bx)\left(b^2(c+dx)^2+2d^2\right)+\cosh(3(a+bx))\left(9b^2(c+dx)^2+2d^2\right)-6bd(c+dx)(\sinh(3(a+bx))-27\sinh}{108b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sinh[a + b*x]^3,x]
```

```
[Out] (-81*(2*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] + (2*d^2 + 9*b^2*(c + d*x)^2)*
Cosh[3*(a + b*x)] - 6*b*d*(c + d*x)*(-27*Sinh[a + b*x] + Sinh[3*(a + b*x)])
)/(108*b^3)
```

Maple [B] time = 0.007, size = 320, normalized size = 2.6

$$\frac{1}{b}\left(\frac{d^2}{b^2}\left(\frac{(bx+a)^2(\sinh(bx+a))^2\cosh(bx+a)}{3} - \frac{2(bx+a)^2\cosh(bx+a)}{3} - \frac{(2bx+2a)\sinh(bx+a)(\cosh(bx+a)}{9}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sinh(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^2*d^2*(1/3*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)-2/3*(b*x+a)^2*cosh(
b*x+a)-2/9*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2+14/9*(b*x+a)*sinh(b*x+a)+2/27*
sinh(b*x+a)^2*cosh(b*x+a)-40/27*cosh(b*x+a))-2/b^2*d^2*a*(-2/3*(b*x+a)*cosh
(b*x+a)+1/3*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)+7/9*sinh(b*x+a)-1/9*sinh(b*x+
a)*cosh(b*x+a)^2)+1/b^2*d^2*a^2*(-2/3+1/3*sinh(b*x+a)^2)*cosh(b*x+a)+2/b*c*
d*(-2/3*(b*x+a)*cosh(b*x+a)+1/3*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)+7/9*sinh(
b*x+a)-1/9*sinh(b*x+a)*cosh(b*x+a)^2)-2/b*c*d*a*(-2/3+1/3*sinh(b*x+a)^2)*co
sh(b*x+a)+c^2*(-2/3+1/3*sinh(b*x+a)^2)*cosh(b*x+a))
```

Maxima [B] time = 1.35653, size = 363, normalized size = 2.95

$$\frac{1}{36}cd\left(\frac{\left(3bxe^{(3a)}-e^{(3a)}\right)e^{(3bx)}}{b^2} - \frac{27\left(bxe^{a}-e^{a}\right)e^{(bx)}}{b^2} - \frac{27(bx+1)e^{(-bx-a)}}{b^2} + \frac{(3bx+1)e^{(-3bx-3a)}}{b^2}\right) + \frac{1}{24}c^2\left(\frac{e^{(3bx+3a)}}{b} - \frac{9}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36*c*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 - 27*(b*x*e^a - e^a)*e^(b
*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 + (3*b*x + 1)*e^(-3*b*x - 3*a)/b^2)
 + 1/24*c^2*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b + e^(-3
*b*x - 3*a)/b) + 1/216*d^2*((9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*a))
*e^(3*b*x)/b^3 - 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 - 81*(b^2
*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 + (9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*
a)/b^3)
```

Fricas [A] time = 2.62494, size = 466, normalized size = 3.79

$$\frac{\left(9\,b^2d^2x^2 + 18\,b^2cdx + 9\,b^2c^2 + 2\,d^2\right)\cosh\left(bx+a\right)^3 + 3\left(9\,b^2d^2x^2 + 18\,b^2cdx + 9\,b^2c^2 + 2\,d^2\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*((9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 2*d^2)*cosh(b*x + a)^3 +
 3*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 2*d^2)*cosh(b*x + a)*sinh(b*
x + a)^2 - 6*(b*d^2*x + b*c*d)*sinh(b*x + a)^3 - 81*(b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2 + 2*d^2)*cosh(b*x + a) + 18*(9*b*d^2*x + 9*b*c*d - (b*d^2*x +
 b*c*d)*cosh(b*x + a)^2)*sinh(b*x + a))/b^3
```

Sympy [A] time = 3.83939, size = 284, normalized size = 2.31

$$\begin{cases} \frac{c^2\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{2c^2\cosh^3(a+bx)}{3b} + \frac{2cdx\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{4cdx\cosh^3(a+bx)}{3b} + \frac{d^2x^2\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{2d^2x^2\cosh^3}{3b} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sinh^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((c**2*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c**2*cosh(a + b*x)**3/
(3*b) + 2*c*d*x*sinh(a + b*x)**2*cosh(a + b*x)/b - 4*c*d*x*cosh(a + b*x)**3
/(3*b) + d**2*x**2*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*d**2*x**2*cosh(a +
b*x)**3/(3*b) - 14*c*d*sinh(a + b*x)**3/(9*b**2) + 4*c*d*sinh(a + b*x)*cosh
(a + b*x)**2/(3*b**2) - 14*d**2*x*sinh(a + b*x)**3/(9*b**2) + 4*d**2*x*sinh
(a + b*x)*cosh(a + b*x)**2/(3*b**2) + 14*d**2*sinh(a + b*x)**2*cosh(a + b*x
)/(9*b**3) - 40*d**2*cosh(a + b*x)**3/(27*b**3), Ne(b, 0)), ((c**2*x + c*d*
x**2 + d**2*x**3/3)*sinh(a)**3, True))
```

Giac [B] time = 1.18285, size = 311, normalized size = 2.53

$$\frac{\left(9\,b^2d^2x^2 + 18\,b^2cdx + 9\,b^2c^2 - 6\,bd^2x - 6\,bcd + 2\,d^2\right)e^{(3\,bx+3\,a)}}{216\,b^3} - \frac{3\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 - 2\,bd^2x - 2\,bcd + 2\,d^2\right)e^{(bx+}}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/216*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 6*b*d^2*x - 6*b*c*d + 2*d
^2)*e^(3*b*x + 3*a)/b^3 - 3/8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*b*d^
2*x - 2*b*c*d + 2*d^2)*e^(b*x + a)/b^3 - 3/8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2 + 2*b*d^2*x + 2*b*c*d + 2*d^2)*e^(-b*x - a)/b^3 + 1/216*(9*b^2*d^2*x
^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 6*b*d^2*x + 6*b*c*d + 2*d^2)*e^(-3*b*x - 3*
a)/b^3
```

3.19 $\int (c+dx)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=75

$$-\frac{d\sinh^3(a+bx)}{9b^2}+\frac{2d\sinh(a+bx)}{3b^2}-\frac{2(c+dx)\cosh(a+bx)}{3b}+\frac{(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}$$

```
[Out] (-2*(c + d*x)*Cosh[a + b*x])/(3*b) + (2*d*Sinh[a + b*x])/(3*b^2) + ((c + d*
x)*Cosh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (d*Sinh[a + b*x]^3)/(9*b^2)
```

Rubi [A] time = 0.0576496, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3310, 3296, 2637}

$$-\frac{d\sinh^3(a+bx)}{9b^2}+\frac{2d\sinh(a+bx)}{3b^2}-\frac{2(c+dx)\cosh(a+bx)}{3b}+\frac{(c+dx)\sinh^2(a+bx)\cosh(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sinh[a + b*x]^3,x]
```

```
[Out] (-2*(c + d*x)*Cosh[a + b*x])/(3*b) + (2*d*Sinh[a + b*x])/(3*b^2) + ((c + d*
x)*Cosh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (d*Sinh[a + b*x]^3)/(9*b^2)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)\sinh^3(a+bx)\,dx &= \frac{(c+dx)\cosh(a+bx)\sinh^2(a+bx)}{3b}-\frac{d\sinh^3(a+bx)}{9b^2}-\frac{2}{3}\int (c+dx)\sinh(a+bx)\,dx\\
&= -\frac{2(c+dx)\cosh(a+bx)}{3b}+\frac{(c+dx)\cosh(a+bx)\sinh^2(a+bx)}{3b}-\frac{d\sinh^3(a+bx)}{9b^2}+\frac{(2d)}{}\\
&= -\frac{2(c+dx)\cosh(a+bx)}{3b}+\frac{2d\sinh(a+bx)}{3b^2}+\frac{(c+dx)\cosh(a+bx)\sinh^2(a+bx)}{3b}-\frac{d\sin}{}
\end{aligned}$$

Mathematica [A] time = 0.183453, size = 59, normalized size = 0.79

$$\frac{-27b(c+dx)\cosh(a+bx)+3b(c+dx)\cosh(3(a+bx))+d(27\sinh(a+bx)-\sinh(3(a+bx)))}{36b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sinh[a + b*x]^3,x]
```

```
[Out] (-27*b*(c + d*x)*Cosh[a + b*x] + 3*b*(c + d*x)*Cosh[3*(a + b*x)] + d*(27*Si
nh[a + b*x] - Sinh[3*(a + b*x)]))/(36*b^2)
```

Maple [A] time = 0.007, size = 115, normalized size = 1.5

$$\frac{1}{b}\left(\frac{d}{b}\left(-\frac{(2\,bx+2\,a)\cosh(bx+a)}{3}+\frac{(bx+a)(\sinh(bx+a))^2\cosh(bx+a)}{3}+\frac{7\sinh(bx+a)}{9}-\frac{\sinh(bx+a)(\cosh(b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sinh(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b*d*(-2/3*(b*x+a)*cosh(b*x+a)+1/3*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)+
7/9*sinh(b*x+a)-1/9*sinh(b*x+a)*cosh(b*x+a)^2)-1/b*d*a*(-2/3+1/3*sinh(b*x+a
)^2)*cosh(b*x+a)+c*(-2/3+1/3*sinh(b*x+a)^2)*cosh(b*x+a))
```

Maxima [B] time = 1.22695, size = 190, normalized size = 2.53

$$\frac{1}{72}\,d\left(\frac{\left(3\,bxe^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx)}}{b^2}-\frac{27\,(bxe^{a}-e^{a})e^{(bx)}}{b^2}-\frac{27\,(bx+1)e^{(-bx-a)}}{b^2}+\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{b^2}\right)+\frac{1}{24}\,c\left(\frac{e^{(3\,bx+3\,a)}}{b}-\frac{9\,e^{(}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/72*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 - 27*(b*x*e^a - e^a)*e^(b*x
)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 + (3*b*x + 1)*e^(-3*b*x - 3*a)/b^2) +
 1/24*c*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b + e^(-3*b*x
 - 3*a)/b)
```

Fricas [A] time = 2.62549, size = 255, normalized size = 3.4

$$\frac{3\,(bdx+bc)\cosh(bx+a)^3+9\,(bdx+bc)\cosh(bx+a)\sinh(bx+a)^2-d\sinh(bx+a)^3-27\,(bdx+bc)\cosh(bx+a)}{36\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/36*(3*(b*d*x + b*c)*cosh(b*x + a)^3 + 9*(b*d*x + b*c)*cosh(b*x + a)*sinh(
b*x + a)^2 - d*sinh(b*x + a)^3 - 27*(b*d*x + b*c)*cosh(b*x + a) - 3*(d*cosh
(b*x + a)^2 - 9*d)*sinh(b*x + a))/b^2
```

Sympy [A] time = 1.8197, size = 126, normalized size = 1.68

$$\begin{cases} \frac{c\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{2c\cosh^3(a+bx)}{3b} + \frac{dx\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{2dx\cosh^3(a+bx)}{3b} - \frac{7d\sinh^3(a+bx)}{9b^2} + \frac{2d\sinh(a+bx)\cosh^2(a+bx)}{3b^2} \\ \left(cx + \frac{dx^2}{2}\right)\sinh^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((c*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c*cosh(a + b*x)**3/(3*b)
+ d*x*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*d*x*cosh(a + b*x)**3/(3*b) - 7*d
*sinh(a + b*x)**3/(9*b**2) + 2*d*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**2), N
e(b, 0)), ((c*x + d*x**2/2)*sinh(a)**3, True))
```

Giac [A] time = 1.18635, size = 132, normalized size = 1.76

$$\frac{(3\,bdx + 3\,bc - d)e^{(3\,bx+3\,a)}}{72\,b^2} - \frac{3\,(bdx + bc - d)e^{(bx+a)}}{8\,b^2} - \frac{3\,(bdx + bc + d)e^{(-bx-a)}}{8\,b^2} + \frac{(3\,bdx + 3\,bc + d)e^{(-3\,bx-3\,a)}}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/72*(3*b*d*x + 3*b*c - d)*e^(3*b*x + 3*a)/b^2 - 3/8*(b*d*x + b*c - d)*e^(b
*x + a)/b^2 - 3/8*(b*d*x + b*c + d)*e^(-b*x - a)/b^2 + 1/72*(3*b*d*x + 3*b*
c + d)*e^(-3*b*x - 3*a)/b^2
```

3.20 $\int \frac{\sinh^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=121

$$\frac{\sinh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{3\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{3\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cosh\left(3a-\frac{3bc}{d}\right)\text{Sl}}{4d}$$

```
[Out] (CoshIntegral[(3*b*c)/d + 3*b*x]*Sinh[3*a - (3*b*c)/d])/(4*d) - (3*CoshInte
gral[(b*c)/d + b*x]*Sinh[a - (b*c)/d])/(4*d) - (3*Cosh[a - (b*c)/d]*SinhInt
egral[(b*c)/d + b*x])/(4*d) + (Cosh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d
 + 3*b*x])/(4*d)
```

Rubi [A] time = 0.282192, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3312, 3303, 3298, 3301}

$$\frac{\sinh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{3\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{3\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cosh\left(3a-\frac{3bc}{d}\right)\text{Sl}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3/(c + d*x),x]
```


```
[Out] (CoshIntegral[(3*b*c)/d + 3*b*x]*Sinh[3*a - (3*b*c)/d])/(4*d) - (3*CoshInte
gral[(b*c)/d + b*x]*Sinh[a - (b*c)/d])/(4*d) - (3*Cosh[a - (b*c)/d]*SinhInt
egral[(b*c)/d + b*x])/(4*d) + (Cosh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d
 + 3*b*x])/(4*d)
```


Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sinh^3(a+bx)}{c+dx}\,dx = i\int \left(\frac{3i\sinh(a+bx)}{4(c+dx)} - \frac{i\sinh(3a+3bx)}{4(c+dx)}\right) dx$$
$$= \frac{1}{4}\int \frac{\sinh(3a+3bx)}{c+dx}\,dx - \frac{3}{4}\int \frac{\sinh(a+bx)}{c+dx}\,dx$$
$$= \frac{1}{4}\cosh\left(3a-\frac{3bc}{d}\right)\int \frac{\sinh\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx - \frac{1}{4}\left(3\cosh\left(a-\frac{bc}{d}\right)\right)\int \frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx + \frac{1}{4}\sinh\Big($$
$$= \frac{\text{Chi}\left(\frac{3bc}{d}+3bx\right)\sinh\left(3a-\frac{3bc}{d}\right)}{4d} - \frac{3\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{4d} - \frac{3\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{4d} +$$

Mathematica [A] time = 0.239217, size = 102, normalized size = 0.84

$$\frac{\sinh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3b(c+dx)}{d}\right) - 3\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(b\left(\frac{c}{d}+x\right)\right) - 3\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(b\left(\frac{c}{d}+x\right)\right) + \cosh\left(3a-\frac{3bc}{d}\right)\text{Shi}\Big(}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3/(c + d*x),x]
```

```
[Out] (CoshIntegral[(3*b*(c + d*x))/d]*Sinh[3*a - (3*b*c)/d] - 3*CoshIntegral[b*(
c/d + x)]*Sinh[a - (b*c)/d] - 3*Cosh[a - (b*c)/d]*SinhIntegral[b*(c/d + x)]
 + Cosh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*(c + d*x))/d])/(4*d)
```

Maple [A] time = 0.083, size = 166, normalized size = 1.4

$$\frac{1}{8d}e^{-3\frac{da-cb}{d}}\text{Ei}\left(1,3bx+3a-3\frac{da-cb}{d}\right) - \frac{3}{8d}e^{-\frac{da-cb}{d}}\text{Ei}\left(1,bx+a-\frac{da-cb}{d}\right) + \frac{3}{8d}e^{\frac{da-cb}{d}}\text{Ei}\left(1,-bx-a-\frac{-da+cb}{d}\right) - \frac{1}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3/(d*x+c),x)
```

```
[Out] 1/8/d*exp(-3*(a*d-b*c)/d)*Ei(1,3*b*x+3*a-3*(a*d-b*c)/d)-3/8/d*exp(-(a*d-b*c
)/d)*Ei(1,b*x+a-(a*d-b*c)/d)+3/8/d*exp((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*c)/
d)-1/8/d*exp(3*(a*d-b*c)/d)*Ei(1,-3*b*x-3*a-3*(-a*d+b*c)/d)
```

Maxima [A] time = 1.39345, size = 158, normalized size = 1.31

$$\frac{e^{\left(-3a+\frac{3bc}{d}\right)}E_1\left(\frac{3(dx+c)b}{d}\right)}{8d} - \frac{3e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{8d} + \frac{3e^{\left(a-\frac{bc}{d}\right)}E_1\left(-\frac{(dx+c)b}{d}\right)}{8d} - \frac{e^{\left(3a-\frac{3bc}{d}\right)}E_1\left(-\frac{3(dx+c)b}{d}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/8*e^(-3*a + 3*b*c/d)*exp_integral_e(1, 3*(d*x + c)*b/d)/d - 3/8*e^(-a + b
*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d + 3/8*e^(a - b*c/d)*exp_integral_e
(1, -(d*x + c)*b/d)/d - 1/8*e^(3*a - 3*b*c/d)*exp_integral_e(1, -3*(d*x + c
```

```
)*b/d)/d
```

Fricas [A] time = 2.53058, size = 400, normalized size = 3.31

$$-\frac{3\left(\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)-\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)-\left(\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)-\mathrm{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{3\,(bc-ad)}{d}\right)+3\left(\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)+\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/8*(3*(Ei((b*d*x + b*c)/d) - Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) -
 (Ei(3*(b*d*x + b*c)/d) - Ei(-3*(b*d*x + b*c)/d))*cosh(-3*(b*c - a*d)/d) +
3*(Ei((b*d*x + b*c)/d) + Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d) - (Ei(3
*(b*d*x + b*c)/d) + Ei(-3*(b*d*x + b*c)/d))*sinh(-3*(b*c - a*d)/d))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20068, size = 153, normalized size = 1.26

$$\frac{\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)}-3\,\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}+3\,\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}-\mathrm{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)e^{\left(-3\,a+\frac{3\,bc}{d}\right)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/8*(Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) - 3*Ei((b*d*x + b*c)/d)*e^(a -
 b*c/d) + 3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - Ei(-3*(b*d*x + b*c)/d)*e^
(-3*a + 3*b*c/d))/d
```

3.21 $\int \frac{\sinh^3(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=145

$$-\frac{3b\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cosh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{3b\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\sinh\left(3a-\right.}{}$$

```
[Out] (-3*b*Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x])/(4*d^2) + (3*b*Cosh[3*
a - (3*b*c)/d]*CoshIntegral[(3*b*c)/d + 3*b*x])/(4*d^2) - Sinh[a + b*x]^3/(
d*(c + d*x)) - (3*b*Sinh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/(4*d^2)
+ (3*b*Sinh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d + 3*b*x])/(4*d^2)
```

Rubi [A] time = 0.262214, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3313, 3303, 3298, 3301}

$$-\frac{3b\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cosh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{3b\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\sinh\left(3a-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] (-3*b*Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x])/(4*d^2) + (3*b*Cosh[3*
a - (3*b*c)/d]*CoshIntegral[(3*b*c)/d + 3*b*x])/(4*d^2) - Sinh[a + b*x]^3/(
d*(c + d*x)) - (3*b*Sinh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/(4*d^2)
+ (3*b*Sinh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d + 3*b*x])/(4*d^2)
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^2}\,dx = -\frac{\sinh^3(a+bx)}{d(c+dx)} - \frac{(3b)\int \left(\frac{\cosh(a+bx)}{4(c+dx)} - \frac{\cosh(3a+3bx)}{4(c+dx)}\right)dx}{d}$$
$$= -\frac{\sinh^3(a+bx)}{d(c+dx)} - \frac{(3b)\int \frac{\cosh(a+bx)}{c+dx}\,dx}{4d} + \frac{(3b)\int \frac{\cosh(3a+3bx)}{c+dx}\,dx}{4d}$$
$$= -\frac{\sinh^3(a+bx)}{d(c+dx)} + \frac{\left(3b\cosh\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\cosh\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx}{4d} - \frac{\left(3b\cosh\left(a-\frac{bc}{d}\right)\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{4d}$$
$$= -\frac{3b\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{4d^2} + \frac{3b\cosh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d^2} - \frac{\sinh^3(a+bx)}{d(c+dx)} - \frac{3b\,\text{s}}{}$$

Mathematica [A] time = 1.34078, size = 160, normalized size = 1.1

$$\frac{6b(c+dx)\left(-\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(b\left(\frac{c}{d}+x\right)\right)+\cosh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3b(c+dx)}{d}\right)-\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(b\left(\frac{c}{d}+x\right)\right)+\sinh\left(3\right.\right.}{8d^2(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] (6*d*Cosh[b*x]*Sinh[a] - 2*d*Cosh[3*b*x]*Sinh[3*a] + 6*d*Cosh[a]*Sinh[b*x]
- 2*d*Cosh[3*a]*Sinh[3*b*x] + 6*b*(c + d*x)*(-(Cosh[a - (b*c)/d]*CoshIntegr
al[b*(c/d + x)]) + Cosh[3*a - (3*b*c)/d]*CoshIntegral[(3*b*(c + d*x))/d] -
Sinh[a - (b*c)/d]*SinhIntegral[b*(c/d + x)] + Sinh[3*a - (3*b*c)/d]*SinhInt
egral[(3*b*(c + d*x))/d]))/(8*d^2*(c + d*x))
```

Maple [A] time = 0.091, size = 271, normalized size = 1.9

$$\frac{b\mathrm{e}^{-3\,bx-3\,a}}{(8\,bdx+8\,cb)\,d} - \frac{3\,b}{8\,d^2}\mathrm{e}^{-3\,\frac{da-cb}{d}}\mathrm{Ei}\left(1,3\,bx+3\,a-3\,\frac{da-cb}{d}\right) - \frac{3\,b\mathrm{e}^{-bx-a}}{8\,d\,(bdx+cb)} + \frac{3\,b}{8\,d^2}\mathrm{e}^{-\frac{da-cb}{d}}\mathrm{Ei}\left(1,bx+a-\frac{da-cb}{d}\right) + $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] 1/8*b*exp(-3*b*x-3*a)/(b*d*x+b*c)/d-3/8*b/d^2*exp(-3*(a*d-b*c)/d)*Ei(1,3*b*
x+3*a-3*(a*d-b*c)/d)-3/8*b*exp(-b*x-a)/d/(b*d*x+b*c)+3/8*b/d^2*exp(-(a*d-b*
c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)+3/8*b/d^2*exp(b*x+a)/(b*c/d+b*x)+3/8*b/d^2*ex
p((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*c)/d)-1/8*b/d^2*exp(3*b*x+3*a)/(b*c/d+b*
x)-3/8*b/d^2*exp(3*(a*d-b*c)/d)*Ei(1,-3*b*x-3*a-3*(-a*d+b*c)/d)
```

Maxima [A] time = 1.38717, size = 196, normalized size = 1.35

$$\frac{e^{\left(-3\,a+\frac{3\,bc}{d}\right)}E_2\left(\frac{3\,(dx+c)b}{d}\right)}{8\,(dx+c)d} - \frac{3\,e^{\left(-a+\frac{bc}{d}\right)}E_2\left(\frac{(dx+c)b}{d}\right)}{8\,(dx+c)d} + \frac{3\,e^{\left(a-\frac{bc}{d}\right)}E_2\left(-\frac{(dx+c)b}{d}\right)}{8\,(dx+c)d} - \frac{e^{\left(3\,a-\frac{3\,bc}{d}\right)}E_2\left(-\frac{3\,(dx+c)b}{d}\right)}{8\,(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/8*e^(-3*a + 3*b*c/d)*exp_integral_e(2, 3*(d*x + c)*b/d)/((d*x + c)*d) - 3
/8*e^(-a + b*c/d)*exp_integral_e(2, (d*x + c)*b/d)/((d*x + c)*d) + 3/8*e^(a
 - b*c/d)*exp_integral_e(2, -(d*x + c)*b/d)/((d*x + c)*d) - 1/8*e^(3*a - 3*
b*c/d)*exp_integral_e(2, -3*(d*x + c)*b/d)/((d*x + c)*d)
```

Fricas [B] time = 2.69591, size = 662, normalized size = 4.57

$$-\frac{2\,d\sinh\left(bx+a\right)^{3}+3\left(\left(bdx+bc\right)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)+\left(bdx+bc\right)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)-3\left(\left(bdx+bc\right)\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)+(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*d*sinh(b*x + a)^3 + 3*((b*d*x + b*c)*Ei((b*d*x + b*c)/d) + (b*d*x +
 b*c)*Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) - 3*((b*d*x + b*c)*Ei(3*(b
*d*x + b*c)/d) + (b*d*x + b*c)*Ei(-3*(b*d*x + b*c)/d))*cosh(-3*(b*c - a*d)/
d) + 6*(d*cosh(b*x + a)^2 - d)*sinh(b*x + a) + 3*((b*d*x + b*c)*Ei((b*d*x +
 b*c)/d) - (b*d*x + b*c)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d) - 3*((b
*d*x + b*c)*Ei(3*(b*d*x + b*c)/d) - (b*d*x + b*c)*Ei(-3*(b*d*x + b*c)/d))*s
inh(-3*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh^{3}\left(a+bx\right)}{\left(c+dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(sinh(a + b*x)**3/(c + d*x)**2, x)
```

Giac [B] time = 1.24623, size = 400, normalized size = 2.76

$$\frac{3\,bdx\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)}-3\,bdx\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}-3\,bdx\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}+3\,bdx\mathrm{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)e^{\left(-3\,a+\frac{3\,bc}{d}\right)}+3\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*b*d*x*Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) - 3*b*d*x*Ei((b*d*x +
b*c)/d)*e^(a - b*c/d) - 3*b*d*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 3*b*d
*x*Ei(-3*(b*d*x + b*c)/d)*e^(-3*a + 3*b*c/d) + 3*b*c*Ei(3*(b*d*x + b*c)/d)*
e^(3*a - 3*b*c/d) - 3*b*c*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) - 3*b*c*Ei(-(b*
d*x + b*c)/d)*e^(-a + b*c/d) + 3*b*c*Ei(-3*(b*d*x + b*c)/d)*e^(-3*a + 3*b*c
```

```
/d) - d*e^(3*b*x + 3*a) + 3*d*e^(b*x + a) - 3*d*e^(-b*x - a) + d*e^(-3*b*x
- 3*a))/(d^3*x + c*d^2)
```

3.22 $\int \frac{\sinh^3(a+bx)}{(c+dx)^3} dx$

Optimal. Leaf size=184

$$\frac{9b^2 \sinh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{3b^2 \sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{3b^2 \cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2 \cosh\left(3a\right.}{}$$

```
[Out] (9*b^2*CoshIntegral[(3*b*c)/d + 3*b*x]*Sinh[3*a - (3*b*c)/d])/(8*d^3) - (3*
b^2*CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*c)/d])/(8*d^3) - (3*b*Cosh[a +
b*x]*Sinh[a + b*x]^2)/(2*d^2*(c + d*x)) - Sinh[a + b*x]^3/(2*d*(c + d*x)^2)
 - (3*b^2*Cosh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/(8*d^3) + (9*b^2*C
osh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

Rubi [A] time = 0.424843, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3314, 3303, 3298, 3301, 3312}

$$\frac{9b^2 \sinh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{3b^2 \sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{3b^2 \cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2 \cosh\left(3a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3/(c + d*x)^3,x]
```

```
[Out] (9*b^2*CoshIntegral[(3*b*c)/d + 3*b*x]*Sinh[3*a - (3*b*c)/d])/(8*d^3) - (3*
b^2*CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*c)/d])/(8*d^3) - (3*b*Cosh[a +
b*x]*Sinh[a + b*x]^2)/(2*d^2*(c + d*x)) - Sinh[a + b*x]^3/(2*d*(c + d*x)^2)
 - (3*b^2*Cosh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/(8*d^3) + (9*b^2*C
osh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx = -\frac{3b\cosh(a+bx)\sinh^2(a+bx)}{2d^2(c+dx)} - \frac{\sinh^3(a+bx)}{2d(c+dx)^2} + \frac{\left(3b^2\right)\int \frac{\sinh(a+bx)}{c+dx}\,dx}{d^2} + \frac{\left(9b^2\right)\int \frac{\sinh^3(a+bx)}{c+dx}\,d$$

$$= -\frac{3b\cosh(a+bx)\sinh^2(a+bx)}{2d^2(c+dx)} - \frac{\sinh^3(a+bx)}{2d(c+dx)^2} + \frac{\left(9ib^2\right)\int \left(\frac{3i\sinh(a+bx)}{4(c+dx)} - \frac{i\sinh(3a+3bx)}{4(c+dx)}\right)dx}{2d^2} +$$

$$= \frac{3b^2\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{d^3} - \frac{3b\cosh(a+bx)\sinh^2(a+bx)}{2d^2(c+dx)} - \frac{\sinh^3(a+bx)}{2d(c+dx)^2} + \frac{3b^2\cosh\left(\right.}{}$$

$$= \frac{3b^2\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{d^3} - \frac{3b\cosh(a+bx)\sinh^2(a+bx)}{2d^2(c+dx)} - \frac{\sinh^3(a+bx)}{2d(c+dx)^2} + \frac{3b^2\cosh\left(\right.}{}$$

$$= \frac{9b^2\text{Chi}\left(\frac{3bc}{d}+3bx\right)\sinh\left(3a-\frac{3bc}{d}\right)}{8d^3} - \frac{3b^2\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{8d^3} - \frac{3b\cosh(a+bx)\sinh^2}{2d^2(c+dx)}$$

Mathematica [A] time = 0.842618, size = 220, normalized size = 1.2

$$6b^2(c+dx)^2\left(3\sinh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3b(c+dx)}{d}\right) - \sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(b\left(\frac{c}{d}+x\right)\right) - \cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(b\left(\frac{c}{d}+x\right)\right) + 3\cosh$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3/(c + d*x)^3,x]
```

```
[Out] (6*d*Cosh[b*x]*(b*(c + d*x)*Cosh[a] + d*Sinh[a]) - 2*d*Cosh[3*b*x]*(3*b*(c
+ d*x)*Cosh[3*a] + d*Sinh[3*a]) + 6*d*(d*Cosh[a] + b*(c + d*x)*Sinh[a])*Sin
h[b*x] - 2*d*(d*Cosh[3*a] + 3*b*(c + d*x)*Sinh[3*a])*Sinh[3*b*x] + 6*b^2*(c
 + d*x)^2*(3*CoshIntegral[(3*b*(c + d*x))/d]*Sinh[3*a - (3*b*c)/d] - CoshIn
tegral[b*(c/d + x)]*Sinh[a - (b*c)/d] - Cosh[a - (b*c)/d]*SinhIntegral[b*(c
/d + x)] + 3*Cosh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*(c + d*x))/d]))/(16*d^
3*(c + d*x)^2)
```

Maple [B] time = 0.089, size = 562, normalized size = 3.1

$$-\frac{3b^3e^{-3bx-3a}x}{16d\left(b^2d^2x^2+2b^2cdx+c^2b^2\right)} - \frac{3b^3e^{-3bx-3a}c}{16d^2\left(b^2d^2x^2+2b^2cdx+c^2b^2\right)} + \frac{b^2e^{-3bx-3a}}{16d\left(b^2d^2x^2+2b^2cdx+c^2b^2\right)} + \frac{9b^2}{16d^3}e^{-3\frac{da-cb}{d}}\text{E}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] -3/16*b^3*exp(-3*b*x-3*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*x-3/16*b^3*ex
p(-3*b*x-3*a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c+1/16*b^2*exp(-3*b*x-3
*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)+9/16*b^2/d^3*exp(-3*(a*d-b*c)/d)*Ei
(1,3*b*x+3*a-3*(a*d-b*c)/d)+3/16*b^3*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x
+b^2*c^2)*x+3/16*b^3*exp(-b*x-a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c-3/
16*b^2*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)-3/16*b^2/d^3*exp(-(a
*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)+3/16*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)^2+3/
16*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)+3/16*b^2/d^3*exp((a*d-b*c)/d)*Ei(1,-b*x-a
-(-a*d+b*c)/d)-1/16*b^2/d^3*exp(3*b*x+3*a)/(b*c/d+b*x)^2-3/16*b^2/d^3*exp(3
*b*x+3*a)/(b*c/d+b*x)-9/16*b^2/d^3*exp(3*(a*d-b*c)/d)*Ei(1,-3*b*x-3*a-3*(-a
*d+b*c)/d)
```

Maxima [A] time = 1.60042, size = 196, normalized size = 1.07

$$\frac{e^{\left(-3a+\frac{3bc}{d}\right)}E_3\left(\frac{3(dx+c)b}{d}\right)}{8(dx+c)^2d}-\frac{3e^{\left(-a+\frac{bc}{d}\right)}E_3\left(\frac{(dx+c)b}{d}\right)}{8(dx+c)^2d}+\frac{3e^{\left(a-\frac{bc}{d}\right)}E_3\left(-\frac{(dx+c)b}{d}\right)}{8(dx+c)^2d}-\frac{e^{\left(3a-\frac{3bc}{d}\right)}E_3\left(-\frac{3(dx+c)b}{d}\right)}{8(dx+c)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*e^(-3*a + 3*b*c/d)*exp_integral_e(3, 3*(d*x + c)*b/d)/((d*x + c)^2*d) -
 3/8*e^(-a + b*c/d)*exp_integral_e(3, (d*x + c)*b/d)/((d*x + c)^2*d) + 3/8*
e^(a - b*c/d)*exp_integral_e(3, -(d*x + c)*b/d)/((d*x + c)^2*d) - 1/8*e^(3*
a - 3*b*c/d)*exp_integral_e(3, -3*(d*x + c)*b/d)/((d*x + c)^2*d)
```

Fricas [B] time = 2.66529, size = 1123, normalized size = 6.1

$$-\frac{2d^2\sinh(bx+a)^3+6\left(bd^2x+bcd\right)\cosh(bx+a)^3+18\left(bd^2x+bcd\right)\cosh(bx+a)\sinh(bx+a)^2-6\left(bd^2x+bcd\right)\cosh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*d^2*sinh(b*x + a)^3 + 6*(b*d^2*x + b*c*d)*cosh(b*x + a)^3 + 18*(b*
d^2*x + b*c*d)*cosh(b*x + a)*sinh(b*x + a)^2 - 6*(b*d^2*x + b*c*d)*cosh(b*x
 + a) + 3*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) - (b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d
) - 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(3*(b*d*x + b*c)/d) - (b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-3*(b*d*x + b*c)/d))*cosh(-3*(b*c - a*d)
/d) + 6*(d^2*cosh(b*x + a)^2 - d^2)*sinh(b*x + a) + 3*((b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d) - 9*((b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2)*Ei(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*Ei(-3*(b*d*x + b*c)/d))*sinh(-3*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2
*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Integral(sinh(a + b*x)**3/(c + d*x)**3, x)
```

Giac [B] time = 1.18637, size = 811, normalized size = 4.41

$$\frac{9\,b^2d^2x^2\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)}-3\,b^2d^2x^2\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}+3\,b^2d^2x^2\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}-9\,b^2d^2x^2\mathrm{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*(9*b^2*d^2*x^2*Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) - 3*b^2*d^2*x^2
*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) + 3*b^2*d^2*x^2*Ei(-(b*d*x + b*c)/d)*e^(
-a + b*c/d) - 9*b^2*d^2*x^2*Ei(-3*(b*d*x + b*c)/d)*e^(-3*a + 3*b*c/d) + 18*
b^2*c*d*x*Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) - 6*b^2*c*d*x*Ei((b*d*x +
 b*c)/d)*e^(a - b*c/d) + 6*b^2*c*d*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) -
18*b^2*c*d*x*Ei(-3*(b*d*x + b*c)/d)*e^(-3*a + 3*b*c/d) + 9*b^2*c^2*Ei(3*(b*
d*x + b*c)/d)*e^(3*a - 3*b*c/d) - 3*b^2*c^2*Ei((b*d*x + b*c)/d)*e^(a - b*c/
d) + 3*b^2*c^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 9*b^2*c^2*Ei(-3*(b*d*x
 + b*c)/d)*e^(-3*a + 3*b*c/d) - 3*b*d^2*x*e^(3*b*x + 3*a) + 3*b*d^2*x*e^(b*
x + a) + 3*b*d^2*x*e^(-b*x - a) - 3*b*d^2*x*e^(-3*b*x - 3*a) - 3*b*c*d*e^(3
*b*x + 3*a) + 3*b*c*d*e^(b*x + a) + 3*b*c*d*e^(-b*x - a) - 3*b*c*d*e^(-3*b*
x - 3*a) - d^2*e^(3*b*x + 3*a) + 3*d^2*e^(b*x + a) - 3*d^2*e^(-b*x - a) + d
^2*e^(-3*b*x - 3*a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

3.23 $\int (c+dx)^3 \operatorname{csch}(a+bx)\, dx$

Optimal. Leaf size=149

$$\frac{6d^2(c+dx)\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{6d^2(c+dx)\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{3d(c+dx)^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{3d(c+dx)^2\text{PolyI}}{b^2}$$

```
[Out] (-2*(c + d*x)^3*ArcTanh[E^(a + b*x)])/b - (3*d*(c + d*x)^2*PolyLog[2, -E^(a
 + b*x)])/b^2 + (3*d*(c + d*x)^2*PolyLog[2, E^(a + b*x)])/b^2 + (6*d^2*(c +
 d*x)*PolyLog[3, -E^(a + b*x)])/b^3 - (6*d^2*(c + d*x)*PolyLog[3, E^(a + b*
x)])/b^3 - (6*d^3*PolyLog[4, -E^(a + b*x)])/b^4 + (6*d^3*PolyLog[4, E^(a +
b*x)])/b^4
```

Rubi [A] time = 0.136457, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4182, 2531, 6609, 2282, 6589}

$$\frac{6d^2(c+dx)\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{6d^2(c+dx)\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{3d(c+dx)^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{3d(c+dx)^2\text{PolyI}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csch[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^3*ArcTanh[E^(a + b*x)])/b - (3*d*(c + d*x)^2*PolyLog[2, -E^(a
 + b*x)])/b^2 + (3*d*(c + d*x)^2*PolyLog[2, E^(a + b*x)])/b^2 + (6*d^2*(c +
 d*x)*PolyLog[3, -E^(a + b*x)])/b^3 - (6*d^2*(c + d*x)*PolyLog[3, E^(a + b*
x)])/b^3 - (6*d^3*PolyLog[4, -E^(a + b*x)])/b^4 + (6*d^3*PolyLog[4, E^(a +
b*x)])/b^4
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3\text{csch}(a+bx)\,dx = -\frac{2(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{(3d)\int (c+dx)^2\log\left(1-e^{a+bx}\right)dx}{b} + \frac{(3d)\int (c+dx)^2\log\left(\right.}{b}$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3d(c+dx)^2\text{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{3d(c+dx)^2\text{Li}_2\left(e^{a+bx}\right)}{b^2} + \frac{\left(6d^2\right.}{}$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3d(c+dx)^2\text{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{3d(c+dx)^2\text{Li}_2\left(e^{a+bx}\right)}{b^2} + \frac{6d^2(}{}$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3d(c+dx)^2\text{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{3d(c+dx)^2\text{Li}_2\left(e^{a+bx}\right)}{b^2} + \frac{6d^2(}{}$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3d(c+dx)^2\text{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{3d(c+dx)^2\text{Li}_2\left(e^{a+bx}\right)}{b^2} + \frac{6d^2(}{}$$

Mathematica [A] time = 2.94047, size = 191, normalized size = 1.28

$$-3d\left(b^2(c+dx)^2\text{PolyLog}(2,-\sinh(a+bx)-\cosh(a+bx))-2bd(c+dx)\text{PolyLog}(3,-\sinh(a+bx)-\cosh(a+bx))\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Csch[a + b*x],x]
```

```
[Out] (-2*b^3*(c + d*x)^3*ArcTanh[Cosh[a + b*x] + Sinh[a + b*x]] - 3*d*(b^2*(c +
d*x)^2*PolyLog[2, -Cosh[a + b*x] - Sinh[a + b*x]] - 2*b*d*(c + d*x)*PolyLog
[3, -Cosh[a + b*x] - Sinh[a + b*x]] + 2*d^2*PolyLog[4, -Cosh[a + b*x] - Sin
h[a + b*x]]) + 3*d*(b^2*(c + d*x)^2*PolyLog[2, Cosh[a + b*x] + Sinh[a + b*x
]] - 2*b*d*(c + d*x)*PolyLog[3, Cosh[a + b*x] + Sinh[a + b*x]] + 2*d^2*Poly
Log[4, Cosh[a + b*x] + Sinh[a + b*x]]))/b^4
```

Maple [B] time = 0.072, size = 541, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csch(b*x+a),x)
```

```
[Out] -6*d^3*polylog(4,-exp(b*x+a))/b^4+6*d^3*polylog(4,exp(b*x+a))/b^4+6/b^2*c*d
^2*polylog(2,exp(b*x+a))*x-3/b*c^2*d*ln(1+exp(b*x+a))*x-3/b^2*c^2*d*ln(1+ex
p(b*x+a))*a+3/b*c^2*d*ln(1-exp(b*x+a))*x+3/b^2*c^2*d*ln(1-exp(b*x+a))*a+6/b
```

```
^2*c^2*d*a*arctanh(exp(b*x+a))-6/b^3*c*d^2*a^2*arctanh(exp(b*x+a))-3/b*c*d^
2*ln(1+exp(b*x+a))*x^2+3/b^3*c*d^2*ln(1+exp(b*x+a))*a^2-6/b^2*c*d^2*polylog
(2,-exp(b*x+a))*x+3/b*c*d^2*ln(1-exp(b*x+a))*x^2-3/b^3*c*d^2*ln(1-exp(b*x+a
))*a^2+6/b^3*d^3*polylog(3,-exp(b*x+a))*x+2/b^4*d^3*a^3*arctanh(exp(b*x+a))
-3/b^2*c^2*d*polylog(2,-exp(b*x+a))+3/b^2*c^2*d*polylog(2,exp(b*x+a))+6/b^3
*c*d^2*polylog(3,-exp(b*x+a))-6/b^3*c*d^2*polylog(3,exp(b*x+a))-1/b^4*d^3*a
^3*ln(1+exp(b*x+a))+1/b^4*d^3*a^3*ln(1-exp(b*x+a))+1/b*d^3*ln(1-exp(b*x+a))
*x^3+3/b^2*d^3*polylog(2,exp(b*x+a))*x^2-6/b^3*d^3*polylog(3,exp(b*x+a))*x-
1/b*d^3*ln(1+exp(b*x+a))*x^3-3/b^2*d^3*polylog(2,-exp(b*x+a))*x^2-2/b*c^3*a
rctanh(exp(b*x+a))
```

Maxima [B] time = 1.50737, size = 450, normalized size = 3.02

$$-c^3\left(\frac{\log\left(e^{(-bx-a)}+1\right)}{b}-\frac{\log\left(e^{(-bx-a)}-1\right)}{b}\right)-\frac{3\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)c^2d}{b^2}+\frac{3\left(bx\log\left(-e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(\right.\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -c^3*(log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b) - 3*(b*x*log(e^(b*
x + a) + 1) + dilog(-e^(b*x + a)))*c^2*d/b^2 + 3*(b*x*log(-e^(b*x + a) + 1)
 + dilog(e^(b*x + a)))*c^2*d/b^2 - 3*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*
dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))*c*d^2/b^3 + 3*(b^2*x^2*lo
g(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))
*c*d^2/b^3 - (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a))
- 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))*d^3/b^4 + (b
^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog
(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))*d^3/b^4
```

Fricas [C] time = 2.78795, size = 986, normalized size = 6.62

$$\frac{6\,d^3\mathrm{polylog}\left(4,\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)-6\,d^3\mathrm{polylog}\left(4,-\cosh\left(bx+a\right)-\sinh\left(bx+a\right)\right)+3\left(b^2d^3x^2+2\,b^2cd^2x+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] (6*d^3*polylog(4, cosh(b*x + a) + sinh(b*x + a)) - 6*d^3*polylog(4, -cosh(b
*x + a) - sinh(b*x + a)) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*dilo
g(cosh(b*x + a) + sinh(b*x + a)) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2
*d)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
+ 3*b^3*c^2*d*x + b^3*c^3)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + (b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cosh(b*x + a) + sinh(b*x +
 a) - 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2 + a^3*d^3)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - 6*(b*d^
3*x + b*c*d^2)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 6*(b*d^3*x + b*c
*d^2)*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^3\operatorname{csch}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csch(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*csch(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \operatorname{csch}(bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csch(b*x + a), x)
```

3.24 $\int (c+dx)^2 \mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=99

$$-\frac{2d(c+dx)\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{2d(c+dx)\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{2d^2\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{2d^2\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-$$

```
[Out] (-2*(c + d*x)^2*ArcTanh[E^(a + b*x)])/b - (2*d*(c + d*x)*PolyLog[2, -E^(a +
 b*x)])/b^2 + (2*d*(c + d*x)*PolyLog[2, E^(a + b*x)])/b^2 + (2*d^2*PolyLog[
3, -E^(a + b*x)])/b^3 - (2*d^2*PolyLog[3, E^(a + b*x)])/b^3
```

Rubi [A] time = 0.0903547, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4182, 2531, 2282, 6589}

$$-\frac{2d(c+dx)\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{2d(c+dx)\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}+\frac{2d^2\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}-\frac{2d^2\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csch[a + b*x],x]
```


```
[Out] (-2*(c + d*x)^2*ArcTanh[E^(a + b*x)])/b - (2*d*(c + d*x)*PolyLog[2, -E^(a +
 b*x)])/b^2 + (2*d*(c + d*x)*PolyLog[2, E^(a + b*x)])/b^2 + (2*d^2*PolyLog[
3, -E^(a + b*x)])/b^3 - (2*d^2*PolyLog[3, E^(a + b*x)])/b^3
```


Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\text{csch}(a+bx)\,dx = -\frac{2(c+dx)^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{(2d)\int (c+dx)\log\left(1-e^{a+bx}\right)dx}{b} + \frac{(2d)\int (c+dx)\log\left(1\right.}{b}$$
$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{2d(c+dx)\text{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{2d(c+dx)\text{Li}_2\left(e^{a+bx}\right)}{b^2} + \frac{\left(2d^2\right)}{}$$
$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{2d(c+dx)\text{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{2d(c+dx)\text{Li}_2\left(e^{a+bx}\right)}{b^2} + \frac{\left(2d^2\right)}{}$$
$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{2d(c+dx)\text{Li}_2\left(-e^{a+bx}\right)}{b^2} + \frac{2d(c+dx)\text{Li}_2\left(e^{a+bx}\right)}{b^2} + \frac{2d^2\text{Li}_3}{}$$

Mathematica [A] time = 2.29854, size = 118, normalized size = 1.19

$$\frac{-\frac{2d\left(b(c+dx)\text{PolyLog}\left(2,-e^{a+bx}\right)-d\text{PolyLog}\left(3,-e^{a+bx}\right)\right)}{b^2}+\frac{2d\left(b(c+dx)\text{PolyLog}\left(2,e^{a+bx}\right)-d\text{PolyLog}\left(3,e^{a+bx}\right)\right)}{b^2}+(c+dx)^2\log\left(1-e^{a+bx}\right)-(c+}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csch[a + b*x],x]
```

```
[Out] ((c + d*x)^2*Log[1 - E^(a + b*x)] - (c + d*x)^2*Log[1 + E^(a + b*x)] - (2*d
*(b*(c + d*x)*PolyLog[2, -E^(a + b*x)] - d*PolyLog[3, -E^(a + b*x)]))/b^2 +
 (2*d*(b*(c + d*x)*PolyLog[2, E^(a + b*x)] - d*PolyLog[3, E^(a + b*x)]))/b^
2)/b
```

Maple [B] time = 0.03, size = 306, normalized size = 3.1

$$-2\,\frac{c^2\text{Artanh}\left(e^{bx+a}\right)}{b}+2\,\frac{d^2\text{polylog}\left(3,-e^{bx+a}\right)}{b^3}-2\,\frac{d^2\text{polylog}\left(3,e^{bx+a}\right)}{b^3}+2\,\frac{cd\ln\left(1-e^{bx+a}\right)a}{b^2}-2\,\frac{cd\ln\left(1+e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csch(b*x+a),x)
```

```
[Out] -2/b*c^2*arctanh(exp(b*x+a))+2*d^2*polylog(3,-exp(b*x+a))/b^3-2*d^2*polylog
(3,exp(b*x+a))/b^3+2/b^2*c*d*ln(1-exp(b*x+a))*a-2/b*c*d*ln(1+exp(b*x+a))*x-
2/b^2*c*d*ln(1+exp(b*x+a))*a+2/b*c*d*ln(1-exp(b*x+a))*x+4/b^2*c*d*a*arctanh
(exp(b*x+a))+1/b*d^2*ln(1-exp(b*x+a))*x^2-1/b^3*d^2*ln(1-exp(b*x+a))*a^2+2/
b^2*d^2*polylog(2,exp(b*x+a))*x-1/b*d^2*ln(1+exp(b*x+a))*x^2+1/b^3*d^2*ln(1
+exp(b*x+a))*a^2-2/b^2*d^2*polylog(2,-exp(b*x+a))*x-2/b^3*d^2*a^2*arctanh(e
xp(b*x+a))-2/b^2*c*d*polylog(2,-exp(b*x+a))+2/b^2*c*d*polylog(2,exp(b*x+a))
```

Maxima [B] time = 1.54884, size = 263, normalized size = 2.66

$$-c^2\left(\frac{\log\left(e^{(-bx-a)}+1\right)}{b}-\frac{\log\left(e^{(-bx-a)}-1\right)}{b}\right)-\frac{2\left(bx\log\left(e^{(bx+a)}+1\right)+\text{Li}_2\left(-e^{(bx+a)}\right)\right)cd}{b^2}+\frac{2\left(bx\log\left(-e^{(bx+a)}+1\right)+\text{Li}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -c^2*(log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b) - 2*(b*x*log(e^(b*
x + a) + 1) + dilog(-e^(b*x + a)))*c*d/b^2 + 2*(b*x*log(-e^(b*x + a) + 1) +
 dilog(e^(b*x + a)))*c*d/b^2 - (b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(
-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))*d^2/b^3 + (b^2*x^2*log(-e^(b*x
+ a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))*d^2/b^3
```

Fricas [C] time = 2.64391, size = 635, normalized size = 6.41

$$-\frac{2\,d^2\mathrm{polylog}\left(3,\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)-2\,d^2\mathrm{polylog}\left(3,-\cosh\left(bx+a\right)-\sinh\left(bx+a\right)\right)-2\left(bd^2x+bcd\right)\mathrm{Li}_2\left(c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*d^2*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 2*d^2*polylog(3, -cosh(
b*x + a) - sinh(b*x + a)) - 2*(b*d^2*x + b*c*d)*dilog(cosh(b*x + a) + sinh(
b*x + a)) + 2*(b*d^2*x + b*c*d)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b^
2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cosh(b*x + a) + sinh(b*x + a) + 1) -
 (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - (
b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cosh(b*x + a) - sinh(
b*x + a) + 1))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2 \operatorname{csch}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csch(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*csch(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \operatorname{csch}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csch(b*x + a), x)
```

3.25 $\int (c+dx)\mathrm{csch}(a+bx)\,dx$

Optimal. Leaf size=50

$$-\frac{d\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{d\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}-\frac{2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] (-2*(c + d*x)*ArcTanh[E^(a + b*x)])/b - (d*PolyLog[2, -E^(a + b*x)])/b^2 +
(d*PolyLog[2, E^(a + b*x)])/b^2
```

Rubi [A] time = 0.04626, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4182, 2279, 2391}

$$-\frac{d\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}+\frac{d\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}-\frac{2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csch[a + b*x],x]
```

```
[Out] (-2*(c + d*x)*ArcTanh[E^(a + b*x)])/b - (d*PolyLog[2, -E^(a + b*x)])/b^2 +
(d*PolyLog[2, E^(a + b*x)])/b^2
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)\mathrm{csch}(a+bx)\,dx &= -\frac{2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{d\int \log\left(1-e^{a+bx}\right)dx}{b}+\frac{d\int \log\left(1+e^{a+bx}\right)dx}{b}\\
&= -\frac{2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{d\,\text{Subst}\left(\int \frac{\log(1-x)}{x}dx,x,e^{a+bx}\right)}{b^2}+\frac{d\,\text{Subst}\left(\int \frac{\log(1+x)}{x}dx,x,\right.}{b^2}\\
&= -\frac{2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{d\text{Li}_2\left(-e^{a+bx}\right)}{b^2}+\frac{d\text{Li}_2\left(e^{a+bx}\right)}{b^2}
\end{aligned}$$

Mathematica [C] time = 0.0785852, size = 174, normalized size = 3.48

$$\frac{d\left(-a\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)-i\left(i\left(\text{PolyLog}\left(2,-e^{i(ia+ibx)}\right)-\text{PolyLog}\left(2,e^{i(ia+ibx)}\right)\right)+(ia+ibx)\left(\log\left(1-e^{i(ia+ibx)}\right)-\log\right.\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csch[a + b*x],x]
```

```
[Out] -((c*Log[Cosh[a/2 + (b*x)/2]])/b) + (c*Log[Sinh[a/2 + (b*x)/2]])/b + (d*(-(
a*Log[Tanh[(a + b*x)/2]]) - I*((I*a + I*b*x)*(Log[1 - E^(I*(I*a + I*b*x))]
- Log[1 + E^(I*(I*a + I*b*x))]) + I*(PolyLog[2, -E^(I*(I*a + I*b*x))] - Pol
yLog[2, E^(I*(I*a + I*b*x))]))))/b^2
```

Maple [A] time = 0.004, size = 60, normalized size = 1.2

$$\frac{1}{b}\left(\frac{d}{b}\left(2\,\text{dilog}\left(e^{-bx-a}\right)-\frac{\text{dilog}\left(e^{-2\,bx-2\,a}\right)}{2}\right)+2\,\frac{da\text{Artanh}\left(e^{bx+a}\right)}{b}-2\,c\text{Artanh}\left(e^{bx+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csch(b*x+a),x)
```

```
[Out] 1/b*(1/b*d*(2*dilog(exp(-b*x-a))-1/2*dilog(exp(-2*b*x-2*a)))+2/b*d*a*arctan
h(exp(b*x+a))-2*c*arctanh(exp(b*x+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\frac{\log\left(e^{(-bx-a)}+1\right)}{b}-\frac{\log\left(e^{(-bx-a)}-1\right)}{b}\right)+2\,d\left(\int\frac{x}{2\left(e^{(bx+a)}+1\right)}\,dx+\int\frac{x}{2\left(e^{(bx+a)}-1\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] -c*(log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b) + 2*d*(integrate(1/2
*x/(e^(b*x + a) + 1), x) + integrate(1/2*x/(e^(b*x + a) - 1), x))
```

Fricas [B] time = 2.63454, size = 340, normalized size = 6.8

$$\frac{d\text{Li}_2\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)-d\text{Li}_2\left(-\cosh\left(bx+a\right)-\sinh\left(bx+a\right)\right)-\left(bdx+bc\right)\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+\right.\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] (d*dilog(cosh(b*x + a) + sinh(b*x + a)) - d*dilog(-cosh(b*x + a) - sinh(b*x
 + a)) - (b*d*x + b*c)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + (b*c - a*d)
*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (b*d*x + a*d)*log(-cosh(b*x + a)
```

```
- sinh(b*x + a) + 1))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)\operatorname{csch}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a),x)
```

```
[Out] Integral((c + d*x)*csch(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\operatorname{csch}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csch(b*x + a), x)
```

3.26 $\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\operatorname{csch}(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[Csch[a + b*x]/(c + d*x), x]
```

Rubi [A] time = 0.0235079, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[a + b*x]/(c + d*x),x]
```

```
[Out] Defer[Int][Csch[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 11.29, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[a + b*x]/(c + d*x),x]
```

```
[Out] Integrate[Csch[a + b*x]/(c + d*x), x]
```

Maple [A] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)/(d*x+c),x)
```

```
[Out] int(csch(b*x+a)/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(csch(b*x + a)/(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(csch(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)/(d*x + c), x)
```

3.27 $\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\operatorname{csch}(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable[Csch[a + b*x]/(c + d*x)^2, x]
```

Rubi [A] time = 0.0220008, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Csch[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 11.1254, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Integrate[Csch[a + b*x]/(c + d*x)^2, x]
```

Maple [A] time = 0.034, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(csch(b*x+a)/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(csch(b*x + a)/(d*x + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(csch(a + b*x)/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)/(d*x + c)^2, x)
```

3.28 $\int (c+dx)^3 \mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=103

$$\frac{3d^2(c+dx)\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}-\frac{3d^3\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^4}+\frac{3d(c+dx)^2\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\coth(a+bx)}{b}-$$

```
[Out] -((c + d*x)^3/b) - ((c + d*x)^3*Coth[a + b*x])/b + (3*d*(c + d*x)^2*Log[1 -
 E^(2*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[2, E^(2*(a + b*x))])/b^3
- (3*d^3*PolyLog[3, E^(2*(a + b*x))])/(2*b^4)
```

Rubi [A] time = 0.227086, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4184, 3716, 2190, 2531, 2282, 6589}

$$\frac{3d^2(c+dx)\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}-\frac{3d^3\text{PolyLog}\left(3,e^{2(a+bx)}\right)}{2b^4}+\frac{3d(c+dx)^2\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\coth(a+bx)}{b}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csch[a + b*x]^2,x]
```

```
[Out] -((c + d*x)^3/b) - ((c + d*x)^3*Coth[a + b*x])/b + (3*d*(c + d*x)^2*Log[1 -
 E^(2*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[2, E^(2*(a + b*x))])/b^3
- (3*d^3*PolyLog[3, E^(2*(a + b*x))])/(2*b^4)
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3\text{csch}^2(a+bx)\,dx = -\frac{(c+dx)^3\coth(a+bx)}{b}+\frac{(3d)\int (c+dx)^2\coth(a+bx)\,dx}{b}$$
$$= -\frac{(c+dx)^3}{b}-\frac{(c+dx)^3\coth(a+bx)}{b}-\frac{(6d)\int \frac{e^{2(a+bx)}(c+dx)^2}{1-e^{2(a+bx)}}\,dx}{b}$$
$$= -\frac{(c+dx)^3}{b}-\frac{(c+dx)^3\coth(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{\left(6d^2\right)\int (c+dx)$$
$$= -\frac{(c+dx)^3}{b}-\frac{(c+dx)^3\coth(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{3d^2(c+dx)\text{Li}_2}{b^3}$$
$$= -\frac{(c+dx)^3}{b}-\frac{(c+dx)^3\coth(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{3d^2(c+dx)\text{Li}_2}{b^3}$$
$$= -\frac{(c+dx)^3}{b}-\frac{(c+dx)^3\coth(a+bx)}{b}+\frac{3d(c+dx)^2\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{3d^2(c+dx)\text{Li}_2}{b^3}$$

Mathematica [A] time = 2.29771, size = 185, normalized size = 1.8

$$-6d^2\left(b(c+dx)\text{PolyLog}\left(2,-e^{-a-bx}\right)+d\text{PolyLog}\left(3,-e^{-a-bx}\right)\right)-6d^2\left(b(c+dx)\text{PolyLog}\left(2,e^{-a-bx}\right)+d\text{PolyLog}\left(3,e$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csch[a + b*x]^2,x]
```

```
[Out] ((-2*b^3*(c + d*x)^3)/(-1 + E^(2*a)) + 3*b^2*d*(c + d*x)^2*Log[1 - E^(-a -
b*x)] + 3*b^2*d*(c + d*x)^2*Log[1 + E^(-a - b*x)] - 6*d^2*(b*(c + d*x)*Poly
Log[2, -E^(-a - b*x)] + d*PolyLog[3, -E^(-a - b*x)]) - 6*d^2*(b*(c + d*x)*P
olyLog[2, E^(-a - b*x)] + d*PolyLog[3, E^(-a - b*x)]) + b^3*(c + d*x)^3*Csc
h[a]*Csch[a + b*x]*Sinh[b*x])/b^4
```

Maple [B] time = 0.059, size = 473, normalized size = 4.6

$$12\,\frac{cd^2a\ln\left(e^{bx+a}\right)}{b^3}+6\,\frac{cd^2\ln\left(1-e^{bx+a}\right)x}{b^2}+6\,\frac{cd^2\ln\left(1-e^{bx+a}\right)a}{b^3}+6\,\frac{cd^2\ln\left(1+e^{bx+a}\right)x}{b^2}-6\,\frac{cd^2a\ln\left(e^{bx+a}-1\right)}{b^3}-12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csch(b*x+a)^2,x)
```

```
[Out] 12*d^2/b^3*c*a*ln(exp(b*x+a))+6*d^2/b^2*c*ln(1-exp(b*x+a))*x+6*d^2/b^3*c*ln
(1-exp(b*x+a))*a+6*d^2/b^2*c*ln(1+exp(b*x+a))*x-6*d^2/b^3*c*a*ln(exp(b*x+a)
-1)-12*d^2/b^2*c*a*x+6*d^3/b^3*a^2*x-6*d^2/b*c*x^2-6*d^2/b^3*c*a^2-6*d/b^2*
c^2*ln(exp(b*x+a))+3*d/b^2*c^2*ln(1+exp(b*x+a))+3*d/b^2*c^2*ln(exp(b*x+a)-1
)+3*d^3/b^2*ln(1-exp(b*x+a))*x^2+6*d^3/b^3*polylog(2,exp(b*x+a))*x+3*d^3/b^
2*ln(1+exp(b*x+a))*x^2+6*d^3/b^3*polylog(2,-exp(b*x+a))*x-3*d^3/b^4*a^2*ln(
1-exp(b*x+a))-6*d^3/b^4*a^2*ln(exp(b*x+a))+3*d^3/b^4*a^2*ln(exp(b*x+a)-1)+6
*d^2/b^3*c*polylog(2,-exp(b*x+a))+6*d^2/b^3*c*polylog(2,exp(b*x+a))+4*d^3/b
^4*a^3-2*d^3/b*x^3-6*d^3/b^4*polylog(3,-exp(b*x+a))-6*d^3/b^4*polylog(3,exp
(b*x+a))-2/b*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/(exp(2*b*x+2*a)-1)
```

Maxima [B] time = 1.99967, size = 432, normalized size = 4.19

$$-3\,c^2 d\left(\frac{2\,xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)}-b}-\frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2}-\frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}\right)+\frac{6\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)cd^2}{b^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -3*c^2*d*(2*x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) - b) - log((e^(b*x + a) +
1)*e^(-a))/b^2 - log((e^(b*x + a) - 1)*e^(-a))/b^2) + 6*(b*x*log(e^(b*x + a
) + 1) + dilog(-e^(b*x + a)))*c*d^2/b^3 + 6*(b*x*log(-e^(b*x + a) + 1) + di
log(e^(b*x + a)))*c*d^2/b^3 + 2*c^3/(b*(e^(-2*b*x - 2*a) - 1)) - 2*(d^3*x^3
 + 3*c*d^2*x^2)/(b*e^(2*b*x + 2*a) - b) + 3*(b^2*x^2*log(e^(b*x + a) + 1) +
 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))*d^3/b^4 + 3*(b^2*x
^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x +
 a)))*d^3/b^4 - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2)/b^4
```

Fricas [C] time = 2.85253, size = 2689, normalized size = 26.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 2*a^3*d^3 + 2*(b^3*d^3*x^3 +
3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*
cosh(b*x + a)^2 + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^
2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cosh(b*x + a)*sinh(b*x + a) + 2*(b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^
3*d^3)*sinh(b*x + a)^2 + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cosh(b*
x + a)^2 - 2*(b*d^3*x + b*c*d^2)*cosh(b*x + a)*sinh(b*x + a) - (b*d^3*x + b
*c*d^2)*sinh(b*x + a)^2)*dilog(cosh(b*x + a) + sinh(b*x + a)) + 6*(b*d^3*x
+ b*c*d^2 - (b*d^3*x + b*c*d^2)*cosh(b*x + a)^2 - 2*(b*d^3*x + b*c*d^2)*cos
h(b*x + a)*sinh(b*x + a) - (b*d^3*x + b*c*d^2)*sinh(b*x + a)^2)*dilog(-cosh
(b*x + a) - sinh(b*x + a)) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - (
b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cosh(b*x + a)^2 - 2*(b^2*d^3*x^2 +
 2*b^2*c*d^2*x + b^2*c^2*d)*cosh(b*x + a)*sinh(b*x + a) - (b^2*d^3*x^2 + 2*
b^2*c*d^2*x + b^2*c^2*d)*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a)
 + 1) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3 - (b^2*c^2*d - 2*a*b*c*d^2 + a
```

```
^2*d^3)*cosh(b*x + a)^2 - 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cosh(b*x +
a)*sinh(b*x + a) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sinh(b*x + a)^2)*log
(cosh(b*x + a) + sinh(b*x + a) - 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*
b*c*d^2 - a^2*d^3 - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*c
osh(b*x + a)^2 - 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*co
sh(b*x + a)*sinh(b*x + a) - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^
2*d^3)*sinh(b*x + a)^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*(d^3*co
sh(b*x + a)^2 + 2*d^3*cosh(b*x + a)*sinh(b*x + a) + d^3*sinh(b*x + a)^2 - d
^3)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 6*(d^3*cosh(b*x + a)^2 + 2*
d^3*cosh(b*x + a)*sinh(b*x + a) + d^3*sinh(b*x + a)^2 - d^3)*polylog(3, -co
sh(b*x + a) - sinh(b*x + a)))/(b^4*cosh(b*x + a)^2 + 2*b^4*cosh(b*x + a)*si
nh(b*x + a) + b^4*sinh(b*x + a)^2 - b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^3 \operatorname{csch}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csch(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*csch(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \operatorname{csch}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csch(b*x + a)^2, x)
```

3.29 $\int (c+dx)^2\operatorname{csch}^2(a+bx)\,dx$

Optimal. Leaf size=74

$$\frac{d^2\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}+\frac{2d(c+dx)\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\coth(a+bx)}{b}-\frac{(c+dx)^2}{b}$$

```
[Out] -((c + d*x)^2/b) - ((c + d*x)^2*Coth[a + b*x])/b + (2*d*(c + d*x)*Log[1 - E
^(2*(a + b*x))])/b^2 + (d^2*PolyLog[2, E^(2*(a + b*x))])/b^3
```

Rubi [A] time = 0.147543, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4184, 3716, 2190, 2279, 2391}

$$\frac{d^2\text{PolyLog}\left(2,e^{2(a+bx)}\right)}{b^3}+\frac{2d(c+dx)\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\coth(a+bx)}{b}-\frac{(c+dx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csch[a + b*x]^2,x]
```

```
[Out] -((c + d*x)^2/b) - ((c + d*x)^2*Coth[a + b*x])/b + (2*d*(c + d*x)*Log[1 - E
^(2*(a + b*x))])/b^2 + (d^2*PolyLog[2, E^(2*(a + b*x))])/b^3
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)^2\text{csch}^2(a+bx)\,dx = -\frac{(c+dx)^2\coth(a+bx)}{b}+\frac{(2d)\int (c+dx)\coth(a+bx)\,dx}{b}$$
$$= -\frac{(c+dx)^2}{b}-\frac{(c+dx)^2\coth(a+bx)}{b}-\frac{(4d)\int \frac{e^{2(a+bx)}(c+dx)}{1-e^{2(a+bx)}}\,dx}{b}$$
$$= -\frac{(c+dx)^2}{b}-\frac{(c+dx)^2\coth(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{\left(2d^2\right)\int \log\left(1-\right.}{b^2}$$
$$= -\frac{(c+dx)^2}{b}-\frac{(c+dx)^2\coth(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2(a+bx)}\right)}{b^2}-\frac{d^2\,\text{Subst}\left(\int \frac{\log(}{}\right.}{}$$
$$= -\frac{(c+dx)^2}{b}-\frac{(c+dx)^2\coth(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2(a+bx)}\right)}{b^2}+\frac{d^2\text{Li}_2\left(e^{2(a+bx)}\right)}{b^3}$$

Mathematica [C] time = 5.39859, size = 198, normalized size = 2.68

$$\text{csch}(a)\left(d^2\left(-\sinh(a)\text{PolyLog}\left(2,e^{-2\left(\tanh^{-1}(\tanh(a))+bx\right)}\right)-b^2x^2\cosh(a)e^{-\tanh^{-1}(\tanh(a))}\sqrt{\text{sech}^2(a)}+i\pi bx\sinh(a)-\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csch[a + b*x]^2,x]
```

```
[Out] (Csch[a]*(-2*b*c*d*(b*x*Cosh[a] - Log[Sinh[a + b*x]]*Sinh[a]) + d^2*(-((b^2
*x^2*Cosh[a]*Sqrt[Sech[a]^2])/E^ArcTanh[Tanh[a]]) + I*b*Pi*x*Sinh[a] - I*Pi
*Log[1 + E^(2*b*x)]*Sinh[a] + 2*b*x*Log[1 - E^(-2*(b*x + ArcTanh[Tanh[a]]))
]*Sinh[a] + I*Pi*Log[Cosh[b*x]]*Sinh[a] + 2*ArcTanh[Tanh[a]]*(b*x + Log[1 -
 E^(-2*(b*x + ArcTanh[Tanh[a]]))] - Log[I*Sinh[b*x + ArcTanh[Tanh[a]]]])*Si
nh[a] - PolyLog[2, E^(-2*(b*x + ArcTanh[Tanh[a]]))]*Sinh[a]) + b^2*(c + d*x
)^2*Csch[a + b*x]*Sinh[b*x]))/b^3
```

Maple [B] time = 0.03, size = 240, normalized size = 3.2

$$-2\,\frac{d^2x^2+2\,cdx+c^2}{b\left(e^{2\,bx+2\,a}-1\right)}-4\,\frac{cd\ln\left(e^{bx+a}\right)}{b^2}+2\,\frac{cd\ln\left(1+e^{bx+a}\right)}{b^2}+2\,\frac{cd\ln\left(e^{bx+a}-1\right)}{b^2}-2\,\frac{d^2x^2}{b}-4\,\frac{ad^2x}{b^2}-2\,\frac{a^2d^2}{b^3}+2\,\frac{d^2\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csch(b*x+a)^2,x)
```

```
[Out] -2/b*(d^2*x^2+2*c*d*x+c^2)/(exp(2*b*x+2*a)-1)-4*d/b^2*c*ln(exp(b*x+a))+2*d/
b^2*c*ln(1+exp(b*x+a))+2*d/b^2*c*ln(exp(b*x+a)-1)-2*d^2/b*x^2-4*d^2/b^2*a*x
-2*d^2/b^3*a^2+2*d^2/b^2*ln(1+exp(b*x+a))*x+2*d^2/b^3*polylog(2,-exp(b*x+a)
)+2*d^2/b^2*ln(1-exp(b*x+a))*x+2*d^2/b^3*ln(1-exp(b*x+a))*a+2*d^2/b^3*polyl
og(2,exp(b*x+a))+4*d^2/b^3*a*ln(exp(b*x+a))-2*d^2/b^3*a*ln(exp(b*x+a)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-2\,d^2\left(\frac{x^2}{be^{(2\,bx+2\,a)}-b}+2\int\frac{x}{2\left(be^{(bx+a)}+b\right)}\,dx-2\int\frac{x}{2\left(be^{(bx+a)}-b\right)}\,dx\right)-2\,cd\left(\frac{2\,xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)}-b}-\frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-}\right.}{b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*d^2*(x^2/(b*e^(2*b*x + 2*a) - b) + 2*integrate(1/2*x/(b*e^(b*x + a) + b)
, x) - 2*integrate(1/2*x/(b*e^(b*x + a) - b), x)) - 2*c*d*(2*x*e^(2*b*x + 2
*a)/(b*e^(2*b*x + 2*a) - b) - log((e^(b*x + a) + 1)*e^(-a))/b^2 - log((e^(b
*x + a) - 1)*e^(-a))/b^2) + 2*c^2/(b*(e^(-2*b*x - 2*a) - 1))
```

Fricas [B] time = 2.78976, size = 1521, normalized size = 20.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d
- a^2*d^2)*cosh(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2
*d^2)*cosh(b*x + a)*sinh(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d
- a^2*d^2)*sinh(b*x + a)^2 - (d^2*cosh(b*x + a)^2 + 2*d^2*cosh(b*x + a)*sin
h(b*x + a) + d^2*sinh(b*x + a)^2 - d^2)*dilog(cosh(b*x + a) + sinh(b*x + a)
) - (d^2*cosh(b*x + a)^2 + 2*d^2*cosh(b*x + a)*sinh(b*x + a) + d^2*sinh(b*x
 + a)^2 - d^2)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*d^2*x + b*c*d - (
b*d^2*x + b*c*d)*cosh(b*x + a)^2 - 2*(b*d^2*x + b*c*d)*cosh(b*x + a)*sinh(b
*x + a) - (b*d^2*x + b*c*d)*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x +
 a) + 1) + (b*c*d - a*d^2 - (b*c*d - a*d^2)*cosh(b*x + a)^2 - 2*(b*c*d - a*
d^2)*cosh(b*x + a)*sinh(b*x + a) - (b*c*d - a*d^2)*sinh(b*x + a)^2)*log(cos
h(b*x + a) + sinh(b*x + a) - 1) + (b*d^2*x + a*d^2 - (b*d^2*x + a*d^2)*cosh
(b*x + a)^2 - 2*(b*d^2*x + a*d^2)*cosh(b*x + a)*sinh(b*x + a) - (b*d^2*x +
a*d^2)*sinh(b*x + a)^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1))/(b^3*cosh(
b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2 - b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \operatorname{csch}^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csch(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*csch(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \operatorname{csch}(bx + a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csch(b*x + a)^2, x)
```

3.30 $\int (c+dx)\text{csch}^2(a+bx)\,dx$

Optimal. Leaf size=29

$$\frac{d\log(\sinh(a+bx))}{b^2}-\frac{(c+dx)\coth(a+bx)}{b}$$

```
[Out] -(((c + d*x)*Coth[a + b*x])/b) + (d*Log[Sinh[a + b*x]])/b^2
```

Rubi [A] time = 0.0303038, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4184, 3475}

$$\frac{d\log(\sinh(a+bx))}{b^2}-\frac{(c+dx)\coth(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csch[a + b*x]^2,x]
```

```
[Out] -(((c + d*x)*Coth[a + b*x])/b) + (d*Log[Sinh[a + b*x]])/b^2
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\text{csch}^2(a+bx)\,dx&=-\frac{(c+dx)\coth(a+bx)}{b}+\frac{d\int \coth(a+bx)\,dx}{b}\\&=-\frac{(c+dx)\coth(a+bx)}{b}+\frac{d\log(\sinh(a+bx))}{b^2}\end{aligned}$$

Mathematica [A] time = 0.099239, size = 52, normalized size = 1.79

$$\frac{d\log(\sinh(a+bx))}{b^2}-\frac{c\coth(a+bx)}{b}-\frac{dx\coth(a)}{b}+\frac{dx\text{csch}(a)\sinh(bx)\text{csch}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csch[a + b*x]^2,x]
```

```
[Out] -((d*x*Coth[a])/b) - (c*Coth[a + b*x])/b + (d*Log[Sinh[a + b*x]])/b^2 + (d*
x*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b
```

Maple [A] time = 0.02, size = 56, normalized size = 1.9

$$-2\frac{dx}{b}-2\frac{da}{b^2}-2\frac{dx+c}{b\left(e^{2bx+2a}-1\right)}+\frac{d\ln\left(e^{2bx+2a}-1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csch(b*x+a)^2,x)
```

```
[Out] -2*d/b*x-2*d/b^2*a-2/b*(d*x+c)/(exp(2*b*x+2*a)-1)+d/b^2*ln(exp(2*b*x+2*a)-1
)
```

Maxima [B] time = 1.14333, size = 123, normalized size = 4.24

$$-d\left(\frac{2\,xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)}-b}-\frac{\log\left(\left(e^{(bx+a)}+1\right)e^{(-a)}\right)}{b^2}-\frac{\log\left(\left(e^{(bx+a)}-1\right)e^{(-a)}\right)}{b^2}\right)+\frac{2\,c}{b\left(e^{(-2\,bx-2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -d*(2*x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) - b) - log((e^(b*x + a) + 1)*e^(
-a))/b^2 - log((e^(b*x + a) - 1)*e^(-a))/b^2) + 2*c/(b*(e^(-2*b*x - 2*a) -
1))
```

Fricas [B] time = 2.57725, size = 431, normalized size = 14.86

$$-\frac{2\,bdx\cosh\left(bx+a\right)^2+4\,bdx\cosh\left(bx+a\right)\sinh\left(bx+a\right)+2\,bdx\sinh\left(bx+a\right)^2+2\,bc-\left(d\cosh\left(bx+a\right)^2+2\,d\cosh\left(bx\right.\right.}{b^2\cosh\left(bx+a\right)^2+2\,b^2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b^2\sinh\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b*d*x*cosh(b*x + a)^2 + 4*b*d*x*cosh(b*x + a)*sinh(b*x + a) + 2*b*d*x*s
inh(b*x + a)^2 + 2*b*c - (d*cosh(b*x + a)^2 + 2*d*cosh(b*x + a)*sinh(b*x +
a) + d*sinh(b*x + a)^2 - d)*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x +
 a))))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*x + a) + b^2*sinh(
b*x + a)^2 - b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)\operatorname{csch}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*csch(a + b*x)**2, x)
```

Giac [B] time = 1.31742, size = 108, normalized size = 3.72

$$-\frac{2\,bdxe^{(2\,bx+2\,a)}-de^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)}-1\right)+2\,bc+d\log\left(e^{(2\,bx+2\,a)}-1\right)}{b^{2}e^{(2\,bx+2\,a)}-b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(2*b*d*x*e^(2*b*x + 2*a) - d*e^(2*b*x + 2*a)*log(e^(2*b*x + 2*a) - 1) + 2*
b*c + d*log(e^(2*b*x + 2*a) - 1))/(b^2*e^(2*b*x + 2*a) - b^2)
```

3.31 $\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^2(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[Csch[a + b*x]^2/(c + d*x), x]
```

Rubi [A] time = 0.0403233, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[a + b*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][Csch[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 17.713, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[a + b*x]^2/(c + d*x),x]
```

```
[Out] Integrate[Csch[a + b*x]^2/(c + d*x), x]
```

Maple [A] time = 0.061, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(bx+a))^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(csch(b*x+a)^2/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$4\,d\int \frac{1}{4\left(bd^2x^2+2\,bcdx+bc^2+\left(bd^2x^2e^a+2\,bcdxe^a+bc^2e^a\right)e^{(bx)}\right)}\,dx-4\,d\int -\frac{1}{4\left(bd^2x^2+2\,bcdx+bc^2-\left(bd^2x^2e^a+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 4*d*integrate(1/4/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2*e^a + 2*b*c*d
*x*e^a + b*c^2*e^a)*e^(b*x)), x) - 4*d*integrate(-1/4/(b*d^2*x^2 + 2*b*c*d*
x + b*c^2 - (b*d^2*x^2*e^a + 2*b*c*d*x*e^a + b*c^2*e^a)*e^(b*x)), x) + 2/(b
*d*x + b*c - (b*d*x*e^(2*a) + b*c*e^(2*a))*e^(2*b*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{csch}\,(bx+a)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^2/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{csch}^2\,(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(csch(a + b*x)**2/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{csch}\,(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2/(d*x + c), x)
```

3.32 $\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable[Csch[a + b*x]^2/(c + d*x)^2, x]
```

Rubi [A] time = 0.0378597, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Csch[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}\,dx=\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 18.6732, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Integrate[Csch[a + b*x]^2/(c + d*x)^2, x]
```

Maple [A] time = 0.072, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(bx+a))^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(csch(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$4\,d\int \frac{1}{2\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+\left(bd^3x^3e^a+3\,bcd^2x^2e^a+3\,bc^2dxe^a+bc^3e^a\right)e^{(bx)}\right)}\,dx-4\,d\int -\frac{}{2\left(bd^3x^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 4*d*integrate(1/2/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3
*x^3*e^a + 3*b*c*d^2*x^2*e^a + 3*b*c^2*d*x*e^a + b*c^3*e^a)*e^(b*x)), x) -
4*d*integrate(-1/2/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 - (b*d^
3*x^3*e^a + 3*b*c*d^2*x^2*e^a + 3*b*c^2*d*x*e^a + b*c^3*e^a)*e^(b*x)), x) +
 2/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 - (b*d^2*x^2*e^(2*a) + 2*b*c*d*x*e^(2*a)
+ b*c^2*e^(2*a))*e^(2*b*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{csch}\,(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{csch}^2\,(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(csch(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{csch}\,(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^2/(d*x + c)^2, x)
```

3.33 $\int (c+dx)^3 \mathbf{csch}^3(a+bx)\,dx$

Optimal. Leaf size=256

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}+\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}+\frac{3d(c+dx)^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}-\frac{3d(c+dx)^2\text{Pol}}{2b}$$

```
[Out] (-6*d^2*(c + d*x)*ArcTanh[E^(a + b*x)])/b^3 + ((c + d*x)^3*ArcTanh[E^(a + b
*x)])/b - (3*d*(c + d*x)^2*Csch[a + b*x])/(2*b^2) - ((c + d*x)^3*Coth[a + b
*x]*Csch[a + b*x])/(2*b) - (3*d^3*PolyLog[2, -E^(a + b*x)])/b^4 + (3*d*(c +
 d*x)^2*PolyLog[2, -E^(a + b*x)])/(2*b^2) + (3*d^3*PolyLog[2, E^(a + b*x)])
/b^4 - (3*d*(c + d*x)^2*PolyLog[2, E^(a + b*x)])/(2*b^2) - (3*d^2*(c + d*x)
*PolyLog[3, -E^(a + b*x)])/b^3 + (3*d^2*(c + d*x)*PolyLog[3, E^(a + b*x)])/
b^3 + (3*d^3*PolyLog[4, -E^(a + b*x)])/b^4 - (3*d^3*PolyLog[4, E^(a + b*x)]
)/b^4
```

Rubi [A] time = 0.273616, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4186, 4182, 2279, 2391, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}+\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}+\frac{3d(c+dx)^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}-\frac{3d(c+dx)^2\text{Pol}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csch[a + b*x]^3,x]
```

```
[Out] (-6*d^2*(c + d*x)*ArcTanh[E^(a + b*x)])/b^3 + ((c + d*x)^3*ArcTanh[E^(a + b
*x)])/b - (3*d*(c + d*x)^2*Csch[a + b*x])/(2*b^2) - ((c + d*x)^3*Coth[a + b
*x]*Csch[a + b*x])/(2*b) - (3*d^3*PolyLog[2, -E^(a + b*x)])/b^4 + (3*d*(c +
 d*x)^2*PolyLog[2, -E^(a + b*x)])/(2*b^2) + (3*d^3*PolyLog[2, E^(a + b*x)])
/b^4 - (3*d*(c + d*x)^2*PolyLog[2, E^(a + b*x)])/(2*b^2) - (3*d^2*(c + d*x)
*PolyLog[3, -E^(a + b*x)])/b^3 + (3*d^2*(c + d*x)*PolyLog[3, E^(a + b*x)])/
b^3 + (3*d^3*PolyLog[4, -E^(a + b*x)])/b^4 - (3*d^3*PolyLog[4, E^(a + b*x)]
)/b^4
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3\operatorname{csch}^3(a+bx)\,dx = -\frac{3d(c+dx)^2\operatorname{csch}(a+bx)}{2b^2} - \frac{(c+dx)^3\coth(a+bx)\operatorname{csch}(a+bx)}{2b} - \frac{1}{2}\int (c+dx)^3\operatorname{csch}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3d(c+dx)^2\operatorname{csch}(a+bx)}{2b^2} -$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3d(c+dx)^2\operatorname{csch}(a+bx)}{2b^2} -$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3d(c+dx)^2\operatorname{csch}(a+bx)}{2b^2} -$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3d(c+dx)^2\operatorname{csch}(a+bx)}{2b^2} -$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{3d(c+dx)^2\operatorname{csch}(a+bx)}{2b^2} -$$

Mathematica [A] time = 10.0834, size = 440, normalized size = 1.72

$$-\frac{-3d\left(b^2(c+dx)^2-2d^2\right)\text{PolyLog}\left(2,-e^{a+bx}\right)+3d\left(b^2(c+dx)^2-2d^2\right)\text{PolyLog}\left(2,e^{a+bx}\right)+6bcd^2\text{PolyLog}\left(3,-e^{a+bx}\right)-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csch[a + b*x]^3,x]
```

```
[Out] -(b^2*(c + d*x)^2*(3*d + b*(c + d*x)*Coth[a + b*x])*Csch[a + b*x] + b^3*c^3
*Log[1 - E^(a + b*x)] - 6*b*c*d^2*Log[1 - E^(a + b*x)] + 3*b^3*c^2*d*x*Log[
1 - E^(a + b*x)] - 6*b*d^3*x*Log[1 - E^(a + b*x)] + 3*b^3*c*d^2*x^2*Log[1 -
 E^(a + b*x)] + b^3*d^3*x^3*Log[1 - E^(a + b*x)] - b^3*c^3*Log[1 + E^(a + b
*x)] + 6*b*c*d^2*Log[1 + E^(a + b*x)] - 3*b^3*c^2*d*x*Log[1 + E^(a + b*x)]
+ 6*b*d^3*x*Log[1 + E^(a + b*x)] - 3*b^3*c*d^2*x^2*Log[1 + E^(a + b*x)] - b
^3*d^3*x^3*Log[1 + E^(a + b*x)] - 3*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2,
 -E^(a + b*x)] + 3*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, E^(a + b*x)] + 6
*b*c*d^2*PolyLog[3, -E^(a + b*x)] + 6*b*d^3*x*PolyLog[3, -E^(a + b*x)] - 6*
b*c*d^2*PolyLog[3, E^(a + b*x)] - 6*b*d^3*x*PolyLog[3, E^(a + b*x)] - 6*d^3
*PolyLog[4, -E^(a + b*x)] + 6*d^3*PolyLog[4, E^(a + b*x)])/(2*b^4)
```

Maple [B] time = 0.06, size = 876, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csch(b*x+a)^3,x)
```

```
[Out] 3*d^3*polylog(4,-exp(b*x+a))/b^4-3*d^3*polylog(4,exp(b*x+a))/b^4-3*d^3*poly
log(2,-exp(b*x+a))/b^4+3*d^3*polylog(2,exp(b*x+a))/b^4-3/b^2*c*d^2*polylog(
2,exp(b*x+a))*x+3/2/b*c^2*d*ln(1+exp(b*x+a))*x+3/2/b^2*c^2*d*ln(1+exp(b*x+a
))*a-3/2/b*c^2*d*ln(1-exp(b*x+a))*x-3/2/b^2*c^2*d*ln(1-exp(b*x+a))*a-3/b^2*
c^2*d*a*arctanh(exp(b*x+a))+3/b^3*c*d^2*a^2*arctanh(exp(b*x+a))+3/2/b*c*d^2
*ln(1+exp(b*x+a))*x^2-3/2/b^3*c*d^2*ln(1+exp(b*x+a))*a^2+3/b^2*c*d^2*polylo
g(2,-exp(b*x+a))*x-3/2/b*c*d^2*ln(1-exp(b*x+a))*x^2+3/2/b^3*c*d^2*ln(1-exp(
b*x+a))*a^2+6/b^4*d^3*a*arctanh(exp(b*x+a))-6/b^3*c*d^2*arctanh(exp(b*x+a))
-3/b^4*d^3*a*ln(1+exp(b*x+a))+3/b^4*d^3*a*ln(1-exp(b*x+a))-3/b^3*d^3*ln(1+e
xp(b*x+a))*x+3/b^3*d^3*ln(1-exp(b*x+a))*x-exp(b*x+a)*(b*d^3*x^3*exp(2*b*x+2
*a)+3*b*c*d^2*x^2*exp(2*b*x+2*a)+3*b*c^2*d*x*exp(2*b*x+2*a)+b*d^3*x^3+3*d^3
*x^2*exp(2*b*x+2*a)+b*c^3*exp(2*b*x+2*a)+3*b*c*d^2*x^2+6*c*d^2*x*exp(2*b*x+
2*a)+3*b*c^2*d*x+3*c^2*d*exp(2*b*x+2*a)-3*d^3*x^2+b*c^3-6*c*d^2*x-3*c^2*d)/
b^2/(exp(2*b*x+2*a)-1)^2-3/b^3*d^3*polylog(3,-exp(b*x+a))*x-1/b^4*d^3*a^3*a
rctanh(exp(b*x+a))+3/2/b^2*c^2*d*polylog(2,-exp(b*x+a))-3/2/b^2*c^2*d*polyl
og(2,exp(b*x+a))-3/b^3*c*d^2*polylog(3,-exp(b*x+a))+3/b^3*c*d^2*polylog(3,e
xp(b*x+a))+1/2/b^4*d^3*a^3*ln(1+exp(b*x+a))-1/2/b^4*d^3*a^3*ln(1-exp(b*x+a)
)-1/2/b*d^3*ln(1-exp(b*x+a))*x^3-3/2/b^2*d^3*polylog(2,exp(b*x+a))*x^2+3/b^
3*d^3*polylog(3,exp(b*x+a))*x+1/2/b*d^3*ln(1+exp(b*x+a))*x^3+3/2/b^2*d^3*po
lylog(2,-exp(b*x+a))*x^2+1/b*c^3*arctanh(exp(b*x+a))
```

Maxima [B] time = 1.97381, size = 817, normalized size = 3.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*c^3*(log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b + 2*(e^(-b*x - a
) + e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))) + 3/
2*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3,
-e^(b*x + a)))*c*d^2/b^3 - 3/2*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog
(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))*c*d^2/b^3 - 3*c*d^2*log(e^(b*x +
 a) + 1)/b^3 + 3*c*d^2*log(e^(b*x + a) - 1)/b^3 - ((b*d^3*x^3*e^(3*a) + 3*c
^2*d*e^(3*a) + 3*(b*c*d^2 + d^3)*x^2*e^(3*a) + 3*(b*c^2*d + 2*c*d^2)*x*e^(3
*a))*e^(3*b*x) + (b*d^3*x^3*e^a - 3*c^2*d*e^a + 3*(b*c*d^2 - d^3)*x^2*e^a +
 3*(b*c^2*d - 2*c*d^2)*x*e^a)*e^(b*x))/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*
x + 2*a) + b^2) + 1/2*(b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b
*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))*d^3
/b^4 - 1/2*(b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) -
6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))*d^3/b^4 + 3/2*(b
^2*c^2*d - 2*d^3)*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^4 - 3/
2*(b^2*c^2*d - 2*d^3)*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^4
```

Fricas [C] time = 3.47511, size = 8979, normalized size = 35.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b^3*d^3*x^3 + b^3*c^3 + 3*b^2*c^2*d + 3*(b^3*c*d^2 + b^2*d^3)*x^2
+ 3*(b^3*c^2*d + 2*b^2*c*d^2)*x)*cosh(b*x + a)^3 + 6*(b^3*d^3*x^3 + b^3*c^3
 + 3*b^2*c^2*d + 3*(b^3*c*d^2 + b^2*d^3)*x^2 + 3*(b^3*c^2*d + 2*b^2*c*d^2)*
x)*cosh(b*x + a)*sinh(b*x + a)^2 + 2*(b^3*d^3*x^3 + b^3*c^3 + 3*b^2*c^2*d +
 3*(b^3*c*d^2 + b^2*d^3)*x^2 + 3*(b^3*c^2*d + 2*b^2*c*d^2)*x)*sinh(b*x + a)
^3 + 2*(b^3*d^3*x^3 + b^3*c^3 - 3*b^2*c^2*d + 3*(b^3*c*d^2 - b^2*d^3)*x^2 +
 3*(b^3*c^2*d - 2*b^2*c*d^2)*x)*cosh(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^
2*x + b^2*c^2*d + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*
x + a)^4 + 4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*x + a
)*sinh(b*x + a)^3 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sinh(
b*x + a)^4 - 2*d^3 - 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*co
sh(b*x + a)^2 - 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3 - 3*(b^2
*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*x + a)^2)*sinh(b*x + a
)^2 + 4*((b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*x + a)^3
- (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*x + a))*sinh(b*x
 + a))*dilog(cosh(b*x + a) + sinh(b*x + a)) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*
x + b^2*c^2*d + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*x
+ a)^4 + 4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*x + a)*
sinh(b*x + a)^3 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sinh(b*
x + a)^4 - 2*d^3 - 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh
(b*x + a)^2 - 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3 - 3*(b^2*d
^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*x + a)^2)*sinh(b*x + a)^
2 + 4*((b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*x + a)^3 -
(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cosh(b*x + a))*sinh(b*x +
 a))*dilog(-cosh(b*x + a) - sinh(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
 + b^3*c^3 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*
c^2*d - 2*b*d^3)*x)*cosh(b*x + a)^4 + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^
3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)*sinh(b*x + a)^
3 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2
*b*d^3)*x)*sinh(b*x + a)^4 - 6*b*c*d^2 - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)^2 - 2*(b^3*
```

```
d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 - 3*(b^3*d^3*x^3 + 3*b^3*c*
d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)^2
+ 3*(b^3*c^2*d - 2*b*d^3)*x)*sinh(b*x + a)^2 + 3*(b^3*c^2*d - 2*b*d^3)*x +
4*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*
b*d^3)*x)*cosh(b*x + a)^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*
c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b
*x + a) + sinh(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d
^2 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cosh
(b*x + a)^4 + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a
)*d^3)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 -
2)*b*c*d^2 - (a^3 - 6*a)*d^3)*sinh(b*x + a)^4 - (a^3 - 6*a)*d^3 - 2*(b^3*c^
3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cosh(b*x + a)^2
- 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3 - 3*(b
^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cosh(b*x +
a)^2)*sinh(b*x + a)^2 + 4*((b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 -
 (a^3 - 6*a)*d^3)*cosh(b*x + a)^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*
b*c*d^2 - (a^3 - 6*a)*d^3)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a)
+ sinh(b*x + a) - 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)^4 + 4*(b^3*d^
3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 +
 3*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*d^3*x^3 +
3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*
c^2*d - 2*b*d^3)*x)*sinh(b*x + a)^4 + (a^3 - 6*a)*d^3 - 2*(b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^
2*d - 2*b*d^3)*x)*cosh(b*x + a)^2 - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*
x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*
d^3)*x)*cosh(b*x + a)^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*sinh(b*x + a)^2 + 3*(b
^3*c^2*d - 2*b*d^3)*x + 4*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)
^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3
- 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a))*sinh(b*x + a))*log(-
cosh(b*x + a) - sinh(b*x + a) + 1) + 6*(d^3*cosh(b*x + a)^4 + 4*d^3*cosh(b*
x + a)*sinh(b*x + a)^3 + d^3*sinh(b*x + a)^4 - 2*d^3*cosh(b*x + a)^2 + d^3
+ 2*(3*d^3*cosh(b*x + a)^2 - d^3)*sinh(b*x + a)^2 + 4*(d^3*cosh(b*x + a)^3
- d^3*cosh(b*x + a))*sinh(b*x + a))*polylog(4, cosh(b*x + a) + sinh(b*x + a
)) - 6*(d^3*cosh(b*x + a)^4 + 4*d^3*cosh(b*x + a)*sinh(b*x + a)^3 + d^3*sin
h(b*x + a)^4 - 2*d^3*cosh(b*x + a)^2 + d^3 + 2*(3*d^3*cosh(b*x + a)^2 - d^3
)*sinh(b*x + a)^2 + 4*(d^3*cosh(b*x + a)^3 - d^3*cosh(b*x + a))*sinh(b*x +
a))*polylog(4, -cosh(b*x + a) - sinh(b*x + a)) - 6*(b*d^3*x + (b*d^3*x + b*
c*d^2)*cosh(b*x + a)^4 + 4*(b*d^3*x + b*c*d^2)*cosh(b*x + a)*sinh(b*x + a)^
3 + (b*d^3*x + b*c*d^2)*sinh(b*x + a)^4 + b*c*d^2 - 2*(b*d^3*x + b*c*d^2)*c
osh(b*x + a)^2 - 2*(b*d^3*x + b*c*d^2 - 3*(b*d^3*x + b*c*d^2)*cosh(b*x + a)
^2)*sinh(b*x + a)^2 + 4*((b*d^3*x + b*c*d^2)*cosh(b*x + a)^3 - (b*d^3*x + b
*c*d^2)*cosh(b*x + a))*sinh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(b*x +
 a)) + 6*(b*d^3*x + (b*d^3*x + b*c*d^2)*cosh(b*x + a)^4 + 4*(b*d^3*x + b*c*
d^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*d^3*x + b*c*d^2)*sinh(b*x + a)^4 +
b*c*d^2 - 2*(b*d^3*x + b*c*d^2)*cosh(b*x + a)^2 - 2*(b*d^3*x + b*c*d^2 - 3*
(b*d^3*x + b*c*d^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4*((b*d^3*x + b*c*d^
2)*cosh(b*x + a)^3 - (b*d^3*x + b*c*d^2)*cosh(b*x + a))*sinh(b*x + a))*poly
log(3, -cosh(b*x + a) - sinh(b*x + a)) + 2*(b^3*d^3*x^3 + b^3*c^3 - 3*b^2*c
^2*d + 3*(b^3*c*d^2 - b^2*d^3)*x^2 + 3*(b^3*d^3*x^3 + b^3*c^3 + 3*b^2*c^2*d
 + 3*(b^3*c*d^2 + b^2*d^3)*x^2 + 3*(b^3*c^2*d + 2*b^2*c*d^2)*x)*cosh(b*x +
a)^2 + 3*(b^3*c^2*d - 2*b^2*c*d^2)*x)*sinh(b*x + a))/(b^4*cosh(b*x + a)^4 +
 4*b^4*cosh(b*x + a)*sinh(b*x + a)^3 + b^4*sinh(b*x + a)^4 - 2*b^4*cosh(b*x
 + a)^2 + b^4 + 2*(3*b^4*cosh(b*x + a)^2 - b^4)*sinh(b*x + a)^2 + 4*(b^4*co
sh(b*x + a)^3 - b^4*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^3 \operatorname{csch}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csch(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**3*csch(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \operatorname{csch}(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csch(b*x + a)^3, x)
```

3.34 $\int (c+dx)^2 \operatorname{csch}^3(a+bx)\,dx$

Optimal. Leaf size=154

$$\frac{d(c+dx)\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{d(c+dx)\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}+\frac{d^2\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{d(c+\cdots}{}$$

```
[Out] ((c + d*x)^2*ArcTanh[E^(a + b*x)])/b - (d^2*ArcTanh[Cosh[a + b*x]])/b^3 - (
d*(c + d*x)*Csch[a + b*x])/b^2 - ((c + d*x)^2*Coth[a + b*x]*Csch[a + b*x])/
(2*b) + (d*(c + d*x)*PolyLog[2, -E^(a + b*x)])/b^2 - (d*(c + d*x)*PolyLog[2
, E^(a + b*x)])/b^2 - (d^2*PolyLog[3, -E^(a + b*x)])/b^3 + (d^2*PolyLog[3,
E^(a + b*x)])/b^3
```

Rubi [A] time = 0.165645, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4186, 3770, 4182, 2531, 2282, 6589}

$$\frac{d(c+dx)\text{PolyLog}\left(2,-e^{a+bx}\right)}{b^2}-\frac{d(c+dx)\text{PolyLog}\left(2,e^{a+bx}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^3}+\frac{d^2\text{PolyLog}\left(3,e^{a+bx}\right)}{b^3}-\frac{d(c+\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csch[a + b*x]^3,x]
```

```
[Out] ((c + d*x)^2*ArcTanh[E^(a + b*x)])/b - (d^2*ArcTanh[Cosh[a + b*x]])/b^3 - (
d*(c + d*x)*Csch[a + b*x])/b^2 - ((c + d*x)^2*Coth[a + b*x]*Csch[a + b*x])/
(2*b) + (d*(c + d*x)*PolyLog[2, -E^(a + b*x)])/b^2 - (d*(c + d*x)*PolyLog[2
, E^(a + b*x)])/b^2 - (d^2*PolyLog[3, -E^(a + b*x)])/b^3 + (d^2*PolyLog[3,
E^(a + b*x)])/b^3
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\text{csch}^3(a+bx)\,dx = -\frac{d(c+dx)\text{csch}(a+bx)}{b^2} - \frac{(c+dx)^2\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{1}{2}\int (c+dx)^2\text{csch}(a$$

$$= \frac{(c+dx)^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{d^2\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{d(c+dx)\text{csch}(a+bx)}{b^2} - (c+d$$

$$= \frac{(c+dx)^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{d^2\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{d(c+dx)\text{csch}(a+bx)}{b^2} - (c+d$$

$$= \frac{(c+dx)^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{d^2\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{d(c+dx)\text{csch}(a+bx)}{b^2} - (c+d$$

$$= \frac{(c+dx)^2\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{d^2\tanh^{-1}(\cosh(a+bx))}{b^3} - \frac{d(c+dx)\text{csch}(a+bx)}{b^2} - (c+d$$

Mathematica [B] time = 10.7472, size = 420, normalized size = 2.73

$$2bd(c+dx)\text{PolyLog}\left(2,-e^{a+bx}\right) - 2bd(c+dx)\text{PolyLog}\left(2,e^{a+bx}\right) - 2d^2\text{PolyLog}\left(3,-e^{a+bx}\right) + 2d^2\text{PolyLog}\left(3,e^{a+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csch[a + b*x]^3,x]
```

```
[Out] -((d*(c + d*x)*Csch[a])/b^2) + ((-c^2 - 2*c*d*x - d^2*x^2)*Csch[a/2 + (b*x)
/2]^2)/(8*b) + (-(b^2*c^2*Log[1 - E^(a + b*x)]) + 2*d^2*Log[1 - E^(a + b*x)
] - 2*b^2*c*d*x*Log[1 - E^(a + b*x)] - b^2*d^2*x^2*Log[1 - E^(a + b*x)] + b
^2*c^2*Log[1 + E^(a + b*x)] - 2*d^2*Log[1 + E^(a + b*x)] + 2*b^2*c*d*x*Log[
1 + E^(a + b*x)] + b^2*d^2*x^2*Log[1 + E^(a + b*x)] + 2*b*d*(c + d*x)*PolyL
og[2, -E^(a + b*x)] - 2*b*d*(c + d*x)*PolyLog[2, E^(a + b*x)] - 2*d^2*PolyL
og[3, -E^(a + b*x)] + 2*d^2*PolyLog[3, E^(a + b*x)])/(2*b^3) + ((-c^2 - 2*c
*d*x - d^2*x^2)*Sech[a/2 + (b*x)/2]^2)/(8*b) + (Csch[a/2]*Csch[a/2 + (b*x)/
2]*(c*d*Sinh[(b*x)/2] + d^2*x*Sinh[(b*x)/2]))/(2*b^2) + (Sech[a/2]*Sech[a/2
 + (b*x)/2]*(c*d*Sinh[(b*x)/2] + d^2*x*Sinh[(b*x)/2]))/(2*b^2)
```

Maple [B] time = 0.044, size = 444, normalized size = 2.9

$$-\frac{e^{bx+a}\left(bd^2x^2e^{2bx+2a}+2bcdxe^{2bx+2a}+bc^2e^{2bx+2a}+bd^2x^2+2d^2xe^{2bx+2a}+2bcdx+2cde^{2bx+2a}+bc^2-2d^2x-2cd\right)}{b^2\left(e^{2bx+2a}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csch(b*x+a)^3,x)
```

```
[Out] -exp(b*x+a)*(b*d^2*x^2*exp(2*b*x+2*a)+2*b*c*d*x*exp(2*b*x+2*a)+b*c^2*exp(2*
b*x+2*a)+b*d^2*x^2+2*d^2*x*exp(2*b*x+2*a)+2*b*c*d*x+2*c*d*exp(2*b*x+2*a)+b*
c^2-2*d^2*x-2*c*d)/b^2/(exp(2*b*x+2*a)-1)^2-d^2*polylog(3,-exp(b*x+a))/b^3+
d^2*polylog(3,exp(b*x+a))/b^3-2/b^3*d^2*arctanh(exp(b*x+a))+1/b*c*d*ln(1+ex
p(b*x+a))*x+1/b^2*c*d*ln(1+exp(b*x+a))*a-1/b*c*d*ln(1-exp(b*x+a))*x-1/b^2*c
*d*ln(1-exp(b*x+a))*a-2/b^2*c*d*a*arctanh(exp(b*x+a))+1/2/b*d^2*ln(1+exp(b*
x+a))*x^2+1/b^2*d^2*polylog(2,-exp(b*x+a))*x-1/2/b*d^2*ln(1-exp(b*x+a))*x^2
-1/b^2*d^2*polylog(2,exp(b*x+a))*x+1/b^2*c*d*polylog(2,-exp(b*x+a))-1/b^2*c
*d*polylog(2,exp(b*x+a))+1/b^3*d^2*a^2*arctanh(exp(b*x+a))+1/b*c^2*arctanh(
exp(b*x+a))-1/2/b^3*d^2*ln(1+exp(b*x+a))*a^2+1/2/b^3*d^2*ln(1-exp(b*x+a))*a
^2
```

Maxima [B] time = 1.84349, size = 531, normalized size = 3.45

$$\frac{1}{2}c^2\left(\frac{\log\left(e^{(-bx-a)}+1\right)}{b}-\frac{\log\left(e^{(-bx-a)}-1\right)}{b}+\frac{2\left(e^{(-bx-a)}+e^{(-3bx-3a)}\right)}{b\left(2e^{(-2bx-2a)}-e^{(-4bx-4a)}-1\right)}\right)+\frac{\left(bx\log\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}\right)\right)cd}{b^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*(log(e^(-b*x - a) + 1)/b - log(e^(-b*x - a) - 1)/b + 2*(e^(-b*x - a
) + e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))) + (b
*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))*c*d/b^2 - (b*x*log(-e^(b*x +
 a) + 1) + dilog(e^(b*x + a)))*c*d/b^2 - ((b*d^2*x^2*e^(3*a) + 2*c*d*e^(3*a
) + 2*(b*c*d + d^2)*x*e^(3*a))*e^(3*b*x) + (b*d^2*x^2*e^a - 2*c*d*e^a + 2*(
b*c*d - d^2)*x*e^a)*e^(b*x))/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) +
 b^2) + 1/2*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*p
olylog(3, -e^(b*x + a)))*d^2/b^3 - 1/2*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b
*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))*d^2/b^3 - d^2*log(e^(b*x
 + a) + 1)/b^3 + d^2*log(e^(b*x + a) - 1)/b^3
```

Fricas [C] time = 3.16852, size = 5252, normalized size = 34.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d + b*d^2)*x)*cosh(b*x
+ a)^3 + 6*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d + b*d^2)*x)*cosh(b
*x + a)*sinh(b*x + a)^2 + 2*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d +
 b*d^2)*x)*sinh(b*x + a)^3 + 2*(b^2*d^2*x^2 + b^2*c^2 - 2*b*c*d + 2*(b^2*c*
```

```
d - b*d^2)*x)*cosh(b*x + a) + 2*((b*d^2*x + b*c*d)*cosh(b*x + a)^4 + 4*(b*d
^2*x + b*c*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*d^2*x + b*c*d)*sinh(b*x +
a)^4 + b*d^2*x + b*c*d - 2*(b*d^2*x + b*c*d)*cosh(b*x + a)^2 - 2*(b*d^2*x +
 b*c*d - 3*(b*d^2*x + b*c*d)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4*((b*d^2*x
 + b*c*d)*cosh(b*x + a)^3 - (b*d^2*x + b*c*d)*cosh(b*x + a))*sinh(b*x + a))
*dilog(cosh(b*x + a) + sinh(b*x + a)) - 2*((b*d^2*x + b*c*d)*cosh(b*x + a)^
4 + 4*(b*d^2*x + b*c*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*d^2*x + b*c*d)*s
inh(b*x + a)^4 + b*d^2*x + b*c*d - 2*(b*d^2*x + b*c*d)*cosh(b*x + a)^2 - 2*
(b*d^2*x + b*c*d - 3*(b*d^2*x + b*c*d)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4
*((b*d^2*x + b*c*d)*cosh(b*x + a)^3 - (b*d^2*x + b*c*d)*cosh(b*x + a))*sinh
(b*x + a))*dilog(-cosh(b*x + a) - sinh(b*x + a)) - (b^2*d^2*x^2 + 2*b^2*c*d
*x + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cosh(b*x + a)^4 + 4*(b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sinh(b*x + a)^4 + b^2*c^2 - 2*
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cosh(b*x + a)^2 - 2*(b^2*d^2*
x^2 + 2*b^2*c*d*x + b^2*c^2 - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^
2)*cosh(b*x + a)^2 - 2*d^2)*sinh(b*x + a)^2 - 2*d^2 + 4*((b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2 - 2*d^2)*cosh(b*x + a)^3 - (b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2 - 2*d^2)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x
 + a) + 1) + ((b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cosh(b*x + a)^4 + 4*(b^
2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*c^2
 - 2*a*b*c*d + (a^2 - 2)*d^2)*sinh(b*x + a)^4 + b^2*c^2 - 2*a*b*c*d + (a^2
- 2)*d^2 - 2*(b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cosh(b*x + a)^2 - 2*(b^2
*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2 - 3*(b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*
cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4*((b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)
*cosh(b*x + a)^3 - (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cosh(b*x + a))*sin
h(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (b^2*d^2*x^2 + 2*b^2*c
*d*x + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cosh(b*x + a)^4 +
4*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cosh(b*x + a)*sinh(b*x
+ a)^3 + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*sinh(b*x + a)^4
+ 2*a*b*c*d - a^2*d^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)
*cosh(b*x + a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - 3*(
b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cosh(b*x + a)^2)*sinh(b*x
+ a)^2 + 4*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cosh(b*x + a)
^3 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cosh(b*x + a))*sinh(
b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) - 2*(d^2*cosh(b*x + a)^4
+ 4*d^2*cosh(b*x + a)*sinh(b*x + a)^3 + d^2*sinh(b*x + a)^4 - 2*d^2*cosh(b*
x + a)^2 + 2*(3*d^2*cosh(b*x + a)^2 - d^2)*sinh(b*x + a)^2 + d^2 + 4*(d^2*c
osh(b*x + a)^3 - d^2*cosh(b*x + a))*sinh(b*x + a))*polylog(3, cosh(b*x + a)
 + sinh(b*x + a)) + 2*(d^2*cosh(b*x + a)^4 + 4*d^2*cosh(b*x + a)*sinh(b*x +
 a)^3 + d^2*sinh(b*x + a)^4 - 2*d^2*cosh(b*x + a)^2 + 2*(3*d^2*cosh(b*x + a
)^2 - d^2)*sinh(b*x + a)^2 + d^2 + 4*(d^2*cosh(b*x + a)^3 - d^2*cosh(b*x +
a))*sinh(b*x + a))*polylog(3, -cosh(b*x + a) - sinh(b*x + a)) + 2*(b^2*d^2*
x^2 + b^2*c^2 - 2*b*c*d + 3*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d +
 b*d^2)*x)*cosh(b*x + a)^2 + 2*(b^2*c*d - b*d^2)*x)*sinh(b*x + a))/(b^3*cos
h(b*x + a)^4 + 4*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 -
2*b^3*cosh(b*x + a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 - b^3)*sinh(b*x + a)
^2 + 4*(b^3*cosh(b*x + a)^3 - b^3*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \operatorname{csch}^3(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csch(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**2*csch(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \operatorname{csch}(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csch(b*x + a)^3, x)
```

3.35 $\int (c+dx)\text{csch}^3(a+bx)\,dx$

Optimal. Leaf size=92

$$\frac{d\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}-\frac{d\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}-\frac{d\text{csch}(a+bx)}{2b^2}+\frac{(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{(c+dx)\coth(a+bx)\text{csc}}{2b}$$

```
[Out] ((c + d*x)*ArcTanh[E^(a + b*x)])/b - (d*Csch[a + b*x])/(2*b^2) - ((c + d*x)
*Coth[a + b*x]*Csch[a + b*x])/(2*b) + (d*PolyLog[2, -E^(a + b*x)])/(2*b^2)
- (d*PolyLog[2, E^(a + b*x)])/(2*b^2)
```

Rubi [A] time = 0.0813309, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4185, 4182, 2279, 2391}

$$\frac{d\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b^2}-\frac{d\text{PolyLog}\left(2,e^{a+bx}\right)}{2b^2}-\frac{d\text{csch}(a+bx)}{2b^2}+\frac{(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b}-\frac{(c+dx)\coth(a+bx)\text{csc}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csch[a + b*x]^3,x]
```


```
[Out] ((c + d*x)*ArcTanh[E^(a + b*x)])/b - (d*Csch[a + b*x])/(2*b^2) - ((c + d*x)
*Coth[a + b*x]*Csch[a + b*x])/(2*b) + (d*PolyLog[2, -E^(a + b*x)])/(2*b^2)
- (d*PolyLog[2, E^(a + b*x)])/(2*b^2)
```


Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)\text{csch}^3(a+bx)\,dx = -\frac{d\text{csch}(a+bx)}{2b^2} - \frac{(c+dx)\coth(a+bx)\text{csch}(a+bx)}{2b} - \frac{1}{2}\int (c+dx)\text{csch}(a+bx)\,dx$$
$$= \frac{(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{d\text{csch}(a+bx)}{2b^2} - \frac{(c+dx)\coth(a+bx)\text{csch}(a+bx)}{2b} + \frac{d\int \log(1$$
$$= \frac{(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{d\text{csch}(a+bx)}{2b^2} - \frac{(c+dx)\coth(a+bx)\text{csch}(a+bx)}{2b} + \frac{d\,\text{Subst}\,(}{}$$
$$= \frac{(c+dx)\tanh^{-1}\left(e^{a+bx}\right)}{b} - \frac{d\text{csch}(a+bx)}{2b^2} - \frac{(c+dx)\coth(a+bx)\text{csch}(a+bx)}{2b} + \frac{d\text{Li}_2\left(-e^{a}\right.}{2b^2}$$

Mathematica [C] time = 2.67384, size = 313, normalized size = 3.4

$$-\frac{d\left(-a\log\left(\tanh\left(\frac{1}{2}(a+bx)\right)\right)-i\left(i\left(\text{PolyLog}\left(2,-e^{i(ia+ibx)}\right)-\text{PolyLog}\left(2,e^{i(ia+ibx)}\right)\right)+(ia+ibx)\left(\log\left(1-e^{i(ia+ibx)}\right)-\text{lo}\right.\right.\right.}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csch[a + b*x]^3,x]
```

```
[Out] -(d*x*Csch[a/2 + (b*x)/2]^2)/(8*b) - (c*Csch[(a + b*x)/2]^2)/(8*b) - (c*Log
[Tanh[(a + b*x)/2]])/(2*b) - (d*(-(a*Log[Tanh[(a + b*x)/2]]) - I*((I*a + I*
b*x)*(Log[1 - E^(I*(I*a + I*b*x))] - Log[1 + E^(I*(I*a + I*b*x))]) + I*(Pol
yLog[2, -E^(I*(I*a + I*b*x))] - PolyLog[2, E^(I*(I*a + I*b*x))]))))/(2*b^2)
 - (d*x*Sech[a/2 + (b*x)/2]^2)/(8*b) - (c*Sech[(a + b*x)/2]^2)/(8*b) + (d*C
sch[a/2]*Csch[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(4*b^2) + (d*Sech[a/2]*Sech[a/2
 + (b*x)/2]*Sinh[(b*x)/2])/(4*b^2)
```

Maple [B] time = 0.033, size = 197, normalized size = 2.1

$$-\frac{e^{bx+a}\left(bdxe^{2bx+2a}+bce^{2bx+2a}+bdx+de^{2bx+2a}+cb-d\right)}{b^2\left(e^{2bx+2a}-1\right)^2}+\frac{c\text{Artanh}\left(e^{bx+a}\right)}{b}+\frac{d\ln\left(1+e^{bx+a}\right)x}{2b}+\frac{d\ln\left(1+e^{bx+a}\right)a}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csch(b*x+a)^3,x)
```

```
[Out] -exp(b*x+a)*(b*d*x*exp(2*b*x+2*a)+b*c*exp(2*b*x+2*a)+b*d*x+d*exp(2*b*x+2*a)
+c*b-d)/b^2/(exp(2*b*x+2*a)-1)^2+1/b*c*arctanh(exp(b*x+a))+1/2/b*d*ln(1+exp
(b*x+a))*x+1/2/b^2*d*ln(1+exp(b*x+a))*a+1/2*d*polylog(2,-exp(b*x+a))/b^2-1/
2/b*d*ln(1-exp(b*x+a))*x-1/2/b^2*d*ln(1-exp(b*x+a))*a-1/2*d*polylog(2,exp(b
*x+a))/b^2-1/b^2*d*a*arctanh(exp(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-d\left(\frac{\left(bxe^{(3a)}+e^{(3a)}\right)e^{(3bx)}+\left(bxe^{a}-e^{a}\right)e^{(bx)}}{b^2e^{(4bx+4a)}-2b^2e^{(2bx+2a)}+b^2}+8\int\frac{x}{16\left(e^{(bx+a)}+1\right)}\,dx+8\int\frac{x}{16\left(e^{(bx+a)}-1\right)}\,dx\right)+\frac{1}{2}c\left(\frac{\log\left(e^{(-bx-a)}+1\right)}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -d*(((b*x*e^(3*a) + e^(3*a))*e^(3*b*x) + (b*x*e^a - e^a)*e^(b*x))/(b^2*e^(4
*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) + 8*integrate(1/16*x/(e^(b*x + a
) + 1), x) + 8*integrate(1/16*x/(e^(b*x + a) - 1), x)) + 1/2*c*(log(e^(-b*x
 - a) + 1)/b - log(e^(-b*x - a) - 1)/b + 2*(e^(-b*x - a) + e^(-3*b*x - 3*a)
)/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1)))
```

Fricas [B] time = 2.83789, size = 2677, normalized size = 29.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*d*x + b*c + d)*cosh(b*x + a)^3 + 6*(b*d*x + b*c + d)*cosh(b*x +
a)*sinh(b*x + a)^2 + 2*(b*d*x + b*c + d)*sinh(b*x + a)^3 + 2*(b*d*x + b*c -
 d)*cosh(b*x + a) + (d*cosh(b*x + a)^4 + 4*d*cosh(b*x + a)*sinh(b*x + a)^3
+ d*sinh(b*x + a)^4 - 2*d*cosh(b*x + a)^2 + 2*(3*d*cosh(b*x + a)^2 - d)*sin
h(b*x + a)^2 + 4*(d*cosh(b*x + a)^3 - d*cosh(b*x + a))*sinh(b*x + a) + d)*d
ilog(cosh(b*x + a) + sinh(b*x + a)) - (d*cosh(b*x + a)^4 + 4*d*cosh(b*x + a
)*sinh(b*x + a)^3 + d*sinh(b*x + a)^4 - 2*d*cosh(b*x + a)^2 + 2*(3*d*cosh(b
*x + a)^2 - d)*sinh(b*x + a)^2 + 4*(d*cosh(b*x + a)^3 - d*cosh(b*x + a))*si
nh(b*x + a) + d)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - ((b*d*x + b*c)*cos
h(b*x + a)^4 + 4*(b*d*x + b*c)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*d*x + b*c
)*sinh(b*x + a)^4 + b*d*x - 2*(b*d*x + b*c)*cosh(b*x + a)^2 - 2*(b*d*x - 3*
(b*d*x + b*c)*cosh(b*x + a)^2 + b*c)*sinh(b*x + a)^2 + b*c + 4*((b*d*x + b*
c)*cosh(b*x + a)^3 - (b*d*x + b*c)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b
*x + a) + sinh(b*x + a) + 1) + ((b*c - a*d)*cosh(b*x + a)^4 + 4*(b*c - a*d)
*cosh(b*x + a)*sinh(b*x + a)^3 + (b*c - a*d)*sinh(b*x + a)^4 - 2*(b*c - a*d
)*cosh(b*x + a)^2 + 2*(3*(b*c - a*d)*cosh(b*x + a)^2 - b*c + a*d)*sinh(b*x
+ a)^2 + b*c - a*d + 4*((b*c - a*d)*cosh(b*x + a)^3 - (b*c - a*d)*cosh(b*x
+ a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1) + ((b*d*x + a*d
)*cosh(b*x + a)^4 + 4*(b*d*x + a*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*d*x
+ a*d)*sinh(b*x + a)^4 + b*d*x - 2*(b*d*x + a*d)*cosh(b*x + a)^2 - 2*(b*d*x
 - 3*(b*d*x + a*d)*cosh(b*x + a)^2 + a*d)*sinh(b*x + a)^2 + a*d + 4*((b*d*x
 + a*d)*cosh(b*x + a)^3 - (b*d*x + a*d)*cosh(b*x + a))*sinh(b*x + a))*log(-
cosh(b*x + a) - sinh(b*x + a) + 1) + 2*(b*d*x + 3*(b*d*x + b*c + d)*cosh(b*
x + a)^2 + b*c - d)*sinh(b*x + a))/(b^2*cosh(b*x + a)^4 + 4*b^2*cosh(b*x +
a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 - 2*b^2*cosh(b*x + a)^2 + 2*(3*b^2
*cosh(b*x + a)^2 - b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^2*cosh(b*x + a)^3 - b^
2*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \operatorname{csch}^3 (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*csch(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)\operatorname{csch}(bx + a)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csch(b*x + a)^3, x)
```

3.36 $\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^3(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[Csch[a + b*x]^3/(c + d*x), x]
```

Rubi [A] time = 0.0396201, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[a + b*x]^3/(c + d*x),x]
```

```
[Out] Defer[Int][Csch[a + b*x]^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 74.2687, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[a + b*x]^3/(c + d*x),x]
```

```
[Out] Integrate[Csch[a + b*x]^3/(c + d*x), x]
```

Maple [A] time = 0.484, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(bx+a))^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(csch(b*x+a)^3/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(bdxe^{(3a)} + (bc-d)e^{(3a)}\right)e^{(3bx)} + (bdxe^{a} + (bc+d)e^{a})e^{(bx)}}{b^2d^2x^2 + 2b^2cdx + b^2c^2 + \left(b^2d^2x^2e^{(4a)} + 2b^2cdxe^{(4a)} + b^2c^2e^{(4a)}\right)e^{(4bx)} - 2\left(b^2d^2x^2e^{(2a)} + 2b^2cdxe^{(2a)} + b^2c^2e^{(2a)}\right)e^{(2bx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] -((b*d*x*e^(3*a) + (b*c - d)*e^(3*a))*e^(3*b*x) + (b*d*x*e^a + (b*c + d)*e^
a)*e^(b*x))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2*e^(4*a) + 2
*b^2*c*d*x*e^(4*a) + b^2*c^2*e^(4*a))*e^(4*b*x) - 2*(b^2*d^2*x^2*e^(2*a) +
2*b^2*c*d*x*e^(2*a) + b^2*c^2*e^(2*a))*e^(2*b*x)) - 8*integrate(1/16*(b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3*e^a + 3*b^2*c*d^2*x^2*e^a + 3*b^2*c^2*
d*x*e^a + b^2*c^3*e^a)*e^(b*x)), x) - 8*integrate(-1/16*(b^2*d^2*x^2 + 2*b^
2*c*d*x + b^2*c^2 - 2*d^2)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
 b^2*c^3 - (b^2*d^3*x^3*e^a + 3*b^2*c*d^2*x^2*e^a + 3*b^2*c^2*d*x*e^a + b^2
*c^3*e^a)*e^(b*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{csch}(bx+a)^3}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^3/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(csch(a + b*x)**3/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3/(d*x + c), x)
```

3.37 $\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable[Csch[a + b*x]^3/(c + d*x)^2, x]
```

Rubi [A] time = 0.0372557, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Csch[a + b*x]^3/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2} dx = \int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 79.6001, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] Integrate[Csch[a + b*x]^3/(c + d*x)^2, x]
```

Maple [A] time = 0.81, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(bx+a))^3}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(csch(b*x+a)^3/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(bdxe^{(3\,a)}+(bc-2\,d)e^{(3\,a)}\right)e^{(3\,bx)}+\left(bdxe^{a}+(bc+2\,d)e^{a}\right)e^{(bx)}}{b^2d^3x^3+3\,b^2cd^2x^2+3\,b^2c^2dx+b^2c^3+\left(b^2d^3x^3e^{(4\,a)}+3\,b^2cd^2x^2e^{(4\,a)}+3\,b^2c^2dxe^{(4\,a)}+b^2c^3e^{(4\,a)}\right)e^{(4\,bx)}-2\left(b^2d^3x^3e^{(2\,a)}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((b*d*x*e^(3*a) + (b*c - 2*d)*e^(3*a))*e^(3*b*x) + (b*d*x*e^a + (b*c + 2*d
)*e^a)*e^(b*x))/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 +
(b^2*d^3*x^3*e^(4*a) + 3*b^2*c*d^2*x^2*e^(4*a) + 3*b^2*c^2*d*x*e^(4*a) + b^
2*c^3*e^(4*a))*e^(4*b*x) - 2*(b^2*d^3*x^3*e^(2*a) + 3*b^2*c*d^2*x^2*e^(2*a)
 + 3*b^2*c^2*d*x*e^(2*a) + b^2*c^3*e^(2*a))*e^(2*b*x)) - 8*integrate(1/16*(
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 6*d^2)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3
 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4*e^a + 4*b^2*c
*d^3*x^3*e^a + 6*b^2*c^2*d^2*x^2*e^a + 4*b^2*c^3*d*x*e^a + b^2*c^4*e^a)*e^(
b*x)), x) - 8*integrate(-1/16*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 6*d^2)
/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c
^4 - (b^2*d^4*x^4*e^a + 4*b^2*c*d^3*x^3*e^a + 6*b^2*c^2*d^2*x^2*e^a + 4*b^2
*c^3*d*x*e^a + b^2*c^4*e^a)*e^(b*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{csch}\left(bx+a\right)^3}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}^3\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(csch(a + b*x)**3/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{csch}\left(bx+a\right)^3}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3/(d*x + c)^2, x)
```

3.38 $\int (c+dx)^{5/2}\sinh(a+bx)\,dx$

Optimal. Leaf size=171

$$-\frac{15\sqrt{\pi}d^{5/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cosh(a+bx)}{4b^3}-\frac{5d(c+dx)^{3/2}\sinh(a+bx)}{2b^2}$$

```
[Out] (15*d^2*Sqrt[c + d*x]*Cosh[a + b*x])/(4*b^3) + ((c + d*x)^(5/2)*Cosh[a + b*
x])/b - (15*d^(5/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/S
qrt[d]])/(16*b^(7/2)) - (15*d^(5/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*
Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d*(c + d*x)^(3/2)*Sinh[a + b*x])
/(2*b^2)
```

Rubi [A] time = 0.370048, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3296, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\pi}d^{5/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cosh(a+bx)}{4b^3}-\frac{5d(c+dx)^{3/2}\sinh(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Sinh[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cosh[a + b*x])/(4*b^3) + ((c + d*x)^(5/2)*Cosh[a + b*
x])/b - (15*d^(5/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/S
qrt[d]])/(16*b^(7/2)) - (15*d^(5/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*
Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d*(c + d*x)^(3/2)*Sinh[a + b*x])
/(2*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (c+dx)^{5/2}\sinh(a+bx)\,dx = \frac{(c+dx)^{5/2}\cosh(a+bx)}{b} - \frac{(5d)\int (c+dx)^{3/2}\cosh(a+bx)\,dx}{2b}$$

$$= \frac{(c+dx)^{5/2}\cosh(a+bx)}{b} - \frac{5d(c+dx)^{3/2}\sinh(a+bx)}{2b^2} + \frac{\left(15d^2\right)\int \sqrt{c+dx}\,\sinh(a+bx}{4b^2}$$

$$= \frac{15d^2\sqrt{c+dx}\cosh(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\cosh(a+bx)}{b} - \frac{5d(c+dx)^{3/2}\sinh(a+bx)}{2b^2} -$$

$$= \frac{15d^2\sqrt{c+dx}\cosh(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\cosh(a+bx)}{b} - \frac{5d(c+dx)^{3/2}\sinh(a+bx)}{2b^2} -$$

$$= \frac{15d^2\sqrt{c+dx}\cosh(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\cosh(a+bx)}{b} - \frac{5d(c+dx)^{3/2}\sinh(a+bx)}{2b^2} -$$

$$= \frac{15d^2\sqrt{c+dx}\cosh(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\cosh(a+bx)}{b} - \frac{15d^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}$$

Mathematica [A] time = 0.058734, size = 108, normalized size = 0.63

$$\frac{d^3e^{-a-\frac{bc}{d}}\left(e^{\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\text{Gamma}\left(\frac{7}{2},\frac{b(c+dx)}{d}\right)-e^{2a}\sqrt{-\frac{b(c+dx)}{d}}\text{Gamma}\left(\frac{7}{2},-\frac{b(c+dx)}{d}\right)\right)}{2b^4\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Sinh[a + b*x],x]
```

```
[Out] (d^3*E^(-a - (b*c)/d)*(-(E^(2*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[7/2, -((b*(
c + d*x))/d)]) + E^((2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[7/2, (b*(c + d*x
))/d]))/(2*b^4*Sqrt[c + d*x])
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{2}}\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*sinh(b*x+a),x)
```

```
[Out] int((d*x+c)^(5/2)*sinh(b*x+a),x)
```

Maxima [B] time = 1.22586, size = 416, normalized size = 2.43

$$\frac{32\,(dx+c)^{\frac{7}{2}}\sinh(bx+a) - \frac{\frac{105\,\sqrt{\pi}d^4\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^4\sqrt{-\frac{b}{d}}} + \frac{105\,\sqrt{\pi}d^4\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^4\sqrt{\frac{b}{d}}} - \frac{2\left(8\,(dx+c)^{\frac{7}{2}}b^3de^{\left(\frac{bc}{d}\right)}+28\,(dx+c)^{\frac{5}{2}}b^2d^2e^{\left(\frac{bc}{d}\right)}+70\,(dx+c)^{\frac{3}{2}}bd^3e^{(\cdots}}{b^4}}{d}}{112\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/112*(32*(d*x + c)^(7/2)*sinh(b*x + a) - (105*sqrt(pi)*d^4*erf(sqrt(d*x +
c)*sqrt(-b/d))*e^(a - b*c/d)/(b^4*sqrt(-b/d)) + 105*sqrt(pi)*d^4*erf(sqrt(d
*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^4*sqrt(b/d)) - 2*(8*(d*x + c)^(7/2)*b^
3*d*e^(b*c/d) + 28*(d*x + c)^(5/2)*b^2*d^2*e^(b*c/d) + 70*(d*x + c)^(3/2)*b
*d^3*e^(b*c/d) + 105*sqrt(d*x + c)*d^4*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^
4 + 2*(8*(d*x + c)^(7/2)*b^3*d*e^a - 28*(d*x + c)^(5/2)*b^2*d^2*e^a + 70*(d
*x + c)^(3/2)*b*d^3*e^a - 105*sqrt(d*x + c)*d^4*e^a)*e^((d*x + c)*b/d - b*c
/d)/b^4)*b/d)/d
```

Fricas [B] time = 2.75247, size = 1179, normalized size = 6.89

$$-\frac{15\,\sqrt{\pi}\left(d^3\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right) - d^3\cosh(bx+a)\sinh\left(-\frac{bc-ad}{d}\right) + \left(d^3\cosh\left(-\frac{bc-ad}{d}\right) - d^3\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh(b\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/16*(15*sqrt(pi)*(d^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d^3*cosh(b*x +
 a)*sinh(-(b*c - a*d)/d) + (d^3*cosh(-(b*c - a*d)/d) - d^3*sinh(-(b*c - a*d
)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 15*sqrt(pi)*(
d^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d^3*cosh(b*x + a)*sinh(-(b*c - a*d
)/d) + (d^3*cosh(-(b*c - a*d)/d) + d^3*sinh(-(b*c - a*d)/d))*sinh(b*x + a))
*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) - 2*(4*b^3*d^2*x^2 + 4*b^3*c^2 +
10*b^2*c*d + 15*b*d^2 + (4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2
+ 2*(4*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^2 + 2*(4*b^3*d^2*x^2 + 4*b^3*c
^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)*sin
h(b*x + a) + (4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*
c*d - 5*b^2*d^2)*x)*sinh(b*x + a)^2 + 2*(4*b^3*c*d + 5*b^2*d^2)*x)*sqrt(d*x
 + c))/(b^4*cosh(b*x + a) + b^4*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*sinh(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37161, size = 313, normalized size = 1.83

$$\frac{\frac{15\sqrt{\pi}d^4\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}b^3}+\frac{15\sqrt{\pi}d^4\operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}b^3}+\frac{2\left(4(dx+c)^{\frac{5}{2}}b^2d-10(dx+c)^{\frac{3}{2}}bd^2+15\sqrt{dx+c}d^3\right)e^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b^3}+\frac{2\left(4(dx+c\right.}{}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/16*(15*sqrt(pi)*d^4*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(sq
rt(b*d)*b^3) + 15*sqrt(pi)*d^4*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c -
a*d)/d)/(sqrt(-b*d)*b^3) + 2*(4*(d*x + c)^(5/2)*b^2*d - 10*(d*x + c)^(3/2)*
b*d^2 + 15*sqrt(d*x + c)*d^3)*e^(((d*x + c)*b - b*c + a*d)/d)/b^3 + 2*(4*(d
*x + c)^(5/2)*b^2*d + 10*(d*x + c)^(3/2)*b*d^2 + 15*sqrt(d*x + c)*d^3)*e^(-
((d*x + c)*b - b*c + a*d)/d)/b^3)/d
```

3.39 $\int (c+dx)^{3/2}\sinh(a+bx)\,dx$

Optimal. Leaf size=146

$$-\frac{3\sqrt{\pi}d^{3/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{3\sqrt{\pi}d^{3/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3d\sqrt{c+dx}\sinh(a+bx)}{2b^2}+\frac{(c+dx)^{3/2}\cosh(a+bx)}{b}$$

```
[Out] ((c + d*x)^(3/2)*Cosh[a + b*x])/b - (3*d^(3/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Er
f[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (3*d^(3/2)*E^(a - (b*c)/d
)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (3*d*Sqrt[c
 + d*x]*Sinh[a + b*x])/(2*b^2)
```

Rubi [A] time = 0.248011, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3296, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\pi}d^{3/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{3\sqrt{\pi}d^{3/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3d\sqrt{c+dx}\sinh(a+bx)}{2b^2}+\frac{(c+dx)^{3/2}\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Sinh[a + b*x],x]
```

```
[Out] ((c + d*x)^(3/2)*Cosh[a + b*x])/b - (3*d^(3/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Er
f[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (3*d^(3/2)*E^(a - (b*c)/d
)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (3*d*Sqrt[c
 + d*x]*Sinh[a + b*x])/(2*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (c+dx)^{3/2}\sinh(a+bx)\,dx = \frac{(c+dx)^{3/2}\cosh(a+bx)}{b} - \frac{(3d)\int \sqrt{c+dx}\cosh(a+bx)\,dx}{2b}$$

$$= \frac{(c+dx)^{3/2}\cosh(a+bx)}{b} - \frac{3d\sqrt{c+dx}\sinh(a+bx)}{2b^2} + \frac{\left(3d^2\right)\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx}{4b^2}$$

$$= \frac{(c+dx)^{3/2}\cosh(a+bx)}{b} - \frac{3d\sqrt{c+dx}\sinh(a+bx)}{2b^2} + \frac{\left(3d^2\right)\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{8b^2} - \frac{\left(3d^2\right)\int \frac{e^{i}}{\sqrt{}}}{8b}$$

$$= \frac{(c+dx)^{3/2}\cosh(a+bx)}{b} - \frac{3d\sqrt{c+dx}\sinh(a+bx)}{2b^2} - \frac{(3d)\,\mathrm{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx, x\right.}{4b^2}$$

$$= \frac{(c+dx)^{3/2}\cosh(a+bx)}{b} - \frac{3d^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} + \frac{3d^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}$$

Mathematica [A] time = 0.0952787, size = 106, normalized size = 0.73

$$\frac{d\sqrt{c+dx}e^{-a-\frac{bc}{d}}\left(\frac{e^{\frac{2bc}{d}}\mathrm{Gamma}\left(\frac{5}{2},\frac{b(c+dx)}{d}\right)}{\sqrt{\frac{b(c+dx)}{d}}} - \frac{e^{2a}\mathrm{Gamma}\left(\frac{5}{2},-\frac{b(c+dx)}{d}\right)}{\sqrt{-\frac{b(c+dx)}{d}}}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Sinh[a + b*x],x]
```

```
[Out] (d*E^(-a - (b*c)/d)*Sqrt[c + d*x]*(-((E^(2*a)*Gamma[5/2, -((b*(c + d*x))/d)
])/Sqrt[-((b*(c + d*x))/d)]) + (E^((2*b*c)/d)*Gamma[5/2, (b*(c + d*x))/d])/
Sqrt[(b*(c + d*x))/d]))/(2*b^2)
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{3}{2}}\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*sinh(b*x+a),x)
```

```
[Out] int((d*x+c)^(3/2)*sinh(b*x+a),x)
```

Maxima [B] time = 1.19344, size = 362, normalized size = 2.48

$$\frac{16(dx+c)^{\frac{5}{2}}\sinh(bx+a) + \frac{\left(\frac{15\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}} - \frac{15\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}} + \frac{2\left(4(dx+c)^{\frac{5}{2}}b^2de^{\left(\frac{bc}{d}\right)}+10(dx+c)^{\frac{3}{2}}bd^2e^{\left(\frac{bc}{d}\right)}+15\sqrt{dx+c}d^3e^{\left(\frac{bc}{d}\right.}\right.}{b^3}}{d}}{40d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/40*(16*(d*x + c)^(5/2)*sinh(b*x + a) + (15*sqrt(pi)*d^3*erf(sqrt(d*x + c)
*sqrt(-b/d))*e^(a - b*c/d)/(b^3*sqrt(-b/d)) - 15*sqrt(pi)*d^3*erf(sqrt(d*x
+ c)*sqrt(b/d))*e^(-a + b*c/d)/(b^3*sqrt(b/d)) + 2*(4*(d*x + c)^(5/2)*b^2*d
*e^(b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(b*c/d) + 15*sqrt(d*x + c)*d^3*e^(b
*c/d))*e^(-a - (d*x + c)*b/d)/b^3 - 2*(4*(d*x + c)^(5/2)*b^2*d*e^a - 10*(d*
x + c)^(3/2)*b*d^2*e^a + 15*sqrt(d*x + c)*d^3*e^a)*e^((d*x + c)*b/d - b*c/d
)/b^3)*b/d)/d
```

Fricas [B] time = 2.64267, size = 894, normalized size = 6.12

$$-\frac{3\sqrt{\pi}\left(d^2\cosh\left(bx+a\right)\cosh\left(-\frac{bc-ad}{d}\right)-d^2\cosh\left(bx+a\right)\sinh\left(-\frac{bc-ad}{d}\right)+\left(d^2\cosh\left(-\frac{bc-ad}{d}\right)-d^2\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh\left(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(3*sqrt(pi)*(d^2*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d^2*cosh(b*x + a
)*sinh(-(b*c - a*d)/d) + (d^2*cosh(-(b*c - a*d)/d) - d^2*sinh(-(b*c - a*d)/
d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 3*sqrt(pi)*(d^2
*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x + a)*sinh(-(b*c - a*d)/d
) + (d^2*cosh(-(b*c - a*d)/d) + d^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sq
rt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) - 2*(2*b^2*d*x + 2*b^2*c + (2*b^2*d*
x + 2*b^2*c - 3*b*d)*cosh(b*x + a)^2 + 2*(2*b^2*d*x + 2*b^2*c - 3*b*d)*cosh
(b*x + a)*sinh(b*x + a) + (2*b^2*d*x + 2*b^2*c - 3*b*d)*sinh(b*x + a)^2 + 3
*b*d)*sqrt(d*x + c))/(b^3*cosh(b*x + a) + b^3*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(c+dx\right)^{\frac{3}{2}}\sinh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*sinh(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(3/2)*sinh(a + b*x), x)
```

Giac [A] time = 1.33518, size = 273, normalized size = 1.87

$$\frac{\frac{3\sqrt{\pi}d^3\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}b^2}-\frac{3\sqrt{\pi}d^3\operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}b^2}+\frac{2\left(2\left(dx+c\right)^{\frac{3}{2}}bd-3\sqrt{dx+c}d^2\right)e^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b^2}+\frac{2\left(2\left(dx+c\right)^{\frac{3}{2}}bd+3\sqrt{dx+c}d^2\right)e^{\left(-\frac{(dx+}{}\right.}}{b^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*(3*sqrt(pi)*d^3*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(sqrt
(b*d)*b^2) - 3*sqrt(pi)*d^3*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c - a*d
)/d)/(sqrt(-b*d)*b^2) + 2*(2*(d*x + c)^(3/2)*b*d - 3*sqrt(d*x + c)*d^2)*e^(
((d*x + c)*b - b*c + a*d)/d)/b^2 + 2*(2*(d*x + c)^(3/2)*b*d + 3*sqrt(d*x +
c)*d^2)*e^(-((d*x + c)*b - b*c + a*d)/d)/b^2)/d
```

3.40 $\int \sqrt{c+dx}\sinh(a+bx)\,dx$

Optimal. Leaf size=123

$$-\frac{\sqrt{\pi}\sqrt{d}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{c+dx}\cosh(a+bx)}{b}$$

```
[Out] (Sqrt[c + d*x]*Cosh[a + b*x])/b - (Sqrt[d]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(S
qrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*b^(3/2)) - (Sqrt[d]*E^(a - (b*c)/d)*Sqrt
[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*b^(3/2))
```

Rubi [A] time = 0.187407, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3296, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{d}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{c+dx}\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Sinh[a + b*x],x]
```

```
[Out] (Sqrt[c + d*x]*Cosh[a + b*x])/b - (Sqrt[d]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(S
qrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*b^(3/2)) - (Sqrt[d]*E^(a - (b*c)/d)*Sqrt
[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*b^(3/2))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sqrt{c+dx}\sinh(a+bx)\,dx = \frac{\sqrt{c+dx}\cosh(a+bx)}{b} - \frac{d\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx}{2b}$$
$$= \frac{\sqrt{c+dx}\cosh(a+bx)}{b} - \frac{d\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{4b} - \frac{d\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{4b}$$
$$= \frac{\sqrt{c+dx}\cosh(a+bx)}{b} - \frac{\text{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{2b} - \frac{\text{Subst}\left(\int e^{-i\left(ia-\frac{ibc}{d}\right)+\frac{bx^2}{d}}\right.}{2b}$$
$$= \frac{\sqrt{c+dx}\cosh(a+bx)}{b} - \frac{\sqrt{d}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{d}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}$$

Mathematica [A] time = 0.0779187, size = 104, normalized size = 0.85

$$\frac{\sqrt{c+dx}e^{-a-\frac{bc}{d}}\left(\frac{e^{2a}\text{Gamma}\left(\frac{3}{2},-\frac{b(c+dx)}{d}\right)}{\sqrt{-\frac{b(c+dx)}{d}}} + \frac{e^{\frac{2bc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{b(c+dx)}{d}\right)}{\sqrt{\frac{b(c+dx)}{d}}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Sinh[a + b*x],x]
```

```
[Out] (E^(-a - (b*c)/d)*Sqrt[c + d*x]*((E^(2*a)*Gamma[3/2, -((b*(c + d*x))/d)])/S
qrt[-((b*(c + d*x))/d)] + (E^((2*b*c)/d)*Gamma[3/2, (b*(c + d*x))/d])/Sqrt[
(b*(c + d*x))/d]))/(2*b)
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \sinh(bx+a)\sqrt{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)*(d*x+c)^(1/2),x)
```

```
[Out] int(sinh(b*x+a)*(d*x+c)^(1/2),x)
```

Maxima [B] time = 1.15584, size = 311, normalized size = 2.53

$$\frac{8(dx+c)^{\frac{3}{2}}\sinh(bx+a) - \frac{\left(\frac{3\sqrt{\pi}d^2\,\text{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} + \frac{3\sqrt{\pi}d^2\,\text{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} - \frac{2\left(2(dx+c)^{\frac{3}{2}}bde^{\left(\frac{bc}{d}\right)}+3\sqrt{dx+c}d^2e^{\left(\frac{bc}{d}\right)}\right)e^{\left(-a-\frac{(dx+c)b}{d}\right)}}{b^2} + \frac{2\left(2(dx+\right.}{}}{d}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(8*(d*x + c)^(3/2)*sinh(b*x + a) - (3*sqrt(pi)*d^2*erf(sqrt(d*x + c)*s
qrt(-b/d))*e^(a - b*c/d)/(b^2*sqrt(-b/d)) + 3*sqrt(pi)*d^2*erf(sqrt(d*x + c
)*sqrt(b/d))*e^(-a + b*c/d)/(b^2*sqrt(b/d)) - 2*(2*(d*x + c)^(3/2)*b*d*e^(b
*c/d) + 3*sqrt(d*x + c)*d^2*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^2 + 2*(2*(d
*x + c)^(3/2)*b*d*e^a - 3*sqrt(d*x + c)*d^2*e^a)*e^((d*x + c)*b/d - b*c/d)/
b^2)*b/d)/d
```

Fricas [B] time = 2.73369, size = 718, normalized size = 5.84

$$-\frac{\sqrt{\pi}\left(d\cosh\left(bx+a\right)\cosh\left(-\frac{bc-ad}{d}\right)-d\cosh\left(bx+a\right)\sinh\left(-\frac{bc-ad}{d}\right)+\left(d\cosh\left(-\frac{bc-ad}{d}\right)-d\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh\left(bx+a\right)\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*(d*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)*sinh
(-(b*c - a*d)/d) + (d*cosh(-(b*c - a*d)/d) - d*sinh(-(b*c - a*d)/d))*sinh(b
*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - sqrt(pi)*(d*cosh(b*x + a)
*cosh(-(b*c - a*d)/d) + d*cosh(b*x + a)*sinh(-(b*c - a*d)/d) + (d*cosh(-(b*
c - a*d)/d) + d*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*
x + c)*sqrt(-b/d)) - 2*(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a)
 + b*sinh(b*x + a)^2 + b)*sqrt(d*x + c))/(b^2*cosh(b*x + a) + b^2*sinh(b*x
+ a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c+dx}\sinh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*sinh(a + b*x), x)
```

Giac [A] time = 1.27523, size = 227, normalized size = 1.85

$$\frac{\frac{\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}b}+\frac{\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}b}+\frac{2\sqrt{dx+c}de^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b}+\frac{2\sqrt{dx+c}de^{\left(-\frac{(dx+c)b-bc+ad}{d}\right)}}{b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(sqrt(b
*d)*b) + sqrt(pi)*d^2*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c - a*d)/d)/(
sqrt(-b*d)*b) + 2*sqrt(d*x + c)*d*e^(((d*x + c)*b - b*c + a*d)/d)/b + 2*sqr
t(d*x + c)*d*e^(-((d*x + c)*b - b*c + a*d)/d)/b)/d
```

3.41 $\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=104

$$\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

```
[Out] -(E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b
]*Sqrt[d]) + (E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]
])/(2*Sqrt[b]*Sqrt[d])
```

Rubi [A] time = 0.134614, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] -(E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b
]*Sqrt[d]) + (E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]
])/(2*Sqrt[b]*Sqrt[d])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx = \frac{1}{2}\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx - \frac{1}{2}\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx$$
$$= -\frac{\text{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d} + \frac{\text{Subst}\left(\int e^{-i\left(ia-\frac{ibc}{d}\right)+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d}$$
$$= -\frac{e^{-a+\frac{bc}{d}}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} + \frac{e^{a-\frac{bc}{d}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

Mathematica [A] time = 0.0376037, size = 104, normalized size = 1.

$$\frac{e^{-a-\frac{bc}{d}}\left(e^{2a}\sqrt{-\frac{b(c+dx)}{d}}\text{Gamma}\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)+e^{\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\text{Gamma}\left(\frac{1}{2},\frac{b(c+dx)}{d}\right)\right)}{2b\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] (E^(-a - (b*c)/d)*(E^(2*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*
x))/d)] + E^((2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (b*(c + d*x))/d]))
/(2*b*Sqrt[c + d*x])
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int \sinh(bx+a)\frac{1}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)/(d*x+c)^(1/2),x)
```

```
[Out] int(sinh(b*x+a)/(d*x+c)^(1/2),x)
```

Maxima [B] time = 1.15108, size = 244, normalized size = 2.35

$$\frac{4\sqrt{dx+c}\sinh(bx+a)+\frac{\left(\frac{\sqrt{\pi}d\,\text{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}}-\frac{\sqrt{\pi}d\,\text{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b\sqrt{\frac{b}{d}}}-\frac{2\sqrt{dx+c}de^{\left(a+\frac{(dx+c)b}{d}-\frac{bc}{d}\right)}}{b}+\frac{2\sqrt{dx+c}de^{\left(-a-\frac{(dx+c)b}{d}+\frac{bc}{d}\right)}}{b}\right)b}{d}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*sqrt(d*x + c)*sinh(b*x + a) + (sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(-b/
d))*e^(a - b*c/d)/(b*sqrt(-b/d)) - sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(b/d))*
e^(-a + b*c/d)/(b*sqrt(b/d)) - 2*sqrt(d*x + c)*d*e^(a + (d*x + c)*b/d - b*c
/d)/b + 2*sqrt(d*x + c)*d*e^(-a - (d*x + c)*b/d + b*c/d)/b)*b/d)/d
```

Fricas [A] time = 2.62937, size = 273, normalized size = 2.62

$$-\frac{\sqrt{\pi}\sqrt{\frac{b}{d}}\left(\cosh\left(-\frac{bc-ad}{d}\right)-\sinh\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)+\sqrt{\pi}\sqrt{-\frac{b}{d}}\left(\cosh\left(-\frac{bc-ad}{d}\right)+\sinh\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(pi)*sqrt(b/d)*(cosh(-(b*c - a*d)/d) - sinh(-(b*c - a*d)/d))*erf(
sqrt(d*x + c)*sqrt(b/d)) + sqrt(pi)*sqrt(-b/d)*(cosh(-(b*c - a*d)/d) + sinh
(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-b/d)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(sinh(a + b*x)/sqrt(c + d*x), x)
```

Giac [A] time = 1.21713, size = 124, normalized size = 1.19

$$\frac{\frac{\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}}-\frac{\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/sqrt(b*d)
 - sqrt(pi)*d*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c - a*d)/d)/sqrt(-b*d
))/d
```

3.42 $\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{\sqrt{\pi}\sqrt{b}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{\pi}\sqrt{b}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sinh(a+bx)}{d\sqrt{c+dx}}$$

```
[Out] (Sqrt[b]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/d^
(3/2) + (Sqrt[b]*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]])/d^(3/2) - (2*Sinh[a + b*x])/(d*Sqrt[c + d*x])
```

Rubi [A] time = 0.200403, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3297, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\sqrt{b}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{\pi}\sqrt{b}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sinh(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (Sqrt[b]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/d^
(3/2) + (Sqrt[b]*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]])/d^(3/2) - (2*Sinh[a + b*x])/(d*Sqrt[c + d*x])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx &= -\frac{2\sinh(a+bx)}{d\sqrt{c+dx}}+\frac{(2b)\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx}{d}\\ &= -\frac{2\sinh(a+bx)}{d\sqrt{c+dx}}+\frac{b\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{d}+\frac{b\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{d}\\ &= -\frac{2\sinh(a+bx)}{d\sqrt{c+dx}}+\frac{(2b)\,\mathrm{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d^2}+\frac{(2b)\,\mathrm{Subst}\left(\int e^{-i\left(ia-\frac{ibc}{d}\right)+\frac{bx^2}{d}}\,dx,\right.}{d^2}\\ &= \frac{\sqrt{b}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{b}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sinh(a+bx)}{d\sqrt{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.148855, size = 120, normalized size = 1.02

$$\frac{e^{-a-\frac{bc}{d}}\left(e^{2a}\sqrt{-\frac{b(c+dx)}{d}}\mathrm{Gamma}\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)-e^{\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\mathrm{Gamma}\left(\frac{1}{2},\frac{b(c+dx)}{d}\right)-2e^{a+\frac{bc}{d}}\sinh(a+bx)\right)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (E^(-a - (b*c)/d)*(E^(2*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*
x))/d)] - E^((2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (b*(c + d*x))/d] -
 2*E^(a + (b*c)/d)*Sinh[a + b*x]))/(d*Sqrt[c + d*x])
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \sinh(bx+a)\,(dx+c)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)/(d*x+c)^(3/2),x)
```

```
[Out] int(sinh(b*x+a)/(d*x+c)^(3/2),x)
```

Maxima [A] time = 1.11129, size = 139, normalized size = 1.18

$$\frac{\frac{\left(\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{\sqrt{\frac{b}{d}}}\right)b}{d}-\frac{2\sinh(bx+a)}{\sqrt{dx+c}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] ((sqrt(pi)*erf(sqrt(d*x + c)*sqrt(-b/d))*e^(a - b*c/d)/sqrt(-b/d) + sqrt(pi
)*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/sqrt(b/d))*b/d - 2*sinh(b*x +
 a)/sqrt(d*x + c))/d
```

Fricas [B] time = 2.76847, size = 814, normalized size = 6.9

$$\frac{\sqrt{\pi}\left((dx+c)\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right)-(dx+c)\cosh(bx+a)\sinh\left(-\frac{bc-ad}{d}\right)+\left((dx+c)\cosh\left(-\frac{bc-ad}{d}\right)-(dx+c)\sinh\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(pi)*((d*x + c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (d*x + c)*cosh(b*
x + a)*sinh(-(b*c - a*d)/d) + ((d*x + c)*cosh(-(b*c - a*d)/d) - (d*x + c)*s
inh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d))
- sqrt(pi)*((d*x + c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + (d*x + c)*cosh(b
*x + a)*sinh(-(b*c - a*d)/d) + ((d*x + c)*cosh(-(b*c - a*d)/d) + (d*x + c)*
sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d
)) - sqrt(d*x + c)*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(
b*x + a)^2 - 1))/((d^2*x + c*d)*cosh(b*x + a) + (d^2*x + c*d)*sinh(b*x + a)
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(a+bx)}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(sinh(a + b*x)/(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx+a)}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)/(d*x + c)^(3/2), x)
```

3.43 $\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{2\sqrt{\pi}b^{3/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2\sqrt{\pi}b^{3/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4b\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh(a+bx)}{3d(c+dx)^{3/2}}$$

```
[Out] (-4*b*Cosh[a + b*x])/(3*d^2*Sqrt[c + d*x]) - (2*b^(3/2)*E^(-a + (b*c)/d)*Sq
rt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) + (2*b^(3/2)*E^(a
- (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) - (2
*Sinh[a + b*x])/(3*d*(c + d*x)^(3/2))
```

Rubi [A] time = 0.250753, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3297, 3308, 2180, 2204, 2205}

$$-\frac{2\sqrt{\pi}b^{3/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2\sqrt{\pi}b^{3/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4b\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (-4*b*Cosh[a + b*x])/(3*d^2*Sqrt[c + d*x]) - (2*b^(3/2)*E^(-a + (b*c)/d)*Sq
rt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) + (2*b^(3/2)*E^(a
- (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) - (2
*Sinh[a + b*x])/(3*d*(c + d*x)^(3/2))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{2\sinh(a+bx)}{3d(c+dx)^{3/2}} + \frac{(2b)\int \frac{\cosh(a+bx)}{(c+dx)^{3/2}}\,dx}{3d}$$

$$= -\frac{4b\cosh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sinh(a+bx)}{3d(c+dx)^{3/2}} + \frac{\left(4b^2\right)\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{4b\cosh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sinh(a+bx)}{3d(c+dx)^{3/2}} + \frac{\left(2b^2\right)\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{3d^2} - \frac{\left(2b^2\right)\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{4b\cosh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sinh(a+bx)}{3d(c+dx)^{3/2}} - \frac{\left(4b^2\right)\text{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{3d^3} + \frac{\left(4b^2\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{4b\cosh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{2\sinh(a+bx)}{3d(c+dx)^{3/2}}$$

Mathematica [A] time = 0.754084, size = 161, normalized size = 1.08

$$\frac{2b\left(\frac{e^a\left(e^{-\frac{bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\text{Gamma}\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)-e^{bx}\right)}{d\sqrt{c+dx}}+\frac{e^{-a-bx}\left(e^{b\left(\frac{c}{d}+x\right)}\sqrt{\frac{b(c+dx)}{d}}\,\text{Gamma}\left(\frac{1}{2},\frac{b(c+dx)}{d}\right)-1\right)}{d\sqrt{c+dx}}\right)}{3d} - \frac{2\sinh(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (2*b*((E^a*(-E^(b*x) + (Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*x))
/d)])/E^((b*c)/d)))/(d*Sqrt[c + d*x]) + (E^(-a - b*x)*(-1 + E^(b*(c/d + x))
*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (b*(c + d*x))/d]))/(d*Sqrt[c + d*x])))/(3
*d) - (2*Sinh[a + b*x])/(3*d*(c + d*x)^(3/2))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \sinh(bx+a)\,(dx+c)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)/(d*x+c)^(5/2),x)
```

```
[Out] int(sinh(b*x+a)/(d*x+c)^(5/2),x)
```

Maxima [A] time = 1.33558, size = 154, normalized size = 1.03

$$-\frac{\frac{\left(\frac{\sqrt{\frac{(dx+c)b}{d}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{\sqrt{-\frac{(dx+c)b}{d}}e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}}\right)b}{d}+\frac{2\sinh(bx+a)}{(dx+c)^{\frac{3}{2}}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*((sqrt((d*x + c)*b/d)*e^(-a + b*c/d)*gamma(-1/2, (d*x + c)*b/d)/sqrt(d
*x + c) + sqrt(-(d*x + c)*b/d)*e^(a - b*c/d)*gamma(-1/2, -(d*x + c)*b/d)/sq
rt(d*x + c))*b/d + 2*sinh(b*x + a)/(d*x + c)^(3/2))/d
```

Fricas [B] time = 2.7866, size = 1226, normalized size = 8.23

$$-\frac{2\sqrt{\pi}\left(\left(bd^2x^2+2\,bcdx+bc^2\right)\cosh\left(bx+a\right)\cosh\left(-\frac{bc-ad}{d}\right)-\left(bd^2x^2+2\,bcdx+bc^2\right)\cosh\left(bx+a\right)\sinh\left(-\frac{bc-ad}{d}\right)+\left(\left(b\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-(b*c
- a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-(b*c - a*d)
/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-(b*c - a*d)/d) - (b*d^2*x^2 +
2*b*c*d*x + b*c^2)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(
d*x + c)*sqrt(b/d)) + 2*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x
+ a)*cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*s
inh(-(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-(b*c - a*d)/d)
 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqr
t(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) + (2*b*d*x + (2*b*d*x + 2*b*c + d)*co
sh(b*x + a)^2 + 2*(2*b*d*x + 2*b*c + d)*cosh(b*x + a)*sinh(b*x + a) + (2*b*
d*x + 2*b*c + d)*sinh(b*x + a)^2 + 2*b*c - d)*sqrt(d*x + c))/((d^4*x^2 + 2*
c*d^3*x + c^2*d^2)*cosh(b*x + a) + (d^4*x^2 + 2*c*d^3*x + c^2*d^2)*sinh(b*x
 + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh\left(a+bx\right)}{\left(c+dx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(sinh(a + b*x)/(c + d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx + a)}{(dx + c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)/(d*x + c)^(5/2), x)
```

3.44 $\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal. Leaf size=174

$$\frac{4\sqrt{\pi}b^{5/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{4\sqrt{\pi}b^{5/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}-\frac{8b^2\sinh(a+bx)}{15d^3\sqrt{c+dx}}-\frac{4b\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}}$$

```
[Out] (-4*b*Cosh[a + b*x])/(15*d^2*(c + d*x)^(3/2)) + (4*b^(5/2)*E^(-a + (b*c)/d)
*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7/2)) + (4*b^(5/2)*E
^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7/2))
 - (2*Sinh[a + b*x])/(5*d*(c + d*x)^(5/2)) - (8*b^2*Sinh[a + b*x])/(15*d^3*
Sqrt[c + d*x])
```

Rubi [A] time = 0.308154, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3297, 3307, 2180, 2204, 2205}

$$\frac{4\sqrt{\pi}b^{5/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{4\sqrt{\pi}b^{5/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}-\frac{8b^2\sinh(a+bx)}{15d^3\sqrt{c+dx}}-\frac{4b\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]/(c + d*x)^(7/2),x]
```

```
[Out] (-4*b*Cosh[a + b*x])/(15*d^2*(c + d*x)^(3/2)) + (4*b^(5/2)*E^(-a + (b*c)/d)
*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7/2)) + (4*b^(5/2)*E
^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7/2))
 - (2*Sinh[a + b*x])/(5*d*(c + d*x)^(5/2)) - (8*b^2*Sinh[a + b*x])/(15*d^3*
Sqrt[c + d*x])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}} + \frac{(2b)\int \frac{\cosh(a+bx)}{(c+dx)^{5/2}}\,dx}{5d}$$

$$= -\frac{4b\cosh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}} + \frac{\left(4b^2\right)\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx}{15d^2}$$

$$= -\frac{4b\cosh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2\sinh(a+bx)}{15d^3\sqrt{c+dx}} + \frac{\left(8b^3\right)\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx}{15d^3}$$

$$= -\frac{4b\cosh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2\sinh(a+bx)}{15d^3\sqrt{c+dx}} + \frac{\left(4b^3\right)\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{15d^3} + \frac{\left(4b^3\right)\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{15d^3}$$

$$= -\frac{4b\cosh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2\sinh(a+bx)}{15d^3\sqrt{c+dx}} + \frac{\left(8b^3\right)\text{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx, x, \sqrt{c+d\right.}{15d^4}$$

$$= -\frac{4b\cosh(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{4b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{4b^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} - \frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}} -$$

Mathematica [A] time = 0.572398, size = 168, normalized size = 0.97

$$\frac{2\left(-b(c+dx)\left(e^{a-\frac{bc}{d}}\left(2d\left(-\frac{b(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)+e^{b\left(\frac{c}{d}+x\right)}(2b(c+dx)+d)\right)+e^{-a-bx}\left(2de^{b\left(\frac{c}{d}+x\right)}\left(\frac{b(c+dx)}{d}\right)^{3/2}\text{Gai}\right.\right.}{15d^3(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]/(c + d*x)^(7/2),x]
```

```
[Out] (2*(-(b*(c + d*x)*(E^(a - (b*c)/d)*(E^(b*(c/d + x))*(d + 2*b*(c + d*x)) + 2
*d*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, -((b*(c + d*x))/d)]) + E^(-a - b*x
)*(d - 2*b*(c + d*x) + 2*d*E^(b*(c/d + x))*((b*(c + d*x))/d)^(3/2)*Gamma[1/
2, (b*(c + d*x))/d]))) - 3*d^2*Sinh[a + b*x]))/(15*d^3*(c + d*x)^(5/2))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \sinh(bx+a)\,(dx+c)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)/(d*x+c)^(7/2),x)
```

```
[Out] int(sinh(b*x+a)/(d*x+c)^(7/2),x)
```

Maxima [A] time = 1.34525, size = 154, normalized size = 0.89

$$-\frac{\frac{\left(\frac{\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}+\frac{\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}\right)b}{d}+\frac{2\sinh(bx+a)}{(dx+c)^{\frac{5}{2}}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/5*((((d*x + c)*b/d)^(3/2)*e^(-a + b*c/d)*gamma(-3/2, (d*x + c)*b/d)/(d*x
 + c)^(3/2) + (-(d*x + c)*b/d)^(3/2)*e^(a - b*c/d)*gamma(-3/2, -(d*x + c)*b
/d)/(d*x + c)^(3/2))*b/d + 2*sinh(b*x + a)/(d*x + c)^(5/2))/d
```

Fricas [B] time = 2.9549, size = 1820, normalized size = 10.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(4*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)
*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3
*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-(b*c - a*d)/d) - (b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*sinh(
b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 4*sqrt(pi)*((b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-(b*c - a*
d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x
+ a)*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
 + b^2*c^3)*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(
d*x + c)*sqrt(-b/d)) + (4*b^2*d^2*x^2 + 4*b^2*c^2 - 2*b*c*d - (4*b^2*d^2*x^
2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^2
- 2*(4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)
*cosh(b*x + a)*sinh(b*x + a) - (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2
 + 2*(4*b^2*c*d + b*d^2)*x)*sinh(b*x + a)^2 + 3*d^2 + 2*(4*b^2*c*d - b*d^2)
*x)*sqrt(d*x + c))/((d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*cosh(b*
x + a) + (d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*sinh(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh (bx + a)}{(dx + c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)/(d*x + c)^(7/2), x)
```

3.45 $\int (c+dx)^{5/2}\sinh^2(a+bx)\,dx$

Optimal. Leaf size=239

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}}+\frac{15d^2\sqrt{c+dx}\sinh(2a+2bx)}{64b^3}-\frac{5d(c+dx)^{3/2}}{8\ldots}$$

```
[Out] (-5*d*(c + d*x)^(3/2))/(16*b^2) - (c + d*x)^(7/2)/(7*d) + (15*d^(5/2)*E^(-2
*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2
56*b^(7/2)) - (15*d^(5/2)*E^(2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt
[b]*Sqrt[c + d*x])/Sqrt[d]])/(256*b^(7/2)) + ((c + d*x)^(5/2)*Cosh[a + b*x]
*Sinh[a + b*x])/(2*b) - (5*d*(c + d*x)^(3/2)*Sinh[a + b*x]^2)/(8*b^2) + (15
*d^2*Sqrt[c + d*x]*Sinh[2*a + 2*b*x])/(64*b^3)
```

Rubi [A] time = 0.448625, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3311, 32, 3312, 3296, 3308, 2180, 2204, 2205}

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}}+\frac{15d^2\sqrt{c+dx}\sinh(2a+2bx)}{64b^3}-\frac{5d(c+dx)^{3/2}}{8\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Sinh[a + b*x]^2,x]
```

```
[Out] (-5*d*(c + d*x)^(3/2))/(16*b^2) - (c + d*x)^(7/2)/(7*d) + (15*d^(5/2)*E^(-2
*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2
56*b^(7/2)) - (15*d^(5/2)*E^(2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt
[b]*Sqrt[c + d*x])/Sqrt[d]])/(256*b^(7/2)) + ((c + d*x)^(5/2)*Cosh[a + b*x]
*Sinh[a + b*x])/(2*b) - (5*d*(c + d*x)^(3/2)*Sinh[a + b*x]^2)/(8*b^2) + (15
*d^2*Sqrt[c + d*x]*Sinh[2*a + 2*b*x])/(64*b^3)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (c+dx)^{5/2}\sinh^2(a+bx)\,dx = \frac{(c+dx)^{5/2}\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{5d(c+dx)^{3/2}\sinh^2(a+bx)}{8b^2} - \frac{1}{2}\int (c+dx)^{5/2}\,dx$$

$$= -\frac{(c+dx)^{7/2}}{7d} + \frac{(c+dx)^{5/2}\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{5d(c+dx)^{3/2}\sinh^2(a+bx)}{8b^2} - \frac{(1}{}$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} - \frac{(c+dx)^{7/2}}{7d} + \frac{(c+dx)^{5/2}\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{5d(c+dx)^{3/2}\sin}{8b^2}$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} - \frac{(c+dx)^{7/2}}{7d} + \frac{(c+dx)^{5/2}\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{5d(c+dx)^{3/2}\sin}{8b^2}$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} - \frac{(c+dx)^{7/2}}{7d} + \frac{(c+dx)^{5/2}\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{5d(c+dx)^{3/2}\sin}{8b^2}$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} - \frac{(c+dx)^{7/2}}{7d} + \frac{(c+dx)^{5/2}\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{5d(c+dx)^{3/2}\sin}{8b^2}$$

$$= -\frac{5d(c+dx)^{3/2}}{16b^2} - \frac{(c+dx)^{7/2}}{7d} + \frac{15d^{5/2}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}} - \frac{15d^{5/2}e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}}{256b^{7/2}}$$

Mathematica [A] time = 6.59064, size = 190, normalized size = 0.79

$$\frac{\sqrt{c+dx}\left(-7\sqrt{2}d^4\sqrt{-\frac{b^2(c+dx)^2}{d^2}}\text{Gamma}\left(\frac{7}{2},-\frac{2b(c+dx)}{d}\right)\left(\sinh\left(2a-\frac{2bc}{d}\right)+\cosh\left(2a-\frac{2bc}{d}\right)\right)-b(c+dx)\left(7\sqrt{2}d^3\text{Gamma}\left(\frac{7}{2}\right.\right.\right.}{448b^3d^2\left(\frac{b(c+dx)}{d}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(5/2)*Sinh[a + b*x]^2,x]
```

```
[Out] (Sqrt[c + d*x]*(-(b*(c + d*x)*(64*b^3*(c + d*x)^3*Sqrt[(b*(c + d*x))/d] + 7
*Sqrt[2]*d^3*Gamma[7/2, (2*b*(c + d*x))/d]*(Cosh[2*a - (2*b*c)/d] - Sinh[2*
a - (2*b*c)/d])) - 7*Sqrt[2]*d^4*Sqrt[-((b^2*(c + d*x)^2)/d^2)]*Gamma[7/2,
 (-2*b*(c + d*x))/d]*(Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d])))/(448
*b^3*d^2*((b*(c + d*x))/d)^(3/2))
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{2}} (\sinh (bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*sinh(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^(5/2)*sinh(b*x+a)^2,x)
```

Maxima [A] time = 1.74935, size = 379, normalized size = 1.59

$$-\frac{512\,(dx+c)^{\frac{7}{2}} + \frac{105\,\sqrt{2}\sqrt{\pi}d^3 \operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}} - \frac{105\,\sqrt{2}\sqrt{\pi}d^3 \operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}} + \frac{28\left(16\,(dx+c)^{\frac{5}{2}}b^2de^{\left(\frac{2\,bc}{d}\right)}+20\,(dx+c)\right)}{}}{3584\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3584*(512*(d*x + c)^(7/2) + 105*sqrt(2)*sqrt(pi)*d^3*erf(sqrt(2)*sqrt(d*
x + c)*sqrt(-b/d))*e^(2*a - 2*b*c/d)/(b^3*sqrt(-b/d)) - 105*sqrt(2)*sqrt(pi
)*d^3*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(-2*a + 2*b*c/d)/(b^3*sqrt(b/d
)) + 28*(16*(d*x + c)^(5/2)*b^2*d*e^(2*b*c/d) + 20*(d*x + c)^(3/2)*b*d^2*e^
(2*b*c/d) + 15*sqrt(d*x + c)*d^3*e^(2*b*c/d))*e^(-2*a - 2*(d*x + c)*b/d)/b^
3 - 28*(16*(d*x + c)^(5/2)*b^2*d*e^(2*a) - 20*(d*x + c)^(3/2)*b*d^2*e^(2*a)
 + 15*sqrt(d*x + c)*d^3*e^(2*a))*e^(2*(d*x + c)*b/d - 2*b*c/d)/b^3)/d
```

Fricas [B] time = 3.19787, size = 2319, normalized size = 9.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3584*(105*sqrt(2)*sqrt(pi)*(d^4*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) -
d^4*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^4*cosh(-2*(b*c - a*d)/d) -
d^4*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^4*cosh(b*x + a)*cosh(-2*
(b*c - a*d)/d) - d^4*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*s
qrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) + 105*sqrt(2)*sqrt(pi)*(d^4*c
osh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + d^4*cosh(b*x + a)^2*sinh(-2*(b*c -
```

```
a*d)/d) + (d^4*cosh(-2*(b*c - a*d)/d) + d^4*sinh(-2*(b*c - a*d)/d))*sinh(b*
x + a)^2 + 2*(d^4*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^4*cosh(b*x + a)*
sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)
*sqrt(-b/d)) - 4*(112*b^3*d^3*x^2 + 112*b^3*c^2*d + 140*b^2*c*d^2 - 7*(16*b
^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^
2*d^3)*x)*cosh(b*x + a)^4 - 28*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^
2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^2*d^3)*x)*cosh(b*x + a)*sinh(b*x + a)^3
 - 7*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*
d^2 - 5*b^2*d^3)*x)*sinh(b*x + a)^4 + 105*b*d^3 + 128*(b^4*d^3*x^3 + 3*b^4*
c*d^2*x^2 + 3*b^4*c^2*d*x + b^4*c^3)*cosh(b*x + a)^2 + 2*(64*b^4*d^3*x^3 +
192*b^4*c*d^2*x^2 + 192*b^4*c^2*d*x + 64*b^4*c^3 - 21*(16*b^3*d^3*x^2 + 16*
b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^2*d^3)*x)*cosh(b
*x + a)^2)*sinh(b*x + a)^2 + 28*(8*b^3*c*d^2 + 5*b^2*d^3)*x - 4*(7*(16*b^3*
d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^2*d
^3)*x)*cosh(b*x + a)^3 - 64*(b^4*d^3*x^3 + 3*b^4*c*d^2*x^2 + 3*b^4*c^2*d*x
+ b^4*c^3)*cosh(b*x + a))*sinh(b*x + a))*sqrt(d*x + c))/(b^4*d*cosh(b*x + a
)^2 + 2*b^4*d*cosh(b*x + a)*sinh(b*x + a) + b^4*d*sinh(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*sinh(b*x+a)**2,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{2}} \sinh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sinh(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^(5/2)*sinh(b*x + a)^2, x)
```

3.46 $\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx$

Optimal. Leaf size=211

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}}-\frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+}{2b}$$

```
[Out] (-3*d*Sqrt[c + d*x])/(16*b^2) - (c + d*x)^(5/2)/(5*d) + (3*d^(3/2)*E^(-2*a
+ (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(64*b
^(5/2)) + (3*d^(3/2)*E^(2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*S
qrt[c + d*x])/Sqrt[d]])/(64*b^(5/2)) + ((c + d*x)^(3/2)*Cosh[a + b*x]*Sinh[
a + b*x])/(2*b) - (3*d*Sqrt[c + d*x]*Sinh[a + b*x]^2)/(8*b^2)
```

Rubi [A] time = 0.320699, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3311, 32, 3312, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}}-\frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Sinh[a + b*x]^2,x]
```

```
[Out] (-3*d*Sqrt[c + d*x])/(16*b^2) - (c + d*x)^(5/2)/(5*d) + (3*d^(3/2)*E^(-2*a
+ (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(64*b
^(5/2)) + (3*d^(3/2)*E^(2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*S
qrt[c + d*x])/Sqrt[d]])/(64*b^(5/2)) + ((c + d*x)^(3/2)*Cosh[a + b*x]*Sinh[
a + b*x])/(2*b) - (3*d*Sqrt[c + d*x]*Sinh[a + b*x]^2)/(8*b^2)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int (c+dx)^{3/2}\sinh^2(a+bx)\,dx = \frac{(c+dx)^{3/2}\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2} - \frac{1}{2}\int (c+dx)^{3/2}\,dx +$$

$$= -\frac{(c+dx)^{5/2}}{5d} + \frac{(c+dx)^{3/2}\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2} - \frac{(3d^2}{}$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} - \frac{(c+dx)^{5/2}}{5d} + \frac{(c+dx)^{3/2}\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{3d\sqrt{c+dx}\sinh^2(a}{8b^2}$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} - \frac{(c+dx)^{5/2}}{5d} + \frac{(c+dx)^{3/2}\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{3d\sqrt{c+dx}\sinh^2(a}{8b^2}$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} - \frac{(c+dx)^{5/2}}{5d} + \frac{(c+dx)^{3/2}\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{3d\sqrt{c+dx}\sinh^2(a}{8b^2}$$

$$= -\frac{3d\sqrt{c+dx}}{16b^2} - \frac{(c+dx)^{5/2}}{5d} + \frac{3d^{3/2}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}} + \frac{3d^{3/2}e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\right.}{64b^{5/2}}$$

Mathematica [A] time = 2.29712, size = 163, normalized size = 0.77

$$\frac{5\sqrt{2}d^3\sqrt{\frac{b(c+dx)}{d}}\operatorname{Gamma}\left(\frac{5}{2},\frac{2b(c+dx)}{d}\right)\left(\sinh\left(2a-\frac{2bc}{d}\right)-\cosh\left(2a-\frac{2bc}{d}\right)\right)+5\sqrt{2}d^3\sqrt{-\frac{b(c+dx)}{d}}\operatorname{Gamma}\left(\frac{5}{2},-\frac{2b(c+dx)}{d}\right)\left(\sinh\right.}{160b^3d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Sinh[a + b*x]^2,x]
```

```
[Out] (-32*b^3*(c + d*x)^3 + 5*Sqrt[2]*d^3*Sqrt[(b*(c + d*x))/d]*Gamma[5/2, (2*b*
(c + d*x))/d]*(-Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d]) + 5*Sqrt[2]*
d^3*Sqrt[-((b*(c + d*x))/d)]*Gamma[5/2, (-2*b*(c + d*x))/d]*(Cosh[2*a - (2*
b*c)/d] + Sinh[2*a - (2*b*c)/d]))/(160*b^3*d*Sqrt[c + d*x])
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{3}{2}} (\sinh (bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*sinh(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^(3/2)*sinh(b*x+a)^2,x)
```

Maxima [A] time = 1.72278, size = 323, normalized size = 1.53

$$-\frac{128\,(dx+c)^{\frac{5}{2}} - \frac{15\,\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} - \frac{15\,\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} + \frac{20\left(4\,(dx+c)^{\frac{3}{2}}bde^{\left(\frac{2\,bc}{d}\right)}+3\,\sqrt{dx+c}d^2e^{\left(\frac{2\,b \cdots}{d}\right)} \cdots\right.}{b^2} \cdots}{640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/640*(128*(d*x + c)^(5/2) - 15*sqrt(2)*sqrt(pi)*d^2*erf(sqrt(2)*sqrt(d*x
+ c)*sqrt(-b/d))*e^(2*a - 2*b*c/d)/(b^2*sqrt(-b/d)) - 15*sqrt(2)*sqrt(pi)*d
^2*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(-2*a + 2*b*c/d)/(b^2*sqrt(b/d))
+ 20*(4*(d*x + c)^(3/2)*b*d*e^(2*b*c/d) + 3*sqrt(d*x + c)*d^2*e^(2*b*c/d))*
e^(-2*a - 2*(d*x + c)*b/d)/b^2 - 20*(4*(d*x + c)^(3/2)*b*d*e^(2*a) - 3*sqrt
(d*x + c)*d^2*e^(2*a))*e^(2*(d*x + c)*b/d - 2*b*c/d)/b^2)/d
```

Fricas [B] time = 3.31488, size = 1789, normalized size = 8.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/640*(15*sqrt(2)*sqrt(pi)*(d^3*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - d^
3*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^3*cosh(-2*(b*c - a*d)/d) - d^
3*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^3*cosh(b*x + a)*cosh(-2*(b
*c - a*d)/d) - d^3*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqr
t(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 15*sqrt(2)*sqrt(pi)*(d^3*cosh
(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + d^3*cosh(b*x + a)^2*sinh(-2*(b*c - a*d
)/d) + (d^3*cosh(-2*(b*c - a*d)/d) + d^3*sinh(-2*(b*c - a*d)/d))*sinh(b*x +
 a)^2 + 2*(d^3*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^3*cosh(b*x + a)*sin
h(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sq
rt(-b/d)) - 4*(20*b^2*d^2*x - 5*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*cosh(b*
x + a)^4 - 20*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*cosh(b*x + a)*sinh(b*x +
a)^3 - 5*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*sinh(b*x + a)^4 + 20*b^2*c*d +
 15*b*d^2 + 32*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*cosh(b*x + a)^2 + 2*(1
6*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c^2 - 15*(4*b^2*d^2*x + 4*b^2*c*d - 3
*b*d^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 4*(5*(4*b^2*d^2*x + 4*b^2*c*d -
3*b*d^2)*cosh(b*x + a)^3 - 16*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*cosh(b*
x + a))*sinh(b*x + a))*sqrt(d*x + c))/(b^3*d*cosh(b*x + a)^2 + 2*b^3*d*cosh
```

```
(b*x + a)*sinh(b*x + a) + b^3*d*sinh(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^{\frac{3}{2}} \sinh^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*sinh(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**(3/2)*sinh(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{3}{2}} \sinh(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(3/2)*sinh(b*x + a)^2, x)
```

3.47 $\int \sqrt{c+dx}\sinh^2(a+bx)\,dx$

Optimal. Leaf size=166

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{c+dx}\sinh(2a+2bx)}{4b}-\frac{(c+dx)^{3/2}}{3d}$$

```
[Out] -(c + d*x)^(3/2)/(3*d) + (Sqrt[d]*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt
[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) - (Sqrt[d]*E^(2*a - (2*b*
c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)
) + (Sqrt[c + d*x]*Sinh[2*a + 2*b*x])/(4*b)
```

Rubi [A] time = 0.295711, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3312, 3296, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{c+dx}\sinh(2a+2bx)}{4b}-\frac{(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Sinh[a + b*x]^2,x]
```



```
[Out] -(c + d*x)^(3/2)/(3*d) + (Sqrt[d]*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt
[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) - (Sqrt[d]*E^(2*a - (2*b*
c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)
) + (Sqrt[c + d*x]*Sinh[2*a + 2*b*x])/(4*b)
```


Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sqrt{c+dx}\sinh^2(a+bx)\,dx = -\int \left(\frac{1}{2}\sqrt{c+dx}-\frac{1}{2}\sqrt{c+dx}\cosh(2a+2bx)\right)dx$$
$$= -\frac{(c+dx)^{3/2}}{3d}+\frac{1}{2}\int \sqrt{c+dx}\cosh(2a+2bx)\,dx$$
$$= -\frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{c+dx}\sinh(2a+2bx)}{4b}-\frac{d\int \frac{\sinh(2a+2bx)}{\sqrt{c+dx}}\,dx}{8b}$$
$$= -\frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{c+dx}\sinh(2a+2bx)}{4b}-\frac{d\int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx}{16b}+\frac{d\int \frac{e^{i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx}{16b}$$
$$= -\frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{c+dx}\sinh(2a+2bx)}{4b}+\frac{\text{Subst}\left(\int e^{i\left(2ia-\frac{2ibc}{d}\right)-\frac{2bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{8b}-\text{Sub}$$
$$= -\frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{d}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{d}e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\sqrt{c+d}$$

Mathematica [A] time = 0.578671, size = 129, normalized size = 0.78

$$\frac{1}{48}\sqrt{c+dx}\left(\frac{3\sqrt{2}e^{2a-\frac{2bc}{d}}\text{Gamma}\left(\frac{3}{2},-\frac{2b(c+dx)}{d}\right)}{b\sqrt{-\frac{b(c+dx)}{d}}}-\frac{3\sqrt{2}e^{\frac{2bc}{d}-2a}\text{Gamma}\left(\frac{3}{2},\frac{2b(c+dx)}{d}\right)}{b\sqrt{\frac{b(c+dx)}{d}}}-\frac{16(c+dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Sinh[a + b*x]^2,x]
```

```
[Out] (Sqrt[c + d*x]*((-16*(c + d*x))/d + (3*Sqrt[2]*E^(2*a - (2*b*c)/d)*Gamma[3/
2, (-2*b*(c + d*x))/d])/(b*Sqrt[-((b*(c + d*x))/d)]) - (3*Sqrt[2]*E^(-2*a +
 (2*b*c)/d)*Gamma[3/2, (2*b*(c + d*x))/d])/(b*Sqrt[(b*(c + d*x))/d])))/48
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int (\sinh(bx+a))^2\sqrt{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2*(d*x+c)^(1/2),x)
```

```
[Out] int(sinh(b*x+a)^2*(d*x+c)^(1/2),x)
```

Maxima [A] time = 1.62864, size = 255, normalized size = 1.54

$$-\frac{\frac{3\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2a-\frac{2bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}} - \frac{3\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2a+\frac{2bc}{d}\right)}}{b\sqrt{\frac{b}{d}}} + 32\,(dx+c)^{\frac{3}{2}} - \frac{12\sqrt{dx+c}de^{\left(2a+\frac{2(dx+c)b}{d}-\frac{2bc}{d}\right)}}{b} + \frac{12\sqrt{dx+c}\cdots}{\cdots}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/96*(3*sqrt(2)*sqrt(pi)*d*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))*e^(2*a -
2*b*c/d)/(b*sqrt(-b/d)) - 3*sqrt(2)*sqrt(pi)*d*erf(sqrt(2)*sqrt(d*x + c)*sq
rt(b/d))*e^(-2*a + 2*b*c/d)/(b*sqrt(b/d)) + 32*(d*x + c)^(3/2) - 12*sqrt(d*
x + c)*d*e^(2*a + 2*(d*x + c)*b/d - 2*b*c/d)/b + 12*sqrt(d*x + c)*d*e^(-2*a
 - 2*(d*x + c)*b/d + 2*b*c/d)/b)/d
```

Fricas [B] time = 2.91236, size = 1438, normalized size = 8.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(3*sqrt(2)*sqrt(pi)*(d^2*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - d^2*
cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^2*cosh(-2*(b*c - a*d)/d) - d^2*
sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^2*cosh(b*x + a)*cosh(-2*(b*c
 - a*d)/d) - d^2*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(
b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) + 3*sqrt(2)*sqrt(pi)*(d^2*cosh(b*
x + a)^2*cosh(-2*(b*c - a*d)/d) + d^2*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d
) + (d^2*cosh(-2*(b*c - a*d)/d) + d^2*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)
^2 + 2*(d^2*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^2*cosh(b*x + a)*sinh(-
2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(
-b/d)) + 4*(3*b*d*cosh(b*x + a)^4 + 12*b*d*cosh(b*x + a)*sinh(b*x + a)^3 +
3*b*d*sinh(b*x + a)^4 - 8*(b^2*d*x + b^2*c)*cosh(b*x + a)^2 - 2*(4*b^2*d*x
- 9*b*d*cosh(b*x + a)^2 + 4*b^2*c)*sinh(b*x + a)^2 - 3*b*d + 4*(3*b*d*cosh(
b*x + a)^3 - 4*(b^2*d*x + b^2*c)*cosh(b*x + a))*sinh(b*x + a))*sqrt(d*x + c
))/(b^2*d*cosh(b*x + a)^2 + 2*b^2*d*cosh(b*x + a)*sinh(b*x + a) + b^2*d*sin
h(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c + dx}\sinh^2(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*sinh(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx + c} \sinh{(bx + a)}^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x + c)*sinh(b*x + a)^2, x)
```

3.48 $\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}-\frac{\sqrt{c+dx}}{d}$$

```
[Out] -(Sqrt[c + d*x]/d) + (E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*
Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d]) + (E^(2*a - (2*b*c)/d)*Sqrt[Pi
/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d])
```

Rubi [A] time = 0.241308, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3312, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}-\frac{\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2/Sqrt[c + d*x],x]
```

```
[Out] -(Sqrt[c + d*x]/d) + (E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*
Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d]) + (E^(2*a - (2*b*c)/d)*Sqrt[Pi
/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx &= -\int \left(\frac{1}{2\sqrt{c+dx}}-\frac{\cosh(2a+2bx)}{2\sqrt{c+dx}}\right)dx\\ &= -\frac{\sqrt{c+dx}}{d}+\frac{1}{2}\int \frac{\cosh(2a+2bx)}{\sqrt{c+dx}}\,dx\\ &= -\frac{\sqrt{c+dx}}{d}+\frac{1}{4}\int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx+\frac{1}{4}\int \frac{e^{i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx\\ &= -\frac{\sqrt{c+dx}}{d}+\frac{\text{Subst}\left(\int e^{i\left(2ia-\frac{2ibc}{d}\right)-\frac{2bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{2d}+\frac{\text{Subst}\left(\int e^{-i\left(2ia-\frac{2ibc}{d}\right)+\frac{2bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{2d}\\ &= -\frac{\sqrt{c+dx}}{d}+\frac{e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}\end{aligned}$$

Mathematica [A] time = 0.130485, size = 142, normalized size = 1.02

$$\frac{e^{2a-\frac{2bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\text{Gamma}\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{c+dx}}-\frac{e^{\frac{2bc}{d}-2a}\sqrt{\frac{b(c+dx)}{d}}\,\text{Gamma}\left(\frac{1}{2},\frac{2b(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{c+dx}}-\frac{\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2/Sqrt[c + d*x],x]
```

```
[Out] -(Sqrt[c + d*x]/d) + (E^(2*a - (2*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/
2, (-2*b*(c + d*x))/d])/(4*Sqrt[2]*b*Sqrt[c + d*x]) - (E^(-2*a + (2*b*c)/d)
*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (2*b*(c + d*x))/d])/(4*Sqrt[2]*b*Sqrt[c +
 d*x])
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int (\sinh(bx+a))^2 \frac{1}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2/(d*x+c)^(1/2),x)
```

```
[Out] int(sinh(b*x+a)^2/(d*x+c)^(1/2),x)
```

Maxima [A] time = 1.68104, size = 144, normalized size = 1.04

$$\frac{\frac{\sqrt{2}\sqrt{\pi}\,\text{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2a-\frac{2bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}+\frac{\sqrt{2}\sqrt{\pi}\,\text{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2a+\frac{2bc}{d}\right)}}{\sqrt{\frac{b}{d}}}-8\sqrt{dx+c}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))*e^(2*a - 2*b*c/
d)/sqrt(-b/d) + sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(-2
*a + 2*b*c/d)/sqrt(b/d) - 8*sqrt(d*x + c))/d
```

Fricas [A] time = 2.82359, size = 369, normalized size = 2.65

$$\frac{\sqrt{2}\sqrt{\pi}\left(d\cosh\left(-\frac{2(bc-ad)}{d}\right)-d\sinh\left(-\frac{2(bc-ad)}{d}\right)\right)\sqrt{\frac{b}{d}}\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)-\sqrt{2}\sqrt{\pi}\left(d\cosh\left(-\frac{2(bc-ad)}{d}\right)+d\sinh\left(-\frac{2(bc-}{d}\right.\right.}{8\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*sqrt(pi)*(d*cosh(-2*(b*c - a*d)/d) - d*sinh(-2*(b*c - a*d)/d))
*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - sqrt(2)*sqrt(pi)*(d*cosh(
-2*(b*c - a*d)/d) + d*sinh(-2*(b*c - a*d)/d))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d
*x + c)*sqrt(-b/d)) - 8*sqrt(d*x + c)*b)/(b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2/(d*x+c)**(1/2),x)
```

```
[Out] Integral(sinh(a + b*x)**2/sqrt(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx+a)^2}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^2/sqrt(d*x + c), x)
```

3.49 $\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}$$

```
[Out] -((Sqrt[b]*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*
x])/Sqrt[d]])/d^(3/2)) + (Sqrt[b]*E^(2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt
[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2) - (2*Sinh[a + b*x]^2)/(d*Sqrt[
c + d*x])
```

Rubi [A] time = 0.253019, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3313, 12, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2/(c + d*x)^(3/2),x]
```

```
[Out] -((Sqrt[b]*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*
x])/Sqrt[d]])/d^(3/2)) + (Sqrt[b]*E^(2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt
[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2) - (2*Sinh[a + b*x]^2)/(d*Sqrt[
c + d*x])
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,dx &= -\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}-\frac{(4ib)\int \frac{i\sinh(2a+2bx)}{2\sqrt{c+dx}}\,dx}{d}\\ &= -\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}+\frac{(2b)\int \frac{\sinh(2a+2bx)}{\sqrt{c+dx}}\,dx}{d}\\ &= -\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}+\frac{b\int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx}{d}-\frac{b\int \frac{e^{i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx}{d}\\ &= -\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}-\frac{(2b)\,\mathrm{Subst}\left(\int e^{i\left(2ia-\frac{2ibc}{d}\right)-\frac{2bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{d^2}+\frac{(2b)\,\mathrm{Subst}\left(\int e^{-i\left(2ia-\frac{2ibc}{d}\right)+\cdots}\right.}{d^2}\\ &= -\frac{\sqrt{b}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{b}e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}\end{aligned}$$

Mathematica [B] time = 4.85797, size = 570, normalized size = 4.01

$$\frac{e^{-\frac{2b(c+dx)}{d}}\left(\sqrt{2}\sqrt{d}e^{\frac{2b(c+dx)}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)\left(\cosh\left(2a-\frac{2bc}{d}\right)+\sinh(2a)\cosh\left(\frac{2bc}{d}\right)\right)+\sqrt{2}\sqrt{d}e^{\frac{2b(c+dx)}{d}}\sqrt{\frac{b(c\cdots}{}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2/(c + d*x)^(3/2),x]
```

```
[Out] (2*Sqrt[d]*E^((2*b*(c + d*x))/d) - Sqrt[d]*Cosh[2*a]*Cosh[(2*b*c)/d] - Sqrt
[d]*E^((4*b*(c + d*x))/d)*Cosh[2*a]*Cosh[(2*b*c)/d] + Sqrt[d]*Cosh[(2*b*c)/
d]*Sinh[2*a] - Sqrt[d]*E^((4*b*(c + d*x))/d)*Cosh[(2*b*c)/d]*Sinh[2*a] + Sq
rt[2]*Sqrt[d]*E^((2*b*(c + d*x))/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-2
*b*(c + d*x))/d]*(Cosh[2*a - (2*b*c)/d] + Cosh[(2*b*c)/d]*Sinh[2*a]) - Sqrt
[d]*Cosh[2*a]*Sinh[(2*b*c)/d] + Sqrt[d]*E^((4*b*(c + d*x))/d)*Cosh[2*a]*Sin
h[(2*b*c)/d] - Sqrt[b]*E^((2*b*(c + d*x))/d)*Sqrt[2*Pi]*Sqrt[c + d*x]*Cosh[
2*a]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]*Sinh[(2*b*c)/d] - Sqrt[b]
*E^((2*b*(c + d*x))/d)*Sqrt[2*Pi]*Sqrt[c + d*x]*Cosh[2*a]*Erfi[(Sqrt[2]*Sqr
t[b]*Sqrt[c + d*x])/Sqrt[d]]*Sinh[(2*b*c)/d] + Sqrt[d]*Sinh[2*a]*Sinh[(2*b*
c)/d] + Sqrt[d]*E^((4*b*(c + d*x))/d)*Sinh[2*a]*Sinh[(2*b*c)/d] + Sqrt[2]*S
qrt[d]*E^((2*b*(c + d*x))/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (2*b*(c + d*x
))/d]*(Cosh[2*a]*Cosh[(2*b*c)/d] - Sinh[2*a]*(Cosh[(2*b*c)/d] + Sinh[(2*b*c
)/d])))/(2*d^(3/2)*E^((2*b*(c + d*x))/d)*Sqrt[c + d*x])
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \left(\sinh \left(bx+a\right)\right)^{2} \left(dx+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2/(d*x+c)^(3/2),x)
```

```
[Out] int(sinh(b*x+a)^2/(d*x+c)^(3/2),x)
```

Maxima [A] time = 1.29964, size = 157, normalized size = 1.11

$$-\frac{\frac{\sqrt{2}\sqrt{\frac{(dx+c)b}{d}}e^{\left(\frac{2\,(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{2\,(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{\sqrt{2}\sqrt{-\frac{(dx+c)b}{d}}e^{\left(-\frac{2\,(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{2\,(dx+c)b}{d}\right)}{\sqrt{dx+c}}-\frac{4}{\sqrt{dx+c}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(sqrt(2)*sqrt((d*x + c)*b/d)*e^(2*(b*c - a*d)/d)*gamma(-1/2, 2*(d*x +
c)*b/d)/sqrt(d*x + c) + sqrt(2)*sqrt(-(d*x + c)*b/d)*e^(-2*(b*c - a*d)/d)*g
amma(-1/2, -2*(d*x + c)*b/d)/sqrt(d*x + c) - 4/sqrt(d*x + c))/d
```

Fricas [B] time = 2.85584, size = 1442, normalized size = 10.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*sqrt(pi)*((d*x + c)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) -
(d*x + c)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((d*x + c)*cosh(-2*(b*c
- a*d)/d) - (d*x + c)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((d*x + c
)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)*sinh(-2*(b
*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))
 + sqrt(2)*sqrt(pi)*((d*x + c)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + (d*
x + c)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((d*x + c)*cosh(-2*(b*c - a
*d)/d) + (d*x + c)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((d*x + c)*c
osh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)*sinh(-2*(b*c
- a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))
+ (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*
(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x +
 a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*sqrt(d*x + c))/((d^2*x + c*d)*cos
h(b*x + a)^2 + 2*(d^2*x + c*d)*cosh(b*x + a)*sinh(b*x + a) + (d^2*x + c*d)*
sinh(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^{2}\left(a+bx\right)}{\left(c+dx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2/(d*x+c)**(3/2),x)
```

```
[Out] Integral(sinh(a + b*x)**2/(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh (bx + a)^2}{(dx + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^2/(d*x + c)^(3/2), x)
```

3.50 $\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=174

$$\frac{2\sqrt{2\pi}b^{3/2}e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2\sqrt{2\pi}b^{3/2}e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}$$

```
[Out] (2*b^(3/2)*E^(-2*a + (2*b*c)/d)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*
x])/Sqrt[d]])/(3*d^(5/2)) + (2*b^(3/2)*E^(2*a - (2*b*c)/d)*Sqrt[2*Pi]*Erfi[
(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) - (8*b*Cosh[a + b*x]*
Sinh[a + b*x])/(3*d^2*Sqrt[c + d*x]) - (2*Sinh[a + b*x]^2)/(3*d*(c + d*x)^(
3/2))
```

Rubi [A] time = 0.321269, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3314, 32, 3312, 3307, 2180, 2204, 2205}

$$\frac{2\sqrt{2\pi}b^{3/2}e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2\sqrt{2\pi}b^{3/2}e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2/(c + d*x)^(5/2),x]
```


```
[Out] (2*b^(3/2)*E^(-2*a + (2*b*c)/d)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*
x])/Sqrt[d]])/(3*d^(5/2)) + (2*b^(3/2)*E^(2*a - (2*b*c)/d)*Sqrt[2*Pi]*Erfi[
(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) - (8*b*Cosh[a + b*x]*
Sinh[a + b*x])/(3*d^2*Sqrt[c + d*x]) - (2*Sinh[a + b*x]^2)/(3*d*(c + d*x)^(
3/2))
```


Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{\left(8b^2\right)\int \frac{1}{\sqrt{c+dx}}\,dx}{3d^2} + \frac{\left(16b^2\right)\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= \frac{16b^2\sqrt{c+dx}}{3d^3} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{\left(16b^2\right)\int \left(\frac{1}{2\sqrt{c+dx}} - \frac{\cosh(2a+2}{2\sqrt{c+dx}}\right)}{3d^2}$$

$$= -\frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{\left(8b^2\right)\int \frac{\cosh(2a+2bx)}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{\left(4b^2\right)\int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx}{3d^2} + \frac{\left(4b^2\right)\int \frac{e^{i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{\left(8b^2\right)\text{Subst}\left(\int e^{i\left(2ia-\frac{2ibc}{d}\right)-\frac{2bx^2}{d}}\,dx, x, \sqrt{c+\right.}{3d^3}$$

$$= \frac{2b^{3/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2b^{3/2}e^{2a-\frac{2bc}{d}}\sqrt{2\pi}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{8b\cosh(a+bx)\sinh(}{3d^2\sqrt{c+dx}}$$

Mathematica [A] time = 1.21472, size = 156, normalized size = 0.9

$$-\frac{2e^{-2\left(a+\frac{bc}{d}\right)}\left(\sqrt{2}e^{4a}d\left(-\frac{b(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)+\sqrt{2}de^{\frac{4bc}{d}}\left(\frac{b(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},\frac{2b(c+dx)}{d}\right)+e^{2\left(a+\frac{bc}{d}\right)}\left(2b(c\right.}{3d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(Sqrt[2]*d*E^(4*a)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, (-2*b*(c + d*x
))/d] + Sqrt[2]*d*E^((4*b*c)/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (2*b*(c
+ d*x))/d] + E^(2*(a + (b*c)/d))*(d*Sinh[a + b*x]^2 + 2*b*(c + d*x)*Sinh[2*
```

```
(a + b*x)])))/(3*d^2*E^(2*(a + (b*c)/d))*(c + d*x)^(3/2))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int (\sinh (bx + a))^2 (dx + c)^{-\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2/(d*x+c)^(5/2),x)

[Out] int(sinh(b*x+a)^2/(d*x+c)^(5/2),x)
```

Maxima [A] time = 1.30908, size = 159, normalized size = 0.91

$$-\frac{\frac{3\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}} e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2}, \frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}} + \frac{3\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}} e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2}, -\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}} - \frac{2}{(dx+c)^{\frac{3}{2}}}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")

[Out] -1/6*(3*sqrt(2)*((d*x + c)*b/d)^(3/2)*e^(2*(b*c - a*d)/d)*gamma(-3/2, 2*(d*
x + c)*b/d)/(d*x + c)^(3/2) + 3*sqrt(2)*(-(d*x + c)*b/d)^(3/2)*e^(-2*(b*c -
 a*d)/d)*gamma(-3/2, -2*(d*x + c)*b/d)/(d*x + c)^(3/2) - 2/(d*x + c)^(3/2))
/d
```

Fricas [B] time = 3.01618, size = 2053, normalized size = 11.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")

[Out] 1/6*(4*sqrt(2)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*co
sh(-2*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh
(-2*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-2*(b*c - a*d)/d
) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2
 + 2*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d)
- (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sin
h(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 4*sqrt(2)*sqrt
(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d
) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d)
+ ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-2*(b*c - a*d)/d) + (b*d^2*x^2 + 2*
b*c*d*x + b*c^2)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b*d^2*x^2 +
2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*
c*d*x + b*c^2)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b
/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d)) - ((4*b*d*x + 4*b*c + d)*cosh(b*x
 + a)^4 + 4*(4*b*d*x + 4*b*c + d)*cosh(b*x + a)*sinh(b*x + a)^3 + (4*b*d*x
```

```
+ 4*b*c + d)*sinh(b*x + a)^4 - 4*b*d*x - 2*d*cosh(b*x + a)^2 + 2*(3*(4*b*d*
x + 4*b*c + d)*cosh(b*x + a)^2 - d)*sinh(b*x + a)^2 - 4*b*c + 4*((4*b*d*x +
 4*b*c + d)*cosh(b*x + a)^3 - d*cosh(b*x + a))*sinh(b*x + a) + d)*sqrt(d*x
+ c))/((d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a)^2 + 2*(d^4*x^2 + 2*c*d
^3*x + c^2*d^2)*cosh(b*x + a)*sinh(b*x + a) + (d^4*x^2 + 2*c*d^3*x + c^2*d^
2)*sinh(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2/(d*x+c)**(5/2),x)
```

```
[Out] Integral(sinh(a + b*x)**2/(c + d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx+a)^2}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^2/(d*x + c)^(5/2), x)
```

3.51 $\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal. Leaf size=220

$$-\frac{8\sqrt{2\pi}b^{5/2}e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8\sqrt{2\pi}b^{5/2}e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}-\frac{32b^2\sinh^2(a+bx)}{15d^3\sqrt{c+dx}}-\frac{8b\sinh(a+bx)\cosh(a+}{15d^2(c+dx)^{3/2}}$$

```
[Out] (-16*b^2)/(15*d^3*Sqrt[c + d*x]) - (8*b^(5/2)*E^(-2*a + (2*b*c)/d)*Sqrt[2*P
i]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7/2)) + (8*b^(5/2)*
E^(2*a - (2*b*c)/d)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]
])/(15*d^(7/2)) - (8*b*Cosh[a + b*x]*Sinh[a + b*x])/(15*d^2*(c + d*x)^(3/2)
) - (2*Sinh[a + b*x]^2)/(5*d*(c + d*x)^(5/2)) - (32*b^2*Sinh[a + b*x]^2)/(1
5*d^3*Sqrt[c + d*x])
```

Rubi [A] time = 0.321564, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3314, 32, 3313, 12, 3308, 2180, 2204, 2205}

$$-\frac{8\sqrt{2\pi}b^{5/2}e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8\sqrt{2\pi}b^{5/2}e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}-\frac{32b^2\sinh^2(a+bx)}{15d^3\sqrt{c+dx}}-\frac{8b\sinh(a+bx)\cosh(a+}{15d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2/(c + d*x)^(7/2),x]
```


```
[Out] (-16*b^2)/(15*d^3*Sqrt[c + d*x]) - (8*b^(5/2)*E^(-2*a + (2*b*c)/d)*Sqrt[2*P
i]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7/2)) + (8*b^(5/2)*
E^(2*a - (2*b*c)/d)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]
])/(15*d^(7/2)) - (8*b*Cosh[a + b*x]*Sinh[a + b*x])/(15*d^2*(c + d*x)^(3/2)
) - (2*Sinh[a + b*x]^2)/(5*d*(c + d*x)^(5/2)) - (32*b^2*Sinh[a + b*x]^2)/(1
5*d^3*Sqrt[c + d*x])
```


Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{8b\cosh(a+bx)\sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{\left(8b^2\right)\int\frac{1}{(c+dx)^{3/2}}\,dx}{15d^2} + \frac{\left(16b^2\right)\int\frac{\sinh^2(a+bx)}{(c+dx)^{3/2}}\,d}{15d^2}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2\sinh^2(a+bx)}{15d^3\sqrt{c+dx}} - \frac{(64i}{}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2\sinh^2(a+bx)}{15d^3\sqrt{c+dx}} + \frac{(32l}{}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2\sinh^2(a+bx)}{15d^3\sqrt{c+dx}} + \frac{(16l}{}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2\sinh^2(a+bx)}{15d^3\sqrt{c+dx}} - \frac{(32l}{}$$

$$= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{8b^{5/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{8b^{5/2}e^{2a-\frac{2bc}{d}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} - \frac{8b\,\mathrm{co}}{}$$

Mathematica [B] time = 9.18661, size = 825, normalized size = 3.75

$$e^{-\frac{2b(c+dx)}{d}}\left(16\sqrt{2}d^2e^{\frac{2b(c+dx)}{d}}\operatorname{Gamma}\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)\left(\cosh\left(2a-\frac{2bc}{d}\right)+\sinh\left(2a-\frac{2bc}{d}\right)\right)\left(-\frac{b(c+dx)}{d}\right)^{5/2}+6d^2e^{\frac{2b(c+dx)}{d}}-16b^2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2/(c + d*x)^(7/2),x]
```

```
[Out] (6*d^2*E^((2*b*(c + d*x))/d) - 16*b^2*c^2*Cosh[2*a - (2*b*c)/d] + 4*b*c*d*C
osh[2*a - (2*b*c)/d] - 3*d^2*Cosh[2*a - (2*b*c)/d] - 16*b^2*c^2*E^((4*b*(c
+ d*x))/d)*Cosh[2*a - (2*b*c)/d] - 4*b*c*d*E^((4*b*(c + d*x))/d)*Cosh[2*a -
 (2*b*c)/d] - 3*d^2*E^((4*b*(c + d*x))/d)*Cosh[2*a - (2*b*c)/d] - 32*b^2*c*
d*x*Cosh[2*a - (2*b*c)/d] + 4*b*d^2*x*Cosh[2*a - (2*b*c)/d] - 32*b^2*c*d*E^
((4*b*(c + d*x))/d)*x*Cosh[2*a - (2*b*c)/d] - 4*b*d^2*E^((4*b*(c + d*x))/d)
*x*Cosh[2*a - (2*b*c)/d] - 16*b^2*d^2*x^2*Cosh[2*a - (2*b*c)/d] - 16*b^2*d^
2*E^((4*b*(c + d*x))/d)*x^2*Cosh[2*a - (2*b*c)/d] + 16*Sqrt[2]*d^2*E^((2*b*
(c + d*x))/d)*((b*(c + d*x))/d)^(5/2)*Gamma[1/2, (2*b*(c + d*x))/d]*(Cosh[2
*a - (2*b*c)/d] - Sinh[2*a - (2*b*c)/d]) + 16*b^2*c^2*Sinh[2*a - (2*b*c)/d]
 - 4*b*c*d*Sinh[2*a - (2*b*c)/d] + 3*d^2*Sinh[2*a - (2*b*c)/d] - 16*b^2*c^2
*E^((4*b*(c + d*x))/d)*Sinh[2*a - (2*b*c)/d] - 4*b*c*d*E^((4*b*(c + d*x))/d
)*Sinh[2*a - (2*b*c)/d] - 3*d^2*E^((4*b*(c + d*x))/d)*Sinh[2*a - (2*b*c)/d]
 + 32*b^2*c*d*x*Sinh[2*a - (2*b*c)/d] - 4*b*d^2*x*Sinh[2*a - (2*b*c)/d] - 3
2*b^2*c*d*E^((4*b*(c + d*x))/d)*x*Sinh[2*a - (2*b*c)/d] - 4*b*d^2*E^((4*b*(
c + d*x))/d)*x*Sinh[2*a - (2*b*c)/d] + 16*b^2*d^2*x^2*Sinh[2*a - (2*b*c)/d]
 - 16*b^2*d^2*E^((4*b*(c + d*x))/d)*x^2*Sinh[2*a - (2*b*c)/d] + 16*Sqrt[2]*
d^2*E^((2*b*(c + d*x))/d)*(-((b*(c + d*x))/d))^(5/2)*Gamma[1/2, (-2*b*(c +
d*x))/d]*(Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d]))/(30*d^3*E^((2*b*(
c + d*x))/d)*(c + d*x)^(5/2))
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int (\sinh (bx + a))^2 (dx + c)^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2/(d*x+c)^(7/2),x)
```

```
[Out] int(sinh(b*x+a)^2/(d*x+c)^(7/2),x)
```

Maxima [A] time = 1.31562, size = 159, normalized size = 0.72

$$-\frac{\frac{5\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}} e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2}, \frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}} + \frac{5\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{5}{2}} e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2}, -\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}} - \frac{1}{(dx+c)^{\frac{5}{2}}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/5*(5*sqrt(2)*((d*x + c)*b/d)^(5/2)*e^(2*(b*c - a*d)/d)*gamma(-5/2, 2*(d*
x + c)*b/d)/(d*x + c)^(5/2) + 5*sqrt(2)*(-(d*x + c)*b/d)^(5/2)*e^(-2*(b*c -
 a*d)/d)*gamma(-5/2, -2*(d*x + c)*b/d)/(d*x + c)^(5/2) - 1/(d*x + c)^(5/2))
/d
```

Fricas [B] time = 3.08432, size = 2974, normalized size = 13.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/30*(16*sqrt(2)*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*
d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) +
 ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-2*(b*c -
a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-2
*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) - (b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-2*(b*c - a*d
)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) + 16*sq
rt(2)*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*c
osh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-2*(b*c - a*d)/d) + (b
^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-2*(b*c - a*d)
/d))*sinh(b*x + a)^2 + 2*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
b^2*c^3)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b
*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d)) + (16*b^2*d^2*x^2
 + (16*b^2*d^2*x^2 + 16*b^2*c^2 + 4*b*c*d + 3*d^2 + 4*(8*b^2*c*d + b*d^2)*x
)*cosh(b*x + a)^4 + 4*(16*b^2*d^2*x^2 + 16*b^2*c^2 + 4*b*c*d + 3*d^2 + 4*(8
*b^2*c*d + b*d^2)*x)*cosh(b*x + a)*sinh(b*x + a)^3 + (16*b^2*d^2*x^2 + 16*b
^2*c^2 + 4*b*c*d + 3*d^2 + 4*(8*b^2*c*d + b*d^2)*x)*sinh(b*x + a)^4 + 16*b^
2*c^2 - 6*d^2*cosh(b*x + a)^2 - 4*b*c*d + 6*((16*b^2*d^2*x^2 + 16*b^2*c^2 +
 4*b*c*d + 3*d^2 + 4*(8*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^2 - d^2)*sinh(b*x
 + a)^2 + 3*d^2 + 4*(8*b^2*c*d - b*d^2)*x + 4*((16*b^2*d^2*x^2 + 16*b^2*c^2
 + 4*b*c*d + 3*d^2 + 4*(8*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^3 - 3*d^2*cosh(
b*x + a))*sinh(b*x + a))*sqrt(d*x + c))/((d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4
*x + c^3*d^3)*cosh(b*x + a)^2 + 2*(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^
3*d^3)*cosh(b*x + a)*sinh(b*x + a) + (d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x +
 c^3*d^3)*sinh(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2/(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx+a)^2}{(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^2/(d*x + c)^(7/2), x)
```

3.52 $\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx$

Optimal. Leaf size=251

$$\frac{32\sqrt{2\pi}b^{7/2}e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}}+\frac{32\sqrt{2\pi}b^{7/2}e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}}-\frac{32b^2\sinh^2(a+bx)}{105d^3(c+dx)^{3/2}}-\frac{128b^3\sinh(a+bx)\,c}{105d^4\sqrt{c+}}$$

```
[Out] (-16*b^2)/(105*d^3*(c + d*x)^(3/2)) + (32*b^(7/2)*E^(-2*a + (2*b*c)/d)*Sqrt
[2*Pi]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(105*d^(9/2)) + (32*b^
(7/2)*E^(2*a - (2*b*c)/d)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/S
qrt[d]])/(105*d^(9/2)) - (8*b*Cosh[a + b*x]*Sinh[a + b*x])/(35*d^2*(c + d*x
)^(5/2)) - (128*b^3*Cosh[a + b*x]*Sinh[a + b*x])/(105*d^4*Sqrt[c + d*x]) -
(2*Sinh[a + b*x]^2)/(7*d*(c + d*x)^(7/2)) - (32*b^2*Sinh[a + b*x]^2)/(105*d
^3*(c + d*x)^(3/2))
```

Rubi [A] time = 0.394982, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3314, 32, 3312, 3307, 2180, 2204, 2205}

$$\frac{32\sqrt{2\pi}b^{7/2}e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}}+\frac{32\sqrt{2\pi}b^{7/2}e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}}-\frac{32b^2\sinh^2(a+bx)}{105d^3(c+dx)^{3/2}}-\frac{128b^3\sinh(a+bx)\,c}{105d^4\sqrt{c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2/(c + d*x)^(9/2),x]
```


```
[Out] (-16*b^2)/(105*d^3*(c + d*x)^(3/2)) + (32*b^(7/2)*E^(-2*a + (2*b*c)/d)*Sqrt
[2*Pi]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(105*d^(9/2)) + (32*b^
(7/2)*E^(2*a - (2*b*c)/d)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/S
qrt[d]])/(105*d^(9/2)) - (8*b*Cosh[a + b*x]*Sinh[a + b*x])/(35*d^2*(c + d*x
)^(5/2)) - (128*b^3*Cosh[a + b*x]*Sinh[a + b*x])/(105*d^4*Sqrt[c + d*x]) -
(2*Sinh[a + b*x]^2)/(7*d*(c + d*x)^(7/2)) - (32*b^2*Sinh[a + b*x]^2)/(105*d
^3*(c + d*x)^(3/2))
```


Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}}\,dx = -\frac{8b\cosh(a+bx)\sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{2\sinh^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{\left(8b^2\right)\int \frac{1}{(c+dx)^{5/2}}\,dx}{35d^2} + \frac{\left(16b^2\right)\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}}\,dx}{35d^2}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cosh(a+bx)\sinh(a+bx)}{105d^4\sqrt{c+dx}} - \frac{2\sinh^2(a}{7d(c+}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} + \frac{256b^4\sqrt{c+dx}}{105d^5} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cosh(a+bx)\sinh(a}{105d^4\sqrt{c+dx}}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cosh(a+bx)\sinh(a+bx)}{105d^4\sqrt{c+dx}} - \frac{2\sinh^2(a}{7d(c+}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cosh(a+bx)\sinh(a+bx)}{105d^4\sqrt{c+dx}} - \frac{2\sinh^2(a}{7d(c+}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cosh(a+bx)\sinh(a+bx)}{105d^4\sqrt{c+dx}} - \frac{2\sinh^2(a}{7d(c+}$$

$$= -\frac{16b^2}{105d^3(c+dx)^{3/2}} + \frac{32b^{7/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}} + \frac{32b^{7/2}e^{2a-\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}} - 8b$$

Mathematica [A] time = 0.562512, size = 222, normalized size = 0.88

$$\frac{2\left(16\sqrt{2}b^3(c+dx)^3e^{2a-\frac{2bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right) - 16\sqrt{2}b^3(c+dx)^3e^{\frac{2bc}{d}-2a}\sqrt{\frac{b(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{1}{2},\frac{2b(c+dx)}{d}\right) - 16b}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2/(c + d*x)^(9/2),x]
```

```
[Out] (2*(-8*b^2*d*(c + d*x)^2 + 16*Sqrt[2]*b^3*E^(2*a - (2*b*c)/d)*(c + d*x)^3*S
qrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-2*b*(c + d*x))/d] - 16*Sqrt[2]*b^3*E^(
-2*a + (2*b*c)/d)*(c + d*x)^3*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (2*b*(c + d*
x))/d] - 15*d^3*Sinh[a + b*x]^2 - 16*b^2*d*(c + d*x)^2*Sinh[a + b*x]^2 - 6*
b*d^2*(c + d*x)*Sinh[2*(a + b*x)] - 32*b^3*(c + d*x)^3*Sinh[2*(a + b*x)]))/
(105*d^4*(c + d*x)^(7/2))
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int (\sinh (bx+a))^2 (dx+c)^{-\frac{9}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2/(d*x+c)^(9/2),x)
```

```
[Out] int(sinh(b*x+a)^2/(d*x+c)^(9/2),x)
```

Maxima [A] time = 1.29399, size = 159, normalized size = 0.63

$$-\frac{\frac{14\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{7}{2}} e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{7}{2}, \frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{7}{2}}} + \frac{14\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{7}{2}} e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{7}{2}, -\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{7}{2}}} - \frac{1}{(dx+c)^{\frac{7}{2}}}}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/7*(14*sqrt(2)*((d*x + c)*b/d)^(7/2)*e^(2*(b*c - a*d)/d)*gamma(-7/2, 2*(d
*x + c)*b/d)/(d*x + c)^(7/2) + 14*sqrt(2)*(-(d*x + c)*b/d)^(7/2)*e^(-2*(b*c
 - a*d)/d)*gamma(-7/2, -2*(d*x + c)*b/d)/(d*x + c)^(7/2) - 1/(d*x + c)^(7/2
))/d
```

Fricas [B] time = 3.57603, size = 3951, normalized size = 15.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/210*(64*sqrt(2)*sqrt(pi)*((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*
x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - (b^
3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*
cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 +
6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(-2*(b*c - a*d)/d) - (b^3*
d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*si
nh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 +
 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)*cosh(-2*(b*c -
```

```
a*d)/d) - (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*
x + b^3*c^4)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)
*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 64*sqrt(2)*sqrt(pi)*((b^3*d^4*x^4 +
 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x +
a)^2*cosh(-2*(b*c - a*d)/d) + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^
2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (
(b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^
4)*cosh(-2*(b*c - a*d)/d) + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*
x^2 + 4*b^3*c^3*d*x + b^3*c^4)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*
((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c
^4)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 +
 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)*sinh(-2*(b*c -
a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d)) +
(64*b^3*d^3*x^3 + 64*b^3*c^3 - 16*b^2*c^2*d + 30*d^3*cosh(b*x + a)^2 - (64*
b^3*d^3*x^3 + 64*b^3*c^3 + 16*b^2*c^2*d + 12*b*c*d^2 + 15*d^3 + 16*(12*b^3*
c*d^2 + b^2*d^3)*x^2 + 4*(48*b^3*c^2*d + 8*b^2*c*d^2 + 3*b*d^3)*x)*cosh(b*x
 + a)^4 - 4*(64*b^3*d^3*x^3 + 64*b^3*c^3 + 16*b^2*c^2*d + 12*b*c*d^2 + 15*d
^3 + 16*(12*b^3*c*d^2 + b^2*d^3)*x^2 + 4*(48*b^3*c^2*d + 8*b^2*c*d^2 + 3*b*
d^3)*x)*cosh(b*x + a)*sinh(b*x + a)^3 - (64*b^3*d^3*x^3 + 64*b^3*c^3 + 16*b
^2*c^2*d + 12*b*c*d^2 + 15*d^3 + 16*(12*b^3*c*d^2 + b^2*d^3)*x^2 + 4*(48*b^
3*c^2*d + 8*b^2*c*d^2 + 3*b*d^3)*x)*sinh(b*x + a)^4 + 12*b*c*d^2 - 15*d^3 +
 16*(12*b^3*c*d^2 - b^2*d^3)*x^2 + 6*(5*d^3 - (64*b^3*d^3*x^3 + 64*b^3*c^3
+ 16*b^2*c^2*d + 12*b*c*d^2 + 15*d^3 + 16*(12*b^3*c*d^2 + b^2*d^3)*x^2 + 4*
(48*b^3*c^2*d + 8*b^2*c*d^2 + 3*b*d^3)*x)*cosh(b*x + a)^2)*sinh(b*x + a)^2
+ 4*(48*b^3*c^2*d - 8*b^2*c*d^2 + 3*b*d^3)*x + 4*(15*d^3*cosh(b*x + a) - (6
4*b^3*d^3*x^3 + 64*b^3*c^3 + 16*b^2*c^2*d + 12*b*c*d^2 + 15*d^3 + 16*(12*b^
3*c*d^2 + b^2*d^3)*x^2 + 4*(48*b^3*c^2*d + 8*b^2*c*d^2 + 3*b*d^3)*x)*cosh(b
*x + a)^3)*sinh(b*x + a))*sqrt(d*x + c))/((d^8*x^4 + 4*c*d^7*x^3 + 6*c^2*d^
6*x^2 + 4*c^3*d^5*x + c^4*d^4)*cosh(b*x + a)^2 + 2*(d^8*x^4 + 4*c*d^7*x^3 +
 6*c^2*d^6*x^2 + 4*c^3*d^5*x + c^4*d^4)*cosh(b*x + a)*sinh(b*x + a) + (d^8*
x^4 + 4*c*d^7*x^3 + 6*c^2*d^6*x^2 + 4*c^3*d^5*x + c^4*d^4)*sinh(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2/(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx+a)^2}{(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^2/(d*x + c)^(9/2), x)
```

3.53 $\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx$

Optimal. Leaf size=381

$$\frac{45\sqrt{\pi}d^{5/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{45\sqrt{\pi}d^{5/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}e^{3a-\frac{3bc}{d}}\text{Er}}{576b^{7}}$$

```
[Out] (-45*d^2*Sqrt[c + d*x]*Cosh[a + b*x])/(16*b^3) - (2*(c + d*x)^(5/2)*Cosh[a
+ b*x])/(3*b) + (5*d^2*Sqrt[c + d*x]*Cosh[3*a + 3*b*x])/(144*b^3) + (45*d^(
5/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^
(7/2)) - (5*d^(5/2)*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sq
rt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (45*d^(5/2)*E^(a - (b*c)/d)*Sqrt[Pi]
*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(7/2)) - (5*d^(5/2)*E^(3*a -
(3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(576*b
^(7/2)) + (5*d*(c + d*x)^(3/2)*Sinh[a + b*x])/(3*b^2) + ((c + d*x)^(5/2)*Co
sh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (5*d*(c + d*x)^(3/2)*Sinh[a + b*x]^3)/
(18*b^2)
```

Rubi [A] time = 1.06608, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3311, 3296, 3307, 2180, 2204, 2205, 3312}

$$\frac{45\sqrt{\pi}d^{5/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{45\sqrt{\pi}d^{5/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}e^{3a-\frac{3bc}{d}}\text{Er}}{576b^{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Sinh[a + b*x]^3,x]
```



```
[Out] (-45*d^2*Sqrt[c + d*x]*Cosh[a + b*x])/(16*b^3) - (2*(c + d*x)^(5/2)*Cosh[a
+ b*x])/(3*b) + (5*d^2*Sqrt[c + d*x]*Cosh[3*a + 3*b*x])/(144*b^3) + (45*d^(
5/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^
(7/2)) - (5*d^(5/2)*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sq
rt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (45*d^(5/2)*E^(a - (b*c)/d)*Sqrt[Pi]
*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(7/2)) - (5*d^(5/2)*E^(3*a -
(3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(576*b
^(7/2)) + (5*d*(c + d*x)^(3/2)*Sinh[a + b*x])/(3*b^2) + ((c + d*x)^(5/2)*Co
sh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (5*d*(c + d*x)^(3/2)*Sinh[a + b*x]^3)/
(18*b^2)
```


Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (c+dx)^{5/2}\sinh^3(a+bx)\,dx = \frac{(c+dx)^{5/2}\cosh(a+bx)\sinh^2(a+bx)}{3b} - \frac{5d(c+dx)^{3/2}\sinh^3(a+bx)}{18b^2} - \frac{2}{3}\int (c+dx)^{5/2}\,\mathrm{s}$$

$$= -\frac{2(c+dx)^{5/2}\cosh(a+bx)}{3b} + \frac{(c+dx)^{5/2}\cosh(a+bx)\sinh^2(a+bx)}{3b} - \frac{5d(c+dx)^{3/2}\sin}{18b^2}$$

$$= -\frac{2(c+dx)^{5/2}\cosh(a+bx)}{3b} + \frac{5d(c+dx)^{3/2}\sinh(a+bx)}{3b^2} + \frac{(c+dx)^{5/2}\cosh(a+bx)\sinh}{3b}$$

$$= -\frac{45d^2\sqrt{c+dx}\cosh(a+bx)}{16b^3} - \frac{2(c+dx)^{5/2}\cosh(a+bx)}{3b} + \frac{5d^2\sqrt{c+dx}\cosh(3a+3bx)}{144b^3}$$

$$= -\frac{45d^2\sqrt{c+dx}\cosh(a+bx)}{16b^3} - \frac{2(c+dx)^{5/2}\cosh(a+bx)}{3b} + \frac{5d^2\sqrt{c+dx}\cosh(3a+3bx)}{144b^3}$$

$$= -\frac{45d^2\sqrt{c+dx}\cosh(a+bx)}{16b^3} - \frac{2(c+dx)^{5/2}\cosh(a+bx)}{3b} + \frac{5d^2\sqrt{c+dx}\cosh(3a+3bx)}{144b^3}$$

$$= -\frac{45d^2\sqrt{c+dx}\cosh(a+bx)}{16b^3} - \frac{2(c+dx)^{5/2}\cosh(a+bx)}{3b} + \frac{5d^2\sqrt{c+dx}\cosh(3a+3bx)}{144b^3}$$

Mathematica [A] time = 9.44604, size = 243, normalized size = 0.64

$$-\frac{d^3\left(\sqrt{3}\sqrt{-\frac{b(c+dx)}{d}}\,\mathrm{Gamma}\left(\frac{7}{2},-\frac{3b(c+dx)}{d}\right)\left(\sinh\left(3a-\frac{3bc}{d}\right)+\cosh\left(3a-\frac{3bc}{d}\right)\right)+\left(\sinh\left(a-\frac{bc}{d}\right)-\cosh\left(a-\frac{bc}{d}\right)\right)\left(\sqrt{\frac{b(c+dx}{d}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Sinh[a + b*x]^3,x]
```

```
[Out] -(d^3*(Sqrt[3]*Sqrt[-((b*(c + d*x))/d)]*Gamma[7/2, (-3*b*(c + d*x))/d]*(Cos
h[3*a - (3*b*c)/d] + Sinh[3*a - (3*b*c)/d]) + (Sqrt[(b*(c + d*x))/d]*(-243*
Gamma[7/2, (b*(c + d*x))/d] + Sqrt[3]*Gamma[7/2, (3*b*(c + d*x))/d]*(Cosh[2
*a - (2*b*c)/d] - Sinh[2*a - (2*b*c)/d])) + 243*Sqrt[-((b*(c + d*x))/d)]*Ga
mma[7/2, -((b*(c + d*x))/d)]*(Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d]
))*(-Cosh[a - (b*c)/d] + Sinh[a - (b*c)/d])))/(648*b^4*Sqrt[c + d*x])
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{2}} \left(\sinh\left(bx+a\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*sinh(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^(5/2)*sinh(b*x+a)^3,x)
```

Maxima [A] time = 1.82584, size = 693, normalized size = 1.82

$$-\frac{\frac{5\sqrt{3}\sqrt{\pi}d^3 \operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}} + \frac{5\sqrt{3}\sqrt{\pi}d^3 \operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}} - \frac{1215\sqrt{\pi}d^3 \operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}} - \frac{1215\sqrt{\pi}d^3 \operatorname{erf}\left(\sqrt{dx+c}\right.}{b^3\sqrt{\frac{b}{d}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/1728*(5*sqrt(3)*sqrt(pi)*d^3*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*
a - 3*b*c/d)/(b^3*sqrt(-b/d)) + 5*sqrt(3)*sqrt(pi)*d^3*erf(sqrt(3)*sqrt(d*x
 + c)*sqrt(b/d))*e^(-3*a + 3*b*c/d)/(b^3*sqrt(b/d)) - 1215*sqrt(pi)*d^3*erf
(sqrt(d*x + c)*sqrt(-b/d))*e^(a - b*c/d)/(b^3*sqrt(-b/d)) - 1215*sqrt(pi)*d
^3*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^3*sqrt(b/d)) + 162*(4*(d*
x + c)^(5/2)*b^2*d*e^(b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(b*c/d) + 15*sqrt
(d*x + c)*d^3*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^3 - 6*(12*(d*x + c)^(5/2)
*b^2*d*e^(3*b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(3*b*c/d) + 5*sqrt(d*x + c)
*d^3*e^(3*b*c/d))*e^(-3*a - 3*(d*x + c)*b/d)/b^3 - 6*(12*(d*x + c)^(5/2)*b^
2*d*e^(3*a) - 10*(d*x + c)^(3/2)*b*d^2*e^(3*a) + 5*sqrt(d*x + c)*d^3*e^(3*a
))*e^(3*(d*x + c)*b/d - 3*b*c/d)/b^3 + 162*(4*(d*x + c)^(5/2)*b^2*d*e^a - 1
0*(d*x + c)^(3/2)*b*d^2*e^a + 15*sqrt(d*x + c)*d^3*e^a)*e^((d*x + c)*b/d -
b*c/d)/b^3)/d
```

Fricas [B] time = 3.42604, size = 4817, normalized size = 12.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/1728*(5*sqrt(3)*sqrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - d
^3*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^3*cosh(-3*(b*c - a*d)/d) - d
^3*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^3*cosh(b*x + a)*cosh(-3*(
b*c - a*d)/d) - d^3*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 +
 3*(d^3*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) - d^3*cosh(b*x + a)^2*sinh(-
3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b
/d)) - 5*sqrt(3)*sqrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + d^3
*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^3*cosh(-3*(b*c - a*d)/d) + d^3
*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^3*cosh(b*x + a)*cosh(-3*(b*
c - a*d)/d) + d^3*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3
*(d^3*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + d^3*cosh(b*x + a)^2*sinh(-3*
(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b
/d)) - 1215*sqrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) - d^3*cosh(b
*x + a)^3*sinh(-(b*c - a*d)/d) + (d^3*cosh(-(b*c - a*d)/d) - d^3*sinh(-(b*c
 - a*d)/d))*sinh(b*x + a)^3 + 3*(d^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d
^3*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d^3*cosh(b*x +
a)^2*cosh(-(b*c - a*d)/d) - d^3*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(
b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 1215*sqrt(pi)*(d^3*cosh(
b*x + a)^3*cosh(-(b*c - a*d)/d) + d^3*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d)
+ (d^3*cosh(-(b*c - a*d)/d) + d^3*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3
*(d^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d^3*cosh(b*x + a)*sinh(-(b*c - a
*d)/d))*sinh(b*x + a)^2 + 3*(d^3*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) + d^3
*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d
*x + c)*sqrt(-b/d)) - 6*(12*b^3*d^2*x^2 + (12*b^3*d^2*x^2 + 12*b^3*c^2 - 10
*b^2*c*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^6 + 6*(12*
b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2
)*x)*cosh(b*x + a)*sinh(b*x + a)^5 + (12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*
c*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x)*sinh(b*x + a)^6 + 12*b^3*c^2
- 27*(4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*
b^2*d^2)*x)*cosh(b*x + a)^4 - 3*(36*b^3*d^2*x^2 + 36*b^3*c^2 - 90*b^2*c*d +
 135*b*d^2 - 5*(12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d + 5*b*d^2 + 2*(12*
b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^2 + 18*(4*b^3*c*d - 5*b^2*d^2)*x)*sin
h(b*x + a)^4 + 10*b^2*c*d + 4*(5*(12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d
+ 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^3 - 27*(4*b^3*d^2*x
^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x)*cosh(
b*x + a))*sinh(b*x + a)^3 + 5*b*d^2 - 27*(4*b^3*d^2*x^2 + 4*b^3*c^2 + 10*b^
2*c*d + 15*b*d^2 + 2*(4*b^3*c*d + 5*b^2*d^2)*x)*cosh(b*x + a)^2 - 3*(36*b^3
*d^2*x^2 + 36*b^3*c^2 - 5*(12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d + 5*b*d
^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^4 + 90*b^2*c*d + 135*b*d^2
 + 54*(4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5
*b^2*d^2)*x)*cosh(b*x + a)^2 + 18*(4*b^3*c*d + 5*b^2*d^2)*x)*sinh(b*x + a)^
2 + 2*(12*b^3*c*d + 5*b^2*d^2)*x + 6*((12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2
*c*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^5 - 18*(4*b^3*
d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x)*
cosh(b*x + a)^3 - 9*(4*b^3*d^2*x^2 + 4*b^3*c^2 + 10*b^2*c*d + 15*b*d^2 + 2*
(4*b^3*c*d + 5*b^2*d^2)*x)*cosh(b*x + a))*sinh(b*x + a))*sqrt(d*x + c))/(b^
4*cosh(b*x + a)^3 + 3*b^4*cosh(b*x + a)^2*sinh(b*x + a) + 3*b^4*cosh(b*x +
a)*sinh(b*x + a)^2 + b^4*sinh(b*x + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{5}{2}} \sinh (bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/2)*sinh(b*x + a)^3, x)
```

3.54 $\int (c+dx)^{3/2}\sinh^3(a+bx)\,dx$

Optimal. Leaf size=325

$$\frac{9\sqrt{\pi}d^{3/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}}-\frac{\sqrt{\frac{\pi}{3}}d^{3/2}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{9\sqrt{\pi}d^{3/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}}+\frac{\sqrt{\frac{\pi}{3}}d^{3/2}e^{3a-\frac{3bc}{d}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}$$

```
[Out] (-2*(c + d*x)^(3/2)*Cosh[a + b*x])/(3*b) + (9*d^(3/2)*E^(-a + (b*c)/d)*Sqrt
[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(32*b^(5/2)) - (d^(3/2)*E^(-3*a
+ (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(96*b
^(5/2)) - (9*d^(3/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/
Sqrt[d]])/(32*b^(5/2)) + (d^(3/2)*E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt
[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (d*Sqrt[c + d*x]*Sinh[a
 + b*x])/b^2 + ((c + d*x)^(3/2)*Cosh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (d*S
qrt[c + d*x]*Sinh[a + b*x]^3)/(6*b^2)
```

Rubi [A] time = 0.799132, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3311, 3296, 3308, 2180, 2204, 2205, 3312}

$$\frac{9\sqrt{\pi}d^{3/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}}-\frac{\sqrt{\frac{\pi}{3}}d^{3/2}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{9\sqrt{\pi}d^{3/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}}+\frac{\sqrt{\frac{\pi}{3}}d^{3/2}e^{3a-\frac{3bc}{d}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Sinh[a + b*x]^3,x]
```

```
[Out] (-2*(c + d*x)^(3/2)*Cosh[a + b*x])/(3*b) + (9*d^(3/2)*E^(-a + (b*c)/d)*Sqrt
[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(32*b^(5/2)) - (d^(3/2)*E^(-3*a
+ (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(96*b
^(5/2)) - (9*d^(3/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/
Sqrt[d]])/(32*b^(5/2)) + (d^(3/2)*E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt
[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (d*Sqrt[c + d*x]*Sinh[a
 + b*x])/b^2 + ((c + d*x)^(3/2)*Cosh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (d*S
qrt[c + d*x]*Sinh[a + b*x]^3)/(6*b^2)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (c+dx)^{3/2}\sinh^3(a+bx)\,dx = \frac{(c+dx)^{3/2}\cosh(a+bx)\sinh^2(a+bx)}{3b} - \frac{d\sqrt{c+dx}\sinh^3(a+bx)}{6b^2} - \frac{2}{3}\int (c+dx)^{3/2}\mathrm{si}$$

$$= -\frac{2(c+dx)^{3/2}\cosh(a+bx)}{3b} + \frac{(c+dx)^{3/2}\cosh(a+bx)\sinh^2(a+bx)}{3b} - \frac{d\sqrt{c+dx}\,\mathrm{sinl}}{6b^2}$$

$$= -\frac{2(c+dx)^{3/2}\cosh(a+bx)}{3b} + \frac{d\sqrt{c+dx}\sinh(a+bx)}{b^2} + \frac{(c+dx)^{3/2}\cosh(a+bx)\,\mathrm{sinh}}{3b}$$

$$= -\frac{2(c+dx)^{3/2}\cosh(a+bx)}{3b} + \frac{d\sqrt{c+dx}\sinh(a+bx)}{b^2} + \frac{(c+dx)^{3/2}\cosh(a+bx)\,\mathrm{sinh}}{3b}$$

$$= -\frac{2(c+dx)^{3/2}\cosh(a+bx)}{3b} + \frac{d\sqrt{c+dx}\sinh(a+bx)}{b^2} + \frac{(c+dx)^{3/2}\cosh(a+bx)\,\mathrm{sinh}}{3b}$$

$$= -\frac{2(c+dx)^{3/2}\cosh(a+bx)}{3b} + \frac{9d^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}} - \frac{d^{3/2}e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\frac{\sqrt{3}\mathrm{v}}{}\right.}{96b^{5/2}}$$

Mathematica [A] time = 3.89138, size = 243, normalized size = 0.75

$$\frac{d^2\left(\sqrt{3}\sqrt{-\frac{b(c+dx)}{d}}\mathrm{Gamma}\left(\frac{5}{2},-\frac{3b(c+dx)}{d}\right)\left(\sinh\left(3a-\frac{3bc}{d}\right)+\cosh\left(3a-\frac{3bc}{d}\right)\right)+\left(\sinh\left(a-\frac{bc}{d}\right)-\cosh\left(a-\frac{bc}{d}\right)\right)\left(81\sqrt{-}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Sinh[a + b*x]^3,x]
```

```
[Out] (d^2*(Sqrt[3]*Sqrt[-((b*(c + d*x))/d)]*Gamma[5/2, (-3*b*(c + d*x))/d]*(Cosh
[3*a - (3*b*c)/d] + Sinh[3*a - (3*b*c)/d]) + (81*Sqrt[-((b*(c + d*x))/d)]*G
amma[5/2, -((b*(c + d*x))/d)]*(Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d
]) + Sqrt[(b*(c + d*x))/d]*(81*Gamma[5/2, (b*(c + d*x))/d] + Sqrt[3]*Gamma[
5/2, (3*b*(c + d*x))/d]*(-Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d])))*
(-Cosh[a - (b*c)/d] + Sinh[a - (b*c)/d])))/(216*b^3*Sqrt[c + d*x])
```

Maple [F] time = 0.078, size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{3}{2}} (\sinh (bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*sinh(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^(3/2)*sinh(b*x+a)^3,x)
```

Maxima [A] time = 1.79347, size = 581, normalized size = 1.79

$$\frac{\frac{\sqrt{3}\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} - \frac{\sqrt{3}\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} - \frac{81\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} + \frac{81\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/288*(sqrt(3)*sqrt(pi)*d^2*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a -
3*b*c/d)/(b^2*sqrt(-b/d)) - sqrt(3)*sqrt(pi)*d^2*erf(sqrt(3)*sqrt(d*x + c)*
sqrt(b/d))*e^(-3*a + 3*b*c/d)/(b^2*sqrt(b/d)) - 81*sqrt(pi)*d^2*erf(sqrt(d*
x + c)*sqrt(-b/d))*e^(a - b*c/d)/(b^2*sqrt(-b/d)) + 81*sqrt(pi)*d^2*erf(sqr
t(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^2*sqrt(b/d)) - 54*(2*(d*x + c)^(3/2
)*b*d*e^(b*c/d) + 3*sqrt(d*x + c)*d^2*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^2
 + 6*(2*(d*x + c)^(3/2)*b*d*e^(3*b*c/d) + sqrt(d*x + c)*d^2*e^(3*b*c/d))*e^
(-3*a - 3*(d*x + c)*b/d)/b^2 + 6*(2*(d*x + c)^(3/2)*b*d*e^(3*a) - sqrt(d*x
+ c)*d^2*e^(3*a))*e^(3*(d*x + c)*b/d - 3*b*c/d)/b^2 - 54*(2*(d*x + c)^(3/2)
*b*d*e^a - 3*sqrt(d*x + c)*d^2*e^a)*e^((d*x + c)*b/d - b*c/d)/b^2)/d
```

Fricas [B] time = 3.30136, size = 3622, normalized size = 11.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/288*(sqrt(3)*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - d^2*
cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^2*cosh(-3*(b*c - a*d)/d) - d^2*
sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-3*(b*c
 - a*d)/d) - d^2*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*
```

```
(d^2*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) - d^2*cosh(b*x + a)^2*sinh(-3*(
b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)
) + sqrt(3)*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + d^2*cosh
(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^2*cosh(-3*(b*c - a*d)/d) + d^2*sinh
(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-3*(b*c - a
*d)/d) + d^2*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d^2
*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + d^2*cosh(b*x + a)^2*sinh(-3*(b*c
- a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))
- 81*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) - d^2*cosh(b*x + a)
^3*sinh(-(b*c - a*d)/d) + (d^2*cosh(-(b*c - a*d)/d) - d^2*sinh(-(b*c - a*d)
/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d^2*cosh
(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d^2*cosh(b*x + a)^2*co
sh(-(b*c - a*d)/d) - d^2*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a
))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 81*sqrt(pi)*(d^2*cosh(b*x + a)^
3*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + (d^2*co
sh(-(b*c - a*d)/d) + d^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cos
h(b*x + a)*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*s
inh(b*x + a)^2 + 3*(d^2*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x
 + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x + c)*s
qrt(-b/d)) - 6*((2*b^2*d*x + 2*b^2*c - b*d)*cosh(b*x + a)^6 + 6*(2*b^2*d*x
+ 2*b^2*c - b*d)*cosh(b*x + a)*sinh(b*x + a)^5 + (2*b^2*d*x + 2*b^2*c - b*d
)*sinh(b*x + a)^6 - 9*(2*b^2*d*x + 2*b^2*c - 3*b*d)*cosh(b*x + a)^4 - 3*(6*
b^2*d*x + 6*b^2*c - 5*(2*b^2*d*x + 2*b^2*c - b*d)*cosh(b*x + a)^2 - 9*b*d)*
sinh(b*x + a)^4 + 2*b^2*d*x + 4*(5*(2*b^2*d*x + 2*b^2*c - b*d)*cosh(b*x + a
)^3 - 9*(2*b^2*d*x + 2*b^2*c - 3*b*d)*cosh(b*x + a))*sinh(b*x + a)^3 + 2*b^
2*c - 9*(2*b^2*d*x + 2*b^2*c + 3*b*d)*cosh(b*x + a)^2 + 3*(5*(2*b^2*d*x + 2
*b^2*c - b*d)*cosh(b*x + a)^4 - 6*b^2*d*x - 6*b^2*c - 18*(2*b^2*d*x + 2*b^2
*c - 3*b*d)*cosh(b*x + a)^2 - 9*b*d)*sinh(b*x + a)^2 + b*d + 6*((2*b^2*d*x
+ 2*b^2*c - b*d)*cosh(b*x + a)^5 - 6*(2*b^2*d*x + 2*b^2*c - 3*b*d)*cosh(b*x
 + a)^3 - 3*(2*b^2*d*x + 2*b^2*c + 3*b*d)*cosh(b*x + a))*sinh(b*x + a))*sqr
t(d*x + c))/(b^3*cosh(b*x + a)^3 + 3*b^3*cosh(b*x + a)^2*sinh(b*x + a) + 3*
b^3*cosh(b*x + a)*sinh(b*x + a)^2 + b^3*sinh(b*x + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^{\frac{3}{2}}\sinh(bx+a)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(3/2)*sinh(b*x + a)^3, x)
```

3.55 $\int \sqrt{c+dx}\sinh^3(a+bx)\,dx$

Optimal. Leaf size=275

$$\frac{3\sqrt{\pi}\sqrt{d}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}+\frac{3\sqrt{\pi}\sqrt{d}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}e^{3a-\frac{3bc}{d}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+}}{\sqrt{d}}\right.}{48b^{3/2}}$$

```
[Out] (-3*Sqrt[c + d*x]*Cosh[a + b*x])/(4*b) + (Sqrt[c + d*x]*Cosh[3*a + 3*b*x])/
(12*b) + (3*Sqrt[d]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/S
qrt[d]])/(16*b^(3/2)) - (Sqrt[d]*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[
3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) + (3*Sqrt[d]*E^(a - (b*c)/
d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) - (Sqrt[d]*
E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]
])/(48*b^(3/2))
```

Rubi [A] time = 0.521909, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3312, 3296, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\pi}\sqrt{d}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}+\frac{3\sqrt{\pi}\sqrt{d}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}e^{3a-\frac{3bc}{d}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+}}{\sqrt{d}}\right.}{48b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Sinh[a + b*x]^3,x]
```


```
[Out] (-3*Sqrt[c + d*x]*Cosh[a + b*x])/(4*b) + (Sqrt[c + d*x]*Cosh[3*a + 3*b*x])/
(12*b) + (3*Sqrt[d]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/S
qrt[d]])/(16*b^(3/2)) - (Sqrt[d]*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[
3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) + (3*Sqrt[d]*E^(a - (b*c)/
d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) - (Sqrt[d]*
E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]
])/(48*b^(3/2))
```


Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \sqrt{c+dx}\sinh^3(a+bx)\,dx = i\int\left(\frac{3}{4}i\sqrt{c+dx}\sinh(a+bx)-\frac{1}{4}i\sqrt{c+dx}\sinh(3a+3bx)\right)dx$$
$$=\frac{1}{4}\int\sqrt{c+dx}\sinh(3a+3bx)\,dx-\frac{3}{4}\int\sqrt{c+dx}\sinh(a+bx)\,dx$$
$$=-\frac{3\sqrt{c+dx}\cosh(a+bx)}{4b}+\frac{\sqrt{c+dx}\cosh(3a+3bx)}{12b}-\frac{d\int\frac{\cosh(3a+3bx)}{\sqrt{c+dx}}\,dx}{24b}+\frac{(3d)\int\frac{\cosh}{\sqrt{}}}{8b}$$
$$=-\frac{3\sqrt{c+dx}\cosh(a+bx)}{4b}+\frac{\sqrt{c+dx}\cosh(3a+3bx)}{12b}-\frac{d\int\frac{e^{-i(3ia+3ibx)}}{\sqrt{c+dx}}\,dx}{48b}-\frac{d\int\frac{e^{i(3ia+3ibx)}}{\sqrt{c+dx}}}{48b}$$
$$=-\frac{3\sqrt{c+dx}\cosh(a+bx)}{4b}+\frac{\sqrt{c+dx}\cosh(3a+3bx)}{12b}-\frac{\text{Subst}\left(\int e^{i\left(3ia-\frac{3ibc}{d}\right)-\frac{3bx^2}{d}}dx,x,\right.}{24b}$$
$$=-\frac{3\sqrt{c+dx}\cosh(a+bx)}{4b}+\frac{\sqrt{c+dx}\cosh(3a+3bx)}{12b}+\frac{3\sqrt{d}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-$$

Mathematica [A] time = 0.291349, size = 209, normalized size = 0.76

$$\frac{\sqrt{c+dx}e^{-3\left(a+\frac{bc}{d}\right)}\left(\sqrt{3}e^{6a}\sqrt{\frac{b(c+dx)}{d}}\text{Gamma}\left(\frac{3}{2},-\frac{3b(c+dx)}{d}\right)-27e^{4a+\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\text{Gamma}\left(\frac{3}{2},-\frac{b(c+dx)}{d}\right)+e^{\frac{4bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\left(\right.\right.}{72b\sqrt{-\frac{b^2(c+dx)^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Sinh[a + b*x]^3,x]
```

```
[Out] (Sqrt[c + d*x]*(Sqrt[3]*E^(6*a)*Sqrt[(b*(c + d*x))/d]*Gamma[3/2, (-3*b*(c +
 d*x))/d] - 27*E^(4*a + (2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[3/2, -((b*(c
 + d*x))/d)] + E^((4*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*(-27*E^(2*a)*Gamma[3/
2, (b*(c + d*x))/d] + Sqrt[3]*E^((2*b*c)/d)*Gamma[3/2, (3*b*(c + d*x))/d]))
)/(72*b*E^(3*(a + (b*c)/d))*Sqrt[-((b^2*(c + d*x)^2)/d^2)])
```

Maple [F] time = 0.078, size = 0, normalized size = 0.

$$\int(\sinh(bx+a))^3\sqrt{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3*(d*x+c)^(1/2),x)
```

```
[Out] int(sinh(b*x+a)^3*(d*x+c)^(1/2),x)
```

Maxima [A] time = 1.88603, size = 450, normalized size = 1.64

$$-\frac{\frac{\sqrt{3}\sqrt{\pi}d\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}}+\frac{\sqrt{3}\sqrt{\pi}d\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b\sqrt{\frac{b}{d}}}-\frac{27\sqrt{\pi}d\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}}-\frac{27\sqrt{\pi}d\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b\sqrt{\frac{b}{d}}}-\cdots}{144d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/144*(sqrt(3)*sqrt(pi)*d*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a - 3
*b*c/d)/(b*sqrt(-b/d)) + sqrt(3)*sqrt(pi)*d*erf(sqrt(3)*sqrt(d*x + c)*sqrt(
b/d))*e^(-3*a + 3*b*c/d)/(b*sqrt(b/d)) - 27*sqrt(pi)*d*erf(sqrt(d*x + c)*sq
rt(-b/d))*e^(a - b*c/d)/(b*sqrt(-b/d)) - 27*sqrt(pi)*d*erf(sqrt(d*x + c)*sq
rt(b/d))*e^(-a + b*c/d)/(b*sqrt(b/d)) - 6*sqrt(d*x + c)*d*e^(3*a + 3*(d*x +
 c)*b/d - 3*b*c/d)/b + 54*sqrt(d*x + c)*d*e^(a + (d*x + c)*b/d - b*c/d)/b +
 54*sqrt(d*x + c)*d*e^(-a - (d*x + c)*b/d + b*c/d)/b - 6*sqrt(d*x + c)*d*e^
(-3*a - 3*(d*x + c)*b/d + 3*b*c/d)/b)/d
```

Fricas [B] time = 2.99826, size = 2966, normalized size = 10.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/144*(sqrt(3)*sqrt(pi)*(d*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - d*cosh
(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d*cosh(-3*(b*c - a*d)/d) - d*sinh(-3*
(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d)
 - d*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d*cosh(b*x
+ a)^2*cosh(-3*(b*c - a*d)/d) - d*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*s
inh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) - sqrt(3)*sqrt
(pi)*(d*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + d*cosh(b*x + a)^3*sinh(-3*
(b*c - a*d)/d) + (d*cosh(-3*(b*c - a*d)/d) + d*sinh(-3*(b*c - a*d)/d))*sinh
(b*x + a)^3 + 3*(d*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) + d*cosh(b*x + a)*s
inh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d*cosh(b*x + a)^2*cosh(-3*(b*c
- a*d)/d) + d*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-
b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d)) - 27*sqrt(pi)*(d*cosh(b*x + a)^3
*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + (d*cosh(-(
b*c - a*d)/d) - d*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d*cosh(b*x + a
)*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a
)^2 + 3*(d*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)^2*sinh(-(
b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 27*s
qrt(pi)*(d*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) + d*cosh(b*x + a)^3*sinh(-(
b*c - a*d)/d) + (d*cosh(-(b*c - a*d)/d) + d*sinh(-(b*c - a*d)/d))*sinh(b*x
+ a)^3 + 3*(d*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d*cosh(b*x + a)*sinh(-(b
```

```
*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) +
 d*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt
(d*x + c)*sqrt(-b/d)) - 6*(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x +
 a)^5 + b*sinh(b*x + a)^6 - 9*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 -
3*b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 - 9*b*cosh(b*x + a))*sinh(b*x
 + a)^3 - 9*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 - 18*b*cosh(b*x + a)
^2 - 3*b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^5 - 6*b*cosh(b*x + a)^3 - 3*
b*cosh(b*x + a))*sinh(b*x + a) + b)*sqrt(d*x + c))/(b^2*cosh(b*x + a)^3 + 3
*b^2*cosh(b*x + a)^2*sinh(b*x + a) + 3*b^2*cosh(b*x + a)*sinh(b*x + a)^2 +
b^2*sinh(b*x + a)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c+dx}\sinh^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*sinh(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx+c}\sinh(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x + c)*sinh(b*x + a)^3, x)
```

3.56 $\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}} \, dx$

Optimal. Leaf size=228

$$\frac{3\sqrt{\pi}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{3}}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{3\sqrt{\pi}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{3}}e^{3a-\frac{3bc}{d}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}$$

```
[Out] (3*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[
b]*Sqrt[d]) - (E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c
+ d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) - (3*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(S
qrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) + (E^(3*a - (3*b*c)/d)*
Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d
])
```

Rubi [A] time = 0.401139, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3312, 3308, 2180, 2204, 2205}

$$\frac{3\sqrt{\pi}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{3}}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{3\sqrt{\pi}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{3}}e^{3a-\frac{3bc}{d}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3/Sqrt[c + d*x],x]
```

```
[Out] (3*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[
b]*Sqrt[d]) - (E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c
+ d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) - (3*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(S
qrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) + (E^(3*a - (3*b*c)/d)*
Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d
])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx &= i\int \left(\frac{3i\sinh(a+bx)}{4\sqrt{c+dx}}-\frac{i\sinh(3a+3bx)}{4\sqrt{c+dx}}\right)dx\\ &= \frac{1}{4}\int \frac{\sinh(3a+3bx)}{\sqrt{c+dx}}\,dx-\frac{3}{4}\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx\\ &= \frac{1}{8}\int \frac{e^{-i(3ia+3ibx)}}{\sqrt{c+dx}}\,dx-\frac{1}{8}\int \frac{e^{i(3ia+3ibx)}}{\sqrt{c+dx}}\,dx-\frac{3}{8}\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx+\frac{3}{8}\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx\\ &= -\frac{\operatorname{Subst}\left(\int e^{i\left(3ia-\frac{3ibc}{d}\right)-\frac{3bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{4d}+\frac{\operatorname{Subst}\left(\int e^{-i\left(3ia-\frac{3ibc}{d}\right)+\frac{3bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{4d}+\frac{3\operatorname{Sub}}{}\\ &= \frac{3e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{3e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{e^{3a-\frac{3bc}{d}}\sqrt{}}{}\end{aligned}$$

Mathematica [A] time = 0.194076, size = 191, normalized size = 0.84

$$\frac{e^{-3\left(a+\frac{bc}{d}\right)}\left(\sqrt{3}e^{6a}\sqrt{-\frac{b(c+dx)}{d}}\text{Gamma}\left(\frac{1}{2},-\frac{3b(c+dx)}{d}\right)-9e^{4a+\frac{2bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\text{Gamma}\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)+e^{\frac{4bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\left(\sqrt{3}e^{\frac{2bc}{d}}\text{Ga}\right.\right.}{24b\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[3]*E^(6*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-3*b*(c + d*x))/d] -
9*E^(4*a + (2*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*x))/d
)] + E^((4*b*c)/d)*Sqrt[(b*(c + d*x))/d]*(-9*E^(2*a)*Gamma[1/2, (b*(c + d*x
))/d] + Sqrt[3]*E^((2*b*c)/d)*Gamma[1/2, (3*b*(c + d*x))/d]))/(24*b*E^(3*(a
 + (b*c)/d))*Sqrt[c + d*x])
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int (\sinh(bx+a))^3\frac{1}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3/(d*x+c)^(1/2),x)
```

```
[Out] int(sinh(b*x+a)^3/(d*x+c)^(1/2),x)
```

Maxima [A] time = 1.70079, size = 240, normalized size = 1.05

$$\frac{\frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}-\frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{\sqrt{\frac{b}{d}}}-\frac{9\sqrt{\pi}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}+\frac{9\sqrt{\pi}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{\sqrt{\frac{b}{d}}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/24*(sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a - 3*b*c
/d)/sqrt(-b/d) - sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d))*e^(-
3*a + 3*b*c/d)/sqrt(b/d) - 9*sqrt(pi)*erf(sqrt(d*x + c)*sqrt(-b/d))*e^(a -
b*c/d)/sqrt(-b/d) + 9*sqrt(pi)*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/
sqrt(b/d))/d
```

Fricas [A] time = 2.72496, size = 595, normalized size = 2.61

$$-\frac{\sqrt{3}\sqrt{\pi}\sqrt{\frac{b}{d}}\left(\cosh\left(-\frac{3(bc-ad)}{d}\right)-\sinh\left(-\frac{3(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)+\sqrt{3}\sqrt{\pi}\sqrt{-\frac{b}{d}}\left(\cosh\left(-\frac{3(bc-ad)}{d}\right)+\sinh\left(-\frac{3(bc-ad)}{d}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(sqrt(3)*sqrt(pi)*sqrt(b/d)*(cosh(-3*(b*c - a*d)/d) - sinh(-3*(b*c -
a*d)/d))*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) + sqrt(3)*sqrt(pi)*sqrt(-b/d)
*(cosh(-3*(b*c - a*d)/d) + sinh(-3*(b*c - a*d)/d))*erf(sqrt(3)*sqrt(d*x + c
)*sqrt(-b/d)) - 9*sqrt(pi)*sqrt(b/d)*(cosh(-(b*c - a*d)/d) - sinh(-(b*c - a
*d)/d))*erf(sqrt(d*x + c)*sqrt(b/d)) - 9*sqrt(pi)*sqrt(-b/d)*(cosh(-(b*c -
a*d)/d) + sinh(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-b/d)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3/(d*x+c)**(1/2),x)
```

```
[Out] Integral(sinh(a + b*x)**3/sqrt(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx+a)^3}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(sinh(b*x + a)^3/sqrt(d*x + c), x)
```

3.57 $\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=246

$$-\frac{3\sqrt{\pi}\sqrt{b}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}+\frac{\sqrt{3\pi}\sqrt{b}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}-\frac{3\sqrt{\pi}\sqrt{b}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}+\frac{\sqrt{3\pi}\sqrt{b}e^{3a-\frac{3bc}{d}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\cdots}{\cdots}\right.}{4d^{3/2}}$$

```
[Out] (-3*Sqrt[b]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])
/(4*d^(3/2)) + (Sqrt[b]*E^(-3*a + (3*b*c)/d)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[b
]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/2)) - (3*Sqrt[b]*E^(a - (b*c)/d)*Sqrt[Pi
]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/2)) + (Sqrt[b]*E^(3*a - (3
*b*c)/d)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/
2)) - (2*Sinh[a + b*x]^3)/(d*Sqrt[c + d*x])
```

Rubi [A] time = 0.453834, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3313, 3307, 2180, 2204, 2205}

$$-\frac{3\sqrt{\pi}\sqrt{b}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}+\frac{\sqrt{3\pi}\sqrt{b}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}-\frac{3\sqrt{\pi}\sqrt{b}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}+\frac{\sqrt{3\pi}\sqrt{b}e^{3a-\frac{3bc}{d}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\cdots}{\cdots}\right.}{4d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3/(c + d*x)^(3/2),x]
```

```
[Out] (-3*Sqrt[b]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])
/(4*d^(3/2)) + (Sqrt[b]*E^(-3*a + (3*b*c)/d)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[b
]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/2)) - (3*Sqrt[b]*E^(a - (b*c)/d)*Sqrt[Pi
]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/2)) + (Sqrt[b]*E^(3*a - (3
*b*c)/d)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/
2)) - (2*Sinh[a + b*x]^3)/(d*Sqrt[c + d*x])
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}}\,dx &= -\frac{2\sinh^3(a+bx)}{d\sqrt{c+dx}} - \frac{(6b)\int \left(\frac{\cosh(a+bx)}{4\sqrt{c+dx}} - \frac{\cosh(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d} \\
&= -\frac{2\sinh^3(a+bx)}{d\sqrt{c+dx}} - \frac{(3b)\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx}{2d} + \frac{(3b)\int \frac{\cosh(3a+3bx)}{\sqrt{c+dx}}\,dx}{2d} \\
&= -\frac{2\sinh^3(a+bx)}{d\sqrt{c+dx}} - \frac{(3b)\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{4d} - \frac{(3b)\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{4d} + \frac{(3b)\int \frac{e^{-i(3ia+3ibx)}}{\sqrt{c+dx}}\,dx}{4d} + \frac{(3b)\int \frac{e^{i(3ia+3}}{\sqrt{c+d}}}{4d} \\
&= -\frac{2\sinh^3(a+bx)}{d\sqrt{c+dx}} + \frac{(3b)\,\mathrm{Subst}\left(\int e^{i\left(3ia-\frac{3ibc}{d}\right)-\frac{3bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{2d^2} - \frac{(3b)\,\mathrm{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\right.}{2d^2} \\
&= -\frac{3\sqrt{b}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} + \frac{\sqrt{b}e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} - \frac{3\sqrt{b}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}
\end{aligned}$$

Mathematica [B] time = 10.2652, size = 2058, normalized size = 8.37

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3/(c + d*x)^(3/2),x]
```

```
[Out] (-3*(Cosh[a]*(-(((-((1 + E^((2*b*(c + d*x))/d))/E^((b*(c + d*x))/d)) + Sqrt
[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*x))/d)] + Sqrt[(b*(c + d*x))/d]
*Gamma[1/2, (b*(c + d*x))/d])*Sinh[(b*c)/d])/(d*Sqrt[c + d*x])) + (Cosh[(b*
c)/d]*(Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*x))/d)] - Sqrt[(b*(c
 + d*x))/d]*Gamma[1/2, (b*(c + d*x))/d] - 2*Sinh[(b*(c + d*x))/d]))/(d*Sqrt
[c + d*x])) + Sinh[a]*((Cosh[(b*c)/d]*(-((1 + E^((2*b*(c + d*x))/d))/E^((b*
(c + d*x))/d)) + Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*x))/d)] +
Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (b*(c + d*x))/d]))/(d*Sqrt[c + d*x]) + (Si
nh[(b*c)/d]*(-(Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*x))/d)]) + S
qrt[(b*(c + d*x))/d]*Gamma[1/2, (b*(c + d*x))/d] + 2*Sinh[(b*(c + d*x))/d])
)/(d*Sqrt[c + d*x]))))/4 + (-(Sinh[3*a]*(-(((1 + 2*Cosh[(2*b*c)/d])*(-((1 +
 E^((6*b*(c + d*x))/d))/E^((3*b*(c + d*x))/d)) + Sqrt[3]*Sqrt[-((b*(c + d*x
))/d)]*Gamma[1/2, (-3*b*(c + d*x))/d] + Sqrt[3]*Sqrt[(b*(c + d*x))/d]*Gamma
[1/2, (3*b*(c + d*x))/d])*Sinh[(b*c)/d])/(d*Sqrt[c + d*x])) + (Cosh[(b*c)/d
]*(-1 + 2*Cosh[(2*b*c)/d])*((Sqrt[b]*Sqrt[6*Pi]*(Erf[(Sqrt[3]*Sqrt[b]*Sqrt[
c + d*x])/Sqrt[d]] + Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/Sqrt[d
] - (2*Sqrt[2]*Sinh[(3*b*(c + d*x))/d])/Sqrt[c + d*x]))/(Sqrt[2]*d))) - Cos
h[3*a]*((Cosh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*(-((1 + E^((6*b*(c + d*x))/
d))/E^((3*b*(c + d*x))/d)) + Sqrt[3]*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-
3*b*(c + d*x))/d] + Sqrt[3]*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (3*b*(c + d*x)
```

```
)/d]))/(d*Sqrt[c + d*x]) - ((1 + 2*Cosh[(2*b*c)/d])*Sinh[(b*c)/d]*((Sqrt[b]
*Sqrt[6*Pi]*(Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + Erfi[(Sqrt[3]*S
qrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/Sqrt[d] - (2*Sqrt[2]*Sinh[(3*b*(c + d*x))/
d])/Sqrt[c + d*x]))/(Sqrt[2]*d)))/8 + (Sinh[3*a]*(-(((1 + 2*Cosh[(2*b*c)/d]
)*(-((1 + E^((6*b*(c + d*x))/d))/E^((3*b*(c + d*x))/d)) + Sqrt[3]*Sqrt[-((b
*(c + d*x))/d)]*Gamma[1/2, (-3*b*(c + d*x))/d] + Sqrt[3]*Sqrt[(b*(c + d*x))
/d]*Gamma[1/2, (3*b*(c + d*x))/d])*Sinh[(b*c)/d])/(d*Sqrt[c + d*x])) + (Cos
h[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*((Sqrt[b]*Sqrt[6*Pi]*(Erf[(Sqrt[3]*Sqrt
[b]*Sqrt[c + d*x])/Sqrt[d]] + Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]
))/Sqrt[d] - (2*Sqrt[2]*Sinh[(3*b*(c + d*x))/d])/Sqrt[c + d*x]))/(Sqrt[2]*d
)) + Cosh[3*a]*((Cosh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*(-((1 + E^((6*b*(c
+ d*x))/d))/E^((3*b*(c + d*x))/d)) + Sqrt[3]*Sqrt[-((b*(c + d*x))/d)]*Gamma
[1/2, (-3*b*(c + d*x))/d] + Sqrt[3]*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (3*b*(
c + d*x))/d]))/(d*Sqrt[c + d*x]) - ((1 + 2*Cosh[(2*b*c)/d])*Sinh[(b*c)/d]*(
(Sqrt[b]*Sqrt[6*Pi]*(Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + Erfi[(S
qrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/Sqrt[d] - (2*Sqrt[2]*Sinh[(3*b*(c
+ d*x))/d])/Sqrt[c + d*x]))/(Sqrt[2]*d)))/8 + (Cosh[3*a]*(-(((1 + 2*Cosh[(2
*b*c)/d])*(-((1 + E^((6*b*(c + d*x))/d))/E^((3*b*(c + d*x))/d)) + Sqrt[3]*S
qrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-3*b*(c + d*x))/d] + Sqrt[3]*Sqrt[(b*(c
 + d*x))/d]*Gamma[1/2, (3*b*(c + d*x))/d])*Sinh[(b*c)/d])/(d*Sqrt[c + d*x])
) + (Cosh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*((Sqrt[b]*Sqrt[6*Pi]*(Erf[(Sqrt
[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/
Sqrt[d]]))/Sqrt[d] - (2*Sqrt[2]*Sinh[(3*b*(c + d*x))/d])/Sqrt[c + d*x]))/(S
qrt[2]*d)) + Sinh[3*a]*((Cosh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*(-((1 + E^(
(6*b*(c + d*x))/d))/E^((3*b*(c + d*x))/d)) + Sqrt[3]*Sqrt[-((b*(c + d*x))/d
)]*Gamma[1/2, (-3*b*(c + d*x))/d] + Sqrt[3]*Sqrt[(b*(c + d*x))/d]*Gamma[1/2
, (3*b*(c + d*x))/d]))/(d*Sqrt[c + d*x]) - ((1 + 2*Cosh[(2*b*c)/d])*Sinh[(b
*c)/d]*((Sqrt[b]*Sqrt[6*Pi]*(Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] +
 Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/Sqrt[d] - (2*Sqrt[2]*Sinh[
(3*b*(c + d*x))/d])/Sqrt[c + d*x]))/(Sqrt[2]*d)))/4
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int (\sinh(bx+a))^3 (dx+c)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3/(d*x+c)^(3/2),x)
```

```
[Out] int(sinh(b*x+a)^3/(d*x+c)^(3/2),x)
```

Maxima [A] time = 1.47258, size = 266, normalized size = 1.08

$$\frac{\frac{\sqrt{3}\sqrt{\frac{(dx+c)b}{d}}e^{\left(\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{3(dx+c)b}{d}\right)}{\sqrt{dx+c}} - \frac{\sqrt{3}\sqrt{-\frac{(dx+c)b}{d}}e^{\left(-\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{3(dx+c)b}{d}\right)}{\sqrt{dx+c}} - \frac{3\sqrt{\frac{(dx+c)b}{d}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}} + \frac{3\sqrt{-\frac{(dx+c)b}{d}}e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{(dx+\cdots}{d}\right)}{\sqrt{dx+c}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(3)*sqrt((d*x + c)*b/d)*e^(3*(b*c - a*d)/d)*gamma(-1/2, 3*(d*x + c
)*b/d)/sqrt(d*x + c) - sqrt(3)*sqrt(-(d*x + c)*b/d)*e^(-3*(b*c - a*d)/d)*ga
mma(-1/2, -3*(d*x + c)*b/d)/sqrt(d*x + c) - 3*sqrt((d*x + c)*b/d)*e^(-a + b
```

```
*c/d)*gamma(-1/2, (d*x + c)*b/d)/sqrt(d*x + c) + 3*sqrt(-(d*x + c)*b/d)*e^(
a - b*c/d)*gamma(-1/2, -(d*x + c)*b/d)/sqrt(d*x + c))/d
```

Fricas [B] time = 3.01363, size = 3312, normalized size = 13.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(3)*sqrt(pi)*((d*x + c)*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - (
d*x + c)*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + ((d*x + c)*cosh(-3*(b*c -
 a*d)/d) - (d*x + c)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((d*x + c)
*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)*sinh(-3*(b*
c - a*d)/d))*sinh(b*x + a)^2 + 3*((d*x + c)*cosh(b*x + a)^2*cosh(-3*(b*c -
a*d)/d) - (d*x + c)*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*
sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) - sqrt(3)*sqrt(pi)*((d*x + c
)*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)^3*sinh(-
3*(b*c - a*d)/d) + ((d*x + c)*cosh(-3*(b*c - a*d)/d) + (d*x + c)*sinh(-3*(b
*c - a*d)/d))*sinh(b*x + a)^3 + 3*((d*x + c)*cosh(b*x + a)*cosh(-3*(b*c - a
*d)/d) + (d*x + c)*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 +
3*((d*x + c)*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + (d*x + c)*cosh(b*x +
a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x
 + c)*sqrt(-b/d)) - 3*sqrt(pi)*((d*x + c)*cosh(b*x + a)^3*cosh(-(b*c - a*d)
/d) - (d*x + c)*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + ((d*x + c)*cosh(-(b*
c - a*d)/d) - (d*x + c)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((d*x + c
)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)*sinh(-(b*c -
 a*d)/d))*sinh(b*x + a)^2 + 3*((d*x + c)*cosh(b*x + a)^2*cosh(-(b*c - a*d)/
d) - (d*x + c)*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/
d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 3*sqrt(pi)*((d*x + c)*cosh(b*x + a)^3*cos
h(-(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + ((d*x
+ c)*cosh(-(b*c - a*d)/d) + (d*x + c)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3
 + 3*((d*x + c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a
)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((d*x + c)*cosh(b*x + a)^2*cosh
(-(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x
 + a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) - (cosh(b*x + a)^6 + 6*cosh
(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 - 1)*sin
h(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - 3*cosh(b*x + a))*
sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sinh(b*x +
a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 + cosh(b*
x + a))*sinh(b*x + a) - 1)*sqrt(d*x + c))/((d^2*x + c*d)*cosh(b*x + a)^3 +
3*(d^2*x + c*d)*cosh(b*x + a)^2*sinh(b*x + a) + 3*(d^2*x + c*d)*cosh(b*x +
a)*sinh(b*x + a)^2 + (d^2*x + c*d)*sinh(b*x + a)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3/(d*x+c)**(3/2),x)
```

```
[Out] Integral(sinh(a + b*x)**3/(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx+a)^3}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^3/(d*x + c)^(3/2), x)
```

3.58 $\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=277

$$\frac{\sqrt{\pi}b^{3/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}-\frac{\sqrt{3\pi}b^{3/2}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}-\frac{\sqrt{\pi}b^{3/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}+\frac{\sqrt{3\pi}b^{3/2}e^{3a-\frac{3bc}{d}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{}}{}\right.}{2d^{5/2}}$$

```
[Out] (b^(3/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2
*d^(5/2)) - (b^(3/2)*E^(-3*a + (3*b*c)/d)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[b]*S
qrt[c + d*x])/Sqrt[d]])/(2*d^(5/2)) - (b^(3/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erf
i[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*d^(5/2)) + (b^(3/2)*E^(3*a - (3*b*c)
/d)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*d^(5/2)) -
 (4*b*Cosh[a + b*x]*Sinh[a + b*x]^2)/(d^2*Sqrt[c + d*x]) - (2*Sinh[a + b*x]
^3)/(3*d*(c + d*x)^(3/2))
```

Rubi [A] time = 0.672687, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3314, 3308, 2180, 2204, 2205, 3312}

$$\frac{\sqrt{\pi}b^{3/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}-\frac{\sqrt{3\pi}b^{3/2}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}-\frac{\sqrt{\pi}b^{3/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}+\frac{\sqrt{3\pi}b^{3/2}e^{3a-\frac{3bc}{d}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{}}{}\right.}{2d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3/(c + d*x)^(5/2),x]
```


```
[Out] (b^(3/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2
*d^(5/2)) - (b^(3/2)*E^(-3*a + (3*b*c)/d)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[b]*S
qrt[c + d*x])/Sqrt[d]])/(2*d^(5/2)) - (b^(3/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erf
i[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*d^(5/2)) + (b^(3/2)*E^(3*a - (3*b*c)
/d)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*d^(5/2)) -
 (4*b*Cosh[a + b*x]*Sinh[a + b*x]^2)/(d^2*Sqrt[c + d*x]) - (2*Sinh[a + b*x]
^3)/(3*d*(c + d*x)^(3/2))
```


Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{4b\cosh(a+bx)\sinh^2(a+bx)}{d^2\sqrt{c+dx}} - \frac{2\sinh^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{\left(8b^2\right)\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx}{d^2} + \frac{\left(12b^2\right)\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}\,d$$

$$= -\frac{4b\cosh(a+bx)\sinh^2(a+bx)}{d^2\sqrt{c+dx}} - \frac{2\sinh^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{\left(12ib^2\right)\int \left(\frac{3i\sinh(a+bx)}{4\sqrt{c+dx}} - \frac{i\sinh(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d^2} + $$

$$= -\frac{4b\cosh(a+bx)\sinh^2(a+bx)}{d^2\sqrt{c+dx}} - \frac{2\sinh^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{\left(8b^2\right)\text{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= -\frac{4b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} + \frac{4b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} - \frac{4b\cosh(a+bx)\sinh^2(a+bx)}{d^2\sqrt{c+dx}} - 2$$

$$= -\frac{4b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} + \frac{4b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} - \frac{4b\cosh(a+bx)\sinh^2(a+bx)}{d^2\sqrt{c+dx}} - 2$$

$$= \frac{b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} - \frac{b^{3/2}e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} - \frac{b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} + b^{3/2}$$

Mathematica [A] time = 3.0608, size = 253, normalized size = 0.91

$$e^{-3\left(a+\frac{bc}{d}\right)}\left(-3\sqrt{3}e^{6a}d\left(-\frac{b(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{3b(c+dx)}{d}\right) + 3de^{4a+\frac{2bc}{d}}\left(-\frac{b(c+dx)}{d}\right)^{3/2}\text{Gamma}\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right) - 3de^{2a+\frac{4bc}{d}}\left(\frac{b(c+d}{d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3/(c + d*x)^(5/2),x]
```

```
[Out] (-3*Sqrt[3]*d*E^(6*a)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, (-3*b*(c + d*x)
)/d] + 3*d*E^(4*a + (2*b*c)/d)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, -((b*(
c + d*x))/d)] - 3*d*E^(2*a + (4*b*c)/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2,
(b*(c + d*x))/d] + 3*Sqrt[3]*d*E^((6*b*c)/d)*((b*(c + d*x))/d)^(3/2)*Gamma[
```

```
1/2, (3*b*(c + d*x))/d] - 4*E^(3*(a + (b*c)/d))*Sinh[a + b*x]^2*(6*b*(c + d
*x)*Cosh[a + b*x] + d*Sinh[a + b*x]))/(6*d^2*E^(3*(a + (b*c)/d))*(c + d*x)^
(3/2))
```

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int (\sinh (bx+a))^3 (dx+c)^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3/(d*x+c)^(5/2),x)
```

```
[Out] int(sinh(b*x+a)^3/(d*x+c)^(5/2),x)
```

Maxima [A] time = 1.42798, size = 265, normalized size = 0.96

$$\frac{3\left(\frac{\sqrt{3}\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}} e^{\left(\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2}, \frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}} - \frac{\sqrt{3}\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}} e^{\left(-\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2}, -\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}} - \frac{\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}} e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{3}{2}, \frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}} + \frac{\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}} e^{\left(a-\frac{bc}{d}\right)}}{(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 3/8*(sqrt(3)*((d*x + c)*b/d)^(3/2)*e^(3*(b*c - a*d)/d)*gamma(-3/2, 3*(d*x +
 c)*b/d)/(d*x + c)^(3/2) - sqrt(3)*(-(d*x + c)*b/d)^(3/2)*e^(-3*(b*c - a*d)
/d)*gamma(-3/2, -3*(d*x + c)*b/d)/(d*x + c)^(3/2) - ((d*x + c)*b/d)^(3/2)*e
^(-a + b*c/d)*gamma(-3/2, (d*x + c)*b/d)/(d*x + c)^(3/2) + (-(d*x + c)*b/d)
^(3/2)*e^(a - b*c/d)*gamma(-3/2, -(d*x + c)*b/d)/(d*x + c)^(3/2))/d
```

Fricas [B] time = 3.15274, size = 4826, normalized size = 17.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(6*sqrt(3)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*
cosh(-3*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*si
nh(-3*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-3*(b*c - a*d)
/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)
^3 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d
) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*s
inh(b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*cosh(-3
*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh(-3*(
b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)
) + 6*sqrt(3)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*cos
h(-3*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*sinh(
-3*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-3*(b*c - a*d)/d)
```

```
 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3
+ 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) +
 (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh
(b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*cosh(-3*(b
*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh(-3*(b*c
 - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))
 - 6*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*cosh(-(b*c -
 a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*sinh(-(b*c - a*d
)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-(b*c - a*d)/d) - (b*d^2*x^2 +
 2*b*c*d*x + b*c^2)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((b*d^2*x^2 +
 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c
*d*x + b*c^2)*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((b*d
^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) - (b*d^2*x
^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a)
)*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 6*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d
*x + b*c^2)*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x +
 b*c^2)*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*
c^2)*cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-(b*c - a*
d)/d))*sinh(b*x + a)^3 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*c
osh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-(
b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*
x + a)^2*cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x +
a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt
(-b/d)) + ((6*b*d*x + 6*b*c + d)*cosh(b*x + a)^6 + 6*(6*b*d*x + 6*b*c + d)*
cosh(b*x + a)*sinh(b*x + a)^5 + (6*b*d*x + 6*b*c + d)*sinh(b*x + a)^6 - 3*(
2*b*d*x + 2*b*c + d)*cosh(b*x + a)^4 - 3*(2*b*d*x - 5*(6*b*d*x + 6*b*c + d)
*cosh(b*x + a)^2 + 2*b*c + d)*sinh(b*x + a)^4 + 4*(5*(6*b*d*x + 6*b*c + d)*
cosh(b*x + a)^3 - 3*(2*b*d*x + 2*b*c + d)*cosh(b*x + a))*sinh(b*x + a)^3 +
6*b*d*x - 3*(2*b*d*x + 2*b*c - d)*cosh(b*x + a)^2 + 3*(5*(6*b*d*x + 6*b*c +
 d)*cosh(b*x + a)^4 - 2*b*d*x - 6*(2*b*d*x + 2*b*c + d)*cosh(b*x + a)^2 - 2
*b*c + d)*sinh(b*x + a)^2 + 6*b*c + 6*((6*b*d*x + 6*b*c + d)*cosh(b*x + a)^
5 - 2*(2*b*d*x + 2*b*c + d)*cosh(b*x + a)^3 - (2*b*d*x + 2*b*c - d)*cosh(b*
x + a))*sinh(b*x + a) - d)*sqrt(d*x + c))/((d^4*x^2 + 2*c*d^3*x + c^2*d^2)*
cosh(b*x + a)^3 + 3*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a)^2*sinh(b*
x + a) + 3*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a)*sinh(b*x + a)^2 +
(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*sinh(b*x + a)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3/(d*x+c)**(5/2),x)
```

```
[Out] Integral(sinh(a + b*x)**3/(c + d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx+a)^3}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^3/(d*x + c)^(5/2), x)
```

3.59 $\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal. Leaf size=331

$$-\frac{\sqrt{\pi}b^{5/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{3\sqrt{3\pi}b^{5/2}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{\sqrt{\pi}b^{5/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{3\sqrt{3\pi}b^{5/2}e^{3a-\frac{3bc}{d}}\text{Erfi}\left(\sqrt{3}\right.}{5d^{7/2}}$$

```
[Out] -(b^(5/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(
5*d^(7/2)) + (3*b^(5/2)*E^(-3*a + (3*b*c)/d)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[b
]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2)) - (b^(5/2)*E^(a - (b*c)/d)*Sqrt[Pi]*
Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2)) + (3*b^(5/2)*E^(3*a - (3
*b*c)/d)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/
2)) - (16*b^2*Sinh[a + b*x])/(5*d^3*Sqrt[c + d*x]) - (4*b*Cosh[a + b*x]*Sin
h[a + b*x]^2)/(5*d^2*(c + d*x)^(3/2)) - (2*Sinh[a + b*x]^3)/(5*d*(c + d*x)^
(5/2)) - (24*b^2*Sinh[a + b*x]^3)/(5*d^3*Sqrt[c + d*x])
```

Rubi [A] time = 0.774647, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3314, 3297, 3307, 2180, 2204, 2205, 3313}

$$-\frac{\sqrt{\pi}b^{5/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{3\sqrt{3\pi}b^{5/2}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{\sqrt{\pi}b^{5/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{3\sqrt{3\pi}b^{5/2}e^{3a-\frac{3bc}{d}}\text{Erfi}\left(\sqrt{3}\right.}{5d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3/(c + d*x)^(7/2),x]
```

```
[Out] -(b^(5/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(
5*d^(7/2)) + (3*b^(5/2)*E^(-3*a + (3*b*c)/d)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[b
]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2)) - (b^(5/2)*E^(a - (b*c)/d)*Sqrt[Pi]*
Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2)) + (3*b^(5/2)*E^(3*a - (3
*b*c)/d)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/
2)) - (16*b^2*Sinh[a + b*x])/(5*d^3*Sqrt[c + d*x]) - (4*b*Cosh[a + b*x]*Sin
h[a + b*x]^2)/(5*d^2*(c + d*x)^(3/2)) - (2*Sinh[a + b*x]^3)/(5*d*(c + d*x)^
(5/2)) - (24*b^2*Sinh[a + b*x]^3)/(5*d^3*Sqrt[c + d*x])
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{4b\cosh(a+bx)\sinh^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2\sinh^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{\left(8b^2\right)\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx}{5d^2} + \frac{\left(12b^2\right)\int \frac{\sinh^3(a+b}{(c+dx)^{3/2}}}{5d^2}$$

$$= -\frac{16b^2\sinh(a+bx)}{5d^3\sqrt{c+dx}} - \frac{4b\cosh(a+bx)\sinh^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2\sinh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2\sinh^3(a+bx)}{5d^3\sqrt{c+dx}}$$

$$= -\frac{16b^2\sinh(a+bx)}{5d^3\sqrt{c+dx}} - \frac{4b\cosh(a+bx)\sinh^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2\sinh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2\sinh^3(a+bx)}{5d^3\sqrt{c+dx}}$$

$$= -\frac{16b^2\sinh(a+bx)}{5d^3\sqrt{c+dx}} - \frac{4b\cosh(a+bx)\sinh^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2\sinh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2\sinh^3(a+bx)}{5d^3\sqrt{c+dx}}$$

$$= \frac{8b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{8b^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{16b^2\sinh(a+bx)}{5d^3\sqrt{c+dx}} - \frac{4b\cosh(a+}{5d^2}$$

$$= -\frac{b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{3b^{5/2}e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{b^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}$$

Mathematica [B] time = 18.1883, size = 3211, normalized size = 9.7

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3/(c + d*x)^(7/2),x]
```

```
[Out] (-3*(Cosh[a]*(-((-2*E^((b*(c + d*x))/d)*(3*d^2 + 2*b*d*(c + d*x) + 4*b^2*(c
 + d*x)^2) + 8*d^2*(-((b*(c + d*x))/d))^(5/2)*Gamma[1/2, -((b*(c + d*x))/d)
] + (-6*d^2 + 4*b*d*(c + d*x) - 8*b^2*(c + d*x)^2 + 8*b*d*E^((b*(c + d*x))/
d)*(c + d*x)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (b*(c + d*x))/d])/E^((b*(c
+ d*x))/d))*Sinh[(b*c)/d])/(30*d^3*(c + d*x)^(5/2)) + (2*Cosh[(b*c)/d]*(-(b
*(c + d*x)*(2*E^((b*(c + d*x))/d)*(d + 2*b*(c + d*x)) + 4*d*(-((b*(c + d*x)
)/d))^(3/2)*Gamma[1/2, -((b*(c + d*x))/d)] + (2*(d - 2*b*(c + d*x) + 2*d*E^
((b*(c + d*x))/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (b*(c + d*x))/d]))/E^(
(b*(c + d*x))/d)))/2 - 3*d^2*Sinh[(b*(c + d*x))/d]))/(15*d^3*(c + d*x)^(5/2
))) + Sinh[a]*((Cosh[(b*c)/d]*(-2*E^((b*(c + d*x))/d)*(3*d^2 + 2*b*d*(c + d
*x) + 4*b^2*(c + d*x)^2) + 8*d^2*(-((b*(c + d*x))/d))^(5/2)*Gamma[1/2, -((b
*(c + d*x))/d)] + (-6*d^2 + 4*b*d*(c + d*x) - 8*b^2*(c + d*x)^2 + 8*b*d*E^(
(b*(c + d*x))/d)*(c + d*x)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (b*(c + d*x))
/d])/E^((b*(c + d*x))/d)))/(30*d^3*(c + d*x)^(5/2)) - (2*Sinh[(b*c)/d]*(-(b
*(c + d*x)*(2*E^((b*(c + d*x))/d)*(d + 2*b*(c + d*x)) + 4*d*(-((b*(c + d*x)
)/d))^(3/2)*Gamma[1/2, -((b*(c + d*x))/d)] + (2*(d - 2*b*(c + d*x) + 2*d*E^
((b*(c + d*x))/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (b*(c + d*x))/d]))/E^(
(b*(c + d*x))/d)))/2 - 3*d^2*Sinh[(b*(c + d*x))/d]))/(15*d^3*(c + d*x)^(5/2
)))))/4 + (-(Sinh[3*a]*(-((1 + 2*Cosh[(2*b*c)/d])*(-2*E^((3*b*(c + d*x))/d)
*(d^2 + 2*b*d*(c + d*x) + 12*b^2*(c + d*x)^2) + 24*Sqrt[3]*d^2*(-((b*(c + d
*x))/d))^(5/2)*Gamma[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) -
 24*b^2*(c + d*x)^2 + 24*Sqrt[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/d
)^(5/2)*Gamma[1/2, (3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d))*Sinh[(b*c)/d]
)/(10*d^3*(c + d*x)^(5/2)) - (2*Cosh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*(-6*
b^(5/2)*Sqrt[3*Pi]*(c + d*x)^(5/2)*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]] - 6*b^(5/2)*Sqrt[3*Pi]*(c + d*x)^(5/2)*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c +
d*x])/Sqrt[d]] + Sqrt[d]*(2*b*d*(c + d*x)*Cosh[(3*b*(c + d*x))/d] + (d^2 +
12*b^2*(c + d*x)^2)*Sinh[(3*b*(c + d*x))/d])))/(5*d^(7/2)*(c + d*x)^(5/2)))
) - Cosh[3*a]*((Cosh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*(-2*E^((3*b*(c + d*x
))/d)*(d^2 + 2*b*d*(c + d*x) + 12*b^2*(c + d*x)^2) + 24*Sqrt[3]*d^2*(-((b*(
c + d*x))/d))^(5/2)*Gamma[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d
*x) - 24*b^2*(c + d*x)^2 + 24*Sqrt[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c + d*
x))/d)^(5/2)*Gamma[1/2, (3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d)))/(10*d^3
*(c + d*x)^(5/2)) + (2*(1 + 2*Cosh[(2*b*c)/d])*Sinh[(b*c)/d]*(-6*b^(5/2)*Sq
rt[3*Pi]*(c + d*x)^(5/2)*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - 6*b
^(5/2)*Sqrt[3*Pi]*(c + d*x)^(5/2)*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]] + Sqrt[d]*(2*b*d*(c + d*x)*Cosh[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(c
+ d*x)^2)*Sinh[(3*b*(c + d*x))/d])))/(5*d^(7/2)*(c + d*x)^(5/2))))/8 + (Sin
h[3*a]*(-((1 + 2*Cosh[(2*b*c)/d])*(-2*E^((3*b*(c + d*x))/d)*(d^2 + 2*b*d*(c
 + d*x) + 12*b^2*(c + d*x)^2) + 24*Sqrt[3]*d^2*(-((b*(c + d*x))/d))^(5/2)*G
amma[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24*b^2*(c + d*x
)^2 + 24*Sqrt[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^(5/2)*Gamma[1/
2, (3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d))*Sinh[(b*c)/d])/(10*d^3*(c + d
*x)^(5/2)) - (2*Cosh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*(-6*b^(5/2)*Sqrt[3*P
i]*(c + d*x)^(5/2)*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - 6*b^(5/2)
*Sqrt[3*Pi]*(c + d*x)^(5/2)*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] +
 Sqrt[d]*(2*b*d*(c + d*x)*Cosh[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(c + d*x)
^2)*Sinh[(3*b*(c + d*x))/d])))/(5*d^(7/2)*(c + d*x)^(5/2))) + Cosh[3*a]*((C
osh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*(-2*E^((3*b*(c + d*x))/d)*(d^2 + 2*b*
d*(c + d*x) + 12*b^2*(c + d*x)^2) + 24*Sqrt[3]*d^2*(-((b*(c + d*x))/d))^(5/
2)*Gamma[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24*b^2*(c +
 d*x)^2 + 24*Sqrt[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^(5/2)*Gamm
a[1/2, (3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d)))/(10*d^3*(c + d*x)^(5/2))
 + (2*(1 + 2*Cosh[(2*b*c)/d])*Sinh[(b*c)/d]*(-6*b^(5/2)*Sqrt[3*Pi]*(c + d*x
)^(5/2)*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - 6*b^(5/2)*Sqrt[3*Pi]
```

```
*(c + d*x)^(5/2)*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + Sqrt[d]*(2
*b*d*(c + d*x)*Cosh[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(c + d*x)^2)*Sinh[(3
*b*(c + d*x))/d])))/(5*d^(7/2)*(c + d*x)^(5/2))))/8 + (Cosh[3*a]*(-((1 + 2*
Cosh[(2*b*c)/d])*(-2*E^((3*b*(c + d*x))/d)*(d^2 + 2*b*d*(c + d*x) + 12*b^2*
(c + d*x)^2) + 24*Sqrt[3]*d^2*(-((b*(c + d*x))/d))^(5/2)*Gamma[1/2, (-3*b*(
c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24*b^2*(c + d*x)^2 + 24*Sqrt[3]*
d^2*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^(5/2)*Gamma[1/2, (3*b*(c + d*x)
)/d])/E^((3*b*(c + d*x))/d))*Sinh[(b*c)/d])/(10*d^3*(c + d*x)^(5/2)) - (2*C
osh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*(-6*b^(5/2)*Sqrt[3*Pi]*(c + d*x)^(5/2
)*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - 6*b^(5/2)*Sqrt[3*Pi]*(c +
d*x)^(5/2)*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + Sqrt[d]*(2*b*d*(
c + d*x)*Cosh[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(c + d*x)^2)*Sinh[(3*b*(c
+ d*x))/d])))/(5*d^(7/2)*(c + d*x)^(5/2))) + Sinh[3*a]*((Cosh[(b*c)/d]*(-1
+ 2*Cosh[(2*b*c)/d])*(-2*E^((3*b*(c + d*x))/d)*(d^2 + 2*b*d*(c + d*x) + 12*
b^2*(c + d*x)^2) + 24*Sqrt[3]*d^2*(-((b*(c + d*x))/d))^(5/2)*Gamma[1/2, (-3
*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24*b^2*(c + d*x)^2 + 24*Sqrt
[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^(5/2)*Gamma[1/2, (3*b*(c +
d*x))/d])/E^((3*b*(c + d*x))/d)))/(10*d^3*(c + d*x)^(5/2)) + (2*(1 + 2*Cosh
[(2*b*c)/d])*Sinh[(b*c)/d]*(-6*b^(5/2)*Sqrt[3*Pi]*(c + d*x)^(5/2)*Erf[(Sqrt
[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - 6*b^(5/2)*Sqrt[3*Pi]*(c + d*x)^(5/2)*
Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + Sqrt[d]*(2*b*d*(c + d*x)*Co
sh[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(c + d*x)^2)*Sinh[(3*b*(c + d*x))/d])
))/(5*d^(7/2)*(c + d*x)^(5/2))))/4
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int (\sinh(bx+a))^3 (dx+c)^{-\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3/(d*x+c)^(7/2),x)
```

```
[Out] int(sinh(b*x+a)^3/(d*x+c)^(7/2),x)
```

Maxima [A] time = 1.42465, size = 266, normalized size = 0.8

$$\frac{3\left(\frac{3\sqrt{3}\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}} e^{\left(\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2}, \frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}} - \frac{3\sqrt{3}\left(-\frac{(dx+c)b}{d}\right)^{\frac{5}{2}} e^{\left(-\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2}, -\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}} - \frac{\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}} e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{5}{2}, \frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}} + \frac{\left(-\frac{(dx+c)b}{d}\right)^{\frac{5}{2}} e^{\left(a- \right.}}{(dx}\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 3/8*(3*sqrt(3)*((d*x + c)*b/d)^(5/2)*e^(3*(b*c - a*d)/d)*gamma(-5/2, 3*(d*x
 + c)*b/d)/(d*x + c)^(5/2) - 3*sqrt(3)*(-(d*x + c)*b/d)^(5/2)*e^(-3*(b*c -
a*d)/d)*gamma(-5/2, -3*(d*x + c)*b/d)/(d*x + c)^(5/2) - ((d*x + c)*b/d)^(5/
2)*e^(-a + b*c/d)*gamma(-5/2, (d*x + c)*b/d)/(d*x + c)^(5/2) + (-(d*x + c)*
b/d)^(5/2)*e^(a - b*c/d)*gamma(-5/2, -(d*x + c)*b/d)/(d*x + c)^(5/2))/d
```

Fricas [B] time = 3.81101, size = 7120, normalized size = 21.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/20*(12*sqrt(3)*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
 b^2*c^3)*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d
^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) +
((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-3*(b*c - a
*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-3*
(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2
*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) - (b^2*d^3*x^3 + 3
*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-3*(b*c - a*d)
/d))*sinh(b*x + a)^2 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
b^2*c^3)*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^
2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*si
nh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) - 12*sqrt(3)*sq
rt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x
+ a)^3*cosh(-3*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*
d*x + b^2*c^3)*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b
^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-3*(b*c - a*d)/d) + (b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-3*(b*c - a*d)/d))*sin
h(b*x + a)^3 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)
*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^
2 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x +
 a)^2*cosh(-3*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-
b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d)) - 4*sqrt(pi)*((b^2*d^3*x^3 + 3*b
^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d
) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)
^3*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
 b^2*c^3)*cosh(-(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2
*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((b^2*d^3*x^3 + 3
*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-(b*c - a*d)/d
) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)
*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) - (b^2*d^3*
x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-(b*c
 - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 4*sqrt(
pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a
)^3*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
 b^2*c^3)*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b^2*c*d^
2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^
2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^
3 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x +
 a)*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
 b^2*c^3)*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((b^2*d^3
*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*cosh(-(b*
c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos
h(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x +
 c)*sqrt(-b/d)) - ((12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2
*c*d + b*d^2)*x)*cosh(b*x + a)^6 + 6*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d
 + d^2 + 2*(12*b^2*c*d + b*d^2)*x)*cosh(b*x + a)*sinh(b*x + a)^5 + (12*b^2*
d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2*c*d + b*d^2)*x)*sinh(b*x +
 a)^6 - 12*b^2*d^2*x^2 - (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(
```

```
4*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^4 - (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*
d - 15*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2*c*d + b*d^2
)*x)*cosh(b*x + a)^2 + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*sinh(b*x + a)^4 - 1
2*b^2*c^2 + 4*(5*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2*c
*d + b*d^2)*x)*cosh(b*x + a)^3 - (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d
^2 + 2*(4*b^2*c*d + b*d^2)*x)*cosh(b*x + a))*sinh(b*x + a)^3 + 2*b*c*d + (4
*b^2*d^2*x^2 + 4*b^2*c^2 - 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d - b*d^2)*x)*cosh(
b*x + a)^2 + (4*b^2*d^2*x^2 + 15*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d
^2 + 2*(12*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^4 + 4*b^2*c^2 - 2*b*c*d - 6*(4
*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*cosh(
b*x + a)^2 + 3*d^2 + 2*(4*b^2*c*d - b*d^2)*x)*sinh(b*x + a)^2 - d^2 - 2*(12
*b^2*c*d - b*d^2)*x + 2*(3*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2
*(12*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^5 - 2*(4*b^2*d^2*x^2 + 4*b^2*c^2 + 2
*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^3 + (4*b^2*d^2*x^2
+ 4*b^2*c^2 - 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d - b*d^2)*x)*cosh(b*x + a))*sin
h(b*x + a))*sqrt(d*x + c))/((d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)
*cosh(b*x + a)^3 + 3*(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*cosh(b
*x + a)^2*sinh(b*x + a) + 3*(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)
*cosh(b*x + a)*sinh(b*x + a)^2 + (d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3
*d^3)*sinh(b*x + a)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3/(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx+a)^3}{(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^3/(d*x + c)^(7/2), x)
```

3.60 $\int (dx)^{3/2} \sinh(fx)\,dx$

Optimal. Leaf size=111

$$-\frac{3\sqrt{\pi}d^{3/2}\text{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}}+\frac{3\sqrt{\pi}d^{3/2}\text{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}}-\frac{3d\sqrt{dx}\sinh(fx)}{2f^2}+\frac{(dx)^{3/2}\cosh(fx)}{f}$$

```
[Out] ((d*x)^(3/2)*Cosh[f*x])/f - (3*d^(3/2)*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqr
t[d]])/(8*f^(5/2)) + (3*d^(3/2)*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])
/(8*f^(5/2)) - (3*d*Sqrt[d*x]*Sinh[f*x])/(2*f^2)
```

Rubi [A] time = 0.158884, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3296, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\pi}d^{3/2}\text{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}}+\frac{3\sqrt{\pi}d^{3/2}\text{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}}-\frac{3d\sqrt{dx}\sinh(fx)}{2f^2}+\frac{(dx)^{3/2}\cosh(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*Sinh[f*x],x]
```

```
[Out] ((d*x)^(3/2)*Cosh[f*x])/f - (3*d^(3/2)*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqr
t[d]])/(8*f^(5/2)) + (3*d^(3/2)*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])
/(8*f^(5/2)) - (3*d*Sqrt[d*x]*Sinh[f*x])/(2*f^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (dx)^{3/2}\sinh(fx)\,dx &= \frac{(dx)^{3/2}\cosh(fx)}{f} - \frac{(3d)\int \sqrt{dx}\cosh(fx)\,dx}{2f}\\
&= \frac{(dx)^{3/2}\cosh(fx)}{f} - \frac{3d\sqrt{dx}\sinh(fx)}{2f^2} + \frac{\left(3d^2\right)\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx}{4f^2}\\
&= \frac{(dx)^{3/2}\cosh(fx)}{f} - \frac{3d\sqrt{dx}\sinh(fx)}{2f^2} - \frac{\left(3d^2\right)\int \frac{e^{-fx}}{\sqrt{dx}}\,dx}{8f^2} + \frac{\left(3d^2\right)\int \frac{e^{fx}}{\sqrt{dx}}\,dx}{8f^2}\\
&= \frac{(dx)^{3/2}\cosh(fx)}{f} - \frac{3d\sqrt{dx}\sinh(fx)}{2f^2} - \frac{(3d)\,\mathrm{Subst}\left(\int e^{-\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{4f^2} + \frac{(3d)\,\mathrm{Subst}\left(\int e^{\frac{fx^2}{d}}\right.}{4f^2}\\
&= \frac{(dx)^{3/2}\cosh(fx)}{f} - \frac{3d^{3/2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} + \frac{3d^{3/2}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} - \frac{3d\sqrt{dx}\sinh(fx)}{2f^2}
\end{aligned}$$

Mathematica [A] time = 0.0128487, size = 50, normalized size = 0.45

$$\frac{d^2\left(\sqrt{-fx}\,\mathrm{Gamma}\left(\frac{5}{2}, -fx\right) + \sqrt{fx}\,\mathrm{Gamma}\left(\frac{5}{2}, fx\right)\right)}{2f^3\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*Sinh[f*x],x]
```

```
[Out] (d^2*(Sqrt[-(f*x)]*Gamma[5/2, -(f*x)] + Sqrt[f*x]*Gamma[5/2, f*x]))/(2*f^3*
Sqrt[d*x])
```

Maple [C] time = 0.052, size = 132, normalized size = 1.2

$$-2\,\frac{(dx)^{3/2}\sqrt{2}\sqrt{\pi}}{x^{3/2}\left(if\right)^{3/2}f}\left(-\frac{\sqrt{x}\sqrt{2}\left(if\right)^{7/2}\left(-14\,fx+21\right)\mathrm{e}^{fx}}{112\,\sqrt{\pi}f^3} + \frac{\sqrt{x}\sqrt{2}\left(if\right)^{7/2}\left(14\,fx+21\right)\mathrm{e}^{-fx}}{112\,\sqrt{\pi}f^3} - \frac{3\,\left(if\right)^{7/2}\sqrt{2}\,\mathrm{Erf}\left(\sqrt{x}\sqrt{f}\right)}{32\,f^{7/2}} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*sinh(f*x),x)
```

```
[Out] -2*(d*x)^(3/2)/x^(3/2)*2^(1/2)/(I*f)^(3/2)*Pi^(1/2)/f*(-1/112/Pi^(1/2)*x^(1
/2)*2^(1/2)*(I*f)^(7/2)*(-14*f*x+21)/f^3*exp(f*x)+1/112/Pi^(1/2)*x^(1/2)*2^
(1/2)*(I*f)^(7/2)*(14*f*x+21)/f^3*exp(-f*x)-3/32*(I*f)^(7/2)*2^(1/2)/f^(7/2
)*erf(x^(1/2)*f^(1/2))+3/32*(I*f)^(7/2)*2^(1/2)/f^(7/2)*erfi(x^(1/2)*f^(1/2
)))
```

Maxima [B] time = 1.15145, size = 236, normalized size = 2.13

$$\frac{16\,(dx)^{\frac{5}{2}}\sinh\left(fx\right) - \frac{f\left(\frac{15\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{f^3\sqrt{\frac{f}{d}}} - \frac{15\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{f^3\sqrt{-\frac{f}{d}}} + \frac{2\left(4\,(dx)^{\frac{5}{2}}df^2 - 10\,(dx)^{\frac{3}{2}}d^2f + 15\sqrt{dx}d^3\right)e^{(fx)}}{f^3} - \frac{2\left(4\,(dx)^{\frac{5}{2}}df^2 + 10\,(dx)^{\frac{3}{2}}d^2f + 15\sqrt{dx}d^3\right)e^{(-fx)}}{f^3}\right)}{d}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*sinh(f*x),x, algorithm="maxima")
```

```
[Out] 1/40*(16*(d*x)^(5/2)*sinh(f*x) - f*(15*sqrt(pi)*d^3*erf(sqrt(d*x)*sqrt(f/d)
)/(f^3*sqrt(f/d)) - 15*sqrt(pi)*d^3*erf(sqrt(d*x)*sqrt(-f/d))/(f^3*sqrt(-f/
d)) + 2*(4*(d*x)^(5/2)*d*f^2 - 10*(d*x)^(3/2)*d^2*f + 15*sqrt(d*x)*d^3)*e^(
f*x)/f^3 - 2*(4*(d*x)^(5/2)*d*f^2 + 10*(d*x)^(3/2)*d^2*f + 15*sqrt(d*x)*d^3
)*e^(-f*x)/f^3)/d)/d
```

Fricas [B] time = 2.75208, size = 467, normalized size = 4.21

$$-\frac{3\sqrt{\pi}\left(d^2\cosh\left(fx\right) + d^2\sinh\left(fx\right)\right)\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right) + 3\sqrt{\pi}\left(d^2\cosh\left(fx\right) + d^2\sinh\left(fx\right)\right)\sqrt{-\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right) - 2\left(2\ldots\right.}{8\left(f^3\cosh\left(fx\right) + \ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*sinh(f*x),x, algorithm="fricas")
```

```
[Out] -1/8*(3*sqrt(pi)*(d^2*cosh(f*x) + d^2*sinh(f*x))*sqrt(f/d)*erf(sqrt(d*x)*sq
rt(f/d)) + 3*sqrt(pi)*(d^2*cosh(f*x) + d^2*sinh(f*x))*sqrt(-f/d)*erf(sqrt(d
*x)*sqrt(-f/d)) - 2*(2*d*f^2*x + (2*d*f^2*x - 3*d*f)*cosh(f*x)^2 + 2*(2*d*f
^2*x - 3*d*f)*cosh(f*x)*sinh(f*x) + (2*d*f^2*x - 3*d*f)*sinh(f*x)^2 + 3*d*f
)*sqrt(d*x))/(f^3*cosh(f*x) + f^3*sinh(f*x))
```

Sympy [C] time = 152.835, size = 133, normalized size = 1.2

$$\frac{7d^{\frac{3}{2}}x^{\frac{3}{2}}\cosh\left(fx\right)\Gamma\left(\frac{7}{4}\right)}{4f\Gamma\left(\frac{11}{4}\right)} - \frac{21d^{\frac{3}{2}}\sqrt{x}\sinh\left(fx\right)\Gamma\left(\frac{7}{4}\right)}{8f^2\Gamma\left(\frac{11}{4}\right)} + \frac{21\sqrt{2}\sqrt{\pi}d^{\frac{3}{2}}e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{7}{4}\right)}{16f^{\frac{5}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*sinh(f*x),x)
```

```
[Out] 7*d**(3/2)*x**(3/2)*cosh(f*x)*gamma(7/4)/(4*f*gamma(11/4)) - 21*d**(3/2)*sq
rt(x)*sinh(f*x)*gamma(7/4)/(8*f**2*gamma(11/4)) + 21*sqrt(2)*sqrt(pi)*d**(3
/2)*exp(-3*I*pi/4)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*pi/4)/sqrt(pi))*g
amma(7/4)/(16*f**(5/2)*gamma(11/4))
```

Giac [A] time = 1.36039, size = 194, normalized size = 1.75

$$\frac{\frac{3\sqrt{\pi}d^3\operatorname{erf}\left(-\frac{\sqrt{df}\sqrt{dx}}{d}\right)}{\sqrt{df}f^2}+\frac{2\left(2\sqrt{dx}d^2fx+3\sqrt{dx}d^2\right)e^{(-fx)}}{f^2}}{8d}-\frac{\frac{3\sqrt{\pi}d^3\operatorname{erf}\left(-\frac{\sqrt{-df}\sqrt{dx}}{d}\right)}{\sqrt{-df}f^2}-\frac{2\left(2\sqrt{dx}d^2fx-3\sqrt{dx}d^2\right)e^{(fx)}}{f^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*sinh(f*x),x, algorithm="giac")
```

```
[Out] 1/8*(3*sqrt(pi)*d^3*erf(-sqrt(d*f)*sqrt(d*x)/d)/(sqrt(d*f)*f^2) + 2*(2*sqrt
(d*x)*d^2*f*x + 3*sqrt(d*x)*d^2)*e^(-f*x)/f^2)/d - 1/8*(3*sqrt(pi)*d^3*erf(
-sqrt(-d*f)*sqrt(d*x)/d)/(sqrt(-d*f)*f^2) - 2*(2*sqrt(d*x)*d^2*f*x - 3*sqrt
(d*x)*d^2)*e^(f*x)/f^2)/d
```

3.61 $\int \sqrt{dx} \sinh(fx) \, dx$

Optimal. Leaf size=92

$$-\frac{\sqrt{\pi}\sqrt{d}\operatorname{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\operatorname{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}}+\frac{\sqrt{dx}\cosh(fx)}{f}$$

```
[Out] (Sqrt[d*x]*Cosh[f*x])/f - (Sqrt[d]*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]
])/(4*f^(3/2)) - (Sqrt[d]*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(4*f^
(3/2))
```

Rubi [A] time = 0.110155, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3296, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{d}\operatorname{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\operatorname{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}}+\frac{\sqrt{dx}\cosh(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*Sinh[f*x],x]
```


```
[Out] (Sqrt[d*x]*Cosh[f*x])/f - (Sqrt[d]*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]
])/(4*f^(3/2)) - (Sqrt[d]*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(4*f^
(3/2))
```


Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{dx}\sinh(fx)\,dx &= \frac{\sqrt{dx}\cosh(fx)}{f} - \frac{d\int \frac{\cosh(fx)}{\sqrt{dx}}\,dx}{2f} \\
&= \frac{\sqrt{dx}\cosh(fx)}{f} - \frac{d\int \frac{e^{-fx}}{\sqrt{dx}}\,dx}{4f} - \frac{d\int \frac{e^{fx}}{\sqrt{dx}}\,dx}{4f} \\
&= \frac{\sqrt{dx}\cosh(fx)}{f} - \frac{\text{Subst}\left(\int e^{-\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{2f} - \frac{\text{Subst}\left(\int e^{\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{2f} \\
&= \frac{\sqrt{dx}\cosh(fx)}{f} - \frac{\sqrt{d}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}} - \frac{\sqrt{d}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0164927, size = 49, normalized size = 0.53

$$\frac{d\left(\sqrt{fx}\,\text{Gamma}\left(\frac{3}{2}, fx\right) - \sqrt{-fx}\,\text{Gamma}\left(\frac{3}{2}, -fx\right)\right)}{2f^2\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*Sinh[f*x],x]
```

```
[Out] (d*(-(Sqrt[-(f*x)]*Gamma[3/2, -(f*x)]) + Sqrt[f*x]*Gamma[3/2, f*x]))/(2*f^2
*Sqrt[d*x])
```

Maple [C] time = 0.024, size = 120, normalized size = 1.3

$$-\frac{\sqrt{\pi}\sqrt{2}}{f}\sqrt{dx}\left(\frac{\sqrt{2}\mathrm{e}^{-fx}}{4\sqrt{\pi}f^2}\sqrt{x}\,(if)^{\frac{5}{2}} + \frac{\sqrt{2}\mathrm{e}^{fx}}{4\sqrt{\pi}f^2}\sqrt{x}\,(if)^{\frac{5}{2}} - \frac{\sqrt{2}}{8}(if)^{\frac{5}{2}}\,\text{Erf}\left(\sqrt{x}\sqrt{f}\right)f^{-\frac{5}{2}} - \frac{\sqrt{2}}{8}(if)^{\frac{5}{2}}\,\text{erfi}\left(\sqrt{x}\sqrt{f}\right)f^{-\frac{5}{2}}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x)*(d*x)^(1/2),x)
```

```
[Out] -Pi^(1/2)*(d*x)^(1/2)/x^(1/2)*2^(1/2)/(I*f)^(1/2)/f*(1/4/Pi^(1/2)*x^(1/2)*2
^(1/2)*(I*f)^(5/2)/f^2*exp(-f*x)+1/4/Pi^(1/2)*x^(1/2)*2^(1/2)*(I*f)^(5/2)/f
^2*exp(f*x)-1/8*(I*f)^(5/2)*2^(1/2)/f^(5/2)*erf(x^(1/2)*f^(1/2))-1/8*(I*f)^
(5/2)*2^(1/2)/f^(5/2)*erfi(x^(1/2)*f^(1/2)))
```

Maxima [B] time = 1.22451, size = 201, normalized size = 2.18

$$\frac{8\,(dx)^{\frac{3}{2}}\sinh(fx) - \frac{f\left(\frac{3\sqrt{\pi}d^2\,\text{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{f^2\sqrt{\frac{f}{d}}} + \frac{3\sqrt{\pi}d^2\,\text{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{f^2\sqrt{-\frac{f}{d}}} + \frac{2\left(2\,(dx)^{\frac{3}{2}}df - 3\sqrt{dx}d^2\right)e^{(fx)}}{f^2} - \frac{2\left(2\,(dx)^{\frac{3}{2}}df + 3\sqrt{dx}d^2\right)e^{(-fx)}}{f^2}\right)}{d}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(8*(d*x)^(3/2)*sinh(f*x) - f*(3*sqrt(pi)*d^2*erf(sqrt(d*x)*sqrt(f/d))/
(f^2*sqrt(f/d)) + 3*sqrt(pi)*d^2*erf(sqrt(d*x)*sqrt(-f/d))/(f^2*sqrt(-f/d))
 + 2*(2*(d*x)^(3/2)*d*f - 3*sqrt(d*x)*d^2)*e^(f*x)/f^2 - 2*(2*(d*x)^(3/2)*d
*f + 3*sqrt(d*x)*d^2)*e^(-f*x)/f^2)/d)/d
```

Fricas [B] time = 2.73524, size = 356, normalized size = 3.87

$$-\frac{\sqrt{\pi}\left(d\cosh\left(fx\right)+d\sinh\left(fx\right)\right)\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)-\sqrt{\pi}\left(d\cosh\left(fx\right)+d\sinh\left(fx\right)\right)\sqrt{-\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)-2\left(f\cosh\left(f\right.\right.}{4\left(f^2\cosh\left(fx\right)+f^2\sinh\left(fx\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*(d*cosh(f*x) + d*sinh(f*x))*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/d
)) - sqrt(pi)*(d*cosh(f*x) + d*sinh(f*x))*sqrt(-f/d)*erf(sqrt(d*x)*sqrt(-f/
d)) - 2*(f*cosh(f*x)^2 + 2*f*cosh(f*x)*sinh(f*x) + f*sinh(f*x)^2 + f)*sqrt(
d*x))/(f^2*cosh(f*x) + f^2*sinh(f*x))
```

Sympy [C] time = 3.04992, size = 99, normalized size = 1.08

$$\frac{5\sqrt{d}\sqrt{x}\cosh\left(fx\right)\Gamma\left(\frac{5}{4}\right)}{4f\Gamma\left(\frac{9}{4}\right)}-\frac{5\sqrt{2}\sqrt{\pi}\sqrt{d}e^{-\frac{i\pi}{4}}C\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{8f^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)*(d*x)**(1/2),x)
```

```
[Out] 5*sqrt(d)*sqrt(x)*cosh(f*x)*gamma(5/4)/(4*f*gamma(9/4)) - 5*sqrt(2)*sqrt(pi
)*sqrt(d)*exp(-I*pi/4)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*pi/4)/sqrt(pi
))*gamma(5/4)/(8*f**(3/2)*gamma(9/4))
```

Giac [A] time = 1.40396, size = 136, normalized size = 1.48

$$\frac{\frac{\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{df}\sqrt{dx}}{d}\right)}{\sqrt{df}f}+\frac{\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{-df}\sqrt{dx}}{d}\right)}{\sqrt{-df}f}+\frac{2\sqrt{dx}de^{(fx)}}{f}+\frac{2\sqrt{dx}de^{(-fx)}}{f}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(pi)*d^2*erf(-sqrt(d*f)*sqrt(d*x)/d)/(sqrt(d*f)*f) + sqrt(pi)*d^2*
erf(-sqrt(-d*f)*sqrt(d*x)/d)/(sqrt(-d*f)*f) + 2*sqrt(d*x)*d*e^(f*x)/f + 2*s
qrt(d*x)*d*e^(-f*x)/f)/d
```

3.62 $\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}-\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] -(Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[d]*Sqrt[f]) + (Sqrt[Pi
]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[d]*Sqrt[f])
```

Rubi [A] time = 0.0761857, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}-\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[f*x]/Sqrt[d*x],x]
```

```
[Out] -(Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[d]*Sqrt[f]) + (Sqrt[Pi
]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[d]*Sqrt[f])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx = -\left(\frac{1}{2}\int \frac{e^{-fx}}{\sqrt{dx}}\,dx\right) + \frac{1}{2}\int \frac{e^{fx}}{\sqrt{dx}}\,dx$$
$$= -\frac{\text{Subst}\left(\int e^{-\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{d} + \frac{\text{Subst}\left(\int e^{\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{d}$$
$$= -\frac{\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}} + \frac{\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [A] time = 0.0087227, size = 47, normalized size = 0.61

$$\frac{\sqrt{-fx}\,\text{Gamma}\left(\frac{1}{2}, -fx\right) + \sqrt{fx}\,\text{Gamma}\left(\frac{1}{2}, fx\right)}{2f\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[f*x]/Sqrt[d*x],x]
```

```
[Out] (Sqrt[-(f*x)]*Gamma[1/2, -(f*x)] + Sqrt[f*x]*Gamma[1/2, f*x])/(2*f*Sqrt[d*x
])
```

Maple [C] time = 0.022, size = 71, normalized size = 0.9

$$-\frac{\sqrt{\pi}\sqrt{2}}{2f}\sqrt{x}\sqrt{if}\left(-\frac{\sqrt{2}}{2}(if)^{\frac{3}{2}}\,\text{Erf}\left(\sqrt{x}\sqrt{f}\right)f^{-\frac{3}{2}} + \frac{\sqrt{2}}{2}(if)^{\frac{3}{2}}\,\text{erfi}\left(\sqrt{x}\sqrt{f}\right)f^{-\frac{3}{2}}\right)\frac{1}{\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x)/(d*x)^(1/2),x)
```

```
[Out] -1/2*Pi^(1/2)/(d*x)^(1/2)*x^(1/2)*2^(1/2)*(I*f)^(1/2)/f*(-1/2*(I*f)^(3/2)*2
^(1/2)/f^(3/2)*erf(x^(1/2)*f^(1/2))+1/2*(I*f)^(3/2)*2^(1/2)/f^(3/2)*erfi(x^
(1/2)*f^(1/2)))
```

Maxima [B] time = 1.21827, size = 157, normalized size = 2.04

$$\frac{4\sqrt{dx}\sinh\left(fx\right) - \frac{\left(\frac{2\sqrt{dx}de^{(fx)}}{f} - \frac{2\sqrt{dx}de^{(-fx)}}{f} + \frac{\sqrt{\pi}d\,\text{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{f\sqrt{\frac{f}{d}}} - \frac{\sqrt{\pi}d\,\text{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{f\sqrt{-\frac{f}{d}}}\right)f}{d}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*sqrt(d*x)*sinh(f*x) - (2*sqrt(d*x)*d*e^(f*x)/f - 2*sqrt(d*x)*d*e^(-f
*x)/f + sqrt(pi)*d*erf(sqrt(d*x)*sqrt(f/d))/(f*sqrt(f/d)) - sqrt(pi)*d*erf(
sqrt(d*x)*sqrt(-f/d))/(f*sqrt(-f/d)))*f/d)/d
```

Fricas [A] time = 2.63551, size = 138, normalized size = 1.79

$$-\frac{\sqrt{\pi}\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)+\sqrt{\pi}\sqrt{-\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(pi)*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/d)) + sqrt(pi)*sqrt(-f/d)*erf
(sqrt(d*x)*sqrt(-f/d)))/f
```

Sympy [C] time = 1.47673, size = 70, normalized size = 0.91

$$\frac{3\sqrt{2}\sqrt{\pi}e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{4\sqrt{d}\sqrt{f}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)**(1/2),x)
```

```
[Out] 3*sqrt(2)*sqrt(pi)*exp(-3*I*pi/4)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*pi
/4)/sqrt(pi))*gamma(3/4)/(4*sqrt(d)*sqrt(f)*gamma(7/4))
```

Giac [A] time = 1.314, size = 82, normalized size = 1.06

$$\frac{\frac{\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{df}\sqrt{dx}}{d}\right)}{\sqrt{df}}-\frac{\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{-df}\sqrt{dx}}{d}\right)}{\sqrt{-df}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(pi)*d*erf(-sqrt(d*f)*sqrt(d*x)/d)/sqrt(d*f) - sqrt(pi)*d*erf(-sqr
t(-d*f)*sqrt(d*x)/d)/sqrt(-d*f))/d
```

3.63 $\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{\pi}\sqrt{f}\operatorname{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{\pi}\sqrt{f}\operatorname{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sinh(fx)}{d\sqrt{dx}}$$

```
[Out] (Sqrt[f]*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (Sqrt[f]*Sqrt
[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/d^(3/2) - (2*Sinh[f*x])/(d*Sqrt[d*x
])
```

Rubi [A] time = 0.112284, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3297, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\sqrt{f}\operatorname{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{\pi}\sqrt{f}\operatorname{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sinh(fx)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[f*x]/(d*x)^(3/2),x]
```

```
[Out] (Sqrt[f]*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (Sqrt[f]*Sqrt
[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/d^(3/2) - (2*Sinh[f*x])/(d*Sqrt[d*x
])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx &= -\frac{2\sinh(fx)}{d\sqrt{dx}}+\frac{(2f)\int\frac{\cosh(fx)}{\sqrt{dx}}\,dx}{d}\\ &= -\frac{2\sinh(fx)}{d\sqrt{dx}}+\frac{f\int\frac{e^{-fx}}{\sqrt{dx}}\,dx}{d}+\frac{f\int\frac{e^{fx}}{\sqrt{dx}}\,dx}{d}\\ &= -\frac{2\sinh(fx)}{d\sqrt{dx}}+\frac{(2f)\,\mathrm{Subst}\left(\int e^{-\frac{fx^2}{d}}\,dx,x,\sqrt{dx}\right)}{d^2}+\frac{(2f)\,\mathrm{Subst}\left(\int e^{\frac{fx^2}{d}}\,dx,x,\sqrt{dx}\right)}{d^2}\\ &= \frac{\sqrt{f}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{f}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\sinh(fx)}{d\sqrt{dx}}\end{aligned}$$

Mathematica [A] time = 0.0198243, size = 49, normalized size = 0.56

$$\frac{x\left(\sqrt{-fx}\,\mathrm{Gamma}\left(\frac{1}{2},-fx\right)-\sqrt{fx}\,\mathrm{Gamma}\left(\frac{1}{2},fx\right)-2\sinh(fx)\right)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[f*x]/(d*x)^(3/2),x]
```

```
[Out] (x*(Sqrt[-(f*x)]*Gamma[1/2, -(f*x)] - Sqrt[f*x]*Gamma[1/2, f*x] - 2*Sinh[f*
x]))/(d*x)^(3/2)
```

Maple [C] time = 0.022, size = 120, normalized size = 1.4

$$-\frac{\sqrt{\pi}\sqrt{2}}{4f}x^{\frac{3}{2}}(if)^{\frac{3}{2}}\left(2\frac{\sqrt{2}\sqrt{if}\,\mathrm{e}^{-fx}}{\sqrt{\pi}\sqrt{x}f}-2\frac{\sqrt{2}\sqrt{if}\,\mathrm{e}^{fx}}{\sqrt{\pi}\sqrt{x}f}+2\frac{\sqrt{2}\sqrt{if}\,\mathrm{Erf}\left(\sqrt{x}\sqrt{f}\right)}{\sqrt{f}}+2\frac{\sqrt{2}\sqrt{if}\,\mathrm{erfi}\left(\sqrt{x}\sqrt{f}\right)}{\sqrt{f}}\right)(dx)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x)/(d*x)^(3/2),x)
```

```
[Out] -1/4*Pi^(1/2)/(d*x)^(3/2)*x^(3/2)*2^(1/2)*(I*f)^(3/2)/f*(2/Pi^(1/2)/x^(1/2)
*2^(1/2)*(I*f)^(1/2)/f*exp(-f*x)-2/Pi^(1/2)/x^(1/2)*2^(1/2)*(I*f)^(1/2)/f*e
xp(f*x)+2*(I*f)^(1/2)*2^(1/2)/f^(1/2)*erf(x^(1/2)*f^(1/2))+2*(I*f)^(1/2)*2^
(1/2)/f^(1/2)*erfi(x^(1/2)*f^(1/2)))
```

Maxima [A] time = 1.13607, size = 100, normalized size = 1.15

$$\frac{\frac{f\left(\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{\sqrt{\frac{f}{d}}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{\sqrt{-\frac{f}{d}}}\right)}{d}-\frac{2\sinh(fx)}{\sqrt{dx}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] (f*(sqrt(pi)*erf(sqrt(d*x)*sqrt(f/d))/sqrt(f/d) + sqrt(pi)*erf(sqrt(d*x)*sq
rt(-f/d))/sqrt(-f/d))/d - 2*sinh(f*x)/sqrt(d*x))/d
```

Fricas [B] time = 2.69863, size = 355, normalized size = 4.08

$$\frac{\sqrt{\pi}\left(dx\cosh\left(fx\right)+dx\sinh\left(fx\right)\right)\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)-\sqrt{\pi}\left(dx\cosh\left(fx\right)+dx\sinh\left(fx\right)\right)\sqrt{-\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)-\sqrt{dx}\left(\cos\right.}{d^2x\cosh\left(fx\right)+d^2x\sinh\left(fx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(pi)*(d*x*cosh(f*x) + d*x*sinh(f*x))*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/d)
) - sqrt(pi)*(d*x*cosh(f*x) + d*x*sinh(f*x))*sqrt(-f/d)*erf(sqrt(d*x)*sqrt(
-f/d)) - sqrt(d*x)*(cosh(f*x)^2 + 2*cosh(f*x)*sinh(f*x) + sinh(f*x)^2 - 1))
/(d^2*x*cosh(f*x) + d^2*x*sinh(f*x))
```

Sympy [C] time = 7.42422, size = 94, normalized size = 1.08

$$\frac{\sqrt{2}\sqrt{\pi}\sqrt{f}e^{-\frac{i\pi}{4}}C\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{2d^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}-\frac{\sinh\left(fx\right)\Gamma\left(\frac{1}{4}\right)}{2d^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)**(3/2),x)
```

```
[Out] sqrt(2)*sqrt(pi)*sqrt(f)*exp(-I*pi/4)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)*exp(
I*pi/4)/sqrt(pi))*gamma(1/4)/(2*d**(3/2)*gamma(5/4)) - sinh(f*x)*gamma(1/4)
/(2*d**(3/2)*sqrt(x)*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh\left(fx\right)}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x)/(d*x)^(3/2), x)
```

3.64 $\int \frac{\sinh(fx)}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{2\sqrt{\pi}f^{3/2}\text{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2\sqrt{\pi}f^{3/2}\text{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4f\cosh(fx)}{3d^2\sqrt{dx}}-\frac{2\sinh(fx)}{3d(dx)^{3/2}}$$

```
[Out] (-4*f*Cosh[f*x])/(3*d^2*Sqrt[d*x]) - (2*f^(3/2)*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[
d*x])/Sqrt[d]])/(3*d^(5/2)) + (2*f^(3/2)*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/
Sqrt[d]])/(3*d^(5/2)) - (2*Sinh[f*x])/(3*d*(d*x)^(3/2))
```

Rubi [A] time = 0.149009, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3297, 3308, 2180, 2204, 2205}

$$-\frac{2\sqrt{\pi}f^{3/2}\text{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2\sqrt{\pi}f^{3/2}\text{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4f\cosh(fx)}{3d^2\sqrt{dx}}-\frac{2\sinh(fx)}{3d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[f*x]/(d*x)^(5/2),x]
```

```
[Out] (-4*f*Cosh[f*x])/(3*d^2*Sqrt[d*x]) - (2*f^(3/2)*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[
d*x])/Sqrt[d]])/(3*d^(5/2)) + (2*f^(3/2)*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/
Sqrt[d]])/(3*d^(5/2)) - (2*Sinh[f*x])/(3*d*(d*x)^(3/2))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(fx)}{(dx)^{5/2}}\,dx &= -\frac{2\sinh(fx)}{3d(dx)^{3/2}} + \frac{(2f)\int \frac{\cosh(fx)}{(dx)^{3/2}}\,dx}{3d} \\
&= -\frac{4f\cosh(fx)}{3d^2\sqrt{dx}} - \frac{2\sinh(fx)}{3d(dx)^{3/2}} + \frac{\left(4f^2\right)\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx}{3d^2} \\
&= -\frac{4f\cosh(fx)}{3d^2\sqrt{dx}} - \frac{2\sinh(fx)}{3d(dx)^{3/2}} - \frac{\left(2f^2\right)\int \frac{e^{-fx}}{\sqrt{dx}}\,dx}{3d^2} + \frac{\left(2f^2\right)\int \frac{e^{fx}}{\sqrt{dx}}\,dx}{3d^2} \\
&= -\frac{4f\cosh(fx)}{3d^2\sqrt{dx}} - \frac{2\sinh(fx)}{3d(dx)^{3/2}} - \frac{\left(4f^2\right)\operatorname{Subst}\left(\int e^{-\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{3d^3} + \frac{\left(4f^2\right)\operatorname{Subst}\left(\int e^{\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{3d^3} \\
&= -\frac{4f\cosh(fx)}{3d^2\sqrt{dx}} - \frac{2f^{3/2}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2f^{3/2}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{2\sinh(fx)}{3d(dx)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0830307, size = 84, normalized size = 0.74

$$-\frac{xe^{-fx}\left(2e^{fx}(-fx)^{3/2}\text{Gamma}\left(\frac{1}{2},-fx\right)-2e^{fx}(fx)^{3/2}\text{Gamma}\left(\frac{1}{2},fx\right)+e^{2fx}+2fxe^{2fx}+2fx-1\right)}{3(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[f*x]/(d*x)^(5/2),x]
```

```
[Out] -(x*(-1 + E^(2*f*x) + 2*f*x + 2*E^(2*f*x)*f*x + 2*E^(f*x)*(-(f*x))^(3/2)*Ga
mma[1/2, -(f*x)] - 2*E^(f*x)*(f*x)^(3/2)*Gamma[1/2, f*x]))/(3*E^(f*x)*(d*x)
^(5/2))
```

Maple [C] time = 0.024, size = 132, normalized size = 1.2

$$-\frac{\sqrt{\pi}\sqrt{2}}{8f}x^{\frac{5}{2}}\left(if\right)^{\frac{5}{2}}\left(-\frac{4\sqrt{2}\left(2fx+1\right)e^{fx}}{3\sqrt{\pi}f}x^{-\frac{3}{2}}\frac{1}{\sqrt{if}}+\frac{4\sqrt{2}\left(-2fx+1\right)e^{-fx}}{3\sqrt{\pi}f}x^{-\frac{3}{2}}\frac{1}{\sqrt{if}}-\frac{8\sqrt{2}}{3}\sqrt{f}\text{Erf}\left(\sqrt{x}\sqrt{f}\right)\frac{1}{\sqrt{if}}+\frac{8\sqrt{2}}{3}\sqrt{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x)/(d*x)^(5/2),x)
```

```
[Out] -1/8*Pi^(1/2)/(d*x)^(5/2)*x^(5/2)*2^(1/2)*(I*f)^(5/2)/f*(-4/3/Pi^(1/2)/x^(3
/2)*2^(1/2)/(I*f)^(1/2)*(2*f*x+1)/f*exp(f*x)+4/3/Pi^(1/2)/x^(3/2)*2^(1/2)/(
I*f)^(1/2)*(-2*f*x+1)/f*exp(-f*x)-8/3/(I*f)^(1/2)*2^(1/2)*f^(1/2)*erf(x^(1/
2)*f^(1/2))+8/3/(I*f)^(1/2)*2^(1/2)*f^(1/2)*erfi(x^(1/2)*f^(1/2)))
```

Maxima [A] time = 1.18749, size = 77, normalized size = 0.68

$$-\frac{\frac{f\left(\frac{\sqrt{fx}\Gamma\left(-\frac{1}{2},fx\right)}{\sqrt{dx}}+\frac{\sqrt{-fx}\Gamma\left(-\frac{1}{2},-fx\right)}{\sqrt{dx}}\right)}{d}+\frac{2\sinh\left(fx\right)}{(dx)^{\frac{3}{2}}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(f*(sqrt(f*x)*gamma(-1/2, f*x)/sqrt(d*x) + sqrt(-f*x)*gamma(-1/2, -f*x
)/sqrt(d*x))/d + 2*sinh(f*x)/(d*x)^(3/2))/d
```

Fricas [B] time = 2.64832, size = 454, normalized size = 3.98

$$-\frac{2\sqrt{\pi}\left(dfx^2\cosh\left(fx\right)+dfx^2\sinh\left(fx\right)\right)\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)+2\sqrt{\pi}\left(dfx^2\cosh\left(fx\right)+dfx^2\sinh\left(fx\right)\right)\sqrt{-\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\right.}{3\left(d^3x^2\cosh\left(fx\right)+d^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(pi)*(d*f*x^2*cosh(f*x) + d*f*x^2*sinh(f*x))*sqrt(f/d)*erf(sqrt
(d*x)*sqrt(f/d)) + 2*sqrt(pi)*(d*f*x^2*cosh(f*x) + d*f*x^2*sinh(f*x))*sqrt(
-f/d)*erf(sqrt(d*x)*sqrt(-f/d)) + ((2*f*x + 1)*cosh(f*x)^2 + 2*(2*f*x + 1)*
cosh(f*x)*sinh(f*x) + (2*f*x + 1)*sinh(f*x)^2 + 2*f*x - 1)*sqrt(d*x))/(d^3*
x^2*cosh(f*x) + d^3*x^2*sinh(f*x))
```

Sympy [C] time = 175.807, size = 129, normalized size = 1.13

$$-\frac{\sqrt{2}\sqrt{\pi}f^{\frac{3}{2}}e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(-\frac{1}{4}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{3}{4}\right)}+\frac{f\cosh\left(fx\right)\Gamma\left(-\frac{1}{4}\right)}{3d^{\frac{5}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}+\frac{\sinh\left(fx\right)\Gamma\left(-\frac{1}{4}\right)}{6d^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)**(5/2),x)
```

```
[Out] -sqrt(2)*sqrt(pi)*f**(3/2)*exp(-3*I*pi/4)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)*
exp(I*pi/4)/sqrt(pi))*gamma(-1/4)/(3*d**(5/2)*gamma(3/4)) + f*cosh(f*x)*gam
ma(-1/4)/(3*d**(5/2)*sqrt(x)*gamma(3/4)) + sinh(f*x)*gamma(-1/4)/(6*d**(5/2
)*x**(3/2)*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(fx\right)}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x)/(d*x)^(5/2), x)
```

3.65 $\int \sqrt{c+dx}\,\mathrm{csch}(a+bx)\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\sqrt{c+dx}\,\text{csch}(a+bx),x\right)$$

```
[Out] Unintegrable[Sqrt[c + d*x]*Csch[a + b*x], x]
```

Rubi [A] time = 0.0281708, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{c+dx}\,\text{csch}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + d*x]*Csch[a + b*x],x]
```

```
[Out] Defer[Int][Sqrt[c + d*x]*Csch[a + b*x], x]
```

Rubi steps

$$\int \sqrt{c+dx}\,\text{csch}(a+bx)\,dx = \int \sqrt{c+dx}\,\text{csch}(a+bx)\,dx$$

Mathematica [A] time = 23.4824, size = 0, normalized size = 0.

$$\int \sqrt{c+dx}\,\text{csch}(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + d*x]*Csch[a + b*x],x]
```

```
[Out] Integrate[Sqrt[c + d*x]*Csch[a + b*x], x]
```

Maple [A] time = 0.042, size = 0, normalized size = 0.

$$\int \text{csch}\,(bx+a)\,\sqrt{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)*(d*x+c)^(1/2),x)
```

```
[Out] int(csch(b*x+a)*(d*x+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx + c}\,\mathrm{csch}\,(bx + a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x + c)*csch(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{dx + c}\,\mathrm{csch}\,(bx + a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x + c)*csch(b*x + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c + dx}\,\mathrm{csch}\,(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*csch(a + b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx + c}\,\mathrm{csch}\,(bx + a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x + c)*csch(b*x + a), x)
```

3.66 $\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}},x\right)$$

```
[Out] Unintegrable[Csch[a + b*x]/Sqrt[c + d*x], x]
```

Rubi [A] time = 0.0297394, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] Defer[Int][Csch[a + b*x]/Sqrt[c + d*x], x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx$$

Mathematica [A] time = 21.7864, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] Integrate[Csch[a + b*x]/Sqrt[c + d*x], x]
```

Maple [A] time = 0.043, size = 0, normalized size = 0.

$$\int \operatorname{csch}(bx+a)\frac{1}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(b*x+a)/(d*x+c)^(1/2),x)
```

```
[Out] int(csch(b*x+a)/(d*x+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(b*x + a)/sqrt(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{csch}(bx+a)}{\sqrt{dx+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)/sqrt(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(csch(a + b*x)/sqrt(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx+a)}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)/sqrt(d*x + c), x)
```

3.67 $\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\,dx$

Optimal. Leaf size=61

$$\frac{9}{8}\text{Unintegrable}\left(\frac{\sinh^{\frac{3}{2}}(x)}{x},x\right)+\frac{3}{8}\text{Unintegrable}\left(\frac{1}{x\sqrt{\sinh(x)}},x\right)-\frac{\sinh^{\frac{3}{2}}(x)}{2x^2}-\frac{3\sqrt{\sinh(x)}\cosh(x)}{4x}$$

```
[Out] (-3*Cosh[x]*Sqrt[Sinh[x]])/(4*x) - Sinh[x]^(3/2)/(2*x^2) + (3*Unintegrable[
1/(x*Sqrt[Sinh[x]]), x])/8 + (9*Unintegrable[Sinh[x]^(3/2)/x, x])/8
```

Rubi [A] time = 0.107143, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sinh[x]^(3/2)/x^3,x]
```

```
[Out] (-3*Cosh[x]*Sqrt[Sinh[x]])/(4*x) - Sinh[x]^(3/2)/(2*x^2) + (3*Defer[Int][1/
(x*Sqrt[Sinh[x]]), x])/8 + (9*Defer[Int][Sinh[x]^(3/2)/x, x])/8
```

Rubi steps

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\,dx=-\frac{3\cosh(x)\sqrt{\sinh(x)}}{4x}-\frac{\sinh^{\frac{3}{2}}(x)}{2x^2}+\frac{3}{8}\int \frac{1}{x\sqrt{\sinh(x)}}\,dx+\frac{9}{8}\int \frac{\sinh^{\frac{3}{2}}(x)}{x}\,dx$$

Mathematica [A] time = 6.01493, size = 0, normalized size = 0.

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[x]^(3/2)/x^3,x]
```

```
[Out] Integrate[Sinh[x]^(3/2)/x^3, x]
```

Maple [A] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}(\sinh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^(3/2)/x^3,x)
```

```
[Out] int(sinh(x)^(3/2)/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(x)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sinh(x)^(3/2)/x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**(3/2)/x**3,x)
```

```
[Out] Integral(sinh(x)**(3/2)/x**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(x)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sinh(x)^(3/2)/x^3, x)
```

3.68 $\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)}-x\sqrt{\sinh(x)}\right) dx$

Optimal. Leaf size=20

$$4\sqrt{\sinh(x)}-\frac{2x\cosh(x)}{\sqrt{\sinh(x)}}$$

```
[Out] (-2*x*Cosh[x])/Sqrt[Sinh[x]] + 4*Sqrt[Sinh[x]]
```

Rubi [A] time = 0.0583554, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3315}

$$4\sqrt{\sinh(x)}-\frac{2x\cosh(x)}{\sqrt{\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sinh[x]^(3/2) - x*Sqrt[Sinh[x]],x]

[Out] (-2*x*Cosh[x])/Sqrt[Sinh[x]] + 4*Sqrt[Sinh[x]]
```

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[((c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\sinh^{\frac{3}{2}}(x)}-x\sqrt{\sinh(x)}\right) dx &= \int \frac{x}{\sinh^{\frac{3}{2}}(x)}\, dx-\int x\sqrt{\sinh(x)}\, dx\\ &= -\frac{2x\cosh(x)}{\sqrt{\sinh(x)}}+4\sqrt{\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.113241, size = 17, normalized size = 0.85

$$\frac{4\sinh(x)-2x\cosh(x)}{\sqrt{\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sinh[x]^(3/2) - x*Sqrt[Sinh[x]],x]

[Out] (-2*x*Cosh[x] + 4*Sinh[x])/Sqrt[Sinh[x]]
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int x\left(\sinh\left(x\right)\right)^{-\frac{3}{2}}-x\sqrt{\sinh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x)
```

```
[Out] int(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int -x\sqrt{\sinh\left(x\right)}+\frac{x}{\sinh\left(x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-x*sqrt(sinh(x)) + x/sinh(x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{x}{\sinh^{\frac{3}{2}}\left(x\right)}\,dx-\int x\sqrt{\sinh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)**(3/2)-x*sinh(x)**(1/2),x)
```

```
[Out] -Integral(-x/sinh(x)**(3/2), x) - Integral(x*sqrt(sinh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -x\sqrt{\sinh\left(x\right)}+\frac{x}{\sinh\left(x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x, algorithm="giac")

[Out] integrate(-x*sqrt(sinh(x)) + x/sinh(x)^(3/2), x)
```

3.69 $\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right) dx$

Optimal. Leaf size=24

$$-\frac{4}{3\sqrt{\sinh(x)}}-\frac{2x\cosh(x)}{3\sinh^{\frac{3}{2}}(x)}$$

```
[Out] (-2*x*Cosh[x])/(3*Sinh[x]^(3/2)) - 4/(3*Sqrt[Sinh[x]])
```

Rubi [A] time = 0.0602881, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {3315}

$$-\frac{4}{3\sqrt{\sinh(x)}}-\frac{2x\cosh(x)}{3\sinh^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sinh[x]^(5/2) + x/(3*Sqrt[Sinh[x]]),x]
```

```
[Out] (-2*x*Cosh[x])/(3*Sinh[x]^(3/2)) - 4/(3*Sqrt[Sinh[x]])
```

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[((c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\sinh^{\frac{5}{2}}(x)}+\frac{x}{3\sqrt{\sinh(x)}}\right) dx &= \frac{1}{3}\int \frac{x}{\sqrt{\sinh(x)}}\, dx+\int \frac{x}{\sinh^{\frac{5}{2}}(x)}\, dx\\ &= -\frac{2x\cosh(x)}{3\sinh^{\frac{3}{2}}(x)}-\frac{4}{3\sqrt{\sinh(x)}}\end{aligned}$$

Mathematica [A] time = 0.0705882, size = 22, normalized size = 0.92

$$\frac{1}{6}\sqrt{\sinh(x)}(-8\text{csch}(x)-4x\coth(x)\text{csch}(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sinh[x]^(5/2) + x/(3*Sqrt[Sinh[x]]),x]
```

```
[Out] ((-8*Csch[x] - 4*x*Coth[x]*Csch[x])*Sqrt[Sinh[x]])/6
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int x\left(\sinh\left(x\right)\right)^{-\frac{5}{2}}+\frac{x}{3}\frac{1}{\sqrt{\sinh\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x)
```

```
[Out] int(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{3\sqrt{\sinh\left(x\right)}}+\frac{x}{\sinh\left(x\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/3*x/sqrt(sinh(x)) + x/sinh(x)^(5/2), x)
```

Fricas [B] time = 2.621, size = 375, normalized size = 15.62

$$-\frac{4\left(\left(x+2\right)\cosh\left(x\right)^{3}+3\left(x+2\right)\cosh\left(x\right)\sinh\left(x\right)^{2}+\left(x+2\right)\sinh\left(x\right)^{3}+\left(x-2\right)\cosh\left(x\right)+\left(3\left(x+2\right)\cosh\left(x\right)^{2}+x-2\right.}{3\left(\cosh\left(x\right)^{4}+4\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+2\left(3\cosh\left(x\right)^{2}-1\right)\sinh\left(x\right)^{2}-2\cosh\left(x\right)^{2}+4\left(\cosh\left(x\right)^{3}-\cosh\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] -4/3*((x + 2)*cosh(x)^3 + 3*(x + 2)*cosh(x)*sinh(x)^2 + (x + 2)*sinh(x)^3 +
 (x - 2)*cosh(x) + (3*(x + 2)*cosh(x)^2 + x - 2)*sinh(x))*sqrt(sinh(x))/(co
sh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 -
 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{3x}{\sinh^{\frac{5}{2}}\left(x\right)}\,dx+\int \frac{x}{\sqrt{\sinh\left(x\right)}}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)**(5/2)+1/3*x/sinh(x)**(1/2),x)
```

```
[Out] (Integral(3*x/sinh(x)**(5/2), x) + Integral(x/sqrt(sinh(x)), x))/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{3\sqrt{\sinh(x)}} + \frac{x}{\sinh(x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/3*x/sqrt(sinh(x)) + x/sinh(x)^(5/2), x)
```

3.70 $\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\sinh(x)}\right)dx$

Optimal. Leaf size=47

$$-\frac{4}{15\sinh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\sinh(x)}}{5}-\frac{2x\cosh(x)}{5\sinh^{\frac{5}{2}}(x)}+\frac{6x\cosh(x)}{5\sqrt{\sinh(x)}}$$

```
[Out] (-2*x*Cosh[x])/(5*Sinh[x]^(5/2)) - 4/(15*Sinh[x]^(3/2)) + (6*x*Cosh[x])/(5*
Sqrt[Sinh[x]]) - (12*Sqrt[Sinh[x]])/5
```

Rubi [A] time = 0.0807323, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {3315}

$$-\frac{4}{15\sinh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\sinh(x)}}{5}-\frac{2x\cosh(x)}{5\sinh^{\frac{5}{2}}(x)}+\frac{6x\cosh(x)}{5\sqrt{\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sinh[x]^(7/2) + (3*x*Sqrt[Sinh[x]])/5,x]
```

```
[Out] (-2*x*Cosh[x])/(5*Sinh[x]^(5/2)) - 4/(15*Sinh[x]^(3/2)) + (6*x*Cosh[x])/(5*
Sqrt[Sinh[x]]) - (12*Sqrt[Sinh[x]])/5
```

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[((c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\sinh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\sinh(x)}\right)dx &= \frac{3}{5}\int x\sqrt{\sinh(x)}\,dx+\int \frac{x}{\sinh^{\frac{7}{2}}(x)}\,dx\\ &= -\frac{2x\cosh(x)}{5\sinh^{\frac{5}{2}}(x)}-\frac{4}{15\sinh^{\frac{3}{2}}(x)}-\frac{3}{5}\int \frac{x}{\sinh^{\frac{3}{2}}(x)}\,dx+\frac{3}{5}\int x\sqrt{\sinh(x)}\,dx\\ &= -\frac{2x\cosh(x)}{5\sinh^{\frac{5}{2}}(x)}-\frac{4}{15\sinh^{\frac{3}{2}}(x)}+\frac{6x\cosh(x)}{5\sqrt{\sinh(x)}}-\frac{12\sqrt{\sinh(x)}}{5}\end{aligned}$$

Mathematica [A] time = 0.116836, size = 33, normalized size = 0.7

$$\frac{46\sinh(x)-18\sinh(3x)-21x\cosh(x)+9x\cosh(3x)}{30\sinh^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sinh[x]^(7/2) + (3*x*Sqrt[Sinh[x]])/5,x]
```

```
[Out] (-21*x*Cosh[x] + 9*x*Cosh[3*x] + 46*Sinh[x] - 18*Sinh[3*x])/(30*Sinh[x]^(5/
2))
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int x(\sinh(x))^{-\frac{7}{2}} + \frac{3x}{5}\sqrt{\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x)
```

```
[Out] int(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3}{5}x\sqrt{\sinh(x)} + \frac{x}{\sinh(x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(3/5*x*sqrt(sinh(x)) + x/sinh(x)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)**(7/2)+3/5*x*sinh(x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3}{5}x\sqrt{\sinh(x)} + \frac{x}{\sinh(x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(3/5*x*sqrt(sinh(x)) + x/sinh(x)^(7/2), x)
```

3.71 $\int \left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)} - x^2\sqrt{\sinh(x)}\right) dx$

Optimal. Leaf size=58

$$-\frac{2x^2\cosh(x)}{\sqrt{\sinh(x)}} + 8x\sqrt{\sinh(x)} - \frac{16i\sqrt{\sinh(x)}E\left(\frac{\pi}{4} - \frac{ix}{2}\middle|2\right)}{\sqrt{i\sinh(x)}}$$

```
[Out] (-2*x^2*Cosh[x])/Sqrt[Sinh[x]] + 8*x*Sqrt[Sinh[x]] - ((16*I)*EllipticE[Pi/4
 - (I/2)*x, 2]*Sqrt[Sinh[x]])/Sqrt[I*Sinh[x]]
```

Rubi [A] time = 0.118467, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3316, 2640, 2639}

$$-\frac{2x^2\cosh(x)}{\sqrt{\sinh(x)}} + 8x\sqrt{\sinh(x)} - \frac{16i\sqrt{\sinh(x)}E\left(\frac{\pi}{4} - \frac{ix}{2}\middle|2\right)}{\sqrt{i\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sinh[x]^(3/2) - x^2*Sqrt[Sinh[x]],x]
```

```
[Out] (-2*x^2*Cosh[x])/Sqrt[Sinh[x]] + 8*x*Sqrt[Sinh[x]] - ((16*I)*EllipticE[Pi/4
 - (I/2)*x, 2]*Sqrt[Sinh[x]])/Sqrt[I*Sinh[x]]
```

Rule 3316

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[((c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1
)), x] + (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n +
 2), x], x] + Dist[(d^2*m*(m - 1))/(b^2*f^2*(n + 1)*(n + 2)), Int[(c + d*x)
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] - Simp[(d*m*(c + d*x)^(m - 1)*(b*
Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; FreeQ[{b, c, d, e,
 f}, x] && LtQ[n, -1] && NeQ[n, -2] && GtQ[m, 1]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int\left(\frac{x^2}{\sinh^{\frac{3}{2}}(x)}-x^2\sqrt{\sinh(x)}\right)dx=\int\frac{x^2}{\sinh^{\frac{3}{2}}(x)}dx-\int x^2\sqrt{\sinh(x)}\,dx$$
$$=-\frac{2x^2\cosh(x)}{\sqrt{\sinh(x)}}+8x\sqrt{\sinh(x)}-8\int\sqrt{\sinh(x)}\,dx$$
$$=-\frac{2x^2\cosh(x)}{\sqrt{\sinh(x)}}+8x\sqrt{\sinh(x)}-\frac{\left(8\sqrt{\sinh(x)}\right)\int\sqrt{i\sinh(x)}\,dx}{\sqrt{i\sinh(x)}}$$
$$=-\frac{2x^2\cosh(x)}{\sqrt{\sinh(x)}}+8x\sqrt{\sinh(x)}-\frac{16iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{\sinh(x)}}{\sqrt{i\sinh(x)}}$$

Mathematica [C] time = 1.2416, size = 68, normalized size = 1.17

$$-\frac{2\left(-8\sqrt{2}(\sinh(x)-\cosh(x))\sqrt{-\sinh(x)(\sinh(x)+\cosh(x))}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};\cosh(2x)+\sinh(2x)\right)+x^2\cosh(x)-4(x\right.}{\sqrt{\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sinh[x]^(3/2) - x^2*Sqrt[Sinh[x]],x]
```

```
[Out] (-2*(x^2*Cosh[x] - 4*(-2 + x)*Sinh[x] - 8*Sqrt[2]*Hypergeometric2F1[-1/4, 1
/2, 3/4, Cosh[2*x] + Sinh[2*x]]*(-Cosh[x] + Sinh[x])*Sqrt[-(Sinh[x]*(Cosh[x
] + Sinh[x]))]))/Sqrt[Sinh[x]]
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int x^2(\sinh(x))^{-\frac{3}{2}}-x^2\sqrt{\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x)
```

```
[Out] int(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int -x^2\sqrt{\sinh(x)}+\frac{x^2}{\sinh(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-x^2*sqrt(sinh(x)) + x^2/sinh(x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{x^2}{\sinh^{\frac{3}{2}}(x)}\,dx - \int x^2\sqrt{\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/sinh(x)**(3/2)-x**2*sinh(x)**(1/2),x)
```

```
[Out] -Integral(-x**2/sinh(x)**(3/2), x) - Integral(x**2*sqrt(sinh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -x^2\sqrt{\sinh(x)} + \frac{x^2}{\sinh(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x^2*sqrt(sinh(x)) + x^2/sinh(x)^(3/2), x)
```

3.72 $\int (c+dx)^m (b\sinh(e+fx))^n \,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left((c+dx)^m(b\sinh(e+fx))^n, x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*(b*Sinh[e + f*x])^n, x]
```

Rubi [A] time = 0.0464853, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m(b\sinh(e+fx))^n \,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*(b*Sinh[e + f*x])^n,x]
```

```
[Out] Defer[Int][(c + d*x)^m*(b*Sinh[e + f*x])^n, x]
```

Rubi steps

$$\int (c+dx)^m(b\sinh(e+fx))^n \,dx = \int (c+dx)^m(b\sinh(e+fx))^n \,dx$$

Mathematica [A] time = 2.99417, size = 0, normalized size = 0.

$$\int (c+dx)^m(b\sinh(e+fx))^n \,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(b*Sinh[e + f*x])^n,x]
```

```
[Out] Integrate[(c + d*x)^m*(b*Sinh[e + f*x])^n, x]
```

Maple [A] time = 0.063, size = 0, normalized size = 0.

$$\int (dx+c)^m \left(b\sinh\left(fx+e\right)\right)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(b*sinh(f*x+e))^n,x)
```

```
[Out] int((d*x+c)^m*(b*sinh(f*x+e))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \left(b\sinh\left(fx+e\right)\right)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*sinh(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*(b*sinh(f*x + e))^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m\left(b\sinh\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*sinh(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*(b*sinh(f*x + e))^n, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sinh\left(e+fx\right)\right)^n(c+dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(b*sinh(f*x+e))**n,x)
```

```
[Out] Integral((b*sinh(e + f*x))**n*(c + d*x)**m, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int(dx+c)^m\left(b\sinh\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*sinh(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*(b*sinh(f*x + e))^n, x)
```

3.73 $\int (c+dx)^m \sinh^3(a+bx)\,dx$

Optimal. Leaf size=237

$$\frac{3^{-m-1}e^{3a-\frac{3bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{3b(c+dx)}{d}\right)}{8b}-\frac{3e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{b(c+}{d}\right.}{8b}$$

```
[Out] (3^(-1 - m)*E^(3*a - (3*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (-3*b*(c + d*x))/d
])/(8*b*(-((b*(c + d*x))/d))^m) - (3*E^(a - (b*c)/d)*(c + d*x)^m*Gamma[1 +
m, -((b*(c + d*x))/d)])/(8*b*(-((b*(c + d*x))/d))^m) - (3*E^(-a + (b*c)/d)*
(c + d*x)^m*Gamma[1 + m, (b*(c + d*x))/d])/(8*b*((b*(c + d*x))/d)^m) + (3^(
-1 - m)*E^(-3*a + (3*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (3*b*(c + d*x))/d])/(
8*b*((b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.317832, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3312, 3308, 2181}

$$\frac{3^{-m-1}e^{3a-\frac{3bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{3b(c+dx)}{d}\right)}{8b}-\frac{3e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{b(c+}{d}\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Sinh[a + b*x]^3,x]
```

```
[Out] (3^(-1 - m)*E^(3*a - (3*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (-3*b*(c + d*x))/d
])/(8*b*(-((b*(c + d*x))/d))^m) - (3*E^(a - (b*c)/d)*(c + d*x)^m*Gamma[1 +
m, -((b*(c + d*x))/d)])/(8*b*(-((b*(c + d*x))/d))^m) - (3*E^(-a + (b*c)/d)*
(c + d*x)^m*Gamma[1 + m, (b*(c + d*x))/d])/(8*b*((b*(c + d*x))/d)^m) + (3^(
-1 - m)*E^(-3*a + (3*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (3*b*(c + d*x))/d])/(
8*b*((b*(c + d*x))/d)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int (c+dx)^m \sinh^3(a+bx)\,dx = i\int \left(\frac{3}{4}i(c+dx)^m \sinh(a+bx) - \frac{1}{4}i(c+dx)^m \sinh(3a+3bx)\right) dx$$
$$= \frac{1}{4}\int (c+dx)^m \sinh(3a+3bx)\,dx - \frac{3}{4}\int (c+dx)^m \sinh(a+bx)\,dx$$
$$= \frac{1}{8}\int e^{-i(3ia+3ibx)}(c+dx)^m\,dx - \frac{1}{8}\int e^{i(3ia+3ibx)}(c+dx)^m\,dx - \frac{3}{8}\int e^{-i(ia+ibx)}(c+dx)^m\,dx +$$
$$= \frac{3^{-1-m}e^{3a-\frac{3bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3b(c+dx)}{d}\right)}{8b} - \frac{3e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma}{8b}$$

Mathematica [A] time = 0.198479, size = 206, normalized size = 0.87

$$\frac{3^{-m-1}e^{-3\left(a+\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(e^{6a}\left(b\left(\frac{c}{d}+x\right)\right)^m \text{Gamma}\left(m+1,-\frac{3b(c+dx)}{d}\right) - 3^{m+2}e^{4a+\frac{2bc}{d}}\left(b\left(\frac{c}{d}+x\right)\right)^m \text{Gamma}\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Sinh[a + b*x]^3,x]
```

```
[Out] (3^(-1 - m)*(c + d*x)^m*(E^(6*a)*(b*(c/d + x))^m*Gamma[1 + m, (-3*b*(c + d*
x))/d] - 3^(2 + m)*E^(4*a + (2*b*c)/d)*(b*(c/d + x))^m*Gamma[1 + m, -((b*(c
 + d*x))/d)] + E^((4*b*c)/d)*(-((b*(c + d*x))/d))^m*(-(3^(2 + m)*E^(2*a)*Ga
mma[1 + m, (b*(c + d*x))/d]) + E^((2*b*c)/d)*Gamma[1 + m, (3*b*(c + d*x))/d
])))/(8*b*E^(3*(a + (b*c)/d))*(-((b^2*(c + d*x)^2)/d^2))^m)
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int (dx+c)^m (\sinh(bx+a))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sinh(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*sinh(b*x+a)^3,x)
```

Maxima [A] time = 1.54627, size = 217, normalized size = 0.92

$$\frac{(dx+c)^{m+1}e^{\left(-3a+\frac{3bc}{d}\right)}E_{-m}\left(\frac{3(dx+c)b}{d}\right)}{8d} - \frac{3(dx+c)^{m+1}e^{\left(-a+\frac{bc}{d}\right)}E_{-m}\left(\frac{(dx+c)b}{d}\right)}{8d} + \frac{3(dx+c)^{m+1}e^{\left(a-\frac{bc}{d}\right)}E_{-m}\left(-\frac{(dx+c)b}{d}\right)}{8d} - \frac{(dx+c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x + c)^(m + 1)*e^(-3*a + 3*b*c/d)*exp_integral_e(-m, 3*(d*x + c)*b/d
)/d - 3/8*(d*x + c)^(m + 1)*e^(-a + b*c/d)*exp_integral_e(-m, (d*x + c)*b/d
)/d + 3/8*(d*x + c)^(m + 1)*e^(a - b*c/d)*exp_integral_e(-m, -(d*x + c)*b/d
)/d - 1/8*(d*x + c)^(m + 1)*e^(3*a - 3*b*c/d)*exp_integral_e(-m, -3*(d*x +
```

```
c)*b/d)/d
```

Fricas [A] time = 2.92055, size = 799, normalized size = 3.37

$$\cosh\left(\frac{dm\log\left(\frac{3b}{d}\right)-3bc+3ad}{d}\right)\Gamma\left(m+1,\frac{3(bdx+bc)}{d}\right)-9\cosh\left(\frac{dm\log\left(\frac{b}{d}\right)-bc+ad}{d}\right)\Gamma\left(m+1,\frac{bdx+bc}{d}\right)-9\cosh\left(\frac{dm\log\left(-\frac{b}{d}\right)+bc-ad}{d}\right)\Gamma$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(cosh((d*m*log(3*b/d) - 3*b*c + 3*a*d)/d)*gamma(m + 1, 3*(b*d*x + b*c)
/d) - 9*cosh((d*m*log(b/d) - b*c + a*d)/d)*gamma(m + 1, (b*d*x + b*c)/d) -
9*cosh((d*m*log(-b/d) + b*c - a*d)/d)*gamma(m + 1, -(b*d*x + b*c)/d) + cosh
((d*m*log(-3*b/d) + 3*b*c - 3*a*d)/d)*gamma(m + 1, -3*(b*d*x + b*c)/d) - ga
mma(m + 1, 3*(b*d*x + b*c)/d)*sinh((d*m*log(3*b/d) - 3*b*c + 3*a*d)/d) + 9*
gamma(m + 1, (b*d*x + b*c)/d)*sinh((d*m*log(b/d) - b*c + a*d)/d) + 9*gamma(
m + 1, -(b*d*x + b*c)/d)*sinh((d*m*log(-b/d) + b*c - a*d)/d) - gamma(m + 1,
 -3*(b*d*x + b*c)/d)*sinh((d*m*log(-3*b/d) + 3*b*c - 3*a*d)/d))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^m \sinh^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sinh(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**m*sinh(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sinh(bx+a)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sinh(b*x + a)^3, x)
```

3.74 $\int (c+dx)^m \sinh^2(a+bx)\,dx$

Optimal. Leaf size=144

$$\frac{2^{-m-3}e^{2a-\frac{2bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2b(c+dx)}{d}\right)}{b}-\frac{2^{-m-3}e^{\frac{2bc}{d}-2a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2b(c\cdot}{\cdot}\right.}{b}$$

```
[Out] -(c + d*x)^(1 + m)/(2*d*(1 + m)) + (2^(-3 - m)*E^(2*a - (2*b*c)/d)*(c + d*x
)^m*Gamma[1 + m, (-2*b*(c + d*x))/d])/(b*(-((b*(c + d*x))/d))^m) - (2^(-3 -
 m)*E^(-2*a + (2*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (2*b*(c + d*x))/d])/(b*((
b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.197365, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3312, 3307, 2181}

$$\frac{2^{-m-3}e^{2a-\frac{2bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2b(c+dx)}{d}\right)}{b}-\frac{2^{-m-3}e^{\frac{2bc}{d}-2a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2b(c\cdot}{\cdot}\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Sinh[a + b*x]^2,x]
```

```
[Out] -(c + d*x)^(1 + m)/(2*d*(1 + m)) + (2^(-3 - m)*E^(2*a - (2*b*c)/d)*(c + d*x
)^m*Gamma[1 + m, (-2*b*(c + d*x))/d])/(b*(-((b*(c + d*x))/d))^m) - (2^(-3 -
 m)*E^(-2*a + (2*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (2*b*(c + d*x))/d])/(b*((
b*(c + d*x))/d)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int (c+dx)^m \sinh^2(a+bx)\,dx = -\int \left(\frac{1}{2}(c+dx)^m - \frac{1}{2}(c+dx)^m \cosh(2a+2bx)\right) dx$$
$$= -\frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{1}{2}\int (c+dx)^m \cosh(2a+2bx)\,dx$$
$$= -\frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{1}{4}\int e^{-i(2ia+2ibx)}(c+dx)^m\,dx + \frac{1}{4}\int e^{i(2ia+2ibx)}(c+dx)^m\,dx$$
$$= -\frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{2^{-3-m}e^{2a-\frac{2bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2b(c+dx)}{d}\right)}{b} - \frac{2^{-3-m}e^{-2a+}}{}$$

Mathematica [A] time = 0.167818, size = 131, normalized size = 0.91

$$\frac{1}{8}(c+dx)^m\left(\frac{2^{-m}e^{2a-\frac{2bc}{d}}\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2b(c+dx)}{d}\right)}{b} - \frac{2^{-m}e^{\frac{2bc}{d}-2a}\left(\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2b(c+dx)}{d}\right)}{b} - \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Sinh[a + b*x]^2,x]
```

```
[Out] ((c + d*x)^m*((-4*(c + d*x))/(d*(1 + m)) + (E^(2*a - (2*b*c)/d)*Gamma[1 + m
, (-2*b*(c + d*x))/d])/(2^m*b*(-((b*(c + d*x))/d))^m) - (E^(-2*a + (2*b*c)/
d)*Gamma[1 + m, (2*b*(c + d*x))/d])/(2^m*b*((b*(c + d*x))/d)^m)))/8
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int (dx+c)^m (\sinh(bx+a))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sinh(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*sinh(b*x+a)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.69236, size = 597, normalized size = 4.15

$$-\frac{(dm+d)\cosh\left(\frac{dm\log\left(\frac{2b}{d}\right)-2bc+2ad}{d}\right)\Gamma\left(m+1,\frac{2(bdx+bc)}{d}\right) - (dm+d)\cosh\left(\frac{dm\log\left(-\frac{2b}{d}\right)+2bc-2ad}{d}\right)\Gamma\left(m+1,-\frac{2(bdx+bc)}{d}\right) - }{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((d*m + d)*cosh((d*m*log(2*b/d) - 2*b*c + 2*a*d)/d)*gamma(m + 1, 2*(b*
d*x + b*c)/d) - (d*m + d)*cosh((d*m*log(-2*b/d) + 2*b*c - 2*a*d)/d)*gamma(m
 + 1, -2*(b*d*x + b*c)/d) - (d*m + d)*gamma(m + 1, 2*(b*d*x + b*c)/d)*sinh(
(d*m*log(2*b/d) - 2*b*c + 2*a*d)/d) + (d*m + d)*gamma(m + 1, -2*(b*d*x + b*
c)/d)*sinh((d*m*log(-2*b/d) + 2*b*c - 2*a*d)/d) + 4*(b*d*x + b*c)*cosh(m*lo
g(d*x + c)) + 4*(b*d*x + b*c)*sinh(m*log(d*x + c)))/(b*d*m + b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^m \sinh^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sinh(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*sinh(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sinh(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sinh(b*x + a)^2, x)
```

3.75 $\int (c+dx)^m \sinh(a+bx)\,dx$

Optimal. Leaf size=110

$$\frac{e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{b(c+dx)}{d}\right)}{2b}+\frac{e^{\frac{bc}{d}-a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{b(c+dx)}{d}\right)}{2b}$$

```
[Out] (E^(a - (b*c)/d)*(c + d*x)^m*Gamma[1 + m, -((b*(c + d*x))/d)])/(2*b*(-((b*(
c + d*x))/d))^m) + (E^(-a + (b*c)/d)*(c + d*x)^m*Gamma[1 + m, (b*(c + d*x))
/d])/(2*b*((b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.0930162, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3308, 2181}

$$\frac{e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{b(c+dx)}{d}\right)}{2b}+\frac{e^{\frac{bc}{d}-a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{b(c+dx)}{d}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Sinh[a + b*x],x]
```

```
[Out] (E^(a - (b*c)/d)*(c + d*x)^m*Gamma[1 + m, -((b*(c + d*x))/d)])/(2*b*(-((b*(
c + d*x))/d))^m) + (E^(-a + (b*c)/d)*(c + d*x)^m*Gamma[1 + m, (b*(c + d*x))
/d])/(2*b*((b*(c + d*x))/d)^m)
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m\sinh(a+bx)\,dx&=\frac{1}{2}\int e^{-i(ia+ibx)}(c+dx)^m\,dx-\frac{1}{2}\int e^{i(ia+ibx)}(c+dx)^m\,dx\\&=\frac{e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{b(c+dx)}{d}\right)}{2b}+\frac{e^{-a+\frac{bc}{d}}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\right.}{2b}\end{aligned}$$

Mathematica [A] time = 0.0546071, size = 101, normalized size = 0.92

$$\frac{e^{-a-\frac{bc}{d}}(c+dx)^m\left(e^{2a}\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{b(c+dx)}{d}\right)+e^{\frac{2bc}{d}}\left(b\left(\frac{c}{d}+x\right)\right)^{-m}\text{Gamma}\left(m+1,\frac{b(c+dx)}{d}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Sinh[a + b*x],x]
```

```
[Out] (E^(-a - (b*c)/d)*(c + d*x)^m*((E^(2*a)*Gamma[1 + m, -((b*(c + d*x))/d)])/(
-((b*(c + d*x))/d))^m + (E^((2*b*c)/d)*Gamma[1 + m, (b*(c + d*x))/d])/(b*(c
/d + x))^m))/(2*b)
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int (dx+c)^m \sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sinh(b*x+a),x)
```

```
[Out] int((d*x+c)^m*sinh(b*x+a),x)
```

Maxima [A] time = 1.28157, size = 107, normalized size = 0.97

$$\frac{(dx+c)^{m+1}e^{\left(-a+\frac{bc}{d}\right)}E_{-m}\left(\frac{(dx+c)b}{d}\right)}{2d}-\frac{(dx+c)^{m+1}e^{\left(a-\frac{bc}{d}\right)}E_{-m}\left(-\frac{(dx+c)b}{d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(d*x + c)^(m + 1)*e^(-a + b*c/d)*exp_integral_e(-m, (d*x + c)*b/d)/d -
1/2*(d*x + c)^(m + 1)*e^(a - b*c/d)*exp_integral_e(-m, -(d*x + c)*b/d)/d
```

Fricas [A] time = 2.77917, size = 377, normalized size = 3.43

$$\frac{\cosh\left(\frac{dm\log\left(\frac{b}{d}\right)-bc+ad}{d}\right)\Gamma\left(m+1,\frac{bdx+bc}{d}\right)+\cosh\left(\frac{dm\log\left(-\frac{b}{d}\right)+bc-ad}{d}\right)\Gamma\left(m+1,-\frac{bdx+bc}{d}\right)-\Gamma\left(m+1,\frac{bdx+bc}{d}\right)\sinh\left(\frac{dm\log\left(\frac{b}{d}\right)-bc\cdots}{d}\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh((d*m*log(b/d) - b*c + a*d)/d)*gamma(m + 1, (b*d*x + b*c)/d) + cos
h((d*m*log(-b/d) + b*c - a*d)/d)*gamma(m + 1, -(b*d*x + b*c)/d) - gamma(m +
 1, (b*d*x + b*c)/d)*sinh((d*m*log(b/d) - b*c + a*d)/d) - gamma(m + 1, -(b*
d*x + b*c)/d)*sinh((d*m*log(-b/d) + b*c - a*d)/d))/b
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sinh(b*x+a),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sinh(b*x + a), x)
```

3.76 $\int (c+dx)^m \operatorname{csch}(a+bx)\, dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\operatorname{csch}(a+bx)(c+dx)^m, x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*Csch[a + b*x], x]
```

Rubi [A] time = 0.020653, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \operatorname{csch}(a+bx)\, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csch[a + b*x],x]
```

```
[Out] Defer[Int][(c + d*x)^m*Csch[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \operatorname{csch}(a+bx)\, dx = \int (c+dx)^m \operatorname{csch}(a+bx)\, dx$$

Mathematica [A] time = 6.42703, size = 0, normalized size = 0.

$$\int (c+dx)^m \operatorname{csch}(a+bx)\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csch[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Csch[a + b*x], x]
```

Maple [A] time = 0.029, size = 0, normalized size = 0.

$$\int (dx+c)^m \operatorname{csch}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csch(b*x+a),x)
```

```
[Out] int((d*x+c)^m*csch(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \operatorname{csch}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csch(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csch(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(dx+c\right)^{m}\operatorname{csch}\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csch(b*x + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(c+dx\right)^{m}\operatorname{csch}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csch(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*csch(a + b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(dx+c\right)^{m}\operatorname{csch}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*csch(b*x + a), x)
```

3.77 $\int (c+dx)^m \mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\text{csch}^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*Csch[a + b*x]^2, x]
```

Rubi [A] time = 0.0369156, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \text{csch}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csch[a + b*x]^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Csch[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \text{csch}^2(a+bx)\,dx = \int (c+dx)^m \text{csch}^2(a+bx)\,dx$$

Mathematica [A] time = 3.77869, size = 0, normalized size = 0.

$$\int (c+dx)^m \text{csch}^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csch[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^m*Csch[a + b*x]^2, x]
```

Maple [A] time = 0.033, size = 0, normalized size = 0.

$$\int (dx+c)^m (\text{csch}(bx+a))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csch(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*csch(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \text{csch}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csch(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m \operatorname{csch}(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csch(b*x + a)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^m \operatorname{csch}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csch(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*csch(a + b*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \operatorname{csch}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*csch(b*x + a)^2, x)
```

3.78 $\int x^{3+m} \sinh(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m+4,bx)}{2b^4}-\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+4,-bx)}{2b^4}$$

```
[Out] -(E^a*x^m*Gamma[4 + m, -(b*x)])/(2*b^4*(-(b*x))^m) + (x^m*Gamma[4 + m, b*x]
)/(2*b^4*E^a*(b*x)^m)
```

Rubi [A] time = 0.0777343, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3308, 2181}

$$\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m+4,bx)}{2b^4}-\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+4,-bx)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 + m)*Sinh[a + b*x],x]
```

```
[Out] -(E^a*x^m*Gamma[4 + m, -(b*x)])/(2*b^4*(-(b*x))^m) + (x^m*Gamma[4 + m, b*x]
)/(2*b^4*E^a*(b*x)^m)
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^{3+m}\sinh(a+bx)\,dx &= \frac{1}{2}\int e^{-i(ia+ibx)}x^{3+m}\,dx-\frac{1}{2}\int e^{i(ia+ibx)}x^{3+m}\,dx\\ &= -\frac{e^a x^m(-bx)^{-m}\Gamma(4+m,-bx)}{2b^4}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(4+m,bx)}{2b^4}\end{aligned}$$

Mathematica [A] time = 0.0258614, size = 54, normalized size = 0.92

$$\frac{e^{-a}x^m\left((bx)^{-m}\text{Gamma}(m+4,bx)-e^{2a}(-bx)^{-m}\text{Gamma}(m+4,-bx)\right)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 + m)*Sinh[a + b*x],x]
```

```
[Out] (x^m*(-((E^(2*a)*Gamma[4 + m, -(b*x)])/(-(b*x))^m) + Gamma[4 + m, b*x]/(b*x
)^m))/(2*b^4*E^a)
```

Maple [C] time = 0.042, size = 73, normalized size = 1.2

$$\frac{x^{4+m}\sinh(a)}{4+m}{}_1F_2(2+\frac{m}{2};\frac{1}{2},3+\frac{m}{2};\frac{x^2b^2}{4})+\frac{bx^{5+m}\cosh(a)}{5+m}{}_1F_2(\frac{5}{2}+\frac{m}{2};\frac{3}{2},\frac{7}{2}+\frac{m}{2};\frac{x^2b^2}{4})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3+m)*sinh(b*x+a),x)
```

```
[Out] 1/(4+m)*x^(4+m)*hypergeom([2+1/2*m],[1/2,3+1/2*m],1/4*x^2*b^2)*sinh(a)+b/(5
+m)*x^(5+m)*hypergeom([5/2+1/2*m],[3/2,7/2+1/2*m],1/4*x^2*b^2)*cosh(a)
```

Maxima [A] time = 1.23068, size = 74, normalized size = 1.25

$$\frac{1}{2}(bx)^{-m-4}x^{m+4}e^{(-a)}\Gamma(m+4,bx)-\frac{1}{2}(-bx)^{-m-4}x^{m+4}e^{a}\Gamma(m+4,-bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x)^(-m - 4)*x^(m + 4)*e^(-a)*gamma(m + 4, b*x) - 1/2*(-b*x)^(-m - 4)
*x^(m + 4)*e^a*gamma(m + 4, -b*x)
```

Fricas [A] time = 2.79044, size = 258, normalized size = 4.37

$$\frac{\cosh((m+3)\log(b)+a)\Gamma(m+4,bx)+\cosh((m+3)\log(-b)-a)\Gamma(m+4,-bx)-\Gamma(m+4,-bx)\sinh((m+3)\log$$
$$2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh((m + 3)*log(b) + a)*gamma(m + 4, b*x) + cosh((m + 3)*log(-b) - a)
*gamma(m + 4, -b*x) - gamma(m + 4, -b*x)*sinh((m + 3)*log(-b) - a) - gamma(
m + 4, b*x)*sinh((m + 3)*log(b) + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3+m)*sinh(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+3}\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 3)*sinh(b*x + a), x)
```

3.79 $\int x^{2+m}\sinh(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+3,-bx)}{2b^3}+\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m+3,bx)}{2b^3}$$

```
[Out] (E^a*x^m*Gamma[3 + m, -(b*x)])/(2*b^3*(-(b*x))^m) + (x^m*Gamma[3 + m, b*x])
/(2*b^3*E^a*(b*x)^m)
```

Rubi [A] time = 0.0730884, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3308, 2181}

$$\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+3,-bx)}{2b^3}+\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m+3,bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)*Sinh[a + b*x],x]
```

```
[Out] (E^a*x^m*Gamma[3 + m, -(b*x)])/(2*b^3*(-(b*x))^m) + (x^m*Gamma[3 + m, b*x])
/(2*b^3*E^a*(b*x)^m)
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^{2+m}\sinh(a+bx)\,dx &= \frac{1}{2}\int e^{-i(ia+ibx)}x^{2+m}\,dx-\frac{1}{2}\int e^{i(ia+ibx)}x^{2+m}\,dx\\ &= \frac{e^a x^m(-bx)^{-m}\Gamma(3+m,-bx)}{2b^3}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(3+m,bx)}{2b^3}\end{aligned}$$

Mathematica [A] time = 0.0192387, size = 53, normalized size = 0.9

$$\frac{e^{-a}x^m\left(e^{2a}(-bx)^{-m}\text{Gamma}(m+3,-bx)+(bx)^{-m}\text{Gamma}(m+3,bx)\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)*Sinh[a + b*x],x]
```

```
[Out] (x^m*((E^(2*a)*Gamma[3 + m, -(b*x)])/(-(b*x))^m + Gamma[3 + m, b*x]/(b*x)^m
))/(2*b^3*E^a)
```

Maple [C] time = 0.048, size = 73, normalized size = 1.2

$$\frac{x^{3+m}\sinh(a)}{3+m}{}_1F_2\left(\frac{3}{2}+\frac{m}{2};\frac{1}{2},\frac{5}{2}+\frac{m}{2};\frac{x^2b^2}{4}\right)+\frac{bx^{4+m}\cosh(a)}{4+m}{}_1F_2\left(2+\frac{m}{2};\frac{3}{2},3+\frac{m}{2};\frac{x^2b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+m)*sinh(b*x+a),x)
```

```
[Out] 1/(3+m)*x^(3+m)*hypergeom([3/2+1/2*m],[1/2,5/2+1/2*m],1/4*x^2*b^2)*sinh(a)+
b/(4+m)*x^(4+m)*hypergeom([2+1/2*m],[3/2,3+1/2*m],1/4*x^2*b^2)*cosh(a)
```

Maxima [A] time = 1.3399, size = 74, normalized size = 1.25

$$\frac{1}{2}(bx)^{-m-3}x^{m+3}e^{(-a)}\Gamma(m+3,bx)-\frac{1}{2}(-bx)^{-m-3}x^{m+3}e^{a}\Gamma(m+3,-bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x)^(-m - 3)*x^(m + 3)*e^(-a)*gamma(m + 3, b*x) - 1/2*(-b*x)^(-m - 3)
*x^(m + 3)*e^a*gamma(m + 3, -b*x)
```

Fricas [A] time = 2.73305, size = 258, normalized size = 4.37

$$\frac{\cosh((m+2)\log(b)+a)\Gamma(m+3,bx)+\cosh((m+2)\log(-b)-a)\Gamma(m+3,-bx)-\Gamma(m+3,-bx)\sinh((m+2)\log(-}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh((m + 2)*log(b) + a)*gamma(m + 3, b*x) + cosh((m + 2)*log(-b) - a)
*gamma(m + 3, -b*x) - gamma(m + 3, -b*x)*sinh((m + 2)*log(-b) - a) - gamma(
m + 3, b*x)*sinh((m + 2)*log(b) + a))/b
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)*sinh(b*x+a),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+2}\sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)*sinh(b*x + a), x)
```

3.80 $\int x^{1+m}\sinh(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{e^{-a}x^m(bx)^{-m}\mathrm{Gamma}(m+2,bx)}{2b^2}-\frac{e^a x^m(-bx)^{-m}\mathrm{Gamma}(m+2,-bx)}{2b^2}$$

```
[Out] -(E^a*x^m*Gamma[2 + m, -(b*x)])/(2*b^2*(-(b*x))^m) + (x^m*Gamma[2 + m, b*x]
)/(2*b^2*E^a*(b*x)^m)
```

Rubi [A] time = 0.0736857, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3308, 2181}

$$\frac{e^{-a}x^m(bx)^{-m}\mathrm{Gamma}(m+2,bx)}{2b^2}-\frac{e^a x^m(-bx)^{-m}\mathrm{Gamma}(m+2,-bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)*Sinh[a + b*x],x]
```

```
[Out] -(E^a*x^m*Gamma[2 + m, -(b*x)])/(2*b^2*(-(b*x))^m) + (x^m*Gamma[2 + m, b*x]
)/(2*b^2*E^a*(b*x)^m)
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^{1+m}\sinh(a+bx)\,dx&=\frac{1}{2}\int e^{-i(ia+ibx)}x^{1+m}\,dx-\frac{1}{2}\int e^{i(ia+ibx)}x^{1+m}\,dx\\&=-\frac{e^a x^m(-bx)^{-m}\Gamma(2+m,-bx)}{2b^2}+\frac{e^{-a}x^m(bx)^{-m}\Gamma(2+m,bx)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.021923, size = 54, normalized size = 0.92

$$\frac{e^{-a}x^m\left((bx)^{-m}\mathrm{Gamma}(m+2,bx)-e^{2a}(-bx)^{-m}\mathrm{Gamma}(m+2,-bx)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)*Sinh[a + b*x],x]
```

```
[Out] (x^m*(-((E^(2*a)*Gamma[2 + m, -(b*x)])/(-(b*x))^m) + Gamma[2 + m, b*x]/(b*x
)^m))/(2*b^2*E^a)
```

Maple [C] time = 0.04, size = 73, normalized size = 1.2

$$\frac{x^{2+m}\sinh(a)}{2+m}{}_1F_2(1+\frac{m}{2};\frac{1}{2},2+\frac{m}{2};\frac{x^2b^2}{4})+\frac{bx^{3+m}\cosh(a)}{3+m}{}_1F_2(\frac{3}{2}+\frac{m}{2};\frac{3}{2},\frac{5}{2}+\frac{m}{2};\frac{x^2b^2}{4})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)*sinh(b*x+a),x)
```

```
[Out] 1/(2+m)*x^(2+m)*hypergeom([1+1/2*m],[1/2,2+1/2*m],1/4*x^2*b^2)*sinh(a)+b/(3
+m)*x^(3+m)*hypergeom([3/2+1/2*m],[3/2,5/2+1/2*m],1/4*x^2*b^2)*cosh(a)
```

Maxima [A] time = 1.29879, size = 74, normalized size = 1.25

$$\frac{1}{2}(bx)^{-m-2}x^{m+2}e^{(-a)}\Gamma(m+2,bx)-\frac{1}{2}(-bx)^{-m-2}x^{m+2}e^{a}\Gamma(m+2,-bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x)^(-m - 2)*x^(m + 2)*e^(-a)*gamma(m + 2, b*x) - 1/2*(-b*x)^(-m - 2)
*x^(m + 2)*e^a*gamma(m + 2, -b*x)
```

Fricas [A] time = 2.74414, size = 258, normalized size = 4.37

$$\frac{\cosh((m+1)\log(b)+a)\Gamma(m+2,bx)+\cosh((m+1)\log(-b)-a)\Gamma(m+2,-bx)-\Gamma(m+2,-bx)\sinh((m+1)\log}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh((m + 1)*log(b) + a)*gamma(m + 2, b*x) + cosh((m + 1)*log(-b) - a)
*gamma(m + 2, -b*x) - gamma(m + 2, -b*x)*sinh((m + 1)*log(-b) - a) - gamma(
m + 2, b*x)*sinh((m + 1)*log(b) + a))/b
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)*sinh(b*x+a),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+1} \sinh (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)*sinh(b*x + a), x)
```

3.81 $\int x^m \sinh(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{e^a x^m (-bx)^{-m} \text{Gamma}(m+1,-bx)}{2b}+\frac{e^{-a} x^m (bx)^{-m} \text{Gamma}(m+1,bx)}{2b}$$

```
[Out] (E^a*x^m*Gamma[1 + m, -(b*x)])/(2*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(
2*b*E^a*(b*x)^m)
```

Rubi [A] time = 0.0701193, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3308, 2181}

$$\frac{e^a x^m (-bx)^{-m} \text{Gamma}(m+1,-bx)}{2b}+\frac{e^{-a} x^m (bx)^{-m} \text{Gamma}(m+1,bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sinh[a + b*x],x]
```

```
[Out] (E^a*x^m*Gamma[1 + m, -(b*x)])/(2*b*(-(b*x))^m) + (x^m*Gamma[1 + m, b*x])/(
2*b*E^a*(b*x)^m)
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^m \sinh(a+bx)\,dx &= \frac{1}{2}\int e^{-i(ia+ibx)} x^m\,dx-\frac{1}{2}\int e^{i(ia+ibx)} x^m\,dx\\ &= \frac{e^a x^m (-bx)^{-m}\Gamma(1+m,-bx)}{2b}+\frac{e^{-a} x^m (bx)^{-m}\Gamma(1+m,bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.016536, size = 53, normalized size = 0.9

$$\frac{e^{-a} x^m \left(e^{2a} (-bx)^{-m} \text{Gamma}(m+1,-bx)+(bx)^{-m} \text{Gamma}(m+1,bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sinh[a + b*x],x]
```

```
[Out] (x^m*((E^(2*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m + Gamma[1 + m, b*x]/(b*x)^m
))/(2*b*E^a)
```

Maple [C] time = 0.034, size = 73, normalized size = 1.2

$$\frac{x^{1+m}\sinh(a)}{1+m}{}_1F_2\left(\frac{1}{2}+\frac{m}{2};\frac{1}{2},\frac{3}{2}+\frac{m}{2};\frac{x^2b^2}{4}\right)+\frac{bx^{2+m}\cosh(a)}{2+m}{}_1F_2\left(1+\frac{m}{2};\frac{3}{2},2+\frac{m}{2};\frac{x^2b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(b*x+a),x)
```

```
[Out] 1/(1+m)*x^(1+m)*hypergeom([1/2+1/2*m],[1/2,3/2+1/2*m],1/4*x^2*b^2)*sinh(a)+
b/(2+m)*x^(2+m)*hypergeom([1+1/2*m],[3/2,2+1/2*m],1/4*x^2*b^2)*cosh(a)
```

Maxima [A] time = 1.31893, size = 74, normalized size = 1.25

$$\frac{1}{2}(bx)^{-m-1}x^{m+1}e^{(-a)}\Gamma(m+1,bx)-\frac{1}{2}(-bx)^{-m-1}x^{m+1}e^{a}\Gamma(m+1,-bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x)^(-m - 1)*x^(m + 1)*e^(-a)*gamma(m + 1, b*x) - 1/2*(-b*x)^(-m - 1)
*x^(m + 1)*e^a*gamma(m + 1, -b*x)
```

Fricas [A] time = 2.70492, size = 225, normalized size = 3.81

$$\frac{\cosh(m\log(b)+a)\Gamma(m+1,bx)+\cosh(m\log(-b)-a)\Gamma(m+1,-bx)-\Gamma(m+1,-bx)\sinh(m\log(-b)-a)-\Gamma(m+1}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(m*log(b) + a)*gamma(m + 1, b*x) + cosh(m*log(-b) - a)*gamma(m + 1
, -b*x) - gamma(m + 1, -b*x)*sinh(m*log(-b) - a) - gamma(m + 1, b*x)*sinh(m
*log(b) + a))/b
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(b*x+a),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sinh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*sinh(b*x + a), x)
```

3.82 $\int x^{-1+m}\sinh(a+bx)\,dx$

Optimal. Leaf size=49

$$\frac{1}{2}e^{-a}x^m(bx)^{-m}\text{Gamma}(m,bx)-\frac{1}{2}e^ax^m(-bx)^{-m}\text{Gamma}(m,-bx)$$

```
[Out] -(E^a*x^m*Gamma[m, -(b*x)])/(2*(-(b*x))^m) + (x^m*Gamma[m, b*x])/(2*E^a*(b*
x)^m)
```

Rubi [A] time = 0.0694422, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3308, 2181}

$$\frac{1}{2}e^{-a}x^m(bx)^{-m}\text{Gamma}(m,bx)-\frac{1}{2}e^ax^m(-bx)^{-m}\text{Gamma}(m,-bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)*Sinh[a + b*x],x]
```

```
[Out] -(E^a*x^m*Gamma[m, -(b*x)])/(2*(-(b*x))^m) + (x^m*Gamma[m, b*x])/(2*E^a*(b*
x)^m)
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^{-1+m}\sinh(a+bx)\,dx&=\frac{1}{2}\int e^{-i(ia+ibx)}x^{-1+m}\,dx-\frac{1}{2}\int e^{i(ia+ibx)}x^{-1+m}\,dx\\&=-\frac{1}{2}e^ax^m(-bx)^{-m}\Gamma(m,-bx)+\frac{1}{2}e^{-a}x^m(bx)^{-m}\Gamma(m,bx)\end{aligned}$$

Mathematica [A] time = 0.022222, size = 49, normalized size = 1.

$$\frac{1}{2}e^{-a}x^m(bx)^{-m}\text{Gamma}(m,bx)-\frac{1}{2}e^ax^m(-bx)^{-m}\text{Gamma}(m,-bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)*Sinh[a + b*x],x]
```

```
[Out] -(E^a*x^m*Gamma[m, -(b*x)])/(2*(-(b*x))^m) + (x^m*Gamma[m, b*x])/(2*E^a*(b*
x)^m)
```

Maple [C] time = 0.041, size = 67, normalized size = 1.4

$$\frac{x^m \sinh(a)}{m} {}_1F_2\left(\frac{m}{2}; \frac{1}{2}, 1+\frac{m}{2}; \frac{x^2b^2}{4}\right) + \frac{bx^{1+m}\cosh(a)}{1+m} {}_1F_2\left(\frac{1}{2}+\frac{m}{2}; \frac{3}{2}, \frac{3}{2}+\frac{m}{2}; \frac{x^2b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*sinh(b*x+a),x)
```

```
[Out] 1/m*x^m*hypergeom([1/2*m],[1/2,1+1/2*m],1/4*x^2*b^2)*sinh(a)+b/(1+m)*x^(1+m
)*hypergeom([1/2+1/2*m],[3/2,3/2+1/2*m],1/4*x^2*b^2)*cosh(a)
```

Maxima [A] time = 1.21694, size = 58, normalized size = 1.18

$$\frac{x^m e^{(-a)}\Gamma(m, bx)}{2(bx)^m} - \frac{x^m e^a\Gamma(m, -bx)}{2(-bx)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^m*e^(-a)*gamma(m, b*x)/(b*x)^m - 1/2*x^m*e^a*gamma(m, -b*x)/(-b*x)^m
```

Fricas [A] time = 2.69251, size = 236, normalized size = 4.82

$$\frac{\cosh((m-1)\log(b)+a)\Gamma(m, bx) + \cosh((m-1)\log(-b)-a)\Gamma(m, -bx) - \Gamma(m, -bx)\sinh((m-1)\log(-b)-a) -}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh((m - 1)*log(b) + a)*gamma(m, b*x) + cosh((m - 1)*log(-b) - a)*gam
ma(m, -b*x) - gamma(m, -b*x)*sinh((m - 1)*log(-b) - a) - gamma(m, b*x)*sinh
((m - 1)*log(b) + a))/b
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*sinh(b*x+a),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-1} \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)*sinh(b*x + a), x)
```

3.83 $\int x^{-2+m}\sinh(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}e^{a}bx^{m}(-bx)^{-m}\text{Gamma}(m-1,-bx)+\frac{1}{2}e^{-a}bx^{m}(bx)^{-m}\text{Gamma}(m-1,bx)$$

```
[Out] (b*E^a*x^m*Gamma[-1 + m, -(b*x)])/(2*(-(b*x))^m) + (b*x^m*Gamma[-1 + m, b*x
])/(2*E^a*(b*x)^m)
```

Rubi [A] time = 0.069492, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3308, 2181}

$$\frac{1}{2}e^{a}bx^{m}(-bx)^{-m}\text{Gamma}(m-1,-bx)+\frac{1}{2}e^{-a}bx^{m}(bx)^{-m}\text{Gamma}(m-1,bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)*Sinh[a + b*x],x]
```

```
[Out] (b*E^a*x^m*Gamma[-1 + m, -(b*x)])/(2*(-(b*x))^m) + (b*x^m*Gamma[-1 + m, b*x
])/(2*E^a*(b*x)^m)
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^{-2+m}\sinh(a+bx)\,dx&=\frac{1}{2}\int e^{-i(ia+ibx)}x^{-2+m}\,dx-\frac{1}{2}\int e^{i(ia+ibx)}x^{-2+m}\,dx\\&=\frac{1}{2}be^{a}x^{m}(-bx)^{-m}\Gamma(-1+m,-bx)+\frac{1}{2}be^{-a}x^{m}(bx)^{-m}\Gamma(-1+m,bx)\end{aligned}$$

Mathematica [A] time = 0.0177609, size = 51, normalized size = 0.93

$$\frac{1}{2}e^{-a}bx^{m}\left(e^{2a}(-bx)^{-m}\text{Gamma}(m-1,-bx)+(bx)^{-m}\text{Gamma}(m-1,bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)*Sinh[a + b*x],x]
```

[Out] (b*x^m*((E^(2*a)*Gamma[-1 + m, -(b*x)])/(-(b*x))^m + Gamma[-1 + m, b*x]/(b*
x)^m))/(2*E^a)

Maple [C] time = 0.046, size = 67, normalized size = 1.2

$$\frac{x^{-1+m}\sinh(a)}{-1+m}{}_1F_2(-\frac{1}{2}+\frac{m}{2};\frac{1}{2},\frac{1}{2}+\frac{m}{2};\frac{x^2b^2}{4})+\frac{bx^m\cosh(a)}{m}{}_1F_2(\frac{m}{2};\frac{3}{2},1+\frac{m}{2};\frac{x^2b^2}{4})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-2+m)*sinh(b*x+a),x)

[Out] 1/(-1+m)*x^(-1+m)*hypergeom([-1/2+1/2*m],[1/2,1/2+1/2*m],1/4*x^2*b^2)*sinh(
a)+b/m*x^m*hypergeom([1/2*m],[3/2,1+1/2*m],1/4*x^2*b^2)*cosh(a)

Maxima [A] time = 1.28518, size = 74, normalized size = 1.35

$$\frac{1}{2}(bx)^{-m+1}x^{m-1}e^{(-a)}\Gamma(m-1,bx)-\frac{1}{2}(-bx)^{-m+1}x^{m-1}e^{a}\Gamma(m-1,-bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-2+m)*sinh(b*x+a),x, algorithm="maxima")

[Out] 1/2*(b*x)^(-m + 1)*x^(m - 1)*e^(-a)*gamma(m - 1, b*x) - 1/2*(-b*x)^(-m + 1)
*x^(m - 1)*e^a*gamma(m - 1, -b*x)

Fricas [A] time = 2.68778, size = 258, normalized size = 4.69

$$\frac{\cosh((m-2)\log(b)+a)\Gamma(m-1,bx)+\cosh((m-2)\log(-b)-a)\Gamma(m-1,-bx)-\Gamma(m-1,-bx)\sinh((m-2)\log(-b}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-2+m)*sinh(b*x+a),x, algorithm="fricas")

[Out] 1/2*(cosh((m - 2)*log(b) + a)*gamma(m - 1, b*x) + cosh((m - 2)*log(-b) - a)
*gamma(m - 1, -b*x) - gamma(m - 1, -b*x)*sinh((m - 2)*log(-b) - a) - gamma(
m - 1, b*x)*sinh((m - 2)*log(b) + a))/b

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-2+m)*sinh(b*x+a),x)

[Out] Exception raised: TypeError

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-2} \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2)*sinh(b*x + a), x)
```

3.84 $\int x^{-3+m}\sinh(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{1}{2}e^{-a}b^2x^m(bx)^{-m}\text{Gamma}(m-2,bx)-\frac{1}{2}e^ab^2x^m(-bx)^{-m}\text{Gamma}(m-2,-bx)$$

```
[Out] -(b^2*E^a*x^m*Gamma[-2 + m, -(b*x)])/(2*(-(b*x))^m) + (b^2*x^m*Gamma[-2 + m
, b*x])/(2*E^a*(b*x)^m)
```

Rubi [A] time = 0.0708014, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3308, 2181}

$$\frac{1}{2}e^{-a}b^2x^m(bx)^{-m}\text{Gamma}(m-2,bx)-\frac{1}{2}e^ab^2x^m(-bx)^{-m}\text{Gamma}(m-2,-bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + m)*Sinh[a + b*x],x]
```

```
[Out] -(b^2*E^a*x^m*Gamma[-2 + m, -(b*x)])/(2*(-(b*x))^m) + (b^2*x^m*Gamma[-2 + m
, b*x])/(2*E^a*(b*x)^m)
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^{-3+m}\sinh(a+bx)\,dx&=\frac{1}{2}\int e^{-i(ia+ibx)}x^{-3+m}\,dx-\frac{1}{2}\int e^{i(ia+ibx)}x^{-3+m}\,dx\\&=-\frac{1}{2}b^2e^ax^m(-bx)^{-m}\Gamma(-2+m,-bx)+\frac{1}{2}b^2e^{-a}x^m(bx)^{-m}\Gamma(-2+m,bx)\end{aligned}$$

Mathematica [A] time = 0.0217781, size = 54, normalized size = 0.92

$$\frac{1}{2}e^{-a}b^2x^m\left((bx)^{-m}\text{Gamma}(m-2,bx)-e^{2a}(-bx)^{-m}\text{Gamma}(m-2,-bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + m)*Sinh[a + b*x],x]
```

```
[Out] (b^2*x^m*(-((E^(2*a)*Gamma[-2 + m, -(b*x)])/(-(b*x))^m) + Gamma[-2 + m, b*x
]/(b*x)^m))/(2*E^a)
```

Maple [C] time = 0.03, size = 71, normalized size = 1.2

$$\frac{x^{-2+m}\sinh(a)}{-2+m}{}_1F_2(-1+\frac{m}{2};\frac{1}{2},\frac{m}{2};\frac{x^2b^2}{4})+\frac{bx^{-1+m}\cosh(a)}{-1+m}{}_1F_2(-\frac{1}{2}+\frac{m}{2};\frac{3}{2},\frac{1}{2}+\frac{m}{2};\frac{x^2b^2}{4})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3+m)*sinh(b*x+a),x)
```

```
[Out] 1/(-2+m)*x^(-2+m)*hypergeom([-1+1/2*m],[1/2,1/2*m],1/4*x^2*b^2)*sinh(a)+b/(
-1+m)*x^(-1+m)*hypergeom([-1/2+1/2*m],[3/2,1/2+1/2*m],1/4*x^2*b^2)*cosh(a)
```

Maxima [A] time = 1.32526, size = 74, normalized size = 1.25

$$\frac{1}{2}(bx)^{-m+2}x^{m-2}e^{(-a)}\Gamma(m-2,bx)-\frac{1}{2}(-bx)^{-m+2}x^{m-2}e^{a}\Gamma(m-2,-bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x)^(-m + 2)*x^(m - 2)*e^(-a)*gamma(m - 2, b*x) - 1/2*(-b*x)^(-m + 2)
*x^(m - 2)*e^a*gamma(m - 2, -b*x)
```

Fricas [A] time = 2.69793, size = 258, normalized size = 4.37

$$\frac{\cosh((m-3)\log(b)+a)\Gamma(m-2,bx)+\cosh((m-3)\log(-b)-a)\Gamma(m-2,-bx)-\Gamma(m-2,-bx)\sinh((m-3)\log}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh((m - 3)*log(b) + a)*gamma(m - 2, b*x) + cosh((m - 3)*log(-b) - a)
*gamma(m - 2, -b*x) - gamma(m - 2, -b*x)*sinh((m - 3)*log(-b) - a) - gamma(
m - 2, b*x)*sinh((m - 3)*log(b) + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+m)*sinh(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-3} \sinh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 3)*sinh(b*x + a), x)
```

3.85 $\int x^{3+m} \sinh^2(a+bx)\,dx$

Optimal. Leaf size=86

$$-\frac{e^{2a}2^{-m-6}x^m(-bx)^{-m}\text{Gamma}(m+4,-2bx)}{b^4}-\frac{e^{-2a}2^{-m-6}x^m(bx)^{-m}\text{Gamma}(m+4,2bx)}{b^4}-\frac{x^{m+4}}{2(m+4)}$$

```
[Out] -x^(4 + m)/(2*(4 + m)) - (2^(-6 - m)*E^(2*a)*x^m*Gamma[4 + m, -2*b*x])/(b^4
*(-(b*x))^m) - (2^(-6 - m)*x^m*Gamma[4 + m, 2*b*x])/(b^4*E^(2*a)*(b*x)^m)
```

Rubi [A] time = 0.158529, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$-\frac{e^{2a}2^{-m-6}x^m(-bx)^{-m}\text{Gamma}(m+4,-2bx)}{b^4}-\frac{e^{-2a}2^{-m-6}x^m(bx)^{-m}\text{Gamma}(m+4,2bx)}{b^4}-\frac{x^{m+4}}{2(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 + m)*Sinh[a + b*x]^2,x]
```

```
[Out] -x^(4 + m)/(2*(4 + m)) - (2^(-6 - m)*E^(2*a)*x^m*Gamma[4 + m, -2*b*x])/(b^4
*(-(b*x))^m) - (2^(-6 - m)*x^m*Gamma[4 + m, 2*b*x])/(b^4*E^(2*a)*(b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^{3+m}\sinh^2(a+bx)\,dx &= -\int\left(\frac{x^{3+m}}{2}-\frac{1}{2}x^{3+m}\cosh(2a+2bx)\right)dx\\
&= -\frac{x^{4+m}}{2(4+m)}+\frac{1}{2}\int x^{3+m}\cosh(2a+2bx)\,dx\\
&= -\frac{x^{4+m}}{2(4+m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{3+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{3+m}\,dx\\
&= -\frac{x^{4+m}}{2(4+m)}-\frac{2^{-6-m}e^{2a}x^m(-bx)^{-m}\Gamma(4+m,-2bx)}{b^4}-\frac{2^{-6-m}e^{-2a}x^m(bx)^{-m}\Gamma(4+m,2bx)}{b^4}
\end{aligned}$$

Mathematica [A] time = 0.126828, size = 79, normalized size = 0.92

$$\frac{1}{64}x^m\left(-\frac{e^{2a}2^{-m}(-bx)^{-m}\text{Gamma}(m+4,-2bx)}{b^4}-\frac{e^{-2a}2^{-m}(bx)^{-m}\text{Gamma}(m+4,2bx)}{b^4}-\frac{32x^4}{m+4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 + m)*Sinh[a + b*x]^2,x]
```

```
[Out] (x^m*((-32*x^4)/(4 + m) - (E^(2*a)*Gamma[4 + m, -2*b*x])/(2^m*b^4*(-(b*x))^
m) - Gamma[4 + m, 2*b*x]/(2^m*b^4*E^(2*a)*(b*x)^m)))/64
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int x^{3+m}\left(\sinh\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3+m)*sinh(b*x+a)^2,x)
```

```
[Out] int(x^(3+m)*sinh(b*x+a)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.74761, size = 427, normalized size = 4.97

$$-\frac{4\,bx\cosh\left((m+3)\log\left(x\right)\right)+(m+4)\cosh\left((m+3)\log\left(2\,b\right)+2\,a\right)\Gamma\left(m+4,2\,bx\right)-(m+4)\cosh\left((m+3)\log\left(-2\,b\right)-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*x*cosh((m + 3)*log(x)) + (m + 4)*cosh((m + 3)*log(2*b) + 2*a)*gam
ma(m + 4, 2*b*x) - (m + 4)*cosh((m + 3)*log(-2*b) - 2*a)*gamma(m + 4, -2*b*
x) - (m + 4)*gamma(m + 4, 2*b*x)*sinh((m + 3)*log(2*b) + 2*a) + (m + 4)*gam
ma(m + 4, -2*b*x)*sinh((m + 3)*log(-2*b) - 2*a) + 4*b*x*sinh((m + 3)*log(x)
))/(b*m + 4*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3+m)*sinh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+3} \sinh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 3)*sinh(b*x + a)^2, x)
```

3.86 $\int x^{2+m} \sinh^2(a + bx) \, dx$

Optimal. Leaf size=85

$$\frac{e^{2a}2^{-m-5}x^m(-bx)^{-m}\text{Gamma}(m+3,-2bx)}{b^3} - \frac{e^{-2a}2^{-m-5}x^m(bx)^{-m}\text{Gamma}(m+3,2bx)}{b^3} - \frac{x^{m+3}}{2(m+3)}$$

```
[Out] -x^(3 + m)/(2*(3 + m)) + (2^(-5 - m)*E^(2*a)*x^m*Gamma[3 + m, -2*b*x])/(b^3
*(-(b*x))^m) - (2^(-5 - m)*x^m*Gamma[3 + m, 2*b*x])/(b^3*E^(2*a)*(b*x)^m)
```

Rubi [A] time = 0.135763, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$\frac{e^{2a}2^{-m-5}x^m(-bx)^{-m}\text{Gamma}(m+3,-2bx)}{b^3} - \frac{e^{-2a}2^{-m-5}x^m(bx)^{-m}\text{Gamma}(m+3,2bx)}{b^3} - \frac{x^{m+3}}{2(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)*Sinh[a + b*x]^2,x]
```

```
[Out] -x^(3 + m)/(2*(3 + m)) + (2^(-5 - m)*E^(2*a)*x^m*Gamma[3 + m, -2*b*x])/(b^3
*(-(b*x))^m) - (2^(-5 - m)*x^m*Gamma[3 + m, 2*b*x])/(b^3*E^(2*a)*(b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^{2+m}\sinh^2(a+bx)\,dx &= -\int \left(\frac{x^{2+m}}{2}-\frac{1}{2}x^{2+m}\cosh(2a+2bx)\right)dx\\
&= -\frac{x^{3+m}}{2(3+m)}+\frac{1}{2}\int x^{2+m}\cosh(2a+2bx)\,dx\\
&= -\frac{x^{3+m}}{2(3+m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{2+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{2+m}\,dx\\
&= -\frac{x^{3+m}}{2(3+m)}+\frac{2^{-5-m}e^{2a}x^m(-bx)^{-m}\Gamma(3+m,-2bx)}{b^3}-\frac{2^{-5-m}e^{-2a}x^m(bx)^{-m}\Gamma(3+m,2bx)}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.123141, size = 78, normalized size = 0.92

$$\frac{1}{32}x^m\left(\frac{e^{2a}2^{-m}(-bx)^{-m}\text{Gamma}(m+3,-2bx)}{b^3}-\frac{e^{-2a}2^{-m}(bx)^{-m}\text{Gamma}(m+3,2bx)}{b^3}-\frac{16x^3}{m+3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)*Sinh[a + b*x]^2,x]
```

```
[Out] (x^m*((-16*x^3)/(3 + m) + (E^(2*a)*Gamma[3 + m, -2*b*x])/(2^m*b^3*(-(b*x))^
m) - Gamma[3 + m, 2*b*x]/(2^m*b^3*E^(2*a)*(b*x)^m)))/32
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int x^{2+m}\left(\sinh\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+m)*sinh(b*x+a)^2,x)
```

```
[Out] int(x^(2+m)*sinh(b*x+a)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.75892, size = 427, normalized size = 5.02

$$-\frac{4\,bx\cosh\left((m+2)\log\left(x\right)\right)+(m+3)\cosh\left((m+2)\log\left(2\,b\right)+2\,a\right)\Gamma\left(m+3,2\,bx\right)-(m+3)\cosh\left((m+2)\log\left(-2\,b\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*x*cosh((m + 2)*log(x)) + (m + 3)*cosh((m + 2)*log(2*b) + 2*a)*gam
ma(m + 3, 2*b*x) - (m + 3)*cosh((m + 2)*log(-2*b) - 2*a)*gamma(m + 3, -2*b*
x) - (m + 3)*gamma(m + 3, 2*b*x)*sinh((m + 2)*log(2*b) + 2*a) + (m + 3)*gam
ma(m + 3, -2*b*x)*sinh((m + 2)*log(-2*b) - 2*a) + 4*b*x*sinh((m + 2)*log(x)
))/(b*m + 3*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)*sinh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+2}\sinh(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)*sinh(b*x + a)^2, x)
```

3.87 $\int x^{1+m} \sinh^2(a+bx)\,dx$

Optimal. Leaf size=86

$$-\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\text{Gamma}(m+2,-2bx)}{b^2}-\frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\text{Gamma}(m+2,2bx)}{b^2}-\frac{x^{m+2}}{2(m+2)}$$

```
[Out] -x^(2 + m)/(2*(2 + m)) - (2^(-4 - m)*E^(2*a)*x^m*Gamma[2 + m, -2*b*x])/(b^2
*(-(b*x))^m) - (2^(-4 - m)*x^m*Gamma[2 + m, 2*b*x])/(b^2*E^(2*a)*(b*x)^m)
```

Rubi [A] time = 0.139944, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$-\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\text{Gamma}(m+2,-2bx)}{b^2}-\frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\text{Gamma}(m+2,2bx)}{b^2}-\frac{x^{m+2}}{2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)*Sinh[a + b*x]^2,x]
```

```
[Out] -x^(2 + m)/(2*(2 + m)) - (2^(-4 - m)*E^(2*a)*x^m*Gamma[2 + m, -2*b*x])/(b^2
*(-(b*x))^m) - (2^(-4 - m)*x^m*Gamma[2 + m, 2*b*x])/(b^2*E^(2*a)*(b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^{1+m}\sinh^2(a+bx)\,dx &= -\int \left(\frac{x^{1+m}}{2}-\frac{1}{2}x^{1+m}\cosh(2a+2bx)\right)dx\\
&= -\frac{x^{2+m}}{2(2+m)}+\frac{1}{2}\int x^{1+m}\cosh(2a+2bx)\,dx\\
&= -\frac{x^{2+m}}{2(2+m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{1+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{1+m}\,dx\\
&= -\frac{x^{2+m}}{2(2+m)}-\frac{2^{-4-m}e^{2a}x^m(-bx)^{-m}\Gamma(2+m,-2bx)}{b^2}-\frac{2^{-4-m}e^{-2a}x^m(bx)^{-m}\Gamma(2+m,2bx)}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.130026, size = 79, normalized size = 0.92

$$\frac{1}{16}x^m\left(-\frac{e^{2a}2^{-m}(-bx)^{-m}\text{Gamma}(m+2,-2bx)}{b^2}-\frac{e^{-2a}2^{-m}(bx)^{-m}\text{Gamma}(m+2,2bx)}{b^2}-\frac{8x^2}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)*Sinh[a + b*x]^2,x]
```

```
[Out] (x^m*((-8*x^2)/(2 + m) - (E^(2*a)*Gamma[2 + m, -2*b*x])/(2^m*b^2*(-(b*x))^m
) - Gamma[2 + m, 2*b*x]/(2^m*b^2*E^(2*a)*(b*x)^m)))/16
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int x^{1+m}\left(\sinh\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)*sinh(b*x+a)^2,x)
```

```
[Out] int(x^(1+m)*sinh(b*x+a)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.7728, size = 427, normalized size = 4.97

$$-\frac{4\,bx\cosh\left((m+1)\log\left(x\right)\right)+(m+2)\cosh\left((m+1)\log\left(2\,b\right)+2\,a\right)\Gamma\left(m+2,2\,bx\right)-(m+2)\cosh\left((m+1)\log\left(-2\,b\right)-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*x*cosh((m + 1)*log(x)) + (m + 2)*cosh((m + 1)*log(2*b) + 2*a)*gam
ma(m + 2, 2*b*x) - (m + 2)*cosh((m + 1)*log(-2*b) - 2*a)*gamma(m + 2, -2*b*
x) - (m + 2)*gamma(m + 2, 2*b*x)*sinh((m + 1)*log(2*b) + 2*a) + (m + 2)*gam
ma(m + 2, -2*b*x)*sinh((m + 1)*log(-2*b) - 2*a) + 4*b*x*sinh((m + 1)*log(x)
))/(b*m + 2*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+1}\sinh^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**(m + 1)*sinh(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m+1}\sinh\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)*sinh(b*x + a)^2, x)
```

3.88 $\int x^m \sinh^2(a+bx)\,dx$

Optimal. Leaf size=85

$$\frac{e^{2a}2^{-m-3}x^m(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}-\frac{e^{-2a}2^{-m-3}x^m(bx)^{-m}\text{Gamma}(m+1,2bx)}{b}-\frac{x^{m+1}}{2(m+1)}$$

```
[Out] -x^(1 + m)/(2*(1 + m)) + (2^(-3 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(
-(b*x))^m) - (2^(-3 - m)*x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m)
```

Rubi [A] time = 0.12687, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3312, 3307, 2181}

$$\frac{e^{2a}2^{-m-3}x^m(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}-\frac{e^{-2a}2^{-m-3}x^m(bx)^{-m}\text{Gamma}(m+1,2bx)}{b}-\frac{x^{m+1}}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sinh[a + b*x]^2,x]
```

```
[Out] -x^(1 + m)/(2*(1 + m)) + (2^(-3 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(
-(b*x))^m) - (2^(-3 - m)*x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^m \sinh^2(a+bx)\,dx &= -\int \left(\frac{x^m}{2}-\frac{1}{2}x^m\cosh(2a+2bx)\right)dx\\
&= -\frac{x^{1+m}}{2(1+m)}+\frac{1}{2}\int x^m\cosh(2a+2bx)\,dx\\
&= -\frac{x^{1+m}}{2(1+m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^m\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^m\,dx\\
&= -\frac{x^{1+m}}{2(1+m)}+\frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b}-\frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.104684, size = 76, normalized size = 0.89

$$\frac{1}{8}x^m\left(\frac{e^{2a}2^{-m}(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}-\frac{e^{-2a}2^{-m}(bx)^{-m}\text{Gamma}(m+1,2bx)}{b}-\frac{4x}{m+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sinh[a + b*x]^2,x]
```

```
[Out] (x^m*((-4*x)/(1 + m) + (E^(2*a)*Gamma[1 + m, -2*b*x])/(2^m*b*(-(b*x))^m) -
Gamma[1 + m, 2*b*x]/(2^m*b*E^(2*a)*(b*x)^m)))/8
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int x^m (\sinh(bx+a))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(b*x+a)^2,x)
```

```
[Out] int(x^m*sinh(b*x+a)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.70444, size = 375, normalized size = 4.41

$$-\frac{4\,bx\cosh(m\log(x))+(m+1)\cosh(m\log(2\,b)+2\,a)\,\Gamma(m+1,2\,bx)-(m+1)\cosh(m\log(-2\,b)-2\,a)\,\Gamma(m+1,-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*x*cosh(m*log(x)) + (m + 1)*cosh(m*log(2*b) + 2*a)*gamma(m + 1, 2*
b*x) - (m + 1)*cosh(m*log(-2*b) - 2*a)*gamma(m + 1, -2*b*x) - (m + 1)*gamma
(m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) + (m + 1)*gamma(m + 1, -2*b*x)*sinh(m
*log(-2*b) - 2*a) + 4*b*x*sinh(m*log(x)))/(b*m + b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sinh^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**m*sinh(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \sinh (bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*sinh(b*x + a)^2, x)
```

3.89 $\int x^{-1+m} \sinh^2(a+bx)\,dx$

Optimal. Leaf size=72

$$e^{2a}\left(-2^{-m-2}\right)x^m(-bx)^{-m}\text{Gamma}(m,-2bx)-e^{-2a}2^{-m-2}x^m(bx)^{-m}\text{Gamma}(m,2bx)-\frac{x^m}{2m}$$

```
[Out] -x^m/(2*m) - (2^(-2 - m)*E^(2*a)*x^m*Gamma[m, -2*b*x])/(-(b*x))^m - (2^(-2
- m)*x^m*Gamma[m, 2*b*x])/(E^(2*a)*(b*x)^m)
```

Rubi [A] time = 0.126916, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$e^{2a}\left(-2^{-m-2}\right)x^m(-bx)^{-m}\text{Gamma}(m,-2bx)-e^{-2a}2^{-m-2}x^m(bx)^{-m}\text{Gamma}(m,2bx)-\frac{x^m}{2m}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)*Sinh[a + b*x]^2,x]
```


```
[Out] -x^m/(2*m) - (2^(-2 - m)*E^(2*a)*x^m*Gamma[m, -2*b*x])/(-(b*x))^m - (2^(-2
- m)*x^m*Gamma[m, 2*b*x])/(E^(2*a)*(b*x)^m)
```


Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^{-1+m}\sinh^2(a+bx)\,dx &= -\int\left(\frac{x^{-1+m}}{2}-\frac{1}{2}x^{-1+m}\cosh(2a+2bx)\right)dx\\ &= -\frac{x^m}{2m}+\frac{1}{2}\int x^{-1+m}\cosh(2a+2bx)\,dx\\ &= -\frac{x^m}{2m}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{-1+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{-1+m}\,dx\\ &= -\frac{x^m}{2m}-2^{-2-m}e^{2a}x^m(-bx)^{-m}\Gamma(m,-2bx)-2^{-2-m}e^{-2a}x^m(bx)^{-m}\Gamma(m,2bx)\end{aligned}$$

Mathematica [A] time = 0.075868, size = 63, normalized size = 0.88

$$-\frac{x^m\left(e^{2a}2^{-m}m(-bx)^{-m}\text{Gamma}(m,-2bx)+e^{-2a}2^{-m}m(bx)^{-m}\text{Gamma}(m,2bx)+2\right)}{4m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)*Sinh[a + b*x]^2,x]
```

```
[Out] -(x^m*(2 + (E^(2*a)*m*Gamma[m, -2*b*x])/(2^m*(-(b*x))^m) + (m*Gamma[m, 2*b*
x])/(2^m*E^(2*a)*(b*x)^m)))/(4*m)
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int x^{-1+m}\left(\sinh\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*sinh(b*x+a)^2,x)
```

```
[Out] int(x^(-1+m)*sinh(b*x+a)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.75443, size = 365, normalized size = 5.07

$$-\frac{4\,bx\cosh\left((m-1)\log\left(x\right)\right)+m\cosh\left((m-1)\log\left(2\,b\right)+2\,a\right)\Gamma\left(m,2\,bx\right)-m\cosh\left((m-1)\log\left(-2\,b\right)-2\,a\right)\Gamma\left(m,-2\,bx\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*x*cosh((m - 1)*log(x)) + m*cosh((m - 1)*log(2*b) + 2*a)*gamma(m,
2*b*x) - m*cosh((m - 1)*log(-2*b) - 2*a)*gamma(m, -2*b*x) - m*gamma(m, 2*b*
x)*sinh((m - 1)*log(2*b) + 2*a) + m*gamma(m, -2*b*x)*sinh((m - 1)*log(-2*b)
 - 2*a) + 4*b*x*sinh((m - 1)*log(x)))/(b*m)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-1}\sinh^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(x**(m - 1)*sinh(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-1} \sinh(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)*sinh(b*x + a)^2, x)
```

3.90 $\int x^{-2+m} \sinh^2(a+bx)\,dx$

Optimal. Leaf size=83

$$e^{2a}b2^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m-1,-2bx)-e^{-2a}b2^{-m-1}x^m(bx)^{-m}\text{Gamma}(m-1,2bx)+\frac{x^{m-1}}{2(1-m)}$$

```
[Out] x^(-1 + m)/(2*(1 - m)) + (2^(-1 - m)*b*E^(2*a)*x^m*Gamma[-1 + m, -2*b*x])/(
-(b*x))^m - (2^(-1 - m)*b*x^m*Gamma[-1 + m, 2*b*x])/(E^(2*a)*(b*x)^m)
```

Rubi [A] time = 0.136826, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$e^{2a}b2^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m-1,-2bx)-e^{-2a}b2^{-m-1}x^m(bx)^{-m}\text{Gamma}(m-1,2bx)+\frac{x^{m-1}}{2(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)*Sinh[a + b*x]^2,x]
```

```
[Out] x^(-1 + m)/(2*(1 - m)) + (2^(-1 - m)*b*E^(2*a)*x^m*Gamma[-1 + m, -2*b*x])/(
-(b*x))^m - (2^(-1 - m)*b*x^m*Gamma[-1 + m, 2*b*x])/(E^(2*a)*(b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^{-2+m}\sinh^2(a+bx)\,dx &= -\int \left(\frac{x^{-2+m}}{2}-\frac{1}{2}x^{-2+m}\cosh(2a+2bx)\right)dx\\
&= \frac{x^{-1+m}}{2(1-m)}+\frac{1}{2}\int x^{-2+m}\cosh(2a+2bx)\,dx\\
&= \frac{x^{-1+m}}{2(1-m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{-2+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{-2+m}\,dx\\
&= \frac{x^{-1+m}}{2(1-m)}+2^{-1-m}be^{2a}x^m(-bx)^{-m}\Gamma(-1+m,-2bx)-2^{-1-m}be^{-2a}x^m(bx)^{-m}\Gamma(-1+m,2bx)
\end{aligned}$$

Mathematica [A] time = 0.100091, size = 72, normalized size = 0.87

$$\frac{1}{2}x^m\left(e^{2a}b2^{-m}(-bx)^{-m}\text{Gamma}(m-1,-2bx)-e^{-2a}b2^{-m}(bx)^{-m}\text{Gamma}(m-1,2bx)+\frac{1}{x-mx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)*Sinh[a + b*x]^2,x]
```

```
[Out] (x^m*((x - m*x)^(-1) + (b*E^(2*a)*Gamma[-1 + m, -2*b*x])/(2^m*(-(b*x))^m) -
 (b*Gamma[-1 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m)))/2
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int x^{-2+m}(\sinh(bx+a))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2+m)*sinh(b*x+a)^2,x)
```

```
[Out] int(x^(-2+m)*sinh(b*x+a)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.79577, size = 424, normalized size = 5.11

$$-\frac{4\,bx\cosh\left((m-2)\log(x)\right)+(m-1)\cosh\left((m-2)\log(2\,b)+2\,a\right)\Gamma\left(m-1,2\,bx\right)-(m-1)\cosh\left((m-2)\log(-2\,b)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*x*cosh((m - 2)*log(x)) + (m - 1)*cosh((m - 2)*log(2*b) + 2*a)*gam
ma(m - 1, 2*b*x) - (m - 1)*cosh((m - 2)*log(-2*b) - 2*a)*gamma(m - 1, -2*b*
x) - (m - 1)*gamma(m - 1, 2*b*x)*sinh((m - 2)*log(2*b) + 2*a) + (m - 1)*gam
ma(m - 1, -2*b*x)*sinh((m - 2)*log(-2*b) - 2*a) + 4*b*x*sinh((m - 2)*log(x)
))/(b*m - b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+m)*sinh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-2} \sinh (bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2)*sinh(b*x + a)^2, x)
```

3.91 $\int x^{-3+m} \sinh^2(a+bx)\,dx$

Optimal. Leaf size=84

$$-e^{2a}b^2 2^{-m}x^m(-bx)^{-m}\text{Gamma}(m-2,-2bx)-e^{-2a}b^2 2^{-m}x^m(bx)^{-m}\text{Gamma}(m-2,2bx)+\frac{x^{m-2}}{2(2-m)}$$

```
[Out] x^(-2 + m)/(2*(2 - m)) - (b^2*E^(2*a)*x^m*Gamma[-2 + m, -2*b*x])/(2^m*(-(b*
x))^m) - (b^2*x^m*Gamma[-2 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m)
```

Rubi [A] time = 0.142652, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3312, 3307, 2181}

$$-e^{2a}b^2 2^{-m}x^m(-bx)^{-m}\text{Gamma}(m-2,-2bx)-e^{-2a}b^2 2^{-m}x^m(bx)^{-m}\text{Gamma}(m-2,2bx)+\frac{x^{m-2}}{2(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + m)*Sinh[a + b*x]^2,x]
```

```
[Out] x^(-2 + m)/(2*(2 - m)) - (b^2*E^(2*a)*x^m*Gamma[-2 + m, -2*b*x])/(2^m*(-(b*
x))^m) - (b^2*x^m*Gamma[-2 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m)
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^{-3+m}\sinh^2(a+bx)\,dx &= -\int\left(\frac{x^{-3+m}}{2}-\frac{1}{2}x^{-3+m}\cosh(2a+2bx)\right)dx\\
&= \frac{x^{-2+m}}{2(2-m)}+\frac{1}{2}\int x^{-3+m}\cosh(2a+2bx)\,dx\\
&= \frac{x^{-2+m}}{2(2-m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{-3+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{-3+m}\,dx\\
&= \frac{x^{-2+m}}{2(2-m)}-2^{-m}b^2e^{2a}x^m(-bx)^{-m}\Gamma(-2+m,-2bx)-2^{-m}b^2e^{-2a}x^m(bx)^{-m}\Gamma(-2+m,2bx)
\end{aligned}$$

Mathematica [A] time = 0.111581, size = 77, normalized size = 0.92

$$x^m\left(e^{2a}b^2\left(-2^{-m}\right)(-bx)^{-m}\text{Gamma}(m-2,-2bx)-e^{-2a}b^22^{-m}(bx)^{-m}\text{Gamma}(m-2,2bx)+\frac{1}{(4-2m)x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + m)*Sinh[a + b*x]^2,x]
```

```
[Out] x^m*(1/((4 - 2*m)*x^2) - (b^2*E^(2*a)*Gamma[-2 + m, -2*b*x])/(2^m*(-(b*x))^
m) - (b^2*Gamma[-2 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int x^{-3+m}\left(\sinh\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3+m)*sinh(b*x+a)^2,x)
```

```
[Out] int(x^(-3+m)*sinh(b*x+a)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.78067, size = 427, normalized size = 5.08

$$-\frac{4\,bx\cosh\left((m-3)\log\left(x\right)\right)+(m-2)\cosh\left((m-3)\log\left(2\,b\right)+2\,a\right)\Gamma\left(m-2,2\,bx\right)-(m-2)\cosh\left((m-3)\log\left(-2\,b\right)-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*x*cosh((m - 3)*log(x)) + (m - 2)*cosh((m - 3)*log(2*b) + 2*a)*gam
ma(m - 2, 2*b*x) - (m - 2)*cosh((m - 3)*log(-2*b) - 2*a)*gamma(m - 2, -2*b*
x) - (m - 2)*gamma(m - 2, 2*b*x)*sinh((m - 3)*log(2*b) + 2*a) + (m - 2)*gam
ma(m - 2, -2*b*x)*sinh((m - 3)*log(-2*b) - 2*a) + 4*b*x*sinh((m - 3)*log(x)
))/(b*m - 2*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+m)*sinh(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{m-3} \sinh (bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 3)*sinh(b*x + a)^2, x)
```

3.92 $\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x\sqrt{\operatorname{csch}(x)}\right)dx$

Optimal. Leaf size=24

$$\frac{2x\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}-\frac{4}{9\operatorname{csch}^{\frac{3}{2}}(x)}$$

```
[Out] -4/(9*Csch[x]^(3/2)) + (2*x*Cosh[x])/(3*Sqrt[Csch[x]])
```

Rubi [A] time = 0.111688, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4187, 4189}

$$\frac{2x\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}-\frac{4}{9\operatorname{csch}^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Csch[x]^(3/2) + (x*Sqrt[Csch[x]])/3,x]
```

```
[Out] -4/(9*Csch[x]^(3/2)) + (2*x*Cosh[x])/(3*Sqrt[Csch[x]])
```

Rule 4187

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(d*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[((c + d*x)*Cos[e + f*x]*(b*C
sc[e + f*x])^(n + 1))/(b*f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x\sqrt{\operatorname{csch}(x)}\right)dx &= \frac{1}{3}\int x\sqrt{\operatorname{csch}(x)}\,dx+\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}\,dx\\ &= -\frac{4}{9\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{2x\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}-\frac{1}{3}\int x\sqrt{\operatorname{csch}(x)}\,dx+\frac{1}{3}\left(\sqrt{\operatorname{csch}(x)}\sqrt{-\sinh(x)}\right)\int \frac{}{\sqrt{-}}\\ &= -\frac{4}{9\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{2x\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}\end{aligned}$$

Mathematica [A] time = 0.0940987, size = 17, normalized size = 0.71

$$\frac{2(3x\coth(x)-2)}{9\operatorname{csch}^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Csch[x]^(3/2) + (x*Sqrt[Csch[x]])/3,x]
```

```
[Out] (2*(-2 + 3*x*Coth[x]))/(9*Csch[x]^(3/2))
```

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int x\left(\operatorname{csch}(x)\right)^{-\frac{3}{2}}+\frac{x}{3}\sqrt{\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/csch(x)^(3/2)+1/3*x*csch(x)^(1/2),x)
```

```
[Out] int(x/csch(x)^(3/2)+1/3*x*csch(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{3}\,x\sqrt{\operatorname{csch}(x)}+\frac{x}{\operatorname{csch}(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)^(3/2)+1/3*x*csch(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/3*x*sqrt(csch(x)) + x/csch(x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)^(3/2)+1/3*x*csch(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{3x}{\operatorname{csch}^{\frac{3}{2}}(x)}\,dx+\int x\sqrt{\operatorname{csch}(x)}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)**(3/2)+1/3*x*csch(x)**(1/2),x)
```

```
[Out] (Integral(3*x/csch(x)**(3/2), x) + Integral(x*sqrt(csch(x)), x))/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{3}\,x\sqrt{\operatorname{csch}(x)} + \frac{x}{\operatorname{csch}(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)^(3/2)+1/3*x*csch(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/3*x*sqrt(csch(x)) + x/csch(x)^(3/2), x)
```

3.93 $\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right)dx$

Optimal. Leaf size=24

$$\frac{2x\cosh(x)}{5\operatorname{csch}^{\frac{3}{2}}(x)}-\frac{4}{25\operatorname{csch}^{\frac{5}{2}}(x)}$$

```
[Out] -4/(25*Csch[x]^(5/2)) + (2*x*Cosh[x])/(5*Csch[x]^(3/2))
```

Rubi [A] time = 0.117555, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4187, 4189}

$$\frac{2x\cosh(x)}{5\operatorname{csch}^{\frac{3}{2}}(x)}-\frac{4}{25\operatorname{csch}^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Csch[x]^(5/2) + (3*x)/(5*Sqrt[Csch[x]]),x]
```

```
[Out] -4/(25*Csch[x]^(5/2)) + (2*x*Cosh[x])/(5*Csch[x]^(3/2))
```

Rule 4187

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(d*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[((c + d*x)*Cos[e + f*x]*(b*C
sc[e + f*x])^(n + 1))/(b*f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{3x}{5\sqrt{\operatorname{csch}(x)}}\right)dx &= \frac{3}{5}\int \frac{x}{\sqrt{\operatorname{csch}(x)}}\,dx+\int \frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)}\,dx\\
&= -\frac{4}{25\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{2x\cosh(x)}{5\operatorname{csch}^{\frac{3}{2}}(x)}-\frac{3}{5}\int \frac{x}{\sqrt{\operatorname{csch}(x)}}\,dx+\frac{3\int x\sqrt{-\sinh(x)}\,dx}{5\sqrt{\operatorname{csch}(x)}\sqrt{-\sinh(x)}}\\
&= -\frac{4}{25\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{2x\cosh(x)}{5\operatorname{csch}^{\frac{3}{2}}(x)}
\end{aligned}$$

Mathematica [A] time = 0.123822, size = 17, normalized size = 0.71

$$\frac{2(5x\coth(x)-2)}{25\operatorname{csch}^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Csch[x]^(5/2) + (3*x)/(5*Sqrt[Csch[x]]),x]
```

```
[Out] (2*(-2 + 5*x*Coth[x]))/(25*Csch[x]^(5/2))
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int x\,(\operatorname{csch}(x))^{-\frac{5}{2}}+\frac{3x}{5}\frac{1}{\sqrt{\operatorname{csch}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x)
```

```
[Out] int(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x}{5\sqrt{\operatorname{csch}(x)}}+\frac{x}{\operatorname{csch}(x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(3/5*x/sqrt(csch(x)) + x/csch(x)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{5x}{\operatorname{csch}^{\frac{5}{2}}(x)}\,dx+\int \frac{3x}{\sqrt{\operatorname{csch}(x)}}\,dx}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)**(5/2)+3/5*x/csch(x)**(1/2),x)
```

```
[Out] (Integral(5*x/csch(x)**(5/2), x) + Integral(3*x/sqrt(csch(x)), x))/5
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x}{5\sqrt{\operatorname{csch}(x)}} + \frac{x}{\operatorname{csch}(x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(3/5*x/sqrt(csch(x)) + x/csch(x)^(5/2), x)
```

3.94 $\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)}-\frac{5}{21}x\sqrt{\operatorname{csch}(x)}\right)dx$

Optimal. Leaf size=47

$$\frac{20}{63\operatorname{csch}^{\frac{3}{2}}(x)}-\frac{4}{49\operatorname{csch}^{\frac{7}{2}}(x)}+\frac{2x\cosh(x)}{7\operatorname{csch}^{\frac{5}{2}}(x)}-\frac{10x\cosh(x)}{21\sqrt{\operatorname{csch}(x)}}$$

```
[Out] -4/(49*Csch[x]^(7/2)) + (2*x*Cosh[x])/(7*Csch[x]^(5/2)) + 20/(63*Csch[x]^(3
/2)) - (10*x*Cosh[x])/(21*Sqrt[Csch[x]])
```

Rubi [A] time = 0.129631, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4187, 4189}

$$\frac{20}{63\operatorname{csch}^{\frac{3}{2}}(x)}-\frac{4}{49\operatorname{csch}^{\frac{7}{2}}(x)}+\frac{2x\cosh(x)}{7\operatorname{csch}^{\frac{5}{2}}(x)}-\frac{10x\cosh(x)}{21\sqrt{\operatorname{csch}(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Csch[x]^(7/2) - (5*x*Sqrt[Csch[x]])/21,x]
```

```
[Out] -4/(49*Csch[x]^(7/2)) + (2*x*Cosh[x])/(7*Csch[x]^(5/2)) + 20/(63*Csch[x]^(3
/2)) - (10*x*Cosh[x])/(21*Sqrt[Csch[x]])
```

Rule 4187

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(d*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[((c + d*x)*Cos[e + f*x]*(b*C
sc[e + f*x])^(n + 1))/(b*f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)}-\frac{5}{21}x\sqrt{\operatorname{csch}(x)}\right)dx &= -\left(\frac{5}{21}\int x\sqrt{\operatorname{csch}(x)}\,dx\right)+\int \frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)}\,dx\\ &= -\frac{4}{49\operatorname{csch}^{\frac{7}{2}}(x)}+\frac{2x\cosh(x)}{7\operatorname{csch}^{\frac{5}{2}}(x)}-\frac{5}{7}\int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)}\,dx-\frac{1}{21}\left(5\sqrt{\operatorname{csch}(x)}\sqrt{-\sinh(x)}\right)\int -\\ &= -\frac{4}{49\operatorname{csch}^{\frac{7}{2}}(x)}+\frac{2x\cosh(x)}{7\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{20}{63\operatorname{csch}^{\frac{3}{2}}(x)}-\frac{10x\cosh(x)}{21\sqrt{\operatorname{csch}(x)}}+\frac{5}{21}\int x\sqrt{\operatorname{csch}(x)}\,dx-\\ &= -\frac{4}{49\operatorname{csch}^{\frac{7}{2}}(x)}+\frac{2x\cosh(x)}{7\operatorname{csch}^{\frac{5}{2}}(x)}+\frac{20}{63\operatorname{csch}^{\frac{3}{2}}(x)}-\frac{10x\cosh(x)}{21\sqrt{\operatorname{csch}(x)}}\end{aligned}$$

Mathematica [A] time = 0.109528, size = 45, normalized size = 0.96

$$\sqrt{\operatorname{csch}(x)}\left(-\frac{13}{42}x\sinh(2x)+\frac{1}{28}x\sinh(4x)+\frac{88}{441}\cosh(2x)-\frac{1}{98}\cosh(4x)-\frac{167}{882}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Csch[x]^(7/2) - (5*x*Sqrt[Csch[x]])/21,x]
```

```
[Out] Sqrt[Csch[x]]*(-167/882 + (88*Cosh[2*x])/441 - Cosh[4*x]/98 - (13*x*Sinh[2*
x])/42 + (x*Sinh[4*x])/28)
```

Maple [F] time = 0.081, size = 0, normalized size = 0.

$$\int x\left(\operatorname{csch}(x)\right)^{-\frac{7}{2}}-\frac{5x}{21}\sqrt{\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x)
```

```
[Out] int(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{5}{21}\,x\sqrt{\operatorname{csch}(x)}+\frac{x}{\operatorname{csch}(x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-5/21*x*sqrt(csch(x)) + x/csch(x)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int -\frac{21x}{\operatorname{csch}^{\frac{7}{2}}(x)}\,dx+\int 5x\sqrt{\operatorname{csch}(x)}\,dx}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)**(7/2)-5/21*x*csch(x)**(1/2),x)
```

```
[Out] -(Integral(-21*x/csch(x)**(7/2), x) + Integral(5*x*sqrt(csch(x)), x))/21
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{5}{21}\,x\sqrt{\operatorname{csch}(x)} + \frac{x}{\operatorname{csch}(x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-5/21*x*sqrt(csch(x)) + x/csch(x)^(7/2), x)
```

3.95 $\int \left(\frac{x^2}{\mathbf{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x^2\sqrt{\mathbf{csch}(x)}\right)dx$

Optimal. Leaf size=76

$$-\frac{16}{27}i\sqrt{i\sinh(x)}\sqrt{\text{csch}(x)}\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)+\frac{2x^2\cosh(x)}{3\sqrt{\text{csch}(x)}}-\frac{8x}{9\text{csch}^{\frac{3}{2}}(x)}+\frac{16\cosh(x)}{27\sqrt{\text{csch}(x)}}$$

```
[Out] (-8*x)/(9*Csch[x]^(3/2)) + (16*Cosh[x])/(27*Sqrt[Csch[x]]) + (2*x^2*Cosh[x]
)/(3*Sqrt[Csch[x]]) - ((16*I)/27)*Sqrt[Csch[x]]*EllipticF[Pi/4 - (I/2)*x, 2
]*Sqrt[I*Sinh[x]]
```

Rubi [A] time = 0.207091, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4188, 4189, 3769, 3771, 2641}

$$\frac{2x^2\cosh(x)}{3\sqrt{\text{csch}(x)}}-\frac{8x}{9\text{csch}^{\frac{3}{2}}(x)}+\frac{16\cosh(x)}{27\sqrt{\text{csch}(x)}}-\frac{16}{27}i\sqrt{i\sinh(x)}\sqrt{\text{csch}(x)}F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Csch[x]^(3/2) + (x^2*Sqrt[Csch[x]])/3,x]
```

```
[Out] (-8*x)/(9*Csch[x]^(3/2)) + (16*Cosh[x])/(27*Sqrt[Csch[x]]) + (2*x^2*Cosh[x]
)/(3*Sqrt[Csch[x]]) - ((16*I)/27)*Sqrt[Csch[x]]*EllipticF[Pi/4 - (I/2)*x, 2
]*Sqrt[I*Sinh[x]]
```

Rule 4188

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(n + 1)/(b^2*n), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n + 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^n, x], x]
+ Simp[((c + d*x)^m*Cos[e + f*x]*(b*Csc[e + f*x])^(n + 1))/(b*f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
```

Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(\frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}\right)dx=\frac{1}{3}\int x^2\sqrt{\operatorname{csch}(x)}\,dx+\int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}\,dx$$
$$=-\frac{8x}{9\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{2x^2\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}-\frac{1}{3}\int x^2\sqrt{\operatorname{csch}(x)}\,dx+\frac{8}{9}\int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(x)}\,dx+\frac{1}{3}\left(\sqrt{\operatorname{csch}\right.$$
$$=-\frac{8x}{9\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{16\cosh(x)}{27\sqrt{\operatorname{csch}(x)}}+\frac{2x^2\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}-\frac{8}{27}\int \sqrt{\operatorname{csch}(x)}\,dx$$
$$=-\frac{8x}{9\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{16\cosh(x)}{27\sqrt{\operatorname{csch}(x)}}+\frac{2x^2\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}-\frac{1}{27}\left(8\sqrt{\operatorname{csch}(x)}\sqrt{i\sinh(x)}\right)\int \frac{1}{\sqrt{i\sinh}}$$
$$=-\frac{8x}{9\operatorname{csch}^{\frac{3}{2}}(x)}+\frac{16\cosh(x)}{27\sqrt{\operatorname{csch}(x)}}+\frac{2x^2\cosh(x)}{3\sqrt{\operatorname{csch}(x)}}-\frac{16}{27}i\sqrt{\operatorname{csch}(x)}F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x}$$

Mathematica [A] time = 0.143932, size = 63, normalized size = 0.83

$$\frac{1}{27}\sqrt{\operatorname{csch}(x)}\left(-16i\sqrt{i\sinh(x)}\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),2\right)+9x^2\sinh(2x)+12x+8\sinh(2x)-12x\cosh(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Csch[x]^(3/2) + (x^2*Sqrt[Csch[x]])/3,x]
```

```
[Out] (Sqrt[Csch[x]]*(12*x - 12*x*Cosh[2*x] - (16*I)*EllipticF[(Pi - (2*I)*x)/4,
2]*Sqrt[I*Sinh[x]] + 8*Sinh[2*x] + 9*x^2*Sinh[2*x]))/27
```

Maple [F] time = 0.071, size = 0, normalized size = 0.

$$\int x^2(\operatorname{csch}(x))^{-\frac{3}{2}}+\frac{x^2}{3}\sqrt{\operatorname{csch}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/csch(x)^(3/2)+1/3*x^2*csch(x)^(1/2),x)
```

```
[Out] int(x^2/csch(x)^(3/2)+1/3*x^2*csch(x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{3}x^2\sqrt{\operatorname{csch}(x)}+\frac{x^2}{\operatorname{csch}(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(x)^(3/2)+1/3*x^2*csch(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/3*x^2*sqrt(csch(x)) + x^2/csch(x)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(x)^(3/2)+1/3*x^2*csch(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{3x^2}{\operatorname{csch}^{\frac{3}{2}}(x)}\,dx + \int x^2\sqrt{\operatorname{csch}(x)}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/csch(x)**(3/2)+1/3*x**2*csch(x)**(1/2),x)
```

```
[Out] (Integral(3*x**2/csch(x)**(3/2), x) + Integral(x**2*sqrt(csch(x)), x))/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{3}\,x^2\sqrt{\operatorname{csch}(x)} + \frac{x^2}{\operatorname{csch}(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/csch(x)^(3/2)+1/3*x^2*csch(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/3*x^2*sqrt(csch(x)) + x^2/csch(x)^(3/2), x)
```

3.96 $\int (c+dx)^3(a+ia\sinh(e+fx))\,dx$

Optimal. Leaf size=98

$$\frac{6iad^2(c+dx)\cosh(e+fx)}{f^3}-\frac{3iad(c+dx)^2\sinh(e+fx)}{f^2}+\frac{ia(c+dx)^3\cosh(e+fx)}{f}+\frac{a(c+dx)^4}{4d}-\frac{6iad^3\sinh(e+fx)}{f^4}$$

```
[Out] (a*(c + d*x)^4)/(4*d) + ((6*I)*a*d^2*(c + d*x)*Cosh[e + f*x])/f^3 + (I*a*(c
 + d*x)^3*Cosh[e + f*x])/f - ((6*I)*a*d^3*Sinh[e + f*x])/f^4 - ((3*I)*a*d*(
c + d*x)^2*Sinh[e + f*x])/f^2
```

Rubi [A] time = 0.139952, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3317, 3296, 2637}

$$\frac{6iad^2(c+dx)\cosh(e+fx)}{f^3}-\frac{3iad(c+dx)^2\sinh(e+fx)}{f^2}+\frac{ia(c+dx)^3\cosh(e+fx)}{f}+\frac{a(c+dx)^4}{4d}-\frac{6iad^3\sinh(e+fx)}{f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) + ((6*I)*a*d^2*(c + d*x)*Cosh[e + f*x])/f^3 + (I*a*(c
 + d*x)^3*Cosh[e + f*x])/f - ((6*I)*a*d^3*Sinh[e + f*x])/f^4 - ((3*I)*a*d*(
c + d*x)^2*Sinh[e + f*x])/f^2
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^3(a+ia\sinh(e+fx))\,dx = \int \left(a(c+dx)^3+ia(c+dx)^3\sinh(e+fx)\right)\,dx$$
$$= \frac{a(c+dx)^4}{4d}+(ia)\int (c+dx)^3\sinh(e+fx)\,dx$$
$$= \frac{a(c+dx)^4}{4d}+\frac{ia(c+dx)^3\cosh(e+fx)}{f}-\frac{(3iad)\int (c+dx)^2\cosh(e+fx)\,dx}{f}$$
$$= \frac{a(c+dx)^4}{4d}+\frac{ia(c+dx)^3\cosh(e+fx)}{f}-\frac{3iad(c+dx)^2\sinh(e+fx)}{f^2}+\frac{(6iad^2)\int}{}$$
$$= \frac{a(c+dx)^4}{4d}+\frac{6iad^2(c+dx)\cosh(e+fx)}{f^3}+\frac{ia(c+dx)^3\cosh(e+fx)}{f}-\frac{3iad(c+}{}$$
$$= \frac{a(c+dx)^4}{4d}+\frac{6iad^2(c+dx)\cosh(e+fx)}{f^3}+\frac{ia(c+dx)^3\cosh(e+fx)}{f}-\frac{6iad^3\sin}{}$$

Mathematica [A] time = 0.832429, size = 128, normalized size = 1.31

$$\frac{a\left(-12id\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2+2\right)\right)\sinh(e+fx)+4if(c+dx)\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2+6\right)\right)\cosh(e+fx)+\right.}{4f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (a*(f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + (4*I)*f*(c + d*x)*(
c^2*f^2 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Cosh[e + f*x] - (12*I)*d*(c^2*f^
2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x]))/(4*f^4)
```

Maple [B] time = 0.01, size = 494, normalized size = 5.

$$\frac{1}{f}\left(\frac{d^3a\left(fx+e\right)^4}{4f^3}-\frac{3idec^2a\cosh\left(fx+e\right)}{f}-\frac{d^3ea\left(fx+e\right)^3}{f^3}-\frac{3id^3ea\left(\left(fx+e\right)^2\cosh\left(fx+e\right)-2\left(fx+e\right)\sinh\left(fx\right.\right.}{f^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+I*a*sinh(f*x+e)),x)
```

```
[Out] 1/f*(1/4/f^3*d^3*a*(f*x+e)^4-3*I*d*e/f*c^2*a*cosh(f*x+e)-1/f^3*d^3*e*a*(f*x
+e)^3-3*I/f^3*d^3*e*a*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sinh(f*x+e)+2*cosh(f
*x+e))+1/f^2*d^2*c*a*(f*x+e)^3-I*d^3*e^3/f^3*a*cosh(f*x+e)+3/2/f^3*d^3*e^2*
a*(f*x+e)^2-6*I/f^2*d^2*e*c*a*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))-3/f^2*d^2*e
*c*a*(f*x+e)^2+3*I*d^2*e^2/f^2*c*a*cosh(f*x+e)+3/2/f*d*c^2*a*(f*x+e)^2+3*I/
f^2*d^2*c*a*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sinh(f*x+e)+2*cosh(f*x+e))-d^3
*e^3/f^3*a*(f*x+e)+I*c^3*a*cosh(f*x+e)+3*d^2*e^2/f^2*c*a*(f*x+e)+3*I/f^3*d^
3*e^2*a*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))-3*d*e/f*c^2*a*(f*x+e)+3*I/f*d*c^2
*a*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))+c^3*a*(f*x+e)+I/f^3*d^3*a*((f*x+e)^3*c
osh(f*x+e)-3*(f*x+e)^2*sinh(f*x+e)+6*(f*x+e)*cosh(f*x+e)-6*sinh(f*x+e)))
```

Maxima [B] time = 1.0986, size = 317, normalized size = 3.23

$$\frac{1}{4}ad^3x^4+acd^2x^3+\frac{3}{2}ac^2dx^2+ac^3x+\frac{3}{2}iac^2d\left(\frac{\left(fxe^e-e^e\right)e^{(fx)}}{f^2}+\frac{\left(fx+1\right)e^{(-fx-e)}}{f^2}\right)+\frac{3}{2}iacd^2\left(\frac{\left(f^2x^2e^e-2fxe^e+2e^e\right.}{f^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 3/2*I*a*c^2*d*((f
*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(-f*x - e)/f^2) + 3/2*I*a*c*d^2*((f
^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x
 - e)/f^3) + 1/2*I*a*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6*f*x*e^e - 6*e^e)
*e^(f*x)/f^4 + (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(-f*x - e)/f^4) + I*a*c^
3*cosh(f*x + e)/f
```

Fricas [B] time = 2.78733, size = 656, normalized size = 6.69

$$\frac{\left(2i\,ad^3f^3x^3 + 2i\,ac^3f^3 + 6i\,ac^2df^2 + 12i\,acd^2f + 12i\,ad^3 + \left(6i\,acd^2f^3 + 6i\,ad^3f^2\right)x^2 + \left(6i\,ac^2df^3 + 12i\,acd^2f^2 + 12i\,ad^3f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*I*a*d^3*f^3*x^3 + 2*I*a*c^3*f^3 + 6*I*a*c^2*d*f^2 + 12*I*a*c*d^2*f +
 12*I*a*d^3 + (6*I*a*c*d^2*f^3 + 6*I*a*d^3*f^2)*x^2 + (6*I*a*c^2*d*f^3 + 12
*I*a*c*d^2*f^2 + 12*I*a*d^3*f)*x + (2*I*a*d^3*f^3*x^3 + 2*I*a*c^3*f^3 - 6*I
*a*c^2*d*f^2 + 12*I*a*c*d^2*f - 12*I*a*d^3 + (6*I*a*c*d^2*f^3 - 6*I*a*d^3*f
^2)*x^2 + (6*I*a*c^2*d*f^3 - 12*I*a*c*d^2*f^2 + 12*I*a*d^3*f)*x)*e^(2*f*x +
 2*e) + (a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^
4*x)*e^(f*x + e))*e^(-f*x - e)/f^4
```

Sympy [A] time = 2.22986, size = 573, normalized size = 5.85

$$ac^3x + \frac{3ac^2dx^2}{2} + acd^2x^3 + \frac{ad^3x^4}{4} + \begin{cases} \frac{\left(\left(2iac^3f^{19}e^{3e} + 6iac^2df^{19}xe^{3e} + 6iac^2df^{18}e^{3e} + 6iacd^2f^{19}x^2e^{3e} + 12iacd^2f^{18}xe^{3e} + 12iacd^2f^{17}e^{3e} + 2iad^3f^{19}x^3e^{3e} + \cdots\right.\right.}{} \\ \frac{x^4\left(iad^3e^{2e} - iad^3\right)e^{-e}}{8} + \frac{x^3\left(iacd^2e^{2e} - iacd^2\right)e^{-e}}{2} + \frac{x^2\left(3iac^2de^{2e} - 3iac^2d\right)e^{-e}}{4} + \frac{x\left(iac^3e^{2e} - iac^3\right)e^{-e}}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+I*a*sinh(f*x+e)),x)
```

```
[Out] a*c**3*x + 3*a*c**2*d*x**2/2 + a*c*d**2*x**3 + a*d**3*x**4/4 + Piecewise(((
(2*I*a*c**3*f**19*exp(3*e) + 6*I*a*c**2*d*f**19*x*exp(3*e) + 6*I*a*c**2*d*f
**18*exp(3*e) + 6*I*a*c*d**2*f**19*x**2*exp(3*e) + 12*I*a*c*d**2*f**18*x*ex
p(3*e) + 12*I*a*c*d**2*f**17*exp(3*e) + 2*I*a*d**3*f**19*x**3*exp(3*e) + 6*
I*a*d**3*f**18*x**2*exp(3*e) + 12*I*a*d**3*f**17*x*exp(3*e) + 12*I*a*d**3*f
**16*exp(3*e))*exp(-f*x) + (2*I*a*c**3*f**19*exp(5*e) + 6*I*a*c**2*d*f**19*
x*exp(5*e) - 6*I*a*c**2*d*f**18*exp(5*e) + 6*I*a*c*d**2*f**19*x**2*exp(5*e)
 - 12*I*a*c*d**2*f**18*x*exp(5*e) + 12*I*a*c*d**2*f**17*exp(5*e) + 2*I*a*d*
*3*f**19*x**3*exp(5*e) - 6*I*a*d**3*f**18*x**2*exp(5*e) + 12*I*a*d**3*f**17
*x*exp(5*e) - 12*I*a*d**3*f**16*exp(5*e))*exp(f*x))*exp(-4*e)/(4*f**20), Ne
(4*f**20*exp(4*e), 0)), (x**4*(I*a*d**3*exp(2*e) - I*a*d**3)*exp(-e)/8 + x*
*3*(I*a*c*d**2*exp(2*e) - I*a*c*d**2)*exp(-e)/2 + x**2*(3*I*a*c**2*d*exp(2*
e) - 3*I*a*c**2*d)*exp(-e)/4 + x*(I*a*c**3*exp(2*e) - I*a*c**3)*exp(-e)/2,
True))
```

Giac [B] time = 1.29727, size = 356, normalized size = 3.63

$$\frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x - \frac{\left(-i\,ad^3f^3x^3 - 3i\,acd^2f^3x^2 - 3i\,ac^2df^3x + 3i\,ad^3f^2x^2 - i\,ac^3f^3 + 6i\,acd^2f^2x + 3\right.}{2\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x - 1/2*(-I*a*d^3*f^3
*x^3 - 3*I*a*c*d^2*f^3*x^2 - 3*I*a*c^2*d*f^3*x + 3*I*a*d^3*f^2*x^2 - I*a*c^
3*f^3 + 6*I*a*c*d^2*f^2*x + 3*I*a*c^2*d*f^2 - 6*I*a*d^3*f*x - 6*I*a*c*d^2*f
 + 6*I*a*d^3)*e^(f*x + e)/f^4 - 1/2*(-I*a*d^3*f^3*x^3 - 3*I*a*c*d^2*f^3*x^2
 - 3*I*a*c^2*d*f^3*x - 3*I*a*d^3*f^2*x^2 - I*a*c^3*f^3 - 6*I*a*c*d^2*f^2*x
- 3*I*a*c^2*d*f^2 - 6*I*a*d^3*f*x - 6*I*a*c*d^2*f - 6*I*a*d^3)*e^(-f*x - e)
/f^4
```

3.97 $\int (c+dx)^2(a+ia\sinh(e+fx))\,dx$

Optimal. Leaf size=74

$$-\frac{2iad(c+dx)\sinh(e+fx)}{f^2}+\frac{ia(c+dx)^2\cosh(e+fx)}{f}+\frac{a(c+dx)^3}{3d}+\frac{2iad^2\cosh(e+fx)}{f^3}$$

```
[Out] (a*(c + d*x)^3)/(3*d) + ((2*I)*a*d^2*Cosh[e + f*x])/f^3 + (I*a*(c + d*x)^2*
Cosh[e + f*x])/f - ((2*I)*a*d*(c + d*x)*Sinh[e + f*x])/f^2
```

Rubi [A] time = 0.0970964, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3317, 3296, 2638}

$$-\frac{2iad(c+dx)\sinh(e+fx)}{f^2}+\frac{ia(c+dx)^2\cosh(e+fx)}{f}+\frac{a(c+dx)^3}{3d}+\frac{2iad^2\cosh(e+fx)}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^3)/(3*d) + ((2*I)*a*d^2*Cosh[e + f*x])/f^3 + (I*a*(c + d*x)^2*
Cosh[e + f*x])/f - ((2*I)*a*d*(c + d*x)*Sinh[e + f*x])/f^2
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2(a+ia\sinh(e+fx))\,dx &= \int \left(a(c+dx)^2+ia(c+dx)^2\sinh(e+fx)\right)\,dx\\
&= \frac{a(c+dx)^3}{3d}+(ia)\int (c+dx)^2\sinh(e+fx)\,dx\\
&= \frac{a(c+dx)^3}{3d}+\frac{ia(c+dx)^2\cosh(e+fx)}{f}-\frac{(2iad)\int (c+dx)\cosh(e+fx)\,dx}{f}\\
&= \frac{a(c+dx)^3}{3d}+\frac{ia(c+dx)^2\cosh(e+fx)}{f}-\frac{2iad(c+dx)\sinh(e+fx)}{f^2}+\frac{\left(2iad^2\right)\int \sin}{f}\\
&= \frac{a(c+dx)^3}{3d}+\frac{2iad^2\cosh(e+fx)}{f^3}+\frac{ia(c+dx)^2\cosh(e+fx)}{f}-\frac{2iad(c+dx)\sinh(e}{f^2}
\end{aligned}$$

Mathematica [A] time = 0.472278, size = 88, normalized size = 1.19

$$\frac{a\left(3i\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2+2\right)\right)\cosh(e+fx)+f^3x\left(3c^2+3cdx+d^2x^2\right)-6idf(c+dx)\sinh(e+fx)\right)}{3f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (a*(f^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) + (3*I)*(c^2*f^2 + 2*c*d*f^2*x + d^2*
(2 + f^2*x^2))*Cosh[e + f*x] - (6*I)*d*f*(c + d*x)*Sinh[e + f*x]))/(3*f^3)
```

Maple [B] time = 0.01, size = 249, normalized size = 3.4

$$\frac{1}{f}\left(\frac{ad^2\left(fx+e\right)^3}{3\,f^2}+\frac{id^2a\left(\left(fx+e\right)^2\cosh\left(fx+e\right)-2\,\left(fx+e\right)\sinh\left(fx+e\right)+2\,\cosh\left(fx+e\right)\right)}{f^2}-\frac{d^2ea\left(fx+e\right)^2}{f^2}-\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+I*a*sinh(f*x+e)),x)
```

```
[Out] 1/f*(1/3/f^2*d^2*a*(f*x+e)^3+I/f^2*d^2*a*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*s
inh(f*x+e)+2*cosh(f*x+e))-1/f^2*d^2*e*a*(f*x+e)^2-2*I/f^2*d^2*e*a*((f*x+e)*
cosh(f*x+e)-sinh(f*x+e))+1/f*d*c*a*(f*x+e)^2+2*I/f*d*c*a*((f*x+e)*cosh(f*x+
e)-sinh(f*x+e))+d^2*e^2/f^2*a*(f*x+e)+I*d^2*e^2/f^2*a*cosh(f*x+e)-2*d*e/f*c
*a*(f*x+e)-2*I*d*e/f*c*a*cosh(f*x+e)+c^2*a*(f*x+e)+I*c^2*a*cosh(f*x+e))
```

Maxima [B] time = 1.06462, size = 190, normalized size = 2.57

$$\frac{1}{3}ad^2x^3+acdx^2+ac^2x+i\,acd\left(\frac{\left(fxe^e-e^e\right)e^{(fx)}}{f^2}+\frac{(fx+1)e^{(-fx-e)}}{f^2}\right)+\frac{1}{2}i\,ad^2\left(\frac{\left(f^2x^2e^e-2\,fxe^e+2\,e^e\right)e^{(fx)}}{f^3}+\frac{\left(f^2x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + I*a*c*d*((f*x*e^e - e^e)*e^(f*x)/f^2
+ (f*x + 1)*e^(-f*x - e)/f^2) + 1/2*I*a*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e
^e)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + I*a*c^2*cosh(f*
x + e)/f
```

Fricas [B] time = 2.64756, size = 402, normalized size = 5.43

$$\frac{\left(3i\,ad^2f^2x^2+3i\,ac^2f^2+6i\,acdf+6i\,ad^2+\left(6i\,acdf^2+6i\,ad^2f\right)x+\left(3i\,ad^2f^2x^2+3i\,ac^2f^2-6i\,acdf+6i\,ad^2+\left(6i\,acc\right.\right.\right.}{6\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*I*a*d^2*f^2*x^2 + 3*I*a*c^2*f^2 + 6*I*a*c*d*f + 6*I*a*d^2 + (6*I*a*c
*d*f^2 + 6*I*a*d^2*f)*x + (3*I*a*d^2*f^2*x^2 + 3*I*a*c^2*f^2 - 6*I*a*c*d*f
+ 6*I*a*d^2 + (6*I*a*c*d*f^2 - 6*I*a*d^2*f)*x)*e^(2*f*x + 2*e) + 2*(a*d^2*f
^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x)*e^(f*x + e))*e^(-f*x - e)/f^3
```

Sympy [A] time = 1.53159, size = 350, normalized size = 4.73

$$ac^2x + acdx^2 + \frac{ad^2x^3}{3} + \begin{cases} \frac{((2iac^2f^{11}e^{2e}+4iacdf^{11}xe^{2e}+4iacdf^{10}e^{2e}+2iad^2f^{11}x^2e^{2e}+4iad^2f^{10}xe^{2e}+4iad^2f^9e^{2e})e^{-fx}+(2iac^2f^{11}e^{4e}+4iacdf^{11}xe^{4e}-4iacdf^{1}\cdots}{4f^{12}} \\ \frac{x^3(iad^2e^{2e}-iad^2)e^{-e}}{6} + \frac{x^2(iacde^{2e}-iacd)e^{-e}}{2} + \frac{x(iac^2e^{2e}-iac^2)e^{-e}}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+I*a*sinh(f*x+e)),x)
```

```
[Out] a*c**2*x + a*c*d*x**2 + a*d**2*x**3/3 + Piecewise((((2*I*a*c**2*f**11*exp(2
*e) + 4*I*a*c*d*f**11*x*exp(2*e) + 4*I*a*c*d*f**10*exp(2*e) + 2*I*a*d**2*f*
*11*x**2*exp(2*e) + 4*I*a*d**2*f**10*x*exp(2*e) + 4*I*a*d**2*f**9*exp(2*e))
*exp(-f*x) + (2*I*a*c**2*f**11*exp(4*e) + 4*I*a*c*d*f**11*x*exp(4*e) - 4*I*
a*c*d*f**10*exp(4*e) + 2*I*a*d**2*f**11*x**2*exp(4*e) - 4*I*a*d**2*f**10*x*
exp(4*e) + 4*I*a*d**2*f**9*exp(4*e))*exp(f*x))*exp(-3*e)/(4*f**12), Ne(4*f*
*12*exp(3*e), 0)), (x**3*(I*a*d**2*exp(2*e) - I*a*d**2)*exp(-e)/6 + x**2*(I
*a*c*d*exp(2*e) - I*a*c*d)*exp(-e)/2 + x*(I*a*c**2*exp(2*e) - I*a*c**2)*exp
(-e)/2, True))
```

Giac [B] time = 1.26303, size = 205, normalized size = 2.77

$$\frac{1}{3}ad^2x^3 + acdx^2 + ac^2x - \frac{(-i\,ad^2f^2x^2 - 2i\,acdf^2x - i\,ac^2f^2 + 2i\,ad^2fx + 2i\,acdf - 2i\,ad^2)e^{(fx+e)}}{2\,f^3} + \frac{(i\,ad^2f^2x^2 + 2i\,acdf^2\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x - 1/2*(-I*a*d^2*f^2*x^2 - 2*I*a*c*d*f^2
*x - I*a*c^2*f^2 + 2*I*a*d^2*f*x + 2*I*a*c*d*f - 2*I*a*d^2)*e^(f*x + e)/f^3
 + 1/2*(I*a*d^2*f^2*x^2 + 2*I*a*c*d*f^2*x + I*a*c^2*f^2 + 2*I*a*d^2*f*x + 2
*I*a*c*d*f + 2*I*a*d^2)*e^(-f*x - e)/f^3
```

3.98 $\int (c+dx)(a+ia\sinh(e+fx))\,dx$

Optimal. Leaf size=50

$$\frac{ia(c+dx)\cosh(e+fx)}{f}+\frac{a(c+dx)^2}{2d}-\frac{iad\sinh(e+fx)}{f^2}$$

```
[Out] (a*(c + d*x)^2)/(2*d) + (I*a*(c + d*x)*Cosh[e + f*x])/f - (I*a*d*Sinh[e + f
*x])/f^2
```

Rubi [A] time = 0.0533599, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3317, 3296, 2637}

$$\frac{ia(c+dx)\cosh(e+fx)}{f}+\frac{a(c+dx)^2}{2d}-\frac{iad\sinh(e+fx)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^2)/(2*d) + (I*a*(c + d*x)*Cosh[e + f*x])/f - (I*a*d*Sinh[e + f
*x])/f^2
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)(a+ia\sinh(e+fx))\,dx &= \int (a(c+dx)+ia(c+dx)\sinh(e+fx))\,dx\\ &= \frac{a(c+dx)^2}{2d}+(ia)\int (c+dx)\sinh(e+fx)\,dx\\ &= \frac{a(c+dx)^2}{2d}+\frac{ia(c+dx)\cosh(e+fx)}{f}-\frac{(iad)\int \cosh(e+fx)\,dx}{f}\\ &= \frac{a(c+dx)^2}{2d}+\frac{ia(c+dx)\cosh(e+fx)}{f}-\frac{iad\sinh(e+fx)}{f^2}\end{aligned}$$

Mathematica [A] time = 0.0842476, size = 48, normalized size = 0.96

$$\frac{a\left(2if(c+dx)\cosh(e+fx)+f^2x(2c+dx)-2id\sinh(e+fx)\right)}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (a*(f^2*x*(2*c + d*x) + (2*I)*f*(c + d*x)*Cosh[e + f*x] - (2*I)*d*Sinh[e +
f*x]))/(2*f^2)
```

Maple [B] time = 0.009, size = 96, normalized size = 1.9

$$\frac{1}{f}\left(\frac{da\left(fx+e\right)^2}{2f}+\frac{ida\left(\left(fx+e\right)\cosh\left(fx+e\right)-\sinh\left(fx+e\right)\right)}{f}-\frac{dea\left(fx+e\right)}{f}-\frac{idea\cosh\left(fx+e\right)}{f}+ca\left(fx+e\right)+ica\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+I*a*sinh(f*x+e)),x)
```

```
[Out] 1/f*(1/2/f*d*a*(f*x+e)^2+I/f*d*a*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))-d*e/f*a*
(f*x+e)-I*d*e/f*a*cosh(f*x+e)+c*a*(f*x+e)+I*c*a*cosh(f*x+e))
```

Maxima [A] time = 1.04286, size = 89, normalized size = 1.78

$$\frac{1}{2}adx^2+acx+\frac{1}{2}iad\left(\frac{\left(fxe^e-e^e\right)e^{(fx)}}{f^2}+\frac{\left(fx+1\right)e^{(-fx-e)}}{f^2}\right)+\frac{iac\cosh\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + 1/2*I*a*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^
(-f*x - e)/f^2) + I*a*c*cosh(f*x + e)/f
```

Fricas [A] time = 2.63207, size = 192, normalized size = 3.84

$$\frac{\left(iadfx+iacf+iad+\left(iadfx+iacf-iad\right)e^{(2fx+2e)}+\left(adf^2x^2+2acf^2x\right)e^{(fx+e)}\right)e^{(-fx-e)}}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*a*d*f*x + I*a*c*f + I*a*d + (I*a*d*f*x + I*a*c*f - I*a*d)*e^(2*f*x +
 2*e) + (a*d*f^2*x^2 + 2*a*c*f^2*x)*e^(f*x + e))*e^(-f*x - e)/f^2
```

Sympy [A] time = 0.943545, size = 177, normalized size = 3.54

$$acx + \frac{adx^2}{2} + \begin{cases} \frac{\left(\left(2iacf^5e^e+2iadf^5xe^e+2iadf^4e^e\right)e^{-fx}+\left(2iacf^5e^{3e}+2iadf^5xe^{3e}-2iadf^4e^{3e}\right)e^{fx}\right)e^{-2e}}{4f^6} & \text{for } 4f^6e^{2e} \neq 0 \\ \frac{x^2\left(iade^{2e}-iad\right)e^{-e}}{4} + \frac{x\left(iace^{2e}-iac\right)e^{-e}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+I*a*sinh(f*x+e)),x)
```

```
[Out] a*c*x + a*d*x**2/2 + Piecewise((((2*I*a*c*f**5*exp(e) + 2*I*a*d*f**5*x*exp(
e) + 2*I*a*d*f**4*exp(e))*exp(-f*x) + (2*I*a*c*f**5*exp(3*e) + 2*I*a*d*f**5
*x*exp(3*e) - 2*I*a*d*f**4*exp(3*e))*exp(f*x))*exp(-2*e)/(4*f**6), Ne(4*f**
6*exp(2*e), 0)), (x**2*(I*a*d*exp(2*e) - I*a*d)*exp(-e)/4 + x*(I*a*c*exp(2*
e) - I*a*c)*exp(-e)/2, True))
```

Giac [A] time = 1.29728, size = 96, normalized size = 1.92

$$\frac{1}{2}adx^2 + acx - \frac{\left(-i\,adfx - i\,acf + i\,ad\right)e^{\left(fx+e\right)}}{2\,f^2} - \frac{\left(-i\,adfx - i\,acf - i\,ad\right)e^{\left(-fx-e\right)}}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*d*x^2 + a*c*x - 1/2*(-I*a*d*f*x - I*a*c*f + I*a*d)*e^(f*x + e)/f^2 -
1/2*(-I*a*d*f*x - I*a*c*f - I*a*d)*e^(-f*x - e)/f^2
```

3.99 $\int \frac{a+ia\sinh(e+fx)}{c+dx}\,dx$

Optimal. Leaf size=70

$$\frac{ia\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{ia\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{a\log(c+dx)}{d}$$

```
[Out] (a*Log[c + d*x])/d + (I*a*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d
+ (I*a*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d
```

Rubi [A] time = 0.161418, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3317, 3303, 3298, 3301}

$$\frac{ia\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{ia\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{a\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[e + f*x])/(c + d*x),x]
```

```
[Out] (a*Log[c + d*x])/d + (I*a*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d
+ (I*a*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{a+ia\sinh(e+fx)}{c+dx}\,dx = \int \left(\frac{a}{c+dx}+\frac{ia\sinh(e+fx)}{c+dx}\right)dx$$
$$= \frac{a\log(c+dx)}{d} + (ia)\int \frac{\sinh(e+fx)}{c+dx}\,dx$$
$$= \frac{a\log(c+dx)}{d} + \left(ia\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\sinh\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx + \left(ia\sinh\left(e-\frac{cf}{d}\right)\right)\int \frac{\cosh\left(\frac{cf}{d}\right.}{c+}$$
$$= \frac{a\log(c+dx)}{d} + \frac{ia\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d} + \frac{ia\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d}$$

Mathematica [A] time = 0.29515, size = 60, normalized size = 0.86

$$\frac{a\left(i\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+i\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+\log(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[e + f*x])/(c + d*x),x]
```

```
[Out] (a*(Log[c + d*x] + I*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] + I*Cosh[e
 - (c*f)/d]*SinhIntegral[f*(c/d + x)]))/d
```

Maple [A] time = 0.045, size = 96, normalized size = 1.4

$$\frac{a\ln(dx+c)}{d}+\frac{\frac{i}{2}a}{d}e^{\frac{cf-de}{d}}\text{Ei}\left(1,fx+e+\frac{cf-de}{d}\right)-\frac{\frac{i}{2}a}{d}e^{-\frac{cf-de}{d}}\text{Ei}\left(1,-fx-e-\frac{cf-de}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))/(d*x+c),x)
```

```
[Out] a*ln(d*x+c)/d+1/2*I*a/d*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*I*a/d*
exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Maxima [A] time = 1.20787, size = 96, normalized size = 1.37

$$\frac{1}{2}ia\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right)+\frac{a\log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*I*a*(e^(-e + c*f/d)*exp_integral_e(1, (d*x + c)*f/d)/d - e^(e - c*f/d)*
exp_integral_e(1, -(d*x + c)*f/d)/d) + a*log(d*x + c)/d
```

Fricas [A] time = 2.77263, size = 162, normalized size = 2.31

$$\frac{i\,a\text{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)} - i\,a\text{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)} + 2\,a\log\left(\frac{dx+c}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(I*a*Ei((d*f*x + c*f)/d)*e^((d*e - c*f)/d) - I*a*Ei(-(d*f*x + c*f)/d)*e
^(-(d*e - c*f)/d) + 2*a*log((d*x + c)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{i\sinh\left(e+fx\right)}{c+dx}\,dx + \int\frac{1}{c+dx}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c),x)
```

```
[Out] a*(Integral(I*sinh(e + f*x)/(c + d*x), x) + Integral(1/(c + d*x), x))
```

Giac [A] time = 1.29067, size = 96, normalized size = 1.37

$$-\frac{i\,a\text{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)} - i\,a\text{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)} - 2\,a\log\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(I*a*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) - I*a*Ei((d*f*x + c*f)/d)*e^(-
c*f/d + e) - 2*a*log(d*x + c))/d
```

3.100 $\int \frac{a+ia\sinh(e+fx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=95

$$\frac{iaf\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{iaf\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{ia\sinh(e+fx)}{d(c+dx)}-\frac{a}{d(c+dx)}$$

```
[Out] -(a/(d*(c + d*x))) + (I*a*f*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/
d^2 - (I*a*Sinh[e + f*x])/(d*(c + d*x)) + (I*a*f*Sinh[e - (c*f)/d]*SinhInte
gral[(c*f)/d + f*x])/d^2
```

Rubi [A] time = 0.189107, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3317, 3297, 3303, 3298, 3301}

$$\frac{iaf\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{iaf\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{ia\sinh(e+fx)}{d(c+dx)}-\frac{a}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[e + f*x])/(c + d*x)^2,x]
```

```
[Out] -(a/(d*(c + d*x))) + (I*a*f*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/
d^2 - (I*a*Sinh[e + f*x])/(d*(c + d*x)) + (I*a*f*Sinh[e - (c*f)/d]*SinhInte
gral[(c*f)/d + f*x])/d^2
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^2}\,dx = \int \left(\frac{a}{(c+dx)^2}+\frac{ia\sinh(e+fx)}{(c+dx)^2}\right)dx$$
$$= -\frac{a}{d(c+dx)}+(ia)\int \frac{\sinh(e+fx)}{(c+dx)^2}\,dx$$
$$= -\frac{a}{d(c+dx)}-\frac{ia\sinh(e+fx)}{d(c+dx)}+\frac{(iaf)\int \frac{\cosh(e+fx)}{c+dx}\,dx}{d}$$
$$= -\frac{a}{d(c+dx)}-\frac{ia\sinh(e+fx)}{d(c+dx)}+\frac{\left(iaf\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\cosh\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx}{d}+\frac{\left(iaf\sinh\left(e-\frac{cf}{d}\right)\right)\int}{d}$$
$$= -\frac{a}{d(c+dx)}+\frac{iaf\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(\frac{cf}{d}+fx\right)}{d^2}-\frac{ia\sinh(e+fx)}{d(c+dx)}+\frac{iaf\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(\frac{cf}{d}\right.}{d^2}$$

Mathematica [A] time = 0.4877, size = 83, normalized size = 0.87

$$\frac{ia\left(f(c+dx)\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\cosh\left(e-\frac{cf}{d}\right)+f(c+dx)\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)-d(\sinh(e+fx)-i)\right)}{d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[e + f*x])/(c + d*x)^2,x]
```

```
[Out] (I*a*(f*(c + d*x)*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] - d*(-I + Sin
h[e + f*x]) + f*(c + d*x)*Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)]))/(d^
2*(c + d*x))
```

Maple [A] time = 0.053, size = 153, normalized size = 1.6

$$-\frac{a}{d(dx+c)}+\frac{\frac{i}{2}fa\mathrm{e}^{-fx-e}}{d\left(dfx+cf\right)}-\frac{\frac{i}{2}fa}{d^2}\mathrm{e}^{\frac{cf-de}{d}}\mathrm{Ei}\left(1,fx+e+\frac{cf-de}{d}\right)-\frac{\frac{i}{2}fa\mathrm{e}^{fx+e}}{d^2}\left(\frac{cf}{d}+fx\right)^{-1}-\frac{\frac{i}{2}fa}{d^2}\mathrm{e}^{-\frac{cf-de}{d}}\mathrm{Ei}\left(1,-fx-e-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))/(d*x+c)^2,x)
```

```
[Out] -a/d/(d*x+c)+1/2*I*a*f*exp(-f*x-e)/d/(d*f*x+c*f)-1/2*I*a*f/d^2*exp((c*f-d*e
)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*I*a*f/d^2*exp(f*x+e)/(c*f/d+f*x)-1/2*I*a*f
/d^2*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Maxima [A] time = 1.22058, size = 119, normalized size = 1.25

$$\frac{1}{2}ia\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right)-\frac{a}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*I*a*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)/((d*x + c)*d) - e^
(e - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d)) - a/(d^2*x + c
*d)
```

Fricas [A] time = 2.7878, size = 290, normalized size = 3.05

$$\frac{\left(-i\,ade^{\left(2\,fx+2\,e\right)}+i\,ad+\left(\left(i\,adfx+i\,acf\right)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)}+\left(i\,adfx+i\,acf\right)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)}-2\,ad\right)e^{\left(fx+e\right)}\right)e^{(-}}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(-I*a*d*e^(2*f*x + 2*e) + I*a*d + ((I*a*d*f*x + I*a*c*f)*Ei((d*f*x + c*
f)/d)*e^((d*e - c*f)/d) + (I*a*d*f*x + I*a*c*f)*Ei(-(d*f*x + c*f)/d)*e^(-(d
*e - c*f)/d) - 2*a*d)*e^(f*x + e))*e^(-f*x - e)/(d^3*x + c*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28447, size = 225, normalized size = 2.37

$$\frac{i\left(dfx\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}+dfx\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)}+cf\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}+cf\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)}-de^{\left(fx+e\right)}+de^{(}}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*I*(d*f*x*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + d*f*x*Ei((d*f*x + c*f)/d)
*e^(-c*f/d + e) + c*f*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + c*f*Ei((d*f*x +
c*f)/d)*e^(-c*f/d + e) - d*e^(f*x + e) + d*e^(-f*x - e))*a/(d^3*x + c*d^2)
- a/((d*x + c)*d)
```

3.101 $\int \frac{a+ia\sinh(e+fx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=131

$$\frac{iaf^2\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{2d^3}+\frac{iaf^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{iaf\cosh(e+fx)}{2d^2(c+dx)}-\frac{ia\sinh(e+fx)}{2d(c+dx)^2}-\frac{a}{2d(c+dx)^2}$$

```
[Out] -a/(2*d*(c + d*x)^2) - ((I/2)*a*f*Cosh[e + f*x])/(d^2*(c + d*x)) + ((I/2)*a
*f^2*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^3 - ((I/2)*a*Sinh[e +
 f*x])/(d*(c + d*x)^2) + ((I/2)*a*f^2*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/
d + f*x])/d^3
```

Rubi [A] time = 0.232228, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3317, 3297, 3303, 3298, 3301}

$$\frac{iaf^2\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{2d^3}+\frac{iaf^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{iaf\cosh(e+fx)}{2d^2(c+dx)}-\frac{ia\sinh(e+fx)}{2d(c+dx)^2}-\frac{a}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[e + f*x])/(c + d*x)^3,x]
```


```
[Out] -a/(2*d*(c + d*x)^2) - ((I/2)*a*f*Cosh[e + f*x])/(d^2*(c + d*x)) + ((I/2)*a
*f^2*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^3 - ((I/2)*a*Sinh[e +
 f*x])/(d*(c + d*x)^2) + ((I/2)*a*f^2*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/
d + f*x])/d^3
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{a+ia\sinh(e+fx)}{(c+dx)^3}\,dx = \int \left(\frac{a}{(c+dx)^3}+\frac{ia\sinh(e+fx)}{(c+dx)^3}\right)dx$$
$$= -\frac{a}{2d(c+dx)^2}+(ia)\int \frac{\sinh(e+fx)}{(c+dx)^3}\,dx$$
$$= -\frac{a}{2d(c+dx)^2}-\frac{ia\sinh(e+fx)}{2d(c+dx)^2}+\frac{(iaf)\int \frac{\cosh(e+fx)}{(c+dx)^2}\,dx}{2d}$$
$$= -\frac{a}{2d(c+dx)^2}-\frac{iaf\cosh(e+fx)}{2d^2(c+dx)}-\frac{ia\sinh(e+fx)}{2d(c+dx)^2}+\frac{\left(iaf^2\right)\int \frac{\sinh(e+fx)}{c+dx}\,dx}{2d^2}$$
$$= -\frac{a}{2d(c+dx)^2}-\frac{iaf\cosh(e+fx)}{2d^2(c+dx)}-\frac{ia\sinh(e+fx)}{2d(c+dx)^2}+\frac{\left(iaf^2\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\sinh\left(\frac{cf}{d}+fx\right)}{c+dx}\,c}{2d^2}$$
$$= -\frac{a}{2d(c+dx)^2}-\frac{iaf\cosh(e+fx)}{2d^2(c+dx)}+\frac{iaf^2\mathrm{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{ia\sinh(e+fx)}{2d(c+dx)^2}+$$

Mathematica [A] time = 0.669285, size = 109, normalized size = 0.83

$$\frac{ia\left(f^2(c+dx)^2\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+f^2(c+dx)^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)-d(f(c+dx)\cosh(e+fx)+\right.}{2d^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[e + f*x])/(c + d*x)^3,x]
```

```
[Out] ((I/2)*a*(f^2*(c + d*x)^2*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] - d*(
f*(c + d*x)*Cosh[e + f*x] + d*(-I + Sinh[e + f*x])) + f^2*(c + d*x)^2*Cosh[
e - (c*f)/d]*SinhIntegral[f*(c/d + x)]))/(d^3*(c + d*x)^2)
```

Maple [B] time = 0.06, size = 303, normalized size = 2.3

$$-\frac{a}{2d\,(dx+c)^2}-\frac{\frac{i}{4}af^3\mathrm{e}^{-fx-e}x}{d\left(d^2f^2x^2+2\,cdf^2x+c^2f^2\right)}-\frac{\frac{i}{4}af^3\mathrm{e}^{-fx-e}c}{d^2\left(d^2f^2x^2+2\,cdf^2x+c^2f^2\right)}+\frac{\frac{i}{4}af^2\mathrm{e}^{-fx-e}}{d\left(d^2f^2x^2+2\,cdf^2x+c^2f^2\right)}+\frac{\frac{i}{4}af^2}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))/(d*x+c)^3,x)
```

```
[Out] -1/2*a/d/(d*x+c)^2-1/4*I*a*f^3*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f
^2)*x-1/4*I*a*f^3*exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c+1/4*I
*a*f^2*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+1/4*I*a*f^2/d^3*exp(
(c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/4*I*a*f^2/d^3*exp(f*x+e)/(c*f/d+f*x)
^2-1/4*I*a*f^2/d^3*exp(f*x+e)/(c*f/d+f*x)-1/4*I*a*f^2/d^3*exp(-(c*f-d*e)/d)
```

```
*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Maxima [A] time = 1.21992, size = 134, normalized size = 1.02

$$\frac{1}{2}ia\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right)-\frac{a}{2\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*I*a*(e^(-e + c*f/d)*exp_integral_e(3, (d*x + c)*f/d)/((d*x + c)^2*d) -
e^(e - c*f/d)*exp_integral_e(3, -(d*x + c)*f/d)/((d*x + c)^2*d)) - 1/2*a/(d
^3*x^2 + 2*c*d^2*x + c^2*d)
```

Fricas [A] time = 2.72999, size = 471, normalized size = 3.6

$$\frac{\left(-i\,ad^2fx-i\,acdf+i\,ad^2+\left(-i\,ad^2fx-i\,acdf-i\,ad^2\right)e^{(2\,fx+2\,e)}-\left(2\,ad^2-\left(i\,ad^2f^2x^2+2i\,acdf^2x+i\,ac^2f^2\right)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e\right.\right.}{4\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(-I*a*d^2*f*x - I*a*c*d*f + I*a*d^2 + (-I*a*d^2*f*x - I*a*c*d*f - I*a*d
^2)*e^(2*f*x + 2*e) - (2*a*d^2 - (I*a*d^2*f^2*x^2 + 2*I*a*c*d*f^2*x + I*a*c
^2*f^2)*Ei((d*f*x + c*f)/d)*e^((d*e - c*f)/d) - (-I*a*d^2*f^2*x^2 - 2*I*a*c
*d*f^2*x - I*a*c^2*f^2)*Ei(-(d*f*x + c*f)/d)*e^(-(d*e - c*f)/d))*e^(f*x + e
))*e^(-f*x - e)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34846, size = 451, normalized size = 3.44

$$\frac{-i\,ad^2f^2x^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}+i\,ad^2f^2x^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)}-2i\,acdf^2x\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}+2i\,acdf^2x\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(-I*a*d^2*f^2*x^2*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + I*a*d^2*f^2*x^2*
Ei((d*f*x + c*f)/d)*e^(-c*f/d + e) - 2*I*a*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e
^(c*f/d - e) + 2*I*a*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(-c*f/d + e) - I*a*c^2
*f^2*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + I*a*c^2*f^2*Ei((d*f*x + c*f)/d)*e
^(-c*f/d + e) - I*a*d^2*f*x*e^(f*x + e) - I*a*d^2*f*x*e^(-f*x - e) - I*a*c*
d*f*e^(f*x + e) - I*a*c*d*f*e^(-f*x - e) - I*a*d^2*e^(f*x + e) + I*a*d^2*e^
(-f*x - e) - 2*a*d^2)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

3.102 $\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx$

Optimal. Leaf size=245

$$\frac{12ia^2d^2(c+dx)\cosh(e+fx)}{f^3}-\frac{3a^2d^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{4f^3}+\frac{3a^2cd^2x}{4f^2}+\frac{3a^2d(c+dx)^2\sinh^2(e+fx)}{4f^2}-\frac{6ia}{}$$

```
[Out] (3*a^2*c*d^2*x)/(4*f^2) + (3*a^2*d^3*x^2)/(8*f^2) + (3*a^2*(c + d*x)^4)/(8*
d) + ((12*I)*a^2*d^2*(c + d*x)*Cosh[e + f*x])/f^3 + ((2*I)*a^2*(c + d*x)^3*
Cosh[e + f*x])/f - ((12*I)*a^2*d^3*Sinh[e + f*x])/f^4 - ((6*I)*a^2*d*(c + d
*x)^2*Sinh[e + f*x])/f^2 - (3*a^2*d^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x]
)/(4*f^3) - (a^2*(c + d*x)^3*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) + (3*a^2*d^
3*Sinh[e + f*x]^2)/(8*f^4) + (3*a^2*d*(c + d*x)^2*Sinh[e + f*x]^2)/(4*f^2)
```

Rubi [A] time = 0.287126, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3317, 3296, 2637, 3311, 32, 3310}

$$\frac{12ia^2d^2(c+dx)\cosh(e+fx)}{f^3}-\frac{3a^2d^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{4f^3}+\frac{3a^2cd^2x}{4f^2}+\frac{3a^2d(c+dx)^2\sinh^2(e+fx)}{4f^2}-\frac{6ia}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] (3*a^2*c*d^2*x)/(4*f^2) + (3*a^2*d^3*x^2)/(8*f^2) + (3*a^2*(c + d*x)^4)/(8*
d) + ((12*I)*a^2*d^2*(c + d*x)*Cosh[e + f*x])/f^3 + ((2*I)*a^2*(c + d*x)^3*
Cosh[e + f*x])/f - ((12*I)*a^2*d^3*Sinh[e + f*x])/f^4 - ((6*I)*a^2*d*(c + d
*x)^2*Sinh[e + f*x])/f^2 - (3*a^2*d^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x]
)/(4*f^3) - (a^2*(c + d*x)^3*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) + (3*a^2*d^
3*Sinh[e + f*x]^2)/(8*f^4) + (3*a^2*d*(c + d*x)^2*Sinh[e + f*x]^2)/(4*f^2)
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)^3(a+ia\sinh(e+fx))^2\,dx = \int \left(a^2(c+dx)^3+2ia^2(c+dx)^3\sinh(e+fx)-a^2(c+dx)^3\sinh^2(e+fx)\right)dx$$

$$= \frac{a^2(c+dx)^4}{4d}+\left(2ia^2\right)\int (c+dx)^3\sinh(e+fx)\,dx-a^2\int (c+dx)^3\sinh^2(e+fx$$

$$= \frac{a^2(c+dx)^4}{4d}+\frac{2ia^2(c+dx)^3\cosh(e+fx)}{f}-\frac{a^2(c+dx)^3\cosh(e+fx)\sinh(e+}{2f}$$

$$= \frac{3a^2(c+dx)^4}{8d}+\frac{2ia^2(c+dx)^3\cosh(e+fx)}{f}-\frac{6ia^2d(c+dx)^2\sinh(e+fx)}{f^2}-3a$$

$$= \frac{3a^2cd^2x}{4f^2}+\frac{3a^2d^3x^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}+\frac{12ia^2d^2(c+dx)\cosh(e+fx)}{f^3}+2ia^2(c+$$

$$= \frac{3a^2cd^2x}{4f^2}+\frac{3a^2d^3x^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}+\frac{12ia^2d^2(c+dx)\cosh(e+fx)}{f^3}+2ia^2(c+$$

Mathematica [A] time = 1.4905, size = 220, normalized size = 0.9

$$\frac{a^2\left(-2f(c+dx)\left(2c^2f^2+4cdf^2x+d^2\left(2f^2x^2+3\right)\right)\sinh(2(e+fx))-96id\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2+2\right)\right)\sinh(e+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] (a^2*(6*f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + (32*I)*f*(c + d
*x)*(c^2*f^2 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Cosh[e + f*x] + 3*d*(2*c^2*
f^2 + 4*c*d*f^2*x + d^2*(1 + 2*f^2*x^2))*Cosh[2*(e + f*x)] - (96*I)*d*(c^2*
f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x] - 2*f*(c + d*x)*(2*c^2
*f^2 + 4*c*d*f^2*x + d^2*(3 + 2*f^2*x^2))*Sinh[2*(e + f*x)]))/(16*f^4)
```

Maple [B] time = 0.018, size = 1082, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+I*a*sinh(f*x+e))^2,x)
```

```
[Out] 1/f*(3/2/f^3*d^3*e^2*a^2*(f*x+e)^2-1/f^3*d^3*e*a^2*(f*x+e)^3+1/f^3*d^3*e^3*
a^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)+3/2/f*c^2*d*a^2*(f*x+e)^2+1
/f^2*c*d^2*a^2*(f*x+e)^3-3/f^2*c*d^2*a^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*
x+e)-1/6*(f*x+e)^3-1/2*(f*x+e)*cosh(f*x+e)^2+1/4*cosh(f*x+e)*sinh(f*x+e)+1/
4*f*x+1/4*e)-3/f*c^2*d*a^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)
^2-1/4*cosh(f*x+e)^2)+2*I/f^3*d^3*a^2*((f*x+e)^3*cosh(f*x+e)-3*(f*x+e)^2*si
nh(f*x+e)+6*(f*x+e)*cosh(f*x+e)-6*sinh(f*x+e))+3/f^3*d^3*e*a^2*(1/2*(f*x+e)
^2*cosh(f*x+e)*sinh(f*x+e)-1/6*(f*x+e)^3-1/2*(f*x+e)*cosh(f*x+e)^2+1/4*cosh
(f*x+e)*sinh(f*x+e)+1/4*f*x+1/4*e)-3/f^3*d^3*e^2*a^2*(1/2*(f*x+e)*cosh(f*x+
e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)-1/f^3*d^3*e^3*a^2*(f*x+e)-1
/f^3*d^3*a^2*(1/2*(f*x+e)^3*cosh(f*x+e)*sinh(f*x+e)-1/8*(f*x+e)^4-3/4*(f*x+
e)^2*cosh(f*x+e)^2+3/4*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+3/8*(f*x+e)^2-3/8*co
sh(f*x+e)^2)+1/4/f^3*d^3*a^2*(f*x+e)^4+2*I*c^3*a^2*cosh(f*x+e)+6*I/f^2*c*d^
2*e^2*a^2*cosh(f*x+e)-12*I/f^2*c*d^2*e*a^2*((f*x+e)*cosh(f*x+e)-sinh(f*x+e)
)-6*I/f*c^2*d*e*a^2*cosh(f*x+e)+6*I/f*c^2*d*a^2*((f*x+e)*cosh(f*x+e)-sinh(f
*x+e))-2*I/f^3*d^3*e^3*a^2*cosh(f*x+e)+6*I/f^3*d^3*e^2*a^2*((f*x+e)*cosh(f*
x+e)-sinh(f*x+e))+3/f^2*c*d^2*e^2*a^2*(f*x+e)-3/f*c^2*d*e*a^2*(f*x+e)+3/f*c
^2*d*e*a^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)+6/f^2*c*d^2*e*a^2*(1
/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)-6*I/f^3
*d^3*e*a^2*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sinh(f*x+e)+2*cosh(f*x+e))-3/f^
2*c*d^2*e^2*a^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)-3/f^2*c*d^2*e*a
^2*(f*x+e)^2+6*I/f^2*c*d^2*a^2*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sinh(f*x+e)
+2*cosh(f*x+e))-c^3*a^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)+c^3*a^2
*(f*x+e))
```

Maxima [B] time = 1.15161, size = 709, normalized size = 2.89

$$\frac{1}{4}a^2d^3x^4 + a^2cd^2x^3 + \frac{3}{2}a^2c^2dx^2 + \frac{3}{16}\left(4x^2 - \frac{\left(2fxe^{(2e)} - e^{(2e)}\right)e^{(2fx)}}{f^2} + \frac{\left(2fx+1\right)e^{(-2fx-2e)}}{f^2}\right)a^2c^2d + \frac{1}{16}\left(8x^3 - \frac{3\left(2f^2x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 + 3/16*(4*x^2 - (2*f*x*
e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 + (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*a^2*c^
2*d + 1/16*(8*x^3 - 3*(2*f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*
x)/f^3 + 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*a^2*c*d^2 + 1/32*(
4*x^4 - (4*f^3*x^3*e^(2*e) - 6*f^2*x^2*e^(2*e) + 6*f*x*e^(2*e) - 3*e^(2*e))
*e^(2*f*x)/f^4 + (4*f^3*x^3 + 6*f^2*x^2 + 6*f*x + 3)*e^(-2*f*x - 2*e)/f^4)*
a^2*d^3 + 1/8*a^2*c^3*(4*x - e^(2*f*x + 2*e)/f + e^(-2*f*x - 2*e)/f) + a^2*
c^3*x + 3*I*a^2*c^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(-f*x - e)
/f^2) + 3*I*a^2*c*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 + (f^2
*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + I*a^2*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e
^e + 6*f*x*e^e - 6*e^e)*e^(f*x)/f^4 + (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(
-f*x - e)/f^4) + 2*I*a^2*c^3*cosh(f*x + e)/f
```

Fricas [B] time = 3.27347, size = 1283, normalized size = 5.24

$$\frac{\left(4a^2d^3f^3x^3 + 4a^2c^3f^3 + 6a^2c^2df^2 + 6a^2cd^2f + 3a^2d^3 + 6\left(2a^2cd^2f^3 + a^2d^3f^2\right)x^2 + 6\left(2a^2c^2df^3 + 2a^2cd^2f^2 + a^2d^3f\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/32*(4*a^2*d^3*f^3*x^3 + 4*a^2*c^3*f^3 + 6*a^2*c^2*d*f^2 + 6*a^2*c*d^2*f +
 3*a^2*d^3 + 6*(2*a^2*c*d^2*f^3 + a^2*d^3*f^2)*x^2 + 6*(2*a^2*c^2*d*f^3 + 2
*a^2*c*d^2*f^2 + a^2*d^3*f)*x - (4*a^2*d^3*f^3*x^3 + 4*a^2*c^3*f^3 - 6*a^2*
c^2*d*f^2 + 6*a^2*c*d^2*f - 3*a^2*d^3 + 6*(2*a^2*c*d^2*f^3 - a^2*d^3*f^2)*x
^2 + 6*(2*a^2*c^2*d*f^3 - 2*a^2*c*d^2*f^2 + a^2*d^3*f)*x)*e^(4*f*x + 4*e) +
 (32*I*a^2*d^3*f^3*x^3 + 32*I*a^2*c^3*f^3 - 96*I*a^2*c^2*d*f^2 + 192*I*a^2*
c*d^2*f - 192*I*a^2*d^3 + (96*I*a^2*c*d^2*f^3 - 96*I*a^2*d^3*f^2)*x^2 + (96
*I*a^2*c^2*d*f^3 - 192*I*a^2*c*d^2*f^2 + 192*I*a^2*d^3*f)*x)*e^(3*f*x + 3*e
) + 12*(a^2*d^3*f^4*x^4 + 4*a^2*c*d^2*f^4*x^3 + 6*a^2*c^2*d*f^4*x^2 + 4*a^2
*c^3*f^4*x)*e^(2*f*x + 2*e) + (32*I*a^2*d^3*f^3*x^3 + 32*I*a^2*c^3*f^3 + 96
*I*a^2*c^2*d*f^2 + 192*I*a^2*c*d^2*f + 192*I*a^2*d^3 + (96*I*a^2*c*d^2*f^3
+ 96*I*a^2*d^3*f^2)*x^2 + (96*I*a^2*c^2*d*f^3 + 192*I*a^2*c*d^2*f^2 + 192*I
*a^2*d^3*f)*x)*e^(f*x + e))*e^(-2*f*x - 2*e)/f^4
```

Sympy [A] time = 4.62814, size = 1153, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+I*a*sinh(f*x+e))**2,x)
```

```
[Out] 3*a**2*c**3*x/2 + 9*a**2*c**2*d*x**2/4 + 3*a**2*c*d**2*x**3/2 + 3*a**2*d**3
*x**4/8 + Piecewise((((128*a**2*c**3*f**27*exp(4*e) + 384*a**2*c**2*d*f**27
*x*exp(4*e) + 192*a**2*c**2*d*f**26*exp(4*e) + 384*a**2*c*d**2*f**27*x**2*e
xp(4*e) + 384*a**2*c*d**2*f**26*x*exp(4*e) + 192*a**2*c*d**2*f**25*exp(4*e)
 + 128*a**2*d**3*f**27*x**3*exp(4*e) + 192*a**2*d**3*f**26*x**2*exp(4*e) +
192*a**2*d**3*f**25*x*exp(4*e) + 96*a**2*d**3*f**24*exp(4*e))*exp(-2*f*x) +
 (-128*a**2*c**3*f**27*exp(8*e) - 384*a**2*c**2*d*f**27*x*exp(8*e) + 192*a*
*2*c**2*d*f**26*exp(8*e) - 384*a**2*c*d**2*f**27*x**2*exp(8*e) + 384*a**2*c
*d**2*f**26*x*exp(8*e) - 192*a**2*c*d**2*f**25*exp(8*e) - 128*a**2*d**3*f**
27*x**3*exp(8*e) + 192*a**2*d**3*f**26*x**2*exp(8*e) - 192*a**2*d**3*f**25*
x*exp(8*e) + 96*a**2*d**3*f**24*exp(8*e))*exp(2*f*x) + (1024*I*a**2*c**3*f*
*27*exp(5*e) + 3072*I*a**2*c**2*d*f**27*x*exp(5*e) + 3072*I*a**2*c**2*d*f**
26*exp(5*e) + 3072*I*a**2*c*d**2*f**27*x**2*exp(5*e) + 6144*I*a**2*c*d**2*f
**26*x*exp(5*e) + 6144*I*a**2*c*d**2*f**25*exp(5*e) + 1024*I*a**2*d**3*f**2
7*x**3*exp(5*e) + 3072*I*a**2*d**3*f**26*x**2*exp(5*e) + 6144*I*a**2*d**3*f
**25*x*exp(5*e) + 6144*I*a**2*d**3*f**24*exp(5*e))*exp(-f*x) + (1024*I*a**2
*c**3*f**27*exp(7*e) + 3072*I*a**2*c**2*d*f**27*x*exp(7*e) - 3072*I*a**2*c*
*2*d*f**26*exp(7*e) + 3072*I*a**2*c*d**2*f**27*x**2*exp(7*e) - 6144*I*a**2*
c*d**2*f**26*x*exp(7*e) + 6144*I*a**2*c*d**2*f**25*exp(7*e) + 1024*I*a**2*d
**3*f**27*x**3*exp(7*e) - 3072*I*a**2*d**3*f**26*x**2*exp(7*e) + 6144*I*a**
2*d**3*f**25*x*exp(7*e) - 6144*I*a**2*d**3*f**24*exp(7*e))*exp(f*x))*exp(-6
*e)/(1024*f**28), Ne(1024*f**28*exp(6*e), 0)), (x**4*(-a**2*d**3*exp(4*e) +
 4*I*a**2*d**3*exp(3*e) - 4*I*a**2*d**3*exp(e) - a**2*d**3)*exp(-2*e)/16 +
x**3*(-a**2*c*d**2*exp(4*e) + 4*I*a**2*c*d**2*exp(3*e) - 4*I*a**2*c*d**2*ex
p(e) - a**2*c*d**2)*exp(-2*e)/4 + x**2*(-3*a**2*c**2*d*exp(4*e) + 12*I*a**2
*c**2*d*exp(3*e) - 12*I*a**2*c**2*d*exp(e) - 3*a**2*c**2*d)*exp(-2*e)/8 + x
*(-a**2*c**3*exp(4*e) + 4*I*a**2*c**3*exp(3*e) - 4*I*a**2*c**3*exp(e) - a**
2*c**3)*exp(-2*e)/4, True))
```

Giac [B] time = 1.34564, size = 788, normalized size = 3.22

$$\frac{3}{8}\,a^2d^3x^4 + \frac{3}{2}\,a^2cd^2x^3 + \frac{9}{4}\,a^2c^2dx^2 + \frac{3}{2}\,a^2c^3x - \frac{\left(4\,a^2d^3f^3x^3 + 12\,a^2cd^2f^3x^2 + 12\,a^2c^2df^3x - 6\,a^2d^3f^2x^2 + 4\,a^2c^3f^3 - 12\,a\right.}{32\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/8*a^2*d^3*x^4 + 3/2*a^2*c*d^2*x^3 + 9/4*a^2*c^2*d*x^2 + 3/2*a^2*c^3*x - 1
/32*(4*a^2*d^3*f^3*x^3 + 12*a^2*c*d^2*f^3*x^2 + 12*a^2*c^2*d*f^3*x - 6*a^2*
d^3*f^2*x^2 + 4*a^2*c^3*f^3 - 12*a^2*c*d^2*f^2*x - 6*a^2*c^2*d*f^2 + 6*a^2*
d^3*f*x + 6*a^2*c*d^2*f - 3*a^2*d^3)*e^(2*f*x + 2*e)/f^4 + (I*a^2*d^3*f^3*x
^3 + 3*I*a^2*c*d^2*f^3*x^2 + 3*I*a^2*c^2*d*f^3*x - 3*I*a^2*d^3*f^2*x^2 + I*
a^2*c^3*f^3 - 6*I*a^2*c*d^2*f^2*x - 3*I*a^2*c^2*d*f^2 + 6*I*a^2*d^3*f*x + 6
*I*a^2*c*d^2*f - 6*I*a^2*d^3)*e^(f*x + e)/f^4 + (I*a^2*d^3*f^3*x^3 + 3*I*a^
2*c*d^2*f^3*x^2 + 3*I*a^2*c^2*d*f^3*x + 3*I*a^2*d^3*f^2*x^2 + I*a^2*c^3*f^3
 + 6*I*a^2*c*d^2*f^2*x + 3*I*a^2*c^2*d*f^2 + 6*I*a^2*d^3*f*x + 6*I*a^2*c*d^
2*f + 6*I*a^2*d^3)*e^(-f*x - e)/f^4 + 1/32*(4*a^2*d^3*f^3*x^3 + 12*a^2*c*d^
2*f^3*x^2 + 12*a^2*c^2*d*f^3*x + 6*a^2*d^3*f^2*x^2 + 4*a^2*c^3*f^3 + 12*a^2
*c*d^2*f^2*x + 6*a^2*c^2*d*f^2 + 6*a^2*d^3*f*x + 6*a^2*c*d^2*f + 3*a^2*d^3)
*e^(-2*f*x - 2*e)/f^4
```

3.103 $\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx$

Optimal. Leaf size=174

$$\frac{a^2d(c+dx)\sinh^2(e+fx)}{2f^2}-\frac{4ia^2d(c+dx)\sinh(e+fx)}{f^2}+\frac{2ia^2(c+dx)^2\cosh(e+fx)}{f}-\frac{a^2(c+dx)^2\sinh(e+fx)\cosh}{2f}$$

```
[Out] (a^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(2*d) + ((4*I)*a^2*d^2*Cosh[e + f*x
])/f^3 + ((2*I)*a^2*(c + d*x)^2*Cosh[e + f*x])/f - ((4*I)*a^2*d*(c + d*x)*S
inh[e + f*x])/f^2 - (a^2*d^2*Cosh[e + f*x]*Sinh[e + f*x])/(4*f^3) - (a^2*(c
 + d*x)^2*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) + (a^2*d*(c + d*x)*Sinh[e + f*
x]^2)/(2*f^2)
```

Rubi [A] time = 0.197184, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3317, 3296, 2638, 3311, 32, 2635, 8}

$$\frac{a^2d(c+dx)\sinh^2(e+fx)}{2f^2}-\frac{4ia^2d(c+dx)\sinh(e+fx)}{f^2}+\frac{2ia^2(c+dx)^2\cosh(e+fx)}{f}-\frac{a^2(c+dx)^2\sinh(e+fx)\cosh}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + I*a*Sinh[e + f*x])^2,x]
```


```
[Out] (a^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(2*d) + ((4*I)*a^2*d^2*Cosh[e + f*x
])/f^3 + ((2*I)*a^2*(c + d*x)^2*Cosh[e + f*x])/f - ((4*I)*a^2*d*(c + d*x)*S
inh[e + f*x])/f^2 - (a^2*d^2*Cosh[e + f*x]*Sinh[e + f*x])/(4*f^3) - (a^2*(c
 + d*x)^2*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) + (a^2*d*(c + d*x)*Sinh[e + f*
x]^2)/(2*f^2)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (c+dx)^2(a+ia\sinh(e+fx))^2\,dx = \int \left(a^2(c+dx)^2+2ia^2(c+dx)^2\sinh(e+fx)-a^2(c+dx)^2\sinh^2(e+fx)\right)dx$$

$$= \frac{a^2(c+dx)^3}{3d}+\left(2ia^2\right)\int (c+dx)^2\sinh(e+fx)\,dx-a^2\int (c+dx)^2\sinh^2(e+fx)\,dx$$

$$= \frac{a^2(c+dx)^3}{3d}+\frac{2ia^2(c+dx)^2\cosh(e+fx)}{f}-\frac{a^2(c+dx)^2\cosh(e+fx)\sinh(e+fx)}{2f}$$

$$= \frac{a^2(c+dx)^3}{2d}+\frac{2ia^2(c+dx)^2\cosh(e+fx)}{f}-\frac{4ia^2d(c+dx)\sinh(e+fx)}{f^2}-\frac{a^2d^2\cos}{}$$

$$= \frac{a^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{2d}+\frac{4ia^2d^2\cosh(e+fx)}{f^3}+\frac{2ia^2(c+dx)^2\cosh(e+fx)}{f}-\frac{4ia^2d}{}$$

Mathematica [A] time = 0.713293, size = 189, normalized size = 1.09

$$\frac{a^2\left(16i\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2+2\right)\right)\cosh(e+fx)-2c^2f^2\sinh(2(e+fx))+12c^2f^3x-4cdf^2x\sinh(2(e+fx))-32ic}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] (a^2*(12*c^2*f^3*x + 12*c*d*f^3*x^2 + 4*d^2*f^3*x^3 + (16*I)*(c^2*f^2 + 2*c
*d*f^2*x + d^2*(2 + f^2*x^2))*Cosh[e + f*x] + 2*d*f*(c + d*x)*Cosh[2*(e + f
*x)] - (32*I)*c*d*f*Sinh[e + f*x] - (32*I)*d^2*f*x*Sinh[e + f*x] - d^2*Sinh
[2*(e + f*x)] - 2*c^2*f^2*Sinh[2*(e + f*x)] - 4*c*d*f^2*x*Sinh[2*(e + f*x)]
 - 2*d^2*f^2*x^2*Sinh[2*(e + f*x)]))/(8*f^3)
```

Maple [B] time = 0.014, size = 550, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+I*a*sinh(f*x+e))^2,x)
```

```
[Out] 1/f*(1/3/f^2*d^2*a^2*(f*x+e)^3-4*I/f^2*d^2*e*a^2*((f*x+e)*cosh(f*x+e)-sinh(
f*x+e))-1/f^2*d^2*a^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)-1/6*(f*x+e)^3-
1/2*(f*x+e)*cosh(f*x+e)^2+1/4*cosh(f*x+e)*sinh(f*x+e)+1/4*f*x+1/4*e)-1/f^2*
d^2*e*a^2*(f*x+e)^2-4*I/f*c*d*e*a^2*cosh(f*x+e)+2/f^2*d^2*e*a^2*(1/2*(f*x+e
)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)+1/f^2*d^2*e^2*a^
2*(f*x+e)+4*I/f*c*d*a^2*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))-1/f^2*d^2*e^2*a^2
*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)+1/f*c*d*a^2*(f*x+e)^2+2*I/f^2*
d^2*a^2*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sinh(f*x+e)+2*cosh(f*x+e))-2/f*c*d
*a^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)-
2/f*c*d*e*a^2*(f*x+e)+2*I*c^2*a^2*cosh(f*x+e)+2/f*c*d*e*a^2*(1/2*cosh(f*x+e
)*sinh(f*x+e)-1/2*f*x-1/2*e)+c^2*a^2*(f*x+e)+2*I/f^2*d^2*e^2*a^2*cosh(f*x+e
)-c^2*a^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e))
```

Maxima [B] time = 1.26859, size = 440, normalized size = 2.53

$$\frac{1}{3}a^2d^2x^3 + a^2cdx^2 + \frac{1}{8}\left(4x^2 - \frac{\left(2fxe^{(2e)} - e^{(2e)}\right)e^{(2fx)}}{f^2} + \frac{(2fx+1)e^{(-2fx-2e)}}{f^2}\right)a^2cd + \frac{1}{48}\left(8x^3 - \frac{3\left(2f^2x^2e^{(2e)} - 2fx\right.}{f^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*d^2*x^3 + a^2*c*d*x^2 + 1/8*(4*x^2 - (2*f*x*e^(2*e) - e^(2*e))*e^(2
*f*x)/f^2 + (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*a^2*c*d + 1/48*(8*x^3 - 3*(2*
f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*x)/f^3 + 3*(2*f^2*x^2 + 2
*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*a^2*d^2 + 1/8*a^2*c^2*(4*x - e^(2*f*x + 2*e
)/f + e^(-2*f*x - 2*e)/f) + a^2*c^2*x + 2*I*a^2*c*d*((f*x*e^e - e^e)*e^(f*x
)/f^2 + (f*x + 1)*e^(-f*x - e)/f^2) + I*a^2*d^2*((f^2*x^2*e^e - 2*f*x*e^e +
 2*e^e)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + 2*I*a^2*c^2
*cosh(f*x + e)/f
```

Fricas [B] time = 3.04527, size = 768, normalized size = 4.41

$$\frac{\left(2a^2d^2f^2x^2 + 2a^2c^2f^2 + 2a^2cdf + a^2d^2 + 2\left(2a^2cdf^2 + a^2d^2f\right)x - \left(2a^2d^2f^2x^2 + 2a^2c^2f^2 - 2a^2cdf + a^2d^2 + 2\left(2a^2\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(2*a^2*d^2*f^2*x^2 + 2*a^2*c^2*f^2 + 2*a^2*c*d*f + a^2*d^2 + 2*(2*a^2*
c*d*f^2 + a^2*d^2*f)*x - (2*a^2*d^2*f^2*x^2 + 2*a^2*c^2*f^2 - 2*a^2*c*d*f +
 a^2*d^2 + 2*(2*a^2*c*d*f^2 - a^2*d^2*f)*x)*e^(4*f*x + 4*e) + (16*I*a^2*d^2
*f^2*x^2 + 16*I*a^2*c^2*f^2 - 32*I*a^2*c*d*f + 32*I*a^2*d^2 + (32*I*a^2*c*d
*f^2 - 32*I*a^2*d^2*f)*x)*e^(3*f*x + 3*e) + 8*(a^2*d^2*f^3*x^3 + 3*a^2*c*d*
f^3*x^2 + 3*a^2*c^2*f^3*x)*e^(2*f*x + 2*e) + (16*I*a^2*d^2*f^2*x^2 + 16*I*a
^2*c^2*f^2 + 32*I*a^2*c*d*f + 32*I*a^2*d^2 + (32*I*a^2*c*d*f^2 + 32*I*a^2*d
^2*f)*x)*e^(f*x + e))*e^(-2*f*x - 2*e)/f^3
```

Sympy [A] time = 3.20624, size = 706, normalized size = 4.06

$$\frac{3a^2c^2x}{2}+\frac{3a^2cdx^2}{2}+\frac{a^2d^2x^3}{2}+\begin{cases}\frac{\left(\left(32a^2c^2f^{17}e^{3e}+64a^2cdf^{17}xe^{3e}+32a^2cdf^{16}e^{3e}+32a^2d^2f^{17}x^2e^{3e}+32a^2d^2f^{16}xe^{3e}+16a^2d^2f^{15}e^{3e}\right)e^{-2fx}+\left(-32a^2c^2f^{17}e^{7e}\right.\right.}{} \\ \frac{x^3\left(-a^2d^2e^{4e}+4ia^2d^2e^{3e}-4ia^2d^2e^{e}-a^2d^2\right)e^{-2e}}{12}+\frac{x^2\left(-a^2cde^{4e}+4ia^2cde^{3e}-4ia^2cde^{e}-a^2cd\right)e^{-2e}}{4}+\frac{x\left(-a^2c^2e^{4e}+4ia^2c^2e^{3e}\right.}{4}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+I*a*sinh(f*x+e))**2,x)
```

```
[Out] 3*a**2*c**2*x/2 + 3*a**2*c*d*x**2/2 + a**2*d**2*x**3/2 + Piecewise((((32*a*
*2*c**2*f**17*exp(3*e) + 64*a**2*c*d*f**17*x*exp(3*e) + 32*a**2*c*d*f**16*e
xp(3*e) + 32*a**2*d**2*f**17*x**2*exp(3*e) + 32*a**2*d**2*f**16*x*exp(3*e)
+ 16*a**2*d**2*f**15*exp(3*e))*exp(-2*f*x) + (-32*a**2*c**2*f**17*exp(7*e)
- 64*a**2*c*d*f**17*x*exp(7*e) + 32*a**2*c*d*f**16*exp(7*e) - 32*a**2*d**2*
f**17*x**2*exp(7*e) + 32*a**2*d**2*f**16*x*exp(7*e) - 16*a**2*d**2*f**15*ex
p(7*e))*exp(2*f*x) + (256*I*a**2*c**2*f**17*exp(4*e) + 512*I*a**2*c*d*f**17
*x*exp(4*e) + 512*I*a**2*c*d*f**16*exp(4*e) + 256*I*a**2*d**2*f**17*x**2*ex
p(4*e) + 512*I*a**2*d**2*f**16*x*exp(4*e) + 512*I*a**2*d**2*f**15*exp(4*e))
*exp(-f*x) + (256*I*a**2*c**2*f**17*exp(6*e) + 512*I*a**2*c*d*f**17*x*exp(6
*e) - 512*I*a**2*c*d*f**16*exp(6*e) + 256*I*a**2*d**2*f**17*x**2*exp(6*e) -
 512*I*a**2*d**2*f**16*x*exp(6*e) + 512*I*a**2*d**2*f**15*exp(6*e))*exp(f*x
))*exp(-5*e)/(256*f**18), Ne(256*f**18*exp(5*e), 0)), (x**3*(-a**2*d**2*exp
(4*e) + 4*I*a**2*d**2*exp(3*e) - 4*I*a**2*d**2*exp(e) - a**2*d**2)*exp(-2*e
)/12 + x**2*(-a**2*c*d*exp(4*e) + 4*I*a**2*c*d*exp(3*e) - 4*I*a**2*c*d*exp(
e) - a**2*c*d)*exp(-2*e)/4 + x*(-a**2*c**2*exp(4*e) + 4*I*a**2*c**2*exp(3*e
) - 4*I*a**2*c**2*exp(e) - a**2*c**2)*exp(-2*e)/4, True))
```

Giac [B] time = 1.26013, size = 455, normalized size = 2.61

$$\frac{1}{2}\,a^2d^2x^3+\frac{3}{2}\,a^2cdx^2+\frac{3}{2}\,a^2c^2x-\frac{\left(2\,a^2d^2f^2x^2+4\,a^2cdf^2x+2\,a^2c^2f^2-2\,a^2d^2fx-2\,a^2cdf+a^2d^2\right)e^{\left(2\,fx+2\,e\right)}}{16\,f^3}+\frac{\left(i\,a^2d^2f^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*d^2*x^3 + 3/2*a^2*c*d*x^2 + 3/2*a^2*c^2*x - 1/16*(2*a^2*d^2*f^2*x^2
 + 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^2 - 2*a^2*d^2*f*x - 2*a^2*c*d*f + a^2*d^2)
*e^(2*f*x + 2*e)/f^3 + (I*a^2*d^2*f^2*x^2 + 2*I*a^2*c*d*f^2*x + I*a^2*c^2*f
^2 - 2*I*a^2*d^2*f*x - 2*I*a^2*c*d*f + 2*I*a^2*d^2)*e^(f*x + e)/f^3 - (-I*a
^2*d^2*f^2*x^2 - 2*I*a^2*c*d*f^2*x - I*a^2*c^2*f^2 - 2*I*a^2*d^2*f*x - 2*I*
a^2*c*d*f - 2*I*a^2*d^2)*e^(-f*x - e)/f^3 + 1/16*(2*a^2*d^2*f^2*x^2 + 4*a^2
*c*d*f^2*x + 2*a^2*c^2*f^2 + 2*a^2*d^2*f*x + 2*a^2*c*d*f + a^2*d^2)*e^(-2*f
*x - 2*e)/f^3
```

3.104 $\int (c+dx)(a+ia\sinh(e+fx))^2\,dx$

Optimal. Leaf size=122

$$\frac{2ia^2(c+dx)\cosh(e+fx)}{f}-\frac{a^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{a^2(c+dx)^2}{2d}+\frac{1}{2}a^2cx+\frac{a^2d\sinh^2(e+fx)}{4f^2}-2ia^2$$

```
[Out] (a^2*c*x)/2 + (a^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) + ((2*I)*a^2*(c + d*x
)*Cosh[e + f*x])/f - ((2*I)*a^2*d*Sinh[e + f*x])/f^2 - (a^2*(c + d*x)*Cosh[
e + f*x]*Sinh[e + f*x])/(2*f) + (a^2*d*Sinh[e + f*x]^2)/(4*f^2)
```

Rubi [A] time = 0.105015, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3317, 3296, 2637, 3310}

$$\frac{2ia^2(c+dx)\cosh(e+fx)}{f}-\frac{a^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{a^2(c+dx)^2}{2d}+\frac{1}{2}a^2cx+\frac{a^2d\sinh^2(e+fx)}{4f^2}-2ia^2$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + I*a*Sinh[e + f*x])^2,x]
```


```
[Out] (a^2*c*x)/2 + (a^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) + ((2*I)*a^2*(c + d*x
)*Cosh[e + f*x])/f - ((2*I)*a^2*d*Sinh[e + f*x])/f^2 - (a^2*(c + d*x)*Cosh[
e + f*x]*Sinh[e + f*x])/(2*f) + (a^2*d*Sinh[e + f*x]^2)/(4*f^2)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)(a+ia\sinh(e+fx))^2\,dx = \int \left(a^2(c+dx)+2ia^2(c+dx)\sinh(e+fx)-a^2(c+dx)\sinh^2(e+fx)\right)dx$$
$$= \frac{a^2(c+dx)^2}{2d}+\left(2ia^2\right)\int (c+dx)\sinh(e+fx)\,dx - a^2\int (c+dx)\sinh^2(e+fx)\,dx$$
$$= \frac{a^2(c+dx)^2}{2d}+\frac{2ia^2(c+dx)\cosh(e+fx)}{f}-\frac{a^2(c+dx)\cosh(e+fx)\sinh(e+fx)}{2f}+$$
$$= \frac{1}{2}a^2cx+\frac{1}{4}a^2dx^2+\frac{a^2(c+dx)^2}{2d}+\frac{2ia^2(c+dx)\cosh(e+fx)}{f}-\frac{2ia^2d\sinh(e+fx)}{f^2}-$$

Mathematica [A] time = 1.20878, size = 86, normalized size = 0.7

$$\frac{a^2(-2(3(e+fx)(-2cf+de-dfx)+f(c+dx)\sinh(2(e+fx))+8id\sinh(e+fx))+16if(c+dx)\cosh(e+fx)+d\cosh}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] (a^2*((16*I)*f*(c + d*x)*Cosh[e + f*x] + d*Cosh[2*(e + f*x)] - 2*(3*(e + f*
x)*(d*e - 2*c*f - d*f*x) + (8*I)*d*Sinh[e + f*x] + f*(c + d*x)*Sinh[2*(e +
f*x)])))/(8*f^2)
```

Maple [A] time = 0.015, size = 215, normalized size = 1.8

$$\frac{1}{f}\left(\frac{da^2\left(fx+e\right)^2}{2f}+\frac{2\,ida^2\left(\left(fx+e\right)\cosh\left(fx+e\right)-\sinh\left(fx+e\right)\right)}{f}-\frac{da^2}{f}\left(\frac{\left(fx+e\right)\cosh\left(fx+e\right)\sinh\left(fx+e\right)}{2}-\frac{(fx+}{4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+I*a*sinh(f*x+e))^2,x)
```

```
[Out] 1/f*(1/2/f*d*a^2*(f*x+e)^2+2*I/f*d*a^2*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))-1/
f*d*a^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^
2)-d*e/f*a^2*(f*x+e)-2*I*d*e/f*a^2*cosh(f*x+e)+d*e/f*a^2*(1/2*cosh(f*x+e)*s
inh(f*x+e)-1/2*f*x-1/2*e)+c*a^2*(f*x+e)+2*I*c*a^2*cosh(f*x+e)-c*a^2*(1/2*co
sh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e))
```

Maxima [A] time = 1.18573, size = 225, normalized size = 1.84

$$\frac{1}{2}a^2dx^2+\frac{1}{16}\left(4x^2-\frac{\left(2fxe^{(2e)}-e^{(2e)}\right)e^{(2fx)}}{f^2}+\frac{\left(2fx+1\right)e^{(-2fx-2e)}}{f^2}\right)a^2d+\frac{1}{8}a^2c\left(4x-\frac{e^{(2fx+2e)}}{f}+\frac{e^{(-2fx-2e)}}{f}\right)+a^2cx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*d*x^2 + 1/16*(4*x^2 - (2*f*x*e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 + (2*
f*x + 1)*e^(-2*f*x - 2*e)/f^2)*a^2*d + 1/8*a^2*c*(4*x - e^(2*f*x + 2*e)/f +
 e^(-2*f*x - 2*e)/f) + a^2*c*x + I*a^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*
```

```
x + 1)*e^(-f*x - e)/f^2) + 2*I*a^2*c*cosh(f*x + e)/f
```

Fricas [A] time = 2.9555, size = 389, normalized size = 3.19

$$\frac{\left(2\,a^2dfx + 2\,a^2cf + a^2d - \left(2\,a^2dfx + 2\,a^2cf - a^2d\right)e^{\left(4\,fx+4\,e\right)} + \left(16i\,a^2dfx + 16i\,a^2cf - 16i\,a^2d\right)e^{\left(3\,fx+3\,e\right)} + 12\,\left(a^2df^2x\right.\right.}{16\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(2*a^2*d*f*x + 2*a^2*c*f + a^2*d - (2*a^2*d*f*x + 2*a^2*c*f - a^2*d)*e
^(4*f*x + 4*e) + (16*I*a^2*d*f*x + 16*I*a^2*c*f - 16*I*a^2*d)*e^(3*f*x + 3*
e) + 12*(a^2*d*f^2*x^2 + 2*a^2*c*f^2*x)*e^(2*f*x + 2*e) + (16*I*a^2*d*f*x +
 16*I*a^2*c*f + 16*I*a^2*d)*e^(f*x + e))*e^(-2*f*x - 2*e)/f^2
```

Sympy [A] time = 2.06403, size = 364, normalized size = 2.98

$$\frac{3a^2cx}{2} + \frac{3a^2dx^2}{4} + \begin{cases} \frac{\left(\left(32a^2cf^9e^{2e}+32a^2df^9xe^{2e}+16a^2df^8e^{2e}\right)e^{-2fx}+\left(-32a^2cf^9e^{6e}-32a^2df^9xe^{6e}+16a^2df^8e^{6e}\right)e^{2fx}+\left(256ia^2cf^9e^{3e}+256ia^2df^9xe^{3e}+256\right.\right.}{256f^{10}} \\ \frac{x^2\left(-a^2de^{4e}+4ia^2de^{3e}-4ia^2de^{e}-a^2d\right)e^{-2e}}{8} + \frac{x\left(-a^2ce^{4e}+4ia^2ce^{3e}-4ia^2ce^{e}-a^2c\right)e^{-2e}}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+I*a*sinh(f*x+e))**2,x)
```

```
[Out] 3*a**2*c*x/2 + 3*a**2*d*x**2/4 + Piecewise((((32*a**2*c*f**9*exp(2*e) + 32*
a**2*d*f**9*x*exp(2*e) + 16*a**2*d*f**8*exp(2*e))*exp(-2*f*x) + (-32*a**2*c
*f**9*exp(6*e) - 32*a**2*d*f**9*x*exp(6*e) + 16*a**2*d*f**8*exp(6*e))*exp(2
*f*x) + (256*I*a**2*c*f**9*exp(3*e) + 256*I*a**2*d*f**9*x*exp(3*e) + 256*I*
a**2*d*f**8*exp(3*e))*exp(-f*x) + (256*I*a**2*c*f**9*exp(5*e) + 256*I*a**2*
d*f**9*x*exp(5*e) - 256*I*a**2*d*f**8*exp(5*e))*exp(f*x))*exp(-4*e)/(256*f*
*10), Ne(256*f**10*exp(4*e), 0)), (x**2*(-a**2*d*exp(4*e) + 4*I*a**2*d*exp(
3*e) - 4*I*a**2*d*exp(e) - a**2*d)*exp(-2*e)/8 + x*(-a**2*c*exp(4*e) + 4*I*
a**2*c*exp(3*e) - 4*I*a**2*c*exp(e) - a**2*c)*exp(-2*e)/4, True))
```

Giac [A] time = 1.24014, size = 215, normalized size = 1.76

$$\frac{3}{4}\,a^2dx^2 + \frac{3}{2}\,a^2cx - \frac{\left(2\,a^2dfx + 2\,a^2cf - a^2d\right)e^{\left(2\,fx+2\,e\right)}}{16\,f^2} + \frac{\left(i\,a^2dfx + i\,a^2cf - i\,a^2d\right)e^{\left(fx+e\right)}}{f^2} + \frac{\left(i\,a^2dfx + i\,a^2cf + i\,a^2d\right)e}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/4*a^2*d*x^2 + 3/2*a^2*c*x - 1/16*(2*a^2*d*f*x + 2*a^2*c*f - a^2*d)*e^(2*f
*x + 2*e)/f^2 + (I*a^2*d*f*x + I*a^2*c*f - I*a^2*d)*e^(f*x + e)/f^2 + (I*a^
2*d*f*x + I*a^2*c*f + I*a^2*d)*e^(-f*x - e)/f^2 + 1/16*(2*a^2*d*f*x + 2*a^2
*c*f + a^2*d)*e^(-2*f*x - 2*e)/f^2
```

3.105 $\int \frac{(a+ia\sinh(e+fx))^2}{c+dx}\,dx$

Optimal. Leaf size=149

$$\frac{2ia^2\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d}-\frac{a^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{2d}-\frac{a^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{2d}+\frac{2ia^2\cosh}{}$$

```
[Out] -(a^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (3*a^2
*Log[c + d*x])/(2*d) + ((2*I)*a^2*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f
)/d])/d + ((2*I)*a^2*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d - (a^
2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```

Rubi [A] time = 0.352419, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3318, 3312, 3303, 3298, 3301}

$$\frac{2ia^2\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d}-\frac{a^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{2d}-\frac{a^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{2d}+\frac{2ia^2\cosh}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[e + f*x])^2/(c + d*x),x]
```


```
[Out] -(a^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (3*a^2
*Log[c + d*x])/(2*d) + ((2*I)*a^2*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f
)/d])/d + ((2*I)*a^2*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d - (a^
2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```


Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+ia\sinh(e+fx))^2}{c+dx}\,dx &= \left(4a^2\right)\int \frac{\sin^4\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)}{c+dx}\,dx\\
&= \left(4a^2\right)\int \left(\frac{3}{8(c+dx)}-\frac{\cosh(2e+2fx)}{8(c+dx)}+\frac{i\sinh(e+fx)}{2(c+dx)}\right)dx\\
&= \frac{3a^2\log(c+dx)}{2d}+\left(2ia^2\right)\int \frac{\sinh(e+fx)}{c+dx}\,dx-\frac{1}{2}a^2\int \frac{\cosh(2e+2fx)}{c+dx}\,dx\\
&= \frac{3a^2\log(c+dx)}{2d}-\frac{1}{2}\left(a^2\cosh\left(2e-\frac{2cf}{d}\right)\right)\int \frac{\cosh\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx+\left(2ia^2\cosh\left(e-\frac{cf}{d}\right.\right.\\
&= -\frac{a^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{3a^2\log(c+dx)}{2d}+\frac{2ia^2\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\right.}{d}
\end{aligned}$$

Mathematica [A] time = 0.395477, size = 117, normalized size = 0.79

$$-\frac{a^2\left(-4i\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+\text{Chi}\left(\frac{2f(c+dx)}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)+\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2f(c+dx)}{d}\right)-4i\cosh\left(e-\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[e + f*x])^2/(c + d*x),x]
```

```
[Out] -(a^2*(Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*f*(c + d*x))/d] - 3*Log[c + d*
x] - (4*I)*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] - (4*I)*Cosh[e - (c*
f)/d]*SinhIntegral[f*(c/d + x)] + Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(
c + d*x))/d]))/(2*d)
```

Maple [A] time = 0.119, size = 193, normalized size = 1.3

$$\frac{-ia^2}{d}e^{-\frac{cf-de}{d}}\text{Ei}\left(1,-fx-e-\frac{cf-de}{d}\right)+\frac{3a^2\ln(dx+c)}{2d}+\frac{a^2}{4d}e^{2\frac{cf-de}{d}}\text{Ei}\left(1,2fx+2e+2\frac{cf-de}{d}\right)+\frac{a^2}{4d}e^{-2\frac{cf-de}{d}}\text{Ei}\left(1,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))^2/(d*x+c),x)
```

```
[Out] -I*a^2/d*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)+3/2*a^2*ln(d*x+c)/d+1/4
*a^2/d*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)+1/4*a^2/d*exp(-2*(c
*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)+I*a^2/d*exp((c*f-d*e)/d)*Ei(1,f*x
+e+(c*f-d*e)/d)
```

Maxima [A] time = 1.38847, size = 203, normalized size = 1.36

$$\frac{1}{4}a^2\left(\frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_1\left(\frac{2(dx+c)f}{d}\right)}{d}+\frac{e^{\left(2e-\frac{2cf}{d}\right)}E_1\left(-\frac{2(dx+c)f}{d}\right)}{d}+\frac{2\log(dx+c)}{d}\right)+ia^2\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E_1\left(-\frac{(dx\right.}{}\right)}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*a^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/d + e^(2*e -
 2*c*f/d)*exp_integral_e(1, -2*(d*x + c)*f/d)/d + 2*log(d*x + c)/d) + I*a^2
*(e^(-e + c*f/d)*exp_integral_e(1, (d*x + c)*f/d)/d - e^(e - c*f/d)*exp_int
egral_e(1, -(d*x + c)*f/d)/d) + a^2*log(d*x + c)/d
```

Fricas [A] time = 2.81089, size = 309, normalized size = 2.07

$$-\frac{a^2\mathrm{Ei}\left(\frac{2\left(dfx+cf\right)}{d}\right)e^{\left(\frac{2\left(de-cf\right)}{d}\right)}-4i\,a^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)}+4i\,a^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)}+a^2\mathrm{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)e^{\left(-\frac{2\left(de-cf\right)}{d}\right)}-6\,a^2\,\mathrm{l}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/4*(a^2*Ei(2*(d*f*x + c*f)/d)*e^(2*(d*e - c*f)/d) - 4*I*a^2*Ei((d*f*x + c
*f)/d)*e^((d*e - c*f)/d) + 4*I*a^2*Ei(-(d*f*x + c*f)/d)*e^(-(d*e - c*f)/d)
+ a^2*Ei(-2*(d*f*x + c*f)/d)*e^(-2*(d*e - c*f)/d) - 6*a^2*log((d*x + c)/d))
/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int -\frac{\sinh^2\left(e+fx\right)}{c+dx}\,dx+\int \frac{2i\sinh\left(e+fx\right)}{c+dx}\,dx+\int \frac{1}{c+dx}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))**2/(d*x+c),x)
```

```
[Out] a**2*(Integral(-sinh(e + f*x)**2/(c + d*x), x) + Integral(2*I*sinh(e + f*x)
/(c + d*x), x) + Integral(1/(c + d*x), x))
```

Giac [A] time = 1.3336, size = 188, normalized size = 1.26

$$-\frac{a^2\mathrm{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)e^{\left(\frac{2\,cf}{d}-2\,e\right)}+4i\,a^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}-4i\,a^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)}+a^2\mathrm{Ei}\left(\frac{2\left(dfx+cf\right)}{d}\right)e^{\left(-\frac{2\,cf}{d}+2\,e\right)}-6\,a^2\,\mathrm{lo}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

```
[Out] -1/4*(a^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) + 4*I*a^2*Ei(-(d*f*x + c
*f)/d)*e^(c*f/d - e) - 4*I*a^2*Ei((d*f*x + c*f)/d)*e^(-c*f/d + e) + a^2*Ei(
2*(d*f*x + c*f)/d)*e^(-2*c*f/d + 2*e) - 6*a^2*log(d*x + c))/d
```

3.106 $\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^2}\,dx$

Optimal. Leaf size=170

$$-\frac{a^2f\,\mathrm{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2}+\frac{2ia^2f\,\mathrm{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{2ia^2f\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{a^2f}{}$$

```
[Out] (-4*a^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^4)/(d*(c + d*x)) + ((2*I)*a^2*f*Cosh
[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d^2 - (a^2*f*CoshIntegral[(2*c*f
)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d^2 + ((2*I)*a^2*f*Sinh[e - (c*f)/d]*Si
nhIntegral[(c*f)/d + f*x])/d^2 - (a^2*f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[
(2*c*f)/d + 2*f*x])/d^2
```

Rubi [A] time = 0.339995, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3318, 3313, 3303, 3298, 3301}

$$-\frac{a^2f\,\mathrm{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2}+\frac{2ia^2f\,\mathrm{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{2ia^2f\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{a^2f}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] (-4*a^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^4)/(d*(c + d*x)) + ((2*I)*a^2*f*Cosh
[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d^2 - (a^2*f*CoshIntegral[(2*c*f
)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d^2 + ((2*I)*a^2*f*Sinh[e - (c*f)/d]*Si
nhIntegral[(c*f)/d + f*x])/d^2 - (a^2*f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[
(2*c*f)/d + 2*f*x])/d^2
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^2}\,dx &= \left(4a^2\right)\int \frac{\sin^4\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)}{(c+dx)^2}\,dx\\ &= -\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)}+\frac{\left(8ia^2f\right)\int\left(\frac{\cosh(e+fx)}{4(c+dx)}+\frac{i\sinh(2e+2fx)}{8(c+dx)}\right)dx}{d}\\ &= -\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)}+\frac{\left(2ia^2f\right)\int\frac{\cosh(e+fx)}{c+dx}dx}{d}-\frac{\left(a^2f\right)\int\frac{\sinh(2e+2fx)}{c+dx}dx}{d}\\ &= -\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)}-\frac{\left(a^2f\cosh\left(2e-\frac{2cf}{d}\right)\right)\int\frac{\sinh\left(\frac{2cf}{d}+2fx\right)}{c+dx}dx}{d}+\frac{\left(2ia^2f\cosh\left(e-\ldots\right)\ldots\right)}{\ldots}\\ &= -\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)}+\frac{2ia^2f\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(\frac{cf}{d}+fx\right)}{d^2}-\frac{a^2f\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)\sin\ldots}{d^2}\end{aligned}$$

Mathematica [A] time = 0.661156, size = 214, normalized size = 1.26

$$\frac{a^2\left(-2f(c+dx)\mathrm{Chi}\left(\frac{2f(c+dx)}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)+4if(c+dx)\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\cosh\left(e-\frac{cf}{d}\right)+4idfx\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}\ldots\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] (a^2*(-3*d + d*Cosh[2*(e + f*x)] + (4*I)*f*(c + d*x)*Cosh[e - (c*f)/d]*Cosh
Integral[f*(c/d + x)] - 2*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x))/d]*Sinh[
2*e - (2*c*f)/d] - (4*I)*d*Sinh[e + f*x] + (4*I)*c*f*Sinh[e - (c*f)/d]*Sinh
Integral[f*(c/d + x)] + (4*I)*d*f*x*Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d +
 x)] - 2*c*f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d] - 2*d*f*
x*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d]))/(2*d^2*(c + d*x))
```

Maple [A] time = 0.141, size = 313, normalized size = 1.8

$$\frac{-ia^2fe^{fx+e}}{d^2}\left(\frac{cf}{d}+fx\right)^{-1}-\frac{ia^2f}{d^2}e^{-\frac{cf-de}{d}}\mathrm{Ei}\left(1,-fx-e-\frac{cf-de}{d}\right)-\frac{3a^2}{2d(dx+c)}+\frac{fa^2e^{-2fx-2e}}{4d\left(dfx+cf\right)}-\frac{fa^2}{2d^2}e^{2\frac{cf-de}{d}}\mathrm{Ei}\left(1,2fx\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))^2/(d*x+c)^2,x)
```

```
[Out] -I*a^2*f/d^2*exp(f*x+e)/(c*f/d+f*x)-I*a^2*f/d^2*exp(-(c*f-d*e)/d)*Ei(1,-f*x
-e-(c*f-d*e)/d)-3/2*a^2/d/(d*x+c)+1/4*a^2*f*exp(-2*f*x-2*e)/d/(d*f*x+c*f)-1
/2*a^2*f/d^2*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)+1/4*f*a^2/d^2
*exp(2*f*x+2*e)/(c*f/d+f*x)+1/2*f*a^2/d^2*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2
*e-2*(c*f-d*e)/d)+I*a^2*f*exp(-f*x-e)/d/(d*f*x+c*f)-I*a^2*f/d^2*exp((c*f-d*
e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)
```

Maxima [A] time = 1.39495, size = 247, normalized size = 1.45

$$\frac{1}{4}a^2\left(\frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_2\left(\frac{2(dx+c)f}{d}\right)}{(dx+c)d}+\frac{e^{\left(2e-\frac{2cf}{d}\right)}E_2\left(-\frac{2(dx+c)f}{d}\right)}{(dx+c)d}-\frac{2}{d^2x+cd}\right)+i\,a^2\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(2, 2*(d*x + c)*f/d)/((d*x + c)*d
) + e^(2*e - 2*c*f/d)*exp_integral_e(2, -2*(d*x + c)*f/d)/((d*x + c)*d) - 2
/(d^2*x + c*d)) + I*a^2*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)/((
d*x + c)*d) - e^(e - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d)
) - a^2/(d^2*x + c*d)
```

Fricas [A] time = 2.7084, size = 581, normalized size = 3.42

$$\frac{\left(a^2de^{\left(4fx+4e\right)}-4i\,a^2de^{\left(3fx+3e\right)}+4i\,a^2de^{\left(fx+e\right)}+a^2d-\left(6\,a^2d+2\left(a^2dfx+a^2cf\right)\mathrm{Ei}\left(\frac{2\left(dfx+cf\right)}{d}\right)e^{\left(\frac{2\left(de-cf\right)}{d}\right)}-\left(4i\,a^2dfx\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(a^2*d*e^(4*f*x + 4*e) - 4*I*a^2*d*e^(3*f*x + 3*e) + 4*I*a^2*d*e^(f*x +
 e) + a^2*d - (6*a^2*d + 2*(a^2*d*f*x + a^2*c*f)*Ei(2*(d*f*x + c*f)/d)*e^(2
*(d*e - c*f)/d) - (4*I*a^2*d*f*x + 4*I*a^2*c*f)*Ei((d*f*x + c*f)/d)*e^((d*e
 - c*f)/d) - (4*I*a^2*d*f*x + 4*I*a^2*c*f)*Ei(-(d*f*x + c*f)/d)*e^(-(d*e -
c*f)/d) - 2*(a^2*d*f*x + a^2*c*f)*Ei(-2*(d*f*x + c*f)/d)*e^(-2*(d*e - c*f)/
d))*e^(2*f*x + 2*e))*e^(-2*f*x - 2*e)/(d^3*x + c*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))**2/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.10966, size = 485, normalized size = 2.85

$$\frac{2\,a^2dfx\mathrm{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)e^{\left(\frac{2\,cf}{d}-2\,e\right)}+4i\,a^2dfx\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}+4i\,a^2dfx\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)}-2\,a^2dfx\mathrm{Ei}\left(\frac{2\left(dfx+cf\right)}{d}\right)e^{\left(\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*a^2*d*f*x*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) + 4*I*a^2*d*f*x*E
i(-(d*f*x + c*f)/d)*e^(c*f/d - e) + 4*I*a^2*d*f*x*Ei((d*f*x + c*f)/d)*e^(-c
*f/d + e) - 2*a^2*d*f*x*Ei(2*(d*f*x + c*f)/d)*e^(-2*c*f/d + 2*e) + 2*a^2*c*
f*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) + 4*I*a^2*c*f*Ei(-(d*f*x + c*f)/
d)*e^(c*f/d - e) + 4*I*a^2*c*f*Ei((d*f*x + c*f)/d)*e^(-c*f/d + e) - 2*a^2*c
*f*Ei(2*(d*f*x + c*f)/d)*e^(-2*c*f/d + 2*e) + a^2*d*e^(2*f*x + 2*e) - 4*I*a
^2*d*e^(f*x + e) + 4*I*a^2*d*e^(-f*x - e) + a^2*d*e^(-2*f*x - 2*e))/(d^3*x
+ c*d^2) - 3/2*a^2/((d*x + c)*d)
```

3.107 $\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^3}\,dx$

Optimal. Leaf size=236

$$\frac{ia^2f^2\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^3}-\frac{a^2f^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{d^3}-\frac{a^2f^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d^3}+\cdots$$

```
[Out] (-2*a^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^4)/(d*(c + d*x)^2) - (a^2*f^2*Cosh[2
*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/d^3 + (I*a^2*f^2*CoshInteg
ral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^3 - (4*a^2*f*Cosh[e/2 + (I/4)*Pi +
(f*x)/2]^3*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/(d^2*(c + d*x)) + (I*a^2*f^2*Cos
h[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^3 - (a^2*f^2*Sinh[2*e - (2*c*
f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d^3
```

Rubi [A] time = 0.530018, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3318, 3314, 3312, 3303, 3298, 3301}

$$\frac{ia^2f^2\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^3}-\frac{a^2f^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{d^3}-\frac{a^2f^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d^3}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[e + f*x])^2/(c + d*x)^3,x]
```


```
[Out] (-2*a^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^4)/(d*(c + d*x)^2) - (a^2*f^2*Cosh[2
*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/d^3 + (I*a^2*f^2*CoshInteg
ral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^3 - (4*a^2*f*Cosh[e/2 + (I/4)*Pi +
(f*x)/2]^3*Sinh[e/2 + (I/4)*Pi + (f*x)/2])/(d^2*(c + d*x)) + (I*a^2*f^2*Cos
h[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^3 - (a^2*f^2*Sinh[2*e - (2*c*
f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d^3
```


Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(a+ia\sinh(e+fx))^2}{(c+dx)^3}\,dx = \left(4a^2\right)\int \frac{\sin^4\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)}{(c+dx)^3}\,dx$$

$$= -\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)^2} - \frac{4a^2f\cosh^3\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d^2(c+dx)} + \frac{\left(6a^2f^2\right)\int \frac{\sin}{}}{}$$

$$= -\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)^2} - \frac{4a^2f\cosh^3\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d^2(c+dx)} - \frac{\left(6a^2f^2\right)\int\left(\frac{}{2}\right.}{}$$

$$= -\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)^2} - \frac{4a^2f\cosh^3\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d^2(c+dx)} - \frac{\left(3ia^2f^2\right)\int \frac{\sin}{}}{d^2}$$

$$= -\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)^2} - \frac{4a^2f\cosh^3\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d^2(c+dx)} - \frac{\left(a^2f^2\cosh\left(\right.\right.}{}$$

$$= -\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{d(c+dx)^2} - \frac{a^2f^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{d^3} + \frac{ia^2f^2\text{Chi}\left(\frac{cf}{d}+fx\right)}{d^3}$$

Mathematica [A] time = 2.35465, size = 198, normalized size = 0.84

$$\frac{a^2\left(4if^2\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)-4f^2\text{Chi}\left(\frac{2f(c+dx)}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)-4f^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2f(c+dx)}{d}\right)+4if^2\cos\right.}{4d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[e + f*x])^2/(c + d*x)^3,x]
```

```
[Out] (a^2*(-4*f^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*f*(c + d*x))/d] + (4*I)*
f^2*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] + (d*(-3*d - (4*I)*f*(c + d
*x)*Cosh[e + f*x] + d*Cosh[2*(e + f*x)] - (4*I)*d*Sinh[e + f*x] + 2*c*f*Sin
h[2*(e + f*x)] + 2*d*f*x*Sinh[2*(e + f*x)]))/(c + d*x)^2 + (4*I)*f^2*Cosh[e
 - (c*f)/d]*SinhIntegral[f*(c/d + x)] - 4*f^2*Sinh[2*e - (2*c*f)/d]*SinhInt
egral[(2*f*(c + d*x))/d]))/(4*d^3)
```

Maple [B] time = 0.154, size = 625, normalized size = 2.7

$$\frac{-\frac{i}{2}a^2f^2e^{fx+e}}{d^3}\left(\frac{cf}{d}+fx\right)^{-2}-\frac{\frac{i}{2}a^2f^2e^{fx+e}}{d^3}\left(\frac{cf}{d}+fx\right)^{-1}-\frac{\frac{i}{2}a^2f^2}{d^3}e^{-\frac{cf-de}{d}}\text{Ei}\left(1,-fx-e-\frac{cf-de}{d}\right)-\frac{3a^2}{4d(dx+c)^2}-\frac{}{4d(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))^2/(d*x+c)^3,x)
```

```
[Out] -1/2*I*a^2*f^2/d^3*exp(f*x+e)/(c*f/d+f*x)^2-1/2*I*a^2*f^2/d^3*exp(f*x+e)/(c
*f/d+f*x)-1/2*I*a^2*f^2/d^3*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)-3/4*
a^2/d/(d*x+c)^2-1/4*a^2*f^3*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*
f^2)*x-1/4*a^2*f^3*exp(-2*f*x-2*e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c+
1/8*a^2*f^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+1/2*a^2*f^2
/d^3*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)+1/8*f^2*a^2/d^3*exp(2
*f*x+2*e)/(c*f/d+f*x)^2+1/4*f^2*a^2/d^3*exp(2*f*x+2*e)/(c*f/d+f*x)+1/2*f^2*
a^2/d^3*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)-1/2*I*a^2*f^3*ex
p(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x-1/2*I*a^2*f^3*exp(-f*x-e)/d
^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c+1/2*I*a^2*f^2*exp(-f*x-e)/d/(d^2*f^2
*x^2+2*c*d*f^2*x+c^2*f^2)+1/2*I*a^2*f^2/d^3*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*
f-d*e)/d)
```

Maxima [A] time = 1.39689, size = 277, normalized size = 1.17

$$-\frac{1}{4}a^2\left(\frac{1}{d^3x^2+2\,cd^2x+c^2d}-\frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_3\left(\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^2d}-\frac{e^{\left(2\,e-\frac{2\,cf}{d}\right)}E_3\left(-\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^2d}\right)+i\,a^2\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E}{(dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a^2*(1/(d^3*x^2 + 2*c*d^2*x + c^2*d) - e^(-2*e + 2*c*f/d)*exp_integral
_e(3, 2*(d*x + c)*f/d)/((d*x + c)^2*d) - e^(2*e - 2*c*f/d)*exp_integral_e(3
, -2*(d*x + c)*f/d)/((d*x + c)^2*d)) + I*a^2*(e^(-e + c*f/d)*exp_integral_e
(3, (d*x + c)*f/d)/((d*x + c)^2*d) - e^(e - c*f/d)*exp_integral_e(3, -(d*x
+ c)*f/d)/((d*x + c)^2*d)) - 1/2*a^2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Fricas [B] time = 2.90521, size = 959, normalized size = 4.06

$$-\frac{\left(2\,a^2d^2fx+2\,a^2cdf-a^2d^2-\left(2\,a^2d^2fx+2\,a^2cdf+a^2d^2\right)e^{(4\,fx+4\,e)}-\left(-4i\,a^2d^2fx-4i\,a^2cdf-4i\,a^2d^2\right)e^{(3\,fx+3\,e)}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a^2*d^2*f*x + 2*a^2*c*d*f - a^2*d^2 - (2*a^2*d^2*f*x + 2*a^2*c*d*f
+ a^2*d^2)*e^(4*f*x + 4*e) - (-4*I*a^2*d^2*f*x - 4*I*a^2*c*d*f - 4*I*a^2*d^
```

```
2)*e^(3*f*x + 3*e) + (6*a^2*d^2 + 4*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^
2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d)*e^(2*(d*e - c*f)/d) - (4*I*a^2*d^2*f^2*x^2
 + 8*I*a^2*c*d*f^2*x + 4*I*a^2*c^2*f^2)*Ei((d*f*x + c*f)/d)*e^((d*e - c*f)/
d) - (-4*I*a^2*d^2*f^2*x^2 - 8*I*a^2*c*d*f^2*x - 4*I*a^2*c^2*f^2)*Ei(-(d*f*
x + c*f)/d)*e^(-(d*e - c*f)/d) + 4*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2
*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d)*e^(-2*(d*e - c*f)/d))*e^(2*f*x + 2*e) - (-
4*I*a^2*d^2*f*x - 4*I*a^2*c*d*f + 4*I*a^2*d^2)*e^(f*x + e))*e^(-2*f*x - 2*e
)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))**2/(d*x+c)**3,x)

[Out] Timed out
```

Giac [B] time = 1.40146, size = 953, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")

[Out] -1/8*(4*a^2*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) + 4*I*a^2*
d^2*f^2*x^2*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) - 4*I*a^2*d^2*f^2*x^2*Ei((d*
f*x + c*f)/d)*e^(-c*f/d + e) + 4*a^2*d^2*f^2*x^2*Ei(2*(d*f*x + c*f)/d)*e^(-
2*c*f/d + 2*e) + 8*a^2*c*d*f^2*x*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) +
 8*I*a^2*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) - 8*I*a^2*c*d*f^2*x*E
i((d*f*x + c*f)/d)*e^(-c*f/d + e) + 8*a^2*c*d*f^2*x*Ei(2*(d*f*x + c*f)/d)*e
^(-2*c*f/d + 2*e) + 4*a^2*c^2*f^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e)
+ 4*I*a^2*c^2*f^2*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) - 4*I*a^2*c^2*f^2*Ei((
d*f*x + c*f)/d)*e^(-c*f/d + e) + 4*a^2*c^2*f^2*Ei(2*(d*f*x + c*f)/d)*e^(-2*
c*f/d + 2*e) - 2*a^2*d^2*f*x*e^(2*f*x + 2*e) + 4*I*a^2*d^2*f*x*e^(f*x + e)
+ 4*I*a^2*d^2*f*x*e^(-f*x - e) + 2*a^2*d^2*f*x*e^(-2*f*x - 2*e) - 2*a^2*c*d
*f*e^(2*f*x + 2*e) + 4*I*a^2*c*d*f*e^(f*x + e) + 4*I*a^2*c*d*f*e^(-f*x - e)
 + 2*a^2*c*d*f*e^(-2*f*x - 2*e) - a^2*d^2*e^(2*f*x + 2*e) + 4*I*a^2*d^2*e^(
f*x + e) - 4*I*a^2*d^2*e^(-f*x - e) - a^2*d^2*e^(-2*f*x - 2*e) + 6*a^2*d^2)
/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

3.108 $\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx$

Optimal. Leaf size=132

$$-\frac{12d^2(c+dx)\text{PolyLog}\left(2,-ie^{e+fx}\right)}{af^3}+\frac{12d^3\text{PolyLog}\left(3,-ie^{e+fx}\right)}{af^4}-\frac{6d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{af^2}+\frac{(c+dx)^3\tanh\left(\frac{e}{2}+\right.}{af}$$

```
[Out] (c + d*x)^3/(a*f) - (6*d*(c + d*x)^2*Log[1 + I*E^(e + f*x)])/(a*f^2) - (12*
d^2*(c + d*x)*PolyLog[2, (-I)*E^(e + f*x)])/(a*f^3) + (12*d^3*PolyLog[3, (-
I)*E^(e + f*x)])/(a*f^4) + ((c + d*x)^3*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(a*
f)
```

Rubi [A] time = 0.302292, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3318, 4184, 3716, 2190, 2531, 2282, 6589}

$$-\frac{12d^2(c+dx)\text{PolyLog}\left(2,-ie^{e+fx}\right)}{af^3}+\frac{12d^3\text{PolyLog}\left(3,-ie^{e+fx}\right)}{af^4}-\frac{6d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{af^2}+\frac{(c+dx)^3\tanh\left(\frac{e}{2}+\right.}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (c + d*x)^3/(a*f) - (6*d*(c + d*x)^2*Log[1 + I*E^(e + f*x)])/(a*f^2) - (12*
d^2*(c + d*x)*PolyLog[2, (-I)*E^(e + f*x)])/(a*f^3) + (12*d^3*PolyLog[3, (-
I)*E^(e + f*x)])/(a*f^4) + ((c + d*x)^3*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(a*
f)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^3}{a+ia\sinh(e+fx)}\,dx &= \frac{\int (c+dx)^3\csc^2\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)dx}{2a}\\
&= \frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}-\frac{(3d)\int (c+dx)^2\coth\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)dx}{af}\\
&= \frac{(c+dx)^3}{af}+\frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}-\frac{(6id)\int \frac{e^{2\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)^2}{1+ie^{2\left(\frac{e}{2}+\frac{fx}{2}\right)}}dx}{af}\\
&= \frac{(c+dx)^3}{af}-\frac{6d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{af^2}+\frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}+\frac{\left(12d^2\right)\int (c+dx)\,\mathrm{l}}{}\\
&= \frac{(c+dx)^3}{af}-\frac{6d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{af^2}-\frac{12d^2(c+dx)\mathrm{Li}_2\left(-ie^{e+fx}\right)}{af^3}+\frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{i}{}\right.}{af}\\
&= \frac{(c+dx)^3}{af}-\frac{6d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{af^2}-\frac{12d^2(c+dx)\mathrm{Li}_2\left(-ie^{e+fx}\right)}{af^3}+\frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{i}{}\right.}{af}\\
&= \frac{(c+dx)^3}{af}-\frac{6d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{af^2}-\frac{12d^2(c+dx)\mathrm{Li}_2\left(-ie^{e+fx}\right)}{af^3}+\frac{12d^3\mathrm{Li}_3\left(-ie^{e+fx}\right)}{af^4}+
\end{aligned}
$$

Mathematica [A] time = 2.89412, size = 206, normalized size = 1.56

$$
\frac{2\left(\frac{3de^e\left(-\frac{2ide^{-e}\left(e^e-i\right)\left(f(c+dx)\mathrm{PolyLog}\left(2,ie^{-e-fx}\right)+d\mathrm{PolyLog}\left(3,ie^{-e-fx}\right)\right)}{f^3}+\frac{\left(e^{-e}+i\right)(c+dx)^2\log\left(1-ie^{-e-fx}\right)}{f}+\frac{e^{-e}(c+dx)^3}{3d}\right)}{-1-ie^e}+\frac{(c+dx)^3\sinh\left(\frac{fx}{2}\right)}{\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\right.\right.}\right.}{af}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (2*((3*d*E^e*((c + d*x)^3/(3*d*E^e) + ((I + E^(-e))*(c + d*x)^2*Log[1 - I*E
^(-e - f*x)])/f - ((2*I)*d*(-I + E^e)*(f*(c + d*x)*PolyLog[2, I*E^(-e - f*x
)] + d*PolyLog[3, I*E^(-e - f*x)]))/(E^e*f^3)))/(-1 - I*E^e) + ((c + d*x)^3
*Sinh[(f*x)/2])/((Cosh[e/2] + I*Sinh[e/2])*(Cosh[(e + f*x)/2] + I*Sinh[(e +
 f*x)/2]))))/(a*f)
```

Maple [B] time = 0.112, size = 435, normalized size = 3.3

$$\frac{2i\left(d^3x^3+3cd^2x^2+3c^2dx+c^3\right)}{fa\left(e^{fx+e}-i\right)}-12\frac{cd^2\text{polylog}\left(2,-ie^{fx+e}\right)}{af^3}-4\frac{d^3e^3}{f^4a}+6\frac{d\ln\left(e^{fx+e}\right)c^2}{af^2}+6\frac{d^3e^2\ln\left(1+ie^{fx+e}\right)}{f^4a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] 2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/f/a/(exp(f*x+e)-I)-12*d^2/f^3/a*c*p
olylog(2,-I*exp(f*x+e))-4*d^3/f^4/a*e^3+6*d/f^2/a*ln(exp(f*x+e))*c^2+6*d^3/
f^4/a*e^2*ln(1+I*exp(f*x+e))-12*d^2/f^2/a*c*ln(1+I*exp(f*x+e))*x-12*d^2/f^3
/a*c*ln(1+I*exp(f*x+e))*e+12*d^2/f^2/a*c*e*x+2*d^3/f/a*x^3-6*d^3/f^3/a*e^2*
x-12*d^2/f^3/a*c*e*ln(exp(f*x+e))+6*d^2/f/a*c*x^2+6*d^2/f^3/a*c*e^2-6*d^3/f
^4/a*e^2*ln(exp(f*x+e)-I)+12*d^2/f^3/a*c*e*ln(exp(f*x+e)-I)-12*d^3/f^3/a*po
lylog(2,-I*exp(f*x+e))*x+12*d^3*polylog(3,-I*exp(f*x+e))/a/f^4-6*d^3/f^2/a*
ln(1+I*exp(f*x+e))*x^2-6*d/f^2/a*ln(exp(f*x+e)-I)*c^2+6*d^3/f^4/a*e^2*ln(ex
p(f*x+e))
```

Maxima [B] time = 1.68317, size = 320, normalized size = 2.42

$$6c^2d\left(\frac{xe^{(fx+e)}}{afe^{(fx+e)}-iaf}-\frac{\log\left(\left(e^{(fx+e)}-i\right)e^{(-e)}\right)}{af^2}\right)-\frac{2c^3}{\left(iae^{(-fx-e)}-a\right)f}+\frac{2id^3x^3+6icd^2x^2}{afe^{(fx+e)}-iaf}-\frac{12\left(fx\log\left(ie^{(fx+e)}+1\right)+\right.}{af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 6*c^2*d*(x*e^(f*x + e)/(a*f*e^(f*x + e) - I*a*f) - log((e^(f*x + e) - I)*e^
(-e))/(a*f^2)) - 2*c^3/((I*a*e^(-f*x - e) - a)*f) + (2*I*d^3*x^3 + 6*I*c*d^
2*x^2)/(a*f*e^(f*x + e) - I*a*f) - 12*(f*x*log(I*e^(f*x + e) + 1) + dilog(-
I*e^(f*x + e)))*c*d^2/(a*f^3) - 6*(f^2*x^2*log(I*e^(f*x + e) + 1) + 2*f*x*d
ilog(-I*e^(f*x + e)) - 2*polylog(3, -I*e^(f*x + e)))*d^3/(a*f^4) + 2*(d^3*f
^3*x^3 + 3*c*d^2*f^3*x^2)/(a*f^4)
```

Fricas [C] time = 2.72744, size = 863, normalized size = 6.54

$$\frac{-2id^3e^3+6icd^2e^2f-6ic^2def^2+2ic^3f^3+\left(12id^3fx+12icd^2f-12\left(d^3fx+cd^2f\right)e^{(fx+e)}\right)\text{Li}_2\left(-ie^{(fx+e)}\right)+2\left(d^3f^3x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] (-2*I*d^3*e^3 + 6*I*c*d^2*e^2*f - 6*I*c^2*d*e*f^2 + 2*I*c^3*f^3 + (12*I*d^3
*f*x + 12*I*c*d^2*f - 12*(d^3*f*x + c*d^2*f)*e^(f*x + e))*dilog(-I*e^(f*x +
 e)) + 2*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + d^3*e^3 - 3*c*d^2
*e^2*f + 3*c^2*d*e*f^2)*e^(f*x + e) + (6*I*d^3*e^2 - 12*I*c*d^2*e*f + 6*I*c
^2*d*f^2 - 6*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*e^(f*x + e))*log(e^(f*x +
e) - I) + (6*I*d^3*f^2*x^2 + 12*I*c*d^2*f^2*x - 6*I*d^3*e^2 + 12*I*c*d^2*e*
f - 6*(d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*e^(f*x + e))*lo
g(I*e^(f*x + e) + 1) + (12*d^3*e^(f*x + e) - 12*I*d^3)*polylog(3, -I*e^(f*x
 + e)))/(a*f^4*e^(f*x + e) - I*a*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^3}{i\,a\sinh\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(I*a*sinh(f*x + e) + a), x)
```

3.109 $\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}\,dx$

Optimal. Leaf size=101

$$-\frac{4d^2\text{PolyLog}\left(2,-ie^{e+fx}\right)}{af^3}-\frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{af^2}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{af}+\frac{(c+dx)^2}{af}$$

```
[Out] (c + d*x)^2/(a*f) - (4*d*(c + d*x)*Log[1 + I*E^(e + f*x)])/(a*f^2) - (4*d^2
*PolyLog[2, (-I)*E^(e + f*x)])/(a*f^3) + ((c + d*x)^2*Tanh[e/2 + (I/4)*Pi +
 (f*x)/2])/(a*f)
```

Rubi [A] time = 0.218708, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3318, 4184, 3716, 2190, 2279, 2391}

$$-\frac{4d^2\text{PolyLog}\left(2,-ie^{e+fx}\right)}{af^3}-\frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{af^2}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{af}+\frac{(c+dx)^2}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (c + d*x)^2/(a*f) - (4*d*(c + d*x)*Log[1 + I*E^(e + f*x)])/(a*f^2) - (4*d^2
*PolyLog[2, (-I)*E^(e + f*x)])/(a*f^3) + ((c + d*x)^2*Tanh[e/2 + (I/4)*Pi +
 (f*x)/2])/(a*f)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a+ia\sinh(e+fx)}\,dx = \frac{\int (c+dx)^2\csc^2\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)dx}{2a}$$
$$= \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af} - \frac{(2d)\int (c+dx)\coth\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)dx}{af}$$
$$= \frac{(c+dx)^2}{af} + \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af} - \frac{(4id)\int \frac{e^{2\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)}{1+ie^{2\left(\frac{e}{2}+\frac{fx}{2}\right)}}dx}{af}$$
$$= \frac{(c+dx)^2}{af} - \frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{af^2} + \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{\left(4d^2\right)\int \log\left(1+ie\right.}{af^2}$$
$$= \frac{(c+dx)^2}{af} - \frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{af^2} + \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{\left(4d^2\right)\text{Subst}\left(\int \frac{\log}{}\right.}{}$$
$$= \frac{(c+dx)^2}{af} - \frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{af^2} - \frac{4d^2\text{Li}_2\left(-ie^{e+fx}\right)}{af^3} + \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}$$

Mathematica [A] time = 2.21886, size = 150, normalized size = 1.49

$$\frac{2\left(2d^2\text{PolyLog}\left(2,ie^{-e-fx}\right)+\frac{f^2(c+dx)^2\sinh\left(\frac{fx}{2}\right)}{\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}+\frac{if(c+dx)\left(f(c+dx)+2d(1+ie^e)\log\left(1-ie^{-e-fx}\right)\right)}{e^e-i}\right)}{af^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (2*((I*f*(c + d*x)*(f*(c + d*x) + 2*d*(1 + I*E^e)*Log[1 - I*E^(-e - f*x)]))
/(-I + E^e) + 2*d^2*PolyLog[2, I*E^(-e - f*x)] + (f^2*(c + d*x)^2*Sinh[(f*x
)/2])/((Cosh[e/2] + I*Sinh[e/2])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))
))/(a*f^3)
```

Maple [B] time = 0.056, size = 227, normalized size = 2.3

$$\frac{2i\left(d^2x^2+2cdx+c^2\right)}{fa\left(e^{fx+e}-i\right)} - 4\frac{d\ln\left(e^{fx+e}-i\right)c}{af^2} + 4\frac{d\ln\left(e^{fx+e}\right)c}{af^2} + 2\frac{d^2x^2}{fa} + 4\frac{d^2ex}{af^2} + 2\frac{d^2e^2}{f^3a} - 4\frac{d^2\ln\left(1+ie^{fx+e}\right)x}{af^2} - 4\frac{d^2\text{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] 2*I*(d^2*x^2+2*c*d*x+c^2)/f/a/(exp(f*x+e)-I)-4*d/f^2/a*ln(exp(f*x+e)-I)*c+4
*d/f^2/a*ln(exp(f*x+e))*c+2*d^2/f/a*x^2+4*d^2/f^2/a*e*x+2*d^2/f^3/a*e^2-4*d
^2/f^2/a*ln(1+I*exp(f*x+e))*x-4*d^2/f^3/a*ln(1+I*exp(f*x+e))*e-4*d^2*polylo
g(2,-I*exp(f*x+e))/a/f^3+4*d^2/f^3/a*e*ln(exp(f*x+e)-I)-4*d^2/f^3/a*e*ln(ex
p(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\frac{2i\,x^2}{afe^{\left(fx+e\right)}-iaf}-4i\int\frac{x}{afe^{\left(fx+e\right)}-iaf}\,dx\right)+4\,cd\left(\frac{xe^{\left(fx+e\right)}}{afe^{\left(fx+e\right)}-iaf}-\frac{\log\left(\left(e^{\left(fx+e\right)}-i\right)e^{\left(-e\right)}\right)}{af^2}\right)-\frac{2\,c^2}{\left(i\,ae^{\left(-fx-e\right)}-a\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] d^2*(2*I*x^2/(a*f*e^(f*x + e) - I*a*f) - 4*I*integrate(x/(a*f*e^(f*x + e) -
 I*a*f), x)) + 4*c*d*(x*e^(f*x + e)/(a*f*e^(f*x + e) - I*a*f) - log((e^(f*x
 + e) - I)*e^(-e))/(a*f^2)) - 2*c^2/((I*a*e^(-f*x - e) - a)*f)
```

Fricas [B] time = 2.69656, size = 483, normalized size = 4.78

$$\frac{2i\,d^2e^2-4i\,cdef+2i\,c^2f^2-\left(4\,d^2e^{\left(fx+e\right)}-4i\,d^2\right)\mathrm{Li}_2\left(-i\,e^{\left(fx+e\right)}\right)+2\left(d^2f^2x^2+2\,cdf^2x-d^2e^2+2\,cdef\right)e^{\left(fx+e\right)}+\left(-4i\,\cdots\right.}{af^3e^{\left(fx+e\right)}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] (2*I*d^2*e^2 - 4*I*c*d*e*f + 2*I*c^2*f^2 - (4*d^2*e^(f*x + e) - 4*I*d^2)*di
log(-I*e^(f*x + e)) + 2*(d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*e
^(f*x + e) + (-4*I*d^2*e + 4*I*c*d*f + 4*(d^2*e - c*d*f)*e^(f*x + e))*log(e
^(f*x + e) - I) + (4*I*d^2*f*x + 4*I*d^2*e - 4*(d^2*f*x + d^2*e)*e^(f*x + e
))*log(I*e^(f*x + e) + 1))/(a*f^3*e^(f*x + e) - I*a*f^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{i\,a\sinh\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(I*a*sinh(f*x + e) + a), x)
```

3.110 $\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx$

Optimal. Leaf size=63

$$\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{af}-\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{af^2}$$

```
[Out] (-2*d*Log[Cosh[e/2 + (I/4)*Pi + (f*x)/2]])/(a*f^2) + ((c + d*x)*Tanh[e/2 +
(I/4)*Pi + (f*x)/2])/(a*f)
```

Rubi [A] time = 0.0739995, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3318, 4184, 3475}

$$\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{af}-\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\right)}{af^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (-2*d*Log[Cosh[e/2 + (I/4)*Pi + (f*x)/2]])/(a*f^2) + ((c + d*x)*Tanh[e/2 +
(I/4)*Pi + (f*x)/2])/(a*f)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{a+ia\sinh(e+fx)}\,dx &= \frac{\int (c+dx)\csc^2\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)dx}{2a}\\ &= \frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}-\frac{d\int \coth\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)dx}{af}\\ &= -\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{af^2}+\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}\end{aligned}$$

Mathematica [B] time = 0.450552, size = 185, normalized size = 2.94

$$\frac{2cf\sinh\left(\frac{fx}{2}\right)+idfx\cosh\left(e+\frac{fx}{2}\right)-id\sinh\left(e+\frac{fx}{2}\right)\log(\cosh(e+fx))+2d\sinh\left(e+\frac{fx}{2}\right)\tan^{-1}\left(\sinh\left(\frac{fx}{2}\right)\operatorname{sech}\left(e+\frac{f}{2}\right.\right.}{af^2\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(e+fx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (I*d*f*x*Cosh[e + (f*x)/2] + Cosh[(f*x)/2]*((-2*I)*d*ArcTan[Sech[e + (f*x)/
2]*Sinh[(f*x)/2]] - d*Log[Cosh[e + f*x]]) + 2*c*f*Sinh[(f*x)/2] + d*f*x*Sin
h[(f*x)/2] + 2*d*ArcTan[Sech[e + (f*x)/2]*Sinh[(f*x)/2]]*Sinh[e + (f*x)/2]
- I*d*Log[Cosh[e + f*x]]*Sinh[e + (f*x)/2])/(a*f^2*(Cosh[e/2] + I*Sinh[e/2]
)*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))
```

Maple [A] time = 0.056, size = 66, normalized size = 1.1

$$2\,\frac{dx}{af}+2\,\frac{de}{af^2}+\frac{2\,i\,(dx+c)}{af\left(e^{fx+e}-i\right)}-2\,\frac{d\ln\left(e^{fx+e}-i\right)}{af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] 2*d/a/f*x+2*d/a/f^2*e+2*I*(d*x+c)/f/a/(exp(f*x+e)-I)-2*d/a/f^2*ln(exp(f*x+e
)-I)
```

Maxima [A] time = 1.06809, size = 101, normalized size = 1.6

$$2\,d\left(\frac{xe^{(fx+e)}}{afe^{(fx+e)}-i\,af}-\frac{\log\left(\left(e^{(fx+e)}-i\right)e^{(-e)}\right)}{af^2}\right)-\frac{2\,c}{\left(i\,ae^{(-fx-e)}-a\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*d*(x*e^(f*x + e)/(a*f*e^(f*x + e) - I*a*f) - log((e^(f*x + e) - I)*e^(-e)
)/(a*f^2)) - 2*c/((I*a*e^(-f*x - e) - a)*f)
```

Fricas [A] time = 3.1936, size = 151, normalized size = 2.4

$$\frac{2\,dfxe^{(fx+e)}+2i\,cf-\left(2\,de^{(fx+e)}-2i\,d\right)\log\left(e^{(fx+e)}-i\right)}{af^2e^{(fx+e)}-i\,af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] (2*d*f*x*e^(f*x + e) + 2*I*c*f - (2*d*e^(f*x + e) - 2*I*d)*log(e^(f*x + e)
- I))/(a*f^2*e^(f*x + e) - I*a*f^2)
```

Sympy [A] time = 0.689824, size = 56, normalized size = 0.89

$$\frac{2dx}{af} - \frac{2d\log\left(e^{fx} - ie^{-e}\right)}{af^2} + \frac{(2ic + 2idx)\,e^{-e}}{af\left(e^{fx} - ie^{-e}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] 2*d*x/(a*f) - 2*d*log(exp(f*x) - I*exp(-e))/(a*f**2) + (2*I*c + 2*I*d*x)*ex
p(-e)/(a*f*(exp(f*x) - I*exp(-e)))
```

Giac [A] time = 1.2167, size = 97, normalized size = 1.54

$$\frac{2\,dfxe^{(fx+e)} - 2\,de^{(fx+e)}\log\left(e^{(fx+e)} - i\right) + 2i\,cf + 2i\,d\log\left(e^{(fx+e)} - i\right)}{af^2e^{(fx+e)} - i\,af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] (2*d*f*x*e^(f*x + e) - 2*d*e^(f*x + e)*log(e^(f*x + e) - I) + 2*I*c*f + 2*I
*d*log(e^(f*x + e) - I))/(a*f^2*e^(f*x + e) - I*a*f^2)
```

3.111 $\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{1}{(c+dx)(a+ia\sinh(e+fx))},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)*(a + I*a*Sinh[e + f*x])), x]
```

Rubi [A] time = 0.0641974, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + I*a*Sinh[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + I*a*Sinh[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx$$

Mathematica [A] time = 20.6237, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + I*a*Sinh[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)*(a + I*a*Sinh[e + f*x])), x]
```

Maple [A] time = 0.098, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(a+ia\sinh\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$2id\int \frac{1}{-iad^2fx^2 - 2iacdfx - iac^2f + \left(ad^2fx^2e^e + 2acdfxe^e + ac^2fe^e\right)e^{(fx)}}\,dx + \frac{2i}{-iadfx - iacf + \left(adfxe^e + acfe^e\right)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*I*d*integrate(1/(-I*a*d^2*f*x^2 - 2*I*a*c*d*f*x - I*a*c^2*f + (a*d^2*f*x^
2*e^e + 2*a*c*d*f*x*e^e + a*c^2*f*e^e)*e^(f*x)), x) + 2*I/(-I*a*d*f*x - I*a
*c*f + (a*d*f*x*e^e + a*c*f*e^e)*e^(f*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(-iadfx - iacf + \left(adfx + acf\right)e^{(fx+e)}\right)\text{integral}\left(\frac{2id}{-iad^2fx^2 - 2iacdfx - iac^2f + \left(ad^2fx^2 + 2acdfx + ac^2f\right)e^{(fx+e)}}, x\right) + 2i}{-iadfx - iacf + \left(adfx + acf\right)e^{(fx+e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] ((-I*a*d*f*x - I*a*c*f + (a*d*f*x + a*c*f)*e^(f*x + e))*integral(2*I*d/(-I*
a*d^2*f*x^2 - 2*I*a*c*d*f*x - I*a*c^2*f + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^
2*f)*e^(f*x + e)), x) + 2*I)/(-I*a*d*f*x - I*a*c*f + (a*d*f*x + a*c*f)*e^(f
*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)\left(ia\sinh\left(fx + e\right) + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(I*a*sinh(f*x + e) + a)), x)
```

3.112 $\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a+ia\sinh(e+fx))},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])), x]
```

Rubi [A] time = 0.0597691, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx=\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx$$

Mathematica [A] time = 21.0866, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])), x]
```

Maple [A] time = 0.164, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(a+ia\sinh\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$4i\,d\int \frac{1}{-i\,ad^3fx^3-3i\,acd^2fx^2-3i\,ac^2dfx-i\,ac^3f+\left(ad^3fx^3e^e+3\,acd^2fx^2e^e+3\,ac^2dfxe^e+ac^3fe^e\right)e^{(fx)}}\,dx+\frac{}{-i\,ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 4*I*d*integrate(1/(-I*a*d^3*f*x^3 - 3*I*a*c*d^2*f*x^2 - 3*I*a*c^2*d*f*x - I
*a*c^3*f + (a*d^3*f*x^3*e^e + 3*a*c*d^2*f*x^2*e^e + 3*a*c^2*d*f*x*e^e + a*c
^3*f*e^e)*e^(f*x)), x) + 2*I/(-I*a*d^2*f*x^2 - 2*I*a*c*d*f*x - I*a*c^2*f +
(a*d^2*f*x^2*e^e + 2*a*c*d*f*x*e^e + a*c^2*f*e^e)*e^(f*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(-i\,ad^2fx^2-2i\,acdfx-i\,ac^2f+\left(ad^2fx^2+2\,acdfx+ac^2f\right)e^{(fx+e)}\right)\mathrm{integral}\left(\frac{4i\,d}{-i\,ad^3fx^3-3i\,acd^2fx^2-3i\,ac^2dfx-i\,ac^3f+\left(ad^3fx^3+3\right.}\right.}{-i\,ad^2fx^2-2i\,acdfx-i\,ac^2f+\left(ad^2fx^2+2\,acdfx+ac^2f\right)e^{(fx+e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] ((-I*a*d^2*f*x^2 - 2*I*a*c*d*f*x - I*a*c^2*f + (a*d^2*f*x^2 + 2*a*c*d*f*x +
 a*c^2*f)*e^(f*x + e))*integral(4*I*d/(-I*a*d^3*f*x^3 - 3*I*a*c*d^2*f*x^2 -
 3*I*a*c^2*d*f*x - I*a*c^3*f + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f
*x + a*c^3*f)*e^(f*x + e)), x) + 2*I)/(-I*a*d^2*f*x^2 - 2*I*a*c*d*f*x - I*a
*c^2*f + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*e^(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(i\,a\sinh\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(I*a*sinh(f*x + e) + a)), x)
```

3.113 $\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx$

Optimal. Leaf size=305

$$-\frac{4d^2(c+dx)\text{PolyLog}\left(2,-ie^{e+fx}\right)}{a^2f^3}+\frac{4d^3\text{PolyLog}\left(3,-ie^{e+fx}\right)}{a^2f^4}-\frac{2d^2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{a^2f^3}-\frac{2d(c+dx)^2\log\left(1+\right.}{a^2f^2}$$

```
[Out] (c + d*x)^3/(3*a^2*f) - (2*d*(c + d*x)^2*Log[1 + I*E^(e + f*x)])/(a^2*f^2)
+ (4*d^3*Log[Cosh[e/2 + (I/4)*Pi + (f*x)/2]])/(a^2*f^4) - (4*d^2*(c + d*x)*
PolyLog[2, (-I)*E^(e + f*x)])/(a^2*f^3) + (4*d^3*PolyLog[3, (-I)*E^(e + f*x
)])/(a^2*f^4) + (d*(c + d*x)^2*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2)/(2*a^2*f^2
) - (2*d^2*(c + d*x)*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(a^2*f^3) + ((c + d*x)
^3*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*a^2*f) + ((c + d*x)^3*Sech[e/2 + (I/4
)*Pi + (f*x)/2]^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(6*a^2*f)
```

Rubi [A] time = 0.397799, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3318, 4186, 4184, 3475, 3716, 2190, 2531, 2282, 6589}

$$-\frac{4d^2(c+dx)\text{PolyLog}\left(2,-ie^{e+fx}\right)}{a^2f^3}+\frac{4d^3\text{PolyLog}\left(3,-ie^{e+fx}\right)}{a^2f^4}-\frac{2d^2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{a^2f^3}-\frac{2d(c+dx)^2\log\left(1+\right.}{a^2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + I*a*Sinh[e + f*x])^2,x]
```


```
[Out] (c + d*x)^3/(3*a^2*f) - (2*d*(c + d*x)^2*Log[1 + I*E^(e + f*x)])/(a^2*f^2)
+ (4*d^3*Log[Cosh[e/2 + (I/4)*Pi + (f*x)/2]])/(a^2*f^4) - (4*d^2*(c + d*x)*
PolyLog[2, (-I)*E^(e + f*x)])/(a^2*f^3) + (4*d^3*PolyLog[3, (-I)*E^(e + f*x
)])/(a^2*f^4) + (d*(c + d*x)^2*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2)/(2*a^2*f^2
) - (2*d^2*(c + d*x)*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(a^2*f^3) + ((c + d*x)
^3*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*a^2*f) + ((c + d*x)^3*Sech[e/2 + (I/4
)*Pi + (f*x)/2]^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(6*a^2*f)
```


Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
```

```
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2}\,dx = \frac{\int (c+dx)^3\csc^4\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)dx}{4a^2}$$

$$= \frac{d(c+dx)^2\operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2a^2f^2} + \frac{(c+dx)^3\operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6a^2f} - \frac{\int(c}{}$$

$$= \frac{d(c+dx)^2\operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2a^2f^2} - \frac{2d^2(c+dx)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{a^2f^3} + \frac{(c+dx)^3\tanh\left(\frac{e}{2}+\right.}{3a^2f}$$

$$= \frac{(c+dx)^3}{3a^2f} + \frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{a^2f^4} + \frac{d(c+dx)^2\operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2a^2f^2} - \frac{2d^2(c+dx)}{}$$

$$= \frac{(c+dx)^3}{3a^2f} - \frac{2d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{a^2f^2} + \frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{a^2f^4} + \frac{d(c+dx)^2\operatorname{sech}}{2a}$$

$$= \frac{(c+dx)^3}{3a^2f} - \frac{2d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{a^2f^2} + \frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{a^2f^4} - \frac{4d^2(c+dx)\mathrm{Li}_2}{a^2f^3}$$

$$= \frac{(c+dx)^3}{3a^2f} - \frac{2d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{a^2f^2} + \frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{a^2f^4} - \frac{4d^2(c+dx)\mathrm{Li}_2}{a^2f^3}$$

$$= \frac{(c+dx)^3}{3a^2f} - \frac{2d(c+dx)^2\log\left(1+ie^{e+fx}\right)}{a^2f^2} + \frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{a^2f^4} - \frac{4d^2(c+dx)\mathrm{Li}_2}{a^2f^3}$$

Mathematica [A] time = 6.24652, size = 443, normalized size = 1.45

$$\frac{\frac{2d\left(-6cd(1+ie^e)\mathrm{PolyLog}\left(2,ie^{-e-fx}\right)-6d^2(1+ie^e)\left(x\mathrm{PolyLog}\left(2,ie^{-e-fx}\right)+\frac{\mathrm{PolyLog}\left(3,ie^{-e-fx}\right)}{f}\right)+\frac{3(1+ie^e)(2d^2-c^2f^2)\left(fx-\log\left(-e^{e+fx}+i\right)\right)}{f}+3c^2f^2x+6cd(1+ie^e)fx\log\left(1-ie^{-}\right.}{-1-ie^e}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] ((2*d*(-6*d^2*x + 3*c^2*f^2*x + 3*c*d*f^2*x^2 + d^2*f^2*x^3 + 6*c*d*(1 + I*
E^e)*f*x*Log[1 - I*E^(-e - f*x)] + 3*d^2*(1 + I*E^e)*f*x^2*Log[1 - I*E^(-e
- f*x)] + (3*(1 + I*E^e)*(2*d^2 - c^2*f^2)*(f*x - Log[I - E^(e + f*x)]))/f
- 6*c*d*(1 + I*E^e)*PolyLog[2, I*E^(-e - f*x)] - 6*d^2*(1 + I*E^e)*(x*PolyL
og[2, I*E^(-e - f*x)] + PolyLog[3, I*E^(-e - f*x)]/f)))/(-1 - I*E^e) + ((c
+ d*x)*(3*d*f*(c + d*x)*Cosh[(f*x)/2] + (6*I)*d^2*Cosh[e + (f*x)/2] + I*(c^
2*f^2 + 2*c*d*f^2*x + d^2*(-6 + f^2*x^2))*Cosh[e + (3*f*x)/2] + 3*(c^2*f^2
+ 2*c*d*f^2*x + d^2*(-4 + f^2*x^2))*Sinh[(f*x)/2] + (3*I)*d*f*(c + d*x)*Sin
h[e + (f*x)/2]))/((Cosh[e/2] + I*Sinh[e/2])*(Cosh[(e + f*x)/2] + I*Sinh[(e
+ f*x)/2])^3))/(3*a^2*f^3)
```

Maple [B] time = 0.141, size = 723, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+I*a*sinh(f*x+e))^2,x)
```

```
[Out] 2/3*(6*I*d^3*x-3*I*f*d^3*x^2*exp(2*f*x+2*e)-I*f^2*d^3*x^3+3*f^2*d^3*x^3*exp
(f*x+e)-3*f*d^3*x^2*exp(f*x+e)-3*f*c^2*d*exp(f*x+e)-3*I*f^2*c*d^2*x^2+9*f^2
*c*d^2*x^2*exp(f*x+e)+9*f^2*c^2*d*x*exp(f*x+e)-6*f*c*d^2*x*exp(f*x+e)-3*I*f
^2*c^2*d*x-6*I*c*d^2*exp(2*f*x+2*e)-I*f^2*c^3-6*I*f*c*d^2*x*exp(2*f*x+2*e)-
6*I*d^3*x*exp(2*f*x+2*e)+6*I*c*d^2-12*d^3*x*exp(f*x+e)-12*c*d^2*exp(f*x+e)+
3*f^2*c^3*exp(f*x+e)-3*I*f*c^2*d*exp(2*f*x+2*e))/(exp(f*x+e)-I)^3/f^3/a^2-4
*d^2/f^2/a^2*ln(1+I*exp(f*x+e))*c*x-4*d^2/f^3/a^2*ln(1+I*exp(f*x+e))*c*e+4*
d^2/f^2/a^2*c*e*x+4*d^2/f^3/a^2*ln(exp(f*x+e)-I)*c*e-4*d^2/f^3/a^2*ln(exp(f
*x+e))*c*e+4*d^3/f^4/a^2*ln(exp(f*x+e)-I)+4*d^3*polylog(3,-I*exp(f*x+e))/a^
2/f^4-4*d^3/f^4/a^2*ln(exp(f*x+e))-4*d^2/f^3/a^2*c*polylog(2,-I*exp(f*x+e))
-2*d^3/f^3/a^2*e^2*x+2*d^2/f/a^2*c*x^2+2*d^2/f^3/a^2*c*e^2-2*d^3/f^2/a^2*ln
(1+I*exp(f*x+e))*x^2-4*d^3/f^3/a^2*polylog(2,-I*exp(f*x+e))*x-2*d^3/f^4/a^2
*ln(exp(f*x+e)-I)*e^2+2*d/f^2/a^2*ln(exp(f*x+e))*c^2-2*d/f^2/a^2*ln(exp(f*x
+e)-I)*c^2+2*d^3/f^4/a^2*ln(exp(f*x+e))*e^2+2/3*d^3/f/a^2*x^3+2*d^3/f^4/a^2
*ln(1+I*exp(f*x+e))*e^2-4/3*d^3/f^4/a^2*e^3
```

Maxima [B] time = 1.99941, size = 856, normalized size = 2.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] c^2*d*(3*(2*f*x*e^(3*f*x + 3*e) + (-6*I*f*x*e^(2*e) - 2*I*e^(2*e))*e^(2*f*x
) - 2*e^(f*x + e))/(3*a^2*f^2*e^(3*f*x + 3*e) - 9*I*a^2*f^2*e^(2*f*x + 2*e)
 - 9*a^2*f^2*e^(f*x + e) + 3*I*a^2*f^2) - 2*log(-I*(I*e^(f*x + e) + 1)*e^(-
e))/(a^2*f^2)) + c^3*(6*e^(-f*x - e)/((9*a^2*e^(-f*x - e) - 9*I*a^2*e^(-2*f
*x - 2*e) - 3*a^2*e^(-3*f*x - 3*e) + 3*I*a^2)*f) + 2*I/((9*a^2*e^(-f*x - e)
 - 9*I*a^2*e^(-2*f*x - 2*e) - 3*a^2*e^(-3*f*x - 3*e) + 3*I*a^2)*f)) + (-2*I
*d^3*f^2*x^3 - 6*I*c*d^2*f^2*x^2 + 12*I*d^3*x + 12*I*c*d^2 - (6*I*d^3*f*x^2
*e^(2*e) + 12*I*c*d^2*e^(2*e) + (12*I*c*d^2*f*e^(2*e) + 12*I*d^3*e^(2*e))*x
)*e^(2*f*x) + 6*(d^3*f^2*x^3*e^e - 4*c*d^2*e^e + (3*c*d^2*f^2*e^e - d^3*f*e
^e)*x^2 - 2*(c*d^2*f*e^e + 2*d^3*e^e)*x)*e^(f*x))/(3*a^2*f^3*e^(3*f*x + 3*e
) - 9*I*a^2*f^3*e^(2*f*x + 2*e) - 9*a^2*f^3*e^(f*x + e) + 3*I*a^2*f^3) - 4*
(f*x*log(I*e^(f*x + e) + 1) + dilog(-I*e^(f*x + e)))*c*d^2/(a^2*f^3) - 4*d^
3*x/(a^2*f^3) - 2*(f^2*x^2*log(I*e^(f*x + e) + 1) + 2*f*x*dilog(-I*e^(f*x +
 e)) - 2*polylog(3, -I*e^(f*x + e)))*d^3/(a^2*f^4) + 4*d^3*log(e^(f*x + e)
- I)/(a^2*f^4) + 2/3*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2)/(a^2*f^4)
```

Fricas [C] time = 2.60388, size = 2147, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] (2*I*d^3*e^3 + 6*I*c^2*d*e*f^2 - 2*I*c^3*f^3 - 12*I*d^3*e + (-6*I*c*d^2*e^2
 + 12*I*c*d^2)*f + (-12*I*d^3*f*x - 12*I*c*d^2*f - 12*(d^3*f*x + c*d^2*f)*e
^(3*f*x + 3*e) + (36*I*d^3*f*x + 36*I*c*d^2*f)*e^(2*f*x + 2*e) + 36*(d^3*f*
x + c*d^2*f)*e^(f*x + e))*dilog(-I*e^(f*x + e)) + 2*(d^3*f^3*x^3 + 3*c*d^2*
```

```
f^3*x^2 + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - 6*d^3*e + 3*(c^2*d*f^3
- 2*d^3*f)*x)*e^(3*f*x + 3*e) + (-6*I*d^3*f^3*x^3 - 6*I*d^3*e^3 + 36*I*d^3*
e + (-18*I*c^2*d*e - 6*I*c^2*d)*f^2 + (-18*I*c*d^2*f^3 - 6*I*d^3*f^2)*x^2 +
 (18*I*c*d^2*e^2 - 12*I*c*d^2)*f + (-18*I*c^2*d*f^3 - 12*I*c*d^2*f^2 + 24*I
*d^3*f)*x)*e^(2*f*x + 2*e) - 6*(d^3*f^2*x^2 + d^3*e^3 - c^3*f^3 - 6*d^3*e +
 (3*c^2*d*e + c^2*d)*f^2 - (3*c*d^2*e^2 - 4*c*d^2)*f + 2*(c*d^2*f^2 - d^3*f
)*x)*e^(f*x + e) + (-6*I*d^3*e^2 + 12*I*c*d^2*e*f - 6*I*c^2*d*f^2 + 12*I*d^
3 - 6*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 - 2*d^3)*e^(3*f*x + 3*e) + (18*I*d
^3*e^2 - 36*I*c*d^2*e*f + 18*I*c^2*d*f^2 - 36*I*d^3)*e^(2*f*x + 2*e) + 18*(
d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 - 2*d^3)*e^(f*x + e))*log(e^(f*x + e) - I
) + (-6*I*d^3*f^2*x^2 - 12*I*c*d^2*f^2*x + 6*I*d^3*e^2 - 12*I*c*d^2*e*f - 6
*(d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*e^(3*f*x + 3*e) + (1
8*I*d^3*f^2*x^2 + 36*I*c*d^2*f^2*x - 18*I*d^3*e^2 + 36*I*c*d^2*e*f)*e^(2*f*
x + 2*e) + 18*(d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*e^(f*x
+ e))*log(I*e^(f*x + e) + 1) + (12*d^3*e^(3*f*x + 3*e) - 36*I*d^3*e^(2*f*x
+ 2*e) - 36*d^3*e^(f*x + e) + 12*I*d^3)*polylog(3, -I*e^(f*x + e)))/(3*a^2*
f^4*e^(3*f*x + 3*e) - 9*I*a^2*f^4*e^(2*f*x + 2*e) - 9*a^2*f^4*e^(f*x + e) +
 3*I*a^2*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+I*a*sinh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^3}{\left(i\,a\sinh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(I*a*sinh(f*x + e) + a)^2, x)
```

3.114 $\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx$

Optimal. Leaf size=241

$$-\frac{4d^2\text{PolyLog}\left(2,-ie^{e+fx}\right)}{3a^2f^3}-\frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{3a^2f^2}+\frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3a^2f^2}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\right.}{3a^2f}$$

```
[Out] (c + d*x)^2/(3*a^2*f) - (4*d*(c + d*x)*Log[1 + I*E^(e + f*x)])/(3*a^2*f^2)
- (4*d^2*PolyLog[2, (-I)*E^(e + f*x)])/(3*a^2*f^3) + (d*(c + d*x)*Sech[e/2
+ (I/4)*Pi + (f*x)/2]^2)/(3*a^2*f^2) - (2*d^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2
])/(3*a^2*f^3) + ((c + d*x)^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*a^2*f) + (
(c + d*x)^2*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2]
)/(6*a^2*f)
```

Rubi [A] time = 0.277598, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3318, 4186, 3767, 8, 4184, 3716, 2190, 2279, 2391}

$$-\frac{4d^2\text{PolyLog}\left(2,-ie^{e+fx}\right)}{3a^2f^3}-\frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{3a^2f^2}+\frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3a^2f^2}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\right.}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] (c + d*x)^2/(3*a^2*f) - (4*d*(c + d*x)*Log[1 + I*E^(e + f*x)])/(3*a^2*f^2)
- (4*d^2*PolyLog[2, (-I)*E^(e + f*x)])/(3*a^2*f^3) + (d*(c + d*x)*Sech[e/2
+ (I/4)*Pi + (f*x)/2]^2)/(3*a^2*f^2) - (2*d^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2
])/(3*a^2*f^3) + ((c + d*x)^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*a^2*f) + (
(c + d*x)^2*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2]
)/(6*a^2*f)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+ia\sinh(e+fx))^2}\,dx = \frac{\int (c+dx)^2\csc^4\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)dx}{4a^2}$$
$$= \frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6a^2f} - \ldots$$
$$= \frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\ldots\right.}{\ldots}$$
$$= \frac{(c+dx)^2}{3a^2f} + \frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f^2} - \frac{2d^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f^3} + \frac{(c+dx)^2\tanh\ldots}{3a\ldots}$$
$$= \frac{(c+dx)^2}{3a^2f} - \frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{3a^2f^2} + \frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f^2} - \frac{2d^2\tanh\left(\frac{e}{2}\ldots\right.}{3a^2\ldots}$$
$$= \frac{(c+dx)^2}{3a^2f} - \frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{3a^2f^2} + \frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f^2} - \frac{2d^2\tanh\left(\frac{e}{2}\ldots\right.}{3a^2\ldots}$$
$$= \frac{(c+dx)^2}{3a^2f} - \frac{4d(c+dx)\log\left(1+ie^{e+fx}\right)}{3a^2f^2} - \frac{4d^2\text{Li}_2\left(-ie^{e+fx}\right)}{3a^2f^3} + \frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\ldots\right.}{3a^2f^2}$$

Mathematica [A] time = 3.55659, size = 269, normalized size = 1.12

$$\frac{4d^2\text{PolyLog}\left(2,ie^{-e-fx}\right)+\frac{i\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2-2\right)\right)\cosh\left(e+\frac{3fx}{2}\right)+\sinh\left(\frac{fx}{2}\right)\left(3c^2f^2+6cdf^2x+d^2\left(3f^2x^2-4\right)\right)+2idf(c+dx)\sinh\left(e+\frac{fx}{2}\right)+2df(c+\ldots}{\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)^3}}{3a^2f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] (((2*I)*f*(c + d*x)*(f*(c + d*x) + 2*d*(1 + I*E^e)*Log[1 - I*E^(-e - f*x)])
)/(-I + E^e) + 4*d^2*PolyLog[2, I*E^(-e - f*x)] + (2*d*f*(c + d*x)*Cosh[(f*
x)/2] + (2*I)*d^2*Cosh[e + (f*x)/2] + I*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-2 +
f^2*x^2))*Cosh[e + (3*f*x)/2] + (3*c^2*f^2 + 6*c*d*f^2*x + d^2*(-4 + 3*f^2*
x^2))*Sinh[(f*x)/2] + (2*I)*d*f*(c + d*x)*Sinh[e + (f*x)/2])/((Cosh[e/2] +
I*Sinh[e/2])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^3))/(3*a^2*f^3)
```

Maple [A] time = 0.113, size = 374, normalized size = 1.6

$$\frac{-2\,if^2d^2x^2-4\,fd^2xe^{fx+e}-4\,fcde^{fx+e}-8\,d^2e^{fx+e}+4\,id^2-4\,id^2e^{2\,fx+2\,e}-2\,if^2c^2-4\,if^2cdx-4\,ifd^2xe^{2\,fx+2\,e}-4\,ifc\ldots}{3\left(e^{fx+e}-i\right)^3f^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x)
```

```
[Out] 2/3*(-I*f^2*d^2*x^2-2*f*d^2*x*exp(f*x+e)-2*f*c*d*exp(f*x+e)-4*d^2*exp(f*x+e
)+2*I*d^2-2*I*d^2*exp(2*f*x+2*e)-I*f^2*c^2-2*I*f^2*c*d*x-2*I*f*d^2*x*exp(2*
```

```
f*x+2*e)-2*I*f*c*d*exp(2*f*x+2*e)+3*f^2*d^2*x^2*exp(f*x+e)+6*f^2*c*d*x*exp(
f*x+e)+3*f^2*c^2*exp(f*x+e))/(exp(f*x+e)-I)^3/f^3/a^2-4/3*d/f^2/a^2*ln(exp(
f*x+e)-I)*c+4/3*d/f^2/a^2*ln(exp(f*x+e))*c+2/3*d^2/f/a^2*x^2+4/3*d^2/f^2/a^
2*e*x+2/3*d^2/f^3/a^2*e^2-4/3*d^2/f^2/a^2*ln(1+I*exp(f*x+e))*x-4/3*d^2/f^3/
a^2*ln(1+I*exp(f*x+e))*e-4/3*d^2*polylog(2,-I*exp(f*x+e))/a^2/f^3+4/3*d^2/f
^3/a^2*e*ln(exp(f*x+e)-I)-4/3*d^2/f^3/a^2*e*ln(exp(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\frac{-2i f^2x^2 - \left(4i fxe^{(2e)} + 4i e^{(2e)}\right)e^{(2fx)} + 2\left(3f^2x^2e^e - 2fxe^e - 4e^e\right)e^{(fx)} + 4i}{3a^2f^3e^{(3fx+3e)} - 9i a^2f^3e^{(2fx+2e)} - 9a^2f^3e^{(fx+e)} + 3i a^2f^3} - 4i\int\frac{x}{3\left(a^2fe^{(fx+e)} - i a^2f\right)}\,dx\right) + \frac{2}{3}cd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] d^2*((-2*I*f^2*x^2 - (4*I*f*x*e^(2*e) + 4*I*e^(2*e))*e^(2*f*x) + 2*(3*f^2*x
^2*e^e - 2*f*x*e^e - 4*e^e)*e^(f*x) + 4*I)/(3*a^2*f^3*e^(3*f*x + 3*e) - 9*I
*a^2*f^3*e^(2*f*x + 2*e) - 9*a^2*f^3*e^(f*x + e) + 3*I*a^2*f^3) - 4*I*integ
rate(1/3*x/(a^2*f*e^(f*x + e) - I*a^2*f), x)) + 2/3*c*d*(3*(2*f*x*e^(3*f*x
+ 3*e) + (-6*I*f*x*e^(2*e) - 2*I*e^(2*e))*e^(2*f*x) - 2*e^(f*x + e))/(3*a^2
*f^2*e^(3*f*x + 3*e) - 9*I*a^2*f^2*e^(2*f*x + 2*e) - 9*a^2*f^2*e^(f*x + e)
+ 3*I*a^2*f^2) - 2*log(-I*(I*e^(f*x + e) + 1)*e^(-e))/(a^2*f^2)) + c^2*(6*e
^(-f*x - e)/((9*a^2*e^(-f*x - e) - 9*I*a^2*e^(-2*f*x - 2*e) - 3*a^2*e^(-3*f
*x - 3*e) + 3*I*a^2)*f) + 2*I/((9*a^2*e^(-f*x - e) - 9*I*a^2*e^(-2*f*x - 2*
e) - 3*a^2*e^(-3*f*x - 3*e) + 3*I*a^2)*f))
```

Fricas [B] time = 2.50375, size = 1162, normalized size = 4.82

$$\frac{-2i d^2e^2 + 4i cdef - 2i c^2f^2 + 4i d^2 - \left(4d^2e^{(3fx+3e)} - 12i d^2e^{(2fx+2e)} - 12d^2e^{(fx+e)} + 4i d^2\right)\mathrm{Li}_2\left(-i e^{(fx+e)}\right) + 2\left(d^2f^2x^2 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] (-2*I*d^2*e^2 + 4*I*c*d*e*f - 2*I*c^2*f^2 + 4*I*d^2 - (4*d^2*e^(3*f*x + 3*e
) - 12*I*d^2*e^(2*f*x + 2*e) - 12*d^2*e^(f*x + e) + 4*I*d^2)*dilog(-I*e^(f*
x + e)) + 2*(d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*e^(3*f*x + 3*
e) + (-6*I*d^2*f^2*x^2 + 6*I*d^2*e^2 - 4*I*d^2 + (-12*I*c*d*e - 4*I*c*d)*f
+ (-12*I*c*d*f^2 - 4*I*d^2*f)*x)*e^(2*f*x + 2*e) + 2*(3*d^2*e^2 + 3*c^2*f^2
 - 2*d^2*f*x - 4*d^2 - 2*(3*c*d*e + c*d)*f)*e^(f*x + e) + (4*I*d^2*e - 4*I*
c*d*f + 4*(d^2*e - c*d*f)*e^(3*f*x + 3*e) + (-12*I*d^2*e + 12*I*c*d*f)*e^(2
*f*x + 2*e) - 12*(d^2*e - c*d*f)*e^(f*x + e))*log(e^(f*x + e) - I) + (-4*I*
d^2*f*x - 4*I*d^2*e - 4*(d^2*f*x + d^2*e)*e^(3*f*x + 3*e) + (12*I*d^2*f*x +
 12*I*d^2*e)*e^(2*f*x + 2*e) + 12*(d^2*f*x + d^2*e)*e^(f*x + e))*log(I*e^(f
*x + e) + 1))/(3*a^2*f^3*e^(3*f*x + 3*e) - 9*I*a^2*f^3*e^(2*f*x + 2*e) - 9*
a^2*f^3*e^(f*x + e) + 3*I*a^2*f^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+I*a*sinh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{\left(i\,a\sinh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(I*a*sinh(f*x + e) + a)^2, x)
```

3.115 $\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx$

Optimal. Leaf size=158

$$\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3a^2f}+\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{6a^2f}+\frac{d\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{6a^2f^2}-\frac{2d\log\left(\cosh\right.}{3a}$$

```
[Out] (-2*d*Log[Cosh[e/2 + (I/4)*Pi + (f*x)/2]])/(3*a^2*f^2) + (d*Sech[e/2 + (I/4
)*Pi + (f*x)/2]^2)/(6*a^2*f^2) + ((c + d*x)*Tanh[e/2 + (I/4)*Pi + (f*x)/2])
/(3*a^2*f) + ((c + d*x)*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2*Tanh[e/2 + (I/4)*P
i + (f*x)/2])/(6*a^2*f)
```

Rubi [A] time = 0.109139, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3318, 4185, 4184, 3475}

$$\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{3a^2f}+\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{6a^2f}+\frac{d\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)}{6a^2f^2}-\frac{2d\log\left(\cosh\right.}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + I*a*Sinh[e + f*x])^2,x]
```


```
[Out] (-2*d*Log[Cosh[e/2 + (I/4)*Pi + (f*x)/2]])/(3*a^2*f^2) + (d*Sech[e/2 + (I/4
)*Pi + (f*x)/2]^2)/(6*a^2*f^2) + ((c + d*x)*Tanh[e/2 + (I/4)*Pi + (f*x)/2])
/(3*a^2*f) + ((c + d*x)*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2*Tanh[e/2 + (I/4)*P
i + (f*x)/2])/(6*a^2*f)
```


Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{c+dx}{(a+ia\sinh(e+fx))^2}\,dx = \frac{\int (c+dx)\csc^4\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)dx}{4a^2}$$

$$= \frac{d\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6a^2f^2} + \frac{(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6a^2f} - \frac{\int (c+dx)c}{}$$

$$= \frac{d\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6a^2f^2} + \frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{ta}}{6a^2f}$$

$$= -\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{3a^2f^2} + \frac{d\text{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6a^2f^2} + \frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2f}$$

Mathematica [A] time = 1.08094, size = 241, normalized size = 1.53

$$\frac{\left(\sinh\left(\frac{1}{2}(e+fx)\right)-i\cosh\left(\frac{1}{2}(e+fx)\right)\right)\left(\cosh\left(\frac{3}{2}(e+fx)\right)\left(2cf+2d\tan^{-1}\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right)-id\log(\cosh(e+fx)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] (((-I)*Cosh[(e + f*x)/2] + Sinh[(e + f*x)/2])*(d*Cosh[(e + f*x)/2]*(-2*I +
3*e + 3*f*x - 6*ArcTan[Tanh[(e + f*x)/2]] + (3*I)*Log[Cosh[e + f*x]]) + Cos
h[(3*(e + f*x))/2]*(-(d*e) + 2*c*f + d*f*x + 2*d*ArcTan[Tanh[(e + f*x)/2]]
- I*d*Log[Cosh[e + f*x]]) + (2*I)*((-I)*d + 2*d*e - 3*c*f - d*f*x - 4*d*Arc
Tan[Tanh[(e + f*x)/2]] + d*Cosh[e + f*x]*(e + f*x - 2*ArcTan[Tanh[(e + f*x)
/2]] + I*Log[Cosh[e + f*x]]) + (2*I)*d*Log[Cosh[e + f*x]])*Sinh[(e + f*x)/2
]))/(6*a^2*f^2*(-I + Sinh[e + f*x])^2)
```

Maple [A] time = 0.103, size = 113, normalized size = 0.7

$$\frac{2\,dx}{3\,a^2f}+\frac{2\,de}{3\,f^2a^2}-\frac{\frac{2i}{3}\left(3\,ifdx\mathrm{e}^{fx+e}+3\,ifc\mathrm{e}^{fx+e}-id\mathrm{e}^{fx+e}+dfx+d\mathrm{e}^{2\,fx+2\,e}+cf\right)}{\left(\mathrm{e}^{fx+e}-i\right)^3f^2a^2}-\frac{2\,d\ln\left(\mathrm{e}^{fx+e}-i\right)}{3\,f^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+I*a*sinh(f*x+e))^2,x)
```

```
[Out] 2/3*d/f/a^2*x+2/3*d/f^2/a^2*e-2/3*I*(3*I*f*d*x*exp(f*x+e)+3*I*f*c*exp(f*x+e
)-I*d*exp(f*x+e)+d*f*x+d*exp(2*f*x+2*e)+c*f)/(exp(f*x+e)-I)^3/f^2/a^2-2/3*d
/f^2/a^2*ln(exp(f*x+e)-I)
```

Maxima [B] time = 1.19648, size = 346, normalized size = 2.19

$$\frac{1}{3}d\left(\frac{3\left(2\,fxe^{(3\,fx+3\,e)}+\left(-6i\,fxe^{(2\,e)}-2i\,e^{(2\,e)}\right)e^{(2\,fx)}-2\,e^{(fx+e)}\right)}{3\,a^2f^2e^{(3\,fx+3\,e)}-9i\,a^2f^2e^{(2\,fx+2\,e)}-9\,a^2f^2e^{(fx+e)}+3i\,a^2f^2}-\frac{2\,\log\left(-i\left(i\,e^{(fx+e)}+1\right)e^{(-e)}\right)}{a^2f^2}\right)+c\left(\frac{}{\left(9\,a^2e^{(-fx-e)}-9\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*d*(3*(2*f*x*e^(3*f*x + 3*e) + (-6*I*f*x*e^(2*e) - 2*I*e^(2*e))*e^(2*f*x
) - 2*e^(f*x + e))/(3*a^2*f^2*e^(3*f*x + 3*e) - 9*I*a^2*f^2*e^(2*f*x + 2*e)
 - 9*a^2*f^2*e^(f*x + e) + 3*I*a^2*f^2) - 2*log(-I*(I*e^(f*x + e) + 1)*e^(-
e))/(a^2*f^2)) + c*(6*e^(-f*x - e)/((9*a^2*e^(-f*x - e) - 9*I*a^2*e^(-2*f*x
 - 2*e) - 3*a^2*e^(-3*f*x - 3*e) + 3*I*a^2)*f) + 2*I/((9*a^2*e^(-f*x - e) -
 9*I*a^2*e^(-2*f*x - 2*e) - 3*a^2*e^(-3*f*x - 3*e) + 3*I*a^2)*f))
```

Fricas [A] time = 2.53945, size = 398, normalized size = 2.52

$$\frac{2\,dfxe^{\left(3\,fx+3\,e\right)} - 2i\,cf + \left(-6i\,dfx - 2i\,d\right)e^{\left(2\,fx+2\,e\right)} + 2\left(3\,cf - d\right)e^{\left(fx+e\right)} - \left(2\,de^{\left(3\,fx+3\,e\right)} - 6i\,de^{\left(2\,fx+2\,e\right)} - 6\,de^{\left(fx+e\right)} + 2i\,d\right)}{3\,a^2f^2e^{\left(3\,fx+3\,e\right)} - 9i\,a^2f^2e^{\left(2\,fx+2\,e\right)} - 9\,a^2f^2e^{\left(fx+e\right)} + 3i\,a^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] (2*d*f*x*e^(3*f*x + 3*e) - 2*I*c*f + (-6*I*d*f*x - 2*I*d)*e^(2*f*x + 2*e) +
 2*(3*c*f - d)*e^(f*x + e) - (2*d*e^(3*f*x + 3*e) - 6*I*d*e^(2*f*x + 2*e) -
 6*d*e^(f*x + e) + 2*I*d)*log(e^(f*x + e) - I))/(3*a^2*f^2*e^(3*f*x + 3*e)
- 9*I*a^2*f^2*e^(2*f*x + 2*e) - 9*a^2*f^2*e^(f*x + e) + 3*I*a^2*f^2)
```

Sympy [A] time = 8.02081, size = 173, normalized size = 1.09

$$\frac{\frac{2ide^{e}e^{-2fx}}{3a^2f^2} - \frac{2ice^{3e}+2idxe^{3e}}{3a^2f} - \frac{\left(6cfe^{2e}+6dfxe^{2e}+2de^{2e}\right)e^{-fx}}{3a^2f^2}}{-ie^{3e} - 3e^{2e}e^{-fx} + 3ie^{e}e^{-2fx} + e^{-3fx}} - \frac{2dx}{3a^2f} - \frac{2d\log\left(ie^{e} + e^{-fx}\right)}{3a^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*sinh(f*x+e))**2,x)
```

```
[Out] (2*I*d*exp(e)*exp(-2*f*x)/(3*a**2*f**2) - (2*I*c*exp(3*e) + 2*I*d*x*exp(3*e
))/(3*a**2*f) - (6*c*f*exp(2*e) + 6*d*f*x*exp(2*e) + 2*d*exp(2*e))*exp(-f*x
)/(3*a**2*f**2))/(-I*exp(3*e) - 3*exp(2*e)*exp(-f*x) + 3*I*exp(e)*exp(-2*f*
x) + exp(-3*f*x)) - 2*d*x/(3*a**2*f) - 2*d*log(I*exp(e) + exp(-f*x))/(3*a**
2*f**2)
```

Giac [A] time = 1.32547, size = 285, normalized size = 1.8

$$\frac{2\,dfxe^{\left(3\,fx+3\,e\right)} - 6i\,dfxe^{\left(2\,fx+2\,e\right)} + 6\,cfe^{\left(fx+e\right)} - 2\,de^{\left(3\,fx+3\,e\right)}\log\left(e^{\left(fx+e\right)} - i\right) + 6i\,de^{\left(2\,fx+2\,e\right)}\log\left(e^{\left(fx+e\right)} - i\right) + 6\,de^{\left(fx+e\right)}\,\mathrm{l}}{3\,a^2f^2e^{\left(3\,fx+3\,e\right)} - 9i\,a^2f^2e^{\left(2\,fx+2\,e\right)} - 9\,a^2f^2e^{\left(fx+e\right)} + 3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] (2*d*f*x*e^(3*f*x + 3*e) - 6*I*d*f*x*e^(2*f*x + 2*e) + 6*c*f*e^(f*x + e) -
2*d*e^(3*f*x + 3*e)*log(e^(f*x + e) - I) + 6*I*d*e^(2*f*x + 2*e)*log(e^(f*x
 + e) - I) + 6*d*e^(f*x + e)*log(e^(f*x + e) - I) - 2*I*c*f - 2*I*d*e^(2*f*
x + 2*e) - 2*d*e^(f*x + e) - 2*I*d*log(e^(f*x + e) - I))/(3*a^2*f^2*e^(3*f*
x + 3*e) - 9*I*a^2*f^2*e^(2*f*x + 2*e) - 9*a^2*f^2*e^(f*x + e) + 3*I*a^2*f^
2)
```

3.116 $\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{1}{(c+dx)(a+ia\sinh(e+fx))^2},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)*(a + I*a*Sinh[e + f*x])^2), x]
```

Rubi [A] time = 0.0604981, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + I*a*Sinh[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + I*a*Sinh[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx=\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx$$

Mathematica [A] time = 36.1759, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a+ia\sinh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + I*a*Sinh[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)*(a + I*a*Sinh[e + f*x])^2), x]
```

Maple [A] time = 0.823, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(a+ia\sinh\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{-2i\,d^2f^2x^2 - 4i\,cdf^2x - 2i\,c^2f^2 + 4i\,d^2 + \left(2i\,d^2fxe^{(2e)} + \cdots\right.}{3i\,a^2d^3f^3x^3 + 9i\,a^2cd^2f^3x^2 + 9i\,a^2c^2df^3x + 3i\,a^2c^3f^3 + 3\left(a^2d^3f^3x^3e^{(3e)} + 3\,a^2cd^2f^3x^2e^{(3e)} + 3\,a^2c^2df^3xe^{(3e)} + a^2c^3f^3 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (-2*I*d^2*f^2*x^2 - 4*I*c*d*f^2*x - 2*I*c^2*f^2 + 4*I*d^2 + (2*I*d^2*f*x*e^
(2*e) + 2*I*c*d*f*e^(2*e) - 4*I*d^2*e^(2*e))*e^(2*f*x) + 2*(3*d^2*f^2*x^2*e
^e + 3*c^2*f^2*e^e + c*d*f*e^e - 4*d^2*e^e + (6*c*d*f^2*e^e + d^2*f*e^e)*x)
*e^(f*x))/(3*I*a^2*d^3*f^3*x^3 + 9*I*a^2*c*d^2*f^3*x^2 + 9*I*a^2*c^2*d*f^3*
x + 3*I*a^2*c^3*f^3 + 3*(a^2*d^3*f^3*x^3*e^(3*e) + 3*a^2*c*d^2*f^3*x^2*e^(3
*e) + 3*a^2*c^2*d*f^3*x*e^(3*e) + a^2*c^3*f^3*e^(3*e))*e^(3*f*x) + (-9*I*a^
2*d^3*f^3*x^3*e^(2*e) - 27*I*a^2*c*d^2*f^3*x^2*e^(2*e) - 27*I*a^2*c^2*d*f^3
*x*e^(2*e) - 9*I*a^2*c^3*f^3*e^(2*e))*e^(2*f*x) - 9*(a^2*d^3*f^3*x^3*e^e +
3*a^2*c*d^2*f^3*x^2*e^e + 3*a^2*c^2*d*f^3*x*e^e + a^2*c^3*f^3*e^e)*e^(f*x))
 - integrate(2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 6*d^3)/(3*a^2*d^4
*f^3*x^4 + 12*a^2*c*d^3*f^3*x^3 + 18*a^2*c^2*d^2*f^3*x^2 + 12*a^2*c^3*d*f^3
*x + 3*a^2*c^4*f^3 + (3*I*a^2*d^4*f^3*x^4*e^e + 12*I*a^2*c*d^3*f^3*x^3*e^e
+ 18*I*a^2*c^2*d^2*f^3*x^2*e^e + 12*I*a^2*c^3*d*f^3*x*e^e + 3*I*a^2*c^4*f^3
*e^e)*e^(f*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] (-2*I*d^2*f^2*x^2 - 4*I*c*d*f^2*x - 2*I*c^2*f^2 + 4*I*d^2 + (2*I*d^2*f*x +
2*I*c*d*f - 4*I*d^2)*e^(2*f*x + 2*e) + 2*(3*d^2*f^2*x^2 + 3*c^2*f^2 + c*d*f
 - 4*d^2 + (6*c*d*f^2 + d^2*f)*x)*e^(f*x + e) + (3*I*a^2*d^3*f^3*x^3 + 9*I*
a^2*c*d^2*f^3*x^2 + 9*I*a^2*c^2*d*f^3*x + 3*I*a^2*c^3*f^3 + 3*(a^2*d^3*f^3*
x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e^(3*f*x + 3*e
) + (-9*I*a^2*d^3*f^3*x^3 - 27*I*a^2*c*d^2*f^3*x^2 - 27*I*a^2*c^2*d*f^3*x -
 9*I*a^2*c^3*f^3)*e^(2*f*x + 2*e) - 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^
2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e^(f*x + e))*integral((2*I*d^3*f^2*x^2
 + 4*I*c*d^2*f^2*x + 2*I*c^2*d*f^2 - 12*I*d^3)/(-3*I*a^2*d^4*f^3*x^4 - 12*I
*a^2*c*d^3*f^3*x^3 - 18*I*a^2*c^2*d^2*f^3*x^2 - 12*I*a^2*c^3*d*f^3*x - 3*I*
a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*
x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^(f*x + e)), x))/(3*I*a^2*d^3*f^3*x
^3 + 9*I*a^2*c*d^2*f^3*x^2 + 9*I*a^2*c^2*d*f^3*x + 3*I*a^2*c^3*f^3 + 3*(a^2
*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e^(3*
f*x + 3*e) + (-9*I*a^2*d^3*f^3*x^3 - 27*I*a^2*c*d^2*f^3*x^2 - 27*I*a^2*c^2*
d*f^3*x - 9*I*a^2*c^3*f^3)*e^(2*f*x + 2*e) - 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d
^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e^(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(ia\sinh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(I*a*sinh(f*x + e) + a)^2), x)
```

3.117 $\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])^2), x]
```

Rubi [A] time = 0.0574974, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx$$

Mathematica [A] time = 37.9065, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a+ia\sinh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + I*a*Sinh[e + f*x])^2), x]
```

Maple [A] time = 1.103, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(a+ia\sinh\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (-2*I*d^2*f^2*x^2 - 4*I*c*d*f^2*x - 2*I*c^2*f^2 + 12*I*d^2 + (4*I*d^2*f*x*e
^(2*e) + 4*I*c*d*f*e^(2*e) - 12*I*d^2*e^(2*e))*e^(2*f*x) + 2*(3*d^2*f^2*x^2
*e^e + 3*c^2*f^2*e^e + 2*c*d*f*e^e - 12*d^2*e^e + 2*(3*c*d*f^2*e^e + d^2*f*
e^e)*x)*e^(f*x))/(3*I*a^2*d^4*f^3*x^4 + 12*I*a^2*c*d^3*f^3*x^3 + 18*I*a^2*c
^2*d^2*f^3*x^2 + 12*I*a^2*c^3*d*f^3*x + 3*I*a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^
4*e^(3*e) + 4*a^2*c*d^3*f^3*x^3*e^(3*e) + 6*a^2*c^2*d^2*f^3*x^2*e^(3*e) + 4
*a^2*c^3*d*f^3*x*e^(3*e) + a^2*c^4*f^3*e^(3*e))*e^(3*f*x) + (-9*I*a^2*d^4*f
^3*x^4*e^(2*e) - 36*I*a^2*c*d^3*f^3*x^3*e^(2*e) - 54*I*a^2*c^2*d^2*f^3*x^2*
e^(2*e) - 36*I*a^2*c^3*d*f^3*x*e^(2*e) - 9*I*a^2*c^4*f^3*e^(2*e))*e^(2*f*x)
 - 9*(a^2*d^4*f^3*x^4*e^e + 4*a^2*c*d^3*f^3*x^3*e^e + 6*a^2*c^2*d^2*f^3*x^2
*e^e + 4*a^2*c^3*d*f^3*x*e^e + a^2*c^4*f^3*e^e)*e^(f*x)) - integrate(4*(d^3
*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 12*d^3)/(3*a^2*d^5*f^3*x^5 + 15*a^2*
c*d^4*f^3*x^4 + 30*a^2*c^2*d^3*f^3*x^3 + 30*a^2*c^3*d^2*f^3*x^2 + 15*a^2*c^
4*d*f^3*x + 3*a^2*c^5*f^3 + (3*I*a^2*d^5*f^3*x^5*e^e + 15*I*a^2*c*d^4*f^3*x
^4*e^e + 30*I*a^2*c^2*d^3*f^3*x^3*e^e + 30*I*a^2*c^3*d^2*f^3*x^2*e^e + 15*I
*a^2*c^4*d*f^3*x*e^e + 3*I*a^2*c^5*f^3*e^e)*e^(f*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] (-2*I*d^2*f^2*x^2 - 4*I*c*d*f^2*x - 2*I*c^2*f^2 + 12*I*d^2 + (4*I*d^2*f*x +
 4*I*c*d*f - 12*I*d^2)*e^(2*f*x + 2*e) + 2*(3*d^2*f^2*x^2 + 3*c^2*f^2 + 2*c
*d*f - 12*d^2 + 2*(3*c*d*f^2 + d^2*f)*x)*e^(f*x + e) + (3*I*a^2*d^4*f^3*x^4
 + 12*I*a^2*c*d^3*f^3*x^3 + 18*I*a^2*c^2*d^2*f^3*x^2 + 12*I*a^2*c^3*d*f^3*x
 + 3*I*a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d
^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^(3*f*x + 3*e) + (-9*I*a^2*d
^4*f^3*x^4 - 36*I*a^2*c*d^3*f^3*x^3 - 54*I*a^2*c^2*d^2*f^3*x^2 - 36*I*a^2*c
^3*d*f^3*x - 9*I*a^2*c^4*f^3)*e^(2*f*x + 2*e) - 9*(a^2*d^4*f^3*x^4 + 4*a^2*
c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^
(f*x + e))*integral((4*I*d^3*f^2*x^2 + 8*I*c*d^2*f^2*x + 4*I*c^2*d*f^2 - 48
*I*d^3)/(-3*I*a^2*d^5*f^3*x^5 - 15*I*a^2*c*d^4*f^3*x^4 - 30*I*a^2*c^2*d^3*f
^3*x^3 - 30*I*a^2*c^3*d^2*f^3*x^2 - 15*I*a^2*c^4*d*f^3*x - 3*I*a^2*c^5*f^3
+ 3*(a^2*d^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3*x^3 + 10*a^
2*c^3*d^2*f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3)*e^(f*x + e)), x))/(3*I
*a^2*d^4*f^3*x^4 + 12*I*a^2*c*d^3*f^3*x^3 + 18*I*a^2*c^2*d^2*f^3*x^2 + 12*I
*a^2*c^3*d*f^3*x + 3*I*a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x
^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^(3*f*x + 3*
e) + (-9*I*a^2*d^4*f^3*x^4 - 36*I*a^2*c*d^3*f^3*x^3 - 54*I*a^2*c^2*d^2*f^3*
x^2 - 36*I*a^2*c^3*d*f^3*x - 9*I*a^2*c^4*f^3)*e^(2*f*x + 2*e) - 9*(a^2*d^4*
f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x +
 a^2*c^4*f^3)*e^(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+I*a*sinh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(i\,a\sinh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(I*a*sinh(f*x + e) + a)^2), x)
```

3.118 $\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx$

Optimal. Leaf size=181

$$-\frac{16x^3\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{96x^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{f^3}-\frac{384x\sqrt{a+ia\sinh(e+fx)}}{f^4}+\frac{768\tanh\left(\frac{e}{2}\right.}{}$$

```
[Out] (-384*x*Sqrt[a + I*a*Sinh[e + f*x]])/f^4 - (16*x^3*Sqrt[a + I*a*Sinh[e + f*
x]])/f^2 + (768*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])
/f^5 + (96*x^2*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/
f^3 + (2*x^4*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/f
```

Rubi [A] time = 0.212391, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3319, 3296, 2638}

$$-\frac{16x^3\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{96x^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{f^3}-\frac{384x\sqrt{a+ia\sinh(e+fx)}}{f^4}+\frac{768\tanh\left(\frac{e}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] (-384*x*Sqrt[a + I*a*Sinh[e + f*x]])/f^4 - (16*x^3*Sqrt[a + I*a*Sinh[e + f*
x]])/f^2 + (768*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])
/f^5 + (96*x^2*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/
f^3 + (2*x^4*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/f
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^4\sqrt{a+ia\sinh(e+fx)}\,dx = \left(\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int x^4\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)dx$$

$$= \frac{2x^4\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f} - \frac{\left(8\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+\right.}{f}$$

$$= -\frac{16x^3\sqrt{a+ia\sinh(e+fx)}}{f^2} + \frac{2x^4\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f} - \frac{(48i\operatorname{csc}}{}$$

$$= -\frac{16x^3\sqrt{a+ia\sinh(e+fx)}}{f^2} + \frac{96x^2\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f^3} + \frac{2x^4\sqrt{a}}{}$$

$$= -\frac{384x\sqrt{a+ia\sinh(e+fx)}}{f^4} - \frac{16x^3\sqrt{a+ia\sinh(e+fx)}}{f^2} + \frac{96x^2\sqrt{a+ia\sinh(e+fx)}}{f^3}$$

$$= -\frac{384x\sqrt{a+ia\sinh(e+fx)}}{f^4} - \frac{16x^3\sqrt{a+ia\sinh(e+fx)}}{f^2} + \frac{768\sqrt{a+ia\sinh(e+fx)}\,\mathrm{t}}{f^5}$$

Mathematica [A] time = 0.23611, size = 141, normalized size = 0.78

$$\frac{2\sqrt{a+ia\sinh(e+fx)}\left(\left(f^4x^4-8if^3x^3+48f^2x^2-192ifx+384\right)\sinh\left(\frac{1}{2}(e+fx)\right)+i\left(f^4x^4+8if^3x^3+48f^2x^2+192\right.\right.}{f^5\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] (2*(I*(384 + (192*I)*f*x + 48*f^2*x^2 + (8*I)*f^3*x^3 + f^4*x^4)*Cosh[(e +
f*x)/2] + (384 - (192*I)*f*x + 48*f^2*x^2 - (8*I)*f^3*x^3 + f^4*x^4)*Sinh[(
e + f*x)/2])*Sqrt[a + I*a*Sinh[e + f*x]])/(f^5*(Cosh[(e + f*x)/2] + I*Sinh[
(e + f*x)/2]))
```

Maple [A] time = 0.087, size = 174, normalized size = 1.

$$\frac{i\sqrt{2}\left(ix^4f^4+f^4x^4e^{fx+e}+8\,ix^3f^3-8\,f^3x^3e^{fx+e}+48\,ix^2f^2+48\,f^2x^2e^{fx+e}+192\,ixf-192\,fxe^{fx+e}+384\,i+384\,e^{fx+}\right.}{\left(ie^{2\,fx+2\,e}-i+2\,e^{fx+e}\right)f^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+I*a*sinh(f*x+e))^(1/2),x)
```

```
[Out] I*2^(1/2)*(a*(I*exp(2*f*x+2*e)-I+2*exp(f*x+e))*exp(-f*x-e))^(1/2)/(I*exp(2*
f*x+2*e)-I+2*exp(f*x+e))*(I*x^4*f^4+f^4*x^4*exp(f*x+e)+8*I*x^3*f^3-8*f^3*x^
3*exp(f*x+e)+48*I*x^2*f^2+48*f^2*x^2*exp(f*x+e)+192*I*x*f-192*f*x*exp(f*x+e
)+384*I+384*exp(f*x+e))*(exp(f*x+e)-I)/f^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i\,a\sinh\left(fx+e\right)+a}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)*x^4, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \sqrt{a \left(i \sinh \left(e + fx\right) + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+I*a*sinh(f*x+e))**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(a*(I*sinh(e + f*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i\, a \sinh \left(fx + e\right) + a} x^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)*x^4, x)
```

3.119 $\int x^3\sqrt{a+ia\sinh(e+fx)}\,dx$

Optimal. Leaf size=136

$$-\frac{12x^2\sqrt{a+ia\sinh(e+fx)}}{f^2}-\frac{96\sqrt{a+ia\sinh(e+fx)}}{f^4}+\frac{48x\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{f^3}+\frac{2x^3\tanh\left(\frac{e}{2}\right.}{}$$

```
[Out] (-96*Sqrt[a + I*a*Sinh[e + f*x]])/f^4 - (12*x^2*Sqrt[a + I*a*Sinh[e + f*x]]
)/f^2 + (48*x*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/f
^3 + (2*x^3*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/f
```

Rubi [A] time = 0.172217, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3319, 3296, 2638}

$$-\frac{12x^2\sqrt{a+ia\sinh(e+fx)}}{f^2}-\frac{96\sqrt{a+ia\sinh(e+fx)}}{f^4}+\frac{48x\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{f^3}+\frac{2x^3\tanh\left(\frac{e}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + I*a*Sinh[e + f*x]],x]
```



```
[Out] (-96*Sqrt[a + I*a*Sinh[e + f*x]])/f^4 - (12*x^2*Sqrt[a + I*a*Sinh[e + f*x]]
)/f^2 + (48*x*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/f
^3 + (2*x^3*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/f
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^3\sqrt{a+ia\sinh(e+fx)}\,dx = \left(\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int x^3\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)dx$$

$$= \frac{2x^3\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f} - \frac{\left(6\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+f\right.}{f}$$

$$= -\frac{12x^2\sqrt{a+ia\sinh(e+fx)}}{f^2} + \frac{2x^3\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f} - \frac{\left(24i\operatorname{csch}\left(\right.\right.}{}$$

$$= -\frac{12x^2\sqrt{a+ia\sinh(e+fx)}}{f^2} + \frac{48x\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f^3} + \frac{2x^3\sqrt{a+i}}{}$$

$$= -\frac{96\sqrt{a+ia\sinh(e+fx)}}{f^4} - \frac{12x^2\sqrt{a+ia\sinh(e+fx)}}{f^2} + \frac{48x\sqrt{a+ia\sinh(e+fx)}\tanh\left(\right.}{f^3}$$

Mathematica [A] time = 0.300401, size = 125, normalized size = 0.92

$$\frac{2\sqrt{a+ia\sinh(e+fx)}\left(\left(f^3x^3-6if^2x^2+24fx-48i\right)\sinh\left(\frac{1}{2}(e+fx)\right)+i\left(f^3x^3+6if^2x^2+24fx+48i\right)\cosh\left(\frac{1}{2}(e+fx)\right.\right.}{f^4\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] (2*(I*(48*I + 24*f*x + (6*I)*f^2*x^2 + f^3*x^3)*Cosh[(e + f*x)/2] + (-48*I
+ 24*f*x - (6*I)*f^2*x^2 + f^3*x^3)*Sinh[(e + f*x)/2])*Sqrt[a + I*a*Sinh[e
+ f*x]])/(f^4*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))
```

Maple [A] time = 0.062, size = 151, normalized size = 1.1

$$\frac{i\sqrt{2}\left(ix^3f^3+f^3x^3e^{fx+e}+6\,ix^2f^2-6\,f^2x^2e^{fx+e}+24\,ixf+24\,fxe^{fx+e}+48\,i-48\,e^{fx+e}\right)\left(e^{fx+e}-i\right)}{\left(ie^{2\,fx+2\,e}-i+2\,e^{fx+e}\right)f^4}\sqrt{a\left(ie^{2\,fx+2\,e}-i+2\,e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+I*a*sinh(f*x+e))^(1/2),x)
```

```
[Out] I*2^(1/2)*(a*(I*exp(2*f*x+2*e)-I+2*exp(f*x+e))*exp(-f*x-e))^(1/2)/(I*exp(2*
f*x+2*e)-I+2*exp(f*x+e))*(I*x^3*f^3+f^3*x^3*exp(f*x+e)+6*I*x^2*f^2-6*f^2*x^
2*exp(f*x+e)+24*I*x*f+24*f*x*exp(f*x+e)+48*I-48*exp(f*x+e))*(exp(f*x+e)-I)/
f^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{i\,a\sinh\left(fx+e\right)+a}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)*x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \sqrt{a \left(i \sinh \left(e+fx\right)+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+I*a*sinh(f*x+e))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*(I*sinh(e + f*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i a \sinh \left(fx+e\right)+a}\, x^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)*x^3, x)
```

3.120 $\int x^2\sqrt{a+ia\sinh(e+fx)}\,dx$

Optimal. Leaf size=111

$$-\frac{8x\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{16\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{f^3}+\frac{2x^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{f}$$

```
[Out] (-8*x*Sqrt[a + I*a*Sinh[e + f*x]])/f^2 + (16*Sqrt[a + I*a*Sinh[e + f*x]]*Ta
nh[e/2 + (I/4)*Pi + (f*x)/2])/f^3 + (2*x^2*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh
[e/2 + (I/4)*Pi + (f*x)/2])/f
```

Rubi [A] time = 0.141345, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3319, 3296, 2638}

$$-\frac{8x\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{16\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{f^3}+\frac{2x^2\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] (-8*x*Sqrt[a + I*a*Sinh[e + f*x]])/f^2 + (16*Sqrt[a + I*a*Sinh[e + f*x]]*Ta
nh[e/2 + (I/4)*Pi + (f*x)/2])/f^3 + (2*x^2*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh
[e/2 + (I/4)*Pi + (f*x)/2])/f
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^2\sqrt{a+ia\sinh(e+fx)}\,dx = \left(\text{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int x^2\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)dx$$
$$= \frac{2x^2\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f} - \frac{\left(4\text{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+\right.}{f}$$
$$= -\frac{8x\sqrt{a+ia\sinh(e+fx)}}{f^2} + \frac{2x^2\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f} - \frac{\left(8i\text{csch}\left(\right.\right.}{}$$
$$= -\frac{8x\sqrt{a+ia\sinh(e+fx)}}{f^2} + \frac{16\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f^3} + \frac{2x^2\sqrt{a+i}}{}$$

Mathematica [A] time = 0.223754, size = 105, normalized size = 0.95

$$\frac{2\sqrt{a+ia\sinh(e+fx)}\left(\left(f^2x^2-4ifx+8\right)\sinh\left(\frac{1}{2}(e+fx)\right)+i\left(f^2x^2+4ifx+8\right)\cosh\left(\frac{1}{2}(e+fx)\right)\right)}{f^3\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] (2*(I*(8 + (4*I)*f*x + f^2*x^2)*Cosh[(e + f*x)/2] + (8 - (4*I)*f*x + f^2*x^
2)*Sinh[(e + f*x)/2])*Sqrt[a + I*a*Sinh[e + f*x]])/(f^3*(Cosh[(e + f*x)/2]
+ I*Sinh[(e + f*x)/2]))
```

Maple [A] time = 0.06, size = 128, normalized size = 1.2

$$\frac{i\sqrt{2}\left(ix^2f^2+f^2x^2e^{fx+e}+4\,ixf-4\,fxe^{fx+e}+8\,i+8\,e^{fx+e}\right)\left(e^{fx+e}-i\right)}{\left(ie^{2\,fx+2\,e}-i+2\,e^{fx+e}\right)f^3}\sqrt{a\left(ie^{2\,fx+2\,e}-i+2\,e^{fx+e}\right)e^{-fx-e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+I*a*sinh(f*x+e))^(1/2),x)
```

```
[Out] I*2^(1/2)*(a*(I*exp(2*f*x+2*e)-I+2*exp(f*x+e))*exp(-f*x-e))^(1/2)/(I*exp(2*
f*x+2*e)-I+2*exp(f*x+e))*(I*x^2*f^2+f^2*x^2*exp(f*x+e)+4*I*x*f-4*f*x*exp(f*
x+e)+8*I+8*exp(f*x+e))*(exp(f*x+e)-I)/f^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{i\,a\sinh\left(fx+e\right)+a}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)*x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{a \left(i \sinh \left(e + f x\right) + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+I*a*sinh(f*x+e))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*(I*sinh(e + f*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i\, a \sinh \left(f x + e\right) + a} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)*x^2, x)
```

3.121 $\int x\sqrt{a+ia\sinh(e+fx)}\,dx$

Optimal. Leaf size=66

$$\frac{2x\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{f}-\frac{4\sqrt{a+ia\sinh(e+fx)}}{f^2}$$

```
[Out] (-4*Sqrt[a + I*a*Sinh[e + f*x]])/f^2 + (2*x*Sqrt[a + I*a*Sinh[e + f*x]]*Tan
h[e/2 + (I/4)*Pi + (f*x)/2])/f
```

Rubi [A] time = 0.0753615, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3319, 3296, 2638}

$$\frac{2x\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{f}-\frac{4\sqrt{a+ia\sinh(e+fx)}}{f^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] (-4*Sqrt[a + I*a*Sinh[e + f*x]])/f^2 + (2*x*Sqrt[a + I*a*Sinh[e + f*x]]*Tan
h[e/2 + (I/4)*Pi + (f*x)/2])/f
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+ia\sinh(e+fx)}\,dx &= \left(\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int x\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)dx\\ &= \frac{2x\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f}-\frac{\left(2\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+ \dots}{f}\\ &= -\frac{4\sqrt{a+ia\sinh(e+fx)}}{f^2}+\frac{2x\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f}\end{aligned}$$

Mathematica [A] time = 0.16385, size = 87, normalized size = 1.32

$$\frac{2\sqrt{a+ia\sinh(e+fx)}\left((fx-2i)\sinh\left(\frac{1}{2}(e+fx)\right)+(-2+ifx)\cosh\left(\frac{1}{2}(e+fx)\right)\right)}{f^2\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] (2*((-2 + I*f*x)*Cosh[(e + f*x)/2] + (-2*I + f*x)*Sinh[(e + f*x)/2])*Sqrt[a
 + I*a*Sinh[e + f*x]])/(f^2*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))
```

Maple [A] time = 0.057, size = 105, normalized size = 1.6

$$\frac{i\sqrt{2}\left(ixf+fx\mathrm{e}^{fx+e}+2i-2\,\mathrm{e}^{fx+e}\right)\left(\mathrm{e}^{fx+e}-i\right)}{\left(i\mathrm{e}^{2\,fx+2\,e}-i+2\,\mathrm{e}^{fx+e}\right)f^2}\sqrt{a\left(i\mathrm{e}^{2\,fx+2\,e}-i+2\,\mathrm{e}^{fx+e}\right)\mathrm{e}^{-fx-e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+I*a*sinh(f*x+e))^(1/2),x)
```

```
[Out] I*2^(1/2)*(a*(I*exp(2*f*x+2*e)-I+2*exp(f*x+e))*exp(-f*x-e))^(1/2)/(I*exp(2*
f*x+2*e)-I+2*exp(f*x+e))*(I*x*f+f*x*exp(f*x+e)+2*I-2*exp(f*x+e))*(exp(f*x+e
)-I)/f^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{i\,a\sinh\left(fx+e\right)+a}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a\left(i\sinh\left(e+fx\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(f*x+e))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*(I*sinh(e + f*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i\,a\sinh\left(fx+e\right)+a}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)*x, x)
```

3.122 $\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x}\,dx$

Optimal. Leaf size=125

$$i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\text{Chi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}+i\cosh\left(\frac{1}{4}(2e-i\pi)\right)\text{Shi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\right.$$

```
[Out] I*CoshIntegral[(f*x)/2]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[(2*e - I*Pi)/4]
*Sqrt[a + I*a*Sinh[e + f*x]] + I*Cosh[(2*e - I*Pi)/4]*Sech[e/2 + (I/4)*Pi +
 (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*SinhIntegral[(f*x)/2]
```

Rubi [A] time = 0.145539, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3319, 3303, 3298, 3301}

$$i\sinh\left(\frac{1}{4}(2e-i\pi)\right)\text{Chi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}+i\cosh\left(\frac{1}{4}(2e-i\pi)\right)\text{Shi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Sinh[e + f*x]]/x,x]
```

```
[Out] I*CoshIntegral[(f*x)/2]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[(2*e - I*Pi)/4]
*Sqrt[a + I*a*Sinh[e + f*x]] + I*Cosh[(2*e - I*Pi)/4]*Sech[e/2 + (I/4)*Pi +
 (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*SinhIntegral[(f*x)/2]
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x}\,dx = \left(\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int \frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)}{x}\,dx$$
$$= \left(\cosh\left(\frac{1}{4}(2e-i\pi)\right)\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int \frac{\sinh\left(\frac{fx}{2}\right)}{x}\,dx + \left(\operatorname{csch}\right.$$
$$= i\text{Chi}\left(\frac{fx}{2}\right)\operatorname{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e-i\pi)\right)\sqrt{a+ia\sinh(e+fx)} + i\cosh\left(\frac{1}{4}(2e\right.$$

Mathematica [A] time = 0.158604, size = 96, normalized size = 0.77

$$\frac{\sqrt{a+ia\sinh(e+fx)}\left(\text{Chi}\left(\frac{fx}{2}\right)\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)+\left(\sinh\left(\frac{e}{2}\right)+i\cosh\left(\frac{e}{2}\right)\right)\text{Shi}\left(\frac{fx}{2}\right)\right)}{\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Sinh[e + f*x]]/x,x]
```

```
[Out] (Sqrt[a + I*a*Sinh[e + f*x]]*(CoshIntegral[(f*x)/2]*(Cosh[e/2] + I*Sinh[e/2
]) + (I*Cosh[e/2] + Sinh[e/2])*SinhIntegral[(f*x)/2]))/(Cosh[(e + f*x)/2] +
 I*Sinh[(e + f*x)/2])
```

Maple [F] time = 0.305, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt{a+ia\sinh\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))^(1/2)/x,x)
```

```
[Out] int((a+I*a*sinh(f*x+e))^(1/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{i\,a\sinh\left(fx+e\right)+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(i\sinh\left(e+fx\right)+1\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a*(I*sinh(e + f*x) + 1))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{i\,a\sinh\left(fx+e\right)+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)/x, x)
```

3.123 $\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^2}\,dx$

Optimal. Leaf size=149

$$\frac{1}{2}f\sinh\left(\frac{1}{4}(2e+i\pi)\right)\operatorname{Chi}\left(\frac{fx}{2}\right)\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}+\frac{1}{2}f\cosh\left(\frac{1}{4}(2e+i\pi)\right)\operatorname{Shi}\left(\frac{fx}{2}\right)\operatorname{sech}\left(\frac{e}{2}\right.$$

```
[Out] -(Sqrt[a + I*a*Sinh[e + f*x]]/x) + (f*CoshIntegral[(f*x)/2]*Sech[e/2 + (I/4
)*Pi + (f*x)/2]*Sinh[(2*e + I*Pi)/4]*Sqrt[a + I*a*Sinh[e + f*x]])/2 + (f*Co
sh[(2*e + I*Pi)/4]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x
]]*SinhIntegral[(f*x)/2])/2
```

Rubi [A] time = 0.175202, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3319, 3297, 3303, 3298, 3301}

$$\frac{1}{2}f\sinh\left(\frac{1}{4}(2e+i\pi)\right)\operatorname{Chi}\left(\frac{fx}{2}\right)\operatorname{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}+\frac{1}{2}f\cosh\left(\frac{1}{4}(2e+i\pi)\right)\operatorname{Shi}\left(\frac{fx}{2}\right)\operatorname{sech}\left(\frac{e}{2}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Sinh[e + f*x]]/x^2,x]
```

```
[Out] -(Sqrt[a + I*a*Sinh[e + f*x]]/x) + (f*CoshIntegral[(f*x)/2]*Sech[e/2 + (I/4
)*Pi + (f*x)/2]*Sinh[(2*e + I*Pi)/4]*Sqrt[a + I*a*Sinh[e + f*x]])/2 + (f*Co
sh[(2*e + I*Pi)/4]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x
]]*SinhIntegral[(f*x)/2])/2
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^2}dx = \left(\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int \frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)}{x^2}dx$$

$$= -\frac{\sqrt{a+ia\sinh(e+fx)}}{x}+\frac{1}{2}\left(f\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int \frac{\cosh\left(\frac{e}{2}-\frac{i\pi}{4}\right.}{x}$$

$$= -\frac{\sqrt{a+ia\sinh(e+fx)}}{x}-\frac{1}{2}\left(if\cosh\left(\frac{1}{4}(2e+i\pi)\right)\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+f\right.$$

$$= -\frac{\sqrt{a+ia\sinh(e+fx)}}{x}+\frac{1}{2}f\operatorname{Chi}\left(\frac{fx}{2}\right)\operatorname{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e+i\pi)\right)\sqrt{a+ia\sin}$$

Mathematica [A] time = 0.25353, size = 133, normalized size = 0.89

$$\frac{\sqrt{a+ia\sinh(e+fx)}\left(fx\operatorname{Chi}\left(\frac{fx}{2}\right)\left(\sinh\left(\frac{e}{2}\right)+i\cosh\left(\frac{e}{2}\right)\right)+fx\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)\operatorname{Shi}\left(\frac{fx}{2}\right)-2\left(\cosh\left(\frac{1}{2}(e+fx)\right)+\right.\right.}{2x\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Sinh[e + f*x]]/x^2,x]
```

```
[Out] (Sqrt[a + I*a*Sinh[e + f*x]]*(f*x*CoshIntegral[(f*x)/2]*(I*Cosh[e/2] + Sinh
[e/2]) - 2*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]) + f*x*(Cosh[e/2] + I*S
inh[e/2])*SinhIntegral[(f*x)/2]))/(2*x*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x
)/2]))
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt{a+ia\sinh\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))^(1/2)/x^2,x)
```

```
[Out] int((a+I*a*sinh(f*x+e))^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{i\,a\sinh\left(fx+e\right)+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)/x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(i\sinh\left(e+fx\right)+1\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a*(I*sinh(e + f*x) + 1))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{i\,a\sinh\left(fx+e\right)+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)/x^2, x)
```

3.124 $\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^3}\,dx$

Optimal. Leaf size=204

$$\frac{1}{8}if^2\sinh\left(\frac{1}{4}(2e-i\pi)\right)\text{Chi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}+\frac{1}{8}if^2\cosh\left(\frac{1}{4}(2e-i\pi)\right)\text{Shi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}\right.$$

```
[Out] -Sqrt[a + I*a*Sinh[e + f*x]]/(2*x^2) + (I/8)*f^2*CoshIntegral[(f*x)/2]*Sech
[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[(2*e - I*Pi)/4]*Sqrt[a + I*a*Sinh[e + f*x]]
 + (I/8)*f^2*Cosh[(2*e - I*Pi)/4]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I
*a*Sinh[e + f*x]]*SinhIntegral[(f*x)/2] - (f*Sqrt[a + I*a*Sinh[e + f*x]]*Ta
nh[e/2 + (I/4)*Pi + (f*x)/2])/(4*x)
```

Rubi [A] time = 0.198161, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3319, 3297, 3303, 3298, 3301}

$$\frac{1}{8}if^2\sinh\left(\frac{1}{4}(2e-i\pi)\right)\text{Chi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}+\frac{1}{8}if^2\cosh\left(\frac{1}{4}(2e-i\pi)\right)\text{Shi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Sinh[e + f*x]]/x^3,x]
```

```
[Out] -Sqrt[a + I*a*Sinh[e + f*x]]/(2*x^2) + (I/8)*f^2*CoshIntegral[(f*x)/2]*Sech
[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[(2*e - I*Pi)/4]*Sqrt[a + I*a*Sinh[e + f*x]]
 + (I/8)*f^2*Cosh[(2*e - I*Pi)/4]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I
*a*Sinh[e + f*x]]*SinhIntegral[(f*x)/2] - (f*Sqrt[a + I*a*Sinh[e + f*x]]*Ta
nh[e/2 + (I/4)*Pi + (f*x)/2])/(4*x)
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\sinh(e+fx)}}{x^3}\,dx = \left(\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int \frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)}{x^3}\,dx$$

$$= -\frac{\sqrt{a+ia\sinh(e+fx)}}{2x^2}+\frac{1}{4}\left(f\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int \frac{\cosh\left(\frac{e}{2}-\right.}{x^2}$$

$$= -\frac{\sqrt{a+ia\sinh(e+fx)}}{2x^2}-\frac{f\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{4x}-\frac{1}{8}\left(if^2\operatorname{csch}\left(\frac{e}{2}-\right.\right.$$

$$= -\frac{\sqrt{a+ia\sinh(e+fx)}}{2x^2}-\frac{f\sqrt{a+ia\sinh(e+fx)}\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{4x}+\frac{1}{8}\left(f^2\cosh\left(\frac{1}{4}(2\right.\right.$$

$$= -\frac{\sqrt{a+ia\sinh(e+fx)}}{2x^2}+\frac{1}{8}if^2\operatorname{Chi}\left(\frac{fx}{2}\right)\operatorname{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e-i\pi)\right)\sqrt{a+i}$$

Mathematica [A] time = 0.339556, size = 170, normalized size = 0.83

$$\frac{\sqrt{a+ia\sinh(e+fx)}\left(f^2x^2\operatorname{Chi}\left(\frac{fx}{2}\right)\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)+f^2x^2\left(\sinh\left(\frac{e}{2}\right)+i\cosh\left(\frac{e}{2}\right)\right)\operatorname{Shi}\left(\frac{fx}{2}\right)-2fx\sinh\left(\frac{1}{2}(e\right.\right.}{8x^2\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Sinh[e + f*x]]/x^3,x]
```

```
[Out] (Sqrt[a + I*a*Sinh[e + f*x]]*(-4*Cosh[(e + f*x)/2] - (2*I)*f*x*Cosh[(e + f*
x)/2] + f^2*x^2*CoshIntegral[(f*x)/2]*(Cosh[e/2] + I*Sinh[e/2]) - (4*I)*Sin
h[(e + f*x)/2] - 2*f*x*Sinh[(e + f*x)/2] + f^2*x^2*(I*Cosh[e/2] + Sinh[e/2]
)*SinhIntegral[(f*x)/2]))/(8*x^2*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{x^3}\sqrt{a+ia\sinh\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))^(1/2)/x^3,x)
```

```
[Out] int((a+I*a*sinh(f*x+e))^(1/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{i a \sinh \left(f x+e\right)+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)/x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a \left(i \sinh \left(e+f x\right)+1\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a*(I*sinh(e + f*x) + 1))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{i a \sinh \left(f x+e\right)+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)/x^3, x)
```

3.125 $\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx$

Optimal. Leaf size=377

$$-\frac{16ax^2\sqrt{a+ia\sinh(e+fx)}}{f^2}-\frac{8ax^2\cosh^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{3f^2}-\frac{1280a\sqrt{a+ia\sinh(e+fx)}}{9f^4}+\frac{640ax}{}$$

```
[Out] (-1280*a*Sqrt[a + I*a*Sinh[e + f*x]])/(9*f^4) - (16*a*x^2*Sqrt[a + I*a*Sinh
[e + f*x]])/f^2 - (64*a*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*a*Sinh[
e + f*x]])/(27*f^4) - (8*a*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*
a*Sinh[e + f*x]])/(3*f^2) + (32*a*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[e/2
 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]])/(9*f^3) + (4*a*x^3*Cosh
[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh
[e + f*x]])/(3*f) + (640*a*x*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*P
i + (f*x)/2])/(9*f^3) + (8*a*x^3*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/
4)*Pi + (f*x)/2])/(3*f)
```

Rubi [A] time = 0.351429, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3319, 3311, 3296, 2638, 3310}

$$-\frac{16ax^2\sqrt{a+ia\sinh(e+fx)}}{f^2}-\frac{8ax^2\cosh^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{3f^2}-\frac{1280a\sqrt{a+ia\sinh(e+fx)}}{9f^4}+\frac{640ax}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + I*a*Sinh[e + f*x])^(3/2),x]
```


```
[Out] (-1280*a*Sqrt[a + I*a*Sinh[e + f*x]])/(9*f^4) - (16*a*x^2*Sqrt[a + I*a*Sinh
[e + f*x]])/f^2 - (64*a*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*a*Sinh[
e + f*x]])/(27*f^4) - (8*a*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*
a*Sinh[e + f*x]])/(3*f^2) + (32*a*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[e/2
 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]])/(9*f^3) + (4*a*x^3*Cosh
[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh
[e + f*x]])/(3*f) + (640*a*x*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*P
i + (f*x)/2])/(9*f^3) + (8*a*x^3*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/
4)*Pi + (f*x)/2])/(3*f)
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int x^3(a+ia\sinh(e+fx))^{3/2}\,dx = -\left(\left(2a\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int x^3\sinh^3\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)dx\right)$$

$$= -\frac{8ax^2\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{3f^2}+\frac{4ax^3\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\right.}{3f}$$

$$= -\frac{64a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{27f^4}-\frac{8ax^2\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\,\mathrm{si}}}{3f^2}$$

$$= -\frac{16ax^2\sqrt{a+ia\sinh(e+fx)}}{f^2}-\frac{64a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{27f^4}-\frac{8ax^2\,\mathrm{c}}{}$$

$$= -\frac{128a\sqrt{a+ia\sinh(e+fx)}}{9f^4}-\frac{16ax^2\sqrt{a+ia\sinh(e+fx)}}{f^2}-\frac{64a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{}}{27f^4}$$

$$= -\frac{1280a\sqrt{a+ia\sinh(e+fx)}}{9f^4}-\frac{16ax^2\sqrt{a+ia\sinh(e+fx)}}{f^2}-\frac{64a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{27f^4}$$

Mathematica [A] time = 1.40348, size = 269, normalized size = 0.71

$$-\frac{a(\sinh(e+fx)-i)\sqrt{a+ia\sinh(e+fx)}\left(-81if^3x^3\sinh\left(\frac{1}{2}(e+fx)\right)+9if^3x^3\sinh\left(\frac{3}{2}(e+fx)\right)-486f^2x^2\sinh\left(\frac{1}{2}(e+f\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] -(a*(-I + Sinh[e + f*x])*Sqrt[a + I*a*Sinh[e + f*x]]*(81*(48*I + 24*f*x + (
6*I)*f^2*x^2 + f^3*x^3)*Cosh[(e + f*x)/2] + (-16*I + 24*f*x - (18*I)*f^2*x^
2 + 9*f^3*x^3)*Cosh[(3*(e + f*x))/2] - 3888*Sinh[(e + f*x)/2] - (1944*I)*f*
x*Sinh[(e + f*x)/2] - 486*f^2*x^2*Sinh[(e + f*x)/2] - (81*I)*f^3*x^3*Sinh[(
e + f*x)/2] - 16*Sinh[(3*(e + f*x))/2] + (24*I)*f*x*Sinh[(3*(e + f*x))/2] -
 18*f^2*x^2*Sinh[(3*(e + f*x))/2] + (9*I)*f^3*x^3*Sinh[(3*(e + f*x))/2]))/(
27*f^4*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^3)
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int x^{3} \left(a+ia\sinh\left(fx+e\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+I*a*sinh(f*x+e))^(3/2),x)

[Out] int(x^3*(a+I*a*sinh(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(i\,a\sinh\left(fx+e\right)+a\right)^{\frac{3}{2}} x^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)*x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+I*a*sinh(f*x+e))**(3/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(i\,a\sinh\left(fx+e\right)+a\right)^{\frac{3}{2}} x^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)*x^3, x)
```

3.126 $\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx$

Optimal. Leaf size=303

$$-\frac{32ax\sqrt{a+ia\sinh(e+fx)}}{3f^2}+\frac{32a\sinh^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{27f^3}+\frac{224a\tanh\left(\frac{e}{2}+\frac{fx}{2}\right.}{}$$

```
[Out] (-32*a*x*Sqrt[a + I*a*Sinh[e + f*x]])/(3*f^2) - (16*a*x*Cosh[e/2 + (I/4)*Pi
 + (f*x)/2]^2*Sqrt[a + I*a*Sinh[e + f*x]])/(9*f^2) + (4*a*x^2*Cosh[e/2 + (I
/4)*Pi + (f*x)/2]*Sinh[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]
])/(3*f) + (224*a*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2
])/(9*f^3) + (8*a*x^2*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*
x)/2])/(3*f) + (32*a*Sinh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*a*Sinh[e +
 f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(27*f^3)
```

Rubi [A] time = 0.25098, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3319, 3311, 3296, 2638, 2633}

$$-\frac{32ax\sqrt{a+ia\sinh(e+fx)}}{3f^2}+\frac{32a\sinh^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{27f^3}+\frac{224a\tanh\left(\frac{e}{2}+\frac{fx}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + I*a*Sinh[e + f*x])^(3/2),x]
```


```
[Out] (-32*a*x*Sqrt[a + I*a*Sinh[e + f*x]])/(3*f^2) - (16*a*x*Cosh[e/2 + (I/4)*Pi
 + (f*x)/2]^2*Sqrt[a + I*a*Sinh[e + f*x]])/(9*f^2) + (4*a*x^2*Cosh[e/2 + (I
/4)*Pi + (f*x)/2]*Sinh[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]
])/(3*f) + (224*a*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2
])/(9*f^3) + (8*a*x^2*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*
x)/2])/(3*f) + (32*a*Sinh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*a*Sinh[e +
 f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(27*f^3)
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int x^2(a+ia\sinh(e+fx))^{3/2}\,dx = -\left(\left(2a\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int x^2\sinh^3\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)dx\right)$$

$$= -\frac{16ax\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^2} + \frac{4ax^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\right.}{3f}$$

$$= -\frac{16ax\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^2} + \frac{4ax^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\right.}{3f}$$

$$= -\frac{32ax\sqrt{a+ia\sinh(e+fx)}}{3f^2} - \frac{16ax\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^2} + \frac{4ax^2}{}$$

$$= -\frac{32ax\sqrt{a+ia\sinh(e+fx)}}{3f^2} - \frac{16ax\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^2} + \frac{4ax^2}{}$$

Mathematica [A] time = 1.1252, size = 173, normalized size = 0.57

$$-\frac{a(\sinh(e+fx)-i)\sqrt{a+ia\sinh(e+fx)}\left(81\left(f^2x^2+4ifx+8\right)\cosh\left(\frac{1}{2}(e+fx)\right)+\left(9f^2x^2-12ifx+8\right)\cosh\left(\frac{3}{2}(e+fx)\right)\right.}{27f^3\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] -(a*(81*(8 + (4*I)*f*x + f^2*x^2)*Cosh[(e + f*x)/2] + (8 - (12*I)*f*x + 9*f
^2*x^2)*Cosh[(3*(e + f*x))/2] + (2*I)*(-4*(80 - (42*I)*f*x + 9*f^2*x^2) + (
8 + (12*I)*f*x + 9*f^2*x^2)*Cosh[e + f*x])*Sinh[(e + f*x)/2])*(-I + Sinh[e
+ f*x])*Sqrt[a + I*a*Sinh[e + f*x]])/(27*f^3*(Cosh[(e + f*x)/2] + I*Sinh[(e
 + f*x)/2])^3)
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int x^2\left(a+ia\sinh\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+I*a*sinh(f*x+e))^(3/2),x)
```

```
[Out] int(x^2*(a+I*a*sinh(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(i a \sinh \left(f x + e\right) + a\right)^{\frac{3}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)*x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+I*a*sinh(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(i a \sinh \left(f x + e\right) + a\right)^{\frac{3}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)*x^2, x)
```

3.127 $\int x(a+ia\sinh(e+fx))^{3/2}\,dx$

Optimal. Leaf size=185

$$-\frac{16a\sqrt{a+ia\sinh(e+fx)}}{3f^2}-\frac{8a\cosh^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^2}+\frac{8ax\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx}}{3f}$$

```
[Out] (-16*a*Sqrt[a + I*a*Sinh[e + f*x]])/(3*f^2) - (8*a*Cosh[e/2 + (I/4)*Pi + (f
*x)/2]^2*Sqrt[a + I*a*Sinh[e + f*x]])/(9*f^2) + (4*a*x*Cosh[e/2 + (I/4)*Pi
+ (f*x)/2]*Sinh[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]])/(3*f
) + (8*a*x*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*f
)
```

Rubi [A] time = 0.133372, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3319, 3310, 3296, 2638}

$$-\frac{16a\sqrt{a+ia\sinh(e+fx)}}{3f^2}-\frac{8a\cosh^2\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^2}+\frac{8ax\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] (-16*a*Sqrt[a + I*a*Sinh[e + f*x]])/(3*f^2) - (8*a*Cosh[e/2 + (I/4)*Pi + (f
*x)/2]^2*Sqrt[a + I*a*Sinh[e + f*x]])/(9*f^2) + (4*a*x*Cosh[e/2 + (I/4)*Pi
+ (f*x)/2]*Sinh[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]])/(3*f
) + (8*a*x*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*f
)
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x(a+ia\sinh(e+fx))^{3/2}\,dx = -\left(\left(2a\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int x\sinh^3\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)dx\right)$$
$$= -\frac{8a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^2}+\frac{4ax\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\right.}{3f}$$
$$= -\frac{8a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^2}+\frac{4ax\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\right.}{3f}$$
$$= -\frac{16a\sqrt{a+ia\sinh(e+fx)}}{3f^2}-\frac{8a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{9f^2}+\frac{4ax\cos}{}$$

Mathematica [A] time = 0.776093, size = 138, normalized size = 0.75

$$-\frac{a(\sinh(e+fx)-i)\sqrt{a+ia\sinh(e+fx)}\left(27(fx+2i)\cosh\left(\frac{1}{2}(e+fx)\right)+(3fx-2i)\cosh\left(\frac{3}{2}(e+fx)\right)+2i\sinh\left(\frac{1}{2}(e\right.\right.}{9f^2\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] -(a*(27*(2*I + f*x)*Cosh[(e + f*x)/2] + (-2*I + 3*f*x)*Cosh[(3*(e + f*x))/2
] + (2*I)*(28*I - 12*f*x + (2*I + 3*f*x)*Cosh[e + f*x])*Sinh[(e + f*x)/2])*
(-I + Sinh[e + f*x])*Sqrt[a + I*a*Sinh[e + f*x]])/(9*f^2*(Cosh[(e + f*x)/2]
 + I*Sinh[(e + f*x)/2])^3)
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int x\left(a+ia\sinh\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+I*a*sinh(f*x+e))^(3/2),x)
```

```
[Out] int(x*(a+I*a*sinh(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(i\,a\sinh\left(fx+e\right)+a\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(f*x+e))**(3/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(i\,a\sinh\left(fx+e\right)+a\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)*x, x)
```

3.128 $\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x}\,dx$

Optimal. Leaf size=261

$$\frac{3}{2}ia\sinh\left(\frac{1}{4}(2e-i\pi)\right)\text{Chi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}+\frac{1}{2}ia\sinh\left(\frac{1}{4}(6e+i\pi)\right)\text{Chi}\left(\frac{3fx}{2}\right)\text{sech}$$

```
[Out] ((3*I)/2)*a*CoshIntegral[(f*x)/2]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[(2*e
- I*Pi)/4]*Sqrt[a + I*a*Sinh[e + f*x]] + (I/2)*a*CoshIntegral[(3*f*x)/2]*Se
ch[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[(6*e + I*Pi)/4]*Sqrt[a + I*a*Sinh[e + f*x
]] + ((3*I)/2)*a*Cosh[(2*e - I*Pi)/4]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a
 + I*a*Sinh[e + f*x]]*SinhIntegral[(f*x)/2] + (I/2)*a*Cosh[(6*e + I*Pi)/4]*
Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*SinhIntegral[(3*
f*x)/2]
```

Rubi [A] time = 0.273379, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3319, 3312, 3303, 3298, 3301}

$$\frac{3}{2}ia\sinh\left(\frac{1}{4}(2e-i\pi)\right)\text{Chi}\left(\frac{fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}+\frac{1}{2}ia\sinh\left(\frac{1}{4}(6e+i\pi)\right)\text{Chi}\left(\frac{3fx}{2}\right)\text{sech}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[e + f*x])^(3/2)/x,x]
```


```
[Out] ((3*I)/2)*a*CoshIntegral[(f*x)/2]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[(2*e
- I*Pi)/4]*Sqrt[a + I*a*Sinh[e + f*x]] + (I/2)*a*CoshIntegral[(3*f*x)/2]*Se
ch[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[(6*e + I*Pi)/4]*Sqrt[a + I*a*Sinh[e + f*x
]] + ((3*I)/2)*a*Cosh[(2*e - I*Pi)/4]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a
 + I*a*Sinh[e + f*x]]*SinhIntegral[(f*x)/2] + (I/2)*a*Cosh[(6*e + I*Pi)/4]*
Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*SinhIntegral[(3*
f*x)/2]
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x}dx = -\left(\left(2a\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int \frac{\sinh^3\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)}{x}dx\right)$$

$$= -\left(\left(2ia\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int \left(\frac{3i\sinh\left(\frac{1}{4}(2e-i\pi)+\frac{fx}{2}\right)}{4x}+\frac{i\sinh\right.$$

$$= \frac{1}{2}\left(a\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int \frac{\sinh\left(\frac{1}{4}(6e+i\pi)+\frac{3fx}{2}\right)}{x}dx+\frac{1}{2}\left(3ac\right.$$

$$= \frac{1}{2}\left(3a\cosh\left(\frac{1}{4}(2e-i\pi)\right)\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int \frac{\sinh\left(\frac{fx}{2}\right)}{x}dx+\frac{1}{2}$$

$$= \frac{3}{2}ia\text{Chi}\left(\frac{fx}{2}\right)\operatorname{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sinh\left(\frac{1}{4}(2e-i\pi)\right)\sqrt{a+ia\sinh(e+fx)}+\frac{1}{2}ia\text{Chi}\left(\frac{3}{}\right.$$

Mathematica [A] time = 0.784664, size = 146, normalized size = 0.56

$$\frac{a\sqrt{a+ia\sinh(e+fx)}\left(3\text{Chi}\left(\frac{fx}{2}\right)\left(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right)\right)-\text{Chi}\left(\frac{3fx}{2}\right)\left(\cosh\left(\frac{3e}{2}\right)-i\sinh\left(\frac{3e}{2}\right)\right)+\left(\sinh\left(\frac{e}{2}\right)+i\cosh\left(\frac{e}{2}\right)\right.}{2\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[e + f*x])^(3/2)/x,x]
```

```
[Out] (a*Sqrt[a + I*a*Sinh[e + f*x]]*(3*CoshIntegral[(f*x)/2]*(Cosh[e/2] + I*Sinh
[e/2]) - CoshIntegral[(3*f*x)/2]*(Cosh[(3*e)/2] - I*Sinh[(3*e)/2]) + (I*Cos
h[e/2] + Sinh[e/2])*(3*SinhIntegral[(f*x)/2] + (1 + (2*I)*Sinh[e])*SinhInte
gral[(3*f*x)/2])))/(2*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+ia\sinh\left(fx+e\right)\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))^(3/2)/x,x)
```

```
[Out] int((a+I*a*sinh(f*x+e))^(3/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ia\sinh\left(fx+e\right)+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ia\sinh\left(fx+e\right)+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)/x, x)
```

3.129 $\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x^2}\,dx$

Optimal. Leaf size=302

$$-\frac{3}{4}af\sinh\left(\frac{1}{4}(6e-i\pi)\right)\mathrm{Chi}\left(\frac{3fx}{2}\right)\mathrm{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}+\frac{3}{4}af\sinh\left(\frac{1}{4}(2e+i\pi)\right)\mathrm{Chi}\left(\frac{fx}{2}\right)\mathrm{sech}\left(\right.$$

```
[Out] (-2*a*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*a*Sinh[e + f*x]])/x - (3*
a*f*CoshIntegral[(3*f*x)/2]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[(6*e - I*Pi
)/4]*Sqrt[a + I*a*Sinh[e + f*x]])/4 + (3*a*f*CoshIntegral[(f*x)/2]*Sech[e/2
 + (I/4)*Pi + (f*x)/2]*Sinh[(2*e + I*Pi)/4]*Sqrt[a + I*a*Sinh[e + f*x]])/4
+ (3*a*f*Cosh[(2*e + I*Pi)/4]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*S
inh[e + f*x]]*SinhIntegral[(f*x)/2])/4 - (3*a*f*Cosh[(6*e - I*Pi)/4]*Sech[e
/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*SinhIntegral[(3*f*x)/2
])/4
```

Rubi [A] time = 0.289703, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3319, 3313, 3303, 3298, 3301}

$$-\frac{3}{4}af\sinh\left(\frac{1}{4}(6e-i\pi)\right)\mathrm{Chi}\left(\frac{3fx}{2}\right)\mathrm{sech}\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(e+fx)}+\frac{3}{4}af\sinh\left(\frac{1}{4}(2e+i\pi)\right)\mathrm{Chi}\left(\frac{fx}{2}\right)\mathrm{sech}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[e + f*x])^(3/2)/x^2,x]
```

```
[Out] (-2*a*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*a*Sinh[e + f*x]])/x - (3*
a*f*CoshIntegral[(3*f*x)/2]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[(6*e - I*Pi
)/4]*Sqrt[a + I*a*Sinh[e + f*x]])/4 + (3*a*f*CoshIntegral[(f*x)/2]*Sech[e/2
 + (I/4)*Pi + (f*x)/2]*Sinh[(2*e + I*Pi)/4]*Sqrt[a + I*a*Sinh[e + f*x]])/4
+ (3*a*f*Cosh[(2*e + I*Pi)/4]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*S
inh[e + f*x]]*SinhIntegral[(f*x)/2])/4 - (3*a*f*Cosh[(6*e - I*Pi)/4]*Sech[e
/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*SinhIntegral[(3*f*x)/2
])/4
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(a+ia\sinh(e+fx))^{3/2}}{x^2}\,dx = -\left(\left(2a\text{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}\right)\int \frac{\sinh^3\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)}{x^2}\,dx\right)$$

$$= -\frac{2a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{x} + \left(3af\text{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\,\text{si}}\right.$$

$$= -\frac{2a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{x} + \frac{1}{4}\left(3af\text{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia}\right.$$

$$= -\frac{2a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{x} + \frac{1}{4}\left(3iaf\cosh\left(\frac{1}{4}(6e-i\pi)\right)\text{csch}\left(\frac{e}{2}\right.\right.$$

$$= -\frac{2a\cosh^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\sqrt{a+ia\sinh(e+fx)}}{x} - \frac{3}{4}af\text{Chi}\left(\frac{3fx}{2}\right)\text{sech}\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right.$$

Mathematica [A] time = 0.973868, size = 243, normalized size = 0.8

$$\frac{a(\sinh(e+fx)-i)\sqrt{a+ia\sinh(e+fx)}\left(-3fx\text{Chi}\left(\frac{fx}{2}\right)\left(\cosh\left(\frac{e}{2}\right)-i\sinh\left(\frac{e}{2}\right)\right)-3fx\text{Chi}\left(\frac{3fx}{2}\right)\left(\cosh\left(\frac{3e}{2}\right)+i\sinh\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[e + f*x])^(3/2)/x^2,x]
```

```
[Out] (a*(-I + Sinh[e + f*x])*Sqrt[a + I*a*Sinh[e + f*x]]*((-6*I)*Cosh[(e + f*x)/
2] + (2*I)*Cosh[(3*(e + f*x))/2] - 3*f*x*CoshIntegral[(f*x)/2]*(Cosh[e/2] -
 I*Sinh[e/2]) - 3*f*x*CoshIntegral[(3*f*x)/2]*(Cosh[(3*e)/2] + I*Sinh[(3*e)
/2]) + 6*Sinh[(e + f*x)/2] + 2*Sinh[(3*(e + f*x))/2] + (3*I)*f*x*Cosh[e/2]*
SinhIntegral[(f*x)/2] - 3*f*x*Sinh[e/2]*SinhIntegral[(f*x)/2] - (3*I)*f*x*C
osh[(3*e)/2]*SinhIntegral[(3*f*x)/2] - 3*f*x*Sinh[(3*e)/2]*SinhIntegral[(3*
f*x)/2]))/(4*x*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^3)
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a + ia\sinh\left(fx+e\right)\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))^(3/2)/x^2,x)
```

```
[Out] int((a+I*a*sinh(f*x+e))^(3/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(i\,a\sinh\left(fx+e\right)+a\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)/x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(i\,a\sinh\left(fx+e\right)+a\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)/x^2, x)
```

3.130 $\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx$

Optimal. Leaf size=638

$$-\frac{128a^2x^2\sqrt{a+ia\sinh(c+dx)}}{5d^2}-\frac{48a^2x^2\cosh^4\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{25d^2}-\frac{64a^2x^2\cosh^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia}}{15d^2}$$

```
[Out] (-265216*a^2*Sqrt[a + I*a*Sinh[c + d*x]])/(1125*d^4) - (128*a^2*x^2*Sqrt[a
+ I*a*Sinh[c + d*x]])/(5*d^2) - (17408*a^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^2
*Sqrt[a + I*a*Sinh[c + d*x]])/(3375*d^4) - (64*a^2*x^2*Cosh[c/2 + (I/4)*Pi
+ (d*x)/2]^2*Sqrt[a + I*a*Sinh[c + d*x]])/(15*d^2) - (384*a^2*Cosh[c/2 + (I
/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/(625*d^4) - (48*a^2*x^2*Co
sh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/(25*d^2) + (870
4*a^2*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[
a + I*a*Sinh[c + d*x]])/(1125*d^3) + (32*a^2*x^3*Cosh[c/2 + (I/4)*Pi + (d*x
)/2]*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]])/(15*d) + (
192*a^2*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*S
qrt[a + I*a*Sinh[c + d*x]])/(125*d^3) + (8*a^2*x^3*Cosh[c/2 + (I/4)*Pi + (d
*x)/2]^3*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]])/(5*d)
+ (132608*a^2*x*Sqrt[a + I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])
/(1125*d^3) + (64*a^2*x^3*Sqrt[a + I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi +
 (d*x)/2])/(15*d)
```

Rubi [A] time = 0.639757, antiderivative size = 638, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3319, 3311, 3296, 2638, 3310}

$$-\frac{128a^2x^2\sqrt{a+ia\sinh(c+dx)}}{5d^2}-\frac{48a^2x^2\cosh^4\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{25d^2}-\frac{64a^2x^2\cosh^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia}}{15d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + I*a*Sinh[c + d*x])^(5/2),x]
```


```
[Out] (-265216*a^2*Sqrt[a + I*a*Sinh[c + d*x]])/(1125*d^4) - (128*a^2*x^2*Sqrt[a
+ I*a*Sinh[c + d*x]])/(5*d^2) - (17408*a^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^2
*Sqrt[a + I*a*Sinh[c + d*x]])/(3375*d^4) - (64*a^2*x^2*Cosh[c/2 + (I/4)*Pi
+ (d*x)/2]^2*Sqrt[a + I*a*Sinh[c + d*x]])/(15*d^2) - (384*a^2*Cosh[c/2 + (I
/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/(625*d^4) - (48*a^2*x^2*Co
sh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/(25*d^2) + (870
4*a^2*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[
a + I*a*Sinh[c + d*x]])/(1125*d^3) + (32*a^2*x^3*Cosh[c/2 + (I/4)*Pi + (d*x
)/2]*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]])/(15*d) + (
192*a^2*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*S
qrt[a + I*a*Sinh[c + d*x]])/(125*d^3) + (8*a^2*x^3*Cosh[c/2 + (I/4)*Pi + (d
*x)/2]^3*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]])/(5*d)
+ (132608*a^2*x*Sqrt[a + I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])
/(1125*d^3) + (64*a^2*x^3*Sqrt[a + I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi +
 (d*x)/2])/(15*d)
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
```

```
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int x^3(a+ia\sinh(c+dx))^{5/2}\,dx = \left(4a^2\operatorname{csch}\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\right)\int x^3\sinh^5\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)dx$$

$$= -\frac{48a^2x^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{25d^2}+\frac{8a^2x^3\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sin}{}$$

$$= -\frac{64a^2x^2\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{15d^2}-\frac{384a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a}}{625d^4}$$

$$= -\frac{17408a^2\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{3375d^4}-\frac{64a^2x^2\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{15d^2}$$

$$= -\frac{128a^2x^2\sqrt{a+ia\sinh(c+dx)}}{5d^2}-\frac{17408a^2\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{3375d^4}$$

$$= -\frac{34816a^2\sqrt{a+ia\sinh(c+dx)}}{1125d^4}-\frac{128a^2x^2\sqrt{a+ia\sinh(c+dx)}}{5d^2}-\frac{17408a^2\cosh^2\left(\frac{c}{2}\right.}{}$$

$$= -\frac{265216a^2\sqrt{a+ia\sinh(c+dx)}}{1125d^4}-\frac{128a^2x^2\sqrt{a+ia\sinh(c+dx)}}{5d^2}-\frac{17408a^2\cosh^2\left(\right.}{}$$

Mathematica [B] time = 7.43947, size = 2918, normalized size = 4.57

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (2*(((-1/135000 - I/135000)*Cosh[5*(c/2 + (d*x)/2)])/d^3 + ((1/135000 + I/1
35000)*Sinh[5*(c/2 + (d*x)/2)])/d^3)*(1296*I - (3240*I)*c + (4050*I)*c^2 -
(3375*I)*c^3 + (6480*I)*(c/2 + (d*x)/2) - (16200*I)*c*(c/2 + (d*x)/2) + (20
250*I)*c^2*(c/2 + (d*x)/2) + (16200*I)*(c/2 + (d*x)/2)^2 - (40500*I)*c*(c/2
 + (d*x)/2)^2 + (27000*I)*(c/2 + (d*x)/2)^3 - 50000*Cosh[2*(c/2 + (d*x)/2)]
 + 75000*c*Cosh[2*(c/2 + (d*x)/2)] - 56250*c^2*Cosh[2*(c/2 + (d*x)/2)] + 28
125*c^3*Cosh[2*(c/2 + (d*x)/2)] - 150000*(c/2 + (d*x)/2)*Cosh[2*(c/2 + (d*x
)/2)] + 225000*c*(c/2 + (d*x)/2)*Cosh[2*(c/2 + (d*x)/2)] - 168750*c^2*(c/2
+ (d*x)/2)*Cosh[2*(c/2 + (d*x)/2)] - 225000*(c/2 + (d*x)/2)^2*Cosh[2*(c/2 +
 (d*x)/2)] + 337500*c*(c/2 + (d*x)/2)^2*Cosh[2*(c/2 + (d*x)/2)] - 225000*(c
/2 + (d*x)/2)^3*Cosh[2*(c/2 + (d*x)/2)] - (8100000*I)*Cosh[4*(c/2 + (d*x)/2
)] + (4050000*I)*c*Cosh[4*(c/2 + (d*x)/2)] - (1012500*I)*c^2*Cosh[4*(c/2 +
(d*x)/2)] + (168750*I)*c^3*Cosh[4*(c/2 + (d*x)/2)] - (8100000*I)*(c/2 + (d*
x)/2)*Cosh[4*(c/2 + (d*x)/2)] + (4050000*I)*c*(c/2 + (d*x)/2)*Cosh[4*(c/2 +
 (d*x)/2)] - (1012500*I)*c^2*(c/2 + (d*x)/2)*Cosh[4*(c/2 + (d*x)/2)] - (405
0000*I)*(c/2 + (d*x)/2)^2*Cosh[4*(c/2 + (d*x)/2)] + (2025000*I)*c*(c/2 + (d
*x)/2)^2*Cosh[4*(c/2 + (d*x)/2)] - (1350000*I)*(c/2 + (d*x)/2)^3*Cosh[4*(c/
2 + (d*x)/2)] + 8100000*Cosh[6*(c/2 + (d*x)/2)] + 4050000*c*Cosh[6*(c/2 + (
d*x)/2)] + 1012500*c^2*Cosh[6*(c/2 + (d*x)/2)] + 168750*c^3*Cosh[6*(c/2 + (
d*x)/2)] - 8100000*(c/2 + (d*x)/2)*Cosh[6*(c/2 + (d*x)/2)] - 4050000*c*(c/2
 + (d*x)/2)*Cosh[6*(c/2 + (d*x)/2)] - 1012500*c^2*(c/2 + (d*x)/2)*Cosh[6*(c
/2 + (d*x)/2)] + 4050000*(c/2 + (d*x)/2)^2*Cosh[6*(c/2 + (d*x)/2)] + 202500
0*c*(c/2 + (d*x)/2)^2*Cosh[6*(c/2 + (d*x)/2)] - 1350000*(c/2 + (d*x)/2)^3*C
osh[6*(c/2 + (d*x)/2)] + (50000*I)*Cosh[8*(c/2 + (d*x)/2)] + (75000*I)*c*Co
sh[8*(c/2 + (d*x)/2)] + (56250*I)*c^2*Cosh[8*(c/2 + (d*x)/2)] + (28125*I)*c
^3*Cosh[8*(c/2 + (d*x)/2)] - (150000*I)*(c/2 + (d*x)/2)*Cosh[8*(c/2 + (d*x)
/2)] - (225000*I)*c*(c/2 + (d*x)/2)*Cosh[8*(c/2 + (d*x)/2)] - (168750*I)*c^
2*(c/2 + (d*x)/2)*Cosh[8*(c/2 + (d*x)/2)] + (225000*I)*(c/2 + (d*x)/2)^2*Co
sh[8*(c/2 + (d*x)/2)] + (337500*I)*c*(c/2 + (d*x)/2)^2*Cosh[8*(c/2 + (d*x)/
2)] - (225000*I)*(c/2 + (d*x)/2)^3*Cosh[8*(c/2 + (d*x)/2)] - 1296*Cosh[10*(
c/2 + (d*x)/2)] - 3240*c*Cosh[10*(c/2 + (d*x)/2)] - 4050*c^2*Cosh[10*(c/2 +
 (d*x)/2)] - 3375*c^3*Cosh[10*(c/2 + (d*x)/2)] + 6480*(c/2 + (d*x)/2)*Cosh[
10*(c/2 + (d*x)/2)] + 16200*c*(c/2 + (d*x)/2)*Cosh[10*(c/2 + (d*x)/2)] + 20
250*c^2*(c/2 + (d*x)/2)*Cosh[10*(c/2 + (d*x)/2)] - 16200*(c/2 + (d*x)/2)^2*
Cosh[10*(c/2 + (d*x)/2)] - 40500*c*(c/2 + (d*x)/2)^2*Cosh[10*(c/2 + (d*x)/2
)] + 27000*(c/2 + (d*x)/2)^3*Cosh[10*(c/2 + (d*x)/2)] - 50000*Sinh[2*(c/2 +
 (d*x)/2)] + 75000*c*Sinh[2*(c/2 + (d*x)/2)] - 56250*c^2*Sinh[2*(c/2 + (d*x
)/2)] + 28125*c^3*Sinh[2*(c/2 + (d*x)/2)] - 150000*(c/2 + (d*x)/2)*Sinh[2*(
c/2 + (d*x)/2)] + 225000*c*(c/2 + (d*x)/2)*Sinh[2*(c/2 + (d*x)/2)] - 168750
*c^2*(c/2 + (d*x)/2)*Sinh[2*(c/2 + (d*x)/2)] - 225000*(c/2 + (d*x)/2)^2*Sin
h[2*(c/2 + (d*x)/2)] + 337500*c*(c/2 + (d*x)/2)^2*Sinh[2*(c/2 + (d*x)/2)] -
 225000*(c/2 + (d*x)/2)^3*Sinh[2*(c/2 + (d*x)/2)] - (8100000*I)*Sinh[4*(c/2
 + (d*x)/2)] + (4050000*I)*c*Sinh[4*(c/2 + (d*x)/2)] - (1012500*I)*c^2*Sinh
[4*(c/2 + (d*x)/2)] + (168750*I)*c^3*Sinh[4*(c/2 + (d*x)/2)] - (8100000*I)*
(c/2 + (d*x)/2)*Sinh[4*(c/2 + (d*x)/2)] + (4050000*I)*c*(c/2 + (d*x)/2)*Sin
h[4*(c/2 + (d*x)/2)] - (1012500*I)*c^2*(c/2 + (d*x)/2)*Sinh[4*(c/2 + (d*x)/
2)] - (4050000*I)*(c/2 + (d*x)/2)^2*Sinh[4*(c/2 + (d*x)/2)] + (2025000*I)*c
*(c/2 + (d*x)/2)^2*Sinh[4*(c/2 + (d*x)/2)] - (1350000*I)*(c/2 + (d*x)/2)^3*
Sinh[4*(c/2 + (d*x)/2)] + 8100000*Sinh[6*(c/2 + (d*x)/2)] + 4050000*c*Sinh[
6*(c/2 + (d*x)/2)] + 1012500*c^2*Sinh[6*(c/2 + (d*x)/2)] + 168750*c^3*Sinh[
6*(c/2 + (d*x)/2)] - 8100000*(c/2 + (d*x)/2)*Sinh[6*(c/2 + (d*x)/2)] - 4050
000*c*(c/2 + (d*x)/2)*Sinh[6*(c/2 + (d*x)/2)] - 1012500*c^2*(c/2 + (d*x)/2)
*Sinh[6*(c/2 + (d*x)/2)] + 4050000*(c/2 + (d*x)/2)^2*Sinh[6*(c/2 + (d*x)/2)
] + 2025000*c*(c/2 + (d*x)/2)^2*Sinh[6*(c/2 + (d*x)/2)] - 1350000*(c/2 + (d
*x)/2)^3*Sinh[6*(c/2 + (d*x)/2)] + (50000*I)*Sinh[8*(c/2 + (d*x)/2)] + (750
00*I)*c*Sinh[8*(c/2 + (d*x)/2)] + (56250*I)*c^2*Sinh[8*(c/2 + (d*x)/2)] + (
28125*I)*c^3*Sinh[8*(c/2 + (d*x)/2)] - (150000*I)*(c/2 + (d*x)/2)*Sinh[8*(c
```

```
/2 + (d*x)/2)] - (225000*I)*c*(c/2 + (d*x)/2)*Sinh[8*(c/2 + (d*x)/2)] - (16
8750*I)*c^2*(c/2 + (d*x)/2)*Sinh[8*(c/2 + (d*x)/2)] + (225000*I)*(c/2 + (d*
x)/2)^2*Sinh[8*(c/2 + (d*x)/2)] + (337500*I)*c*(c/2 + (d*x)/2)^2*Sinh[8*(c/
2 + (d*x)/2)] - (225000*I)*(c/2 + (d*x)/2)^3*Sinh[8*(c/2 + (d*x)/2)] - 1296
*Sinh[10*(c/2 + (d*x)/2)] - 3240*c*Sinh[10*(c/2 + (d*x)/2)] - 4050*c^2*Sinh
[10*(c/2 + (d*x)/2)] - 3375*c^3*Sinh[10*(c/2 + (d*x)/2)] + 6480*(c/2 + (d*x
)/2)*Sinh[10*(c/2 + (d*x)/2)] + 16200*c*(c/2 + (d*x)/2)*Sinh[10*(c/2 + (d*x
)/2)] + 20250*c^2*(c/2 + (d*x)/2)*Sinh[10*(c/2 + (d*x)/2)] - 16200*(c/2 + (
d*x)/2)^2*Sinh[10*(c/2 + (d*x)/2)] - 40500*c*(c/2 + (d*x)/2)^2*Sinh[10*(c/2
 + (d*x)/2)] + 27000*(c/2 + (d*x)/2)^3*Sinh[10*(c/2 + (d*x)/2)])*(a + I*a*S
inh[c + d*x])^(5/2))/(d*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2])^5)
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int x^3 (a + ia \sinh (dx + c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+I*a*sinh(d*x+c))^(5/2),x)
```

```
[Out] int(x^3*(a+I*a*sinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (i\, a \sinh (dx + c) + a)^{\frac{5}{2}} x^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)*x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\sinh{(dx+c)}+a)^{\frac{5}{2}}x^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)*x^3, x)
```

3.131 $\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx$

Optimal. Leaf size=506

$$-\frac{256a^2x\sqrt{a+ia\sinh(c+dx)}}{15d^2}+\frac{64a^2\sinh^4\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{125d^3}+\frac{2432a^2\sinh^2\left(\right.}{}$$

```
[Out] (-256*a^2*x*Sqrt[a + I*a*Sinh[c + d*x]])/(15*d^2) - (128*a^2*x*Cosh[c/2 + (
I/4)*Pi + (d*x)/2]^2*Sqrt[a + I*a*Sinh[c + d*x]])/(45*d^2) - (32*a^2*x*Cosh
[c/2 + (I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/(25*d^2) + (32*a^
2*x^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a
+ I*a*Sinh[c + d*x]])/(15*d) + (8*a^2*x^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3*
Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]])/(5*d) + (9536*a
^2*Sqrt[a + I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(225*d^3) +
(64*a^2*x^2*Sqrt[a + I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(15
*d) + (2432*a^2*Sinh[c/2 + (I/4)*Pi + (d*x)/2]^2*Sqrt[a + I*a*Sinh[c + d*x]
]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(675*d^3) + (64*a^2*Sinh[c/2 + (I/4)*Pi +
 (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(12
5*d^3)
```

Rubi [A] time = 0.389703, antiderivative size = 506, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3319, 3311, 3296, 2638, 2633}

$$-\frac{256a^2x\sqrt{a+ia\sinh(c+dx)}}{15d^2}+\frac{64a^2\sinh^4\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{125d^3}+\frac{2432a^2\sinh^2\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + I*a*Sinh[c + d*x])^(5/2),x]
```


```
[Out] (-256*a^2*x*Sqrt[a + I*a*Sinh[c + d*x]])/(15*d^2) - (128*a^2*x*Cosh[c/2 + (
I/4)*Pi + (d*x)/2]^2*Sqrt[a + I*a*Sinh[c + d*x]])/(45*d^2) - (32*a^2*x*Cosh
[c/2 + (I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/(25*d^2) + (32*a^
2*x^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a
+ I*a*Sinh[c + d*x]])/(15*d) + (8*a^2*x^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3*
Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]])/(5*d) + (9536*a
^2*Sqrt[a + I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(225*d^3) +
(64*a^2*x^2*Sqrt[a + I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(15
*d) + (2432*a^2*Sinh[c/2 + (I/4)*Pi + (d*x)/2]^2*Sqrt[a + I*a*Sinh[c + d*x]
]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(675*d^3) + (64*a^2*Sinh[c/2 + (I/4)*Pi +
 (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(12
5*d^3)
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
```

```
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int x^2(a+ia\sinh(c+dx))^{5/2}\,dx = \left(4a^2\operatorname{csch}\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\right)\int x^2\sinh^5\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)dx$$

$$= -\frac{32a^2x\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{25d^2}+\frac{8a^2x^2\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}\right.}{5d}$$

$$= -\frac{128a^2x\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{45d^2}-\frac{32a^2x\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+i}}{25d^2}$$

$$= -\frac{128a^2x\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{45d^2}-\frac{32a^2x\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+i}}{25d^2}$$

$$= -\frac{256a^2x\sqrt{a+ia\sinh(c+dx)}}{15d^2}-\frac{128a^2x\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{45d^2}-\frac{32}{}$$

$$= -\frac{256a^2x\sqrt{a+ia\sinh(c+dx)}}{15d^2}-\frac{128a^2x\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{45d^2}-\frac{32}{}$$

Mathematica [A] time = 1.80694, size = 300, normalized size = 0.59

$$\frac{a^2\sqrt{a+ia\sinh(c+dx)}\left(33750d^2x^2\sinh\left(\frac{1}{2}(c+dx)\right)-5625d^2x^2\sinh\left(\frac{3}{2}(c+dx)\right)-675d^2x^2\sinh\left(\frac{5}{2}(c+dx)\right)-675id^2x^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[a + I*a*Sinh[c + d*x]]*((33750*I)*(8 + (4*I)*d*x + d^2*x^2)*Cosh[
(c + d*x)/2] + 625*(8*I + 12*d*x + (9*I)*d^2*x^2)*Cosh[(3*(c + d*x))/2] - (
216*I)*Cosh[(5*(c + d*x))/2] + 540*d*x*Cosh[(5*(c + d*x))/2] - (675*I)*d^2*
x^2*Cosh[(5*(c + d*x))/2] + 270000*Sinh[(c + d*x)/2] - (135000*I)*d*x*Sinh[
(c + d*x)/2] + 33750*d^2*x^2*Sinh[(c + d*x)/2] - 5000*Sinh[(3*(c + d*x))/2]
```

```
 - (7500*I)*d*x*Sinh[(3*(c + d*x))/2] - 5625*d^2*x^2*Sinh[(3*(c + d*x))/2]
- 216*Sinh[(5*(c + d*x))/2] + (540*I)*d*x*Sinh[(5*(c + d*x))/2] - 675*d^2*x
^2*Sinh[(5*(c + d*x))/2]))/(6750*d^3*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/
2]))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int x^{2} \left(a + ia \sinh \left(dx + c\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+I*a*sinh(d*x+c))^(5/2),x)
```

```
[Out] int(x^2*(a+I*a*sinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(i\, a \sinh \left(dx + c\right) + a\right)^{\frac{5}{2}} x^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)*x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i a \sinh (dx + c) + a)^{\frac{5}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)*x^2, x)
```

3.132 $\int x(a+ia\sinh(c+dx))^{5/2}\,dx$

Optimal. Leaf size=312

$$-\frac{128a^2\sqrt{a+ia\sinh(c+dx)}}{15d^2}-\frac{16a^2\cosh^4\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{25d^2}-\frac{64a^2\cosh^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sin}}{45d^2}$$

```
[Out] (-128*a^2*Sqrt[a + I*a*Sinh[c + d*x]])/(15*d^2) - (64*a^2*Cosh[c/2 + (I/4)*
Pi + (d*x)/2]^2*Sqrt[a + I*a*Sinh[c + d*x]])/(45*d^2) - (16*a^2*Cosh[c/2 +
(I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/(25*d^2) + (32*a^2*x*Cos
h[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sin
h[c + d*x]])/(15*d) + (8*a^2*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3*Sinh[c/2 +
(I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]])/(5*d) + (64*a^2*x*Sqrt[a +
 I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(15*d)
```

Rubi [A] time = 0.208157, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3319, 3310, 3296, 2638}

$$-\frac{128a^2\sqrt{a+ia\sinh(c+dx)}}{15d^2}-\frac{16a^2\cosh^4\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{25d^2}-\frac{64a^2\cosh^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sin}}{45d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + I*a*Sinh[c + d*x])^(5/2),x]
```


```
[Out] (-128*a^2*Sqrt[a + I*a*Sinh[c + d*x]])/(15*d^2) - (64*a^2*Cosh[c/2 + (I/4)*
Pi + (d*x)/2]^2*Sqrt[a + I*a*Sinh[c + d*x]])/(45*d^2) - (16*a^2*Cosh[c/2 +
(I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/(25*d^2) + (32*a^2*x*Cos
h[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sin
h[c + d*x]])/(15*d) + (8*a^2*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3*Sinh[c/2 +
(I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]])/(5*d) + (64*a^2*x*Sqrt[a +
 I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(15*d)
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x(a+ia\sinh(c+dx))^{5/2}\,dx = \left(4a^2\operatorname{csch}\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\right)\int x\sinh^5\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)dx$$
$$= -\frac{16a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{25d^2}+\frac{8a^2x\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\right.}{5d}$$
$$= -\frac{64a^2\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{45d^2}-\frac{16a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sin}}{25d^2}$$
$$= -\frac{64a^2\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{45d^2}-\frac{16a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sin}}{25d^2}$$
$$= -\frac{128a^2\sqrt{a+ia\sinh(c+dx)}}{15d^2}-\frac{64a^2\cosh^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{45d^2}-\frac{16a^2\operatorname{co}}{}$$

Mathematica [A] time = 1.38736, size = 218, normalized size = 0.7

$$\frac{a^2(\sinh(c+dx)-i)^2\sqrt{a+ia\sinh(c+dx)}\left(-2250dx\sinh\left(\frac{1}{2}(c+dx)\right)+4500i\sinh\left(\frac{1}{2}(c+dx)\right)+375dx\sinh\left(\frac{3}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (a^2*(-I + Sinh[c + d*x])^2*Sqrt[a + I*a*Sinh[c + d*x]]*(2250*(2 - I*d*x)*C
osh[(c + d*x)/2] + (-250 - (375*I)*d*x)*Cosh[(3*(c + d*x))/2] - 18*Cosh[(5*
(c + d*x))/2] + (45*I)*d*x*Cosh[(5*(c + d*x))/2] + (4500*I)*Sinh[(c + d*x)/
2] - 2250*d*x*Sinh[(c + d*x)/2] + (250*I)*Sinh[(3*(c + d*x))/2] + 375*d*x*S
inh[(3*(c + d*x))/2] - (18*I)*Sinh[(5*(c + d*x))/2] + 45*d*x*Sinh[(5*(c + d
*x))/2]))/(450*d^2*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^5)
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int x\left(a+ia\sinh(dx+c)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+I*a*sinh(d*x+c))^(5/2),x)
```

```
[Out] int(x*(a+I*a*sinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(i\,a\sinh(dx+c)+a\right)^{\frac{5}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)*x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\sinh(dx+c)+a)^{\frac{5}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)*x, x)
```

3.133 $\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x}\,dx$

Optimal. Leaf size=403

$$-\frac{1}{4}ia^2\sinh\left(\frac{5c}{2}-\frac{i\pi}{4}\right)\mathrm{Chi}\left(\frac{5dx}{2}\right)\mathrm{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}+\frac{5}{2}ia^2\sinh\left(\frac{1}{4}(2c-i\pi)\right)\mathrm{Chi}\left(\frac{dx}{2}\right)\mathrm{sech}\left(\frac{c}{2}\right.$$

```
[Out] (-I/4)*a^2*CoshIntegral[(5*d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(5*c
)/2 - (I/4)*Pi]*Sqrt[a + I*a*Sinh[c + d*x]] + ((5*I)/2)*a^2*CoshIntegral[(d
*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(2*c - I*Pi)/4]*Sqrt[a + I*a*Sin
h[c + d*x]] + ((5*I)/4)*a^2*CoshIntegral[(3*d*x)/2]*Sech[c/2 + (I/4)*Pi + (
d*x)/2]*Sinh[(6*c + I*Pi)/4]*Sqrt[a + I*a*Sinh[c + d*x]] + ((5*I)/2)*a^2*Co
sh[(2*c - I*Pi)/4]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x
]]*SinhIntegral[(d*x)/2] + ((5*I)/4)*a^2*Cosh[(6*c + I*Pi)/4]*Sech[c/2 + (I
/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(3*d*x)/2] - (I/
4)*a^2*Cosh[(5*c)/2 - (I/4)*Pi]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a
*Sinh[c + d*x]]*SinhIntegral[(5*d*x)/2]
```

Rubi [A] time = 0.417715, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3319, 3312, 3303, 3298, 3301}

$$-\frac{1}{4}ia^2\sinh\left(\frac{5c}{2}-\frac{i\pi}{4}\right)\mathrm{Chi}\left(\frac{5dx}{2}\right)\mathrm{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}+\frac{5}{2}ia^2\sinh\left(\frac{1}{4}(2c-i\pi)\right)\mathrm{Chi}\left(\frac{dx}{2}\right)\mathrm{sech}\left(\frac{c}{2}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[c + d*x])^(5/2)/x,x]
```

```
[Out] (-I/4)*a^2*CoshIntegral[(5*d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(5*c
)/2 - (I/4)*Pi]*Sqrt[a + I*a*Sinh[c + d*x]] + ((5*I)/2)*a^2*CoshIntegral[(d
*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(2*c - I*Pi)/4]*Sqrt[a + I*a*Sin
h[c + d*x]] + ((5*I)/4)*a^2*CoshIntegral[(3*d*x)/2]*Sech[c/2 + (I/4)*Pi + (
d*x)/2]*Sinh[(6*c + I*Pi)/4]*Sqrt[a + I*a*Sinh[c + d*x]] + ((5*I)/2)*a^2*Co
sh[(2*c - I*Pi)/4]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x
]]*SinhIntegral[(d*x)/2] + ((5*I)/4)*a^2*Cosh[(6*c + I*Pi)/4]*Sech[c/2 + (I
/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(3*d*x)/2] - (I/
4)*a^2*Cosh[(5*c)/2 - (I/4)*Pi]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a
*Sinh[c + d*x]]*SinhIntegral[(5*d*x)/2]
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x}\,dx = \left(4a^2\operatorname{csch}\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\right)\int \frac{\sinh^5\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)}{x}\,dx$$

$$= -\left(\left(4ia^2\operatorname{csch}\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\right)\int\left(\frac{5i\sinh\left(\frac{1}{4}(2c-i\pi)+\frac{dx}{2}\right)}{8x}+ \cdots\right.\right.$$

$$= -\left(\frac{1}{4}\left(a^2\operatorname{csch}\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\right)\int\frac{\sinh\left(\frac{1}{4}(10c-i\pi)+\frac{5dx}{2}\right)}{x}\,dx\right) + \cdots$$

$$= -\left(\frac{1}{4}\left(a^2\cosh\left(\frac{5c}{2}-\frac{i\pi}{4}\right)\operatorname{csch}\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\right)\int\frac{\sinh\left(\frac{5dx}{2}\right)}{x}\,dx \cdots\right.$$

$$= -\frac{1}{4}ia^2\operatorname{Chi}\left(\frac{5dx}{2}\right)\operatorname{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{5c}{2}-\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}+\frac{5}{2}ia^2\mathrm{Cl}\cdots$$

Mathematica [A] time = 1.49706, size = 242, normalized size = 0.6

$$\frac{a^2(\sinh(c+dx)-i)^2\sqrt{a+ia\sinh(c+dx)}\left(i\sinh\left(\frac{5c}{2}\right)\operatorname{Chi}\left(\frac{5dx}{2}\right)+\cosh\left(\frac{5c}{2}\right)\operatorname{Chi}\left(\frac{5dx}{2}\right)-10\left(\cosh\left(\frac{c}{2}\right)+i\sinh\left(\frac{c}{2}\right)\right)\mathrm{C}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(5/2)/x,x]
```

```
[Out] (a^2*(-I + Sinh[c + d*x])^2*Sqrt[a + I*a*Sinh[c + d*x]]*(Cosh[(5*c)/2]*Cosh
Integral[(5*d*x)/2] - 10*CoshIntegral[(d*x)/2]*(Cosh[c/2] + I*Sinh[c/2]) +
5*CoshIntegral[(3*d*x)/2]*(Cosh[(3*c)/2] - I*Sinh[(3*c)/2]) + I*CoshIntegra
l[(5*d*x)/2]*Sinh[(5*c)/2] - (10*I)*Cosh[c/2]*SinhIntegral[(d*x)/2] - 10*Si
nh[c/2]*SinhIntegral[(d*x)/2] - (5*I)*Cosh[(3*c)/2]*SinhIntegral[(3*d*x)/2]
 + 5*Sinh[(3*c)/2]*SinhIntegral[(3*d*x)/2] + I*Cosh[(5*c)/2]*SinhIntegral[(
5*d*x)/2] + Sinh[(5*c)/2]*SinhIntegral[(5*d*x)/2]))/(4*(Cosh[(c + d*x)/2] +
 I*Sinh[(c + d*x)/2])^5)
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x} (a + ia \sinh (dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(d*x+c))^(5/2)/x,x)
```

```
[Out] int((a+I*a*sinh(d*x+c))^(5/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \sinh (dx + c) + a)^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \sinh (dx + c) + a)^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)/x, x)
```

3.134 $\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^2}\,dx$

Optimal. Leaf size=444

$$-\frac{5}{8}a^2d\sinh\left(\frac{5c}{2}+\frac{i\pi}{4}\right)\text{Chi}\left(\frac{5dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}-\frac{15}{8}a^2d\sinh\left(\frac{1}{4}(6c-i\pi)\right)\text{Chi}\left(\frac{3dx}{2}\right)\text{se}$$

```
[Out] (-4*a^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/x - (
5*a^2*d*CoshIntegral[(5*d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(5*c)/2
 + (I/4)*Pi]*Sqrt[a + I*a*Sinh[c + d*x]])/8 - (15*a^2*d*CoshIntegral[(3*d*x
)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(6*c - I*Pi)/4]*Sqrt[a + I*a*Sinh[
c + d*x]])/8 + (5*a^2*d*CoshIntegral[(d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2
]*Sinh[(2*c + I*Pi)/4]*Sqrt[a + I*a*Sinh[c + d*x]])/4 + (5*a^2*d*Cosh[(2*c
+ I*Pi)/4]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhI
ntegral[(d*x)/2])/4 - (15*a^2*d*Cosh[(6*c - I*Pi)/4]*Sech[c/2 + (I/4)*Pi +
(d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(3*d*x)/2])/8 - (5*a^2*d*
Cosh[(5*c)/2 + (I/4)*Pi]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c
 + d*x]]*SinhIntegral[(5*d*x)/2])/8
```

Rubi [A] time = 0.436698, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3319, 3313, 3303, 3298, 3301}

$$-\frac{5}{8}a^2d\sinh\left(\frac{5c}{2}+\frac{i\pi}{4}\right)\text{Chi}\left(\frac{5dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}-\frac{15}{8}a^2d\sinh\left(\frac{1}{4}(6c-i\pi)\right)\text{Chi}\left(\frac{3dx}{2}\right)\text{se}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[c + d*x])^(5/2)/x^2,x]
```


```
[Out] (-4*a^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/x - (
5*a^2*d*CoshIntegral[(5*d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(5*c)/2
 + (I/4)*Pi]*Sqrt[a + I*a*Sinh[c + d*x]])/8 - (15*a^2*d*CoshIntegral[(3*d*x
)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(6*c - I*Pi)/4]*Sqrt[a + I*a*Sinh[
c + d*x]])/8 + (5*a^2*d*CoshIntegral[(d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2
]*Sinh[(2*c + I*Pi)/4]*Sqrt[a + I*a*Sinh[c + d*x]])/4 + (5*a^2*d*Cosh[(2*c
+ I*Pi)/4]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhI
ntegral[(d*x)/2])/4 - (15*a^2*d*Cosh[(6*c - I*Pi)/4]*Sech[c/2 + (I/4)*Pi +
(d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(3*d*x)/2])/8 - (5*a^2*d*
Cosh[(5*c)/2 + (I/4)*Pi]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c
 + d*x]]*SinhIntegral[(5*d*x)/2])/8
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
```

```
1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^2}\,dx = \left(4a^2\text{csch}\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\right)\int \frac{\sinh^5\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)}{x^2}\,dx$$

$$= -\frac{4a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x} + \left(10a^2d\text{csch}\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sin\right.$$

$$= -\frac{4a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x} - \frac{1}{8}\left(5a^2d\text{csch}\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\,\text{si}\right.$$

$$= -\frac{4a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x} + \frac{1}{8}\left(5ia^2d\cosh\left(\frac{5c}{2}+\frac{i\pi}{4}\right)\text{csch}\left(\frac{c}{2}-\frac{i\pi}{4}\right.\right.$$

$$= -\frac{4a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x} - \frac{5}{8}a^2d\text{Chi}\left(\frac{5dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\text{s}$$

Mathematica [A] time = 2.43392, size = 347, normalized size = 0.78

$$\frac{a^2(\sinh(c+dx)-i)^2\sqrt{a+ia\sinh(c+dx)}\left(5dx\sinh\left(\frac{5c}{2}\right)\text{Chi}\left(\frac{5dx}{2}\right)+5idx\cosh\left(\frac{5c}{2}\right)\text{Chi}\left(\frac{5dx}{2}\right)-10idx\left(\cosh\left(\frac{c}{2}\right)-i\sinh\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(5/2)/x^2,x]
```

```
[Out] (a^2*(-I + Sinh[c + d*x])^2*Sqrt[a + I*a*Sinh[c + d*x]]*(20*Cosh[(c + d*x)/
2] - 10*Cosh[(3*(c + d*x))/2] - 2*Cosh[(5*(c + d*x))/2] + (5*I)*d*x*Cosh[(5
*c)/2]*CoshIntegral[(5*d*x)/2] - (10*I)*d*x*CoshIntegral[(d*x)/2]*(Cosh[c/2
] - I*Sinh[c/2]) + 15*d*x*CoshIntegral[(3*d*x)/2]*((-I)*Cosh[(3*c)/2] + Sin
h[(3*c)/2]) + 5*d*x*CoshIntegral[(5*d*x)/2]*Sinh[(5*c)/2] + (20*I)*Sinh[(c
```

```
+ d*x)/2] + (10*I)*Sinh[(3*(c + d*x))/2] - (2*I)*Sinh[(5*(c + d*x))/2] - 10
*d*x*Cosh[c/2]*SinhIntegral[(d*x)/2] - (10*I)*d*x*Sinh[c/2]*SinhIntegral[(d
*x)/2] + 15*d*x*Cosh[(3*c)/2]*SinhIntegral[(3*d*x)/2] - (15*I)*d*x*Sinh[(3*
c)/2]*SinhIntegral[(3*d*x)/2] + 5*d*x*Cosh[(5*c)/2]*SinhIntegral[(5*d*x)/2]
 + (5*I)*d*x*Sinh[(5*c)/2]*SinhIntegral[(5*d*x)/2]))/(8*x*(Cosh[(c + d*x)/2
] + I*Sinh[(c + d*x)/2])^5)
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (a + ia \sinh (dx + c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(d*x+c))^(5/2)/x^2,x)
```

```
[Out] int((a+I*a*sinh(d*x+c))^(5/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a \sinh (dx + c) + a)^{\frac{5}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)/x^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))**(5/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\sinh{(dx+c)}+a)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)/x^2, x)
```

3.135 $\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^3}\,dx$

Optimal. Leaf size=536

$$-\frac{25}{32}ia^2d^2\sinh\left(\frac{5c}{2}-\frac{i\pi}{4}\right)\text{Chi}\left(\frac{5dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}+\frac{5}{16}ia^2d^2\sinh\left(\frac{1}{4}(2c-i\pi)\right)\text{Chi}\left(\frac{dx}{2}\right.$$

```
[Out] (-2*a^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/x^2 -
 ((25*I)/32)*a^2*d^2*CoshIntegral[(5*d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]
*Sinh[(5*c)/2 - (I/4)*Pi]*Sqrt[a + I*a*Sinh[c + d*x]] + ((5*I)/16)*a^2*d^2*
CoshIntegral[(d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(2*c - I*Pi)/4]*S
qrt[a + I*a*Sinh[c + d*x]] + ((45*I)/32)*a^2*d^2*CoshIntegral[(3*d*x)/2]*Se
ch[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(6*c + I*Pi)/4]*Sqrt[a + I*a*Sinh[c + d*x
]] - (5*a^2*d*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3*Sinh[c/2 + (I/4)*Pi + (d*x)/
2]*Sqrt[a + I*a*Sinh[c + d*x]])/x + ((5*I)/16)*a^2*d^2*Cosh[(2*c - I*Pi)/4]
*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(d
*x)/2] + ((45*I)/32)*a^2*d^2*Cosh[(6*c + I*Pi)/4]*Sech[c/2 + (I/4)*Pi + (d*
x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(3*d*x)/2] - ((25*I)/32)*a^2
*d^2*Cosh[(5*c)/2 - (I/4)*Pi]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*S
inh[c + d*x]]*SinhIntegral[(5*d*x)/2]
```

Rubi [A] time = 0.638273, antiderivative size = 536, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3319, 3314, 3312, 3303, 3298, 3301}

$$-\frac{25}{32}ia^2d^2\sinh\left(\frac{5c}{2}-\frac{i\pi}{4}\right)\text{Chi}\left(\frac{5dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}+\frac{5}{16}ia^2d^2\sinh\left(\frac{1}{4}(2c-i\pi)\right)\text{Chi}\left(\frac{dx}{2}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[c + d*x])^(5/2)/x^3,x]
```

```
[Out] (-2*a^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/x^2 -
 ((25*I)/32)*a^2*d^2*CoshIntegral[(5*d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]
*Sinh[(5*c)/2 - (I/4)*Pi]*Sqrt[a + I*a*Sinh[c + d*x]] + ((5*I)/16)*a^2*d^2*
CoshIntegral[(d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(2*c - I*Pi)/4]*S
qrt[a + I*a*Sinh[c + d*x]] + ((45*I)/32)*a^2*d^2*CoshIntegral[(3*d*x)/2]*Se
ch[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(6*c + I*Pi)/4]*Sqrt[a + I*a*Sinh[c + d*x
]] - (5*a^2*d*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3*Sinh[c/2 + (I/4)*Pi + (d*x)/
2]*Sqrt[a + I*a*Sinh[c + d*x]])/x + ((5*I)/16)*a^2*d^2*Cosh[(2*c - I*Pi)/4]
*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(d
*x)/2] + ((45*I)/32)*a^2*d^2*Cosh[(6*c + I*Pi)/4]*Sech[c/2 + (I/4)*Pi + (d*
x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(3*d*x)/2] - ((25*I)/32)*a^2
*d^2*Cosh[(5*c)/2 - (I/4)*Pi]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*S
inh[c + d*x]]*SinhIntegral[(5*d*x)/2]
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(a+ia\sinh(c+dx))^{5/2}}{x^3}\,dx = \left(4a^2\operatorname{csch}\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}\right)\int \frac{\sinh^5\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)}{x^3}\,dx$$

$$= -\frac{2a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x^2} - \frac{5a^2d\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}\right.}{x}$$

$$= -\frac{2a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x^2} - \frac{5a^2d\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}\right.}{x}$$

$$= -\frac{2a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x^2} - \frac{5a^2d\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}\right.}{x}$$

$$= -\frac{2a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x^2} - \frac{5a^2d\cosh^3\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}+\frac{i\pi}{4}\right.}{x}$$

$$= -\frac{2a^2\cosh^4\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\sqrt{a+ia\sinh(c+dx)}}{x^2} - \frac{25}{32}ia^2d^2\operatorname{Chi}\left(\frac{5dx}{2}\right)\operatorname{sech}\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right.$$

Mathematica [B] time = 7.68558, size = 4751, normalized size = 8.86

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(5/2)/x^3,x]
```

```
[Out] (2*((1/128 + I/128)*Cosh[5*(c/2 + (d*x)/2)] - (1/128 + I/128)*Sinh[5*(c/2 +
 (d*x)/2)])*(a + I*a*Sinh[c + d*x])^(5/2)*((-4*I)*d^3 - (10*I)*c*d^3 + (20*
I)*d^3*(c/2 + (d*x)/2) + 20*d^3*Cosh[2*(c/2 + (d*x)/2)] + 30*c*d^3*Cosh[2*(
c/2 + (d*x)/2)] - 60*d^3*(c/2 + (d*x)/2)*Cosh[2*(c/2 + (d*x)/2)] + (40*I)*d
^3*Cosh[4*(c/2 + (d*x)/2)] + (20*I)*c*d^3*Cosh[4*(c/2 + (d*x)/2)] - (40*I)*
d^3*(c/2 + (d*x)/2)*Cosh[4*(c/2 + (d*x)/2)] - 40*d^3*Cosh[6*(c/2 + (d*x)/2)
] + 20*c*d^3*Cosh[6*(c/2 + (d*x)/2)] - 40*d^3*(c/2 + (d*x)/2)*Cosh[6*(c/2 +
 (d*x)/2)] - (20*I)*d^3*Cosh[8*(c/2 + (d*x)/2)] + (30*I)*c*d^3*Cosh[8*(c/2
+ (d*x)/2)] - (60*I)*d^3*(c/2 + (d*x)/2)*Cosh[8*(c/2 + (d*x)/2)] + 4*d^3*Co
sh[10*(c/2 + (d*x)/2)] - 10*c*d^3*Cosh[10*(c/2 + (d*x)/2)] + 20*d^3*(c/2 +
(d*x)/2)*Cosh[10*(c/2 + (d*x)/2)] - (10*I)*c^2*d^3*Cosh[c/2 - 5*(c/2 + (d*x
)/2)]*CoshIntegral[(d*x)/2] + (40*I)*c*d^3*(c/2 + (d*x)/2)*Cosh[c/2 - 5*(c/
2 + (d*x)/2)]*CoshIntegral[(d*x)/2] - (40*I)*d^3*(c/2 + (d*x)/2)^2*Cosh[c/2
 - 5*(c/2 + (d*x)/2)]*CoshIntegral[(d*x)/2] + 10*c^2*d^3*Cosh[c/2 + 5*(c/2
+ (d*x)/2)]*CoshIntegral[(d*x)/2] - 40*c*d^3*(c/2 + (d*x)/2)*Cosh[c/2 + 5*(
c/2 + (d*x)/2)]*CoshIntegral[(d*x)/2] + 40*d^3*(c/2 + (d*x)/2)^2*Cosh[c/2 +
 5*(c/2 + (d*x)/2)]*CoshIntegral[(d*x)/2] - 45*c^2*d^3*Cosh[(3*c)/2 - 5*(c/
2 + (d*x)/2)]*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)] + 180*c*d^3*(c/2 +
 (d*x)/2)*Cosh[(3*c)/2 - 5*(c/2 + (d*x)/2)]*CoshIntegral[(-3*c)/2 + 3*(c/2
+ (d*x)/2)] - 180*d^3*(c/2 + (d*x)/2)^2*Cosh[(3*c)/2 - 5*(c/2 + (d*x)/2)]*C
oshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)] + (45*I)*c^2*d^3*Cosh[(3*c)/2 + 5
*(c/2 + (d*x)/2)]*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)] - (180*I)*c*d^
3*(c/2 + (d*x)/2)*Cosh[(3*c)/2 + 5*(c/2 + (d*x)/2)]*CoshIntegral[(-3*c)/2 +
 3*(c/2 + (d*x)/2)] + (180*I)*d^3*(c/2 + (d*x)/2)^2*Cosh[(3*c)/2 + 5*(c/2 +
 (d*x)/2)]*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)] + (25*I)*c^2*d^3*Cosh
[(5*c)/2 - 5*(c/2 + (d*x)/2)]*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)] -
(100*I)*c*d^3*(c/2 + (d*x)/2)*Cosh[(5*c)/2 - 5*(c/2 + (d*x)/2)]*CoshIntegra
l[(-5*c)/2 + 5*(c/2 + (d*x)/2)] + (100*I)*d^3*(c/2 + (d*x)/2)^2*Cosh[(5*c)/
2 - 5*(c/2 + (d*x)/2)]*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)] - 25*c^2*
d^3*Cosh[(5*c)/2 + 5*(c/2 + (d*x)/2)]*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x
)/2)] + 100*c*d^3*(c/2 + (d*x)/2)*Cosh[(5*c)/2 + 5*(c/2 + (d*x)/2)]*CoshInt
egral[(-5*c)/2 + 5*(c/2 + (d*x)/2)] - 100*d^3*(c/2 + (d*x)/2)^2*Cosh[(5*c)/
2 + 5*(c/2 + (d*x)/2)]*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)] + 20*d^3*
Sinh[2*(c/2 + (d*x)/2)] + 30*c*d^3*Sinh[2*(c/2 + (d*x)/2)] - 60*d^3*(c/2 +
(d*x)/2)*Sinh[2*(c/2 + (d*x)/2)] + (40*I)*d^3*Sinh[4*(c/2 + (d*x)/2)] + (20
*I)*c*d^3*Sinh[4*(c/2 + (d*x)/2)] - (40*I)*d^3*(c/2 + (d*x)/2)*Sinh[4*(c/2
+ (d*x)/2)] - 40*d^3*Sinh[6*(c/2 + (d*x)/2)] + 20*c*d^3*Sinh[6*(c/2 + (d*x)
/2)] - 40*d^3*(c/2 + (d*x)/2)*Sinh[6*(c/2 + (d*x)/2)] - (20*I)*d^3*Sinh[8*(
c/2 + (d*x)/2)] + (30*I)*c*d^3*Sinh[8*(c/2 + (d*x)/2)] - (60*I)*d^3*(c/2 +
(d*x)/2)*Sinh[8*(c/2 + (d*x)/2)] + 4*d^3*Sinh[10*(c/2 + (d*x)/2)] - 10*c*d^
3*Sinh[10*(c/2 + (d*x)/2)] + 20*d^3*(c/2 + (d*x)/2)*Sinh[10*(c/2 + (d*x)/2)
] + (10*I)*c^2*d^3*CoshIntegral[(d*x)/2]*Sinh[c/2 - 5*(c/2 + (d*x)/2)] - (4
0*I)*c*d^3*(c/2 + (d*x)/2)*CoshIntegral[(d*x)/2]*Sinh[c/2 - 5*(c/2 + (d*x)/
2)] + (40*I)*d^3*(c/2 + (d*x)/2)^2*CoshIntegral[(d*x)/2]*Sinh[c/2 - 5*(c/2
+ (d*x)/2)] + 45*c^2*d^3*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)]*Sinh[(3
*c)/2 - 5*(c/2 + (d*x)/2)] - 180*c*d^3*(c/2 + (d*x)/2)*CoshIntegral[(-3*c)/
2 + 3*(c/2 + (d*x)/2)]*Sinh[(3*c)/2 - 5*(c/2 + (d*x)/2)] + 180*d^3*(c/2 + (
d*x)/2)^2*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)]*Sinh[(3*c)/2 - 5*(c/2
+ (d*x)/2)] - (25*I)*c^2*d^3*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)]*Sin
h[(5*c)/2 - 5*(c/2 + (d*x)/2)] + (100*I)*c*d^3*(c/2 + (d*x)/2)*CoshIntegral
[(-5*c)/2 + 5*(c/2 + (d*x)/2)]*Sinh[(5*c)/2 - 5*(c/2 + (d*x)/2)] - (100*I)*
d^3*(c/2 + (d*x)/2)^2*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)]*Sinh[(5*c)
/2 - 5*(c/2 + (d*x)/2)] + 10*c^2*d^3*CoshIntegral[(d*x)/2]*Sinh[c/2 + 5*(c/
2 + (d*x)/2)] - 40*c*d^3*(c/2 + (d*x)/2)*CoshIntegral[(d*x)/2]*Sinh[c/2 + 5
*(c/2 + (d*x)/2)] + 40*d^3*(c/2 + (d*x)/2)^2*CoshIntegral[(d*x)/2]*Sinh[c/2
```

```
 + 5*(c/2 + (d*x)/2)] + (45*I)*c^2*d^3*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*
x)/2)]*Sinh[(3*c)/2 + 5*(c/2 + (d*x)/2)] - (180*I)*c*d^3*(c/2 + (d*x)/2)*Co
shIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)]*Sinh[(3*c)/2 + 5*(c/2 + (d*x)/2)]
 + (180*I)*d^3*(c/2 + (d*x)/2)^2*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)]*
Sinh[(3*c)/2 + 5*(c/2 + (d*x)/2)] - 25*c^2*d^3*CoshIntegral[(-5*c)/2 + 5*(c
/2 + (d*x)/2)]*Sinh[(5*c)/2 + 5*(c/2 + (d*x)/2)] + 100*c*d^3*(c/2 + (d*x)/2
)*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)]*Sinh[(5*c)/2 + 5*(c/2 + (d*x)/
2)] - 100*d^3*(c/2 + (d*x)/2)^2*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)]*
Sinh[(5*c)/2 + 5*(c/2 + (d*x)/2)] + (10*I)*c^2*d^3*Cosh[c/2 - 5*(c/2 + (d*x
)/2)]*SinhIntegral[(d*x)/2] - (40*I)*c*d^3*(c/2 + (d*x)/2)*Cosh[c/2 - 5*(c/
2 + (d*x)/2)]*SinhIntegral[(d*x)/2] + (40*I)*d^3*(c/2 + (d*x)/2)^2*Cosh[c/2
 - 5*(c/2 + (d*x)/2)]*SinhIntegral[(d*x)/2] + 10*c^2*d^3*Cosh[c/2 + 5*(c/2
+ (d*x)/2)]*SinhIntegral[(d*x)/2] - 40*c*d^3*(c/2 + (d*x)/2)*Cosh[c/2 + 5*(
c/2 + (d*x)/2)]*SinhIntegral[(d*x)/2] + 40*d^3*(c/2 + (d*x)/2)^2*Cosh[c/2 +
 5*(c/2 + (d*x)/2)]*SinhIntegral[(d*x)/2] - (10*I)*c^2*d^3*Sinh[c/2 - 5*(c/
2 + (d*x)/2)]*SinhIntegral[(d*x)/2] + (40*I)*c*d^3*(c/2 + (d*x)/2)*Sinh[c/2
 - 5*(c/2 + (d*x)/2)]*SinhIntegral[(d*x)/2] - (40*I)*d^3*(c/2 + (d*x)/2)^2*
Sinh[c/2 - 5*(c/2 + (d*x)/2)]*SinhIntegral[(d*x)/2] + 10*c^2*d^3*Sinh[c/2 +
 5*(c/2 + (d*x)/2)]*SinhIntegral[(d*x)/2] - 40*c*d^3*(c/2 + (d*x)/2)*Sinh[c
/2 + 5*(c/2 + (d*x)/2)]*SinhIntegral[(d*x)/2] + 40*d^3*(c/2 + (d*x)/2)^2*Si
nh[c/2 + 5*(c/2 + (d*x)/2)]*SinhIntegral[(d*x)/2] + (25*I)*c^2*d^3*Cosh[(5*
c)/2 - 5*(c/2 + (d*x)/2)]*SinhIntegral[(5*c)/2 - 5*(c/2 + (d*x)/2)] - (100*
I)*c*d^3*(c/2 + (d*x)/2)*Cosh[(5*c)/2 - 5*(c/2 + (d*x)/2)]*SinhIntegral[(5*
c)/2 - 5*(c/2 + (d*x)/2)] + (100*I)*d^3*(c/2 + (d*x)/2)^2*Cosh[(5*c)/2 - 5*
(c/2 + (d*x)/2)]*SinhIntegral[(5*c)/2 - 5*(c/2 + (d*x)/2)] + 25*c^2*d^3*Cos
h[(5*c)/2 + 5*(c/2 + (d*x)/2)]*SinhIntegral[(5*c)/2 - 5*(c/2 + (d*x)/2)] -
100*c*d^3*(c/2 + (d*x)/2)*Cosh[(5*c)/2 + 5*(c/2 + (d*x)/2)]*SinhIntegral[(5
*c)/2 - 5*(c/2 + (d*x)/2)] + 100*d^3*(c/2 + (d*x)/2)^2*Cosh[(5*c)/2 + 5*(c/
2 + (d*x)/2)]*SinhIntegral[(5*c)/2 - 5*(c/2 + (d*x)/2)] - (25*I)*c^2*d^3*Si
nh[(5*c)/2 - 5*(c/2 + (d*x)/2)]*SinhIntegral[(5*c)/2 - 5*(c/2 + (d*x)/2)] +
 (100*I)*c*d^3*(c/2 + (d*x)/2)*Sinh[(5*c)/2 - 5*(c/2 + (d*x)/2)]*SinhIntegr
al[(5*c)/2 - 5*(c/2 + (d*x)/2)] - (100*I)*d^3*(c/2 + (d*x)/2)^2*Sinh[(5*c)/
2 - 5*(c/2 + (d*x)/2)]*SinhIntegral[(5*c)/2 - 5*(c/2 + (d*x)/2)] + 25*c^2*d
^3*Sinh[(5*c)/2 + 5*(c/2 + (d*x)/2)]*SinhIntegral[(5*c)/2 - 5*(c/2 + (d*x)/
2)] - 100*c*d^3*(c/2 + (d*x)/2)*Sinh[(5*c)/2 + 5*(c/2 + (d*x)/2)]*SinhInteg
ral[(5*c)/2 - 5*(c/2 + (d*x)/2)] + 100*d^3*(c/2 + (d*x)/2)^2*Sinh[(5*c)/2 +
 5*(c/2 + (d*x)/2)]*SinhIntegral[(5*c)/2 - 5*(c/2 + (d*x)/2)] - 45*c^2*d^3*
Cosh[(3*c)/2 - 5*(c/2 + (d*x)/2)]*SinhIntegral[(3*c)/2 - 3*(c/2 + (d*x)/2)]
 + 180*c*d^3*(c/2 + (d*x)/2)*Cosh[(3*c)/2 - 5*(c/2 + (d*x)/2)]*SinhIntegral
[(3*c)/2 - 3*(c/2 + (d*x)/2)] - 180*d^3*(c/2 + (d*x)/2)^2*Cosh[(3*c)/2 - 5*
(c/2 + (d*x)/2)]*SinhIntegral[(3*c)/2 - 3*(c/2 + (d*x)/2)] - (45*I)*c^2*d^3
*Cosh[(3*c)/2 + 5*(c/2 + (d*x)/2)]*SinhIntegral[(3*c)/2 - 3*(c/2 + (d*x)/2)
] + (180*I)*c*d^3*(c/2 + (d*x)/2)*Cosh[(3*c)/2 + 5*(c/2 + (d*x)/2)]*SinhInt
egral[(3*c)/2 - 3*(c/2 + (d*x)/2)] - (180*I)*d^3*(c/2 + (d*x)/2)^2*Cosh[(3*
c)/2 + 5*(c/2 + (d*x)/2)]*SinhIntegral[(3*c)/2 - 3*(c/2 + (d*x)/2)] + 45*c^
2*d^3*Sinh[(3*c)/2 - 5*(c/2 + (d*x)/2)]*SinhIntegral[(3*c)/2 - 3*(c/2 + (d*
x)/2)] - 180*c*d^3*(c/2 + (d*x)/2)*Sinh[(3*c)/2 - 5*(c/2 + (d*x)/2)]*SinhIn
tegral[(3*c)/2 - 3*(c/2 + (d*x)/2)] + 180*d^3*(c/2 + (d*x)/2)^2*Sinh[(3*c)/
2 - 5*(c/2 + (d*x)/2)]*SinhIntegral[(3*c)/2 - 3*(c/2 + (d*x)/2)] - (45*I)*c
^2*d^3*Sinh[(3*c)/2 + 5*(c/2 + (d*x)/2)]*SinhIntegral[(3*c)/2 - 3*(c/2 + (d
*x)/2)] + (180*I)*c*d^3*(c/2 + (d*x)/2)*Sinh[(3*c)/2 + 5*(c/2 + (d*x)/2)]*S
inhIntegral[(3*c)/2 - 3*(c/2 + (d*x)/2)] - (180*I)*d^3*(c/2 + (d*x)/2)^2*Si
nh[(3*c)/2 + 5*(c/2 + (d*x)/2)]*SinhIntegral[(3*c)/2 - 3*(c/2 + (d*x)/2)]))
/(d*(-c + 2*(c/2 + (d*x)/2))^2*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2]
)^5)
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} (a + ia \sinh (dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(d*x+c))^(5/2)/x^3,x)
```

```
[Out] int((a+I*a*sinh(d*x+c))^(5/2)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \sinh (dx + c) + a)^{\frac{5}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)/x^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))**(5/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \sinh (dx + c) + a)^{\frac{5}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)/x^3, x)
```

3.136 $\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}\,dx$

Optimal. Leaf size=493

$$\frac{12ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}-\frac{12ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}-\frac{48ix\cosh(}{}$$

```
[Out] ((4*I)*x^3*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x
)/2])/(f*Sqrt[a + I*a*Sinh[e + f*x]]) + ((12*I)*x^2*Cosh[e/2 + (I/4)*Pi + (
f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e
 + f*x]]) - ((12*I)*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e -
 I*Pi)/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((48*I)*x*Cosh[e/
2 + (I/4)*Pi + (f*x)/2]*PolyLog[3, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^3*Sqr
t[a + I*a*Sinh[e + f*x]]) + ((48*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLo
g[3, E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^3*Sqrt[a + I*a*Sinh[e + f*x]]) + ((9
6*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[4, -E^((2*e - I*Pi)/4 + (f*x)/2
)])/(f^4*Sqrt[a + I*a*Sinh[e + f*x]]) - ((96*I)*Cosh[e/2 + (I/4)*Pi + (f*x)
/2]*PolyLog[4, E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^4*Sqrt[a + I*a*Sinh[e + f*
x]])
```

Rubi [A] time = 0.260732, antiderivative size = 493, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3319, 4182, 2531, 6609, 2282, 6589}

$$\frac{12ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}-\frac{12ix^2\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}-\frac{48ix\cosh(}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] ((4*I)*x^3*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x
)/2])/(f*Sqrt[a + I*a*Sinh[e + f*x]]) + ((12*I)*x^2*Cosh[e/2 + (I/4)*Pi + (
f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e
 + f*x]]) - ((12*I)*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e -
 I*Pi)/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((48*I)*x*Cosh[e/
2 + (I/4)*Pi + (f*x)/2]*PolyLog[3, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^3*Sqr
t[a + I*a*Sinh[e + f*x]]) + ((48*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLo
g[3, E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^3*Sqrt[a + I*a*Sinh[e + f*x]]) + ((9
6*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[4, -E^((2*e - I*Pi)/4 + (f*x)/2
)])/(f^4*Sqrt[a + I*a*Sinh[e + f*x]]) - ((96*I)*Cosh[e/2 + (I/4)*Pi + (f*x)
/2]*PolyLog[4, E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^4*Sqrt[a + I*a*Sinh[e + f*
x]])
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\int x^3\operatorname{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)dx}{\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{4ix^3\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} - \frac{\left(6\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\right)\int x^2\log\left(1-e^{-i\left(\frac{ie}{2}\right.}\right.}{f\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{4ix^3\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} + \frac{12ix^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{Li}_2\left(-e^{\frac{1}{4}(2e-i\pi)+}\right.}{f^2\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{4ix^3\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} + \frac{12ix^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{Li}_2\left(-e^{\frac{1}{4}(2e-i\pi)+}\right.}{f^2\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{4ix^3\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} + \frac{12ix^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{Li}_2\left(-e^{\frac{1}{4}(2e-i\pi)+}\right.}{f^2\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{4ix^3\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} + \frac{12ix^2\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\operatorname{Li}_2\left(-e^{\frac{1}{4}(2e-i\pi)+}\right.}{f^2\sqrt{a+ia\sinh(e+fx)}}$$

Mathematica [A] time = 1.27041, size = 331, normalized size = 0.67

$$\frac{(1-i)(-1)^{3/4}\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)\left(-6f^2x^2\text{PolyLog}\left(2,-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)+6f^2x^2\text{PolyLog}\left(2,(-1)^{3/4}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] ((1 - I)*(-1)^(3/4)*((2*I)*e^3*ArcTan[(-1)^(1/4)*E^((e + f*x)/2)] + e^3*Log
[1 - (-1)^(3/4)*E^((e + f*x)/2)] + f^3*x^3*Log[1 - (-1)^(3/4)*E^((e + f*x)/
2)] - e^3*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] - f^3*x^3*Log[1 + (-1)^(3/4)*
E^((e + f*x)/2)] - 6*f^2*x^2*PolyLog[2, -((-1)^(3/4)*E^((e + f*x)/2))] + 6*
f^2*x^2*PolyLog[2, (-1)^(3/4)*E^((e + f*x)/2)] + 24*f*x*PolyLog[3, -((-1)^(
3/4)*E^((e + f*x)/2))] - 24*f*x*PolyLog[3, (-1)^(3/4)*E^((e + f*x)/2)] - 48
*PolyLog[4, -((-1)^(3/4)*E^((e + f*x)/2))] + 48*PolyLog[4, (-1)^(3/4)*E^((e
 + f*x)/2)])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))/(f^4*Sqrt[a + I*a*S
inh[e + f*x]])
```

Maple [F] time = 0.201, size = 0, normalized size = 0.

$$\int x^3\frac{1}{\sqrt{a+ia\sinh\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+I*a*sinh(f*x+e))^(1/2),x)
```

```
[Out] int(x^3/(a+I*a*sinh(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{i a \sinh(fx + e) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(I*a*sinh(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{2i\sqrt{\frac{1}{2}}\sqrt{i ae^{(2fx+2e)} + 2 ae^{(fx+e)} - i a}x^3e^{\left(\frac{1}{2}fx+\frac{1}{2}e\right)}}{ae^{(2fx+2e)} - 2i ae^{(fx+e)} - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-2*I*sqrt(1/2)*sqrt(I*a*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*x
^3*e^(1/2*f*x + 1/2*e)/(a*e^(2*f*x + 2*e) - 2*I*a*e^(f*x + e) - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a\left(i\sinh(e + fx) + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+I*a*sinh(f*x+e))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a*(I*sinh(e + f*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{i a \sinh(fx + e) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(I*a*sinh(f*x + e) + a), x)
```

3.137 $\int \frac{x^2}{\sqrt{a+ia\sinh(e+fx)}}\,dx$

Optimal. Leaf size=349

$$\frac{8ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}-\frac{8ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}-\frac{16i\cosh\left(\frac{e}{2}+\frac{f}{\cdot}\right.}{f^3}$$

```
[Out] ((4*I)*x^2*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x
)/2])/(f*Sqrt[a + I*a*Sinh[e + f*x]]) + ((8*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x
)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e +
f*x]]) - ((8*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e - I*Pi)
/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((16*I)*Cosh[e/2 + (I/4
)*Pi + (f*x)/2]*PolyLog[3, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^3*Sqrt[a + I*
a*Sinh[e + f*x]]) + ((16*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[3, E^((2
*e - I*Pi)/4 + (f*x)/2)])/(f^3*Sqrt[a + I*a*Sinh[e + f*x]])
```

Rubi [A] time = 0.2019, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3319, 4182, 2531, 2282, 6589}

$$\frac{8ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}-\frac{8ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}-\frac{16i\cosh\left(\frac{e}{2}+\frac{f}{\cdot}\right.}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] ((4*I)*x^2*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x
)/2])/(f*Sqrt[a + I*a*Sinh[e + f*x]]) + ((8*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x
)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e +
f*x]]) - ((8*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e - I*Pi)
/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((16*I)*Cosh[e/2 + (I/4
)*Pi + (f*x)/2]*PolyLog[3, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^3*Sqrt[a + I*
a*Sinh[e + f*x]]) + ((16*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[3, E^((2
*e - I*Pi)/4 + (f*x)/2)])/(f^3*Sqrt[a + I*a*Sinh[e + f*x]])
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\int x^2\text{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)dx}{\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{4ix^2\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} - \frac{\left(4\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\right)\int x\log\left(1-e^{-i\cdots}\right.}{f\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{4ix^2\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} + \frac{8ix\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{Li}_2\left(-e^{\frac{1}{4}(2e-i\pi)+\cdots}\right.}{f^2\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{4ix^2\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} + \frac{8ix\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{Li}_2\left(-e^{\frac{1}{4}(2e-i\pi)+\cdots}\right.}{f^2\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{4ix^2\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} + \frac{8ix\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{Li}_2\left(-e^{\frac{1}{4}(2e-i\pi)+\cdots}\right.}{f^2\sqrt{a+ia\sinh(e+fx)}}$$

Mathematica [A] time = 0.973604, size = 276, normalized size = 0.79

$$\frac{(1+i)(-1)^{3/4}\left(\sinh\left(\frac{1}{2}(e+fx)\right)-i\cosh\left(\frac{1}{2}(e+fx)\right)\right)\left(-4fx\text{PolyLog}\left(2,-(-1)^{3/4}e^{\frac{1}{2}(e+fx)}\right)+4fx\text{PolyLog}\left(2,(-1)^{3/4}e\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] ((1 + I)*(-1)^(3/4)*((-2*I)*e^2*ArcTan[(-1)^(1/4)*E^((e + f*x)/2)] - e^2*Lo
g[1 - (-1)^(3/4)*E^((e + f*x)/2)] + f^2*x^2*Log[1 - (-1)^(3/4)*E^((e + f*x)
/2)] + e^2*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] - f^2*x^2*Log[1 + (-1)^(3/4)
*E^((e + f*x)/2)] - 4*f*x*PolyLog[2, -((-1)^(3/4)*E^((e + f*x)/2))] + 4*f*x
*PolyLog[2, (-1)^(3/4)*E^((e + f*x)/2)] + 8*PolyLog[3, -((-1)^(3/4)*E^((e +
 f*x)/2))] - 8*PolyLog[3, (-1)^(3/4)*E^((e + f*x)/2)])*((-I)*Cosh[(e + f*x)
```

```
/2] + Sinh[(e + f*x)/2]))/(f^3*Sqrt[a + I*a*Sinh[e + f*x]])
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt{a + ia\sinh(fx + e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+I*a*sinh(f*x+e))^(1/2),x)
```

```
[Out] int(x^2/(a+I*a*sinh(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{i\,a\sinh(fx + e) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(I*a*sinh(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{2i\sqrt{\frac{1}{2}}\sqrt{i\,ae^{(2\,fx+2\,e)} + 2\,ae^{(fx+e)} - i\,a}x^2e^{\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}}{ae^{(2\,fx+2\,e)} - 2i\,ae^{(fx+e)} - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-2*I*sqrt(1/2)*sqrt(I*a*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*x
^2*e^(1/2*f*x + 1/2*e)/(a*e^(2*f*x + 2*e) - 2*I*a*e^(f*x + e) - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a\left(i\sinh(e + fx) + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+I*a*sinh(f*x+e))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a*(I*sinh(e + f*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{i\,a\sinh\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(I*a*sinh(f*x + e) + a), x)
```

3.138 $\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx$

Optimal. Leaf size=207

$$\frac{4i\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}-\frac{4i\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}+\frac{4ix\cosh\left(\frac{e}{2}+\frac{fx}{2}\right.}{f\sqrt{a+}}$$

```
[Out] ((4*I)*x*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x)/
2])/(f*Sqrt[a + I*a*Sinh[e + f*x]]) + ((4*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]
*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e + f*x]
]) - ((4*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e - I*Pi)/4 + (
f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e + f*x]])
```

Rubi [A] time = 0.104719, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3319, 4182, 2279, 2391}

$$\frac{4i\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}-\frac{4i\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{f^2\sqrt{a+ia\sinh(e+fx)}}+\frac{4ix\cosh\left(\frac{e}{2}+\frac{fx}{2}\right.}{f\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + I*a*Sinh[e + f*x]],x]
```


```
[Out] ((4*I)*x*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x)/
2])/(f*Sqrt[a + I*a*Sinh[e + f*x]]) + ((4*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]
*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e + f*x]
]) - ((4*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e - I*Pi)/4 + (
f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e + f*x]])
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+ia\sinh(e+fx)}}\,dx = \frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\int x\text{csch}\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)dx}{\sqrt{a+ia\sinh(e+fx)}}$$
$$= \frac{4ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} - \frac{\left(2\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\right)\int\log\left(1-e^{-i\left(\frac{ie}{2}+\right.}\right.}{f\sqrt{a+ia\sinh(e+fx)}}$$
$$= \frac{4ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} - \frac{\left(4\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\right)\text{Subst}\left(\int\frac{\log(1-x}{x}\right.}{f^2\sqrt{a+ia\sinh(e+f}}$$
$$= \frac{4ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{f\sqrt{a+ia\sinh(e+fx)}} + \frac{4i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\text{Li}_2\left(-e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right.}{f^2\sqrt{a+ia\sinh(e+fx)}}$$

Mathematica [A] time = 0.505013, size = 221, normalized size = 1.07

$$\frac{\sqrt{2}\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)\left(-2i\left(\text{PolyLog}\left(2,-\sqrt[4]{-1}e^{-\frac{e}{2}-\frac{fx}{2}}\right)-\text{PolyLog}\left(2,\sqrt[4]{-1}e^{-\frac{e}{2}-\frac{fx}{2}}\right)\right)-\frac{1}{2}(2ie+2\right.}{f^2\sqrt{a+ia\sin}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] (Sqrt[2]*(-2*e*ArcTan[(I + Tanh[(e + f*x)/4])/Sqrt[2]] + I*Pi*ArcTan[(I + T
anh[(e + f*x)/4])/Sqrt[2]] - (((2*I)*e + Pi + (2*I)*f*x)*(Log[1 - (-1)^(1/4
)*E^(-e/2 - (f*x)/2)] - Log[1 + (-1)^(1/4)*E^(-e/2 - (f*x)/2)]))/2 - (2*I)*
(PolyLog[2, -((-1)^(1/4)*E^(-e/2 - (f*x)/2))] - PolyLog[2, (-1)^(1/4)*E^(-e
/2 - (f*x)/2)]))*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))/(f^2*Sqrt[a + I
*a*Sinh[e + f*x]])
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{a+ia\sinh(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

```
[Out] int(x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{i\,a\sinh(fx+e)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(I*a*sinh(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{2i\sqrt{\frac{1}{2}}\sqrt{i\,ae^{\left(2\,fx+2\,e\right)}+2\,ae^{\left(fx+e\right)}-i\,a}\,xe^{\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}}{ae^{\left(2\,fx+2\,e\right)}-2i\,ae^{\left(fx+e\right)}-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-2*I*sqrt(1/2)*sqrt(I*a*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*x
*e^(1/2*f*x + 1/2*e)/(a*e^(2*f*x + 2*e) - 2*I*a*e^(f*x + e) - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a\left(i\sinh\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(f*x+e))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a*(I*sinh(e + f*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{i\,a\sinh\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(I*a*sinh(f*x + e) + a), x)
```

3.139 $\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{a+ia\sinh(e+fx)}},x\right)$$

```
[Out] Unintegrable[1/(x*Sqrt[a + I*a*Sinh[e + f*x]]), x]
```

Rubi [A] time = 0.0776774, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[a + I*a*Sinh[e + f*x]]),x]
```

```
[Out] Defer[Int][1/(x*Sqrt[a + I*a*Sinh[e + f*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx=\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx$$

Mathematica [A] time = 3.84339, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+ia\sinh(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[a + I*a*Sinh[e + f*x]]),x]
```

```
[Out] Integrate[1/(x*Sqrt[a + I*a*Sinh[e + f*x]]), x]
```

Maple [A] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{1}{x}\frac{1}{\sqrt{a+ia\sinh\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

```
[Out] int(1/x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{i\,a\sinh\left(fx+e\right)+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(I*a*sinh(f*x + e) + a)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{2i\sqrt{\frac{1}{2}}\sqrt{i\,ae^{\left(2\,fx+2\,e\right)}+2\,ae^{\left(fx+e\right)}-i\,a}e^{\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}}{axe^{\left(2\,fx+2\,e\right)}-2i\,axe^{\left(fx+e\right)}-ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-2*I*sqrt(1/2)*sqrt(I*a*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*e
^(1/2*f*x + 1/2*e)/(a*x*e^(2*f*x + 2*e) - 2*I*a*x*e^(f*x + e) - a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a\left(i\sinh\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a*(I*sinh(e + f*x) + 1))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{i\,a\sinh\left(fx+e\right)+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(I*a*sinh(f*x + e) + a)*x), x)
```

3.140 $\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}},x\right)$$

```
[Out] Unintegrable[1/(x^2*Sqrt[a + I*a*Sinh[e + f*x]]), x]
```

Rubi [A] time = 0.0757779, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[a + I*a*Sinh[e + f*x]]),x]
```

```
[Out] Defer[Int][1/(x^2*Sqrt[a + I*a*Sinh[e + f*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx=\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx$$

Mathematica [A] time = 3.95241, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a+ia\sinh(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[a + I*a*Sinh[e + f*x]]),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[a + I*a*Sinh[e + f*x]]), x]
```

Maple [A] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\frac{1}{\sqrt{a+ia\sinh\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x)
```

```
[Out] int(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{i\,a\sinh\left(fx+e\right)+a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(I*a*sinh(f*x + e) + a)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{2i\sqrt{\frac{1}{2}}\sqrt{i\,ae^{\left(2\,fx+2\,e\right)}+2\,ae^{\left(fx+e\right)}-i\,a}e^{\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}}{ax^2e^{\left(2\,fx+2\,e\right)}-2i\,ax^2e^{\left(fx+e\right)}-ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-2*I*sqrt(1/2)*sqrt(I*a*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*e
^(1/2*f*x + 1/2*e)/(a*x^2*e^(2*f*x + 2*e) - 2*I*a*x^2*e^(f*x + e) - a*x^2),
 x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a\left(i\sinh\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+I*a*sinh(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a*(I*sinh(e + f*x) + 1))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{i\,a\sinh\left(fx+e\right)+a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(I*a*sinh(f*x + e) + a)*x^2), x)
```

3.141 $\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=807

$$\frac{\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)x^3}{2af\sqrt{i\sinh(e+fx)a+a}}+\frac{i\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)x^3}{af\sqrt{i\sinh(e+fx)a+a}}+\frac{3i\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e}\right.}{af^2\sqrt{i\sinh(e+fx)a+a}}$$

```
[Out] (3*x^2)/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((24*I)*x*ArcTanh[E^((2*e - I
*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/(a*f^3*Sqrt[a + I*a*Sinh
[e + f*x]]) + (I*x^3*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)
*Pi + (f*x)/2])/(a*f*Sqrt[a + I*a*Sinh[e + f*x]]) - ((24*I)*Cosh[e/2 + (I/4
)*Pi + (f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^4*Sqrt[a +
I*a*Sinh[e + f*x]]) + ((3*I)*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2,
-E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) + ((24*
I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])
/(a*f^4*Sqrt[a + I*a*Sinh[e + f*x]]) - ((3*I)*x^2*Cosh[e/2 + (I/4)*Pi + (f*
x)/2]*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sinh[e
+ f*x]]) - ((12*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[3, -E^((2*e - I
*Pi)/4 + (f*x)/2)])/(a*f^3*Sqrt[a + I*a*Sinh[e + f*x]]) + ((12*I)*x*Cosh[e/
2 + (I/4)*Pi + (f*x)/2]*PolyLog[3, E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^3*Sq
rt[a + I*a*Sinh[e + f*x]]) + ((24*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog
[4, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^4*Sqrt[a + I*a*Sinh[e + f*x]]) - (
(24*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[4, E^((2*e - I*Pi)/4 + (f*x)/
2)])/(a*f^4*Sqrt[a + I*a*Sinh[e + f*x]]) + (x^3*Tanh[e/2 + (I/4)*Pi + (f*x)
/2])/(2*a*f*Sqrt[a + I*a*Sinh[e + f*x]])
```

Rubi [A] time = 0.437724, antiderivative size = 807, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3319, 4186, 4182, 2279, 2391, 2531, 6609, 2282, 6589}

$$\frac{\tanh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)x^3}{2af\sqrt{i\sinh(e+fx)a+a}}+\frac{i\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)x^3}{af\sqrt{i\sinh(e+fx)a+a}}+\frac{3i\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{1}{4}(2e}\right.}{af^2\sqrt{i\sinh(e+fx)a+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + I*a*Sinh[e + f*x])^(3/2),x]
```


```
[Out] (3*x^2)/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((24*I)*x*ArcTanh[E^((2*e - I
*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/(a*f^3*Sqrt[a + I*a*Sinh
[e + f*x]]) + (I*x^3*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)
*Pi + (f*x)/2])/(a*f*Sqrt[a + I*a*Sinh[e + f*x]]) - ((24*I)*Cosh[e/2 + (I/4
)*Pi + (f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^4*Sqrt[a +
I*a*Sinh[e + f*x]]) + ((3*I)*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2,
-E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) + ((24*
I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])
/(a*f^4*Sqrt[a + I*a*Sinh[e + f*x]]) - ((3*I)*x^2*Cosh[e/2 + (I/4)*Pi + (f*
x)/2]*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sinh[e
+ f*x]]) - ((12*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[3, -E^((2*e - I
*Pi)/4 + (f*x)/2)])/(a*f^3*Sqrt[a + I*a*Sinh[e + f*x]]) + ((12*I)*x*Cosh[e/
2 + (I/4)*Pi + (f*x)/2]*PolyLog[3, E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^3*Sq
rt[a + I*a*Sinh[e + f*x]]) + ((24*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog
[4, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^4*Sqrt[a + I*a*Sinh[e + f*x]]) - (
(24*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[4, E^((2*e - I*Pi)/4 + (f*x)/
2)])/(a*f^4*Sqrt[a + I*a*Sinh[e + f*x]]) + (x^3*Tanh[e/2 + (I/4)*Pi + (f*x)
```

```
/2])/(2*a*f*Sqrt[a + I*a*Sinh[e + f*x]])
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3}{(a+ia\sinh(e+fx))^{3/2}}\,dx = -\frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\int x^3\text{csch}^3\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)dx}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{3x^2}{af^2\sqrt{a+ia\sinh(e+fx)}} + \frac{x^3\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+ia\sinh(e+fx)}} + \frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\int x^3\text{csch}}{4a\sqrt{a+ia\sinh(e}}$$

$$= \frac{3x^2}{af^2\sqrt{a+ia\sinh(e+fx)}} - \frac{24ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}} + \frac{ix^3\tanh}{}$$

$$= \frac{3x^2}{af^2\sqrt{a+ia\sinh(e+fx)}} - \frac{24ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}} + \frac{ix^3\tanh}{}$$

$$= \frac{3x^2}{af^2\sqrt{a+ia\sinh(e+fx)}} - \frac{24ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}} + \frac{ix^3\tanh}{}$$

$$= \frac{3x^2}{af^2\sqrt{a+ia\sinh(e+fx)}} - \frac{24ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}} + \frac{ix^3\tanh}{}$$

$$= \frac{3x^2}{af^2\sqrt{a+ia\sinh(e+fx)}} - \frac{24ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}} + \frac{ix^3\tanh}{}$$

Mathematica [A] time = 3.23932, size = 546, normalized size = 0.68

$$\frac{\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)\left(\left(\frac{1}{2}-\frac{i}{2}\right)(-1)^{3/4}\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)^2\left(-6\left(f^2x^2-8\right)\text{Poly}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] ((Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])*(f^2*x^2*(6 + I*f*x)*(Cosh[(e +
f*x)/2] + I*Sinh[(e + f*x)/2]) + (1/2 - I/2)*(-1)^(3/4)*(-48*e*ArcTanh[(-1)
^(3/4)*E^((e + f*x)/2)] + 2*e^3*ArcTanh[(-1)^(3/4)*E^((e + f*x)/2)] - 24*e*
Log[1 - (-1)^(3/4)*E^((e + f*x)/2)] + e^3*Log[1 - (-1)^(3/4)*E^((e + f*x)/2
)] - 24*f*x*Log[1 - (-1)^(3/4)*E^((e + f*x)/2)] + f^3*x^3*Log[1 - (-1)^(3/4
)*E^((e + f*x)/2)] + 24*e*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] - e^3*Log[1 +
 (-1)^(3/4)*E^((e + f*x)/2)] + 24*f*x*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] -
 f^3*x^3*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] - 6*(-8 + f^2*x^2)*PolyLog[2,
-((-1)^(3/4)*E^((e + f*x)/2))] + 6*(-8 + f^2*x^2)*PolyLog[2, (-1)^(3/4)*E^(
```

```
(e + f*x)/2)] + 24*f*x*PolyLog[3, -((-1)^(3/4)*E^((e + f*x)/2))] - 24*f*x*P
olyLog[3, (-1)^(3/4)*E^((e + f*x)/2)] - 48*PolyLog[4, -((-1)^(3/4)*E^((e +
f*x)/2))] + 48*PolyLog[4, (-1)^(3/4)*E^((e + f*x)/2)])*(Cosh[(e + f*x)/2] +
 I*Sinh[(e + f*x)/2])^2 + 2*f^3*x^3*Sinh[(e + f*x)/2]))/(2*f^4*(a + I*a*Sin
h[e + f*x])^(3/2))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int x^{3} \left(a + ia \sinh \left(fx + e\right)\right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+I*a*sinh(f*x+e))^(3/2),x)
```

```
[Out] int(x^3/(a+I*a*sinh(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\left(i\, a \sinh \left(fx + e\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\frac{1}{2}}\left(\left(-i\, f x^{3} - 6 i\, x^{2}\right)e^{\left(2\, fx + 2\, e\right)} + \left(f x^{3} - 6\, x^{2}\right)e^{\left(fx + e\right)}\right)\sqrt{i\, a e^{\left(2\, fx + 2\, e\right)} + 2\, a e^{\left(fx + e\right)} - i\, a}\, e^{\left(-\frac{1}{2}\, fx - \frac{1}{2}\, e\right)} + \left(a^{2} f^{2} e^{\left(3\, fx + 3\, e\right)} - 3 i\, a^{2} f^{2} e^{\cdots}\right.}{a^{2} f^{2} e^{\left(3\, fx + 3\, e\right)} - 3 i\, a^{2} f^{2} e^{\left(2\, fx + 2\, e\right)} - 3\, a^{2} f^{2} e^{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(1/2)*((-I*f*x^3 - 6*I*x^2)*e^(2*f*x + 2*e) + (f*x^3 - 6*x^2)*e^(f*x +
 e))*sqrt(I*a*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*e^(-1/2*f*x - 1/2*e)
 + (a^2*f^2*e^(3*f*x + 3*e) - 3*I*a^2*f^2*e^(2*f*x + 2*e) - 3*a^2*f^2*e^(f*
x + e) + I*a^2*f^2)*integral(sqrt(1/2)*(-I*f^2*x^3 + 24*I*x)*sqrt(I*a*e^(2*
f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*e^(1/2*f*x + 1/2*e)/(2*a^2*f^2*e^(2*f*x
 + 2*e) - 4*I*a^2*f^2*e^(f*x + e) - 2*a^2*f^2), x))/(a^2*f^2*e^(3*f*x + 3*e
) - 3*I*a^2*f^2*e^(2*f*x + 2*e) - 3*a^2*f^2*e^(f*x + e) + I*a^2*f^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a\left(i\sinh\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+I*a*sinh(f*x+e))**(3/2),x)
```

```
[Out] Integral(x**3/(a*(I*sinh(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(i\,a\sinh\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

3.142 $\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=506

$$\frac{2ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}}-\frac{2ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}}-\frac{4i\cosh\left(\frac{e}{2}+\frac{fx}{2}\right.}{af^3}$$

```
[Out] (2*x)/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - (4*ArcTan[Sinh[e/2 + (I/4)*Pi +
 (f*x)/2]]*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/(a*f^3*Sqrt[a + I*a*Sinh[e + f*x
]]) + (I*x^2*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f
*x)/2])/(a*f*Sqrt[a + I*a*Sinh[e + f*x]]) + ((2*I)*x*Cosh[e/2 + (I/4)*Pi +
(f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sin
h[e + f*x]]) - ((2*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e -
 I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((4*I)*Cosh[e/2
 + (I/4)*Pi + (f*x)/2]*PolyLog[3, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^3*Sq
rt[a + I*a*Sinh[e + f*x]]) + ((4*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[
3, E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^3*Sqrt[a + I*a*Sinh[e + f*x]]) + (x^
2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(2*a*f*Sqrt[a + I*a*Sinh[e + f*x]])
```

Rubi [A] time = 0.311229, antiderivative size = 506, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3319, 4186, 3770, 4182, 2531, 2282, 6589}

$$\frac{2ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}}-\frac{2ix\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}}-\frac{4i\cosh\left(\frac{e}{2}+\frac{fx}{2}\right.}{af^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] (2*x)/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - (4*ArcTan[Sinh[e/2 + (I/4)*Pi +
 (f*x)/2]]*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/(a*f^3*Sqrt[a + I*a*Sinh[e + f*x
]]) + (I*x^2*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f
*x)/2])/(a*f*Sqrt[a + I*a*Sinh[e + f*x]]) + ((2*I)*x*Cosh[e/2 + (I/4)*Pi +
(f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sin
h[e + f*x]]) - ((2*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e -
 I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((4*I)*Cosh[e/2
 + (I/4)*Pi + (f*x)/2]*PolyLog[3, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^3*Sq
rt[a + I*a*Sinh[e + f*x]]) + ((4*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[
3, E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^3*Sqrt[a + I*a*Sinh[e + f*x]]) + (x^
2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(2*a*f*Sqrt[a + I*a*Sinh[e + f*x]])
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
```

```
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{(a+ia\sinh(e+fx))^{3/2}}\,dx = -\frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\int x^2\mathrm{csch}^3\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)dx}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{2x}{af^2\sqrt{a+ia\sinh(e+fx)}} + \frac{x^2\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+ia\sinh(e+fx)}} + \frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\int x^2\mathrm{csch}\left(\frac{e}{2}\right.}{4a\sqrt{a+ia\sinh(e+f}}$$

$$= \frac{2x}{af^2\sqrt{a+ia\sinh(e+fx)}} - \frac{4\tan^{-1}\left(\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}} + \frac{ix^2\tanh}{}$$

$$= \frac{2x}{af^2\sqrt{a+ia\sinh(e+fx)}} - \frac{4\tan^{-1}\left(\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}} + \frac{ix^2\tanh}{}$$

$$= \frac{2x}{af^2\sqrt{a+ia\sinh(e+fx)}} - \frac{4\tan^{-1}\left(\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}} + \frac{ix^2\tanh}{}$$

$$= \frac{2x}{af^2\sqrt{a+ia\sinh(e+fx)}} - \frac{4\tan^{-1}\left(\sinh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af^3\sqrt{a+ia\sinh(e+fx)}} + \frac{ix^2\tanh}{}$$

Mathematica [A] time = 1.76413, size = 384, normalized size = 0.76

$$\frac{\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)\left(-\left(\frac{1}{2}-\frac{i}{2}\right)(-1)^{3/4}\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)^2\left(4fx\mathrm{PolyLog}\left(2,-(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] ((Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])*(f*x*(4 + I*f*x)*(Cosh[(e + f*x)
/2] + I*Sinh[(e + f*x)/2]) - (1/2 - I/2)*(-1)^(3/4)*(-16*ArcTanh[(-1)^(3/4)
*E^((e + f*x)/2)] + 2*e^2*ArcTanh[(-1)^(3/4)*E^((e + f*x)/2)] + e^2*Log[1 -
 (-1)^(3/4)*E^((e + f*x)/2)] - f^2*x^2*Log[1 - (-1)^(3/4)*E^((e + f*x)/2)]
- e^2*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] + f^2*x^2*Log[1 + (-1)^(3/4)*E^((
e + f*x)/2)] + 4*f*x*PolyLog[2, -((-1)^(3/4)*E^((e + f*x)/2))] - 4*f*x*Poly
Log[2, (-1)^(3/4)*E^((e + f*x)/2)] - 8*PolyLog[3, -((-1)^(3/4)*E^((e + f*x)
/2))] + 8*PolyLog[3, (-1)^(3/4)*E^((e + f*x)/2)])*(Cosh[(e + f*x)/2] + I*Si
nh[(e + f*x)/2])^2 + 2*f^2*x^2*Sinh[(e + f*x)/2]))/(2*f^3*(a + I*a*Sinh[e +
 f*x])^(3/2))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int x^2\left(a+ia\sinh\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+I*a*sinh(f*x+e))^(3/2),x)
```

```
[Out] int(x^2/(a+I*a*sinh(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(i a \sinh\left(f x + e\right) + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\frac{1}{2}}\left(\left(-i f x^2 - 4 i x\right)e^{\left(2 f x + 2 e\right)} + \left(f x^2 - 4 x\right)e^{\left(f x + e\right)}\right)\sqrt{i a e^{\left(2 f x + 2 e\right)} + 2 a e^{\left(f x + e\right)} - i a}\, e^{\left(-\frac{1}{2} f x - \frac{1}{2} e\right)} + \left(a^2 f^2 e^{\left(3 f x + 3 e\right)} - 3 i a^2 f^2}{a^2 f^2 e^{\left(3 f x + 3 e\right)} - 3 i a^2 f^2 e^{\left(2 f x + 2 e\right)} - 3 a^2 f^2 e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(1/2)*((-I*f*x^2 - 4*I*x)*e^(2*f*x + 2*e) + (f*x^2 - 4*x)*e^(f*x + e))
*sqrt(I*a*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*e^(-1/2*f*x - 1/2*e) + (
a^2*f^2*e^(3*f*x + 3*e) - 3*I*a^2*f^2*e^(2*f*x + 2*e) - 3*a^2*f^2*e^(f*x +
e) + I*a^2*f^2)*integral(sqrt(1/2)*(-I*f^2*x^2 + 8*I)*sqrt(I*a*e^(2*f*x + 2
*e) + 2*a*e^(f*x + e) - I*a)*e^(1/2*f*x + 1/2*e)/(2*a^2*f^2*e^(2*f*x + 2*e)
 - 4*I*a^2*f^2*e^(f*x + e) - 2*a^2*f^2), x))/(a^2*f^2*e^(3*f*x + 3*e) - 3*I
*a^2*f^2*e^(2*f*x + 2*e) - 3*a^2*f^2*e^(f*x + e) + I*a^2*f^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a\left(i \sinh\left(e + f x\right) + 1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+I*a*sinh(f*x+e))**(3/2),x)
```

```
[Out] Integral(x**2/(a*(I*sinh(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(i a \sinh\left(f x + e\right) + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

3.143 $\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=288

$$\frac{i\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}}-\frac{i\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}}+\frac{1}{af^2\sqrt{a+ia\sinh}}$$

```
[Out] 1/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) + (I*x*ArcTanh[E^((2*e - I*Pi)/4 + (f
*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/(a*f*Sqrt[a + I*a*Sinh[e + f*x]]) +
 (I*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)
])/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - (I*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*
PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]
]) + (x*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(2*a*f*Sqrt[a + I*a*Sinh[e + f*x]])
```

Rubi [A] time = 0.169333, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3319, 4185, 4182, 2279, 2391}

$$\frac{i\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}}-\frac{i\cosh\left(\frac{e}{2}+\frac{fx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{fx}{2}+\frac{1}{4}(2e-i\pi)}\right)}{af^2\sqrt{a+ia\sinh(e+fx)}}+\frac{1}{af^2\sqrt{a+ia\sinh}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + I*a*Sinh[e + f*x])^(3/2),x]
```


```
[Out] 1/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) + (I*x*ArcTanh[E^((2*e - I*Pi)/4 + (f
*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/(a*f*Sqrt[a + I*a*Sinh[e + f*x]]) +
 (I*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)
])/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - (I*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*
PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]
]) + (x*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(2*a*f*Sqrt[a + I*a*Sinh[e + f*x]])
```


Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x}{(a+ia\sinh(e+fx))^{3/2}}\,dx = -\frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\int x\operatorname{csch}^3\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)dx}{2a\sqrt{a+ia\sinh(e+fx)}}$$

$$= \frac{1}{af^2\sqrt{a+ia\sinh(e+fx)}} + \frac{x\tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2af\sqrt{a+ia\sinh(e+fx)}} + \frac{\sinh\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right)\int x\operatorname{csch}\left(\frac{e}{2}-\ldots\right.}{4a\sqrt{a+ia\sinh(e+f\ldots}}$$

$$= \frac{1}{af^2\sqrt{a+ia\sinh(e+fx)}} + \frac{ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af\sqrt{a+ia\sinh(e+fx)}} + \frac{x\tanh\left(\frac{e}{2}+\ldots\right.}{2af\sqrt{a+ia\,\mathrm{si}\ldots}}$$

$$= \frac{1}{af^2\sqrt{a+ia\sinh(e+fx)}} + \frac{ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af\sqrt{a+ia\sinh(e+fx)}} + \frac{x\tanh\left(\frac{e}{2}+\ldots\right.}{2af\sqrt{a+ia\,\mathrm{si}\ldots}}$$

$$= \frac{1}{af^2\sqrt{a+ia\sinh(e+fx)}} + \frac{ix\tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af\sqrt{a+ia\sinh(e+fx)}} + \frac{i\cosh\left(\frac{e}{2}+\frac{i\pi}{4}\ldots\right.}{af^2\sqrt{\ldots}}$$

Mathematica [A] time = 0.852845, size = 332, normalized size = 1.15

$$\frac{\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{i\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right)^2\left(-2\mathrm{PolyLog}\left(2,-\sqrt[4]{-1}e^{-\frac{e}{2}-\frac{fx}{2}}\right)+2\mathrm{PolyLog}\left(2,\sqrt[4]{-1}e^{-\frac{e}{2}-\frac{fx}{2}}\right)+\frac{1}{2}i(2ie\ldots\right.}{\sqrt{2}}\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] ((Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])*((2 + I*f*x)*(Cosh[(e + f*x)/2]
+ I*Sinh[(e + f*x)/2]) - Sqrt[2]*e*ArcTan[(I + Tanh[(e + f*x)/4])/Sqrt[2]]*
(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^2 + (I*(Pi*ArcTan[(I + Tanh[(e +
f*x)/4])/Sqrt[2]] + (I/2)*((2*I)*e + Pi + (2*I)*f*x)*(Log[1 - (-1)^(1/4)*E^
(-e/2 - (f*x)/2)] - Log[1 + (-1)^(1/4)*E^(-e/2 - (f*x)/2)]) - 2*PolyLog[2,
-((-1)^(1/4)*E^(-e/2 - (f*x)/2))] + 2*PolyLog[2, (-1)^(1/4)*E^(-e/2 - (f*x)
/2)])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^2)/Sqrt[2] + 2*f*x*Sinh[(e
+ f*x)/2]))/(2*f^2*(a + I*a*Sinh[e + f*x])^(3/2))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int x\left(a+ia\sinh\left(fx+e\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+I*a*sinh(f*x+e))^(3/2),x)
```

```
[Out] int(x/(a+I*a*sinh(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(i\,a\sinh\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\frac{1}{2}}\left(\left(-i\,fx-2i\right)e^{\left(2\,fx+2\,e\right)}+\left(fx-2\right)e^{\left(fx+e\right)}\right)\sqrt{i\,ae^{\left(2\,fx+2\,e\right)}+2\,ae^{\left(fx+e\right)}-i\,ae}^{\left(-\frac{1}{2}\,fx-\frac{1}{2}\,e\right)}+\left(a^2f^2e^{\left(3\,fx+3\,e\right)}-3i\,a^2f^2e^{\left(2\,fx-\right.}\right.}{a^2f^2e^{\left(3\,fx+3\,e\right)}-3i\,a^2f^2e^{\left(2\,fx+2\,e\right)}-3\,a^2f^2e^{\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(1/2)*((-I*f*x - 2*I)*e^(2*f*x + 2*e) + (f*x - 2)*e^(f*x + e))*sqrt(I*
a*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*e^(-1/2*f*x - 1/2*e) + (a^2*f^2*
e^(3*f*x + 3*e) - 3*I*a^2*f^2*e^(2*f*x + 2*e) - 3*a^2*f^2*e^(f*x + e) + I*a
^2*f^2)*integral(-I*sqrt(1/2)*sqrt(I*a*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) -
I*a)*x*e^(1/2*f*x + 1/2*e)/(2*a^2*e^(2*f*x + 2*e) - 4*I*a^2*e^(f*x + e) - 2
*a^2), x))/(a^2*f^2*e^(3*f*x + 3*e) - 3*I*a^2*f^2*e^(2*f*x + 2*e) - 3*a^2*f
^2*e^(f*x + e) + I*a^2*f^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a\left(i\sinh\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(f*x+e))**(3/2),x)
```

```
[Out] Integral(x/(a*(I*sinh(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(ia\sinh\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

3.144 $\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{x(a+ia\sinh(e+fx))^{3/2}},x\right)$$

```
[Out] Unintegrable[1/(x*(a + I*a*Sinh[e + f*x])^(3/2)), x]
```

Rubi [A] time = 0.0876549, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + I*a*Sinh[e + f*x])^(3/2)),x]
```

```
[Out] Defer[Int][1/(x*(a + I*a*Sinh[e + f*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx=\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx$$

Mathematica [A] time = 21.6249, size = 0, normalized size = 0.

$$\int \frac{1}{x(a+ia\sinh(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + I*a*Sinh[e + f*x])^(3/2)),x]
```

```
[Out] Integrate[1/(x*(a + I*a*Sinh[e + f*x])^(3/2)), x]
```

Maple [A] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+ia\sinh\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+I*a*sinh(f*x+e))^(3/2),x)
```

```
[Out] int(1/x/(a+I*a*sinh(f*x+e))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(i\,a\sinh\left(fx+e\right)+a\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*sinh(f*x + e) + a)^(3/2)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\frac{1}{2}}\left(\left(-i\,fx+2i\right)e^{\left(2\,fx+2\,e\right)}+\left(fx+2\right)e^{\left(fx+e\right)}\right)\sqrt{i\,ae^{\left(2\,fx+2\,e\right)}+2\,ae^{\left(fx+e\right)}-i\,a}\,e^{\left(-\frac{1}{2}\,fx-\frac{1}{2}\,e\right)}+\left(a^2f^2x^2e^{\left(3\,fx+3\,e\right)}-3i\,a^2f^2x^2e^{\left(2\,f\right.}\right.}{a^2f^2x^2e^{\left(3\,fx+3\,e\right)}-3i\,a^2f^2x^2e^{\left(2\,fx+2\,e\right)}-3\,a^2f^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(1/2)*((-I*f*x + 2*I)*e^(2*f*x + 2*e) + (f*x + 2)*e^(f*x + e))*sqrt(I*
a*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*e^(-1/2*f*x - 1/2*e) + (a^2*f^2*
x^2*e^(3*f*x + 3*e) - 3*I*a^2*f^2*x^2*e^(2*f*x + 2*e) - 3*a^2*f^2*x^2*e^(f*
x + e) + I*a^2*f^2*x^2)*integral(sqrt(1/2)*(-I*f^2*x^2 + 8*I)*sqrt(I*a*e^(2
*f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*e^(1/2*f*x + 1/2*e)/(2*a^2*f^2*x^3*e^(
2*f*x + 2*e) - 4*I*a^2*f^2*x^3*e^(f*x + e) - 2*a^2*f^2*x^3), x))/(a^2*f^2*x
^2*e^(3*f*x + 3*e) - 3*I*a^2*f^2*x^2*e^(2*f*x + 2*e) - 3*a^2*f^2*x^2*e^(f*x
 + e) + I*a^2*f^2*x^2)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a\left(i\sinh\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(x*(a*(I*sinh(e + f*x) + 1))**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(i\,a\sinh\left(fx+e\right)+a\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*sinh(f*x + e) + a)^(3/2)*x), x)
```

3.145 $\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}},x\right)$$

```
[Out] Unintegrable[1/(x^2*(a + I*a*Sinh[e + f*x])^(3/2)), x]
```

Rubi [A] time = 0.085883, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(a + I*a*Sinh[e + f*x])^(3/2)),x]
```

```
[Out] Defer[Int][1/(x^2*(a + I*a*Sinh[e + f*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx=\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx$$

Mathematica [A] time = 24.0865, size = 0, normalized size = 0.

$$\int \frac{1}{x^2(a+ia\sinh(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(a + I*a*Sinh[e + f*x])^(3/2)),x]
```

```
[Out] Integrate[1/(x^2*(a + I*a*Sinh[e + f*x])^(3/2)), x]
```

Maple [A] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(a+ia\sinh\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x)
```

```
[Out] int(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(i a \sinh\left(fx+e\right)+a\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*sinh(f*x + e) + a)^(3/2)*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\frac{1}{2}}\left(\left(-i fx+4i\right)e^{\left(2fx+2e\right)}+\left(fx+4\right)e^{\left(fx+e\right)}\right)\sqrt{i a e^{\left(2fx+2e\right)}+2 a e^{\left(fx+e\right)}-i a}\, e^{\left(-\frac{1}{2}fx-\frac{1}{2}e\right)}+\left(a^2f^2x^3e^{\left(3fx+3e\right)}-3i a^2f^2x^3e^{\left(2f\right.}\right.}{a^2f^2x^3e^{\left(3fx+3e\right)}-3i a^2f^2x^3e^{\left(2fx+2e\right)}-3a^2f^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(1/2)*((-I*f*x + 4*I)*e^(2*f*x + 2*e) + (f*x + 4)*e^(f*x + e))*sqrt(I*
a*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*e^(-1/2*f*x - 1/2*e) + (a^2*f^2*
x^3*e^(3*f*x + 3*e) - 3*I*a^2*f^2*x^3*e^(2*f*x + 2*e) - 3*a^2*f^2*x^3*e^(f*
x + e) + I*a^2*f^2*x^3)*integral(sqrt(1/2)*(-I*f^2*x^2 + 24*I)*sqrt(I*a*e^(
2*f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*e^(1/2*f*x + 1/2*e)/(2*a^2*f^2*x^4*e^
(2*f*x + 2*e) - 4*I*a^2*f^2*x^4*e^(f*x + e) - 2*a^2*f^2*x^4), x))/(a^2*f^2*
x^3*e^(3*f*x + 3*e) - 3*I*a^2*f^2*x^3*e^(2*f*x + 2*e) - 3*a^2*f^2*x^3*e^(f*
x + e) + I*a^2*f^2*x^3)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a\left(i\sinh\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+I*a*sinh(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a*(I*sinh(e + f*x) + 1))**(3/2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(i a \sinh\left(fx+e\right)+a\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*sinh(f*x + e) + a)^(3/2)*x^2), x)
```

3.146 $\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}} \, dx$

Optimal. Leaf size=1016

result too large to display

```
[Out] -(1/(a^2*d^4*Sqrt[a + I*a*Sinh[c + d*x]])) + (9*x^2)/(8*a^2*d^2*Sqrt[a + I*
a*Sinh[c + d*x]]) - ((10*I)*x*ArcTanh[E^((2*c - I*Pi)/4 + (d*x)/2)]*Cosh[c/
2 + (I/4)*Pi + (d*x)/2])/(a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (((3*I)/8)
*x^3*ArcTanh[E^((2*c - I*Pi)/4 + (d*x)/2)]*Cosh[c/2 + (I/4)*Pi + (d*x)/2])/
(a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) - ((10*I)*Cosh[c/2 + (I/4)*Pi + (d*x)/2
]*PolyLog[2, -E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^4*Sqrt[a + I*a*Sinh[c +
 d*x]]) + (((9*I)/8)*x^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, -E^((2*c
 - I*Pi)/4 + (d*x)/2)])/(a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) + ((10*I)*Cos
h[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*
d^4*Sqrt[a + I*a*Sinh[c + d*x]]) - (((9*I)/8)*x^2*Cosh[c/2 + (I/4)*Pi + (d*
x)/2]*PolyLog[2, E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^2*Sqrt[a + I*a*Sinh[
c + d*x]]) - (((9*I)/2)*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[3, -E^((2*
c - I*Pi)/4 + (d*x)/2)])/(a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (((9*I)/2)
*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[3, E^((2*c - I*Pi)/4 + (d*x)/2)])
/(a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + ((9*I)*Cosh[c/2 + (I/4)*Pi + (d*x)
/2]*PolyLog[4, -E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^4*Sqrt[a + I*a*Sinh[c
 + d*x]]) - ((9*I)*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[4, E^((2*c - I*Pi
)/4 + (d*x)/2)])/(a^2*d^4*Sqrt[a + I*a*Sinh[c + d*x]]) + (x^2*Sech[c/2 + (I
/4)*Pi + (d*x)/2]^2)/(4*a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - (x*Tanh[c/2
+ (I/4)*Pi + (d*x)/2])/(2*a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (3*x^3*Tan
h[c/2 + (I/4)*Pi + (d*x)/2])/(16*a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) + (x^3*
Sech[c/2 + (I/4)*Pi + (d*x)/2]^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(8*a^2*d*S
qrt[a + I*a*Sinh[c + d*x]])
```

Rubi [A] time = 0.676576, antiderivative size = 1016, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3319, 4186, 4185, 4182, 2279, 2391, 2531, 6609, 2282, 6589}

$$\frac{\operatorname{sech}^2\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)x^3}{8a^2d\sqrt{i\sinh(c+dx)a+a}}+\frac{3\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)x^3}{16a^2d\sqrt{i\sinh(c+dx)a+a}}+\frac{3i\tanh^{-1}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right.}{8a^2d\sqrt{i\sinh(c+dx)a+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + I*a*Sinh[c + d*x])^(5/2),x]
```


```
[Out] -(1/(a^2*d^4*Sqrt[a + I*a*Sinh[c + d*x]])) + (9*x^2)/(8*a^2*d^2*Sqrt[a + I*
a*Sinh[c + d*x]]) - ((10*I)*x*ArcTanh[E^((2*c - I*Pi)/4 + (d*x)/2)]*Cosh[c/
2 + (I/4)*Pi + (d*x)/2])/(a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (((3*I)/8)
*x^3*ArcTanh[E^((2*c - I*Pi)/4 + (d*x)/2)]*Cosh[c/2 + (I/4)*Pi + (d*x)/2])/
(a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) - ((10*I)*Cosh[c/2 + (I/4)*Pi + (d*x)/2
]*PolyLog[2, -E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^4*Sqrt[a + I*a*Sinh[c +
 d*x]]) + (((9*I)/8)*x^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, -E^((2*c
 - I*Pi)/4 + (d*x)/2)])/(a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) + ((10*I)*Cos
h[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*
d^4*Sqrt[a + I*a*Sinh[c + d*x]]) - (((9*I)/8)*x^2*Cosh[c/2 + (I/4)*Pi + (d*
x)/2]*PolyLog[2, E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^2*Sqrt[a + I*a*Sinh[
c + d*x]]) - (((9*I)/2)*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[3, -E^((2*
c - I*Pi)/4 + (d*x)/2)])/(a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (((9*I)/2)
*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[3, E^((2*c - I*Pi)/4 + (d*x)/2)])
```

```
/(a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + ((9*I)*Cosh[c/2 + (I/4)*Pi + (d*x)
/2]*PolyLog[4, -E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^4*Sqrt[a + I*a*Sinh[c
 + d*x]]) - ((9*I)*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[4, E^((2*c - I*Pi
)/4 + (d*x)/2)])/(a^2*d^4*Sqrt[a + I*a*Sinh[c + d*x]]) + (x^2*Sech[c/2 + (I
/4)*Pi + (d*x)/2]^2)/(4*a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - (x*Tanh[c/2
+ (I/4)*Pi + (d*x)/2])/(2*a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (3*x^3*Tan
h[c/2 + (I/4)*Pi + (d*x)/2])/(16*a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) + (x^3*
Sech[c/2 + (I/4)*Pi + (d*x)/2]^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(8*a^2*d*S
qrt[a + I*a*Sinh[c + d*x]])
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \frac{\sinh\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\int x^3\operatorname{csch}^5\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)dx}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$= \frac{x^2\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}} + \frac{x^3\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}} - \frac{\left(3\sinh\left(\frac{c}{2}\right.\right.}{}$$

$$= -\frac{1}{a^2d^4\sqrt{a+ia\sinh(c+dx)}} + \frac{9x^2}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} + \frac{x^2\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{4a^2d^2\sqrt{a+ia\sinh(c+dx}}$$

$$= -\frac{1}{a^2d^4\sqrt{a+ia\sinh(c+dx)}} + \frac{9x^2}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{10ix\tanh^{-1}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right.}{a^2d^3\sqrt{a+ia\sin}}$$

$$= -\frac{1}{a^2d^4\sqrt{a+ia\sinh(c+dx)}} + \frac{9x^2}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{10ix\tanh^{-1}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right.}{a^2d^3\sqrt{a+ia\sin}}$$

$$= -\frac{1}{a^2d^4\sqrt{a+ia\sinh(c+dx)}} + \frac{9x^2}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{10ix\tanh^{-1}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right.}{a^2d^3\sqrt{a+ia\sin}}$$

$$= -\frac{1}{a^2d^4\sqrt{a+ia\sinh(c+dx)}} + \frac{9x^2}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{10ix\tanh^{-1}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right.}{a^2d^3\sqrt{a+ia\sin}}$$

$$= -\frac{1}{a^2d^4\sqrt{a+ia\sinh(c+dx)}} + \frac{9x^2}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{10ix\tanh^{-1}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right.}{a^2d^3\sqrt{a+ia\sin}}$$

Mathematica [A] time = 4.53243, size = 1200, normalized size = 1.18

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(-48*Cosh[(c + d*x)/2] + (8*I)*c
*Cosh[(c + d*x)/2] + 70*c^2*Cosh[(c + d*x)/2] - (11*I)*c^3*Cosh[(c + d*x)/2
] - (8*I)*(c + d*x)*Cosh[(c + d*x)/2] - 140*c*(c + d*x)*Cosh[(c + d*x)/2] +
 (33*I)*c^2*(c + d*x)*Cosh[(c + d*x)/2] + 70*(c + d*x)^2*Cosh[(c + d*x)/2]
- (33*I)*c*(c + d*x)^2*Cosh[(c + d*x)/2] + (11*I)*(c + d*x)^3*Cosh[(c + d*x
)/2] + 16*Cosh[(3*(c + d*x))/2] + (8*I)*c*Cosh[(3*(c + d*x))/2] - 18*c^2*Co
sh[(3*(c + d*x))/2] - (3*I)*c^3*Cosh[(3*(c + d*x))/2] - (8*I)*(c + d*x)*Cos
h[(3*(c + d*x))/2] + 36*c*(c + d*x)*Cosh[(3*(c + d*x))/2] + (9*I)*c^2*(c +
d*x)*Cosh[(3*(c + d*x))/2] - 18*(c + d*x)^2*Cosh[(3*(c + d*x))/2] - (9*I)*c
*(c + d*x)^2*Cosh[(3*(c + d*x))/2] + (3*I)*(c + d*x)^3*Cosh[(3*(c + d*x))/2
] + (1 - I)*(-1)^(3/4)*(-160*c*ArcTanh[(-1)^(3/4)*E^((c + d*x)/2)] + 6*c^3*
ArcTanh[(-1)^(3/4)*E^((c + d*x)/2)] - 80*c*Log[1 - (-1)^(3/4)*E^((c + d*x)/
2)] + 3*c^3*Log[1 - (-1)^(3/4)*E^((c + d*x)/2)] - 80*d*x*Log[1 - (-1)^(3/4)
*E^((c + d*x)/2)] + 3*d^3*x^3*Log[1 - (-1)^(3/4)*E^((c + d*x)/2)] + 80*c*Lo
g[1 + (-1)^(3/4)*E^((c + d*x)/2)] - 3*c^3*Log[1 + (-1)^(3/4)*E^((c + d*x)/2
)] + 80*d*x*Log[1 + (-1)^(3/4)*E^((c + d*x)/2)] - 3*d^3*x^3*Log[1 + (-1)^(3
/4)*E^((c + d*x)/2)] - 2*(-80 + 9*d^2*x^2)*PolyLog[2, -((-1)^(3/4)*E^((c +
d*x)/2))] + 2*(-80 + 9*d^2*x^2)*PolyLog[2, (-1)^(3/4)*E^((c + d*x)/2)] + 72
*d*x*PolyLog[3, -((-1)^(3/4)*E^((c + d*x)/2))] - 72*d*x*PolyLog[3, (-1)^(3/
4)*E^((c + d*x)/2)] - 144*PolyLog[4, -((-1)^(3/4)*E^((c + d*x)/2))] + 144*P
olyLog[4, (-1)^(3/4)*E^((c + d*x)/2)])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x
)/2])^4 - (48*I)*Sinh[(c + d*x)/2] + 8*c*Sinh[(c + d*x)/2] + (70*I)*c^2*Sin
h[(c + d*x)/2] - 11*c^3*Sinh[(c + d*x)/2] - 8*(c + d*x)*Sinh[(c + d*x)/2] -
 (140*I)*c*(c + d*x)*Sinh[(c + d*x)/2] + 33*c^2*(c + d*x)*Sinh[(c + d*x)/2]
 + (70*I)*(c + d*x)^2*Sinh[(c + d*x)/2] - 33*c*(c + d*x)^2*Sinh[(c + d*x)/2
] + 11*(c + d*x)^3*Sinh[(c + d*x)/2] - (16*I)*Sinh[(3*(c + d*x))/2] - 8*c*S
inh[(3*(c + d*x))/2] + (18*I)*c^2*Sinh[(3*(c + d*x))/2] + 3*c^3*Sinh[(3*(c
+ d*x))/2] + 8*(c + d*x)*Sinh[(3*(c + d*x))/2] - (36*I)*c*(c + d*x)*Sinh[(3
*(c + d*x))/2] - 9*c^2*(c + d*x)*Sinh[(3*(c + d*x))/2] + (18*I)*(c + d*x)^2
*Sinh[(3*(c + d*x))/2] + 9*c*(c + d*x)^2*Sinh[(3*(c + d*x))/2] - 3*(c + d*x
)^3*Sinh[(3*(c + d*x))/2]))/(32*d^4*(a + I*a*Sinh[c + d*x])^(5/2))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int x^{3} \left(a + ia \sinh \left(dx + c\right)\right)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+I*a*sinh(d*x+c))^(5/2),x)
```

```
[Out] int(x^3/(a+I*a*sinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\left(i\, a \sinh \left(dx + c\right) + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{\frac{1}{2}}\left(\left(-3i\,d^3x^3 - 18i\,d^2x^2 + 8i\,dx + 16i\right)e^{(4\,dx+4\,c)} - \left(11\,d^3x^3 + 70\,d^2x^2 - 8\,dx - 48\right)e^{(3\,dx+3\,c)} + \left(-11i\,d^3x^3 + 70i\,d^2x^2 + 8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] (sqrt(1/2)*((-3*I*d^3*x^3 - 18*I*d^2*x^2 + 8*I*d*x + 16*I)*e^(4*d*x + 4*c)
- (11*d^3*x^3 + 70*d^2*x^2 - 8*d*x - 48)*e^(3*d*x + 3*c) + (-11*I*d^3*x^3 +
 70*I*d^2*x^2 + 8*I*d*x - 48*I)*e^(2*d*x + 2*c) - (3*d^3*x^3 - 18*d^2*x^2 -
 8*d*x + 16)*e^(d*x + c))*sqrt(I*a*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - I*a)
*e^(-1/2*d*x - 1/2*c) + (8*a^3*d^4*e^(5*d*x + 5*c) - 40*I*a^3*d^4*e^(4*d*x
+ 4*c) - 80*a^3*d^4*e^(3*d*x + 3*c) + 80*I*a^3*d^4*e^(2*d*x + 2*c) + 40*a^3
*d^4*e^(d*x + c) - 8*I*a^3*d^4)*integral(sqrt(1/2)*(-3*I*d^2*x^3 + 80*I*x)*
sqrt(I*a*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - I*a)*e^(1/2*d*x + 1/2*c)/(16*a
^3*d^2*e^(2*d*x + 2*c) - 32*I*a^3*d^2*e^(d*x + c) - 16*a^3*d^2), x))/(8*a^3
*d^4*e^(5*d*x + 5*c) - 40*I*a^3*d^4*e^(4*d*x + 4*c) - 80*a^3*d^4*e^(3*d*x +
 3*c) + 80*I*a^3*d^4*e^(2*d*x + 2*c) + 40*a^3*d^4*e^(d*x + c) - 8*I*a^3*d^4
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(i\,a\sinh{(dx + c)} + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

3.147 $\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=689

$$\frac{3ix\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}}-\frac{3ix\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}}-\frac{3i\cosh\left(\frac{c}{2}+\frac{dx}{2}\cdots\right.}{2a^2d\cdots}$$

```
[Out] (3*x)/(4*a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - (5*ArcTan[Sinh[c/2 + (I/4)*
Pi + (d*x)/2]]*Cosh[c/2 + (I/4)*Pi + (d*x)/2])/(3*a^2*d^3*Sqrt[a + I*a*Sinh
[c + d*x]]) + (((3*I)/8)*x^2*ArcTanh[E^((2*c - I*Pi)/4 + (d*x)/2)]*Cosh[c/2
 + (I/4)*Pi + (d*x)/2])/(a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) + (((3*I)/4)*x*
Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, -E^((2*c - I*Pi)/4 + (d*x)/2)])/(
a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - (((3*I)/4)*x*Cosh[c/2 + (I/4)*Pi + (
d*x)/2]*PolyLog[2, E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^2*Sqrt[a + I*a*Sin
h[c + d*x]]) - (((3*I)/2)*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[3, -E^((2*
c - I*Pi)/4 + (d*x)/2)])/(a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (((3*I)/2)
*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[3, E^((2*c - I*Pi)/4 + (d*x)/2)])/(
a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (x*Sech[c/2 + (I/4)*Pi + (d*x)/2]^2)
/(6*a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - Tanh[c/2 + (I/4)*Pi + (d*x)/2]/(
6*a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (3*x^2*Tanh[c/2 + (I/4)*Pi + (d*x)
/2])/(16*a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) + (x^2*Sech[c/2 + (I/4)*Pi + (d
*x)/2]^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(8*a^2*d*Sqrt[a + I*a*Sinh[c + d*x
]])
```

Rubi [A] time = 0.460092, antiderivative size = 689, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3319, 4186, 3768, 3770, 4182, 2531, 2282, 6589}

$$\frac{3ix\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}}-\frac{3ix\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}}-\frac{3i\cosh\left(\frac{c}{2}+\frac{dx}{2}\cdots\right.}{2a^2d\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (3*x)/(4*a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - (5*ArcTan[Sinh[c/2 + (I/4)*
Pi + (d*x)/2]]*Cosh[c/2 + (I/4)*Pi + (d*x)/2])/(3*a^2*d^3*Sqrt[a + I*a*Sinh
[c + d*x]]) + (((3*I)/8)*x^2*ArcTanh[E^((2*c - I*Pi)/4 + (d*x)/2)]*Cosh[c/2
 + (I/4)*Pi + (d*x)/2])/(a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) + (((3*I)/4)*x*
Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, -E^((2*c - I*Pi)/4 + (d*x)/2)])/(
a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - (((3*I)/4)*x*Cosh[c/2 + (I/4)*Pi + (
d*x)/2]*PolyLog[2, E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^2*Sqrt[a + I*a*Sin
h[c + d*x]]) - (((3*I)/2)*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[3, -E^((2*
c - I*Pi)/4 + (d*x)/2)])/(a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (((3*I)/2)
*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[3, E^((2*c - I*Pi)/4 + (d*x)/2)])/(
a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (x*Sech[c/2 + (I/4)*Pi + (d*x)/2]^2)
/(6*a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - Tanh[c/2 + (I/4)*Pi + (d*x)/2]/(
6*a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (3*x^2*Tanh[c/2 + (I/4)*Pi + (d*x)
/2])/(16*a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) + (x^2*Sech[c/2 + (I/4)*Pi + (d
*x)/2]^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(8*a^2*d*Sqrt[a + I*a*Sinh[c + d*x
]])
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \frac{\sinh\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\int x^2\operatorname{csch}^5\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)dx}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$= \frac{x\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{6a^2d^2\sqrt{a+ia\sinh(c+dx)}} + \frac{x^2\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}} - \frac{\left(3\sinh\left(\frac{c}{2}-\right.\right.}{1}$$

$$= \frac{3x}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}} + \frac{x\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{6a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{6a^2d^3\sqrt{a+ia\sinh(c+dx)}} +$$

$$= \frac{3x}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{5\tan^{-1}\left(\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{3a^2d^3\sqrt{a+ia\sinh(c+dx)}} + \frac{3ix^2\,\mathrm{ta}}{}$$

$$= \frac{3x}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{5\tan^{-1}\left(\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{3a^2d^3\sqrt{a+ia\sinh(c+dx)}} + \frac{3ix^2\,\mathrm{ta}}{}$$

$$= \frac{3x}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{5\tan^{-1}\left(\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{3a^2d^3\sqrt{a+ia\sinh(c+dx)}} + \frac{3ix^2\,\mathrm{ta}}{}$$

$$= \frac{3x}{4a^2d^2\sqrt{a+ia\sinh(c+dx)}} - \frac{5\tan^{-1}\left(\sinh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{3a^2d^3\sqrt{a+ia\sinh(c+dx)}} + \frac{3ix^2\,\mathrm{ta}}{}$$

Mathematica [A] time = 2.28438, size = 482, normalized size = 0.7

$$\frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(\left(-\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^4\left(36dx\mathrm{PolyLog}\left(2,-(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(4*d*x*(4 + (3*I)*d*x)*(Cosh[(c
+ d*x)/2] + I*Sinh[(c + d*x)/2]) + (-8*I + 36*d*x + (9*I)*d^2*x^2)*(Cosh[(c
 + d*x)/2] + I*Sinh[(c + d*x)/2])^3 - (1/2 - I/2)*(-1)^(3/4)*(-160*ArcTanh[
(-1)^(3/4)*E^((c + d*x)/2)] + 18*c^2*ArcTanh[(-1)^(3/4)*E^((c + d*x)/2)] +
9*c^2*Log[1 - (-1)^(3/4)*E^((c + d*x)/2)] - 9*d^2*x^2*Log[1 - (-1)^(3/4)*E^
((c + d*x)/2)] - 9*c^2*Log[1 + (-1)^(3/4)*E^((c + d*x)/2)] + 9*d^2*x^2*Log[
1 + (-1)^(3/4)*E^((c + d*x)/2)] + 36*d*x*PolyLog[2, -((-1)^(3/4)*E^((c + d*
x)/2))] - 36*d*x*PolyLog[2, (-1)^(3/4)*E^((c + d*x)/2)] - 72*PolyLog[3, -((
-1)^(3/4)*E^((c + d*x)/2))] + 72*PolyLog[3, (-1)^(3/4)*E^((c + d*x)/2)])*(C
osh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^4 + 24*d^2*x^2*Sinh[(c + d*x)/2] +
2*(-8 + 9*d^2*x^2)*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2*Sinh[(c + d*
x)/2]))/(48*d^3*(a + I*a*Sinh[c + d*x])^(5/2))
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int x^2\left(a+ia\sinh(dx+c)\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+I*a*sinh(d*x+c))^(5/2),x)
```

```
[Out] int(x^2/(a+I*a*sinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(i a \sinh(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\frac{1}{2}}\left(\left(-9i\,d^2x^2-36i\,dx+8i\right)e^{(4\,dx+4\,c)}-\left(33\,d^2x^2+140\,dx-8\right)e^{(3\,dx+3\,c)}+\left(-33i\,d^2x^2+140i\,dx+8i\right)e^{(2\,dx+2\,c)}-\left(9\,d^2x^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] (sqrt(1/2)*((-9*I*d^2*x^2 - 36*I*d*x + 8*I)*e^(4*d*x + 4*c) - (33*d^2*x^2 +
 140*d*x - 8)*e^(3*d*x + 3*c) + (-33*I*d^2*x^2 + 140*I*d*x + 8*I)*e^(2*d*x
+ 2*c) - (9*d^2*x^2 - 36*d*x - 8)*e^(d*x + c))*sqrt(I*a*e^(2*d*x + 2*c) + 2
*a*e^(d*x + c) - I*a)*e^(-1/2*d*x - 1/2*c) + (24*a^3*d^3*e^(5*d*x + 5*c) -
120*I*a^3*d^3*e^(4*d*x + 4*c) - 240*a^3*d^3*e^(3*d*x + 3*c) + 240*I*a^3*d^3
*e^(2*d*x + 2*c) + 120*a^3*d^3*e^(d*x + c) - 24*I*a^3*d^3)*integral(sqrt(1/
2)*(-9*I*d^2*x^2 + 80*I)*sqrt(I*a*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - I*a)*
e^(1/2*d*x + 1/2*c)/(48*a^3*d^2*e^(2*d*x + 2*c) - 96*I*a^3*d^2*e^(d*x + c)
- 48*a^3*d^2), x))/(24*a^3*d^3*e^(5*d*x + 5*c) - 120*I*a^3*d^3*e^(4*d*x + 4
*c) - 240*a^3*d^3*e^(3*d*x + 3*c) + 240*I*a^3*d^3*e^(2*d*x + 2*c) + 120*a^3
*d^3*e^(d*x + c) - 24*I*a^3*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(i a \sinh(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

3.148 $\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=416

$$\frac{3i\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}}-\frac{3i\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}}+\frac{3}{8a^2d^2\sqrt{a+ia\,\text{s}}}$$

```
[Out] 3/(8*a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) + (((3*I)/8)*x*ArcTanh[E^((2*c -
I*Pi)/4 + (d*x)/2)]*Cosh[c/2 + (I/4)*Pi + (d*x)/2])/(a^2*d*Sqrt[a + I*a*Sin
h[c + d*x]]) + (((3*I)/8)*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, -E^((2*
c - I*Pi)/4 + (d*x)/2)])/(a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - (((3*I)/8)
*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, E^((2*c - I*Pi)/4 + (d*x)/2)])/(
a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) + Sech[c/2 + (I/4)*Pi + (d*x)/2]^2/(12
*a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) + (3*x*Tanh[c/2 + (I/4)*Pi + (d*x)/2]
)/(16*a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) + (x*Sech[c/2 + (I/4)*Pi + (d*x)/2
]^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(8*a^2*d*Sqrt[a + I*a*Sinh[c + d*x]])
```

Rubi [A] time = 0.242906, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3319, 4185, 4182, 2279, 2391}

$$\frac{3i\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,-e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}}-\frac{3i\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\text{PolyLog}\left(2,e^{\frac{dx}{2}+\frac{1}{4}(2c-i\pi)}\right)}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}}+\frac{3}{8a^2d^2\sqrt{a+ia\,\text{s}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] 3/(8*a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) + (((3*I)/8)*x*ArcTanh[E^((2*c -
I*Pi)/4 + (d*x)/2)]*Cosh[c/2 + (I/4)*Pi + (d*x)/2])/(a^2*d*Sqrt[a + I*a*Sin
h[c + d*x]]) + (((3*I)/8)*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, -E^((2*
c - I*Pi)/4 + (d*x)/2)])/(a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - (((3*I)/8)
*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, E^((2*c - I*Pi)/4 + (d*x)/2)])/(
a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) + Sech[c/2 + (I/4)*Pi + (d*x)/2]^2/(12
*a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) + (3*x*Tanh[c/2 + (I/4)*Pi + (d*x)/2]
)/(16*a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) + (x*Sech[c/2 + (I/4)*Pi + (d*x)/2
]^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(8*a^2*d*Sqrt[a + I*a*Sinh[c + d*x]])
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \frac{\sinh\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)\int x\operatorname{csch}^5\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)dx}{4a^2\sqrt{a+ia\sinh(c+dx)}}$$

$$= \frac{\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{12a^2d^2\sqrt{a+ia\sinh(c+dx)}} + \frac{x\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}} - \frac{\left(3\sinh\left(\frac{c}{2}-\right.\right.}{1}$$

$$= \frac{3}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} + \frac{\operatorname{sech}^2\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{12a^2d^2\sqrt{a+ia\sinh(c+dx)}} + \frac{3x\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{16a^2d\sqrt{a+ia\sinh(c+dx)}}$$

$$= \frac{3}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} + \frac{3ix\tanh^{-1}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}} + \frac{\operatorname{sech}^2\left(\right.}{12a^2d^2\sqrt{a\,}}$$

$$= \frac{3}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} + \frac{3ix\tanh^{-1}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}} + \frac{\operatorname{sech}^2\left(\right.}{12a^2d^2\sqrt{a\,}}$$

$$= \frac{3}{8a^2d^2\sqrt{a+ia\sinh(c+dx)}} + \frac{3ix\tanh^{-1}\left(e^{\frac{1}{4}(2c-i\pi)+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{8a^2d\sqrt{a+ia\sinh(c+dx)}} + \frac{3i\cosh\left(\frac{c}{2}\right.}{8a^2}$$

Mathematica [A] time = 1.57683, size = 411, normalized size = 0.99

$$\frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(\frac{9i\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^4\left(-2\text{PolyLog}\left(2,-\sqrt[4]{-1}e^{-\frac{c}{2}-\frac{dx}{2}}\right)+2\text{PolyLog}\left(2,\sqrt[4]{-1}e^{-\frac{c}{2}-\frac{dx}{2}}\right)+\frac{1}{2}i(2ic\right.}{\sqrt{2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(4*(2 + (3*I)*d*x)*(Cosh[(c + d*
x)/2] + I*Sinh[(c + d*x)/2]) + 9*(2 + I*d*x)*(Cosh[(c + d*x)/2] + I*Sinh[(c
 + d*x)/2])^3 - 9*Sqrt[2]*c*ArcTan[(I + Tanh[(c + d*x)/4])/Sqrt[2]]*(Cosh[(
c + d*x)/2] + I*Sinh[(c + d*x)/2])^4 + ((9*I)*(Pi*ArcTan[(I + Tanh[(c + d*x
```

```
)/4])/Sqrt[2]] + (I/2)*((2*I)*c + Pi + (2*I)*d*x)*(Log[1 - (-1)^(1/4)*E^(-c
/2 - (d*x)/2)] - Log[1 + (-1)^(1/4)*E^(-c/2 - (d*x)/2)]) - 2*PolyLog[2, -((
-1)^(1/4)*E^(-c/2 - (d*x)/2))] + 2*PolyLog[2, (-1)^(1/4)*E^(-c/2 - (d*x)/2)
])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^4)/Sqrt[2] + 24*d*x*Sinh[(c +
d*x)/2] + 18*d*x*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2*Sinh[(c + d*x)
/2]))/(48*d^2*(a + I*a*Sinh[c + d*x])^(5/2))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int x\left(a+ia\sinh\left(dx+c\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+I*a*sinh(d*x+c))^(5/2),x)
```

```
[Out] int(x/(a+I*a*sinh(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(i\,a\sinh\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\frac{1}{2}}\left((-9i\,dx-18i)e^{(4\,dx+4\,c)}-(33\,dx+70)e^{(3\,dx+3\,c)}+(-33i\,dx+70i)e^{(2\,dx+2\,c)}-9\,(dx-2)e^{(dx+c)}\right)\sqrt{i\,ae^{(2\,dx+2\,c)}+2\,ae^{(dx}}}{24\,a^3d^2e^{(5\,dx+5\,c)}-120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] (sqrt(1/2)*((-9*I*d*x - 18*I)*e^(4*d*x + 4*c) - (33*d*x + 70)*e^(3*d*x + 3*
c) + (-33*I*d*x + 70*I)*e^(2*d*x + 2*c) - 9*(d*x - 2)*e^(d*x + c))*sqrt(I*a
*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - I*a)*e^(-1/2*d*x - 1/2*c) + (24*a^3*d^
2*e^(5*d*x + 5*c) - 120*I*a^3*d^2*e^(4*d*x + 4*c) - 240*a^3*d^2*e^(3*d*x +
3*c) + 240*I*a^3*d^2*e^(2*d*x + 2*c) + 120*a^3*d^2*e^(d*x + c) - 24*I*a^3*d
^2)*integral(-3*I*sqrt(1/2)*sqrt(I*a*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - I*
a)*x*e^(1/2*d*x + 1/2*c)/(16*a^3*e^(2*d*x + 2*c) - 32*I*a^3*e^(d*x + c) - 1
6*a^3), x))/(24*a^3*d^2*e^(5*d*x + 5*c) - 120*I*a^3*d^2*e^(4*d*x + 4*c) - 2
40*a^3*d^2*e^(3*d*x + 3*c) + 240*I*a^3*d^2*e^(2*d*x + 2*c) + 120*a^3*d^2*e^
(d*x + c) - 24*I*a^3*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(i\,a\sinh(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

3.149 $\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{x(a+ia\sinh(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable[1/(x*(a + I*a*Sinh[c + d*x])^(5/2)), x]
```

Rubi [A] time = 0.0900751, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(a + I*a*Sinh[c + d*x])^(5/2)),x]
```

```
[Out] Defer[Int][1/(x*(a + I*a*Sinh[c + d*x])^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx=\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx$$

Mathematica [A] time = 35.33, size = 0, normalized size = 0.

$$\int \frac{1}{x(a+ia\sinh(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(a + I*a*Sinh[c + d*x])^(5/2)),x]
```

```
[Out] Integrate[1/(x*(a + I*a*Sinh[c + d*x])^(5/2)), x]
```

Maple [A] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x}\left(a+ia\sinh(dx+c)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+I*a*sinh(d*x+c))^(5/2),x)
```

```
[Out] int(1/x/(a+I*a*sinh(d*x+c))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(i\,a\sinh\left(dx+c\right)+a\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*sinh(d*x + c) + a)^(5/2)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\sqrt{\frac{1}{2}}\left(\left(-9i\,d^3x^3+18i\,d^2x^2+8i\,dx-48i\right)e^{(4\,dx+4\,c)}-\left(33\,d^3x^3-70\,d^2x^2-8\,dx+144\right)e^{(3\,dx+3\,c)}+\left(-33i\,d^3x^3-70i\,d^2x^2+8i\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] (sqrt(1/2)*((-9*I*d^3*x^3 + 18*I*d^2*x^2 + 8*I*d*x - 48*I)*e^(4*d*x + 4*c)
- (33*d^3*x^3 - 70*d^2*x^2 - 8*d*x + 144)*e^(3*d*x + 3*c) + (-33*I*d^3*x^3
- 70*I*d^2*x^2 + 8*I*d*x + 144*I)*e^(2*d*x + 2*c) - (9*d^3*x^3 + 18*d^2*x^2
 - 8*d*x - 48)*e^(d*x + c))*sqrt(I*a*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - I*
a)*e^(-1/2*d*x - 1/2*c) + (24*a^3*d^4*x^4*e^(5*d*x + 5*c) - 120*I*a^3*d^4*x
^4*e^(4*d*x + 4*c) - 240*a^3*d^4*x^4*e^(3*d*x + 3*c) + 240*I*a^3*d^4*x^4*e^
(2*d*x + 2*c) + 120*a^3*d^4*x^4*e^(d*x + c) - 24*I*a^3*d^4*x^4)*integral(sq
rt(1/2)*(-9*I*d^4*x^4 + 80*I*d^2*x^2 - 384*I)*sqrt(I*a*e^(2*d*x + 2*c) + 2*
a*e^(d*x + c) - I*a)*e^(1/2*d*x + 1/2*c)/(48*a^3*d^4*x^5*e^(2*d*x + 2*c) -
96*I*a^3*d^4*x^5*e^(d*x + c) - 48*a^3*d^4*x^5), x))/(24*a^3*d^4*x^4*e^(5*d*
x + 5*c) - 120*I*a^3*d^4*x^4*e^(4*d*x + 4*c) - 240*a^3*d^4*x^4*e^(3*d*x + 3
*c) + 240*I*a^3*d^4*x^4*e^(2*d*x + 2*c) + 120*a^3*d^4*x^4*e^(d*x + c) - 24*
I*a^3*d^4*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(i\,a\sinh\left(dx+c\right)+a\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(1/((I*a*sinh(d*x + c) + a)^(5/2)*x), x)
```

3.150 $\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x},x\right)$$

```
[Out] Unintegrable[(a + I*a*Sinh[e + f*x])^(1/3)/x, x]
```

Rubi [A] time = 0.0733574, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + I*a*Sinh[e + f*x])^(1/3)/x,x]
```

```
[Out] Defer[Int][(a + I*a*Sinh[e + f*x])^(1/3)/x, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx = \int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx$$

Mathematica [A] time = 3.97186, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+ia\sinh(e+fx)}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + I*a*Sinh[e + f*x])^(1/3)/x,x]
```

```
[Out] Integrate[(a + I*a*Sinh[e + f*x])^(1/3)/x, x]
```

Maple [A] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{1}{x}\sqrt[3]{a+ia\sinh\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))^(1/3)/x,x)
```

```
[Out] int((a+I*a*sinh(f*x+e))^(1/3)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(i\, a \sinh\left(fx+e\right)+a\right)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/3)/x,x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(1/3)/x, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/3)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a\left(i \sinh\left(e+fx\right)+1\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))**(1/3)/x,x)
```

```
[Out] Integral((a*(I*sinh(e + f*x) + 1))**(1/3)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(i\, a \sinh\left(fx+e\right)+a\right)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/3)/x,x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(1/3)/x, x)
```

3.151 $\int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left((c+dx)^m(a+ia\sinh(e+fx))^n,x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^n, x]
```

Rubi [A] time = 0.0514336, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^n,x]
```

```
[Out] Defer[Int][(c + d*x)^m*(a + I*a*Sinh[e + f*x])^n, x]
```

Rubi steps

$$\int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx = \int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx$$

Mathematica [A] time = 3.91554, size = 0, normalized size = 0.

$$\int (c+dx)^m(a+ia\sinh(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^n,x]
```

```
[Out] Integrate[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^n, x]
```

Maple [A] time = 0.048, size = 0, normalized size = 0.

$$\int (dx+c)^m\left(a+ia\sinh\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x)
```

```
[Out] int((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m\left(i\,a\sinh\left(fx+e\right)+a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*(I*a*sinh(f*x + e) + a)^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m\left(\frac{1}{2}\left(i\,ae^{(2\,fx+2\,e)}+2\,ae^{(fx+e)}-i\,a\right)e^{(-fx-e)}\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*(1/2*(I*a*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*e^(
-f*x - e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+I*a*sinh(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m\left(i\,a\sinh\left(fx+e\right)+a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*(I*a*sinh(f*x + e) + a)^n, x)
```

3.152 $\int (c+dx)^m (a+ia\sinh(e+fx))^3\,dx$

Optimal. Leaf size=410

$$-\frac{ia^3 3^{-m-1} e^{3e-\frac{3cf}{d}} (c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1,-\frac{3f(c+dx)}{d}\right)}{8f}-\frac{3a^3 2^{-m-3} e^{2e-\frac{2cf}{d}} (c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(\right.}{f}$$

```
[Out] (5*a^3*(c + d*x)^(1 + m))/(2*d*(1 + m)) - ((I/8)*3^(-1 - m)*a^3*E^(3*e - (3
*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-3*f*(c + d*x))/d])/(f*(-((f*(c + d*x))/
d))^m) - (3*2^(-3 - m)*a^3*E^(2*e - (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2
*f*(c + d*x))/d])/(f*(-((f*(c + d*x))/d))^m) + (((15*I)/8)*a^3*E^(e - (c*f)
/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m
) + (((15*I)/8)*a^3*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))
/d])/(f*((f*(c + d*x))/d)^m) + (3*2^(-3 - m)*a^3*E^(-2*e + (2*c*f)/d)*(c +
d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) - ((I/8)*3^
(-1 - m)*a^3*E^(-3*e + (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (3*f*(c + d*x))/
d])/(f*((f*(c + d*x))/d)^m)
```

Rubi [A] time = 0.602034, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3318, 3312, 3307, 2181, 3308}

$$-\frac{ia^3 3^{-m-1} e^{3e-\frac{3cf}{d}} (c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1,-\frac{3f(c+dx)}{d}\right)}{8f}-\frac{3a^3 2^{-m-3} e^{2e-\frac{2cf}{d}} (c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^3,x]
```

```
[Out] (5*a^3*(c + d*x)^(1 + m))/(2*d*(1 + m)) - ((I/8)*3^(-1 - m)*a^3*E^(3*e - (3
*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-3*f*(c + d*x))/d])/(f*(-((f*(c + d*x))/
d))^m) - (3*2^(-3 - m)*a^3*E^(2*e - (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2
*f*(c + d*x))/d])/(f*(-((f*(c + d*x))/d))^m) + (((15*I)/8)*a^3*E^(e - (c*f)
/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m
) + (((15*I)/8)*a^3*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))
/d])/(f*((f*(c + d*x))/d)^m) + (3*2^(-3 - m)*a^3*E^(-2*e + (2*c*f)/d)*(c +
d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) - ((I/8)*3^
(-1 - m)*a^3*E^(-3*e + (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (3*f*(c + d*x))/
d])/(f*((f*(c + d*x))/d)^m)
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\int (c+dx)^m(a+ia\sinh(e+fx))^3\,dx = \left(8a^3\right)\int (c+dx)^m\sin^6\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)dx$$

$$= \left(8a^3\right)\int \left(\frac{5}{16}(c+dx)^m-\frac{3}{16}(c+dx)^m\cosh(2e+2fx)+\frac{15}{32}i(c+dx)^m\sinh(e+$$

$$= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)}-\frac{1}{4}\left(ia^3\right)\int (c+dx)^m\sinh(3e+3fx)\,dx+\frac{1}{4}\left(15ia^3\right)\int (c+dx$$

$$= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)}-\frac{1}{8}\left(ia^3\right)\int e^{-i(3ie+3ifx)}(c+dx)^m\,dx+\frac{1}{8}\left(ia^3\right)\int e^{i(3ie+3ifx)}(c+$$

$$= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)}-\frac{i3^{-1-m}a^3e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3f(c+dx)}{d}\right)}{8f}-$$

Mathematica [A] time = 1.58512, size = 339, normalized size = 0.83

$$\frac{1}{24}a^3(c+dx)^m\left(-\frac{i3^{-m}e^{3e-\frac{3cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{3f(c+dx)}{d}\right)}{f}-\frac{9\,2^{-m}e^{2e-\frac{2cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\right.}{f}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^3,x]
```

```
[Out] (a^3*(c + d*x)^m*((60*(c + d*x))/(d*(1 + m)) - (I*E^(3*e - (3*c*f)/d)*Gamma
[1 + m, (-3*f*(c + d*x))/d])/(3^m*f*(-((f*(c + d*x))/d))^m) - (9*E^(2*e - (
2*c*f)/d)*Gamma[1 + m, (-2*f*(c + d*x))/d])/(2^m*f*(-((f*(c + d*x))/d))^m)
+ ((45*I)*E^(e - (c*f)/d)*Gamma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d
*x))/d))^m) + ((45*I)*E^(-e + (c*f)/d)*Gamma[1 + m, (f*(c + d*x))/d])/(f*((
f*(c + d*x))/d)^m) + (9*E^(-2*e + (2*c*f)/d)*Gamma[1 + m, (2*f*(c + d*x))/d
])/(2^m*f*((f*(c + d*x))/d)^m) - (I*E^(-3*e + (3*c*f)/d)*Gamma[1 + m, (3*f*
(c + d*x))/d])/(3^m*f*((f*(c + d*x))/d)^m)))/24
```

Maple [F] time = 0.12, size = 0, normalized size = 0.

$$\int (dx+c)^m \left(a+ia\sinh\left(fx+e\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x)
```

```
[Out] int((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.8322, size = 857, normalized size = 2.09

$$\left(-i\,a^3dm-i\,a^3d\right)e^{\left(-\frac{dm\log\left(\frac{3f}{d}\right)+3de-3cf}{d}\right)}\Gamma\left(m+1,\frac{3\left(dfx+cf\right)}{d}\right)+9\left(a^3dm+a^3d\right)e^{\left(-\frac{dm\log\left(\frac{2f}{d}\right)+2de-2cf}{d}\right)}\Gamma\left(m+1,\frac{2\left(dfx+cf\right)}{d}\right)+\left(45i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/24*((-I*a^3*d*m - I*a^3*d)*e^(-(d*m*log(3*f/d) + 3*d*e - 3*c*f)/d)*gamma(
m + 1, 3*(d*f*x + c*f)/d) + 9*(a^3*d*m + a^3*d)*e^(-(d*m*log(2*f/d) + 2*d*e
 - 2*c*f)/d)*gamma(m + 1, 2*(d*f*x + c*f)/d) + (45*I*a^3*d*m + 45*I*a^3*d)*
e^(-(d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f*x + c*f)/d) + (45*I*a^3
*d*m + 45*I*a^3*d)*e^(-(d*m*log(-f/d) - d*e + c*f)/d)*gamma(m + 1, -(d*f*x
+ c*f)/d) - 9*(a^3*d*m + a^3*d)*e^(-(d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d)*ga
mma(m + 1, -2*(d*f*x + c*f)/d) + (-I*a^3*d*m - I*a^3*d)*e^(-(d*m*log(-3*f/d
) - 3*d*e + 3*c*f)/d)*gamma(m + 1, -3*(d*f*x + c*f)/d) + 60*(a^3*d*f*x + a^
3*c*f)*(d*x + c)^m)/(d*f*m + d*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+I*a*sinh(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(i a \sinh \left(f x + e\right) + a\right)^{3} (d x + c)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^3*(d*x + c)^m, x)
```

3.153 $\int (c+dx)^m (a+ia\sinh(e+fx))^2\,dx$

Optimal. Leaf size=268

$$-\frac{a^2 2^{-m-3} e^{2e-\frac{2cf}{d}} (c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f}+\frac{ia^2 e^{e-\frac{cf}{d}} (c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1,-\frac{f}{}\right.}{f}$$

```
[Out] (3*a^2*(c + d*x)^(1 + m))/(2*d*(1 + m)) - (2^(-3 - m)*a^2*E^(2*e - (2*c*f)/
d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c + d*x))/d])/(f*(-((f*(c + d*x))/d))^m)
 + (I*a^2*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(f*
(-((f*(c + d*x))/d))^m) + (I*a^2*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m,
(f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) + (2^(-3 - m)*a^2*E^(-2*e + (2*c*
f)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m)
```

Rubi [A] time = 0.361244, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3318, 3312, 3307, 2181, 3308}

$$-\frac{a^2 2^{-m-3} e^{2e-\frac{2cf}{d}} (c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f}+\frac{ia^2 e^{e-\frac{cf}{d}} (c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1,-\frac{f}{}\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^2,x]
```


```
[Out] (3*a^2*(c + d*x)^(1 + m))/(2*d*(1 + m)) - (2^(-3 - m)*a^2*E^(2*e - (2*c*f)/
d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c + d*x))/d])/(f*(-((f*(c + d*x))/d))^m)
 + (I*a^2*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(f*
(-((f*(c + d*x))/d))^m) + (I*a^2*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m,
(f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) + (2^(-3 - m)*a^2*E^(-2*e + (2*c*
f)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m)
```


Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
```

```
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\int (c+dx)^m (a+ia\sinh(e+fx))^2\,dx = \left(4a^2\right)\int (c+dx)^m \sin^4\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right)dx$$
$$= \left(4a^2\right)\int \left(\frac{3}{8}(c+dx)^m - \frac{1}{8}(c+dx)^m\cosh(2e+2fx)+\frac{1}{2}i(c+dx)^m\sinh(e+fx$$
$$= \frac{3a^2(c+dx)^{1+m}}{2d(1+m)} + \left(2ia^2\right)\int (c+dx)^m\sinh(e+fx)\,dx - \frac{1}{2}a^2\int (c+dx)^m\cosh($$
$$= \frac{3a^2(c+dx)^{1+m}}{2d(1+m)} + \left(ia^2\right)\int e^{-i(ie+ifx)}(c+dx)^m\,dx - \left(ia^2\right)\int e^{i(ie+ifx)}(c+dx)^m\,dx$$
$$= \frac{3a^2(c+dx)^{1+m}}{2d(1+m)} - \frac{2^{-3-m}a^2e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f} +$$

Mathematica [A] time = 0.799054, size = 229, normalized size = 0.85

$$\frac{1}{8}a^2(c+dx)^m\left(-\frac{2^{-m}e^{2e-\frac{2cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f}+\frac{8ie^{e-\frac{cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right.}{f}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] (a^2*(c + d*x)^m*((12*(c + d*x))/(d*(1 + m)) - (E^(2*e - (2*c*f)/d)*Gamma[1
 + m, (-2*f*(c + d*x))/d])/(2^m*f*(-((f*(c + d*x))/d))^m) + ((8*I)*E^(e - (
c*f)/d)*Gamma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) + ((8*
I)*E^(-e + (c*f)/d)*Gamma[1 + m, (f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m)
+ (E^(-2*e + (2*c*f)/d)*Gamma[1 + m, (2*f*(c + d*x))/d])/(2^m*f*((f*(c + d*
x))/d)^m)))/8
```

Maple [F] time = 0.101, size = 0, normalized size = 0.

$$\int (dx+c)^m\left(a+ia\sinh\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+I*a*sinh(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m*(a+I*a*sinh(f*x+e))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.6165, size = 581, normalized size = 2.17

$$\frac{\left(a^2dm + a^2d\right)e^{\left(-\frac{dm\log\left(\frac{2f}{d}\right)+2de-2cf}{d}\right)}\Gamma\left(m+1, \frac{2\left(dfx+cf\right)}{d}\right) + \left(8i\,a^2dm + 8i\,a^2d\right)e^{\left(-\frac{dm\log\left(\frac{f}{d}\right)+de-cf}{d}\right)}\Gamma\left(m+1, \frac{dfx+cf}{d}\right) + \left(8i\,a^2dm + \right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((a^2*d*m + a^2*d)*e^(-(d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)*gamma(m + 1,
 2*(d*f*x + c*f)/d) + (8*I*a^2*d*m + 8*I*a^2*d)*e^(-(d*m*log(f/d) + d*e - c
*f)/d)*gamma(m + 1, (d*f*x + c*f)/d) + (8*I*a^2*d*m + 8*I*a^2*d)*e^(-(d*m*l
og(-f/d) - d*e + c*f)/d)*gamma(m + 1, -(d*f*x + c*f)/d) - (a^2*d*m + a^2*d)
*e^(-(d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d)*gamma(m + 1, -2*(d*f*x + c*f)/d)
+ 12*(a^2*d*f*x + a^2*c*f)*(d*x + c)^m)/(d*f*m + d*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+I*a*sinh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(i\,a\sinh\left(fx+e\right)+a\right)^2(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^2*(d*x + c)^m, x)
```

3.154 $\int (c+dx)^m (a+ia\sinh(e+fx))\,dx$

Optimal. Leaf size=135

$$\frac{iae^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}+\frac{iae^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+\frac{a}{}$$

```
[Out] (a*(c + d*x)^(1 + m))/(d*(1 + m)) + ((I/2)*a*E^(e - (c*f)/d)*(c + d*x)^m*Ga
mma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) + ((I/2)*a*E^(-e
 + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(f*((f*(c + d*x))/d)
^m)
```

Rubi [A] time = 0.153541, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3317, 3308, 2181}

$$\frac{iae^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}+\frac{iae^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+\frac{a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + I*a*Sinh[e + f*x]),x]
```


```
[Out] (a*(c + d*x)^(1 + m))/(d*(1 + m)) + ((I/2)*a*E^(e - (c*f)/d)*(c + d*x)^m*Ga
mma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) + ((I/2)*a*E^(-e
 + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(f*((f*(c + d*x))/d)
^m)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule steps

$$\int (c+dx)^m(a+ia\sinh(e+fx))\,dx = \int \left(a(c+dx)^m + ia(c+dx)^m\sinh(e+fx)\right)dx$$
$$= \frac{a(c+dx)^{1+m}}{d(1+m)} + (ia)\int (c+dx)^m\sinh(e+fx)\,dx$$
$$= \frac{a(c+dx)^{1+m}}{d(1+m)} + \frac{1}{2}(ia)\int e^{-i(ie+ifx)}(c+dx)^m\,dx - \frac{1}{2}(ia)\int e^{i(ie+ifx)}(c+dx)^m\,dx$$
$$= \frac{a(c+dx)^{1+m}}{d(1+m)} + \frac{iae^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{2f} + \frac{iae^{-e+\frac{cf}{d}}(c+d}{}$$

Mathematica [A] time = 0.55988, size = 207, normalized size = 1.53

$$-\frac{ae^{-\frac{cf}{d}-e}(c+dx)^m(\sinh(e+fx)-i)\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}\left(de^{2e}(m+1)\left(f\left(\frac{c}{d}+x\right)\right)^m\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)+d(m+1)e^{\frac{2cf}{d}}\left(\right.\right.}{2df(m+1)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] -(a*E^(-e - (c*f)/d)*(c + d*x)^m*((-2*I)*E^(e + (c*f)/d)*f*(c + d*x)*(-((f^
2*(c + d*x)^2)/d^2))^m + d*E^(2*e)*(1 + m)*(f*(c/d + x))^m*Gamma[1 + m, -((
f*(c + d*x))/d)] + d*E^((2*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 +
 m, (f*(c + d*x))/d])*(-I + Sinh[e + f*x]))/(2*d*f*(1 + m)*(-((f^2*(c + d*x
)^2)/d^2))^m*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])^2)
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int (dx+c)^m\left(a+ia\sinh\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+I*a*sinh(f*x+e)),x)
```

```
[Out] int((d*x+c)^m*(a+I*a*sinh(f*x+e)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.54164, size = 301, normalized size = 2.23

$$\frac{(i\,adm+i\,ad)e^{\left(-\frac{dm\log\left(\frac{f}{d}\right)+de-cf}{d}\right)}\Gamma\left(m+1,\frac{dfx+cf}{d}\right)+(i\,adm+i\,ad)e^{\left(-\frac{dm\log\left(-\frac{f}{d}\right)-de+cf}{d}\right)}\Gamma\left(m+1,-\frac{dfx+cf}{d}\right)+2\left(adfx+acf\right)(}{2\left(dfm+df\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*((I*a*d*m + I*a*d)*e^(-(d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f*
x + c*f)/d) + (I*a*d*m + I*a*d)*e^(-(d*m*log(-f/d) - d*e + c*f)/d)*gamma(m
+ 1, -(d*f*x + c*f)/d) + 2*(a*d*f*x + a*c*f)*(d*x + c)^m)/(d*f*m + d*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+I*a*sinh(f*x+e)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(i\,a\sinh\left(fx+e\right)+a\right)(dx+c)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)*(d*x + c)^m, x)
```

3.155 $\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{(c+dx)^m}{a+ia\sinh(e+fx)},x\right)$$

```
[Out] Unintegrable[(c + d*x)^m/(a + I*a*Sinh[e + f*x]), x]
```

Rubi [A] time = 0.0561594, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] Defer[Int][(c + d*x)^m/(a + I*a*Sinh[e + f*x]), x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx = \int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx$$

Mathematica [A] time = 4.27846, size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{a+ia\sinh(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] Integrate[(c + d*x)^m/(a + I*a*Sinh[e + f*x]), x]
```

Maple [A] time = 0.034, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{a+ia\sinh\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] int((d*x+c)^m/(a+I*a*sinh(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{i a \sinh(fx+e)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m/(I*a*sinh(f*x + e) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(afe^{(fx+e)}-iaf\right)\mathrm{integral}\left(-\frac{2i(dx+c)^m dm}{-iadfx-iacf+(adfx+acf)e^{(fx+e)}},x\right)+2i(dx+c)^m}{afe^{(fx+e)}-iaf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] ((a*f*e^(f*x + e) - I*a*f)*integral(-2*I*(d*x + c)^m*d*m/(-I*a*d*f*x - I*a*
c*f + (a*d*f*x + a*c*f)*e^(f*x + e)), x) + 2*I*(d*x + c)^m)/(a*f*e^(f*x + e
) - I*a*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{i a \sinh(fx+e)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(I*a*sinh(f*x + e) + a), x)
```

3.156 $\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2},x\right)$$

```
[Out] Unintegrable[(c + d*x)^m/(a + I*a*Sinh[e + f*x])^2, x]
```

Rubi [A] time = 0.0572106, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m/(a + I*a*Sinh[e + f*x])^2, x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx=\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx$$

Mathematica [A] time = 15.7454, size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{(a+ia\sinh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] Integrate[(c + d*x)^m/(a + I*a*Sinh[e + f*x])^2, x]
```

Maple [A] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{\left(a+ia\sinh\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{\left(ia\sinh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m/(I*a*sinh(f*x + e) + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(-2i\,d^2f^2x^2-4i\,cdf^2x-2i\,c^2f^2+2i\,d^2m^2-2i\,d^2m+\left(-2i\,d^2fmx-2i\,d^2m^2+\left(-2i\,cdf+2i\,d^2\right)m\right)e^{\left(2fx+2e\right)}+2\left(3\,d^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] ((-2*I*d^2*f^2*x^2 - 4*I*c*d*f^2*x - 2*I*c^2*f^2 + 2*I*d^2*m^2 - 2*I*d^2*m
+ (-2*I*d^2*f*m*x - 2*I*d^2*m^2 + (-2*I*c*d*f + 2*I*d^2)*m)*e^(2*f*x + 2*e)
 + 2*(3*d^2*f^2*x^2 + 3*c^2*f^2 - 2*d^2*m^2 - (c*d*f - 2*d^2)*m + (6*c*d*f^
2 - d^2*f*m)*x)*e^(f*x + e))*(d*x + c)^m + (3*I*a^2*d^2*f^3*x^2 + 6*I*a^2*c
*d*f^3*x + 3*I*a^2*c^2*f^3 + 3*(a^2*d^2*f^3*x^2 + 2*a^2*c*d*f^3*x + a^2*c^2
*f^3)*e^(3*f*x + 3*e) + (-9*I*a^2*d^2*f^3*x^2 - 18*I*a^2*c*d*f^3*x - 9*I*a^
2*c^2*f^3)*e^(2*f*x + 2*e) - 9*(a^2*d^2*f^3*x^2 + 2*a^2*c*d*f^3*x + a^2*c^2
*f^3)*e^(f*x + e))*integral((-2*I*d^3*f^2*m*x^2 - 4*I*c*d^2*f^2*m*x + 2*I*d
^3*m^3 - 6*I*d^3*m^2 + (-2*I*c^2*d*f^2 + 4*I*d^3)*m)*(d*x + c)^m/(-3*I*a^2*
d^3*f^3*x^3 - 9*I*a^2*c*d^2*f^3*x^2 - 9*I*a^2*c^2*d*f^3*x - 3*I*a^2*c^3*f^3
 + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f
^3)*e^(f*x + e)), x))/(3*I*a^2*d^2*f^3*x^2 + 6*I*a^2*c*d*f^3*x + 3*I*a^2*c^
2*f^3 + 3*(a^2*d^2*f^3*x^2 + 2*a^2*c*d*f^3*x + a^2*c^2*f^3)*e^(3*f*x + 3*e)
 + (-9*I*a^2*d^2*f^3*x^2 - 18*I*a^2*c*d*f^3*x - 9*I*a^2*c^2*f^3)*e^(2*f*x +
 2*e) - 9*(a^2*d^2*f^3*x^2 + 2*a^2*c*d*f^3*x + a^2*c^2*f^3)*e^(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+I*a*sinh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{\left(ia\sinh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(I*a*sinh(f*x + e) + a)^2, x)
```

3.157 $\int (c+dx)^3(a+b\sinh(e+fx))\,dx$

Optimal. Leaf size=89

$$\frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\cosh(e+fx)}{f^3}-\frac{3bd(c+dx)^2\sinh(e+fx)}{f^2}+\frac{b(c+dx)^3\cosh(e+fx)}{f}-\frac{6bd^3\sinh(e+fx)}{f^4}$$

```
[Out] (a*(c + d*x)^4)/(4*d) + (6*b*d^2*(c + d*x)*Cosh[e + f*x])/f^3 + (b*(c + d*x
)^3*Cosh[e + f*x])/f - (6*b*d^3*Sinh[e + f*x])/f^4 - (3*b*d*(c + d*x)^2*Sin
h[e + f*x])/f^2
```

Rubi [A] time = 0.142754, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3317, 3296, 2637}

$$\frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\cosh(e+fx)}{f^3}-\frac{3bd(c+dx)^2\sinh(e+fx)}{f^2}+\frac{b(c+dx)^3\cosh(e+fx)}{f}-\frac{6bd^3\sinh(e+fx)}{f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Sinh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) + (6*b*d^2*(c + d*x)*Cosh[e + f*x])/f^3 + (b*(c + d*x
)^3*Cosh[e + f*x])/f - (6*b*d^3*Sinh[e + f*x])/f^4 - (3*b*d*(c + d*x)^2*Sin
h[e + f*x])/f^2
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^3(a+b\sinh(e+fx))\,dx = \int \left(a(c+dx)^3 + b(c+dx)^3\sinh(e+fx)\right)dx$$
$$= \frac{a(c+dx)^4}{4d} + b\int (c+dx)^3\sinh(e+fx)\,dx$$
$$= \frac{a(c+dx)^4}{4d} + \frac{b(c+dx)^3\cosh(e+fx)}{f} - \frac{(3bd)\int (c+dx)^2\cosh(e+fx)\,dx}{f}$$
$$= \frac{a(c+dx)^4}{4d} + \frac{b(c+dx)^3\cosh(e+fx)}{f} - \frac{3bd(c+dx)^2\sinh(e+fx)}{f^2} + \frac{\left(6bd^2\right)\int (c+$$
$$= \frac{a(c+dx)^4}{4d} + \frac{6bd^2(c+dx)\cosh(e+fx)}{f^3} + \frac{b(c+dx)^3\cosh(e+fx)}{f} - \frac{3bd(c+dx)^2 \mathrm{s}}{f^2}$$
$$= \frac{a(c+dx)^4}{4d} + \frac{6bd^2(c+dx)\cosh(e+fx)}{f^3} + \frac{b(c+dx)^3\cosh(e+fx)}{f} - \frac{6bd^3\sinh(e+}{f^4}$$

Mathematica [A] time = 0.45088, size = 123, normalized size = 1.38

$$\frac{1}{4}ax\left(6c^2dx+4c^3+4cd^2x^2+d^3x^3\right) - \frac{3bd\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2+2\right)\right)\sinh(e+fx)}{f^4} + \frac{b(c+dx)\left(c^2f^2+2cdf^2x+d^2\right.}{f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*Sinh[e + f*x]),x]
```

```
[Out] (a*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 + (b*(c + d*x)*(c^2*f^2
 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Cosh[e + f*x])/f^3 - (3*b*d*(c^2*f^2 +
2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x])/f^4
```

Maple [B] time = 0.01, size = 482, normalized size = 5.4

$$\frac{1}{f}\left(\frac{d^3a\left(fx+e\right)^4}{4f^3} + \frac{d^3b\left(\left(fx+e\right)^3\cosh\left(fx+e\right) - 3\left(fx+e\right)^2\sinh\left(fx+e\right) + 6\left(fx+e\right)\cosh\left(fx+e\right) - 6\sinh\left(fx+e\right.\right.}{f^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*sinh(f*x+e)),x)
```

```
[Out] 1/f*(1/4/f^3*d^3*a*(f*x+e)^4+1/f^3*d^3*b*((f*x+e)^3*cosh(f*x+e)-3*(f*x+e)^2
*sinh(f*x+e)+6*(f*x+e)*cosh(f*x+e)-6*sinh(f*x+e))-1/f^3*d^3*e*a*(f*x+e)^3-3
/f^3*d^3*e*b*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sinh(f*x+e)+2*cosh(f*x+e))+3/
2/f^3*d^3*e^2*a*(f*x+e)^2+3/f^3*d^3*e^2*b*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))
-d^3*e^3/f^3*a*(f*x+e)-1/f^3*d^3*e^3*b*cosh(f*x+e)+1/f^2*d^2*c*a*(f*x+e)^3+
3/f^2*c*d^2*b*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sinh(f*x+e)+2*cosh(f*x+e))-3
/f^2*d^2*e*c*a*(f*x+e)^2-6/f^2*c*d^2*e*b*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))+
3*d^2*e^2/f^2*c*a*(f*x+e)+3/f^2*c*d^2*e^2*b*cosh(f*x+e)+3/2/f*d*c^2*a*(f*x+
e)^2+3/f*c^2*d*b*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))-3*d*e/f*c^2*a*(f*x+e)-3/
f*c^2*d*e*b*cosh(f*x+e)+c^3*a*(f*x+e)+b*c^3*cosh(f*x+e))
```

Maxima [B] time = 1.31172, size = 316, normalized size = 3.55

$$\frac{1}{4}ad^3x^4 + acd^2x^3 + \frac{3}{2}ac^2dx^2 + ac^3x + \frac{3}{2}bc^2d\left(\frac{\left(fxe^e - e^e\right)e^{(fx)}}{f^2} + \frac{\left(fx+1\right)e^{(-fx-e)}}{f^2}\right) + \frac{3}{2}bcd^2\left(\frac{\left(f^2x^2e^e - 2fxe^e + 2e^e\right)e^{(fx}}{f^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 3/2*b*c^2*d*((f*x
*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(-f*x - e)/f^2) + 3/2*b*c*d^2*((f^2*x
^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e
)/f^3) + 1/2*b*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6*f*x*e^e - 6*e^e)*e^(f*
x)/f^4 + (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(-f*x - e)/f^4) + b*c^3*cosh(f
*x + e)/f
```

Fricas [A] time = 2.42436, size = 365, normalized size = 4.1

$$\frac{ad^3f^4x^4 + 4\,acd^2f^4x^3 + 6\,ac^2df^4x^2 + 4\,ac^3f^4x + 4\left(bd^3f^3x^3 + 3\,bcd^2f^3x^2 + bc^3f^3 + 6\,bcd^2f + 3\left(bc^2df^3 + 2\,bd^3f\right)x\right)}{4\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x
+ 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + b*c^3*f^3 + 6*b*c*d^2*f + 3*(b*c^2
*d*f^3 + 2*b*d^3*f)*x)*cosh(f*x + e) - 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x
+ b*c^2*d*f^2 + 2*b*d^3)*sinh(f*x + e))/f^4
```

Sympy [A] time = 1.93265, size = 264, normalized size = 2.97

$$\begin{cases} ac^3x + \frac{3ac^2dx^2}{2} + acd^2x^3 + \frac{ad^3x^4}{4} + \frac{bc^3\cosh\left(e+fx\right)}{f} + \frac{3bc^2dx\cosh\left(e+fx\right)}{f} - \frac{3bc^2d\sinh\left(e+fx\right)}{f^2} + \frac{3bcd^2x^2\cosh\left(e+fx\right)}{f} - \frac{6bcd^2x\sinh\left(e+f\right.}{f^2} \\ (a + b\sinh(e))\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*sinh(f*x+e)),x)
```

```
[Out] Piecewise((a*c**3*x + 3*a*c**2*d*x**2/2 + a*c*d**2*x**3 + a*d**3*x**4/4 + b
*c**3*cosh(e + f*x)/f + 3*b*c**2*d*x*cosh(e + f*x)/f - 3*b*c**2*d*sinh(e +
f*x)/f**2 + 3*b*c*d**2*x**2*cosh(e + f*x)/f - 6*b*c*d**2*x*sinh(e + f*x)/f*
*2 + 6*b*c*d**2*cosh(e + f*x)/f**3 + b*d**3*x**3*cosh(e + f*x)/f - 3*b*d**3
*x**2*sinh(e + f*x)/f**2 + 6*b*d**3*x*cosh(e + f*x)/f**3 - 6*b*d**3*sinh(e
+ f*x)/f**4, Ne(f, 0)), ((a + b*sinh(e))*(c**3*x + 3*c**2*d*x**2/2 + c*d**2
*x**3 + d**3*x**4/4), True))
```

Giac [B] time = 1.26699, size = 351, normalized size = 3.94

$$\frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x + \frac{\left(bd^3f^3x^3 + 3\,bcd^2f^3x^2 + 3\,bc^2df^3x - 3\,bd^3f^2x^2 + bc^3f^3 - 6\,bcd^2f^2x - 3\,bc^2df\right.}{2\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 1/2*(b*d^3*f^3*x^
3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x - 3*b*d^3*f^2*x^2 + b*c^3*f^3 - 6*b
*c*d^2*f^2*x - 3*b*c^2*d*f^2 + 6*b*d^3*f*x + 6*b*c*d^2*f - 6*b*d^3)*e^(f*x
+ e)/f^4 + 1/2*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + 3*b*d
^3*f^2*x^2 + b*c^3*f^3 + 6*b*c*d^2*f^2*x + 3*b*c^2*d*f^2 + 6*b*d^3*f*x + 6*
b*c*d^2*f + 6*b*d^3)*e^(-f*x - e)/f^4
```

3.158 $\int (c+dx)^2(a+b\sinh(e+fx))\,dx$

Optimal. Leaf size=67

$$\frac{a(c+dx)^3}{3d}-\frac{2bd(c+dx)\sinh(e+fx)}{f^2}+\frac{b(c+dx)^2\cosh(e+fx)}{f}+\frac{2bd^2\cosh(e+fx)}{f^3}$$

```
[Out] (a*(c + d*x)^3)/(3*d) + (2*b*d^2*Cosh[e + f*x])/f^3 + (b*(c + d*x)^2*Cosh[e
 + f*x])/f - (2*b*d*(c + d*x)*Sinh[e + f*x])/f^2
```

Rubi [A] time = 0.0957637, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3317, 3296, 2638}

$$\frac{a(c+dx)^3}{3d}-\frac{2bd(c+dx)\sinh(e+fx)}{f^2}+\frac{b(c+dx)^2\cosh(e+fx)}{f}+\frac{2bd^2\cosh(e+fx)}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Sinh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^3)/(3*d) + (2*b*d^2*Cosh[e + f*x])/f^3 + (b*(c + d*x)^2*Cosh[e
 + f*x])/f - (2*b*d*(c + d*x)*Sinh[e + f*x])/f^2
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2(a+b\sinh(e+fx))\,dx &= \int \left(a(c+dx)^2+b(c+dx)^2\sinh(e+fx)\right)dx\\
&= \frac{a(c+dx)^3}{3d}+b\int (c+dx)^2\sinh(e+fx)\,dx\\
&= \frac{a(c+dx)^3}{3d}+\frac{b(c+dx)^2\cosh(e+fx)}{f}-\frac{(2bd)\int (c+dx)\cosh(e+fx)\,dx}{f}\\
&= \frac{a(c+dx)^3}{3d}+\frac{b(c+dx)^2\cosh(e+fx)}{f}-\frac{2bd(c+dx)\sinh(e+fx)}{f^2}+\frac{\left(2bd^2\right)\int \sin}{}\\
&= \frac{a(c+dx)^3}{3d}+\frac{2bd^2\cosh(e+fx)}{f^3}+\frac{b(c+dx)^2\cosh(e+fx)}{f}-\frac{2bd(c+dx)\sinh(e}{f^2}
\end{aligned}$$

Mathematica [A] time = 0.308752, size = 83, normalized size = 1.24

$$\frac{1}{3}ax\left(3c^2+3cdx+d^2x^2\right)+\frac{b\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2+2\right)\right)\cosh(e+fx)}{f^3}-\frac{2bd(c+dx)\sinh(e+fx)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*Sinh[e + f*x]),x]
```

```
[Out] (a*x*(3*c^2 + 3*c*d*x + d^2*x^2))/3 + (b*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2 +
f^2*x^2))*Cosh[e + f*x])/f^3 - (2*b*d*(c + d*x)*Sinh[e + f*x])/f^2
```

Maple [B] time = 0.007, size = 240, normalized size = 3.6

$$\frac{1}{f}\left(\frac{ad^2\left(fx+e\right)^3}{3f^2}+\frac{d^2b\left(\left(fx+e\right)^2\cosh\left(fx+e\right)-2\left(fx+e\right)\sinh\left(fx+e\right)+2\cosh\left(fx+e\right)\right)}{f^2}-\frac{d^2ea\left(fx+e\right)^2}{f^2}-2\frac{d^2e}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*sinh(f*x+e)),x)
```

```
[Out] 1/f*(1/3/f^2*d^2*a*(f*x+e)^3+1/f^2*d^2*b*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*s
inh(f*x+e)+2*cosh(f*x+e))-1/f^2*d^2*e*a*(f*x+e)^2-2/f^2*d^2*e*b*((f*x+e)*co
sh(f*x+e)-sinh(f*x+e))+d^2*e^2/f^2*a*(f*x+e)+1/f^2*d^2*e^2*b*cosh(f*x+e)+1/
f*d*c*a*(f*x+e)^2+2/f*c*d*b*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))-2*d*e/f*c*a*(
f*x+e)-2/f*c*d*e*b*cosh(f*x+e)+c^2*a*(f*x+e)+b*c^2*cosh(f*x+e))
```

Maxima [B] time = 1.1858, size = 188, normalized size = 2.81

$$\frac{1}{3}ad^2x^3+acdx^2+ac^2x+bcd\left(\frac{\left(fxe^e-e^e\right)e^{(fx)}}{f^2}+\frac{\left(fx+1\right)e^{(-fx-e)}}{f^2}\right)+\frac{1}{2}bd^2\left(\frac{\left(f^2x^2e^e-2fxe^e+2e^e\right)e^{(fx)}}{f^3}+\frac{\left(f^2x^2+2fx\right.}{f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + b*c*d*((f*x*e^e - e^e)*e^(f*x)/f^2 +
(f*x + 1)*e^(-f*x - e)/f^2) + 1/2*b*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*
e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + b*c^2*cosh(f*x + e)
/f
```

Fricas [A] time = 2.4565, size = 231, normalized size = 3.45

$$\frac{ad^2f^3x^3+3acdf^3x^2+3ac^2f^3x+3\left(bd^2f^2x^2+2bcdf^2x+bc^2f^2+2bd^2\right)\cosh\left(fx+e\right)-6\left(bd^2fx+bcdf\right)\sinh\left(fx+\right.}{3f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/3*(a*d^2*f^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x + 3*(b*d^2*f^2*x^2 + 2
*b*c*d*f^2*x + b*c^2*f^2 + 2*b*d^2)*cosh(f*x + e) - 6*(b*d^2*f*x + b*c*d*f)
*sinh(f*x + e))/f^3
```

Sympy [A] time = 0.879971, size = 151, normalized size = 2.25

$$\begin{cases} ac^2x + acdx^2 + \frac{ad^2x^3}{3} + \frac{bc^2\cosh(e+fx)}{f} + \frac{2bcdx\cosh(e+fx)}{f} - \frac{2bcd\sinh(e+fx)}{f^2} + \frac{bd^2x^2\cosh(e+fx)}{f} - \frac{2bd^2x\sinh(e+fx)}{f^2} + \frac{2bd^2\cosh(e \\ (a + b\sinh(e))\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*sinh(f*x+e)),x)
```

```
[Out] Piecewise((a*c**2*x + a*c*d*x**2 + a*d**2*x**3/3 + b*c**2*cosh(e + f*x)/f +
 2*b*c*d*x*cosh(e + f*x)/f - 2*b*c*d*sinh(e + f*x)/f**2 + b*d**2*x**2*cosh(
e + f*x)/f - 2*b*d**2*x*sinh(e + f*x)/f**2 + 2*b*d**2*cosh(e + f*x)/f**3, N
e(f, 0)), ((a + b*sinh(e))*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Giac [B] time = 1.32828, size = 200, normalized size = 2.99

$$\frac{1}{3}ad^2x^3 + acdx^2 + ac^2x + \frac{\left(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2 - 2\,bd^2fx - 2\,bcdf + 2\,bd^2\right)e^{(fx+e)}}{2\,f^3} + \frac{\left(bd^2f^2x^2 + 2\,bcdf^2x + b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + 1/2*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x +
b*c^2*f^2 - 2*b*d^2*f*x - 2*b*c*d*f + 2*b*d^2)*e^(f*x + e)/f^3 + 1/2*(b*d^2
*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2 + 2*b*d^2*f*x + 2*b*c*d*f + 2*b*d^2)*e
^(-f*x - e)/f^3
```

3.159 $\int (c+dx)(a+b\sinh(e+fx))\,dx$

Optimal. Leaf size=45

$$\frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\cosh(e+fx)}{f}-\frac{bd\sinh(e+fx)}{f^2}$$

```
[Out] (a*(c + d*x)^2)/(2*d) + (b*(c + d*x)*Cosh[e + f*x])/f - (b*d*Sinh[e + f*x])
/f^2
```

Rubi [A] time = 0.047893, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3317, 3296, 2637}

$$\frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\cosh(e+fx)}{f}-\frac{bd\sinh(e+fx)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Sinh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^2)/(2*d) + (b*(c + d*x)*Cosh[e + f*x])/f - (b*d*Sinh[e + f*x])
/f^2
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)(a+b\sinh(e+fx))\,dx &= \int (a(c+dx)+b(c+dx)\sinh(e+fx))\,dx\\
&= \frac{a(c+dx)^2}{2d}+b\int (c+dx)\sinh(e+fx)\,dx\\
&= \frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\cosh(e+fx)}{f}-\frac{(bd)\int \cosh(e+fx)\,dx}{f}\\
&= \frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\cosh(e+fx)}{f}-\frac{bd\sinh(e+fx)}{f^2}
\end{aligned}$$

Mathematica [A] time = 0.111022, size = 43, normalized size = 0.96

$$\frac{1}{2}ax(2c+dx)+\frac{b(c+dx)\cosh(e+fx)}{f}-\frac{bd\sinh(e+fx)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Sinh[e + f*x]),x]
```

```
[Out] (a*x*(2*c + d*x))/2 + (b*(c + d*x)*Cosh[e + f*x])/f - (b*d*Sinh[e + f*x])/f
^2
```

Maple [B] time = 0.007, size = 91, normalized size = 2.

$$\frac{1}{f}\left(\frac{da\left(fx+e\right)^2}{2f}+\frac{bd\left(\left(fx+e\right)\cosh\left(fx+e\right)-\sinh\left(fx+e\right)\right)}{f}-\frac{dea\left(fx+e\right)}{f}-\frac{deb\cosh\left(fx+e\right)}{f}+ca\left(fx+e\right)+cb\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*sinh(f*x+e)),x)
```

```
[Out] 1/f*(1/2/f*d*a*(f*x+e)^2+1/f*d*b*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))-d*e/f*a*
(f*x+e)-d*e/f*b*cosh(f*x+e)+c*a*(f*x+e)+c*b*cosh(f*x+e))
```

Maxima [A] time = 1.30236, size = 88, normalized size = 1.96

$$\frac{1}{2}adx^2+acx+\frac{1}{2}bd\left(\frac{\left(fxe^e-e^e\right)e^{(fx)}}{f^2}+\frac{\left(fx+1\right)e^{(-fx-e)}}{f^2}\right)+\frac{bc\cosh\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + 1/2*b*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(-
f*x - e)/f^2) + b*c*cosh(f*x + e)/f
```

Fricas [A] time = 2.46015, size = 128, normalized size = 2.84

$$\frac{adf^2x^2+2\,acf^2x-2\,bd\sinh\left(fx+e\right)+2\left(bdfx+bcf\right)\cosh\left(fx+e\right)}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*f^2*x^2 + 2*a*c*f^2*x - 2*b*d*sinh(f*x + e) + 2*(b*d*f*x + b*c*f)*
cosh(f*x + e))/f^2
```

Sympy [A] time = 0.36472, size = 68, normalized size = 1.51

$$\begin{cases} acx + \frac{adx^2}{2} + \frac{bc\cosh(e+fx)}{f} + \frac{bdx\cosh(e+fx)}{f} - \frac{bd\sinh(e+fx)}{f^2} & \text{for } f \neq 0 \\ (a + b\sinh(e))\left(cx + \frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sinh(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x + a*d*x**2/2 + b*c*cosh(e + f*x)/f + b*d*x*cosh(e + f*x)/f
 - b*d*sinh(e + f*x)/f**2, Ne(f, 0)), ((a + b*sinh(e))*(c*x + d*x**2/2), Tr
ue))
```

Giac [A] time = 1.28085, size = 89, normalized size = 1.98

$$\frac{1}{2}adx^2 + acx + \frac{(bdfx + bcf - bd)e^{(fx+e)}}{2f^2} + \frac{(bdfx + bcf + bd)e^{(-fx-e)}}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + 1/2*(b*d*f*x + b*c*f - b*d)*e^(f*x + e)/f^2 + 1/2*(b*
d*f*x + b*c*f + b*d)*e^(-f*x - e)/f^2
```

3.160 $\int \frac{a+b\sinh(e+fx)}{c+dx}\,dx$

Optimal. Leaf size=64

$$\frac{a\log(c+dx)}{d}+\frac{b\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{b\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}$$

```
[Out] (a*Log[c + d*x])/d + (b*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d +
(b*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d
```

Rubi [A] time = 0.124337, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3317, 3303, 3298, 3301}

$$\frac{a\log(c+dx)}{d}+\frac{b\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{b\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x])/(c + d*x),x]
```

```
[Out] (a*Log[c + d*x])/d + (b*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d +
(b*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh(e+fx)}{c+dx}\,dx &= \int \left(\frac{a}{c+dx}+\frac{b\sinh(e+fx)}{c+dx}\right)dx\\ &= \frac{a\log(c+dx)}{d}+b\int \frac{\sinh(e+fx)}{c+dx}\,dx\\ &= \frac{a\log(c+dx)}{d}+\left(b\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\sinh\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx+\left(b\sinh\left(e-\frac{cf}{d}\right)\right)\int \frac{\cosh\left(\frac{cf}{d}+fx\right)}{c+dx}\\ &= \frac{a\log(c+dx)}{d}+\frac{b\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{b\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.144335, size = 57, normalized size = 0.89

$$\frac{a\log(c+dx)+b\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+b\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x])/(c + d*x),x]
```

```
[Out] (a*Log[c + d*x] + b*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] + b*Cosh[e
- (c*f)/d]*SinhIntegral[f*(c/d + x)])/d
```

Maple [A] time = 0.023, size = 94, normalized size = 1.5

$$\frac{a\ln(dx+c)}{d}+\frac{b}{2d}e^{\frac{cf-de}{d}}\text{Ei}\left(1,fx+e+\frac{cf-de}{d}\right)-\frac{b}{2d}e^{-\frac{cf-de}{d}}\text{Ei}\left(1,-fx-e-\frac{cf-de}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e))/(d*x+c),x)
```

```
[Out] a*ln(d*x+c)/d+1/2*b/d*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*b/d*exp(
-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Maxima [A] time = 1.42502, size = 96, normalized size = 1.5

$$\frac{1}{2}b\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right)+\frac{a\log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*b*(e^(-e + c*f/d)*exp_integral_e(1, (d*x + c)*f/d)/d - e^(e - c*f/d)*ex
p_integral_e(1, -(d*x + c)*f/d)/d) + a*log(d*x + c)/d
```

Fricas [A] time = 2.4936, size = 230, normalized size = 3.59

$$\frac{\left(b\mathrm{Ei}\left(\frac{dfx+cf}{d}\right) - b\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{de-cf}{d}\right) + 2\,a\log\left(dx+c\right) - \left(b\mathrm{Ei}\left(\frac{dfx+cf}{d}\right) + b\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\sinh\left(-\frac{de-cf}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*((b*Ei((d*f*x + c*f)/d) - b*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d)
+ 2*a*log(d*x + c) - (b*Ei((d*f*x + c*f)/d) + b*Ei(-(d*f*x + c*f)/d))*sinh(
-(d*e - c*f)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\sinh\left(e + fx\right)}{c + dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c),x)
```

```
[Out] Integral((a + b*sinh(e + f*x))/(c + d*x), x)
```

Giac [A] time = 1.24907, size = 95, normalized size = 1.48

$$-\frac{b\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)} - b\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)} - 2\,a\log\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(b*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) - b*Ei((d*f*x + c*f)/d)*e^(-c*f/
d + e) - 2*a*log(d*x + c))/d
```

3.161 $\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=87

$$-\frac{a}{d(c+dx)}+\frac{bf\,\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{bf\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{b\sinh(e+fx)}{d(c+dx)}$$

```
[Out] -(a/(d*(c + d*x))) + (b*f*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d^
2 - (b*Sinh[e + f*x])/(d*(c + d*x)) + (b*f*Sinh[e - (c*f)/d]*SinhIntegral[(
c*f)/d + f*x])/d^2
```

Rubi [A] time = 0.169289, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3317, 3297, 3303, 3298, 3301}

$$-\frac{a}{d(c+dx)}+\frac{bf\,\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{bf\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{b\sinh(e+fx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x])/(c + d*x)^2,x]
```

```
[Out] -(a/(d*(c + d*x))) + (b*f*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d^
2 - (b*Sinh[e + f*x])/(d*(c + d*x)) + (b*f*Sinh[e - (c*f)/d]*SinhIntegral[(
c*f)/d + f*x])/d^2
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sinh(e+fx)}{(c+dx)^2}\,dx &= \int \left(\frac{a}{(c+dx)^2}+\frac{b\sinh(e+fx)}{(c+dx)^2}\right)dx \\
&= -\frac{a}{d(c+dx)}+b\int \frac{\sinh(e+fx)}{(c+dx)^2}\,dx \\
&= -\frac{a}{d(c+dx)}-\frac{b\sinh(e+fx)}{d(c+dx)}+\frac{(bf)\int \frac{\cosh(e+fx)}{c+dx}\,dx}{d} \\
&= -\frac{a}{d(c+dx)}-\frac{b\sinh(e+fx)}{d(c+dx)}+\frac{\left(bf\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\cosh\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx}{d}+\frac{\left(bf\sinh\left(e-\frac{cf}{d}\right)\right)\int}{d} \\
&= -\frac{a}{d(c+dx)}+\frac{bf\cosh\left(e-\frac{cf}{d}\right)\operatorname{Chi}\left(\frac{cf}{d}+fx\right)}{d^2}-\frac{b\sinh(e+fx)}{d(c+dx)}+\frac{bf\sinh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(\frac{cf}{d}\right.}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.366188, size = 71, normalized size = 0.82

$$\frac{-\frac{d(a+b\sinh(e+fx))}{c+dx}+bf\operatorname{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\cosh\left(e-\frac{cf}{d}\right)+bf\sinh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(f\left(\frac{c}{d}+x\right)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x])/(c + d*x)^2,x]
```

```
[Out] (b*f*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] - (d*(a + b*Sinh[e + f*x])
)/(c + d*x) + b*f*Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)])/d^2
```

Maple [A] time = 0.022, size = 149, normalized size = 1.7

$$-\frac{a}{d\,(dx+c)}+\frac{fb\mathrm{e}^{-fx-e}}{2\,d\left(dfx+cf\right)}-\frac{fb}{2\,d^2}\mathrm{e}^{\frac{cf-de}{d}}\operatorname{Ei}\left(1,fx+e+\frac{cf-de}{d}\right)-\frac{fb\mathrm{e}^{fx+e}}{2\,d^2}\left(\frac{cf}{d}+fx\right)^{-1}-\frac{fb}{2\,d^2}\mathrm{e}^{-\frac{cf-de}{d}}\operatorname{Ei}\left(1,-fx-e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e))/(d*x+c)^2,x)
```

```
[Out] -a/d/(d*x+c)+1/2*b*f*exp(-f*x-e)/d/(d*f*x+c*f)-1/2*b*f/d^2*exp((c*f-d*e)/d)
*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*f*b/d^2*exp(f*x+e)/(c*f/d+f*x)-1/2*f*b/d^2*exp
(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Maxima [A] time = 1.45451, size = 119, normalized size = 1.37

$$\frac{1}{2}\,b\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right)-\frac{a}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)/((d*x + c)*d) - e^(e
 - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d)) - a/(d^2*x + c*d
)
```

Fricas [A] time = 2.47105, size = 351, normalized size = 4.03

$$-\frac{2\,bd\sinh\left(fx+e\right)+2\,ad-\left(\left(bdfx+bcf\right)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+\left(bdfx+bcf\right)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{de-cf}{d}\right)+\left(\left(bdfx+bcf\right)\mathrm{Ei}\left(\right.\right.}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*sinh(f*x + e) + 2*a*d - ((b*d*f*x + b*c*f)*Ei((d*f*x + c*f)/d)
+ (b*d*f*x + b*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) + ((b*d*f*x
+ b*c*f)*Ei((d*f*x + c*f)/d) - (b*d*f*x + b*c*f)*Ei(-(d*f*x + c*f)/d))*sinh
(-(d*e - c*f)/d))/(d^3*x + c*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3074, size = 225, normalized size = 2.59

$$\frac{\left(dfx\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}+dfx\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)}+cf\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}+cf\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)}-de^{\left(fx+e\right)}+de^{\left(-fx-\right.}}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(d*f*x*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + d*f*x*Ei((d*f*x + c*f)/d)*e
^(-c*f/d + e) + c*f*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + c*f*Ei((d*f*x + c*
f)/d)*e^(-c*f/d + e) - d*e^(f*x + e) + d*e^(-f*x - e))*b/(d^3*x + c*d^2) -
a/((d*x + c)*d)
```

3.162 $\int \frac{a+b\sinh(e+fx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=123

$$-\frac{a}{2d(c+dx)^2}+\frac{bf^2\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{2d^3}+\frac{bf^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{bf\cosh(e+fx)}{2d^2(c+dx)}-\frac{b\sinh(e+f}{2d(c+dx)^2}$$

```
[Out] -a/(2*d*(c + d*x)^2) - (b*f*Cosh[e + f*x])/(2*d^2*(c + d*x)) + (b*f^2*CoshI
ntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/(2*d^3) - (b*Sinh[e + f*x])/(2*d*
(c + d*x)^2) + (b*f^2*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/(2*d^3
)
```

Rubi [A] time = 0.205068, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3317, 3297, 3303, 3298, 3301}

$$-\frac{a}{2d(c+dx)^2}+\frac{bf^2\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{2d^3}+\frac{bf^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{bf\cosh(e+fx)}{2d^2(c+dx)}-\frac{b\sinh(e+f}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x])/(c + d*x)^3,x]
```

```
[Out] -a/(2*d*(c + d*x)^2) - (b*f*Cosh[e + f*x])/(2*d^2*(c + d*x)) + (b*f^2*CoshI
ntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/(2*d^3) - (b*Sinh[e + f*x])/(2*d*
(c + d*x)^2) + (b*f^2*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/(2*d^3
)
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{a+b\sinh(e+fx)}{(c+dx)^3}\,dx = \int \left(\frac{a}{(c+dx)^3}+\frac{b\sinh(e+fx)}{(c+dx)^3}\right)dx$$
$$= -\frac{a}{2d(c+dx)^2}+b\int \frac{\sinh(e+fx)}{(c+dx)^3}\,dx$$
$$= -\frac{a}{2d(c+dx)^2}-\frac{b\sinh(e+fx)}{2d(c+dx)^2}+\frac{(bf)\int \frac{\cosh(e+fx)}{(c+dx)^2}\,dx}{2d}$$
$$= -\frac{a}{2d(c+dx)^2}-\frac{bf\cosh(e+fx)}{2d^2(c+dx)}-\frac{b\sinh(e+fx)}{2d(c+dx)^2}+\frac{\left(bf^2\right)\int \frac{\sinh(e+fx)}{c+dx}\,dx}{2d^2}$$
$$= -\frac{a}{2d(c+dx)^2}-\frac{bf\cosh(e+fx)}{2d^2(c+dx)}-\frac{b\sinh(e+fx)}{2d(c+dx)^2}+\frac{\left(bf^2\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\sinh\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx}{2d^2}+$$
$$= -\frac{a}{2d(c+dx)^2}-\frac{bf\cosh(e+fx)}{2d^2(c+dx)}+\frac{bf^2\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{2d^3}-\frac{b\sinh(e+fx)}{2d(c+dx)^2}+\frac{bf^2\,\text{c}}{}$$

Mathematica [A] time = 0.639493, size = 95, normalized size = 0.77

$$\frac{-\frac{d(d(a+b\sinh(e+fx))+bf(c+dx)\cosh(e+fx))}{(c+dx)^2}+bf^2\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+bf^2\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x])/(c + d*x)^3,x]
```

```
[Out] (b*f^2*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] - (d*(b*f*(c + d*x)*Cosh
[e + f*x] + d*(a + b*Sinh[e + f*x])))/(c + d*x)^2 + b*f^2*Cosh[e - (c*f)/d]
*SinhIntegral[f*(c/d + x)])/(2*d^3)
```

Maple [B] time = 0.024, size = 296, normalized size = 2.4

$$-\frac{a}{2\,d\,(dx+c)^2}-\frac{bf^3\mathrm{e}^{-fx-e}x}{4\,d\left(d^2f^2x^2+2\,dcf^2x+c^2f^2\right)}-\frac{bf^3\mathrm{e}^{-fx-e}c}{4\,d^2\left(d^2f^2x^2+2\,dcf^2x+c^2f^2\right)}+\frac{f^2b\mathrm{e}^{-fx-e}}{4\,d\left(d^2f^2x^2+2\,dcf^2x+c^2f^2\right)}+\frac{f^2b}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e))/(d*x+c)^3,x)
```

```
[Out] -1/2*a/d/(d*x+c)^2-1/4*b*f^3*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2
)*x-1/4*b*f^3*exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c+1/4*b*f^2
*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+1/4*b*f^2/d^3*exp((c*f-d*e
)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/4*f^2*b/d^3*exp(f*x+e)/(c*f/d+f*x)^2-1/4*f^2
*b/d^3*exp(f*x+e)/(c*f/d+f*x)-1/4*f^2*b/d^3*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(
c*f-d*e)/d)
```

Maxima [A] time = 1.46596, size = 134, normalized size = 1.09

$$\frac{1}{2}b\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right)-\frac{a}{2\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*(e^(-e + c*f/d)*exp_integral_e(3, (d*x + c)*f/d)/((d*x + c)^2*d) - e^
(e - c*f/d)*exp_integral_e(3, -(d*x + c)*f/d)/((d*x + c)^2*d)) - 1/2*a/(d^3
*x^2 + 2*c*d^2*x + c^2*d)
```

Fricas [B] time = 2.45782, size = 572, normalized size = 4.65

$$-\frac{2\,bd^2\sinh\left(fx+e\right)+2\,ad^2+2\left(bd^2fx+bcdf\right)\cosh\left(fx+e\right)-\left(\left(bd^2f^2x^2+2\,bcdf^2x+bc^2f^2\right)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-\left(bd^2f^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*d^2*sinh(f*x + e) + 2*a*d^2 + 2*(b*d^2*f*x + b*c*d*f)*cosh(f*x +
e) - ((b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*Ei((d*f*x + c*f)/d) - (b*
d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e -
 c*f)/d) + ((b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*Ei((d*f*x + c*f)/d)
 + (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*sinh(-
(d*e - c*f)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24726, size = 441, normalized size = 3.59

$$-\frac{bd^2f^2x^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}-bd^2f^2x^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)}+2\,bcdf^2x\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}-2\,bcdf^2x\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*(b*d^2*f^2*x^2*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) - b*d^2*f^2*x^2*Ei((
d*f*x + c*f)/d)*e^(-c*f/d + e) + 2*b*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^(c*f/
d - e) - 2*b*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(-c*f/d + e) + b*c^2*f^2*Ei(-(
d*f*x + c*f)/d)*e^(c*f/d - e) - b*c^2*f^2*Ei((d*f*x + c*f)/d)*e^(-c*f/d + e
) + b*d^2*f*x*e^(f*x + e) + b*d^2*f*x*e^(-f*x - e) + b*c*d*f*e^(f*x + e) +
b*c*d*f*e^(-f*x - e) + b*d^2*e^(f*x + e) - b*d^2*e^(-f*x - e) + 2*a*d^2)/(d
^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

3.163 $\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx$

Optimal. Leaf size=250

$$\frac{a^2(c+dx)^4}{4d}+\frac{12abd^2(c+dx)\cosh(e+fx)}{f^3}-\frac{6abd(c+dx)^2\sinh(e+fx)}{f^2}+\frac{2ab(c+dx)^3\cosh(e+fx)}{f}-\frac{12abd^3\sinh}{f^4}$$

```
[Out] (-3*b^2*c*d^2*x)/(4*f^2) - (3*b^2*d^3*x^2)/(8*f^2) + (a^2*(c + d*x)^4)/(4*d
) - (b^2*(c + d*x)^4)/(8*d) + (12*a*b*d^2*(c + d*x)*Cosh[e + f*x])/f^3 + (2
*a*b*(c + d*x)^3*Cosh[e + f*x])/f - (12*a*b*d^3*Sinh[e + f*x])/f^4 - (6*a*b
*d*(c + d*x)^2*Sinh[e + f*x])/f^2 + (3*b^2*d^2*(c + d*x)*Cosh[e + f*x]*Sinh
[e + f*x])/(4*f^3) + (b^2*(c + d*x)^3*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) -
(3*b^2*d^3*Sinh[e + f*x]^2)/(8*f^4) - (3*b^2*d*(c + d*x)^2*Sinh[e + f*x]^2)
/(4*f^2)
```

Rubi [A] time = 0.289963, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3317, 3296, 2637, 3311, 32, 3310}

$$\frac{a^2(c+dx)^4}{4d}+\frac{12abd^2(c+dx)\cosh(e+fx)}{f^3}-\frac{6abd(c+dx)^2\sinh(e+fx)}{f^2}+\frac{2ab(c+dx)^3\cosh(e+fx)}{f}-\frac{12abd^3\sinh}{f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Sinh[e + f*x])^2,x]
```


```
[Out] (-3*b^2*c*d^2*x)/(4*f^2) - (3*b^2*d^3*x^2)/(8*f^2) + (a^2*(c + d*x)^4)/(4*d
) - (b^2*(c + d*x)^4)/(8*d) + (12*a*b*d^2*(c + d*x)*Cosh[e + f*x])/f^3 + (2
*a*b*(c + d*x)^3*Cosh[e + f*x])/f - (12*a*b*d^3*Sinh[e + f*x])/f^4 - (6*a*b
*d*(c + d*x)^2*Sinh[e + f*x])/f^2 + (3*b^2*d^2*(c + d*x)*Cosh[e + f*x]*Sinh
[e + f*x])/(4*f^3) + (b^2*(c + d*x)^3*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) -
(3*b^2*d^3*Sinh[e + f*x]^2)/(8*f^4) - (3*b^2*d*(c + d*x)^2*Sinh[e + f*x]^2)
/(4*f^2)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
```

```
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)^3(a+b\sinh(e+fx))^2\,dx = \int \left(a^2(c+dx)^3+2ab(c+dx)^3\sinh(e+fx)+b^2(c+dx)^3\sinh^2(e+fx)\right)dx$$

$$= \frac{a^2(c+dx)^4}{4d}+(2ab)\int (c+dx)^3\sinh(e+fx)\,dx+b^2\int (c+dx)^3\sinh^2(e+fx)\,dx$$

$$= \frac{a^2(c+dx)^4}{4d}+\frac{2ab(c+dx)^3\cosh(e+fx)}{f}+\frac{b^2(c+dx)^3\cosh(e+fx)\sinh(e+fx)}{2f}$$

$$= \frac{a^2(c+dx)^4}{4d}-\frac{b^2(c+dx)^4}{8d}+\frac{2ab(c+dx)^3\cosh(e+fx)}{f}-\frac{6abd(c+dx)^2\sinh(e+f}{f^2}$$

$$= -\frac{3b^2cd^2x}{4f^2}-\frac{3b^2d^3x^2}{8f^2}+\frac{a^2(c+dx)^4}{4d}-\frac{b^2(c+dx)^4}{8d}+\frac{12abd^2(c+dx)\cosh(e+fx)}{f^3}$$

$$= -\frac{3b^2cd^2x}{4f^2}-\frac{3b^2d^3x^2}{8f^2}+\frac{a^2(c+dx)^4}{4d}-\frac{b^2(c+dx)^4}{8d}+\frac{12abd^2(c+dx)\cosh(e+fx)}{f^3}$$

Mathematica [A] time = 1.38262, size = 235, normalized size = 0.94

$$2\left(f^4x\left(2a^2-b^2\right)\left(6c^2dx+4c^3+4cd^2x^2+d^3x^3\right)-48abd\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2+2\right)\right)\sinh(e+fx)+b^2f(c+dx)\left(2c\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] (32*a*b*f*(c + d*x)*(c^2*f^2 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Cosh[e + f*
x] - 3*b^2*d*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(1 + 2*f^2*x^2))*Cosh[2*(e + f*
x)] + 2*((2*a^2 - b^2)*f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) -
48*a*b*d*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x] + b^2*f*
(c + d*x)*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(3 + 2*f^2*x^2))*Sinh[2*(e + f*x)]
))/(16*f^4)
```

Maple [B] time = 0.014, size = 1061, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*sinh(f*x+e))^2,x)
```

```
[Out] 1/f*(c^3*b^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)+3/2/f^3*d^3*e^2*a^
2*(f*x+e)^2-1/f^3*d^3*e*a^2*(f*x+e)^3+3/2/f*c^2*d*a^2*(f*x+e)^2+1/f^2*c*d^2
*a^2*(f*x+e)^3-1/f^3*d^3*e^3*a^2*(f*x+e)+2/f^3*d^3*a*b*((f*x+e)^3*cosh(f*x+
e)-3*(f*x+e)^2*sinh(f*x+e)+6*(f*x+e)*cosh(f*x+e)-6*sinh(f*x+e))-3/f^3*d^3*e
*b^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)-1/6*(f*x+e)^3-1/2*(f*x+e)*cosh(
f*x+e)^2+1/4*cosh(f*x+e)*sinh(f*x+e)+1/4*f*x+1/4*e)+3/f^3*d^3*e^2*b^2*(1/2*
(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)+3/f^2*c*d^
2*b^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)-1/6*(f*x+e)^3-1/2*(f*x+e)*cosh
(f*x+e)^2+1/4*cosh(f*x+e)*sinh(f*x+e)+1/4*f*x+1/4*e)-1/f^3*d^3*e^3*b^2*(1/2
*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)+3/f*c^2*d*b^2*(1/2*(f*x+e)*cosh(f*x
+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)+1/f^3*d^3*b^2*(1/2*(f*x+e)
^3*cosh(f*x+e)*sinh(f*x+e)-1/8*(f*x+e)^4-3/4*(f*x+e)^2*cosh(f*x+e)^2+3/4*(f
*x+e)*cosh(f*x+e)*sinh(f*x+e)+3/8*(f*x+e)^2-3/8*cosh(f*x+e)^2)+2*c^3*a*b*co
sh(f*x+e)+1/4/f^3*d^3*a^2*(f*x+e)^4+3/f^2*c*d^2*e^2*b^2*(1/2*cosh(f*x+e)*si
nh(f*x+e)-1/2*f*x-1/2*e)-6/f^2*c*d^2*e*b^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*
x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)+6/f*c^2*d*a*b*((f*x+e)*cosh(f*x+e)-si
nh(f*x+e))-2/f^3*d^3*e^3*a*b*cosh(f*x+e)-3/f*c^2*d*e*b^2*(1/2*cosh(f*x+e)*s
inh(f*x+e)-1/2*f*x-1/2*e)+6/f^3*d^3*e^2*a*b*((f*x+e)*cosh(f*x+e)-sinh(f*x+e
))+6/f^2*c*d^2*a*b*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sinh(f*x+e)+2*cosh(f*x+
e))-6/f^3*d^3*e*a*b*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sinh(f*x+e)+2*cosh(f*x
+e))+6/f^2*c*d^2*e^2*a*b*cosh(f*x+e)-12/f^2*c*d^2*e*a*b*((f*x+e)*cosh(f*x+e
)-sinh(f*x+e))-6/f*c^2*d*e*a*b*cosh(f*x+e)+3/f^2*c*d^2*e^2*a^2*(f*x+e)-3/f*
c^2*d*e*a^2*(f*x+e)-3/f^2*c*d^2*e*a^2*(f*x+e)^2+c^3*a^2*(f*x+e))
```

Maxima [B] time = 1.3345, size = 702, normalized size = 2.81

$$\frac{1}{4}a^2d^3x^4 + a^2cd^2x^3 + \frac{3}{2}a^2c^2dx^2 - \frac{3}{16}\left(4x^2 - \frac{\left(2fxe^{(2e)} - e^{(2e)}\right)e^{(2fx)}}{f^2} + \frac{\left(2fx+1\right)e^{(-2fx-2e)}}{f^2}\right)b^2c^2d - \frac{1}{16}\left(8x^3 - \frac{3\left(2 \cdots\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 - 3/16*(4*x^2 - (2*f*x*
e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 + (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*c^
2*d - 1/16*(8*x^3 - 3*(2*f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*
x)/f^3 + 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*b^2*c*d^2 - 1/32*(
4*x^4 - (4*f^3*x^3*e^(2*e) - 6*f^2*x^2*e^(2*e) + 6*f*x*e^(2*e) - 3*e^(2*e))
*e^(2*f*x)/f^4 + (4*f^3*x^3 + 6*f^2*x^2 + 6*f*x + 3)*e^(-2*f*x - 2*e)/f^4)*
b^2*d^3 - 1/8*b^2*c^3*(4*x - e^(2*f*x + 2*e)/f + e^(-2*f*x - 2*e)/f) + a^2*
c^3*x + 3*a*b*c^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(-f*x - e)/f
^2) + 3*a*b*c*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 + (f^2*x^2
 + 2*f*x + 2)*e^(-f*x - e)/f^3) + a*b*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6
*f*x*e^e - 6*e^e)*e^(f*x)/f^4 + (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(-f*x -
 e)/f^4) + 2*a*b*c^3*cosh(f*x + e)/f
```

Fricas [A] time = 2.4951, size = 880, normalized size = 3.52

$$\frac{2\left(2a^2-b^2\right)d^3f^4x^4 + 8\left(2a^2-b^2\right)cd^2f^4x^3 + 12\left(2a^2-b^2\right)c^2df^4x^2 + 8\left(2a^2-b^2\right)c^3f^4x - 3\left(2b^2d^3f^2x^2 + 4b^2cd^2f^2x \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(2*a^2 - b^2)*d^3*f^4*x^4 + 8*(2*a^2 - b^2)*c*d^2*f^4*x^3 + 12*(2*a
^2 - b^2)*c^2*d*f^4*x^2 + 8*(2*a^2 - b^2)*c^3*f^4*x - 3*(2*b^2*d^3*f^2*x^2
+ 4*b^2*c*d^2*f^2*x + 2*b^2*c^2*d*f^2 + b^2*d^3)*cosh(f*x + e)^2 - 3*(2*b^2
*d^3*f^2*x^2 + 4*b^2*c*d^2*f^2*x + 2*b^2*c^2*d*f^2 + b^2*d^3)*sinh(f*x + e)
^2 + 32*(a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 + a*b*c^3*f^3 + 6*a*b*c*d^2*
f + 3*(a*b*c^2*d*f^3 + 2*a*b*d^3*f)*x)*cosh(f*x + e) - 4*(24*a*b*d^3*f^2*x^
2 + 48*a*b*c*d^2*f^2*x + 24*a*b*c^2*d*f^2 + 48*a*b*d^3 - (2*b^2*d^3*f^3*x^3
 + 6*b^2*c*d^2*f^3*x^2 + 2*b^2*c^3*f^3 + 3*b^2*c*d^2*f + 3*(2*b^2*c^2*d*f^3
 + b^2*d^3*f)*x)*cosh(f*x + e))*sinh(f*x + e))/f^4
```

Sympy [A] time = 5.31346, size = 779, normalized size = 3.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*sinh(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**3*x + 3*a**2*c**2*d*x**2/2 + a**2*c*d**2*x**3 + a**2*d**
3*x**4/4 + 2*a*b*c**3*cosh(e + f*x)/f + 6*a*b*c**2*d*x*cosh(e + f*x)/f - 6*
a*b*c**2*d*sinh(e + f*x)/f**2 + 6*a*b*c*d**2*x**2*cosh(e + f*x)/f - 12*a*b*
c*d**2*x*sinh(e + f*x)/f**2 + 12*a*b*c*d**2*cosh(e + f*x)/f**3 + 2*a*b*d**3
*x**3*cosh(e + f*x)/f - 6*a*b*d**3*x**2*sinh(e + f*x)/f**2 + 12*a*b*d**3*x*
cosh(e + f*x)/f**3 - 12*a*b*d**3*sinh(e + f*x)/f**4 + b**2*c**3*x*sinh(e +
f*x)**2/2 - b**2*c**3*x*cosh(e + f*x)**2/2 + b**2*c**3*sinh(e + f*x)*cosh(e
 + f*x)/(2*f) + 3*b**2*c**2*d*x**2*sinh(e + f*x)**2/4 - 3*b**2*c**2*d*x**2*
cosh(e + f*x)**2/4 + 3*b**2*c**2*d*x*sinh(e + f*x)*cosh(e + f*x)/(2*f) - 3*
b**2*c**2*d*sinh(e + f*x)**2/(4*f**2) + b**2*c*d**2*x**3*sinh(e + f*x)**2/2
 - b**2*c*d**2*x**3*cosh(e + f*x)**2/2 + 3*b**2*c*d**2*x**2*sinh(e + f*x)*c
osh(e + f*x)/(2*f) - 3*b**2*c*d**2*x*sinh(e + f*x)**2/(4*f**2) - 3*b**2*c*d
**2*x*cosh(e + f*x)**2/(4*f**2) + 3*b**2*c*d**2*sinh(e + f*x)*cosh(e + f*x)
/(4*f**3) + b**2*d**3*x**4*sinh(e + f*x)**2/8 - b**2*d**3*x**4*cosh(e + f*x
)**2/8 + b**2*d**3*x**3*sinh(e + f*x)*cosh(e + f*x)/(2*f) - 3*b**2*d**3*x**
2*sinh(e + f*x)**2/(8*f**2) - 3*b**2*d**3*x**2*cosh(e + f*x)**2/(8*f**2) +
3*b**2*d**3*x*sinh(e + f*x)*cosh(e + f*x)/(4*f**3) - 3*b**2*d**3*sinh(e + f
*x)**2/(8*f**4), Ne(f, 0)), ((a + b*sinh(e))**2*(c**3*x + 3*c**2*d*x**2/2 +
 c*d**2*x**3 + d**3*x**4/4), True))
```

Giac [B] time = 1.26267, size = 813, normalized size = 3.25

$$\frac{1}{4}a^2d^3x^4 - \frac{1}{8}b^2d^3x^4 + a^2cd^2x^3 - \frac{1}{2}b^2cd^2x^3 + \frac{3}{2}a^2c^2dx^2 - \frac{3}{4}b^2c^2dx^2 + a^2c^3x - \frac{1}{2}b^2c^3x + \frac{(4\,b^2d^3f^3x^3 + 12\,b^2cd^2f^3x^2 + 1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/4*a^2*d^3*x^4 - 1/8*b^2*d^3*x^4 + a^2*c*d^2*x^3 - 1/2*b^2*c*d^2*x^3 + 3/2
*a^2*c^2*d*x^2 - 3/4*b^2*c^2*d*x^2 + a^2*c^3*x - 1/2*b^2*c^3*x + 1/32*(4*b^
2*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x - 6*b^2*d^3*f^2*x
^2 + 4*b^2*c^3*f^3 - 12*b^2*c*d^2*f^2*x - 6*b^2*c^2*d*f^2 + 6*b^2*d^3*f*x +
 6*b^2*c*d^2*f - 3*b^2*d^3)*e^(2*f*x + 2*e)/f^4 + (a*b*d^3*f^3*x^3 + 3*a*b*
```

```
c*d^2*f^3*x^2 + 3*a*b*c^2*d*f^3*x - 3*a*b*d^3*f^2*x^2 + a*b*c^3*f^3 - 6*a*b
*c*d^2*f^2*x - 3*a*b*c^2*d*f^2 + 6*a*b*d^3*f*x + 6*a*b*c*d^2*f - 6*a*b*d^3)
*e^(f*x + e)/f^4 + (a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 + 3*a*b*c^2*d*f^3
*x + 3*a*b*d^3*f^2*x^2 + a*b*c^3*f^3 + 6*a*b*c*d^2*f^2*x + 3*a*b*c^2*d*f^2
+ 6*a*b*d^3*f*x + 6*a*b*c*d^2*f + 6*a*b*d^3)*e^(-f*x - e)/f^4 - 1/32*(4*b^2
*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x + 6*b^2*d^3*f^2*x^
2 + 4*b^2*c^3*f^3 + 12*b^2*c*d^2*f^2*x + 6*b^2*c^2*d*f^2 + 6*b^2*d^3*f*x +
6*b^2*c*d^2*f + 3*b^2*d^3)*e^(-2*f*x - 2*e)/f^4
```

3.164 $\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx$

Optimal. Leaf size=182

$$\frac{a^2(c+dx)^3}{3d}-\frac{4abd(c+dx)\sinh(e+fx)}{f^2}+\frac{2ab(c+dx)^2\cosh(e+fx)}{f}+\frac{4abd^2\cosh(e+fx)}{f^3}-\frac{b^2d(c+dx)\sinh^2(e+fx)}{2f^2}$$

```
[Out] -(b^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(3*d) - (b^2*(c + d*x)^3)/(6*d) +
(4*a*b*d^2*Cosh[e + f*x])/f^3 + (2*a*b*(c + d*x)^2*Cosh[e + f*x])/f - (4*a*
b*d*(c + d*x)*Sinh[e + f*x])/f^2 + (b^2*d^2*Cosh[e + f*x]*Sinh[e + f*x])/(4
*f^3) + (b^2*(c + d*x)^2*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) - (b^2*d*(c + d
*x)*Sinh[e + f*x]^2)/(2*f^2)
```

Rubi [A] time = 0.202018, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {3317, 3296, 2638, 3311, 32, 2635, 8}

$$\frac{a^2(c+dx)^3}{3d}-\frac{4abd(c+dx)\sinh(e+fx)}{f^2}+\frac{2ab(c+dx)^2\cosh(e+fx)}{f}+\frac{4abd^2\cosh(e+fx)}{f^3}-\frac{b^2d(c+dx)\sinh^2(e+fx)}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Sinh[e + f*x])^2,x]
```



```
[Out] -(b^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(3*d) - (b^2*(c + d*x)^3)/(6*d) +
(4*a*b*d^2*Cosh[e + f*x])/f^3 + (2*a*b*(c + d*x)^2*Cosh[e + f*x])/f - (4*a*
b*d*(c + d*x)*Sinh[e + f*x])/f^2 + (b^2*d^2*Cosh[e + f*x]*Sinh[e + f*x])/(4
*f^3) + (b^2*(c + d*x)^2*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) - (b^2*d*(c + d
*x)*Sinh[e + f*x]^2)/(2*f^2)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (c+dx)^2(a+b\sinh(e+fx))^2\,dx = \int \left(a^2(c+dx)^2+2ab(c+dx)^2\sinh(e+fx)+b^2(c+dx)^2\sinh^2(e+fx)\right)dx$$
$$= \frac{a^2(c+dx)^3}{3d}+(2ab)\int (c+dx)^2\sinh(e+fx)\,dx+b^2\int (c+dx)^2\sinh^2(e+fx)$$
$$= \frac{a^2(c+dx)^3}{3d}+\frac{2ab(c+dx)^2\cosh(e+fx)}{f}+\frac{b^2(c+dx)^2\cosh(e+fx)\sinh(e+f}{2f}$$
$$= \frac{a^2(c+dx)^3}{3d}-\frac{b^2(c+dx)^3}{6d}+\frac{2ab(c+dx)^2\cosh(e+fx)}{f}-\frac{4abd(c+dx)\sinh(e+}{f^2}$$
$$= -\frac{b^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{3d}-\frac{b^2(c+dx)^3}{6d}+\frac{4abd^2\cosh(e+fx)}{f^3}+\frac{2ab(c+dx)^2\cosh}{f}$$

Mathematica [A] time = 0.752984, size = 249, normalized size = 1.37

$$\frac{24a^2c^2f^3x+24a^2cdf^3x^2+8a^2d^2f^3x^3+48ab\left(c^2f^2+2cdf^2x+d^2\left(f^2x^2+2\right)\right)\cosh(e+fx)-96abcdf\sinh(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] (24*a^2*c^2*f^3*x - 12*b^2*c^2*f^3*x + 24*a^2*c*d*f^3*x^2 - 12*b^2*c*d*f^3*
x^2 + 8*a^2*d^2*f^3*x^3 - 4*b^2*d^2*f^3*x^3 + 48*a*b*(c^2*f^2 + 2*c*d*f^2*x
 + d^2*(2 + f^2*x^2))*Cosh[e + f*x] - 6*b^2*d*f*(c + d*x)*Cosh[2*(e + f*x)]
 - 96*a*b*c*d*f*Sinh[e + f*x] - 96*a*b*d^2*f*x*Sinh[e + f*x] + 3*b^2*d^2*Si
nh[2*(e + f*x)] + 6*b^2*c^2*f^2*Sinh[2*(e + f*x)] + 12*b^2*c*d*f^2*x*Sinh[2
*(e + f*x)] + 6*b^2*d^2*f^2*x^2*Sinh[2*(e + f*x)])/(24*f^3)
```

Maple [B] time = 0.016, size = 535, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*sinh(f*x+e))^2,x)
```

```
[Out] 1/f*(1/3/f^2*d^2*a^2*(f*x+e)^3+2/f^2*d^2*a*b*((f*x+e)^2*cosh(f*x+e)-2*(f*x+
e)*sinh(f*x+e)+2*cosh(f*x+e))+1/f^2*d^2*b^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh
(f*x+e)-1/6*(f*x+e)^3-1/2*(f*x+e)*cosh(f*x+e)^2+1/4*cosh(f*x+e)*sinh(f*x+e)
+1/4*f*x+1/4*e)-1/f^2*d^2*e*a^2*(f*x+e)^2-4/f^2*d^2*e*a*b*((f*x+e)*cosh(f*x
+e)-sinh(f*x+e))-2/f^2*d^2*e*b^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(
f*x+e)^2-1/4*cosh(f*x+e)^2)+1/f^2*d^2*e^2*a^2*(f*x+e)+2/f^2*d^2*e^2*a*b*cos
h(f*x+e)+1/f^2*d^2*e^2*b^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)+1/f*
c*d*a^2*(f*x+e)^2+4/f*c*d*a*b*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))+2/f*c*d*b^2
*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)-2/f*
c*d*e*a^2*(f*x+e)-4/f*c*d*e*a*b*cosh(f*x+e)-2/f*c*d*e*b^2*(1/2*cosh(f*x+e)*
sinh(f*x+e)-1/2*f*x-1/2*e)+c^2*a^2*(f*x+e)+2*c^2*a*b*cosh(f*x+e)+c^2*b^2*(1
/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e))
```

Maxima [A] time = 1.27115, size = 435, normalized size = 2.39

$$\frac{1}{3}a^2d^2x^3 + a^2cdx^2 - \frac{1}{8}\left(4x^2 - \frac{\left(2fxe^{(2e)} - e^{(2e)}\right)e^{(2fx)}}{f^2} + \frac{(2fx+1)e^{(-2fx-2e)}}{f^2}\right)b^2cd - \frac{1}{48}\left(8x^3 - \frac{3\left(2f^2x^2e^{(2e)} - 2fxe^{(2e}\right.}{f^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*d^2*x^3 + a^2*c*d*x^2 - 1/8*(4*x^2 - (2*f*x*e^(2*e) - e^(2*e))*e^(2
*f*x)/f^2 + (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*c*d - 1/48*(8*x^3 - 3*(2*
f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*x)/f^3 + 3*(2*f^2*x^2 + 2
*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*b^2*d^2 - 1/8*b^2*c^2*(4*x - e^(2*f*x + 2*e
)/f + e^(-2*f*x - 2*e)/f) + a^2*c^2*x + 2*a*b*c*d*((f*x*e^e - e^e)*e^(f*x)/
f^2 + (f*x + 1)*e^(-f*x - e)/f^2) + a*b*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e
^e)*e^(f*x)/f^3 + (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + 2*a*b*c^2*cosh(
f*x + e)/f
```

Fricas [A] time = 2.47877, size = 541, normalized size = 2.97

$$\frac{2\left(2a^2 - b^2\right)d^2f^3x^3 + 6\left(2a^2 - b^2\right)cdf^3x^2 + 6\left(2a^2 - b^2\right)c^2f^3x - 3\left(b^2d^2fx + b^2cdf\right)\cosh\left(fx+e\right)^2 - 3\left(b^2d^2fx + b^2cdf\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*(2*a^2 - b^2)*d^2*f^3*x^3 + 6*(2*a^2 - b^2)*c*d*f^3*x^2 + 6*(2*a^2
- b^2)*c^2*f^3*x - 3*(b^2*d^2*f*x + b^2*c*d*f)*cosh(f*x + e)^2 - 3*(b^2*d^2
*f*x + b^2*c*d*f)*sinh(f*x + e)^2 + 24*(a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x +
 a*b*c^2*f^2 + 2*a*b*d^2)*cosh(f*x + e) - 3*(16*a*b*d^2*f*x + 16*a*b*c*d*f
- (2*b^2*d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*f^2 + b^2*d^2)*cosh(f*x
+ e))*sinh(f*x + e))/f^3
```

Sympy [A] time = 2.35746, size = 456, normalized size = 2.51

$$\begin{cases} a^2c^2x + a^2cdx^2 + \frac{a^2d^2x^3}{3} + \frac{2abc^2\cosh(e+fx)}{f} + \frac{4abcdx\cosh(e+fx)}{f} - \frac{4abcd\sinh(e+fx)}{f^2} + \frac{2abd^2x^2\cosh(e+fx)}{f} - \frac{4abd^2x\sinh(e+fx)}{f^2} + \frac{4abd}{} \\ (a+b\sinh(e))^2\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*sinh(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**2*x + a**2*c*d*x**2 + a**2*d**2*x**3/3 + 2*a*b*c**2*cosh
(e + f*x)/f + 4*a*b*c*d*x*cosh(e + f*x)/f - 4*a*b*c*d*sinh(e + f*x)/f**2 +
2*a*b*d**2*x**2*cosh(e + f*x)/f - 4*a*b*d**2*x*sinh(e + f*x)/f**2 + 4*a*b*d
**2*cosh(e + f*x)/f**3 + b**2*c**2*x*sinh(e + f*x)**2/2 - b**2*c**2*x*cosh(
e + f*x)**2/2 + b**2*c**2*sinh(e + f*x)*cosh(e + f*x)/(2*f) + b**2*c*d*x**2
*sinh(e + f*x)**2/2 - b**2*c*d*x**2*cosh(e + f*x)**2/2 + b**2*c*d*x*sinh(e
+ f*x)*cosh(e + f*x)/f - b**2*c*d*sinh(e + f*x)**2/(2*f**2) + b**2*d**2*x**
3*sinh(e + f*x)**2/6 - b**2*d**2*x**3*cosh(e + f*x)**2/6 + b**2*d**2*x**2*s
inh(e + f*x)*cosh(e + f*x)/(2*f) - b**2*d**2*x*sinh(e + f*x)**2/(4*f**2) -
b**2*d**2*x*cosh(e + f*x)**2/(4*f**2) + b**2*d**2*sinh(e + f*x)*cosh(e + f*
x)/(4*f**3), Ne(f, 0)), ((a + b*sinh(e))**2*(c**2*x + c*d*x**2 + d**2*x**3/
3), True))
```

Giac [B] time = 1.2962, size = 470, normalized size = 2.58

$$\frac{1}{3}\,a^2d^2x^3-\frac{1}{6}\,b^2d^2x^3+a^2cdx^2-\frac{1}{2}\,b^2cdx^2+a^2c^2x-\frac{1}{2}\,b^2c^2x+\frac{\left(2\,b^2d^2f^2x^2+4\,b^2cdf^2x+2\,b^2c^2f^2-2\,b^2d^2fx-2\,b^2c\cdots\right.}{16\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*a^2*d^2*x^3 - 1/6*b^2*d^2*x^3 + a^2*c*d*x^2 - 1/2*b^2*c*d*x^2 + a^2*c^2
*x - 1/2*b^2*c^2*x + 1/16*(2*b^2*d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*
f^2 - 2*b^2*d^2*f*x - 2*b^2*c*d*f + b^2*d^2)*e^(2*f*x + 2*e)/f^3 + (a*b*d^2
*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2 - 2*a*b*d^2*f*x - 2*a*b*c*d*f + 2*
a*b*d^2)*e^(f*x + e)/f^3 + (a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2
 + 2*a*b*d^2*f*x + 2*a*b*c*d*f + 2*a*b*d^2)*e^(-f*x - e)/f^3 - 1/16*(2*b^2*
d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*f^2 + 2*b^2*d^2*f*x + 2*b^2*c*d*f
 + b^2*d^2)*e^(-2*f*x - 2*e)/f^3
```

3.165 $\int (c+dx)(a+b\sinh(e+fx))^2\,dx$

Optimal. Leaf size=116

$$\frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\cosh(e+fx)}{f}-\frac{2abd\sinh(e+fx)}{f^2}+\frac{b^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{2f}-\frac{1}{2}b^2cx-\frac{b^2d\sinh}{4}$$

```
[Out] -(b^2*c*x)/2 - (b^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) + (2*a*b*(c + d*x)*C
osh[e + f*x])/f - (2*a*b*d*Sinh[e + f*x])/f^2 + (b^2*(c + d*x)*Cosh[e + f*x
]*Sinh[e + f*x])/(2*f) - (b^2*d*Sinh[e + f*x]^2)/(4*f^2)
```

Rubi [A] time = 0.105548, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3317, 3296, 2637, 3310}

$$\frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\cosh(e+fx)}{f}-\frac{2abd\sinh(e+fx)}{f^2}+\frac{b^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{2f}-\frac{1}{2}b^2cx-\frac{b^2d\sinh}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Sinh[e + f*x])^2,x]
```


```
[Out] -(b^2*c*x)/2 - (b^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) + (2*a*b*(c + d*x)*C
osh[e + f*x])/f - (2*a*b*d*Sinh[e + f*x])/f^2 + (b^2*(c + d*x)*Cosh[e + f*x
]*Sinh[e + f*x])/(2*f) - (b^2*d*Sinh[e + f*x]^2)/(4*f^2)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)(a+b\sinh(e+fx))^2\,dx = \int \left(a^2(c+dx)+2ab(c+dx)\sinh(e+fx)+b^2(c+dx)\sinh^2(e+fx)\right)dx$$
$$= \frac{a^2(c+dx)^2}{2d} + (2ab)\int (c+dx)\sinh(e+fx)\,dx + b^2\int (c+dx)\sinh^2(e+fx)\,dx$$
$$= \frac{a^2(c+dx)^2}{2d} + \frac{2ab(c+dx)\cosh(e+fx)}{f} + \frac{b^2(c+dx)\cosh(e+fx)\sinh(e+fx)}{2f}$$
$$= -\frac{1}{2}b^2cx - \frac{1}{4}b^2dx^2 + \frac{a^2(c+dx)^2}{2d} + \frac{2ab(c+dx)\cosh(e+fx)}{f} - \frac{2abd\sinh(e+fx}{f^2}$$

Mathematica [A] time = 0.745308, size = 98, normalized size = 0.84

$$-\frac{2\left(2a^2-b^2\right)(e+fx)(d(e-fx)-2cf)-16abf(c+dx)\cosh(e+fx)+16abd\sinh(e+fx)-2b^2f(c+dx)\sinh(2(e+}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] -(2*(2*a^2 - b^2)*(e + f*x)*(-2*c*f + d*(e - f*x)) - 16*a*b*f*(c + d*x)*Cos
h[e + f*x] + b^2*d*Cosh[2*(e + f*x)] + 16*a*b*d*Sinh[e + f*x] - 2*b^2*f*(c
+ d*x)*Sinh[2*(e + f*x)])/(8*f^2)
```

Maple [A] time = 0.014, size = 208, normalized size = 1.8

$$\frac{1}{f}\left(\frac{da^2\left(fx+e\right)^2}{2f}+2\,\frac{dab\left(\left(fx+e\right)\cosh\left(fx+e\right)-\sinh\left(fx+e\right)\right)}{f}+\frac{db^2}{f}\left(\frac{\left(fx+e\right)\cosh\left(fx+e\right)\sinh\left(fx+e\right)}{2}-\frac{(f}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*sinh(f*x+e))^2,x)
```

```
[Out] 1/f*(1/2/f*d*a^2*(f*x+e)^2+2/f*d*a*b*((f*x+e)*cosh(f*x+e)-sinh(f*x+e))+1/f*
d*b^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)
-d*e/f*a^2*(f*x+e)-2*d*e/f*a*b*cosh(f*x+e)-d*e/f*b^2*(1/2*cosh(f*x+e)*sinh(
f*x+e)-1/2*f*x-1/2*e)+c*a^2*(f*x+e)+2*c*a*b*cosh(f*x+e)+c*b^2*(1/2*cosh(f*x
+e)*sinh(f*x+e)-1/2*f*x-1/2*e))
```

Maxima [A] time = 1.25498, size = 221, normalized size = 1.91

$$\frac{1}{2}\,a^2dx^2-\frac{1}{16}\left(4\,x^2-\frac{\left(2\,fxe^{(2\,e)}-e^{(2\,e)}\right)e^{(2\,fx)}}{f^2}+\frac{\left(2\,fx+1\right)e^{(-2\,fx-2\,e)}}{f^2}\right)b^2d-\frac{1}{8}\,b^2c\left(4\,x-\frac{e^{(2\,fx+2\,e)}}{f}+\frac{e^{(-2\,fx-2\,e)}}{f}\right)+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*d*x^2 - 1/16*(4*x^2 - (2*f*x*e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 + (2*
f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*d - 1/8*b^2*c*(4*x - e^(2*f*x + 2*e)/f +
 e^(-2*f*x - 2*e)/f) + a^2*c*x + a*b*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x
```

```
+ 1)*e^(-f*x - e)/f^2) + 2*a*b*c*cosh(f*x + e)/f
```

Fricas [A] time = 2.29991, size = 294, normalized size = 2.53

$$\frac{2\left(2a^2-b^2\right)df^2x^2+4\left(2a^2-b^2\right)cf^2x-b^2d\cosh\left(fx+e\right)^2-b^2d\sinh\left(fx+e\right)^2+16\left(abdfx+abcf\right)\cosh\left(fx+e\right)-4\left(\right.}{8f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2 - b^2)*d*f^2*x^2 + 4*(2*a^2 - b^2)*c*f^2*x - b^2*d*cosh(f*x +
 e)^2 - b^2*d*sinh(f*x + e)^2 + 16*(a*b*d*f*x + a*b*c*f)*cosh(f*x + e) - 4*
(4*a*b*d - (b^2*d*f*x + b^2*c*f)*cosh(f*x + e))*sinh(f*x + e))/f^2
```

Sympy [A] time = 0.935067, size = 219, normalized size = 1.89

$$\begin{cases} a^2cx+\frac{a^2dx^2}{2}+\frac{2abc\cosh\left(e+fx\right)}{f}+\frac{2abdx\cosh\left(e+fx\right)}{f}-\frac{2abd\sinh\left(e+fx\right)}{f^2}+\frac{b^2cx\sinh^2\left(e+fx\right)}{2}-\frac{b^2cx\cosh^2\left(e+fx\right)}{2}+\frac{b^2c\sinh\left(e+fx\right)\cosh\left(e+f\right.}{2f} \\ \left(a+b\sinh\left(e\right)\right)^2\left(cx+\frac{dx^2}{2}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sinh(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c*x + a**2*d*x**2/2 + 2*a*b*c*cosh(e + f*x)/f + 2*a*b*d*x*c
osh(e + f*x)/f - 2*a*b*d*sinh(e + f*x)/f**2 + b**2*c*x*sinh(e + f*x)**2/2 -
 b**2*c*x*cosh(e + f*x)**2/2 + b**2*c*sinh(e + f*x)*cosh(e + f*x)/(2*f) + b
**2*d*x**2*sinh(e + f*x)**2/4 - b**2*d*x**2*cosh(e + f*x)**2/4 + b**2*d*x*s
inh(e + f*x)*cosh(e + f*x)/(2*f) - b**2*d*sinh(e + f*x)**2/(4*f**2), Ne(f,
0)), ((a + b*sinh(e))**2*(c*x + d*x**2/2), True))
```

Giac [A] time = 1.23683, size = 220, normalized size = 1.9

$$\frac{1}{2}a^2dx^2-\frac{1}{4}b^2dx^2+a^2cx-\frac{1}{2}b^2cx+\frac{\left(2b^2dfx+2b^2cf-b^2d\right)e^{\left(2fx+2e\right)}}{16f^2}+\frac{\left(abdfx+abcf-abd\right)e^{\left(fx+e\right)}}{f^2}+\frac{\left(abdfx+abc\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*d*x^2 - 1/4*b^2*d*x^2 + a^2*c*x - 1/2*b^2*c*x + 1/16*(2*b^2*d*f*x +
 2*b^2*c*f - b^2*d)*e^(2*f*x + 2*e)/f^2 + (a*b*d*f*x + a*b*c*f - a*b*d)*e^(
f*x + e)/f^2 + (a*b*d*f*x + a*b*c*f + a*b*d)*e^(-f*x - e)/f^2 - 1/16*(2*b^2
*d*f*x + 2*b^2*c*f + b^2*d)*e^(-2*f*x - 2*e)/f^2
```

3.166 $\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx$

Optimal. Leaf size=156

$$\frac{a^2\log(c+dx)}{d}+\frac{2ab\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{b^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\right.}{2d}$$

```
[Out] (b^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (a^2*Lo
g[c + d*x])/d - (b^2*Log[c + d*x])/(2*d) + (2*a*b*CoshIntegral[(c*f)/d + f*
x]*Sinh[e - (c*f)/d])/d + (2*a*b*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f
*x])/d + (b^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```

Rubi [A] time = 0.325149, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3317, 3303, 3298, 3301, 3312}

$$\frac{a^2\log(c+dx)}{d}+\frac{2ab\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{b^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x])^2/(c + d*x),x]
```

```
[Out] (b^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (a^2*Lo
g[c + d*x])/d - (b^2*Log[c + d*x])/(2*d) + (2*a*b*CoshIntegral[(c*f)/d + f*
x]*Sinh[e - (c*f)/d])/d + (2*a*b*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f
*x])/d + (b^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{(a+b\sinh(e+fx))^2}{c+dx}\,dx = \int \left(\frac{a^2}{c+dx}+\frac{2ab\sinh(e+fx)}{c+dx}+\frac{b^2\sinh^2(e+fx)}{c+dx}\right)dx$$
$$= \frac{a^2\log(c+dx)}{d}+(2ab)\int \frac{\sinh(e+fx)}{c+dx}\,dx+b^2\int \frac{\sinh^2(e+fx)}{c+dx}\,dx$$
$$= \frac{a^2\log(c+dx)}{d}-b^2\int \left(\frac{1}{2(c+dx)}-\frac{\cosh(2e+2fx)}{2(c+dx)}\right)dx+\left(2ab\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\sinh\left(\frac{c\cdots}{d}\right.}{c+\cdots}$$
$$= \frac{a^2\log(c+dx)}{d}-\frac{b^2\log(c+dx)}{2d}+\frac{2ab\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\cdots}{d}$$
$$= \frac{a^2\log(c+dx)}{d}-\frac{b^2\log(c+dx)}{2d}+\frac{2ab\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\cdots}{d}$$
$$= \frac{b^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{a^2\log(c+dx)}{d}-\frac{b^2\log(c+dx)}{2d}+\frac{2ab\text{Chi}\left(\frac{cf}{d}+fx\right)\cdots}{d}$$

Mathematica [A] time = 0.297739, size = 134, normalized size = 0.86

$$\frac{2a^2\log(c+dx)+4ab\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+4ab\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+b^2\text{Chi}\left(\frac{2f(c+dx)}{d}\right)\cosh\left(2e-\frac{2cf}{d}\cdots\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x])^2/(c + d*x),x]
```

```
[Out] (b^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*f*(c + d*x))/d] + 2*a^2*Log[c +
d*x] - b^2*Log[c + d*x] + 4*a*b*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d]
 + 4*a*b*Cosh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] + b^2*Sinh[2*e - (2*c*
f)/d]*SinhIntegral[(2*f*(c + d*x))/d])/(2*d)
```

Maple [A] time = 0.108, size = 201, normalized size = 1.3

$$-\frac{ab}{d}e^{-\frac{cf-de}{d}}\text{Ei}\left(1,-fx-e-\frac{cf-de}{d}\right)+\frac{a^2\ln(dx+c)}{d}-\frac{b^2\ln(dx+c)}{2d}-\frac{b^2}{4d}e^{2\frac{cf-de}{d}}\text{Ei}\left(1,2fx+2e+2\frac{cf-de}{d}\right)-\frac{b^2}{4d}e^{-2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e))^2/(d*x+c),x)
```

```
[Out] -a*b/d*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)+a^2*ln(d*x+c)/d-1/2*b^2*l
n(d*x+c)/d-1/4*b^2/d*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)-1/4*b
^2/d*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)+a*b/d*exp((c*f-d*e)
/d)*Ei(1,f*x+e+(c*f-d*e)/d)
```

Maxima [A] time = 1.59282, size = 200, normalized size = 1.28

$$-\frac{1}{4}b^2\left(\frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_1\left(\frac{2(dx+c)f}{d}\right)}{d}+\frac{e^{\left(2e-\frac{2cf}{d}\right)}E_1\left(-\frac{2(dx+c)f}{d}\right)}{d}+\frac{2\log(dx+c)}{d}\right)+ab\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E_1\left(-\frac{(dx}{}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/d + e^(2*e
- 2*c*f/d)*exp_integral_e(1, -2*(d*x + c)*f/d)/d + 2*log(d*x + c)/d) + a*b*
(e^(-e + c*f/d)*exp_integral_e(1, (d*x + c)*f/d)/d - e^(e - c*f/d)*exp_inte
gral_e(1, -(d*x + c)*f/d)/d) + a^2*log(d*x + c)/d
```

Fricas [A] time = 2.50693, size = 483, normalized size = 3.1

$$\frac{4\left(ab\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-ab\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{de-cf}{d}\right)+\left(b^2\mathrm{Ei}\left(\frac{2(dfx+cf)}{d}\right)+b^2\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)\right)\cosh\left(-\frac{2(de-cf)}{d}\right)+2\left(2a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(4*(a*b*Ei((d*f*x + c*f)/d) - a*b*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*
f)/d) + (b^2*Ei(2*(d*f*x + c*f)/d) + b^2*Ei(-2*(d*f*x + c*f)/d))*cosh(-2*(d
*e - c*f)/d) + 2*(2*a^2 - b^2)*log(d*x + c) - 4*(a*b*Ei((d*f*x + c*f)/d) +
a*b*Ei(-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d) - (b^2*Ei(2*(d*f*x + c*f)/d)
 - b^2*Ei(-2*(d*f*x + c*f)/d))*sinh(-2*(d*e - c*f)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sinh\left(e+fx\right)\right)^2}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))**2/(d*x+c),x)
```

```
[Out] Integral((a + b*sinh(e + f*x))**2/(c + d*x), x)
```

Giac [A] time = 1.29421, size = 200, normalized size = 1.28

$$\frac{b^2\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)e^{\left(\frac{2cf}{d}-2e\right)}-4ab\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)}+4ab\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)}+b^2\mathrm{Ei}\left(\frac{2(dfx+cf)}{d}\right)e^{\left(-\frac{2cf}{d}+2e\right)}+4a^2\mathrm{lo}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(b^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) - 4*a*b*Ei(-(d*f*x + c*f)
/d)*e^(c*f/d - e) + 4*a*b*Ei((d*f*x + c*f)/d)*e^(-c*f/d + e) + b^2*Ei(2*(d*
f*x + c*f)/d)*e^(-2*c*f/d + 2*e) + 4*a^2*log(d*x + c) - 2*b^2*log(d*x + c))
/d
```

3.167 $\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2}\,dx$

Optimal. Leaf size=183

$$-\frac{a^2}{d(c+dx)}+\frac{2abf\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{2abf\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{2ab\sinh(e+fx)}{d(c+dx)}+\frac{b^2f\text{Chi}\left(2x\right.}{}$$

```
[Out] -(a^2/(d*(c + d*x))) + (2*a*b*f*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*
x])/d^2 + (b^2*f*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d^2
 - (2*a*b*Sinh[e + f*x])/(d*(c + d*x)) - (b^2*Sinh[e + f*x]^2)/(d*(c + d*x)
) + (2*a*b*f*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^2 + (b^2*f*Co
sh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d^2
```

Rubi [A] time = 0.350092, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {3317, 3297, 3303, 3298, 3301, 3313, 12}

$$-\frac{a^2}{d(c+dx)}+\frac{2abf\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{2abf\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{2ab\sinh(e+fx)}{d(c+dx)}+\frac{b^2f\text{Chi}\left(2x\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x])^2/(c + d*x)^2,x]
```



```
[Out] -(a^2/(d*(c + d*x))) + (2*a*b*f*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*
x])/d^2 + (b^2*f*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d^2
 - (2*a*b*Sinh[e + f*x])/(d*(c + d*x)) - (b^2*Sinh[e + f*x]^2)/(d*(c + d*x)
) + (2*a*b*f*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^2 + (b^2*f*Co
sh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d^2
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^2}\,dx = \int \left(\frac{a^2}{(c+dx)^2}+\frac{2ab\sinh(e+fx)}{(c+dx)^2}+\frac{b^2\sinh^2(e+fx)}{(c+dx)^2}\right)dx$$

$$= -\frac{a^2}{d(c+dx)}+(2ab)\int \frac{\sinh(e+fx)}{(c+dx)^2}\,dx+b^2\int \frac{\sinh^2(e+fx)}{(c+dx)^2}\,dx$$

$$= -\frac{a^2}{d(c+dx)}-\frac{2ab\sinh(e+fx)}{d(c+dx)}-\frac{b^2\sinh^2(e+fx)}{d(c+dx)}+\frac{(2abf)\int \frac{\cosh(e+fx)}{c+dx}\,dx}{d}-\frac{\left(2ib^2f\right)\int \frac{i\mathrm{s}}{}}{}$$

$$= -\frac{a^2}{d(c+dx)}-\frac{2ab\sinh(e+fx)}{d(c+dx)}-\frac{b^2\sinh^2(e+fx)}{d(c+dx)}+\frac{\left(b^2f\right)\int \frac{\sinh(2e+2fx)}{c+dx}\,dx}{d}+\frac{\left(2abf\cosh\right.}{}$$

$$= -\frac{a^2}{d(c+dx)}+\frac{2abf\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(\frac{cf}{d}+fx\right)}{d^2}-\frac{2ab\sinh(e+fx)}{d(c+dx)}-\frac{b^2\sinh^2(e+fx)}{d(c+dx)}+$$

$$= -\frac{a^2}{d(c+dx)}+\frac{2abf\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(\frac{cf}{d}+fx\right)}{d^2}+\frac{b^2f\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2}-$$

Mathematica [A] time = 0.641991, size = 232, normalized size = 1.27

$$\frac{-2a^2d+4abf(c+dx)\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\cosh\left(e-\frac{cf}{d}\right)+4abcf\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+4abdfx\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] (-2*a^2*d + b^2*d - b^2*d*Cosh[2*(e + f*x)] + 4*a*b*f*(c + d*x)*Cosh[e - (c
*f)/d]*CoshIntegral[f*(c/d + x)] + 2*b^2*f*(c + d*x)*CoshIntegral[(2*f*(c +
 d*x))/d]*Sinh[2*e - (2*c*f)/d] - 4*a*b*d*Sinh[e + f*x] + 4*a*b*c*f*Sinh[e
- (c*f)/d]*SinhIntegral[f*(c/d + x)] + 4*a*b*d*f*x*Sinh[e - (c*f)/d]*SinhIn
tegral[f*(c/d + x)] + 2*b^2*c*f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c
+ d*x))/d] + 2*b^2*d*f*x*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))
```

```
/d])/(2*d^2*(c + d*x))
```

Maple [A] time = 0.128, size = 319, normalized size = 1.7

$$-\frac{abf\,e^{fx+e}}{d^2}\left(\frac{cf}{d}+fx\right)^{-1}-\frac{abf}{d^2}e^{-\frac{cf-de}{d}}\mathrm{Ei}\left(1,-fx-e-\frac{cf-de}{d}\right)-\frac{a^2}{d\,(dx+c)}+\frac{b^2}{2\,d\,(dx+c)}-\frac{fb^2e^{-2\,fx-2\,e}}{4\,d\left(dfx+cf\right)}+\frac{fb^2}{2\,d^2}e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e))^2/(d*x+c)^2,x)
```

```
[Out] -a*b*f/d^2*exp(f*x+e)/(c*f/d+f*x)-a*b*f/d^2*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(
c*f-d*e)/d)-a^2/d/(d*x+c)+1/2*b^2/d/(d*x+c)-1/4*b^2*f*exp(-2*f*x-2*e)/d/(d*
f*x+c*f)+1/2*b^2*f/d^2*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)-1/4
*f*b^2/d^2*exp(2*f*x+2*e)/(c*f/d+f*x)-1/2*f*b^2/d^2*exp(-2*(c*f-d*e)/d)*Ei(
1,-2*f*x-2*e-2*(c*f-d*e)/d)+a*b*f*exp(-f*x-e)/d/(d*f*x+c*f)-a*b*f/d^2*exp((
c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)
```

Maxima [A] time = 1.50236, size = 244, normalized size = 1.33

$$-\frac{1}{4}b^2\left(\frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_2\left(\frac{2\,(dx+c)f}{d}\right)}{(dx+c)d}+\frac{e^{\left(2\,e-\frac{2\,cf}{d}\right)}E_2\left(-\frac{2\,(dx+c)f}{d}\right)}{(dx+c)d}-\frac{2}{d^2x+cd}\right)+ab\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(2, 2*(d*x + c)*f/d)/((d*x + c)*
d) + e^(2*e - 2*c*f/d)*exp_integral_e(2, -2*(d*x + c)*f/d)/((d*x + c)*d) -
2/(d^2*x + c*d)) + a*b*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)/((d
*x + c)*d) - e^(e - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d))
 - a^2/(d^2*x + c*d)
```

Fricas [A] time = 2.56421, size = 778, normalized size = 4.25

$$-\frac{b^2d\cosh\left(fx+e\right)^2+b^2d\sinh\left(fx+e\right)^2+4\,abd\sinh\left(fx+e\right)+\left(2\,a^2-b^2\right)d-2\left(\left(abdfx+abcf\right)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+\left(ab\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*d*cosh(f*x + e)^2 + b^2*d*sinh(f*x + e)^2 + 4*a*b*d*sinh(f*x + e)
 + (2*a^2 - b^2)*d - 2*((a*b*d*f*x + a*b*c*f)*Ei((d*f*x + c*f)/d) + (a*b*d*
f*x + a*b*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) - ((b^2*d*f*x + b
^2*c*f)*Ei(2*(d*f*x + c*f)/d) - (b^2*d*f*x + b^2*c*f)*Ei(-2*(d*f*x + c*f)/d
))*cosh(-2*(d*e - c*f)/d) + 2*((a*b*d*f*x + a*b*c*f)*Ei((d*f*x + c*f)/d) -
(a*b*d*f*x + a*b*c*f)*Ei(-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d) + ((b^2*d*
```

```
f*x + b^2*c*f)*Ei(2*(d*f*x + c*f)/d) + (b^2*d*f*x + b^2*c*f)*Ei(-2*(d*f*x +
 c*f)/d))*sinh(-2*(d*e - c*f)/d))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\sinh\left(e + fx\right)\right)^2}{(c + dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))**2/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sinh(e + f*x))**2/(c + d*x)**2, x)
```

Giac [A] time = 2.0866, size = 487, normalized size = 2.66

$$-\frac{2\,b^2 dfx\mathrm{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)e^{\left(\frac{2cf}{d}-2e\right)} - 4\,abdfx\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)} - 4\,abdfx\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)} - 2\,b^2dfx\mathrm{Ei}\left(\frac{2\left(dfx+cf\right)}{d}\right)e^{\left(\cdots\right.}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*b^2*d*f*x*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) - 4*a*b*d*f*x*Ei
(-(d*f*x + c*f)/d)*e^(c*f/d - e) - 4*a*b*d*f*x*Ei((d*f*x + c*f)/d)*e^(-c*f/
d + e) - 2*b^2*d*f*x*Ei(2*(d*f*x + c*f)/d)*e^(-2*c*f/d + 2*e) + 2*b^2*c*f*E
i(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) - 4*a*b*c*f*Ei(-(d*f*x + c*f)/d)*e^
(c*f/d - e) - 4*a*b*c*f*Ei((d*f*x + c*f)/d)*e^(-c*f/d + e) - 2*b^2*c*f*Ei(2
*(d*f*x + c*f)/d)*e^(-2*c*f/d + 2*e) + b^2*d*e^(2*f*x + 2*e) + 4*a*b*d*e^(f
*x + e) - 4*a*b*d*e^(-f*x - e) + b^2*d*e^(-2*f*x - 2*e))/(d^3*x + c*d^2) -
1/2*(2*a^2 - b^2)/((d*x + c)*d)
```

3.168 $\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{a^2}{2d(c+dx)^2}+\frac{abf^2\mathrm{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^3}+\frac{abf^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(xf+\frac{cf}{d}\right)}{d^3}-\frac{abf\cosh(e+fx)}{d^2(c+dx)}-\frac{ab\sinh(e}{d(c+}$$

```
[Out] -a^2/(2*d*(c + d*x)^2) - (a*b*f*Cosh[e + f*x])/(d^2*(c + d*x)) + (b^2*f^2*C
osh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/d^3 + (a*b*f^2*CoshIn
tegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^3 - (a*b*Sinh[e + f*x])/(d*(c +
d*x)^2) - (b^2*f*Cosh[e + f*x]*Sinh[e + f*x])/(d^2*(c + d*x)) - (b^2*Sinh[e
 + f*x]^2)/(2*d*(c + d*x)^2) + (a*b*f^2*Cosh[e - (c*f)/d]*SinhIntegral[(c*f
)/d + f*x])/d^3 + (b^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2
*f*x])/d^3
```

Rubi [A] time = 0.446764, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3317, 3297, 3303, 3298, 3301, 3314, 31, 3312}

$$-\frac{a^2}{2d(c+dx)^2}+\frac{abf^2\mathrm{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^3}+\frac{abf^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(xf+\frac{cf}{d}\right)}{d^3}-\frac{abf\cosh(e+fx)}{d^2(c+dx)}-\frac{ab\sinh(e}{d(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x])^2/(c + d*x)^3,x]
```

```
[Out] -a^2/(2*d*(c + d*x)^2) - (a*b*f*Cosh[e + f*x])/(d^2*(c + d*x)) + (b^2*f^2*C
osh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/d^3 + (a*b*f^2*CoshIn
tegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^3 - (a*b*Sinh[e + f*x])/(d*(c +
d*x)^2) - (b^2*f*Cosh[e + f*x]*Sinh[e + f*x])/(d^2*(c + d*x)) - (b^2*Sinh[e
 + f*x]^2)/(2*d*(c + d*x)^2) + (a*b*f^2*Cosh[e - (c*f)/d]*SinhIntegral[(c*f
)/d + f*x])/d^3 + (b^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2
*f*x])/d^3
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{(a+b\sinh(e+fx))^2}{(c+dx)^3}\,dx = \int \left(\frac{a^2}{(c+dx)^3}+\frac{2ab\sinh(e+fx)}{(c+dx)^3}+\frac{b^2\sinh^2(e+fx)}{(c+dx)^3}\right)dx$$

$$= -\frac{a^2}{2d(c+dx)^2}+(2ab)\int \frac{\sinh(e+fx)}{(c+dx)^3}\,dx+b^2\int \frac{\sinh^2(e+fx)}{(c+dx)^3}\,dx$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{ab\sinh(e+fx)}{d(c+dx)^2}-\frac{b^2f\cosh(e+fx)\sinh(e+fx)}{d^2(c+dx)}-\frac{b^2\sinh^2(e+fx)}{2d(c+dx)^2}+\frac{(ab}{}$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{abf\cosh(e+fx)}{d^2(c+dx)}+\frac{b^2f^2\log(c+dx)}{d^3}-\frac{ab\sinh(e+fx)}{d(c+dx)^2}-\frac{b^2f\cosh(e+f}{d^2(c+}$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{abf\cosh(e+fx)}{d^2(c+dx)}-\frac{ab\sinh(e+fx)}{d(c+dx)^2}-\frac{b^2f\cosh(e+fx)\sinh(e+fx)}{d^2(c+dx)}-\frac{b^2}{}$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{abf\cosh(e+fx)}{d^2(c+dx)}+\frac{abf^2\mathrm{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d^3}-\frac{ab\sinh(e+fx)}{d(c+dx)^2}-$$

$$= -\frac{a^2}{2d(c+dx)^2}-\frac{abf\cosh(e+fx)}{d^2(c+dx)}+\frac{b^2f^2\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)}{d^3}+\frac{abf^2\mathrm{Chi}\left(\frac{cf}{d}\right.}{}$$

Mathematica [A] time = 0.960048, size = 395, normalized size = 1.63

$$\frac{-2a^2d^2 + 4abc^2f^2\cosh\left(e - \frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d} + x\right)\right) + 4abf^2(c + dx)^2\text{Chi}\left(f\left(\frac{c}{d} + x\right)\right)\sinh\left(e - \frac{cf}{d}\right) + 4abd^2f^2x^2\cosh\left(e - \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x])^2/(c + d*x)^3,x]
```

```
[Out] (-2*a^2*d^2 + b^2*d^2 - 4*a*b*c*d*f*Cosh[e + f*x] - 4*a*b*d^2*f*x*Cosh[e +
f*x] - b^2*d^2*Cosh[2*(e + f*x)] + 4*b^2*f^2*(c + d*x)^2*Cosh[2*e - (2*c*f)
/d]*CoshIntegral[(2*f*(c + d*x))/d] + 4*a*b*f^2*(c + d*x)^2*CoshIntegral[f*
(c/d + x)]*Sinh[e - (c*f)/d] - 4*a*b*d^2*Sinh[e + f*x] - 2*b^2*c*d*f*Sinh[2
*(e + f*x)] - 2*b^2*d^2*f*x*Sinh[2*(e + f*x)] + 4*a*b*c^2*f^2*Cosh[e - (c*f
)/d]*SinhIntegral[f*(c/d + x)] + 8*a*b*c*d*f^2*x*Cosh[e - (c*f)/d]*SinhInte
gral[f*(c/d + x)] + 4*a*b*d^2*f^2*x^2*Cosh[e - (c*f)/d]*SinhIntegral[f*(c/d
 + x)] + 4*b^2*c^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d
] + 8*b^2*c*d*f^2*x*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d] +
 4*b^2*d^2*f^2*x^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d])/(
4*d^3*(c + d*x)^2)
```

Maple [B] time = 0.135, size = 626, normalized size = 2.6

$$-\frac{abf^2 e^{fx+e}}{2d^3}\left(\frac{cf}{d} + fx\right)^{-2} - \frac{abf^2 e^{fx+e}}{2d^3}\left(\frac{cf}{d} + fx\right)^{-1} - \frac{abf^2}{2d^3}e^{-\frac{cf-de}{d}}\text{Ei}\left(1, -fx - e - \frac{cf - de}{d}\right) - \frac{a^2}{2d(dx+c)^2} + \frac{b^2}{4d(dx-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e))^2/(d*x+c)^3,x)
```

```
[Out] -1/2*a*b*f^2/d^3*exp(f*x+e)/(c*f/d+f*x)^2-1/2*a*b*f^2/d^3*exp(f*x+e)/(c*f/d
+f*x)-1/2*a*b*f^2/d^3*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)-1/2*a^2/d/
(d*x+c)^2+1/4*b^2/d/(d*x+c)^2+1/4*b^2*f^3*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*
c*d*f^2*x+c^2*f^2)*x+1/4*b^2*f^3*exp(-2*f*x-2*e)/d^2/(d^2*f^2*x^2+2*c*d*f^2
*x+c^2*f^2)*c-1/8*b^2*f^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^
2)-1/2*b^2*f^2/d^3*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)-1/8*f^2
*b^2/d^3*exp(2*f*x+2*e)/(c*f/d+f*x)^2-1/4*f^2*b^2/d^3*exp(2*f*x+2*e)/(c*f/d
+f*x)-1/2*f^2*b^2/d^3*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)-1/
2*a*b*f^3*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x-1/2*a*b*f^3*exp
(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c+1/2*a*b*f^2*exp(-f*x-e)/d/
(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+1/2*a*b*f^2/d^3*exp((c*f-d*e)/d)*Ei(1,f*x
+e+(c*f-d*e)/d)
```

Maxima [A] time = 1.58655, size = 274, normalized size = 1.13

$$\frac{1}{4}b^2\left(\frac{1}{d^3x^2 + 2cd^2x + c^2d} - \frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_3\left(\frac{2(dx+c)f}{d}\right)}{(dx+c)^2 d} - \frac{e^{\left(2e-\frac{2cf}{d}\right)}E_3\left(-\frac{2(dx+c)f}{d}\right)}{(dx+c)^2 d}\right) + ab\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2 d} - \frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(\right.}{(dx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*(1/(d^3*x^2 + 2*c*d^2*x + c^2*d) - e^(-2*e + 2*c*f/d)*exp_integral_
e(3, 2*(d*x + c)*f/d)/((d*x + c)^2*d) - e^(2*e - 2*c*f/d)*exp_integral_e(3,
 -2*(d*x + c)*f/d)/((d*x + c)^2*d)) + a*b*(e^(-e + c*f/d)*exp_integral_e(3,
 (d*x + c)*f/d)/((d*x + c)^2*d) - e^(e - c*f/d)*exp_integral_e(3, -(d*x + c
)*f/d)/((d*x + c)^2*d)) - 1/2*a^2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Fricas [B] time = 2.47013, size = 1237, normalized size = 5.11

$$-\frac{b^2d^2\cosh\left(fx+e\right)^2+b^2d^2\sinh\left(fx+e\right)^2+\left(2a^2-b^2\right)d^2+4\left(abd^2fx+abcdf\right)\cosh\left(fx+e\right)-2\left(\left(abd^2f^2x^2+2\,abcd\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*d^2*cosh(f*x + e)^2 + b^2*d^2*sinh(f*x + e)^2 + (2*a^2 - b^2)*d^2
 + 4*(a*b*d^2*f*x + a*b*c*d*f)*cosh(f*x + e) - 2*((a*b*d^2*f^2*x^2 + 2*a*b*
c*d*f^2*x + a*b*c^2*f^2)*Ei((d*f*x + c*f)/d) - (a*b*d^2*f^2*x^2 + 2*a*b*c*d
*f^2*x + a*b*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) - 2*((b^2*
d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d) + (b^2*d
^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d))*cosh(-2
*(d*e - c*f)/d) + 4*(a*b*d^2 + (b^2*d^2*f*x + b^2*c*d*f)*cosh(f*x + e))*sin
h(f*x + e) + 2*((a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*Ei((d*f*x
 + c*f)/d) + (a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*Ei(-(d*f*x +
 c*f)/d))*sinh(-(d*e - c*f)/d) + 2*((b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^
2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d) - (b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2
*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d))*sinh(-2*(d*e - c*f)/d))/(d^5*x^2 + 2*c*d^
4*x + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\sinh\left(e+fx\right)\right)^2}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))**2/(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sinh(e + f*x))**2/(c + d*x)**3, x)
```

Giac [B] time = 1.30969, size = 948, normalized size = 3.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^2*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) - 4*a*b*d^2
*f^2*x^2*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + 4*a*b*d^2*f^2*x^2*Ei((d*f*x +
```

```
 c*f)/d)*e^(-c*f/d + e) + 4*b^2*d^2*f^2*x^2*Ei(2*(d*f*x + c*f)/d)*e^(-2*c*f
/d + 2*e) + 8*b^2*c*d*f^2*x*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) - 8*a*
b*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + 8*a*b*c*d*f^2*x*Ei((d*f*x
+ c*f)/d)*e^(-c*f/d + e) + 8*b^2*c*d*f^2*x*Ei(2*(d*f*x + c*f)/d)*e^(-2*c*f/
d + 2*e) + 4*b^2*c^2*f^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) - 4*a*b*c
^2*f^2*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + 4*a*b*c^2*f^2*Ei((d*f*x + c*f)/
d)*e^(-c*f/d + e) + 4*b^2*c^2*f^2*Ei(2*(d*f*x + c*f)/d)*e^(-2*c*f/d + 2*e)
- 2*b^2*d^2*f*x*e^(2*f*x + 2*e) - 4*a*b*d^2*f*x*e^(f*x + e) - 4*a*b*d^2*f*x
*e^(-f*x - e) + 2*b^2*d^2*f*x*e^(-2*f*x - 2*e) - 2*b^2*c*d*f*e^(2*f*x + 2*e
) - 4*a*b*c*d*f*e^(f*x + e) - 4*a*b*c*d*f*e^(-f*x - e) + 2*b^2*c*d*f*e^(-2*
f*x - 2*e) - b^2*d^2*e^(2*f*x + 2*e) - 4*a*b*d^2*e^(f*x + e) + 4*a*b*d^2*e^
(-f*x - e) - b^2*d^2*e^(-2*f*x - 2*e) - 4*a^2*d^2 + 2*b^2*d^2)/(d^5*x^2 + 2
*c*d^4*x + c^2*d^3)
```

3.169 $\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx$

Optimal. Leaf size=404

$$-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^3\sqrt{a^2+b^2}}+\frac{6d^2(c+dx)\text{PolyLog}\left(3,-\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^3\sqrt{a^2+b^2}}+\frac{3d(c+dx)^2\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2\sqrt{a^2+b^2}}-\frac{3d}{}$$

```
[Out] ((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^
2]*f) - ((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/(Sqrt[
a^2 + b^2]*f) + (3*d*(c + d*x)^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2
 + b^2]))])/(Sqrt[a^2 + b^2]*f^2) - (3*d*(c + d*x)^2*PolyLog[2, -((b*E^(e +
 f*x))/(a + Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^2) - (6*d^2*(c + d*x)*Po
lyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^3) +
 (6*d^2*(c + d*x)*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(Sq
rt[a^2 + b^2]*f^3) + (6*d^3*PolyLog[4, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^
2]))])/(Sqrt[a^2 + b^2]*f^4) - (6*d^3*PolyLog[4, -((b*E^(e + f*x))/(a + Sqr
t[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^4)
```

Rubi [A] time = 0.825822, antiderivative size = 404, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {3322, 2264, 2190, 2531, 6609, 2282, 6589}

$$-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^3\sqrt{a^2+b^2}}+\frac{6d^2(c+dx)\text{PolyLog}\left(3,-\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^3\sqrt{a^2+b^2}}+\frac{3d(c+dx)^2\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2\sqrt{a^2+b^2}}-\frac{3d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*Sinh[e + f*x]),x]
```


```
[Out] ((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^
2]*f) - ((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/(Sqrt[
a^2 + b^2]*f) + (3*d*(c + d*x)^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2
 + b^2]))])/(Sqrt[a^2 + b^2]*f^2) - (3*d*(c + d*x)^2*PolyLog[2, -((b*E^(e +
 f*x))/(a + Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^2) - (6*d^2*(c + d*x)*Po
lyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^3) +
 (6*d^2*(c + d*x)*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(Sq
rt[a^2 + b^2]*f^3) + (6*d^3*PolyLog[4, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^
2]))])/(Sqrt[a^2 + b^2]*f^4) - (6*d^3*PolyLog[4, -((b*E^(e + f*x))/(a + Sqr
t[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^4)
```


Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a+b\sinh(e+fx)}\,dx = 2\int \frac{e^{e+fx}(c+dx)^3}{-b+2ae^{e+fx}+be^{2(e+fx)}}\,dx$$

$$= \frac{(2b)\int \frac{e^{e+fx}(c+dx)^3}{2a-2\sqrt{a^2+b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2+b^2}} - \frac{(2b)\int \frac{e^{e+fx}(c+dx)^3}{2a+2\sqrt{a^2+b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2+b^2}}$$

$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(3d)\int(c+dx)^2\log\left(1+\frac{2be^{e+f}}{2a-2\sqrt{a^2}}\right.}{\sqrt{a^2+b^2}f}$$

$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} + \frac{3d(c+dx)^2\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2} - \ldots$$

$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} + \frac{3d(c+dx)^2\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2} - \ldots$$

$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} + \frac{3d(c+dx)^2\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2} - \ldots$$

$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} + \frac{3d(c+dx)^2\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2} - \ldots$$

Mathematica [A] time = 0.260614, size = 318, normalized size = 0.79

$$\frac{\frac{3d\left(f^2(c+dx)^2\text{PolyLog}\left(2,\frac{be^{e+fx}}{\sqrt{a^2+b^2}-a}\right)-2df(c+dx)\text{PolyLog}\left(3,\frac{be^{e+fx}}{\sqrt{a^2+b^2}-a}\right)+2d^2\text{PolyLog}\left(4,\frac{be^{e+fx}}{\sqrt{a^2+b^2}-a}\right)\right)}{f^3} - \frac{3d\left(f^2(c+dx)^2\text{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)-2df(c+dx)\text{PolyL}\ldots\right.}{f^3}\ldots}{f\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*Sinh[e + f*x]),x]
```

```
[Out] ((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])] - (c + d*x)^3*L
og[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])] + (3*d*(f^2*(c + d*x)^2*PolyL
og[2, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*f*(c + d*x)*PolyLog[3,
(b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] + 2*d^2*PolyLog[4, (b*E^(e + f*x))/
(-a + Sqrt[a^2 + b^2])]))/f^3 - (3*d*(f^2*(c + d*x)^2*PolyLog[2, -((b*E^(e
+ f*x))/(a + Sqrt[a^2 + b^2]))] - 2*d*f*(c + d*x)*PolyLog[3, -((b*E^(e + f*
x))/(a + Sqrt[a^2 + b^2]))] + 2*d^2*PolyLog[4, -((b*E^(e + f*x))/(a + Sqrt[
a^2 + b^2]))]))/f^3)/(Sqrt[a^2 + b^2]*f)
```

Maple [F] time = 0.168, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^3}{a+b\sinh(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*sinh(f*x+e)),x)
```

```
[Out] int((d*x+c)^3/(a+b*sinh(f*x+e)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.72142, size = 2412, normalized size = 5.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] (6*b*d^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(f*x + e) + a*sinh(f*x + e
) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))/b) - 6*b*d^3
*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(f*x + e) + a*sinh(f*x + e) - (b*c
osh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))/b) + 3*(b*d^3*f^2*x^
2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x
+ e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^
2)/b^2) - b)/b + 1) - 3*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*sqr
t((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x +
 e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b*d^3*e^3 - 3*b
*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt((a^2 + b^2)/b^2)*log(2*b*c
osh(f*x + e) + 2*b*sinh(f*x + e) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b*d^
3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt((a^2 + b^2)/b^2
)*log(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) - 2*b*sqrt((a^2 + b^2)/b^2) + 2
*a) + (b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 - 3*
b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(f*x + e
) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/
b^2) - b)/b) - (b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3
*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cos
h(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^
2 + b^2)/b^2) - b)/b) - 6*(b*d^3*f*x + b*c*d^2*f)*sqrt((a^2 + b^2)/b^2)*pol
ylog(3, (a*cosh(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x
+ e))*sqrt((a^2 + b^2)/b^2))/b) + 6*(b*d^3*f*x + b*c*d^2*f)*sqrt((a^2 + b^2
)/b^2)*polylog(3, (a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e) + b
*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))/b))/((a^2 + b^2)*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*sinh(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^3}{b\sinh\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*sinh(f*x + e) + a), x)
```

3.170 $\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx$

Optimal. Leaf size=296

$$\frac{2d(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2\sqrt{a^2+b^2}}-\frac{2d(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^2\sqrt{a^2+b^2}}-\frac{2d^2\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^3\sqrt{a^2+b^2}}+\frac{2d^2\text{PolyLo}}{f^3}$$

```
[Out] ((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^
2]*f) - ((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/(Sqrt[
a^2 + b^2]*f) + (2*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 +
 b^2]))])/(Sqrt[a^2 + b^2]*f^2) - (2*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x
))/(a + Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^2) - (2*d^2*PolyLog[3, -((b*
E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^3) + (2*d^2*PolyLo
g[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^3)
```

Rubi [A] time = 0.668979, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3322, 2264, 2190, 2531, 2282, 6589}

$$\frac{2d(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2\sqrt{a^2+b^2}}-\frac{2d(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^2\sqrt{a^2+b^2}}-\frac{2d^2\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^3\sqrt{a^2+b^2}}+\frac{2d^2\text{PolyLo}}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*Sinh[e + f*x]),x]
```

```
[Out] ((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^
2]*f) - ((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/(Sqrt[
a^2 + b^2]*f) + (2*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 +
 b^2]))])/(Sqrt[a^2 + b^2]*f^2) - (2*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x
))/(a + Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^2) - (2*d^2*PolyLog[3, -((b*
E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^3) + (2*d^2*PolyLo
g[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^3)
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx = 2\int \frac{e^{e+fx}(c+dx)^2}{-b+2ae^{e+fx}+be^{2(e+fx)}}\,dx$$

$$= \frac{(2b)\int \frac{e^{e+fx}(c+dx)^2}{2a-2\sqrt{a^2+b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2+b^2}} - \frac{(2b)\int \frac{e^{e+fx}(c+dx)^2}{2a+2\sqrt{a^2+b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2+b^2}}$$

$$= \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(2d)\int (c+dx)\log\left(1+\frac{2be^{e+fx}}{2a-2\sqrt{a^2+\dots}}\right)}{\sqrt{a^2+b^2}f}$$

$$= \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} + \frac{2d(c+dx)\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2} - \dots$$

$$= \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} + \frac{2d(c+dx)\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2} - \dots$$

$$= \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} + \frac{2d(c+dx)\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2} - \dots$$

Mathematica [A] time = 0.155571, size = 233, normalized size = 0.79

$$\frac{\frac{2d\left(f(c+dx)\text{PolyLog}\left(2,\frac{be^{e+fx}}{\sqrt{a^2+b^2}-a}\right)-d\text{PolyLog}\left(3,\frac{be^{e+fx}}{\sqrt{a^2+b^2}-a}\right)\right)}{f^2} - \frac{2d\left(f(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)-d\text{PolyLog}\left(3,-\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)\right)}{f^2} + (c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+\dots}}\right.\dots}{f\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*Sinh[e + f*x]),x]
```

```
[Out] ((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])] - (c + d*x)^2*L
og[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])] + (2*d*(f*(c + d*x)*PolyLog[2
, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] - d*PolyLog[3, (b*E^(e + f*x))/(-
a + Sqrt[a^2 + b^2])]))/f^2 - (2*d*(f*(c + d*x)*PolyLog[2, -((b*E^(e + f*x)
)/(a + Sqrt[a^2 + b^2]))] - d*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 +
b^2]))]))/f^2)/(Sqrt[a^2 + b^2]*f)
```

Maple [F] time = 0.12, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{a+b\sinh(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*sinh(f*x+e)),x)
```

```
[Out] int((d*x+c)^2/(a+b*sinh(f*x+e)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.5205, size = 1733, normalized size = 5.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(2*b*d^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(f*x + e) + a*sinh(f*x +
e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))/b) - 2*b*d^
2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(f*x + e) + a*sinh(f*x + e) - (b*
cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))/b) - 2*(b*d^2*f*x +
 b*c*d*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + e) + a*sinh(f*x + e) +
(b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(
b*d^2*f*x + b*c*d*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + e) + a*sinh(
f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b
 + 1) + (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(2*b
*cosh(f*x + e) + 2*b*sinh(f*x + e) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b*
d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(f*x +
 e) + 2*b*sinh(f*x + e) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b*d^2*f^2*x^2
 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*sqrt((a^2 + b^2)/b^2)*log(-(a*c
osh(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((
a^2 + b^2)/b^2) - b)/b) + (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*
```

```
c*d*e*f)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(f*x + e) + a*sinh(f*x + e) - (b
*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b))/((a^2 + b^
2)*f^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*sinh(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{b\sinh\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*sinh(f*x + e) + a), x)
```

3.171 $\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx$

Optimal. Leaf size=187

$$\frac{d\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2\sqrt{a^2+b^2}}-\frac{d\text{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^2\sqrt{a^2+b^2}}+\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{f\sqrt{a^2+b^2}}-\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{f\sqrt{a^2+b^2}}$$

```
[Out] ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]
*f) - ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/(Sqrt[a^2
+ b^2]*f) + (d*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/(Sqrt[
a^2 + b^2]*f^2) - (d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/
(Sqrt[a^2 + b^2]*f^2)
```

Rubi [A] time = 0.369, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3322, 2264, 2190, 2279, 2391}

$$\frac{d\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2\sqrt{a^2+b^2}}-\frac{d\text{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^2\sqrt{a^2+b^2}}+\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{f\sqrt{a^2+b^2}}-\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{f\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Sinh[e + f*x]),x]
```

```
[Out] ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]
*f) - ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/(Sqrt[a^2
+ b^2]*f) + (d*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/(Sqrt[
a^2 + b^2]*f^2) - (d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/
(Sqrt[a^2 + b^2]*f^2)
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx = 2\int \frac{e^{e+fx}(c+dx)}{-b+2ae^{e+fx}+be^{2(e+fx)}}\,dx$$
$$= \frac{(2b)\int \frac{e^{e+fx}(c+dx)}{2a-2\sqrt{a^2+b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2+b^2}} - \frac{(2b)\int \frac{e^{e+fx}(c+dx)}{2a+2\sqrt{a^2+b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2+b^2}}$$
$$= \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{d\int \log\left(1+\frac{2be^{e+fx}}{2a-2\sqrt{a^2+b^2}}\right)dx}{\sqrt{a^2+b^2}f} + \frac{d\int$$
$$= \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{d\,\text{Subst}\left(\int \frac{\log\left(1+\frac{2bx}{2a-2\sqrt{a^2+b^2}}\right)}{x}dx, x,\right.}{\sqrt{a^2+b^2}f^2}$$
$$= \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} + \frac{d\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2} - \frac{d\text{Li}_2\left(-\frac{be^{e+}}{a+\sqrt{a^2}}\right.}{\sqrt{a^2+b^2}f}$$

Mathematica [A] time = 0.0365069, size = 142, normalized size = 0.76

$$\frac{d\text{PolyLog}\left(2,\frac{be^{e+fx}}{\sqrt{a^2+b^2}-a}\right)-d\text{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)+f(c+dx)\left(\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)-\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)\right)}{f^2\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*Sinh[e + f*x]),x]
```

```
[Out] (f*(c + d*x)*(Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])] - Log[1 + (b*E
^(e + f*x))/(a + Sqrt[a^2 + b^2])]) + d*PolyLog[2, (b*E^(e + f*x))/(-a + Sq
rt[a^2 + b^2])] - d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(
Sqrt[a^2 + b^2]*f^2)
```

Maple [B] time = 0.054, size = 393, normalized size = 2.1

$$-2\,\frac{c}{f\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\,\frac{2\,b{e}^{fx+e}+2\,a}{\sqrt{a^2+b^2}}\right)+\frac{dx}{f}\ln\left(\left(-b{e}^{fx+e}+\sqrt{a^2+b^2}-a\right)\left(-a+\sqrt{a^2+b^2}\right)^{-1}\right)\frac{1}{\sqrt{a^2+b^2}}+\frac{de}{f^2}\ln\left(\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*sinh(f*x+e)),x)
```

```
[Out] -2/f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(f*x+e)+2*a)/(a^2+b^2)^(1/2))+1/
f*d/(a^2+b^2)^(1/2)*ln((-b*exp(f*x+e)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2
```

```
)))*x+1/f^2*d/(a^2+b^2)^(1/2)*ln((-b*exp(f*x+e)+(a^2+b^2)^(1/2)-a)/(-a+(a^2
+b^2)^(1/2)))*e-1/f*d/(a^2+b^2)^(1/2)*ln((b*exp(f*x+e)+(a^2+b^2)^(1/2)+a)/(
a+(a^2+b^2)^(1/2)))*x-1/f^2*d/(a^2+b^2)^(1/2)*ln((b*exp(f*x+e)+(a^2+b^2)^(1
/2)+a)/(a+(a^2+b^2)^(1/2)))*e+1/f^2*d/(a^2+b^2)^(1/2)*dilog((-b*exp(f*x+e)+
(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/f^2*d/(a^2+b^2)^(1/2)*dilog((b*e
xp(f*x+e)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/f^2*d*e/(a^2+b^2)^(1/2)
*arctanh(1/2*(2*b*exp(f*x+e)+2*a)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.54519, size = 1127, normalized size = 6.03

$$bd\sqrt{\frac{a^2+b^2}{b^2}}\mathrm{Li}_2\left(\frac{a\cosh(fx+e)+a\sinh(fx+e)+\left(b\cosh(fx+e)+b\sinh(fx+e)\right)\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}+1\right)-bd\sqrt{\frac{a^2+b^2}{b^2}}\mathrm{Li}_2\left(\frac{a\cosh(fx+e)+a\sinh(fx+e)-(b\,\mathrm{c}\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] (b*d*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + e) + a*sinh(f*x + e) + (b*co
sh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - b*d*sqrt
((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x +
e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b*d*e - b*c*f)*s
qrt((a^2 + b^2)/b^2)*log(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) + 2*b*sqrt((
a^2 + b^2)/b^2) + 2*a) - (b*d*e - b*c*f)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh
(f*x + e) + 2*b*sinh(f*x + e) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b*d*f*x
 + b*d*e)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(f*x + e) + a*sinh(f*x + e) + (
b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b*d*f*x
 + b*d*e)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(f*x + e) + a*sinh(f*x + e) - (
b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b))/((a^2 + b
^2)*f^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sinh(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx+c}{b\sinh\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*sinh(f*x + e) + a), x)
```

3.172 $\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)(a+b\sinh(e+fx))},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)*(a + b*Sinh[e + f*x])), x]
```

Rubi [A] time = 0.0635657, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*Sinh[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + b*Sinh[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx$$

Mathematica [A] time = 0.944279, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*Sinh[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)*(a + b*Sinh[e + f*x])), x]
```

Maple [A] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(a+b\sinh\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*sinh(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)/(a+b*sinh(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(b\sinh\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x + c)*(b*sinh(f*x + e) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{adx+ac+(bdx+bc)\sinh\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d*x + a*c + (b*d*x + b*c)*sinh(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sinh(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(b\sinh\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(b*sinh(f*x + e) + a)), x)
```

3.173 $\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a+b\sinh(e+fx))},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)^2*(a + b*Sinh[e + f*x])), x]
```

Rubi [A] time = 0.0595487, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*Sinh[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + b*Sinh[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx$$

Mathematica [A] time = 0.938603, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*Sinh[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + b*Sinh[e + f*x])), x]
```

Maple [A] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(a+b\sinh\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(b\sinh\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x + c)^2*(b*sinh(f*x + e) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{ad^2x^2+2\,acdx+ac^2+\left(bd^2x^2+2\,bcdx+bc^2\right)\sinh\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)
*sinh(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*sinh(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(b\sinh\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(b*sinh(f*x + e) + a)), x)
```

3.174 $\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx$

Optimal. Leaf size=549

$$\frac{2ad(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2\left(a^2+b^2\right)^{3/2}}-\frac{2ad(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^2\left(a^2+b^2\right)^{3/2}}+\frac{2d^2\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^3\left(a^2+b^2\right)}+\frac{2d^2\text{Poly}}{}$$

```
[Out] -((c + d*x)^2/((a^2 + b^2)*f)) + (2*d*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a
- Sqrt[a^2 + b^2])])/((a^2 + b^2)*f^2) + (a*(c + d*x)^2*Log[1 + (b*E^(e + f
*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*f) + (2*d*(c + d*x)*Log[1 +
 (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*f^2) - (a*(c + d*x)^2
*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*f) + (2
*d^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*f^3
) + (2*a*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/
((a^2 + b^2)^(3/2)*f^2) + (2*d^2*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2
 + b^2]))])/((a^2 + b^2)*f^3) - (2*a*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x
))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*f^2) - (2*a*d^2*PolyLog[3, -
((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*f^3) + (2*a*d^
2*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*
f^3) - (b*(c + d*x)^2*Cosh[e + f*x])/((a^2 + b^2)*f*(a + b*Sinh[e + f*x]))
```

Rubi [A] time = 1.03665, antiderivative size = 549, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3324, 3322, 2264, 2190, 2531, 2282, 6589, 5561, 2279, 2391}

$$\frac{2ad(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2\left(a^2+b^2\right)^{3/2}}-\frac{2ad(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^2\left(a^2+b^2\right)^{3/2}}+\frac{2d^2\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^3\left(a^2+b^2\right)}+\frac{2d^2\text{Poly}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*Sinh[e + f*x])^2,x]
```


```
[Out] -((c + d*x)^2/((a^2 + b^2)*f)) + (2*d*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a
- Sqrt[a^2 + b^2])])/((a^2 + b^2)*f^2) + (a*(c + d*x)^2*Log[1 + (b*E^(e + f
*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*f) + (2*d*(c + d*x)*Log[1 +
 (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*f^2) - (a*(c + d*x)^2
*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*f) + (2
*d^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*f^3
) + (2*a*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/
((a^2 + b^2)^(3/2)*f^2) + (2*d^2*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2
 + b^2]))])/((a^2 + b^2)*f^3) - (2*a*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x
))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*f^2) - (2*a*d^2*PolyLog[3, -
((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*f^3) + (2*a*d^
2*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*
f^3) - (b*(c + d*x)^2*Cosh[e + f*x])/((a^2 + b^2)*f*(a + b*Sinh[e + f*x]))
```


Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
```

```
2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2}\,dx = -\frac{b(c+dx)^2\cosh(e+fx)}{\left(a^2+b^2\right)f(a+b\sinh(e+fx))} + \frac{a\int \frac{(c+dx)^2}{a+b\sinh(e+fx)}\,dx}{a^2+b^2} + \frac{(2bd)\int \frac{(c+dx)\cosh(e+fx)}{a+b\sinh(e+fx)}\,dx}{\left(a^2+b^2\right)f}$$

$$= -\frac{(c+dx)^2}{\left(a^2+b^2\right)f} - \frac{b(c+dx)^2\cosh(e+fx)}{\left(a^2+b^2\right)f(a+b\sinh(e+fx))} + \frac{(2a)\int \frac{e^{e+fx}(c+dx)^2}{-b+2ae^{e+fx}+be^{2(e+fx)}}\,dx}{a^2+b^2} + \frac{(2bd)\int}{}$$

$$= -\frac{(c+dx)^2}{\left(a^2+b^2\right)f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)f^2} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)f^2} - \frac{b(c}{\left(a^2+b\right.}$$

$$= -\frac{(c+dx)^2}{\left(a^2+b^2\right)f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)f^2} + \frac{a(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}f} + \frac{2d(c+}{}$$

$$= -\frac{(c+dx)^2}{\left(a^2+b^2\right)f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)f^2} + \frac{a(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}f} + \frac{2d(c+}{}$$

$$= -\frac{(c+dx)^2}{\left(a^2+b^2\right)f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)f^2} + \frac{a(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}f} + \frac{2d(c+}{}$$

$$= -\frac{(c+dx)^2}{\left(a^2+b^2\right)f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)f^2} + \frac{a(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}f} + \frac{2d(c+}{}$$

Mathematica [A] time = 1.93532, size = 428, normalized size = 0.78

$$-\frac{a\left(-2df(c+dx)\text{PolyLog}\left(2,\frac{be^{e+fx}}{\sqrt{a^2+b^2}-a}\right)+2df(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)+2d^2\text{PolyLog}\left(3,\frac{be^{e+fx}}{\sqrt{a^2+b^2}-a}\right)-2d^2\text{PolyLog}\left(3,-\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)-f^2(c+dx)^2\log\left(\frac{be^{e+f}}{a-\sqrt{a^2+}}\right.\right.}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] (-(f^2*(c + d*x)^2) + 2*d*f*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2
 + b^2])] + 2*d*f*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])]
+ 2*d^2*PolyLog[2, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] + 2*d^2*PolyLog[
2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))] - (a*(-(f^2*(c + d*x)^2*Log[1
+ (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])]) + f^2*(c + d*x)^2*Log[1 + (b*E^(e
 + f*x))/(a + Sqrt[a^2 + b^2])] - 2*d*f*(c + d*x)*PolyLog[2, (b*E^(e + f*x)
)/(-a + Sqrt[a^2 + b^2])] + 2*d*f*(c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a
 + Sqrt[a^2 + b^2]))] + 2*d^2*PolyLog[3, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b
^2])] - 2*d^2*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a
^2 + b^2] - (b*f^2*(c + d*x)^2*Cosh[e + f*x])/(a + b*Sinh[e + f*x]))/((a^2
```

```
+ b^2)*f^3)
```

Maple [F] time = 0.29, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{\left(a+b\sinh\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*sinh(f*x+e))^2,x)

[Out] int((d*x+c)^2/(a+b*sinh(f*x+e))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [C] time = 3.52775, size = 8910, normalized size = 16.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="fricas")

[Out] -(2*(a^2*b + b^3)*d^2*e^2 - 4*(a^2*b + b^3)*c*d*e*f + 2*(a^2*b + b^3)*c^2*f
^2 + 2*((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*c*d*f^2*x - (a^2*b + b^
3)*d^2*e^2 + 2*(a^2*b + b^3)*c*d*e*f)*cosh(f*x + e)^2 + 2*((a^2*b + b^3)*d^
2*f^2*x^2 + 2*(a^2*b + b^3)*c*d*f^2*x - (a^2*b + b^3)*d^2*e^2 + 2*(a^2*b +
b^3)*c*d*e*f)*sinh(f*x + e)^2 + 2*(a*b^2*d^2*cosh(f*x + e)^2 + a*b^2*d^2*si
nh(f*x + e)^2 + 2*a^2*b*d^2*cosh(f*x + e) - a*b^2*d^2 + 2*(a*b^2*d^2*cosh(f
*x + e) + a^2*b*d^2)*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cos
h(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^
2 + b^2)/b^2))/b) - 2*(a*b^2*d^2*cosh(f*x + e)^2 + a*b^2*d^2*sinh(f*x + e)^
2 + 2*a^2*b*d^2*cosh(f*x + e) - a*b^2*d^2 + 2*(a*b^2*d^2*cosh(f*x + e) + a^
2*b*d^2)*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(f*x + e) +
 a*sinh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2
))/b) + 2*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*c*d*f^2*x - 2*(a^3 +
 a*b^2)*d^2*e^2 + 4*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*cosh(f*x
 + e) - 2*((a^2*b + b^3)*d^2*cosh(f*x + e)^2 + (a^2*b + b^3)*d^2*sinh(f*x +
 e)^2 + 2*(a^3 + a*b^2)*d^2*cosh(f*x + e) - (a^2*b + b^3)*d^2 + 2*((a^2*b +
 b^3)*d^2*cosh(f*x + e) + (a^3 + a*b^2)*d^2)*sinh(f*x + e) - (a*b^2*d^2*f*x
 + a*b^2*c*d*f - (a*b^2*d^2*f*x + a*b^2*c*d*f)*cosh(f*x + e)^2 - (a*b^2*d^2
*f*x + a*b^2*c*d*f)*sinh(f*x + e)^2 - 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*cosh(
f*x + e) - 2*(a^2*b*d^2*f*x + a^2*b*c*d*f + (a*b^2*d^2*f*x + a*b^2*c*d*f)*c
```

```
osh(f*x + e))*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))*dilog((a*cosh(f*x + e)
+ a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^
2) - b)/b + 1) - 2*((a^2*b + b^3)*d^2*cosh(f*x + e)^2 + (a^2*b + b^3)*d^2*s
inh(f*x + e)^2 + 2*(a^3 + a*b^2)*d^2*cosh(f*x + e) - (a^2*b + b^3)*d^2 + 2*
((a^2*b + b^3)*d^2*cosh(f*x + e) + (a^3 + a*b^2)*d^2)*sinh(f*x + e) + (a*b^
2*d^2*f*x + a*b^2*c*d*f - (a*b^2*d^2*f*x + a*b^2*c*d*f)*cosh(f*x + e)^2 - (
a*b^2*d^2*f*x + a*b^2*c*d*f)*sinh(f*x + e)^2 - 2*(a^2*b*d^2*f*x + a^2*b*c*d
*f)*cosh(f*x + e) - 2*(a^2*b*d^2*f*x + a^2*b*c*d*f + (a*b^2*d^2*f*x + a*b^2
*c*d*f)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))*dilog((a*cosh(
f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2
+ b^2)/b^2) - b)/b + 1) - (2*(a^2*b + b^3)*d^2*e - 2*(a^2*b + b^3)*c*d*f -
2*((a^2*b + b^3)*d^2*e - (a^2*b + b^3)*c*d*f)*cosh(f*x + e)^2 - 2*((a^2*b +
 b^3)*d^2*e - (a^2*b + b^3)*c*d*f)*sinh(f*x + e)^2 - 4*((a^3 + a*b^2)*d^2*e
 - (a^3 + a*b^2)*c*d*f)*cosh(f*x + e) - 4*((a^3 + a*b^2)*d^2*e - (a^3 + a*b
^2)*c*d*f + ((a^2*b + b^3)*d^2*e - (a^2*b + b^3)*c*d*f)*cosh(f*x + e))*sinh
(f*x + e) + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2 - (a*b^2*d^2*e
^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(f*x + e)^2 - (a*b^2*d^2*e^2 - 2*
a*b^2*c*d*e*f + a*b^2*c^2*f^2)*sinh(f*x + e)^2 - 2*(a^2*b*d^2*e^2 - 2*a^2*b
*c*d*e*f + a^2*b*c^2*f^2)*cosh(f*x + e) - 2*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*
f + a^2*b*c^2*f^2 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(
f*x + e))*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))*log(2*b*cosh(f*x + e) + 2*b
*sinh(f*x + e) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (2*(a^2*b + b^3)*d^2*e
- 2*(a^2*b + b^3)*c*d*f - 2*((a^2*b + b^3)*d^2*e - (a^2*b + b^3)*c*d*f)*cos
h(f*x + e)^2 - 2*((a^2*b + b^3)*d^2*e - (a^2*b + b^3)*c*d*f)*sinh(f*x + e)^
2 - 4*((a^3 + a*b^2)*d^2*e - (a^3 + a*b^2)*c*d*f)*cosh(f*x + e) - 4*((a^3 +
 a*b^2)*d^2*e - (a^3 + a*b^2)*c*d*f + ((a^2*b + b^3)*d^2*e - (a^2*b + b^3)*
c*d*f)*cosh(f*x + e))*sinh(f*x + e) - (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*
b^2*c^2*f^2 - (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(f*x +
e)^2 - (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*sinh(f*x + e)^2 -
2*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(f*x + e) - 2*(a^2*
b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*
e*f + a*b^2*c^2*f^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))*l
og(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
 + (2*(a^2*b + b^3)*d^2*f*x + 2*(a^2*b + b^3)*d^2*e - 2*((a^2*b + b^3)*d^2*
f*x + (a^2*b + b^3)*d^2*e)*cosh(f*x + e)^2 - 2*((a^2*b + b^3)*d^2*f*x + (a^
2*b + b^3)*d^2*e)*sinh(f*x + e)^2 - 4*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2
)*d^2*e)*cosh(f*x + e) - 4*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*d^2*e + (
(a^2*b + b^3)*d^2*f*x + (a^2*b + b^3)*d^2*e)*cosh(f*x + e))*sinh(f*x + e) +
 (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f -
 (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f)*
cosh(f*x + e)^2 - (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 +
2*a*b^2*c*d*e*f)*sinh(f*x + e)^2 - 2*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*c*d*f^2*x
 - a^2*b*d^2*e^2 + 2*a^2*b*c*d*e*f)*cosh(f*x + e) - 2*(a^2*b*d^2*f^2*x^2 +
2*a^2*b*c*d*f^2*x - a^2*b*d^2*e^2 + 2*a^2*b*c*d*e*f + (a*b^2*d^2*f^2*x^2 +
2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f)*cosh(f*x + e))*sinh(f*
x + e))*sqrt((a^2 + b^2)/b^2))*log(-(a*cosh(f*x + e) + a*sinh(f*x + e) + (b
*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b) + (2*(a^2*b
 + b^3)*d^2*f*x + 2*(a^2*b + b^3)*d^2*e - 2*((a^2*b + b^3)*d^2*f*x + (a^2*b
 + b^3)*d^2*e)*cosh(f*x + e)^2 - 2*((a^2*b + b^3)*d^2*f*x + (a^2*b + b^3)*d
^2*e)*sinh(f*x + e)^2 - 4*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*d^2*e)*cos
h(f*x + e) - 4*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*d^2*e + ((a^2*b + b^3
)*d^2*f*x + (a^2*b + b^3)*d^2*e)*cosh(f*x + e))*sinh(f*x + e) - (a*b^2*d^2*
f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f - (a*b^2*d^2*
f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f)*cosh(f*x + e
)^2 - (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*
e*f)*sinh(f*x + e)^2 - 2*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*c*d*f^2*x - a^2*b*d^2
*e^2 + 2*a^2*b*c*d*e*f)*cosh(f*x + e) - 2*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*c*d*
f^2*x - a^2*b*d^2*e^2 + 2*a^2*b*c*d*e*f + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*
f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f)*cosh(f*x + e))*sinh(f*x + e))*sqrt
```

```
((a^2 + b^2)/b^2))*log(-(a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x +
e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*((a^3 + a*b^2)*d^2*
f^2*x^2 + 2*(a^3 + a*b^2)*c*d*f^2*x - 2*(a^3 + a*b^2)*d^2*e^2 + 4*(a^3 + a*
b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2 + 2*((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^
2*b + b^3)*c*d*f^2*x - (a^2*b + b^3)*d^2*e^2 + 2*(a^2*b + b^3)*c*d*e*f)*cos
h(f*x + e))*sinh(f*x + e))/((a^4*b + 2*a^2*b^3 + b^5)*f^3*cosh(f*x + e)^2 +
 (a^4*b + 2*a^2*b^3 + b^5)*f^3*sinh(f*x + e)^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4
)*f^3*cosh(f*x + e) - (a^4*b + 2*a^2*b^3 + b^5)*f^3 + 2*((a^4*b + 2*a^2*b^3
 + b^5)*f^3*cosh(f*x + e) + (a^5 + 2*a^3*b^2 + a*b^4)*f^3)*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*sinh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^2}{\left(b\sinh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*sinh(f*x + e) + a)^2, x)
```

3.175 $\int \frac{c+dx}{(a+b\sinh(e+fx))^2} dx$

Optimal. Leaf size=254

$$\frac{ad\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2\left(a^2+b^2\right)^{3/2}}-\frac{ad\text{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^2\left(a^2+b^2\right)^{3/2}}+\frac{a(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{f\left(a^2+b^2\right)^{3/2}}-\frac{a(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{f\left(a^2+b^2\right)^{3/2}}$$

```
[Out] (a*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(
3/2)*f) - (a*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/((a^
2 + b^2)^(3/2)*f) + (d*Log[a + b*Sinh[e + f*x]])/((a^2 + b^2)*f^2) + (a*d*P
olyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*f^2
) - (a*d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)
^(3/2)*f^2) - (b*(c + d*x)*Cosh[e + f*x])/((a^2 + b^2)*f*(a + b*Sinh[e + f*
x]))
```

Rubi [A] time = 0.442247, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3324, 3322, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{ad\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2\left(a^2+b^2\right)^{3/2}}-\frac{ad\text{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^2\left(a^2+b^2\right)^{3/2}}+\frac{a(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{f\left(a^2+b^2\right)^{3/2}}-\frac{a(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{f\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Sinh[e + f*x])^2,x]
```


```
[Out] (a*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(
3/2)*f) - (a*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/((a^
2 + b^2)^(3/2)*f) + (d*Log[a + b*Sinh[e + f*x]])/((a^2 + b^2)*f^2) + (a*d*P
olyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*f^2
) - (a*d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)
^(3/2)*f^2) - (b*(c + d*x)*Cosh[e + f*x])/((a^2 + b^2)*f*(a + b*Sinh[e + f*
x]))
```


Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
```

```
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{c+dx}{(a+b\sinh(e+fx))^2}\,dx = -\frac{b(c+dx)\cosh(e+fx)}{(a^2+b^2)\,f(a+b\sinh(e+fx))} + \frac{a\int \frac{c+dx}{a+b\sinh(e+fx)}\,dx}{a^2+b^2} + \frac{(bd)\int \frac{\cosh(e+fx)}{a+b\sinh(e+fx)}\,dx}{(a^2+b^2)\,f}$$

$$= -\frac{b(c+dx)\cosh(e+fx)}{(a^2+b^2)\,f(a+b\sinh(e+fx))} + \frac{(2a)\int \frac{e^{e+fx}(c+dx)}{-b+2ae^{e+fx}+be^{2(e+fx)}}\,dx}{a^2+b^2} + \frac{d\,\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx,x,\right.}{(a^2+b^2)}$$

$$= \frac{d\log(a+b\sinh(e+fx))}{(a^2+b^2)\,f^2} - \frac{b(c+dx)\cosh(e+fx)}{(a^2+b^2)\,f(a+b\sinh(e+fx))} + \frac{(2ab)\int \frac{e^{e+fx}(c+dx)}{2a-2\sqrt{a^2+b^2}+2be^{e+fx}}}{(a^2+b^2)^{3/2}}$$

$$= \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,f} - \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,f} + \frac{d\log(a+b\sinh(e+fx))}{(a^2+b^2)\,f^2}$$

$$= \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,f} - \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,f} + \frac{d\log(a+b\sinh(e+fx))}{(a^2+b^2)\,f^2}$$

$$= \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,f} - \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,f} + \frac{d\log(a+b\sinh(e+fx))}{(a^2+b^2)\,f^2}$$

Mathematica [A] time = 1.07478, size = 194, normalized size = 0.76

$$\frac{\frac{a\left(d\,\mathrm{PolyLog}\left(2,\frac{be^{e+fx}}{\sqrt{a^2+b^2}-a}\right)-d\,\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)+f(c+dx)\left(\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)-\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)\right)\right)}{\sqrt{a^2+b^2}} - \frac{bf(c+dx)\cosh(e+fx)}{a+b\sinh(e+fx)} + d\log(a+b\sinh(}{f^2\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] (d*Log[a + b*Sinh[e + f*x]] + (a*(f*(c + d*x)*(Log[1 + (b*E^(e + f*x))/(a -
 Sqrt[a^2 + b^2])] - Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])]) + d*Po
lyLog[2, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] - d*PolyLog[2, -((b*E^(e +
 f*x))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2] - (b*f*(c + d*x)*Cosh[e +
f*x])/(a + b*Sinh[e + f*x]))/((a^2 + b^2)*f^2)
```

Maple [B] time = 0.103, size = 519, normalized size = 2.

$$2\,\frac{(dx+c)\left(ae^{fx+e}-b\right)}{f\left(a^2+b^2\right)\left(be^{2\,fx+2\,e}+2\,ae^{fx+e}-b\right)} - 2\,\frac{d\ln\left(e^{fx+e}\right)}{\left(a^2+b^2\right)f^2} + \frac{d\ln\left(be^{2\,fx+2\,e}+2\,ae^{fx+e}-b\right)}{\left(a^2+b^2\right)f^2} - 2\,\frac{ac}{\left(a^2+b^2\right)^{3/2}f}\mathrm{Artanh}\left(1/\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*sinh(f*x+e))^2,x)
```

```
[Out] 2*(d*x+c)*(a*exp(f*x+e)-b)/f/(a^2+b^2)/(b*exp(2*f*x+2*e)+2*a*exp(f*x+e)-b)-
2/(a^2+b^2)/f^2*d*ln(exp(f*x+e))+1/(a^2+b^2)/f^2*d*ln(b*exp(2*f*x+2*e)+2*a*
exp(f*x+e)-b)-2/(a^2+b^2)^(3/2)/f*a*c*arctanh(1/2*(2*b*exp(f*x+e)+2*a)/(a^2
+b^2)^(1/2))+1/(a^2+b^2)^(3/2)/f*d*a*ln((-b*exp(f*x+e)+(a^2+b^2)^(1/2)-a)/(
-a+(a^2+b^2)^(1/2)))*x+1/(a^2+b^2)^(3/2)/f^2*d*a*ln((-b*exp(f*x+e)+(a^2+b^2
```

```
)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*e-1/(a^2+b^2)^(3/2)/f*d*a*ln((b*exp(f*x+e)
+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/(a^2+b^2)^(3/2)/f^2*d*a*ln((b*
exp(f*x+e)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*e+1/(a^2+b^2)^(3/2)/f^2*
d*a*dilog((-b*exp(f*x+e)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/(a^2+b^
2)^(3/2)/f^2*d*a*dilog((b*exp(f*x+e)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))
)+2/(a^2+b^2)^(3/2)/f^2*a*d*e*arctanh(1/2*(2*b*exp(f*x+e)+2*a)/(a^2+b^2)^(1
/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 3.115, size = 4073, normalized size = 16.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="fricas")

[Out] (2*(a^2*b + b^3)*d*e - 2*(a^2*b + b^3)*c*f - 2*((a^2*b + b^3)*d*f*x + (a^2*
b + b^3)*d*e)*cosh(f*x + e)^2 - 2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e)
*sinh(f*x + e)^2 + (a*b^2*d*cosh(f*x + e)^2 + a*b^2*d*sinh(f*x + e)^2 + 2*a
^2*b*d*cosh(f*x + e) - a*b^2*d + 2*(a*b^2*d*cosh(f*x + e) + a^2*b*d)*sinh(f
*x + e))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + e) + a*sinh(f*x + e) + (
b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (a*b
^2*d*cosh(f*x + e)^2 + a*b^2*d*sinh(f*x + e)^2 + 2*a^2*b*d*cosh(f*x + e) -
a*b^2*d + 2*(a*b^2*d*cosh(f*x + e) + a^2*b*d)*sinh(f*x + e))*sqrt((a^2 + b^
2)/b^2)*dilog((a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e) + b*sin
h(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (a*b^2*d*f*x + a*b^2*d*e -
(a*b^2*d*f*x + a*b^2*d*e)*cosh(f*x + e)^2 - (a*b^2*d*f*x + a*b^2*d*e)*sinh(
f*x + e)^2 - 2*(a^2*b*d*f*x + a^2*b*d*e)*cosh(f*x + e) - 2*(a^2*b*d*f*x + a
^2*b*d*e + (a*b^2*d*f*x + a*b^2*d*e)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^
2 + b^2)/b^2)*log(-(a*cosh(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) +
b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b) + (a*b^2*d*f*x + a*b^2*d*e -
 (a*b^2*d*f*x + a*b^2*d*e)*cosh(f*x + e)^2 - (a*b^2*d*f*x + a*b^2*d*e)*sinh
(f*x + e)^2 - 2*(a^2*b*d*f*x + a^2*b*d*e)*cosh(f*x + e) - 2*(a^2*b*d*f*x +
a^2*b*d*e + (a*b^2*d*f*x + a*b^2*d*e)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a
^2 + b^2)/b^2)*log(-(a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e) +
 b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*((a^3 + a*b^2)*d*f*x +
2*(a^3 + a*b^2)*d*e - (a^3 + a*b^2)*c*f)*cosh(f*x + e) + ((a^2*b + b^3)*d*c
osh(f*x + e)^2 + (a^2*b + b^3)*d*sinh(f*x + e)^2 + 2*(a^3 + a*b^2)*d*cosh(f
*x + e) - (a^2*b + b^3)*d + 2*((a^2*b + b^3)*d*cosh(f*x + e) + (a^3 + a*b^2
)*d)*sinh(f*x + e) - (a*b^2*d*e - a*b^2*c*f - (a*b^2*d*e - a*b^2*c*f)*cosh(
f*x + e)^2 - (a*b^2*d*e - a*b^2*c*f)*sinh(f*x + e)^2 - 2*(a^2*b*d*e - a^2*b
*c*f)*cosh(f*x + e) - 2*(a^2*b*d*e - a^2*b*c*f + (a*b^2*d*e - a*b^2*c*f)*co
sh(f*x + e))*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))*log(2*b*cosh(f*x + e) +
2*b*sinh(f*x + e) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + ((a^2*b + b^3)*d*cos
h(f*x + e)^2 + (a^2*b + b^3)*d*sinh(f*x + e)^2 + 2*(a^3 + a*b^2)*d*cosh(f*x
```

```
 + e) - (a^2*b + b^3)*d + 2*((a^2*b + b^3)*d*cosh(f*x + e) + (a^3 + a*b^2)*
d)*sinh(f*x + e) + (a*b^2*d*e - a*b^2*c*f - (a*b^2*d*e - a*b^2*c*f)*cosh(f*
x + e)^2 - (a*b^2*d*e - a*b^2*c*f)*sinh(f*x + e)^2 - 2*(a^2*b*d*e - a^2*b*c
*f)*cosh(f*x + e) - 2*(a^2*b*d*e - a^2*b*c*f + (a*b^2*d*e - a*b^2*c*f)*cosh
(f*x + e))*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))*log(2*b*cosh(f*x + e) + 2*
b*sinh(f*x + e) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*((a^3 + a*b^2)*d*f*x
 + 2*(a^3 + a*b^2)*d*e - (a^3 + a*b^2)*c*f + 2*((a^2*b + b^3)*d*f*x + (a^2*
b + b^3)*d*e)*cosh(f*x + e))*sinh(f*x + e))/((a^4*b + 2*a^2*b^3 + b^5)*f^2*
cosh(f*x + e)^2 + (a^4*b + 2*a^2*b^3 + b^5)*f^2*sinh(f*x + e)^2 + 2*(a^5 +
2*a^3*b^2 + a*b^4)*f^2*cosh(f*x + e) - (a^4*b + 2*a^2*b^3 + b^5)*f^2 + 2*((
a^4*b + 2*a^2*b^3 + b^5)*f^2*cosh(f*x + e) + (a^5 + 2*a^3*b^2 + a*b^4)*f^2)
*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sinh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{\left(b \sinh\left(fx + e\right) + a\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*sinh(f*x + e) + a)^2, x)
```

3.176 $\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2} dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)(a+b\sinh(e+fx))^2},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)*(a + b*Sinh[e + f*x])^2), x]
```

Rubi [A] time = 0.0615198, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*Sinh[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + b*Sinh[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2} dx = \int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2} dx$$

Mathematica [A] time = 53.3547, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a+b\sinh(e+fx))^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*Sinh[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)*(a + b*Sinh[e + f*x])^2), x]
```

Maple [A] time = 0.283, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(a+b\sinh\left(fx+e\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(ae^{(fx+e)}-b\right)}{a^2bcf+b^3cf+\left(a^2bdf+b^3df\right)x-\left(a^2bcfe^{(2e)}+b^3cfe^{(2e)}+\left(a^2bdfe^{(2e)}+b^3dfe^{(2e)}\right)x\right)e^{(2fx)}-2\left(a^3cfe^e+ab^2cfe^e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2*(a*e^(f*x + e) - b)/(a^2*b*c*f + b^3*c*f + (a^2*b*d*f + b^3*d*f)*x - (a^
2*b*c*f*e^(2*e) + b^3*c*f*e^(2*e) + (a^2*b*d*f*e^(2*e) + b^3*d*f*e^(2*e))*x
)*e^(2*f*x) - 2*(a^3*c*f*e^e + a*b^2*c*f*e^e + (a^3*d*f*e^e + a*b^2*d*f*e^e
)*x)*e^(f*x)) + integrate(2*(b*d - (a*d*f*x*e^e + (c*f*e^e + d*e^e)*a)*e^(f
*x))/(a^2*b*c^2*f + b^3*c^2*f + (a^2*b*d^2*f + b^3*d^2*f)*x^2 + 2*(a^2*b*c*
d*f + b^3*c*d*f)*x - (a^2*b*c^2*f*e^(2*e) + b^3*c^2*f*e^(2*e) + (a^2*b*d^2*
f*e^(2*e) + b^3*d^2*f*e^(2*e))*x^2 + 2*(a^2*b*c*d*f*e^(2*e) + b^3*c*d*f*e^(
2*e))*x)*e^(2*f*x) - 2*(a^3*c^2*f*e^e + a*b^2*c^2*f*e^e + (a^3*d^2*f*e^e +
a*b^2*d^2*f*e^e)*x^2 + 2*(a^3*c*d*f*e^e + a*b^2*c*d*f*e^e)*x)*e^(f*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^2dx+a^2c+\left(b^2dx+b^2c\right)\sinh\left(fx+e\right)^2+2\left(abdx+abc\right)\sinh\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d*x + a^2*c + (b^2*d*x + b^2*c)*sinh(f*x + e)^2 + 2*(a*b*d*
x + a*b*c)*sinh(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sinh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(b\sinh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="giac")

[Out] integrate(1/((d*x + c)*(b*sinh(f*x + e) + a)^2), x)
```

3.177 $\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2},x\right)$$

```
[Out] Unintegrable[1/((c + d*x)^2*(a + b*Sinh[e + f*x])^2), x]
```

Rubi [A] time = 0.0579459, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*Sinh[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + b*Sinh[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2}\,dx$$

Mathematica [A] time = 55.645, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a+b\sinh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*Sinh[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + b*Sinh[e + f*x])^2), x]
```

Maple [A] time = 0.47, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(a+b\sinh\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(ae^{(f}\right.}{a^2bc^2f+b^3c^2f+\left(a^2bd^2f+b^3d^2f\right)x^2+2\left(a^2bcdf+b^3cdf\right)x-\left(a^2bc^2fe^{(2e)}+b^3c^2fe^{(2e)}+\left(a^2bd^2fe^{(2e)}+b^3d^2fe^{(2e)}\right)x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2*(a*e^(f*x + e) - b)/(a^2*b*c^2*f + b^3*c^2*f + (a^2*b*d^2*f + b^3*d^2*f)
*x^2 + 2*(a^2*b*c*d*f + b^3*c*d*f)*x - (a^2*b*c^2*f*e^(2*e) + b^3*c^2*f*e^(
2*e) + (a^2*b*d^2*f*e^(2*e) + b^3*d^2*f*e^(2*e))*x^2 + 2*(a^2*b*c*d*f*e^(2*
e) + b^3*c*d*f*e^(2*e))*x)*e^(2*f*x) - 2*(a^3*c^2*f*e^e + a*b^2*c^2*f*e^e +
 (a^3*d^2*f*e^e + a*b^2*d^2*f*e^e)*x^2 + 2*(a^3*c*d*f*e^e + a*b^2*c*d*f*e^e
)*x)*e^(f*x)) + integrate(2*(2*b*d - (a*d*f*x*e^e + (c*f*e^e + 2*d*e^e)*a)*
e^(f*x))/(a^2*b*c^3*f + b^3*c^3*f + (a^2*b*d^3*f + b^3*d^3*f)*x^3 + 3*(a^2*
b*c*d^2*f + b^3*c*d^2*f)*x^2 + 3*(a^2*b*c^2*d*f + b^3*c^2*d*f)*x - (a^2*b*c
^3*f*e^(2*e) + b^3*c^3*f*e^(2*e) + (a^2*b*d^3*f*e^(2*e) + b^3*d^3*f*e^(2*e)
)*x^3 + 3*(a^2*b*c*d^2*f*e^(2*e) + b^3*c*d^2*f*e^(2*e))*x^2 + 3*(a^2*b*c^2*
d*f*e^(2*e) + b^3*c^2*d*f*e^(2*e))*x)*e^(2*f*x) - 2*(a^3*c^3*f*e^e + a*b^2*
c^3*f*e^e + (a^3*d^3*f*e^e + a*b^2*d^3*f*e^e)*x^3 + 3*(a^3*c*d^2*f*e^e + a*
b^2*c*d^2*f*e^e)*x^2 + 3*(a^3*c^2*d*f*e^e + a*b^2*c^2*d*f*e^e)*x)*e^(f*x)),
 x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^2d^2x^2+2\,a^2cdx+a^2c^2+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\sinh\left(fx+e\right)^2+2\left(abd^2x^2+2\,abcdx+abc^2\right)\sinh\left(fx+e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*sinh(f*x + e)^2 + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*sinh
(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*sinh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2\left(b\sinh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(b*sinh(f*x + e) + a)^2), x)
```

3.178 $\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=544

$$\frac{3a^2f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2d^2\left(a^2+b^2\right)^{5/2}}-\frac{3a^2f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{2d^2\left(a^2+b^2\right)^{5/2}}-\frac{f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2d^2\left(a^2+b^2\right)^{3/2}}+\frac{f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{2d^2\left(a^2+b^2\right)^{3/2}}-$$

```
[Out] (3*a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(2*(a^2 +
b^2)^(5/2)*d) - ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/
(2*(a^2 + b^2)^(3/2)*d) - (3*a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])])/(2*(a^2 + b^2)^(5/2)*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2])])/(2*(a^2 + b^2)^(3/2)*d) + (3*a*f*Log[a + b*Sinh[c
 + d*x]])/(2*(a^2 + b^2)^2*d^2) + (3*a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/(2*(a^2 + b^2)^(5/2)*d^2) - (f*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(2*(a^2 + b^2)^(3/2)*d^2) - (3*a^2*f*PolyLo
g[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(2*(a^2 + b^2)^(5/2)*d^2) +
 (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(2*(a^2 + b^2)^(3
/2)*d^2) - (b*(e + f*x)*Cosh[c + d*x])/(2*(a^2 + b^2)*d*(a + b*Sinh[c + d*x
])^2) - f/(2*(a^2 + b^2)*d^2*(a + b*Sinh[c + d*x])) - (3*a*b*(e + f*x)*Cosh
[c + d*x])/(2*(a^2 + b^2)^2*d*(a + b*Sinh[c + d*x]))
```

Rubi [A] time = 2.08106, antiderivative size = 544, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {3325, 3324, 3322, 2264, 2190, 2279, 2391, 2668, 31, 6742, 32}

$$\frac{3a^2f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2d^2\left(a^2+b^2\right)^{5/2}}-\frac{3a^2f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{2d^2\left(a^2+b^2\right)^{5/2}}-\frac{f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2d^2\left(a^2+b^2\right)^{3/2}}+\frac{f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{2d^2\left(a^2+b^2\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(a + b*Sinh[c + d*x])^3,x]
```


```
[Out] (3*a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(2*(a^2 +
b^2)^(5/2)*d) - ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/
(2*(a^2 + b^2)^(3/2)*d) - (3*a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])])/(2*(a^2 + b^2)^(5/2)*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2])])/(2*(a^2 + b^2)^(3/2)*d) + (3*a*f*Log[a + b*Sinh[c
 + d*x]])/(2*(a^2 + b^2)^2*d^2) + (3*a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/(2*(a^2 + b^2)^(5/2)*d^2) - (f*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(2*(a^2 + b^2)^(3/2)*d^2) - (3*a^2*f*PolyLo
g[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(2*(a^2 + b^2)^(5/2)*d^2) +
 (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(2*(a^2 + b^2)^(3
/2)*d^2) - (b*(e + f*x)*Cosh[c + d*x])/(2*(a^2 + b^2)*d*(a + b*Sinh[c + d*x
])^2) - f/(2*(a^2 + b^2)*d^2*(a + b*Sinh[c + d*x])) - (3*a*b*(e + f*x)*Cosh
[c + d*x])/(2*(a^2 + b^2)^2*d*(a + b*Sinh[c + d*x]))
```


Rule 3325

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> -Simp[(b*(c + d*x)^m*Cos[e + f*x]*(a + b*Sin[e + f*x])^(n + 1
))/(f*(n + 1)*(a^2 - b^2)), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m*(a +
b*Sin[e + f*x])^(n + 1), x], x] - Dist[(b*(n + 2))/((n + 1)*(a^2 - b^2)), I
nt[(c + d*x)^m*Sin[e + f*x]*(a + b*Sin[e + f*x])^(n + 1), x], x] + Dist[(b*
d*m)/(f*(n + 1)*(a^2 - b^2)), Int[(c + d*x)^(m - 1)*Cos[e + f*x]*(a + b*Sin
```

[e + f*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^2, 0] && ILtQ[n, -2] && IGtQ[m, 0]

Rule 3324

Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e + f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x], x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 3322

Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 2264

Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 2668

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

Rule 31

Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{e+fx}{(a+b\sinh(c+dx))^3}\,dx = -\frac{b(e+fx)\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2} + \frac{a\int \frac{e+fx}{(a+b\sinh(c+dx))^2}\,dx}{a^2+b^2} - \frac{b\int \frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx}{2\left(a^2+b^2\right)} + \frac{(bf)}{}$$

$$= -\frac{b(e+fx)\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2} - \frac{ab(e+fx)\cosh(c+dx)}{\left(a^2+b^2\right)^2 d(a+b\sinh(c+dx))} + \frac{a^2\int \frac{e+fx}{a+b\sinh(c+dx)}}{\left(a^2+b^2\right)^2}$$

$$= -\frac{b(e+fx)\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2} - \frac{f}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))} - \frac{ab(e+fx)\,\mathrm{c}}{\left(a^2+b^2\right)^2 d(a+}$$

$$= \frac{af\log(a+b\sinh(c+dx))}{\left(a^2+b^2\right)^2 d^2} - \frac{b(e+fx)\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2} - \frac{f}{2\left(a^2+b^2\right)d^2(a+b\sinh}$$

$$= \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}d} - \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}d} + \frac{af\log(a+b\sinh(c+dx))}{\left(a^2+b^2\right)^2 d^2}$$

$$= \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}d} - \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d} - \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}d}$$

$$= \frac{3a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{5/2}d} - \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d} - \frac{3a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+}}\right)}{2\left(a^2+b^2\right)^{5/2}d}$$

$$= \frac{3a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{5/2}d} - \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d} - \frac{3a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+}}\right)}{2\left(a^2+b^2\right)^{5/2}d}$$

$$= \frac{3a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{5/2}d} - \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d} - \frac{3a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+}}\right)}{2\left(a^2+b^2\right)^{5/2}d}$$

Mathematica [A] time = 9.85403, size = 836, normalized size = 1.54

$$\frac{-bde\cosh(c+dx)+bcf\cosh(c+dx)-bf(c+dx)\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))^2} - \frac{6\sqrt{a^2+b^2}f\tan^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)a^2-4\sqrt{-a^2-b^2}de\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] -(-3*a*Sqrt[-(a^2 + b^2)^2]*f*(c + d*x) + 6*a^2*Sqrt[a^2 + b^2]*f*ArcTan[(a
 + b*E^(c + d*x))/Sqrt[-a^2 - b^2]] - 4*a^2*Sqrt[-a^2 - b^2]*d*e*ArcTanh[(a
```

```
 + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*b^2*Sqrt[-a^2 - b^2]*d*e*ArcTanh[(a
+ b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 6*a^2*Sqrt[-a^2 - b^2]*f*ArcTanh[(a + b
*E^(c + d*x))/Sqrt[a^2 + b^2]] + 4*a^2*Sqrt[-a^2 - b^2]*c*f*ArcTanh[(a + b*
E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*b^2*Sqrt[-a^2 - b^2]*c*f*ArcTanh[(a + b*E
^(c + d*x))/Sqrt[a^2 + b^2]] + 2*a^2*Sqrt[-a^2 - b^2]*f*(c + d*x)*Log[1 + (
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - b^2*Sqrt[-a^2 - b^2]*f*(c + d*x)*Lo
g[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*a^2*Sqrt[-a^2 - b^2]*f*(c
+ d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + b^2*Sqrt[-a^2 - b^2
]*f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3*a*Sqrt[-(a
^2 + b^2)^2]*f*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + Sqrt[-a^2
- b^2]*(2*a^2 - b^2)*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] +
 Sqrt[-a^2 - b^2]*(-2*a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2]))])/(2*(-(a^2 + b^2)^2)^(3/2)*d^2) + (-(b*d*e*Cosh[c + d*x]) + b*
c*f*Cosh[c + d*x] - b*f*(c + d*x)*Cosh[c + d*x])/(2*(a^2 + b^2)*d^2*(a + b*
Sinh[c + d*x])^2) + (-(a^2*f) - b^2*f - 3*a*b*d*e*Cosh[c + d*x] + 3*a*b*c*f
*Cosh[c + d*x] - 3*a*b*f*(c + d*x)*Cosh[c + d*x])/(2*(a^2 + b^2)^2*d^2*(a +
 b*Sinh[c + d*x]))
```

Maple [B] time = 0.192, size = 1232, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(a+b*sinh(d*x+c))^3,x)
```

```
[Out] (2*a^2*b*d*f*x*exp(3*d*x+3*c)-b^3*d*f*x*exp(3*d*x+3*c)+6*a^3*d*f*x*exp(2*d*
x+2*c)+2*a^2*b*d*e*exp(3*d*x+3*c)-3*a*b^2*d*f*x*exp(2*d*x+2*c)-b^3*d*e*exp(
3*d*x+3*c)+6*a^3*d*e*exp(2*d*x+2*c)-10*a^2*b*d*f*x*exp(d*x+c)-a^2*b*f*exp(3
*d*x+3*c)-3*a*b^2*d*e*exp(2*d*x+2*c)-b^3*d*f*x*exp(d*x+c)-b^3*f*exp(3*d*x+3
*c)-2*a^3*f*exp(2*d*x+2*c)-10*a^2*b*d*e*exp(d*x+c)+3*a*b^2*d*f*x-2*a*b^2*f*
exp(2*d*x+2*c)-b^3*d*e*exp(d*x+c)+a^2*b*f*exp(d*x+c)+3*a*b^2*d*e+b^3*f*exp(
d*x+c))/d^2/(a^2+b^2)^2/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)^2-1/d^2/(a^2+b^
2)^(5/2)*b^2*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+3/2/d^2/
(a^2+b^2)^2*a*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-3/d^2/(a^2+b^2)^2*a*f
*ln(exp(d*x+c))-2/d/(a^2+b^2)^(5/2)*a^2*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/
(a^2+b^2)^(1/2))+2/d^2/(a^2+b^2)^(5/2)*a^2*f*c*arctanh(1/2*(2*b*exp(d*x+c)+
2*a)/(a^2+b^2)^(1/2))+1/d/(a^2+b^2)^(5/2)*a^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)
^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d^2/(a^2+b^2)^(5/2)*a^2*f*ln((-b*exp(d*
x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d/(a^2+b^2)^(5/2)*a^2*f*l
n((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2/(a^2+b^2)^(
5/2)*a^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d^2
/(a^2+b^2)^(5/2)*a^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2
)^(1/2)))-1/d^2/(a^2+b^2)^(5/2)*a^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a
)/(a+(a^2+b^2)^(1/2)))+1/d/(a^2+b^2)^(5/2)*b^2*e*arctanh(1/2*(2*b*exp(d*x+c
)+2*a)/(a^2+b^2)^(1/2))-1/2/d/(a^2+b^2)^(5/2)*b^2*f*ln((-b*exp(d*x+c)+(a^2+
b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/2/d^2/(a^2+b^2)^(5/2)*b^2*f*ln((-b*
exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/2/d/(a^2+b^2)^(5/2)
*b^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/2/d^2/(
a^2+b^2)^(5/2)*b^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)
))*c-1/2/d^2/(a^2+b^2)^(5/2)*b^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
(-a+(a^2+b^2)^(1/2)))+1/2/d^2/(a^2+b^2)^(5/2)*b^2*f*dilog((b*exp(d*x+c)+(a^
2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.71103, size = 13728, normalized size = 25.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*((a^3*b^2 + a*b^4)*d*f*x + (a^3*b^2 + a*b^4)*c*f)*cosh(d*x + c)^4 +
 6*((a^3*b^2 + a*b^4)*d*f*x + (a^3*b^2 + a*b^4)*c*f)*sinh(d*x + c)^4 + 2*((
10*a^4*b + 11*a^2*b^3 + b^5)*d*f*x - (2*a^4*b + a^2*b^3 - b^5)*d*e + (a^4*b
 + 2*a^2*b^3 + b^5 + 12*(a^4*b + a^2*b^3)*c)*f)*cosh(d*x + c)^3 + 2*((10*a^
4*b + 11*a^2*b^3 + b^5)*d*f*x - (2*a^4*b + a^2*b^3 - b^5)*d*e + (a^4*b + 2*
a^2*b^3 + b^5 + 12*(a^4*b + a^2*b^3)*c)*f + 12*((a^3*b^2 + a*b^4)*d*f*x + (
a^3*b^2 + a*b^4)*c*f)*cosh(d*x + c))*sinh(d*x + c)^3 - 6*(a^3*b^2 + a*b^4)*
d*e + 6*(a^3*b^2 + a*b^4)*c*f + 2*(3*(2*a^5 + a^3*b^2 - a*b^4)*d*f*x - 3*(2
*a^5 + a^3*b^2 - a*b^4)*d*e + 2*(a^5 + 2*a^3*b^2 + a*b^4 + 3*(2*a^5 + a^3*b
^2 - a*b^4)*c)*f)*cosh(d*x + c)^2 + 2*(3*(2*a^5 + a^3*b^2 - a*b^4)*d*f*x -
3*(2*a^5 + a^3*b^2 - a*b^4)*d*e + 18*((a^3*b^2 + a*b^4)*d*f*x + (a^3*b^2 +
a*b^4)*c*f)*cosh(d*x + c)^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4 + 3*(2*a^5 + a^3*b
^2 - a*b^4)*c)*f + 3*((10*a^4*b + 11*a^2*b^3 + b^5)*d*f*x - (2*a^4*b + a^2*
b^3 - b^5)*d*e + (a^4*b + 2*a^2*b^3 + b^5 + 12*(a^4*b + a^2*b^3)*c)*f)*cosh
(d*x + c))*sinh(d*x + c)^2 - ((2*a^2*b^3 - b^5)*f*cosh(d*x + c)^4 + (2*a^2*
b^3 - b^5)*f*sinh(d*x + c)^4 + 4*(2*a^3*b^2 - a*b^4)*f*cosh(d*x + c)^3 + 2*
(4*a^4*b - 4*a^2*b^3 + b^5)*f*cosh(d*x + c)^2 + 4*((2*a^2*b^3 - b^5)*f*cosh
(d*x + c) + (2*a^3*b^2 - a*b^4)*f)*sinh(d*x + c)^3 - 4*(2*a^3*b^2 - a*b^4)*
f*cosh(d*x + c) + 2*(3*(2*a^2*b^3 - b^5)*f*cosh(d*x + c)^2 + 6*(2*a^3*b^2 -
 a*b^4)*f*cosh(d*x + c) + (4*a^4*b - 4*a^2*b^3 + b^5)*f)*sinh(d*x + c)^2 +
(2*a^2*b^3 - b^5)*f + 4*((2*a^2*b^3 - b^5)*f*cosh(d*x + c)^3 + 3*(2*a^3*b^2
 - a*b^4)*f*cosh(d*x + c)^2 + (4*a^4*b - 4*a^2*b^3 + b^5)*f*cosh(d*x + c) -
 (2*a^3*b^2 - a*b^4)*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(
d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2) - b)/b + 1) + ((2*a^2*b^3 - b^5)*f*cosh(d*x + c)^4 + (2*a^2*b^3
 - b^5)*f*sinh(d*x + c)^4 + 4*(2*a^3*b^2 - a*b^4)*f*cosh(d*x + c)^3 + 2*(4*
a^4*b - 4*a^2*b^3 + b^5)*f*cosh(d*x + c)^2 + 4*((2*a^2*b^3 - b^5)*f*cosh(d*
x + c) + (2*a^3*b^2 - a*b^4)*f)*sinh(d*x + c)^3 - 4*(2*a^3*b^2 - a*b^4)*f*c
osh(d*x + c) + 2*(3*(2*a^2*b^3 - b^5)*f*cosh(d*x + c)^2 + 6*(2*a^3*b^2 - a*
b^4)*f*cosh(d*x + c) + (4*a^4*b - 4*a^2*b^3 + b^5)*f)*sinh(d*x + c)^2 + (2*
a^2*b^3 - b^5)*f + 4*((2*a^2*b^3 - b^5)*f*cosh(d*x + c)^3 + 3*(2*a^3*b^2 -
a*b^4)*f*cosh(d*x + c)^2 + (4*a^4*b - 4*a^2*b^3 + b^5)*f*cosh(d*x + c) - (2
*a^3*b^2 - a*b^4)*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x
 + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2) - b)/b + 1) - (((2*a^2*b^3 - b^5)*d*f*x + (2*a^2*b^3 - b^5)*c*f)*c
osh(d*x + c)^4 + ((2*a^2*b^3 - b^5)*d*f*x + (2*a^2*b^3 - b^5)*c*f)*sinh(d*x
 + c)^4 + (2*a^2*b^3 - b^5)*d*f*x + 4*((2*a^3*b^2 - a*b^4)*d*f*x + (2*a^3*b
^2 - a*b^4)*c*f)*cosh(d*x + c)^3 + 4*((2*a^3*b^2 - a*b^4)*d*f*x + (2*a^3*b^
2 - a*b^4)*c*f + ((2*a^2*b^3 - b^5)*d*f*x + (2*a^2*b^3 - b^5)*c*f)*cosh(d*x
```

```
+ c))*sinh(d*x + c)^3 + (2*a^2*b^3 - b^5)*c*f + 2*((4*a^4*b - 4*a^2*b^3 +
b^5)*d*f*x + (4*a^4*b - 4*a^2*b^3 + b^5)*c*f)*cosh(d*x + c)^2 + 2*((4*a^4*b
 - 4*a^2*b^3 + b^5)*d*f*x + (4*a^4*b - 4*a^2*b^3 + b^5)*c*f + 3*((2*a^2*b^3
 - b^5)*d*f*x + (2*a^2*b^3 - b^5)*c*f)*cosh(d*x + c)^2 + 6*((2*a^3*b^2 - a*
b^4)*d*f*x + (2*a^3*b^2 - a*b^4)*c*f)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*((
2*a^3*b^2 - a*b^4)*d*f*x + (2*a^3*b^2 - a*b^4)*c*f)*cosh(d*x + c) - 4*((2*a
^3*b^2 - a*b^4)*d*f*x - ((2*a^2*b^3 - b^5)*d*f*x + (2*a^2*b^3 - b^5)*c*f)*c
osh(d*x + c)^3 + (2*a^3*b^2 - a*b^4)*c*f - 3*((2*a^3*b^2 - a*b^4)*d*f*x + (
2*a^3*b^2 - a*b^4)*c*f)*cosh(d*x + c)^2 - ((4*a^4*b - 4*a^2*b^3 + b^5)*d*f*
x + (4*a^4*b - 4*a^2*b^3 + b^5)*c*f)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (((2*a^2*b^3 - b^5)*d*f*x
+ (2*a^2*b^3 - b^5)*c*f)*cosh(d*x + c)^4 + ((2*a^2*b^3 - b^5)*d*f*x + (2*a^
2*b^3 - b^5)*c*f)*sinh(d*x + c)^4 + (2*a^2*b^3 - b^5)*d*f*x + 4*((2*a^3*b^2
 - a*b^4)*d*f*x + (2*a^3*b^2 - a*b^4)*c*f)*cosh(d*x + c)^3 + 4*((2*a^3*b^2
- a*b^4)*d*f*x + (2*a^3*b^2 - a*b^4)*c*f + ((2*a^2*b^3 - b^5)*d*f*x + (2*a^
2*b^3 - b^5)*c*f)*cosh(d*x + c))*sinh(d*x + c)^3 + (2*a^2*b^3 - b^5)*c*f +
2*((4*a^4*b - 4*a^2*b^3 + b^5)*d*f*x + (4*a^4*b - 4*a^2*b^3 + b^5)*c*f)*cos
h(d*x + c)^2 + 2*((4*a^4*b - 4*a^2*b^3 + b^5)*d*f*x + (4*a^4*b - 4*a^2*b^3
+ b^5)*c*f + 3*((2*a^2*b^3 - b^5)*d*f*x + (2*a^2*b^3 - b^5)*c*f)*cosh(d*x +
 c)^2 + 6*((2*a^3*b^2 - a*b^4)*d*f*x + (2*a^3*b^2 - a*b^4)*c*f)*cosh(d*x +
c))*sinh(d*x + c)^2 - 4*((2*a^3*b^2 - a*b^4)*d*f*x + (2*a^3*b^2 - a*b^4)*c*
f)*cosh(d*x + c) - 4*((2*a^3*b^2 - a*b^4)*d*f*x - ((2*a^2*b^3 - b^5)*d*f*x
+ (2*a^2*b^3 - b^5)*c*f)*cosh(d*x + c)^3 + (2*a^3*b^2 - a*b^4)*c*f - 3*((2*
a^3*b^2 - a*b^4)*d*f*x + (2*a^3*b^2 - a*b^4)*c*f)*cosh(d*x + c)^2 - ((4*a^4
*b - 4*a^2*b^3 + b^5)*d*f*x + (4*a^4*b - 4*a^2*b^3 + b^5)*c*f)*cosh(d*x + c
))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x
+ c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) -
2*((2*a^4*b + a^2*b^3 - b^5)*d*f*x - (10*a^4*b + 11*a^2*b^3 + b^5)*d*e + (a
^4*b + 2*a^2*b^3 + b^5 + 12*(a^4*b + a^2*b^3)*c)*f)*cosh(d*x + c) - (3*(a^3
*b^2 + a*b^4)*f*cosh(d*x + c)^4 + 3*(a^3*b^2 + a*b^4)*f*sinh(d*x + c)^4 + 1
2*(a^4*b + a^2*b^3)*f*cosh(d*x + c)^3 + 6*(2*a^5 + a^3*b^2 - a*b^4)*f*cosh(
d*x + c)^2 + 12*((a^3*b^2 + a*b^4)*f*cosh(d*x + c) + (a^4*b + a^2*b^3)*f)*s
inh(d*x + c)^3 - 12*(a^4*b + a^2*b^3)*f*cosh(d*x + c) + 6*(3*(a^3*b^2 + a*b
^4)*f*cosh(d*x + c)^2 + 6*(a^4*b + a^2*b^3)*f*cosh(d*x + c) + (2*a^5 + a^3*
b^2 - a*b^4)*f)*sinh(d*x + c)^2 + 3*(a^3*b^2 + a*b^4)*f + 12*((a^3*b^2 + a*
b^4)*f*cosh(d*x + c)^3 + 3*(a^4*b + a^2*b^3)*f*cosh(d*x + c)^2 + (2*a^5 + a
^3*b^2 - a*b^4)*f*cosh(d*x + c) - (a^4*b + a^2*b^3)*f)*sinh(d*x + c) - (((2
*a^2*b^3 - b^5)*d*e - (2*a^2*b^3 - b^5)*c*f)*cosh(d*x + c)^4 + ((2*a^2*b^3
 - b^5)*d*e - (2*a^2*b^3 - b^5)*c*f)*sinh(d*x + c)^4 + 4*((2*a^3*b^2 - a*b^4
)*d*e - (2*a^3*b^2 - a*b^4)*c*f)*cosh(d*x + c)^3 + 4*((2*a^3*b^2 - a*b^4)*d
*e - (2*a^3*b^2 - a*b^4)*c*f + ((2*a^2*b^3 - b^5)*d*e - (2*a^2*b^3 - b^5)*c
*f)*cosh(d*x + c))*sinh(d*x + c)^3 + (2*a^2*b^3 - b^5)*d*e - (2*a^2*b^3 - b
^5)*c*f + 2*((4*a^4*b - 4*a^2*b^3 + b^5)*d*e - (4*a^4*b - 4*a^2*b^3 + b^5)*
c*f)*cosh(d*x + c)^2 + 2*((4*a^4*b - 4*a^2*b^3 + b^5)*d*e - (4*a^4*b - 4*a^
2*b^3 + b^5)*c*f + 3*((2*a^2*b^3 - b^5)*d*e - (2*a^2*b^3 - b^5)*c*f)*cosh(d
*x + c)^2 + 6*((2*a^3*b^2 - a*b^4)*d*e - (2*a^3*b^2 - a*b^4)*c*f)*cosh(d*x
+ c))*sinh(d*x + c)^2 - 4*((2*a^3*b^2 - a*b^4)*d*e - (2*a^3*b^2 - a*b^4)*c*
f)*cosh(d*x + c) + 4*(((2*a^2*b^3 - b^5)*d*e - (2*a^2*b^3 - b^5)*c*f)*cosh(
d*x + c)^3 - (2*a^3*b^2 - a*b^4)*d*e + (2*a^3*b^2 - a*b^4)*c*f + 3*((2*a^3*
b^2 - a*b^4)*d*e - (2*a^3*b^2 - a*b^4)*c*f)*cosh(d*x + c)^2 + ((4*a^4*b - 4
*a^2*b^3 + b^5)*d*e - (4*a^4*b - 4*a^2*b^3 + b^5)*c*f)*cosh(d*x + c))*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c)
+ 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (3*(a^3*b^2 + a*b^4)*f*cosh(d*x + c)^4
 + 3*(a^3*b^2 + a*b^4)*f*sinh(d*x + c)^4 + 12*(a^4*b + a^2*b^3)*f*cosh(d*x
+ c)^3 + 6*(2*a^5 + a^3*b^2 - a*b^4)*f*cosh(d*x + c)^2 + 12*((a^3*b^2 + a*b
^4)*f*cosh(d*x + c) + (a^4*b + a^2*b^3)*f)*sinh(d*x + c)^3 - 12*(a^4*b + a^
2*b^3)*f*cosh(d*x + c) + 6*(3*(a^3*b^2 + a*b^4)*f*cosh(d*x + c)^2 + 6*(a^4*
b + a^2*b^3)*f*cosh(d*x + c) + (2*a^5 + a^3*b^2 - a*b^4)*f)*sinh(d*x + c)^2
```

```
 + 3*(a^3*b^2 + a*b^4)*f + 12*((a^3*b^2 + a*b^4)*f*cosh(d*x + c)^3 + 3*(a^4
*b + a^2*b^3)*f*cosh(d*x + c)^2 + (2*a^5 + a^3*b^2 - a*b^4)*f*cosh(d*x + c)
 - (a^4*b + a^2*b^3)*f)*sinh(d*x + c) + (((2*a^2*b^3 - b^5)*d*e - (2*a^2*b^
3 - b^5)*c*f)*cosh(d*x + c)^4 + ((2*a^2*b^3 - b^5)*d*e - (2*a^2*b^3 - b^5)*
c*f)*sinh(d*x + c)^4 + 4*((2*a^3*b^2 - a*b^4)*d*e - (2*a^3*b^2 - a*b^4)*c*f
)*cosh(d*x + c)^3 + 4*((2*a^3*b^2 - a*b^4)*d*e - (2*a^3*b^2 - a*b^4)*c*f +
((2*a^2*b^3 - b^5)*d*e - (2*a^2*b^3 - b^5)*c*f)*cosh(d*x + c))*sinh(d*x + c
)^3 + (2*a^2*b^3 - b^5)*d*e - (2*a^2*b^3 - b^5)*c*f + 2*((4*a^4*b - 4*a^2*b
^3 + b^5)*d*e - (4*a^4*b - 4*a^2*b^3 + b^5)*c*f)*cosh(d*x + c)^2 + 2*((4*a^
4*b - 4*a^2*b^3 + b^5)*d*e - (4*a^4*b - 4*a^2*b^3 + b^5)*c*f + 3*((2*a^2*b^
3 - b^5)*d*e - (2*a^2*b^3 - b^5)*c*f)*cosh(d*x + c)^2 + 6*((2*a^3*b^2 - a*b
^4)*d*e - (2*a^3*b^2 - a*b^4)*c*f)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*((2*a
^3*b^2 - a*b^4)*d*e - (2*a^3*b^2 - a*b^4)*c*f)*cosh(d*x + c) + 4*(((2*a^2*b
^3 - b^5)*d*e - (2*a^2*b^3 - b^5)*c*f)*cosh(d*x + c)^3 - (2*a^3*b^2 - a*b^4
)*d*e + (2*a^3*b^2 - a*b^4)*c*f + 3*((2*a^3*b^2 - a*b^4)*d*e - (2*a^3*b^2 -
 a*b^4)*c*f)*cosh(d*x + c)^2 + ((4*a^4*b - 4*a^2*b^3 + b^5)*d*e - (4*a^4*b
- 4*a^2*b^3 + b^5)*c*f)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2
*a) - 2*((2*a^4*b + a^2*b^3 - b^5)*d*f*x - 12*((a^3*b^2 + a*b^4)*d*f*x + (a
^3*b^2 + a*b^4)*c*f)*cosh(d*x + c)^3 - (10*a^4*b + 11*a^2*b^3 + b^5)*d*e -
3*((10*a^4*b + 11*a^2*b^3 + b^5)*d*f*x - (2*a^4*b + a^2*b^3 - b^5)*d*e + (a
^4*b + 2*a^2*b^3 + b^5 + 12*(a^4*b + a^2*b^3)*c)*f)*cosh(d*x + c)^2 + (a^4*
b + 2*a^2*b^3 + b^5 + 12*(a^4*b + a^2*b^3)*c)*f - 2*(3*(2*a^5 + a^3*b^2 - a
*b^4)*d*f*x - 3*(2*a^5 + a^3*b^2 - a*b^4)*d*e + 2*(a^5 + 2*a^3*b^2 + a*b^4
+ 3*(2*a^5 + a^3*b^2 - a*b^4)*c)*f)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b^2
 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d^2*cosh(d*x + c)^4 + (a^6*b^2 + 3*a^4*b^4
+ 3*a^2*b^6 + b^8)*d^2*sinh(d*x + c)^4 + 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 +
 a*b^7)*d^2*cosh(d*x + c)^3 + 2*(2*a^8 + 5*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 -
b^8)*d^2*cosh(d*x + c)^2 - 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d^2*co
sh(d*x + c) + 4*((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d^2*cosh(d*x + c)
+ (a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d^2)*sinh(d*x + c)^3 + (a^6*b^2 +
 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d^2 + 2*(3*(a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 +
 b^8)*d^2*cosh(d*x + c)^2 + 6*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d^2*c
osh(d*x + c) + (2*a^8 + 5*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 - b^8)*d^2)*sinh(d*
x + c)^2 + 4*((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d^2*cosh(d*x + c)^3 +
 3*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d^2*cosh(d*x + c)^2 + (2*a^8 + 5
*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 - b^8)*d^2*cosh(d*x + c) - (a^7*b + 3*a^5*b^
3 + 3*a^3*b^5 + a*b^7)*d^2)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx+e}{(b\sinh(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(b*sinh(d*x + c) + a)^3, x)
```

3.179 $\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3} dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(e+fx)(a+b\sinh(c+dx))^3},x\right)$$

```
[Out] Unintegrable[1/((e + f*x)*(a + b*Sinh[c + d*x])^3), x]
```

Rubi [A] time = 0.0622738, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((e + f*x)*(a + b*Sinh[c + d*x])^3),x]
```

```
[Out] Defer[Int][1/((e + f*x)*(a + b*Sinh[c + d*x])^3), x]
```

Rubi steps

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3} dx = \int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3} dx$$

Mathematica [A] time = 104.981, size = 0, normalized size = 0.

$$\int \frac{1}{(e+fx)(a+b\sinh(c+dx))^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((e + f*x)*(a + b*Sinh[c + d*x])^3),x]
```

```
[Out] Integrate[1/((e + f*x)*(a + b*Sinh[c + d*x])^3), x]
```

Maple [A] time = 0.61, size = 0, normalized size = 0.

$$\int \frac{1}{(fx+e)(a+b\sinh(dx+c))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x)
```

```
[Out] int(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (3*a*b^2*d*f*x + 3*a*b^2*d*e + ((2*d*e + f)*a^2*b*e^(3*c) - (d*e - f)*b^3*e
^(3*c) + (2*a^2*b*d*f*e^(3*c) - b^3*d*f*e^(3*c))*x)*e^(3*d*x) + (2*(3*d*e +
 f)*a^3*e^(2*c) - (3*d*e - 2*f)*a*b^2*e^(2*c) + 3*(2*a^3*d*f*e^(2*c) - a*b^
2*d*f*e^(2*c))*x)*e^(2*d*x) - ((10*d*e + f)*a^2*b*e^c + (d*e + f)*b^3*e^c +
 (10*a^2*b*d*f*e^c + b^3*d*f*e^c)*x)*e^(d*x))/(a^4*b^2*d^2*e^2 + 2*a^2*b^4*
d^2*e^2 + b^6*d^2*e^2 + (a^4*b^2*d^2*f^2 + 2*a^2*b^4*d^2*f^2 + b^6*d^2*f^2)
*x^2 + 2*(a^4*b^2*d^2*e*f + 2*a^2*b^4*d^2*e*f + b^6*d^2*e*f)*x + (a^4*b^2*d
^2*e^2*e^(4*c) + 2*a^2*b^4*d^2*e^2*e^(4*c) + b^6*d^2*e^2*e^(4*c) + (a^4*b^2
*d^2*f^2*e^(4*c) + 2*a^2*b^4*d^2*f^2*e^(4*c) + b^6*d^2*f^2*e^(4*c))*x^2 + 2
*(a^4*b^2*d^2*e*f*e^(4*c) + 2*a^2*b^4*d^2*e*f*e^(4*c) + b^6*d^2*e*f*e^(4*c)
)*x)*e^(4*d*x) + 4*(a^5*b*d^2*e^2*e^(3*c) + 2*a^3*b^3*d^2*e^2*e^(3*c) + a*b
^5*d^2*e^2*e^(3*c) + (a^5*b*d^2*f^2*e^(3*c) + 2*a^3*b^3*d^2*f^2*e^(3*c) + a
*b^5*d^2*f^2*e^(3*c))*x^2 + 2*(a^5*b*d^2*e*f*e^(3*c) + 2*a^3*b^3*d^2*e*f*e^
(3*c) + a*b^5*d^2*e*f*e^(3*c))*x)*e^(3*d*x) + 2*(2*a^6*d^2*e^2*e^(2*c) + 3*
a^4*b^2*d^2*e^2*e^(2*c) - b^6*d^2*e^2*e^(2*c) + (2*a^6*d^2*f^2*e^(2*c) + 3*
a^4*b^2*d^2*f^2*e^(2*c) - b^6*d^2*f^2*e^(2*c))*x^2 + 2*(2*a^6*d^2*e*f*e^(2*
c) + 3*a^4*b^2*d^2*e*f*e^(2*c) - b^6*d^2*e*f*e^(2*c))*x)*e^(2*d*x) - 4*(a^5
*b*d^2*e^2*e^c + 2*a^3*b^3*d^2*e^2*e^c + a*b^5*d^2*e^2*e^c + (a^5*b*d^2*f^2
*e^c + 2*a^3*b^3*d^2*f^2*e^c + a*b^5*d^2*f^2*e^c)*x^2 + 2*(a^5*b*d^2*e*f*e^
c + 2*a^3*b^3*d^2*e*f*e^c + a*b^5*d^2*e*f*e^c)*x)*e^(d*x)) + integrate((3*a
*b*d*f^2*x + 3*a*b*d*e*f - ((2*d^2*e^2 + 3*d*e*f + 2*f^2)*a^2*e^c - (d^2*e^
2 - 2*f^2)*b^2*e^c + (2*a^2*d^2*f^2*e^c - b^2*d^2*f^2*e^c)*x^2 - (2*b^2*d^2
*e*f*e^c - (4*d^2*e*f + 3*d*f^2)*a^2*e^c)*x)*e^(d*x))/(a^4*b*d^2*e^3 + 2*a^
2*b^3*d^2*e^3 + b^5*d^2*e^3 + (a^4*b*d^2*f^3 + 2*a^2*b^3*d^2*f^3 + b^5*d^2*
f^3)*x^3 + 3*(a^4*b*d^2*e*f^2 + 2*a^2*b^3*d^2*e*f^2 + b^5*d^2*e*f^2)*x^2 +
3*(a^4*b*d^2*e^2*f + 2*a^2*b^3*d^2*e^2*f + b^5*d^2*e^2*f)*x - (a^4*b*d^2*e^
3*e^(2*c) + 2*a^2*b^3*d^2*e^3*e^(2*c) + b^5*d^2*e^3*e^(2*c) + (a^4*b*d^2*f^
3*e^(2*c) + 2*a^2*b^3*d^2*f^3*e^(2*c) + b^5*d^2*f^3*e^(2*c))*x^3 + 3*(a^4*b
*d^2*e*f^2*e^(2*c) + 2*a^2*b^3*d^2*e*f^2*e^(2*c) + b^5*d^2*e*f^2*e^(2*c))*x
^2 + 3*(a^4*b*d^2*e^2*f*e^(2*c) + 2*a^2*b^3*d^2*e^2*f*e^(2*c) + b^5*d^2*e^2
*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^5*d^2*e^3*e^c + 2*a^3*b^2*d^2*e^3*e^c + a*b
^4*d^2*e^3*e^c + (a^5*d^2*f^3*e^c + 2*a^3*b^2*d^2*f^3*e^c + a*b^4*d^2*f^3*e
^c)*x^3 + 3*(a^5*d^2*e*f^2*e^c + 2*a^3*b^2*d^2*e*f^2*e^c + a*b^4*d^2*e*f^2*
e^c)*x^2 + 3*(a^5*d^2*e^2*f*e^c + 2*a^3*b^2*d^2*e^2*f*e^c + a*b^4*d^2*e^2*f
*e^c)*x)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^3fx + a^3e + \left(b^3fx + b^3e\right)\sinh\left(dx+c\right)^3 + 3\left(ab^2fx + ab^2e\right)\sinh\left(dx+c\right)^2 + 3\left(a^2bfx + a^2be\right)\sinh\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(a^3*f*x + a^3*e + (b^3*f*x + b^3*e)*sinh(d*x + c)^3 + 3*(a*b^2*
f*x + a*b^2*e)*sinh(d*x + c)^2 + 3*(a^2*b*f*x + a^2*b*e)*sinh(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x+e)/(a+b*sinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(fx+e)(b\sinh(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((f*x + e)*(b*sinh(d*x + c) + a)^3), x)
```

3.180 $\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3},x\right)$$

```
[Out] Unintegrable[1/((e + f*x)^2*(a + b*Sinh[c + d*x])^3), x]
```

Rubi [A] time = 0.0611184, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((e + f*x)^2*(a + b*Sinh[c + d*x])^3),x]
```

```
[Out] Defer[Int][1/((e + f*x)^2*(a + b*Sinh[c + d*x])^3), x]
```

Rubi steps

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx = \int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx$$

Mathematica [A] time = 98.0677, size = 0, normalized size = 0.

$$\int \frac{1}{(e+fx)^2(a+b\sinh(c+dx))^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((e + f*x)^2*(a + b*Sinh[c + d*x])^3),x]
```

```
[Out] Integrate[1/((e + f*x)^2*(a + b*Sinh[c + d*x])^3), x]
```

Maple [A] time = 0.79, size = 0, normalized size = 0.

$$\int \frac{1}{\left(fx+e\right)^2\left(a+b\sinh\left(dx+c\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x)
```

```
[Out] int(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (3*a*b^2*d*f*x + 3*a*b^2*d*e + (2*(d*e + f)*a^2*b*e^(3*c) - (d*e - 2*f)*b^3
*e^(3*c) + (2*a^2*b*d*f*e^(3*c) - b^3*d*f*e^(3*c))*x)*e^(3*d*x) + (2*(3*d*e
 + 2*f)*a^3*e^(2*c) - (3*d*e - 4*f)*a*b^2*e^(2*c) + 3*(2*a^3*d*f*e^(2*c) -
a*b^2*d*f*e^(2*c))*x)*e^(2*d*x) - (2*(5*d*e + f)*a^2*b*e^c + (d*e + 2*f)*b^
3*e^c + (10*a^2*b*d*f*e^c + b^3*d*f*e^c)*x)*e^(d*x))/(a^4*b^2*d^2*e^3 + 2*a
^2*b^4*d^2*e^3 + b^6*d^2*e^3 + (a^4*b^2*d^2*f^3 + 2*a^2*b^4*d^2*f^3 + b^6*d
^2*f^3)*x^3 + 3*(a^4*b^2*d^2*e*f^2 + 2*a^2*b^4*d^2*e*f^2 + b^6*d^2*e*f^2)*x
^2 + 3*(a^4*b^2*d^2*e^2*f + 2*a^2*b^4*d^2*e^2*f + b^6*d^2*e^2*f)*x + (a^4*b
^2*d^2*e^3*e^(4*c) + 2*a^2*b^4*d^2*e^3*e^(4*c) + b^6*d^2*e^3*e^(4*c) + (a^4
*b^2*d^2*f^3*e^(4*c) + 2*a^2*b^4*d^2*f^3*e^(4*c) + b^6*d^2*f^3*e^(4*c))*x^3
 + 3*(a^4*b^2*d^2*e*f^2*e^(4*c) + 2*a^2*b^4*d^2*e*f^2*e^(4*c) + b^6*d^2*e*f
^2*e^(4*c))*x^2 + 3*(a^4*b^2*d^2*e^2*f*e^(4*c) + 2*a^2*b^4*d^2*e^2*f*e^(4*c
) + b^6*d^2*e^2*f*e^(4*c))*x)*e^(4*d*x) + 4*(a^5*b*d^2*e^3*e^(3*c) + 2*a^3*
b^3*d^2*e^3*e^(3*c) + a*b^5*d^2*e^3*e^(3*c) + (a^5*b*d^2*f^3*e^(3*c) + 2*a^
3*b^3*d^2*f^3*e^(3*c) + a*b^5*d^2*f^3*e^(3*c))*x^3 + 3*(a^5*b*d^2*e*f^2*e^(
3*c) + 2*a^3*b^3*d^2*e*f^2*e^(3*c) + a*b^5*d^2*e*f^2*e^(3*c))*x^2 + 3*(a^5*
b*d^2*e^2*f*e^(3*c) + 2*a^3*b^3*d^2*e^2*f*e^(3*c) + a*b^5*d^2*e^2*f*e^(3*c)
)*x)*e^(3*d*x) + 2*(2*a^6*d^2*e^3*e^(2*c) + 3*a^4*b^2*d^2*e^3*e^(2*c) - b^6
*d^2*e^3*e^(2*c) + (2*a^6*d^2*f^3*e^(2*c) + 3*a^4*b^2*d^2*f^3*e^(2*c) - b^6
*d^2*f^3*e^(2*c))*x^3 + 3*(2*a^6*d^2*e*f^2*e^(2*c) + 3*a^4*b^2*d^2*e*f^2*e^
(2*c) - b^6*d^2*e*f^2*e^(2*c))*x^2 + 3*(2*a^6*d^2*e^2*f*e^(2*c) + 3*a^4*b^2
*d^2*e^2*f*e^(2*c) - b^6*d^2*e^2*f*e^(2*c))*x)*e^(2*d*x) - 4*(a^5*b*d^2*e^3
*e^c + 2*a^3*b^3*d^2*e^3*e^c + a*b^5*d^2*e^3*e^c + (a^5*b*d^2*f^3*e^c + 2*a
^3*b^3*d^2*f^3*e^c + a*b^5*d^2*f^3*e^c)*x^3 + 3*(a^5*b*d^2*e*f^2*e^c + 2*a^
3*b^3*d^2*e*f^2*e^c + a*b^5*d^2*e*f^2*e^c)*x^2 + 3*(a^5*b*d^2*e^2*f*e^c + 2
*a^3*b^3*d^2*e^2*f*e^c + a*b^5*d^2*e^2*f*e^c)*x)*e^(d*x)) + integrate((6*a*
b*d*f^2*x + 6*a*b*d*e*f - (2*(d^2*e^2 + 3*d*e*f + 3*f^2)*a^2*e^c - (d^2*e^2
 - 6*f^2)*b^2*e^c + (2*a^2*d^2*f^2*e^c - b^2*d^2*f^2*e^c)*x^2 - 2*(b^2*d^2*
e*f*e^c - (2*d^2*e*f + 3*d*f^2)*a^2*e^c)*x)*e^(d*x))/(a^4*b*d^2*e^4 + 2*a^2
*b^3*d^2*e^4 + b^5*d^2*e^4 + (a^4*b*d^2*f^4 + 2*a^2*b^3*d^2*f^4 + b^5*d^2*f
^4)*x^4 + 4*(a^4*b*d^2*e*f^3 + 2*a^2*b^3*d^2*e*f^3 + b^5*d^2*e*f^3)*x^3 + 6
*(a^4*b*d^2*e^2*f^2 + 2*a^2*b^3*d^2*e^2*f^2 + b^5*d^2*e^2*f^2)*x^2 + 4*(a^4
*b*d^2*e^3*f + 2*a^2*b^3*d^2*e^3*f + b^5*d^2*e^3*f)*x - (a^4*b*d^2*e^4*e^(2
*c) + 2*a^2*b^3*d^2*e^4*e^(2*c) + b^5*d^2*e^4*e^(2*c) + (a^4*b*d^2*f^4*e^(2
*c) + 2*a^2*b^3*d^2*f^4*e^(2*c) + b^5*d^2*f^4*e^(2*c))*x^4 + 4*(a^4*b*d^2*e
*f^3*e^(2*c) + 2*a^2*b^3*d^2*e*f^3*e^(2*c) + b^5*d^2*e*f^3*e^(2*c))*x^3 + 6
*(a^4*b*d^2*e^2*f^2*e^(2*c) + 2*a^2*b^3*d^2*e^2*f^2*e^(2*c) + b^5*d^2*e^2*f
^2*e^(2*c))*x^2 + 4*(a^4*b*d^2*e^3*f*e^(2*c) + 2*a^2*b^3*d^2*e^3*f*e^(2*c)
+ b^5*d^2*e^3*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^5*d^2*e^4*e^c + 2*a^3*b^2*d^2*
e^4*e^c + a*b^4*d^2*e^4*e^c + (a^5*d^2*f^4*e^c + 2*a^3*b^2*d^2*f^4*e^c + a*
b^4*d^2*f^4*e^c)*x^4 + 4*(a^5*d^2*e*f^3*e^c + 2*a^3*b^2*d^2*e*f^3*e^c + a*b
^4*d^2*e*f^3*e^c)*x^3 + 6*(a^5*d^2*e^2*f^2*e^c + 2*a^3*b^2*d^2*e^2*f^2*e^c
+ a*b^4*d^2*e^2*f^2*e^c)*x^2 + 4*(a^5*d^2*e^3*f*e^c + 2*a^3*b^2*d^2*e^3*f*e
^c + a*b^4*d^2*e^3*f*e^c)*x)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^3f^2x^2 + 2\,a^3efx + a^3e^2 + \left(b^3f^2x^2 + 2\,b^3efx + b^3e^2\right)\sinh\left(dx+c\right)^3 + 3\left(ab^2f^2x^2 + 2\,ab^2efx + ab^2e^2\right)\sinh\left(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(a^3*f^2*x^2 + 2*a^3*e*f*x + a^3*e^2 + (b^3*f^2*x^2 + 2*b^3*e*f*
x + b^3*e^2)*sinh(d*x + c)^3 + 3*(a*b^2*f^2*x^2 + 2*a*b^2*e*f*x + a*b^2*e^2
)*sinh(d*x + c)^2 + 3*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x + a^2*b*e^2)*sinh(d*x
+ c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x+e)**2/(a+b*sinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(fx+e\right)^2 \left(b\sinh\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x+e)^2/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((f*x + e)^2*(b*sinh(d*x + c) + a)^3), x)
```

3.181 $\int (c+dx)^m (a+b\sinh(e+fx))^n \, dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left((c+dx)^m(a+b\sinh(e+fx))^n,x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*(a + b*Sinh[e + f*x])^n, x]
```

Rubi [A] time = 0.0546259, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m(a+b\sinh(e+fx))^n \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*(a + b*Sinh[e + f*x])^n,x]
```

```
[Out] Defer[Int][(c + d*x)^m*(a + b*Sinh[e + f*x])^n, x]
```

Rubi steps

$$\int (c+dx)^m(a+b\sinh(e+fx))^n \, dx = \int (c+dx)^m(a+b\sinh(e+fx))^n \, dx$$

Mathematica [A] time = 4.44785, size = 0, normalized size = 0.

$$\int (c+dx)^m(a+b\sinh(e+fx))^n \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(a + b*Sinh[e + f*x])^n,x]
```

```
[Out] Integrate[(c + d*x)^m*(a + b*Sinh[e + f*x])^n, x]
```

Maple [A] time = 0.042, size = 0, normalized size = 0.

$$\int (dx+c)^m \left(a+b\sinh\left(fx+e\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*sinh(f*x+e))^n,x)
```

```
[Out] int((d*x+c)^m*(a+b*sinh(f*x+e))^n,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \left(b\sinh\left(fx+e\right)+a\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*(b*sinh(f*x + e) + a)^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m\left(b\sinh\left(fx+e\right)+a\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*(b*sinh(f*x + e) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*sinh(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m\left(b\sinh\left(fx+e\right)+a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*(b*sinh(f*x + e) + a)^n, x)
```

3.182 $\int (c+dx)^m (a+b\sinh(e+fx))^3\,dx$

Optimal. Leaf size=543

$$\frac{3a^2be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}+\frac{3a^2be^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+$$

```
[Out] (a^3*(c + d*x)^(1 + m))/(d*(1 + m)) - (3*a*b^2*(c + d*x)^(1 + m))/(2*d*(1 +
 m)) + (3^(-1 - m)*b^3*E^(3*e - (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-3*f*(
c + d*x))/d])/(8*f*(-((f*(c + d*x))/d))^m) + (3*2^(-3 - m)*a*b^2*E^(2*e - (
2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c + d*x))/d])/(f*(-((f*(c + d*x))
/d))^m) + (3*a^2*b*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))
/d)])/(2*f*(-((f*(c + d*x))/d))^m) - (3*b^3*E^(e - (c*f)/d)*(c + d*x)^m*Gam
ma[1 + m, -((f*(c + d*x))/d)])/(8*f*(-((f*(c + d*x))/d))^m) + (3*a^2*b*E^(-
e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c + d*x))
/d)^m) - (3*b^3*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])
/(8*f*((f*(c + d*x))/d)^m) - (3*2^(-3 - m)*a*b^2*E^(-2*e + (2*c*f)/d)*(c +
d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) + (3^(-1 -
m)*b^3*E^(-3*e + (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (3*f*(c + d*x))/d])/(8
*f*((f*(c + d*x))/d)^m)
```

Rubi [A] time = 0.807379, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3317, 3308, 2181, 3312, 3307}

$$\frac{3a^2be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}+\frac{3a^2be^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + b*Sinh[e + f*x])^3,x]
```


```
[Out] (a^3*(c + d*x)^(1 + m))/(d*(1 + m)) - (3*a*b^2*(c + d*x)^(1 + m))/(2*d*(1 +
 m)) + (3^(-1 - m)*b^3*E^(3*e - (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-3*f*(
c + d*x))/d])/(8*f*(-((f*(c + d*x))/d))^m) + (3*2^(-3 - m)*a*b^2*E^(2*e - (
2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c + d*x))/d])/(f*(-((f*(c + d*x))
/d))^m) + (3*a^2*b*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))
/d)])/(2*f*(-((f*(c + d*x))/d))^m) - (3*b^3*E^(e - (c*f)/d)*(c + d*x)^m*Gam
ma[1 + m, -((f*(c + d*x))/d)])/(8*f*(-((f*(c + d*x))/d))^m) + (3*a^2*b*E^(-
e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c + d*x))
/d)^m) - (3*b^3*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])
/(8*f*((f*(c + d*x))/d)^m) - (3*2^(-3 - m)*a*b^2*E^(-2*e + (2*c*f)/d)*(c +
d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) + (3^(-1 -
m)*b^3*E^(-3*e + (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (3*f*(c + d*x))/d])/(8
*f*((f*(c + d*x))/d)^m)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\int (c+dx)^m(a+b\sinh(e+fx))^3\,dx = \int \left(a^3(c+dx)^m + 3a^2b(c+dx)^m\sinh(e+fx) + 3ab^2(c+dx)^m\sinh^2(e+fx) + \right.$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \left(3a^2b\right)\int (c+dx)^m\sinh(e+fx)\,dx + \left(3ab^2\right)\int (c+dx)^m\sinh$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \frac{1}{2}\left(3a^2b\right)\int e^{-i(ie+ifx)}(c+dx)^m\,dx - \frac{1}{2}\left(3a^2b\right)\int e^{i(ie+ifx)}(c+d$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} - \frac{3ab^2(c+dx)^{1+m}}{2d(1+m)} + \frac{3a^2be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\right.}{2f}$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} - \frac{3ab^2(c+dx)^{1+m}}{2d(1+m)} + \frac{3a^2be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\right.}{2f}$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)} - \frac{3ab^2(c+dx)^{1+m}}{2d(1+m)} + \frac{3^{-1-m}b^3e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1\right.}{8f}$$

Mathematica [A] time = 1.74719, size = 448, normalized size = 0.83

$$\frac{2^{-m-3}3^{-m-1}e^{-3\left(\frac{cf}{d}+e\right)}(c+dx)^m\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}\left(2^me^{\frac{3cf}{d}}\left(b^3d(m+1)e^{\frac{3cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^m\text{Gamma}\left(m+1,\frac{3f(c+dx)}{d}\right)+4ae^{3e}f3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + b*Sinh[e + f*x])^3,x]
```

```
[Out] (2^(-3 - m)*3^(-1 - m)*(c + d*x)^m*(2^m*b^3*d*E^(6*e)*(1 + m)*((f*(c + d*x)
)/d)^m*Gamma[1 + m, (-3*f*(c + d*x))/d] + 3^(2 + m)*a*b^2*d*E^(5*e + (c*f)/
d)*(1 + m)*(f*(c/d + x))^m*Gamma[1 + m, (-2*f*(c + d*x))/d] - 2^m*3^(2 + m)
```

```
*b*(-4*a^2 + b^2)*d*E^(4*e + (2*c*f)/d)*(1 + m)*((f*(c + d*x))/d)^m*Gamma[1
 + m, -((f*(c + d*x))/d)] - 2^m*3^(2 + m)*b*(-4*a^2 + b^2)*d*E^(2*e + (4*c*
f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (f*(c + d*x))/d] - 3^(2 +
 m)*a*b^2*d*E^(e + (5*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (
2*f*(c + d*x))/d] + 2^m*E^((3*c*f)/d)*(4*3^(1 + m)*a*(2*a^2 - 3*b^2)*E^(3*e
)*f*(c + d*x)*(-((f^2*(c + d*x)^2)/d^2))^m + b^3*d*E^((3*c*f)/d)*(1 + m)*(-
((f*(c + d*x))/d))^m*Gamma[1 + m, (3*f*(c + d*x))/d])))/(d*E^(3*(e + (c*f)/
d))*f*(1 + m)*(-((f^2*(c + d*x)^2)/d^2))^m)
```

Maple [F] time = 0.108, size = 0, normalized size = 0.

$$\int (dx+c)^m \left(a+b\sinh\left(fx+e\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*sinh(f*x+e))^3,x)

[Out] int((d*x+c)^m*(a+b*sinh(f*x+e))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 2.96735, size = 1885, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e))^3,x, algorithm="fricas")

[Out] 1/24*((b^3*d*m + b^3*d)*cosh((d*m*log(3*f/d) + 3*d*e - 3*c*f)/d)*gamma(m +
1, 3*(d*f*x + c*f)/d) - 9*(a*b^2*d*m + a*b^2*d)*cosh((d*m*log(2*f/d) + 2*d*
e - 2*c*f)/d)*gamma(m + 1, 2*(d*f*x + c*f)/d) + 9*((4*a^2*b - b^3)*d*m + (4
*a^2*b - b^3)*d)*cosh((d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f*x + c
*f)/d) + 9*((4*a^2*b - b^3)*d*m + (4*a^2*b - b^3)*d)*cosh((d*m*log(-f/d) -
d*e + c*f)/d)*gamma(m + 1, -(d*f*x + c*f)/d) + 9*(a*b^2*d*m + a*b^2*d)*cosh
((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d)*gamma(m + 1, -2*(d*f*x + c*f)/d) + (b
^3*d*m + b^3*d)*cosh((d*m*log(-3*f/d) - 3*d*e + 3*c*f)/d)*gamma(m + 1, -3*(
d*f*x + c*f)/d) - (b^3*d*m + b^3*d)*gamma(m + 1, 3*(d*f*x + c*f)/d)*sinh((d
*m*log(3*f/d) + 3*d*e - 3*c*f)/d) + 9*(a*b^2*d*m + a*b^2*d)*gamma(m + 1, 2*
(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d) - 9*((4*a^2*b - b
^3)*d*m + (4*a^2*b - b^3)*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh((d*m*log(f/
d) + d*e - c*f)/d) - 9*((4*a^2*b - b^3)*d*m + (4*a^2*b - b^3)*d)*gamma(m +
1, -(d*f*x + c*f)/d)*sinh((d*m*log(-f/d) - d*e + c*f)/d) - 9*(a*b^2*d*m + a
```

```
*b^2*d)*gamma(m + 1, -2*(d*f*x + c*f)/d)*sinh((d*m*log(-2*f/d) - 2*d*e + 2*
c*f)/d) - (b^3*d*m + b^3*d)*gamma(m + 1, -3*(d*f*x + c*f)/d)*sinh((d*m*log(
-3*f/d) - 3*d*e + 3*c*f)/d) + 12*((2*a^3 - 3*a*b^2)*d*f*x + (2*a^3 - 3*a*b^
2)*c*f)*cosh(m*log(d*x + c)) + 12*((2*a^3 - 3*a*b^2)*d*f*x + (2*a^3 - 3*a*b
^2)*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*sinh(f*x+e))**3,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)+a\right)^{3}(dx+c)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e) + a)^3*(d*x + c)^m, x)
```

3.183 $\int (c+dx)^m (a+b\sinh(e+fx))^2\,dx$

Optimal. Leaf size=281

$$\frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{f}+\frac{abe^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{f}+\frac{b^22}{}$$

```
[Out] (a^2*(c + d*x)^(1 + m))/(d*(1 + m)) - (b^2*(c + d*x)^(1 + m))/(2*d*(1 + m))
 + (2^(-3 - m)*b^2*E^(2*e - (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c +
d*x))/d])/(f*(-((f*(c + d*x))/d))^m) + (a*b*E^(e - (c*f)/d)*(c + d*x)^m*Gam
ma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) + (a*b*E^(-e + (c
*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) -
 (2^(-3 - m)*b^2*E^(-2*e + (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*
x))/d])/(f*((f*(c + d*x))/d)^m)
```

Rubi [A] time = 0.374911, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3317, 3308, 2181, 3312, 3307}

$$\frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{f}+\frac{abe^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{f}+\frac{b^22}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + b*Sinh[e + f*x])^2,x]
```


```
[Out] (a^2*(c + d*x)^(1 + m))/(d*(1 + m)) - (b^2*(c + d*x)^(1 + m))/(2*d*(1 + m))
 + (2^(-3 - m)*b^2*E^(2*e - (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c +
d*x))/d])/(f*(-((f*(c + d*x))/d))^m) + (a*b*E^(e - (c*f)/d)*(c + d*x)^m*Gam
ma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) + (a*b*E^(-e + (c
*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) -
 (2^(-3 - m)*b^2*E^(-2*e + (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*
x))/d])/(f*((f*(c + d*x))/d)^m)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\int (c+dx)^m(a+b\sinh(e+fx))^2\,dx = \int \left(a^2(c+dx)^m + 2ab(c+dx)^m\sinh(e+fx) + b^2(c+dx)^m\sinh^2(e+fx)\right)dx$$
$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + (2ab)\int (c+dx)^m\sinh(e+fx)\,dx + b^2\int (c+dx)^m\sinh^2(e+$$
$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + (ab)\int e^{-i(ie+ifx)}(c+dx)^m\,dx - (ab)\int e^{i(ie+ifx)}(c+dx)^m\,dx - b$$
$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} - \frac{b^2(c+dx)^{1+m}}{2d(1+m)} + \frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c}{}\right.}{f}$$
$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} - \frac{b^2(c+dx)^{1+m}}{2d(1+m)} + \frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c}{}\right.}{f}$$
$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} - \frac{b^2(c+dx)^{1+m}}{2d(1+m)} + \frac{2^{-3-m}b^2e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+\right.}{f}$$

Mathematica [A] time = 0.764251, size = 254, normalized size = 0.9

$$\frac{(c+dx)^m\left(8abd(m+1)e^{e-\frac{cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)+8abd(m+1)e^{\frac{cf}{d}-e}\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] ((c + d*x)^m*(8*a^2*f*(c + d*x) - 4*b^2*f*(c + d*x) + (b^2*d*E^(2*e - (2*c*
f)/d)*(1 + m)*Gamma[1 + m, (-2*f*(c + d*x))/d])/(2^m*(-((f*(c + d*x))/d))^m
) + (8*a*b*d*E^(e - (c*f)/d)*(1 + m)*Gamma[1 + m, -((f*(c + d*x))/d)])/(-((
f*(c + d*x))/d))^m + (8*a*b*d*E^(-e + (c*f)/d)*(1 + m)*Gamma[1 + m, (f*(c +
 d*x))/d])/((f*(c + d*x))/d)^m - (b^2*d*E^(-2*e + (2*c*f)/d)*(1 + m)*Gamma[
1 + m, (2*f*(c + d*x))/d])/(2^m*((f*(c + d*x))/d)^m)))/(8*d*f*(1 + m))
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int (dx+c)^m\left(a+b\sinh\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*sinh(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m*(a+b*sinh(f*x+e))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.49271, size = 1191, normalized size = 4.24

$$-\frac{\left(b^2dm + b^2d\right)\cosh\left(\frac{dm\log\left(\frac{2f}{d}\right)+2de-2cf}{d}\right)\Gamma\left(m+1,\frac{2\left(dfx+cf\right)}{d}\right) - 8\left(abdm+abd\right)\cosh\left(\frac{dm\log\left(\frac{f}{d}\right)+de-cf}{d}\right)\Gamma\left(m+1,\frac{dfx+cf}{d}\right) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/8*((b^2*d*m + b^2*d)*cosh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)*gamma(m +
1, 2*(d*f*x + c*f)/d) - 8*(a*b*d*m + a*b*d)*cosh((d*m*log(f/d) + d*e - c*f)
/d)*gamma(m + 1, (d*f*x + c*f)/d) - 8*(a*b*d*m + a*b*d)*cosh((d*m*log(-f/d)
 - d*e + c*f)/d)*gamma(m + 1, -(d*f*x + c*f)/d) - (b^2*d*m + b^2*d)*cosh((d
*m*log(-2*f/d) - 2*d*e + 2*c*f)/d)*gamma(m + 1, -2*(d*f*x + c*f)/d) - (b^2*
d*m + b^2*d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) + 2*d*e -
 2*c*f)/d) + 8*(a*b*d*m + a*b*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh((d*m*lo
g(f/d) + d*e - c*f)/d) + 8*(a*b*d*m + a*b*d)*gamma(m + 1, -(d*f*x + c*f)/d)
*sinh((d*m*log(-f/d) - d*e + c*f)/d) + (b^2*d*m + b^2*d)*gamma(m + 1, -2*(d
*f*x + c*f)/d)*sinh((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d) - 4*((2*a^2 - b^2)
*d*f*x + (2*a^2 - b^2)*c*f)*cosh(m*log(d*x + c)) - 4*((2*a^2 - b^2)*d*f*x +
 (2*a^2 - b^2)*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*sinh(f*x+e))**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)+a\right)^{2}(dx+c)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e) + a)^2*(d*x + c)^m, x)
```

3.184 $\int (c+dx)^m(a+b\sinh(e+fx))\,dx$

Optimal. Leaf size=131

$$\frac{be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}+\frac{be^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+\frac{a(c+}{d(m}$$

```
[Out] (a*(c + d*x)^(1 + m))/(d*(1 + m)) + (b*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1
+ m, -((f*(c + d*x))/d)])/(2*f*(-((f*(c + d*x))/d))^m) + (b*E^(-e + (c*f)/d
)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c + d*x))/d)^m)
```

Rubi [A] time = 0.148223, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3317, 3308, 2181}

$$\frac{be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}+\frac{be^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+\frac{a(c+}{d(m}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + b*Sinh[e + f*x]),x]
```



```
[Out] (a*(c + d*x)^(1 + m))/(d*(1 + m)) + (b*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1
+ m, -((f*(c + d*x))/d)])/(2*f*(-((f*(c + d*x))/d))^m) + (b*E^(-e + (c*f)/d
)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c + d*x))/d)^m)
```


Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int (c+dx)^m(a+b\sinh(e+fx))\,dx = \int \left(a(c+dx)^m + b(c+dx)^m\sinh(e+fx)\right)\,dx$$
$$= \frac{a(c+dx)^{1+m}}{d(1+m)} + b\int (c+dx)^m\sinh(e+fx)\,dx$$
$$= \frac{a(c+dx)^{1+m}}{d(1+m)} + \frac{1}{2}b\int e^{-i(ie+ifx)}(c+dx)^m\,dx - \frac{1}{2}b\int e^{i(ie+ifx)}(c+dx)^m\,dx$$
$$= \frac{a(c+dx)^{1+m}}{d(1+m)} + \frac{be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{2f} + \frac{be^{-e+\frac{cf}{d}}(c+ \ldots}{\ldots}$$

Mathematica [A] time = 0.191807, size = 118, normalized size = 0.9

$$\frac{1}{2}(c+dx)^m\left(\frac{be^{\frac{cf}{d}-e}\left(f\left(\frac{c}{d}+x\right)\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{f} + \frac{be^{e-\frac{cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{f} + \frac{2a(c+ \ldots}{d(m \ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + b*Sinh[e + f*x]),x]
```

```
[Out] ((c + d*x)^m*((2*a*(c + d*x))/(d*(1 + m)) + (b*E^(e - (c*f)/d)*Gamma[1 + m,
 -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) + (b*E^(-e + (c*f)/d)*Gamm
a[1 + m, (f*(c + d*x))/d])/(f*(f*(c/d + x))^m)))/2
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int (dx+c)^m\left(a+b\sinh\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*sinh(f*x+e)),x)
```

```
[Out] int((d*x+c)^m*(a+b*sinh(f*x+e)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.73051, size = 585, normalized size = 4.47

$$\frac{(bdm+bd)\cosh\left(\frac{dm\log\left(\frac{f}{d}\right)+de-cf}{d}\right)\Gamma\left(m+1,\frac{dfx+cf}{d}\right) + (bdm+bd)\cosh\left(\frac{dm\log\left(-\frac{f}{d}\right)-de+cf}{d}\right)\Gamma\left(m+1,-\frac{dfx+cf}{d}\right) - (bdm+ \ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*((b*d*m + b*d)*cosh((d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f*x +
 c*f)/d) + (b*d*m + b*d)*cosh((d*m*log(-f/d) - d*e + c*f)/d)*gamma(m + 1, -
(d*f*x + c*f)/d) - (b*d*m + b*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh((d*m*lo
g(f/d) + d*e - c*f)/d) - (b*d*m + b*d)*gamma(m + 1, -(d*f*x + c*f)/d)*sinh(
(d*m*log(-f/d) - d*e + c*f)/d) + 2*(a*d*f*x + a*c*f)*cosh(m*log(d*x + c)) +
 2*(a*d*f*x + a*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*sinh(f*x+e)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh (fx+e)+a)(dx+c)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e) + a)*(d*x + c)^m, x)
```

3.185 $\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(c+dx)^m}{a+b\sinh(e+fx)},x\right)$$

```
[Out] Unintegrable[(c + d*x)^m/(a + b*Sinh[e + f*x]), x]
```

Rubi [A] time = 0.0567216, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + b*Sinh[e + f*x]),x]
```

```
[Out] Defer[Int][(c + d*x)^m/(a + b*Sinh[e + f*x]), x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx = \int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx$$

Mathematica [A] time = 1.19354, size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{a+b\sinh(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + b*Sinh[e + f*x]),x]
```

```
[Out] Integrate[(c + d*x)^m/(a + b*Sinh[e + f*x]), x]
```

Maple [A] time = 0.03, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{a+b\sinh\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+b*sinh(f*x+e)),x)
```

```
[Out] int((d*x+c)^m/(a+b*sinh(f*x+e)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{b\sinh\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m/(b*sinh(f*x + e) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx+c)^m}{b\sinh\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m/(b*sinh(f*x + e) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{a+b\sinh\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+b*sinh(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**m/(a + b*sinh(e + f*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{b\sinh\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(b*sinh(f*x + e) + a), x)
```

3.186 $\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(c+dx)^m}{(a+b\sinh(e+fx))^2},x\right)$$

```
[Out] Unintegrable[(c + d*x)^m/(a + b*Sinh[e + f*x])^2, x]
```

Rubi [A] time = 0.055234, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m/(a + b*Sinh[e + f*x])^2, x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx=\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx$$

Mathematica [A] time = 5.30235, size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{(a+b\sinh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] Integrate[(c + d*x)^m/(a + b*Sinh[e + f*x])^2, x]
```

Maple [A] time = 0.068, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{\left(a+b\sinh\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+b*sinh(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m/(a+b*sinh(f*x+e))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{\left(b\sinh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m/(b*sinh(f*x + e) + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(dx+c)^m}{b^2\sinh\left(fx+e\right)^2+2\,ab\sinh\left(fx+e\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m/(b^2*sinh(f*x + e)^2 + 2*a*b*sinh(f*x + e) + a^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{\left(a+b\sinh\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+b*sinh(f*x+e))**2,x)
```

```
[Out] Integral((c + d*x)**m/(a + b*sinh(e + f*x))**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{\left(b\sinh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(b*sinh(f*x + e) + a)^2, x)
```

3.187 $\int \frac{(e+fx)^3 \sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=163

$$-\frac{12if^2(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{12if^3\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^4}-\frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}+\frac{i(e+fx)^3\tanh\left(\right.}{ad}$$

```
[Out] (I*(e + f*x)^3)/(a*d) - ((I/4)*(e + f*x)^4)/(a*f) - ((6*I)*f*(e + f*x)^2*Lo
g[1 + I*E^(c + d*x)])/(a*d^2) - ((12*I)*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c
+ d*x)])/(a*d^3) + ((12*I)*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^4) + (I*(
e + f*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rubi [A] time = 0.356154, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {5557, 32, 3318, 4184, 3716, 2190, 2531, 2282, 6589}

$$-\frac{12if^2(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{12if^3\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^4}-\frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2}+\frac{i(e+fx)^3\tanh\left(\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (I*(e + f*x)^3)/(a*d) - ((I/4)*(e + f*x)^4)/(a*f) - ((6*I)*f*(e + f*x)^2*Lo
g[1 + I*E^(c + d*x)])/(a*d^2) - ((12*I)*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c
+ d*x)])/(a*d^3) + ((12*I)*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^4) + (I*(
e + f*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rule 5557

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3 \sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = i\int \frac{(e+fx)^3}{a+ia\sinh(c+dx)}\,dx - \frac{i\int (e+fx)^3\,dx}{a}$$

$$= -\frac{i(e+fx)^4}{4af} + \frac{i\int (e+fx)^3 \csc^2\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{idx}{2}\right)dx}{2a}$$

$$= -\frac{i(e+fx)^4}{4af} + \frac{i(e+fx)^3 \tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(3if)\int (e+fx)^2 \coth\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)dx}{ad}$$

$$= \frac{i(e+fx)^3}{ad} - \frac{i(e+fx)^4}{4af} + \frac{i(e+fx)^3 \tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{(6f)\int \frac{e^{2\left(\frac{c}{2}+\frac{dx}{2}\right)}(e+fx)^2}{1+ie^{2\left(\frac{c}{2}+\frac{dx}{2}\right)}}dx}{ad}$$

$$= \frac{i(e+fx)^3}{ad} - \frac{i(e+fx)^4}{4af} - \frac{6if(e+fx)^2 \log\left(1+ie^{c+dx}\right)}{ad^2} + \frac{i(e+fx)^3 \tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

$$= \frac{i(e+fx)^3}{ad} - \frac{i(e+fx)^4}{4af} - \frac{6if(e+fx)^2 \log\left(1+ie^{c+dx}\right)}{ad^2} - \frac{12if^2(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{ad^3} +$$

$$= \frac{i(e+fx)^3}{ad} - \frac{i(e+fx)^4}{4af} - \frac{6if(e+fx)^2 \log\left(1+ie^{c+dx}\right)}{ad^2} - \frac{12if^2(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{ad^3} +$$

$$= \frac{i(e+fx)^3}{ad} - \frac{i(e+fx)^4}{4af} - \frac{6if(e+fx)^2 \log\left(1+ie^{c+dx}\right)}{ad^2} - \frac{12if^2(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{ad^3} +$$

Mathematica [A] time = 3.27641, size = 232, normalized size = 1.42

$$\frac{-\frac{8\left(6i(-e^c+i)f^2\left(d(e+fx)\text{PolyLog}\left(2,ie^{-c-dx}\right)+f\text{PolyLog}\left(3,ie^{-c-dx}\right)\right)+3(1+ie^c)d^2f(e+fx)^2\log\left(1-ie^{-c-dx}\right)+d^3(e+fx)^3\right)}{(e^c-i)d^4} + \frac{8i\sinh\left(\frac{dx}{2}\right)(e+}{d\left(\cosh\left(\frac{c}{2}\right)+i\sinh\left(\frac{c}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(c\right.\right.}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-I)*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) - (8*(d^3*(e + f*x)^3 +
 3*d^2*(1 + I*E^c)*f*(e + f*x)^2*Log[1 - I*E^(-c - d*x)] + (6*I)*(I - E^c)*
f^2*(d*(e + f*x)*PolyLog[2, I*E^(-c - d*x)] + f*PolyLog[3, I*E^(-c - d*x)])
))/(d^4*(-I + E^c)) + ((8*I)*(e + f*x)^3*Sinh[(d*x)/2])/(d*(Cosh[c/2] + I*S
inh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])))/(4*a)
```

Maple [B] time = 0.103, size = 501, normalized size = 3.1

$$\frac{12\,if^3\text{polylog}\left(3,-ie^{dx+c}\right)}{ad^4} - \frac{6\,if^3\ln\left(1+ie^{dx+c}\right)x^2}{ad^2} - \frac{12\,if^3\text{polylog}\left(2,-ie^{dx+c}\right)x}{ad^3} + \frac{6\,ief^2c^2}{ad^3} - 2\,\frac{x^3f^3+3\,ef^2x^2+3}{da\left(e^{dx+c}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 12*I*f^3*polylog(3,-I*exp(d*x+c))/a/d^4-6*I/d^2/a*f^3*ln(1+I*exp(d*x+c))*x^
2-12*I/d^3/a*f^3*polylog(2,-I*exp(d*x+c))*x+6*I/d^3/a*e*f^2*c^2-2*(f^3*x^3+
```

```
3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(exp(d*x+c)-I)-6*I/d^2/a*ln(exp(d*x+c)-I)*e^
2*f+12*I/d^3/a*e*f^2*c*ln(exp(d*x+c)-I)-4*I/d^4/a*f^3*c^3-12*I/d^3/a*e*f^2*
polylog(2,-I*exp(d*x+c))+6*I/d^4/a*f^3*c^2*ln(1+I*exp(d*x+c))+12*I/d^2/a*e*
f^2*c*x-6*I/d^3/a*f^3*c^2*x-1/4*I/a*x^4*f^3-12*I/d^3/a*e*f^2*c*ln(exp(d*x+c
))-I/a*e^3*x-6*I/d^4/a*f^3*c^2*ln(exp(d*x+c)-I)+2*I/d/a*f^3*x^3-12*I/d^3/a*
e*f^2*ln(1+I*exp(d*x+c))*c-I/a*e*f^2*x^3-12*I/d^2/a*e*f^2*ln(1+I*exp(d*x+c)
)*x+6*I/d^4/a*f^3*c^2*ln(exp(d*x+c))+6*I/d/a*e*f^2*x^2-3/2*I/a*e^2*f*x^2+6*
I/d^2/a*ln(exp(d*x+c))*e^2*f
```

Maxima [B] time = 1.7308, size = 428, normalized size = 2.63

$$\frac{3}{2}e^2f\left(\frac{-i\,dx^2+\left(dx^2e^c-4\,xe^c\right)e^{(dx)}}{i\,ade^{(dx+c)}+ad}-\frac{4i\,\log\left(\left(e^{(dx+c)}-i\right)e^{(-c)}\right)}{ad^2}\right)-e^3\left(\frac{i\,(dx+c)}{ad}+\frac{2}{\left(ae^{(-dx-c)}+i\,a\right)d}\right)-\frac{df^3x^4+24\,ef^2x^2+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 3/2*e^2*f*((-I*d*x^2 + (d*x^2*e^c - 4*x*e^c)*e^(d*x))/(I*a*d*e^(d*x + c) +
a*d) - 4*I*log((e^(d*x + c) - I)*e^(-c))/(a*d^2)) - e^3*(I*(d*x + c)/(a*d)
+ 2/((a*e^(-d*x - c) + I*a)*d)) - 1/4*(d*f^3*x^4 + 24*e*f^2*x^2 + 4*(d*e*f^
2 + 2*f^3)*x^3 + (I*d*f^3*x^4*e^c + 4*I*d*e*f^2*x^3*e^c)*e^(d*x))/(a*d*e^(d
*x + c) - I*a*d) - 12*I*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c))
)*e*f^2/(a*d^3) - 6*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d
*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a*d^4) + (2*I*d^3*f^3*x^3 + 6
*I*d^3*e*f^2*x^2)/(a*d^4)
```

Fricas [C] time = 2.48288, size = 1096, normalized size = 6.72

$$-\frac{d^4f^3x^4+4\,d^4ef^2x^3+6\,d^4e^2fx^2+4\,d^4e^3x+8\,d^3e^3-24\,cd^2e^2f+24\,c^2def^2-8\,c^3f^3+\left(48\,df^3x+48\,def^2-\left(-48i\,df^3x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(d^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*d^4*e^2*f*x^2 + 4*d^4*e^3*x + 8*d^3*e^3
 - 24*c*d^2*e^2*f + 24*c^2*d*e*f^2 - 8*c^3*f^3 + (48*d*f^3*x + 48*d*e*f^2 -
 (-48*I*d*f^3*x - 48*I*d*e*f^2)*e^(d*x + c))*dilog(-I*e^(d*x + c)) - (-I*d^
4*f^3*x^4 + 24*I*c*d^2*e^2*f - 24*I*c^2*d*e*f^2 + 8*I*c^3*f^3 + (-4*I*d^4*e
*f^2 + 8*I*d^3*f^3)*x^3 + (-6*I*d^4*e^2*f + 24*I*d^3*e*f^2)*x^2 + (-4*I*d^4
*e^3 + 24*I*d^3*e^2*f)*x)*e^(d*x + c) + (24*d^2*e^2*f - 48*c*d*e*f^2 + 24*c
^2*f^3 - (-24*I*d^2*e^2*f + 48*I*c*d*e*f^2 - 24*I*c^2*f^3)*e^(d*x + c))*log
(e^(d*x + c) - I) + (24*d^2*f^3*x^2 + 48*d^2*e*f^2*x + 48*c*d*e*f^2 - 24*c^
2*f^3 - (-24*I*d^2*f^3*x^2 - 48*I*d^2*e*f^2*x - 48*I*c*d*e*f^2 + 24*I*c^2*f
^3)*e^(d*x + c))*log(I*e^(d*x + c) + 1) - (48*I*f^3*e^(d*x + c) + 48*f^3)*p
olylog(3, -I*e^(d*x + c)))/(4*a*d^4*e^(d*x + c) - 4*I*a*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \sinh(dx+c)}{ia\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sinh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.188 $\int \frac{(e+fx)^2 \sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=130

$$-\frac{4if^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}-\frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}+\frac{i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}+\frac{i(e+fx)^2}{ad}-\frac{i(e+fx)^3}{3af}$$

```
[Out] (I*(e + f*x)^2)/(a*d) - ((I/3)*(e + f*x)^3)/(a*f) - ((4*I)*f*(e + f*x)*Log[
1 + I*E^(c + d*x)])/(a*d^2) - ((4*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d
^3) + (I*(e + f*x)^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rubi [A] time = 0.272064, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {5557, 32, 3318, 4184, 3716, 2190, 2279, 2391}

$$-\frac{4if^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}-\frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}+\frac{i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}+\frac{i(e+fx)^2}{ad}-\frac{i(e+fx)^3}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (I*(e + f*x)^2)/(a*d) - ((I/3)*(e + f*x)^3)/(a*f) - ((4*I)*f*(e + f*x)*Log[
1 + I*E^(c + d*x)])/(a*d^2) - ((4*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d
^3) + (I*(e + f*x)^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rule 5557

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e+fx)^2\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx &= i\int \frac{(e+fx)^2}{a+ia\sinh(c+dx)}\,dx - \frac{i\int (e+fx)^2\,dx}{a}\\
&= -\frac{i(e+fx)^3}{3af} + \frac{i\int (e+fx)^2\csc^2\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{idx}{2}\right)dx}{2a}\\
&= -\frac{i(e+fx)^3}{3af} + \frac{i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(2if)\int (e+fx)\coth\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)dx}{ad}\\
&= \frac{i(e+fx)^2}{ad} - \frac{i(e+fx)^3}{3af} + \frac{i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{(4f)\int \frac{e^{2\left(\frac{c}{2}+\frac{dx}{2}\right)}(e+fx)}{1+ie^{2\left(\frac{c}{2}+\frac{dx}{2}\right)}}\,dx}{ad}\\
&= \frac{i(e+fx)^2}{ad} - \frac{i(e+fx)^3}{3af} - \frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2} + \frac{i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}\\
&= \frac{i(e+fx)^2}{ad} - \frac{i(e+fx)^3}{3af} - \frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2} + \frac{i(e+fx)^2\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}\\
&= \frac{i(e+fx)^2}{ad} - \frac{i(e+fx)^3}{3af} - \frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2} - \frac{4if^2\mathrm{Li}_2\left(-ie^{c+dx}\right)}{ad^3} + \frac{i(e+fx)^2
\end{aligned}
$$

Mathematica [A] time = 2.45513, size = 188, normalized size = 1.45

$$
\frac{\frac{3\left(4(1+ie^c)f^2\mathrm{PolyLog}\left(2,ie^{-c-dx}\right)-2d(e+fx)\left(d(e+fx)+2(1+ie^c)f\log\left(1-ie^{-c-dx}\right)\right)\right)}{(e^c-i)d^3} + \frac{6i\sinh\left(\frac{dx}{2}\right)(e+fx)^2}{d\left(\cosh\left(\frac{c}{2}\right)+i\sinh\left(\frac{c}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)} - ix\left(3e^2}{3a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-I)*x*(3*e^2 + 3*e*f*x + f^2*x^2) + (3*(-2*d*(e + f*x)*(d*(e + f*x) + 2*(
1 + I*E^c)*f*Log[1 - I*E^(-c - d*x)]) + 4*(1 + I*E^c)*f^2*PolyLog[2, I*E^(-
c - d*x)]))/(d^3*(-I + E^c)) + ((6*I)*(e + f*x)^2*Sinh[(d*x)/2])/(d*(Cosh[c
/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])))/(3*a)
```

Maple [B] time = 0.069, size = 269, normalized size = 2.1

$$-\frac{\frac{i}{3}x^3f^2}{a}-\frac{iefx^2}{a}-\frac{ie^2x}{a}-2\frac{x^2f^2+2efx+e^2}{da\left(e^{dx+c}-i\right)}-\frac{4i\ln\left(e^{dx+c}-i\right)ef}{ad^2}+\frac{4i\ln\left(e^{dx+c}\right)ef}{ad^2}+\frac{2if^2x^2}{da}+\frac{4if^2cx}{ad^2}+\frac{2if^2c^2}{ad^3}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -1/3*I/a*x^3*f^2-I/a*e*f*x^2-I/a*e^2*x-2*(f^2*x^2+2*e*f*x+e^2)/d/a/(exp(d*x
+c)-I)-4*I/a/d^2*ln(exp(d*x+c)-I)*e*f+4*I/a/d^2*ln(exp(d*x+c))*e*f+2*I/a/d*
f^2*x^2+4*I/a/d^2*f^2*c*x+2*I/a/d^3*f^2*c^2-4*I/a/d^2*f^2*ln(1+I*exp(d*x+c)
)*x-4*I/a/d^3*f^2*ln(1+I*exp(d*x+c))*c-4*I*f^2*polylog(2,-I*exp(d*x+c))/a/d
^3+4*I/a/d^3*f^2*c*ln(exp(d*x+c)-I)-4*I/a/d^3*f^2*c*ln(exp(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}f^2\left(\frac{2idx^3e^{(dx+c)}+2dx^3+12x^2}{ade^{(dx+c)}-iad}-24\int\frac{x}{ade^{(dx+c)}-iad}\,dx\right)+ef\left(\frac{-idx^2+\left(dx^2e^c-4xe^c\right)e^{(dx)}}{iade^{(dx+c)}+ad}-\frac{4i\log\left(\left(e^{(dx+c)}-i\right)e^{\cdots}\right)}{ad^2}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*f^2*((2*I*d*x^3*e^(d*x + c) + 2*d*x^3 + 12*x^2)/(a*d*e^(d*x + c) - I*a
*d) - 24*integrate(x/(a*d*e^(d*x + c) - I*a*d), x)) + e*f*((-I*d*x^2 + (d*x
^2*e^c - 4*x*e^c)*e^(d*x))/(I*a*d*e^(d*x + c) + a*d) - 4*I*log((e^(d*x + c)
 - I)*e^(-c))/(a*d^2)) - e^2*(I*(d*x + c)/(a*d) + 2/((a*e^(-d*x - c) + I*a)
*d))
```

Fricas [B] time = 2.46449, size = 649, normalized size = 4.99

$$-\frac{d^3f^2x^3+3d^3efx^2+3d^3e^2x+6d^2e^2-12cdef+6c^2f^2-\left(-12if^2e^{(dx+c)}-12f^2\right)\mathrm{Li}_2\left(-ie^{(dx+c)}\right)-\left(-id^3f^2x^3+12icde\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d^3*e^2*x + 6*d^2*e^2 - 12*c*d*e*f + 6*c^
2*f^2 - (-12*I*f^2*e^(d*x + c) - 12*f^2)*dilog(-I*e^(d*x + c)) - (-I*d^3*f^
2*x^3 + 12*I*c*d*e*f - 6*I*c^2*f^2 + (-3*I*d^3*e*f + 6*I*d^2*f^2)*x^2 + (-3
*I*d^3*e^2 + 12*I*d^2*e*f)*x)*e^(d*x + c) + (12*d*e*f - 12*c*f^2 - (-12*I*d
```

```
*e*f + 12*I*c*f^2)*e^(d*x + c))*log(e^(d*x + c) - I) + (12*d*f^2*x + 12*c*f
^2 - (-12*I*d*f^2*x - 12*I*c*f^2)*e^(d*x + c))*log(I*e^(d*x + c) + 1))/(3*a
*d^3*e^(d*x + c) - 3*I*a*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \sinh(dx+c)}{i a \sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sinh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.189 $\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=90

$$-\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{ad^2}+\frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}-\frac{iex}{a}-\frac{ifx^2}{2a}$$

```
[Out] ((-I)*e*x)/a - ((I/2)*f*x^2)/a - ((2*I)*f*Log[Cosh[c/2 + (I/4)*Pi + (d*x)/2
]])/(a*d^2) + (I*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rubi [A] time = 0.112382, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5557, 3318, 4184, 3475}

$$-\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{ad^2}+\frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}-\frac{iex}{a}-\frac{ifx^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-I)*e*x)/a - ((I/2)*f*x^2)/a - ((2*I)*f*Log[Cosh[c/2 + (I/4)*Pi + (d*x)/2
]])/(a*d^2) + (I*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rule 5557

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx = i\int \frac{e+fx}{a+ia\sinh(c+dx)}\,dx - \frac{i\int (e+fx)\,dx}{a}$$
$$= -\frac{iex}{a} - \frac{ifx^2}{2a} + \frac{i\int (e+fx)\csc^2\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{idx}{2}\right)dx}{2a}$$
$$= -\frac{iex}{a} - \frac{ifx^2}{2a} + \frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(if)\int \coth\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right)dx}{ad}$$
$$= -\frac{iex}{a} - \frac{ifx^2}{2a} - \frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2} + \frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}$$

Mathematica [B] time = 0.637151, size = 239, normalized size = 2.66

$$\frac{-i\cosh\left(\frac{dx}{2}\right)\left(2f\log(\cosh(c+dx))+4if\tan^{-1}\left(\sinh\left(\frac{dx}{2}\right)\text{sech}\left(c+\frac{dx}{2}\right)\right)+d^2x(2e+fx)\right)+2d^2ex\sinh\left(c+\frac{dx}{2}\right)+d^2}{2ad^2\left(\cosh\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (-2*d*f*x*Cosh[c + (d*x)/2] - I*Cosh[(d*x)/2]*(d^2*x*(2*e + f*x) + (4*I)*f*
ArcTan[Sech[c + (d*x)/2]*Sinh[(d*x)/2]] + 2*f*Log[Cosh[c + d*x]]) + (4*I)*d
*e*Sinh[(d*x)/2] + (2*I)*d*f*x*Sinh[(d*x)/2] + 2*d^2*e*x*Sinh[c + (d*x)/2]
+ d^2*f*x^2*Sinh[c + (d*x)/2] + (4*I)*f*ArcTan[Sech[c + (d*x)/2]*Sinh[(d*x)
/2]]*Sinh[c + (d*x)/2] + 2*f*Log[Cosh[c + d*x]]*Sinh[c + (d*x)/2])/(2*a*d^2
*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]))
```

Maple [A] time = 0.079, size = 86, normalized size = 1.

$$\frac{-\frac{i}{2}fx^2}{a} - \frac{iex}{a} + \frac{2ifx}{da} + \frac{2ifc}{ad^2} - 2\frac{fx+e}{da\left(e^{dx+c}-i\right)} - \frac{2if\ln\left(e^{dx+c}-i\right)}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -1/2*I*f*x^2/a-I*e*x/a+2*I*f/a/d*x+2*I*f/a/d^2*c-2*(f*x+e)/d/a/(exp(d*x+c)-
I)-2*I*f/a/d^2*ln(exp(d*x+c)-I)
```

Maxima [A] time = 1.26702, size = 146, normalized size = 1.62

$$\frac{1}{2}f\left(\frac{-i\,dx^2+\left(dx^2e^c-4\,xe^c\right)e^{(dx)}}{i\,ade^{(dx+c)}+ad} - \frac{4i\log\left(\left(e^{(dx+c)}-i\right)e^{(-c)}\right)}{ad^2}\right) - e\left(\frac{i\,(dx+c)}{ad} + \frac{2}{\left(ae^{(-dx-c)}+i\,a\right)d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*f*((-I*d*x^2 + (d*x^2*e^c - 4*x*e^c)*e^(d*x))/(I*a*d*e^(d*x + c) + a*d)
 - 4*I*log((e^(d*x + c) - I)*e^(-c))/(a*d^2)) - e*(I*(d*x + c)/(a*d) + 2/((
a*e^(-d*x - c) + I*a)*d))
```

Fricas [A] time = 2.53768, size = 235, normalized size = 2.61

$$-\frac{d^2fx^2+2\,d^2ex+4\,de-\left(-i\,d^2fx^2+\left(-2i\,d^2e+4i\,df\right)x\right)e^{(dx+c)}-\left(-4i\,fe^{(dx+c)}-4\,f\right)\log\left(e^{(dx+c)}-i\right)}{2\,ad^2e^{(dx+c)}-2i\,ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(d^2*f*x^2 + 2*d^2*e*x + 4*d*e - (-I*d^2*f*x^2 + (-2*I*d^2*e + 4*I*d*f)*x)
*e^(d*x + c) - (-4*I*f*e^(d*x + c) - 4*f)*log(e^(d*x + c) - I))/(2*a*d^2*e^
(d*x + c) - 2*I*a*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21546, size = 180, normalized size = 2.

$$-\frac{i\,d^2fx^2e^{(dx+2\,c)}+d^2fx^2e^c+2i\,d^2xe^{(dx+2\,c+1)}-4i\,dfxe^{(dx+2\,c)}+2\,d^2xe^{(c+1)}+4i\,fe^{(dx+2\,c)}\log\left(e^{(dx+c)}-i\right)+4\,fe^c\log\left(e^{(dx+}\cdots\right.}{2\left(ad^2e^{(dx+2\,c)}-i\,ad^2e^c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(I*d^2*f*x^2*e^(d*x + 2*c) + d^2*f*x^2*e^c + 2*I*d^2*x*e^(d*x + 2*c +
1) - 4*I*d*f*x*e^(d*x + 2*c) + 2*d^2*x*e^(c + 1) + 4*I*f*e^(d*x + 2*c)*log(
e^(d*x + c) - I) + 4*f*e^c*log(e^(d*x + c) - I) + 4*d*e^(c + 1))/(a*d^2*e^(
d*x + 2*c) - I*a*d^2*e^c)
```

3.190 $\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=35

$$-\frac{\cosh(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{ix}{a}$$

```
[Out] ((-I)*x)/a - Cosh[c + d*x]/(d*(a + I*a*Sinh[c + d*x]))
```

Rubi [A] time = 0.0438023, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2735, 2648}

$$-\frac{\cosh(c+dx)}{d(a+ia\sinh(c+dx))}-\frac{ix}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-I)*x)/a - Cosh[c + d*x]/(d*(a + I*a*Sinh[c + d*x]))
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\frac{ix}{a}+i\int \frac{1}{a+ia\sinh(c+dx)}\,dx\\ &= -\frac{ix}{a}-\frac{\cosh(c+dx)}{d(a+ia\sinh(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.206753, size = 61, normalized size = 1.74

$$\frac{i\cosh(c+dx)\left(1-\frac{\sinh^{-1}(\sinh(c+dx))(\sinh(c+dx)-i)}{\sqrt{\cosh^2(c+dx)}}\right)}{ad(\sinh(c+dx)-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (I*Cosh[c + d*x]*(1 - (ArcSinh[Sinh[c + d*x]]*(-I + Sinh[c + d*x]))/Sqrt[Co
sh[c + d*x]^2]))/(a*d*(-I + Sinh[c + d*x]))
```

Maple [A] time = 0.026, size = 67, normalized size = 1.9

$$\frac{-i}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{2i}{da}\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}+\frac{i}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I/d/a*ln(tanh(1/2*d*x+1/2*c)+1)+2*I/d/a/(-I+tanh(1/2*d*x+1/2*c))+I/d/a*ln(
tanh(1/2*d*x+1/2*c)-1)
```

Maxima [A] time = 1.1521, size = 49, normalized size = 1.4

$$-\frac{i(dx+c)}{ad}-\frac{2}{\left(ae^{(-dx-c)}+i\,a\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -I*(d*x + c)/(a*d) - 2/((a*e^(-d*x - c) + I*a)*d)
```

Fricas [A] time = 2.32153, size = 78, normalized size = 2.23

$$\frac{-i\,dxe^{(dx+c)}-dx-2}{ade^{(dx+c)}-i\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (-I*d*x*e^(d*x + c) - d*x - 2)/(a*d*e^(d*x + c) - I*a*d)
```

Sympy [A] time = 0.352585, size = 24, normalized size = 0.69

$$-\frac{ix}{a}-\frac{2e^{-c}}{ad\left(e^{dx}-ie^{-c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I*x/a - 2*exp(-c)/(a*d*(exp(d*x) - I*exp(-c)))
```

Giac [A] time = 1.23488, size = 46, normalized size = 1.31

$$-\frac{i(dx+c)}{ad}-\frac{2i}{ad\left(i\,e^{(dx+c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -I*(d*x + c)/(a*d) - 2*I/(a*d*(I*e^(d*x + c) + 1))
```

3.191 $\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Sinh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0504119, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sinh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Sinh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Mathematica [A] time = 48.143, size = 0, normalized size = 0.

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Sinh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Maple [A] time = 0.091, size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)}{(fx+e)(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-2f\int \frac{1}{-i\,adf^2x^2-2i\,adefx-i\,ade^2+\left(adf^2x^2e^c+2\,adefxe^c+ade^2e^c\right)e^{(dx)}}\,dx-\frac{2}{-i\,adfx-i\,ade+\left(adfxe^c+adee^c\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*f*integrate(1/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2
*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) - 2/(-I*a*d*f*x - I*a*d*
e + (a*d*f*x*e^c + a*d*e*e^c)*e^(d*x)) - I*log(f*x + e)/(a*f)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(-i\,adfx-i\,ade+\left(adfx+ade\right)e^{(dx+c)}\right)\mathrm{integral}\left(-\frac{dfx+de-\left(-i\,dfx-i\,de\right)e^{(dx+c)}+2\,f}{-i\,adf^2x^2-2i\,adefx-i\,ade^2+\left(adf^2x^2+2\,adefx+ade^2\right)e^{(dx+c)}},x\right)-2}{-i\,adfx-i\,ade+\left(adfx+ade\right)e^{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))*integral(-(d*f*x +
d*e - (-I*d*f*x - I*d*e)*e^(d*x + c) + 2*f)/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x
 - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c)), x) - 2)/
(-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(dx+c\right)}{\left(fx+e\right)\left(i\,a\sinh\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

3.192 $\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Sinh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0511999, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sinh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Sinh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Mathematica [A] time = 41.715, size = 0, normalized size = 0.

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Sinh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Maple [A] time = 0.102, size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)}{(fx+e)^2(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-4f\int \frac{1}{-i\,adf^3x^3 - 3i\,adef^2x^2 - 3i\,ade^2fx - i\,ade^3 + \left(adf^3x^3e^c + 3\,adef^2x^2e^c + 3\,ade^2fxe^c + ade^3e^c\right)e^{(dx)}}\,dx + \frac{\ldots}{2\left(-i\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -4*f*integrate(1/(-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x - I*
a*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c + a*d*
e^3*e^c)*e^(d*x)), x) + 1/2*(2*d*f*x + 2*d*e + (2*I*d*f*x*e^c + 2*I*d*e*e^c
)*e^(d*x) - 4*f)/(-I*a*d*f^3*x^2 - 2*I*a*d*e*f^2*x - I*a*d*e^2*f + (a*d*f^3
*x^2*e^c + 2*a*d*e*f^2*x*e^c + a*d*e^2*f*e^c)*e^(d*x))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(-i\,adf^2x^2 - 2i\,adefx - i\,ade^2 + \left(adf^2x^2 + 2\,adefx + ade^2\right)e^{(dx+c)}\right)\text{integral}\left(-\frac{dfx+de-\left(-i\,dfx-i\,de\right)e^{(dx\ldots}}{-i\,adf^3x^3-3i\,adef^2x^2-3i\,ade^2fx-i\,ade^3+\left(adf^3x^3+3\ldots\right.}\right.}{-i\,adf^2x^2 - 2i\,adefx - i\,ade^2 + \left(adf^2x^2 + 2\,adefx + ade^2\right)e^{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x +
 a*d*e^2)*e^(d*x + c))*integral(-(d*f*x + d*e - (-I*d*f*x - I*d*e)*e^(d*x +
 c) + 4*f)/(-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x - I*a*d*e^
3 + (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3)*e^(d*x + c)),
 x) - 2)/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d
*e*f*x + a*d*e^2)*e^(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(dx+c\right)}{\left(fx+e\right)^2\left(i\,a\sinh\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")

[Out] integrate(sinh(d*x + c)/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

3.193 $\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=241

$$\frac{12f^2(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}-\frac{12f^3\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^4}-\frac{6if^2(e+fx)\cosh(c+dx)}{ad^3}+\frac{6f(e+fx)^2\log\left(1+ie\right.}{ad^2}$$

```
[Out] -((e + f*x)^3/(a*d)) + (e + f*x)^4/(4*a*f) - ((6*I)*f^2*(e + f*x)*Cosh[c +
d*x])/(a*d^3) - (I*(e + f*x)^3*Cosh[c + d*x])/(a*d) + (6*f*(e + f*x)^2*Log[
1 + I*E^(c + d*x)])/(a*d^2) + (12*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)
])/(a*d^3) - (12*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^4) + ((6*I)*f^3*Sin
h[c + d*x])/(a*d^4) + ((3*I)*f*(e + f*x)^2*Sinh[c + d*x])/(a*d^2) - ((e + f
*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rubi [A] time = 0.526142, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {5557, 3296, 2637, 32, 3318, 4184, 3716, 2190, 2531, 2282, 6589}

$$\frac{12f^2(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}-\frac{12f^3\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^4}-\frac{6if^2(e+fx)\cosh(c+dx)}{ad^3}+\frac{6f(e+fx)^2\log\left(1+ie\right.}{ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```



```
[Out] -((e + f*x)^3/(a*d)) + (e + f*x)^4/(4*a*f) - ((6*I)*f^2*(e + f*x)*Cosh[c +
d*x])/(a*d^3) - (I*(e + f*x)^3*Cosh[c + d*x])/(a*d) + (6*f*(e + f*x)^2*Log[
1 + I*E^(c + d*x)])/(a*d^2) + (12*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)
])/(a*d^3) - (12*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^4) + ((6*I)*f^3*Sin
h[c + d*x])/(a*d^4) + ((3*I)*f*(e + f*x)^2*Sinh[c + d*x])/(a*d^2) - ((e + f
*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```


Rule 5557

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = i\int \frac{(e+fx)^3\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx - \frac{i\int (e+fx)^3\sinh(c+dx)\,dx}{a}$$

$$= -\frac{i(e+fx)^3\cosh(c+dx)}{ad} + \frac{\int (e+fx)^3\,dx}{a} + \frac{(3if)\int (e+fx)^2\cosh(c+dx)\,dx}{ad} - \int \frac{}{a}$$

$$= \frac{(e+fx)^4}{4af} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + \frac{3if(e+fx)^2\sinh(c+dx)}{ad^2} - \frac{\int (e+fx)^3\csc^2\left(\right.}{}$$

$$= \frac{(e+fx)^4}{4af} - \frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + \frac{3if(e+fx)^2\sinh}{ad^2}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} - \frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + 6if$$

$$= -\frac{(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} - \frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + 6f($$

$$= -\frac{(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} - \frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + 6f($$

$$= -\frac{(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} - \frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + 6f($$

$$= -\frac{(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} - \frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + 6f($$

Mathematica [B] time = 6.5947, size = 857, normalized size = 3.56

$$\frac{if^3x^4\sinh\left(c+\frac{dx}{2}\right)d^4+4ief^2x^3\sinh\left(c+\frac{dx}{2}\right)d^4+6ie^2fx^2\sinh\left(c+\frac{dx}{2}\right)d^4+4ie^3x\sinh\left(c+\frac{dx}{2}\right)d^4-10e^3\sinh\left(\frac{dx}{2}\right)d^3-10f^3x^3\sinh\left(\frac{dx}{2}\right)d^3-30ef^2x^2\sinh\left(\frac{dx}{2}\right)d^3-30e}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (((-8*I)*(d^3*(e + f*x)^3 + 3*d^2*(1 + I*E^c)*f*(e + f*x)^2*Log[1 - I*E^(-c
 - d*x)] + (6*I)*(I - E^c)*f^2*(d*(e + f*x)*PolyLog[2, I*E^(-c - d*x)] + f*
PolyLog[3, I*E^(-c - d*x)]])))/(-I + E^c) + ((12*f^3 + 6*d^2*f*(e + f*x)^2 +
 d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))*Cosh[(d*x)/2] - (2*I)*d
*(e + f*x)*(6*f^2 + d^2*(e + f*x)^2)*Cosh[c + (d*x)/2] - (2*I)*d*(e + f*x)*
(6*f^2 + d^2*(e + f*x)^2)*Cosh[c + (3*d*x)/2] - 6*d^2*e^2*f*Cosh[2*c + (3*d
*x)/2] - 12*f^3*Cosh[2*c + (3*d*x)/2] - 12*d^2*e*f^2*x*Cosh[2*c + (3*d*x)/2
] - 6*d^2*f^3*x^2*Cosh[2*c + (3*d*x)/2] - 10*d^3*e^3*Sinh[(d*x)/2] - 12*d*e
*f^2*Sinh[(d*x)/2] - 30*d^3*e^2*f*x*Sinh[(d*x)/2] - 12*d*f^3*x*Sinh[(d*x)/2
] - 30*d^3*e*f^2*x^2*Sinh[(d*x)/2] - 10*d^3*f^3*x^3*Sinh[(d*x)/2] + (6*I)*d
^2*e^2*f*Sinh[c + (d*x)/2] + (12*I)*f^3*Sinh[c + (d*x)/2] + (4*I)*d^4*e^3*x
*Sinh[c + (d*x)/2] + (12*I)*d^2*e*f^2*x*Sinh[c + (d*x)/2] + (6*I)*d^4*e^2*f
*x^2*Sinh[c + (d*x)/2] + (6*I)*d^2*f^3*x^2*Sinh[c + (d*x)/2] + (4*I)*d^4*e*
f^2*x^3*Sinh[c + (d*x)/2] + I*d^4*f^3*x^4*Sinh[c + (d*x)/2] + (6*I)*d^2*e^2
*f*Sinh[c + (3*d*x)/2] + (12*I)*f^3*Sinh[c + (3*d*x)/2] + (12*I)*d^2*e*f^2*
x*Sinh[c + (3*d*x)/2] + (6*I)*d^2*f^3*x^2*Sinh[c + (3*d*x)/2] + 2*d^3*e^3*S
inh[2*c + (3*d*x)/2] + 12*d*e*f^2*Sinh[2*c + (3*d*x)/2] + 6*d^3*e^2*f*x*Sin
```

```
h[2*c + (3*d*x)/2] + 12*d*f^3*x*Sinh[2*c + (3*d*x)/2] + 6*d^3*e*f^2*x^2*Sin
h[2*c + (3*d*x)/2] + 2*d^3*f^3*x^3*Sinh[2*c + (3*d*x)/2])/((Cosh[c/2] + I*S
inh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])))/(4*a*d^4)
```

Maple [B] time = 0.183, size = 688, normalized size = 2.9

$$12\,\frac{ef^2\ln\left(1+i\mathrm{e}^{dx+c}\right)x}{ad^2}+12\,\frac{ef^2\ln\left(1+i\mathrm{e}^{dx+c}\right)c}{ad^3}+12\,\frac{ef^2c\ln\left(\mathrm{e}^{dx+c}\right)}{ad^3}-12\,\frac{ef^2c\ln\left(\mathrm{e}^{dx+c}-i\right)}{ad^3}-12\,\frac{ef^2cx}{ad^2}+4\,\frac{c^3f^3}{ad^4}-2\,\frac{x^3}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 12*f^2/d^2/a*e*ln(1+I*exp(d*x+c))*x+12*f^2/d^3/a*e*ln(1+I*exp(d*x+c))*c+12*
f^2/d^3/a*e*c*ln(exp(d*x+c))-12*f^2/d^3/a*e*c*ln(exp(d*x+c)-I)-12*f^2/d^2/a
*e*c*x+4*f^3/d^4/a*c^3-2*f^3/d/a*x^3-1/2*I*(d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d
^3*e^2*f*x-3*d^2*f^3*x^2+d^3*e^3-6*d^2*e*f^2*x-3*d^2*e^2*f+6*d*f^3*x+6*d*e*
f^2-6*f^3)/a/d^4*exp(d*x+c)-2*I*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(ex
p(d*x+c)-I)-12*f^3*polylog(3,-I*exp(d*x+c))/a/d^4+6*f^3/d^3/a*c^2*x-6*f/d^2
/a*ln(exp(d*x+c))*e^2+6*f^3/d^2/a*ln(1+I*exp(d*x+c))*x^2-6*f^3/d^4/a*ln(1+I
*exp(d*x+c))*c^2-6*f^2/d^3/a*e*c^2+6*f/d^2/a*ln(exp(d*x+c)-I)*e^2+6*f^3/d^4
/a*c^2*ln(exp(d*x+c)-I)-6*f^2/d/a*e*x^2-6*f^3/d^4/a*c^2*ln(exp(d*x+c))+12*f
^3/d^3/a*polylog(2,-I*exp(d*x+c))*x+12*f^2/d^3/a*e*polylog(2,-I*exp(d*x+c))
+1/a*e*f^2*x^3+3/2/a*e^2*f*x^2-1/2*I*(d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2
*f*x+3*d^2*f^3*x^2+d^3*e^3+6*d^2*e*f^2*x+3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2+6*
f^3)/a/d^4*exp(-d*x-c)+1/4/a*x^4*f^3+1/a*e^3*x
```

Maxima [B] time = 1.98274, size = 906, normalized size = 3.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -3/4*e^2*f*(4*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) - (-2*I*d^2*x^2*e^c -
 2*I*d*x*e^c - (2*I*d*x*e^(3*c) - 2*I*e^(3*c))*e^(2*d*x) + 2*(d^2*x^2*e^(2*
c) - 3*d*x*e^(2*c) + e^(2*c))*e^(d*x) - 2*(d*x + 1)*e^(-d*x) - 2*I*e^c)/(a*
d^2*e^(d*x + 2*c) - I*a*d^2*e^c) - 8*log((e^(d*x + c) - I)*e^(-c))/(a*d^2))
 + 1/4*e^3*(4*(d*x + c)/(a*d) + 2*(-5*I*e^(-d*x - c) + 1)/((I*a*e^(-d*x - c
) + a*e^(-2*d*x - 2*c))*d) - 2*I*e^(-d*x - c)/(a*d)) + 1/4*(-I*d^4*f^3*x^4
- 12*I*d*e*f^2 - (4*I*d^4*e*f^2 + 10*I*d^3*f^3)*x^3 - 12*I*f^3 - (30*I*d^3*
e*f^2 + 6*I*d^2*f^3)*x^2 - (12*I*d^2*e*f^2 + 12*I*d*f^3)*x - (2*I*d^3*f^3*x
^3*e^(2*c) + (6*I*d^3*e*f^2 - 6*I*d^2*f^3)*x^2*e^(2*c) + (-12*I*d^2*e*f^2 +
 12*I*d*f^3)*x*e^(2*c) + (12*I*d*e*f^2 - 12*I*f^3)*e^(2*c))*e^(2*d*x) + (d^
4*f^3*x^4*e^c + 2*(2*d^4*e*f^2 - d^3*f^3)*x^3*e^c - 6*(d^3*e*f^2 - d^2*f^3)
*x^2*e^c + 12*(d^2*e*f^2 - d*f^3)*x*e^c - 12*(d*e*f^2 - f^3)*e^c)*e^(d*x))/
(a*d^4*e^(d*x + c) - I*a*d^4) + 12*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e
^(d*x + c)))*e*f^2/(a*d^3) + 6*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilo
g(-I*e^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a*d^4) - 2*(d^3*f^3*
x^3 + 3*d^3*e*f^2*x^2)/(a*d^4)
```

Fricas [C] time = 2.6279, size = 1904, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -(2*d^3*f^3*x^3 + 2*d^3*e^3 + 6*d^2*e^2*f + 12*d*e*f^2 + 12*f^3 + 6*(d^3*e*
f^2 + d^2*f^3)*x^2 + 6*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*x - (48*(d*f^3*x
 + d*e*f^2)*e^(2*d*x + 2*c) + (-48*I*d*f^3*x - 48*I*d*e*f^2)*e^(d*x + c))*d
ilog(-I*e^(d*x + c)) - (-2*I*d^3*f^3*x^3 - 2*I*d^3*e^3 + 6*I*d^2*e^2*f - 12
*I*d*e*f^2 + 12*I*f^3 + (-6*I*d^3*e*f^2 + 6*I*d^2*f^3)*x^2 + (-6*I*d^3*e^2*
f + 12*I*d^2*e*f^2 - 12*I*d*f^3)*x)*e^(3*d*x + 3*c) - (d^4*f^3*x^4 - 2*d^3*
e^3 - 6*(4*c - 1)*d^2*e^2*f + 12*(2*c^2 - 1)*d*e*f^2 - 4*(2*c^3 - 3)*f^3 +
2*(2*d^4*e*f^2 - 5*d^3*f^3)*x^3 + 6*(d^4*e^2*f - 5*d^3*e*f^2 + d^2*f^3)*x^2
 + 2*(2*d^4*e^3 - 15*d^3*e^2*f + 6*d^2*e*f^2 - 6*d*f^3)*x)*e^(2*d*x + 2*c)
- (-I*d^4*f^3*x^4 - 10*I*d^3*e^3 + (24*I*c - 6*I)*d^2*e^2*f + (-24*I*c^2 -
12*I)*d*e*f^2 + (8*I*c^3 - 12*I)*f^3 + (-4*I*d^4*e*f^2 - 2*I*d^3*f^3)*x^3 +
 (-6*I*d^4*e^2*f - 6*I*d^3*e*f^2 - 6*I*d^2*f^3)*x^2 + (-4*I*d^4*e^3 - 6*I*d
^3*e^2*f - 12*I*d^2*e*f^2 - 12*I*d*f^3)*x)*e^(d*x + c) - (24*(d^2*e^2*f - 2
*c*d*e*f^2 + c^2*f^3)*e^(2*d*x + 2*c) + (-24*I*d^2*e^2*f + 48*I*c*d*e*f^2 -
 24*I*c^2*f^3)*e^(d*x + c))*log(e^(d*x + c) - I) - (24*(d^2*f^3*x^2 + 2*d^2
*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*e^(2*d*x + 2*c) + (-24*I*d^2*f^3*x^2 - 48
*I*d^2*e*f^2*x - 48*I*c*d*e*f^2 + 24*I*c^2*f^3)*e^(d*x + c))*log(I*e^(d*x +
 c) + 1) + 48*(f^3*e^(2*d*x + 2*c) - I*f^3*e^(d*x + c))*polylog(3, -I*e^(d*
x + c)))/(4*a*d^4*e^(2*d*x + 2*c) - 4*I*a*d^4*e^(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \sinh(dx+c)^2}{i\,a\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sinh(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

3.194 $\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=184

$$\frac{4f^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{4f(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}+\frac{2if(e+fx)\sinh(c+dx)}{ad^2}-\frac{2if^2\cosh(c+dx)}{ad^3}-\frac{i(e+fx)^2\cosh}{ad}$$

```
[Out] -((e + f*x)^2/(a*d)) + (e + f*x)^3/(3*a*f) - ((2*I)*f^2*Cosh[c + d*x])/(a*d
^3) - (I*(e + f*x)^2*Cosh[c + d*x])/(a*d) + (4*f*(e + f*x)*Log[1 + I*E^(c +
 d*x)])/(a*d^2) + (4*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^3) + ((2*I)*f*(
e + f*x)*Sinh[c + d*x])/(a*d^2) - ((e + f*x)^2*Tanh[c/2 + (I/4)*Pi + (d*x)/
2])/(a*d)
```

Rubi [A] time = 0.390758, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {5557, 3296, 2638, 32, 3318, 4184, 3716, 2190, 2279, 2391}

$$\frac{4f^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{4f(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}+\frac{2if(e+fx)\sinh(c+dx)}{ad^2}-\frac{2if^2\cosh(c+dx)}{ad^3}-\frac{i(e+fx)^2\cosh}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```


```
[Out] -((e + f*x)^2/(a*d)) + (e + f*x)^3/(3*a*f) - ((2*I)*f^2*Cosh[c + d*x])/(a*d
^3) - (I*(e + f*x)^2*Cosh[c + d*x])/(a*d) + (4*f*(e + f*x)*Log[1 + I*E^(c +
 d*x)])/(a*d^2) + (4*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^3) + ((2*I)*f*(
e + f*x)*Sinh[c + d*x])/(a*d^2) - ((e + f*x)^2*Tanh[c/2 + (I/4)*Pi + (d*x)/
2])/(a*d)
```


Rule 5557

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = i\int \frac{(e+fx)^2\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx - \frac{i\int (e+fx)^2\sinh(c+dx)\,dx}{a}$$

$$= -\frac{i(e+fx)^2\cosh(c+dx)}{ad} + \frac{\int (e+fx)^2\,dx}{a} + \frac{(2if)\int (e+fx)\cosh(c+dx)\,dx}{ad} - \int \frac{(}{a+ia}$$

$$= \frac{(e+fx)^3}{3af} - \frac{i(e+fx)^2\cosh(c+dx)}{ad} + \frac{2if(e+fx)\sinh(c+dx)}{ad^2} - \frac{\int (e+fx)^2\csc^2\left(\frac{1}{2}(ic\right.}{2a}$$

$$= \frac{(e+fx)^3}{3af} - \frac{2if^2\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2\cosh(c+dx)}{ad} + \frac{2if(e+fx)\sinh(c+dx)}{ad^2} - \frac{(}{}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} - \frac{2if^2\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2\cosh(c+dx)}{ad} + \frac{2if(e+fx)\sin}{ad^2}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} - \frac{2if^2\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2\cosh(c+dx)}{ad} + \frac{4f(e+fx)\log}{ad^2}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} - \frac{2if^2\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2\cosh(c+dx)}{ad} + \frac{4f(e+fx)\log}{ad^2}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} - \frac{2if^2\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2\cosh(c+dx)}{ad} + \frac{4f(e+fx)\log}{ad^2}$$

Mathematica [A] time = 3.4604, size = 260, normalized size = 1.41

$$\frac{\frac{6\left(\frac{d(e+fx)\left(2(e^c-i)f\log\left(1-ie^{-c-dx}\right)-id(e+fx)\right)}{e^c-i}-2f^2\text{PolyLog}\left(2,ie^{-c-dx}\right)\right)}{d^3} - \frac{3i\cosh(dx)\left(\cosh(c)\left(d^2(e+fx)^2+2f^2\right)-2df\sinh(c)(e+fx)\right)}{d^3} - \frac{3i\sinh(dx)\left(\sinh(c)\left(d^2(e+f\right.\right.}{}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2) + (6*((d*(e + f*x)*((-I)*d*(e + f*x) + 2*(-I
 + E^c)*f*Log[1 - I*E^(-c - d*x)]))/(-I + E^c) - 2*f^2*PolyLog[2, I*E^(-c -
 d*x)]))/d^3 - ((3*I)*Cosh[d*x]*((2*f^2 + d^2*(e + f*x)^2)*Cosh[c] - 2*d*f*
(e + f*x)*Sinh[c]))/d^3 - ((3*I)*(-2*d*f*(e + f*x)*Cosh[c] + (2*f^2 + d^2*(
e + f*x)^2)*Sinh[c])*Sinh[d*x])/d^3 - (6*(e + f*x)^2*Sinh[(d*x)/2])/(d*(Cos
h[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])))/(3*a)
```

Maple [B] time = 0.13, size = 374, normalized size = 2.

$$\frac{x^3f^2}{3a} + \frac{efx^2}{a} + \frac{e^2x}{a} - \frac{2i\left(x^2f^2+2efx+e^2\right)}{da\left(e^{dx+c}-i\right)} - \frac{\frac{i}{2}\left(f^2x^2d^2+2d^2efx+d^2e^2-2df^2x-2efd+2f^2\right)e^{dx+c}}{ad^3} - \frac{\frac{i}{2}\left(f^2x^2d^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 1/3/a*x^3*f^2+1/a*e*f*x^2+1/a*e^2*x-2*I*(f^2*x^2+2*e*f*x+e^2)/d/a/(exp(d*x+
c)-I)-1/2*I*(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2-2*d*f^2*x-2*d*e*f+2*f^2)/a/d^3
*exp(d*x+c)-1/2*I*(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2+2*d*f^2*x+2*d*e*f+2*f^2)
/a/d^3*exp(-d*x-c)+4*f/d^2/a*ln(exp(d*x+c)-I)*e-4*f/d^2/a*ln(exp(d*x+c))*e-
2*f^2*x^2/a/d-4*f^2/d^2/a*c*x-2*f^2/d^3/a*c^2+4*f^2/d^2/a*ln(1+I*exp(d*x+c)
)*x+4*f^2/d^3/a*ln(1+I*exp(d*x+c))*c+4*f^2*polylog(2,-I*exp(d*x+c))/a/d^3-4
*f^2/d^3/a*c*ln(exp(d*x+c)-I)+4*f^2/d^3/a*c*ln(exp(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,ef\left(\frac{4\,xe^{(dx+c)}}{ade^{(dx+c)}-i\,ad}-\frac{-2i\,d^2x^2e^c-2i\,dxe^c-\left(2i\,dxe^{(3\,c)}-2i\,e^{(3\,c)}\right)e^{(2\,dx)}+2\left(d^2x^2e^{(2\,c)}-3\,dxe^{(2\,c)}+e^{(2\,c)}\right)e^{(dx)}-2\,(dx}{ad^2e^{(dx+2\,c)}-i\,ad^2e^c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/2*e*f*(4*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) - (-2*I*d^2*x^2*e^c - 2
*I*d*x*e^c - (2*I*d*x*e^(3*c) - 2*I*e^(3*c))*e^(2*d*x) + 2*(d^2*x^2*e^(2*c)
 - 3*d*x*e^(2*c) + e^(2*c))*e^(d*x) - 2*(d*x + 1)*e^(-d*x) - 2*I*e^c)/(a*d^
2*e^(d*x + 2*c) - I*a*d^2*e^c) - 8*log((e^(d*x + c) - I)*e^(-c))/(a*d^2)) +
 1/12*f^2*((-4*I*d^3*x^3 - 30*I*d^2*x^2 - 12*I*d*x - (6*I*d^2*x^2*e^(2*c) -
 12*I*d*x*e^(2*c) + 12*I*e^(2*c))*e^(2*d*x) + 2*(2*d^3*x^3*e^c - 3*d^2*x^2*
e^c + 6*d*x*e^c - 6*e^c)*e^(d*x) - 12*I)/(a*d^3*e^(d*x + c) - I*a*d^3) + 48
*I*integrate(x/(a*d*e^(d*x + c) - I*a*d), x)) + 1/4*e^2*(4*(d*x + c)/(a*d)
+ 2*(-5*I*e^(-d*x - c) + 1)/((I*a*e^(-d*x - c) + a*e^(-2*d*x - 2*c))*d) - 2
*I*e^(-d*x - c)/(a*d))
```

Fricas [B] time = 2.72295, size = 1123, normalized size = 6.1

$$-\frac{3\,d^2f^2x^2+3\,d^2e^2+6\,def+6\,f^2+6\left(d^2ef+df^2\right)x-24\left(f^2e^{(2\,dx+2\,c)}-i\,f^2e^{(dx+c)}\right)\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)-\left(-3i\,d^2f^2x^2-3i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -(3*d^2*f^2*x^2 + 3*d^2*e^2 + 6*d*e*f + 6*f^2 + 6*(d^2*e*f + d*f^2)*x - 24*
(f^2*e^(2*d*x + 2*c) - I*f^2*e^(d*x + c))*dilog(-I*e^(d*x + c)) - (-3*I*d^2
*f^2*x^2 - 3*I*d^2*e^2 + 6*I*d*e*f - 6*I*f^2 + (-6*I*d^2*e*f + 6*I*d*f^2)*x
)*e^(3*d*x + 3*c) - (2*d^3*f^2*x^3 - 3*d^2*e^2 - 6*(4*c - 1)*d*e*f + 6*(2*c
^2 - 1)*f^2 + 3*(2*d^3*e*f - 5*d^2*f^2)*x^2 + 6*(d^3*e^2 - 5*d^2*e*f + d*f^
2)*x)*e^(2*d*x + 2*c) - (-2*I*d^3*f^2*x^3 - 15*I*d^2*e^2 + (24*I*c - 6*I)*d
*e*f + (-12*I*c^2 - 6*I)*f^2 + (-6*I*d^3*e*f - 3*I*d^2*f^2)*x^2 + (-6*I*d^3
*e^2 - 6*I*d^2*e*f - 6*I*d*f^2)*x)*e^(d*x + c) - (24*(d*e*f - c*f^2)*e^(2*d
*x + 2*c) + (-24*I*d*e*f + 24*I*c*f^2)*e^(d*x + c))*log(e^(d*x + c) - I) -
(24*(d*f^2*x + c*f^2)*e^(2*d*x + 2*c) + (-24*I*d*f^2*x - 24*I*c*f^2)*e^(d*x
 + c))*log(I*e^(d*x + c) + 1))/(6*a*d^3*e^(2*d*x + 2*c) - 6*I*a*d^3*e^(d*x
+ c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \sinh(dx+c)^2}{i\,a\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sinh(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

3.195 $\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=119

$$\frac{if\sinh(c+dx)}{ad^2}+\frac{2f\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{ad^2}-\frac{i(e+fx)\cosh(c+dx)}{ad}-\frac{(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}+\frac{ex}{a}+\frac{fx^2}{2a}$$

```
[Out] (e*x)/a + (f*x^2)/(2*a) - (I*(e + f*x)*Cosh[c + d*x])/(a*d) + (2*f*Log[Cosh
[c/2 + (I/4)*Pi + (d*x)/2]])/(a*d^2) + (I*f*Sinh[c + d*x])/(a*d^2) - ((e +
f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rubi [A] time = 0.177117, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {5557, 3296, 2637, 3318, 4184, 3475}

$$\frac{if\sinh(c+dx)}{ad^2}+\frac{2f\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{ad^2}-\frac{i(e+fx)\cosh(c+dx)}{ad}-\frac{(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)}{ad}+\frac{ex}{a}+\frac{fx^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (e*x)/a + (f*x^2)/(2*a) - (I*(e + f*x)*Cosh[c + d*x])/(a*d) + (2*f*Log[Cosh
[c/2 + (I/4)*Pi + (d*x)/2]])/(a*d^2) + (I*f*Sinh[c + d*x])/(a*d^2) - ((e +
f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rule 5557

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}dx = i\int \frac{(e+fx)\sinh(c+dx)}{a+ia\sinh(c+dx)}dx - \frac{i\int (e+fx)\sinh(c+dx)\,dx}{a}$$
$$= -\frac{i(e+fx)\cosh(c+dx)}{ad} + \frac{\int (e+fx)\,dx}{a} + \frac{(if)\int \cosh(c+dx)\,dx}{ad} - \int \frac{e+fx}{a+ia\sinh(c+dx}$$
$$= \frac{ex}{a} + \frac{fx^2}{2a} - \frac{i(e+fx)\cosh(c+dx)}{ad} + \frac{if\sinh(c+dx)}{ad^2} - \frac{\int (e+fx)\csc^2\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{idx}{2}\right)}{2a}$$
$$= \frac{ex}{a} + \frac{fx^2}{2a} - \frac{i(e+fx)\cosh(c+dx)}{ad} + \frac{if\sinh(c+dx)}{ad^2} - \frac{(e+fx)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{f}{}$$
$$= \frac{ex}{a} + \frac{fx^2}{2a} - \frac{i(e+fx)\cosh(c+dx)}{ad} + \frac{2f\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2} + \frac{if\sinh(c+dx)}{ad^2} - \frac{(e}{}$$

Mathematica [A] time = 1.08925, size = 238, normalized size = 2.

$$\left(\sinh\left(\frac{1}{2}(c+dx)\right) - i\cosh\left(\frac{1}{2}(c+dx)\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)\left(c^2(-f) - 2id(e+fx)\cosh(c+dx) + 2cde + 2if\sinh(c+dx)\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sinh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (((-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])*(Sinh[(c + d*x)/2]*(I*(2*I +
c + d*x)*(2*d*e - c*f + d*f*x) - 4*f*ArcTan[Tanh[(c + d*x)/2]] + 2*d*(e + f
*x)*Cosh[c + d*x] + (2*I)*f*Log[Cosh[c + d*x]] - 2*f*Sinh[c + d*x]) + Cosh[
(c + d*x)/2]*(2*c*d*e - (2*I)*c*f - c^2*f + 2*d^2*e*x - (2*I)*d*f*x + d^2*f
*x^2 + (4*I)*f*ArcTan[Tanh[(c + d*x)/2]] - (2*I)*d*(e + f*x)*Cosh[c + d*x]
+ 2*f*Log[Cosh[c + d*x]] + (2*I)*f*Sinh[c + d*x])))/(2*a*d^2*(-I + Sinh[c +
 d*x]))
```

Maple [A] time = 0.106, size = 134, normalized size = 1.1

$$\frac{fx^2}{2a} + \frac{ex}{a} - \frac{\frac{i}{2}\left(dfx+de-f\right)e^{dx+c}}{ad^2} - \frac{\frac{i}{2}\left(dfx+de+f\right)e^{-dx-c}}{ad^2} - 2\frac{fx}{da} - 2\frac{cf}{ad^2} - \frac{2i\left(fx+e\right)}{da\left(e^{dx+c}-i\right)} + 2\frac{f\ln\left(e^{dx+c}-i\right)}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 1/2*f*x^2/a+e*x/a-1/2*I*(d*f*x+d*e-f)/a/d^2*exp(d*x+c)-1/2*I*(d*f*x+d*e+f)/
a/d^2*exp(-d*x-c)-2*f*x/a/d-2*f/a/d^2*c-2*I*(f*x+e)/d/a/(exp(d*x+c)-I)+2*f/
```

```
a/d^2*ln(exp(d*x+c)-I)
```

Maxima [B] time = 1.3415, size = 325, normalized size = 2.73

$$-\frac{1}{4}f\left(\frac{4\,xe^{(dx+c)}}{ade^{(dx+c)}-i\,ad}-\frac{-2i\,d^2x^2e^c-2i\,dxe^c-\left(2i\,dxe^{(3\,c)}-2i\,e^{(3\,c)}\right)e^{(2\,dx)}+2\left(d^2x^2e^{(2\,c)}-3\,dxe^{(2\,c)}+e^{(2\,c)}\right)e^{(dx)}-2\,(dx}{ad^2e^{(dx+2\,c)}-i\,ad^2e^c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*f*(4*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) - (-2*I*d^2*x^2*e^c - 2*I
*d*x*e^c - (2*I*d*x*e^(3*c) - 2*I*e^(3*c))*e^(2*d*x) + 2*(d^2*x^2*e^(2*c) -
 3*d*x*e^(2*c) + e^(2*c))*e^(d*x) - 2*(d*x + 1)*e^(-d*x) - 2*I*e^c)/(a*d^2*
e^(d*x + 2*c) - I*a*d^2*e^c) - 8*log((e^(d*x + c) - I)*e^(-c))/(a*d^2)) + 1
/4*e*(4*(d*x + c)/(a*d) + 2*(-5*I*e^(-d*x - c) + 1)/((I*a*e^(-d*x - c) + a*
e^(-2*d*x - 2*c))*d) - 2*I*e^(-d*x - c)/(a*d))
```

Fricas [A] time = 2.66552, size = 416, normalized size = 3.5

$$-\frac{dfx+de-\left(-i\,dfx-i\,de+i\,f\right)e^{(3\,dx+3\,c)}-\left(d^2fx^2-de+\left(2\,d^2e-5\,df\right)x+f\right)e^{(2\,dx+2\,c)}-\left(-i\,d^2fx^2-5i\,de+\left(-2i\,d^2e\right.\right.}{2\,ad^2e^{(2\,dx+2\,c)}-2i\,ad^2e^{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(d*f*x + d*e - (-I*d*f*x - I*d*e + I*f)*e^(3*d*x + 3*c) - (d^2*f*x^2 - d*e
 + (2*d^2*e - 5*d*f)*x + f)*e^(2*d*x + 2*c) - (-I*d^2*f*x^2 - 5*I*d*e + (-2
*I*d^2*e - I*d*f)*x - I*f)*e^(d*x + c) - 4*(f*e^(2*d*x + 2*c) - I*f*e^(d*x
+ c))*log(e^(d*x + c) - I) + f)/(2*a*d^2*e^(2*d*x + 2*c) - 2*I*a*d^2*e^(d*x
 + c))
```

Sympy [A] time = 2.36023, size = 449, normalized size = 3.77

$$\begin{cases}\frac{\left(\left(-2ia^3d^5ee^c-2ia^3d^5fxe^c-2ia^3d^4fe^c\right)e^{-dx}+\left(-2ia^3d^5ee^{3c}-2ia^3d^5fxe^{3c}+2ia^3d^4fe^{3c}\right)e^{dx}\right)e^{-2c}}{4a^4d^6} & \text{for } 4a^4d^6e^{2c}\neq 0\\ \frac{x^2\left(ife^{6c}+4fe^{5c}-7ife^{4c}-8fe^{3c}+7ife^{2c}+4fe^c-if\right)}{-4ae^{5c}+16iae^{4c}+24ae^{3c}-16iae^{2c}-4ae^c}+\frac{x\left(iee^{8c}+6ee^{7c}-16iee^{6c}-26ee^{5c}+30iee^{4c}+26ee^{3c}-16iee^{2c}-6ee^c+ie\right)}{-2ae^{7c}+12iae^{6c}+30ae^{5c}-40iae^{4c}-30ae^{3c}+12iae^{2c}+2ae^c} & \text{otherwise}\end{cases}+\frac{fx^2}{2a}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise((((-2*I*a**3*d**5*e*exp(c) - 2*I*a**3*d**5*f*x*exp(c) - 2*I*a**3*
d**4*f*exp(c))*exp(-d*x) + (-2*I*a**3*d**5*e*exp(3*c) - 2*I*a**3*d**5*f*x*e
xp(3*c) + 2*I*a**3*d**4*f*exp(3*c))*exp(d*x))*exp(-2*c)/(4*a**4*d**6), Ne(4
*a**4*d**6*exp(2*c), 0)), (x**2*(I*f*exp(6*c) + 4*f*exp(5*c) - 7*I*f*exp(4*
c) - 8*f*exp(3*c) + 7*I*f*exp(2*c) + 4*f*exp(c) - I*f)/(-4*a*exp(5*c) + 16*
I*a*exp(4*c) + 24*a*exp(3*c) - 16*I*a*exp(2*c) - 4*a*exp(c)) + x*(I*e*exp(8
*c) + 6*e*exp(7*c) - 16*I*e*exp(6*c) - 26*e*exp(5*c) + 30*I*e*exp(4*c) + 26
*e*exp(3*c) - 16*I*e*exp(2*c) - 6*e*exp(c) + I*e)/(-2*a*exp(7*c) + 12*I*a*e
```

```
xp(6*c) + 30*a*exp(5*c) - 40*I*a*exp(4*c) - 30*a*exp(3*c) + 12*I*a*exp(2*c)
 + 2*a*exp(c)), True)) + f*x**2/(2*a) + x*(d*e - 2*f)/(a*d) - (2*I*e + 2*I*
f*x)*exp(-c)/(a*d*(exp(d*x) - I*exp(-c))) + 2*f*log(exp(d*x) - I*exp(-c))/(
a*d**2)
```

Giac [B] time = 1.52809, size = 367, normalized size = 3.08

$$d^2fx^2e^{(2\,dx+3\,c)} - i\,d^2fx^2e^{(dx+2\,c)} - i\,dfxe^{(3\,dx+4\,c)} + 2\,d^2xe^{(2\,dx+3\,c+1)} - 5\,dfxe^{(2\,dx+3\,c)} - 2i\,d^2xe^{(dx+2\,c+1)} - i\,dfxe^{(dx+2\,c)} - df$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(d^2*f*x^2*e^(2*d*x + 3*c) - I*d^2*f*x^2*e^(d*x + 2*c) - I*d*f*x*e^(3*d
*x + 4*c) + 2*d^2*x*e^(2*d*x + 3*c + 1) - 5*d*f*x*e^(2*d*x + 3*c) - 2*I*d^2
*x*e^(d*x + 2*c + 1) - I*d*f*x*e^(d*x + 2*c) - d*f*x*e^c + 4*f*e^(2*d*x + 3
*c)*log(e^(d*x + c) - I) - 4*I*f*e^(d*x + 2*c)*log(e^(d*x + c) - I) - I*d*e
^(3*d*x + 4*c + 1) + I*f*e^(3*d*x + 4*c) - d*e^(2*d*x + 3*c + 1) + f*e^(2*d
*x + 3*c) - 5*I*d*e^(d*x + 2*c + 1) - I*f*e^(d*x + 2*c) - d*e^(c + 1) - f*e
^c)/(a*d^2*e^(2*d*x + 3*c) - I*a*d^2*e^(d*x + 2*c))
```

3.196 $\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=52

$$-\frac{i\cosh(c+dx)}{ad}-\frac{i\cosh(c+dx)}{ad(1+i\sinh(c+dx))}+\frac{x}{a}$$

```
[Out] x/a - (I*Cosh[c + d*x])/(a*d) - (I*Cosh[c + d*x])/(a*d*(1 + I*Sinh[c + d*x]
))
```

Rubi [A] time = 0.08879, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2746, 12, 2735, 2648}

$$-\frac{i\cosh(c+dx)}{ad}-\frac{i\cosh(c+dx)}{ad(1+i\sinh(c+dx))}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] x/a - (I*Cosh[c + d*x])/(a*d) - (I*Cosh[c + d*x])/(a*d*(1 + I*Sinh[c + d*x]
))
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\cosh(c+dx)}{ad} + \frac{i\int \frac{a\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx}{a}$$
$$= -\frac{i\cosh(c+dx)}{ad} + i\int \frac{\sinh(c+dx)}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{x}{a} - \frac{i\cosh(c+dx)}{ad} - \int \frac{1}{a+ia\sinh(c+dx)}\,dx$$
$$= \frac{x}{a} - \frac{i\cosh(c+dx)}{ad} - \frac{i\cosh(c+dx)}{d(a+ia\sinh(c+dx))}$$

Mathematica [A] time = 0.220904, size = 59, normalized size = 1.13

$$\frac{\cosh(c+dx)\left(\frac{\sinh^{-1}(\sinh(c+dx))}{\sqrt{\cosh^2(c+dx)}} + \frac{-2-i\sinh(c+dx)}{\sinh(c+dx)-i}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (Cosh[c + d*x]*(ArcSinh[Sinh[c + d*x]]/Sqrt[Cosh[c + d*x]^2] + (-2 - I*Sinh
[c + d*x])/(-I + Sinh[c + d*x])))/(a*d)
```

Maple [B] time = 0.036, size = 107, normalized size = 2.1

$$-2\,\frac{1}{da\left(-i+\tanh\left(1/2\,dx+c/2\right)\right)} - \frac{i}{da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} + \frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) + \frac{i}{da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -2/d/a/(-I+tanh(1/2*d*x+1/2*c))-I/d/a/(tanh(1/2*d*x+1/2*c)+1)+1/d/a*ln(tanh
(1/2*d*x+1/2*c)+1)+I/d/a/(tanh(1/2*d*x+1/2*c)-1)-1/d/a*ln(tanh(1/2*d*x+1/2*
c)-1)
```

Maxima [A] time = 1.00892, size = 100, normalized size = 1.92

$$\frac{dx+c}{ad} + \frac{-5i\,e^{(-dx-c)}+1}{2\left(i\,ae^{(-dx-c)}+ae^{(-2\,dx-2\,c)}\right)d} - \frac{i\,e^{(-dx-c)}}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(a*d) + 1/2*(-5*I*e^(-d*x - c) + 1)/((I*a*e^(-d*x - c) + a*e^(-2*
d*x - 2*c))*d) - 1/2*I*e^(-d*x - c)/(a*d)
```

Fricas [A] time = 2.46326, size = 178, normalized size = 3.42

$$\frac{(2\,dx-1)e^{(2\,dx+2\,c)}+(-2i\,dx-5i)e^{(dx+c)}-i\,e^{(3\,dx+3\,c)}-1}{2\,ade^{(2\,dx+2\,c)}-2i\,ade^{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((2*d*x - 1)*e^(2*d*x + 2*c) + (-2*I*d*x - 5*I)*e^(d*x + c) - I*e^(3*d*x +
3*c) - 1)/(2*a*d*e^(2*d*x + 2*c) - 2*I*a*d*e^(d*x + c))
```

Sympy [A] time = 0.747901, size = 102, normalized size = 1.96

$$\begin{cases}\frac{\left(-2iade^{2c}e^{dx}-2iade^{-dx}\right)e^{-c}}{4a^2d^2} & \text{for } 4a^2d^2e^c\neq 0\\ x\left(-\frac{\left(ie^{2c}-2e^c-i\right)e^{-c}}{2a}-\frac{1}{a}\right) & \text{otherwise}\end{cases}+\frac{x}{a}-\frac{2ie^{-c}}{ad\left(e^{dx}-ie^{-c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise(((-2*I*a*d*exp(2*c)*exp(d*x) - 2*I*a*d*exp(-d*x))*exp(-c)/(4*a**2
*d**2), Ne(4*a**2*d**2*exp(c), 0)), (x*(-(I*exp(2*c) - 2*exp(c) - I)*exp(-c
)/(2*a) - 1/a), True)) + x/a - 2*I*exp(-c)/(a*d*(exp(d*x) - I*exp(-c)))
```

Giac [A] time = 1.41128, size = 89, normalized size = 1.71

$$\frac{dx+c}{ad}-\frac{i\,e^{(dx+c)}}{2\,ad}+\frac{\left(5\,e^{(dx+c)}-i\right)e^{(-dx-c)}}{2\,ad\left(i\,e^{(dx+c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (d*x + c)/(a*d) - 1/2*I*e^(d*x + c)/(a*d) + 1/2*(5*e^(d*x + c) - I)*e^(-d*x
 - c)/(a*d*(I*e^(d*x + c) + 1))
```

3.197 $\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))} \, dx$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Sinh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0778347, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Sinh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Sinh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))} \, dx = \int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))} \, dx$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.128, size = 0, normalized size = 0.

$$\int \frac{(\sinh(dx+c))^2}{(fx+e)(a+ia\sinh(dx+c))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-2if\int\frac{1}{-iadf^2x^2-2iadefx-iade^2+\left(adf^2x^2e^c+2adefxe^c+ade^2e^c\right)e^{(dx)}}\,dx-\frac{ie^{\left(-c+\frac{de}{f}\right)}E_1\left(\frac{(fx+e)d}{f}\right)}{2af}+\frac{ie^{\left(c-\frac{de}{f}\right)}E_1(}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*I*f*integrate(1/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x
^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) - 1/2*I*e^(-c + d*e/f)
*exp_integral_e(1, (f*x + e)*d/f)/(a*f) + 1/2*I*e^(c - d*e/f)*exp_integral_
e(1, -(f*x + e)*d/f)/(a*f) - 2*I/(-I*a*d*f*x - I*a*d*e + (a*d*f*x*e^c + a*d
*e*e^c)*e^(d*x)) + log(f*x + e)/(a*f)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(-iadfx-iade+\left(adfx+ade\right)e^{(dx+c)}\right)\mathrm{integral}\left(\frac{dfx+de+\left(-idfx-ide\right)e^{(3dx+3c)}+\left(dfx+de\right)e^{(2dx+2c)}+\left(-idfx-ide-4if\right)e^{(dx+c)}}{2\left(adf^2x^2+2adefx+ade^2\right)e^{(2dx+2c)}+\left(-2iadf^2x^2-4iadefx-2iade^2\right)e^{(dx+c)}},x\right)-2i}{-iadfx-iade+\left(adfx+ade\right)e^{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))*integral((d*f*x + d
*e + (-I*d*f*x - I*d*e)*e^(3*d*x + 3*c) + (d*f*x + d*e)*e^(2*d*x + 2*c) + (
-I*d*f*x - I*d*e - 4*I*f)*e^(d*x + c))/(2*(a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*
e^2)*e^(2*d*x + 2*c) + (-2*I*a*d*f^2*x^2 - 4*I*a*d*e*f*x - 2*I*a*d*e^2)*e^(
d*x + c)), x) - 2*I)/(-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh(dx+c)^2}{(fx+e)(ia\sinh(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^2/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

3.198 $\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} dx$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Sinh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0751888, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

```
[In] Int[Sinh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Sinh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} dx = \int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} dx$$

Mathematica [F] time = 180.002, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.218, size = 0, normalized size = 0.

$$\int \frac{(\sinh(dx+c))^2}{(fx+e)^2(a+ia\sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-4if\int \frac{1}{-iadf^3x^3 - 3iadef^2x^2 - 3iade^2fx - iade^3 + \left(adf^3x^3e^c + 3adef^2x^2e^c + 3ade^2fxe^c + ade^3e^c\right)e^{(dx)}}\,dx + \frac{\cdots}{4\left(-iad\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -4*I*f*integrate(1/(-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x -
I*a*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c + a*
d*e^3*e^c)*e^(d*x)), x) + 1/4*(4*I*d*f*x + 4*I*d*e - 4*(d*f*x*e^c + d*e*e^c
)*e^(d*x) - 8*I*f)/(-I*a*d*f^3*x^2 - 2*I*a*d*e*f^2*x - I*a*d*e^2*f + (a*d*f
^3*x^2*e^c + 2*a*d*e*f^2*x*e^c + a*d*e^2*f*e^c)*e^(d*x)) - 1/2*I*e^(-c + d*
e/f)*exp_integral_e(2, (f*x + e)*d/f)/((f*x + e)*a*f) + 1/2*I*e^(c - d*e/f)
*exp_integral_e(2, -(f*x + e)*d/f)/((f*x + e)*a*f)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(-iadf^2x^2 - 2iadefx - iade^2 + \left(adf^2x^2 + 2adefx + ade^2\right)e^{(dx+c)}\right)\mathrm{integral}\left(\frac{dfx+de+\left(-idfx-ide\right)e^{(3dx+3c)}+\left(dfx+de\right)e^{(2dx\cdots}}{2\left(adf^3x^3+3adef^2x^2+3ade^2fx+ade^3\right)e^{(2dx+2c)}+\left(-2iadf\cdots\right.}\right.\cdots}{-iadf^2x^2 - 2iadefx - iade^2 + \left(adf^2x^2 + 2adefx + ade^2\right)e^{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] ((-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x +
 a*d*e^2)*e^(d*x + c))*integral((d*f*x + d*e + (-I*d*f*x - I*d*e)*e^(3*d*x
+ 3*c) + (d*f*x + d*e)*e^(2*d*x + 2*c) + (-I*d*f*x - I*d*e - 8*I*f)*e^(d*x
+ c))/(2*(a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3)*e^(2*d*x
 + 2*c) + (-2*I*a*d*f^3*x^3 - 6*I*a*d*e*f^2*x^2 - 6*I*a*d*e^2*f*x - 2*I*a*d
*e^3)*e^(d*x + c)), x) - 2*I)/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 +
 (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh (dx + c)^2}{(fx + e)^2(i a \sinh (dx + c) + a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^2/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

3.199 $\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=393

$$\frac{12if^2(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}-\frac{12if^3\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^4}+\frac{6f^2(e+fx)\cosh(c+dx)}{ad^3}-\frac{3if^2(e+fx)\sinh(c+dx}{4ad^3}$$

```
[Out] (((3*I)/4)*e*f^2*x)/(a*d^2) + (((3*I)/8)*f^3*x^2)/(a*d^2) - (I*(e + f*x)^3)
/(a*d) + (((3*I)/8)*(e + f*x)^4)/(a*f) + (6*f^2*(e + f*x)*Cosh[c + d*x])/(a
*d^3) + ((e + f*x)^3*Cosh[c + d*x])/(a*d) + ((6*I)*f*(e + f*x)^2*Log[1 + I*
E^(c + d*x)])/(a*d^2) + ((12*I)*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])
/(a*d^3) - ((12*I)*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^4) - (6*f^3*Sinh[
c + d*x])/(a*d^4) - (3*f*(e + f*x)^2*Sinh[c + d*x])/(a*d^2) - (((3*I)/4)*f^
2*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(a*d^3) - ((I/2)*(e + f*x)^3*Cosh[
c + d*x]*Sinh[c + d*x])/(a*d) + (((3*I)/8)*f^3*Sinh[c + d*x]^2)/(a*d^4) + (
((3*I)/4)*f*(e + f*x)^2*Sinh[c + d*x]^2)/(a*d^2) - (I*(e + f*x)^3*Tanh[c/2
+ (I/4)*Pi + (d*x)/2])/(a*d)
```

Rubi [A] time = 0.702415, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.419, Rules used = {5557, 3311, 32, 3310, 3296, 2637, 3318, 4184, 3716, 2190, 2531, 2282, 6589}

$$\frac{12if^2(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}-\frac{12if^3\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^4}+\frac{6f^2(e+fx)\cosh(c+dx)}{ad^3}-\frac{3if^2(e+fx)\sinh(c+dx}{4ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (((3*I)/4)*e*f^2*x)/(a*d^2) + (((3*I)/8)*f^3*x^2)/(a*d^2) - (I*(e + f*x)^3)
/(a*d) + (((3*I)/8)*(e + f*x)^4)/(a*f) + (6*f^2*(e + f*x)*Cosh[c + d*x])/(a
*d^3) + ((e + f*x)^3*Cosh[c + d*x])/(a*d) + ((6*I)*f*(e + f*x)^2*Log[1 + I*
E^(c + d*x)])/(a*d^2) + ((12*I)*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])
/(a*d^3) - ((12*I)*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^4) - (6*f^3*Sinh[
c + d*x])/(a*d^4) - (3*f*(e + f*x)^2*Sinh[c + d*x])/(a*d^2) - (((3*I)/4)*f^
2*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(a*d^3) - ((I/2)*(e + f*x)^3*Cosh[
c + d*x]*Sinh[c + d*x])/(a*d) + (((3*I)/8)*f^3*Sinh[c + d*x]^2)/(a*d^4) + (
((3*I)/4)*f*(e + f*x)^2*Sinh[c + d*x]^2)/(a*d^2) - (I*(e + f*x)^3*Tanh[c/2
+ (I/4)*Pi + (d*x)/2])/(a*d)
```

Rule 5557

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
```

```
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = i\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx - \frac{i\int (e+fx)^3\sinh^2(c+dx)\,dx}{a}$$

$$= -\frac{i(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{2ad} + \frac{3if(e+fx)^2\sinh^2(c+dx)}{4ad^2} + \frac{i\int (e+fx)^3\,dx}{2a} + \cdots$$

$$= \frac{i(e+fx)^4}{8af} + \frac{(e+fx)^3\cosh(c+dx)}{ad} - \frac{3if^2(e+fx)\cosh(c+dx)\sinh(c+dx)}{4ad^3} - \frac{i(e+fx\cdots}{\cdots}$$

$$= \frac{3ief^2x}{4ad^2} + \frac{3if^3x^2}{8ad^2} + \frac{3i(e+fx)^4}{8af} + \frac{(e+fx)^3\cosh(c+dx)}{ad} - \frac{3f(e+fx)^2\sinh(c+dx)}{ad^2} - \cdots$$

$$= \frac{3ief^2x}{4ad^2} + \frac{3if^3x^2}{8ad^2} + \frac{3i(e+fx)^4}{8af} + \frac{6f^2(e+fx)\cosh(c+dx)}{ad^3} + \frac{(e+fx)^3\cosh(c+dx)}{ad} - \cdots$$

$$= \frac{3ief^2x}{4ad^2} + \frac{3if^3x^2}{8ad^2} - \frac{i(e+fx)^3}{ad} + \frac{3i(e+fx)^4}{8af} + \frac{6f^2(e+fx)\cosh(c+dx)}{ad^3} + \frac{(e+fx)^3\cos\cdots}{ad\cdots}$$

$$= \frac{3ief^2x}{4ad^2} + \frac{3if^3x^2}{8ad^2} - \frac{i(e+fx)^3}{ad} + \frac{3i(e+fx)^4}{8af} + \frac{6f^2(e+fx)\cosh(c+dx)}{ad^3} + \frac{(e+fx)^3\cos\cdots}{ad\cdots}$$

$$= \frac{3ief^2x}{4ad^2} + \frac{3if^3x^2}{8ad^2} - \frac{i(e+fx)^3}{ad} + \frac{3i(e+fx)^4}{8af} + \frac{6f^2(e+fx)\cosh(c+dx)}{ad^3} + \frac{(e+fx)^3\cos\cdots}{ad\cdots}$$

$$= \frac{3ief^2x}{4ad^2} + \frac{3if^3x^2}{8ad^2} - \frac{i(e+fx)^3}{ad} + \frac{3i(e+fx)^4}{8af} + \frac{6f^2(e+fx)\cosh(c+dx)}{ad^3} + \frac{(e+fx)^3\cos\cdots}{ad\cdots}$$

$$= \frac{3ief^2x}{4ad^2} + \frac{3if^3x^2}{8ad^2} - \frac{i(e+fx)^3}{ad} + \frac{3i(e+fx)^4}{8af} + \frac{6f^2(e+fx)\cosh(c+dx)}{ad^3} + \frac{(e+fx)^3\cos\cdots}{ad\cdots}$$

Mathematica [A] time = 7.31088, size = 376, normalized size = 0.96

$$-\frac{192if^2\left(d(e+fx)\text{PolyLog}\left(2,ie^{-c-dx}\right)+f\text{PolyLog}\left(3,ie^{-c-dx}\right)\right)}{d^4}-\frac{6if^2(e+fx)\sinh(2(c+dx))}{d^3}+\frac{96f^2(e+fx)\cosh(c+dx)}{d^3}+\frac{96if(e+fx)^2\log\left(1-ie^{-c-dx}\right)}{d^2}-$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((24*I)*e^3*x + (36*I)*e^2*f*x^2 + (24*I)*e*f^2*x^3 + (6*I)*f^3*x^4 + (32*(
e + f*x)^3)/(d*(-I + E^c)) + (96*f^2*(e + f*x)*Cosh[c + d*x])/d^3 + (16*(e
+ f*x)^3*Cosh[c + d*x])/d + ((3*I)*f^3*Cosh[2*(c + d*x)])/d^4 + ((6*I)*f*(e
 + f*x)^2*Cosh[2*(c + d*x)])/d^2 + ((96*I)*f*(e + f*x)^2*Log[1 - I*E^(-c -
d*x)])/d^2 - ((192*I)*f^2*(d*(e + f*x)*PolyLog[2, I*E^(-c - d*x)] + f*PolyL
og[3, I*E^(-c - d*x)]))/d^4 - ((32*I)*(e + f*x)^3*Sinh[(d*x)/2])/(d*(Cosh[c
/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])) - (96*f^3*Sin
h[c + d*x])/d^4 - (48*f*(e + f*x)^2*Sinh[c + d*x])/d^2 - ((6*I)*f^2*(e + f*
x)*Sinh[2*(c + d*x)])/d^3 - ((4*I)*(e + f*x)^3*Sinh[2*(c + d*x)])/d)/(16*a)
```

Maple [B] time = 0.155, size = 928, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 2*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(exp(d*x+c)-I)+6*I/d^2/a*f^3*ln(1
+I*exp(d*x+c))*x^2-6*I/d^2/a*ln(exp(d*x+c))*e^2*f-6*I/d/a*e*f^2*x^2+6*I/d^2
/a*ln(exp(d*x+c)-I)*e^2*f+6*I/d^3/a*f^3*c^2*x+6*I/d^4/a*f^3*c^2*ln(exp(d*x+
c)-I)+12*I/d^3/a*e*f^2*polylog(2,-I*exp(d*x+c))-6*I/d^4/a*f^3*c^2*ln(exp(d*
x+c))-6*I/d^3/a*e*f^2*c^2+12*I/d^3/a*f^3*polylog(2,-I*exp(d*x+c))*x-6*I/d^4
/a*f^3*c^2*ln(1+I*exp(d*x+c))-12*I/d^2/a*e*f^2*c*x+12*I/d^3/a*e*f^2*ln(1+I*
exp(d*x+c))*c+12*I/d^2/a*e*f^2*ln(1+I*exp(d*x+c))*x+12*I/d^3/a*e*f^2*c*ln(e
xp(d*x+c))-12*I*f^3*polylog(3,-I*exp(d*x+c))/a/d^4-2*I/d/a*f^3*x^3+4*I/d^4/
a*f^3*c^3-1/32*I*(4*d^3*f^3*x^3+12*d^3*e*f^2*x^2+12*d^3*e^2*f*x-6*d^2*f^3*x
^2+4*d^3*e^3-12*d^2*e*f^2*x-6*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2-3*f^3)/a/d^4*ex
p(2*d*x+2*c)-12*I/d^3/a*e*f^2*c*ln(exp(d*x+c)-I)+3/8*I/a*x^4*f^3+3/2*I/a*e^
3*x+1/32*I*(4*d^3*f^3*x^3+12*d^3*e*f^2*x^2+12*d^3*e^2*f*x+6*d^2*f^3*x^2+4*d
^3*e^3+12*d^2*e*f^2*x+6*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2+3*f^3)/a/d^4*exp(-2*d
*x-2*c)+1/2*(d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x+3*d^2*f^3*x^2+d^3*e^
3+6*d^2*e*f^2*x+3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2+6*f^3)/a/d^4*exp(-d*x-c)+1/
2*(d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x-3*d^2*f^3*x^2+d^3*e^3-6*d^2*e*
f^2*x-3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2-6*f^3)/a/d^4*exp(d*x+c)+3/2*I/a*e*f^2
*x^3+9/4*I/a*e^2*f*x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [C] time = 2.78172, size = 2449, normalized size = 6.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] (4*d^3*f^3*x^3 + 4*d^3*e^3 + 6*d^2*e^2*f + 6*d*e*f^2 + 3*f^3 + 6*(2*d^3*e*f
^2 + d^2*f^3)*x^2 + 6*(2*d^3*e^2*f + 2*d^2*e*f^2 + d*f^3)*x + ((384*I*d*f^3
*x + 384*I*d*e*f^2)*e^(3*d*x + 3*c) + 384*(d*f^3*x + d*e*f^2)*e^(2*d*x + 2*
c))*dilog(-I*e^(d*x + c)) + (-4*I*d^3*f^3*x^3 - 4*I*d^3*e^3 + 6*I*d^2*e^2*f
 - 6*I*d*e*f^2 + 3*I*f^3 + (-12*I*d^3*e*f^2 + 6*I*d^2*f^3)*x^2 + (-12*I*d^3
*e^2*f + 12*I*d^2*e*f^2 - 6*I*d*f^3)*x)*e^(5*d*x + 5*c) + 3*(4*d^3*f^3*x^3
+ 4*d^3*e^3 - 14*d^2*e^2*f + 30*d*e*f^2 - 31*f^3 + 2*(6*d^3*e*f^2 - 7*d^2*f
^3)*x^2 + 2*(6*d^3*e^2*f - 14*d^2*e*f^2 + 15*d*f^3)*x)*e^(4*d*x + 4*c) + (1
2*I*d^4*f^3*x^4 - 16*I*d^3*e^3 + (-192*I*c + 48*I)*d^2*e^2*f + (192*I*c^2 -
 96*I)*d*e*f^2 + (-64*I*c^3 + 96*I)*f^3 + (48*I*d^4*e*f^2 - 80*I*d^3*f^3)*x
^3 + (72*I*d^4*e^2*f - 240*I*d^3*e*f^2 + 48*I*d^2*f^3)*x^2 + (48*I*d^4*e^3
- 240*I*d^3*e^2*f + 96*I*d^2*e*f^2 - 96*I*d*f^3)*x)*e^(3*d*x + 3*c) + 4*(3*
d^4*f^3*x^4 + 20*d^3*e^3 - 12*(4*c - 1)*d^2*e^2*f + 24*(2*c^2 + 1)*d*e*f^2
- 8*(2*c^3 - 3)*f^3 + 4*(3*d^4*e*f^2 + d^3*f^3)*x^3 + 6*(3*d^4*e^2*f + 2*d^
3*e*f^2 + 2*d^2*f^3)*x^2 + 12*(d^4*e^3 + d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)
*x)*e^(2*d*x + 2*c) + (-12*I*d^3*f^3*x^3 - 12*I*d^3*e^3 - 42*I*d^2*e^2*f -
90*I*d*e*f^2 - 93*I*f^3 + (-36*I*d^3*e*f^2 - 42*I*d^2*f^3)*x^2 + (-36*I*d^3
*e^2*f - 84*I*d^2*e*f^2 - 90*I*d*f^3)*x)*e^(d*x + c) + ((192*I*d^2*e^2*f -
384*I*c*d*e*f^2 + 192*I*c^2*f^3)*e^(3*d*x + 3*c) + 192*(d^2*e^2*f - 2*c*d*e
*f^2 + c^2*f^3)*e^(2*d*x + 2*c))*log(e^(d*x + c) - I) + ((192*I*d^2*f^3*x^2
 + 384*I*d^2*e*f^2*x + 384*I*c*d*e*f^2 - 192*I*c^2*f^3)*e^(3*d*x + 3*c) + 1
92*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*e^(2*d*x + 2*c))*l
og(I*e^(d*x + c) + 1) + (-384*I*f^3*e^(3*d*x + 3*c) - 384*f^3*e^(2*d*x + 2*
c))*polylog(3, -I*e^(d*x + c)))/(32*a*d^4*e^(3*d*x + 3*c) - 32*I*a*d^4*e^(2
*d*x + 2*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3}\sinh\left(dx+c\right)^{3}}{i\,a\sinh\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sinh(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

3.200 $\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=287

$$\frac{4if^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}+\frac{if(e+fx)\sinh^2(c+dx)}{2ad^2}-\frac{2f(e+fx)\sinh(c+dx)}{ad^2}+\frac{2f^2\cos}{}$$

```
[Out] ((I/4)*f^2*x)/(a*d^2) - (I*(e + f*x)^2)/(a*d) + ((I/2)*(e + f*x)^3)/(a*f) +
 (2*f^2*Cosh[c + d*x])/(a*d^3) + ((e + f*x)^2*Cosh[c + d*x])/(a*d) + ((4*I)
*f*(e + f*x)*Log[1 + I*E^(c + d*x)])/(a*d^2) + ((4*I)*f^2*PolyLog[2, (-I)*E
^(c + d*x)])/(a*d^3) - (2*f*(e + f*x)*Sinh[c + d*x])/(a*d^2) - ((I/4)*f^2*C
osh[c + d*x]*Sinh[c + d*x])/(a*d^3) - ((I/2)*(e + f*x)^2*Cosh[c + d*x]*Sinh
[c + d*x])/(a*d) + ((I/2)*f*(e + f*x)*Sinh[c + d*x]^2)/(a*d^2) - (I*(e + f*
x)^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rubi [A] time = 0.546854, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.419, Rules used = {5557, 3311, 32, 2635, 8, 3296, 2638, 3318, 4184, 3716, 2190, 2279, 2391}

$$\frac{4if^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2}+\frac{if(e+fx)\sinh^2(c+dx)}{2ad^2}-\frac{2f(e+fx)\sinh(c+dx)}{ad^2}+\frac{2f^2\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```


```
[Out] ((I/4)*f^2*x)/(a*d^2) - (I*(e + f*x)^2)/(a*d) + ((I/2)*(e + f*x)^3)/(a*f) +
 (2*f^2*Cosh[c + d*x])/(a*d^3) + ((e + f*x)^2*Cosh[c + d*x])/(a*d) + ((4*I)
*f*(e + f*x)*Log[1 + I*E^(c + d*x)])/(a*d^2) + ((4*I)*f^2*PolyLog[2, (-I)*E
^(c + d*x)])/(a*d^3) - (2*f*(e + f*x)*Sinh[c + d*x])/(a*d^2) - ((I/4)*f^2*C
osh[c + d*x]*Sinh[c + d*x])/(a*d^3) - ((I/2)*(e + f*x)^2*Cosh[c + d*x]*Sinh
[c + d*x])/(a*d) + ((I/2)*f*(e + f*x)*Sinh[c + d*x]^2)/(a*d^2) - (I*(e + f*
x)^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```


Rule 5557

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = i\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx - \frac{i\int (e+fx)^2\sinh^2(c+dx)\,dx}{a}$$

$$= -\frac{i(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{2ad} + \frac{if(e+fx)\sinh^2(c+dx)}{2ad^2} + \frac{i\int (e+fx)^2\,dx}{2a} + \int$$

$$= \frac{i(e+fx)^3}{6af} + \frac{(e+fx)^2\cosh(c+dx)}{ad} - \frac{if^2\cosh(c+dx)\sinh(c+dx)}{4ad^3} - \frac{i(e+fx)^2\cosh(c}{2}$$

$$= \frac{if^2x}{4ad^2} + \frac{i(e+fx)^3}{2af} + \frac{(e+fx)^2\cosh(c+dx)}{ad} - \frac{2f(e+fx)\sinh(c+dx)}{ad^2} - \frac{if^2\cosh(c+d}{4a}$$

$$= \frac{if^2x}{4ad^2} + \frac{i(e+fx)^3}{2af} + \frac{2f^2\cosh(c+dx)}{ad^3} + \frac{(e+fx)^2\cosh(c+dx)}{ad} - \frac{2f(e+fx)\sinh(c+}{ad^2}$$

$$= \frac{if^2x}{4ad^2} - \frac{i(e+fx)^2}{ad} + \frac{i(e+fx)^3}{2af} + \frac{2f^2\cosh(c+dx)}{ad^3} + \frac{(e+fx)^2\cosh(c+dx)}{ad} - 2f(e+f$$

$$= \frac{if^2x}{4ad^2} - \frac{i(e+fx)^2}{ad} + \frac{i(e+fx)^3}{2af} + \frac{2f^2\cosh(c+dx)}{ad^3} + \frac{(e+fx)^2\cosh(c+dx)}{ad} + 4if(e+$$

$$= \frac{if^2x}{4ad^2} - \frac{i(e+fx)^2}{ad} + \frac{i(e+fx)^3}{2af} + \frac{2f^2\cosh(c+dx)}{ad^3} + \frac{(e+fx)^2\cosh(c+dx)}{ad} + 4if(e+$$

$$= \frac{if^2x}{4ad^2} - \frac{i(e+fx)^2}{ad} + \frac{i(e+fx)^3}{2af} + \frac{2f^2\cosh(c+dx)}{ad^3} + \frac{(e+fx)^2\cosh(c+dx)}{ad} + 4if(e+$$

Mathematica [B] time = 5.13595, size = 1661, normalized size = 5.79

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-6*I)*d^2*e^2*E^c*Cosh[(3*d*x)/2] + 6*d^2*e^2*E^(4*c)*Cosh[(3*d*x)/2] - (
14*I)*d*e*E^c*f*Cosh[(3*d*x)/2] - 14*d*e*E^(4*c)*f*Cosh[(3*d*x)/2] - (15*I)
*E^c*f^2*Cosh[(3*d*x)/2] + 15*E^(4*c)*f^2*Cosh[(3*d*x)/2] - (12*I)*d^2*e*E^
c*f*x*Cosh[(3*d*x)/2] + 12*d^2*e*E^(4*c)*f*x*Cosh[(3*d*x)/2] - (14*I)*d*E^c
*f^2*x*Cosh[(3*d*x)/2] - 14*d*E^(4*c)*f^2*x*Cosh[(3*d*x)/2] - (6*I)*d^2*E^c
*f^2*x^2*Cosh[(3*d*x)/2] + 6*d^2*E^(4*c)*f^2*x^2*Cosh[(3*d*x)/2] + 2*d^2*e^
2*Cosh[(5*d*x)/2] - (2*I)*d^2*e^2*E^(5*c)*Cosh[(5*d*x)/2] + 2*d*e*f*Cosh[(5
*d*x)/2] + (2*I)*d*e*E^(5*c)*f*Cosh[(5*d*x)/2] + f^2*Cosh[(5*d*x)/2] - I*E^
(5*c)*f^2*Cosh[(5*d*x)/2] + 4*d^2*e*f*x*Cosh[(5*d*x)/2] - (4*I)*d^2*e*E^(5*
c)*f*x*Cosh[(5*d*x)/2] + 2*d*f^2*x*Cosh[(5*d*x)/2] + (2*I)*d*E^(5*c)*f^2*x*
Cosh[(5*d*x)/2] + 2*d^2*f^2*x^2*Cosh[(5*d*x)/2] - (2*I)*d^2*E^(5*c)*f^2*x^2
*Cosh[(5*d*x)/2] + 8*E^(2*c)*Cosh[(d*x)/2]*(2*(1 - I*E^c)*f^2 + 2*d*(1 + I*
```

```
E^c)*f*(e + f*x) + d^2*(5 - I*E^c)*(e + f*x)^2 + d^3*(1 + I*E^c)*x*(3*e^2 +
 3*e*f*x + f^2*x^2) + 8*d*(1 + I*E^c)*f*(e + f*x)*Log[1 - I*E^(-c - d*x)])
- 40*d^2*e^2*E^(2*c)*Sinh[(d*x)/2] - (8*I)*d^2*e^2*E^(3*c)*Sinh[(d*x)/2] -
16*d*e*E^(2*c)*f*Sinh[(d*x)/2] + (16*I)*d*e*E^(3*c)*f*Sinh[(d*x)/2] - 16*E^
(2*c)*f^2*Sinh[(d*x)/2] - (16*I)*E^(3*c)*f^2*Sinh[(d*x)/2] - 24*d^3*e^2*E^(
2*c)*x*Sinh[(d*x)/2] + (24*I)*d^3*e^2*E^(3*c)*x*Sinh[(d*x)/2] - 80*d^2*e*E^
(2*c)*f*x*Sinh[(d*x)/2] - (16*I)*d^2*e*E^(3*c)*f*x*Sinh[(d*x)/2] - 16*d*E^(
2*c)*f^2*x*Sinh[(d*x)/2] + (16*I)*d*E^(3*c)*f^2*x*Sinh[(d*x)/2] - 24*d^3*e*
E^(2*c)*f*x^2*Sinh[(d*x)/2] + (24*I)*d^3*e*E^(3*c)*f*x^2*Sinh[(d*x)/2] - 40
*d^2*E^(2*c)*f^2*x^2*Sinh[(d*x)/2] - (8*I)*d^2*E^(3*c)*f^2*x^2*Sinh[(d*x)/2
] - 8*d^3*E^(2*c)*f^2*x^3*Sinh[(d*x)/2] + (8*I)*d^3*E^(3*c)*f^2*x^3*Sinh[(d
*x)/2] - 64*d*e*E^(2*c)*f*Log[1 - I*E^(-c - d*x)]*Sinh[(d*x)/2] + (64*I)*d*
e*E^(3*c)*f*Log[1 - I*E^(-c - d*x)]*Sinh[(d*x)/2] - 64*d*E^(2*c)*f^2*x*Log[
1 - I*E^(-c - d*x)]*Sinh[(d*x)/2] + (64*I)*d*E^(3*c)*f^2*x*Log[1 - I*E^(-c
- d*x)]*Sinh[(d*x)/2] + 64*E^(2*c)*f^2*PolyLog[2, I*E^(-c - d*x)]*((-1 - I*
E^c)*Cosh[(d*x)/2] + (1 - I*E^c)*Sinh[(d*x)/2]) + (6*I)*d^2*e^2*E^c*Sinh[(3
*d*x)/2] + 6*d^2*e^2*E^(4*c)*Sinh[(3*d*x)/2] + (14*I)*d*e*E^c*f*Sinh[(3*d*x
)/2] - 14*d*e*E^(4*c)*f*Sinh[(3*d*x)/2] + (15*I)*E^c*f^2*Sinh[(3*d*x)/2] +
15*E^(4*c)*f^2*Sinh[(3*d*x)/2] + (12*I)*d^2*e*E^c*f*x*Sinh[(3*d*x)/2] + 12*
d^2*e*E^(4*c)*f*x*Sinh[(3*d*x)/2] + (14*I)*d*E^c*f^2*x*Sinh[(3*d*x)/2] - 14
*d*E^(4*c)*f^2*x*Sinh[(3*d*x)/2] + (6*I)*d^2*E^c*f^2*x^2*Sinh[(3*d*x)/2] +
6*d^2*E^(4*c)*f^2*x^2*Sinh[(3*d*x)/2] - 2*d^2*e^2*Sinh[(5*d*x)/2] - (2*I)*d
^2*e^2*E^(5*c)*Sinh[(5*d*x)/2] - 2*d*e*f*Sinh[(5*d*x)/2] + (2*I)*d*e*E^(5*c
)*f*Sinh[(5*d*x)/2] - f^2*Sinh[(5*d*x)/2] - I*E^(5*c)*f^2*Sinh[(5*d*x)/2] -
 4*d^2*e*f*x*Sinh[(5*d*x)/2] - (4*I)*d^2*e*E^(5*c)*f*x*Sinh[(5*d*x)/2] - 2*
d*f^2*x*Sinh[(5*d*x)/2] + (2*I)*d*E^(5*c)*f^2*x*Sinh[(5*d*x)/2] - 2*d^2*f^2
*x^2*Sinh[(5*d*x)/2] - (2*I)*d^2*E^(5*c)*f^2*x^2*Sinh[(5*d*x)/2])/(16*a*d^3
*E^(2*c)*((-I + E^c)*Cosh[(d*x)/2] + (I + E^c)*Sinh[(d*x)/2]))
```

Maple [A] time = 0.128, size = 508, normalized size = 1.8

$$\frac{-2\,i f^2c^2}{ad^3}+\frac{\frac{i}{16}\left(2\,f^2x^2d^2+4\,d^2efx+2\,d^2e^2+2\,df^2x+2\,efd+f^2\right)\mathrm{e}^{-2\,dx-2\,c}}{ad^3}-\frac{2\,if^2x^2}{da}-\frac{4\,i\ln\left(\mathrm{e}^{dx+c}\right)ef}{ad^2}+\frac{\left(f^2x^2d^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -2*I/a/d^3*f^2*c^2+1/16*I*(2*d^2*f^2*x^2+4*d^2*e*f*x+2*d^2*e^2+2*d*f^2*x+2*
d*e*f+f^2)/a/d^3*exp(-2*d*x-2*c)-2*I/a/d*f^2*x^2-4*I/a/d^2*ln(exp(d*x+c))*e
*f+1/2*(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2-2*d*f^2*x-2*d*e*f+2*f^2)/a/d^3*exp(
d*x+c)+1/2*(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2+2*d*f^2*x+2*d*e*f+2*f^2)/a/d^3*
exp(-d*x-c)+4*I/a/d^3*f^2*ln(1+I*exp(d*x+c))*c+2*(f^2*x^2+2*e*f*x+e^2)/d/a/
(exp(d*x+c)-I)-4*I/a/d^3*f^2*c*ln(exp(d*x+c)-I)-1/16*I*(2*d^2*f^2*x^2+4*d^2
*e*f*x+2*d^2*e^2-2*d*f^2*x-2*d*e*f+f^2)/a/d^3*exp(2*d*x+2*c)+4*I/a/d^3*f^2*
c*ln(exp(d*x+c))+3/2*I/a*e^2*x-4*I/a/d^2*f^2*c*x+1/2*I/a*x^3*f^2+4*I*f^2*po
lylog(2,-I*exp(d*x+c))/a/d^3+3/2*I/a*e*f*x^2+4*I/a/d^2*ln(exp(d*x+c)-I)*e*f
+4*I/a/d^2*f^2*ln(1+I*exp(d*x+c))*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.67821, size = 1419, normalized size = 4.94

$$\frac{2\,d^2f^2x^2+2\,d^2e^2+2\,def+f^2+2\left(2\,d^2ef+df^2\right)x+\left(64i\,f^2e^{(3\,dx+3\,c)}+64\,f^2e^{(2\,dx+2\,c)}\right)\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)+\left(-2i\,d^2f^2x^2-2i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] (2*d^2*f^2*x^2 + 2*d^2*e^2 + 2*d*e*f + f^2 + 2*(2*d^2*e*f + d*f^2)*x + (64*
I*f^2*e^(3*d*x + 3*c) + 64*f^2*e^(2*d*x + 2*c))*dilog(-I*e^(d*x + c)) + (-2
*I*d^2*f^2*x^2 - 2*I*d^2*e^2 + 2*I*d*e*f - I*f^2 + (-4*I*d^2*e*f + 2*I*d*f^
2)*x)*e^(5*d*x + 5*c) + (6*d^2*f^2*x^2 + 6*d^2*e^2 - 14*d*e*f + 15*f^2 + 2*
(6*d^2*e*f - 7*d*f^2)*x)*e^(4*d*x + 4*c) + (8*I*d^3*f^2*x^3 - 8*I*d^2*e^2 +
 (-64*I*c + 16*I)*d*e*f + (32*I*c^2 - 16*I)*f^2 + (24*I*d^3*e*f - 40*I*d^2*
f^2)*x^2 + (24*I*d^3*e^2 - 80*I*d^2*e*f + 16*I*d*f^2)*x)*e^(3*d*x + 3*c) +
8*(d^3*f^2*x^3 + 5*d^2*e^2 - 2*(4*c - 1)*d*e*f + 2*(2*c^2 + 1)*f^2 + (3*d^3
*e*f + d^2*f^2)*x^2 + (3*d^3*e^2 + 2*d^2*e*f + 2*d*f^2)*x)*e^(2*d*x + 2*c)
+ (-6*I*d^2*f^2*x^2 - 6*I*d^2*e^2 - 14*I*d*e*f - 15*I*f^2 + (-12*I*d^2*e*f
- 14*I*d*f^2)*x)*e^(d*x + c) + ((64*I*d*e*f - 64*I*c*f^2)*e^(3*d*x + 3*c) +
 64*(d*e*f - c*f^2)*e^(2*d*x + 2*c))*log(e^(d*x + c) - I) + ((64*I*d*f^2*x
+ 64*I*c*f^2)*e^(3*d*x + 3*c) + 64*(d*f^2*x + c*f^2)*e^(2*d*x + 2*c))*log(I
*e^(d*x + c) + 1))/(16*a*d^3*e^(3*d*x + 3*c) - 16*I*a*d^3*e^(2*d*x + 2*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^2\sinh\left(dx+c\right)^3}{i\,a\sinh\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sinh(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

3.201 $\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=175

$$\frac{if\sinh^2(c+dx)}{4ad^2}-\frac{f\sinh(c+dx)}{ad^2}+\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{ad^2}+\frac{(e+fx)\cosh(c+dx)}{ad}-\frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\right.}{ad}$$

```
[Out] (((3*I)/2)*e*x)/a + (((3*I)/4)*f*x^2)/a + ((e + f*x)*Cosh[c + d*x])/(a*d) +
 ((2*I)*f*Log[Cosh[c/2 + (I/4)*Pi + (d*x)/2]])/(a*d^2) - (f*Sinh[c + d*x])/
(a*d^2) - ((I/2)*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(a*d) + ((I/4)*f*Si
nh[c + d*x]^2)/(a*d^2) - (I*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rubi [A] time = 0.262787, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {5557, 3310, 3296, 2637, 3318, 4184, 3475}

$$\frac{if\sinh^2(c+dx)}{4ad^2}-\frac{f\sinh(c+dx)}{ad^2}+\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{ad^2}+\frac{(e+fx)\cosh(c+dx)}{ad}-\frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{dx}{2}+\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (((3*I)/2)*e*x)/a + (((3*I)/4)*f*x^2)/a + ((e + f*x)*Cosh[c + d*x])/(a*d) +
 ((2*I)*f*Log[Cosh[c/2 + (I/4)*Pi + (d*x)/2]])/(a*d^2) - (f*Sinh[c + d*x])/
(a*d^2) - ((I/2)*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(a*d) + ((I/4)*f*Si
nh[c + d*x]^2)/(a*d^2) - (I*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rule 5557

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = i\int \frac{(e+fx)\sinh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx - \frac{i\int (e+fx)\sinh^2(c+dx)\,dx}{a}$$

$$= -\frac{i(e+fx)\cosh(c+dx)\sinh(c+dx)}{2ad} + \frac{if\sinh^2(c+dx)}{4ad^2} + \frac{i\int (e+fx)\,dx}{2a} + \frac{\int (e+fx)\sin}{}$$

$$= \frac{iex}{2a} + \frac{ifx^2}{4a} + \frac{(e+fx)\cosh(c+dx)}{ad} - \frac{i(e+fx)\cosh(c+dx)\sinh(c+dx)}{2ad} + \frac{if\sinh^2(c+}{4ad^2}$$

$$= \frac{3iex}{2a} + \frac{3ifx^2}{4a} + \frac{(e+fx)\cosh(c+dx)}{ad} - \frac{f\sinh(c+dx)}{ad^2} - \frac{i(e+fx)\cosh(c+dx)\sinh(c+}{2ad}$$

$$= \frac{3iex}{2a} + \frac{3ifx^2}{4a} + \frac{(e+fx)\cosh(c+dx)}{ad} - \frac{f\sinh(c+dx)}{ad^2} - \frac{i(e+fx)\cosh(c+dx)\sinh(c+}{2ad}$$

$$= \frac{3iex}{2a} + \frac{3ifx^2}{4a} + \frac{(e+fx)\cosh(c+dx)}{ad} + \frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2} - \frac{f\sinh(c+dx)}{ad^2} -$$

Mathematica [A] time = 1.80156, size = 325, normalized size = 1.86

$$\frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)\left(2\left(-3c^2f-d(e+fx)\sinh(2(c+dx))+6cde+4if\sinh(c+d\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(Cosh[(c + d*x)/2]*((-8*I)*d*(e
+ f*x)*Cosh[c + d*x] + f*Cosh[2*(c + d*x)] + 2*(6*c*d*e - (4*I)*c*f - 3*c^2
*f + 6*d^2*e*x - (4*I)*d*f*x + 3*d^2*f*x^2 + (8*I)*f*ArcTan[Tanh[(c + d*x)/
2]] + 4*f*Log[Cosh[c + d*x]] + (4*I)*f*Sinh[c + d*x] - d*(e + f*x)*Sinh[2*(
c + d*x)])) + Sinh[(c + d*x)/2]*(8*d*(e + f*x)*Cosh[c + d*x] + I*(f*Cosh[2*
(c + d*x)] + 2*((8*I)*d*e + 6*c*d*e - (4*I)*c*f - 3*c^2*f + 6*d^2*e*x + (4*
I)*d*f*x + 3*d^2*f*x^2 + (8*I)*f*ArcTan[Tanh[(c + d*x)/2]] + 4*f*Log[Cosh[c
 + d*x]] + (4*I)*f*Sinh[c + d*x] - d*(e + f*x)*Sinh[2*(c + d*x)])))))/(8*a*
d^2*(-I + Sinh[c + d*x]))
```

Maple [A] time = 0.109, size = 197, normalized size = 1.1

$$\frac{\frac{3i}{4}fx^2}{a}+\frac{\frac{3i}{2}ex}{a}-\frac{\frac{i}{16}\left(2dfx+2de-f\right)\mathrm{e}^{2dx+2c}}{ad^2}+\frac{\left(dfx+de-f\right)\mathrm{e}^{dx+c}}{2ad^2}+\frac{\left(dfx+de+f\right)\mathrm{e}^{-dx-c}}{2ad^2}+\frac{\frac{i}{16}\left(2dfx+2de+f\right.}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 3/4*I*f*x^2/a+3/2*I*e*x/a-1/16*I*(2*d*f*x+2*d*e-f)/a/d^2*exp(2*d*x+2*c)+1/2
*(d*f*x+d*e-f)/a/d^2*exp(d*x+c)+1/2*(d*f*x+d*e+f)/a/d^2*exp(-d*x-c)+1/16*I*
(2*d*f*x+2*d*e+f)/a/d^2*exp(-2*d*x-2*c)-2*I*f/a/d*x-2*I*f/a/d^2*c+2*(f*x+e)
/d/a/(exp(d*x+c)-I)+2*I*f/a/d^2*ln(exp(d*x+c)-I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.55633, size = 586, normalized size = 3.35

$$\frac{2dfx+2de+\left(-2idfx-2ide+if\right)e^{(5dx+5c)}+\left(6dfx+6de-7f\right)e^{(4dx+4c)}+\left(12id^2fx^2-8ide+\left(24id^2e-40idf\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*d*f*x + 2*d*e + (-2*I*d*f*x - 2*I*d*e + I*f)*e^(5*d*x + 5*c) + (6*d*f*x
+ 6*d*e - 7*f)*e^(4*d*x + 4*c) + (12*I*d^2*f*x^2 - 8*I*d*e + (24*I*d^2*e -
40*I*d*f)*x + 8*I*f)*e^(3*d*x + 3*c) + 4*(3*d^2*f*x^2 + 10*d*e + 2*(3*d^2*e
 + d*f)*x + 2*f)*e^(2*d*x + 2*c) + (-6*I*d*f*x - 6*I*d*e - 7*I*f)*e^(d*x +
c) - 32*(-I*f*e^(3*d*x + 3*c) - f*e^(2*d*x + 2*c))*log(e^(d*x + c) - I) + f
)/(16*a*d^2*e^(3*d*x + 3*c) - 16*I*a*d^2*e^(2*d*x + 2*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.43709, size = 479, normalized size = 2.74

$$\frac{12i\,d^2fx^2e^{(3\,dx+4\,c)} + 12\,d^2fx^2e^{(2\,dx+3\,c)} - 2i\,dfxe^{(5\,dx+6\,c)} + 6\,dfxe^{(4\,dx+5\,c)} + 24i\,d^2xe^{(3\,dx+4\,c+1)} - 40i\,dfxe^{(3\,dx+4\,c)} + 24\,d^2x\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (12*I*d^2*f*x^2*e^(3*d*x + 4*c) + 12*d^2*f*x^2*e^(2*d*x + 3*c) - 2*I*d*f*x*
e^(5*d*x + 6*c) + 6*d*f*x*e^(4*d*x + 5*c) + 24*I*d^2*x*e^(3*d*x + 4*c + 1)
- 40*I*d*f*x*e^(3*d*x + 4*c) + 24*d^2*x*e^(2*d*x + 3*c + 1) + 8*d*f*x*e^(2*
d*x + 3*c) - 6*I*d*f*x*e^(d*x + 2*c) + 2*d*f*x*e^c + 32*I*f*e^(3*d*x + 4*c)
*log(e^(d*x + c) - I) + 32*f*e^(2*d*x + 3*c)*log(e^(d*x + c) - I) - 2*I*d*e
^(5*d*x + 6*c + 1) + I*f*e^(5*d*x + 6*c) + 6*d*e^(4*d*x + 5*c + 1) - 7*f*e^
(4*d*x + 5*c) - 8*I*d*e^(3*d*x + 4*c + 1) + 8*I*f*e^(3*d*x + 4*c) + 40*d*e^
(2*d*x + 3*c + 1) + 8*f*e^(2*d*x + 3*c) - 6*I*d*e^(d*x + 2*c + 1) - 7*I*f*e
^(d*x + 2*c) + 2*d*e^(c + 1) + f*e^c)/(16*a*d^2*e^(3*d*x + 4*c) - 16*I*a*d^
2*e^(2*d*x + 3*c))
```

3.202 $\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)} dx$

Optimal. Leaf size=83

$$\frac{2\cosh(c+dx)}{ad} - \frac{\sinh^2(c+dx)\cosh(c+dx)}{d(a+ia\sinh(c+dx))} - \frac{3i\sinh(c+dx)\cosh(c+dx)}{2ad} + \frac{3ix}{2a}$$

```
[Out] (((3*I)/2)*x)/a + (2*Cosh[c + d*x])/(a*d) - (((3*I)/2)*Cosh[c + d*x]*Sinh[c
 + d*x])/(a*d) - (Cosh[c + d*x]*Sinh[c + d*x]^2)/(d*(a + I*a*Sinh[c + d*x])
)
```

Rubi [A] time = 0.0795302, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2767, 2734}

$$\frac{2\cosh(c+dx)}{ad} - \frac{\sinh^2(c+dx)\cosh(c+dx)}{d(a+ia\sinh(c+dx))} - \frac{3i\sinh(c+dx)\cosh(c+dx)}{2ad} + \frac{3ix}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (((3*I)/2)*x)/a + (2*Cosh[c + d*x])/(a*d) - (((3*I)/2)*Cosh[c + d*x]*Sinh[c
 + d*x])/(a*d) - (Cosh[c + d*x]*Sinh[c + d*x]^2)/(d*(a + I*a*Sinh[c + d*x])
)
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(c+dx)}{a+ia\sinh(c+dx)} dx &= -\frac{\cosh(c+dx)\sinh^2(c+dx)}{d(a+ia\sinh(c+dx))} + \frac{\int \sinh(c+dx)(2a-3ia\sinh(c+dx)) dx}{a^2} \\ &= \frac{3ix}{2a} + \frac{2\cosh(c+dx)}{ad} - \frac{3i\cosh(c+dx)\sinh(c+dx)}{2ad} - \frac{\cosh(c+dx)\sinh^2(c+dx)}{d(a+ia\sinh(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.163607, size = 109, normalized size = 1.31

$$\frac{\left(3\sqrt{1+i\sinh(c+dx)}\sinh^{-1}(\sinh(c+dx)) + \sqrt{1-i\sinh(c+dx)}\left(-i\sinh^2(c+dx)+\sinh(c+dx)-4i\right)\right)\cosh(c+dx}{2ad\sqrt{1-i\sinh(c+dx)}(\sinh(c+dx)-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (Cosh[c + d*x]*(3*ArcSinh[Sinh[c + d*x]]*Sqrt[1 + I*Sinh[c + d*x]] + Sqrt[1
 - I*Sinh[c + d*x]]*(-4*I + Sinh[c + d*x] - I*Sinh[c + d*x]^2)))/(2*a*d*Sqr
t[1 - I*Sinh[c + d*x]]*(-I + Sinh[c + d*x]))
```

Maple [B] time = 0.041, size = 196, normalized size = 2.4

$$\frac{-2i}{da}\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}+\frac{\frac{i}{2}}{da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+\frac{\frac{3i}{2}}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{1}{da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -2*I/d/a/(-I+tanh(1/2*d*x+1/2*c))+1/2*I/d/a/(tanh(1/2*d*x+1/2*c)+1)^2+3/2*I
/d/a*ln(tanh(1/2*d*x+1/2*c)+1)+1/d/a/(tanh(1/2*d*x+1/2*c)+1)-1/2*I/d/a/(tan
h(1/2*d*x+1/2*c)+1)-3/2*I/d/a*ln(tanh(1/2*d*x+1/2*c)-1)-1/2*I/d/a/(tanh(1/2
*d*x+1/2*c)-1)^2-1/d/a/(tanh(1/2*d*x+1/2*c)-1)-1/2*I/d/a/(tanh(1/2*d*x+1/2*
c)-1)
```

Maxima [A] time = 1.02886, size = 132, normalized size = 1.59

$$\frac{3i\,(dx+c)}{2\,ad}+\frac{3i\,e^{(-dx-c)}+20\,e^{(-2\,dx-2\,c)}+1}{8\left(i\,ae^{(-2\,dx-2\,c)}+ae^{(-3\,dx-3\,c)}\right)d}+\frac{i\left(-4i\,e^{(-dx-c)}+e^{(-2\,dx-2\,c)}\right)}{8\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 3/2*I*(d*x + c)/(a*d) + 1/8*(3*I*e^(-d*x - c) + 20*e^(-2*d*x - 2*c) + 1)/((
I*a*e^(-2*d*x - 2*c) + a*e^(-3*d*x - 3*c))*d) + 1/8*I*(-4*I*e^(-d*x - c) +
e^(-2*d*x - 2*c))/(a*d)
```

Fricas [A] time = 2.59988, size = 243, normalized size = 2.93

$$\frac{(12i\,dx-4i)e^{(3\,dx+3\,c)}+4\,(3\,dx+5)e^{(2\,dx+2\,c)}-i\,e^{(5\,dx+5\,c)}+3\,e^{(4\,dx+4\,c)}-3i\,e^{(dx+c)}+1}{8\,ade^{(3\,dx+3\,c)}-8i\,ade^{(2\,dx+2\,c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((12*I*d*x - 4*I)*e^(3*d*x + 3*c) + 4*(3*d*x + 5)*e^(2*d*x + 2*c) - I*e^(5*
d*x + 5*c) + 3*e^(4*d*x + 4*c) - 3*I*e^(d*x + c) + 1)/(8*a*d*e^(3*d*x + 3*c
) - 8*I*a*d*e^(2*d*x + 2*c))
```

Sympy [A] time = 1.39073, size = 180, normalized size = 2.17

$$\begin{cases} \frac{\left(-32ia^{3}d^{3}e^{5c}e^{2dx}+128a^{3}d^{3}e^{4c}e^{dx}+128a^{3}d^{3}e^{2c}e^{-dx}+32ia^{3}d^{3}e^{c}e^{-2dx}\right)e^{-3c}}{256a^{4}d^{4}} & \text{for } 256a^{4}d^{4}e^{3c} \neq 0 \\ x\left(-\frac{\left(ie^{4c}-2e^{3c}-6ie^{2c}+2e^{c}+i\right)e^{-2c}}{4a}-\frac{3i}{2a}\right) & \text{otherwise} \end{cases} + \frac{3ix}{2a} + \frac{2e^{c}}{ad\left(ie^{c}+e^{-dx}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise(((-32*I*a**3*d**3*exp(5*c)*exp(2*d*x) + 128*a**3*d**3*exp(4*c)*ex
p(d*x) + 128*a**3*d**3*exp(2*c)*exp(-d*x) + 32*I*a**3*d**3*exp(c)*exp(-2*d*
x))*exp(-3*c)/(256*a**4*d**4), Ne(256*a**4*d**4*exp(3*c), 0)), (x*(-(I*exp(
4*c) - 2*exp(3*c) - 6*I*exp(2*c) + 2*exp(c) + I)*exp(-2*c)/(4*a) - 3*I/(2*a
)), True)) + 3*I*x/(2*a) + 2*exp(c)/(a*d*(I*exp(c) + exp(-d*x)))
```

Giac [A] time = 1.47046, size = 127, normalized size = 1.53

$$\frac{3i\,(dx+c)}{2\,ad} + \frac{\left(20\,e^{(2\,dx+2\,c)} - 3i\,e^{(dx+c)} + 1\right)e^{(-2\,dx-2\,c)}}{8\,ad\left(e^{(dx+c)} - i\right)} - \frac{i\,ade^{(2\,dx+2\,c)} - 4\,ade^{(dx+c)}}{8\,a^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/2*I*(d*x + c)/(a*d) + 1/8*(20*e^(2*d*x + 2*c) - 3*I*e^(d*x + c) + 1)*e^(-
2*d*x - 2*c)/(a*d*(e^(d*x + c) - I)) - 1/8*(I*a*d*e^(2*d*x + 2*c) - 4*a*d*e
^(d*x + c))/(a^2*d^2)
```

3.203 $\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Sinh[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0780933, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sinh[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Sinh[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.003, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.137, size = 0, normalized size = 0.

$$\int \frac{(\sinh(dx+c))^3}{(fx+e)(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(-i\,adfx-i\,ade+\left(adfx+ade\right)e^{(dx+c)}\right)\text{integral}\left(-\frac{dfx+de-\left(-i\,dfx-i\,de\right)e^{(5\,dx+5\,c)}-\left(dfx+de\right)e^{(4\,dx+4\,c)}-\left(4i\,dfx+4i\,de\right)e^{(3\,dx+3\,c)}-4\,\left(dfx+de-\right.}{4\left(adf^2x^2+2\,adefx+ade^2\right)e^{(3\,dx+3\,c)}+\left(-4i\,adf^2x^2-8i\,adefx-4i\,ade\right.}\right.}{-i\,adfx-i\,ade+\left(adfx+ade\right)e^{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))*integral(-(d*f*x +
d*e - (-I*d*f*x - I*d*e)*e^(5*d*x + 5*c) - (d*f*x + d*e)*e^(4*d*x + 4*c) -
(4*I*d*f*x + 4*I*d*e)*e^(3*d*x + 3*c) - 4*(d*f*x + d*e + 2*f)*e^(2*d*x + 2*
c) - (I*d*f*x + I*d*e)*e^(d*x + c))/(4*(a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2
)*e^(3*d*x + 3*c) + (-4*I*a*d*f^2*x^2 - 8*I*a*d*e*f*x - 4*I*a*d*e^2)*e^(2*d
*x + 2*c)), x) + 2)/(-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(dx+c\right)^3}{\left(fx+e\right)\left(i\,a\sinh\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^3/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

3.204 $\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Sinh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0786479, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sinh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Sinh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.002, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.157, size = 0, normalized size = 0.

$$\int \frac{(\sinh(dx+c))^3}{(fx+e)^2(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(-i\,adf^2x^2-2i\,adefx-i\,ade^2+\left(adf^2x^2+2\,adefx+ade^2\right)e^{(dx+c)}\right)\text{integral}\left(-\frac{dfx+de-\left(-i\,dfx-i\,de\right)e^{(5\,dx+5\,c)}-\left(dfx+de\right)e^{(4\,dx+4\,c)}\ldots}{4\left(adf^3x^3+3\,adef^2x^2+3\,ade^2fx+ade^3\right)e^{(3\,d\ldots}}\right.}{-i\,adf^2x^2-2i\,adefx-i\,ade^2+\left(adf^2x^2+2\,adefx+ade^2\right)e^{(\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] ((-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x +
 a*d*e^2)*e^(d*x + c))*integral(-(d*f*x + d*e - (-I*d*f*x - I*d*e)*e^(5*d*x
 + 5*c) - (d*f*x + d*e)*e^(4*d*x + 4*c) - (4*I*d*f*x + 4*I*d*e)*e^(3*d*x +
3*c) - 4*(d*f*x + d*e + 4*f)*e^(2*d*x + 2*c) - (I*d*f*x + I*d*e)*e^(d*x + c
))/(4*(a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3)*e^(3*d*x +
3*c) + (-4*I*a*d*f^3*x^3 - 12*I*a*d*e*f^2*x^2 - 12*I*a*d*e^2*f*x - 4*I*a*d*
e^3)*e^(2*d*x + 2*c)), x) + 2)/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2
+ (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(dx+c\right)^3}{\left(fx+e\right)^2\left(i\,a\sinh\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^3/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

3.205 $\int \frac{(e+fx)^3\mathbf{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=313

$$\frac{12if^2(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{6f^2(e+fx)\text{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3}-\frac{6f^2(e+fx)\text{PolyLog}\left(3,e^{c+dx}\right)}{ad^3}-\frac{3f(e+fx)^2\text{Po}}{}$$

```
[Out] ((-I)*(e + f*x)^3)/(a*d) - (2*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) + ((6
*I)*f*(e + f*x)^2*Log[1 + I*E^(c + d*x)])/(a*d^2) - (3*f*(e + f*x)^2*PolyLo
g[2, -E^(c + d*x)])/(a*d^2) + ((12*I)*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c +
d*x)])/(a*d^3) + (3*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)])/(a*d^2) + (6*f^2
*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) - ((12*I)*f^3*PolyLog[3, (-I)*
E^(c + d*x)])/(a*d^4) - (6*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a*d^3) -
 (6*f^3*PolyLog[4, -E^(c + d*x)])/(a*d^4) + (6*f^3*PolyLog[4, E^(c + d*x)])
/(a*d^4) - (I*(e + f*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rubi [A] time = 0.482013, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {5575, 4182, 2531, 6609, 2282, 6589, 3318, 4184, 3716, 2190}

$$\frac{12if^2(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{6f^2(e+fx)\text{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3}-\frac{6f^2(e+fx)\text{PolyLog}\left(3,e^{c+dx}\right)}{ad^3}-\frac{3f(e+fx)^2\text{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```


```
[Out] ((-I)*(e + f*x)^3)/(a*d) - (2*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) + ((6
*I)*f*(e + f*x)^2*Log[1 + I*E^(c + d*x)])/(a*d^2) - (3*f*(e + f*x)^2*PolyLo
g[2, -E^(c + d*x)])/(a*d^2) + ((12*I)*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c +
d*x)])/(a*d^3) + (3*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)])/(a*d^2) + (6*f^2
*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) - ((12*I)*f^3*PolyLog[3, (-I)*
E^(c + d*x)])/(a*d^4) - (6*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a*d^3) -
 (6*f^3*PolyLog[4, -E^(c + d*x)])/(a*d^4) + (6*f^3*PolyLog[4, E^(c + d*x)])
/(a*d^4) - (I*(e + f*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```


Rule 5575

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a +
b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\left(i\int \frac{(e+fx)^3}{a+ia\sinh(c+dx)}\,dx\right) + \frac{\int (e+fx)^3\text{csch}(c+dx)\,dx}{a}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{i\int (e+fx)^3\csc^2\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{idx}{2}\right)dx}{2a} - \frac{(3f)\int (e+fx)^2\,\text{lo}}{ad}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{3f(e+fx)^2\text{Li}_2\left(-e^{c+dx}\right)}{ad^2} + \frac{3f(e+fx)^2\text{Li}_2\left(e^{c+dx}\right)}{ad^2} - \frac{i(e+f}{}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{3f(e+fx)^2\text{Li}_2\left(-e^{c+dx}\right)}{ad^2} + \frac{3f(e+fx)^2\text{Li}_2\left(e^{c+}\right.}{ad^2}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2} - \frac{3f(e+fx)^2\text{Li}_2}{ad^2}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2} - \frac{3f(e+fx)^2\text{Li}_2}{ad^2}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2} - \frac{3f(e+fx)^2\text{Li}_2}{ad^2}$$

$$= -\frac{i(e+fx)^3}{ad} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{6if(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad^2} - \frac{3f(e+fx)^2\text{Li}_2}{ad^2}$$

Mathematica [A] time = 5.59883, size = 363, normalized size = 1.16

$$-3f\left(d^2(e+fx)^2\text{PolyLog}(2,-\sinh(c+dx)-\cosh(c+dx)) - 2df(e+fx)\text{PolyLog}(3,-\sinh(c+dx)-\cosh(c+dx)) + \right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((2*d^3*(e + f*x)^3)/(-I + E^c) - 2*d^3*(e + f*x)^3*ArcTanh[Cosh[c + d*x] +
 Sinh[c + d*x]] + (6*I)*d^2*f*(e + f*x)^2*Log[1 - I*E^(-c - d*x)] - (12*I)*
f^2*(d*(e + f*x)*PolyLog[2, I*E^(-c - d*x)] + f*PolyLog[3, I*E^(-c - d*x)])
 - 3*f*(d^2*(e + f*x)^2*PolyLog[2, -Cosh[c + d*x] - Sinh[c + d*x]] - 2*d*f*
(e + f*x)*PolyLog[3, -Cosh[c + d*x] - Sinh[c + d*x]] + 2*f^2*PolyLog[4, -Co
sh[c + d*x] - Sinh[c + d*x]]) + 3*f*(d^2*(e + f*x)^2*PolyLog[2, Cosh[c + d*
x] + Sinh[c + d*x]] - 2*d*f*(e + f*x)*PolyLog[3, Cosh[c + d*x] + Sinh[c + d
*x]] + 2*f^2*PolyLog[4, Cosh[c + d*x] + Sinh[c + d*x]]) - ((2*I)*d^3*(e + f
*x)^3*Sinh[(d*x)/2])/((Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh
[(c + d*x)/2]))))/(a*d^4)
```

Maple [B] time = 0.268, size = 1034, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 2*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(exp(d*x+c)-I)+6*I/d^2/a*f^3*ln(1
+I*exp(d*x+c))*x^2-6*I/d^2/a*ln(exp(d*x+c))*e^2*f-6*I/d/a*e*f^2*x^2+6*I/d^2
/a*ln(exp(d*x+c)-I)*e^2*f+6*I/d^3/a*f^3*c^2*x+6*I/d^4/a*f^3*c^2*ln(exp(d*x+
c)-I)+12*I/d^3/a*e*f^2*polylog(2,-I*exp(d*x+c))-6*I/d^4/a*f^3*c^2*ln(exp(d*
x+c))-6*I/d^3/a*e*f^2*c^2+12*I/d^3/a*f^3*polylog(2,-I*exp(d*x+c))*x-6*I/d^4
/a*f^3*c^2*ln(1+I*exp(d*x+c))-12*I/d^2/a*e*f^2*c*x+12*I/d^3/a*e*f^2*ln(1+I*
exp(d*x+c))*c+12*I/d^2/a*e*f^2*ln(1+I*exp(d*x+c))*x+12*I/d^3/a*e*f^2*c*ln(e
xp(d*x+c))-12*I*f^3*polylog(3,-I*exp(d*x+c))/a/d^4-6*f^3*polylog(4,-exp(d*x
+c))/a/d^4+6*f^3*polylog(4,exp(d*x+c))/a/d^4-2*I/d/a*f^3*x^3+4*I/d^4/a*f^3*
c^3+3/d^3/a*e*f^2*c^2*ln(exp(d*x+c)-1)-12*I/d^3/a*e*f^2*c*ln(exp(d*x+c)-I)-
3/d^2/a*e^2*f*c*ln(exp(d*x+c)-1)+3/d^2/a*ln(1-exp(d*x+c))*c*e^2*f+3/d/a*ln(
1-exp(d*x+c))*e^2*f*x-3/d/a*ln(exp(d*x+c)+1)*e^2*f*x-3/d^3/a*e*f^2*c^2*ln(1
-exp(d*x+c))+3/d/a*e*f^2*ln(1-exp(d*x+c))*x^2+6/d^2/a*e*f^2*polylog(2,exp(d
*x+c))*x-3/d/a*e*f^2*ln(exp(d*x+c)+1)*x^2-6/d^2/a*e*f^2*polylog(2,-exp(d*x+
c))*x+1/d/a*e^3*ln(exp(d*x+c)-1)-1/d/a*e^3*ln(exp(d*x+c)+1)+3/d^2/a*f^3*pol
ylog(2,exp(d*x+c))*x^2-6/d^3/a*f^3*polylog(3,exp(d*x+c))*x+3/d^2/a*e^2*f*po
lylog(2,exp(d*x+c))-3/d^2/a*e^2*f*polylog(2,-exp(d*x+c))-6/d^3/a*e*f^2*poly
log(3,exp(d*x+c))+6/d^3/a*e*f^2*polylog(3,-exp(d*x+c))-1/d^4/a*f^3*c^3*ln(e
xp(d*x+c)-1)-1/d/a*f^3*ln(exp(d*x+c)+1)*x^3-3/d^2/a*f^3*polylog(2,-exp(d*x+
c))*x^2+6/d^3/a*f^3*polylog(3,-exp(d*x+c))*x+1/d/a*f^3*ln(1-exp(d*x+c))*x^3
+1/d^4/a*f^3*ln(1-exp(d*x+c))*c^3
```

Maxima [B] time = 1.96947, size = 783, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -e^3*(log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d) - 2/((a*e^(
-d*x - c) + I*a)*d)) - 6*I*e^2*f*x/(a*d) - 3*(d*x*log(e^(d*x + c) + 1) + di
log(-e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(
d*x + c)))*e^2*f/(a*d^2) + 6*I*e^2*f*log(I*e^(d*x + c) + 1)/(a*d^2) + 2*(f^
3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x)/(a*d*e^(d*x + c) - I*a*d) - 3*(d^2*x^2*log
(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))
*e*f^2/(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)
) - 2*polylog(3, e^(d*x + c)))*e*f^2/(a*d^3) + 12*I*(d*x*log(I*e^(d*x + c)
+ 1) + dilog(-I*e^(d*x + c)))*e*f^2/(a*d^3) - (d^3*x^3*log(e^(d*x + c) + 1)
 + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polyl
og(4, -e^(d*x + c)))*f^3/(a*d^4) + (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x
^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x
 + c)))*f^3/(a*d^4) + 6*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*
e^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a*d^4) - (2*I*d^3*f^3*x^3
 + 6*I*d^3*e*f^2*x^2)/(a*d^4)
```

Fricas [C] time = 2.82403, size = 2367, normalized size = 7.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*d^3*e^3 - 6*c*d^2*e^2*f + 6*c^2*d*e*f^2 - 2*c^3*f^3 + (12*d*f^3*x + 12*d
*e*f^2 + (12*I*d*f^3*x + 12*I*d*e*f^2)*e^(d*x + c))*dilog(-I*e^(d*x + c)) +
 (3*I*d^2*f^3*x^2 + 6*I*d^2*e*f^2*x + 3*I*d^2*e^2*f - 3*(d^2*f^3*x^2 + 2*d^
2*e*f^2*x + d^2*e^2*f)*e^(d*x + c))*dilog(-e^(d*x + c)) + (-3*I*d^2*f^3*x^2
 - 6*I*d^2*e*f^2*x - 3*I*d^2*e^2*f + 3*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e
^2*f)*e^(d*x + c))*dilog(e^(d*x + c)) + (-2*I*d^3*f^3*x^3 - 6*I*d^3*e*f^2*x
^2 - 6*I*d^3*e^2*f*x - 6*I*c*d^2*e^2*f + 6*I*c^2*d*e*f^2 - 2*I*c^3*f^3)*e^(
d*x + c) + (I*d^3*f^3*x^3 + 3*I*d^3*e*f^2*x^2 + 3*I*d^3*e^2*f*x + I*d^3*e^3
 - (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + d^3*e^3)*e^(d*x + c))*l
og(e^(d*x + c) + 1) + (6*d^2*e^2*f - 12*c*d*e*f^2 + 6*c^2*f^3 + (6*I*d^2*e^
2*f - 12*I*c*d*e*f^2 + 6*I*c^2*f^3)*e^(d*x + c))*log(e^(d*x + c) - I) + (-I
*d^3*e^3 + 3*I*c*d^2*e^2*f - 3*I*c^2*d*e*f^2 + I*c^3*f^3 + (d^3*e^3 - 3*c*d
^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*e^(d*x + c))*log(e^(d*x + c) - 1) + (6*
d^2*f^3*x^2 + 12*d^2*e*f^2*x + 12*c*d*e*f^2 - 6*c^2*f^3 + (6*I*d^2*f^3*x^2
+ 12*I*d^2*e*f^2*x + 12*I*c*d*e*f^2 - 6*I*c^2*f^3)*e^(d*x + c))*log(I*e^(d*
x + c) + 1) + (-I*d^3*f^3*x^3 - 3*I*d^3*e*f^2*x^2 - 3*I*d^3*e^2*f*x - 3*I*c
*d^2*e^2*f + 3*I*c^2*d*e*f^2 - I*c^3*f^3 + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 +
 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*e^(d*x + c))*log(
-e^(d*x + c) + 1) - 6*(f^3*e^(d*x + c) - I*f^3)*polylog(4, -e^(d*x + c)) +
6*(f^3*e^(d*x + c) - I*f^3)*polylog(4, e^(d*x + c)) + (-12*I*f^3*e^(d*x + c
) - 12*f^3)*polylog(3, -I*e^(d*x + c)) + (-6*I*d*f^3*x - 6*I*d*e*f^2 + 6*(d
*f^3*x + d*e*f^2)*e^(d*x + c))*polylog(3, -e^(d*x + c)) + (6*I*d*f^3*x + 6*
I*d*e*f^2 - 6*(d*f^3*x + d*e*f^2)*e^(d*x + c))*polylog(3, e^(d*x + c)))/(a*
d^4*e^(d*x + c) - I*a*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c)}{i\,a\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*csch(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.206 $\int \frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=224

$$-\frac{2f(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}+\frac{2f(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}+\frac{4if^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{2f^2\text{PolyLog}\left(3,-e^{c\right)}{ad^3}$$

```
[Out] ((-I)*(e + f*x)^2)/(a*d) - (2*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) + ((4
*I)*f*(e + f*x)*Log[1 + I*E^(c + d*x)])/(a*d^2) - (2*f*(e + f*x)*PolyLog[2,
 -E^(c + d*x)])/(a*d^2) + ((4*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^3)
+ (2*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) + (2*f^2*PolyLog[3, -E^(c
 + d*x)])/(a*d^3) - (2*f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) - (I*(e + f*x)^
2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rubi [A] time = 0.345685, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {5575, 4182, 2531, 2282, 6589, 3318, 4184, 3716, 2190, 2279, 2391}

$$-\frac{2f(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}+\frac{2f(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}+\frac{4if^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{2f^2\text{PolyLog}\left(3,-e^{c\right)}{ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```


```
[Out] ((-I)*(e + f*x)^2)/(a*d) - (2*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) + ((4
*I)*f*(e + f*x)*Log[1 + I*E^(c + d*x)])/(a*d^2) - (2*f*(e + f*x)*PolyLog[2,
 -E^(c + d*x)])/(a*d^2) + ((4*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^3)
+ (2*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) + (2*f^2*PolyLog[3, -E^(c
 + d*x)])/(a*d^3) - (2*f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) - (I*(e + f*x)^
2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```


Rule 5575

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a +
b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\left(i\int \frac{(e+fx)^2}{a+ia\sinh(c+dx)}\,dx\right) + \frac{\int (e+fx)^2\text{csch}(c+dx)\,dx}{a}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{i\int (e+fx)^2\csc^2\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{idx}{2}\right)dx}{2a} - \frac{(2f)\int (e+fx)}{}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2f(e+fx)\text{Li}_2\left(-e^{c+dx}\right)}{ad^2} + \frac{2f(e+fx)\text{Li}_2\left(e^{c+dx}\right)}{ad^2} - \frac{i(e+}{}$$

$$= -\frac{i(e+fx)^2}{ad} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2f(e+fx)\text{Li}_2\left(-e^{c+dx}\right)}{ad^2} + \frac{2f(e+fx)\text{Li}_2\left(e^{\right.}}{ad^2}$$

$$= -\frac{i(e+fx)^2}{ad} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2} - \frac{2f(e+fx)\text{Li}}{ad^2}$$

$$= -\frac{i(e+fx)^2}{ad} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2} - \frac{2f(e+fx)\text{Li}}{ad^2}$$

$$= -\frac{i(e+fx)^2}{ad} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{4if(e+fx)\log\left(1+ie^{c+dx}\right)}{ad^2} - \frac{2f(e+fx)\text{Li}}{ad^2}$$

Mathematica [A] time = 4.15301, size = 275, normalized size = 1.23

$$\frac{\frac{2d(e+fx)\left(2(e^c-i)f\log\left(1-ie^{-c-dx}\right)-id(e+fx)\right)-4(e^c-i)f^2\text{PolyLog}\left(2,ie^{-c-dx}\right)}{-1-ie^c} - 2df(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right) + 2df(e+fx)\text{PolyLog}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (d^2*(e + f*x)^2*Log[1 - E^(c + d*x)] - d^2*(e + f*x)^2*Log[1 + E^(c + d*x)
] + (2*d*(e + f*x)*((-I)*d*(e + f*x) + 2*(-I + E^c)*f*Log[1 - I*E^(-c - d*x
)]) - 4*(-I + E^c)*f^2*PolyLog[2, I*E^(-c - d*x)])/(-1 - I*E^c) - 2*d*f*(e
+ f*x)*PolyLog[2, -E^(c + d*x)] + 2*d*f*(e + f*x)*PolyLog[2, E^(c + d*x)] +
 2*f^2*PolyLog[3, -E^(c + d*x)] - 2*f^2*PolyLog[3, E^(c + d*x)] - ((2*I)*d^
2*(e + f*x)^2*Sinh[(d*x)/2])/((Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2]
+ I*Sinh[(c + d*x)/2])))/(a*d^3)
```

Maple [B] time = 0.131, size = 573, normalized size = 2.6

$$2\,\frac{x^2f^2+2\,efx+e^2}{da\left(e^{dx+c}-i\right)} + \frac{4\,i\ln\left(e^{dx+c}-i\right)ef}{ad^2} - \frac{4\,i\ln\left(e^{dx+c}\right)ef}{ad^2} - \frac{4\,if^2cx}{ad^2} + \frac{4\,if^2\ln\left(1+ie^{dx+c}\right)x}{ad^2} + \frac{4\,if^2\ln\left(1+ie^{dx+c}\right)c}{ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 2*(f^2*x^2+2*e*f*x+e^2)/d/a/(exp(d*x+c)-I)+4*I/a/d^2*ln(exp(d*x+c)-I)*e*f-4
*I/a/d^2*ln(exp(d*x+c))*e*f-4*I/a/d^2*f^2*c*x+4*I/a/d^2*f^2*ln(1+I*exp(d*x+
c))*x+4*I/a/d^3*f^2*ln(1+I*exp(d*x+c))*c-4*I/a/d^3*f^2*c*ln(exp(d*x+c)-I)+4
*I/a/d^3*f^2*c*ln(exp(d*x+c))+2/a/d^2*ln(1-exp(d*x+c))*c*e*f-2/a/d*ln(exp(d
```

```
*x+c)+1)*e*f*x+2/a/d*ln(1-exp(d*x+c))*e*f*x-2/a/d^2*e*f*c*ln(exp(d*x+c)-1)-
2*f^2*polylog(3,exp(d*x+c))/a/d^3+2*f^2*polylog(3,-exp(d*x+c))/a/d^3+4*I*f^
2*polylog(2,-I*exp(d*x+c))/a/d^3+1/a/d*e^2*ln(exp(d*x+c)-1)-1/a/d*e^2*ln(ex
p(d*x+c)+1)-2/a/d^2*f^2*polylog(2,-exp(d*x+c))*x-1/a/d^3*f^2*c^2*ln(1-exp(d
*x+c))+1/a/d*f^2*ln(1-exp(d*x+c))*x^2+1/a/d^3*f^2*c^2*ln(exp(d*x+c)-1)-2/a/
d^2*e*f*polylog(2,-exp(d*x+c))+2/a/d^2*e*f*polylog(2,exp(d*x+c))-2*I/a/d*f^
2*x^2-2*I/a/d^3*f^2*c^2+2/a/d^2*f^2*polylog(2,exp(d*x+c))*x-1/a/d*f^2*ln(ex
p(d*x+c)+1)*x^2
```

Maxima [A] time = 1.85616, size = 468, normalized size = 2.09

$$-e^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}-\frac{\log\left(e^{(-dx-c)}-1\right)}{ad}-\frac{2}{\left(ae^{(-dx-c)}+ia\right)d}\right)-\frac{2if^2x^2}{ad}-\frac{4iefx}{ad}+\frac{2\left(f^2x^2+2efx\right)}{ade^{(dx+c)}-iad}-\frac{2\left(dx\log\left(e^{(dx+c)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -e^2*(log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d) - 2/((a*e^(
-d*x - c) + I*a)*d)) - 2*I*f^2*x^2/(a*d) - 4*I*e*f*x/(a*d) + 2*(f^2*x^2 + 2
*e*f*x)/(a*d*e^(d*x + c) - I*a*d) - 2*(d*x*log(e^(d*x + c) + 1) + dilog(-e^
(d*x + c)))*e*f/(a*d^2) + 2*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c))
)*e*f/(a*d^2) + 4*I*e*f*log(I*e^(d*x + c) + 1)/(a*d^2) - (d^2*x^2*log(e^(d*
x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2/(
a*d^3) + (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*poly
log(3, e^(d*x + c)))*f^2/(a*d^3) + 4*I*(d*x*log(I*e^(d*x + c) + 1) + dilog(
-I*e^(d*x + c)))*f^2/(a*d^3)
```

Fricas [C] time = 2.71007, size = 1368, normalized size = 6.11

$$\frac{2d^2e^2-4cdef+2c^2f^2+\left(4if^2e^{(dx+c)}+4f^2\right)\mathrm{Li}_2\left(-ie^{(dx+c)}\right)+\left(2idf^2x+2idef-2\left(df^2x+def\right)e^{(dx+c)}\right)\mathrm{Li}_2\left(-e^{(dx+c)}\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*d^2*e^2 - 4*c*d*e*f + 2*c^2*f^2 + (4*I*f^2*e^(d*x + c) + 4*f^2)*dilog(-I
*e^(d*x + c)) + (2*I*d*f^2*x + 2*I*d*e*f - 2*(d*f^2*x + d*e*f)*e^(d*x + c))
*dilog(-e^(d*x + c)) + (-2*I*d*f^2*x - 2*I*d*e*f + 2*(d*f^2*x + d*e*f)*e^(d
*x + c))*dilog(e^(d*x + c)) + (-2*I*d^2*f^2*x^2 - 4*I*d^2*e*f*x - 4*I*c*d*e
*f + 2*I*c^2*f^2)*e^(d*x + c) + (I*d^2*f^2*x^2 + 2*I*d^2*e*f*x + I*d^2*e^2
- (d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*e^(d*x + c))*log(e^(d*x + c) + 1) +
 (4*d*e*f - 4*c*f^2 + (4*I*d*e*f - 4*I*c*f^2)*e^(d*x + c))*log(e^(d*x + c)
- I) + (-I*d^2*e^2 + 2*I*c*d*e*f - I*c^2*f^2 + (d^2*e^2 - 2*c*d*e*f + c^2*f
^2)*e^(d*x + c))*log(e^(d*x + c) - 1) + (4*d*f^2*x + 4*c*f^2 + (4*I*d*f^2*x
 + 4*I*c*f^2)*e^(d*x + c))*log(I*e^(d*x + c) + 1) + (-I*d^2*f^2*x^2 - 2*I*d
^2*e*f*x - 2*I*c*d*e*f + I*c^2*f^2 + (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f
 - c^2*f^2)*e^(d*x + c))*log(-e^(d*x + c) + 1) + 2*(f^2*e^(d*x + c) - I*f^2
)*polylog(3, -e^(d*x + c)) - 2*(f^2*e^(d*x + c) - I*f^2)*polylog(3, e^(d*x
+ c)))/(a*d^3*e^(d*x + c) - I*a*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)}{i\,a\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*csch(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.207 $\int \frac{(e+fx)\mathbf{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=126

$$-\frac{f\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}+\frac{f\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}+\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{ad^2}-\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{i(e+fx)\text{ta}}{}$$

```
[Out] (-2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d) + ((2*I)*f*Log[Cosh[c/2 + (I/4)*P
i + (d*x)/2]])/(a*d^2) - (f*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (f*PolyLog[
2, E^(c + d*x)])/(a*d^2) - (I*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*
d)
```

Rubi [A] time = 0.147479, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5575, 4182, 2279, 2391, 3318, 4184, 3475}

$$-\frac{f\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}+\frac{f\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}+\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{ad^2}-\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{i(e+fx)\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```


```
[Out] (-2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d) + ((2*I)*f*Log[Cosh[c/2 + (I/4)*P
i + (d*x)/2]])/(a*d^2) - (f*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (f*PolyLog[
2, E^(c + d*x)])/(a*d^2) - (I*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*
d)
```


Rule 5575

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a +
b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\left(i\int \frac{e+fx}{a+ia\sinh(c+dx)}\,dx\right)+\frac{\int (e+fx)\operatorname{csch}(c+dx)\,dx}{a}$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{i\int (e+fx)\csc^2\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{idx}{2}\right)dx}{2a}-\frac{f\int \log\left(1-e^{c+dx}\right.}{ad}$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{i(e+fx)\tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}-\frac{f\,\mathrm{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e\right.}{ad^2}$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}-\frac{f\mathrm{Li}_2\left(-e^{c+dx}\right)}{ad^2}+\frac{f\mathrm{Li}_2\left(e^{c+\right.}}{ad^2}$$

Mathematica [B] time = 1.25237, size = 345, normalized size = 2.74

$$\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(f\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(\mathrm{PolyLog}\left(2,-e^{-c-dx}\right)-\mathrm{PolyLog}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(f*(c + d*x)*(Cosh[(c + d*x)/2]
+ I*Sinh[(c + d*x)/2]) - 2*f*ArcTan[Tanh[(c + d*x)/2]]*(Cosh[(c + d*x)/2] +
 I*Sinh[(c + d*x)/2]) + I*f*Log[Cosh[c + d*x]]*(Cosh[(c + d*x)/2] + I*Sinh[
(c + d*x)/2]) + d*e*Log[Tanh[(c + d*x)/2]]*(Cosh[(c + d*x)/2] + I*Sinh[(c +
 d*x)/2]) - c*f*Log[Tanh[(c + d*x)/2]]*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x
)/2]) + f*((c + d*x)*(Log[1 - E^(-c - d*x)] - Log[1 + E^(-c - d*x)]) + Poly
Log[2, -E^(-c - d*x)] - PolyLog[2, E^(-c - d*x)])*(Cosh[(c + d*x)/2] + I*Si
nh[(c + d*x)/2]) - (2*I)*d*(e + f*x)*Sinh[(c + d*x)/2]))/(d^2*(a + I*a*Sinh
[c + d*x]))
```

Maple [A] time = 0.132, size = 211, normalized size = 1.7

$$2\,\frac{fx+e}{da\left(e^{dx+c}-i\right)}+\frac{f\mathrm{polylog}\left(2,e^{dx+c}\right)}{ad^2}-\frac{f\mathrm{polylog}\left(2,-e^{dx+c}\right)}{ad^2}-\frac{2\,if\ln\left(e^{dx+c}\right)}{ad^2}+\frac{e\ln\left(e^{dx+c}-1\right)}{da}-\frac{e\ln\left(e^{dx+c}+1\right)}{da}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 2*(f*x+e)/d/a/(exp(d*x+c)-I)+f*polylog(2,exp(d*x+c))/a/d^2-f*polylog(2,-exp
(d*x+c))/a/d^2-2*I/d^2/a*f*ln(exp(d*x+c))+1/d/a*e*ln(exp(d*x+c)-1)-1/d/a*e*
ln(exp(d*x+c)+1)+1/d/a*ln(1-exp(d*x+c))*f*x+1/d^2/a*ln(1-exp(d*x+c))*c*f-1/
d/a*ln(exp(d*x+c)+1)*f*x-1/d^2/a*f*c*ln(exp(d*x+c)-1)+2*I*f/a/d^2*ln(exp(d*
x+c)-I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2f\left(\frac{xe^{(dx+c)}}{iade^{(dx+c)}+ad}+\frac{i\log\left(\left(e^{(dx+c)}-i\right)e^{(-c)}\right)}{ad^2}+\int\frac{x}{2\left(ae^{(dx+c)}+a\right)}\,dx+\int\frac{x}{2\left(ae^{(dx+c)}-a\right)}\,dx\right)-e\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}-\frac{\mathrm{lo}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*f*(x*e^(d*x + c)/(I*a*d*e^(d*x + c) + a*d) + I*log((e^(d*x + c) - I)*e^(-
c))/(a*d^2) + integrate(1/2*x/(a*e^(d*x + c) + a), x) + integrate(1/2*x/(a*
e^(d*x + c) - a), x)) - e*(log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) -
 1)/(a*d) - 2/((a*e^(-d*x - c) + I*a)*d))
```

Fricas [B] time = 2.62454, size = 548, normalized size = 4.35

$$\frac{-2idfxe^{(dx+c)}+2de-\left(fe^{(dx+c)}-if\right)\mathrm{Li}_2\left(-e^{(dx+c)}\right)+\left(fe^{(dx+c)}-if\right)\mathrm{Li}_2\left(e^{(dx+c)}\right)+\left(idfx+ide-\left(dfx+de\right)e^{(dx+c)}\right)\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (-2*I*d*f*x*e^(d*x + c) + 2*d*e - (f*e^(d*x + c) - I*f)*dilog(-e^(d*x + c))
 + (f*e^(d*x + c) - I*f)*dilog(e^(d*x + c)) + (I*d*f*x + I*d*e - (d*f*x + d
*e)*e^(d*x + c))*log(e^(d*x + c) + 1) + (2*I*f*e^(d*x + c) + 2*f)*log(e^(d*
x + c) - I) + (-I*d*e + I*c*f + (d*e - c*f)*e^(d*x + c))*log(e^(d*x + c) -
1) + (-I*d*f*x - I*c*f + (d*f*x + c*f)*e^(d*x + c))*log(-e^(d*x + c) + 1))/
(a*d^2*e^(d*x + c) - I*a*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)\operatorname{csch}\left(dx+c\right)}{i\,a\sinh\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*csch(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.208 $\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=41

$$-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{\cosh(c+dx)}{d(a+ia\sinh(c+dx))}$$

```
[Out] -(ArcTanh[Cosh[c + d*x]]/(a*d)) + Cosh[c + d*x]/(d*(a + I*a*Sinh[c + d*x]))
```

Rubi [A] time = 0.0599134, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2747, 3770, 2648}

$$-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{\cosh(c+dx)}{d(a+ia\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] -(ArcTanh[Cosh[c + d*x]]/(a*d)) + Cosh[c + d*x]/(d*(a + I*a*Sinh[c + d*x]))
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\left(i\int \frac{1}{a+ia\sinh(c+dx)}\,dx\right)+\frac{\int \operatorname{csch}(c+dx)\,dx}{a}\\ &= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{\cosh(c+dx)}{d(a+ia\sinh(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.0642838, size = 52, normalized size = 1.27

$$-\frac{\operatorname{sech}(c+dx)\left(i\sinh(c+dx)+\sqrt{\cosh^2(c+dx)}\tanh^{-1}\left(\sqrt{\cosh^2(c+dx)}\right)-1\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] -((Sech[c + d*x]*(-1 + ArcTanh[Sqrt[Cosh[c + d*x]^2]]*Sqrt[Cosh[c + d*x]^2]
 + I*Sinh[c + d*x]))/(a*d))
```

Maple [A] time = 0.037, size = 42, normalized size = 1.

$$\frac{-2i}{da}\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}+\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -2*I/d/a/(-I+tanh(1/2*d*x+1/2*c))+1/d/a*ln(tanh(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.08649, size = 84, normalized size = 2.05

$$-\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}+\frac{\log\left(e^{(-dx-c)}-1\right)}{ad}+\frac{2}{\left(ae^{(-dx-c)}+i\,a\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -log(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d) + 2/((a*e^(-d*x
- c) + I*a)*d)
```

Fricas [A] time = 2.49623, size = 154, normalized size = 3.76

$$-\frac{\left(e^{(dx+c)}-i\right)\log\left(e^{(dx+c)}+1\right)-\left(e^{(dx+c)}-i\right)\log\left(e^{(dx+c)}-1\right)-2}{ade^{(dx+c)}-i\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((e^(d*x + c) - I)*log(e^(d*x + c) + 1) - (e^(d*x + c) - I)*log(e^(d*x + c
) - 1) - 2)/(a*d*e^(d*x + c) - I*a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{csch}(c+dx)}{i\sinh(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Integral(csch(c + d*x)/(I*sinh(c + d*x) + 1), x)/a
```

Giac [A] time = 1.24345, size = 72, normalized size = 1.76

$$-\frac{\log\left(e^{(dx+c)}+1\right)}{ad}+\frac{\log\left(\left|e^{(dx+c)}-1\right|\right)}{ad}+\frac{2}{ad\left(e^{(dx+c)}-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -log(e^(d*x + c) + 1)/(a*d) + log(abs(e^(d*x + c) - 1))/(a*d) + 2/(a*d*(e^(
d*x + c) - I))
```

3.209 $\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Csch[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0501509, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]

[Out] Defer[Int][Csch[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Mathematica [A] time = 46.1088, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]

[Out] Integrate[Csch[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Maple [A] time = 0.742, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx+c)}{(fx+e)(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)

[Out] int(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$2f\int \frac{1}{-i\,adf^2x^2-2i\,adefx-i\,ade^2+\left(adf^2x^2e^c+2\,adefxe^c+ade^2e^c\right)e^{(dx)}}\,dx+\frac{2}{-i\,adfx-i\,ade+\left(adfxe^c+adee^c\right)e^{(dx)}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*f*integrate(1/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2*
e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) + 2/(-I*a*d*f*x - I*a*d*e
 + (a*d*f*x*e^c + a*d*e*e^c)*e^(d*x)) + 2*integrate(1/2/(a*f*x + a*e + (a*f
*x*e^c + a*e*e^c)*e^(d*x)), x) + 2*integrate(-1/2/(a*f*x + a*e - (a*f*x*e^c
 + a*e*e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(-i\,adfx-i\,ade+\left(adfx+ade\right)e^{(dx+c)}\right)\mathrm{integral}\left(\frac{2\left(dfx+de+f\right)e^{(2\,dx+2\,c)}+\left(-2i\,dfx-2i\,de\right)e^{(dx+c)}-2\,f}{i\,adf^2x^2+2i\,adefx+i\,ade^2+\left(adf^2x^2+2\,adefx+ade^2\right)e^{(3\,dx+3\,c)}+\left(-i\,adf^2x^2-2i\,adefx-i\,ade^2\right)e^{(2\,dx\cdots}}\cdots\right.}{-i\,adfx-i\,ade+\left(adfx+ade\right)e^{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))*integral((2*(d*f*x
+ d*e + f)*e^(2*d*x + 2*c) + (-2*I*d*f*x - 2*I*d*e)*e^(d*x + c) - 2*f)/(I*a
*d*f^2*x^2 + 2*I*a*d*e*f*x + I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e
^2)*e^(3*d*x + 3*c) + (-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2)*e^(2*d*x
 + 2*c) - (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c)), x) + 2)/(-I*a
*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx+c)}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(csch(d*x + c)/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

3.210 $\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Csch[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.050252, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Csch[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Mathematica [A] time = 57.7485, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Csch[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Maple [A] time = 1.383, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx+c)}{\left(fx+e\right)^2(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$4f\int \frac{1}{-i\,adf^3x^3 - 3i\,adef^2x^2 - 3i\,ade^2fx - i\,ade^3 + \left(adf^3x^3e^c + 3\,adef^2x^2e^c + 3\,ade^2fxe^c + ade^3e^c\right)e^{(dx)}}\,dx + \frac{}{-i\,adf^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 4*f*integrate(1/(-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x - I*a
*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c + a*d*e
^3*e^c)*e^(d*x)), x) + 2/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d
*f^2*x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)) + 2*integrate(1/2/(a
*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2*e^c + 2*a*e*f*x*e^c + a*e^2*e^c)*
e^(d*x)), x) + 2*integrate(-1/2/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 - (a*f^2*x^2
*e^c + 2*a*e*f*x*e^c + a*e^2*e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(-i\,adf^2x^2 - 2i\,adefx - i\,ade^2 + \left(adf^2x^2 + 2\,adefx + ade^2\right)e^{(dx+c)}\right)\mathrm{integral}\left(\frac{}{i\,adf^3x^3+3i\,adef^2x^2+3i\,ade^2fx+i\,ade^3+\left(adf^3x^3+3\,ad\right.}\right.}{-i\,adf^2x^2 - 2i\,adefx - i\,ade^2 + \left(adf^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x +
 a*d*e^2)*e^(d*x + c))*integral((2*(d*f*x + d*e + 2*f)*e^(2*d*x + 2*c) + (-
2*I*d*f*x - 2*I*d*e)*e^(d*x + c) - 4*f)/(I*a*d*f^3*x^3 + 3*I*a*d*e*f^2*x^2
+ 3*I*a*d*e^2*f*x + I*a*d*e^3 + (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*
f*x + a*d*e^3)*e^(3*d*x + 3*c) + (-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*
a*d*e^2*f*x - I*a*d*e^3)*e^(2*d*x + 2*c) - (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 +
 3*a*d*e^2*f*x + a*d*e^3)*e^(d*x + c)), x) + 2)/(-I*a*d*f^2*x^2 - 2*I*a*d*e
*f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.211 $\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=419

$$\frac{12f^2(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{3f^2(e+fx)\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3}-\frac{6if^2(e+fx)\text{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3}+\frac{6if^2(e+$$

```
[Out] (-2*(e + f*x)^3)/(a*d) + ((2*I)*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (
(e + f*x)^3*Coth[c + d*x])/(a*d) + (6*f*(e + f*x)^2*Log[1 + I*E^(c + d*x)])
/(a*d^2) + (3*f*(e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a*d^2) + ((3*I)*f*(e
 + f*x)^2*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (12*f^2*(e + f*x)*PolyLog[2,
(-I)*E^(c + d*x)])/(a*d^3) - ((3*I)*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)])/
(a*d^2) + (3*f^2*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a*d^3) - ((6*I)*f^
2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) - (12*f^3*PolyLog[3, (-I)*E^(
c + d*x)])/(a*d^4) + ((6*I)*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a*d^3)
- (3*f^3*PolyLog[3, E^(2*(c + d*x))])/(2*a*d^4) + ((6*I)*f^3*PolyLog[4, -E^
(c + d*x)])/(a*d^4) - ((6*I)*f^3*PolyLog[4, E^(c + d*x)])/(a*d^4) - ((e + f
*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rubi [A] time = 0.805277, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {5575, 4184, 3716, 2190, 2531, 2282, 6589, 4182, 6609, 3318}

$$\frac{12f^2(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{3f^2(e+fx)\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3}-\frac{6if^2(e+fx)\text{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3}+\frac{6if^2(e+$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```


```
[Out] (-2*(e + f*x)^3)/(a*d) + ((2*I)*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (
(e + f*x)^3*Coth[c + d*x])/(a*d) + (6*f*(e + f*x)^2*Log[1 + I*E^(c + d*x)])
/(a*d^2) + (3*f*(e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a*d^2) + ((3*I)*f*(e
 + f*x)^2*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (12*f^2*(e + f*x)*PolyLog[2,
(-I)*E^(c + d*x)])/(a*d^3) - ((3*I)*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)])/
(a*d^2) + (3*f^2*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a*d^3) - ((6*I)*f^
2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) - (12*f^3*PolyLog[3, (-I)*E^(
c + d*x)])/(a*d^4) + ((6*I)*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a*d^3)
- (3*f^3*PolyLog[3, E^(2*(c + d*x))])/(2*a*d^4) + ((6*I)*f^3*PolyLog[4, -E^
(c + d*x)])/(a*d^4) - ((6*I)*f^3*PolyLog[4, E^(c + d*x)])/(a*d^4) - ((e + f
*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```


Rule 5575

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a +
b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
```

```
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
```

```
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\left(i\int \frac{(e+fx)^3\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx\right)+\frac{\int (e+fx)^3\text{csch}^2(c+dx)\,dx}{a}$$

$$= -\frac{(e+fx)^3\coth(c+dx)}{ad}-\frac{i\int (e+fx)^3\text{csch}(c+dx)\,dx}{a}+\frac{(3f)\int (e+fx)^2\coth(c+d\ldots}{ad}$$

$$= -\frac{(e+fx)^3}{ad}+\frac{2i(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^3\coth(c+dx)}{ad}-\frac{\int (e+fx)^3\csc^2\ldots}{\ldots}$$

$$= -\frac{(e+fx)^3}{ad}+\frac{2i(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^3\coth(c+dx)}{ad}+\frac{3f(e+fx)^2\log\ldots}{ad^2}$$

$$= -\frac{2(e+fx)^3}{ad}+\frac{2i(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^3\coth(c+dx)}{ad}+\frac{3f(e+fx)^2\log\ldots}{ad\ldots}$$

$$= -\frac{2(e+fx)^3}{ad}+\frac{2i(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^3\coth(c+dx)}{ad}+\frac{6f(e+fx)^2\log\ldots}{ad^2}$$

$$= -\frac{2(e+fx)^3}{ad}+\frac{2i(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^3\coth(c+dx)}{ad}+\frac{6f(e+fx)^2\log\ldots}{ad^2}$$

$$= -\frac{2(e+fx)^3}{ad}+\frac{2i(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^3\coth(c+dx)}{ad}+\frac{6f(e+fx)^2\log\ldots}{ad^2}$$

$$= -\frac{2(e+fx)^3}{ad}+\frac{2i(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^3\coth(c+dx)}{ad}+\frac{6f(e+fx)^2\log\ldots}{ad^2}$$

Mathematica [B] time = 17.2794, size = 1042, normalized size = 2.49

$$-\frac{2i\left(d^3(e+fx)^3+3d^2(1+ie^c)f\log\left(1-ie^{-c-dx}\right)(e+fx)^2+6i(i-e^c)f^2\left(d(e+fx)\text{PolyLog}\left(2,ie^{-c-dx}\right)+f\text{PolyLog}\ldots\right.\right.}{ad^4(-i+e^c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-2*I)*(d^3*(e + f*x)^3 + 3*d^2*(1 + I*E^c)*f*(e + f*x)^2*Log[1 - I*E^(-c
- d*x)] + (6*I)*(I - E^c)*f^2*(d*(e + f*x)*PolyLog[2, I*E^(-c - d*x)] + f*P
olyLog[3, I*E^(-c - d*x)])))/(a*d^4*(-I + E^c)) + (-(d^3*(e + f*x)^3*(-1 +
Coth[c])) + I*d^2*e^2*(d*e + (3*I)*f)*(d*x - Log[1 - Cosh[c + d*x] - Sinh[c
 + d*x]]) + 3*d^2*e*f*(I*d*e + 2*f)*x*Log[1 + Cosh[c + d*x] - Sinh[c + d*x]
] + 3*d^2*f^2*(I*d*e + f)*x^2*Log[1 + Cosh[c + d*x] - Sinh[c + d*x]] + I*d^
3*f^3*x^3*Log[1 + Cosh[c + d*x] - Sinh[c + d*x]] + 3*d^2*e*f*((-I)*d*e + 2*
f)*x*Log[1 - Cosh[c + d*x] + Sinh[c + d*x]] + 3*d^2*f^2*((-I)*d*e + f)*x^2*
Log[1 - Cosh[c + d*x] + Sinh[c + d*x]] - I*d^3*f^3*x^3*Log[1 - Cosh[c + d*x
] + Sinh[c + d*x]] - I*d^2*e^2*(d*e - (3*I)*f)*(d*x - Log[1 + Cosh[c + d*x]
 + Sinh[c + d*x]]) + (3*I)*d*e*(d*e + (2*I)*f)*f*PolyLog[2, Cosh[c + d*x] -
 Sinh[c + d*x]] - (3*I)*d*e*(d*e - (2*I)*f)*f*PolyLog[2, -Cosh[c + d*x] + S
inh[c + d*x]] + (6*I)*(d*e + I*f)*f^2*(d*x*PolyLog[2, Cosh[c + d*x] - Sinh[
```

```
c + d*x]] + PolyLog[3, Cosh[c + d*x] - Sinh[c + d*x]]) - 6*f^2*(I*d*e + f)*
(d*x*PolyLog[2, -Cosh[c + d*x] + Sinh[c + d*x]] + PolyLog[3, -Cosh[c + d*x]
 + Sinh[c + d*x]]) + (3*I)*f^3*(d^2*x^2*PolyLog[2, Cosh[c + d*x] - Sinh[c +
 d*x]] + 2*(d*x*PolyLog[3, Cosh[c + d*x] - Sinh[c + d*x]] + PolyLog[4, Cosh
[c + d*x] - Sinh[c + d*x]])) - (3*I)*f^3*(d^2*x^2*PolyLog[2, -Cosh[c + d*x]
 + Sinh[c + d*x]] + 2*(d*x*PolyLog[3, -Cosh[c + d*x] + Sinh[c + d*x]] + Pol
yLog[4, -Cosh[c + d*x] + Sinh[c + d*x]])))/(a*d^4) + (Sech[c/2]*Sech[c/2 +
(d*x)/2]*(-(e^3*Sinh[(d*x)/2]) - 3*e^2*f*x*Sinh[(d*x)/2] - 3*e*f^2*x^2*Sinh
[(d*x)/2] - f^3*x^3*Sinh[(d*x)/2]))/(2*a*d) + (Csch[c/2]*Csch[c/2 + (d*x)/2
]*(e^3*Sinh[(d*x)/2] + 3*e^2*f*x*Sinh[(d*x)/2] + 3*e*f^2*x^2*Sinh[(d*x)/2]
+ f^3*x^3*Sinh[(d*x)/2]))/(2*a*d) - (2*(e^3*Sinh[(d*x)/2] + 3*e^2*f*x*Sinh[
(d*x)/2] + 3*e*f^2*x^2*Sinh[(d*x)/2] + f^3*x^3*Sinh[(d*x)/2]))/(a*d*(Cosh[c
/2] + I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2]))
```

Maple [B] time = 0.266, size = 1535, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 12*f^2/d^2/a*e*ln(1+I*exp(d*x+c))*x+12*f^2/d^3/a*e*ln(1+I*exp(d*x+c))*c+24*
f^2/d^3/a*e*c*ln(exp(d*x+c))-12*f^2/d^3/a*e*c*ln(exp(d*x+c)-I)-24*f^2/d^2/a
*e*c*x+8*f^3/d^4/a*c^3-4*f^3/d/a*x^3-6*f^3*polylog(3,-exp(d*x+c))/a/d^4-6*f
^3*polylog(3,exp(d*x+c))/a/d^4+6*I*f^3*polylog(4,-exp(d*x+c))/a/d^4-6*I*f^3
*polylog(4,exp(d*x+c))/a/d^4-12*f^3*polylog(3,-I*exp(d*x+c))/a/d^4-2*I*(f^3
*x^3*exp(2*d*x+2*c)+3*e*f^2*x^2*exp(2*d*x+2*c)+3*e^2*f*x*exp(2*d*x+2*c)-2*f
^3*x^3-I*exp(d*x+c)*f^3*x^3+e^3*exp(2*d*x+2*c)-6*e*f^2*x^2-3*I*exp(d*x+c)*e
*f^2*x^2-6*e^2*f*x-3*I*exp(d*x+c)*e^2*f*x-2*e^3-I*exp(d*x+c)*e^3)/(exp(2*d*
x+2*c)-1)/(exp(d*x+c)-I)/d/a+12*f^3/d^3/a*c^2*x-12*f/d^2/a*ln(exp(d*x+c))*e
^2+6*f^3/d^2/a*ln(1+I*exp(d*x+c))*x^2-6*f^3/d^4/a*ln(1+I*exp(d*x+c))*c^2-12
*f^2/d^3/a*e*c^2+6*f/d^2/a*ln(exp(d*x+c)-I)*e^2+6*f^3/d^4/a*c^2*ln(exp(d*x+
c)-I)-12*f^2/d/a*e*x^2-12*f^3/d^4/a*c^2*ln(exp(d*x+c))+12*f^3/d^3/a*polylog
(2,-I*exp(d*x+c))*x+12*f^2/d^3/a*e*polylog(2,-I*exp(d*x+c))+I/d/a*e^3*ln(ex
p(d*x+c)+1)+6/d^3/a*f^3*polylog(2,-exp(d*x+c))*x+6/d^3/a*f^3*polylog(2,exp(
d*x+c))*x+3/d^2/a*f^3*ln(exp(d*x+c)+1)*x^2+3/d^2/a*f^3*ln(1-exp(d*x+c))*x^2
-3/d^4/a*f^3*c^2*ln(1-exp(d*x+c))+3/d^2/a*e^2*f*ln(exp(d*x+c)-1)+3/d^2/a*e^
2*f*ln(exp(d*x+c)+1)+6/d^3/a*e*f^2*polylog(2,exp(d*x+c))+6/d^3/a*e*f^2*poly
log(2,-exp(d*x+c))+3/d^4/a*f^3*c^2*ln(exp(d*x+c)-1)-I/d/a*e^3*ln(exp(d*x+c)
-1)+3*I/d^2/a*e^2*f*c*ln(exp(d*x+c)-1)-3*I/d^3/a*e*f^2*c^2*ln(exp(d*x+c)-1)
-3*I/d^2/a*ln(1-exp(d*x+c))*c*e^2*f+6*I/d^2/a*e*f^2*polylog(2,-exp(d*x+c))*
x-3*I/d/a*ln(1-exp(d*x+c))*e^2*f*x+3*I/d/a*ln(exp(d*x+c)+1)*e^2*f*x+3*I/d^3
/a*e*f^2*c^2*ln(1-exp(d*x+c))-3*I/d/a*e*f^2*ln(1-exp(d*x+c))*x^2-6*I/d^2/a*
e*f^2*polylog(2,exp(d*x+c))*x+3*I/d/a*e*f^2*ln(exp(d*x+c)+1)*x^2+6/d^2/a*e*
f^2*ln(1-exp(d*x+c))*x-6/d^3/a*e*f^2*c*ln(exp(d*x+c)-1)+6/d^3/a*e*f^2*ln(1-
exp(d*x+c))*c+6/d^2/a*e*f^2*ln(exp(d*x+c)+1)*x+I/d^4/a*f^3*c^3*ln(exp(d*x+c
)-1)+I/d/a*f^3*ln(exp(d*x+c)+1)*x^3-3*I/d^2/a*e^2*f*polylog(2,exp(d*x+c))+3
*I/d^2/a*e^2*f*polylog(2,-exp(d*x+c))+6*I/d^3/a*e*f^2*polylog(3,exp(d*x+c))
-6*I/d^3/a*e*f^2*polylog(3,-exp(d*x+c))+6*I/d^3/a*f^3*polylog(3,exp(d*x+c))
*x+3*I/d^2/a*f^3*polylog(2,-exp(d*x+c))*x^2-6*I/d^3/a*f^3*polylog(3,-exp(d*
x+c))*x-I/d/a*f^3*ln(1-exp(d*x+c))*x^3-I/d^4/a*f^3*ln(1-exp(d*x+c))*c^3-3*I
/d^2/a*f^3*polylog(2,exp(d*x+c))*x^2
```

Maxima [B] time = 2.22416, size = 1268, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -e^3*(4*(e^(-d*x - c) - I*e^(-2*d*x - 2*c) + 2*I)/((2*a*e^(-d*x - c) - 2*I*
a*e^(-2*d*x - 2*c) - 2*a*e^(-3*d*x - 3*c) + 2*I*a)*d) - I*log(e^(-d*x - c)
+ 1)/(a*d) + I*log(e^(-d*x - c) - 1)/(a*d)) - 12*e^2*f*x/(a*d) + 3*e^2*f*lo
g(e^(d*x + c) + 1)/(a*d^2) + 6*e^2*f*log(e^(d*x + c) - I)/(a*d^2) + 3*e^2*f
*log(e^(d*x + c) - 1)/(a*d^2) + (4*I*f^3*x^3 + 12*I*e*f^2*x^2 + 12*I*e^2*f*
x - (2*I*f^3*x^3*e^(2*c) + 6*I*e*f^2*x^2*e^(2*c) + 6*I*e^2*f*x*e^(2*c))*e^(
2*d*x) - 2*(f^3*x^3*e^c + 3*e*f^2*x^2*e^c + 3*e^2*f*x*e^c)*e^(d*x))/(a*d*e^
(3*d*x + 3*c) - I*a*d*e^(2*d*x + 2*c) - a*d*e^(d*x + c) + I*a*d) + 12*(d*x*
log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*e*f^2/(a*d^3) + I*(d^3*x^3*
log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^
(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*f^3/(a*d^4) - I*(d^3*x^3*log(-e^(d
*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c))
 + 6*polylog(4, e^(d*x + c)))*f^3/(a*d^4) + 6*(d^2*x^2*log(I*e^(d*x + c) +
1) + 2*d*x*dilog(-I*e^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a*d^4
) - 3*(-I*d*e^2*f - 2*e*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)
))/(a*d^3) + 3*(-I*d*e^2*f + 2*e*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^
(d*x + c)))/(a*d^3) + (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x +
 c)) - 2*polylog(3, -e^(d*x + c)))*(3*I*d*e*f^2 + 3*f^3)/(a*d^4) - (d^2*x^2
*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c
)))*(3*I*d*e*f^2 - 3*f^3)/(a*d^4) - 1/4*(I*d^4*f^3*x^4 + (4*I*d*e*f^2 + 4*f
^3)*d^3*x^3 + (6*I*d^2*e^2*f + 12*d*e*f^2)*d^2*x^2)/(a*d^4) + 1/4*(I*d^4*f^
3*x^4 + (4*I*d*e*f^2 - 4*f^3)*d^3*x^3 + (6*I*d^2*e^2*f - 12*d*e*f^2)*d^2*x^
2)/(a*d^4) - 2*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2)/(a*d^4)
```

Fricas [C] time = 3.03668, size = 5860, normalized size = 13.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] (4*I*d^3*e^3 - 12*I*c*d^2*e^2*f + 12*I*c^2*d*e*f^2 - 4*I*c^3*f^3 + (12*I*d*
f^3*x + 12*I*d*e*f^2 + 12*(d*f^3*x + d*e*f^2)*e^(3*d*x + 3*c) + (-12*I*d*f^
3*x - 12*I*d*e*f^2)*e^(2*d*x + 2*c) - 12*(d*f^3*x + d*e*f^2)*e^(d*x + c))*d
ilog(-I*e^(d*x + c)) - (3*d^2*f^3*x^2 + 3*d^2*e^2*f - 6*I*d*e*f^2 + (6*d^2*
e*f^2 - 6*I*d*f^3)*x - (3*I*d^2*f^3*x^2 + 3*I*d^2*e^2*f + 6*d*e*f^2 - 6*(-I
*d^2*e*f^2 - d*f^3)*x)*e^(3*d*x + 3*c) - (3*d^2*f^3*x^2 + 3*d^2*e^2*f - 6*I
*d*e*f^2 + (6*d^2*e*f^2 - 6*I*d*f^3)*x)*e^(2*d*x + 2*c) - (-3*I*d^2*f^3*x^2
 - 3*I*d^2*e^2*f - 6*d*e*f^2 - 6*(I*d^2*e*f^2 + d*f^3)*x)*e^(d*x + c))*dilo
g(-e^(d*x + c)) + (3*d^2*f^3*x^2 + 3*d^2*e^2*f + 6*I*d*e*f^2 + (6*d^2*e*f^2
 + 6*I*d*f^3)*x + (-3*I*d^2*f^3*x^2 - 3*I*d^2*e^2*f + 6*d*e*f^2 - 6*(I*d^2*
e*f^2 - d*f^3)*x)*e^(3*d*x + 3*c) - (3*d^2*f^3*x^2 + 3*d^2*e^2*f + 6*I*d*e*
f^2 + (6*d^2*e*f^2 + 6*I*d*f^3)*x)*e^(2*d*x + 2*c) + (3*I*d^2*f^3*x^2 + 3*I
*d^2*e^2*f - 6*d*e*f^2 - 6*(-I*d^2*e*f^2 + d*f^3)*x)*e^(d*x + c))*dilog(e^(
d*x + c)) - 4*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*
f - 3*c^2*d*e*f^2 + c^3*f^3)*e^(3*d*x + 3*c) + (2*I*d^3*f^3*x^3 + 6*I*d^3*e
```

```
*f^2*x^2 + 6*I*d^3*e^2*f*x - 2*I*d^3*e^3 + 12*I*c*d^2*e^2*f - 12*I*c^2*d*e*
f^2 + 4*I*c^3*f^3)*e^(2*d*x + 2*c) + 2*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d
^3*e^2*f*x - d^3*e^3 + 6*c*d^2*e^2*f - 6*c^2*d*e*f^2 + 2*c^3*f^3)*e^(d*x +
c) - (d^3*f^3*x^3 + d^3*e^3 - 3*I*d^2*e^2*f + (3*d^3*e*f^2 - 3*I*d^2*f^3)*x
^2 + 3*(d^3*e^2*f - 2*I*d^2*e*f^2)*x - (I*d^3*f^3*x^3 + I*d^3*e^3 + 3*d^2*e
^2*f - 3*(-I*d^3*e*f^2 - d^2*f^3)*x^2 + (3*I*d^3*e^2*f + 6*d^2*e*f^2)*x)*e^
(3*d*x + 3*c) - (d^3*f^3*x^3 + d^3*e^3 - 3*I*d^2*e^2*f + (3*d^3*e*f^2 - 3*I
*d^2*f^3)*x^2 + 3*(d^3*e^2*f - 2*I*d^2*e*f^2)*x)*e^(2*d*x + 2*c) - (-I*d^3*
f^3*x^3 - I*d^3*e^3 - 3*d^2*e^2*f - 3*(I*d^3*e*f^2 + d^2*f^3)*x^2 + (-3*I*d
^3*e^2*f - 6*d^2*e*f^2)*x)*e^(d*x + c))*log(e^(d*x + c) + 1) + (6*I*d^2*e^2
*f - 12*I*c*d*e*f^2 + 6*I*c^2*f^3 + 6*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*e
^(3*d*x + 3*c) + (-6*I*d^2*e^2*f + 12*I*c*d*e*f^2 - 6*I*c^2*f^3)*e^(2*d*x +
 2*c) - 6*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*e^(d*x + c))*log(e^(d*x + c)
- I) + (d^3*e^3 - (3*c - 3*I)*d^2*e^2*f + 3*(c^2 - 2*I*c)*d*e*f^2 - (c^3 -
3*I*c^2)*f^3 + (-I*d^3*e^3 - 3*(-I*c - 1)*d^2*e^2*f + (-3*I*c^2 - 6*c)*d*e*
f^2 + (I*c^3 + 3*c^2)*f^3)*e^(3*d*x + 3*c) - (d^3*e^3 - (3*c - 3*I)*d^2*e^2
*f + 3*(c^2 - 2*I*c)*d*e*f^2 - (c^3 - 3*I*c^2)*f^3)*e^(2*d*x + 2*c) + (I*d^
3*e^3 - 3*(I*c + 1)*d^2*e^2*f + (3*I*c^2 + 6*c)*d*e*f^2 + (-I*c^3 - 3*c^2)*
f^3)*e^(d*x + c))*log(e^(d*x + c) - 1) + (6*I*d^2*f^3*x^2 + 12*I*d^2*e*f^2*
x + 12*I*c*d*e*f^2 - 6*I*c^2*f^3 + 6*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e
*f^2 - c^2*f^3)*e^(3*d*x + 3*c) + (-6*I*d^2*f^3*x^2 - 12*I*d^2*e*f^2*x - 12
*I*c*d*e*f^2 + 6*I*c^2*f^3)*e^(2*d*x + 2*c) - 6*(d^2*f^3*x^2 + 2*d^2*e*f^2*
x + 2*c*d*e*f^2 - c^2*f^3)*e^(d*x + c))*log(I*e^(d*x + c) + 1) + (d^3*f^3*x
^3 + 3*c*d^2*e^2*f - 3*(c^2 - 2*I*c)*d*e*f^2 + (c^3 - 3*I*c^2)*f^3 + (3*d^3
*e*f^2 + 3*I*d^2*f^3)*x^2 + 3*(d^3*e^2*f + 2*I*d^2*e*f^2)*x + (-I*d^3*f^3*x
^3 - 3*I*c*d^2*e^2*f + (3*I*c^2 + 6*c)*d*e*f^2 + (-I*c^3 - 3*c^2)*f^3 - 3*(
I*d^3*e*f^2 - d^2*f^3)*x^2 + (-3*I*d^3*e^2*f + 6*d^2*e*f^2)*x)*e^(3*d*x + 3
*c) - (d^3*f^3*x^3 + 3*c*d^2*e^2*f - 3*(c^2 - 2*I*c)*d*e*f^2 + (c^3 - 3*I*c
^2)*f^3 + (3*d^3*e*f^2 + 3*I*d^2*f^3)*x^2 + 3*(d^3*e^2*f + 2*I*d^2*e*f^2)*x
)*e^(2*d*x + 2*c) + (I*d^3*f^3*x^3 + 3*I*c*d^2*e^2*f + (-3*I*c^2 - 6*c)*d*e
*f^2 + (I*c^3 + 3*c^2)*f^3 - 3*(-I*d^3*e*f^2 + d^2*f^3)*x^2 + (3*I*d^3*e^2*
f - 6*d^2*e*f^2)*x)*e^(d*x + c))*log(-e^(d*x + c) + 1) + (6*I*f^3*e^(3*d*x
+ 3*c) + 6*f^3*e^(2*d*x + 2*c) - 6*I*f^3*e^(d*x + c) - 6*f^3)*polylog(4, -e
^(d*x + c)) + (-6*I*f^3*e^(3*d*x + 3*c) - 6*f^3*e^(2*d*x + 2*c) + 6*I*f^3*e
^(d*x + c) + 6*f^3)*polylog(4, e^(d*x + c)) - (12*f^3*e^(3*d*x + 3*c) - 12*
I*f^3*e^(2*d*x + 2*c) - 12*f^3*e^(d*x + c) + 12*I*f^3)*polylog(3, -I*e^(d*x
 + c)) + (6*d*f^3*x + 6*d*e*f^2 - 6*I*f^3 + (-6*I*d*f^3*x - 6*I*d*e*f^2 - 6
*f^3)*e^(3*d*x + 3*c) - 6*(d*f^3*x + d*e*f^2 - I*f^3)*e^(2*d*x + 2*c) + (6*
I*d*f^3*x + 6*I*d*e*f^2 + 6*f^3)*e^(d*x + c))*polylog(3, -e^(d*x + c)) - (6
*d*f^3*x + 6*d*e*f^2 + 6*I*f^3 - (6*I*d*f^3*x + 6*I*d*e*f^2 - 6*f^3)*e^(3*d
*x + 3*c) - 6*(d*f^3*x + d*e*f^2 + I*f^3)*e^(2*d*x + 2*c) - (-6*I*d*f^3*x -
 6*I*d*e*f^2 + 6*f^3)*e^(d*x + c))*polylog(3, e^(d*x + c)))/(a*d^4*e^(3*d*x
 + 3*c) - I*a*d^4*e^(2*d*x + 2*c) - a*d^4*e^(d*x + c) + I*a*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.212 $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=296

$$\frac{2if(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}-\frac{2if(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}+\frac{4f^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{f^2\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3}$$

```
[Out] (-2*(e + f*x)^2)/(a*d) + ((2*I)*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (
(e + f*x)^2*Coth[c + d*x])/(a*d) + (4*f*(e + f*x)*Log[1 + I*E^(c + d*x)])/(
a*d^2) + (2*f*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a*d^2) + ((2*I)*f*(e + f
*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (4*f^2*PolyLog[2, (-I)*E^(c + d*x)]
)/(a*d^3) - ((2*I)*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) + (f^2*Poly
Log[2, E^(2*(c + d*x))])/(a*d^3) - ((2*I)*f^2*PolyLog[3, -E^(c + d*x)])/(a*
d^3) + ((2*I)*f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) - ((e + f*x)^2*Tanh[c/2
+ (I/4)*Pi + (d*x)/2])/(a*d)
```

Rubi [A] time = 0.583369, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {5575, 4184, 3716, 2190, 2279, 2391, 4182, 2531, 2282, 6589, 3318}

$$\frac{2if(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}-\frac{2if(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}+\frac{4f^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{f^2\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```


```
[Out] (-2*(e + f*x)^2)/(a*d) + ((2*I)*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (
(e + f*x)^2*Coth[c + d*x])/(a*d) + (4*f*(e + f*x)*Log[1 + I*E^(c + d*x)])/(
a*d^2) + (2*f*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a*d^2) + ((2*I)*f*(e + f
*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (4*f^2*PolyLog[2, (-I)*E^(c + d*x)]
)/(a*d^3) - ((2*I)*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) + (f^2*Poly
Log[2, E^(2*(c + d*x))])/(a*d^3) - ((2*I)*f^2*PolyLog[3, -E^(c + d*x)])/(a*
d^3) + ((2*I)*f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) - ((e + f*x)^2*Tanh[c/2
+ (I/4)*Pi + (d*x)/2])/(a*d)
```


Rule 5575

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a +
b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
```

```
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(e+fx)^2\mathrm{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\left(i\int \frac{(e+fx)^2\mathrm{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx\right)+\frac{\int (e+fx)^2\mathrm{csch}^2(c+dx)\,dx}{a}$$
$$= -\frac{(e+fx)^2\coth(c+dx)}{ad}-\frac{i\int (e+fx)^2\mathrm{csch}(c+dx)\,dx}{a}+\frac{(2f)\int (e+fx)\coth(c+dx)\,dx}{ad}$$
$$= -\frac{(e+fx)^2}{ad}+\frac{2i(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^2\coth(c+dx)}{ad}-\frac{\int (e+fx)^2\csc^2\left(\frac{1}{2}\right.}{2a}$$
$$= -\frac{(e+fx)^2}{ad}+\frac{2i(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^2\coth(c+dx)}{ad}+\frac{2f(e+fx)\log\left(1-\right.}{ad^2}$$
$$= -\frac{2(e+fx)^2}{ad}+\frac{2i(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^2\coth(c+dx)}{ad}+\frac{2f(e+fx)\log\left(1\right.}{ad^2}$$
$$= -\frac{2(e+fx)^2}{ad}+\frac{2i(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^2\coth(c+dx)}{ad}+\frac{4f(e+fx)\log\left(1\right.}{ad^2}$$
$$= -\frac{2(e+fx)^2}{ad}+\frac{2i(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^2\coth(c+dx)}{ad}+\frac{4f(e+fx)\log\left(1\right.}{ad^2}$$
$$= -\frac{2(e+fx)^2}{ad}+\frac{2i(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)^2\coth(c+dx)}{ad}+\frac{4f(e+fx)\log\left(1\right.}{ad^2}$$

Mathematica [B] time = 13.0869, size = 715, normalized size = 2.42

$$\frac{-2\left(e^{2c}-1\right)f(f+ide)\mathrm{PolyLog}\left(2,-e^{-c-dx}\right)+2i\left(e^{2c}-1\right)f(de+if)\mathrm{PolyLog}\left(2,e^{-c-dx}\right)-2i\left(e^{2c}-1\right)f^2\left(dx\mathrm{PolyLog}\left(2,\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (2*((d*(e + f*x)*((-I)*d*(e + f*x) + 2*(-I + E^c)*f*Log[1 - I*E^(-c - d*x)]
))/(-I + E^c) - 2*f^2*PolyLog[2, I*E^(-c - d*x)]))/(a*d^3) + (-2*d^2*(e + f
*x)^2 + 2*d*(-1 + E^(2*c))*f*((-I)*d*e + f)*x*Log[1 - E^(-c - d*x)] - I*d^2
*(-1 + E^(2*c))*f^2*x^2*Log[1 - E^(-c - d*x)] + 2*d*(-1 + E^(2*c))*f*(I*d*e
 + f)*x*Log[1 + E^(-c - d*x)] + I*d^2*(-1 + E^(2*c))*f^2*x^2*Log[1 + E^(-c
- d*x)] + I*d*e*(-1 + E^(2*c))*(d*e + (2*I)*f)*(d*x - Log[1 - E^(c + d*x)])
 + d*e*(1 - E^(2*c))*(I*d*e + 2*f)*(d*x - Log[1 + E^(c + d*x)]) - 2*(-1 + E
^(2*c))*f*(I*d*e + f)*PolyLog[2, -E^(-c - d*x)] + (2*I)*(-1 + E^(2*c))*(d*e
 + I*f)*f*PolyLog[2, E^(-c - d*x)] - (2*I)*(-1 + E^(2*c))*f^2*(d*x*PolyLog[
2, -E^(-c - d*x)] + PolyLog[3, -E^(-c - d*x)]) + (2*I)*(-1 + E^(2*c))*f^2*(
d*x*PolyLog[2, E^(-c - d*x)] + PolyLog[3, E^(-c - d*x)]))/(a*d^3*(-1 + E^(2
*c))) + (Sech[c/2]*Sech[c/2 + (d*x)/2]*(-(e^2*Sinh[(d*x)/2]) - 2*e*f*x*Sinh
[(d*x)/2] - f^2*x^2*Sinh[(d*x)/2]))/(2*a*d) + (Csch[c/2]*Csch[c/2 + (d*x)/2
]*(e^2*Sinh[(d*x)/2] + 2*e*f*x*Sinh[(d*x)/2] + f^2*x^2*Sinh[(d*x)/2]))/(2*a
*d) - (2*(e^2*Sinh[(d*x)/2] + 2*e*f*x*Sinh[(d*x)/2] + f^2*x^2*Sinh[(d*x)/2]
))/(a*d*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)
/2]))
```

Maple [B] time = 0.151, size = 847, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -2*I/d^2/a*f^2*polylog(2,exp(d*x+c))*x+2*I/d^2/a*f^2*polylog(2,-exp(d*x+c))
*x-I/d^3/a*f^2*c^2*ln(exp(d*x+c)-1)+2*I/d^2/a*e*f*polylog(2,-exp(d*x+c))-2*
I/d^2/a*e*f*polylog(2,exp(d*x+c))+I/d^3/a*f^2*c^2*ln(1-exp(d*x+c))+I/d/a*f^
2*ln(exp(d*x+c)+1)*x^2-I/d/a*f^2*ln(1-exp(d*x+c))*x^2-2*I*(f^2*x^2*exp(2*d*
x+2*c)+2*e*f*x*exp(2*d*x+2*c)+e^2*exp(2*d*x+2*c)-2*x^2*f^2-I*exp(d*x+c)*f^2
*x^2-4*e*f*x-2*I*exp(d*x+c)*e*f*x-2*e^2-I*exp(d*x+c)*e^2)/(exp(2*d*x+2*c)-1
)/(exp(d*x+c)-I)/d/a+2*f^2*polylog(2,-exp(d*x+c))/a/d^3+2*f^2*polylog(2,exp
(d*x+c))/a/d^3-4*f^2*x^2/a/d+2*I*f^2*polylog(3,exp(d*x+c))/a/d^3-2*I*f^2*po
lylog(3,-exp(d*x+c))/a/d^3+4*f^2*polylog(2,-I*exp(d*x+c))/a/d^3-2*I/d/a*ln(
1-exp(d*x+c))*e*f*x+2*I/d/a*ln(exp(d*x+c)+1)*e*f*x-2*I/d^2/a*ln(1-exp(d*x+c
))*c*e*f+2*I/d^2/a*e*f*c*ln(exp(d*x+c)-1)-4*f^2/d^3/a*c^2+4*f/d^2/a*ln(exp(
d*x+c)-I)*e-8*f/d^2/a*ln(exp(d*x+c))*e-8*f^2/d^2/a*c*x+4*f^2/d^2/a*ln(1+I*e
xp(d*x+c))*x+4*f^2/d^3/a*ln(1+I*exp(d*x+c))*c-4*f^2/d^3/a*c*ln(exp(d*x+c)-I
)+8*f^2/d^3/a*c*ln(exp(d*x+c))+2/d^2/a*f^2*ln(exp(d*x+c)+1)*x+2/d^2/a*e*f*l
n(exp(d*x+c)-1)+2/d^2/a*e*f*ln(exp(d*x+c)+1)-2/d^3/a*f^2*c*ln(exp(d*x+c)-1)
+2/d^2/a*f^2*ln(1-exp(d*x+c))*x+2/d^3/a*f^2*ln(1-exp(d*x+c))*c+I/d/a*e^2*ln
(exp(d*x+c)+1)-I/d/a*e^2*ln(exp(d*x+c)-1)
```

Maxima [B] time = 2.0813, size = 814, normalized size = 2.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -e^2*(4*(e^(-d*x - c) - I*e^(-2*d*x - 2*c) + 2*I)/((2*a*e^(-d*x - c) - 2*I*
a*e^(-2*d*x - 2*c) - 2*a*e^(-3*d*x - 3*c) + 2*I*a)*d) - I*log(e^(-d*x - c)
+ 1)/(a*d) + I*log(e^(-d*x - c) - 1)/(a*d)) - 2*f^2*x^2/(a*d) - 8*e*f*x/(a*
d) + (4*I*f^2*x^2 + 8*I*e*f*x - (2*I*f^2*x^2*e^(2*c) + 4*I*e*f*x*e^(2*c))*e
^(2*d*x) - 2*(f^2*x^2*e^c + 2*e*f*x*e^c)*e^(d*x))/(a*d*e^(3*d*x + 3*c) - I*
a*d*e^(2*d*x + 2*c) - a*d*e^(d*x + c) + I*a*d) + 2*e*f*log(e^(d*x + c) + 1)
/(a*d^2) + 4*e*f*log(e^(d*x + c) - I)/(a*d^2) + 2*e*f*log(e^(d*x + c) - 1)/
(a*d^2) + I*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*p
olylog(3, -e^(d*x + c)))*f^2/(a*d^3) - I*(d^2*x^2*log(-e^(d*x + c) + 1) + 2
*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*f^2/(a*d^3) + 4*(d*x*l
og(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*f^2/(a*d^3) + (2*I*d*e*f + 2
*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a*d^3) - (2*I*d*e*f
 - 2*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a*d^3) - 1/3*(I
*d^3*f^2*x^3 + (3*I*d*e*f + 3*f^2)*d^2*x^2)/(a*d^3) + 1/3*(I*d^3*f^2*x^3 +
(3*I*d*e*f - 3*f^2)*d^2*x^2)/(a*d^3)
```

Fricas [C] time = 2.76264, size = 3237, normalized size = 10.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] (4*I*d^2*e^2 - 8*I*c*d*e*f + 4*I*c^2*f^2 + (4*f^2*e^(3*d*x + 3*c) - 4*I*f^2
*e^(2*d*x + 2*c) - 4*f^2*e^(d*x + c) + 4*I*f^2)*dilog(-I*e^(d*x + c)) - (2*
d*f^2*x + 2*d*e*f - 2*I*f^2 - (2*I*d*f^2*x + 2*I*d*e*f + 2*f^2)*e^(3*d*x +
3*c) - 2*(d*f^2*x + d*e*f - I*f^2)*e^(2*d*x + 2*c) - (-2*I*d*f^2*x - 2*I*d*
e*f - 2*f^2)*e^(d*x + c))*dilog(-e^(d*x + c)) + (2*d*f^2*x + 2*d*e*f + 2*I*
f^2 + (-2*I*d*f^2*x - 2*I*d*e*f + 2*f^2)*e^(3*d*x + 3*c) - 2*(d*f^2*x + d*e
*f + I*f^2)*e^(2*d*x + 2*c) + (2*I*d*f^2*x + 2*I*d*e*f - 2*f^2)*e^(d*x + c)
)*dilog(e^(d*x + c)) - 4*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*
e^(3*d*x + 3*c) + (2*I*d^2*f^2*x^2 + 4*I*d^2*e*f*x - 2*I*d^2*e^2 + 8*I*c*d*
e*f - 4*I*c^2*f^2)*e^(2*d*x + 2*c) + 2*(d^2*f^2*x^2 + 2*d^2*e*f*x - d^2*e^2
 + 4*c*d*e*f - 2*c^2*f^2)*e^(d*x + c) - (d^2*f^2*x^2 + d^2*e^2 - 2*I*d*e*f
+ (2*d^2*e*f - 2*I*d*f^2)*x - (I*d^2*f^2*x^2 + I*d^2*e^2 + 2*d*e*f - 2*(-I*
d^2*e*f - d*f^2)*x)*e^(3*d*x + 3*c) - (d^2*f^2*x^2 + d^2*e^2 - 2*I*d*e*f +
(2*d^2*e*f - 2*I*d*f^2)*x)*e^(2*d*x + 2*c) - (-I*d^2*f^2*x^2 - I*d^2*e^2 -
2*d*e*f - 2*(I*d^2*e*f + d*f^2)*x)*e^(d*x + c))*log(e^(d*x + c) + 1) + (4*I
*d*e*f - 4*I*c*f^2 + 4*(d*e*f - c*f^2)*e^(3*d*x + 3*c) + (-4*I*d*e*f + 4*I*
c*f^2)*e^(2*d*x + 2*c) - 4*(d*e*f - c*f^2)*e^(d*x + c))*log(e^(d*x + c) - I
) + (d^2*e^2 - (2*c - 2*I)*d*e*f + (c^2 - 2*I*c)*f^2 + (-I*d^2*e^2 - 2*(-I*
c - 1)*d*e*f + (-I*c^2 - 2*c)*f^2)*e^(3*d*x + 3*c) - (d^2*e^2 - (2*c - 2*I)
*d*e*f + (c^2 - 2*I*c)*f^2)*e^(2*d*x + 2*c) + (I*d^2*e^2 - 2*(I*c + 1)*d*e*
f + (I*c^2 + 2*c)*f^2)*e^(d*x + c))*log(e^(d*x + c) - 1) + (4*I*d*f^2*x + 4
*I*c*f^2 + 4*(d*f^2*x + c*f^2)*e^(3*d*x + 3*c) + (-4*I*d*f^2*x - 4*I*c*f^2)
*e^(2*d*x + 2*c) - 4*(d*f^2*x + c*f^2)*e^(d*x + c))*log(I*e^(d*x + c) + 1)
+ (d^2*f^2*x^2 + 2*c*d*e*f - (c^2 - 2*I*c)*f^2 + (2*d^2*e*f + 2*I*d*f^2)*x
+ (-I*d^2*f^2*x^2 - 2*I*c*d*e*f + (I*c^2 + 2*c)*f^2 - 2*(I*d^2*e*f - d*f^2)
*x)*e^(3*d*x + 3*c) - (d^2*f^2*x^2 + 2*c*d*e*f - (c^2 - 2*I*c)*f^2 + (2*d^2
*e*f + 2*I*d*f^2)*x)*e^(2*d*x + 2*c) + (I*d^2*f^2*x^2 + 2*I*c*d*e*f + (-I*c
^2 - 2*c)*f^2 - 2*(-I*d^2*e*f + d*f^2)*x)*e^(d*x + c))*log(-e^(d*x + c) + 1
) + (-2*I*f^2*e^(3*d*x + 3*c) - 2*f^2*e^(2*d*x + 2*c) + 2*I*f^2*e^(d*x + c)
 + 2*f^2)*polylog(3, -e^(d*x + c)) + (2*I*f^2*e^(3*d*x + 3*c) + 2*f^2*e^(2*
d*x + 2*c) - 2*I*f^2*e^(d*x + c) - 2*f^2)*polylog(3, e^(d*x + c)))/(a*d^3*e
^(3*d*x + 3*c) - I*a*d^3*e^(2*d*x + 2*c) - a*d^3*e^(d*x + c) + I*a*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")

[Out] Timed out
```

3.213 $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=163

$$\frac{if\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}-\frac{if\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}+\frac{f\log(\sinh(c+dx))}{ad^2}+\frac{2f\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{ad^2}+\frac{2i(e+fx)\tanh^{-}}{ad}$$

```
[Out] ((2*I)*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d) - ((e + f*x)*Coth[c + d*x])/(a
*d) + (2*f*Log[Cosh[c/2 + (I/4)*Pi + (d*x)/2]])/(a*d^2) + (f*Log[Sinh[c + d
*x]])/(a*d^2) + (I*f*PolyLog[2, -E^(c + d*x)])/(a*d^2) - (I*f*PolyLog[2, E^
(c + d*x)])/(a*d^2) - ((e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rubi [A] time = 0.228043, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {5575, 4184, 3475, 4182, 2279, 2391, 3318}

$$\frac{if\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}-\frac{if\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}+\frac{f\log(\sinh(c+dx))}{ad^2}+\frac{2f\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{ad^2}+\frac{2i(e+fx)\tanh^{-}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((2*I)*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d) - ((e + f*x)*Coth[c + d*x])/(a
*d) + (2*f*Log[Cosh[c/2 + (I/4)*Pi + (d*x)/2]])/(a*d^2) + (f*Log[Sinh[c + d
*x]])/(a*d^2) + (I*f*PolyLog[2, -E^(c + d*x)])/(a*d^2) - (I*f*PolyLog[2, E^
(c + d*x)])/(a*d^2) - ((e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rule 5575

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a +
b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\left(i\int \frac{(e+fx)\text{csch}(c+dx)}{a+ia\sinh(c+dx)}\,dx\right)+\frac{\int (e+fx)\text{csch}^2(c+dx)\,dx}{a}$$

$$= -\frac{(e+fx)\coth(c+dx)}{ad}-\frac{i\int (e+fx)\text{csch}(c+dx)\,dx}{a}+\frac{f\int \coth(c+dx)\,dx}{ad}-\int \frac{\ldots}{a+\ldots}$$

$$= \frac{2i(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)\coth(c+dx)}{ad}+\frac{f\log(\sinh(c+dx))}{ad^2}-\frac{\int (e+fx)\ldots}{\ldots}$$

$$= \frac{2i(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)\coth(c+dx)}{ad}+\frac{f\log(\sinh(c+dx))}{ad^2}-\frac{(e+fx)\text{ta}\ldots}{\ldots}$$

$$= \frac{2i(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}-\frac{(e+fx)\coth(c+dx)}{ad}+\frac{2f\log\left(\cosh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)\right)}{ad^2}+\frac{f\ldots}{\ldots}$$

Mathematica [B] time = 5.43604, size = 454, normalized size = 2.79

$$\frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(-2if\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(\text{PolyLog}\left(2,-e^{-c-dx}\right)-\text{PolyL}\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(-(d*(e + f*x)*Cosh[(c + d*x)/2]
*(I + Coth[(c + d*x)/2])) + (4*I)*f*ArcTan[Tanh[(c + d*x)/2]]*(Cosh[(c + d*
x)/2] + I*Sinh[(c + d*x)/2]) + 2*f*Log[Cosh[c + d*x]]*(Cosh[(c + d*x)/2] +
I*Sinh[(c + d*x)/2]) + 2*f*Log[Sinh[c + d*x]]*(Cosh[(c + d*x)/2] + I*Sinh[(
c + d*x)/2]) + (2*I)*c*f*Log[Tanh[(c + d*x)/2]]*(Cosh[(c + d*x)/2] + I*Sinh
[(c + d*x)/2]) - (2*I)*f*((c + d*x)*(Log[1 - E^(-c - d*x)] - Log[1 + E^(-c
- d*x)]) + PolyLog[2, -E^(-c - d*x)] - PolyLog[2, E^(-c - d*x)])*(Cosh[(c +
 d*x)/2] + I*Sinh[(c + d*x)/2]) - 4*d*(e + f*x)*Sinh[(c + d*x)/2] + 2*f*(c
+ d*x)*((-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]) + 2*d*e*Log[Tanh[(c + d
*x)/2]]*((-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]) - I*d*(e + f*x)*Sinh[(
c + d*x)/2]*(-I + Tanh[(c + d*x)/2])))/(2*d^2*(a + I*a*Sinh[c + d*x]))
```

Maple [B] time = 0.151, size = 316, normalized size = 1.9

$$\frac{-2 i \left(f x \mathrm{e}^{2 d x+2 c}+e \mathrm{e}^{2 d x+2 c}-2 f x-i \mathrm{e}^{d x+c} f x-2 e-i \mathrm{e}^{d x+c} e\right)}{\left(\mathrm{e}^{2 d x+2 c}-1\right) \left(\mathrm{e}^{d x+c}-i\right) d a}-\frac{i f \,\mathrm{polylog}\left(2, \mathrm{e}^{d x+c}\right)}{a d^{2}}+\frac{i f \,\mathrm{polylog}\left(2, -\mathrm{e}^{d x+c}\right)}{a d^{2}}-4 \frac{f \ln \left(\mathrm{e}^{d\ldots}\right.}{a d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -2*I*(f*x*exp(2*d*x+2*c)+e*exp(2*d*x+2*c)-2*f*x-I*exp(d*x+c)*f*x-2*e-I*exp(
d*x+c)*e)/(exp(2*d*x+2*c)-1)/(exp(d*x+c)-I)/d/a-I*f*polylog(2,exp(d*x+c))/a
/d^2+I*f*polylog(2,-exp(d*x+c))/a/d^2-4/d^2/a*f*ln(exp(d*x+c))+1/d^2/a*f*ln
(exp(d*x+c)-1)+1/d^2/a*f*ln(exp(d*x+c)+1)-I/d/a*e*ln(exp(d*x+c)-1)+I/d/a*e*
ln(exp(d*x+c)+1)-I/d/a*ln(1-exp(d*x+c))*f*x-I/d^2/a*ln(1-exp(d*x+c))*c*f+I/
d/a*ln(exp(d*x+c)+1)*f*x+2*f/a/d^2*ln(exp(d*x+c)-I)+I/d^2/a*f*c*ln(exp(d*x+
c)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\left(4 i d \int \frac{x}{4 \left(a d e^{(d x+c)}+a d\right)}\,dx+4 i d \int \frac{x}{4 \left(a d e^{(d x+c)}-a d\right)}\,dx+\frac{4 \left(x e^{(3 d x+3 c)}-i x\right)}{2 a d e^{(3 d x+3 c)}-2 i a d e^{(2 d x+2 c)}-2 a d e^{(d x+c)}+2 i a d}+\frac{2 (d x\ldots}{a d\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(4*I*d*integrate(1/4*x/(a*d*e^(d*x + c) + a*d), x) + 4*I*d*integrate(1/4*x
/(a*d*e^(d*x + c) - a*d), x) + 4*(x*e^(3*d*x + 3*c) - I*x)/(2*a*d*e^(3*d*x
+ 3*c) - 2*I*a*d*e^(2*d*x + 2*c) - 2*a*d*e^(d*x + c) + 2*I*a*d) + 2*(d*x +
c)/(a*d^2) - 2*log((e^(d*x + c) - I)*e^(-c))/(a*d^2) - log(e^(d*x + c) + 1)
/(a*d^2) - log(e^(d*x + c) - 1)/(a*d^2))*f - e*(4*(e^(-d*x - c) - I*e^(-2*d
*x - 2*c) + 2*I)/((2*a*e^(-d*x - c) - 2*I*a*e^(-2*d*x - 2*c) - 2*a*e^(-3*d*
x - 3*c) + 2*I*a)*d) - I*log(e^(-d*x - c) + 1)/(a*d) + I*log(e^(-d*x - c) -
 1)/(a*d))
```

Fricas [B] time = 2.59178, size = 1272, normalized size = 7.8

$$\frac{4 i d e-2 i c f+\left(i f e^{(3 d x+3 c)}+f e^{(2 d x+2 c)}-i f e^{(d x+c)}-f\right) \mathrm{Li}_2\left(-e^{(d x+c)}\right)+\left(-i f e^{(3 d x+3 c)}-f e^{(2 d x+2 c)}+i f e^{(d x+c)}+f\right) \mathrm{Li}_2\left(e^{(\ldots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (4*I*d*e - 2*I*c*f + (I*f*e^(3*d*x + 3*c) + f*e^(2*d*x + 2*c) - I*f*e^(d*x
+ c) - f)*dilog(-e^(d*x + c)) + (-I*f*e^(3*d*x + 3*c) - f*e^(2*d*x + 2*c) +
 I*f*e^(d*x + c) + f)*dilog(e^(d*x + c)) - 2*(2*d*f*x + c*f)*e^(3*d*x + 3*c
) + (2*I*d*f*x - 2*I*d*e + 2*I*c*f)*e^(2*d*x + 2*c) + 2*(d*f*x - d*e + c*f)
*e^(d*x + c) - (d*f*x + d*e - (I*d*f*x + I*d*e + f)*e^(3*d*x + 3*c) - (d*f*
x + d*e - I*f)*e^(2*d*x + 2*c) - (-I*d*f*x - I*d*e - f)*e^(d*x + c) - I*f)*
log(e^(d*x + c) + 1) + (2*f*e^(3*d*x + 3*c) - 2*I*f*e^(2*d*x + 2*c) - 2*f*e
^(d*x + c) + 2*I*f)*log(e^(d*x + c) - I) + (d*e - (c - I)*f + (-I*d*e + (I*
c + 1)*f)*e^(3*d*x + 3*c) - (d*e - (c - I)*f)*e^(2*d*x + 2*c) + (I*d*e + (-
```

```
I*c - 1)*f)*e^(d*x + c))*log(e^(d*x + c) - 1) + (d*f*x + c*f + (-I*d*f*x -
I*c*f)*e^(3*d*x + 3*c) - (d*f*x + c*f)*e^(2*d*x + 2*c) + (I*d*f*x + I*c*f)*
e^(d*x + c))*log(-e^(d*x + c) + 1))/(a*d^2*e^(3*d*x + 3*c) - I*a*d^2*e^(2*d
*x + 2*c) - a*d^2*e^(d*x + c) + I*a*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\operatorname{csch}(dx+c)^2}{i\,a\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*csch(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

3.214 $\int \frac{\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=57

$$-\frac{2\coth(c+dx)}{ad}+\frac{i\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}$$

```
[Out] (I*ArcTanh[Cosh[c + d*x]])/(a*d) - (2*Coth[c + d*x])/(a*d) + Coth[c + d*x]/
(d*(a + I*a*Sinh[c + d*x]))
```

Rubi [A] time = 0.0856202, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2768, 2748, 3767, 8, 3770}

$$-\frac{2\coth(c+dx)}{ad}+\frac{i\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (I*ArcTanh[Cosh[c + d*x]])/(a*d) - (2*Coth[c + d*x])/(a*d) + Coth[c + d*x]/
(d*(a + I*a*Sinh[c + d*x]))
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))} - \frac{\int \operatorname{csch}^2(c+dx)(-2a+ia\sinh(c+dx))\,dx}{a^2}\\ &= \frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))} - \frac{i\int \operatorname{csch}(c+dx)\,dx}{a} + \frac{2\int \operatorname{csch}^2(c+dx)\,dx}{a}\\ &= \frac{i\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))} - \frac{(2i)\operatorname{Subst}(\int 1\,dx, x, -i\coth(c+dx))}{ad}\\ &= \frac{i\tanh^{-1}(\cosh(c+dx))}{ad} - \frac{2\coth(c+dx)}{ad} + \frac{\coth(c+dx)}{d(a+ia\sinh(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.205343, size = 61, normalized size = 1.07

$$-\frac{\operatorname{sech}(c+dx)\left(2\sinh(c+dx)+\operatorname{csch}(c+dx)-i\sqrt{\cosh^2(c+dx)}\tanh^{-1}\left(\sqrt{\cosh^2(c+dx)}\right)+i\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] -((Sech[c + d*x]*(I - I*ArcTanh[Sqrt[Cosh[c + d*x]^2]]*Sqrt[Cosh[c + d*x]^2
] + Csch[c + d*x] + 2*Sinh[c + d*x]))/(a*d))
```

Maple [A] time = 0.04, size = 79, normalized size = 1.4

$$-\frac{1}{2da}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-\frac{i}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\frac{1}{da\left(-i+\tanh\left(1/2\,dx+c/2\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -1/2/d/a*tanh(1/2*d*x+1/2*c)-1/2/d/a/tanh(1/2*d*x+1/2*c)-I/d/a*ln(tanh(1/2*
d*x+1/2*c))-2/d/a/(-I+tanh(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.07214, size = 149, normalized size = 2.61

$$-\frac{4\left(e^{(-dx-c)}-ie^{(-2dx-2c)}+2i\right)}{\left(2ae^{(-dx-c)}-2iae^{(-2dx-2c)}-2ae^{(-3dx-3c)}+2ia\right)d}+\frac{i\log\left(e^{(-dx-c)}+1\right)}{ad}-\frac{i\log\left(e^{(-dx-c)}-1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -4*(e^(-d*x - c) - I*e^(-2*d*x - 2*c) + 2*I)/((2*a*e^(-d*x - c) - 2*I*a*e^(
-2*d*x - 2*c) - 2*a*e^(-3*d*x - 3*c) + 2*I*a)*d) + I*log(e^(-d*x - c) + 1)/
(a*d) - I*log(e^(-d*x - c) - 1)/(a*d)
```

Fricas [B] time = 2.56622, size = 378, normalized size = 6.63

$$\frac{\left(ie^{(3\,dx+3\,c)}+e^{(2\,dx+2\,c)}-ie^{(dx+c)}-1\right)\log\left(e^{(dx+c)}+1\right)+\left(-ie^{(3\,dx+3\,c)}-e^{(2\,dx+2\,c)}+ie^{(dx+c)}+1\right)\log\left(e^{(dx+c)}-1\right)-2ie^{(2\,dx+2}}{ade^{(3\,dx+3\,c)}-iade^{(2\,dx+2\,c)}-ade^{(dx+c)}+iad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((I*e^(3*d*x + 3*c) + e^(2*d*x + 2*c) - I*e^(d*x + c) - 1)*log(e^(d*x + c)
+ 1) + (-I*e^(3*d*x + 3*c) - e^(2*d*x + 2*c) + I*e^(d*x + c) + 1)*log(e^(d*
x + c) - 1) - 2*I*e^(2*d*x + 2*c) - 2*e^(d*x + c) + 4*I)/(a*d*e^(3*d*x + 3*
c) - I*a*d*e^(2*d*x + 2*c) - a*d*e^(d*x + c) + I*a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2282, size = 128, normalized size = 2.25

$$\frac{i\log\left(e^{(dx+c)}+1\right)}{ad}-\frac{i\log\left(\left|e^{(dx+c)}-1\right|\right)}{ad}+\frac{2\left(e^{(2\,dx+2\,c)}-ie^{(dx+c)}-2\right)}{ad\left(ie^{(3\,dx+3\,c)}+e^{(2\,dx+2\,c)}-ie^{(dx+c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] I*log(e^(d*x + c) + 1)/(a*d) - I*log(abs(e^(d*x + c) - 1))/(a*d) + 2*(e^(2*
d*x + 2*c) - I*e^(d*x + c) - 2)/(a*d*(I*e^(3*d*x + 3*c) + e^(2*d*x + 2*c) -
 I*e^(d*x + c) - 1))
```

3.215 $\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Csch[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.075928, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Csch[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Mathematica [A] time = 131.951, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Csch[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Maple [A] time = 0.571, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx+c))^2}{(fx+e)(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-4if\int\frac{1}{-2iadf^2x^2-4iadefx-2iade^2+2\left(adf^2x^2e^c+2adefxe^c+ade^2e^c\right)e^{(dx)}}\,dx-\frac{}{2iadfx+2iade+2\left(adfxe^{(3c)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -4*I*f*integrate(1/(-2*I*a*d*f^2*x^2 - 4*I*a*d*e*f*x - 2*I*a*d*e^2 + 2*(a*d
*f^2*x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) - 4*(I*e^(2*d*x
+ 2*c) + e^(d*x + c) - 2*I)/(2*I*a*d*f*x + 2*I*a*d*e + 2*(a*d*f*x*e^(3*c) +
 a*d*e*e^(3*c))*e^(3*d*x) + (-2*I*a*d*f*x*e^(2*c) - 2*I*a*d*e*e^(2*c))*e^(2
*d*x) - 2*(a*d*f*x*e^c + a*d*e*e^c)*e^(d*x)) - 4*integrate(-1/4*(I*d*f*x +
I*d*e + f)/(a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2 - (a*d*f^2*x^2*e^c + 2*a*d*
e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) - 4*integrate(1/4*(I*d*f*x + I*d*e -
f)/(a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2 + (a*d*f^2*x^2*e^c + 2*a*d*e*f*x*e^
c + a*d*e^2*e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(iadfx+iade+\left(adfx+ade\right)e^{(3dx+3c)}+\left(-iadfx-iade\right)e^{(2dx+2c)}-\left(adfx+ade\right)e^{(dx+c)}\right)\mathrm{integral}\left(\frac{}{iadf^2x^2+2iadefx+iade^2+}\right.}{iadfx+iade+\left(adfx+ade\right)e^{(3dx+3c)}+\left(-iadf\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((I*a*d*f*x + I*a*d*e + (a*d*f*x + a*d*e)*e^(3*d*x + 3*c) + (-I*a*d*f*x - I
*a*d*e)*e^(2*d*x + 2*c) - (a*d*f*x + a*d*e)*e^(d*x + c))*integral(((-2*I*d*
f*x - 2*I*d*e - 2*I*f)*e^(2*d*x + 2*c) - 2*(d*f*x + d*e + f)*e^(d*x + c) +
4*I*f)/(I*a*d*f^2*x^2 + 2*I*a*d*e*f*x + I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*
f*x + a*d*e^2)*e^(3*d*x + 3*c) + (-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^
2)*e^(2*d*x + 2*c) - (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c)), x)
 - 2*I*e^(2*d*x + 2*c) - 2*e^(d*x + c) + 4*I)/(I*a*d*f*x + I*a*d*e + (a*d*f
*x + a*d*e)*e^(3*d*x + 3*c) + (-I*a*d*f*x - I*a*d*e)*e^(2*d*x + 2*c) - (a*d
*f*x + a*d*e)*e^(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.216 $\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

[Out] Unintegrable[Csch[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]

Rubi [A] time = 0.0744917, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

[In] Int[Csch[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]

[Out] Defer[Int][Csch[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

[In] Integrate[Csch[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]

[Out] $Aborted

Maple [A] time = 1.329, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx+c))^2}{(fx+e)^2(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)

[Out] int(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-4if\int\frac{1}{-iadf^3x^3-3iadef^2x^2-3iade^2fx-iade^3+\left(adf^3x^3e^c+3adef^2x^2e^c+3ade^2fxe^c+ade^3e^c\right)e^{(dx)}}\,dx-\frac{}{2iad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -4*I*f*integrate(1/(-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x -
I*a*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c + a*
d*e^3*e^c)*e^(d*x)), x) - 4*(I*e^(2*d*x + 2*c) + e^(d*x + c) - 2*I)/(2*I*a*
d*f^2*x^2 + 4*I*a*d*e*f*x + 2*I*a*d*e^2 + 2*(a*d*f^2*x^2*e^(3*c) + 2*a*d*e*
f*x*e^(3*c) + a*d*e^2*e^(3*c))*e^(3*d*x) + (-2*I*a*d*f^2*x^2*e^(2*c) - 4*I*
a*d*e*f*x*e^(2*c) - 2*I*a*d*e^2*e^(2*c))*e^(2*d*x) - 2*(a*d*f^2*x^2*e^c + 2
*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)) - 4*integrate(-1/4*(I*d*f*x + I*d*e
+ 2*f)/(a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3 - (a*d*f^3*
x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c + a*d*e^3*e^c)*e^(d*x)),
x) - 4*integrate(1/4*(I*d*f*x + I*d*e - 2*f)/(a*d*f^3*x^3 + 3*a*d*e*f^2*x^2
 + 3*a*d*e^2*f*x + a*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d
*e^2*f*x*e^c + a*d*e^3*e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(iadf^2x^2+2iadefx+iade^2+\left(adf^2x^2+2adefx+ade^2\right)e^{(3dx+3c)}+\left(-iadf^2x^2-2iadefx-iade^2\right)e^{(2dx+2c)}-\left(adf^2\right.\right.}{iadf^2x^2+2iadefx+iade}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] ((I*a*d*f^2*x^2 + 2*I*a*d*e*f*x + I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x +
a*d*e^2)*e^(3*d*x + 3*c) + (-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2)*e^(
2*d*x + 2*c) - (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))*integral(
((-2*I*d*f*x - 2*I*d*e - 4*I*f)*e^(2*d*x + 2*c) - 2*(d*f*x + d*e + 2*f)*e^(
d*x + c) + 8*I*f)/(I*a*d*f^3*x^3 + 3*I*a*d*e*f^2*x^2 + 3*I*a*d*e^2*f*x + I*
a*d*e^3 + (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3)*e^(3*d*
x + 3*c) + (-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x - I*a*d*e^
3)*e^(2*d*x + 2*c) - (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e
^3)*e^(d*x + c)), x) - 2*I*e^(2*d*x + 2*c) - 2*e^(d*x + c) + 4*I)/(I*a*d*f^
2*x^2 + 2*I*a*d*e*f*x + I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e
^(3*d*x + 3*c) + (-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2)*e^(2*d*x + 2*
c) - (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.217 $\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=546

$$-\frac{12if^2(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}-\frac{3if^2(e+fx)\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3}-\frac{9f^2(e+fx)\text{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3}+\frac{9f^2(e$$

```
[Out] ((2*I)*(e + f*x)^3)/(a*d) - (6*f^2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d^3)
+ (3*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) + (I*(e + f*x)^3*Coth[c + d*x]
)/(a*d) - (3*f*(e + f*x)^2*Csch[c + d*x])/(2*a*d^2) - ((e + f*x)^3*Coth[c +
 d*x]*Csch[c + d*x])/(2*a*d) - ((6*I)*f*(e + f*x)^2*Log[1 + I*E^(c + d*x)])
/(a*d^2) - ((3*I)*f*(e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a*d^2) - (3*f^3*
PolyLog[2, -E^(c + d*x)])/(a*d^4) + (9*f*(e + f*x)^2*PolyLog[2, -E^(c + d*x
)])/(2*a*d^2) - ((12*I)*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^3)
 + (3*f^3*PolyLog[2, E^(c + d*x)])/(a*d^4) - (9*f*(e + f*x)^2*PolyLog[2, E^
(c + d*x)])/(2*a*d^2) - ((3*I)*f^2*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(
a*d^3) - (9*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) + ((12*I)*f^3*P
olyLog[3, (-I)*E^(c + d*x)])/(a*d^4) + (9*f^2*(e + f*x)*PolyLog[3, E^(c + d
*x)])/(a*d^3) + (((3*I)/2)*f^3*PolyLog[3, E^(2*(c + d*x))])/(a*d^4) + (9*f^
3*PolyLog[4, -E^(c + d*x)])/(a*d^4) - (9*f^3*PolyLog[4, E^(c + d*x)])/(a*d^
4) + (I*(e + f*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

Rubi [A] time = 1.21183, antiderivative size = 546, normalized size of antiderivative = 1., number of steps used = 40, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.419, Rules used = {5575, 4186, 4182, 2279, 2391, 2531, 6609, 2282, 6589, 4184, 3716, 2190, 3318}

$$-\frac{12if^2(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}-\frac{3if^2(e+fx)\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3}-\frac{9f^2(e+fx)\text{PolyLog}\left(3,-e^{c+dx}\right)}{ad^3}+\frac{9f^2(e$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```


```
[Out] ((2*I)*(e + f*x)^3)/(a*d) - (6*f^2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d^3)
+ (3*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) + (I*(e + f*x)^3*Coth[c + d*x]
)/(a*d) - (3*f*(e + f*x)^2*Csch[c + d*x])/(2*a*d^2) - ((e + f*x)^3*Coth[c +
 d*x]*Csch[c + d*x])/(2*a*d) - ((6*I)*f*(e + f*x)^2*Log[1 + I*E^(c + d*x)])
/(a*d^2) - ((3*I)*f*(e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a*d^2) - (3*f^3*
PolyLog[2, -E^(c + d*x)])/(a*d^4) + (9*f*(e + f*x)^2*PolyLog[2, -E^(c + d*x
)])/(2*a*d^2) - ((12*I)*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^3)
 + (3*f^3*PolyLog[2, E^(c + d*x)])/(a*d^4) - (9*f*(e + f*x)^2*PolyLog[2, E^
(c + d*x)])/(2*a*d^2) - ((3*I)*f^2*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(
a*d^3) - (9*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) + ((12*I)*f^3*P
olyLog[3, (-I)*E^(c + d*x)])/(a*d^4) + (9*f^2*(e + f*x)*PolyLog[3, E^(c + d
*x)])/(a*d^3) + (((3*I)/2)*f^3*PolyLog[3, E^(2*(c + d*x))])/(a*d^4) + (9*f^
3*PolyLog[4, -E^(c + d*x)])/(a*d^4) - (9*f^3*PolyLog[4, E^(c + d*x)])/(a*d^
4) + (I*(e + f*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```


Rule 5575

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a +
b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
```

```
GtQ[n, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\left(i\int \frac{(e+fx)^3\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx\right)+\frac{\int (e+fx)^3\text{csch}^3(c+dx)\,dx}{a}$$

$$= -\frac{3f(e+fx)^2\text{csch}(c+dx)}{2ad^2}-\frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{2ad}-\frac{i\int (e+fx)^3\text{csch}^2(c}{a}$$

$$= -\frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{i(e+fx)^3\coth(c+dx)}{ad}-\frac{3f}{}$$

$$= \frac{i(e+fx)^3}{ad}-\frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{3(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{i(e+fx)^3\coth(c}{ad}$$

$$= \frac{i(e+fx)^3}{ad}-\frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{3(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{i(e+fx)^3\coth(c}{ad}$$

$$= \frac{2i(e+fx)^3}{ad}-\frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{3(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{i(e+fx)^3\coth}{ad}$$

$$= \frac{2i(e+fx)^3}{ad}-\frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{3(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{i(e+fx)^3\coth}{ad}$$

$$= \frac{2i(e+fx)^3}{ad}-\frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{3(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{i(e+fx)^3\coth}{ad}$$

$$= \frac{2i(e+fx)^3}{ad}-\frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{3(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{i(e+fx)^3\coth}{ad}$$

$$= \frac{2i(e+fx)^3}{ad}-\frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{3(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{i(e+fx)^3\coth}{ad}$$

Mathematica [B] time = 68.6696, size = 2479, normalized size = 4.54

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (-3*e^3*Log[Tanh[(c + d*x)/2]])/(2*a*d) + (3*e*f^2*Log[Tanh[(c + d*x)/2]])/
(a*d^3) - (9*e^2*f*(-(c*Log[Tanh[(c + d*x)/2]]) - I*((I*c + I*d*x)*(Log[1 -
 E^(I*(I*c + I*d*x))] - Log[1 + E^(I*(I*c + I*d*x))]) + I*(PolyLog[2, -E^(I
*(I*c + I*d*x))] - PolyLog[2, E^(I*(I*c + I*d*x))]))))/(2*a*d^2) + (3*f^3*(
-(c*Log[Tanh[(c + d*x)/2]]) - I*((I*c + I*d*x)*(Log[1 - E^(I*(I*c + I*d*x))
] - Log[1 + E^(I*(I*c + I*d*x))]) + I*(PolyLog[2, -E^(I*(I*c + I*d*x))] - P
olyLog[2, E^(I*(I*c + I*d*x))]))))/(a*d^4) - (2*(d^3*(e + f*x)^3 + 3*d^2*(1
 + I*E^c)*f*(e + f*x)^2*Log[1 - I*E^(-c - d*x)] + (6*I)*(I - E^c)*f^2*(d*(e
 + f*x)*PolyLog[2, I*E^(-c - d*x)] + f*PolyLog[3, I*E^(-c - d*x)])))/(a*d^4
*(-I + E^c)) + ((I/2)*E^c*f^3*Csch[c]*((2*d^3*x^3)/E^(2*c) - 3*d^2*(1 - E^(
-2*c))*x^2*Log[1 - E^(-c - d*x)] - 3*d^2*(1 - E^(-2*c))*x^2*Log[1 + E^(-c -
 d*x)] + 6*(1 - E^(-2*c))*(d*x*PolyLog[2, -E^(-c - d*x)] + PolyLog[3, -E^(-
c - d*x)]) + 6*(1 - E^(-2*c))*(d*x*PolyLog[2, E^(-c - d*x)] + PolyLog[3, E^
(-c - d*x)])))/(a*d^4) + (9*e*f^2*(d^2*x^2*ArcTanh[Cosh[c + d*x] + Sinh[c +
 d*x]] + d*x*PolyLog[2, -Cosh[c + d*x] - Sinh[c + d*x]] - d*x*PolyLog[2, Co
sh[c + d*x] + Sinh[c + d*x]] - PolyLog[3, -Cosh[c + d*x] - Sinh[c + d*x]] +
 PolyLog[3, Cosh[c + d*x] + Sinh[c + d*x]]))/(a*d^3) - (3*f^3*(-2*d^3*x^3*A
rcTanh[Cosh[c + d*x] + Sinh[c + d*x]] - 3*d^2*x^2*PolyLog[2, -Cosh[c + d*x]
 - Sinh[c + d*x]] + 3*d^2*x^2*PolyLog[2, Cosh[c + d*x] + Sinh[c + d*x]] + 6
```

```
*d*x*PolyLog[3, -Cosh[c + d*x] - Sinh[c + d*x]] - 6*d*x*PolyLog[3, Cosh[c +
 d*x] + Sinh[c + d*x]] - 6*PolyLog[4, -Cosh[c + d*x] - Sinh[c + d*x]] + 6*P
olyLog[4, Cosh[c + d*x] + Sinh[c + d*x]]))/(2*a*d^4) + ((3*I)*e^2*f*Csch[c]
*(-(d*x*Cosh[c]) + Log[Cosh[d*x]*Sinh[c] + Cosh[c]*Sinh[d*x]]*Sinh[c]))/(a*
d^2*(-Cosh[c]^2 + Sinh[c]^2)) + (Csch[c]*Csch[c + d*x]^2*(3*e^2*f*Cosh[(d*x
)/2] + 6*e*f^2*x*Cosh[(d*x)/2] + 3*f^3*x^2*Cosh[(d*x)/2] + 3*e^2*f*Cosh[(3*
d*x)/2] + 6*e*f^2*x*Cosh[(3*d*x)/2] + 3*f^3*x^2*Cosh[(3*d*x)/2] + (5*I)*d*e
^3*Cosh[c - (d*x)/2] + (15*I)*d*e^2*f*x*Cosh[c - (d*x)/2] + (15*I)*d*e*f^2*
x^2*Cosh[c - (d*x)/2] + (5*I)*d*f^3*x^3*Cosh[c - (d*x)/2] - I*d*e^3*Cosh[c
+ (d*x)/2] - (3*I)*d*e^2*f*x*Cosh[c + (d*x)/2] - (3*I)*d*e*f^2*x^2*Cosh[c +
 (d*x)/2] - I*d*f^3*x^3*Cosh[c + (d*x)/2] - 3*e^2*f*Cosh[2*c + (d*x)/2] - 6
*e*f^2*x*Cosh[2*c + (d*x)/2] - 3*f^3*x^2*Cosh[2*c + (d*x)/2] + I*d*e^3*Cosh
[c + (3*d*x)/2] + (3*I)*d*e^2*f*x*Cosh[c + (3*d*x)/2] + (3*I)*d*e*f^2*x^2*C
osh[c + (3*d*x)/2] + I*d*f^3*x^3*Cosh[c + (3*d*x)/2] - 3*e^2*f*Cosh[2*c + (
3*d*x)/2] - 6*e*f^2*x*Cosh[2*c + (3*d*x)/2] - 3*f^3*x^2*Cosh[2*c + (3*d*x)/
2] - (3*I)*d*e^3*Cosh[3*c + (3*d*x)/2] - (9*I)*d*e^2*f*x*Cosh[3*c + (3*d*x)
/2] - (9*I)*d*e*f^2*x^2*Cosh[3*c + (3*d*x)/2] - (3*I)*d*f^3*x^3*Cosh[3*c +
(3*d*x)/2] - (4*I)*d*e^3*Cosh[c + (5*d*x)/2] - (12*I)*d*e^2*f*x*Cosh[c + (5
*d*x)/2] - (12*I)*d*e*f^2*x^2*Cosh[c + (5*d*x)/2] - (4*I)*d*f^3*x^3*Cosh[c
+ (5*d*x)/2] + (2*I)*d*e^3*Cosh[3*c + (5*d*x)/2] + (6*I)*d*e^2*f*x*Cosh[3*c
 + (5*d*x)/2] + (6*I)*d*e*f^2*x^2*Cosh[3*c + (5*d*x)/2] + (2*I)*d*f^3*x^3*C
osh[3*c + (5*d*x)/2] - d*e^3*Sinh[(d*x)/2] - 3*d*e^2*f*x*Sinh[(d*x)/2] - 3*
d*e*f^2*x^2*Sinh[(d*x)/2] - d*f^3*x^3*Sinh[(d*x)/2] - d*e^3*Sinh[(3*d*x)/2]
 - 3*d*e^2*f*x*Sinh[(3*d*x)/2] - 3*d*e*f^2*x^2*Sinh[(3*d*x)/2] - d*f^3*x^3*
Sinh[(3*d*x)/2] + (3*I)*e^2*f*Sinh[c - (d*x)/2] + (6*I)*e*f^2*x*Sinh[c - (d
*x)/2] + (3*I)*f^3*x^2*Sinh[c - (d*x)/2] + (3*I)*e^2*f*Sinh[c + (d*x)/2] +
(6*I)*e*f^2*x*Sinh[c + (d*x)/2] + (3*I)*f^3*x^2*Sinh[c + (d*x)/2] - 3*d*e^3
*Sinh[2*c + (d*x)/2] - 9*d*e^2*f*x*Sinh[2*c + (d*x)/2] - 9*d*e*f^2*x^2*Sinh
[2*c + (d*x)/2] - 3*d*f^3*x^3*Sinh[2*c + (d*x)/2] + (3*I)*e^2*f*Sinh[c + (3
*d*x)/2] + (6*I)*e*f^2*x*Sinh[c + (3*d*x)/2] + (3*I)*f^3*x^2*Sinh[c + (3*d*
x)/2] - d*e^3*Sinh[2*c + (3*d*x)/2] - 3*d*e^2*f*x*Sinh[2*c + (3*d*x)/2] - 3
*d*e*f^2*x^2*Sinh[2*c + (3*d*x)/2] - d*f^3*x^3*Sinh[2*c + (3*d*x)/2] - (3*I
)*e^2*f*Sinh[3*c + (3*d*x)/2] - (6*I)*e*f^2*x*Sinh[3*c + (3*d*x)/2] - (3*I)
*f^3*x^2*Sinh[3*c + (3*d*x)/2] + 2*d*e^3*Sinh[2*c + (5*d*x)/2] + 6*d*e^2*f*
x*Sinh[2*c + (5*d*x)/2] + 6*d*e*f^2*x^2*Sinh[2*c + (5*d*x)/2] + 2*d*f^3*x^3
*Sinh[2*c + (5*d*x)/2]))/(8*a*d^2*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c/2 + (d*
x)/2] + I*Sinh[c/2 + (d*x)/2])) - ((3*I)*e*f^2*Csch[c]*Sech[c]*(-((d^2*x^2)
/E^ArcTanh[Tanh[c]]) + (I*(-(d*x*(-Pi + (2*I)*ArcTanh[Tanh[c]])) - Pi*Log[1
 + E^(2*d*x)] - 2*(I*d*x + I*ArcTanh[Tanh[c]])*Log[1 - E^((2*I)*(I*d*x + I*
ArcTanh[Tanh[c]]))] + Pi*Log[Cosh[d*x]] + (2*I)*ArcTanh[Tanh[c]]*Log[I*Sinh
[d*x + ArcTanh[Tanh[c]]]] + I*PolyLog[2, E^((2*I)*(I*d*x + I*ArcTanh[Tanh[c
]]))])*Tanh[c])/Sqrt[1 - Tanh[c]^2]))/(a*d^3*Sqrt[Sech[c]^2*(Cosh[c]^2 - Si
nh[c]^2)])
```

Maple [B] time = 0.256, size = 2058, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -6*I/a/d^2*ln(1-exp(d*x+c))*e*f^2*x-6*I/a/d^2*ln(exp(d*x+c)+1)*e*f^2*x+24*I
/a/d^2*c*e*f^2*x-6*I/a/d^3*ln(1-exp(d*x+c))*c*e*f^2+6*I/a/d^3*e*f^2*c*ln(ex
p(d*x+c)-1)-24*I/a/d^3*e*f^2*c*ln(exp(d*x+c))+6*I/d^4/a*f^3*c^2*ln(1+I*exp(
d*x+c))-6*I/d^2/a*ln(exp(d*x+c)-I)*e^2*f-12*I/d^3/a*f^3*polylog(2,-I*exp(d*
x+c))*x-6*I/d^2/a*f^3*ln(1+I*exp(d*x+c))*x^2-12*I/d^3/a*e*f^2*polylog(2,-I*
```

```
exp(d*x+c))-6*I/d^4/a*f^3*c^2*ln(exp(d*x+c)-I)-12*I/d^2/a*e*f^2*ln(1+I*exp(
d*x+c))*x-12*I/d^3/a*e*f^2*ln(1+I*exp(d*x+c))*c+12*I/d^3/a*e*f^2*c*ln(exp(d
*x+c)-I)-3*f^3*polylog(2,-exp(d*x+c))/a/d^4+3*f^3*polylog(2,exp(d*x+c))/a/d
^4+9*f^3*polylog(4,-exp(d*x+c))/a/d^4-9*f^3*polylog(4,exp(d*x+c))/a/d^4+12*
I*f^3*polylog(3,-I*exp(d*x+c))/a/d^4-(-3*I*f^3*x^2*exp(3*d*x+3*c)-3*I*d*e^3
*exp(3*d*x+3*c)-3*I*e^2*f*exp(3*d*x+3*c)+4*d*e^3+3*I*d*e*f^2*x^2*exp(d*x+c)
-5*d*f^3*x^3*exp(2*d*x+2*c)-6*e*f^2*x*exp(2*d*x+2*c)+3*d*f^3*x^3*exp(4*d*x+
4*c)+6*e*f^2*x*exp(4*d*x+4*c)+I*d*f^3*x^3*exp(d*x+c)+9*d*e*f^2*x^2*exp(4*d*
x+4*c)+9*d*e^2*f*x*exp(4*d*x+4*c)-6*I*e*f^2*x*exp(3*d*x+3*c)-3*I*d*f^3*x^3*
exp(3*d*x+3*c)+4*d*f^3*x^3+3*I*d*e^2*f*x*exp(d*x+c)-15*d*e*f^2*x^2*exp(2*d*
x+2*c)-15*d*e^2*f*x*exp(2*d*x+2*c)+12*d*e*f^2*x^2+12*d*e^2*f*x+3*I*exp(d*x+
c)*e^2*f+3*I*f^3*x^2*exp(d*x+c)+6*I*e*f^2*x*exp(d*x+c)+I*d*e^3*exp(d*x+c)-9
*I*d*e*f^2*x^2*exp(3*d*x+3*c)-9*I*d*e^2*f*x*exp(3*d*x+3*c)-3*f^3*x^2*exp(2*
d*x+2*c)-3*e^2*f*exp(2*d*x+2*c)+3*f^3*x^2*exp(4*d*x+4*c)+3*d*e^3*exp(4*d*x+
4*c)+3*e^2*f*exp(4*d*x+4*c)-5*d*e^3*exp(2*d*x+2*c))/(exp(2*d*x+2*c)-1)^2/d^
2/(exp(d*x+c)-I)/a-9/2/d^3/a*e*f^2*c^2*ln(exp(d*x+c)-1)+9/2/d^2/a*e^2*f*c*l
n(exp(d*x+c)-1)-9/2/d^2/a*ln(1-exp(d*x+c))*c*e^2*f-9/2/d/a*ln(1-exp(d*x+c))
*e^2*f*x+9/2/d/a*ln(exp(d*x+c)+1)*e^2*f*x+9/2/d^3/a*e*f^2*c^2*ln(1-exp(d*x+
c))-9/2/d/a*e*f^2*ln(1-exp(d*x+c))*x^2-9/d^2/a*e*f^2*polylog(2,exp(d*x+c))*
x+9/2/d/a*e*f^2*ln(exp(d*x+c)+1)*x^2+9/d^2/a*e*f^2*polylog(2,-exp(d*x+c))*x
-3*I/a/d^4*f^3*c^2*ln(exp(d*x+c)-1)-12*I/a/d^3*f^3*c^2*x+12*I/a/d*e*f^2*x^2
+3*I/a/d^4*f^3*c^2*ln(1-exp(d*x+c))-3*I/a/d^2*f^3*ln(1-exp(d*x+c))*x^2-3*I/
a/d^2*f^3*ln(exp(d*x+c)+1)*x^2-6*I/a/d^3*f^3*polylog(2,-exp(d*x+c))*x-6*I/a
/d^3*f^3*polylog(2,exp(d*x+c))*x+12*I/a/d^4*f^3*c^2*ln(exp(d*x+c))-3*I/a/d^
2*e^2*f*ln(exp(d*x+c)-1)-3*I/a/d^2*e^2*f*ln(exp(d*x+c)+1)+12*I/a/d^2*e^2*f*
ln(exp(d*x+c))+12*I/a/d^3*c^2*e*f^2-6*I/a/d^3*e*f^2*polylog(2,exp(d*x+c))-6
*I/a/d^3*e*f^2*polylog(2,-exp(d*x+c))+3/a/d^3*e*f^2*ln(exp(d*x+c)-1)-3/a/d^
3*e*f^2*ln(exp(d*x+c)+1)+3/a/d^3*f^3*ln(1-exp(d*x+c))*x-3/2/d/a*e^3*ln(exp(
d*x+c)-1)+3/2/d/a*e^3*ln(exp(d*x+c)+1)-3/a/d^3*f^3*ln(exp(d*x+c)+1)*x+3/a/d
^4*f^3*c*ln(1-exp(d*x+c))-3/a/d^4*f^3*c*ln(exp(d*x+c)-1)+4*I/a/d*f^3*x^3-8*
I/a/d^4*f^3*c^3+6*I/a/d^4*f^3*polylog(3,exp(d*x+c))+6*I/a/d^4*f^3*polylog(3
,-exp(d*x+c))-9/2/d^2/a*f^3*polylog(2,exp(d*x+c))*x^2+9/d^3/a*f^3*polylog(3
,exp(d*x+c))*x-9/2/d^2/a*e^2*f*polylog(2,exp(d*x+c))+9/2/d^2/a*e^2*f*polylo
g(2,-exp(d*x+c))+9/d^3/a*e*f^2*polylog(3,exp(d*x+c))-9/d^3/a*e*f^2*polylog(
3,-exp(d*x+c))+3/2/d^4/a*f^3*c^3*ln(exp(d*x+c)-1)+3/2/d/a*f^3*ln(exp(d*x+c)
+1)*x^3+9/2/d^2/a*f^3*polylog(2,-exp(d*x+c))*x^2-9/d^3/a*f^3*polylog(3,-exp
(d*x+c))*x-3/2/d/a*f^3*ln(1-exp(d*x+c))*x^3-3/2/d^4/a*f^3*ln(1-exp(d*x+c))*
c^3
```

Maxima [B] time = 2.74902, size = 1777, normalized size = 3.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/2*e^3*(16*(-I*e^(-d*x - c) - 5*e^(-2*d*x - 2*c) + 3*I*e^(-3*d*x - 3*c) +
 3*e^(-4*d*x - 4*c) + 4)/((8*a*e^(-d*x - c) - 16*I*a*e^(-2*d*x - 2*c) - 16*
a*e^(-3*d*x - 3*c) + 8*I*a*e^(-4*d*x - 4*c) + 8*a*e^(-5*d*x - 5*c) + 8*I*a)
*d) - 3*log(e^(-d*x - c) + 1)/(a*d) + 3*log(e^(-d*x - c) - 1)/(a*d)) + 6*I*
e^2*f*x/(a*d) - 6*I*e^2*f*log(I*e^(d*x + c) + 1)/(a*d^2) - (4*d*f^3*x^3 + 1
2*d*e*f^2*x^2 + 12*d*e^2*f*x + 3*(d*f^3*x^3*e^(4*c) + e^2*f*e^(4*c) + (3*d*
e*f^2 + f^3)*x^2*e^(4*c) + (3*d*e^2*f + 2*e*f^2)*x*e^(4*c))*e^(4*d*x) + (-3
*I*d*f^3*x^3*e^(3*c) - 3*I*e^2*f*e^(3*c) + (-9*I*d*e*f^2 - 3*I*f^3)*x^2*e^(
3*c) + (-9*I*d*e^2*f - 6*I*e*f^2)*x*e^(3*c))*e^(3*d*x) - (5*d*f^3*x^3*e^(2*
```

```
c) + 3*e^2*f*e^(2*c) + 3*(5*d*e*f^2 + f^3)*x^2*e^(2*c) + 3*(5*d*e^2*f + 2*e
*f^2)*x*e^(2*c))*e^(2*d*x) + (I*d*f^3*x^3*e^c + 3*I*e^2*f*e^c + (3*I*d*e*f^
2 + 3*I*f^3)*x^2*e^c + (3*I*d*e^2*f + 6*I*e*f^2)*x*e^c)*e^(d*x))/(a*d^2*e^(
5*d*x + 5*c) - I*a*d^2*e^(4*d*x + 4*c) - 2*a*d^2*e^(3*d*x + 3*c) + 2*I*a*d^
2*e^(2*d*x + 2*c) + a*d^2*e^(d*x + c) - I*a*d^2) - 12*I*(d*x*log(I*e^(d*x +
 c) + 1) + dilog(-I*e^(d*x + c)))*e*f^2/(a*d^3) + 3/2*(d^3*x^3*log(e^(d*x +
 c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) +
 6*polylog(4, -e^(d*x + c)))*f^3/(a*d^4) - 3/2*(d^3*x^3*log(-e^(d*x + c) +
1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polyl
og(4, e^(d*x + c)))*f^3/(a*d^4) - 6*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d
*x*dilog(-I*e^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a*d^4) - 3*(-
I*d*e^2*f + e*f^2)*x/(a*d^2) - 3*(-I*d*e^2*f - e*f^2)*x/(a*d^2) + 3*(-I*d*e
^2*f - e*f^2)*log(e^(d*x + c) + 1)/(a*d^3) + 3*(-I*d*e^2*f + e*f^2)*log(e^(
d*x + c) - 1)/(a*d^3) - 3/2*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^
(d*x + c)) - 2*polylog(3, e^(d*x + c)))*(3*d*e*f^2 + 2*I*f^3)/(a*d^4) + 3/2
*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -
e^(d*x + c)))*(3*d*e*f^2 - 2*I*f^3)/(a*d^4) + 1/2*(9*d^2*e^2*f - 12*I*d*e*f
^2 - 6*f^3)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a*d^4) - 1/2*
(9*d^2*e^2*f + 12*I*d*e*f^2 - 6*f^3)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(
d*x + c)))/(a*d^4) + 1/8*(3*d^4*f^3*x^4 + 4*(3*d*e*f^2 + 2*I*f^3)*d^3*x^3 +
 (18*d^2*e^2*f + 24*I*d*e*f^2 - 12*f^3)*d^2*x^2)/(a*d^4) - 1/8*(3*d^4*f^3*x
^4 + 4*(3*d*e*f^2 - 2*I*f^3)*d^3*x^3 + (18*d^2*e^2*f - 24*I*d*e*f^2 - 12*f^
3)*d^2*x^2)/(a*d^4) + (2*I*d^3*f^3*x^3 + 6*I*d^3*e*f^2*x^2)/(a*d^4)
```

Fricas [C] time = 3.72538, size = 10116, normalized size = 18.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -(8*d^3*e^3 - 24*c*d^2*e^2*f + 24*c^2*d*e*f^2 - 8*c^3*f^3 + (24*d*f^3*x + 2
4*d*e*f^2 - (-24*I*d*f^3*x - 24*I*d*e*f^2)*e^(5*d*x + 5*c) + 24*(d*f^3*x +
d*e*f^2)*e^(4*d*x + 4*c) - (48*I*d*f^3*x + 48*I*d*e*f^2)*e^(3*d*x + 3*c) -
48*(d*f^3*x + d*e*f^2)*e^(2*d*x + 2*c) - (-24*I*d*f^3*x - 24*I*d*e*f^2)*e^(
d*x + c))*dilog(-I*e^(d*x + c)) - (-9*I*d^2*f^3*x^2 - 9*I*d^2*e^2*f - 12*d*
e*f^2 + 6*I*f^3 - 6*(3*I*d^2*e*f^2 + 2*d*f^3)*x + (9*d^2*f^3*x^2 + 9*d^2*e^
2*f - 12*I*d*e*f^2 - 6*f^3 + (18*d^2*e*f^2 - 12*I*d*f^3)*x)*e^(5*d*x + 5*c)
 + (-9*I*d^2*f^3*x^2 - 9*I*d^2*e^2*f - 12*d*e*f^2 + 6*I*f^3 - 6*(3*I*d^2*e*
f^2 + 2*d*f^3)*x)*e^(4*d*x + 4*c) - (18*d^2*f^3*x^2 + 18*d^2*e^2*f - 24*I*d
*e*f^2 - 12*f^3 + (36*d^2*e*f^2 - 24*I*d*f^3)*x)*e^(3*d*x + 3*c) + (18*I*d^
2*f^3*x^2 + 18*I*d^2*e^2*f + 24*d*e*f^2 - 12*I*f^3 - 12*(-3*I*d^2*e*f^2 - 2
*d*f^3)*x)*e^(2*d*x + 2*c) + (9*d^2*f^3*x^2 + 9*d^2*e^2*f - 12*I*d*e*f^2 -
6*f^3 + (18*d^2*e*f^2 - 12*I*d*f^3)*x)*e^(d*x + c))*dilog(-e^(d*x + c)) - (
9*I*d^2*f^3*x^2 + 9*I*d^2*e^2*f - 12*d*e*f^2 - 6*I*f^3 - 6*(-3*I*d^2*e*f^2
+ 2*d*f^3)*x - (9*d^2*f^3*x^2 + 9*d^2*e^2*f + 12*I*d*e*f^2 - 6*f^3 + (18*d^
2*e*f^2 + 12*I*d*f^3)*x)*e^(5*d*x + 5*c) + (9*I*d^2*f^3*x^2 + 9*I*d^2*e^2*f
 - 12*d*e*f^2 - 6*I*f^3 - 6*(-3*I*d^2*e*f^2 + 2*d*f^3)*x)*e^(4*d*x + 4*c) +
 (18*d^2*f^3*x^2 + 18*d^2*e^2*f + 24*I*d*e*f^2 - 12*f^3 + (36*d^2*e*f^2 + 2
4*I*d*f^3)*x)*e^(3*d*x + 3*c) + (-18*I*d^2*f^3*x^2 - 18*I*d^2*e^2*f + 24*d*
e*f^2 + 12*I*f^3 - 12*(3*I*d^2*e*f^2 - 2*d*f^3)*x)*e^(2*d*x + 2*c) - (9*d^2
*f^3*x^2 + 9*d^2*e^2*f + 12*I*d*e*f^2 - 6*f^3 + (18*d^2*e*f^2 + 12*I*d*f^3)
*x)*e^(d*x + c))*dilog(e^(d*x + c)) - (8*I*d^3*f^3*x^3 + 24*I*d^3*e*f^2*x^2
 + 24*I*d^3*e^2*f*x + 24*I*c*d^2*e^2*f - 24*I*c^2*d*e*f^2 + 8*I*c^3*f^3)*e^
(5*d*x + 5*c) - 2*(d^3*f^3*x^3 - 3*d^3*e^3 + 3*(4*c - 1)*d^2*e^2*f - 12*c^2
```

```
*d*e*f^2 + 4*c^3*f^3 + 3*(d^3*e*f^2 - d^2*f^3)*x^2 + 3*(d^3*e^2*f - 2*d^2*e
*f^2)*x)*e^(4*d*x + 4*c) - (-10*I*d^3*f^3*x^3 + 6*I*d^3*e^3 + (-48*I*c + 6*
I)*d^2*e^2*f + 48*I*c^2*d*e*f^2 - 16*I*c^3*f^3 + (-30*I*d^3*e*f^2 + 6*I*d^2
*f^3)*x^2 + (-30*I*d^3*e^2*f + 12*I*d^2*e*f^2)*x)*e^(3*d*x + 3*c) + 2*(3*d^
3*f^3*x^3 - 5*d^3*e^3 + 3*(8*c - 1)*d^2*e^2*f - 24*c^2*d*e*f^2 + 8*c^3*f^3
+ 3*(3*d^3*e*f^2 - d^2*f^3)*x^2 + 3*(3*d^3*e^2*f - 2*d^2*e*f^2)*x)*e^(2*d*x
 + 2*c) - (6*I*d^3*f^3*x^3 - 2*I*d^3*e^3 + (24*I*c - 6*I)*d^2*e^2*f - 24*I*
c^2*d*e*f^2 + 8*I*c^3*f^3 + (18*I*d^3*e*f^2 - 6*I*d^2*f^3)*x^2 + (18*I*d^3*
e^2*f - 12*I*d^2*e*f^2)*x)*e^(d*x + c) - (-3*I*d^3*f^3*x^3 - 3*I*d^3*e^3 -
6*d^2*e^2*f + 6*I*d*e*f^2 - 3*(3*I*d^3*e*f^2 + 2*d^2*f^3)*x^2 + (-9*I*d^3*e
^2*f - 12*d^2*e*f^2 + 6*I*d*f^3)*x + (3*d^3*f^3*x^3 + 3*d^3*e^3 - 6*I*d^2*e
^2*f - 6*d*e*f^2 + (9*d^3*e*f^2 - 6*I*d^2*f^3)*x^2 + 3*(3*d^3*e^2*f - 4*I*d
^2*e*f^2 - 2*d*f^3)*x)*e^(5*d*x + 5*c) + (-3*I*d^3*f^3*x^3 - 3*I*d^3*e^3 -
6*d^2*e^2*f + 6*I*d*e*f^2 - 3*(3*I*d^3*e*f^2 + 2*d^2*f^3)*x^2 + (-9*I*d^3*e
^2*f - 12*d^2*e*f^2 + 6*I*d*f^3)*x)*e^(4*d*x + 4*c) - (6*d^3*f^3*x^3 + 6*d^
3*e^3 - 12*I*d^2*e^2*f - 12*d*e*f^2 + (18*d^3*e*f^2 - 12*I*d^2*f^3)*x^2 + 6
*(3*d^3*e^2*f - 4*I*d^2*e*f^2 - 2*d*f^3)*x)*e^(3*d*x + 3*c) + (6*I*d^3*f^3*
x^3 + 6*I*d^3*e^3 + 12*d^2*e^2*f - 12*I*d*e*f^2 - 6*(-3*I*d^3*e*f^2 - 2*d^2
*f^3)*x^2 + (18*I*d^3*e^2*f + 24*d^2*e*f^2 - 12*I*d*f^3)*x)*e^(2*d*x + 2*c)
 + (3*d^3*f^3*x^3 + 3*d^3*e^3 - 6*I*d^2*e^2*f - 6*d*e*f^2 + (9*d^3*e*f^2 -
6*I*d^2*f^3)*x^2 + 3*(3*d^3*e^2*f - 4*I*d^2*e*f^2 - 2*d*f^3)*x)*e^(d*x + c)
)*log(e^(d*x + c) + 1) + (12*d^2*e^2*f - 24*c*d*e*f^2 + 12*c^2*f^3 - (-12*I
*d^2*e^2*f + 24*I*c*d*e*f^2 - 12*I*c^2*f^3)*e^(5*d*x + 5*c) + 12*(d^2*e^2*f
 - 2*c*d*e*f^2 + c^2*f^3)*e^(4*d*x + 4*c) - (24*I*d^2*e^2*f - 48*I*c*d*e*f^
2 + 24*I*c^2*f^3)*e^(3*d*x + 3*c) - 24*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*
e^(2*d*x + 2*c) - (-12*I*d^2*e^2*f + 24*I*c*d*e*f^2 - 12*I*c^2*f^3)*e^(d*x
+ c))*log(e^(d*x + c) - I) - (3*I*d^3*e^3 - 3*(3*I*c + 2)*d^2*e^2*f + (9*I*
c^2 + 12*c - 6*I)*d*e*f^2 + (-3*I*c^3 - 6*c^2 + 6*I*c)*f^3 - (3*d^3*e^3 - (
9*c - 6*I)*d^2*e^2*f + 3*(3*c^2 - 4*I*c - 2)*d*e*f^2 - (3*c^3 - 6*I*c^2 - 6
*c)*f^3)*e^(5*d*x + 5*c) + (3*I*d^3*e^3 - 3*(3*I*c + 2)*d^2*e^2*f + (9*I*c^
2 + 12*c - 6*I)*d*e*f^2 + (-3*I*c^3 - 6*c^2 + 6*I*c)*f^3)*e^(4*d*x + 4*c) +
 (6*d^3*e^3 - (18*c - 12*I)*d^2*e^2*f + 6*(3*c^2 - 4*I*c - 2)*d*e*f^2 - (6*
c^3 - 12*I*c^2 - 12*c)*f^3)*e^(3*d*x + 3*c) + (-6*I*d^3*e^3 - 6*(-3*I*c - 2
)*d^2*e^2*f + (-18*I*c^2 - 24*c + 12*I)*d*e*f^2 + (6*I*c^3 + 12*c^2 - 12*I*
c)*f^3)*e^(2*d*x + 2*c) - (3*d^3*e^3 - (9*c - 6*I)*d^2*e^2*f + 3*(3*c^2 - 4
*I*c - 2)*d*e*f^2 - (3*c^3 - 6*I*c^2 - 6*c)*f^3)*e^(d*x + c))*log(e^(d*x +
c) - 1) + (12*d^2*f^3*x^2 + 24*d^2*e*f^2*x + 24*c*d*e*f^2 - 12*c^2*f^3 - (-
12*I*d^2*f^3*x^2 - 24*I*d^2*e*f^2*x - 24*I*c*d*e*f^2 + 12*I*c^2*f^3)*e^(5*d
*x + 5*c) + 12*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*e^(4*d
*x + 4*c) - (24*I*d^2*f^3*x^2 + 48*I*d^2*e*f^2*x + 48*I*c*d*e*f^2 - 24*I*c^
2*f^3)*e^(3*d*x + 3*c) - 24*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^
2*f^3)*e^(2*d*x + 2*c) - (-12*I*d^2*f^3*x^2 - 24*I*d^2*e*f^2*x - 24*I*c*d*e
*f^2 + 12*I*c^2*f^3)*e^(d*x + c))*log(I*e^(d*x + c) + 1) - (3*I*d^3*f^3*x^3
 + 9*I*c*d^2*e^2*f + (-9*I*c^2 - 12*c)*d*e*f^2 + (3*I*c^3 + 6*c^2 - 6*I*c)*
f^3 - 3*(-3*I*d^3*e*f^2 + 2*d^2*f^3)*x^2 + (9*I*d^3*e^2*f - 12*d^2*e*f^2 -
6*I*d*f^3)*x - (3*d^3*f^3*x^3 + 9*c*d^2*e^2*f - 3*(3*c^2 - 4*I*c)*d*e*f^2 +
 (3*c^3 - 6*I*c^2 - 6*c)*f^3 + (9*d^3*e*f^2 + 6*I*d^2*f^3)*x^2 + 3*(3*d^3*e
^2*f + 4*I*d^2*e*f^2 - 2*d*f^3)*x)*e^(5*d*x + 5*c) + (3*I*d^3*f^3*x^3 + 9*I
*c*d^2*e^2*f + (-9*I*c^2 - 12*c)*d*e*f^2 + (3*I*c^3 + 6*c^2 - 6*I*c)*f^3 -
3*(-3*I*d^3*e*f^2 + 2*d^2*f^3)*x^2 + (9*I*d^3*e^2*f - 12*d^2*e*f^2 - 6*I*d*
f^3)*x)*e^(4*d*x + 4*c) + (6*d^3*f^3*x^3 + 18*c*d^2*e^2*f - 6*(3*c^2 - 4*I*
c)*d*e*f^2 + (6*c^3 - 12*I*c^2 - 12*c)*f^3 + (18*d^3*e*f^2 + 12*I*d^2*f^3)*
x^2 + 6*(3*d^3*e^2*f + 4*I*d^2*e*f^2 - 2*d*f^3)*x)*e^(3*d*x + 3*c) + (-6*I*
d^3*f^3*x^3 - 18*I*c*d^2*e^2*f + (18*I*c^2 + 24*c)*d*e*f^2 + (-6*I*c^3 - 12
*c^2 + 12*I*c)*f^3 - 6*(3*I*d^3*e*f^2 - 2*d^2*f^3)*x^2 + (-18*I*d^3*e^2*f +
 24*d^2*e*f^2 + 12*I*d*f^3)*x)*e^(2*d*x + 2*c) - (3*d^3*f^3*x^3 + 9*c*d^2*e
^2*f - 3*(3*c^2 - 4*I*c)*d*e*f^2 + (3*c^3 - 6*I*c^2 - 6*c)*f^3 + (9*d^3*e*f
^2 + 6*I*d^2*f^3)*x^2 + 3*(3*d^3*e^2*f + 4*I*d^2*e*f^2 - 2*d*f^3)*x)*e^(d*x
 + c))*log(-e^(d*x + c) + 1) - (18*f^3*e^(5*d*x + 5*c) - 18*I*f^3*e^(4*d*x
```

```
+ 4*c) - 36*f^3*e^(3*d*x + 3*c) + 36*I*f^3*e^(2*d*x + 2*c) + 18*f^3*e^(d*x
+ c) - 18*I*f^3)*polylog(4, -e^(d*x + c)) + (18*f^3*e^(5*d*x + 5*c) - 18*I*
f^3*e^(4*d*x + 4*c) - 36*f^3*e^(3*d*x + 3*c) + 36*I*f^3*e^(2*d*x + 2*c) + 1
8*f^3*e^(d*x + c) - 18*I*f^3)*polylog(4, e^(d*x + c)) - (24*I*f^3*e^(5*d*x
+ 5*c) + 24*f^3*e^(4*d*x + 4*c) - 48*I*f^3*e^(3*d*x + 3*c) - 48*f^3*e^(2*d*
x + 2*c) + 24*I*f^3*e^(d*x + c) + 24*f^3)*polylog(3, -I*e^(d*x + c)) - (18*
I*d*f^3*x + 18*I*d*e*f^2 + 12*f^3 - 6*(3*d*f^3*x + 3*d*e*f^2 - 2*I*f^3)*e^(
5*d*x + 5*c) + (18*I*d*f^3*x + 18*I*d*e*f^2 + 12*f^3)*e^(4*d*x + 4*c) + 12*
(3*d*f^3*x + 3*d*e*f^2 - 2*I*f^3)*e^(3*d*x + 3*c) + (-36*I*d*f^3*x - 36*I*d
*e*f^2 - 24*f^3)*e^(2*d*x + 2*c) - 6*(3*d*f^3*x + 3*d*e*f^2 - 2*I*f^3)*e^(d
*x + c))*polylog(3, -e^(d*x + c)) - (-18*I*d*f^3*x - 18*I*d*e*f^2 + 12*f^3
+ 6*(3*d*f^3*x + 3*d*e*f^2 + 2*I*f^3)*e^(5*d*x + 5*c) + (-18*I*d*f^3*x - 18
*I*d*e*f^2 + 12*f^3)*e^(4*d*x + 4*c) - 12*(3*d*f^3*x + 3*d*e*f^2 + 2*I*f^3)
*e^(3*d*x + 3*c) + (36*I*d*f^3*x + 36*I*d*e*f^2 - 24*f^3)*e^(2*d*x + 2*c) +
 6*(3*d*f^3*x + 3*d*e*f^2 + 2*I*f^3)*e^(d*x + c))*polylog(3, e^(d*x + c)))/
(2*a*d^4*e^(5*d*x + 5*c) - 2*I*a*d^4*e^(4*d*x + 4*c) - 4*a*d^4*e^(3*d*x + 3
*c) + 4*I*a*d^4*e^(2*d*x + 2*c) + 2*a*d^4*e^(d*x + c) - 2*I*a*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.218 $\int \frac{(e+fx)^2\mathbf{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=368

$$\frac{3f(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}-\frac{3f(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}-\frac{4if^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}-\frac{if^2\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3}$$

```
[Out] ((2*I)*(e + f*x)^2)/(a*d) + (3*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (f
^2*ArcTanh[Cosh[c + d*x]])/(a*d^3) + (I*(e + f*x)^2*Coth[c + d*x])/(a*d) -
(f*(e + f*x)*Csch[c + d*x])/(a*d^2) - ((e + f*x)^2*Coth[c + d*x]*Csch[c + d
*x])/(2*a*d) - ((4*I)*f*(e + f*x)*Log[1 + I*E^(c + d*x)])/(a*d^2) - ((2*I)*
f*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a*d^2) + (3*f*(e + f*x)*PolyLog[2, -
E^(c + d*x)])/(a*d^2) - ((4*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^3) -
(3*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) - (I*f^2*PolyLog[2, E^(2*(c
 + d*x))])/(a*d^3) - (3*f^2*PolyLog[3, -E^(c + d*x)])/(a*d^3) + (3*f^2*Poly
Log[3, E^(c + d*x)])/(a*d^3) + (I*(e + f*x)^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2
])/(a*d)
```

Rubi [A] time = 0.855921, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.419, Rules used = {5575, 4186, 3770, 4182, 2531, 2282, 6589, 4184, 3716, 2190, 2279, 2391, 3318}

$$\frac{3f(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}-\frac{3f(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}-\frac{4if^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}-\frac{if^2\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```


```
[Out] ((2*I)*(e + f*x)^2)/(a*d) + (3*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (f
^2*ArcTanh[Cosh[c + d*x]])/(a*d^3) + (I*(e + f*x)^2*Coth[c + d*x])/(a*d) -
(f*(e + f*x)*Csch[c + d*x])/(a*d^2) - ((e + f*x)^2*Coth[c + d*x]*Csch[c + d
*x])/(2*a*d) - ((4*I)*f*(e + f*x)*Log[1 + I*E^(c + d*x)])/(a*d^2) - ((2*I)*
f*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a*d^2) + (3*f*(e + f*x)*PolyLog[2, -
E^(c + d*x)])/(a*d^2) - ((4*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^3) -
(3*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) - (I*f^2*PolyLog[2, E^(2*(c
 + d*x))])/(a*d^3) - (3*f^2*PolyLog[3, -E^(c + d*x)])/(a*d^3) + (3*f^2*Poly
Log[3, E^(c + d*x)])/(a*d^3) + (I*(e + f*x)^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2
])/(a*d)
```


Rule 5575

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a +
b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
```

```
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(e+fx)^2\mathrm{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\left(i\int \frac{(e+fx)^2\mathrm{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx\right) + \frac{\int (e+fx)^2\mathrm{csch}^3(c+dx)\,dx}{a}$$

$$= -\frac{f(e+fx)\mathrm{csch}(c+dx)}{ad^2} - \frac{(e+fx)^2\coth(c+dx)\mathrm{csch}(c+dx)}{2ad} - \frac{i\int (e+fx)^2\mathrm{csch}^2(c+\ldots}{a}$$

$$= \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{i(e+fx)^2\coth(c+dx)}{ad} - \frac{f(e+\ldots}{}$$

$$= \frac{i(e+fx)^2}{ad} + \frac{3(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{i(e+fx)^2\coth(c+\ldots}{ad}$$

$$= \frac{i(e+fx)^2}{ad} + \frac{3(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{i(e+fx)^2\coth(c+\ldots}{ad}$$

$$= \frac{2i(e+fx)^2}{ad} + \frac{3(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{i(e+fx)^2\coth(c\ldots}{ad}$$

$$= \frac{2i(e+fx)^2}{ad} + \frac{3(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{i(e+fx)^2\coth(c\ldots}{ad}$$

$$= \frac{2i(e+fx)^2}{ad} + \frac{3(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{i(e+fx)^2\coth(c\ldots}{ad}$$

$$= \frac{2i(e+fx)^2}{ad} + \frac{3(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{i(e+fx)^2\coth(c\ldots}{ad}$$

Mathematica [B] time = 17.0436, size = 1378, normalized size = 3.74

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (-2*d*(e + f*x)*(d*(e + f*x) + 2*(1 + I*E^c)*f*Log[1 - I*E^(-c - d*x)]) + 4
*(1 + I*E^c)*f^2*PolyLog[2, I*E^(-c - d*x)])/(a*d^3*(-I + E^c)) + ((2*I)*d^
2*(e + f*x)^2*(-1 + Coth[c]) + (3*d^2*e^2 + (4*I)*d*e*f - 2*f^2)*(d*x - Log
[1 - Cosh[c + d*x] - Sinh[c + d*x]]) + 2*d*(3*d*e - (2*I)*f)*f*x*Log[1 + Co
sh[c + d*x] - Sinh[c + d*x]] + 3*d^2*f^2*x^2*Log[1 + Cosh[c + d*x] - Sinh[c
 + d*x]] - 2*d*(3*d*e + (2*I)*f)*f*x*Log[1 - Cosh[c + d*x] + Sinh[c + d*x]]
 - 3*d^2*f^2*x^2*Log[1 - Cosh[c + d*x] + Sinh[c + d*x]] - (3*d^2*e^2 - (4*I
)*d*e*f - 2*f^2)*(d*x - Log[1 + Cosh[c + d*x] + Sinh[c + d*x]]) + 2*(3*d*e
+ (2*I)*f)*f*PolyLog[2, Cosh[c + d*x] - Sinh[c + d*x]] - 2*(3*d*e - (2*I)*f
)*f*PolyLog[2, -Cosh[c + d*x] + Sinh[c + d*x]] + 6*f^2*(d*x*PolyLog[2, Cosh
[c + d*x] - Sinh[c + d*x]] + PolyLog[3, Cosh[c + d*x] - Sinh[c + d*x]]) - 6
*f^2*(d*x*PolyLog[2, -Cosh[c + d*x] + Sinh[c + d*x]] + PolyLog[3, -Cosh[c +
 d*x] + Sinh[c + d*x]]))/(2*a*d^3) + (Csch[c]*Csch[c + d*x]^2*(2*e*f*Cosh[(
d*x)/2] + 2*f^2*x*Cosh[(d*x)/2] + 2*e*f*Cosh[(3*d*x)/2] + 2*f^2*x*Cosh[(3*d
*x)/2] + (5*I)*d*e^2*Cosh[c - (d*x)/2] + (10*I)*d*e*f*x*Cosh[c - (d*x)/2] +
 (5*I)*d*f^2*x^2*Cosh[c - (d*x)/2] - I*d*e^2*Cosh[c + (d*x)/2] - (2*I)*d*e*
f*x*Cosh[c + (d*x)/2] - I*d*f^2*x^2*Cosh[c + (d*x)/2] - 2*e*f*Cosh[2*c + (d
*x)/2] - 2*f^2*x*Cosh[2*c + (d*x)/2] + I*d*e^2*Cosh[c + (3*d*x)/2] + (2*I)*
d*e*f*x*Cosh[c + (3*d*x)/2] + I*d*f^2*x^2*Cosh[c + (3*d*x)/2] - 2*e*f*Cosh[
2*c + (3*d*x)/2] - 2*f^2*x*Cosh[2*c + (3*d*x)/2] - (3*I)*d*e^2*Cosh[3*c + (
3*d*x)/2] - (6*I)*d*e*f*x*Cosh[3*c + (3*d*x)/2] - (3*I)*d*f^2*x^2*Cosh[3*c
+ (3*d*x)/2] - (4*I)*d*e^2*Cosh[c + (5*d*x)/2] - (8*I)*d*e*f*x*Cosh[c + (5*
d*x)/2] - (4*I)*d*f^2*x^2*Cosh[c + (5*d*x)/2] + (2*I)*d*e^2*Cosh[3*c + (5*d
*x)/2] + (4*I)*d*e*f*x*Cosh[3*c + (5*d*x)/2] + (2*I)*d*f^2*x^2*Cosh[3*c + (
5*d*x)/2] - d*e^2*Sinh[(d*x)/2] - 2*d*e*f*x*Sinh[(d*x)/2] - d*f^2*x^2*Sinh[
(d*x)/2] - d*e^2*Sinh[(3*d*x)/2] - 2*d*e*f*x*Sinh[(3*d*x)/2] - d*f^2*x^2*Si
nh[(3*d*x)/2] + (2*I)*e*f*Sinh[c - (d*x)/2] + (2*I)*f^2*x*Sinh[c - (d*x)/2]
 + (2*I)*e*f*Sinh[c + (d*x)/2] + (2*I)*f^2*x*Sinh[c + (d*x)/2] - 3*d*e^2*Si
nh[2*c + (d*x)/2] - 6*d*e*f*x*Sinh[2*c + (d*x)/2] - 3*d*f^2*x^2*Sinh[2*c +
(d*x)/2] + (2*I)*e*f*Sinh[c + (3*d*x)/2] + (2*I)*f^2*x*Sinh[c + (3*d*x)/2]
- d*e^2*Sinh[2*c + (3*d*x)/2] - 2*d*e*f*x*Sinh[2*c + (3*d*x)/2] - d*f^2*x^2
*Sinh[2*c + (3*d*x)/2] - (2*I)*e*f*Sinh[3*c + (3*d*x)/2] - (2*I)*f^2*x*Sinh
[3*c + (3*d*x)/2] + 2*d*e^2*Sinh[2*c + (5*d*x)/2] + 4*d*e*f*x*Sinh[2*c + (5
*d*x)/2] + 2*d*f^2*x^2*Sinh[2*c + (5*d*x)/2]))/(8*a*d^2*(Cosh[c/2] + I*Sinh
[c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2]))
```

Maple [B] time = 0.192, size = 1107, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -3/a/d^2*ln(1-exp(d*x+c))*c*e*f+3/a/d*ln(exp(d*x+c)+1)*e*f*x-3/a/d*ln(1-exp
(d*x+c))*e*f*x+3/a/d^2*e*f*c*ln(exp(d*x+c)-1)-4*I/a/d^2*ln(exp(d*x+c)-I)*e*
f-4*I/a/d^2*f^2*ln(1+I*exp(d*x+c))*x-4*I/a/d^3*f^2*ln(1+I*exp(d*x+c))*c+4*I
/a/d^3*f^2*c*ln(exp(d*x+c)-I)-4*I*f^2*polylog(2,-I*exp(d*x+c))/a/d^3+3*f^2*
polylog(3,exp(d*x+c))/a/d^3-3*f^2*polylog(3,-exp(d*x+c))/a/d^3-1/a/d^3*f^2*
ln(exp(d*x+c)+1)+1/a/d^3*f^2*ln(exp(d*x+c)-1)+4*I/a/d^3*c^2*f^2-2*I/a/d^3*f
^2*polylog(2,-exp(d*x+c))-2*I/a/d^3*f^2*polylog(2,exp(d*x+c))+4*I/a/d*f^2*x
^2-3/2/a/d*e^2*ln(exp(d*x+c)-1)+3/2/a/d*e^2*ln(exp(d*x+c)+1)-2*I/a/d^2*f*e*
ln(exp(d*x+c)+1)+2*I/a/d^3*f^2*c*ln(exp(d*x+c)-1)-8*I/a/d^3*f^2*c*ln(exp(d*
x+c))-2*I/a/d^3*ln(1-exp(d*x+c))*c*f^2+8*I/a/d^2*c*f^2*x-2*I/a/d^2*ln(exp(d
*x+c)+1)*f^2*x-2*I/a/d^2*ln(1-exp(d*x+c))*f^2*x+8*I/a/d^2*e*f*ln(exp(d*x+c)
)-2*I/a/d^2*f*e*ln(exp(d*x+c)-1)+3/a/d^2*f^2*polylog(2,-exp(d*x+c))*x+3/2/a
/d^3*f^2*c^2*ln(1-exp(d*x+c))-3/2/a/d*f^2*ln(1-exp(d*x+c))*x^2-3/2/a/d^3*f^
```

```
2*c^2*ln(exp(d*x+c)-1)+3/a/d^2*e*f*polylog(2,-exp(d*x+c))-3/a/d^2*e*f*polyl
og(2,exp(d*x+c))-3/a/d^2*f^2*polylog(2,exp(d*x+c))*x+3/2/a/d*f^2*ln(exp(d*x
+c)+1)*x^2-(6*d*e*f*x*exp(4*d*x+4*c)+4*d*e^2-5*d*e^2*exp(2*d*x+2*c)+2*I*d*e
*f*x*exp(d*x+c)+2*I*exp(d*x+c)*e*f-10*d*e*f*x*exp(2*d*x+2*c)-3*I*d*e^2*exp(
3*d*x+3*c)+I*d*f^2*x^2*exp(d*x+c)+8*d*e*f*x+4*d*f^2*x^2+3*d*e^2*exp(4*d*x+4
*c)+2*f^2*x*exp(4*d*x+4*c)+2*e*f*exp(4*d*x+4*c)-2*f^2*x*exp(2*d*x+2*c)-2*e*
f*exp(2*d*x+2*c)+3*d*f^2*x^2*exp(4*d*x+4*c)+I*d*e^2*exp(d*x+c)-3*I*d*f^2*x^
2*exp(3*d*x+3*c)-2*I*e*f*exp(3*d*x+3*c)-6*I*d*e*f*x*exp(3*d*x+3*c)+2*I*f^2*
x*exp(d*x+c)-2*I*f^2*x*exp(3*d*x+3*c)-5*d*f^2*x^2*exp(2*d*x+2*c))/(exp(2*d*
x+2*c)-1)^2/d^2/(exp(d*x+c)-I)/a
```

Maxima [B] time = 2.44375, size = 1165, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/2*e^2*(16*(-I*e^(-d*x - c) - 5*e^(-2*d*x - 2*c) + 3*I*e^(-3*d*x - 3*c) +
 3*e^(-4*d*x - 4*c) + 4)/((8*a*e^(-d*x - c) - 16*I*a*e^(-2*d*x - 2*c) - 16*
a*e^(-3*d*x - 3*c) + 8*I*a*e^(-4*d*x - 4*c) + 8*a*e^(-5*d*x - 5*c) + 8*I*a)
*d) - 3*log(e^(-d*x - c) + 1)/(a*d) + 3*log(e^(-d*x - c) - 1)/(a*d)) + 2*I*
f^2*x^2/(a*d) + 4*I*e*f*x/(a*d) - (4*d*f^2*x^2 + 8*d*e*f*x + (3*d*f^2*x^2*e
^(4*c) + 2*e*f*e^(4*c) + 2*(3*d*e*f + f^2)*x*e^(4*c))*e^(4*d*x) + (-3*I*d*f
^2*x^2*e^(3*c) - 2*I*e*f*e^(3*c) + (-6*I*d*e*f - 2*I*f^2)*x*e^(3*c))*e^(3*d
*x) - (5*d*f^2*x^2*e^(2*c) + 2*e*f*e^(2*c) + 2*(5*d*e*f + f^2)*x*e^(2*c))*e
^(2*d*x) + (I*d*f^2*x^2*e^c + 2*I*e*f*e^c + (2*I*d*e*f + 2*I*f^2)*x*e^c)*e^
(d*x))/(a*d^2*e^(5*d*x + 5*c) - I*a*d^2*e^(4*d*x + 4*c) - 2*a*d^2*e^(3*d*x
+ 3*c) + 2*I*a*d^2*e^(2*d*x + 2*c) + a*d^2*e^(d*x + c) - I*a*d^2) - 4*I*e*f
*log(I*e^(d*x + c) + 1)/(a*d^2) + 3/2*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x
*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2/(a*d^3) - 3/2*(d^2*x
^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x +
 c)))*f^2/(a*d^3) - 4*I*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c))
)*f^2/(a*d^3) + (2*I*d*e*f + f^2)*x/(a*d^2) + (2*I*d*e*f - f^2)*x/(a*d^2) +
 (3*d*e*f - 2*I*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a*d^
3) - (3*d*e*f + 2*I*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(
a*d^3) - (2*I*d*e*f + f^2)*log(e^(d*x + c) + 1)/(a*d^3) - (2*I*d*e*f - f^2)
*log(e^(d*x + c) - 1)/(a*d^3) + 1/2*(d^3*f^2*x^3 + (3*d*e*f + 2*I*f^2)*d^2*
x^2)/(a*d^3) - 1/2*(d^3*f^2*x^3 + (3*d*e*f - 2*I*f^2)*d^2*x^2)/(a*d^3)
```

Fricas [C] time = 2.95854, size = 5378, normalized size = 14.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -(8*d^2*e^2 - 16*c*d*e*f + 8*c^2*f^2 - (-8*I*f^2*e^(5*d*x + 5*c) - 8*f^2*e^
(4*d*x + 4*c) + 16*I*f^2*e^(3*d*x + 3*c) + 16*f^2*e^(2*d*x + 2*c) - 8*I*f^2
*e^(d*x + c) - 8*f^2)*dilog(-I*e^(d*x + c)) - (-6*I*d*f^2*x - 6*I*d*e*f - 4
*f^2 + 2*(3*d*f^2*x + 3*d*e*f - 2*I*f^2)*e^(5*d*x + 5*c) + (-6*I*d*f^2*x -
```

```
6*I*d*e*f - 4*f^2)*e^(4*d*x + 4*c) - 4*(3*d*f^2*x + 3*d*e*f - 2*I*f^2)*e^(3
*d*x + 3*c) + (12*I*d*f^2*x + 12*I*d*e*f + 8*f^2)*e^(2*d*x + 2*c) + 2*(3*d*
f^2*x + 3*d*e*f - 2*I*f^2)*e^(d*x + c))*dilog(-e^(d*x + c)) - (6*I*d*f^2*x
+ 6*I*d*e*f - 4*f^2 - 2*(3*d*f^2*x + 3*d*e*f + 2*I*f^2)*e^(5*d*x + 5*c) + (
6*I*d*f^2*x + 6*I*d*e*f - 4*f^2)*e^(4*d*x + 4*c) + 4*(3*d*f^2*x + 3*d*e*f +
 2*I*f^2)*e^(3*d*x + 3*c) + (-12*I*d*f^2*x - 12*I*d*e*f + 8*f^2)*e^(2*d*x +
 2*c) - 2*(3*d*f^2*x + 3*d*e*f + 2*I*f^2)*e^(d*x + c))*dilog(e^(d*x + c)) -
 (8*I*d^2*f^2*x^2 + 16*I*d^2*e*f*x + 16*I*c*d*e*f - 8*I*c^2*f^2)*e^(5*d*x +
 5*c) - 2*(d^2*f^2*x^2 - 3*d^2*e^2 + 2*(4*c - 1)*d*e*f - 4*c^2*f^2 + 2*(d^2
*e*f - d*f^2)*x)*e^(4*d*x + 4*c) - (-10*I*d^2*f^2*x^2 + 6*I*d^2*e^2 + (-32*
I*c + 4*I)*d*e*f + 16*I*c^2*f^2 + (-20*I*d^2*e*f + 4*I*d*f^2)*x)*e^(3*d*x +
 3*c) + 2*(3*d^2*f^2*x^2 - 5*d^2*e^2 + 2*(8*c - 1)*d*e*f - 8*c^2*f^2 + 2*(3
*d^2*e*f - d*f^2)*x)*e^(2*d*x + 2*c) - (6*I*d^2*f^2*x^2 - 2*I*d^2*e^2 + (16
*I*c - 4*I)*d*e*f - 8*I*c^2*f^2 + (12*I*d^2*e*f - 4*I*d*f^2)*x)*e^(d*x + c)
 - (-3*I*d^2*f^2*x^2 - 3*I*d^2*e^2 - 4*d*e*f + 2*I*f^2 - 2*(3*I*d^2*e*f + 2
*d*f^2)*x + (3*d^2*f^2*x^2 + 3*d^2*e^2 - 4*I*d*e*f - 2*f^2 + (6*d^2*e*f - 4
*I*d*f^2)*x)*e^(5*d*x + 5*c) + (-3*I*d^2*f^2*x^2 - 3*I*d^2*e^2 - 4*d*e*f +
2*I*f^2 - 2*(3*I*d^2*e*f + 2*d*f^2)*x)*e^(4*d*x + 4*c) - (6*d^2*f^2*x^2 + 6
*d^2*e^2 - 8*I*d*e*f - 4*f^2 + (12*d^2*e*f - 8*I*d*f^2)*x)*e^(3*d*x + 3*c)
+ (6*I*d^2*f^2*x^2 + 6*I*d^2*e^2 + 8*d*e*f - 4*I*f^2 - 4*(-3*I*d^2*e*f - 2*
d*f^2)*x)*e^(2*d*x + 2*c) + (3*d^2*f^2*x^2 + 3*d^2*e^2 - 4*I*d*e*f - 2*f^2
+ (6*d^2*e*f - 4*I*d*f^2)*x)*e^(d*x + c))*log(e^(d*x + c) + 1) + (8*d*e*f -
 8*c*f^2 - (-8*I*d*e*f + 8*I*c*f^2)*e^(5*d*x + 5*c) + 8*(d*e*f - c*f^2)*e^(
4*d*x + 4*c) - (16*I*d*e*f - 16*I*c*f^2)*e^(3*d*x + 3*c) - 16*(d*e*f - c*f^
2)*e^(2*d*x + 2*c) - (-8*I*d*e*f + 8*I*c*f^2)*e^(d*x + c))*log(e^(d*x + c)
- I) - (3*I*d^2*e^2 - 2*(3*I*c + 2)*d*e*f + (3*I*c^2 + 4*c - 2*I)*f^2 - (3*
d^2*e^2 - (6*c - 4*I)*d*e*f + (3*c^2 - 4*I*c - 2)*f^2)*e^(5*d*x + 5*c) + (3
*I*d^2*e^2 - 2*(3*I*c + 2)*d*e*f + (3*I*c^2 + 4*c - 2*I)*f^2)*e^(4*d*x + 4*
c) + (6*d^2*e^2 - (12*c - 8*I)*d*e*f + 2*(3*c^2 - 4*I*c - 2)*f^2)*e^(3*d*x
+ 3*c) + (-6*I*d^2*e^2 - 4*(-3*I*c - 2)*d*e*f + (-6*I*c^2 - 8*c + 4*I)*f^2)
*e^(2*d*x + 2*c) - (3*d^2*e^2 - (6*c - 4*I)*d*e*f + (3*c^2 - 4*I*c - 2)*f^2
)*e^(d*x + c))*log(e^(d*x + c) - 1) + (8*d*f^2*x + 8*c*f^2 - (-8*I*d*f^2*x
- 8*I*c*f^2)*e^(5*d*x + 5*c) + 8*(d*f^2*x + c*f^2)*e^(4*d*x + 4*c) - (16*I*
d*f^2*x + 16*I*c*f^2)*e^(3*d*x + 3*c) - 16*(d*f^2*x + c*f^2)*e^(2*d*x + 2*c
) - (-8*I*d*f^2*x - 8*I*c*f^2)*e^(d*x + c))*log(I*e^(d*x + c) + 1) - (3*I*d
^2*f^2*x^2 + 6*I*c*d*e*f + (-3*I*c^2 - 4*c)*f^2 - 2*(-3*I*d^2*e*f + 2*d*f^2
)*x - (3*d^2*f^2*x^2 + 6*c*d*e*f - (3*c^2 - 4*I*c)*f^2 + (6*d^2*e*f + 4*I*d
*f^2)*x)*e^(5*d*x + 5*c) + (3*I*d^2*f^2*x^2 + 6*I*c*d*e*f + (-3*I*c^2 - 4*c
)*f^2 - 2*(-3*I*d^2*e*f + 2*d*f^2)*x)*e^(4*d*x + 4*c) + (6*d^2*f^2*x^2 + 12
*c*d*e*f - 2*(3*c^2 - 4*I*c)*f^2 + (12*d^2*e*f + 8*I*d*f^2)*x)*e^(3*d*x + 3
*c) + (-6*I*d^2*f^2*x^2 - 12*I*c*d*e*f + (6*I*c^2 + 8*c)*f^2 - 4*(3*I*d^2*e
*f - 2*d*f^2)*x)*e^(2*d*x + 2*c) - (3*d^2*f^2*x^2 + 6*c*d*e*f - (3*c^2 - 4*
I*c)*f^2 + (6*d^2*e*f + 4*I*d*f^2)*x)*e^(d*x + c))*log(-e^(d*x + c) + 1) +
(6*f^2*e^(5*d*x + 5*c) - 6*I*f^2*e^(4*d*x + 4*c) - 12*f^2*e^(3*d*x + 3*c) +
 12*I*f^2*e^(2*d*x + 2*c) + 6*f^2*e^(d*x + c) - 6*I*f^2)*polylog(3, -e^(d*x
 + c)) - (6*f^2*e^(5*d*x + 5*c) - 6*I*f^2*e^(4*d*x + 4*c) - 12*f^2*e^(3*d*x
 + 3*c) + 12*I*f^2*e^(2*d*x + 2*c) + 6*f^2*e^(d*x + c) - 6*I*f^2)*polylog(3
, e^(d*x + c)))/(2*a*d^3*e^(5*d*x + 5*c) - 2*I*a*d^3*e^(4*d*x + 4*c) - 4*a*
d^3*e^(3*d*x + 3*c) + 4*I*a*d^3*e^(2*d*x + 2*c) + 2*a*d^3*e^(d*x + c) - 2*I
*a*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)^3}{i a \sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*csch(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

3.219 $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=214

$$\frac{3f\text{PolyLog}\left(2,-e^{c+dx}\right)}{2ad^2}-\frac{3f\text{PolyLog}\left(2,e^{c+dx}\right)}{2ad^2}-\frac{f\operatorname{csch}(c+dx)}{2ad^2}-\frac{if\log(\sinh(c+dx))}{ad^2}-\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{ad^2}$$

```
[Out] (3*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d) + (I*(e + f*x)*Coth[c + d*x])/(a*d
) - (f*Csch[c + d*x])/(2*a*d^2) - ((e + f*x)*Coth[c + d*x]*Csch[c + d*x])/(
2*a*d) - ((2*I)*f*Log[Cosh[c/2 + (I/4)*Pi + (d*x)/2]])/(a*d^2) - (I*f*Log[S
inh[c + d*x]])/(a*d^2) + (3*f*PolyLog[2, -E^(c + d*x)])/(2*a*d^2) - (3*f*Po
lyLog[2, E^(c + d*x)])/(2*a*d^2) + (I*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)
/2])/(a*d)
```

Rubi [A] time = 0.367409, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {5575, 4185, 4182, 2279, 2391, 4184, 3475, 3318}

$$\frac{3f\text{PolyLog}\left(2,-e^{c+dx}\right)}{2ad^2}-\frac{3f\text{PolyLog}\left(2,e^{c+dx}\right)}{2ad^2}-\frac{f\operatorname{csch}(c+dx)}{2ad^2}-\frac{if\log(\sinh(c+dx))}{ad^2}-\frac{2if\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}+\frac{i\pi}{4}\right)\right)}{ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```


```
[Out] (3*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d) + (I*(e + f*x)*Coth[c + d*x])/(a*d
) - (f*Csch[c + d*x])/(2*a*d^2) - ((e + f*x)*Coth[c + d*x]*Csch[c + d*x])/(
2*a*d) - ((2*I)*f*Log[Cosh[c/2 + (I/4)*Pi + (d*x)/2]])/(a*d^2) - (I*f*Log[S
inh[c + d*x]])/(a*d^2) + (3*f*PolyLog[2, -E^(c + d*x)])/(2*a*d^2) - (3*f*Po
lyLog[2, E^(c + d*x)])/(2*a*d^2) + (I*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)
/2])/(a*d)
```


Rule 5575

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a +
b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\left(i\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx\right)+\frac{\int (e+fx)\text{csch}^3(c+dx)\,dx}{a}$$

$$= -\frac{f\text{csch}(c+dx)}{2ad^2}-\frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2ad}-\frac{i\int (e+fx)\text{csch}^2(c+dx)\,dx}{a}-\frac{\int (\ldots}{}$$

$$= \frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{i(e+fx)\coth(c+dx)}{ad}-\frac{f\text{csch}(c+dx)}{2ad^2}-\frac{(e+fx)\coth(c+dx\ldots}{2ad}$$

$$= \frac{3(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{i(e+fx)\coth(c+dx)}{ad}-\frac{f\text{csch}(c+dx)}{2ad^2}-\frac{(e+fx)\coth(c+\ldots}{2ad}$$

$$= \frac{3(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{i(e+fx)\coth(c+dx)}{ad}-\frac{f\text{csch}(c+dx)}{2ad^2}-\frac{(e+fx)\coth(c+\ldots}{2ad}$$

$$= \frac{3(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad}+\frac{i(e+fx)\coth(c+dx)}{ad}-\frac{f\text{csch}(c+dx)}{2ad^2}-\frac{(e+fx)\coth(c+\ldots}{2ad}$$

Mathematica [B] time = 2.88705, size = 541, normalized size = 2.53

$$\frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(-12f\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(\text{PolyLog}\left(2,-e^{-c-dx}\right)-\text{PolyLog}\ldots\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*((2*I)*(I*f + 2*d*(e + f*x))*Cos
h[(c + d*x)/2]*(I + Coth[(c + d*x)/2]) - d*(e + f*x)*(I + Coth[(c + d*x)/2]
)*Csch[(c + d*x)/2] - 8*f*(c + d*x)*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2
]) + 16*f*ArcTan[Tanh[(c + d*x)/2]]*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2
]) - 12*d*e*Log[Tanh[(c + d*x)/2]]*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]
) + 12*c*f*Log[Tanh[(c + d*x)/2]]*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])
 - 12*f*((c + d*x)*(Log[1 - E^(-c - d*x)] - Log[1 + E^(-c - d*x)]) + PolyLo
g[2, -E^(-c - d*x)] - PolyLog[2, E^(-c - d*x)])*(Cosh[(c + d*x)/2] + I*Sinh
[(c + d*x)/2]) + (16*I)*d*(e + f*x)*Sinh[(c + d*x)/2] + 8*f*Log[Cosh[c + d*
x]]*((-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]) + 8*f*Log[Sinh[c + d*x]]*(
(-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]) + 2*(f + (2*I)*d*(e + f*x))*(Co
sh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*Tanh[(c + d*x)/2] - I*d*(e + f*x)*Se
ch[(c + d*x)/2]*(-I + Tanh[(c + d*x)/2])))/(8*d^2*(a + I*a*Sinh[c + d*x]))
```

Maple [B] time = 0.184, size = 423, normalized size = 2.

$$-\frac{-3\,idfx\mathrm{e}^{3\,dx+3\,c}-5\,dfx\mathrm{e}^{2\,dx+2\,c}+3\,dfx\mathrm{e}^{4\,dx+4\,c}+idfx\mathrm{e}^{dx+c}-5\,de\mathrm{e}^{2\,dx+2\,c}+3\,de\mathrm{e}^{4\,dx+4\,c}-3\,ide\mathrm{e}^{3\,dx+3\,c}+4\,dfx+fe}{\left(\mathrm{e}^{2\,dx+2\,c}-1\right)^{2}d^{2}\left(\mathrm{e}^{dx+c}-i\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -(-3*I*d*f*x*exp(3*d*x+3*c)-5*d*f*x*exp(2*d*x+2*c)+3*d*f*x*exp(4*d*x+4*c)+I
*d*f*x*exp(d*x+c)-5*d*e*exp(2*d*x+2*c)+3*d*e*exp(4*d*x+4*c)-3*I*d*e*exp(3*d
*x+3*c)+4*d*f*x+f*exp(4*d*x+4*c)+I*exp(d*x+c)*f-I*exp(3*d*x+3*c)*f+4*d*e-f*
exp(2*d*x+2*c)+I*d*e*exp(d*x+c))/(exp(2*d*x+2*c)-1)^2/d^2/(exp(d*x+c)-I)/a-
3/2*f*polylog(2,exp(d*x+c))/a/d^2+3/2*f*polylog(2,-exp(d*x+c))/a/d^2-2*I*f/
a/d^2*ln(exp(d*x+c)-I)+4*I/d^2/a*f*ln(exp(d*x+c))-I/d^2/a*f*ln(exp(d*x+c)-1
)-I/d^2/a*f*ln(exp(d*x+c)+1)-3/2/d/a*e*ln(exp(d*x+c)-1)+3/2/d/a*e*ln(exp(d*
x+c)+1)-3/2/d/a*ln(1-exp(d*x+c))*f*x-3/2/d^2/a*ln(1-exp(d*x+c))*c*f+3/2/d/a
*ln(exp(d*x+c)+1)*f*x+3/2/d^2/a*f*c*ln(exp(d*x+c)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\left(24\,d\int\frac{x}{16\left(ade^{(dx+c)}+ad\right)}\,dx+24\,d\int\frac{x}{16\left(ade^{(dx+c)}-ad\right)}\,dx+\frac{8\left(2\,dxe^{(5\,dx+5\,c)}+2i\,dx+\left(i\,dxe^{(4\,c)}+i\,e^{(4\,c)}\right)e^{(4\,dx)}-\right.}{8i\,ad^{2}e^{(5\,dx+5\,c)}+8\,ad^{2}e^{(4\,dx+4\,c)}-16}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(24*d*integrate(1/16*x/(a*d*e^(d*x + c) + a*d), x) + 24*d*integrate(1/16*x
/(a*d*e^(d*x + c) - a*d), x) + 8*(2*d*x*e^(5*d*x + 5*c) + 2*I*d*x + (I*d*x*
e^(4*c) + I*e^(4*c))*e^(4*d*x) - (d*x*e^(3*c) - e^(3*c))*e^(3*d*x) + (-I*d*
x*e^(2*c) - I*e^(2*c))*e^(2*d*x) + (d*x*e^c - e^c)*e^(d*x))/(8*I*a*d^2*e^(5
*d*x + 5*c) + 8*a*d^2*e^(4*d*x + 4*c) - 16*I*a*d^2*e^(3*d*x + 3*c) - 16*a*d
^2*e^(2*d*x + 2*c) + 8*I*a*d^2*e^(d*x + c) + 8*a*d^2) - 2*I*(d*x + c)/(a*d^
2) + 2*I*log((e^(d*x + c) - I)*e^(-c))/(a*d^2) + I*log(e^(d*x + c) + 1)/(a*
d^2) + I*log(e^(d*x + c) - 1)/(a*d^2))*f - 1/2*e*(16*(-I*e^(-d*x - c) - 5*e
^(-2*d*x - 2*c) + 3*I*e^(-3*d*x - 3*c) + 3*e^(-4*d*x - 4*c) + 4)/((8*a*e^(-
```

```
d*x - c) - 16*I*a*e^(-2*d*x - 2*c) - 16*a*e^(-3*d*x - 3*c) + 8*I*a*e^(-4*d*
x - 4*c) + 8*a*e^(-5*d*x - 5*c) + 8*I*a)*d) - 3*log(e^(-d*x - c) + 1)/(a*d)
 + 3*log(e^(-d*x - c) - 1)/(a*d))
```

Fricas [B] time = 2.77223, size = 2163, normalized size = 10.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(8*d*e - 4*c*f - (3*f*e^(5*d*x + 5*c) - 3*I*f*e^(4*d*x + 4*c) - 6*f*e^(3*d
*x + 3*c) + 6*I*f*e^(2*d*x + 2*c) + 3*f*e^(d*x + c) - 3*I*f)*dilog(-e^(d*x
+ c)) + (3*f*e^(5*d*x + 5*c) - 3*I*f*e^(4*d*x + 4*c) - 6*f*e^(3*d*x + 3*c)
+ 6*I*f*e^(2*d*x + 2*c) + 3*f*e^(d*x + c) - 3*I*f)*dilog(e^(d*x + c)) - (8*
I*d*f*x + 4*I*c*f)*e^(5*d*x + 5*c) - 2*(d*f*x - 3*d*e + (2*c - 1)*f)*e^(4*d
*x + 4*c) - (-10*I*d*f*x + 6*I*d*e + (-8*I*c + 2*I)*f)*e^(3*d*x + 3*c) + 2*
(3*d*f*x - 5*d*e + (4*c - 1)*f)*e^(2*d*x + 2*c) - (6*I*d*f*x - 2*I*d*e + (4
*I*c - 2*I)*f)*e^(d*x + c) - (-3*I*d*f*x - 3*I*d*e + (3*d*f*x + 3*d*e - 2*I
*f)*e^(5*d*x + 5*c) + (-3*I*d*f*x - 3*I*d*e - 2*f)*e^(4*d*x + 4*c) - 2*(3*d
*f*x + 3*d*e - 2*I*f)*e^(3*d*x + 3*c) + (6*I*d*f*x + 6*I*d*e + 4*f)*e^(2*d*
x + 2*c) + (3*d*f*x + 3*d*e - 2*I*f)*e^(d*x + c) - 2*f)*log(e^(d*x + c) + 1
) - (-4*I*f*e^(5*d*x + 5*c) - 4*f*e^(4*d*x + 4*c) + 8*I*f*e^(3*d*x + 3*c) +
 8*f*e^(2*d*x + 2*c) - 4*I*f*e^(d*x + c) - 4*f)*log(e^(d*x + c) - I) - (3*I
*d*e + (-3*I*c - 2)*f - (3*d*e - (3*c - 2*I)*f)*e^(5*d*x + 5*c) + (3*I*d*e
+ (-3*I*c - 2)*f)*e^(4*d*x + 4*c) + (6*d*e - (6*c - 4*I)*f)*e^(3*d*x + 3*c)
 - 2*(3*I*d*e + (-3*I*c - 2)*f)*e^(2*d*x + 2*c) - (3*d*e - (3*c - 2*I)*f)*e
^(d*x + c))*log(e^(d*x + c) - 1) - (3*I*d*f*x + 3*I*c*f - 3*(d*f*x + c*f)*e
^(5*d*x + 5*c) + (3*I*d*f*x + 3*I*c*f)*e^(4*d*x + 4*c) + 6*(d*f*x + c*f)*e^
(3*d*x + 3*c) + (-6*I*d*f*x - 6*I*c*f)*e^(2*d*x + 2*c) - 3*(d*f*x + c*f)*e^
(d*x + c))*log(-e^(d*x + c) + 1))/(2*a*d^2*e^(5*d*x + 5*c) - 2*I*a*d^2*e^(4
*d*x + 4*c) - 4*a*d^2*e^(3*d*x + 3*c) + 4*I*a*d^2*e^(2*d*x + 2*c) + 2*a*d^2
*e^(d*x + c) - 2*I*a*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\,\mathrm{csch}\,(dx+c)^3}{i\,a\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*csch(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

3.220 $\int \frac{\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=87

$$\frac{2i\coth(c+dx)}{ad}+\frac{3\tanh^{-1}(\cosh(c+dx))}{2ad}-\frac{3\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}+\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{d(a+ia\sinh(c+dx))}$$

```
[Out] (3*ArcTanh[Cosh[c + d*x]])/(2*a*d) + ((2*I)*Coth[c + d*x])/(a*d) - (3*Coth[
c + d*x]*Csch[c + d*x])/(2*a*d) + (Coth[c + d*x]*Csch[c + d*x])/(d*(a + I*a
*Sinh[c + d*x]))
```

Rubi [A] time = 0.12336, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2768, 2748, 3768, 3770, 3767, 8}

$$\frac{2i\coth(c+dx)}{ad}+\frac{3\tanh^{-1}(\cosh(c+dx))}{2ad}-\frac{3\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}+\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{d(a+ia\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Cosh[c + d*x]])/(2*a*d) + ((2*I)*Coth[c + d*x])/(a*d) - (3*Coth[
c + d*x]*Csch[c + d*x])/(2*a*d) + (Coth[c + d*x]*Csch[c + d*x])/(d*(a + I*a
*Sinh[c + d*x]))
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\text{csch}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))} - \frac{\int \text{csch}^3(c+dx)(-3a+2ia\sinh(c+dx))\,dx}{a^2}$$
$$= \frac{\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))} - \frac{(2i)\int \text{csch}^2(c+dx)\,dx}{a} + \frac{3\int \text{csch}^3(c+dx)\,dx}{a}$$
$$= -\frac{3\coth(c+dx)\text{csch}(c+dx)}{2ad} + \frac{\coth(c+dx)\text{csch}(c+dx)}{d(a+ia\sinh(c+dx))} - \frac{3\int \text{csch}(c+dx)\,dx}{2a} - \frac{2\,\text{Sul}}{}$$
$$= \frac{3\tanh^{-1}(\cosh(c+dx))}{2ad} + \frac{2i\coth(c+dx)}{ad} - \frac{3\coth(c+dx)\text{csch}(c+dx)}{2ad} + \frac{\coth(c+dx)}{d(a+ia\sin}$$

Mathematica [A] time = 0.41949, size = 90, normalized size = 1.03

$$\frac{4i\tanh(c+dx)+4i\text{csch}(2(c+dx))-3\text{sech}(c+dx)+\text{csch}^2(c+dx)(-\text{sech}(c+dx))+3\sqrt{\cosh^2(c+dx)}\text{sech}(c+dx}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((4*I)*Csch[2*(c + d*x)] - 3*Sech[c + d*x] + 3*ArcTanh[Sqrt[Cosh[c + d*x]^2
]]*Sqrt[Cosh[c + d*x]^2]*Sech[c + d*x] - Csch[c + d*x]^2*Sech[c + d*x] + (4
*I)*Tanh[c + d*x])/(2*a*d)
```

Maple [A] time = 0.049, size = 119, normalized size = 1.4

$$\frac{2i}{da}\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}+\frac{\frac{i}{2}}{da}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{8da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}-\frac{1}{8da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-2}+\frac{\frac{i}{2}}{da}\left(\tanh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 2*I/d/a/(-I+tanh(1/2*d*x+1/2*c))+1/2*I/d/a*tanh(1/2*d*x+1/2*c)+1/8/d/a*tanh
(1/2*d*x+1/2*c)^2-1/8/d/a/tanh(1/2*d*x+1/2*c)^2+1/2*I/d/a/tanh(1/2*d*x+1/2*
c)-3/2/d/a*ln(tanh(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.15247, size = 213, normalized size = 2.45

$$-\frac{8\left(-ie^{(-dx-c)}-5e^{(-2dx-2c)}+3ie^{(-3dx-3c)}+3e^{(-4dx-4c)}+4\right)}{\left(8ae^{(-dx-c)}-16iae^{(-2dx-2c)}-16ae^{(-3dx-3c)}+8iae^{(-4dx-4c)}+8ae^{(-5dx-5c)}+8ia\right)d}+\frac{3\log\left(e^{(-dx-c)}+1\right)}{2ad}-\frac{3\log\left(e\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -8*(-I*e^(-d*x - c) - 5*e^(-2*d*x - 2*c) + 3*I*e^(-3*d*x - 3*c) + 3*e^(-4*d
*x - 4*c) + 4)/((8*a*e^(-d*x - c) - 16*I*a*e^(-2*d*x - 2*c) - 16*a*e^(-3*d*
x - 3*c) + 8*I*a*e^(-4*d*x - 4*c) + 8*a*e^(-5*d*x - 5*c) + 8*I*a)*d) + 3/2*
log(e^(-d*x - c) + 1)/(a*d) - 3/2*log(e^(-d*x - c) - 1)/(a*d)
```

Fricas [B] time = 2.6035, size = 640, normalized size = 7.36

$$\frac{\left(3\,e^{(5\,dx+5\,c)} - 3i\,e^{(4\,dx+4\,c)} - 6\,e^{(3\,dx+3\,c)} + 6i\,e^{(2\,dx+2\,c)} + 3\,e^{(dx+c)} - 3i\right)\log\left(e^{(dx+c)} + 1\right) - \left(3\,e^{(5\,dx+5\,c)} - 3i\,e^{(4\,dx+4\,c)} - 6\,e^{(3\,dx+}\right.}{2\,ade^{(5\,dx+5\,c)} - 2i\,ade^{(4\,dx+4\,c)} - 4\,ade^{(3\,dx+3\,c)} + 4i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((3*e^(5*d*x + 5*c) - 3*I*e^(4*d*x + 4*c) - 6*e^(3*d*x + 3*c) + 6*I*e^(2*d*
x + 2*c) + 3*e^(d*x + c) - 3*I)*log(e^(d*x + c) + 1) - (3*e^(5*d*x + 5*c) -
 3*I*e^(4*d*x + 4*c) - 6*e^(3*d*x + 3*c) + 6*I*e^(2*d*x + 2*c) + 3*e^(d*x +
 c) - 3*I)*log(e^(d*x + c) - 1) - 6*e^(4*d*x + 4*c) + 6*I*e^(3*d*x + 3*c) +
 10*e^(2*d*x + 2*c) - 2*I*e^(d*x + c) - 8)/(2*a*d*e^(5*d*x + 5*c) - 2*I*a*d
*e^(4*d*x + 4*c) - 4*a*d*e^(3*d*x + 3*c) + 4*I*a*d*e^(2*d*x + 2*c) + 2*a*d*
e^(d*x + c) - 2*I*a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21453, size = 142, normalized size = 1.63

$$\frac{3\,\log\left(e^{(dx+c)} + 1\right)}{2\,ad} - \frac{3\,\log\left(\left|e^{(dx+c)} - 1\right|\right)}{2\,ad} - \frac{e^{(3\,dx+3\,c)} - 2i\,e^{(2\,dx+2\,c)} + e^{(dx+c)} + 2i}{ad\left(e^{(2\,dx+2\,c)} - 1\right)^2} - \frac{2i}{ad\left(i\,e^{(dx+c)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/2*log(e^(d*x + c) + 1)/(a*d) - 3/2*log(abs(e^(d*x + c) - 1))/(a*d) - (e^(
3*d*x + 3*c) - 2*I*e^(2*d*x + 2*c) + e^(d*x + c) + 2*I)/(a*d*(e^(2*d*x + 2*
c) - 1)^2) - 2*I/(a*d*(I*e^(d*x + c) + 1))
```

3.221 $\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Csch[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0749999, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]

[Out] Defer[Int][Csch[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.023, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Csch[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]

[Out] $Aborted
```

Maple [A] time = 1.332, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx+c))^3}{(fx+e)(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)

[Out] int(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -8*f*integrate(1/(-4*I*a*d*f^2*x^2 - 8*I*a*d*e*f*x - 4*I*a*d*e^2 + 4*(a*d*f
^2*x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) - 8*(4*d*f*x + 4*d
*e + (3*d*f*x*e^(4*c) + (3*d*e - f)*e^(4*c))*e^(4*d*x) + (-3*I*d*f*x*e^(3*c
) + (-3*I*d*e + I*f)*e^(3*c))*e^(3*d*x) - (5*d*f*x*e^(2*c) + (5*d*e - f)*e^
(2*c))*e^(2*d*x) + (I*d*f*x*e^c + (I*d*e - I*f)*e^c)*e^(d*x))/(-8*I*a*d^2*f
^2*x^2 - 16*I*a*d^2*e*f*x - 8*I*a*d^2*e^2 + 8*(a*d^2*f^2*x^2*e^(5*c) + 2*a*
d^2*e*f*x*e^(5*c) + a*d^2*e^2*e^(5*c))*e^(5*d*x) + (-8*I*a*d^2*f^2*x^2*e^(4
*c) - 16*I*a*d^2*e*f*x*e^(4*c) - 8*I*a*d^2*e^2*e^(4*c))*e^(4*d*x) - 16*(a*d
^2*f^2*x^2*e^(3*c) + 2*a*d^2*e*f*x*e^(3*c) + a*d^2*e^2*e^(3*c))*e^(3*d*x) +
 (16*I*a*d^2*f^2*x^2*e^(2*c) + 32*I*a*d^2*e*f*x*e^(2*c) + 16*I*a*d^2*e^2*e^
(2*c))*e^(2*d*x) + 8*(a*d^2*f^2*x^2*e^c + 2*a*d^2*e*f*x*e^c + a*d^2*e^2*e^c
)*e^(d*x)) - 8*integrate(1/16*(3*d^2*f^2*x^2 + 3*d^2*e^2 + 2*I*d*e*f - 2*f^
2 + (6*d^2*e*f + 2*I*d*f^2)*x)/(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2
*e^2*f*x + a*d^2*e^3 + (a*d^2*f^3*x^3*e^c + 3*a*d^2*e*f^2*x^2*e^c + 3*a*d^2
*e^2*f*x*e^c + a*d^2*e^3*e^c)*e^(d*x)), x) - 8*integrate(-1/16*(3*d^2*f^2*x
^2 + 3*d^2*e^2 - 2*I*d*e*f - 2*f^2 + (6*d^2*e*f - 2*I*d*f^2)*x)/(a*d^2*f^3*
x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 - (a*d^2*f^3*x^3*e^c
+ 3*a*d^2*e*f^2*x^2*e^c + 3*a*d^2*e^2*f*x*e^c + a*d^2*e^3*e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(4*d*f*x + 4*d*e + (3*d*f*x + 3*d*e - f)*e^(4*d*x + 4*c) - (3*I*d*f*x + 3*
I*d*e - I*f)*e^(3*d*x + 3*c) - (5*d*f*x + 5*d*e - f)*e^(2*d*x + 2*c) - (-I*
d*f*x - I*d*e + I*f)*e^(d*x + c) - (-I*a*d^2*f^2*x^2 - 2*I*a*d^2*e*f*x - I*
a*d^2*e^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*e^(5*d*x + 5*c) + (
-I*a*d^2*f^2*x^2 - 2*I*a*d^2*e*f*x - I*a*d^2*e^2)*e^(4*d*x + 4*c) - 2*(a*d^
2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*e^(3*d*x + 3*c) + (2*I*a*d^2*f^2*x^2
 + 4*I*a*d^2*e*f*x + 2*I*a*d^2*e^2)*e^(2*d*x + 2*c) + (a*d^2*f^2*x^2 + 2*a*
d^2*e*f*x + a*d^2*e^2)*e^(d*x + c))*integral((4*d*f^2*x + 4*d*e*f - (3*d^2*
f^2*x^2 + 3*d^2*e^2 + 2*d*e*f - 2*f^2 + 2*(3*d^2*e*f + d*f^2)*x)*e^(2*d*x +
 2*c) + (3*I*d^2*f^2*x^2 + 3*I*d^2*e^2 + 2*I*d*e*f - 2*I*f^2 + (6*I*d^2*e*f
 + 2*I*d*f^2)*x)*e^(d*x + c))/(I*a*d^2*f^3*x^3 + 3*I*a*d^2*e*f^2*x^2 + 3*I*
a*d^2*e^2*f*x + I*a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*
e^2*f*x + a*d^2*e^3)*e^(3*d*x + 3*c) + (-I*a*d^2*f^3*x^3 - 3*I*a*d^2*e*f^2*
x^2 - 3*I*a*d^2*e^2*f*x - I*a*d^2*e^3)*e^(2*d*x + 2*c) - (a*d^2*f^3*x^3 + 3
*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^(d*x + c)), x))/(-I*a*d^2
*f^2*x^2 - 2*I*a*d^2*e*f*x - I*a*d^2*e^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x +
 a*d^2*e^2)*e^(5*d*x + 5*c) + (-I*a*d^2*f^2*x^2 - 2*I*a*d^2*e*f*x - I*a*d^2
*e^2)*e^(4*d*x + 4*c) - 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*e^(3*
d*x + 3*c) + (2*I*a*d^2*f^2*x^2 + 4*I*a*d^2*e*f*x + 2*I*a*d^2*e^2)*e^(2*d*x
 + 2*c) + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*e^(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.222 $\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Csch[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0739772, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Csch[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.047, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Csch[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] $Aborted
```

Maple [A] time = 2.257, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx+c))^3}{(fx+e)^2(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -8*f*integrate(1/(-2*I*a*d*f^3*x^3 - 6*I*a*d*e*f^2*x^2 - 6*I*a*d*e^2*f*x -
2*I*a*d*e^3 + 2*(a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c
+ a*d*e^3*e^c)*e^(d*x)), x) - 8*(4*d*f*x + 4*d*e + (3*d*f*x*e^(4*c) + (3*d*
e - 2*f)*e^(4*c))*e^(4*d*x) + (-3*I*d*f*x*e^(3*c) + (-3*I*d*e + 2*I*f)*e^(3
*c))*e^(3*d*x) - (5*d*f*x*e^(2*c) + (5*d*e - 2*f)*e^(2*c))*e^(2*d*x) + (I*d
*f*x*e^c + (I*d*e - 2*I*f)*e^c)*e^(d*x))/(-8*I*a*d^2*f^3*x^3 - 24*I*a*d^2*e
*f^2*x^2 - 24*I*a*d^2*e^2*f*x - 8*I*a*d^2*e^3 + 8*(a*d^2*f^3*x^3*e^(5*c) +
3*a*d^2*e*f^2*x^2*e^(5*c) + 3*a*d^2*e^2*f*x*e^(5*c) + a*d^2*e^3*e^(5*c))*e^
(5*d*x) + (-8*I*a*d^2*f^3*x^3*e^(4*c) - 24*I*a*d^2*e*f^2*x^2*e^(4*c) - 24*I
*a*d^2*e^2*f*x*e^(4*c) - 8*I*a*d^2*e^3*e^(4*c))*e^(4*d*x) - 16*(a*d^2*f^3*x
^3*e^(3*c) + 3*a*d^2*e*f^2*x^2*e^(3*c) + 3*a*d^2*e^2*f*x*e^(3*c) + a*d^2*e^
3*e^(3*c))*e^(3*d*x) + (16*I*a*d^2*f^3*x^3*e^(2*c) + 48*I*a*d^2*e*f^2*x^2*e
^(2*c) + 48*I*a*d^2*e^2*f*x*e^(2*c) + 16*I*a*d^2*e^3*e^(2*c))*e^(2*d*x) + 8
*(a*d^2*f^3*x^3*e^c + 3*a*d^2*e*f^2*x^2*e^c + 3*a*d^2*e^2*f*x*e^c + a*d^2*e
^3*e^c)*e^(d*x)) - 8*integrate(1/16*(3*d^2*f^2*x^2 + 3*d^2*e^2 + 4*I*d*e*f
- 6*f^2 + (6*d^2*e*f + 4*I*d*f^2)*x)/(a*d^2*f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6
*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4 + (a*d^2*f^4*x^4*e^c + 4*a
*d^2*e*f^3*x^3*e^c + 6*a*d^2*e^2*f^2*x^2*e^c + 4*a*d^2*e^3*f*x*e^c + a*d^2*
e^4*e^c)*e^(d*x)), x) - 8*integrate(-1/16*(3*d^2*f^2*x^2 + 3*d^2*e^2 - 4*I*
d*e*f - 6*f^2 + (6*d^2*e*f - 4*I*d*f^2)*x)/(a*d^2*f^4*x^4 + 4*a*d^2*e*f^3*x
^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4 - (a*d^2*f^4*x^4*e^c
 + 4*a*d^2*e*f^3*x^3*e^c + 6*a*d^2*e^2*f^2*x^2*e^c + 4*a*d^2*e^3*f*x*e^c +
a*d^2*e^4*e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -(4*d*f*x + 4*d*e + (3*d*f*x + 3*d*e - 2*f)*e^(4*d*x + 4*c) - (3*I*d*f*x +
3*I*d*e - 2*I*f)*e^(3*d*x + 3*c) - (5*d*f*x + 5*d*e - 2*f)*e^(2*d*x + 2*c)
- (-I*d*f*x - I*d*e + 2*I*f)*e^(d*x + c) - (-I*a*d^2*f^3*x^3 - 3*I*a*d^2*e*
f^2*x^2 - 3*I*a*d^2*e^2*f*x - I*a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*
x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^(5*d*x + 5*c) + (-I*a*d^2*f^3*x^3 - 3*
I*a*d^2*e*f^2*x^2 - 3*I*a*d^2*e^2*f*x - I*a*d^2*e^3)*e^(4*d*x + 4*c) - 2*(a
*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^(3*d*x +
3*c) + (2*I*a*d^2*f^3*x^3 + 6*I*a*d^2*e*f^2*x^2 + 6*I*a*d^2*e^2*f*x + 2*I*a
*d^2*e^3)*e^(2*d*x + 2*c) + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^
2*f*x + a*d^2*e^3)*e^(d*x + c))*integral((8*d*f^2*x + 8*d*e*f - (3*d^2*f^2*
x^2 + 3*d^2*e^2 + 4*d*e*f - 6*f^2 + 2*(3*d^2*e*f + 2*d*f^2)*x)*e^(2*d*x + 2
*c) + (3*I*d^2*f^2*x^2 + 3*I*d^2*e^2 + 4*I*d*e*f - 6*I*f^2 + (6*I*d^2*e*f +
 4*I*d*f^2)*x)*e^(d*x + c))/(I*a*d^2*f^4*x^4 + 4*I*a*d^2*e*f^3*x^3 + 6*I*a*
```

```
d^2*e^2*f^2*x^2 + 4*I*a*d^2*e^3*f*x + I*a*d^2*e^4 + (a*d^2*f^4*x^4 + 4*a*d^
2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4)*e^(3*d*x +
 3*c) + (-I*a*d^2*f^4*x^4 - 4*I*a*d^2*e*f^3*x^3 - 6*I*a*d^2*e^2*f^2*x^2 - 4
*I*a*d^2*e^3*f*x - I*a*d^2*e^4)*e^(2*d*x + 2*c) - (a*d^2*f^4*x^4 + 4*a*d^2*
e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4)*e^(d*x + c))
, x))/(-I*a*d^2*f^3*x^3 - 3*I*a*d^2*e*f^2*x^2 - 3*I*a*d^2*e^2*f*x - I*a*d^2
*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^
(5*d*x + 5*c) + (-I*a*d^2*f^3*x^3 - 3*I*a*d^2*e*f^2*x^2 - 3*I*a*d^2*e^2*f*x
 - I*a*d^2*e^3)*e^(4*d*x + 4*c) - 2*(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*
a*d^2*e^2*f*x + a*d^2*e^3)*e^(3*d*x + 3*c) + (2*I*a*d^2*f^3*x^3 + 6*I*a*d^2
*e*f^2*x^2 + 6*I*a*d^2*e^2*f*x + 2*I*a*d^2*e^3)*e^(2*d*x + 2*c) + (a*d^2*f^
3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")

[Out] Timed out
```

3.223 $\int \frac{(e+fx)^3 \sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=453

$$\frac{6af^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{6af^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{3af(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}$$

```
[Out] (e + f*x)^4/(4*b*f) - (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2])])/(b*Sqrt[a^2 + b^2]*d) + (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d) - (3*a*f*(e + f*x)^2*PolyLog[2,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2) + (3*a*
f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt
[a^2 + b^2]*d^2) + (6*a*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (6*a*f^2*(e + f*x)*PolyLog[3, -(
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (6*a*f^3
*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d
^4) + (6*a*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqr
t[a^2 + b^2]*d^4)
```

Rubi [A] time = 0.794095, antiderivative size = 453, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5557, 32, 3322, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{6af^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{6af^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{3af(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (e + f*x)^4/(4*b*f) - (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2])])/(b*Sqrt[a^2 + b^2]*d) + (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d) - (3*a*f*(e + f*x)^2*PolyLog[2,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2) + (3*a*
f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt
[a^2 + b^2]*d^2) + (6*a*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (6*a*f^2*(e + f*x)*PolyLog[3, -(
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (6*a*f^3
*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d
^4) + (6*a*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqr
t[a^2 + b^2]*d^4)
```


Rule *5557*

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\,dx}{b} - \frac{a\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{(2a)\int \frac{e^{c+dx}(e+fx)^3}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{b}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{(2a)\int \frac{e^{c+dx}(e+fx)^3}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\sqrt{a^2+b^2}} + \frac{(2a)\int \frac{e^{c+dx}(e+fx)^3}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\sqrt{a^2+b^2}}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{(3af)\int (e\cdots}{\cdots}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} - \frac{3af(e+f\cdots}{b\cdots}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} - \frac{3af(e+f\cdots}{b\cdots}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} - \frac{3af(e+f\cdots}{b\cdots}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} - \frac{3af(e+f\cdots}{b\cdots}$$

Mathematica [A] time = 2.31197, size = 607, normalized size = 1.34

$$\frac{a\left(-3d^2f(e+fx)^2\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+3d^2f(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+6def^2\text{PolyLog}\left(3,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)-6\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(4*b) + (a*(2*d^3*e^3*ArcTa
nh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2])] - 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2])] - d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3*d^3*
e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^3*f^3*x^3*Log[
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyLog[2,
 (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*d*e*f^2*PolyLog[3, (b*E^(c + d
*x))/(-a + Sqrt[a^2 + b^2])] + 6*d*f^3*x*PolyLog[3, (b*E^(c + d*x))/(-a + S
qrt[a^2 + b^2])] - 6*d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2]))] - 6*d*f^3*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6
*f^3*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*f^3*PolyLog[4,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(b*Sqrt[a^2 + b^2]*d^4)
```

Maple [F] time = 0.241, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^3\sinh\left(dx+c\right)}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.12158, size = 2677, normalized size = 5.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*((a^2 + b^2)*d^4*f^3*x^4 + 4*(a^2 + b^2)*d^4*e*f^2*x^3 + 6*(a^2 + b^2)*
d^4*e^2*f*x^2 + 4*(a^2 + b^2)*d^4*e^3*x - 24*a*b*f^3*sqrt((a^2 + b^2)/b^2)*
polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*a*b*f^3*sqrt((a^2 + b^2)/b^2)*polyl
og(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2))/b) - 12*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2*x + a
*b*d^2*e^2*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c
) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) +
 12*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2*x + a*b*d^2*e^2*f)*sqrt((a^2 + b^2)/
b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 4*(a*b*d^3*e^3 - 3*a*b*c*d^2*e
^2*f + 3*a*b*c^2*d*e*f^2 - a*b*c^3*f^3)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(
d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 4*(a*b*d^
3*e^3 - 3*a*b*c*d^2*e^2*f + 3*a*b*c^2*d*e*f^2 - a*b*c^3*f^3)*sqrt((a^2 + b^
2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^
2) + 2*a) - 4*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*e^2*f*x +
3*a*b*c*d^2*e^2*f - 3*a*b*c^2*d*e*f^2 + a*b*c^3*f^3)*sqrt((a^2 + b^2)/b^2)*
log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2
 + 3*a*b*d^3*e^2*f*x + 3*a*b*c*d^2*e^2*f - 3*a*b*c^2*d*e*f^2 + a*b*c^3*f^3)
*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*
x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 24*(a*b*d*f^3*x +
 a*b*d*e*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*
x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 24
*(a*b*d*f^3*x + a*b*d*e*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x +
 c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2))/b))/((a^2*b + b^3)*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \sinh(dx+c)}{b \sinh(dx+c)+a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.224 $\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=337

$$-\frac{2af(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}+\frac{2af(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2\sqrt{a^2+b^2}}+\frac{2af^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{2af^2\text{Pol}}{}$$

```
[Out] (e + f*x)^3/(3*b*f) - (a*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2])])/(b*Sqrt[a^2 + b^2]*d) + (a*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d) - (2*a*f*(e + f*x)*PolyLog[2, -
((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2) + (2*a*f*
(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2
 + b^2]*d^2) + (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))
])/(b*Sqrt[a^2 + b^2]*d^3) - (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3)
```

Rubi [A] time = 0.708701, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5557, 32, 3322, 2264, 2190, 2531, 2282, 6589}

$$-\frac{2af(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}+\frac{2af(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2\sqrt{a^2+b^2}}+\frac{2af^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{2af^2\text{Pol}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (e + f*x)^3/(3*b*f) - (a*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2])])/(b*Sqrt[a^2 + b^2]*d) + (a*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d) - (2*a*f*(e + f*x)*PolyLog[2, -
((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2) + (2*a*f*
(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2
 + b^2]*d^2) + (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))
])/(b*Sqrt[a^2 + b^2]*d^3) - (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3)
```


Rule 5557

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\,dx}{b} - \frac{a\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{(2a)\int \frac{e^{c+dx}(e+fx)^2}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{b}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{(2a)\int \frac{e^{c+dx}(e+fx)^2}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\sqrt{a^2+b^2}} + \frac{(2a)\int \frac{e^{c+dx}(e+fx)^2}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\sqrt{a^2+b^2}}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{(2af)\int (e+$$

$$= \frac{(e+fx)^3}{3bf} - \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} - \frac{2af(e+fx)}{b\sqrt{a^2}}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} - \frac{2af(e+fx)}{b\sqrt{a^2}}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} - \frac{2af(e+fx)}{b\sqrt{a^2}}$$

Mathematica [A] time = 1.69056, size = 366, normalized size = 1.09

$$\frac{a\left(-2df(e+fx)\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+2df(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+2f^2\text{PolyLog}\left(3,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)-2f^2\text{PolyLo}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2))/(3*b) + (a*(2*d^2*e^2*ArcTanh[(a + b*E^(c +
 d*x))/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*
d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, (b*
E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))] + 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + S
qrt[a^2 + b^2])] - 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)]))/(b*Sqrt[a^2 + b^2]*d^3)
```

Maple [F] time = 0.179, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2\sinh(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.74246, size = 1913, normalized size = 5.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*((a^2 + b^2)*d^3*f^2*x^3 + 3*(a^2 + b^2)*d^3*e*f*x^2 + 3*(a^2 + b^2)*d^
3*e^2*x + 6*a*b*f^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*s
inh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b
) - 6*a*b*f^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*
x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*
(a*b*d*f^2*x + a*b*d*e*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*
sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
 b)/b + 1) + 6*(a*b*d*f^2*x + a*b*d*e*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cos
h(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b + 1) + 3*(a*b*d^2*e^2 - 2*a*b*c*d*e*f + a*b*c^2*f^2)*s
qrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((
a^2 + b^2)/b^2) + 2*a) - 3*(a*b*d^2*e^2 - 2*a*b*c*d*e*f + a*b*c^2*f^2)*sqrt
((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2
 + b^2)/b^2) + 2*a) - 3*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f
- a*b*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c
) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 3*(
a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*f^2)*sqrt((a^2
+ b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b))/((a^2*b + b^3)*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.225 $\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=220

$$-\frac{af\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}+\frac{af\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2\sqrt{a^2+b^2}}-\frac{a(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd\sqrt{a^2+b^2}}+\frac{a(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+\right.}{bd\sqrt{a^2+b^2}}$$

```
[Out] (e*x)/b + (f*x^2)/(2*b) - (a*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2])])/(b*Sqrt[a^2 + b^2]*d) + (a*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d) - (a*f*PolyLog[2, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2) + (a*f*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2)
```

Rubi [A] time = 0.412666, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5557, 3322, 2264, 2190, 2279, 2391}

$$-\frac{af\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}+\frac{af\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2\sqrt{a^2+b^2}}-\frac{a(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd\sqrt{a^2+b^2}}+\frac{a(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+\right.}{bd\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (e*x)/b + (f*x^2)/(2*b) - (a*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2])])/(b*Sqrt[a^2 + b^2]*d) + (a*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d) - (a*f*PolyLog[2, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2) + (a*f*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2)
```


Rule 5557

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\,dx}{b} - \frac{a\int \frac{e+fx}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{(2a)\int \frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{b}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{(2a)\int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\sqrt{a^2+b^2}} + \frac{(2a)\int \frac{e^{c+dx}(e+fx)}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\sqrt{a^2+b^2}}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{(af)\int \log\left(1+ \cdots\right.}{b\sqrt{a^2 \cdots}}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{(af)\,\mathrm{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} - \frac{af\,\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+ \cdots}}\right.}{b\sqrt{a^2+b^2}d^2 \cdots}$$

Mathematica [A] time = 0.699166, size = 163, normalized size = 0.74

$$\frac{x(2e+fx)}{2b} - \frac{a\left(f\,\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) - f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) + d(e+fx)\left(\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right) - \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1 \cdots\right.\right.\right.}{bd^2\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (x*(2*e + f*x))/(2*b) - (a*(d*(e + f*x)*(Log[1 + (b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2])] - Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]) + f*PolyLog[
2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2]))]))/(b*Sqrt[a^2 + b^2]*d^2)
```

Maple [B] time = 0.069, size = 440, normalized size = 2.

$$\frac{fx^2}{2b}+\frac{ex}{b}+2\frac{ae}{bd\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\,\frac{2\,be^{dx+c}+2\,a}{\sqrt{a^2+b^2}}\right)-\frac{afx}{bd}\ln\left(\left(-be^{dx+c}+\sqrt{a^2+b^2}-a\right)\left(-a+\sqrt{a^2+b^2}\right)^{-1}\right)\frac{1}{\sqrt{a^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/2*f*x^2/b+e*x/b+2*a/b/d*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a
)/(a^2+b^2)^(1/2))-a/b/d*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2
)-a)/(-a+(a^2+b^2)^(1/2)))*x-a/b/d^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a
^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+a/b/d*f/(a^2+b^2)^(1/2)*ln((b*exp(
d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+a/b/d^2*f/(a^2+b^2)^(1/2)*
ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-a/b/d^2*f/(a^2+b
^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+a/b
/d^2*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^
(1/2)))-2*a/b/d^2*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2
+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.53014, size = 1243, normalized size = 5.65

$$\frac{\left(a^2+b^2\right)d^2fx^2+2\left(a^2+b^2\right)d^2ex-2\,abf\sqrt{\frac{a^2+b^2}{b^2}}\mathrm{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}+1\right)+2\,ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + b^2)*d^2*f*x^2 + 2*(a^2 + b^2)*d^2*e*x - 2*a*b*f*sqrt((a^2 + b^
2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*a*b*f*sqrt((a^2 + b^2)/b^
2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a*b*d*e - a*b*c*f)*sqrt((a^2
+ b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2
)/b^2) + 2*a) - 2*(a*b*d*e - a*b*c*f)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*
x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(a*b*d*f*
x + a*b*c*f)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
+ (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(a*
b*d*f*x + a*b*c*f)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x
 + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b))/
```

```
((a^2*b + b^3)*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")

[Out] integrate((f*x + e)*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.226 $\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=54

$$\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd\sqrt{a^2+b^2}}+\frac{x}{b}$$

```
[Out] x/b + (2*a*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b*Sqrt[a^2
+ b^2]*d)
```

Rubi [A] time = 0.0748817, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2735, 2660, 618, 204}

$$\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd\sqrt{a^2+b^2}}+\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] x/b + (2*a*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b*Sqrt[a^2
+ b^2]*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{x}{b} - \frac{a\int \frac{1}{a+b\sinh(c+dx)}\,dx}{b}$$
$$= \frac{x}{b} + \frac{(2ia)\,\mathrm{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{bd}$$
$$= \frac{x}{b} - \frac{(4ia)\,\mathrm{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx, x, -2ib+2a\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{bd}$$
$$= \frac{x}{b} + \frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d}$$

Mathematica [A] time = 0.105599, size = 64, normalized size = 1.19

$$\frac{-\frac{2a\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{d\sqrt{-a^2-b^2}}+\frac{c}{d}+x}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(Sqrt[-
a^2 - b^2]*d))/b
```

Maple [A] time = 0.003, size = 87, normalized size = 1.6

$$\frac{1}{bd}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-2\,\frac{a}{bd\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\,\frac{2\,a\tanh(1/2\,dx+c/2)-2\,b}{\sqrt{a^2+b^2}}\right)-\frac{1}{bd}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/d/b*ln(tanh(1/2*d*x+1/2*c)+1)-2/d*a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*ta
nh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-1/d/b*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.47331, size = 473, normalized size = 8.76

$$\frac{\left(a^2+b^2\right)dx+\sqrt{a^2+b^2}a\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2+b^2+2\left(b^2\cosh(dx+c)+ab\right)\sinh(dx+c)+2\,\sqrt{a^2+b^2}(b\cosh(dx}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)-b}\right.}{\left(a^2b+b^3\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((a^2 + b^2)*d*x + sqrt(a^2 + b^2)*a*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*
x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*
sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/
(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x
+ c) + a)*sinh(d*x + c) - b)))/((a^2*b + b^3)*d)
```

Sympy [A] time = 169.228, size = 371, normalized size = 6.87

$$\begin{cases} \tilde{\infty}x & \text{for } a=0\wedge b=0\wedge d=0 \\ \frac{x}{b} & \text{for } a=0 \\ -\frac{b^2dx\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{-b^3d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+ib^2d\sqrt{b^2}}+\frac{2b^2}{-b^3d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+ib^2d\sqrt{b^2}}+\frac{ibdx\sqrt{b^2}}{-b^3d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+ib^2d\sqrt{b^2}} & \text{for } a=-\sqrt{-b^2} \\ \frac{b^2dx\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{b^3d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+ib^2d\sqrt{b^2}}-\frac{2b^2}{b^3d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+ib^2d\sqrt{b^2}}+\frac{ibdx\sqrt{b^2}}{b^3d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+ib^2d\sqrt{b^2}} & \text{for } a=\sqrt{-b^2} \\ \frac{\cosh(c+dx)}{ad} & \text{for } b=0 \\ \frac{x\sinh(c)}{a+b\sinh(c)} & \text{for } d=0 \\ \frac{a\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{bd\sqrt{a^2+b^2}}-\frac{a\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{bd\sqrt{a^2+b^2}}+\frac{x}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/b, Eq(a, 0)), (-b**2*
d*x*tanh(c/2 + d*x/2)/(-b**3*d*tanh(c/2 + d*x/2) + I*b**2*d*sqrt(b**2)) + 2
*b**2/(-b**3*d*tanh(c/2 + d*x/2) + I*b**2*d*sqrt(b**2)) + I*b*d*x*sqrt(b**2
)/(-b**3*d*tanh(c/2 + d*x/2) + I*b**2*d*sqrt(b**2)), Eq(a, -sqrt(-b**2))),
(b**2*d*x*tanh(c/2 + d*x/2)/(b**3*d*tanh(c/2 + d*x/2) + I*b**2*d*sqrt(b**2)
) - 2*b**2/(b**3*d*tanh(c/2 + d*x/2) + I*b**2*d*sqrt(b**2)) + I*b*d*x*sqrt(
b**2)/(b**3*d*tanh(c/2 + d*x/2) + I*b**2*d*sqrt(b**2)), Eq(a, sqrt(-b**2)))
, (cosh(c + d*x)/(a*d), Eq(b, 0)), (x*sinh(c)/(a + b*sinh(c)), Eq(d, 0)), (
a*log(tanh(c/2 + d*x/2) - b/a - sqrt(a**2 + b**2)/a)/(b*d*sqrt(a**2 + b**2)
) - a*log(tanh(c/2 + d*x/2) - b/a + sqrt(a**2 + b**2)/a)/(b*d*sqrt(a**2 + b
**2)) + x/b, True))
```

Giac [A] time = 1.29623, size = 115, normalized size = 2.13

$$-\frac{a\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}bd}+\frac{dx+c}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -a*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c) +
 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b*d) + (d*x + c)/(b*d)
```

3.227 $\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Sinh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0490424, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sinh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Sinh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A] time = 9.2245, size = 0, normalized size = 0.

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Sinh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-2\,a\int -\frac{e^{(dx+c)}}{b^2fx+b^2e-\left(b^2fxe^{(2\,c)}+b^2ee^{(2\,c)}\right)e^{(2\,dx)}-2\left(abfxe^c+abee^c\right)e^{(dx)}}\,dx+\frac{\log\left(fx+e\right)}{bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*a*integrate(-e^(d*x + c)/(b^2*f*x + b^2*e - (b^2*f*x*e^(2*c) + b^2*e*e^(
2*c))*e^(2*d*x) - 2*(a*b*f*x*e^c + a*b*e*e^c)*e^(d*x)), x) + log(f*x + e)/(
b*f)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh\left(dx+c\right)}{afx+ae+\left(bfx+be\right)\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sinh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(dx+c\right)}{\left(fx+e\right)\left(b\sinh\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

3.228 $\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+b\sinh(c+dx)} dx$

Optimal. Leaf size=551

$$-\frac{6a^2f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3\sqrt{a^2+b^2}}+\frac{6a^2f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^3\sqrt{a^2+b^2}}+\frac{3a^2f(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c}}{a-\sqrt{a}}\right.}{b^2d^2\sqrt{a^2+b^2}}$$

```
[Out] -(a*(e + f*x)^4)/(4*b^2*f) + (6*f^2*(e + f*x)*Cosh[c + d*x])/(b*d^3) + ((e
+ f*x)^3*Cosh[c + d*x])/(b*d) + (a^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])])/(b^2*Sqrt[a^2 + b^2]*d) - (a^2*(e + f*x)^3*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*Sqrt[a^2 + b^2]*d) + (3*a^2*f*(e
 + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*Sqrt[a
^2 + b^2]*d^2) - (3*a^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^2) - (6*a^2*f^2*(e + f*x)*PolyLog[3
, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^3) + (6
*a^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b
^2*Sqrt[a^2 + b^2]*d^3) + (6*a^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^4) - (6*a^2*f^3*PolyLog[4, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^4) - (6*f^3*Sinh[c
 + d*x])/(b*d^4) - (3*f*(e + f*x)^2*Sinh[c + d*x])/(b*d^2)
```

Rubi [A] time = 1.02733, antiderivative size = 551, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {5557, 3296, 2637, 32, 3322, 2264, 2190, 2531, 6609, 2282, 6589}

$$-\frac{6a^2f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3\sqrt{a^2+b^2}}+\frac{6a^2f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^3\sqrt{a^2+b^2}}+\frac{3a^2f(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c}}{a-\sqrt{a}}\right.}{b^2d^2\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -(a*(e + f*x)^4)/(4*b^2*f) + (6*f^2*(e + f*x)*Cosh[c + d*x])/(b*d^3) + ((e
+ f*x)^3*Cosh[c + d*x])/(b*d) + (a^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])])/(b^2*Sqrt[a^2 + b^2]*d) - (a^2*(e + f*x)^3*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*Sqrt[a^2 + b^2]*d) + (3*a^2*f*(e
 + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*Sqrt[a
^2 + b^2]*d^2) - (3*a^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^2) - (6*a^2*f^2*(e + f*x)*PolyLog[3
, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^3) + (6
*a^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b
^2*Sqrt[a^2 + b^2]*d^3) + (6*a^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^4) - (6*a^2*f^3*PolyLog[4, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^4) - (6*f^3*Sinh[c
 + d*x])/(b*d^4) - (3*f*(e + f*x)^2*Sinh[c + d*x])/(b*d^2)
```


Rule 5557

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)^3\cosh(c+dx)}{bd} - \frac{a\int (e+fx)^3\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}\,dx}{b^2} - \frac{(3f)\int (e+fx)^2 \mathrm{c}}{bd}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{(e+fx)^3\cosh(c+dx)}{bd} - \frac{3f(e+fx)^2\sinh(c+dx)}{bd^2} + \frac{\left(2a^2\right)\int \frac{e^{c+dx}(}{-b+2ae^{c+d}}}{b^2}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cosh(c+dx)}{bd^3} + \frac{(e+fx)^3\cosh(c+dx)}{bd} - \frac{3f(e+fx)^2\sinh}{bd^2}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cosh(c+dx)}{bd^3} + \frac{(e+fx)^3\cosh(c+dx)}{bd} + \frac{a^2(e+fx)^3\log}{b^2\sqrt{a^2}}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cosh(c+dx)}{bd^3} + \frac{(e+fx)^3\cosh(c+dx)}{bd} + \frac{a^2(e+fx)^3\log}{b^2\sqrt{a^2}}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cosh(c+dx)}{bd^3} + \frac{(e+fx)^3\cosh(c+dx)}{bd} + \frac{a^2(e+fx)^3\log}{b^2\sqrt{a^2}}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cosh(c+dx)}{bd^3} + \frac{(e+fx)^3\cosh(c+dx)}{bd} + \frac{a^2(e+fx)^3\log}{b^2\sqrt{a^2}}$$

$$= -\frac{a(e+fx)^4}{4b^2f} + \frac{6f^2(e+fx)\cosh(c+dx)}{bd^3} + \frac{(e+fx)^3\cosh(c+dx)}{bd} + \frac{a^2(e+fx)^3\log}{b^2\sqrt{a^2}}$$

Mathematica [A] time = 2.96971, size = 979, normalized size = 1.78

$$\frac{-a\sqrt{a^2+b^2}f^3x^4d^4 - 4a\sqrt{a^2+b^2}ef^2x^3d^4 - 6a\sqrt{a^2+b^2}e^2fx^2d^4 - 4a\sqrt{a^2+b^2}e^3xd^4 - 8a^2e^3\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)d^3 + 4b\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-4*a*Sqrt[a^2 + b^2]*d^4*e^3*x - 6*a*Sqrt[a^2 + b^2]*d^4*e^2*f*x^2 - 4*a*S
qrt[a^2 + b^2]*d^4*e*f^2*x^3 - a*Sqrt[a^2 + b^2]*d^4*f^3*x^4 - 8*a^2*d^3*e^
3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 4*b*Sqrt[a^2 + b^2]*d^3*e^
3*Cosh[c + d*x] + 24*b*Sqrt[a^2 + b^2]*d*e*f^2*Cosh[c + d*x] + 12*b*Sqrt[a^
2 + b^2]*d^3*e^2*f*x*Cosh[c + d*x] + 24*b*Sqrt[a^2 + b^2]*d*f^3*x*Cosh[c +
d*x] + 12*b*Sqrt[a^2 + b^2]*d^3*e*f^2*x^2*Cosh[c + d*x] + 4*b*Sqrt[a^2 + b^
2]*d^3*f^3*x^3*Cosh[c + d*x] + 12*a^2*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(
```

```
a - Sqrt[a^2 + b^2])] + 12*a^2*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])] + 4*a^2*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2])] - 12*a^2*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
] - 12*a^2*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 4
*a^2*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 12*a^2*d^
2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 12*a^2
*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2
4*a^2*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 24*a^2*d
*f^3*x*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 24*a^2*d*e*f^2*
PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 24*a^2*d*f^3*x*PolyL
og[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 24*a^2*f^3*PolyLog[4, (b*
E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 24*a^2*f^3*PolyLog[4, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))] - 12*b*Sqrt[a^2 + b^2]*d^2*e^2*f*Sinh[c + d*x]
- 24*b*Sqrt[a^2 + b^2]*f^3*Sinh[c + d*x] - 24*b*Sqrt[a^2 + b^2]*d^2*e*f^2*x
*Sinh[c + d*x] - 12*b*Sqrt[a^2 + b^2]*d^2*f^3*x^2*Sinh[c + d*x])/(4*b^2*Sqr
t[a^2 + b^2]*d^4)
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3}\left(\sinh\left(dx+c\right)\right)^{2}}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)

[Out] int((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [C] time = 3.19243, size = 5847, normalized size = 10.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")

[Out] 1/4*(2*(a^2*b + b^3)*d^3*f^3*x^3 + 2*(a^2*b + b^3)*d^3*e^3 + 6*(a^2*b + b^3
)*d^2*e^2*f + 12*(a^2*b + b^3)*d*e*f^2 + 12*(a^2*b + b^3)*f^3 + 6*((a^2*b +
 b^3)*d^3*e*f^2 + (a^2*b + b^3)*d^2*f^3)*x^2 + 2*((a^2*b + b^3)*d^3*f^3*x^3
 + (a^2*b + b^3)*d^3*e^3 - 3*(a^2*b + b^3)*d^2*e^2*f + 6*(a^2*b + b^3)*d*e*
f^2 - 6*(a^2*b + b^3)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^2*b + b^3)*d^2*
```

```
f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^2*b + b^3)*d^2*e*f^2 + 2*(a^2*
b + b^3)*d*f^3)*x)*cosh(d*x + c)^2 + 2*((a^2*b + b^3)*d^3*f^3*x^3 + (a^2*b
+ b^3)*d^3*e^3 - 3*(a^2*b + b^3)*d^2*e^2*f + 6*(a^2*b + b^3)*d*e*f^2 - 6*(a
^2*b + b^3)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^2*b + b^3)*d^2*f^3)*x^2 +
 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^2*b + b^3)*d^2*e*f^2 + 2*(a^2*b + b^3)*d
*f^3)*x)*sinh(d*x + c)^2 + 12*((a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a
^2*b*d^2*e^2*f)*cosh(d*x + c) + (a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x +
a^2*b*d^2*e^2*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c
) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2) - b)/b + 1) - 12*((a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2*b*d^2
*e^2*f)*cosh(d*x + c) + (a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2*b*d^
2*e^2*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*si
nh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b
)/b + 1) - 4*((a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 -
a^2*b*c^3*f^3)*cosh(d*x + c) + (a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2
*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*
b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 4*
((a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3
)*cosh(d*x + c) + (a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^
2 - a^2*b*c^3*f^3)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x +
c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 4*((a^2*b*d^3*f
^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f
- 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*cosh(d*x + c) + (a^2*b*d^3*f^3*x^3 +
 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*
b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a
*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b) - 4*((a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 +
 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^
3*f^3)*cosh(d*x + c) + (a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b
*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*s
inh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
 - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 24*(
a^2*b*f^3*cosh(d*x + c) + a^2*b*f^3*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*po
lylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2))/b) - 24*(a^2*b*f^3*cosh(d*x + c) + a^2*b*f^3*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d
*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2
4*((a^2*b*d*f^3*x + a^2*b*d*e*f^2)*cosh(d*x + c) + (a^2*b*d*f^3*x + a^2*b*d
*e*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) +
a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
)/b) + 24*((a^2*b*d*f^3*x + a^2*b*d*e*f^2)*cosh(d*x + c) + (a^2*b*d*f^3*x +
 a^2*b*d*e*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x
 + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2))/b) + 6*((a^2*b + b^3)*d^3*e^2*f + 2*(a^2*b + b^3)*d^2*e*f^2 + 2*(
a^2*b + b^3)*d*f^3)*x - ((a^3 + a*b^2)*d^4*f^3*x^4 + 4*(a^3 + a*b^2)*d^4*e*
f^2*x^3 + 6*(a^3 + a*b^2)*d^4*e^2*f*x^2 + 4*(a^3 + a*b^2)*d^4*e^3*x)*cosh(d
*x + c) - ((a^3 + a*b^2)*d^4*f^3*x^4 + 4*(a^3 + a*b^2)*d^4*e*f^2*x^3 + 6*(a
^3 + a*b^2)*d^4*e^2*f*x^2 + 4*(a^3 + a*b^2)*d^4*e^3*x - 4*((a^2*b + b^3)*d^
3*f^3*x^3 + (a^2*b + b^3)*d^3*e^3 - 3*(a^2*b + b^3)*d^2*e^2*f + 6*(a^2*b +
b^3)*d*e*f^2 - 6*(a^2*b + b^3)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^2*b +
b^3)*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^2*b + b^3)*d^2*e*f^2
+ 2*(a^2*b + b^3)*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c))/((a^2*b^2 + b^4)*
d^4*cosh(d*x + c) + (a^2*b^2 + b^4)*d^4*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

3.229 $\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=407

$$\frac{2a^2 f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2\sqrt{a^2+b^2}}-\frac{2a^2 f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^2\sqrt{a^2+b^2}}-\frac{2a^2 f^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3\sqrt{a^2+b^2}}+\frac{2a^2}{}$$

```
[Out] -(a*(e + f*x)^3)/(3*b^2*f) + (2*f^2*Cosh[c + d*x])/(b*d^3) + ((e + f*x)^2*C
osh[c + d*x])/(b*d) + (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2])])/(b^2*Sqrt[a^2 + b^2]*d) - (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2])])/(b^2*Sqrt[a^2 + b^2]*d) + (2*a^2*f*(e + f*x)*Po
lyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^
2) - (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/(b^2*Sqrt[a^2 + b^2]*d^2) - (2*a^2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^3) + (2*a^2*f^2*PolyLog[3, -((b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^3) - (2*f*(e
+ f*x)*Sinh[c + d*x])/(b*d^2)
```

Rubi [A] time = 0.853208, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5557, 3296, 2638, 32, 3322, 2264, 2190, 2531, 2282, 6589}

$$\frac{2a^2 f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2\sqrt{a^2+b^2}}-\frac{2a^2 f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^2\sqrt{a^2+b^2}}-\frac{2a^2 f^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3\sqrt{a^2+b^2}}+\frac{2a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -(a*(e + f*x)^3)/(3*b^2*f) + (2*f^2*Cosh[c + d*x])/(b*d^3) + ((e + f*x)^2*C
osh[c + d*x])/(b*d) + (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2])])/(b^2*Sqrt[a^2 + b^2]*d) - (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2])])/(b^2*Sqrt[a^2 + b^2]*d) + (2*a^2*f*(e + f*x)*Po
lyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^
2) - (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/(b^2*Sqrt[a^2 + b^2]*d^2) - (2*a^2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^3) + (2*a^2*f^2*PolyLog[3, -((b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^3) - (2*f*(e
+ f*x)*Sinh[c + d*x])/(b*d^2)
```


Rule 5557

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)^2\cosh(c+dx)}{bd} - \frac{a\int (e+fx)^2\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}\,dx}{b^2} - \frac{(2f)\int (e+fx)\,\mathrm{co}}{bd}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{(e+fx)^2\cosh(c+dx)}{bd} - \frac{2f(e+fx)\sinh(c+dx)}{bd^2} + \frac{\left(2a^2\right)\int \frac{e^{c+dx}(e}{-b+2ae^{c+dx}}}{b^2}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cosh(c+dx)}{bd^3} + \frac{(e+fx)^2\cosh(c+dx)}{bd} - \frac{2f(e+fx)\sinh(c+dx)}{bd^2}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cosh(c+dx)}{bd^3} + \frac{(e+fx)^2\cosh(c+dx)}{bd} + \frac{a^2(e+fx)^2\log\left(1+\frac{be}{a-\sqrt{}}\right.}{b^2\sqrt{a^2+b^2}d}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cosh(c+dx)}{bd^3} + \frac{(e+fx)^2\cosh(c+dx)}{bd} + \frac{a^2(e+fx)^2\log\left(1+\frac{be}{a-\sqrt{}}\right.}{b^2\sqrt{a^2+b^2}d}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cosh(c+dx)}{bd^3} + \frac{(e+fx)^2\cosh(c+dx)}{bd} + \frac{a^2(e+fx)^2\log\left(1+\frac{be}{a-\sqrt{}}\right.}{b^2\sqrt{a^2+b^2}d}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cosh(c+dx)}{bd^3} + \frac{(e+fx)^2\cosh(c+dx)}{bd} + \frac{a^2(e+fx)^2\log\left(1+\frac{be}{a-\sqrt{}}\right.}{b^2\sqrt{a^2+b^2}d}$$

Mathematica [A] time = 3.03193, size = 453, normalized size = 1.11

$$\frac{\frac{3a^2\left(2df(e+fx)\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)-2df(e+fx)\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)-2f^2\mathrm{PolyLog}\left(3,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+2f^2\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)-2d^2e^2\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right.}{d^3\sqrt{a^2+b^2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-(a*x*(3*e^2 + 3*e*f*x + f^2*x^2)) + (3*a^2*(-2*d^2*e^2*ArcTanh[(a + b*E^(
c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] -
 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^2*f^2*x^2*L
og[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2,
(b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, -((b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a
+ Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2]))]))/(Sqrt[a^2 + b^2]*d^3) + (3*b*Cosh[d*x]*((2*f^2 + d^2*(e + f*x)^2)*C
osh[c] - 2*d*f*(e + f*x)*Sinh[c]))/d^3 + (3*b*(-2*d*f*(e + f*x)*Cosh[c] + (
2*f^2 + d^2*(e + f*x)^2)*Sinh[c])*Sinh[d*x])/d^3)/(3*b^2)
```

Maple [F] time = 0.088, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^2\left(\sinh\left(dx+c\right)\right)^2}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)

[Out] int((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [C] time = 2.81087, size = 3897, normalized size = 9.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")

[Out] 1/6*(3*(a^2*b + b^3)*d^2*f^2*x^2 + 3*(a^2*b + b^3)*d^2*e^2 + 6*(a^2*b + b^3
)*d*e*f + 6*(a^2*b + b^3)*f^2 + 3*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3
)*d^2*e^2 - 2*(a^2*b + b^3)*d*e*f + 2*(a^2*b + b^3)*f^2 + 2*((a^2*b + b^3)*
d^2*e*f - (a^2*b + b^3)*d*f^2)*x)*cosh(d*x + c)^2 + 3*((a^2*b + b^3)*d^2*f^
2*x^2 + (a^2*b + b^3)*d^2*e^2 - 2*(a^2*b + b^3)*d*e*f + 2*(a^2*b + b^3)*f^2
 + 2*((a^2*b + b^3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x)*sinh(d*x + c)^2 + 12*
((a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c) + (a^2*b*d*f^2*x + a^2*b*d*e*f
)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x
+ c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1
) - 12*((a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c) + (a^2*b*d*f^2*x + a^2*
b*d*e*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*si
nh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b
)/b + 1) - 6*((a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x +
c) + (a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2
+ b^2)/b^2) + 2*a) + 6*((a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*c
osh(d*x + c) + (a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*sinh(d*x +
 c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*
sqrt((a^2 + b^2)/b^2) + 2*a) + 6*((a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x +
2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*cosh(d*x + c) + (a^2*b*d^2*f^2*x^2 + 2*a^2
*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 6*((a^2*b*d^2*f^2*x^2 + 2*a^2*
b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*cosh(d*x + c) + (a^2*b*d^2*f
^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 12*(a^2*b*f^2*co
sh(d*x + c) + a^2*b*f^2*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*
cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2))/b) + 12*(a^2*b*f^2*cosh(d*x + c) + a^2*b*f^2*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*((a^2*b + b
```

```
^3)*d^2*e*f + (a^2*b + b^3)*d*f^2)*x - 2*((a^3 + a*b^2)*d^3*f^2*x^3 + 3*(a^
3 + a*b^2)*d^3*e*f*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*x)*cosh(d*x + c) - 2*((a^3
 + a*b^2)*d^3*f^2*x^3 + 3*(a^3 + a*b^2)*d^3*e*f*x^2 + 3*(a^3 + a*b^2)*d^3*e
^2*x - 3*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 - 2*(a^2*b + b^
3)*d*e*f + 2*(a^2*b + b^3)*f^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^2*b + b^3)*d
*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))/((a^2*b^2 + b^4)*d^3*cosh(d*x + c) +
 (a^2*b^2 + b^4)*d^3*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

3.230 $\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=264

$$\frac{a^2 f \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2\sqrt{a^2+b^2}} - \frac{a^2 f \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^2\sqrt{a^2+b^2}} + \frac{a^2(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^2d\sqrt{a^2+b^2}} - \frac{a^2(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+\right.}{b^2d\sqrt{a^2+b^2}}$$

```
[Out] -((a*e*x)/b^2) - (a*f*x^2)/(2*b^2) + ((e + f*x)*Cosh[c + d*x])/(b*d) + (a^2
*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*Sqrt[a^2 +
b^2]*d) - (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b
^2*Sqrt[a^2 + b^2]*d) + (a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^2) - (a^2*f*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^2) - (f*Sinh[c + d*x])/(b*d^
2)
```

Rubi [A] time = 0.486714, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5557, 3296, 2637, 3322, 2264, 2190, 2279, 2391}

$$\frac{a^2 f \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2\sqrt{a^2+b^2}} - \frac{a^2 f \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^2\sqrt{a^2+b^2}} + \frac{a^2(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^2d\sqrt{a^2+b^2}} - \frac{a^2(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+\right.}{b^2d\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -((a*e*x)/b^2) - (a*f*x^2)/(2*b^2) + ((e + f*x)*Cosh[c + d*x])/(b*d) + (a^2
*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*Sqrt[a^2 +
b^2]*d) - (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b
^2*Sqrt[a^2 + b^2]*d) + (a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^2) - (a^2*f*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(b^2*Sqrt[a^2 + b^2]*d^2) - (f*Sinh[c + d*x])/(b*d^
2)
```


Rule 5557

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)\cosh(c+dx)}{bd} - \frac{a\int (e+fx)\,dx}{b^2} + \frac{a^2\int \frac{e+fx}{a+b\sinh(c+dx)}\,dx}{b^2} - \frac{f\int \cosh(c+dx)\,dx}{bd}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx)\cosh(c+dx)}{bd} - \frac{f\sinh(c+dx)}{bd^2} + \frac{\left(2a^2\right)\int \frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{b^2}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx)\cosh(c+dx)}{bd} - \frac{f\sinh(c+dx)}{bd^2} + \frac{\left(2a^2\right)\int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{b\sqrt{a^2+b^2}}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx)\cosh(c+dx)}{bd} + \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d} - \frac{a^2(e+fx)\log}{b^2\sqrt{}}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx)\cosh(c+dx)}{bd} + \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d} - \frac{a^2(e+fx)\log}{b^2\sqrt{}}$$

$$= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx)\cosh(c+dx)}{bd} + \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d} - \frac{a^2(e+fx)\log}{b^2\sqrt{}}$$

Mathematica [A] time = 2.18564, size = 299, normalized size = 1.13

$$\frac{2a^2\left(f\text{PolyLog}\left(2,\frac{b(\sinh(c+dx)+\cosh(c+dx))}{\sqrt{a^2+b^2}-a}\right)-f\text{PolyLog}\left(2,-\frac{b(\sinh(c+dx)+\cosh(c+dx))}{\sqrt{a^2+b^2}+a}\right)-2de\tanh^{-1}\left(\frac{a+b\sinh(c+dx)+b\cosh(c+dx)}{\sqrt{a^2+b^2}}\right)+f(c+dx)\log\left(\frac{b(\sinh(c+dx)+\cosh(c\cdots}{a-\sqrt{a^2+b^2}}\right.\right.}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(c + d*x)*(c*f - d*(2*e + f*x)) + 2*b*d*(e + f*x)*Cosh[c + d*x] + (2*a^2
*(-2*d*e*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] +
 2*c*f*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + f
*(c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2]
)] - f*(c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2
+ b^2])] + f*PolyLog[2, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2
+ b^2])] - f*PolyLog[2, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2
 + b^2]))]))/Sqrt[a^2 + b^2] - 2*b*f*Sinh[c + d*x])/(2*b^2*d^2)
```

Maple [B] time = 0.071, size = 510, normalized size = 1.9

$$-\frac{afx^2}{2b^2}-\frac{aex}{b^2}+\frac{\left(dfx+de-f\right)e^{dx+c}}{2d^2b}+\frac{\left(dfx+de+f\right)e^{-dx-c}}{2d^2b}-2\frac{a^2e}{b^2d\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\frac{2be^{dx+c}+2a}{\sqrt{a^2+b^2}}\right)+\frac{a^2fx}{b^2d}\ln\left(\left(\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/2*a*f*x^2/b^2-a*e*x/b^2+1/2*(d*f*x+d*e-f)/d^2/b*exp(d*x+c)+1/2*(d*f*x+d*
e+f)/d^2/b*exp(-d*x-c)-2*a^2/b^2/d*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d
*x+c)+2*a)/(a^2+b^2)^(1/2))+a^2/b^2/d*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(
a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+a^2/b^2/d^2*f/(a^2+b^2)^(1/2)*ln(
(-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-a^2/b^2/d*f/(a^2+
b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-a^2/b
^2/d^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(
1/2)))*c+a^2/b^2/d^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))-a^2/b^2/d^2*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)
+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2*a^2/b^2/d^2*f*c/(a^2+b^2)^(1/2)*
arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.661, size = 2272, normalized size = 8.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + ((a^2*b + b^3)*d*f*x + (a^2*
b + b^3)*d*e - (a^2*b + b^3)*f)*cosh(d*x + c)^2 + ((a^2*b + b^3)*d*f*x + (a
^2*b + b^3)*d*e - (a^2*b + b^3)*f)*sinh(d*x + c)^2 + 2*(a^2*b*f*cosh(d*x +
c) + a^2*b*f*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) +
a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
 - b)/b + 1) - 2*(a^2*b*f*cosh(d*x + c) + a^2*b*f*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*((a^2*b*d*e - a^2*b*c
*f)*cosh(d*x + c) + (a^2*b*d*e - a^2*b*c*f)*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2)
 + 2*a) + 2*((a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c) + (a^2*b*d*e - a^2*b*c*f
)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x
 + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((a^2*b*d*f*x + a^2*b*c*f)*cos
h(d*x + c) + (a^2*b*d*f*x + a^2*b*c*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*((a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x +
 c) + (a^2*b*d*f*x + a^2*b*c*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(
a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqr
t((a^2 + b^2)/b^2) - b)/b) + (a^2*b + b^3)*f - ((a^3 + a*b^2)*d^2*f*x^2 + 2
*(a^3 + a*b^2)*d^2*e*x)*cosh(d*x + c) - ((a^3 + a*b^2)*d^2*f*x^2 + 2*(a^3 +
 a*b^2)*d^2*e*x - 2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e - (a^2*b + b^3
)*f)*cosh(d*x + c))*sinh(d*x + c))/((a^2*b^2 + b^4)*d^2*cosh(d*x + c) + (a^
2*b^2 + b^4)*d^2*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

3.231 $\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=71

$$-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{ax}{b^2}+\frac{\cosh(c+dx)}{bd}$$

```
[Out] -((a*x)/b^2) - (2*a^2*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(
b^2*Sqrt[a^2 + b^2]*d) + Cosh[c + d*x]/(b*d)
```

Rubi [A] time = 0.127736, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2746, 12, 2735, 2660, 618, 204}

$$-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{ax}{b^2}+\frac{\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a*x)/b^2) - (2*a^2*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(
b^2*Sqrt[a^2 + b^2]*d) + Cosh[c + d*x]/(b*d)
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{\cosh(c+dx)}{bd} - \frac{\int \frac{a\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b} \\
&= \frac{\cosh(c+dx)}{bd} - \frac{a\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b} \\
&= -\frac{ax}{b^2} + \frac{\cosh(c+dx)}{bd} + \frac{a^2\int \frac{1}{a+b\sinh(c+dx)}\,dx}{b^2} \\
&= -\frac{ax}{b^2} + \frac{\cosh(c+dx)}{bd} - \frac{(2ia^2)\,\mathrm{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{b^2d} \\
&= -\frac{ax}{b^2} + \frac{\cosh(c+dx)}{bd} + \frac{(4ia^2)\,\mathrm{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx, x, -2ib+2a\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{b^2d} \\
&= -\frac{ax}{b^2} - \frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d} + \frac{\cosh(c+dx)}{bd}
\end{aligned}$$

Mathematica [A] time = 0.274514, size = 74, normalized size = 1.04

$$\frac{b\cosh(c+dx) - a\left(-\frac{2a\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + c + dx\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-(a*(c + d*x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sq
rt[-a^2 - b^2])) + b*Cosh[c + d*x])/(b^2*d)
```

Maple [A] time = 0.03, size = 132, normalized size = 1.9

$$\frac{1}{bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} - \frac{a}{db^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) + 2\,\frac{a^2}{db^2\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\,\frac{2\,a\tanh(1/2\,dx+c/2)-2\,b}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/d/b/(tanh(1/2*d*x+1/2*c)+1)-1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)+2/d*a^2/b
^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2
```

```
))-1/d/b/(tanh(1/2*d*x+1/2*c)-1)+1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.52095, size = 822, normalized size = 11.58

$$-\frac{2\left(a^3+ab^2\right)dx\cosh\left(dx+c\right)-a^2b-b^3-\left(a^2b+b^3\right)\cosh\left(dx+c\right)^2-\left(a^2b+b^3\right)\sinh\left(dx+c\right)^2-2\left(a^2\cosh\left(dx+c\right)+a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^3 + a*b^2)*d*x*cosh(d*x + c) - a^2*b - b^3 - (a^2*b + b^3)*cosh(
d*x + c)^2 - (a^2*b + b^3)*sinh(d*x + c)^2 - 2*(a^2*cosh(d*x + c) + a^2*sin
h(d*x + c))*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2
+ 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x
+ c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d
*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)
*sinh(d*x + c) - b)) + 2*((a^3 + a*b^2)*d*x - (a^2*b + b^3)*cosh(d*x + c))*
sinh(d*x + c))/((a^2*b^2 + b^4)*d*cosh(d*x + c) + (a^2*b^2 + b^4)*d*sinh(d*
x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16792, size = 161, normalized size = 2.27

$$\frac{a^2\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^2d}-\frac{(dx+c)a}{b^2d}+\frac{e^{(dx+c)}}{2\,bd}+\frac{e^{(-dx-c)}}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] a^2*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c)
+ 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2*d) - (d*x + c)*a/(b^2*d) +
 1/2*e^(d*x + c)/(b*d) + 1/2*e^(-d*x - c)/(b*d)
```

3.232 $\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Sinh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0755898, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sinh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Sinh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A] time = 136.201, size = 0, normalized size = 0.

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Sinh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A] time = 0.088, size = 0, normalized size = 0.

$$\int \frac{(\sinh(dx+c))^2}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$2\,a^2 \int -\frac{e^{(dx+c)}}{b^3 fx + b^3 e - \left(b^3 fx e^{(2\,c)} + b^3 e e^{(2\,c)}\right) e^{(2\,dx)} - 2\left(ab^2 fx e^c + ab^2 e e^c\right) e^{(dx)}}\,dx + \frac{e^{\left(-c+\frac{de}{f}\right)} E_1\left(\frac{(fx+e)d}{f}\right)}{2\,bf} - \frac{e^{\left(c-\frac{de}{f}\right)} E_1\left(-\right.}{2\,bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*a^2*integrate(-e^(d*x + c)/(b^3*f*x + b^3*e - (b^3*f*x*e^(2*c) + b^3*e*e^
(2*c))*e^(2*d*x) - 2*(a*b^2*f*x*e^c + a*b^2*e*e^c)*e^(d*x)), x) + 1/2*e^(-c
 + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b*f) - 1/2*e^(c - d*e/f)*exp_in
tegral_e(1, -(f*x + e)*d/f)/(b*f) - a*log(f*x + e)/(b^2*f)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh\left(dx+c\right)^2}{afx + ae + \left(bfx + be\right)\sinh\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sinh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(dx+c\right)^2}{\left(fx+e\right)\left(b\sinh\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

3.233 $\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=712

$$\frac{6a^3f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3\sqrt{a^2+b^2}}-\frac{6a^3f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^3\sqrt{a^2+b^2}}-\frac{3a^3f(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2\sqrt{a^2+b^2}}$$

```
[Out] (-3*e*f^2*x)/(4*b*d^2) - (3*f^3*x^2)/(8*b*d^2) + (a^2*(e + f*x)^4)/(4*b^3*f
) - (e + f*x)^4/(8*b*f) - (6*a*f^2*(e + f*x)*Cosh[c + d*x])/(b^2*d^3) - (a*
(e + f*x)^3*Cosh[c + d*x])/(b^2*d) - (a^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2])])/(b^3*Sqrt[a^2 + b^2]*d) + (a^3*(e + f*x)^3*Log[1
 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*Sqrt[a^2 + b^2]*d) - (3*a^3
*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*S
qrt[a^2 + b^2]*d^2) + (3*a^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]*d^2) + (6*a^3*f^2*(e + f*x)*Poly
Log[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]*d^3)
 - (6*a^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/(b^3*Sqrt[a^2 + b^2]*d^3) - (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]*d^4) + (6*a^3*f^3*PolyLog[4, -((b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]*d^4) + (6*a*f^3
*Sinh[c + d*x])/(b^2*d^4) + (3*a*f*(e + f*x)^2*Sinh[c + d*x])/(b^2*d^2) + (
3*f^2*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^3) + ((e + f*x)^3*Cosh[
c + d*x]*Sinh[c + d*x])/(2*b*d) - (3*f^3*Sinh[c + d*x]^2)/(8*b*d^4) - (3*f*
(e + f*x)^2*Sinh[c + d*x]^2)/(4*b*d^2)
```

Rubi [A] time = 1.23456, antiderivative size = 712, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {5557, 3311, 32, 3310, 3296, 2637, 3322, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{6a^3f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3\sqrt{a^2+b^2}}-\frac{6a^3f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^3\sqrt{a^2+b^2}}-\frac{3a^3f(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (-3*e*f^2*x)/(4*b*d^2) - (3*f^3*x^2)/(8*b*d^2) + (a^2*(e + f*x)^4)/(4*b^3*f
) - (e + f*x)^4/(8*b*f) - (6*a*f^2*(e + f*x)*Cosh[c + d*x])/(b^2*d^3) - (a*
(e + f*x)^3*Cosh[c + d*x])/(b^2*d) - (a^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2])])/(b^3*Sqrt[a^2 + b^2]*d) + (a^3*(e + f*x)^3*Log[1
 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*Sqrt[a^2 + b^2]*d) - (3*a^3
*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*S
qrt[a^2 + b^2]*d^2) + (3*a^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]*d^2) + (6*a^3*f^2*(e + f*x)*Poly
Log[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]*d^3)
 - (6*a^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/(b^3*Sqrt[a^2 + b^2]*d^3) - (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]*d^4) + (6*a^3*f^3*PolyLog[4, -((b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]*d^4) + (6*a*f^3
*Sinh[c + d*x])/(b^2*d^4) + (3*a*f*(e + f*x)^2*Sinh[c + d*x])/(b^2*d^2) + (
3*f^2*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^3) + ((e + f*x)^3*Cosh[
c + d*x]*Sinh[c + d*x])/(2*b*d) - (3*f^3*Sinh[c + d*x]^2)/(8*b*d^4) - (3*f*
(e + f*x)^2*Sinh[c + d*x]^2)/(4*b*d^2)
```

Rule 5557

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{3f(e+fx)^2\sinh^2(c+dx)}{4bd^2} - \frac{a\int (e+fx)^3\sinh}{b^2}$$

$$= -\frac{(e+fx)^4}{8bf} - \frac{a(e+fx)^3\cosh(c+dx)}{b^2d} + \frac{3f^2(e+fx)\cosh(c+dx)\sinh(c+dx)}{4bd^3} + \frac{(e}{}$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} - \frac{(e+fx)^4}{8bf} - \frac{a(e+fx)^3\cosh(c+dx)}{b^2d} + \frac{3af(e+fx}{b}$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} - \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3} - \frac{a(e+fx}{}$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} - \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3} - \frac{a(e+fx}{}$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} - \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3} - \frac{a(e+fx}{}$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} - \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3} - \frac{a(e+fx}{}$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} - \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3} - \frac{a(e+fx}{}$$

$$= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} - \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3} - \frac{a(e+fx}{}$$

Mathematica [A] time = 4.58408, size = 1407, normalized size = 1.98

$$4a^2\sqrt{a^2+b^2}f^3x^4d^4 - 2b^2\sqrt{a^2+b^2}f^3x^4d^4 + 16a^2\sqrt{a^2+b^2}ef^2x^3d^4 - 8b^2\sqrt{a^2+b^2}ef^2x^3d^4 + 24a^2\sqrt{a^2+b^2}e^2fx^2d^4 - 1$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (16*a^2*Sqrt[a^2 + b^2]*d^4*e^3*x - 8*b^2*Sqrt[a^2 + b^2]*d^4*e^3*x + 24*a^
2*Sqrt[a^2 + b^2]*d^4*e^2*f*x^2 - 12*b^2*Sqrt[a^2 + b^2]*d^4*e^2*f*x^2 + 16
*a^2*Sqrt[a^2 + b^2]*d^4*e*f^2*x^3 - 8*b^2*Sqrt[a^2 + b^2]*d^4*e*f^2*x^3 +
4*a^2*Sqrt[a^2 + b^2]*d^4*f^3*x^4 - 2*b^2*Sqrt[a^2 + b^2]*d^4*f^3*x^4 + 32*
a^3*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 16*a*b*Sqrt[a^2
+ b^2]*d^3*e^3*Cosh[c + d*x] - 96*a*b*Sqrt[a^2 + b^2]*d*e*f^2*Cosh[c + d*x]
 - 48*a*b*Sqrt[a^2 + b^2]*d^3*e^2*f*x*Cosh[c + d*x] - 96*a*b*Sqrt[a^2 + b^2
]*d*f^3*x*Cosh[c + d*x] - 48*a*b*Sqrt[a^2 + b^2]*d^3*e*f^2*x^2*Cosh[c + d*x
] - 16*a*b*Sqrt[a^2 + b^2]*d^3*f^3*x^3*Cosh[c + d*x] - 6*b^2*Sqrt[a^2 + b^2
]*d^2*e^2*f*Cosh[2*(c + d*x)] - 3*b^2*Sqrt[a^2 + b^2]*f^3*Cosh[2*(c + d*x)]
 - 12*b^2*Sqrt[a^2 + b^2]*d^2*e*f^2*x*Cosh[2*(c + d*x)] - 6*b^2*Sqrt[a^2 +
b^2]*d^2*f^3*x^2*Cosh[2*(c + d*x)] - 48*a^3*d^3*e^2*f*x*Log[1 + (b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2])] - 48*a^3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])] - 16*a^3*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2])] + 48*a^3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
```

```
+ b^2])] + 48*a^3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
])] + 16*a^3*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 4
8*a^3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
+ 48*a^3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
]))] + 96*a^3*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] +
96*a^3*d*f^3*x*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 96*a^3*
d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 96*a^3*d*f^3
*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 96*a^3*f^3*PolyLo
g[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 96*a^3*f^3*PolyLog[4, -((b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 48*a*b*Sqrt[a^2 + b^2]*d^2*e^2*f*Sinh
[c + d*x] + 96*a*b*Sqrt[a^2 + b^2]*f^3*Sinh[c + d*x] + 96*a*b*Sqrt[a^2 + b^
2]*d^2*e*f^2*x*Sinh[c + d*x] + 48*a*b*Sqrt[a^2 + b^2]*d^2*f^3*x^2*Sinh[c +
d*x] + 4*b^2*Sqrt[a^2 + b^2]*d^3*e^3*Sinh[2*(c + d*x)] + 6*b^2*Sqrt[a^2 + b
^2]*d*e*f^2*Sinh[2*(c + d*x)] + 12*b^2*Sqrt[a^2 + b^2]*d^3*e^2*f*x*Sinh[2*(
c + d*x)] + 6*b^2*Sqrt[a^2 + b^2]*d*f^3*x*Sinh[2*(c + d*x)] + 12*b^2*Sqrt[a
^2 + b^2]*d^3*e*f^2*x^2*Sinh[2*(c + d*x)] + 4*b^2*Sqrt[a^2 + b^2]*d^3*f^3*x
^3*Sinh[2*(c + d*x)])/(16*b^3*Sqrt[a^2 + b^2]*d^4)
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 (\sinh(dx+c))^3}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 4.02698, size = 11113, normalized size = 15.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/32*(4*(a^2*b^2 + b^4)*d^3*f^3*x^3 + 4*(a^2*b^2 + b^4)*d^3*e^3 + 6*(a^2*b
^2 + b^4)*d^2*e^2*f + 6*(a^2*b^2 + b^4)*d*e*f^2 - (4*(a^2*b^2 + b^4)*d^3*f^
3*x^3 + 4*(a^2*b^2 + b^4)*d^3*e^3 - 6*(a^2*b^2 + b^4)*d^2*e^2*f + 6*(a^2*b^
2 + b^4)*d*e*f^2 - 3*(a^2*b^2 + b^4)*f^3 + 6*(2*(a^2*b^2 + b^4)*d^3*e*f^2 -
```

```
(a^2*b^2 + b^4)*d^2*f^3)*x^2 + 6*(2*(a^2*b^2 + b^4)*d^3*e^2*f - 2*(a^2*b^2
 + b^4)*d^2*e*f^2 + (a^2*b^2 + b^4)*d*f^3)*x)*cosh(d*x + c)^4 - (4*(a^2*b^2
 + b^4)*d^3*f^3*x^3 + 4*(a^2*b^2 + b^4)*d^3*e^3 - 6*(a^2*b^2 + b^4)*d^2*e^2
*f + 6*(a^2*b^2 + b^4)*d*e*f^2 - 3*(a^2*b^2 + b^4)*f^3 + 6*(2*(a^2*b^2 + b^
4)*d^3*e*f^2 - (a^2*b^2 + b^4)*d^2*f^3)*x^2 + 6*(2*(a^2*b^2 + b^4)*d^3*e^2*
f - 2*(a^2*b^2 + b^4)*d^2*e*f^2 + (a^2*b^2 + b^4)*d*f^3)*x)*sinh(d*x + c)^4
 + 3*(a^2*b^2 + b^4)*f^3 + 16*((a^3*b + a*b^3)*d^3*f^3*x^3 + (a^3*b + a*b^3
)*d^3*e^3 - 3*(a^3*b + a*b^3)*d^2*e^2*f + 6*(a^3*b + a*b^3)*d*e*f^2 - 6*(a^
3*b + a*b^3)*f^3 + 3*((a^3*b + a*b^3)*d^3*e*f^2 - (a^3*b + a*b^3)*d^2*f^3)*
x^2 + 3*((a^3*b + a*b^3)*d^3*e^2*f - 2*(a^3*b + a*b^3)*d^2*e*f^2 + 2*(a^3*b
 + a*b^3)*d*f^3)*x)*cosh(d*x + c)^3 + 4*(4*(a^3*b + a*b^3)*d^3*f^3*x^3 + 4*
(a^3*b + a*b^3)*d^3*e^3 - 12*(a^3*b + a*b^3)*d^2*e^2*f + 24*(a^3*b + a*b^3)
*d*e*f^2 - 24*(a^3*b + a*b^3)*f^3 + 12*((a^3*b + a*b^3)*d^3*e*f^2 - (a^3*b
+ a*b^3)*d^2*f^3)*x^2 + 12*((a^3*b + a*b^3)*d^3*e^2*f - 2*(a^3*b + a*b^3)*d
^2*e*f^2 + 2*(a^3*b + a*b^3)*d*f^3)*x - (4*(a^2*b^2 + b^4)*d^3*f^3*x^3 + 4*
(a^2*b^2 + b^4)*d^3*e^3 - 6*(a^2*b^2 + b^4)*d^2*e^2*f + 6*(a^2*b^2 + b^4)*d
*e*f^2 - 3*(a^2*b^2 + b^4)*f^3 + 6*(2*(a^2*b^2 + b^4)*d^3*e*f^2 - (a^2*b^2
+ b^4)*d^2*f^3)*x^2 + 6*(2*(a^2*b^2 + b^4)*d^3*e^2*f - 2*(a^2*b^2 + b^4)*d^
2*e*f^2 + (a^2*b^2 + b^4)*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(2*(
a^2*b^2 + b^4)*d^3*e*f^2 + (a^2*b^2 + b^4)*d^2*f^3)*x^2 - 4*((2*a^4 + a^2*b
^2 - b^4)*d^4*f^3*x^4 + 4*(2*a^4 + a^2*b^2 - b^4)*d^4*e*f^2*x^3 + 6*(2*a^4
+ a^2*b^2 - b^4)*d^4*e^2*f*x^2 + 4*(2*a^4 + a^2*b^2 - b^4)*d^4*e^3*x)*cosh(
d*x + c)^2 - 2*(2*(2*a^4 + a^2*b^2 - b^4)*d^4*f^3*x^4 + 8*(2*a^4 + a^2*b^2
- b^4)*d^4*e*f^2*x^3 + 12*(2*a^4 + a^2*b^2 - b^4)*d^4*e^2*f*x^2 + 8*(2*a^4
+ a^2*b^2 - b^4)*d^4*e^3*x + 3*(4*(a^2*b^2 + b^4)*d^3*f^3*x^3 + 4*(a^2*b^2
+ b^4)*d^3*e^3 - 6*(a^2*b^2 + b^4)*d^2*e^2*f + 6*(a^2*b^2 + b^4)*d*e*f^2 -
3*(a^2*b^2 + b^4)*f^3 + 6*(2*(a^2*b^2 + b^4)*d^3*e*f^2 - (a^2*b^2 + b^4)*d^
2*f^3)*x^2 + 6*(2*(a^2*b^2 + b^4)*d^3*e^2*f - 2*(a^2*b^2 + b^4)*d^2*e*f^2 +
 (a^2*b^2 + b^4)*d*f^3)*x)*cosh(d*x + c)^2 - 24*((a^3*b + a*b^3)*d^3*f^3*x^
3 + (a^3*b + a*b^3)*d^3*e^3 - 3*(a^3*b + a*b^3)*d^2*e^2*f + 6*(a^3*b + a*b^
3)*d*e*f^2 - 6*(a^3*b + a*b^3)*f^3 + 3*((a^3*b + a*b^3)*d^3*e*f^2 - (a^3*b
+ a*b^3)*d^2*f^3)*x^2 + 3*((a^3*b + a*b^3)*d^3*e^2*f - 2*(a^3*b + a*b^3)*d^
2*e*f^2 + 2*(a^3*b + a*b^3)*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 96*(
(a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*f)*cosh(d*x + c)^2
 + 2*(a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*f)*cosh(d*x +
 c)*sinh(d*x + c) + (a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^
2*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh
(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/
b + 1) - 96*((a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*f)*co
sh(d*x + c)^2 + 2*(a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*
f)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x +
 a^3*b*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2) - b)/b + 1) - 32*((a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^
2*d*e*f^2 - a^3*b*c^3*f^3)*cosh(d*x + c)^2 + 2*(a^3*b*d^3*e^3 - 3*a^3*b*c*d
^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c)
 + (a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f
^3)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh
(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 32*((a^3*b*d^3*e^3 - 3*a^3*b
*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*cosh(d*x + c)^2 + 2*(a^
3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*co
sh(d*x + c)*sinh(d*x + c) + (a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*
c^2*d*e*f^2 - a^3*b*c^3*f^3)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b
*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 32*
((a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b
*c*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*cosh(d*x + c)^2 + 2*(a^
3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d
^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c)
 + (a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3
```

```
*b*c*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*sinh(d*x + c)^2)*sqrt
((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 32*((a^3*b*d^3*f^3*x^3
 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^2*e^2*f - 3*a^
3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*cosh(d*x + c)^2 + 2*(a^3*b*d^3*f^3*x^3 + 3
*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^2*e^2*f - 3*a^3*b*
c^2*d*e*f^2 + a^3*b*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d^3*f^3*x
^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^2*e^2*f - 3*
a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*l
og(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2) - b)/b) + 192*(a^3*b*f^3*cosh(d*x + c)^2 + 2*a^3*b*
f^3*cosh(d*x + c)*sinh(d*x + c) + a^3*b*f^3*sinh(d*x + c)^2)*sqrt((a^2 + b^
2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 192*(a^3*b*f^3*cosh(d*x + c)^2
 + 2*a^3*b*f^3*cosh(d*x + c)*sinh(d*x + c) + a^3*b*f^3*sinh(d*x + c)^2)*sqr
t((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 192*((a^3*b*d*f^3*x
 + a^3*b*d*e*f^2)*cosh(d*x + c)^2 + 2*(a^3*b*d*f^3*x + a^3*b*d*e*f^2)*cosh(
d*x + c)*sinh(d*x + c) + (a^3*b*d*f^3*x + a^3*b*d*e*f^2)*sinh(d*x + c)^2)*s
qrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 192*((a^3*b*d*f^3
*x + a^3*b*d*e*f^2)*cosh(d*x + c)^2 + 2*(a^3*b*d*f^3*x + a^3*b*d*e*f^2)*cos
h(d*x + c)*sinh(d*x + c) + (a^3*b*d*f^3*x + a^3*b*d*e*f^2)*sinh(d*x + c)^2)
*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*(2*(a^2*b^2 +
 b^4)*d^3*e^2*f + 2*(a^2*b^2 + b^4)*d^2*e*f^2 + (a^2*b^2 + b^4)*d*f^3)*x +
16*((a^3*b + a*b^3)*d^3*f^3*x^3 + (a^3*b + a*b^3)*d^3*e^3 + 3*(a^3*b + a*b^
3)*d^2*e^2*f + 6*(a^3*b + a*b^3)*d*e*f^2 + 6*(a^3*b + a*b^3)*f^3 + 3*((a^3*
b + a*b^3)*d^3*e*f^2 + (a^3*b + a*b^3)*d^2*f^3)*x^2 + 3*((a^3*b + a*b^3)*d^
3*e^2*f + 2*(a^3*b + a*b^3)*d^2*e*f^2 + 2*(a^3*b + a*b^3)*d*f^3)*x)*cosh(d*
x + c) + 4*(4*(a^3*b + a*b^3)*d^3*f^3*x^3 + 4*(a^3*b + a*b^3)*d^3*e^3 + 12*
(a^3*b + a*b^3)*d^2*e^2*f + 24*(a^3*b + a*b^3)*d*e*f^2 + 24*(a^3*b + a*b^3)
*f^3 - (4*(a^2*b^2 + b^4)*d^3*f^3*x^3 + 4*(a^2*b^2 + b^4)*d^3*e^3 - 6*(a^2*
b^2 + b^4)*d^2*e^2*f + 6*(a^2*b^2 + b^4)*d*e*f^2 - 3*(a^2*b^2 + b^4)*f^3 +
6*(2*(a^2*b^2 + b^4)*d^3*e*f^2 - (a^2*b^2 + b^4)*d^2*f^3)*x^2 + 6*(2*(a^2*b
^2 + b^4)*d^3*e^2*f - 2*(a^2*b^2 + b^4)*d^2*e*f^2 + (a^2*b^2 + b^4)*d*f^3)*
x)*cosh(d*x + c)^3 + 12*((a^3*b + a*b^3)*d^3*e*f^2 + (a^3*b + a*b^3)*d^2*f^
3)*x^2 + 12*((a^3*b + a*b^3)*d^3*f^3*x^3 + (a^3*b + a*b^3)*d^3*e^3 - 3*(a^3
*b + a*b^3)*d^2*e^2*f + 6*(a^3*b + a*b^3)*d*e*f^2 - 6*(a^3*b + a*b^3)*f^3 +
 3*((a^3*b + a*b^3)*d^3*e*f^2 - (a^3*b + a*b^3)*d^2*f^3)*x^2 + 3*((a^3*b +
a*b^3)*d^3*e^2*f - 2*(a^3*b + a*b^3)*d^2*e*f^2 + 2*(a^3*b + a*b^3)*d*f^3)*x
)*cosh(d*x + c)^2 + 12*((a^3*b + a*b^3)*d^3*e^2*f + 2*(a^3*b + a*b^3)*d^2*e
*f^2 + 2*(a^3*b + a*b^3)*d*f^3)*x - 2*((2*a^4 + a^2*b^2 - b^4)*d^4*f^3*x^4
+ 4*(2*a^4 + a^2*b^2 - b^4)*d^4*e*f^2*x^3 + 6*(2*a^4 + a^2*b^2 - b^4)*d^4*e
^2*f*x^2 + 4*(2*a^4 + a^2*b^2 - b^4)*d^4*e^3*x)*cosh(d*x + c))*sinh(d*x + c
))/((a^2*b^3 + b^5)*d^4*cosh(d*x + c)^2 + 2*(a^2*b^3 + b^5)*d^4*cosh(d*x +
c)*sinh(d*x + c) + (a^2*b^3 + b^5)*d^4*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^3 \sinh\left(dx+c\right)^3}{b\sinh\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sinh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

3.234 $\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=522

$$-\frac{2a^3f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2\sqrt{a^2+b^2}}+\frac{2a^3f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2\sqrt{a^2+b^2}}+\frac{2a^3f^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3\sqrt{a^2+b^2}}-\frac{2a^3f}{}$$

```
[Out] -(f^2*x)/(4*b*d^2) + (a^2*(e + f*x)^3)/(3*b^3*f) - (e + f*x)^3/(6*b*f) - (2
*a*f^2*Cosh[c + d*x])/(b^2*d^3) - (a*(e + f*x)^2*Cosh[c + d*x])/(b^2*d) - (
a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*Sqrt[a
^2 + b^2]*d) + (a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
])])/(b^3*Sqrt[a^2 + b^2]*d) - (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]*d^2) + (2*a^3*f*(e + f*x)
*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]
*d^2) + (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b
^3*Sqrt[a^2 + b^2]*d^3) - (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]*d^3) + (2*a*f*(e + f*x)*Sinh[c + d*x])
/(b^2*d^2) + (f^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^3) + ((e + f*x)^2*Cos
h[c + d*x]*Sinh[c + d*x])/(2*b*d) - (f*(e + f*x)*Sinh[c + d*x]^2)/(2*b*d^2)
```

Rubi [A] time = 1.04363, antiderivative size = 522, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {5557, 3311, 32, 2635, 8, 3296, 2638, 3322, 2264, 2190, 2531, 2282, 6589}

$$-\frac{2a^3f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2\sqrt{a^2+b^2}}+\frac{2a^3f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2\sqrt{a^2+b^2}}+\frac{2a^3f^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3\sqrt{a^2+b^2}}-\frac{2a^3f}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -(f^2*x)/(4*b*d^2) + (a^2*(e + f*x)^3)/(3*b^3*f) - (e + f*x)^3/(6*b*f) - (2
*a*f^2*Cosh[c + d*x])/(b^2*d^3) - (a*(e + f*x)^2*Cosh[c + d*x])/(b^2*d) - (
a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*Sqrt[a
^2 + b^2]*d) + (a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
])])/(b^3*Sqrt[a^2 + b^2]*d) - (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]*d^2) + (2*a^3*f*(e + f*x)
*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]
*d^2) + (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b
^3*Sqrt[a^2 + b^2]*d^3) - (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]*d^3) + (2*a*f*(e + f*x)*Sinh[c + d*x])
/(b^2*d^2) + (f^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^3) + ((e + f*x)^2*Cos
h[c + d*x]*Sinh[c + d*x])/(2*b*d) - (f*(e + f*x)*Sinh[c + d*x]^2)/(2*b*d^2)
```


Rule 5557

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{f(e+fx)\sinh^2(c+dx)}{2bd^2} - \frac{a\int (e+fx)^2\sinh(c+ \ldots}{b^2}$$

$$= -\frac{(e+fx)^3}{6bf} - \frac{a(e+fx)^2\cosh(c+dx)}{b^2d} + \frac{f^2\cosh(c+dx)\sinh(c+dx)}{4bd^3} + \frac{(e+fx)^2\cosh(\ldots}{}$$

$$= -\frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} - \frac{(e+fx)^3}{6bf} - \frac{a(e+fx)^2\cosh(c+dx)}{b^2d} + \frac{2af(e+fx)\sinh(c+dx)}{b^2d^2} \ldots$$

$$= -\frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} - \frac{(e+fx)^3}{6bf} - \frac{2af^2\cosh(c+dx)}{b^2d^3} - \frac{a(e+fx)^2\cosh(c+dx)}{b^2d} + \frac{2af \ldots}{}$$

$$= -\frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} - \frac{(e+fx)^3}{6bf} - \frac{2af^2\cosh(c+dx)}{b^2d^3} - \frac{a(e+fx)^2\cosh(c+dx)}{b^2d} - \frac{a^3(e \ldots}{}$$

$$= -\frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} - \frac{(e+fx)^3}{6bf} - \frac{2af^2\cosh(c+dx)}{b^2d^3} - \frac{a(e+fx)^2\cosh(c+dx)}{b^2d} - \frac{a^3(e \ldots}{}$$

$$= -\frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} - \frac{(e+fx)^3}{6bf} - \frac{2af^2\cosh(c+dx)}{b^2d^3} - \frac{a(e+fx)^2\cosh(c+dx)}{b^2d} - \frac{a^3(e \ldots}{}$$

$$= -\frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} - \frac{(e+fx)^3}{6bf} - \frac{2af^2\cosh(c+dx)}{b^2d^3} - \frac{a(e+fx)^2\cosh(c+dx)}{b^2d} - \frac{a^3(e \ldots}{}$$

Mathematica [A] time = 4.35971, size = 740, normalized size = 1.42

$$-\frac{48a^3f(e+fx)\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)}{d^2\sqrt{a^2+b^2}} + \frac{48a^3f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2\sqrt{a^2+b^2}} + \frac{48a^3f^2\text{PolyLog}\left(3,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)}{d^3\sqrt{a^2+b^2}} - \frac{48a^3f^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^3\sqrt{a^2+b^2}} + \frac{48a^3e^2 \ldots}{} \ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (24*a^2*e^2*x - 12*b^2*e^2*x + 24*a^2*e*f*x^2 - 12*b^2*e*f*x^2 + 8*a^2*f^2*
x^3 - 4*b^2*f^2*x^3 + (48*a^3*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^
2]])/(Sqrt[a^2 + b^2]*d) - (24*a*b*e^2*Cosh[c + d*x])/d - (48*a*b*f^2*Cosh[
c + d*x])/d^3 - (48*a*b*e*f*x*Cosh[c + d*x])/d - (24*a*b*f^2*x^2*Cosh[c + d
*x])/d - (6*b^2*e*f*Cosh[2*(c + d*x)])/d^2 - (6*b^2*f^2*x*Cosh[2*(c + d*x)]
)/d^2 - (48*a^3*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(Sqrt
[a^2 + b^2]*d) - (24*a^3*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])])/(Sqrt[a^2 + b^2]*d) + (48*a^3*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])])/(Sqrt[a^2 + b^2]*d) + (24*a^3*f^2*x^2*Log[1 + (b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]*d) - (48*a^3*f*(e + f*x)*PolyLog
[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]*d^2) + (48*a^
3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(Sqrt[a
^2 + b^2]*d^2) + (48*a^3*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^
2])])/(Sqrt[a^2 + b^2]*d^3) - (48*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*d^3) + (48*a*b*e*f*Sinh[c + d*x])/d^2
 + (48*a*b*f^2*x*Sinh[c + d*x])/d^2 + (6*b^2*e^2*Sinh[2*(c + d*x)])/d + (3*
b^2*f^2*Sinh[2*(c + d*x)])/d^3 + (12*b^2*e*f*x*Sinh[2*(c + d*x)])/d + (6*b^
2*f^2*x^2*Sinh[2*(c + d*x)])/d)/(24*b^3)
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2}\left(\sinh\left(dx+c\right)\right)^{3}}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.25167, size = 7156, normalized size = 13.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(6*(a^2*b^2 + b^4)*d^2*f^2*x^2 + 6*(a^2*b^2 + b^4)*d^2*e^2 - 3*(2*(a^
2*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^2*b^2 + b^4)*d^2*e^2 - 2*(a^2*b^2 + b^4)*d*
```

```
e*f + (a^2*b^2 + b^4)*f^2 + 2*(2*(a^2*b^2 + b^4)*d^2*e*f - (a^2*b^2 + b^4)*
d*f^2)*x)*cosh(d*x + c)^4 - 3*(2*(a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^2*b^2 +
 b^4)*d^2*e^2 - 2*(a^2*b^2 + b^4)*d*e*f + (a^2*b^2 + b^4)*f^2 + 2*(2*(a^2*b
^2 + b^4)*d^2*e*f - (a^2*b^2 + b^4)*d*f^2)*x)*sinh(d*x + c)^4 + 6*(a^2*b^2
+ b^4)*d*e*f + 24*((a^3*b + a*b^3)*d^2*f^2*x^2 + (a^3*b + a*b^3)*d^2*e^2 -
2*(a^3*b + a*b^3)*d*e*f + 2*(a^3*b + a*b^3)*f^2 + 2*((a^3*b + a*b^3)*d^2*e*
f - (a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c)^3 + 12*(2*(a^3*b + a*b^3)*d^2*f
^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e^2 - 4*(a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a
*b^3)*f^2 + 4*((a^3*b + a*b^3)*d^2*e*f - (a^3*b + a*b^3)*d*f^2)*x - (2*(a^2
*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^2*b^2 + b^4)*d^2*e^2 - 2*(a^2*b^2 + b^4)*d*e
*f + (a^2*b^2 + b^4)*f^2 + 2*(2*(a^2*b^2 + b^4)*d^2*e*f - (a^2*b^2 + b^4)*d
*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^2*b^2 + b^4)*f^2 - 8*((2*a^4
 + a^2*b^2 - b^4)*d^3*f^2*x^3 + 3*(2*a^4 + a^2*b^2 - b^4)*d^3*e*f*x^2 + 3*(
2*a^4 + a^2*b^2 - b^4)*d^3*e^2*x)*cosh(d*x + c)^2 - 2*(4*(2*a^4 + a^2*b^2 -
 b^4)*d^3*f^2*x^3 + 12*(2*a^4 + a^2*b^2 - b^4)*d^3*e*f*x^2 + 12*(2*a^4 + a^
2*b^2 - b^4)*d^3*e^2*x + 9*(2*(a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^2*b^2 + b^
4)*d^2*e^2 - 2*(a^2*b^2 + b^4)*d*e*f + (a^2*b^2 + b^4)*f^2 + 2*(2*(a^2*b^2
+ b^4)*d^2*e*f - (a^2*b^2 + b^4)*d*f^2)*x)*cosh(d*x + c)^2 - 36*((a^3*b + a
*b^3)*d^2*f^2*x^2 + (a^3*b + a*b^3)*d^2*e^2 - 2*(a^3*b + a*b^3)*d*e*f + 2*(
a^3*b + a*b^3)*f^2 + 2*((a^3*b + a*b^3)*d^2*e*f - (a^3*b + a*b^3)*d*f^2)*x)
*cosh(d*x + c))*sinh(d*x + c)^2 + 96*((a^3*b*d*f^2*x + a^3*b*d*e*f)*cosh(d*
x + c)^2 + 2*(a^3*b*d*f^2*x + a^3*b*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a
^3*b*d*f^2*x + a^3*b*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a
*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b + 1) - 96*((a^3*b*d*f^2*x + a^3*b*d*e*f)*cosh(d*x
+ c)^2 + 2*(a^3*b*d*f^2*x + a^3*b*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a^3
*b*d*f^2*x + a^3*b*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*c
osh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((
a^2 + b^2)/b^2) - b)/b + 1) - 48*((a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*
c^2*f^2)*cosh(d*x + c)^2 + 2*(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f
^2)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*
c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b
*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 48*((a^3*b*d^2*e^2 - 2*
a^3*b*c*d*e*f + a^3*b*c^2*f^2)*cosh(d*x + c)^2 + 2*(a^3*b*d^2*e^2 - 2*a^3*b
*c*d*e*f + a^3*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d^2*e^2 - 2*
a^3*b*c*d*e*f + a^3*b*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2
*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 4
8*((a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2
)*cosh(d*x + c)^2 + 2*(a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*
e*f - a^3*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d^2*f^2*x^2 + 2*a
^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^
2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 48*((a^3*b*d^2*f^2*x^2 + 2
*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*cosh(d*x + c)^2 + 2*(a^
3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*cosh
(d*x + c)*sinh(d*x + c) + (a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*
c*d*e*f - a^3*b*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cos
h(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b) - 96*(a^3*b*f^2*cosh(d*x + c)^2 + 2*a^3*b*f^2*cosh(d*
x + c)*sinh(d*x + c) + a^3*b*f^2*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*pol
ylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2))/b) + 96*(a^3*b*f^2*cosh(d*x + c)^2 + 2*a^3*b*f
^2*cosh(d*x + c)*sinh(d*x + c) + a^3*b*f^2*sinh(d*x + c)^2)*sqrt((a^2 + b^2
)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*(2*(a^2*b^2 + b^4)*d^2*e*f +
(a^2*b^2 + b^4)*d*f^2)*x + 24*((a^3*b + a*b^3)*d^2*f^2*x^2 + (a^3*b + a*b^3
)*d^2*e^2 + 2*(a^3*b + a*b^3)*d*e*f + 2*(a^3*b + a*b^3)*f^2 + 2*((a^3*b + a
*b^3)*d^2*e*f + (a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c) + 4*(6*(a^3*b + a*b
^3)*d^2*f^2*x^2 + 6*(a^3*b + a*b^3)*d^2*e^2 + 12*(a^3*b + a*b^3)*d*e*f - 3*
```

```
(2*(a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^2*b^2 + b^4)*d^2*e^2 - 2*(a^2*b^2 + b
^4)*d*e*f + (a^2*b^2 + b^4)*f^2 + 2*(2*(a^2*b^2 + b^4)*d^2*e*f - (a^2*b^2 +
 b^4)*d*f^2)*x)*cosh(d*x + c)^3 + 12*(a^3*b + a*b^3)*f^2 + 18*((a^3*b + a*b
^3)*d^2*f^2*x^2 + (a^3*b + a*b^3)*d^2*e^2 - 2*(a^3*b + a*b^3)*d*e*f + 2*(a^
3*b + a*b^3)*f^2 + 2*((a^3*b + a*b^3)*d^2*e*f - (a^3*b + a*b^3)*d*f^2)*x)*c
osh(d*x + c)^2 + 12*((a^3*b + a*b^3)*d^2*e*f + (a^3*b + a*b^3)*d*f^2)*x - 4
*((2*a^4 + a^2*b^2 - b^4)*d^3*f^2*x^3 + 3*(2*a^4 + a^2*b^2 - b^4)*d^3*e*f*x
^2 + 3*(2*a^4 + a^2*b^2 - b^4)*d^3*e^2*x)*cosh(d*x + c))*sinh(d*x + c))/((a
^2*b^3 + b^5)*d^3*cosh(d*x + c)^2 + 2*(a^2*b^3 + b^5)*d^3*cosh(d*x + c)*sin
h(d*x + c) + (a^2*b^3 + b^5)*d^3*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sinh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

3.235 $\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=335

$$-\frac{a^3 f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2\sqrt{a^2+b^2}}+\frac{a^3 f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2\sqrt{a^2+b^2}}-\frac{a^3(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3d\sqrt{a^2+b^2}}+\frac{a^3(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right.}{b^3d\sqrt{a^2+b^2}}$$

```
[Out] (a^2*e*x)/b^3 - (e*x)/(2*b) + (a^2*f*x^2)/(2*b^3) - (f*x^2)/(4*b) - (a*(e +
 f*x)*Cosh[c + d*x])/(b^2*d) - (a^3*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2])])/(b^3*Sqrt[a^2 + b^2]*d) + (a^3*(e + f*x)*Log[1 + (b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*Sqrt[a^2 + b^2]*d) - (a^3*f*PolyLog[2
, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]*d^2) + (a
^3*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 +
b^2]*d^2) + (a*f*Sinh[c + d*x])/(b^2*d^2) + ((e + f*x)*Cosh[c + d*x]*Sinh[c
 + d*x])/(2*b*d) - (f*Sinh[c + d*x]^2)/(4*b*d^2)
```

Rubi [A] time = 0.593089, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5557, 3310, 3296, 2637, 3322, 2264, 2190, 2279, 2391}

$$-\frac{a^3 f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2\sqrt{a^2+b^2}}+\frac{a^3 f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2\sqrt{a^2+b^2}}-\frac{a^3(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3d\sqrt{a^2+b^2}}+\frac{a^3(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right.}{b^3d\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*e*x)/b^3 - (e*x)/(2*b) + (a^2*f*x^2)/(2*b^3) - (f*x^2)/(4*b) - (a*(e +
 f*x)*Cosh[c + d*x])/(b^2*d) - (a^3*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2])])/(b^3*Sqrt[a^2 + b^2]*d) + (a^3*(e + f*x)*Log[1 + (b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*Sqrt[a^2 + b^2]*d) - (a^3*f*PolyLog[2
, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 + b^2]*d^2) + (a
^3*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*Sqrt[a^2 +
b^2]*d^2) + (a*f*Sinh[c + d*x])/(b^2*d^2) + ((e + f*x)*Cosh[c + d*x]*Sinh[c
 + d*x])/(2*b*d) - (f*Sinh[c + d*x]^2)/(4*b*d^2)
```

Rule 5557

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sinh[c
 + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{f\sinh^2(c+dx)}{4bd^2} - \frac{a\int (e+fx)\sinh(c+dx)\,dx}{b^2} + \frac{a^2}{}$$

$$= -\frac{ex}{2b} - \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} + \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{f\sinh^2(c+}{4bd^2}$$

$$= \frac{a^2ex}{b^3} - \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} - \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} + \frac{af\sinh(c+dx)}{b^2d^2} + \frac{(e+fx)\cosh(c}{}$$

$$= \frac{a^2ex}{b^3} - \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} - \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} + \frac{af\sinh(c+dx)}{b^2d^2} + \frac{(e+fx)\cosh(c}{}$$

$$= \frac{a^2ex}{b^3} - \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} - \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} - \frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d} + \frac{a^3}{}$$

$$= \frac{a^2ex}{b^3} - \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} - \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} - \frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d} + \frac{a^3}{}$$

$$= \frac{a^2ex}{b^3} - \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} - \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} - \frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d} + \frac{a^3}{}$$

Mathematica [A] time = 2.68747, size = 307, normalized size = 0.92

$$\frac{8a^3\left(-f\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-f(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)+f(c+dx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)-2cf\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right.\right.}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*(2*a^2 - b^2)*(c + d*x)*(c*f - d*(2*e + f*x)) - 8*a*b*d*(e + f*x)*Cosh[
c + d*x] - b^2*f*Cosh[2*(c + d*x)] + (8*a^3*(2*d*e*ArcTanh[(a + b*E^(c + d*
x))/Sqrt[a^2 + b^2]] - 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] -
 f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*L
og[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - f*PolyLog[2, (b*E^(c + d*x)
)/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))]))/Sqrt[a^2 + b^2] + 8*a*b*f*Sinh[c + d*x] + 2*b^2*d*(e + f*x)*Sinh[
2*(c + d*x)])/(8*b^3*d^2)
```

Maple [A] time = 0.067, size = 589, normalized size = 1.8

$$\frac{a^2fx^2}{2b^3} - \frac{fx^2}{4b} + \frac{a^2ex}{b^3} - \frac{ex}{2b} + \frac{(2dfx+2de-f)e^{2dx+2c}}{16d^2b} - \frac{a(dfx+de-f)e^{dx+c}}{2b^2d^2} - \frac{a(dfx+de+f)e^{-dx-c}}{2b^2d^2} - \frac{(2dfx+2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/2*a^2*f*x^2/b^3-1/4*f*x^2/b+a^2*e*x/b^3-1/2*e*x/b+1/16*(2*d*f*x+2*d*e-f)/
d^2/b*exp(2*d*x+2*c)-1/2*a*(d*f*x+d*e-f)/b^2/d^2*exp(d*x+c)-1/2*a*(d*f*x+d*
e+f)/b^2/d^2*exp(-d*x-c)-1/16*(2*d*f*x+2*d*e+f)/d^2/b*exp(-2*d*x-2*c)+2*a^3
/b^3/d*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-
a^3/b^3/d*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b
^2)^(1/2)))*x-a^3/b^3/d^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/
2)-a)/(-a+(a^2+b^2)^(1/2)))*c+a^3/b^3/d*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+
(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+a^3/b^3/d^2*f/(a^2+b^2)^(1/2)*ln(
(b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-a^3/b^3/d^2*f/(a^2+
b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+a^
3/b^3/d^2*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+
b^2)^(1/2)))-2*a^3/b^3/d^2*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+
2*a)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.90955, size = 3992, normalized size = 11.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*((2*(a^2*b^2 + b^4)*d*f*x + 2*(a^2*b^2 + b^4)*d*e - (a^2*b^2 + b^4)*f)
*cosh(d*x + c)^4 + (2*(a^2*b^2 + b^4)*d*f*x + 2*(a^2*b^2 + b^4)*d*e - (a^2*
b^2 + b^4)*f)*sinh(d*x + c)^4 - 2*(a^2*b^2 + b^4)*d*f*x - 8*((a^3*b + a*b^3
)*d*f*x + (a^3*b + a*b^3)*d*e - (a^3*b + a*b^3)*f)*cosh(d*x + c)^3 - 4*(2*(
a^3*b + a*b^3)*d*f*x + 2*(a^3*b + a*b^3)*d*e - 2*(a^3*b + a*b^3)*f - (2*(a^
2*b^2 + b^4)*d*f*x + 2*(a^2*b^2 + b^4)*d*e - (a^2*b^2 + b^4)*f)*cosh(d*x +
c))*sinh(d*x + c)^3 - 2*(a^2*b^2 + b^4)*d*e + 4*((2*a^4 + a^2*b^2 - b^4)*d^
2*f*x^2 + 2*(2*a^4 + a^2*b^2 - b^4)*d^2*e*x)*cosh(d*x + c)^2 + 2*(2*(2*a^4
+ a^2*b^2 - b^4)*d^2*f*x^2 + 4*(2*a^4 + a^2*b^2 - b^4)*d^2*e*x + 3*(2*(a^2*
b^2 + b^4)*d*f*x + 2*(a^2*b^2 + b^4)*d*e - (a^2*b^2 + b^4)*f)*cosh(d*x + c)
^2 - 12*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e - (a^3*b + a*b^3)*f)*c
osh(d*x + c))*sinh(d*x + c)^2 - 16*(a^3*b*f*cosh(d*x + c)^2 + 2*a^3*b*f*cos
h(d*x + c)*sinh(d*x + c) + a^3*b*f*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*d
ilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 16*(a^3*b*f*cosh(d*x + c)^2 + 2*a^3*
b*f*cosh(d*x + c)*sinh(d*x + c) + a^3*b*f*sinh(d*x + c)^2)*sqrt((a^2 + b^2)
/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 16*((a^3*b*d*e - a^3*b*c*f)*c
osh(d*x + c)^2 + 2*(a^3*b*d*e - a^3*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a
^3*b*d*e - a^3*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d
*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 16*((a^3*b
*d*e - a^3*b*c*f)*cosh(d*x + c)^2 + 2*(a^3*b*d*e - a^3*b*c*f)*cosh(d*x + c)
*sinh(d*x + c) + (a^3*b*d*e - a^3*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/
```

```
b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2)
+ 2*a) - 16*((a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x + c)^2 + 2*(a^3*b*d*f*x + a
^3*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d*f*x + a^3*b*c*f)*sinh(d*x
+ c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 16*((a^3*b
*d*f*x + a^3*b*c*f)*cosh(d*x + c)^2 + 2*(a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x
+ c)*sinh(d*x + c) + (a^3*b*d*f*x + a^3*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 +
 b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*s
inh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (a^2*b^2 + b^4)*f - 8*((a^3*b
 + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e + (a^3*b + a*b^3)*f)*cosh(d*x + c) -
4*(2*(a^3*b + a*b^3)*d*f*x - (2*(a^2*b^2 + b^4)*d*f*x + 2*(a^2*b^2 + b^4)*d
*e - (a^2*b^2 + b^4)*f)*cosh(d*x + c)^3 + 2*(a^3*b + a*b^3)*d*e + 6*((a^3*b
 + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e - (a^3*b + a*b^3)*f)*cosh(d*x + c)^2
+ 2*(a^3*b + a*b^3)*f - 2*((2*a^4 + a^2*b^2 - b^4)*d^2*f*x^2 + 2*(2*a^4 + a
^2*b^2 - b^4)*d^2*e*x)*cosh(d*x + c))*sinh(d*x + c))/((a^2*b^3 + b^5)*d^2*c
osh(d*x + c)^2 + 2*(a^2*b^3 + b^5)*d^2*cosh(d*x + c)*sinh(d*x + c) + (a^2*b
^3 + b^5)*d^2*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sinh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

3.236 $\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3d\sqrt{a^2+b^2}}+\frac{x\left(2a^2-b^2\right)}{2b^3}-\frac{a\cosh(c+dx)}{b^2d}+\frac{\sinh(c+dx)\cosh(c+dx)}{2bd}$$

```
[Out] ((2*a^2 - b^2)*x)/(2*b^3) + (2*a^3*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a
^2 + b^2]])/(b^3*Sqrt[a^2 + b^2]*d) - (a*Cosh[c + d*x])/(b^2*d) + (Cosh[c +
 d*x]*Sinh[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.220769, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2793, 3023, 2735, 2660, 618, 204}

$$\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3d\sqrt{a^2+b^2}}+\frac{x\left(2a^2-b^2\right)}{2b^3}-\frac{a\cosh(c+dx)}{b^2d}+\frac{\sinh(c+dx)\cosh(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((2*a^2 - b^2)*x)/(2*b^3) + (2*a^3*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a
^2 + b^2]])/(b^3*Sqrt[a^2 + b^2]*d) - (a*Cosh[c + d*x])/(b^2*d) + (Cosh[c +
 d*x]*Sinh[c + d*x])/(2*b*d)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{\int \frac{a+b\sinh(c+dx)+2a\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{2b}$$

$$= -\frac{a\cosh(c+dx)}{b^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{i\int \frac{-iab+i(2a^2-b^2)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx}{2b^2}$$

$$= \frac{(2a^2-b^2)\,x}{2b^3} - \frac{a\cosh(c+dx)}{b^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{a^3\int \frac{1}{a+b\sinh(c+dx)}\,dx}{b^3}$$

$$= \frac{(2a^2-b^2)\,x}{2b^3} - \frac{a\cosh(c+dx)}{b^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} + \frac{(2ia^3)\,\mathrm{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx, x\right.}{b^3d}$$

$$= \frac{(2a^2-b^2)\,x}{2b^3} - \frac{a\cosh(c+dx)}{b^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{(4ia^3)\,\mathrm{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx\right.}{b^3}$$

$$= \frac{(2a^2-b^2)\,x}{2b^3} + \frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}\,d} - \frac{a\cosh(c+dx)}{b^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd}$$

Mathematica [A] time = 0.305172, size = 101, normalized size = 0.94

$$\frac{-2\left(b^2-2a^2\right)(c+dx) - \frac{8a^3\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 4ab\cosh(c+dx) + b^2\sinh(2(c+dx))}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*(-2*a^2 + b^2)*(c + d*x) - (8*a^3*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt
[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 4*a*b*Cosh[c + d*x] + b^2*Sinh[2*(c + d*x
)])/(4*b^3*d)
```

Maple [B] time = 0.03, size = 262, normalized size = 2.5

$$-\frac{1}{2\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+\frac{1}{2\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{a}{db^{2}}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+\frac{a^{2}}{db^{3}}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)-1/d/b^2/
(tanh(1/2*d*x+1/2*c)+1)*a+1/d/b^3*ln(tanh(1/2*d*x+1/2*c)+1)*a^2-1/2/d/b*ln(
tanh(1/2*d*x+1/2*c)+1)-2/d*a^3/b^3/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/
2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))+1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/
b/(tanh(1/2*d*x+1/2*c)-1)+1/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*a-1/d/b^3*ln(tanh
(1/2*d*x+1/2*c)-1)*a^2+1/2/d/b*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.42845, size = 1434, normalized size = 13.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(2*a^4 + a^2*b^2 - b^4)*d*x*cosh(d*x + c)^2 + (a^2*b^2 + b^4)*cosh(d
*x + c)^4 + (a^2*b^2 + b^4)*sinh(d*x + c)^4 - a^2*b^2 - b^4 - 4*(a^3*b + a*
b^3)*cosh(d*x + c)^3 - 4*(a^3*b + a*b^3 - (a^2*b^2 + b^4)*cosh(d*x + c))*si
nh(d*x + c)^3 + 2*(2*(2*a^4 + a^2*b^2 - b^4)*d*x + 3*(a^2*b^2 + b^4)*cosh(d
*x + c)^2 - 6*(a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*(a^3*cosh(
d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*x + c)^2)*sqrt(
a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x
+ c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2
 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sin
h(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) -
b)) - 4*(a^3*b + a*b^3)*cosh(d*x + c) - 4*(a^3*b + a*b^3 - 2*(2*a^4 + a^2*b
^2 - b^4)*d*x*cosh(d*x + c) - (a^2*b^2 + b^4)*cosh(d*x + c)^3 + 3*(a^3*b +
a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^2*b^3 + b^5)*d*cosh(d*x + c)^2 +
 2*(a^2*b^3 + b^5)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2*b^3 + b^5)*d*sinh(d
*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20252, size = 217, normalized size = 2.03

$$-\frac{a^3\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^3d}+\frac{\left(2\,a^2-b^2\right)(dx+c)}{2\,b^3d}-\frac{\left(4\,abe^{(dx+c)}+b^2\right)e^{(-2\,dx-2\,c)}}{8\,b^3d}+\frac{bde^{(2\,dx+2\,c)}-4\,ade^{(dx+c)}}{8\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -a^3*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c)
 + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3*d) + 1/2*(2*a^2 - b^2)*(d
*x + c)/(b^3*d) - 1/8*(4*a*b*e^(d*x + c) + b^2)*e^(-2*d*x - 2*c)/(b^3*d) +
1/8*(b*d*e^(2*d*x + 2*c) - 4*a*d*e^(d*x + c))/(b^2*d^2)
```

3.237 $\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Sinh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0765824, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sinh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]

[Out] Defer[Int][Sinh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.002, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]

[Out] $Aborted
```

Maple [A] time = 0.091, size = 0, normalized size = 0.

$$\int \frac{(\sinh(dx+c))^3}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)

[Out] int(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-2\,a^{3}\int -\frac{e^{(dx+c)}}{b^{4}fx+b^{4}e-\left(b^{4}fxe^{(2c)}+b^{4}ee^{(2c)}\right)e^{(2dx)}-2\left(ab^{3}fxe^{c}+ab^{3}ee^{c}\right)e^{(dx)}}\,dx-\frac{e^{\left(-2c+\frac{2de}{f}\right)}E_{1}\left(\frac{2\left(fx+e\right)d}{f}\right)}{4\,bf}-\frac{ae^{\left(-c+\frac{de}{f}\right)}E_{1}}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*a^3*integrate(-e^(d*x + c)/(b^4*f*x + b^4*e - (b^4*f*x*e^(2*c) + b^4*e*e
^(2*c))*e^(2*d*x) - 2*(a*b^3*f*x*e^c + a*b^3*e*e^c)*e^(d*x)), x) - 1/4*e^(-
2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b*f) - 1/2*a*e^(-c + d*e
/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^2*f) + 1/2*a*e^(c - d*e/f)*exp_inte
gral_e(1, -(f*x + e)*d/f)/(b^2*f) - 1/4*e^(2*c - 2*d*e/f)*exp_integral_e(1,
 -2*(f*x + e)*d/f)/(b*f) + 1/2*(2*a^2 - b^2)*log(f*x + e)/(b^3*f)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh\left(dx+c\right)^{3}}{afx+ae+\left(bfx+be\right)\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sinh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.238 $\int \frac{(e+fx)^3 \operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=605

$$\frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}}-\frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^3\sqrt{a^2+b^2}}-\frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}}$$

```
[Out] (-2*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (b*(e + f*x)^3*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) + (b*(e + f*x)^3*Lo
g[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) - (3*f*
(e + f*x)^2*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (3*f*(e + f*x)^2*PolyLog[2,
 E^(c + d*x)])/(a*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) + (3*b*f*(e + f*x)^2*PolyLog
[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) + (6
*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) - (6*f^2*(e + f*x)*PolyLog
[3, E^(c + d*x)])/(a*d^3) + (6*b*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^3) - (6*b*f^2*(e + f*x)*Poly
Log[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^3) -
 (6*f^3*PolyLog[4, -E^(c + d*x)])/(a*d^4) + (6*f^3*PolyLog[4, E^(c + d*x)])
/(a*d^4) - (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(
a*Sqrt[a^2 + b^2]*d^4) + (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^4)
```

Rubi [A] time = 0.964937, antiderivative size = 605, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5575, 4182, 2531, 6609, 2282, 6589, 3322, 2264, 2190}

$$\frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}}-\frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^3\sqrt{a^2+b^2}}-\frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (-2*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (b*(e + f*x)^3*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) + (b*(e + f*x)^3*Lo
g[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) - (3*f*
(e + f*x)^2*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (3*f*(e + f*x)^2*PolyLog[2,
 E^(c + d*x)])/(a*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) + (3*b*f*(e + f*x)^2*PolyLog
[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) + (6
*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) - (6*f^2*(e + f*x)*PolyLog
[3, E^(c + d*x)])/(a*d^3) + (6*b*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^3) - (6*b*f^2*(e + f*x)*Poly
Log[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^3) -
 (6*f^3*PolyLog[4, -E^(c + d*x)])/(a*d^4) + (6*f^3*PolyLog[4, E^(c + d*x)])
/(a*d^4) - (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(
a*Sqrt[a^2 + b^2]*d^4) + (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^4)
```


Rule 5575

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
```

```
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a +
b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\mathrm{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\mathrm{csch}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{(2b)\int \frac{e^{c+dx}(e+fx)^3}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{a} - \frac{(3f)\int (e+fx)^2\log\left(1-e^{c\cdots}\right.}{ad}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{3f(e+fx)^2\mathrm{Li}_2\left(-e^{c+dx}\right)}{ad^2} + \frac{3f(e+fx)^2\mathrm{Li}_2\left(e^{c+dx}\right)}{ad^2} - \frac{(2b\cdots}{}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c\cdots}}{a+\sqrt{a\cdots}}\right.}{a\sqrt{a^2+b^2}d}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c\cdots}}{a+\sqrt{a\cdots}}\right.}{a\sqrt{a^2+b^2}d}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c\cdots}}{a+\sqrt{a\cdots}}\right.}{a\sqrt{a^2+b^2}d}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c\cdots}}{a+\sqrt{a\cdots}}\right.}{a\sqrt{a^2+b^2}d}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c\cdots}}{a+\sqrt{a\cdots}}\right.}{a\sqrt{a^2+b^2}d}$$

Mathematica [A] time = 3.11956, size = 757, normalized size = 1.25

$$\frac{b\left(-3d^2f(e+fx)^2\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+3d^2f(e+fx)^2\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+6def^2\mathrm{PolyLog}\left(3,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)-6def^2\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+6df^3x\mathrm{PolyLog}\cdots\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*d^3*(e + f*x)^3*ArcTanh[Cosh[c + d*x] + Sinh[c + d*x]] + (b*(2*d^3*e^3*
ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 3*d^3*e^2*f*x*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])] - 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])] - d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2])] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3
*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^3*f^3*x^3
*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyL
og[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog
[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*d*e*f^2*PolyLog[3, (b*E^(
c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*d*f^3*x*PolyLog[3, (b*E^(c + d*x))/(-
a + Sqrt[a^2 + b^2])] - 6*d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))] - 6*d*f^3*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
] - 6*f^3*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*f^3*PolyLo
```

```
g[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]]))/Sqrt[a^2 + b^2] - 3*f*(d^2
*(e + f*x)^2*PolyLog[2, -Cosh[c + d*x] - Sinh[c + d*x]] - 2*d*f*(e + f*x)*P
olyLog[3, -Cosh[c + d*x] - Sinh[c + d*x]] + 2*f^2*PolyLog[4, -Cosh[c + d*x]
 - Sinh[c + d*x]]) + 3*f*(d^2*(e + f*x)^2*PolyLog[2, Cosh[c + d*x] + Sinh[c
 + d*x]] - 2*d*f*(e + f*x)*PolyLog[3, Cosh[c + d*x] + Sinh[c + d*x]] + 2*f^
2*PolyLog[4, Cosh[c + d*x] + Sinh[c + d*x]]))/(a*d^4)
```

Maple [F] time = 0.438, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x)

[Out] int((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [C] time = 3.05163, size = 3903, normalized size = 6.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")

[Out] -(6*b^2*f^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x
+ c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*b^
2*f^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) -
 (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*(a^2 + b
^2)*f^3*polylog(4, cosh(d*x + c) + sinh(d*x + c)) + 6*(a^2 + b^2)*f^3*polyl
og(4, -cosh(d*x + c) - sinh(d*x + c)) + 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^
2*x + b^2*d^2*e^2*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(
d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b
 + 1) - 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*sqrt((a^2 +
 b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^2*d^3*e^3 - 3*b^2*c*d
^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*sqrt((a^2 + b^2)/b^2)*log(2*b*c
osh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^2*
d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*sqrt((a^2 +
b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/
```

```
b^2) + 2*a) + (b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x +
3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*sqrt((a^2 + b^2)/b^2)*
log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 +
 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*s
qrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 6*(b^2*d*f^3*x + b^
2*d*e*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x +
 c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*(b^
2*d*f^3*x + b^2*d*e*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c)
+ a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2))/b) - 3*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^
2)*d^2*e^2*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) + 3*((a^2 + b^2)*d^2*f^3
*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*dilog(-cosh(d*x +
 c) - sinh(d*x + c)) + ((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x
^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + (a^2 + b^2)*d^3*e^3)*log(cosh(d*x + c) + s
inh(d*x + c) + 1) - ((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a
^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*log(cosh(d*x + c) + sinh(d*x +
 c) - 1) - ((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2
+ b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2
+ (a^2 + b^2)*c^3*f^3)*log(-cosh(d*x + c) - sinh(d*x + c) + 1) + 6*((a^2 +
b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*polylog(3, cosh(d*x + c) + sinh(d*x + c
)) - 6*((a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*polylog(3, -cosh(d*x + c
) - sinh(d*x + c)))/((a^3 + a*b^2)*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)/(a+b*sinh(d*x+c)),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")

[Out] Timed out
```

3.239 $\int \frac{(e+fx)^2\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=433

$$-\frac{2bf(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}}+\frac{2bf(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2\sqrt{a^2+b^2}}+\frac{2bf^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}}-\frac{2bf^2\text{Pol}}{}$$

```
[Out] (-2*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (b*(e + f*x)^2*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) + (b*(e + f*x)^2*Lo
g[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) - (2*f*
(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (2*f*(e + f*x)*PolyLog[2, E^(
c + d*x)])/(a*d^2) - (2*b*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) + (2*b*f*(e + f*x)*PolyLog[2, -((b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) + (2*f^2*Pol
yLog[3, -E^(c + d*x)])/(a*d^3) - (2*f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) +
(2*b*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2
+ b^2]*d^3) - (2*b*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/(a*Sqrt[a^2 + b^2]*d^3)
```

Rubi [A] time = 0.814057, antiderivative size = 433, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5575, 4182, 2531, 2282, 6589, 3322, 2264, 2190}

$$-\frac{2bf(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}}+\frac{2bf(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2\sqrt{a^2+b^2}}+\frac{2bf^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}}-\frac{2bf^2\text{Pol}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (-2*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (b*(e + f*x)^2*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) + (b*(e + f*x)^2*Lo
g[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) - (2*f*
(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (2*f*(e + f*x)*PolyLog[2, E^(
c + d*x)])/(a*d^2) - (2*b*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) + (2*b*f*(e + f*x)*PolyLog[2, -((b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) + (2*f^2*Pol
yLog[3, -E^(c + d*x)])/(a*d^3) - (2*f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) +
(2*b*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2
+ b^2]*d^3) - (2*b*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/(a*Sqrt[a^2 + b^2]*d^3)
```


Rule *5575*

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a +
b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
```

```
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{csch}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{(2b)\int \frac{e^{c+dx}(e+fx)^2}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{a} - \frac{(2f)\int (e+fx)\log\left(1-e^{c+dx}\right)}{ad}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2f(e+fx)\text{Li}_2\left(-e^{c+dx}\right)}{ad^2} + \frac{2f(e+fx)\text{Li}_2\left(e^{c+dx}\right)}{ad^2} - \frac{\left(2b^2\right)\int}{}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$

Mathematica [A] time = 2.12001, size = 454, normalized size = 1.05

$$\frac{\frac{b\left(-2df(e+fx)\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+2df(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+2f^2\text{PolyLog}\left(3,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)-2f^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+2d^2e^2\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-2}{\sqrt{a^2+b^2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (d^2*(e + f*x)^2*Log[1 - E^(c + d*x)] - d^2*(e + f*x)^2*Log[1 + E^(c + d*x)
] - 2*d*f*(e + f*x)*PolyLog[2, -E^(c + d*x)] + 2*d*f*(e + f*x)*PolyLog[2, E
^(c + d*x)] + 2*f^2*PolyLog[3, -E^(c + d*x)] - 2*f^2*PolyLog[3, E^(c + d*x)
] + (b*(2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*d^2*e*f*
x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2])] - 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
 + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 2
*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*f^2*PolyLog[3,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2])/(a*d^3)
```

Maple [F] time = 0.334, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^2\text{csch}\left(dx+c\right)}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.68204, size = 2674, normalized size = 6.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*b^2*f^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x +
 c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*b^2
*f^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) -
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*(a^2 + b^
2)*f^2*polylog(3, cosh(d*x + c) + sinh(d*x + c)) + 2*(a^2 + b^2)*f^2*polylo
g(3, -cosh(d*x + c) - sinh(d*x + c)) - 2*(b^2*d*f^2*x + b^2*d*e*f)*sqrt((a^
2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^2*d*f^2*x + b^2*
d*e*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b^2*d
^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*
x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*d^2*e^
2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c
) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*d^2*f^2*x^2
 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log
(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2) - b)/b) + (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*
c*d*e*f - b^2*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh
(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/
b) + 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*dilog(cosh(d*x + c) + sinh
(d*x + c)) - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*dilog(-cosh(d*x +
c) - sinh(d*x + c)) - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x +
(a^2 + b^2)*d^2*e^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a^2 + b^2)*
d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*log(cosh(d*x + c) +
sinh(d*x + c) - 1) + ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2
*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*log(-cosh(d*x + c) - sinh(d*x +
 c) + 1))/((a^3 + a*b^2)*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.240 $\int \frac{(e+fx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=261

$$-\frac{bf\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}}+\frac{bf\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2\sqrt{a^2+b^2}}-\frac{f\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}+\frac{f\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}-\frac{b(e+f\cdots}{}$$

```
[Out] (-2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d) - (b*(e + f*x)*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) + (b*(e + f*x)*Log[1 +
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) - (f*PolyLog[
2, -E^(c + d*x)])/(a*d^2) + (f*PolyLog[2, E^(c + d*x)])/(a*d^2) - (b*f*Poly
Log[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) +
 (b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b
^2]*d^2)
```

Rubi [A] time = 0.459786, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5575, 4182, 2279, 2391, 3322, 2264, 2190}

$$-\frac{bf\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}}+\frac{bf\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2\sqrt{a^2+b^2}}-\frac{f\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}+\frac{f\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}-\frac{b(e+f\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d) - (b*(e + f*x)*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) + (b*(e + f*x)*Log[1 +
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*Sqrt[a^2 + b^2]*d) - (f*PolyLog[
2, -E^(c + d*x)])/(a*d^2) + (f*PolyLog[2, E^(c + d*x)])/(a*d^2) - (b*f*Poly
Log[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) +
 (b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b
^2]*d^2)
```

Rule 5575

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a +
b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\operatorname{csch}(c+dx)\,dx}{a} - \frac{b\int \frac{e+fx}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{(2b)\int \frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{a} - \frac{f\int \log\left(1-e^{c+dx}\right)dx}{ad} + \frac{f\int \log$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\left(2b^2\right)\int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{a\sqrt{a^2+b^2}} + \frac{\left(2b^2\right)\int \frac{e^{c+dx}(e+fx)}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{a\sqrt{a^2+b^2}} -$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} -$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} -$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} -$$

Mathematica [A] time = 1.95028, size = 306, normalized size = 1.17

$$\frac{b\left(-f\operatorname{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-f(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)+f(c+dx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)-2cf\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)\right.}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (d*e*Log[Tanh[(c + d*x)/2]] - c*f*Log[Tanh[(c + d*x)/2]] + f*((c + d*x)*(Lo
g[1 - E^(-c - d*x)] - Log[1 + E^(-c - d*x)]) + PolyLog[2, -E^(-c - d*x)] -
PolyLog[2, E^(-c - d*x)]) + (b*(2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2
+ b^2]] - 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - f*(c + d*x)*
Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(
c + d*x))/(a + Sqrt[a^2 + b^2])] - f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[
a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/Sqr
t[a^2 + b^2])/(a*d^2)
```

Maple [B] time = 0.112, size = 532, normalized size = 2.

$$\frac{e\ln\left(e^{dx+c}-1\right)}{da}+2\frac{eb}{da\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\frac{2\,be^{dx+c}+2\,a}{\sqrt{a^2+b^2}}\right)-\frac{e\ln\left(e^{dx+c}+1\right)}{da}-\frac{f\,\mathrm{dilog}\left(e^{dx+c}\right)}{ad^2}-\frac{bfx}{da}\ln\left(\left(-be^{dx+c}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/d/a*e*ln(exp(d*x+c)-1)+2/d*e*b/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x
+c)+2*a)/(a^2+b^2)^(1/2))-1/d/a*e*ln(exp(d*x+c)+1)-1/d^2*f/a*dilog(exp(d*x+
c))-1/d*f*b/a/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2
+b^2)^(1/2)))*x-1/d^2*f*b/a/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/
2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d*f*b/a/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a
^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2*f*b/a/(a^2+b^2)^(1/2)*ln((b*e
xp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d^2*f*b/a/(a^2+b^2)^(
1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2*f*
b/a/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/
2)))-1/d/a*ln(exp(d*x+c)+1)*f*x-1/d^2*f/a*dilog(exp(d*x+c)+1)-1/d^2/a*f*c*l
n(exp(d*x+c)-1)-2/d^2*f*c*b/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2
*a)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.54721, size = 1621, normalized size = 6.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(b^2*f*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - b^2*f
*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (a^2 + b^2)*
f*dilog(cosh(d*x + c) + sinh(d*x + c)) + (a^2 + b^2)*f*dilog(-cosh(d*x + c)
 - sinh(d*x + c)) - (b^2*d*e - b^2*c*f)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(
d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^2*d*e
- b^2*c*f)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c)
- 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^2*d*f*x + b^2*c*f)*sqrt((a^2 + b^2)
/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^2*d*f*x + b^2*c*f)*sqrt((a^2 +
b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + ((a^2 + b^2)*d*f*x + (a^2 + b^
2)*d*e)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - ((a^2 + b^2)*d*e - (a^2 +
b^2)*c*f)*log(cosh(d*x + c) + sinh(d*x + c) - 1) - ((a^2 + b^2)*d*f*x + (a^
2 + b^2)*c*f)*log(-cosh(d*x + c) - sinh(d*x + c) + 1))/((a^3 + a*b^2)*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.241 $\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=64

$$\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

```
[Out] -(ArcTanh[Cosh[c + d*x]]/(a*d)) + (2*b*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sq
rt[a^2 + b^2]])/(a*Sqrt[a^2 + b^2]*d)
```

Rubi [A] time = 0.0878481, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2747, 3770, 2660, 618, 204}

$$\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(ArcTanh[Cosh[c + d*x]]/(a*d)) + (2*b*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sq
rt[a^2 + b^2]])/(a*Sqrt[a^2 + b^2]*d)
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{\int \operatorname{csch}(c+dx)\,dx}{a}-\frac{b\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a}\\ &= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{(2ib)\,\text{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{ad}\\ &= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad}-\frac{(4ib)\,\text{Subst}\left(\int \frac{1}{-4\left(a^2+b^2\right)-x^2}\,dx,x,-2ib+2a\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{ad}\\ &= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}\,d}\end{aligned}$$

Mathematica [A] time = 0.0823074, size = 69, normalized size = 1.08

$$\frac{\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-\frac{2b\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((-2*b*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
 + Log[Tanh[(c + d*x)/2]])/(a*d)
```

Maple [A] time = 0.003, size = 65, normalized size = 1.

$$-2\,\frac{b}{da\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\,\frac{2\,a\tanh\left(1/2\,dx+c/2\right)-2\,b}{\sqrt{a^2+b^2}}\right)+\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] -2/d*b/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2
)^(1/2))+1/d/a*ln(tanh(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.66225, size = 591, normalized size = 9.23

$$\frac{\sqrt{a^2+b^2}b\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2+b^2+2\left(b^2\cosh(dx+c)+ab\right)\sinh(dx+c)+2\,\sqrt{a^2+b^2}(b\cosh(dx+c)+b\sinh(dx+c)+}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)-b}\right.}{\left(a^3+ab^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*b*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*c
osh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*
sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2
 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x
 + c) - b)) - (a^2 + b^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (a^2 + b
^2)*log(cosh(d*x + c) + sinh(d*x + c) - 1))/((a^3 + a*b^2)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A] time = 1.58671, size = 144, normalized size = 2.25

$$-\frac{b\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}ad}-\frac{\log\left(e^{(dx+c)}+1\right)}{ad}+\frac{\log\left(\left|e^{(dx+c)}-1\right|\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -b*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c) +
 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*d) - log(e^(d*x + c) + 1)/(a*
d) + log(abs(e^(d*x + c) - 1))/(a*d)
```

3.242 $\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Csch[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0487392, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]

[Out] Defer[Int][Csch[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A] time = 5.92489, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]

[Out] Integrate[Csch[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)

[Out] int(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(csch(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(dx+c)}{afx+ae+(bfx+be)\sinh(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(csch(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(csch(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.243 $\int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)} \, dx$

Optimal. Leaf size=745

$$-\frac{6b^2f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3\sqrt{a^2+b^2}}+\frac{6b^2f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^3\sqrt{a^2+b^2}}+\frac{3b^2f(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2\sqrt{a^2+b^2}}$$

```
[Out] -((e + f*x)^3/(a*d)) + (2*b*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a^2*d) - ((e
 + f*x)^3*Coth[c + d*x])/(a*d) + (b^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2 + b^2]*d) - (b^2*(e + f*x)^3*Log[1 + (
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2 + b^2]*d) + (3*f*(e +
f*x)^2*Log[1 - E^(2*(c + d*x))])/(a*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2, -E
^(c + d*x)])/(a^2*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)])/(a^2*d
^2) + (3*b^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/(a^2*Sqrt[a^2 + b^2]*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^2) + (3*f^2*(e + f
*x)*PolyLog[2, E^(2*(c + d*x))])/(a*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, -E
^(c + d*x)])/(a^2*d^3) + (6*b*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a^2*d
^3) - (6*b^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/(a^2*Sqrt[a^2 + b^2]*d^3) + (6*b^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^3) - (3*f^3*PolyLo
g[3, E^(2*(c + d*x))])/(2*a*d^4) + (6*b*f^3*PolyLog[4, -E^(c + d*x)])/(a^2*
d^4) - (6*b*f^3*PolyLog[4, E^(c + d*x)])/(a^2*d^4) + (6*b^2*f^3*PolyLog[4,
-((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^4) - (6*b
^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2
+ b^2]*d^4)
```

Rubi [A] time = 1.31073, antiderivative size = 745, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {5575, 4184, 3716, 2190, 2531, 2282, 6589, 4182, 6609, 3322, 2264}

$$-\frac{6b^2f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3\sqrt{a^2+b^2}}+\frac{6b^2f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^3\sqrt{a^2+b^2}}+\frac{3b^2f(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((e + f*x)^3/(a*d)) + (2*b*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a^2*d) - ((e
 + f*x)^3*Coth[c + d*x])/(a*d) + (b^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2 + b^2]*d) - (b^2*(e + f*x)^3*Log[1 + (
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2 + b^2]*d) + (3*f*(e +
f*x)^2*Log[1 - E^(2*(c + d*x))])/(a*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2, -E
^(c + d*x)])/(a^2*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)])/(a^2*d
^2) + (3*b^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/(a^2*Sqrt[a^2 + b^2]*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^2) + (3*f^2*(e + f
*x)*PolyLog[2, E^(2*(c + d*x))])/(a*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, -E
^(c + d*x)])/(a^2*d^3) + (6*b*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a^2*d
^3) - (6*b^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/(a^2*Sqrt[a^2 + b^2]*d^3) + (6*b^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^3) - (3*f^3*PolyLo
g[3, E^(2*(c + d*x))])/(2*a*d^4) + (6*b*f^3*PolyLog[4, -E^(c + d*x)])/(a^2*
d^4) - (6*b*f^3*PolyLog[4, E^(c + d*x)])/(a^2*d^4) + (6*b^2*f^3*PolyLog[4,
```

```
-((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^4) - (6*b
^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2
+ b^2]*d^4)
```

Rule 5575

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a +
b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
```

```
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\operatorname{csch}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^3\coth(c+dx)}{ad} - \frac{b\int (e+fx)^3\operatorname{csch}(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}\,dx}{a^2} + \frac{(3f)\int$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{\left(2b^2\right)\int \frac{e^{c+dx}(e+f}{-b+2ae^{c+dx}+b}}{a^2}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{3f(e+fx)^2\log\left(1\right.}{ad^2}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{b^2(e+fx)^3\log\left(1\right.}{a^2\sqrt{a^2+b}}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{b^2(e+fx)^3\log\left(1\right.}{a^2\sqrt{a^2+b}}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{b^2(e+fx)^3\log\left(1\right.}{a^2\sqrt{a^2+b}}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{b^2(e+fx)^3\log\left(1\right.}{a^2\sqrt{a^2+b}}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{b^2(e+fx)^3\log\left(1\right.}{a^2\sqrt{a^2+b}}$$

Mathematica [A] time = 18.5958, size = 1353, normalized size = 1.82

$$\left(-2e^3\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)d^3+f^3x^3\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^3+3ef^2x^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^3+3e^2fx\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^3-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b^2*(-2*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 3*d^3*e^2*f
*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 +
 (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2])] - 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])] - 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
])] - d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3*d^2*f*
(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e
 + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*e*f^2*
PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*d*f^3*x*PolyLog[3, (
b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*d*e*f^2*PolyLog[3, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))] + 6*d*f^3*x*PolyLog[3, -((b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2]))] + 6*f^3*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]
)] - 6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^2*Sqrt
[a^2 + b^2]*d^4) - (a*d^3*(e + f*x)^3*(-1 + Coth[c]) - d^2*e^2*(b*d*e - 3*a
*f)*(d*x - Log[1 - Cosh[c + d*x] - Sinh[c + d*x]]) - 3*d^2*e*f*(b*d*e + 2*a
*f)*x*Log[1 + Cosh[c + d*x] - Sinh[c + d*x]] - 3*d^2*f^2*(b*d*e + a*f)*x^2*
Log[1 + Cosh[c + d*x] - Sinh[c + d*x]] - b*d^3*f^3*x^3*Log[1 + Cosh[c + d*x
] - Sinh[c + d*x]] + 3*d^2*e*f*(b*d*e - 2*a*f)*x*Log[1 - Cosh[c + d*x] + Si
nh[c + d*x]] + 3*d^2*f^2*(b*d*e - a*f)*x^2*Log[1 - Cosh[c + d*x] + Sinh[c +
 d*x]] + b*d^3*f^3*x^3*Log[1 - Cosh[c + d*x] + Sinh[c + d*x]] + d^2*e^2*(b*
d*e + 3*a*f)*(d*x - Log[1 + Cosh[c + d*x] + Sinh[c + d*x]]) - 3*d*e*f*(b*d*
e - 2*a*f)*PolyLog[2, Cosh[c + d*x] - Sinh[c + d*x]] + 3*d*e*f*(b*d*e + 2*a
*f)*PolyLog[2, -Cosh[c + d*x] + Sinh[c + d*x]] + 6*f^2*(-(b*d*e) + a*f)*(d*
x*PolyLog[2, Cosh[c + d*x] - Sinh[c + d*x]] + PolyLog[3, Cosh[c + d*x] - Si
nh[c + d*x]]) + 6*f^2*(b*d*e + a*f)*(d*x*PolyLog[2, -Cosh[c + d*x] + Sinh[c
 + d*x]] + PolyLog[3, -Cosh[c + d*x] + Sinh[c + d*x]]) - 3*b*f^3*(d^2*x^2*P
olyLog[2, Cosh[c + d*x] - Sinh[c + d*x]] + 2*(d*x*PolyLog[3, Cosh[c + d*x]
- Sinh[c + d*x]] + PolyLog[4, Cosh[c + d*x] - Sinh[c + d*x]])) + 3*b*f^3*(d
^2*x^2*PolyLog[2, -Cosh[c + d*x] + Sinh[c + d*x]] + 2*(d*x*PolyLog[3, -Cosh
[c + d*x] + Sinh[c + d*x]] + PolyLog[4, -Cosh[c + d*x] + Sinh[c + d*x]]))))/
(a^2*d^4) + (Sech[c/2]*Sech[c/2 + (d*x)/2]*(-(e^3*Sinh[(d*x)/2]) - 3*e^2*f*
x*Sinh[(d*x)/2] - 3*e*f^2*x^2*Sinh[(d*x)/2] - f^3*x^3*Sinh[(d*x)/2]))/(2*a*
d) + (Csch[c/2]*Csch[c/2 + (d*x)/2]*(e^3*Sinh[(d*x)/2] + 3*e^2*f*x*Sinh[(d*
x)/2] + 3*e*f^2*x^2*Sinh[(d*x)/2] + f^3*x^3*Sinh[(d*x)/2]))/(2*a*d)
```

Maple [F] time = 0.692, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3}\left(\operatorname{csch}\left(dx+c\right)\right)^{2}}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 4.12939, size = 13974, normalized size = 18.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*(a^3 + a*b^2)*d^3*e^3 - 6*(a^3 + a*b^2)*c*d^2*e^2*f + 6*(a^3 + a*b^2)*c
^2*d*e*f^2 - 2*(a^3 + a*b^2)*c^3*f^3 + 2*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^
3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*
d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*cosh(d*x +
 c)^2 + 4*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a
^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2
*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) + 2*((a^3 + a
*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2
*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 + (a^3 + a
*b^2)*c^3*f^3)*sinh(d*x + c)^2 + 3*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b
^3*d^2*e^2*f - (b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*cosh(d
*x + c)^2 - 2*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*cosh(d*
x + c)*sinh(d*x + c) - (b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f
)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*
x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b +
 1) - 3*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f - (b^3*d^2*f^3
*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*cosh(d*x + c)^2 - 2*(b^3*d^2*f^3*
x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3
*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt((a^
2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*d^3*e^3 - 3*b^3*
c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3 - (b^3*d^3*e^3 - 3*b^3*c*d^2*
e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*cosh(d*x + c)^2 - 2*(b^3*d^3*e^3 -
 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*cosh(d*x + c)*sinh(d*
x + c) - (b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3
)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d
*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*d^3*e^3 - 3*b^3*c*d^2*e^2
*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3 - (b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3
*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*cosh(d*x + c)^2 - 2*(b^3*d^3*e^3 - 3*b^3*c*
d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) -
(b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*sinh(d*
x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) -
 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2
+ 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3 -
 (b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e
^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3)*cosh(d*x + c)^2 - 2*(b^3*d^3*f^3*x^
3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2
```

```
*d*e*f^2 + b^3*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d^3*f^3*x^3 + 3*
b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f
^2 + b^3*c^3*f^3)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x +
 c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2) - b)/b) - (b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x
 + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3 - (b^3*d^3*f^3*x^3 +
 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*
e*f^2 + b^3*c^3*f^3)*cosh(d*x + c)^2 - 2*(b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2
*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*
f^3)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 +
 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3)*s
inh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x +
c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 6*
(b^3*f^3*cosh(d*x + c)^2 + 2*b^3*f^3*cosh(d*x + c)*sinh(d*x + c) + b^3*f^3*
sinh(d*x + c)^2 - b^3*f^3)*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c
) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2))/b) + 6*(b^3*f^3*cosh(d*x + c)^2 + 2*b^3*f^3*cosh(d*x + c)*sinh(d*x +
c) + b^3*f^3*sinh(d*x + c)^2 - b^3*f^3)*sqrt((a^2 + b^2)/b^2)*polylog(4, (a
*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2))/b) - 6*(b^3*d*f^3*x + b^3*d*e*f^2 - (b^3*d*f^3*x + b^3*d
*e*f^2)*cosh(d*x + c)^2 - 2*(b^3*d*f^3*x + b^3*d*e*f^2)*cosh(d*x + c)*sinh(
d*x + c) - (b^3*d*f^3*x + b^3*d*e*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^
2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*(b^3*d*f^3*x + b^3*d*e*f^2 - (b^3
*d*f^3*x + b^3*d*e*f^2)*cosh(d*x + c)^2 - 2*(b^3*d*f^3*x + b^3*d*e*f^2)*cos
h(d*x + c)*sinh(d*x + c) - (b^3*d*f^3*x + b^3*d*e*f^2)*sinh(d*x + c)^2)*sqr
t((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 3*((a^2*b + b^3)*d^
2*f^3*x^2 + (a^2*b + b^3)*d^2*e^2*f - 2*(a^3 + a*b^2)*d*e*f^2 - ((a^2*b + b
^3)*d^2*f^3*x^2 + (a^2*b + b^3)*d^2*e^2*f - 2*(a^3 + a*b^2)*d*e*f^2 + 2*((a
^2*b + b^3)*d^2*e*f^2 - (a^3 + a*b^2)*d*f^3)*x)*cosh(d*x + c)^2 - 2*((a^2*b
 + b^3)*d^2*f^3*x^2 + (a^2*b + b^3)*d^2*e^2*f - 2*(a^3 + a*b^2)*d*e*f^2 + 2
*((a^2*b + b^3)*d^2*e*f^2 - (a^3 + a*b^2)*d*f^3)*x)*cosh(d*x + c)*sinh(d*x
+ c) - ((a^2*b + b^3)*d^2*f^3*x^2 + (a^2*b + b^3)*d^2*e^2*f - 2*(a^3 + a*b^
2)*d*e*f^2 + 2*((a^2*b + b^3)*d^2*e*f^2 - (a^3 + a*b^2)*d*f^3)*x)*sinh(d*x
+ c)^2 + 2*((a^2*b + b^3)*d^2*e*f^2 - (a^3 + a*b^2)*d*f^3)*x)*dilog(cosh(d*
x + c) + sinh(d*x + c)) + 3*((a^2*b + b^3)*d^2*f^3*x^2 + (a^2*b + b^3)*d^2*
e^2*f + 2*(a^3 + a*b^2)*d*e*f^2 - ((a^2*b + b^3)*d^2*f^3*x^2 + (a^2*b + b^3
)*d^2*e^2*f + 2*(a^3 + a*b^2)*d*e*f^2 + 2*((a^2*b + b^3)*d^2*e*f^2 + (a^3 +
 a*b^2)*d*f^3)*x)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d^2*f^3*x^2 + (a^2*b +
 b^3)*d^2*e^2*f + 2*(a^3 + a*b^2)*d*e*f^2 + 2*((a^2*b + b^3)*d^2*e*f^2 + (a
^3 + a*b^2)*d*f^3)*x)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d^2*f^3*
x^2 + (a^2*b + b^3)*d^2*e^2*f + 2*(a^3 + a*b^2)*d*e*f^2 + 2*((a^2*b + b^3)*
d^2*e*f^2 + (a^3 + a*b^2)*d*f^3)*x)*sinh(d*x + c)^2 + 2*((a^2*b + b^3)*d^2*
e*f^2 + (a^3 + a*b^2)*d*f^3)*x)*dilog(-cosh(d*x + c) - sinh(d*x + c)) + ((a
^2*b + b^3)*d^3*f^3*x^3 + (a^2*b + b^3)*d^3*e^3 + 3*(a^3 + a*b^2)*d^2*e^2*f
 + 3*((a^2*b + b^3)*d^3*e*f^2 + (a^3 + a*b^2)*d^2*f^3)*x^2 - ((a^2*b + b^3)
*d^3*f^3*x^3 + (a^2*b + b^3)*d^3*e^3 + 3*(a^3 + a*b^2)*d^2*e^2*f + 3*((a^2*
b + b^3)*d^3*e*f^2 + (a^3 + a*b^2)*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*
f + 2*(a^3 + a*b^2)*d^2*e*f^2)*x)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d^3*f^
3*x^3 + (a^2*b + b^3)*d^3*e^3 + 3*(a^3 + a*b^2)*d^2*e^2*f + 3*((a^2*b + b^3
)*d^3*e*f^2 + (a^3 + a*b^2)*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f + 2*(
a^3 + a*b^2)*d^2*e*f^2)*x)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d^3
*f^3*x^3 + (a^2*b + b^3)*d^3*e^3 + 3*(a^3 + a*b^2)*d^2*e^2*f + 3*((a^2*b +
b^3)*d^3*e*f^2 + (a^3 + a*b^2)*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f +
2*(a^3 + a*b^2)*d^2*e*f^2)*x)*sinh(d*x + c)^2 + 3*((a^2*b + b^3)*d^3*e^2*f
+ 2*(a^3 + a*b^2)*d^2*e*f^2)*x)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - ((
a^2*b + b^3)*d^3*e^3 - 3*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d^2*e^2*f + 3*((a^
2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 - ((a^2*b + b^3)*c^3 + 3*(a^3 +
```

```
 a*b^2)*c^2)*f^3 - ((a^2*b + b^3)*d^3*e^3 - 3*(a^3 + a*b^2 + (a^2*b + b^3)*
c)*d^2*e^2*f + 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 - ((a^2*b
+ b^3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d
^3*e^3 - 3*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d^2*e^2*f + 3*((a^2*b + b^3)*c^2
 + 2*(a^3 + a*b^2)*c)*d*e*f^2 - ((a^2*b + b^3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f
^3)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d^3*e^3 - 3*(a^3 + a*b^2 +
 (a^2*b + b^3)*c)*d^2*e^2*f + 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e
*f^2 - ((a^2*b + b^3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3)*sinh(d*x + c)^2)*log(
cosh(d*x + c) + sinh(d*x + c) - 1) - ((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b
+ b^3)*c*d^2*e^2*f - 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 + ((
a^2*b + b^3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 -
(a^3 + a*b^2)*d^2*f^3)*x^2 - ((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3)*c
*d^2*e^2*f - 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 + ((a^2*b +
b^3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^3 + a
*b^2)*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^3 + a*b^2)*d^2*e*f^2
)*x)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3)*c*d^2
*e^2*f - 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 + ((a^2*b + b^3)
*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^3 + a*b^2
)*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^3 + a*b^2)*d^2*e*f^2)*x)
*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3)
*c*d^2*e^2*f - 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 + ((a^2*b
+ b^3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^3 +
 a*b^2)*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^3 + a*b^2)*d^2*e*f
^2)*x)*sinh(d*x + c)^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^3 + a*b^2)*d^2*e
*f^2)*x)*log(-cosh(d*x + c) - sinh(d*x + c) + 1) + 6*((a^2*b + b^3)*f^3*cos
h(d*x + c)^2 + 2*(a^2*b + b^3)*f^3*cosh(d*x + c)*sinh(d*x + c) + (a^2*b + b
^3)*f^3*sinh(d*x + c)^2 - (a^2*b + b^3)*f^3)*polylog(4, cosh(d*x + c) + sin
h(d*x + c)) - 6*((a^2*b + b^3)*f^3*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*f^3*co
sh(d*x + c)*sinh(d*x + c) + (a^2*b + b^3)*f^3*sinh(d*x + c)^2 - (a^2*b + b^
3)*f^3)*polylog(4, -cosh(d*x + c) - sinh(d*x + c)) + 6*((a^2*b + b^3)*d*f^3
*x + (a^2*b + b^3)*d*e*f^2 - (a^3 + a*b^2)*f^3 - ((a^2*b + b^3)*d*f^3*x + (
a^2*b + b^3)*d*e*f^2 - (a^3 + a*b^2)*f^3)*cosh(d*x + c)^2 - 2*((a^2*b + b^3
)*d*f^3*x + (a^2*b + b^3)*d*e*f^2 - (a^3 + a*b^2)*f^3)*cosh(d*x + c)*sinh(d
*x + c) - ((a^2*b + b^3)*d*f^3*x + (a^2*b + b^3)*d*e*f^2 - (a^3 + a*b^2)*f^
3)*sinh(d*x + c)^2)*polylog(3, cosh(d*x + c) + sinh(d*x + c)) - 6*((a^2*b +
 b^3)*d*f^3*x + (a^2*b + b^3)*d*e*f^2 + (a^3 + a*b^2)*f^3 - ((a^2*b + b^3)*
d*f^3*x + (a^2*b + b^3)*d*e*f^2 + (a^3 + a*b^2)*f^3)*cosh(d*x + c)^2 - 2*((
a^2*b + b^3)*d*f^3*x + (a^2*b + b^3)*d*e*f^2 + (a^3 + a*b^2)*f^3)*cosh(d*x
+ c)*sinh(d*x + c) - ((a^2*b + b^3)*d*f^3*x + (a^2*b + b^3)*d*e*f^2 + (a^3
+ a*b^2)*f^3)*sinh(d*x + c)^2)*polylog(3, -cosh(d*x + c) - sinh(d*x + c)))/
((a^4 + a^2*b^2)*d^4*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*d^4*cosh(d*x + c)*
sinh(d*x + c) + (a^4 + a^2*b^2)*d^4*sinh(d*x + c)^2 - (a^4 + a^2*b^2)*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.244 $\int \frac{(e+fx)^2\mathbf{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=535

$$\frac{2b^2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2\sqrt{a^2+b^2}}-\frac{2b^2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2\sqrt{a^2+b^2}}-\frac{2b^2f^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3\sqrt{a^2+b^2}}+\frac{2b^2f^2\text{P}}{}$$

```
[Out] -((e + f*x)^2/(a*d)) + (2*b*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d) - ((e
 + f*x)^2*Coth[c + d*x])/(a*d) + (b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2 + b^2]*d) - (b^2*(e + f*x)^2*Log[1 + (
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2 + b^2]*d) + (2*f*(e +
f*x)*Log[1 - E^(2*(c + d*x))])/(a*d^2) + (2*b*f*(e + f*x)*PolyLog[2, -E^(c
+ d*x)])/(a^2*d^2) - (2*b*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^2*d^2) +
(2*b^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a
^2*Sqrt[a^2 + b^2]*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^2) + (f^2*PolyLog[2, E^(2*(c
 + d*x))])/(a*d^3) - (2*b*f^2*PolyLog[3, -E^(c + d*x)])/(a^2*d^3) + (2*b*f^
2*PolyLog[3, E^(c + d*x)])/(a^2*d^3) - (2*b^2*f^2*PolyLog[3, -((b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^3) + (2*b^2*f^2*PolyLog
[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^3)
```

Rubi [A] time = 1.07963, antiderivative size = 535, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5575, 4184, 3716, 2190, 2279, 2391, 4182, 2531, 2282, 6589, 3322, 2264}

$$\frac{2b^2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2\sqrt{a^2+b^2}}-\frac{2b^2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2\sqrt{a^2+b^2}}-\frac{2b^2f^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3\sqrt{a^2+b^2}}+\frac{2b^2f^2\text{P}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -((e + f*x)^2/(a*d)) + (2*b*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d) - ((e
 + f*x)^2*Coth[c + d*x])/(a*d) + (b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2 + b^2]*d) - (b^2*(e + f*x)^2*Log[1 + (
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*Sqrt[a^2 + b^2]*d) + (2*f*(e +
f*x)*Log[1 - E^(2*(c + d*x))])/(a*d^2) + (2*b*f*(e + f*x)*PolyLog[2, -E^(c
+ d*x)])/(a^2*d^2) - (2*b*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^2*d^2) +
(2*b^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a
^2*Sqrt[a^2 + b^2]*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^2) + (f^2*PolyLog[2, E^(2*(c
 + d*x))])/(a*d^3) - (2*b*f^2*PolyLog[3, -E^(c + d*x)])/(a^2*d^3) + (2*b*f^
2*PolyLog[3, E^(c + d*x)])/(a^2*d^3) - (2*b^2*f^2*PolyLog[3, -((b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^3) + (2*b^2*f^2*PolyLog
[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^2 + b^2]*d^3)
```


Rule 5575

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a +
b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{csch}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^2\coth(c+dx)}{ad} - \frac{b\int (e+fx)^2\text{csch}(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}\,dx}{a^2} + \frac{(2f)\int}{}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{\left(2b^2\right)\int \frac{e^{c+dx}(e+f}{-b+2ae^{c+dx}+b}}{a^2}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{2f(e+fx)\log\left(1-\right.}{ad^2}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{b^2(e+fx)^2\log\left(1+\right.}{a^2\sqrt{a^2+b}}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{b^2(e+fx)^2\log\left(1+\right.}{a^2\sqrt{a^2+b}}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{b^2(e+fx)^2\log\left(1+\right.}{a^2\sqrt{a^2+b}}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{b^2(e+fx)^2\log\left(1+\right.}{a^2\sqrt{a^2+b}}$$

Mathematica [A] time = 16.1467, size = 795, normalized size = 1.49

$$\frac{\left(-2e^2\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)d^2 + f^2x^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^2 + 2efx\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^2 - f^2x^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)d^2 - 2efx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(((2*a*d^2*(e + f*x)^2)/(-1 + E^(2*c)) + 2*d*f*(b*d*e - a*f)*x*Log[1 - E^(
-c - d*x)] + b*d^2*f^2*x^2*Log[1 - E^(-c - d*x)] - 2*d*f*(b*d*e + a*f)*x*Lo
g[1 + E^(-c - d*x)] - b*d^2*f^2*x^2*Log[1 + E^(-c - d*x)] - d*e*(b*d*e - 2*
a*f)*(d*x - Log[1 - E^(c + d*x)]) + d*e*(b*d*e + 2*a*f)*(d*x - Log[1 + E^(c
 + d*x)]) + 2*f*(b*d*e + a*f)*PolyLog[2, -E^(-c - d*x)] + 2*f*(-(b*d*e) + a
*f)*PolyLog[2, E^(-c - d*x)] + 2*b*f^2*(d*x*PolyLog[2, -E^(-c - d*x)] + Pol
yLog[3, -E^(-c - d*x)]) - 2*b*f^2*(d*x*PolyLog[2, E^(-c - d*x)] + PolyLog[3
, E^(-c - d*x)]))/(a^2*d^3)) + (b^2*(-2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))
/Sqrt[a^2 + b^2]] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*d^2*e*
f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c +
 d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))] - 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^
2 + b^2])] + 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(
a^2*Sqrt[a^2 + b^2]*d^3) + (Sech[c/2]*Sech[c/2 + (d*x)/2]*(-(e^2*Sinh[(d*x)
/2]) - 2*e*f*x*Sinh[(d*x)/2] - f^2*x^2*Sinh[(d*x)/2]))/(2*a*d) + (Csch[c/2]
*Csch[c/2 + (d*x)/2]*(e^2*Sinh[(d*x)/2] + 2*e*f*x*Sinh[(d*x)/2] + f^2*x^2*S
inh[(d*x)/2]))/(2*a*d)
```

Maple [F] time = 0.743, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \left(\mathrm{csch}\left(dx+c\right)\right)^{2}}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.49746, size = 8581, normalized size = 16.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*(a^3 + a*b^2)*d^2*e^2 - 4*(a^3 + a*b^2)*c*d*e*f + 2*(a^3 + a*b^2)*c^2*f
^2 + 2*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*
```

```
b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*cosh(d*x + c)^2 + 4*((a^3 + a*b^2)*d^
2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*
b^2)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + 2*((a^3 + a*b^2)*d^2*f^2*x^2 +
2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2
)*sinh(d*x + c)^2 + 2*(b^3*d*f^2*x + b^3*d*e*f - (b^3*d*f^2*x + b^3*d*e*f)*
cosh(d*x + c)^2 - 2*(b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)*sinh(d*x + c) -
 (b^3*d*f^2*x + b^3*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*
cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2) - b)/b + 1) - 2*(b^3*d*f^2*x + b^3*d*e*f - (b^3*d*f^2*x +
b^3*d*e*f)*cosh(d*x + c)^2 - 2*(b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)*sinh
(d*x + c) - (b^3*d*f^2*x + b^3*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2
)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^
3*c^2*f^2 - (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)^2 - 2
*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) -
(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^
2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^
2) + 2*a) + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2 - (b^3*d^2*e^2 - 2*b
^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)^2 - 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f
+ b^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d^2*e^2 - 2*b^3*c*d*e*f +
 b^3*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c)
+ 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*d^2*f^2*x^2 +
 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2 - (b^3*d^2*f^2*x^2 + 2*b^3*d
^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x + c)^2 - 2*(b^3*d^2*f^2*x^
2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x +
 c) - (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*sin
h(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
 + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^3
*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2 - (b^3*d^2*f^2
*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x + c)^2 - 2*(
b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x +
 c)*sinh(d*x + c) - (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^
3*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a
*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b) + 2*(b^3*f^2*cosh(d*x + c)^2 + 2*b^3*f^2*cosh(d*x + c)*sinh(d*x + c
) + b^3*f^2*sinh(d*x + c)^2 - b^3*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*
cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2))/b) - 2*(b^3*f^2*cosh(d*x + c)^2 + 2*b^3*f^2*cosh(d*x + c)
*sinh(d*x + c) + b^3*f^2*sinh(d*x + c)^2 - b^3*f^2)*sqrt((a^2 + b^2)/b^2)*p
olylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3
)*d*e*f - (a^3 + a*b^2)*f^2 - ((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*e*f
- (a^3 + a*b^2)*f^2)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d*f^2*x + (a^2*b +
b^3)*d*e*f - (a^3 + a*b^2)*f^2)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3
)*d*f^2*x + (a^2*b + b^3)*d*e*f - (a^3 + a*b^2)*f^2)*sinh(d*x + c)^2)*dilog
(cosh(d*x + c) + sinh(d*x + c)) + 2*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*
d*e*f + (a^3 + a*b^2)*f^2 - ((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*e*f +
(a^3 + a*b^2)*f^2)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^
3)*d*e*f + (a^3 + a*b^2)*f^2)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*
d*f^2*x + (a^2*b + b^3)*d*e*f + (a^3 + a*b^2)*f^2)*sinh(d*x + c)^2)*dilog(-
cosh(d*x + c) - sinh(d*x + c)) + ((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3)
*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f - ((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^
3)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + (a^3 + a*b^
2)*d*f^2)*x)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3)
*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + (a^3 + a*b^2)
*d*f^2)*x)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*
b + b^3)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + (a^3
+ a*b^2)*d*f^2)*x)*sinh(d*x + c)^2 + 2*((a^2*b + b^3)*d^2*e*f + (a^3 + a*b^
2)*d*f^2)*x)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - ((a^2*b + b^3)*d^2*e^
```

```
2 - 2*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d*e*f + ((a^2*b + b^3)*c^2 + 2*(a^3 +
 a*b^2)*c)*f^2 - ((a^2*b + b^3)*d^2*e^2 - 2*(a^3 + a*b^2 + (a^2*b + b^3)*c)
*d*e*f + ((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*f^2)*cosh(d*x + c)^2 - 2*(
(a^2*b + b^3)*d^2*e^2 - 2*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d*e*f + ((a^2*b +
 b^3)*c^2 + 2*(a^3 + a*b^2)*c)*f^2)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b +
 b^3)*d^2*e^2 - 2*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d*e*f + ((a^2*b + b^3)*c^
2 + 2*(a^3 + a*b^2)*c)*f^2)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x +
 c) - 1) - ((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*c*d*e*f - ((a^2*b +
 b^3)*c^2 + 2*(a^3 + a*b^2)*c)*f^2 - ((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b
+ b^3)*c*d*e*f - ((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*f^2 + 2*((a^2*b +
b^3)*d^2*e*f - (a^3 + a*b^2)*d*f^2)*x)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d
^2*f^2*x^2 + 2*(a^2*b + b^3)*c*d*e*f - ((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)
*c)*f^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^3 + a*b^2)*d*f^2)*x)*cosh(d*x + c)*
sinh(d*x + c) - ((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*c*d*e*f - ((a^
2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*f^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^3 +
 a*b^2)*d*f^2)*x)*sinh(d*x + c)^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^3 + a*b^2
)*d*f^2)*x)*log(-cosh(d*x + c) - sinh(d*x + c) + 1) - 2*((a^2*b + b^3)*f^2*
cosh(d*x + c)^2 + 2*(a^2*b + b^3)*f^2*cosh(d*x + c)*sinh(d*x + c) + (a^2*b
+ b^3)*f^2*sinh(d*x + c)^2 - (a^2*b + b^3)*f^2)*polylog(3, cosh(d*x + c) +
sinh(d*x + c)) + 2*((a^2*b + b^3)*f^2*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*f^2
*cosh(d*x + c)*sinh(d*x + c) + (a^2*b + b^3)*f^2*sinh(d*x + c)^2 - (a^2*b +
 b^3)*f^2)*polylog(3, -cosh(d*x + c) - sinh(d*x + c)))/((a^4 + a^2*b^2)*d^3
*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*d^3*cosh(d*x + c)*sinh(d*x + c) + (a^4
 + a^2*b^2)*d^3*sinh(d*x + c)^2 - (a^4 + a^2*b^2)*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.245 $\int \frac{(e+fx)\mathrm{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=306

$$\frac{b^2 f \mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2\sqrt{a^2+b^2}}-\frac{b^2 f \mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2\sqrt{a^2+b^2}}+\frac{bf\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^2}-\frac{bf\mathrm{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^2}+\frac{b^2(e+f}{}$$

```
[Out] (2*b*(e + f*x)*ArcTanh[E^(c + d*x)])/(a^2*d) - ((e + f*x)*Coth[c + d*x])/(a
*d) + (b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*S
qrt[a^2 + b^2]*d) - (b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2])])/(a^2*Sqrt[a^2 + b^2]*d) + (f*Log[Sinh[c + d*x]])/(a*d^2) + (b*f*Pol
yLog[2, -E^(c + d*x)])/(a^2*d^2) - (b*f*PolyLog[2, E^(c + d*x)])/(a^2*d^2)
+ (b^2*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^
2 + b^2]*d^2) - (b^2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/(a^2*Sqrt[a^2 + b^2]*d^2)
```

Rubi [A] time = 0.565233, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5575, 4184, 3475, 4182, 2279, 2391, 3322, 2264, 2190}

$$\frac{b^2 f \mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2\sqrt{a^2+b^2}}-\frac{b^2 f \mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2\sqrt{a^2+b^2}}+\frac{bf\mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^2}-\frac{bf\mathrm{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^2}+\frac{b^2(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (2*b*(e + f*x)*ArcTanh[E^(c + d*x)])/(a^2*d) - ((e + f*x)*Coth[c + d*x])/(a
*d) + (b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*S
qrt[a^2 + b^2]*d) - (b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2])])/(a^2*Sqrt[a^2 + b^2]*d) + (f*Log[Sinh[c + d*x]])/(a*d^2) + (b*f*Pol
yLog[2, -E^(c + d*x)])/(a^2*d^2) - (b*f*PolyLog[2, E^(c + d*x)])/(a^2*d^2)
+ (b^2*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*Sqrt[a^
2 + b^2]*d^2) - (b^2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/(a^2*Sqrt[a^2 + b^2]*d^2)
```


Rule 5575

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a +
b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}dx = \frac{\int (e+fx)\text{csch}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$= -\frac{(e+fx)\coth(c+dx)}{ad} - \frac{b\int (e+fx)\text{csch}(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{e+fx}{a+b\sinh(c+dx)}dx}{a^2} + \frac{f\int \coth(\ldots}{a\ldots}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{f\log(\sinh(c+dx))}{ad^2} + \frac{\left(2b^2\right)\int \frac{e^{\ldots}}{-b+2a\ldots}}{\ldots}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{f\log(\sinh(c+dx))}{ad^2} + \frac{\left(2b^3\right)\int \frac{e^{\ldots}}{2a-2\sqrt{\ldots}}}{a^2\sqrt{\ldots}}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} - \frac{b^2(e\ldots}{\ldots}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} - \frac{b^2(e\ldots}{\ldots}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} - \frac{b^2(e\ldots}{\ldots}$$

Mathematica [A] time = 5.601, size = 405, normalized size = 1.32

$$\frac{\frac{2b^2\left(f\text{PolyLog}\left(2,\frac{b(\sinh(c+dx)+\cosh(c+dx))}{\sqrt{a^2+b^2}-a}\right)-f\text{PolyLog}\left(2,-\frac{b(\sinh(c+dx)+\cosh(c+dx))}{\sqrt{a^2+b^2}+a}\right)-2de\tanh^{-1}\left(\frac{a+b\sinh(c+dx)+b\cosh(c+dx)}{\sqrt{a^2+b^2}}\right)+f(c+dx)\log\left(\frac{b(\sinh(c+dx)+\cosh(c\ldots}{a-\sqrt{a^2+b^2}}\right.\ldots}{\sqrt{a^2+b^2}}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-(a*d*(e + f*x)*Coth[(c + d*x)/2]) + 2*a*f*Log[Sinh[c + d*x]] - 2*b*d*e*Lo
g[Tanh[(c + d*x)/2]] + 2*b*c*f*Log[Tanh[(c + d*x)/2]] + 2*b*f*(-((c + d*x)*
(Log[1 - E^(-c - d*x)] - Log[1 + E^(-c - d*x)])) - PolyLog[2, -E^(-c - d*x)
] + PolyLog[2, E^(-c - d*x)]) + (2*b^2*(-2*d*e*ArcTanh[(a + b*Cosh[c + d*x]
 + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a + b*Cosh[c + d*x] +
 b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*(Cosh[c + d*x]
+ Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*(Cosh[c +
 d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*(Cosh[c +
d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*(Cosh[c
+ d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2] - a*d*(e
 + f*x)*Tanh[(c + d*x)/2])/(2*a^2*d^2)
```

Maple [B] time = 0.105, size = 626, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] -2/d*(f*x+e)/a/(exp(2*d*x+2*c)-1)-2/d^2/a*f*ln(exp(d*x+c))+1/d^2/a*f*ln(exp
(d*x+c)-1)+1/d^2/a*f*ln(exp(d*x+c)+1)-1/a^2/d*b*e*ln(exp(d*x+c)-1)-2/a^2/d*
```

```
b^2*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/a
^2/d*b*e*ln(exp(d*x+c)+1)+1/a^2/d^2*b*f*c*ln(exp(d*x+c)-1)+2/a^2/d^2*b^2*f*
c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/a^2/d
^2*b*f*dilog(exp(d*x+c))+1/a^2/d*b^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a
^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/a^2/d^2*b^2*f/(a^2+b^2)^(1/2)*ln
((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/a^2/d*b^2*f/(a
^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/
a^2/d^2*b^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b
^2)^(1/2)))*c+1/a^2/d^2*b^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2
)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/a^2/d^2*b^2*f/(a^2+b^2)^(1/2)*dilog((b*e
xp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/a^2/d*b*f*ln(exp(d*x+c)
+1)*x+1/a^2/d^2*b*f*dilog(exp(d*x+c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.89152, size = 4370, normalized size = 14.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*(a^3 + a*b^2)*d*e - 2*(a^3 + a*b^2)*c*f + 2*((a^3 + a*b^2)*d*f*x + (a^3
 + a*b^2)*c*f)*cosh(d*x + c)^2 + 4*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f
)*cosh(d*x + c)*sinh(d*x + c) + 2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)
*sinh(d*x + c)^2 - (b^3*f*cosh(d*x + c)^2 + 2*b^3*f*cosh(d*x + c)*sinh(d*x
+ c) + b^3*f*sinh(d*x + c)^2 - b^3*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d
*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2) - b)/b + 1) + (b^3*f*cosh(d*x + c)^2 + 2*b^3*f*cosh(d*x + c)*sin
h(d*x + c) + b^3*f*sinh(d*x + c)^2 - b^3*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*
cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2) - b)/b + 1) - (b^3*d*e - b^3*c*f - (b^3*d*e - b^3*c*f)*cos
h(d*x + c)^2 - 2*(b^3*d*e - b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*e
 - b^3*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) +
2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*d*e - b^3*c*f -
 (b^3*d*e - b^3*c*f)*cosh(d*x + c)^2 - 2*(b^3*d*e - b^3*c*f)*cosh(d*x + c)*
sinh(d*x + c) - (b^3*d*e - b^3*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*
log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a
) + (b^3*d*f*x + b^3*c*f - (b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^2 - 2*(b^3*d
*f*x + b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*f*x + b^3*c*f)*sinh(d*
x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^3*d*f
*x + b^3*c*f - (b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^2 - 2*(b^3*d*f*x + b^3*c
*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*f*x + b^3*c*f)*sinh(d*x + c)^2)*sq
rt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
 c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + ((a^2*b + b^3)*f*cos
```

```
h(d*x + c)^2 + 2*(a^2*b + b^3)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2*b + b^3
)*f*sinh(d*x + c)^2 - (a^2*b + b^3)*f)*dilog(cosh(d*x + c) + sinh(d*x + c))
 - ((a^2*b + b^3)*f*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*f*cosh(d*x + c)*sinh(
d*x + c) + (a^2*b + b^3)*f*sinh(d*x + c)^2 - (a^2*b + b^3)*f)*dilog(-cosh(d
*x + c) - sinh(d*x + c)) + ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e - ((a^2
*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^3 + a*b^2)*f)*cosh(d*x + c)^2 - 2*
((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^3 + a*b^2)*f)*cosh(d*x + c)*s
inh(d*x + c) - ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^3 + a*b^2)*f)*
sinh(d*x + c)^2 + (a^3 + a*b^2)*f)*log(cosh(d*x + c) + sinh(d*x + c) + 1) -
 ((a^2*b + b^3)*d*e - ((a^2*b + b^3)*d*e - (a^3 + a*b^2 + (a^2*b + b^3)*c)*
f)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d*e - (a^3 + a*b^2 + (a^2*b + b^3)*c)
*f)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d*e - (a^3 + a*b^2 + (a^2*
b + b^3)*c)*f)*sinh(d*x + c)^2 - (a^3 + a*b^2 + (a^2*b + b^3)*c)*f)*log(cos
h(d*x + c) + sinh(d*x + c) - 1) - ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*c*f
- ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*c*f)*cosh(d*x + c)^2 - 2*((a^2*b + b
^3)*d*f*x + (a^2*b + b^3)*c*f)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)
*d*f*x + (a^2*b + b^3)*c*f)*sinh(d*x + c)^2)*log(-cosh(d*x + c) - sinh(d*x
+ c) + 1))/((a^4 + a^2*b^2)*d^2*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*d^2*cos
h(d*x + c)*sinh(d*x + c) + (a^4 + a^2*b^2)*d^2*sinh(d*x + c)^2 - (a^4 + a^2
*b^2)*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")

[Out] Timed out
```

3.246 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d\sqrt{a^2+b^2}}+\frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}-\frac{\coth(c+dx)}{ad}$$

```
[Out] (b*ArcTanh[Cosh[c + d*x]])/(a^2*d) - (2*b^2*ArcTanh[(b - a*Tanh[(c + d*x)/2
])/Sqrt[a^2 + b^2]])/(a^2*Sqrt[a^2 + b^2]*d) - Coth[c + d*x]/(a*d)
```

Rubi [A] time = 0.147012, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2802, 12, 2747, 3770, 2660, 618, 204}

$$-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d\sqrt{a^2+b^2}}+\frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}-\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Cosh[c + d*x]])/(a^2*d) - (2*b^2*ArcTanh[(b - a*Tanh[(c + d*x)/2
])/Sqrt[a^2 + b^2]])/(a^2*Sqrt[a^2 + b^2]*d) - Coth[c + d*x]/(a*d)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{\coth(c+dx)}{ad} - \frac{\int \frac{b\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a} \\
&= -\frac{\coth(c+dx)}{ad} - \frac{b\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a} \\
&= -\frac{\coth(c+dx)}{ad} - \frac{b\int \operatorname{csch}(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a^2} \\
&= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{\coth(c+dx)}{ad} - \frac{\left(2ib^2\right)\operatorname{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{a^2d} \\
&= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{\coth(c+dx)}{ad} + \frac{\left(4ib^2\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^2+b^2\right)-x^2}\,dx, x, -2ib+2a\tan\left(\right.\right.}{a^2d} \\
&= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}\,d} - \frac{\coth(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.734301, size = 100, normalized size = 1.25

$$-\frac{2b\left(\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - \frac{2b\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right) + a\tanh\left(\frac{1}{2}(c+dx)\right) + a\coth\left(\frac{1}{2}(c+dx)\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(a*Coth[(c + d*x)/2] + 2*b*((-2*b*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a
^2 - b^2]])/Sqrt[-a^2 - b^2] + Log[Tanh[(c + d*x)/2]]) + a*Tanh[(c + d*x)/2
])/(2*a^2*d)
```

Maple [A] time = 0.003, size = 105, normalized size = 1.3

$$-\frac{1}{2\,da}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{b^2}{da^2\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\,\frac{2\,a\tanh(1/2\,dx+c/2)-2\,b}{\sqrt{a^2+b^2}}\right)-\frac{1}{2\,da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-\frac{b}{da^2}\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/2/d/a*tanh(1/2*d*x+1/2*c)+2/d/a^2*b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*t
anh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-1/2/d/a/tanh(1/2*d*x+1/2*c)-1/d/a^
2*b*ln(tanh(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.86922, size = 1200, normalized size = 15.

$$-\frac{2\,a^3+2\,ab^2-\left(b^2\cosh(dx+c)^2+2\,b^2\cosh(dx+c)\sinh(dx+c)+b^2\sinh(dx+c)^2-b^2\right)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(dx+c}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*a^3 + 2*a*b^2 - (b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c
) + b^2*sinh(d*x + c)^2 - b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b
^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x +
c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x +
 c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b
*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + (a^2*b + b^3 - (a^2*b + b^3)*cosh
(d*x + c)^2 - 2*(a^2*b + b^3)*cosh(d*x + c)*sinh(d*x + c) - (a^2*b + b^3)*s
inh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a^2*b + b^3 - (a^
2*b + b^3)*cosh(d*x + c)^2 - 2*(a^2*b + b^3)*cosh(d*x + c)*sinh(d*x + c) -
(a^2*b + b^3)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) - 1))/((a^
4 + a^2*b^2)*d*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x
 + c) + (a^4 + a^2*b^2)*d*sinh(d*x + c)^2 - (a^4 + a^2*b^2)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [A] time = 1.62952, size = 177, normalized size = 2.21

$$\frac{b^2 \log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^2d} + \frac{b\log\left(e^{(dx+c)}+1\right)}{a^2d} - \frac{b\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^2d} - \frac{2}{ad\left(e^{(2\,dx+2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c)
+ 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2*d) + b*log(e^(d*x + c) + 1
)/(a^2*d) - b*log(abs(e^(d*x + c) - 1))/(a^2*d) - 2/(a*d*(e^(2*d*x + 2*c) -
 1))
```

3.247 $\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Csch[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0771399, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Csch[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A] time = 117.797, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Csch[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A] time = 0.879, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx+c))^2}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$4b^2\int -\frac{e^{(dx+c)}}{2\left(a^2bfx + a^2be - \left(a^2bfxe^{(2c)} + a^2bee^{(2c)}\right)e^{(2dx)} - 2\left(a^3fxe^c + a^3ee^c\right)e^{(dx)}\right)}\,dx + \frac{2}{adfx + ade - \left(adfxe^{(2c)} + adee^{(2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 4*b^2*integrate(-1/2*e^(d*x + c)/(a^2*b*f*x + a^2*b*e - (a^2*b*f*x*e^(2*c)
+ a^2*b*e*e^(2*c))*e^(2*d*x) - 2*(a^3*f*x*e^c + a^3*e*e^c)*e^(d*x)), x) + 2
/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e*e^(2*c))*e^(2*d*x)) - 4*integr
ate(-1/4*(b*d*f*x + b*d*e + a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2
 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) - 4
*integrate(1/4*(b*d*f*x + b*d*e - a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2
*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)),
x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{csch}\left(dx+c\right)^2}{afx + ae + \left(bfx + be\right)\sinh\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(csch(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2\left(c+dx\right)}{\left(a + b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(csch(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.248 $\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1053

result too large to display

```
[Out] (b*(e + f*x)^3)/(a^2*d) - (6*f^2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d^3) +
((e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (2*b^2*(e + f*x)^3*ArcTanh[E^(c
+ d*x)])/(a^3*d) + (b*(e + f*x)^3*Coth[c + d*x])/(a^2*d) - (3*f*(e + f*x)^2
*Csch[c + d*x])/(2*a*d^2) - ((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x])/(2*a*
d) - (b^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*
Sqrt[a^2 + b^2]*d) + (b^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2])])/(a^3*Sqrt[a^2 + b^2]*d) - (3*b*f*(e + f*x)^2*Log[1 - E^(2*(c + d
*x))])/(a^2*d^2) - (3*f^3*PolyLog[2, -E^(c + d*x)])/(a*d^4) + (3*f*(e + f*x
)^2*PolyLog[2, -E^(c + d*x)])/(2*a*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -
E^(c + d*x)])/(a^3*d^2) + (3*f^3*PolyLog[2, E^(c + d*x)])/(a*d^4) - (3*f*(e
 + f*x)^2*PolyLog[2, E^(c + d*x)])/(2*a*d^2) + (3*b^2*f*(e + f*x)^2*PolyLog
[2, E^(c + d*x)])/(a^3*d^2) - (3*b^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b^2]*d^2) + (3*b^3*f*(e + f*x
)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b
^2]*d^2) - (3*b*f^2*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a^2*d^3) - (3*f
^2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) + (6*b^2*f^2*(e + f*x)*PolyL
og[3, -E^(c + d*x)])/(a^3*d^3) + (3*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/
(a*d^3) - (6*b^2*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a^3*d^3) + (6*b^3*
f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*Sq
rt[a^2 + b^2]*d^3) - (6*b^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b^2]*d^3) + (3*b*f^3*PolyLog[3, E^(2*(
c + d*x))])/(2*a^2*d^4) + (3*f^3*PolyLog[4, -E^(c + d*x)])/(a*d^4) - (6*b^2
*f^3*PolyLog[4, -E^(c + d*x)])/(a^3*d^4) - (3*f^3*PolyLog[4, E^(c + d*x)])/
(a*d^4) + (6*b^2*f^3*PolyLog[4, E^(c + d*x)])/(a^3*d^4) - (6*b^3*f^3*PolyLo
g[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b^2]*d^4) +
 (6*b^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*Sqrt
[a^2 + b^2]*d^4)
```

Rubi [A] time = 1.75162, antiderivative size = 1053, normalized size of antiderivative = 1., number of steps used = 45, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5575, 4186, 4182, 2279, 2391, 2531, 6609, 2282, 6589, 4184, 3716, 2190, 3322, 2264}

$$-\frac{(e+fx)^3\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)b^3}{a^3\sqrt{a^2+b^2}d}+\frac{(e+fx)^3\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)b^3}{a^3\sqrt{a^2+b^2}d}-\frac{3f(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)b^3}{a^3\sqrt{a^2+b^2}d^2}+\frac{3f(e+}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (b*(e + f*x)^3)/(a^2*d) - (6*f^2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d^3) +
((e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (2*b^2*(e + f*x)^3*ArcTanh[E^(c
+ d*x)])/(a^3*d) + (b*(e + f*x)^3*Coth[c + d*x])/(a^2*d) - (3*f*(e + f*x)^2
*Csch[c + d*x])/(2*a*d^2) - ((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x])/(2*a*
d) - (b^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*
Sqrt[a^2 + b^2]*d) + (b^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2])])/(a^3*Sqrt[a^2 + b^2]*d) - (3*b*f*(e + f*x)^2*Log[1 - E^(2*(c + d
*x))])/(a^2*d^2) - (3*f^3*PolyLog[2, -E^(c + d*x)])/(a*d^4) + (3*f*(e + f*x
)^2*PolyLog[2, -E^(c + d*x)])/(2*a*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -
```

E^(c + d*x)])/(a^3*d^2) + (3*f^3*PolyLog[2, E^(c + d*x)])/(a*d^4) - (3*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)])/(2*a*d^2) + (3*b^2*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)])/(a^3*d^2) - (3*b^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b^2]*d^2) + (3*b^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b^2]*d^2) - (3*b*f^2*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a^2*d^3) - (3*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) + (6*b^2*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a^3*d^3) + (3*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a*d^3) - (6*b^2*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a^3*d^3) + (6*b^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b^2]*d^3) - (6*b^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b^2]*d^3) + (3*b*f^3*PolyLog[3, E^(2*(c + d*x))])/(2*a^2*d^4) + (3*f^3*PolyLog[4, -E^(c + d*x)])/(a*d^4) - (6*b^2*f^3*PolyLog[4, -E^(c + d*x)])/(a^3*d^4) - (3*f^3*PolyLog[4, E^(c + d*x)])/(a*d^4) + (6*b^2*f^3*PolyLog[4, E^(c + d*x)])/(a^3*d^4) - (6*b^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b^2]*d^4) + (6*b^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b^2]*d^4)

Rule 5575

Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

Rule 4186

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

Rule 4182

Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 2531

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)

```
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
```

```
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\operatorname{csch}^3(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{3f(e+fx)^2\operatorname{csch}(c+dx)}{2ad^2} - \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{\int (e+fx)^3\operatorname{csch}(c+\cdots}{2a}$$

$$= -\frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{b(e+fx)^3\coth(c+dx)}{a^2d} - 3f\cdots$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^3\tanh\cdots}{a^3d}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^3\tanh\cdots}{a^3d}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^3\tanh\cdots}{a^3d}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^3\tanh\cdots}{a^3d}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^3\tanh\cdots}{a^3d}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^3\tanh\cdots}{a^3d}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^3\tanh\cdots}{a^3d}$$

Mathematica [B] time = 45.3315, size = 2800, normalized size = 2.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(-12*a*b*d^3*e^2*E^(2*c)*f*x - 12*a*b*d^3*e*E^(2*c)*f^2*x^2 - 4*a*b*d^3*E^
(2*c)*f^3*x^3 + 2*a^2*d^3*e^3*ArcTanh[E^(c + d*x)] - 4*b^2*d^3*e^3*ArcTanh[
E^(c + d*x)] - 2*a^2*d^3*e^3*E^(2*c)*ArcTanh[E^(c + d*x)] + 4*b^2*d^3*e^3*E
^(2*c)*ArcTanh[E^(c + d*x)] - 12*a^2*d*e*f^2*ArcTanh[E^(c + d*x)] + 12*a^2*
d*e*E^(2*c)*f^2*ArcTanh[E^(c + d*x)] - 3*a^2*d^3*e^2*f*x*Log[1 - E^(c + d*x
)] + 6*b^2*d^3*e^2*f*x*Log[1 - E^(c + d*x)] + 3*a^2*d^3*e^2*E^(2*c)*f*x*Log
[1 - E^(c + d*x)] - 6*b^2*d^3*e^2*E^(2*c)*f*x*Log[1 - E^(c + d*x)] + 6*a^2*
d*f^3*x*Log[1 - E^(c + d*x)] - 6*a^2*d*E^(2*c)*f^3*x*Log[1 - E^(c + d*x)] -
 3*a^2*d^3*e*f^2*x^2*Log[1 - E^(c + d*x)] + 6*b^2*d^3*e*f^2*x^2*Log[1 - E^(
c + d*x)] + 3*a^2*d^3*e*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] - 6*b^2*d^3*e*
```

```
E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] - a^2*d^3*f^3*x^3*Log[1 - E^(c + d*x)]
 + 2*b^2*d^3*f^3*x^3*Log[1 - E^(c + d*x)] + a^2*d^3*E^(2*c)*f^3*x^3*Log[1 -
 E^(c + d*x)] - 2*b^2*d^3*E^(2*c)*f^3*x^3*Log[1 - E^(c + d*x)] + 3*a^2*d^3*
e^2*f*x*Log[1 + E^(c + d*x)] - 6*b^2*d^3*e^2*f*x*Log[1 + E^(c + d*x)] - 3*a
^2*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(c + d*x)] + 6*b^2*d^3*e^2*E^(2*c)*f*x*Log
[1 + E^(c + d*x)] - 6*a^2*d*f^3*x*Log[1 + E^(c + d*x)] + 6*a^2*d*E^(2*c)*f^
3*x*Log[1 + E^(c + d*x)] + 3*a^2*d^3*e*f^2*x^2*Log[1 + E^(c + d*x)] - 6*b^2
*d^3*e*f^2*x^2*Log[1 + E^(c + d*x)] - 3*a^2*d^3*e*E^(2*c)*f^2*x^2*Log[1 + E
^(c + d*x)] + 6*b^2*d^3*e*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] + a^2*d^3*f^
3*x^3*Log[1 + E^(c + d*x)] - 2*b^2*d^3*f^3*x^3*Log[1 + E^(c + d*x)] - a^2*d
^3*E^(2*c)*f^3*x^3*Log[1 + E^(c + d*x)] + 2*b^2*d^3*E^(2*c)*f^3*x^3*Log[1 +
 E^(c + d*x)] - 6*a*b*d^2*e^2*f*Log[1 - E^(2*(c + d*x))] + 6*a*b*d^2*e^2*E^
(2*c)*f*Log[1 - E^(2*(c + d*x))] - 12*a*b*d^2*e*f^2*x*Log[1 - E^(2*(c + d*x
))] + 12*a*b*d^2*e*E^(2*c)*f^2*x*Log[1 - E^(2*(c + d*x))] - 6*a*b*d^2*f^3*x
^2*Log[1 - E^(2*(c + d*x))] + 6*a*b*d^2*E^(2*c)*f^3*x^2*Log[1 - E^(2*(c + d
*x))] - 3*(-1 + E^(2*c))*f*(-2*b^2*d^2*(e + f*x)^2 + a^2*(-2*f^2 + d^2*(e +
 f*x)^2))*PolyLog[2, -E^(c + d*x)] + 3*(-1 + E^(2*c))*f*(-2*b^2*d^2*(e + f*
x)^2 + a^2*(-2*f^2 + d^2*(e + f*x)^2))*PolyLog[2, E^(c + d*x)] - 6*a*b*d*e*
f^2*PolyLog[2, E^(2*(c + d*x))] + 6*a*b*d*e*E^(2*c)*f^2*PolyLog[2, E^(2*(c
+ d*x))] - 6*a*b*d*f^3*x*PolyLog[2, E^(2*(c + d*x))] + 6*a*b*d*E^(2*c)*f^3*
x*PolyLog[2, E^(2*(c + d*x))] - 6*a^2*d*e*f^2*PolyLog[3, -E^(c + d*x)] + 12
*b^2*d*e*f^2*PolyLog[3, -E^(c + d*x)] + 6*a^2*d*e*E^(2*c)*f^2*PolyLog[3, -E
^(c + d*x)] - 12*b^2*d*e*E^(2*c)*f^2*PolyLog[3, -E^(c + d*x)] - 6*a^2*d*f^3
*x*PolyLog[3, -E^(c + d*x)] + 12*b^2*d*f^3*x*PolyLog[3, -E^(c + d*x)] + 6*a
^2*d*E^(2*c)*f^3*x*PolyLog[3, -E^(c + d*x)] - 12*b^2*d*E^(2*c)*f^3*x*PolyLo
g[3, -E^(c + d*x)] + 6*a^2*d*e*f^2*PolyLog[3, E^(c + d*x)] - 12*b^2*d*e*f^2
*PolyLog[3, E^(c + d*x)] - 6*a^2*d*e*E^(2*c)*f^2*PolyLog[3, E^(c + d*x)] +
12*b^2*d*e*E^(2*c)*f^2*PolyLog[3, E^(c + d*x)] + 6*a^2*d*f^3*x*PolyLog[3, E
^(c + d*x)] - 12*b^2*d*f^3*x*PolyLog[3, E^(c + d*x)] - 6*a^2*d*E^(2*c)*f^3*
x*PolyLog[3, E^(c + d*x)] + 12*b^2*d*E^(2*c)*f^3*x*PolyLog[3, E^(c + d*x)]
+ 3*a*b*f^3*PolyLog[3, E^(2*(c + d*x))] - 3*a*b*E^(2*c)*f^3*PolyLog[3, E^(2
*(c + d*x))] + 6*a^2*f^3*PolyLog[4, -E^(c + d*x)] - 12*b^2*f^3*PolyLog[4, -
E^(c + d*x)] - 6*a^2*E^(2*c)*f^3*PolyLog[4, -E^(c + d*x)] + 12*b^2*E^(2*c)*
f^3*PolyLog[4, -E^(c + d*x)] - 6*a^2*f^3*PolyLog[4, E^(c + d*x)] + 12*b^2*f
^3*PolyLog[4, E^(c + d*x)] + 6*a^2*E^(2*c)*f^3*PolyLog[4, E^(c + d*x)] - 12
*b^2*E^(2*c)*f^3*PolyLog[4, E^(c + d*x)])/(2*a^3*d^4*(-1 + E^(2*c))) + (b^3
*(2*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 3*d^3*e^2*f*x*Lo
g[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 3*d^3*e*f^2*x^2*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] +
 d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e +
f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*
x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*d*e*f^2*PolyL
og[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*d*f^3*x*PolyLog[3, (b*E^(
c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))] - 6*d*f^3*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2]))] - 6*f^3*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] +
6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^3*Sqrt[a^2
+ b^2]*d^4) + (Csch[c]*Csch[c + d*x]^2*(2*b*d*e^3*Cosh[c] + 6*b*d*e^2*f*x*C
osh[c] + 6*b*d*e*f^2*x^2*Cosh[c] + 2*b*d*f^3*x^3*Cosh[c] + 3*a*e^2*f*Cosh[d
*x] + 6*a*e*f^2*x*Cosh[d*x] + 3*a*f^3*x^2*Cosh[d*x] - 3*a*e^2*f*Cosh[2*c +
d*x] - 6*a*e*f^2*x*Cosh[2*c + d*x] - 3*a*f^3*x^2*Cosh[2*c + d*x] - 2*b*d*e^
3*Cosh[c + 2*d*x] - 6*b*d*e^2*f*x*Cosh[c + 2*d*x] - 6*b*d*e*f^2*x^2*Cosh[c
+ 2*d*x] - 2*b*d*f^3*x^3*Cosh[c + 2*d*x] + a*d*e^3*Sinh[d*x] + 3*a*d*e^2*f*
x*Sinh[d*x] + 3*a*d*e*f^2*x^2*Sinh[d*x] + a*d*f^3*x^3*Sinh[d*x] - a*d*e^3*S
inh[2*c + d*x] - 3*a*d*e^2*f*x*Sinh[2*c + d*x] - 3*a*d*e*f^2*x^2*Sinh[2*c +
 d*x] - a*d*f^3*x^3*Sinh[2*c + d*x]))/(4*a^2*d^2)
```

Maple [F] time = 0.806, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3} \left(\operatorname{csch}\left(dx+c\right)\right)^{3}}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.249 $\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=725

$$-\frac{2b^3f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2\sqrt{a^2+b^2}}+\frac{2b^3f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3d^2\sqrt{a^2+b^2}}-\frac{2b^2f(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{a^3d^2}+$$

```
[Out] (b*(e + f*x)^2)/(a^2*d) + ((e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (2*b^2
*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^3*d) - (f^2*ArcTanh[Cosh[c + d*x]])/(
a*d^3) + (b*(e + f*x)^2*Coth[c + d*x])/(a^2*d) - (f*(e + f*x)*Csch[c + d*x]
)/(a*d^2) - ((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x])/(2*a*d) - (b^3*(e + f
*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*Sqrt[a^2 + b^2]*
d) + (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*
Sqrt[a^2 + b^2]*d) - (2*b*f*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a^2*d^2) +
 (f*(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) - (2*b^2*f*(e + f*x)*PolyLo
g[2, -E^(c + d*x)])/(a^3*d^2) - (f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^
2) + (2*b^2*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^3*d^2) - (2*b^3*f*(e +
f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 +
b^2]*d^2) + (2*b^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/(a^3*Sqrt[a^2 + b^2]*d^2) - (b*f^2*PolyLog[2, E^(2*(c + d*x))])/(
a^2*d^3) - (f^2*PolyLog[3, -E^(c + d*x)])/(a*d^3) + (2*b^2*f^2*PolyLog[3, -
E^(c + d*x)])/(a^3*d^3) + (f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) - (2*b^2*f^
2*PolyLog[3, E^(c + d*x)])/(a^3*d^3) + (2*b^3*f^2*PolyLog[3, -((b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b^2]*d^3) - (2*b^3*f^2*PolyLog
[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b^2]*d^3)
```

Rubi [A] time = 1.37234, antiderivative size = 725, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5575, 4186, 3770, 4182, 2531, 2282, 6589, 4184, 3716, 2190, 2279, 2391, 3322, 2264}

$$-\frac{2b^3f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2\sqrt{a^2+b^2}}+\frac{2b^3f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3d^2\sqrt{a^2+b^2}}-\frac{2b^2f(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{a^3d^2}+$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (b*(e + f*x)^2)/(a^2*d) + ((e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (2*b^2
*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^3*d) - (f^2*ArcTanh[Cosh[c + d*x]])/(
a*d^3) + (b*(e + f*x)^2*Coth[c + d*x])/(a^2*d) - (f*(e + f*x)*Csch[c + d*x]
)/(a*d^2) - ((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x])/(2*a*d) - (b^3*(e + f
*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*Sqrt[a^2 + b^2]*
d) + (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*
Sqrt[a^2 + b^2]*d) - (2*b*f*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a^2*d^2) +
 (f*(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) - (2*b^2*f*(e + f*x)*PolyLo
g[2, -E^(c + d*x)])/(a^3*d^2) - (f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^
2) + (2*b^2*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^3*d^2) - (2*b^3*f*(e +
f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 +
b^2]*d^2) + (2*b^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/(a^3*Sqrt[a^2 + b^2]*d^2) - (b*f^2*PolyLog[2, E^(2*(c + d*x))])/(
a^2*d^3) - (f^2*PolyLog[3, -E^(c + d*x)])/(a*d^3) + (2*b^2*f^2*PolyLog[3, -
E^(c + d*x)])/(a^3*d^3) + (f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) - (2*b^2*f^
2*PolyLog[3, E^(c + d*x)])/(a^3*d^3) + (2*b^3*f^2*PolyLog[3, -((b*E^(c + d*
```

```
x))/(a - Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b^2]*d^3) - (2*b^3*f^2*PolyLog
[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b^2]*d^3)
```

Rule 5575

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a +
b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
```

```
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2 \text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2 \text{csch}^3(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2 \text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{f(e+fx)\text{csch}(c+dx)}{ad^2} - \frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{2ad} - \frac{\int (e+fx)^2 \text{csch}(c+dx}{2a}$$

$$= \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{f^2\tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{b(e+fx)^2\coth(c+dx)}{a^2d} - \frac{f(e+}{}$$

$$= \frac{b(e+fx)^2}{a^2d} + \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} - \frac{f^2\tanh^{-1}(\cosh(}{ad^3}$$

$$= \frac{b(e+fx)^2}{a^2d} + \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} - \frac{f^2\tanh^{-1}(\cosh(}{ad^3}$$

$$= \frac{b(e+fx)^2}{a^2d} + \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} - \frac{f^2\tanh^{-1}(\cosh(}{ad^3}$$

$$= \frac{b(e+fx)^2}{a^2d} + \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} - \frac{f^2\tanh^{-1}(\cosh(}{ad^3}$$

$$= \frac{b(e+fx)^2}{a^2d} + \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} - \frac{f^2\tanh^{-1}(\cosh(}{ad^3}$$

$$= \frac{b(e+fx)^2}{a^2d} + \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} - \frac{f^2\tanh^{-1}(\cosh(}{ad^3}$$

Mathematica [B] time = 25.6502, size = 1531, normalized size = 2.11

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (8*a*b*d^2*e*E^(2*c)*f*x + 4*a*b*d^2*E^(2*c)*f^2*x^2 - 2*a^2*d^2*e^2*ArcTan
h[E^(c + d*x)] + 4*b^2*d^2*e^2*ArcTanh[E^(c + d*x)] + 2*a^2*d^2*e^2*E^(2*c)
*ArcTanh[E^(c + d*x)] - 4*b^2*d^2*e^2*E^(2*c)*ArcTanh[E^(c + d*x)] + 4*a^2*
f^2*ArcTanh[E^(c + d*x)] - 4*a^2*E^(2*c)*f^2*ArcTanh[E^(c + d*x)] + 2*a^2*d
^2*e*f*x*Log[1 - E^(c + d*x)] - 4*b^2*d^2*e*f*x*Log[1 - E^(c + d*x)] - 2*a^
2*d^2*e*E^(2*c)*f*x*Log[1 - E^(c + d*x)] + 4*b^2*d^2*e*E^(2*c)*f*x*Log[1 -
E^(c + d*x)] + a^2*d^2*f^2*x^2*Log[1 - E^(c + d*x)] - 2*b^2*d^2*f^2*x^2*Log
[1 - E^(c + d*x)] - a^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + 2*b^2*d^
2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] - 2*a^2*d^2*e*f*x*Log[1 + E^(c + d*x
)] + 4*b^2*d^2*e*f*x*Log[1 + E^(c + d*x)] + 2*a^2*d^2*e*E^(2*c)*f*x*Log[1 +
 E^(c + d*x)] - 4*b^2*d^2*e*E^(2*c)*f*x*Log[1 + E^(c + d*x)] - a^2*d^2*f^2*
x^2*Log[1 + E^(c + d*x)] + 2*b^2*d^2*f^2*x^2*Log[1 + E^(c + d*x)] + a^2*d^2
*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] - 2*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E
^(c + d*x)] + 4*a*b*d*e*f*Log[1 - E^(2*(c + d*x))] - 4*a*b*d*e*E^(2*c)*f*Lo
g[1 - E^(2*(c + d*x))] + 4*a*b*d*f^2*x*Log[1 - E^(2*(c + d*x))] - 4*a*b*d*E
^(2*c)*f^2*x*Log[1 - E^(2*(c + d*x))] + 2*(a^2 - 2*b^2)*d*(-1 + E^(2*c))*f*
(e + f*x)*PolyLog[2, -E^(c + d*x)] - 2*(a^2 - 2*b^2)*d*(-1 + E^(2*c))*f*(e
+ f*x)*PolyLog[2, E^(c + d*x)] + 2*a*b*f^2*PolyLog[2, E^(2*(c + d*x))] - 2*
a*b*E^(2*c)*f^2*PolyLog[2, E^(2*(c + d*x))] + 2*a^2*f^2*PolyLog[3, -E^(c +
d*x)] - 4*b^2*f^2*PolyLog[3, -E^(c + d*x)] - 2*a^2*E^(2*c)*f^2*PolyLog[3, -
```

```
E^(c + d*x)] + 4*b^2*E^(2*c)*f^2*PolyLog[3, -E^(c + d*x)] - 2*a^2*f^2*PolyL
og[3, E^(c + d*x)] + 4*b^2*f^2*PolyLog[3, E^(c + d*x)] + 2*a^2*E^(2*c)*f^2*
PolyLog[3, E^(c + d*x)] - 4*b^2*E^(2*c)*f^2*PolyLog[3, E^(c + d*x)])/(2*a^3
*d^3*(-1 + E^(2*c))) + (b^3*(2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2
 + b^2]] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2
*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*d^2*e*f*x*Log[1
 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-
a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))] + 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])
] - 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^3*Sqrt[
a^2 + b^2]*d^3) + (Csch[c]*Csch[c + d*x]^2*(2*b*d*e^2*Cosh[c] + 4*b*d*e*f*x
*Cosh[c] + 2*b*d*f^2*x^2*Cosh[c] + 2*a*e*f*Cosh[d*x] + 2*a*f^2*x*Cosh[d*x]
- 2*a*e*f*Cosh[2*c + d*x] - 2*a*f^2*x*Cosh[2*c + d*x] - 2*b*d*e^2*Cosh[c +
2*d*x] - 4*b*d*e*f*x*Cosh[c + 2*d*x] - 2*b*d*f^2*x^2*Cosh[c + 2*d*x] + a*d*
e^2*Sinh[d*x] + 2*a*d*e*f*x*Sinh[d*x] + a*d*f^2*x^2*Sinh[d*x] - a*d*e^2*Sin
h[2*c + d*x] - 2*a*d*e*f*x*Sinh[2*c + d*x] - a*d*f^2*x^2*Sinh[2*c + d*x]))/
(4*a^2*d^2)
```

Maple [F] time = 0.655, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 (\operatorname{csch}(dx+c))^3}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 4.14245, size = 22066, normalized size = 30.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(a^3*b + a*b^3)*d^2*e^2 - 8*(a^3*b + a*b^3)*c*d*e*f + 4*(a^3*b + a*
b^3)*c^2*f^2 - 4*((a^3*b + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x
 + 2*(a^3*b + a*b^3)*c*d*e*f - (a^3*b + a*b^3)*c^2*f^2)*cosh(d*x + c)^4 - 4
```

```
*((a^3*b + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x + 2*(a^3*b + a*
b^3)*c*d*e*f - (a^3*b + a*b^3)*c^2*f^2)*sinh(d*x + c)^4 + 2*((a^4 + a^2*b^2
)*d^2*f^2*x^2 + (a^4 + a^2*b^2)*d^2*e^2 + 2*(a^4 + a^2*b^2)*d*e*f + 2*((a^4
 + a^2*b^2)*d^2*e*f + (a^4 + a^2*b^2)*d*f^2)*x)*cosh(d*x + c)^3 + 2*((a^4 +
 a^2*b^2)*d^2*f^2*x^2 + (a^4 + a^2*b^2)*d^2*e^2 + 2*(a^4 + a^2*b^2)*d*e*f +
 2*((a^4 + a^2*b^2)*d^2*e*f + (a^4 + a^2*b^2)*d*f^2)*x - 8*((a^3*b + a*b^3)
*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x + 2*(a^3*b + a*b^3)*c*d*e*f - (a
^3*b + a*b^3)*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*((a^3*b + a*b^3)*
d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x - (a^3*b + a*b^3)*d^2*e^2 + 4*(a^
3*b + a*b^3)*c*d*e*f - 2*(a^3*b + a*b^3)*c^2*f^2)*cosh(d*x + c)^2 + 2*(2*(a
^3*b + a*b^3)*d^2*f^2*x^2 + 4*(a^3*b + a*b^3)*d^2*e*f*x - 2*(a^3*b + a*b^3)
*d^2*e^2 + 8*(a^3*b + a*b^3)*c*d*e*f - 4*(a^3*b + a*b^3)*c^2*f^2 - 12*((a^3
*b + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x + 2*(a^3*b + a*b^3)*c
*d*e*f - (a^3*b + a*b^3)*c^2*f^2)*cosh(d*x + c)^2 + 3*((a^4 + a^2*b^2)*d^2*
f^2*x^2 + (a^4 + a^2*b^2)*d^2*e^2 + 2*(a^4 + a^2*b^2)*d*e*f + 2*((a^4 + a^2
*b^2)*d^2*e*f + (a^4 + a^2*b^2)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^2 +
4*(b^4*d*f^2*x + b^4*d*e*f + (b^4*d*f^2*x + b^4*d*e*f)*cosh(d*x + c)^4 + 4*
(b^4*d*f^2*x + b^4*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*d*f^2*x + b^
4*d*e*f)*sinh(d*x + c)^4 - 2*(b^4*d*f^2*x + b^4*d*e*f)*cosh(d*x + c)^2 - 2*
(b^4*d*f^2*x + b^4*d*e*f - 3*(b^4*d*f^2*x + b^4*d*e*f)*cosh(d*x + c)^2)*sin
h(d*x + c)^2 + 4*((b^4*d*f^2*x + b^4*d*e*f)*cosh(d*x + c)^3 - (b^4*d*f^2*x
 + b^4*d*e*f)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*c
osh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((
a^2 + b^2)/b^2) - b)/b + 1) - 4*(b^4*d*f^2*x + b^4*d*e*f + (b^4*d*f^2*x + b
^4*d*e*f)*cosh(d*x + c)^4 + 4*(b^4*d*f^2*x + b^4*d*e*f)*cosh(d*x + c)*sinh(
d*x + c)^3 + (b^4*d*f^2*x + b^4*d*e*f)*sinh(d*x + c)^4 - 2*(b^4*d*f^2*x + b
^4*d*e*f)*cosh(d*x + c)^2 - 2*(b^4*d*f^2*x + b^4*d*e*f - 3*(b^4*d*f^2*x + b
^4*d*e*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d*f^2*x + b^4*d*e*f)*c
osh(d*x + c)^3 - (b^4*d*f^2*x + b^4*d*e*f)*cosh(d*x + c))*sinh(d*x + c))*sq
rt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b^4*d^2*e^2
 - 2*b^4*c*d*e*f + b^4*c^2*f^2 + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)
*cosh(d*x + c)^4 + 4*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*sinh(d*x
 + c)^4 - 2*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*cosh(d*x + c)^2 - 2*
(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2 - 3*(b^4*d^2*e^2 - 2*b^4*c*d*e*f
 + b^4*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d^2*e^2 - 2*b^4*
c*d*e*f + b^4*c^2*f^2)*cosh(d*x + c)^3 - (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4
*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(
d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(b^4*d^
2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2 + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^
2*f^2)*cosh(d*x + c)^4 + 4*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*cosh
(d*x + c)*sinh(d*x + c)^3 + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*sin
h(d*x + c)^4 - 2*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*cosh(d*x + c)^
2 - 2*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2 - 3*(b^4*d^2*e^2 - 2*b^4*c
*d*e*f + b^4*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d^2*e^2 -
2*b^4*c*d*e*f + b^4*c^2*f^2)*cosh(d*x + c)^3 - (b^4*d^2*e^2 - 2*b^4*c*d*e*f
 + b^4*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b
*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(
b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2 + (b^4*d^2*
f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh(d*x + c)^4 +
4*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh(d*
x + c)*sinh(d*x + c)^3 + (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f
 - b^4*c^2*f^2)*sinh(d*x + c)^4 - 2*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*
b^4*c*d*e*f - b^4*c^2*f^2)*cosh(d*x + c)^2 - 2*(b^4*d^2*f^2*x^2 + 2*b^4*d^2
*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2 - 3*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x
 + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*
d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh(d*x + c)^
3 - (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh(
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d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*si
nh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b
)/b) - 2*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2 +
 (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh(d*x
 + c)^4 + 4*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^
2)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b
^4*c*d*e*f - b^4*c^2*f^2)*sinh(d*x + c)^4 - 2*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*
e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh(d*x + c)^2 - 2*(b^4*d^2*f^2*x^2 +
 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2 - 3*(b^4*d^2*f^2*x^2 + 2*b^4
*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 4*((b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh
(d*x + c)^3 - (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*
f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x +
 c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2) - b)/b) - 4*(b^4*f^2*cosh(d*x + c)^4 + 4*b^4*f^2*cosh(d*x + c)*sinh(
d*x + c)^3 + b^4*f^2*sinh(d*x + c)^4 - 2*b^4*f^2*cosh(d*x + c)^2 + b^4*f^2
+ 2*(3*b^4*f^2*cosh(d*x + c)^2 - b^4*f^2)*sinh(d*x + c)^2 + 4*(b^4*f^2*cosh
(d*x + c)^3 - b^4*f^2*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*p
olylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2))/b) + 4*(b^4*f^2*cosh(d*x + c)^4 + 4*b^4*f^2*
cosh(d*x + c)*sinh(d*x + c)^3 + b^4*f^2*sinh(d*x + c)^4 - 2*b^4*f^2*cosh(d*
x + c)^2 + b^4*f^2 + 2*(3*b^4*f^2*cosh(d*x + c)^2 - b^4*f^2)*sinh(d*x + c)^
2 + 4*(b^4*f^2*cosh(d*x + c)^3 - b^4*f^2*cosh(d*x + c))*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*((a^4 + a^2*b^2)*d
^2*f^2*x^2 + (a^4 + a^2*b^2)*d^2*e^2 - 2*(a^4 + a^2*b^2)*d*e*f + 2*((a^4 +
a^2*b^2)*d^2*e*f - (a^4 + a^2*b^2)*d*f^2)*x)*cosh(d*x + c) + 2*((a^4 - a^2*
b^2 - 2*b^4)*d*f^2*x + ((a^4 - a^2*b^2 - 2*b^4)*d*f^2*x + (a^4 - a^2*b^2 -
2*b^4)*d*e*f + 2*(a^3*b + a*b^3)*f^2)*cosh(d*x + c)^4 + 4*((a^4 - a^2*b^2 -
 2*b^4)*d*f^2*x + (a^4 - a^2*b^2 - 2*b^4)*d*e*f + 2*(a^3*b + a*b^3)*f^2)*co
sh(d*x + c)*sinh(d*x + c)^3 + ((a^4 - a^2*b^2 - 2*b^4)*d*f^2*x + (a^4 - a^2
*b^2 - 2*b^4)*d*e*f + 2*(a^3*b + a*b^3)*f^2)*sinh(d*x + c)^4 + (a^4 - a^2*b
^2 - 2*b^4)*d*e*f + 2*(a^3*b + a*b^3)*f^2 - 2*((a^4 - a^2*b^2 - 2*b^4)*d*f^
2*x + (a^4 - a^2*b^2 - 2*b^4)*d*e*f + 2*(a^3*b + a*b^3)*f^2)*cosh(d*x + c)^
2 - 2*((a^4 - a^2*b^2 - 2*b^4)*d*f^2*x + (a^4 - a^2*b^2 - 2*b^4)*d*e*f + 2*
(a^3*b + a*b^3)*f^2 - 3*((a^4 - a^2*b^2 - 2*b^4)*d*f^2*x + (a^4 - a^2*b^2 -
 2*b^4)*d*e*f + 2*(a^3*b + a*b^3)*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4
*(((a^4 - a^2*b^2 - 2*b^4)*d*f^2*x + (a^4 - a^2*b^2 - 2*b^4)*d*e*f + 2*(a^3
*b + a*b^3)*f^2)*cosh(d*x + c)^3 - ((a^4 - a^2*b^2 - 2*b^4)*d*f^2*x + (a^4
- a^2*b^2 - 2*b^4)*d*e*f + 2*(a^3*b + a*b^3)*f^2)*cosh(d*x + c))*sinh(d*x +
 c))*dilog(cosh(d*x + c) + sinh(d*x + c)) - 2*((a^4 - a^2*b^2 - 2*b^4)*d*f^
2*x + ((a^4 - a^2*b^2 - 2*b^4)*d*f^2*x + (a^4 - a^2*b^2 - 2*b^4)*d*e*f - 2*
(a^3*b + a*b^3)*f^2)*cosh(d*x + c)^4 + 4*((a^4 - a^2*b^2 - 2*b^4)*d*f^2*x +
 (a^4 - a^2*b^2 - 2*b^4)*d*e*f - 2*(a^3*b + a*b^3)*f^2)*cosh(d*x + c)*sinh(
d*x + c)^3 + ((a^4 - a^2*b^2 - 2*b^4)*d*f^2*x + (a^4 - a^2*b^2 - 2*b^4)*d*e
*f - 2*(a^3*b + a*b^3)*f^2)*sinh(d*x + c)^4 + (a^4 - a^2*b^2 - 2*b^4)*d*e*f
 - 2*(a^3*b + a*b^3)*f^2 - 2*((a^4 - a^2*b^2 - 2*b^4)*d*f^2*x + (a^4 - a^2*
b^2 - 2*b^4)*d*e*f - 2*(a^3*b + a*b^3)*f^2)*cosh(d*x + c)^2 - 2*((a^4 - a^2
*b^2 - 2*b^4)*d*f^2*x + (a^4 - a^2*b^2 - 2*b^4)*d*e*f - 2*(a^3*b + a*b^3)*f
^2 - 3*((a^4 - a^2*b^2 - 2*b^4)*d*f^2*x + (a^4 - a^2*b^2 - 2*b^4)*d*e*f - 2
*(a^3*b + a*b^3)*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^4 - a^2*b^2
 - 2*b^4)*d*f^2*x + (a^4 - a^2*b^2 - 2*b^4)*d*e*f - 2*(a^3*b + a*b^3)*f^2)*
cosh(d*x + c)^3 - ((a^4 - a^2*b^2 - 2*b^4)*d*f^2*x + (a^4 - a^2*b^2 - 2*b^4
)*d*e*f - 2*(a^3*b + a*b^3)*f^2)*cosh(d*x + c))*sinh(d*x + c))*dilog(-cosh(
d*x + c) - sinh(d*x + c)) - ((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + (a^4 - a
^2*b^2 - 2*b^4)*d^2*e^2 + ((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + (a^4 - a^2
*b^2 - 2*b^4)*d^2*e^2 - 4*(a^3*b + a*b^3)*d*e*f - 2*(a^4 + a^2*b^2)*f^2 + 2
*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f - 2*(a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x +
c)^4 + 4*((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + (a^4 - a^2*b^2 - 2*b^4)*d^2
```

e^2 - 4(a^3*b + a*b^3)*d*e*f - 2*(a^4 + a^2*b^2)*f^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f - 2*(a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + (a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 - 4*(a^3*b + a*b^3)*d*e*f - 2*(a^4 + a^2*b^2)*f^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f - 2*(a^3*b + a*b^3)*d*f^2)*x)*sinh(d*x + c)^4 - 4*(a^3*b + a*b^3)*d*e*f - 2*(a^4 + a^2*b^2)*f^2 - 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + (a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 - 4*(a^3*b + a*b^3)*d*e*f - 2*(a^4 + a^2*b^2)*f^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f - 2*(a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c)^2 - 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + (a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 - 4*(a^3*b + a*b^3)*d*e*f - 2*(a^4 + a^2*b^2)*f^2 - 3*((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + (a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 - 4*(a^3*b + a*b^3)*d*e*f - 2*(a^4 + a^2*b^2)*f^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f - 2*(a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c)^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f - 2*(a^3*b + a*b^3)*d*f^2)*x)*sinh(d*x + c)^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f - 2*(a^3*b + a*b^3)*d*f^2)*x + 4*(((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + (a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 - 4*(a^3*b + a*b^3)*d*e*f - 2*(a^4 + a^2*b^2)*f^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f - 2*(a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c)^3 - ((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + (a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 - 4*(a^3*b + a*b^3)*d*e*f - 2*(a^4 + a^2*b^2)*f^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f - 2*(a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 + ((a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 + 2*(2*a^3*b + 2*a*b^3 - (a^4 - a^2*b^2 - 2*b^4)*c)*d*e*f - (2*a^4 + 2*a^2*b^2 - (a^4 - a^2*b^2 - 2*b^4)*c^2 + 4*(a^3*b + a*b^3)*c)*f^2)*cosh(d*x + c)^4 + 4*((a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 + 2*(2*a^3*b + 2*a*b^3 - (a^4 - a^2*b^2 - 2*b^4)*c)*d*e*f - (2*a^4 + 2*a^2*b^2 - (a^4 - a^2*b^2 - 2*b^4)*c^2 + 4*(a^3*b + a*b^3)*c)*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 + 2*(2*a^3*b + 2*a*b^3 - (a^4 - a^2*b^2 - 2*b^4)*c)*d*e*f - (2*a^4 + 2*a^2*b^2 - (a^4 - a^2*b^2 - 2*b^4)*c^2 + 4*(a^3*b + a*b^3)*c)*f^2)*sinh(d*x + c)^4 + 2*(2*a^3*b + 2*a*b^3 - (a^4 - a^2*b^2 - 2*b^4)*c)*d*e*f - (2*a^4 + 2*a^2*b^2 - (a^4 - a^2*b^2 - 2*b^4)*c^2 + 4*(a^3*b + a*b^3)*c)*f^2 - 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 + 2*(2*a^3*b + 2*a*b^3 - (a^4 - a^2*b^2 - 2*b^4)*c)*d*e*f - (2*a^4 + 2*a^2*b^2 - (a^4 - a^2*b^2 - 2*b^4)*c^2 + 4*(a^3*b + a*b^3)*c)*f^2)*cosh(d*x + c)^2 - 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 + 2*(2*a^3*b + 2*a*b^3 - (a^4 - a^2*b^2 - 2*b^4)*c)*d*e*f - (2*a^4 + 2*a^2*b^2 - (a^4 - a^2*b^2 - 2*b^4)*c^2 + 4*(a^3*b + a*b^3)*c)*f^2 - 3*((a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 + 2*(2*a^3*b + 2*a*b^3 - (a^4 - a^2*b^2 - 2*b^4)*c)*d*e*f - (2*a^4 + 2*a^2*b^2 - (a^4 - a^2*b^2 - 2*b^4)*c^2 + 4*(a^3*b + a*b^3)*c)*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 + 2*(2*a^3*b + 2*a*b^3 - (a^4 - a^2*b^2 - 2*b^4)*c)*d*e*f - (2*a^4 + 2*a^2*b^2 - (a^4 - a^2*b^2 - 2*b^4)*c^2 + 4*(a^3*b + a*b^3)*c)*f^2)*cosh(d*x + c)^3 - ((a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 + 2*(2*a^3*b + 2*a*b^3 - (a^4 - a^2*b^2 - 2*b^4)*c)*d*e*f - (2*a^4 + 2*a^2*b^2 - (a^4 - a^2*b^2 - 2*b^4)*c^2 + 4*(a^3*b + a*b^3)*c)*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + ((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + 2*(a^4 - a^2*b^2 - 2*b^4)*c*d*e*f + ((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + 2*(a^4 - a^2*b^2 - 2*b^4)*c*d*e*f - ((a^4 - a^2*b^2 - 2*b^4)*c^2 - 4*(a^3*b + a*b^3)*c)*f^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f + 2*(a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c)^4 + 4*((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + 2*(a^4 - a^2*b^2 - 2*b^4)*c*d*e*f - ((a^4 - a^2*b^2 - 2*b^4)*c^2 - 4*(a^3*b + a*b^3)*c)*f^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f + 2*(a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + 2*(a^4 - a^2*b^2 - 2*b^4)*c*d*e*f - ((a^4 - a^2*b^2 - 2*b^4)*c^2 - 4*(a^3*b + a*b^3)*c)*f^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f + 2*(a^3*b + a*b^3)*d*f^2)*x)*sinh(d*x + c)^4 - ((a^4 - a^2*b^2 - 2*b^4)*c^2 - 4*(a^3*b + a*b^3)*c)*f^2 - 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + 2*(a^4 - a^2*b^2 - 2*b^4)*c*d*e*f - ((a^4 - a^2*b^2 - 2*b^4)*c^2 - 4*(a^3*b + a*b^3)*c)*f^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f + 2*(a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c)^2 - 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + 2*(a^4 - a^2*b^2 - 2*b^4)*c*d*e*f - ((a^4 - a^2*

```
b^2 - 2*b^4)*c^2 - 4*(a^3*b + a*b^3)*c)*f^2 - 3*((a^4 - a^2*b^2 - 2*b^4)*d^
2*f^2*x^2 + 2*(a^4 - a^2*b^2 - 2*b^4)*c*d*e*f - ((a^4 - a^2*b^2 - 2*b^4)*c^
2 - 4*(a^3*b + a*b^3)*c)*f^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f + 2*(a^3*
b + a*b^3)*d*f^2)*x)*cosh(d*x + c)^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f +
 2*(a^3*b + a*b^3)*d*f^2)*x)*sinh(d*x + c)^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d
^2*e*f + 2*(a^3*b + a*b^3)*d*f^2)*x + 4*(((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x
^2 + 2*(a^4 - a^2*b^2 - 2*b^4)*c*d*e*f - ((a^4 - a^2*b^2 - 2*b^4)*c^2 - 4*(
a^3*b + a*b^3)*c)*f^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f + 2*(a^3*b + a*b
^3)*d*f^2)*x)*cosh(d*x + c)^3 - ((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + 2*(a
^4 - a^2*b^2 - 2*b^4)*c*d*e*f - ((a^4 - a^2*b^2 - 2*b^4)*c^2 - 4*(a^3*b + a
*b^3)*c)*f^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f + 2*(a^3*b + a*b^3)*d*f^2
)*x)*cosh(d*x + c))*sinh(d*x + c))*log(-cosh(d*x + c) - sinh(d*x + c) + 1)
- 2*((a^4 - a^2*b^2 - 2*b^4)*f^2*cosh(d*x + c)^4 + 4*(a^4 - a^2*b^2 - 2*b^4
)*f^2*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 - a^2*b^2 - 2*b^4)*f^2*sinh(d*x
+ c)^4 - 2*(a^4 - a^2*b^2 - 2*b^4)*f^2*cosh(d*x + c)^2 + (a^4 - a^2*b^2 - 2
*b^4)*f^2 + 2*(3*(a^4 - a^2*b^2 - 2*b^4)*f^2*cosh(d*x + c)^2 - (a^4 - a^2*b
^2 - 2*b^4)*f^2)*sinh(d*x + c)^2 + 4*((a^4 - a^2*b^2 - 2*b^4)*f^2*cosh(d*x
+ c)^3 - (a^4 - a^2*b^2 - 2*b^4)*f^2*cosh(d*x + c))*sinh(d*x + c))*polylog(
3, cosh(d*x + c) + sinh(d*x + c)) + 2*((a^4 - a^2*b^2 - 2*b^4)*f^2*cosh(d*x
 + c)^4 + 4*(a^4 - a^2*b^2 - 2*b^4)*f^2*cosh(d*x + c)*sinh(d*x + c)^3 + (a^
4 - a^2*b^2 - 2*b^4)*f^2*sinh(d*x + c)^4 - 2*(a^4 - a^2*b^2 - 2*b^4)*f^2*co
sh(d*x + c)^2 + (a^4 - a^2*b^2 - 2*b^4)*f^2 + 2*(3*(a^4 - a^2*b^2 - 2*b^4)*
f^2*cosh(d*x + c)^2 - (a^4 - a^2*b^2 - 2*b^4)*f^2)*sinh(d*x + c)^2 + 4*((a^
4 - a^2*b^2 - 2*b^4)*f^2*cosh(d*x + c)^3 - (a^4 - a^2*b^2 - 2*b^4)*f^2*cosh
(d*x + c))*sinh(d*x + c))*polylog(3, -cosh(d*x + c) - sinh(d*x + c)) + 2*((
a^4 + a^2*b^2)*d^2*f^2*x^2 + (a^4 + a^2*b^2)*d^2*e^2 - 2*(a^4 + a^2*b^2)*d*
e*f - 8*((a^3*b + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x + 2*(a^3
*b + a*b^3)*c*d*e*f - (a^3*b + a*b^3)*c^2*f^2)*cosh(d*x + c)^3 + 3*((a^4 +
a^2*b^2)*d^2*f^2*x^2 + (a^4 + a^2*b^2)*d^2*e^2 + 2*(a^4 + a^2*b^2)*d*e*f +
2*((a^4 + a^2*b^2)*d^2*e*f + (a^4 + a^2*b^2)*d*f^2)*x)*cosh(d*x + c)^2 + 2*
((a^4 + a^2*b^2)*d^2*e*f - (a^4 + a^2*b^2)*d*f^2)*x + 4*((a^3*b + a*b^3)*d^
2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x - (a^3*b + a*b^3)*d^2*e^2 + 4*(a^3*
b + a*b^3)*c*d*e*f - 2*(a^3*b + a*b^3)*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c
))/((a^5 + a^3*b^2)*d^3*cosh(d*x + c)^4 + 4*(a^5 + a^3*b^2)*d^3*cosh(d*x +
c)*sinh(d*x + c)^3 + (a^5 + a^3*b^2)*d^3*sinh(d*x + c)^4 - 2*(a^5 + a^3*b^2
)*d^3*cosh(d*x + c)^2 + (a^5 + a^3*b^2)*d^3 + 2*(3*(a^5 + a^3*b^2)*d^3*cosh
(d*x + c)^2 - (a^5 + a^3*b^2)*d^3)*sinh(d*x + c)^2 + 4*((a^5 + a^3*b^2)*d^3
*cosh(d*x + c)^3 - (a^5 + a^3*b^2)*d^3*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.250 $\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=420

$$-\frac{b^3 f \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2\sqrt{a^2+b^2}}+\frac{b^3 f \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3d^2\sqrt{a^2+b^2}}-\frac{b^2 f \operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^3d^2}+\frac{b^2 f \operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^3d^2}+\frac{f}{}$$

```
[Out] ((e + f*x)*ArcTanh[E^(c + d*x)])/(a*d) - (2*b^2*(e + f*x)*ArcTanh[E^(c + d*
x)])/(a^3*d) + (b*(e + f*x)*Coth[c + d*x])/(a^2*d) - (f*Csch[c + d*x])/(2*a
*d^2) - ((e + f*x)*Coth[c + d*x]*Csch[c + d*x])/(2*a*d) - (b^3*(e + f*x)*Lo
g[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*Sqrt[a^2 + b^2]*d) + (b^
3*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*Sqrt[a^2 +
 b^2]*d) - (b*f*Log[Sinh[c + d*x]])/(a^2*d^2) + (f*PolyLog[2, -E^(c + d*x)]
)/(2*a*d^2) - (b^2*f*PolyLog[2, -E^(c + d*x)])/(a^3*d^2) - (f*PolyLog[2, E^
(c + d*x)])/(2*a*d^2) + (b^2*f*PolyLog[2, E^(c + d*x)])/(a^3*d^2) - (b^3*f*
PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b^2]*
d^2) + (b^3*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*Sq
rt[a^2 + b^2]*d^2)
```

Rubi [A] time = 0.72555, antiderivative size = 420, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5575, 4185, 4182, 2279, 2391, 4184, 3475, 3322, 2264, 2190}

$$-\frac{b^3 f \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2\sqrt{a^2+b^2}}+\frac{b^3 f \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3d^2\sqrt{a^2+b^2}}-\frac{b^2 f \operatorname{PolyLog}\left(2,-e^{c+dx}\right)}{a^3d^2}+\frac{b^2 f \operatorname{PolyLog}\left(2,e^{c+dx}\right)}{a^3d^2}+\frac{f}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] ((e + f*x)*ArcTanh[E^(c + d*x)])/(a*d) - (2*b^2*(e + f*x)*ArcTanh[E^(c + d*
x)])/(a^3*d) + (b*(e + f*x)*Coth[c + d*x])/(a^2*d) - (f*Csch[c + d*x])/(2*a
*d^2) - ((e + f*x)*Coth[c + d*x]*Csch[c + d*x])/(2*a*d) - (b^3*(e + f*x)*Lo
g[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*Sqrt[a^2 + b^2]*d) + (b^
3*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*Sqrt[a^2 +
 b^2]*d) - (b*f*Log[Sinh[c + d*x]])/(a^2*d^2) + (f*PolyLog[2, -E^(c + d*x)]
)/(2*a*d^2) - (b^2*f*PolyLog[2, -E^(c + d*x)])/(a^3*d^2) - (f*PolyLog[2, E^
(c + d*x)])/(2*a*d^2) + (b^2*f*PolyLog[2, E^(c + d*x)])/(a^3*d^2) - (b^3*f*
PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2 + b^2]*
d^2) + (b^3*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*Sq
rt[a^2 + b^2]*d^2)
```


Rule 5575

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a +
b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
```

```
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\text{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{csch}^3(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{f\text{csch}(c+dx)}{2ad^2} - \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{2ad} - \frac{\int (e+fx)\text{csch}(c+dx)\,dx}{2a} - \frac{b}{}$$

$$= \frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{b(e+fx)\coth(c+dx)}{a^2d} - \frac{f\text{csch}(c+dx)}{2ad^2} - \frac{(e+fx)\coth(c}{2}$$

$$= \frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} + \frac{b(e+fx)\coth(c+dx)}{a^2d} - \frac{f\text{csc}}{}$$

$$= \frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} + \frac{b(e+fx)\coth(c+dx)}{a^2d} - \frac{f\text{csc}}{}$$

$$= \frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} + \frac{b(e+fx)\coth(c+dx)}{a^2d} - \frac{f\text{csc}}{}$$

$$= \frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} + \frac{b(e+fx)\coth(c+dx)}{a^2d} - \frac{f\text{csc}}{}$$

$$= \frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} + \frac{b(e+fx)\coth(c+dx)}{a^2d} - \frac{f\text{csc}}{}$$

Mathematica [C] time = 7.98397, size = 736, normalized size = 1.75

$$\frac{b^3\left(-f\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-f(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)+f(}{a^3d^2\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((2*b*d*e*Cosh[(c + d*x)/2] - a*f*Cosh[(c + d*x)/2] - 2*b*c*f*Cosh[(c + d*x
)/2] + 2*b*f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(4*a^2*d^2) +
((-(d*e) + c*f - f*(c + d*x))*Csch[(c + d*x)/2]^2)/(8*a*d^2) - (b*f*Log[Sin
h[c + d*x]])/(a^2*d^2) - (e*Log[Tanh[(c + d*x)/2]])/(2*a*d) + (b^2*e*Log[Ta
nh[(c + d*x)/2]])/(a^3*d) + (c*f*Log[Tanh[(c + d*x)/2]])/(2*a*d^2) - (b^2*c
*f*Log[Tanh[(c + d*x)/2]])/(a^3*d^2) + ((I/2)*f*(I*(c + d*x)*(Log[1 - E^(-c
 - d*x)] - Log[1 + E^(-c - d*x)]) + I*(PolyLog[2, -E^(-c - d*x)] - PolyLog[
2, E^(-c - d*x)])))/(a*d^2) - (I*b^2*f*(I*(c + d*x)*(Log[1 - E^(-c - d*x)]
- Log[1 + E^(-c - d*x)]) + I*(PolyLog[2, -E^(-c - d*x)] - PolyLog[2, E^(-c
- d*x)])))/(a^3*d^2) + (b^3*(2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b
^2]] - 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - f*(c + d*x)*Log
[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2])] - f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2
 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^3*S
qrt[a^2 + b^2]*d^2) + ((-(d*e) + c*f - f*(c + d*x))*Sech[(c + d*x)/2]^2)/(8
*a*d^2) + (Sech[(c + d*x)/2]*(2*b*d*e*Sinh[(c + d*x)/2] + a*f*Sinh[(c + d*x
)/2] - 2*b*c*f*Sinh[(c + d*x)/2] + 2*b*f*(c + d*x)*Sinh[(c + d*x)/2]))/(4*a
^2*d^2)
```

Maple [B] time = 0.162, size = 861, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] -(a*d*f*x*exp(3*d*x+3*c)+a*d*e*exp(3*d*x+3*c)-2*b*d*f*x*exp(2*d*x+2*c)+a*d*
f*x*exp(d*x+c)+a*f*exp(3*d*x+3*c)-2*b*d*e*exp(2*d*x+2*c)+a*d*e*exp(d*x+c)+2
*b*d*f*x-a*f*exp(d*x+c)+2*b*d*e)/d^2/a^2/(exp(2*d*x+2*c)-1)^2-1/a^3/d^2*b^3
*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)
))*c+1/a^3/d^2*b^3*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a
+(a^2+b^2)^(1/2)))*c-1/a^3/d*b^2*f*ln(exp(d*x+c)+1)*x+2/a^3/d*b^3*e/(a^2+b^
2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/a^3/d^2*b^3*f*
c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/a^3/d
*b^3*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(
1/2)))*x+1/a^3/d*b^3*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/
(a+(a^2+b^2)^(1/2)))*x+1/2/d/a*ln(exp(d*x+c)+1)*f*x-1/2/d/a*e*ln(exp(d*x+c)
-1)+1/2/d/a*e*ln(exp(d*x+c)+1)+1/2/d^2/a*f*c*ln(exp(d*x+c)-1)-1/a^3/d^2*b^2
*f*dilog(exp(d*x+c))-1/a^3/d^2*b^2*f*dilog(exp(d*x+c)+1)+2/a^2/d^2*b*f*ln(e
xp(d*x+c))+1/2/d^2*f/a*dilog(exp(d*x+c))+1/2/d^2*f/a*dilog(exp(d*x+c)+1)-1/
a^2/d^2*b*f*ln(exp(d*x+c)-1)-1/a^2/d^2*b*f*ln(exp(d*x+c)+1)+1/a^3/d*b^2*e*l
n(exp(d*x+c)-1)-1/a^3/d*b^2*e*ln(exp(d*x+c)+1)-1/a^3/d^2*b^2*f*c*ln(exp(d*x
+c)-1)-1/a^3/d^2*b^3*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))+1/a^3/d^2*b^3*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+
c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.99808, size = 10647, normalized size = 25.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*c*f)*cosh(d*x + c)^4 + 4*((
a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*c*f)*sinh(d*x + c)^4 - 2*((a^4 + a^2
*b^2)*d*f*x + (a^4 + a^2*b^2)*d*e + (a^4 + a^2*b^2)*f)*cosh(d*x + c)^3 - 2*
((a^4 + a^2*b^2)*d*f*x + (a^4 + a^2*b^2)*d*e + (a^4 + a^2*b^2)*f - 8*((a^3*
b + a*b^3)*d*f*x + (a^3*b + a*b^3)*c*f)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*
(a^3*b + a*b^3)*d*e + 4*(a^3*b + a*b^3)*c*f - 4*((a^3*b + a*b^3)*d*f*x - (a
^3*b + a*b^3)*d*e + 2*(a^3*b + a*b^3)*c*f)*cosh(d*x + c)^2 - 2*(2*(a^3*b +
a*b^3)*d*f*x - 2*(a^3*b + a*b^3)*d*e + 4*(a^3*b + a*b^3)*c*f - 12*((a^3*b +
```

a*b^3)*d*f*x + (a^3*b + a*b^3)*c*f)*cosh(d*x + c)^2 + 3*((a^4 + a^2*b^2)*d
*f*x + (a^4 + a^2*b^2)*d*e + (a^4 + a^2*b^2)*f)*cosh(d*x + c))*sinh(d*x + c
)^2 - 2*(b^4*f*cosh(d*x + c)^4 + 4*b^4*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^
4*f*sinh(d*x + c)^4 - 2*b^4*f*cosh(d*x + c)^2 + b^4*f + 2*(3*b^4*f*cosh(d*x
 + c)^2 - b^4*f)*sinh(d*x + c)^2 + 4*(b^4*f*cosh(d*x + c)^3 - b^4*f*cosh(d*
x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sin
h(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)
/b + 1) + 2*(b^4*f*cosh(d*x + c)^4 + 4*b^4*f*cosh(d*x + c)*sinh(d*x + c)^3
 + b^4*f*sinh(d*x + c)^4 - 2*b^4*f*cosh(d*x + c)^2 + b^4*f + 2*(3*b^4*f*cosh
(d*x + c)^2 - b^4*f)*sinh(d*x + c)^2 + 4*(b^4*f*cosh(d*x + c)^3 - b^4*f*cos
h(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a
*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
 - b)/b + 1) + 2*(b^4*d*e - b^4*c*f + (b^4*d*e - b^4*c*f)*cosh(d*x + c)^4 +
4*(b^4*d*e - b^4*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*d*e - b^4*c*f)*s
inh(d*x + c)^4 - 2*(b^4*d*e - b^4*c*f)*cosh(d*x + c)^2 - 2*(b^4*d*e - b^4*c
f - 3(b^4*d*e - b^4*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d*e -
 b^4*c*f)*cosh(d*x + c)^3 - (b^4*d*e - b^4*c*f)*cosh(d*x + c))*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sq
rt((a^2 + b^2)/b^2) + 2*a) - 2*(b^4*d*e - b^4*c*f + (b^4*d*e - b^4*c*f)*cos
h(d*x + c)^4 + 4*(b^4*d*e - b^4*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*d
*e - b^4*c*f)*sinh(d*x + c)^4 - 2*(b^4*d*e - b^4*c*f)*cosh(d*x + c)^2 - 2*(
b^4*d*e - b^4*c*f - 3*(b^4*d*e - b^4*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2
 + 4*((b^4*d*e - b^4*c*f)*cosh(d*x + c)^3 - (b^4*d*e - b^4*c*f)*cosh(d*x + c
))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*
x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(b^4*d*f*x + b^4*c*f + (b^4*d
*f*x + b^4*c*f)*cosh(d*x + c)^4 + 4*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)*sin
h(d*x + c)^3 + (b^4*d*f*x + b^4*c*f)*sinh(d*x + c)^4 - 2*(b^4*d*f*x + b^4*c
*f)*cosh(d*x + c)^2 - 2*(b^4*d*f*x + b^4*c*f - 3*(b^4*d*f*x + b^4*c*f)*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^3 - (
b^4*d*f*x + b^4*c*f)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*lo
g(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))
sqrt((a^2 + b^2)/b^2) - b)/b) + 2(b^4*d*f*x + b^4*c*f + (b^4*d*f*x + b^4*
c*f)*cosh(d*x + c)^4 + 4*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)*sinh(d*x + c)^
3 + (b^4*d*f*x + b^4*c*f)*sinh(d*x + c)^4 - 2*(b^4*d*f*x + b^4*c*f)*cosh(d*
x + c)^2 - 2*(b^4*d*f*x + b^4*c*f - 3*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 4*((b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^3 - (b^4*d*f*x +
 b^4*c*f)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(
d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b) - 2*((a^4 + a^2*b^2)*d*f*x + (a^4 + a^2*b^2)*d*e - (a^4
 + a^2*b^2)*f)*cosh(d*x + c) - ((a^4 - a^2*b^2 - 2*b^4)*f*cosh(d*x + c)^4 +
 4*(a^4 - a^2*b^2 - 2*b^4)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 - a^2*b^2
 - 2*b^4)*f*sinh(d*x + c)^4 - 2*(a^4 - a^2*b^2 - 2*b^4)*f*cosh(d*x + c)^2 +
 2*(3*(a^4 - a^2*b^2 - 2*b^4)*f*cosh(d*x + c)^2 - (a^4 - a^2*b^2 - 2*b^4)*f
)*sinh(d*x + c)^2 + (a^4 - a^2*b^2 - 2*b^4)*f + 4*((a^4 - a^2*b^2 - 2*b^4)*
f*cosh(d*x + c)^3 - (a^4 - a^2*b^2 - 2*b^4)*f*cosh(d*x + c))*sinh(d*x + c))
*dilog(cosh(d*x + c) + sinh(d*x + c)) + ((a^4 - a^2*b^2 - 2*b^4)*f*cosh(d*x
 + c)^4 + 4*(a^4 - a^2*b^2 - 2*b^4)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4
 - a^2*b^2 - 2*b^4)*f*sinh(d*x + c)^4 - 2*(a^4 - a^2*b^2 - 2*b^4)*f*cosh(d*x
 + c)^2 + 2*(3*(a^4 - a^2*b^2 - 2*b^4)*f*cosh(d*x + c)^2 - (a^4 - a^2*b^2 -
 2*b^4)*f)*sinh(d*x + c)^2 + (a^4 - a^2*b^2 - 2*b^4)*f + 4*((a^4 - a^2*b^2
 - 2*b^4)*f*cosh(d*x + c)^3 - (a^4 - a^2*b^2 - 2*b^4)*f*cosh(d*x + c))*sinh(
d*x + c))*dilog(-cosh(d*x + c) - sinh(d*x + c)) + (((a^4 - a^2*b^2 - 2*b^4)
*d*f*x + (a^4 - a^2*b^2 - 2*b^4)*d*e - 2*(a^3*b + a*b^3)*f)*cosh(d*x + c)^4
 + 4*((a^4 - a^2*b^2 - 2*b^4)*d*f*x + (a^4 - a^2*b^2 - 2*b^4)*d*e - 2*(a^3*
b + a*b^3)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 - a^2*b^2 - 2*b^4)*d*f*
x + (a^4 - a^2*b^2 - 2*b^4)*d*e - 2*(a^3*b + a*b^3)*f)*sinh(d*x + c)^4 + (a
^4 - a^2*b^2 - 2*b^4)*d*f*x + (a^4 - a^2*b^2 - 2*b^4)*d*e - 2*((a^4 - a^2*b
^2 - 2*b^4)*d*f*x + (a^4 - a^2*b^2 - 2*b^4)*d*e - 2*(a^3*b + a*b^3)*f)*cosh
(d*x + c)^2 - 2*((a^4 - a^2*b^2 - 2*b^4)*d*f*x + (a^4 - a^2*b^2 - 2*b^4)*d*

```
e - 3*((a^4 - a^2*b^2 - 2*b^4)*d*f*x + (a^4 - a^2*b^2 - 2*b^4)*d*e - 2*(a^3
*b + a*b^3)*f)*cosh(d*x + c)^2 - 2*(a^3*b + a*b^3)*f)*sinh(d*x + c)^2 - 2*(
a^3*b + a*b^3)*f + 4*(((a^4 - a^2*b^2 - 2*b^4)*d*f*x + (a^4 - a^2*b^2 - 2*b
^4)*d*e - 2*(a^3*b + a*b^3)*f)*cosh(d*x + c)^3 - ((a^4 - a^2*b^2 - 2*b^4)*d
*f*x + (a^4 - a^2*b^2 - 2*b^4)*d*e - 2*(a^3*b + a*b^3)*f)*cosh(d*x + c))*si
nh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (((a^4 - a^2*b^2 - 2*
b^4)*d*e + (2*a^3*b + 2*a*b^3 - (a^4 - a^2*b^2 - 2*b^4)*c)*f)*cosh(d*x + c)
^4 + 4*((a^4 - a^2*b^2 - 2*b^4)*d*e + (2*a^3*b + 2*a*b^3 - (a^4 - a^2*b^2 -
 2*b^4)*c)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 - a^2*b^2 - 2*b^4)*d*e
+ (2*a^3*b + 2*a*b^3 - (a^4 - a^2*b^2 - 2*b^4)*c)*f)*sinh(d*x + c)^4 + (a^4
 - a^2*b^2 - 2*b^4)*d*e - 2*((a^4 - a^2*b^2 - 2*b^4)*d*e + (2*a^3*b + 2*a*b
^3 - (a^4 - a^2*b^2 - 2*b^4)*c)*f)*cosh(d*x + c)^2 - 2*((a^4 - a^2*b^2 - 2*
b^4)*d*e - 3*((a^4 - a^2*b^2 - 2*b^4)*d*e + (2*a^3*b + 2*a*b^3 - (a^4 - a^2
*b^2 - 2*b^4)*c)*f)*cosh(d*x + c)^2 + (2*a^3*b + 2*a*b^3 - (a^4 - a^2*b^2 -
 2*b^4)*c)*f)*sinh(d*x + c)^2 + (2*a^3*b + 2*a*b^3 - (a^4 - a^2*b^2 - 2*b^4
)*c)*f + 4*(((a^4 - a^2*b^2 - 2*b^4)*d*e + (2*a^3*b + 2*a*b^3 - (a^4 - a^2*
b^2 - 2*b^4)*c)*f)*cosh(d*x + c)^3 - ((a^4 - a^2*b^2 - 2*b^4)*d*e + (2*a^3*
b + 2*a*b^3 - (a^4 - a^2*b^2 - 2*b^4)*c)*f)*cosh(d*x + c))*sinh(d*x + c))*l
og(cosh(d*x + c) + sinh(d*x + c) - 1) - (((a^4 - a^2*b^2 - 2*b^4)*d*f*x + (
a^4 - a^2*b^2 - 2*b^4)*c*f)*cosh(d*x + c)^4 + 4*((a^4 - a^2*b^2 - 2*b^4)*d*
f*x + (a^4 - a^2*b^2 - 2*b^4)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 -
a^2*b^2 - 2*b^4)*d*f*x + (a^4 - a^2*b^2 - 2*b^4)*c*f)*sinh(d*x + c)^4 + (a^
4 - a^2*b^2 - 2*b^4)*d*f*x + (a^4 - a^2*b^2 - 2*b^4)*c*f - 2*((a^4 - a^2*b^
2 - 2*b^4)*d*f*x + (a^4 - a^2*b^2 - 2*b^4)*c*f)*cosh(d*x + c)^2 - 2*((a^4 -
 a^2*b^2 - 2*b^4)*d*f*x + (a^4 - a^2*b^2 - 2*b^4)*c*f - 3*((a^4 - a^2*b^2 -
 2*b^4)*d*f*x + (a^4 - a^2*b^2 - 2*b^4)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)
^2 + 4*(((a^4 - a^2*b^2 - 2*b^4)*d*f*x + (a^4 - a^2*b^2 - 2*b^4)*c*f)*cosh(
d*x + c)^3 - ((a^4 - a^2*b^2 - 2*b^4)*d*f*x + (a^4 - a^2*b^2 - 2*b^4)*c*f)*
cosh(d*x + c))*sinh(d*x + c))*log(-cosh(d*x + c) - sinh(d*x + c) + 1) - 2*(
(a^4 + a^2*b^2)*d*f*x - 8*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*c*f)*cos
h(d*x + c)^3 + (a^4 + a^2*b^2)*d*e + 3*((a^4 + a^2*b^2)*d*f*x + (a^4 + a^2*
b^2)*d*e + (a^4 + a^2*b^2)*f)*cosh(d*x + c)^2 - (a^4 + a^2*b^2)*f + 4*((a^3
*b + a*b^3)*d*f*x - (a^3*b + a*b^3)*d*e + 2*(a^3*b + a*b^3)*c*f)*cosh(d*x +
 c))*sinh(d*x + c))/((a^5 + a^3*b^2)*d^2*cosh(d*x + c)^4 + 4*(a^5 + a^3*b^2
)*d^2*cosh(d*x + c)*sinh(d*x + c)^3 + (a^5 + a^3*b^2)*d^2*sinh(d*x + c)^4 -
 2*(a^5 + a^3*b^2)*d^2*cosh(d*x + c)^2 + (a^5 + a^3*b^2)*d^2 + 2*(3*(a^5 +
a^3*b^2)*d^2*cosh(d*x + c)^2 - (a^5 + a^3*b^2)*d^2)*sinh(d*x + c)^2 + 4*((a
^5 + a^3*b^2)*d^2*cosh(d*x + c)^3 - (a^5 + a^3*b^2)*d^2*cosh(d*x + c))*sinh
(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.251 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=113

$$\frac{2b^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3d\sqrt{a^2+b^2}}+\frac{\left(a^2-2b^2\right)\tanh^{-1}(\cosh(c+dx))}{2a^3d}+\frac{b\coth(c+dx)}{a^2d}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

```
[Out] ((a^2 - 2*b^2)*ArcTanh[Cosh[c + d*x]])/(2*a^3*d) + (2*b^3*ArcTanh[(b - a*Ta
nh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^3*Sqrt[a^2 + b^2]*d) + (b*Coth[c + d*
x])/(a^2*d) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.404259, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3d\sqrt{a^2+b^2}}+\frac{\left(a^2-2b^2\right)\tanh^{-1}(\cosh(c+dx))}{2a^3d}+\frac{b\coth(c+dx)}{a^2d}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```


```
[Out] ((a^2 - 2*b^2)*ArcTanh[Cosh[c + d*x]])/(2*a^3*d) + (2*b^3*ArcTanh[(b - a*Ta
nh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^3*Sqrt[a^2 + b^2]*d) + (b*Coth[c + d*
x])/(a^2*d) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d)
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
```

```
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} + \frac{i\int \frac{\operatorname{csch}^2(c+dx)\left(2ib+ia\sinh(c+dx)+ib\sinh^2(c+dx)\right)}{a+b\sinh(c+dx)}\,dx}{2a}$$

$$= \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{\int \frac{\operatorname{csch}(c+dx)\left(a^2-2b^2+ab\sinh(c+dx)\right)}{a+b\sinh(c+dx)}\,dx}{2a^2}$$

$$= \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{b^3\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a^3} - \frac{\left(a^2-2b^2\right)\int \operatorname{csch}(c$$

$$= \frac{\left(a^2-2b^2\right)\tanh^{-1}(\cosh(c+dx))}{2a^3d} + \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} + \frac{\left(2ib^3\right)}{}$$

$$= \frac{\left(a^2-2b^2\right)\tanh^{-1}(\cosh(c+dx))}{2a^3d} + \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{\left(4ib^3\right)}{}$$

$$= \frac{\left(a^2-2b^2\right)\tanh^{-1}(\cosh(c+dx))}{2a^3d} + \frac{2b^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}\,d} + \frac{b\coth(c+dx)}{a^2d} - \operatorname{cot}$$

Mathematica [A] time = 1.94352, size = 145, normalized size = 1.28

$$-\frac{4\left(a^2-2b^2\right)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+\frac{16b^3\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)-4ab\tanh}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((16*b^3*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b
^2] - 4*a*b*Coth[(c + d*x)/2] + a^2*Csch[(c + d*x)/2]^2 + 4*(a^2 - 2*b^2)*L
og[Tanh[(c + d*x)/2]] + a^2*Sech[(c + d*x)/2]^2 - 4*a*b*Tanh[(c + d*x)/2])/
(8*a^3*d)
```

Maple [A] time = 0.003, size = 164, normalized size = 1.5

$$\frac{1}{8da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+\frac{b}{2da^2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\frac{b^3}{da^3\sqrt{a^2+b^2}}\operatorname{Artanh}\left(1/2\frac{2a\tanh(1/2dx+c/2)-2b}{\sqrt{a^2+b^2}}\right)-\frac{1}{8da}\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/8/d/a*tanh(1/2*d*x+1/2*c)^2+1/2/d/a^2*tanh(1/2*d*x+1/2*c)*b-2/d/a^3*b^3/(
a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-1
/8/d/a/tanh(1/2*d*x+1/2*c)^2-1/2/d/a*ln(tanh(1/2*d*x+1/2*c))+1/d/a^3*ln(tan
h(1/2*d*x+1/2*c))*b^2+1/2/d*b/a^2/tanh(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.02522, size = 2844, normalized size = 25.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^3*b + 4*a*b^3 + 2*(a^4 + a^2*b^2)*cosh(d*x + c)^3 + 2*(a^4 + a^2*
b^2)*sinh(d*x + c)^3 - 4*(a^3*b + a*b^3)*cosh(d*x + c)^2 - 2*(2*a^3*b + 2*a
*b^3 - 3*(a^4 + a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(b^3*cosh(d*x +
```

```
 c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(d*x + c)^4 - 2*b^3*c
osh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^2 + 4*
(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*lo
g((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2
+ b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*co
sh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
+ 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(a^4
+ a^2*b^2)*cosh(d*x + c) - ((a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^4 + 4*(a^
4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 - a^2*b^2 - 2*b^4
)*sinh(d*x + c)^4 + a^4 - a^2*b^2 - 2*b^4 - 2*(a^4 - a^2*b^2 - 2*b^4)*cosh(
d*x + c)^2 - 2*(a^4 - a^2*b^2 - 2*b^4 - 3*(a^4 - a^2*b^2 - 2*b^4)*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 + 4*((a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^3 - (a^4
 - a^2*b^2 - 2*b^4)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(
d*x + c) + 1) + ((a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^4 + 4*(a^4 - a^2*b^2
 - 2*b^4)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 - a^2*b^2 - 2*b^4)*sinh(d*x
+ c)^4 + a^4 - a^2*b^2 - 2*b^4 - 2*(a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^2
- 2*(a^4 - a^2*b^2 - 2*b^4 - 3*(a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^2)*sin
h(d*x + c)^2 + 4*((a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^3 - (a^4 - a^2*b^2
- 2*b^4)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) -
1) + 2*(a^4 + a^2*b^2 + 3*(a^4 + a^2*b^2)*cosh(d*x + c)^2 - 4*(a^3*b + a*b^
3)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^
5 + a^3*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^5 + a^3*b^2)*d*sinh(d*x +
 c)^4 - 2*(a^5 + a^3*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^5 + a^3*b^2)*d*cosh(d
*x + c)^2 - (a^5 + a^3*b^2)*d)*sinh(d*x + c)^2 + (a^5 + a^3*b^2)*d + 4*((a^
5 + a^3*b^2)*d*cosh(d*x + c)^3 - (a^5 + a^3*b^2)*d*cosh(d*x + c))*sinh(d*x
+ c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2314, size = 248, normalized size = 2.19

$$-\frac{b^3 \log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^3 d} + \frac{\left(a^2-2\,b^2\right)\log\left(e^{(dx+c)}+1\right)}{2\,a^3 d} - \frac{\left(a^2-2\,b^2\right)\log\left(\left|e^{(dx+c)}-1\right|\right)}{2\,a^3 d} - \frac{ae^{(3\,dx+3\,c)}-2\,be^{(2\,dx+2\,c}}{a^2 d\left(e^{(2\,dx+2\,c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c)
 + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3*d) + 1/2*(a^2 - 2*b^2)*lo
g(e^(d*x + c) + 1)/(a^3*d) - 1/2*(a^2 - 2*b^2)*log(abs(e^(d*x + c) - 1))/(a
^3*d) - (a*e^(3*d*x + 3*c) - 2*b*e^(2*d*x + 2*c) + a*e^(d*x + c) + 2*b)/(a^
2*d*(e^(2*d*x + 2*c) - 1)^2)
```

3.252 $\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Csch[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0793883, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Csch[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Csch[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] $Aborted
```

Maple [A] time = 1.941, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx+c))^3}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -8*b^3*integrate(-1/4*e^(d*x + c)/(a^3*b*f*x + a^3*b*e - (a^3*b*f*x*e^(2*c)
 + a^3*b*e*e^(2*c))*e^(2*d*x) - 2*(a^4*f*x*e^c + a^4*e*e^c)*e^(d*x)), x) -
(2*b*d*f*x + 2*b*d*e + (a*d*f*x*e^(3*c) + (d*e - f)*a*e^(3*c))*e^(3*d*x) -
2*(b*d*f*x*e^(2*c) + b*d*e*e^(2*c))*e^(2*d*x) + (a*d*f*x*e^c + (d*e + f)*a*
e^c)*e^(d*x))/(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + a^2*d^2*e^2 + (a^2*d^2*f
^2*x^2*e^(4*c) + 2*a^2*d^2*e*f*x*e^(4*c) + a^2*d^2*e^2*e^(4*c))*e^(4*d*x) -
 2*(a^2*d^2*f^2*x^2*e^(2*c) + 2*a^2*d^2*e*f*x*e^(2*c) + a^2*d^2*e^2*e^(2*c)
)*e^(2*d*x)) - 8*integrate(1/16*(2*b^2*d^2*e^2 + 2*a*b*d*e*f - (d^2*e^2 - 2
*f^2)*a^2 - (a^2*d^2*f^2 - 2*b^2*d^2*f^2)*x^2 - 2*(a^2*d^2*e*f - 2*b^2*d^2*
e*f - a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*
f*x + a^3*d^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*
d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) - 8*integrate(-1/16*(2*b^2*
d^2*e^2 - 2*a*b*d*e*f - (d^2*e^2 - 2*f^2)*a^2 - (a^2*d^2*f^2 - 2*b^2*d^2*f^
2)*x^2 - 2*(a^2*d^2*e*f - 2*b^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 +
3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^
c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d
*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{csch}\left(dx+c\right)^{3}}{afx+ae+\left(bfx+be\right)\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(csch(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.253 $\int \frac{(e+fx)^3 \cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=139

$$\frac{12if^2(e+fx)\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^3}-\frac{6if(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2}-\frac{12if^3\text{PolyLog}\left(4,-ie^{c+dx}\right)}{ad^4}-\frac{2i(e+fx)^3\text{lo}}{a}$$

```
[Out] ((I/4)*(e + f*x)^4)/(a*f) - ((2*I)*(e + f*x)^3*Log[1 + I*E^(c + d*x)])/(a*d
) - ((6*I)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) + ((12*I)*f^
2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^3) - ((12*I)*f^3*PolyLog[4,
(-I)*E^(c + d*x)])/(a*d^4)
```

Rubi [A] time = 0.212735, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {5559, 2190, 2531, 6609, 2282, 6589}

$$\frac{12if^2(e+fx)\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^3}-\frac{6if(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2}-\frac{12if^3\text{PolyLog}\left(4,-ie^{c+dx}\right)}{ad^4}-\frac{2i(e+fx)^3\text{lo}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((I/4)*(e + f*x)^4)/(a*f) - ((2*I)*(e + f*x)^3*Log[1 + I*E^(c + d*x)])/(a*d
) - ((6*I)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) + ((12*I)*f^
2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^3) - ((12*I)*f^3*PolyLog[4,
(-I)*E^(c + d*x)])/(a*d^4)
```

Rule 5559

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + Dist[2, Int[((e + f*x)^m*E^(c + d*x))/(a + b*E^(c + d*x)), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
```

```
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i(e+fx)^4}{4af}+2\int \frac{e^{c+dx}(e+fx)^3}{a+iae^{c+dx}}\,dx$$
$$= \frac{i(e+fx)^4}{4af}-\frac{2i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad}+\frac{(6if)\int (e+fx)^2\log\left(1+ie^{c+dx}\right)dx}{ad}$$
$$= \frac{i(e+fx)^4}{4af}-\frac{2i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad}-\frac{6if(e+fx)^2\text{Li}_2\left(-ie^{c+dx}\right)}{ad^2}+\frac{\left(12if^2\right)\int (e+fx)}{ad}$$
$$= \frac{i(e+fx)^4}{4af}-\frac{2i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad}-\frac{6if(e+fx)^2\text{Li}_2\left(-ie^{c+dx}\right)}{ad^2}+\frac{12if^2(e+fx)\text{Li}_3}{ad^3}$$
$$= \frac{i(e+fx)^4}{4af}-\frac{2i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad}-\frac{6if(e+fx)^2\text{Li}_2\left(-ie^{c+dx}\right)}{ad^2}+\frac{12if^2(e+fx)\text{Li}_3}{ad^3}$$
$$= \frac{i(e+fx)^4}{4af}-\frac{2i(e+fx)^3\log\left(1+ie^{c+dx}\right)}{ad}-\frac{6if(e+fx)^2\text{Li}_2\left(-ie^{c+dx}\right)}{ad^2}+\frac{12if^2(e+fx)\text{Li}_3}{ad^3}$$

Mathematica [A] time = 0.0779278, size = 118, normalized size = 0.85

$$\frac{i\left(-\frac{24f\left(d^2(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)-2df(e+fx)\text{PolyLog}\left(3,-ie^{c+dx}\right)+2f^2\text{PolyLog}\left(4,-ie^{c+dx}\right)\right)}{d^4}-\frac{8(e+fx)^3\log\left(1+ie^{c+dx}\right)}{d}+\frac{(e+fx)^4}{f}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((I/4)*((e + f*x)^4/f - (8*(e + f*x)^3*Log[1 + I*E^(c + d*x)])/d - (24*f*(d
^2*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)] - 2*d*f*(e + f*x)*PolyLog[3, (-
I)*E^(c + d*x)] + 2*f^2*PolyLog[4, (-I)*E^(c + d*x)]))/d^4))/a
```

Maple [B] time = 0.118, size = 635, normalized size = 4.6

$$\frac{ief^2x^3}{a}+\frac{6\,ie^2fcx}{da}-\frac{6\,ief^2c^2x}{ad^2}-\frac{6\,ie^2fc\ln\left(e^{dx+c}\right)}{ad^2}+\frac{6\,ief^2c^2\ln\left(e^{dx+c}\right)}{ad^3}+\frac{6\,ie^2fc\ln\left(e^{dx+c}-i\right)}{ad^2}-\frac{6\,ief^2c^2\ln\left(e^{dx+c}-i\right)}{ad^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] I/a*e*f^2*x^3+6*I/a/d*e^2*f*c*x-6*I/a/d^2*e*f^2*c^2*x-6*I/a/d^2*e^2*f*c*ln(
exp(d*x+c))+6*I/a/d^3*e*f^2*c^2*ln(exp(d*x+c))+6*I/a/d^2*e^2*f*c*ln(exp(d*x
+c)-I)-6*I/a/d^3*e*f^2*c^2*ln(exp(d*x+c)-I)-6*I/a/d*e^2*f*ln(1+I*exp(d*x+c)
)*x-6*I/a/d^2*e^2*f*ln(1+I*exp(d*x+c))*c-6*I/a/d*e*f^2*ln(1+I*exp(d*x+c))*x
^2+6*I/a/d^3*e*f^2*ln(1+I*exp(d*x+c))*c^2-12*I/a/d^2*e*f^2*polylog(2,-I*exp
(d*x+c))*x-12*I*f^3*polylog(4,-I*exp(d*x+c))/a/d^4+1/4*I/a*x^4*f^3+3/2*I/a*
e^2*f*x^2-I/a*e^3*x+2*I/a/d^3*f^3*c^3*x+2*I/a/d^4*f^3*c^3*ln(exp(d*x+c)-I)-
4*I/a/d^3*e*f^2*c^3+3*I/a/d^2*e^2*f*c^2-6*I/a/d^2*e^2*f*polylog(2,-I*exp(d*
x+c))+12*I/a/d^3*e*f^2*polylog(3,-I*exp(d*x+c))-2*I/a/d^4*f^3*c^3*ln(exp(d*
x+c))-2*I/a/d^4*f^3*c^3*ln(1+I*exp(d*x+c))-2*I/a/d*f^3*ln(1+I*exp(d*x+c))*x
^3-6*I/a/d^2*f^3*polylog(2,-I*exp(d*x+c))*x^2+12*I/a/d^3*f^3*polylog(3,-I*e
xp(d*x+c))*x-2*I/a/d*ln(exp(d*x+c)-I)*e^3+3/2*I/a/d^4*f^3*c^4+2*I/a/d*ln(ex
p(d*x+c))*e^3
```

Maxima [B] time = 1.64123, size = 356, normalized size = 2.56

$$-\frac{i e^3 \log\left(i a \sinh\left(dx+c\right)+a\right)}{ad}-\frac{6i\left(dx\log\left(i e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(-i e^{(dx+c)}\right)\right)e^2 f}{ad^2}-\frac{i\left(f^3x^4+4ef^2x^3+6e^2fx^2\right)}{4a}-\frac{6i\left(d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -I*e^3*log(I*a*sinh(d*x + c) + a)/(a*d) - 6*I*(d*x*log(I*e^(d*x + c) + 1) +
 dilog(-I*e^(d*x + c)))*e^2*f/(a*d^2) - 1/4*I*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^
2*f*x^2)/a - 6*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x +
c)) - 2*polylog(3, -I*e^(d*x + c)))*e*f^2/(a*d^3) - 2*I*(d^3*x^3*log(I*e^(d
*x + c) + 1) + 3*d^2*x^2*dilog(-I*e^(d*x + c)) - 6*d*x*polylog(3, -I*e^(d*x
 + c)) + 6*polylog(4, -I*e^(d*x + c)))*f^3/(a*d^4) + 1/2*(I*d^4*f^3*x^4 + 4
*I*d^4*e*f^2*x^3 + 6*I*d^4*e^2*f*x^2)/(a*d^4)
```

Fricas [C] time = 2.17348, size = 752, normalized size = 5.41

$$\frac{i d^4 f^3 x^4+4i d^4 e f^2 x^3+6i d^4 e^2 f x^2+4i d^4 e^3 x+8i c d^3 e^3-12i c^2 d^2 e^2 f+8i c^3 d e f^2-2i c^4 f^3-48i f^3 \mathrm{polylog}\left(4,-i e^{(dx+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(I*d^4*f^3*x^4 + 4*I*d^4*e*f^2*x^3 + 6*I*d^4*e^2*f*x^2 + 4*I*d^4*e^3*x
+ 8*I*c*d^3*e^3 - 12*I*c^2*d^2*e^2*f + 8*I*c^3*d*e*f^2 - 2*I*c^4*f^3 - 48*I
*f^3*polylog(4, -I*e^(d*x + c)) + (-24*I*d^2*f^3*x^2 - 48*I*d^2*e*f^2*x - 2
4*I*d^2*e^2*f)*dilog(-I*e^(d*x + c)) + (-8*I*d^3*e^3 + 24*I*c*d^2*e^2*f - 2
4*I*c^2*d*e*f^2 + 8*I*c^3*f^3)*log(e^(d*x + c) - I) + (-8*I*d^3*f^3*x^3 - 2
4*I*d^3*e*f^2*x^2 - 24*I*d^3*e^2*f*x - 24*I*c*d^2*e^2*f + 24*I*c^2*d*e*f^2
- 8*I*c^3*f^3)*log(I*e^(d*x + c) + 1) + (48*I*d*f^3*x + 48*I*d*e*f^2)*polyl
og(3, -I*e^(d*x + c)))/(a*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \cosh(dx+c)}{i\,a\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.254 $\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=106

$$-\frac{4if(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2}+\frac{4if^2\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^3}-\frac{2i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad}+\frac{i(e+fx)^3}{3af}$$

```
[Out] ((I/3)*(e + f*x)^3)/(a*f) - ((2*I)*(e + f*x)^2*Log[1 + I*E^(c + d*x)])/(a*d
) - ((4*I)*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) + ((4*I)*f^2*P
olyLog[3, (-I)*E^(c + d*x)])/(a*d^3)
```

Rubi [A] time = 0.184023, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {5559, 2190, 2531, 2282, 6589}

$$-\frac{4if(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2}+\frac{4if^2\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^3}-\frac{2i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad}+\frac{i(e+fx)^3}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((I/3)*(e + f*x)^3)/(a*f) - ((2*I)*(e + f*x)^2*Log[1 + I*E^(c + d*x)])/(a*d
) - ((4*I)*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) + ((4*I)*f^2*P
olyLog[3, (-I)*E^(c + d*x)])/(a*d^3)
```

Rule 5559

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + Dist[2, Int[((e + f*x)^m*E^(c + d*x))/(a + b*E^(c + d*x)), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i(e+fx)^3}{3af} + 2\int \frac{e^{c+dx}(e+fx)^2}{a+iae^{c+dx}}\,dx$$
$$= \frac{i(e+fx)^3}{3af} - \frac{2i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad} + \frac{(4if)\int (e+fx)\log\left(1+ie^{c+dx}\right)\,dx}{ad}$$
$$= \frac{i(e+fx)^3}{3af} - \frac{2i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad} - \frac{4if(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{ad^2} + \frac{\left(4if^2\right)\int \text{Li}_2\left(-ie^{c+}\right.}{ad^2}$$
$$= \frac{i(e+fx)^3}{3af} - \frac{2i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad} - \frac{4if(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{ad^2} + \frac{\left(4if^2\right)\text{Subst}\left(\int\right.}{ad}$$
$$= \frac{i(e+fx)^3}{3af} - \frac{2i(e+fx)^2\log\left(1+ie^{c+dx}\right)}{ad} - \frac{4if(e+fx)\text{Li}_2\left(-ie^{c+dx}\right)}{ad^2} + \frac{4if^2\text{Li}_3\left(-ie^{c+dx}\right)}{ad^3}$$

Mathematica [A] time = 0.0498098, size = 94, normalized size = 0.89

$$\frac{i\left(-12df^2(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)+12f^3\text{PolyLog}\left(3,-ie^{c+dx}\right)+d^2(e+fx)^2\left(d(e+fx)-6f\log\left(1+ie^{c+dx}\right)\right)\right)}{3ad^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((I/3)*(d^2*(e + f*x)^2*(d*(e + f*x) - 6*f*Log[1 + I*E^(c + d*x)]) - 12*d*f
^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)] + 12*f^3*PolyLog[3, (-I)*E^(c + d
*x)]))/(a*d^3*f)
```

Maple [B] time = 0.073, size = 393, normalized size = 3.7

$$\frac{-2\,if^2\ln\left(1+ie^{dx+c}\right)x^2}{da} + \frac{2\,iefc^2}{ad^2} - \frac{4\,if^2\text{polylog}\left(2,-ie^{dx+c}\right)x}{ad^2} - \frac{2\,i\ln\left(e^{dx+c}-i\right)e^2}{da} - \frac{2\,if^2c^2x}{ad^2} + \frac{4\,if^2\text{polylog}\left(3,-ie^{d}\right.}{ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -2*I/a/d*f^2*ln(1+I*exp(d*x+c))*x^2+2*I/a/d^2*e*f*c^2-4*I/a/d^2*f^2*polylog
(2,-I*exp(d*x+c))*x-2*I/a/d*ln(exp(d*x+c)-I)*e^2-2*I/a/d^2*f^2*c^2*x+4*I*f^
2*polylog(3,-I*exp(d*x+c))/a/d^3-2*I/a/d^3*f^2*c^2*ln(exp(d*x+c)-I)+2*I/a/d
*ln(exp(d*x+c))*e^2-I/a*e^2*x-4*I/a/d^2*e*f*c*ln(exp(d*x+c))-4*I/a/d*e*f*ln
(1+I*exp(d*x+c))*x+1/3*I/a*x^3*f^2-4*I/a/d^2*e*f*ln(1+I*exp(d*x+c))*c-4*I/a
/d^2*e*f*polylog(2,-I*exp(d*x+c))+2*I/a/d^3*f^2*c^2*ln(exp(d*x+c))-4/3*I/a/
d^3*f^2*c^3+4*I/a/d^2*e*f*c*ln(exp(d*x+c)-I)+I/a*e*f*x^2+4*I/a/d*e*f*c*x+2*
```

```
I/a/d^3*f^2*c^2*ln(1+I*exp(d*x+c))
```

Maxima [A] time = 1.56212, size = 221, normalized size = 2.08

$$-\frac{ie^2 \log (ia \sinh (dx+c)+a)}{ad} - \frac{if^2x^3+3iefx^2}{3a} - \frac{4i\left(dx \log \left(ie^{(dx+c)}+1\right)+\mathrm{Li}_2\left(-ie^{(dx+c)}\right)\right)ef}{ad^2} - \frac{2i\left(d^2x^2 \log \left(ie^{(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -I*e^2*log(I*a*sinh(d*x + c) + a)/(a*d) - 1/3*(I*f^2*x^3 + 3*I*e*f*x^2)/a -
 4*I*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*e*f/(a*d^2) - 2*I
*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x + c)) - 2*polylog(
3, -I*e^(d*x + c)))*f^2/(a*d^3) + 1/3*(2*I*d^3*f^2*x^3 + 6*I*d^3*e*f*x^2)/(
a*d^3)
```

Fricas [C] time = 2.22861, size = 482, normalized size = 4.55

$$\frac{id^3f^2x^3+3id^3efx^2+3id^3e^2x+6icd^2e^2-6ic^2def+2ic^3f^2+12if^2\mathrm{polylog}\left(3,-ie^{(dx+c)}\right)+\left(-12idf^2x-12idef\right)\mathrm{L}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(I*d^3*f^2*x^3 + 3*I*d^3*e*f*x^2 + 3*I*d^3*e^2*x + 6*I*c*d^2*e^2 - 6*I*
c^2*d*e*f + 2*I*c^3*f^2 + 12*I*f^2*polylog(3, -I*e^(d*x + c)) + (-12*I*d*f^
2*x - 12*I*d*e*f)*dilog(-I*e^(d*x + c)) + (-6*I*d^2*e^2 + 12*I*c*d*e*f - 6*
I*c^2*f^2)*log(e^(d*x + c) - I) + (-6*I*d^2*f^2*x^2 - 12*I*d^2*e*f*x - 12*I
*c*d*e*f + 6*I*c^2*f^2)*log(I*e^(d*x + c) + 1))/(a*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^2 \cosh (c+dx)}{i \sinh (c+dx)+1}\,dx + \int \frac{f^2x^2 \cosh (c+dx)}{i \sinh (c+dx)+1}\,dx + \int \frac{2efx \cosh (c+dx)}{i \sinh (c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] (Integral(e**2*cosh(c + d*x)/(I*sinh(c + d*x) + 1), x) + Integral(f**2*x**2
*cosh(c + d*x)/(I*sinh(c + d*x) + 1), x) + Integral(2*e*f*x*cosh(c + d*x)/(
I*sinh(c + d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \cosh (dx+c)}{ia \sinh (dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.255 $\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=73

$$-\frac{2if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2}-\frac{2i(e+fx)\log\left(1+ie^{c+dx}\right)}{ad}+\frac{i(e+fx)^2}{2af}$$

```
[Out] ((I/2)*(e + f*x)^2)/(a*f) - ((2*I)*(e + f*x)*Log[1 + I*E^(c + d*x)])/(a*d)
- ((2*I)*f*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2)
```

Rubi [A] time = 0.111111, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5559, 2190, 2279, 2391}

$$-\frac{2if\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2}-\frac{2i(e+fx)\log\left(1+ie^{c+dx}\right)}{ad}+\frac{i(e+fx)^2}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((I/2)*(e + f*x)^2)/(a*f) - ((2*I)*(e + f*x)*Log[1 + I*E^(c + d*x)])/(a*d)
- ((2*I)*f*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2)
```

Rule 5559

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + Dist[2, Int[((e + f*x)^m*E^(c + d*x))/(a + b*E^(c + d*x)), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx = \frac{i(e+fx)^2}{2af} + 2\int \frac{e^{c+dx}(e+fx)}{a+iae^{c+dx}}\,dx$$
$$= \frac{i(e+fx)^2}{2af} - \frac{2i(e+fx)\log\left(1+ie^{c+dx}\right)}{ad} + \frac{(2if)\int \log\left(1+ie^{c+dx}\right)dx}{ad}$$
$$= \frac{i(e+fx)^2}{2af} - \frac{2i(e+fx)\log\left(1+ie^{c+dx}\right)}{ad} + \frac{(2if)\,\mathrm{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{c+dx}\right)}{ad^2}$$
$$= \frac{i(e+fx)^2}{2af} - \frac{2i(e+fx)\log\left(1+ie^{c+dx}\right)}{ad} - \frac{2if\,\mathrm{Li}_2\left(-ie^{c+dx}\right)}{ad^2}$$

Mathematica [A] time = 0.0258735, size = 66, normalized size = 0.9

$$\frac{i\left(d(e+fx)\left(d(e+fx)-4f\log\left(1+ie^{c+dx}\right)\right)-4f^2\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)\right)}{2ad^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((I/2)*(d*(e + f*x)*(d*(e + f*x) - 4*f*Log[1 + I*E^(c + d*x)]) - 4*f^2*Poly
Log[2, (-I)*E^(c + d*x)]))/(a*d^2*f)
```

Maple [B] time = 0.073, size = 188, normalized size = 2.6

$$\frac{\frac{i}{2}fx^2}{a} - \frac{iex}{a} - \frac{2i\ln\left(e^{dx+c}-i\right)e}{da} + \frac{2i\ln\left(e^{dx+c}\right)e}{da} + \frac{2ifcx}{da} + \frac{ifc^2}{ad^2} - \frac{2if\ln\left(1+ie^{dx+c}\right)x}{da} - \frac{2if\ln\left(1+ie^{dx+c}\right)c}{ad^2} - \frac{2if\mathrm{pc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 1/2*I*f*x^2/a-I*e*x/a-2*I/d/a*ln(exp(d*x+c)-I)*e+2*I/d/a*ln(exp(d*x+c))*e+2
*I/d/a*f*c*x+I/d^2/a*f*c^2-2*I/d/a*f*ln(1+I*exp(d*x+c))*x-2*I/d^2/a*f*ln(1+
I*exp(d*x+c))*c-2*I*f*polylog(2,-I*exp(d*x+c))/a/d^2+2*I/d^2/a*f*c*ln(exp(d
*x+c)-I)-2*I/d^2/a*f*c*ln(exp(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}f\left(-\frac{ix^2}{a} + 4\int \frac{x}{ae^{(dx+c)}-ia}\,dx\right) - \frac{ie\log\left(ia\sinh\left(dx+c\right)+a\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*f*(-I*x^2/a + 4*integrate(x/(a*e^(d*x + c) - I*a), x)) - I*e*log(I*a*si
nh(d*x + c) + a)/(a*d)
```

Fricas [A] time = 2.18812, size = 252, normalized size = 3.45

$$\frac{i\,d^2fx^2+2i\,d^2ex+4i\,cde-2i\,c^2f-4i\,f\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)+\left(-4i\,de+4i\,cf\right)\log\left(e^{(dx+c)}-i\right)+\left(-4i\,dfx-4i\,cf\right)\log\left(i\,e^{(dx}\right.}{2\,ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*d^2*f*x^2 + 2*I*d^2*e*x + 4*I*c*d*e - 2*I*c^2*f - 4*I*f*dilog(-I*e^(
d*x + c)) + (-4*I*d*e + 4*I*c*f)*log(e^(d*x + c) - I) + (-4*I*d*f*x - 4*I*c
*f)*log(I*e^(d*x + c) + 1))/(a*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{e\cosh(c+dx)}{i\sinh(c+dx)+1}\,dx+\int\frac{fx\cosh(c+dx)}{i\sinh(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] (Integral(e*cosh(c + d*x)/(I*sinh(c + d*x) + 1), x) + Integral(f*x*cosh(c +
 d*x)/(I*sinh(c + d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(fx+e\right)\cosh\left(dx+c\right)}{i\,a\sinh\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.256 $\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=23

$$-\frac{i\log(-\sinh(c+dx)+i)}{ad}$$

```
[Out] ((-I)*Log[I - Sinh[c + d*x]])/(a*d)
```

Rubi [A] time = 0.0273148, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2667, 31}

$$-\frac{i\log(-\sinh(c+dx)+i)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-I)*Log[I - Sinh[c + d*x]])/(a*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, ia\sinh(c+dx)\right)}{ad}\\ &= -\frac{i\log(i-\sinh(c+dx))}{ad}\end{aligned}$$

Mathematica [A] time = 0.0145465, size = 23, normalized size = 1.

$$-\frac{i\log(-\sinh(c+dx)+i)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-I)*Log[I - Sinh[c + d*x]])/(a*d)
```

Maple [A] time = 0.013, size = 23, normalized size = 1.

$$\frac{-i\ln\left(a+ia\sinh\left(dx+c\right)\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I/d*ln(a+I*a*sinh(d*x+c))/a
```

Maxima [A] time = 1.12375, size = 27, normalized size = 1.17

$$-\frac{i\log\left(i\,a\sinh\left(dx+c\right)+a\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -I*log(I*a*sinh(d*x + c) + a)/(a*d)
```

Fricas [A] time = 2.18425, size = 57, normalized size = 2.48

$$\frac{i\,dx-2i\,\log\left(e^{(dx+c)}-i\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (I*d*x - 2*I*log(e^(d*x + c) - I))/(a*d)
```

Sympy [A] time = 4.53724, size = 22, normalized size = 0.96

$$\frac{ix}{a}-\frac{2i\log\left(e^{dx}-ie^{-c}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] I*x/a - 2*I*log(exp(d*x) - I*exp(-c))/(a*d)
```

Giac [A] time = 1.23608, size = 45, normalized size = 1.96

$$\frac{i\left(dx+c\right)}{ad}-\frac{2i\,\log\left(i\,e^{(dx+c)}+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] I*(d*x + c)/(a*d) - 2*I*log(I*e^(d*x + c) + 1)/(a*d)
```

3.257 $\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Cosh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0491233, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cosh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Cosh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Mathematica [A] time = 27.5074, size = 0, normalized size = 0.

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cosh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Cosh[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Maple [A] time = 0.088, size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{\left(fx+e\right)(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{i \log\left(fx+e\right)}{af} + 2\int \frac{1}{-i\,afx - i\,ae + \left(afxe^{c} + aee^{c}\right)e^{(dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -I*log(f*x + e)/(a*f) + 2*integrate(1/(-I*a*f*x - I*a*e + (a*f*x*e^c + a*e*
e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-i\,e^{(dx+c)}+1}{-i\,afx - i\,ae + \left(afx + ae\right)e^{(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((-I*e^(d*x + c) + 1)/(-I*a*f*x - I*a*e + (a*f*x + a*e)*e^(d*x + c)
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(dx+c\right)}{\left(fx+e\right)\left(i\,a\sinh\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

3.258 $\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Cosh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0492806, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cosh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Cosh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Mathematica [A] time = 32.4266, size = 0, normalized size = 0.

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cosh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Cosh[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Maple [A] time = 0.09, size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(fx+e)^2\,(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{i}{af^2x+aef}+2\int\frac{1}{-i\,af^2x^2-2i\,aefx-i\,ae^2+\left(af^2x^2e^c+2\,aefxe^c+ae^2e^c\right)e^{(dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] I/(a*f^2*x + a*e*f) + 2*integrate(1/(-I*a*f^2*x^2 - 2*I*a*e*f*x - I*a*e^2 +
 (a*f^2*x^2*e^c + 2*a*e*f*x*e^c + a*e^2*e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-i\,e^{(dx+c)}+1}{-i\,af^2x^2-2i\,aefx-i\,ae^2+\left(af^2x^2+2\,aefx+ae^2\right)e^{(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((-I*e^(d*x + c) + 1)/(-I*a*f^2*x^2 - 2*I*a*e*f*x - I*a*e^2 + (a*f^
2*x^2 + 2*a*e*f*x + a*e^2)*e^(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh(dx+c)}{(fx+e)^2(i\,a\sinh(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

3.259 $\int \frac{(e+fx)^3 \cosh^2(c+dx)}{a+ia\sinh(c+dx)} dx$

Optimal. Leaf size=108

$$-\frac{6if^2(e+fx)\cosh(c+dx)}{ad^3}+\frac{3if(e+fx)^2\sinh(c+dx)}{ad^2}+\frac{6if^3\sinh(c+dx)}{ad^4}-\frac{i(e+fx)^3\cosh(c+dx)}{ad}+\frac{(e+fx)^4}{4af}$$

```
[Out] (e + f*x)^4/(4*a*f) - ((6*I)*f^2*(e + f*x)*Cosh[c + d*x])/(a*d^3) - (I*(e +
 f*x)^3*Cosh[c + d*x])/(a*d) + ((6*I)*f^3*Sinh[c + d*x])/(a*d^4) + ((3*I)*f
*(e + f*x)^2*Sinh[c + d*x])/(a*d^2)
```

Rubi [A] time = 0.163624, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {5563, 32, 3296, 2637}

$$-\frac{6if^2(e+fx)\cosh(c+dx)}{ad^3}+\frac{3if(e+fx)^2\sinh(c+dx)}{ad^2}+\frac{6if^3\sinh(c+dx)}{ad^4}-\frac{i(e+fx)^3\cosh(c+dx)}{ad}+\frac{(e+fx)^4}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (e + f*x)^4/(4*a*f) - ((6*I)*f^2*(e + f*x)*Cosh[c + d*x])/(a*d^3) - (I*(e +
 f*x)^3*Cosh[c + d*x])/(a*d) + ((6*I)*f^3*Sinh[c + d*x])/(a*d^4) + ((3*I)*f
*(e + f*x)^2*Sinh[c + d*x])/(a*d^2)
```

Rule 5563

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cosh[c
+ d*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Si
nh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ
[a^2 + b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int (e+fx)^3\sinh(c+dx)\,dx}{a} + \frac{\int (e+fx)^3\,dx}{a}$$
$$= \frac{(e+fx)^4}{4af} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + \frac{(3if)\int (e+fx)^2\cosh(c+dx)\,dx}{ad}$$
$$= \frac{(e+fx)^4}{4af} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + \frac{3if(e+fx)^2\sinh(c+dx)}{ad^2} - \frac{(6if^2)\int (e+fx)\sin}{ad^2}$$
$$= \frac{(e+fx)^4}{4af} - \frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + \frac{3if(e+fx)^2\sinh(c}{ad^2}$$
$$= \frac{(e+fx)^4}{4af} - \frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + \frac{6if^3\sinh(c+dx)}{ad^4} +$$

Mathematica [A] time = 0.779149, size = 106, normalized size = 0.98

$$\frac{12if\sinh(c+dx)\left(d^2(e+fx)^2+2f^2\right)-4id(e+fx)\cosh(c+dx)\left(d^2(e+fx)^2+6f^2\right)+d^4x\left(6e^2fx+4e^3+4ef^2x^2+f^3x\right.}{4ad^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) - (4*I)*d*(e + f*x)*(6*f
^2 + d^2*(e + f*x)^2)*Cosh[c + d*x] + (12*I)*f*(2*f^2 + d^2*(e + f*x)^2)*Si
nh[c + d*x])/(4*a*d^4)
```

Maple [B] time = 0.045, size = 448, normalized size = 4.2

$$-\frac{1}{d^4a}\left(id^3e^3\cosh(dx+c)+3\,ic^2f^2ed\cosh(dx+c)+if^3\left((dx+c)^3\cosh(dx+c)-3\,(dx+c)^2\sinh(dx+c)+6\,(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -1/d^4/a*(I*d^3*e^3*cosh(d*x+c)+3*I*c^2*f^2*e*d*cosh(d*x+c)+I*f^3*((d*x+c)^
3*cosh(d*x+c)-3*(d*x+c)^2*sinh(d*x+c)+6*(d*x+c)*cosh(d*x+c)-6*sinh(d*x+c))-
6*I*f^2*c*e*d*((d*x+c)*cosh(d*x+c)-sinh(d*x+c))+3*I*f^2*e*d*((d*x+c)^2*cosh
(d*x+c)-2*(d*x+c)*sinh(d*x+c)+2*cosh(d*x+c))+3*I*f*e^2*d^2*((d*x+c)*cosh(d*
x+c)-sinh(d*x+c))-3*I*c*f^3*((d*x+c)^2*cosh(d*x+c)-2*(d*x+c)*sinh(d*x+c)+2*
cosh(d*x+c))-I*f^3*c^3*cosh(d*x+c)+3*I*c^2*f^3*((d*x+c)*cosh(d*x+c)-sinh(d*
x+c))-3*I*c*f*e^2*d^2*cosh(d*x+c)-1/4*f^3*(d*x+c)^4+c*f^3*(d*x+c)^3-f^2*e*d
*(d*x+c)^3-3/2*c^2*f^3*(d*x+c)^2+3*c*d*e*f^2*(d*x+c)^2-3/2*d^2*e^2*f*(d*x+c
)^2+f^3*c^3*(d*x+c)-3*c^2*f^2*e*d*(d*x+c)+3*c*f*e^2*d^2*(d*x+c)-d^3*e^3*(d*
x+c))
```

Maxima [B] time = 2.02196, size = 504, normalized size = 4.67

$$\frac{3}{4}e^2f\left(\frac{4\,xe^{(dx+c)}}{ade^{(dx+c)}-i\,ad}+\frac{-2i\,d^2x^2e^c-2i\,dxe^c-\left(2i\,dxe^{(3\,c)}-2i\,e^{(3\,c)}\right)e^{(2\,dx)}+2\left(d^2x^2e^{(2\,c)}-3\,dxe^{(2\,c)}+e^{(2\,c)}\right)e^{(dx)}-2\,(dx+1}{ad^2e^{(dx+2\,c)}-i\,ad^2e^c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 3/4*e^2*f*(4*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) + (-2*I*d^2*x^2*e^c -
2*I*d*x*e^c - (2*I*d*x*e^(3*c) - 2*I*e^(3*c))*e^(2*d*x) + 2*(d^2*x^2*e^(2*c
) - 3*d*x*e^(2*c) + e^(2*c))*e^(d*x) - 2*(d*x + 1)*e^(-d*x) - 2*I*e^c)/(a*d
^2*e^(d*x + 2*c) - I*a*d^2*e^c)) + 1/4*e^3*(4*(d*x + c)/(a*d) - 2*I*e^(d*x
+ c)/(a*d) - 2*I*e^(-d*x - c)/(a*d)) + 1/4*(4*d^3*x^3*e^c - (6*I*d^2*x^2*e^
(2*c) - 12*I*d*x*e^(2*c) + 12*I*e^(2*c))*e^(d*x) - (6*I*d^2*x^2 + 12*I*d*x
+ 12*I)*e^(-d*x))*e*f^2*e^(-c)/(a*d^3) + 1/4*(d^4*x^4*e^c - (2*I*d^3*x^3*e^
(2*c) - 6*I*d^2*x^2*e^(2*c) + 12*I*d*x*e^(2*c) - 12*I*e^(2*c))*e^(d*x) - (2
*I*d^3*x^3 + 6*I*d^2*x^2 + 12*I*d*x + 12*I)*e^(-d*x))*f^3*e^(-c)/(a*d^4)
```

Fricas [B] time = 2.21788, size = 605, normalized size = 5.6

$$\frac{\left(-2i\,d^3f^3x^3 - 2i\,d^3e^3 - 6i\,d^2e^2f - 12i\,def^2 - 12i\,f^3 + \left(-6i\,d^3ef^2 - 6i\,d^2f^3\right)x^2 + \left(-6i\,d^3e^2f - 12i\,d^2ef^2 - 12i\,df^3\right)x + \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/4*(-2*I*d^3*f^3*x^3 - 2*I*d^3*e^3 - 6*I*d^2*e^2*f - 12*I*d*e*f^2 - 12*I*f
^3 + (-6*I*d^3*e*f^2 - 6*I*d^2*f^3)*x^2 + (-6*I*d^3*e^2*f - 12*I*d^2*e*f^2
- 12*I*d*f^3)*x + (-2*I*d^3*f^3*x^3 - 2*I*d^3*e^3 + 6*I*d^2*e^2*f - 12*I*d*
e*f^2 + 12*I*f^3 + (-6*I*d^3*e*f^2 + 6*I*d^2*f^3)*x^2 + (-6*I*d^3*e^2*f + 1
2*I*d^2*e*f^2 - 12*I*d*f^3)*x)*e^(2*d*x + 2*c) + (d^4*f^3*x^4 + 4*d^4*e*f^2
*x^3 + 6*d^4*e^2*f*x^2 + 4*d^4*e^3*x)*e^(d*x + c))*e^(-d*x - c)/(a*d^4)
```

Sympy [A] time = 2.66624, size = 610, normalized size = 5.65

$$\begin{cases} \frac{\left(\left(-2ia^7d^{19}e^3e^{3c} - 6ia^7d^{19}e^2fxe^{3c} - 6ia^7d^{19}ef^2x^2e^{3c} - 2ia^7d^{19}f^3x^3e^{3c} - 6ia^7d^{18}e^2fe^{3c} - 12ia^7d^{18}ef^2xe^{3c} - 6ia^7d^{18}f^3x^2e^{3c} - 12ia^7d^{17}ef^2e^{3c} - 12ia^7d^{17}f^3xe^{3c} - 12ia^7 \cdots\right.\right.}{\cdots} \\ -\frac{x^4\left(if^3e^{2c} - if^3\right)e^{-c}}{8a} - \frac{x^3\left(ief^2e^{2c} - ief^2\right)e^{-c}}{2a} - \frac{x^2\left(3ie^2fe^{2c} - 3ie^2f\right)e^{-c}}{4a} - \frac{x\left(ie^3e^{2c} - ie^3\right)e^{-c}}{2a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise((((-2*I*a**7*d**19*e**3*exp(3*c) - 6*I*a**7*d**19*e**2*f*x*exp(3*
c) - 6*I*a**7*d**19*e*f**2*x**2*exp(3*c) - 2*I*a**7*d**19*f**3*x**3*exp(3*c
) - 6*I*a**7*d**18*e**2*f*exp(3*c) - 12*I*a**7*d**18*e*f**2*x*exp(3*c) - 6*
I*a**7*d**18*f**3*x**2*exp(3*c) - 12*I*a**7*d**17*e*f**2*exp(3*c) - 12*I*a*
*7*d**17*f**3*x*exp(3*c) - 12*I*a**7*d**16*f**3*exp(3*c))*exp(-d*x) + (-2*I
*a**7*d**19*e**3*exp(5*c) - 6*I*a**7*d**19*e**2*f*x*exp(5*c) - 6*I*a**7*d**
19*e*f**2*x**2*exp(5*c) - 2*I*a**7*d**19*f**3*x**3*exp(5*c) + 6*I*a**7*d**1
8*e**2*f*exp(5*c) + 12*I*a**7*d**18*e*f**2*x*exp(5*c) + 6*I*a**7*d**18*f**3
*x**2*exp(5*c) - 12*I*a**7*d**17*e*f**2*exp(5*c) - 12*I*a**7*d**17*f**3*x*e
xp(5*c) + 12*I*a**7*d**16*f**3*exp(5*c))*exp(d*x))*exp(-4*c)/(4*a**8*d**20)
, Ne(4*a**8*d**20*exp(4*c), 0)), (-x**4*(I*f**3*exp(2*c) - I*f**3)*exp(-c)/
(8*a) - x**3*(I*e*f**2*exp(2*c) - I*e*f**2)*exp(-c)/(2*a) - x**2*(3*I*e**2*
f*exp(2*c) - 3*I*e**2*f)*exp(-c)/(4*a) - x*(I*e**3*exp(2*c) - I*e**3)*exp(-
c)/(2*a), True)) + e**3*x/a + 3*e**2*f*x**2/(2*a) + e*f**2*x**3/a + f**3*x*
```

```
*4/(4*a)
```

Giac [B] time = 1.34559, size = 1084, normalized size = 10.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(d^4*f^3*x^4*e^(2*d*x + 3*c) - I*d^4*f^3*x^4*e^(d*x + 2*c) - 2*I*d^3*f^
3*x^3*e^(3*d*x + 4*c) + 4*d^4*f^2*x^3*e^(2*d*x + 3*c + 1) - 2*d^3*f^3*x^3*e
^(2*d*x + 3*c) - 4*I*d^4*f^2*x^3*e^(d*x + 2*c + 1) - 2*I*d^3*f^3*x^3*e^(d*x
 + 2*c) - 2*d^3*f^3*x^3*e^c - 6*I*d^3*f^2*x^2*e^(3*d*x + 4*c + 1) + 6*I*d^2
*f^3*x^2*e^(3*d*x + 4*c) + 6*d^4*f*x^2*e^(2*d*x + 3*c + 2) - 6*d^3*f^2*x^2*
e^(2*d*x + 3*c + 1) + 6*d^2*f^3*x^2*e^(2*d*x + 3*c) - 6*I*d^4*f*x^2*e^(d*x
+ 2*c + 2) - 6*I*d^3*f^2*x^2*e^(d*x + 2*c + 1) - 6*I*d^2*f^3*x^2*e^(d*x + 2
*c) - 6*d^3*f^2*x^2*e^(c + 1) - 6*d^2*f^3*x^2*e^c - 6*I*d^3*f*x*e^(3*d*x +
4*c + 2) + 12*I*d^2*f^2*x*e^(3*d*x + 4*c + 1) - 12*I*d*f^3*x*e^(3*d*x + 4*c
) + 4*d^4*x*e^(2*d*x + 3*c + 3) - 6*d^3*f*x*e^(2*d*x + 3*c + 2) + 12*d^2*f^
2*x*e^(2*d*x + 3*c + 1) - 12*d*f^3*x*e^(2*d*x + 3*c) - 4*I*d^4*x*e^(d*x + 2
*c + 3) - 6*I*d^3*f*x*e^(d*x + 2*c + 2) - 12*I*d^2*f^2*x*e^(d*x + 2*c + 1)
- 12*I*d*f^3*x*e^(d*x + 2*c) - 6*d^3*f*x*e^(c + 2) - 12*d^2*f^2*x*e^(c + 1)
 - 12*d*f^3*x*e^c - 2*I*d^3*e^(3*d*x + 4*c + 3) + 6*I*d^2*f*e^(3*d*x + 4*c
+ 2) - 12*I*d*f^2*e^(3*d*x + 4*c + 1) + 12*I*f^3*e^(3*d*x + 4*c) - 2*d^3*e^
(2*d*x + 3*c + 3) + 6*d^2*f*e^(2*d*x + 3*c + 2) - 12*d*f^2*e^(2*d*x + 3*c +
 1) + 12*f^3*e^(2*d*x + 3*c) - 2*I*d^3*e^(d*x + 2*c + 3) - 6*I*d^2*f*e^(d*x
 + 2*c + 2) - 12*I*d*f^2*e^(d*x + 2*c + 1) - 12*I*f^3*e^(d*x + 2*c) - 2*d^3
*e^(c + 3) - 6*d^2*f*e^(c + 2) - 12*d*f^2*e^(c + 1) - 12*f^3*e^c)/(a*d^4*e^
(2*d*x + 3*c) - I*a*d^4*e^(d*x + 2*c))
```

3.260 $\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{2if(e+fx)\sinh(c+dx)}{ad^2}-\frac{2if^2\cosh(c+dx)}{ad^3}-\frac{i(e+fx)^2\cosh(c+dx)}{ad}+\frac{(e+fx)^3}{3af}$$

```
[Out] (e + f*x)^3/(3*a*f) - ((2*I)*f^2*Cosh[c + d*x])/(a*d^3) - (I*(e + f*x)^2*Co
sh[c + d*x])/(a*d) + ((2*I)*f*(e + f*x)*Sinh[c + d*x])/(a*d^2)
```

Rubi [A] time = 0.126473, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {5563, 32, 3296, 2638}

$$\frac{2if(e+fx)\sinh(c+dx)}{ad^2}-\frac{2if^2\cosh(c+dx)}{ad^3}-\frac{i(e+fx)^2\cosh(c+dx)}{ad}+\frac{(e+fx)^3}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (e + f*x)^3/(3*a*f) - ((2*I)*f^2*Cosh[c + d*x])/(a*d^3) - (I*(e + f*x)^2*Co
sh[c + d*x])/(a*d) + ((2*I)*f*(e + f*x)*Sinh[c + d*x])/(a*d^2)
```

Rule 5563

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cosh[c
+ d*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Si
nh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ
[a^2 + b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int (e+fx)^2\sinh(c+dx)\,dx}{a} + \frac{\int (e+fx)^2\,dx}{a}$$
$$= \frac{(e+fx)^3}{3af} - \frac{i(e+fx)^2\cosh(c+dx)}{ad} + \frac{(2if)\int (e+fx)\cosh(c+dx)\,dx}{ad}$$
$$= \frac{(e+fx)^3}{3af} - \frac{i(e+fx)^2\cosh(c+dx)}{ad} + \frac{2if(e+fx)\sinh(c+dx)}{ad^2} - \frac{(2if^2)\int \sinh(c+dx)}{ad^2}$$
$$= \frac{(e+fx)^3}{3af} - \frac{2if^2\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2\cosh(c+dx)}{ad} + \frac{2if(e+fx)\sinh(c+dx)}{ad^2}$$

Mathematica [A] time = 0.484384, size = 78, normalized size = 0.95

$$\frac{-3i\cosh(c+dx)\left(d^2(e+fx)^2+2f^2\right)+6idf(e+fx)\sinh(c+dx)+d^3x\left(3e^2+3efx+f^2x^2\right)}{3ad^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2) - (3*I)*(2*f^2 + d^2*(e + f*x)^2)*Cosh[c
 + d*x] + (6*I)*d*f*(e + f*x)*Sinh[c + d*x])/(3*a*d^3)
```

Maple [B] time = 0.042, size = 223, normalized size = 2.7

$$-\frac{1}{d^3a}\Big(if^2\left((dx+c)^2\cosh(dx+c)-2(dx+c)\sinh(dx+c)+2\cosh(dx+c)\right)-2icf^2\left((dx+c)\cosh(dx+c)-\sinh(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -1/d^3/a*(I*f^2*((d*x+c)^2*cosh(d*x+c)-2*(d*x+c)*sinh(d*x+c)+2*cosh(d*x+c))
-2*I*c*f^2*((d*x+c)*cosh(d*x+c)-sinh(d*x+c))+2*I*f*e*d*((d*x+c)*cosh(d*x+c)
-sinh(d*x+c))+I*c^2*f^2*cosh(d*x+c)-2*I*c*d*f*e*cosh(d*x+c)+I*d^2*e^2*cosh(
d*x+c)-1/3*f^2*(d*x+c)^3+c*f^2*(d*x+c)^2-e*f*d*(d*x+c)^2-c^2*f^2*(d*x+c)+2*
c*d*f*e*(d*x+c)-d^2*e^2*(d*x+c))
```

Maxima [B] time = 1.59838, size = 366, normalized size = 4.46

$$\frac{1}{2}ef\left(\frac{4xe^{(dx+c)}}{ade^{(dx+c)}-iad} + \frac{-2id^2x^2e^c-2idxe^c-\left(2idxe^{(3c)}-2ie^{(3c)}\right)e^{(2dx)}+2\left(d^2x^2e^{(2c)}-3dxe^{(2c)}+e^{(2c)}\right)e^{(dx)}-2(dx+1)}{ad^2e^{(dx+2c)}-iad^2e^c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/2*e*f*(4*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) + (-2*I*d^2*x^2*e^c - 2*
I*d*x*e^c - (2*I*d*x*e^(3*c) - 2*I*e^(3*c))*e^(2*d*x) + 2*(d^2*x^2*e^(2*c)
- 3*d*x*e^(2*c) + e^(2*c))*e^(d*x) - 2*(d*x + 1)*e^(-d*x) - 2*I*e^c)/(a*d^2
```

```
*e^(d*x + 2*c) - I*a*d^2*e^c)) + 1/4*e^2*(4*(d*x + c)/(a*d) - 2*I*e^(d*x +
c)/(a*d) - 2*I*e^(-d*x - c)/(a*d)) + 1/12*(4*d^3*x^3*e^c - (6*I*d^2*x^2*e^(
2*c) - 12*I*d*x*e^(2*c) + 12*I*e^(2*c))*e^(d*x) - (6*I*d^2*x^2 + 12*I*d*x +
 12*I)*e^(-d*x))*f^2*e^(-c)/(a*d^3)
```

Fricas [B] time = 2.1649, size = 373, normalized size = 4.55

$$\frac{\left(-3i\,d^2f^2x^2-3i\,d^2e^2-6i\,def-6i\,f^2+\left(-6i\,d^2ef-6i\,df^2\right)x+\left(-3i\,d^2f^2x^2-3i\,d^2e^2+6i\,def-6i\,f^2+\left(-6i\,d^2ef+6i\right.\right.\right.}{6\,ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/6*(-3*I*d^2*f^2*x^2 - 3*I*d^2*e^2 - 6*I*d*e*f - 6*I*f^2 + (-6*I*d^2*e*f -
 6*I*d*f^2)*x + (-3*I*d^2*f^2*x^2 - 3*I*d^2*e^2 + 6*I*d*e*f - 6*I*f^2 + (-6
*I*d^2*e*f + 6*I*d*f^2)*x)*e^(2*d*x + 2*c) + 2*(d^3*f^2*x^3 + 3*d^3*e*f*x^2
 + 3*d^3*e^2*x)*e^(d*x + c))*e^(-d*x - c)/(a*d^3)
```

Sympy [A] time = 1.94693, size = 376, normalized size = 4.59

$$\begin{cases}\frac{\left(\left(-2ia^5d^{11}e^2e^{2c}-4ia^5d^{11}efxe^{2c}-2ia^5d^{11}f^2x^2e^{2c}-4ia^5d^{10}efe^{2c}-4ia^5d^{10}f^2xe^{2c}-4ia^5d^9f^2e^{2c}\right)e^{-dx}+\left(-2ia^5d^{11}e^2e^{4c}-4ia^5d^{11}efxe^{4c}-2ia^5d^{11}f^2x^2e^{4c}+4ia^5d^{10}e\right.\right.}{4a^6d^{12}} \\ -\frac{x^3\left(if^2e^{2c}-if^2\right)e^{-c}}{6a}-\frac{x^2\left(iefe^{2c}-ief\right)e^{-c}}{2a}-\frac{x\left(ie^2e^{2c}-ie^2\right)e^{-c}}{2a}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise((((-2*I*a**5*d**11*e**2*exp(2*c) - 4*I*a**5*d**11*e*f*x*exp(2*c)
- 2*I*a**5*d**11*f**2*x**2*exp(2*c) - 4*I*a**5*d**10*e*f*exp(2*c) - 4*I*a**
5*d**10*f**2*x*exp(2*c) - 4*I*a**5*d**9*f**2*exp(2*c))*exp(-d*x) + (-2*I*a*
*5*d**11*e**2*exp(4*c) - 4*I*a**5*d**11*e*f*x*exp(4*c) - 2*I*a**5*d**11*f**
2*x**2*exp(4*c) + 4*I*a**5*d**10*e*f*exp(4*c) + 4*I*a**5*d**10*f**2*x*exp(4
*c) - 4*I*a**5*d**9*f**2*exp(4*c))*exp(d*x))*exp(-3*c)/(4*a**6*d**12), Ne(4
*a**6*d**12*exp(3*c), 0)), (-x**3*(I*f**2*exp(2*c) - I*f**2)*exp(-c)/(6*a)
- x**2*(I*e*f*exp(2*c) - I*e*f)*exp(-c)/(2*a) - x*(I*e**2*exp(2*c) - I*e**2
)*exp(-c)/(2*a), True)) + e**2*x/a + e*f*x**2/a + f**2*x**3/(3*a)
```

Giac [B] time = 1.2607, size = 648, normalized size = 7.9

$$\frac{2\,d^3f^2x^3e^{(2\,dx+3\,c)}-2i\,d^3f^2x^3e^{(dx+2\,c)}-3i\,d^2f^2x^2e^{(3\,dx+4\,c)}+6\,d^3fx^2e^{(2\,dx+3\,c+1)}-3\,d^2f^2x^2e^{(2\,dx+3\,c)}-6i\,d^3fx^2e^{(dx+2\,c+1)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(2*d^3*f^2*x^3*e^(2*d*x + 3*c) - 2*I*d^3*f^2*x^3*e^(d*x + 2*c) - 3*I*d^
2*f^2*x^2*e^(3*d*x + 4*c) + 6*d^3*f*x^2*e^(2*d*x + 3*c + 1) - 3*d^2*f^2*x^2
```

```
*e^(2*d*x + 3*c) - 6*I*d^3*f*x^2*e^(d*x + 2*c + 1) - 3*I*d^2*f^2*x^2*e^(d*x
 + 2*c) - 3*d^2*f^2*x^2*e^c - 6*I*d^2*f*x*e^(3*d*x + 4*c + 1) + 6*I*d*f^2*x
*e^(3*d*x + 4*c) + 6*d^3*x*e^(2*d*x + 3*c + 2) - 6*d^2*f*x*e^(2*d*x + 3*c +
 1) + 6*d*f^2*x*e^(2*d*x + 3*c) - 6*I*d^3*x*e^(d*x + 2*c + 2) - 6*I*d^2*f*x
*e^(d*x + 2*c + 1) - 6*I*d*f^2*x*e^(d*x + 2*c) - 6*d^2*f*x*e^(c + 1) - 6*d*
f^2*x*e^c - 3*I*d^2*e^(3*d*x + 4*c + 2) + 6*I*d*f*e^(3*d*x + 4*c + 1) - 6*I
*f^2*e^(3*d*x + 4*c) - 3*d^2*e^(2*d*x + 3*c + 2) + 6*d*f*e^(2*d*x + 3*c + 1
) - 6*f^2*e^(2*d*x + 3*c) - 3*I*d^2*e^(d*x + 2*c + 2) - 6*I*d*f*e^(d*x + 2*
c + 1) - 6*I*f^2*e^(d*x + 2*c) - 3*d^2*e^(c + 2) - 6*d*f*e^(c + 1) - 6*f^2*
e^c)/(a*d^3*e^(2*d*x + 3*c) - I*a*d^3*e^(d*x + 2*c))
```

3.261 $\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=56

$$\frac{if\sinh(c+dx)}{ad^2}-\frac{i(e+fx)\cosh(c+dx)}{ad}+\frac{ex}{a}+\frac{fx^2}{2a}$$

```
[Out] (e*x)/a + (f*x^2)/(2*a) - (I*(e + f*x)*Cosh[c + d*x])/(a*d) + (I*f*Sinh[c +
 d*x])/(a*d^2)
```

Rubi [A] time = 0.0715893, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {5563, 3296, 2637}

$$\frac{if\sinh(c+dx)}{ad^2}-\frac{i(e+fx)\cosh(c+dx)}{ad}+\frac{ex}{a}+\frac{fx^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (e*x)/a + (f*x^2)/(2*a) - (I*(e + f*x)*Cosh[c + d*x])/(a*d) + (I*f*Sinh[c +
 d*x])/(a*d^2)
```

Rule 5563

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cosh[c
+ d*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Si
nh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ
[a^2 + b^2, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\frac{i\int (e+fx)\sinh(c+dx)\,dx}{a}+\frac{\int (e+fx)\,dx}{a}\\ &= \frac{ex}{a}+\frac{fx^2}{2a}-\frac{i(e+fx)\cosh(c+dx)}{ad}+\frac{(if)\int \cosh(c+dx)\,dx}{ad}\\ &= \frac{ex}{a}+\frac{fx^2}{2a}-\frac{i(e+fx)\cosh(c+dx)}{ad}+\frac{if\sinh(c+dx)}{ad^2}\end{aligned}$$

Mathematica [A] time = 0.638456, size = 57, normalized size = 1.02

$$-\frac{(c+dx)(cf-2de-dfx)+2id(e+fx)\cosh(c+dx)-2if\sinh(c+dx)}{2ad^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] -((c + d*x)*(-2*d*e + c*f - d*f*x) + (2*I)*d*(e + f*x)*Cosh[c + d*x] - (2*I
)*f*Sinh[c + d*x])/(2*a*d^2)
```

Maple [A] time = 0.04, size = 84, normalized size = 1.5

$$-\frac{1}{ad^2}\left(if\left((dx+c)\cosh(dx+c)-\sinh(dx+c)\right)-icf\cosh(dx+c)+ide\cosh(dx+c)-\frac{f(dx+c)^2}{2}+cf(dx+c)-de(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -1/d^2/a*(I*f*((d*x+c)*cosh(d*x+c)-sinh(d*x+c))-I*c*f*cosh(d*x+c)+I*d*e*cos
h(d*x+c)-1/2*f*(d*x+c)^2+c*f*(d*x+c)-d*e*(d*x+c))
```

Maxima [B] time = 1.50894, size = 254, normalized size = 4.54

$$\frac{1}{4}f\left(\frac{4xe^{(dx+c)}}{ade^{(dx+c)}-iad}+\frac{-2id^2x^2e^c-2idxe^c-\left(2idxe^{(3c)}-2ie^{(3c)}\right)e^{(2dx)}+2\left(d^2x^2e^{(2c)}-3dxe^{(2c)}+e^{(2c)}\right)e^{(dx)}-2(dx+1)e}{ad^2e^{(dx+2c)}-iad^2e^c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*f*(4*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) + (-2*I*d^2*x^2*e^c - 2*I*
d*x*e^c - (2*I*d*x*e^(3*c) - 2*I*e^(3*c))*e^(2*d*x) + 2*(d^2*x^2*e^(2*c) -
3*d*x*e^(2*c) + e^(2*c))*e^(d*x) - 2*(d*x + 1)*e^(-d*x) - 2*I*e^c)/(a*d^2*e
^(d*x + 2*c) - I*a*d^2*e^c)) + 1/4*e*(4*(d*x + c)/(a*d) - 2*I*e^(d*x + c)/(
a*d) - 2*I*e^(-d*x - c)/(a*d))
```

Fricas [A] time = 2.15053, size = 178, normalized size = 3.18

$$\frac{\left(-idfx-ide+\left(-idfx-ide+if\right)e^{(2dx+2c)}+\left(d^2fx^2+2d^2ex\right)e^{(dx+c)}-if\right)e^{(-dx-c)}}{2ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*f*x - I*d*e + (-I*d*f*x - I*d*e + I*f)*e^(2*d*x + 2*c) + (d^2*f*x
^2 + 2*d^2*e*x)*e^(d*x + c) - I*f)*e^(-d*x - c)/(a*d^2)
```

Sympy [A] time = 1.10237, size = 194, normalized size = 3.46

$$\begin{cases} \frac{\left(\left(-2ia^3d^5ee^c-2ia^3d^5fxe^c-2ia^3d^4fe^c\right)e^{-dx}+\left(-2ia^3d^5ee^{3c}-2ia^3d^5fxe^{3c}+2ia^3d^4fe^{3c}\right)e^{dx}\right)e^{-2c}}{4a^4d^6} & \text{for } 4a^4d^6e^{2c} \neq 0 \\ -\frac{x^2\left(ife^{2c}-if\right)e^{-c}}{4a}-\frac{x\left(iee^{2c}-ie\right)e^{-c}}{2a} & \text{otherwise} \end{cases} + \frac{ex}{a} + \frac{fx^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise((((-2*I*a**3*d**5*e*exp(c) - 2*I*a**3*d**5*f*x*exp(c) - 2*I*a**3*
d**4*f*exp(c))*exp(-d*x) + (-2*I*a**3*d**5*e*exp(3*c) - 2*I*a**3*d**5*f*x*e
xp(3*c) + 2*I*a**3*d**4*f*exp(3*c))*exp(d*x))*exp(-2*c)/(4*a**4*d**6), Ne(4
*a**4*d**6*exp(2*c), 0)), (-x**2*(I*f*exp(2*c) - I*f)*exp(-c)/(4*a) - x*(I*
e*exp(2*c) - I*e)*exp(-c)/(2*a), True)) + e*x/a + f*x**2/(2*a)
```

Giac [B] time = 1.16307, size = 312, normalized size = 5.57

$$\frac{d^2fx^2e^{(2dx+3c)} - i\,d^2fx^2e^{(dx+2c)} - i\,dfxe^{(3dx+4c)} + 2\,d^2xe^{(2dx+3c+1)} - dfxe^{(2dx+3c)} - 2i\,d^2xe^{(dx+2c+1)} - i\,dfxe^{(dx+2c)} - d}{2\left(ad^2e^{(2dx+3c)} - i\,ad^2e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(d^2*f*x^2*e^(2*d*x + 3*c) - I*d^2*f*x^2*e^(d*x + 2*c) - I*d*f*x*e^(3*d
*x + 4*c) + 2*d^2*x*e^(2*d*x + 3*c + 1) - d*f*x*e^(2*d*x + 3*c) - 2*I*d^2*x
*e^(d*x + 2*c + 1) - I*d*f*x*e^(d*x + 2*c) - d*f*x*e^c - I*d*e^(3*d*x + 4*c
 + 1) + I*f*e^(3*d*x + 4*c) - d*e^(2*d*x + 3*c + 1) + f*e^(2*d*x + 3*c) - I
*d*e^(d*x + 2*c + 1) - I*f*e^(d*x + 2*c) - d*e^(c + 1) - f*e^c)/(a*d^2*e^(2
*d*x + 3*c) - I*a*d^2*e^(d*x + 2*c))
```

3.262 $\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=22

$$\frac{x}{a}-\frac{i\cosh(c+dx)}{ad}$$

```
[Out] x/a - (I*Cosh[c + d*x])/(a*d)
```

Rubi [A] time = 0.0431233, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2682, 8}

$$\frac{x}{a}-\frac{i\cosh(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] x/a - (I*Cosh[c + d*x])/(a*d)
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\frac{i\cosh(c+dx)}{ad}+\frac{\int 1\,dx}{a}\\ &= \frac{x}{a}-\frac{i\cosh(c+dx)}{ad}\end{aligned}$$

Mathematica [B] time = 0.1447, size = 139, normalized size = 6.32

$$\frac{\cosh^3(c+dx)\left(-i\sqrt{1+i\sinh(c+dx)}\sinh(c+dx)+\sqrt{1+i\sinh(c+dx)}-2\sqrt{1-i\sinh(c+dx)}\sin^{-1}\left(\frac{\sqrt{1-i\sinh(c+dx)}}{\sqrt{2}}\right)\right)}{ad\sqrt{1+i\sinh(c+dx)}(\sinh(c+dx)-i)(\sinh(c+dx)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (Cosh[c + d*x]^3*(-2*ArcSin[Sqrt[1 - I*Sinh[c + d*x]]/Sqrt[2]]*Sqrt[1 - I*S
inh[c + d*x]] + Sqrt[1 + I*Sinh[c + d*x]] - I*Sqrt[1 + I*Sinh[c + d*x]]*Sin
h[c + d*x]))/(a*d*Sqrt[1 + I*Sinh[c + d*x]]*(-I + Sinh[c + d*x])*(I + Sinh[
```

```
c + d*x])^2)
```

Maple [B] time = 0.044, size = 85, normalized size = 3.9

$$\frac{-i}{da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{i}{da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -I/d/a/(tanh(1/2*d*x+1/2*c)+1)+1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)+I/d/a/(tanh(
1/2*d*x+1/2*c)-1)-1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.06201, size = 59, normalized size = 2.68

$$\frac{dx+c}{ad}-\frac{i\,e^{(dx+c)}}{2\,ad}-\frac{i\,e^{(-dx-c)}}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(a*d) - 1/2*I*e^(d*x + c)/(a*d) - 1/2*I*e^(-d*x - c)/(a*d)
```

Fricas [A] time = 2.11257, size = 92, normalized size = 4.18

$$\frac{\left(2\,dxe^{(dx+c)}-i\,e^{(2\,dx+2\,c)}-i\right)e^{(-dx-c)}}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d*x*e^(d*x + c) - I*e^(2*d*x + 2*c) - I)*e^(-d*x - c)/(a*d)
```

Sympy [A] time = 0.433829, size = 82, normalized size = 3.73

$$\begin{cases}\frac{\left(-2iade^{2c}e^{dx}-2iade^{-dx}\right)e^{-c}}{4a^2d^2} & \text{for } 4a^2d^2e^c\neq 0\\ x\left(-\frac{\left(ie^{2c}-2e^c-i\right)e^{-c}}{2a}-\frac{1}{a}\right) & \text{otherwise}\end{cases}+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise(((-2*I*a*d*exp(2*c)*exp(d*x) - 2*I*a*d*exp(-d*x))*exp(-c)/(4*a**2
*d**2), Ne(4*a**2*d**2*exp(c), 0)), (x*(-(I*exp(2*c) - 2*exp(c) - I)*exp(-c
```

```
)/(2*a) - 1/a), True)) + x/a
```

Giac [A] time = 1.22428, size = 47, normalized size = 2.14

$$\frac{2\,dx + 2\,c - i\,e^{(dx+c)} - i\,e^{(-dx-c)}}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*d*x + 2*c - I*e^(d*x + c) - I*e^(-d*x - c))/(a*d)
```

3.263 $\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=76

$$-\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{af}-\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{af}+\frac{\log(e+fx)}{af}$$

```
[Out] Log[e + f*x]/(a*f) - (I*CoshIntegral[(d*e)/f + d*x]*Sinh[c - (d*e)/f])/(a*f
) - (I*Cosh[c - (d*e)/f]*SinhIntegral[(d*e)/f + d*x])/(a*f)
```

Rubi [A] time = 0.204152, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {5563, 31, 3303, 3298, 3301}

$$-\frac{i\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{af}-\frac{i\cosh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{af}+\frac{\log(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Log[e + f*x]/(a*f) - (I*CoshIntegral[(d*e)/f + d*x]*Sinh[c - (d*e)/f])/(a*f
) - (I*Cosh[c - (d*e)/f]*SinhIntegral[(d*e)/f + d*x])/(a*f)
```

Rule 5563

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cosh[c
+ d*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Si
nh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ
[a^2 + b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\sinh(c+dx)}{e+fx}\,dx}{a}+\frac{\int \frac{1}{e+fx}\,dx}{a}$$
$$= \frac{\log(e+fx)}{af}-\frac{\left(i\cosh\left(c-\frac{de}{f}\right)\right)\int \frac{\sinh\left(\frac{de}{f}+dx\right)}{e+fx}\,dx}{a}-\frac{\left(i\sinh\left(c-\frac{de}{f}\right)\right)\int \frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx}\,dx}{a}$$
$$= \frac{\log(e+fx)}{af}-\frac{i\operatorname{Chi}\left(\frac{de}{f}+dx\right)\sinh\left(c-\frac{de}{f}\right)}{af}-\frac{i\cosh\left(c-\frac{de}{f}\right)\operatorname{Shi}\left(\frac{de}{f}+dx\right)}{af}$$

Mathematica [A] time = 0.320749, size = 62, normalized size = 0.82

$$\frac{-i\sinh\left(c-\frac{de}{f}\right)\operatorname{Chi}\left(d\left(\frac{e}{f}+x\right)\right)-i\cosh\left(c-\frac{de}{f}\right)\operatorname{Shi}\left(d\left(\frac{e}{f}+x\right)\right)+\log(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] (Log[e + f*x] - I*CoshIntegral[d*(e/f + x)]*Sinh[c - (d*e)/f] - I*Cosh[c -
(d*e)/f]*SinhIntegral[d*(e/f + x)])/(a*f)
```

Maple [A] time = 0.09, size = 103, normalized size = 1.4

$$\frac{\ln\left(fx+e\right)}{af}+\frac{\frac{i}{2}}{af}e^{\frac{cf-de}{f}}\operatorname{Ei}\left(1,-dx-c-\frac{-cf+de}{f}\right)-\frac{\frac{i}{2}}{af}e^{-\frac{cf-de}{f}}\operatorname{Ei}\left(1,dx+c-\frac{cf-de}{f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] ln(f*x+e)/a/f+1/2*I/a/f*exp((c*f-d*e)/f)*Ei(1,-d*x-c-(-c*f+d*e)/f)-1/2*I/a/
f*exp(-(c*f-d*e)/f)*Ei(1,d*x+c-(c*f-d*e)/f)
```

Maxima [A] time = 1.51494, size = 103, normalized size = 1.36

$$-\frac{ie^{\left(-c+\frac{de}{f}\right)}E_1\left(\frac{(fx+e)d}{f}\right)}{2af}+\frac{ie^{\left(c-\frac{de}{f}\right)}E_1\left(-\frac{(fx+e)d}{f}\right)}{2af}+\frac{\log\left(fx+e\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*I*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(a*f) + 1/2*I*e^(c -
 d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(a*f) + log(f*x + e)/(a*f)
```

Fricas [A] time = 2.18993, size = 159, normalized size = 2.09

$$\frac{i\,\mathrm{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(\frac{de-cf}{f}\right)}-i\,\mathrm{Ei}\left(\frac{dfx+de}{f}\right)e^{\left(-\frac{de-cf}{f}\right)}+2\,\log\left(\frac{fx+e}{f}\right)}{2\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*Ei(-(d*f*x + d*e)/f)*e^((d*e - c*f)/f) - I*Ei((d*f*x + d*e)/f)*e^(-(
d*e - c*f)/f) + 2*log((f*x + e)/f))/(a*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18928, size = 109, normalized size = 1.43

$$-\frac{\left(i\,\mathrm{Ei}\left(\frac{dfx+de}{f}\right)e^{\left(2\,c-\frac{de}{f}\right)}-i\,\mathrm{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(\frac{de}{f}\right)}-2\,e^{c}\log\left(i\,fx+i\,e\right)\right)e^{(-c)}}{2\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(I*Ei((d*f*x + d*e)/f)*e^(2*c - d*e/f) - I*Ei(-(d*f*x + d*e)/f)*e^(d*e
/f) - 2*e^c*log(I*f*x + I*e))*e^(-c)/(a*f)
```

3.264 $\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=103

$$-\frac{id\cosh\left(c-\frac{de}{f}\right)\mathrm{Chi}\left(\frac{de}{f}+dx\right)}{af^2}-\frac{id\sinh\left(c-\frac{de}{f}\right)\mathrm{Shi}\left(\frac{de}{f}+dx\right)}{af^2}+\frac{i\sinh(c+dx)}{af(e+fx)}-\frac{1}{af(e+fx)}$$

```
[Out] -(1/(a*f*(e + f*x))) - (I*d*Cosh[c - (d*e)/f]*CoshIntegral[(d*e)/f + d*x])/
(a*f^2) + (I*Sinh[c + d*x])/(a*f*(e + f*x)) - (I*d*Sinh[c - (d*e)/f]*SinhIn
tegral[(d*e)/f + d*x])/(a*f^2)
```

Rubi [A] time = 0.220979, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {5563, 32, 3297, 3303, 3298, 3301}

$$-\frac{id\cosh\left(c-\frac{de}{f}\right)\mathrm{Chi}\left(\frac{de}{f}+dx\right)}{af^2}-\frac{id\sinh\left(c-\frac{de}{f}\right)\mathrm{Shi}\left(\frac{de}{f}+dx\right)}{af^2}+\frac{i\sinh(c+dx)}{af(e+fx)}-\frac{1}{af(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] -(1/(a*f*(e + f*x))) - (I*d*Cosh[c - (d*e)/f]*CoshIntegral[(d*e)/f + d*x])/
(a*f^2) + (I*Sinh[c + d*x])/(a*f*(e + f*x)) - (I*d*Sinh[c - (d*e)/f]*SinhIn
tegral[(d*e)/f + d*x])/(a*f^2)
```

Rule 5563

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cosh[c
+ d*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Si
nh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ
[a^2 + b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}dx = -\frac{i\int \frac{\sinh(c+dx)}{(e+fx)^2}dx}{a}+\frac{\int \frac{1}{(e+fx)^2}dx}{a}$$

$$= -\frac{1}{af(e+fx)}+\frac{i\sinh(c+dx)}{af(e+fx)}-\frac{(id)\int \frac{\cosh(c+dx)}{e+fx}dx}{af}$$

$$= -\frac{1}{af(e+fx)}+\frac{i\sinh(c+dx)}{af(e+fx)}-\frac{\left(id\cosh\left(c-\frac{de}{f}\right)\right)\int \frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx}dx}{af}-\frac{\left(id\sinh\right.}{}$$

$$= -\frac{1}{af(e+fx)}-\frac{id\cosh\left(c-\frac{de}{f}\right)\mathrm{Chi}\left(\frac{de}{f}+dx\right)}{af^2}+\frac{i\sinh(c+dx)}{af(e+fx)}-\frac{id\sinh\left(c-\right.}{}$$

Mathematica [A] time = 0.528068, size = 85, normalized size = 0.83

$$-\frac{i\left(d(e+fx)\cosh\left(c-\frac{de}{f}\right)\mathrm{Chi}\left(d\left(\frac{e}{f}+x\right)\right)+d(e+fx)\sinh\left(c-\frac{de}{f}\right)\mathrm{Shi}\left(d\left(\frac{e}{f}+x\right)\right)-f(\sinh(c+dx)+i)\right)}{af^2(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] ((-I)*(d*(e + f*x)*Cosh[c - (d*e)/f]*CoshIntegral[d*(e/f + x)] - f*(I + Sin
h[c + d*x]) + d*(e + f*x)*Sinh[c - (d*e)/f]*SinhIntegral[d*(e/f + x)]))/(a*
f^2*(e + f*x))
```

Maple [A] time = 0.095, size = 164, normalized size = 1.6

$$-\frac{1}{af\left(fx+e\right)}+\frac{\frac{i}{2}de^{dx+c}}{af^2}\left(\frac{de}{f}+dx\right)^{-1}+\frac{\frac{i}{2}d}{af^2}e^{\frac{cf-de}{f}}\mathrm{Ei}\left(1,-dx-c-\frac{-cf+de}{f}\right)-\frac{\frac{i}{2}de^{-dx-c}}{af\left(dfx+de\right)}+\frac{\frac{i}{2}d}{af^2}e^{-\frac{cf-de}{f}}\mathrm{Ei}\left(1,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -1/a/f/(f*x+e)+1/2*I*d/a/f^2*exp(d*x+c)/(d*e/f+d*x)+1/2*I*d/a/f^2*exp((c*f-
d*e)/f)*Ei(1,-d*x-c-(-c*f+d*e)/f)-1/2*I/a*d*exp(-d*x-c)/f/(d*f*x+d*e)+1/2*I
/a*d/f^2*exp(-(c*f-d*e)/f)*Ei(1,d*x+c-(c*f-d*e)/f)
```

Maxima [A] time = 1.65693, size = 124, normalized size = 1.2

$$-\frac{1}{af^2x+aef}-\frac{ie^{\left(-c+\frac{de}{f}\right)}E_2\left(\frac{(fx+e)d}{f}\right)}{2(fx+e)af}+\frac{ie^{\left(c-\frac{de}{f}\right)}E_2\left(-\frac{(fx+e)d}{f}\right)}{2(fx+e)af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/(a*f^2*x + a*e*f) - 1/2*I*e^(-c + d*e/f)*exp_integral_e(2, (f*x + e)*d/f
)/((f*x + e)*a*f) + 1/2*I*e^(c - d*e/f)*exp_integral_e(2, -(f*x + e)*d/f)/(
(f*x + e)*a*f)
```

Fricas [A] time = 2.16562, size = 278, normalized size = 2.7

$$\frac{\left(ife^{(2dx+2c)}+\left((-idfx-ide)\mathrm{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(\frac{de-cf}{f}\right)}+(-idfx-ide)\mathrm{Ei}\left(\frac{dfx+de}{f}\right)e^{\left(-\frac{de-cf}{f}\right)}-2f\right)e^{(dx+c)}-if\right)e^{(-dx-c)}}{2\left(af^3x+aef^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/2*(I*f*e^(2*d*x + 2*c) + ((-I*d*f*x - I*d*e)*Ei(-(d*f*x + d*e)/f)*e^((d*e
 - c*f)/f) + (-I*d*f*x - I*d*e)*Ei((d*f*x + d*e)/f)*e^(-(d*e - c*f)/f) - 2*
f)*e^(d*x + c) - I*f)*e^(-d*x - c)/(a*f^3*x + a*e*f^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.265 $\int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=231

$$-\frac{3if^2(e+fx)\sinh^2(c+dx)}{4ad^3}+\frac{6f^2(e+fx)\sinh(c+dx)}{ad^3}-\frac{3f(e+fx)^2\cosh(c+dx)}{ad^2}+\frac{3if(e+fx)^2\sinh(c+dx)\cos}{4ad^2}$$

```
[Out] (((-3*I)/8)*f^3*x)/(a*d^3) - ((I/4)*(e + f*x)^3)/(a*d) - (6*f^3*Cosh[c + d*
x])/(a*d^4) - (3*f*(e + f*x)^2*Cosh[c + d*x])/(a*d^2) + (6*f^2*(e + f*x)*Si
nh[c + d*x])/(a*d^3) + ((e + f*x)^3*Sinh[c + d*x])/(a*d) + (((3*I)/8)*f^3*C
osh[c + d*x]*Sinh[c + d*x])/(a*d^4) + (((3*I)/4)*f*(e + f*x)^2*Cosh[c + d*x
]*Sinh[c + d*x])/(a*d^2) - (((3*I)/4)*f^2*(e + f*x)*Sinh[c + d*x]^2)/(a*d^3
) - ((I/2)*(e + f*x)^3*Sinh[c + d*x]^2)/(a*d)
```

Rubi [A] time = 0.260774, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {5563, 3296, 2638, 5446, 3311, 32, 2635, 8}

$$-\frac{3if^2(e+fx)\sinh^2(c+dx)}{4ad^3}+\frac{6f^2(e+fx)\sinh(c+dx)}{ad^3}-\frac{3f(e+fx)^2\cosh(c+dx)}{ad^2}+\frac{3if(e+fx)^2\sinh(c+dx)\cos}{4ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```


```
[Out] (((-3*I)/8)*f^3*x)/(a*d^3) - ((I/4)*(e + f*x)^3)/(a*d) - (6*f^3*Cosh[c + d*
x])/(a*d^4) - (3*f*(e + f*x)^2*Cosh[c + d*x])/(a*d^2) + (6*f^2*(e + f*x)*Si
nh[c + d*x])/(a*d^3) + ((e + f*x)^3*Sinh[c + d*x])/(a*d) + (((3*I)/8)*f^3*C
osh[c + d*x]*Sinh[c + d*x])/(a*d^4) + (((3*I)/4)*f*(e + f*x)^2*Cosh[c + d*x
]*Sinh[c + d*x])/(a*d^2) - (((3*I)/4)*f^2*(e + f*x)*Sinh[c + d*x]^2)/(a*d^3
) - ((I/2)*(e + f*x)^3*Sinh[c + d*x]^2)/(a*d)
```


Rule 5563

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cosh[c
+ d*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Si
nh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ
[a^2 + b^2, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
```

```
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\frac{i\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)\,dx}{a}+\frac{\int (e+fx)^3\cosh(c+dx)\,dx}{a}\\ &= \frac{(e+fx)^3\sinh(c+dx)}{ad}-\frac{i(e+fx)^3\sinh^2(c+dx)}{2ad}+\frac{(3if)\int (e+fx)^2\sinh^2(c+dx)\,dx}{2ad}-\\ &= -\frac{3f(e+fx)^2\cosh(c+dx)}{ad^2}+\frac{(e+fx)^3\sinh(c+dx)}{ad}+\frac{3if(e+fx)^2\cosh(c+dx)\sinh(c}{4ad^2}\\ &= -\frac{i(e+fx)^3}{4ad}-\frac{3f(e+fx)^2\cosh(c+dx)}{ad^2}+\frac{6f^2(e+fx)\sinh(c+dx)}{ad^3}+\frac{(e+fx)^3\sinh(c}{ad}\\ &= -\frac{3if^3x}{8ad^3}-\frac{i(e+fx)^3}{4ad}-\frac{6f^3\cosh(c+dx)}{ad^4}-\frac{3f(e+fx)^2\cosh(c+dx)}{ad^2}+\frac{6f^2(e+fx)\sinh}{ad^3}\end{aligned}$$

Mathematica [A] time = 1.30041, size = 134, normalized size = 0.58

$$\frac{-96f\cosh(c+dx)\left(d^2(e+fx)^2+2f^2\right)-4id(e+fx)\cosh(2(c+dx))\left(2d^2(e+fx)^2+3f^2\right)+4\sinh(c+dx)\left(8d(e+fx)\right(}{32ad^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (-96*f*(2*f^2 + d^2*(e + f*x)^2)*Cosh[c + d*x] - (4*I)*d*(e + f*x)*(3*f^2 +
 2*d^2*(e + f*x)^2)*Cosh[2*(c + d*x)] + 4*(8*d*(e + f*x)*(6*f^2 + d^2*(e +
f*x)^2) + (3*I)*f*(f^2 + 2*d^2*(e + f*x)^2)*Cosh[c + d*x])*Sinh[c + d*x])/(
32*a*d^4)
```

Maple [B] time = 0.16, size = 429, normalized size = 1.9

$$\frac{-\frac{i}{32}\left(4 f^{3} x^{3} d^{3}+12 d^{3} e f^{2} x^{2}+12 d^{3} e^{2} f x-6 d^{2} f^{3} x^{2}+4 d^{3} e^{3}-12 d^{2} e f^{2} x-6 d^{2} e^{2} f+6 d f^{3} x+6 d e f^{2}-3 f^{3}\right) \mathrm{e}^{2 d x+2 c}}{a d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -1/32*I*(4*d^3*f^3*x^3+12*d^3*e*f^2*x^2+12*d^3*e^2*f*x-6*d^2*f^3*x^2+4*d^3*
e^3-12*d^2*e*f^2*x-6*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2-3*f^3)/a/d^4*exp(2*d*x+2
*c)+1/2*(d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x-3*d^2*f^3*x^2+d^3*e^3-6*
d^2*e*f^2*x-3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2-6*f^3)/a/d^4*exp(d*x+c)-1/2*(d^
3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x+3*d^2*f^3*x^2+d^3*e^3+6*d^2*e*f^2*x
+3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2+6*f^3)/a/d^4*exp(-d*x-c)-1/32*I*(4*d^3*f^3
*x^3+12*d^3*e*f^2*x^2+12*d^3*e^2*f*x+6*d^2*f^3*x^2+4*d^3*e^3+12*d^2*e*f^2*x
+6*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2+3*f^3)/a/d^4*exp(-2*d*x-2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.21988, size = 906, normalized size = 3.92

$$\left(-4 i d^{3} f^{3} x^{3}-4 i d^{3} e^{3}-6 i d^{2} e^{2} f-6 i d e f^{2}-3 i f^{3}+\left(-12 i d^{3} e f^{2}-6 i d^{2} f^{3}\right) x^{2}+\left(-12 i d^{3} e^{2} f-12 i d^{2} e f^{2}-6 i d f^{3}\right) x+\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/32*(-4*I*d^3*f^3*x^3 - 4*I*d^3*e^3 - 6*I*d^2*e^2*f - 6*I*d*e*f^2 - 3*I*f^
3 + (-12*I*d^3*e*f^2 - 6*I*d^2*f^3)*x^2 + (-12*I*d^3*e^2*f - 12*I*d^2*e*f^2
 - 6*I*d*f^3)*x + (-4*I*d^3*f^3*x^3 - 4*I*d^3*e^3 + 6*I*d^2*e^2*f - 6*I*d*e
*f^2 + 3*I*f^3 + (-12*I*d^3*e*f^2 + 6*I*d^2*f^3)*x^2 + (-12*I*d^3*e^2*f + 1
2*I*d^2*e*f^2 - 6*I*d*f^3)*x)*e^(4*d*x + 4*c) + 16*(d^3*f^3*x^3 + d^3*e^3 -
 3*d^2*e^2*f + 6*d*e*f^2 - 6*f^3 + 3*(d^3*e*f^2 - d^2*f^3)*x^2 + 3*(d^3*e^2
*f - 2*d^2*e*f^2 + 2*d*f^3)*x)*e^(3*d*x + 3*c) - 16*(d^3*f^3*x^3 + d^3*e^3
+ 3*d^2*e^2*f + 6*d*e*f^2 + 6*f^3 + 3*(d^3*e*f^2 + d^2*f^3)*x^2 + 3*(d^3*e^
2*f + 2*d^2*e*f^2 + 2*d*f^3)*x)*e^(d*x + c))*e^(-2*d*x - 2*c)/(a*d^4)
```

Sympy [A] time = 4.77379, size = 1061, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise((((-2048*a**9*d**27*e**3*exp(8*c) - 6144*a**9*d**27*e**2*f*x*exp(
8*c) - 6144*a**9*d**27*e*f**2*x**2*exp(8*c) - 2048*a**9*d**27*f**3*x**3*exp
(8*c) - 6144*a**9*d**26*e**2*f*exp(8*c) - 12288*a**9*d**26*e*f**2*x*exp(8*c
) - 6144*a**9*d**26*f**3*x**2*exp(8*c) - 12288*a**9*d**25*e*f**2*exp(8*c) -
 12288*a**9*d**25*f**3*x*exp(8*c) - 12288*a**9*d**24*f**3*exp(8*c))*exp(-d*
x) + (2048*a**9*d**27*e**3*exp(10*c) + 6144*a**9*d**27*e**2*f*x*exp(10*c) +
 6144*a**9*d**27*e*f**2*x**2*exp(10*c) + 2048*a**9*d**27*f**3*x**3*exp(10*c
) - 6144*a**9*d**26*e**2*f*exp(10*c) - 12288*a**9*d**26*e*f**2*x*exp(10*c)
- 6144*a**9*d**26*f**3*x**2*exp(10*c) + 12288*a**9*d**25*e*f**2*exp(10*c) +
 12288*a**9*d**25*f**3*x*exp(10*c) - 12288*a**9*d**24*f**3*exp(10*c))*exp(d
*x) + (-512*I*a**9*d**27*e**3*exp(7*c) - 1536*I*a**9*d**27*e**2*f*x*exp(7*c
) - 1536*I*a**9*d**27*e*f**2*x**2*exp(7*c) - 512*I*a**9*d**27*f**3*x**3*exp
(7*c) - 768*I*a**9*d**26*e**2*f*exp(7*c) - 1536*I*a**9*d**26*e*f**2*x*exp(7
*c) - 768*I*a**9*d**26*f**3*x**2*exp(7*c) - 768*I*a**9*d**25*e*f**2*exp(7*c
) - 768*I*a**9*d**25*f**3*x*exp(7*c) - 384*I*a**9*d**24*f**3*exp(7*c))*exp(
-2*d*x) + (-512*I*a**9*d**27*e**3*exp(11*c) - 1536*I*a**9*d**27*e**2*f*x*ex
p(11*c) - 1536*I*a**9*d**27*e*f**2*x**2*exp(11*c) - 512*I*a**9*d**27*f**3*x
**3*exp(11*c) + 768*I*a**9*d**26*e**2*f*exp(11*c) + 1536*I*a**9*d**26*e*f**
2*x*exp(11*c) + 768*I*a**9*d**26*f**3*x**2*exp(11*c) - 768*I*a**9*d**25*e*f
**2*exp(11*c) - 768*I*a**9*d**25*f**3*x*exp(11*c) + 384*I*a**9*d**24*f**3*e
xp(11*c))*exp(2*d*x))*exp(-9*c)/(4096*a**10*d**28), Ne(4096*a**10*d**28*exp
(9*c), 0)), (-x**4*(I*f**3*exp(4*c) - 2*f**3*exp(3*c) - 2*f**3*exp(c) - I*f
**3)*exp(-2*c)/(16*a) - x**3*(I*e*f**2*exp(4*c) - 2*e*f**2*exp(3*c) - 2*e*f
**2*exp(c) - I*e*f**2)*exp(-2*c)/(4*a) - x**2*(3*I*e**2*f*exp(4*c) - 6*e**2
*f*exp(3*c) - 6*e**2*f*exp(c) - 3*I*e**2*f)*exp(-2*c)/(8*a) - x*(I*e**3*exp
(4*c) - 2*e**3*exp(3*c) - 2*e**3*exp(c) - I*e**3)*exp(-2*c)/(4*a), True))
```

Giac [B] time = 1.28698, size = 1357, normalized size = 5.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (-4*I*d^3*f^3*x^3*e^(5*d*x + 6*c) + 12*d^3*f^3*x^3*e^(4*d*x + 5*c) - 16*I*d
^3*f^3*x^3*e^(3*d*x + 4*c) - 16*d^3*f^3*x^3*e^(2*d*x + 3*c) + 12*I*d^3*f^3*
x^3*e^(d*x + 2*c) - 4*d^3*f^3*x^3*e^c - 12*I*d^3*f^2*x^2*e^(5*d*x + 6*c + 1
) + 6*I*d^2*f^3*x^2*e^(5*d*x + 6*c) + 36*d^3*f^2*x^2*e^(4*d*x + 5*c + 1) -
42*d^2*f^3*x^2*e^(4*d*x + 5*c) - 48*I*d^3*f^2*x^2*e^(3*d*x + 4*c + 1) + 48*
I*d^2*f^3*x^2*e^(3*d*x + 4*c) - 48*d^3*f^2*x^2*e^(2*d*x + 3*c + 1) - 48*d^2
*f^3*x^2*e^(2*d*x + 3*c) + 36*I*d^3*f^2*x^2*e^(d*x + 2*c + 1) + 42*I*d^2*f^
3*x^2*e^(d*x + 2*c) - 12*d^3*f^2*x^2*e^(c + 1) - 6*d^2*f^3*x^2*e^c - 12*I*d
^3*f*x*e^(5*d*x + 6*c + 2) + 12*I*d^2*f^2*x*e^(5*d*x + 6*c + 1) - 6*I*d*f^3
*x*e^(5*d*x + 6*c) + 36*d^3*f*x*e^(4*d*x + 5*c + 2) - 84*d^2*f^2*x*e^(4*d*x
 + 5*c + 1) + 90*d*f^3*x*e^(4*d*x + 5*c) - 48*I*d^3*f*x*e^(3*d*x + 4*c + 2)
 + 96*I*d^2*f^2*x*e^(3*d*x + 4*c + 1) - 96*I*d*f^3*x*e^(3*d*x + 4*c) - 48*d
^3*f*x*e^(2*d*x + 3*c + 2) - 96*d^2*f^2*x*e^(2*d*x + 3*c + 1) - 96*d*f^3*x*
e^(2*d*x + 3*c) + 36*I*d^3*f*x*e^(d*x + 2*c + 2) + 84*I*d^2*f^2*x*e^(d*x +
2*c + 1) + 90*I*d*f^3*x*e^(d*x + 2*c) - 12*d^3*f*x*e^(c + 2) - 12*d^2*f^2*x
*e^(c + 1) - 6*d*f^3*x*e^c - 4*I*d^3*e^(5*d*x + 6*c + 3) + 6*I*d^2*f*e^(5*d
*x + 6*c + 2) - 6*I*d*f^2*e^(5*d*x + 6*c + 1) + 3*I*f^3*e^(5*d*x + 6*c) + 1
2*d^3*e^(4*d*x + 5*c + 3) - 42*d^2*f*e^(4*d*x + 5*c + 2) + 90*d*f^2*e^(4*d*
```

```
x + 5*c + 1) - 93*f^3*e^(4*d*x + 5*c) - 16*I*d^3*e^(3*d*x + 4*c + 3) + 48*I
*d^2*f*e^(3*d*x + 4*c + 2) - 96*I*d*f^2*e^(3*d*x + 4*c + 1) + 96*I*f^3*e^(3
*d*x + 4*c) - 16*d^3*e^(2*d*x + 3*c + 3) - 48*d^2*f*e^(2*d*x + 3*c + 2) - 9
6*d*f^2*e^(2*d*x + 3*c + 1) - 96*f^3*e^(2*d*x + 3*c) + 12*I*d^3*e^(d*x + 2*
c + 3) + 42*I*d^2*f*e^(d*x + 2*c + 2) + 90*I*d*f^2*e^(d*x + 2*c + 1) + 93*I
*f^3*e^(d*x + 2*c) - 4*d^3*e^(c + 3) - 6*d^2*f*e^(c + 2) - 6*d*f^2*e^(c + 1
) - 3*f^3*e^c)/(32*a*d^4*e^(3*d*x + 4*c) - 32*I*a*d^4*e^(2*d*x + 3*c))
```

3.266 $\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=171

$$-\frac{2f(e+fx)\cosh(c+dx)}{ad^2}+\frac{if(e+fx)\sinh(c+dx)\cosh(c+dx)}{2ad^2}-\frac{if^2\sinh^2(c+dx)}{4ad^3}+\frac{2f^2\sinh(c+dx)}{ad^3}-\frac{i(e+fx)^2\text{si}}{2a}$$

```
[Out] ((-I/2)*e*f*x)/(a*d) - ((I/4)*f^2*x^2)/(a*d) - (2*f*(e + f*x)*Cosh[c + d*x]
)/(a*d^2) + (2*f^2*Sinh[c + d*x])/(a*d^3) + ((e + f*x)^2*Sinh[c + d*x])/(a*
d) + ((I/2)*f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(a*d^2) - ((I/4)*f^2*S
inh[c + d*x]^2)/(a*d^3) - ((I/2)*(e + f*x)^2*Sinh[c + d*x]^2)/(a*d)
```

Rubi [A] time = 0.186152, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {5563, 3296, 2637, 5446, 3310}

$$-\frac{2f(e+fx)\cosh(c+dx)}{ad^2}+\frac{if(e+fx)\sinh(c+dx)\cosh(c+dx)}{2ad^2}-\frac{if^2\sinh^2(c+dx)}{4ad^3}+\frac{2f^2\sinh(c+dx)}{ad^3}-\frac{i(e+fx)^2\text{si}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-I/2)*e*f*x)/(a*d) - ((I/4)*f^2*x^2)/(a*d) - (2*f*(e + f*x)*Cosh[c + d*x]
)/(a*d^2) + (2*f^2*Sinh[c + d*x])/(a*d^3) + ((e + f*x)^2*Sinh[c + d*x])/(a*
d) + ((I/2)*f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(a*d^2) - ((I/4)*f^2*S
inh[c + d*x]^2)/(a*d^3) - ((I/2)*(e + f*x)^2*Sinh[c + d*x]^2)/(a*d)
```

Rule 5563

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cosh[c
+ d*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Si
nh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ
[a^2 + b^2, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int(e+fx)^2\cosh(c+dx)\sinh(c+dx)\,dx}{a}+\frac{\int(e+fx)^2\cosh(c+dx)\,dx}{a}$$
$$=\frac{(e+fx)^2\sinh(c+dx)}{ad}-\frac{i(e+fx)^2\sinh^2(c+dx)}{2ad}+\frac{(if)\int(e+fx)\sinh^2(c+dx)\,dx}{ad}$$
$$=-\frac{2f(e+fx)\cosh(c+dx)}{ad^2}+\frac{(e+fx)^2\sinh(c+dx)}{ad}+\frac{if(e+fx)\cosh(c+dx)\sinh(c}{2ad^2}$$
$$=-\frac{iefx}{2ad}-\frac{if^2x^2}{4ad}-\frac{2f(e+fx)\cosh(c+dx)}{ad^2}+\frac{2f^2\sinh(c+dx)}{ad^3}+\frac{(e+fx)^2\sinh(c+}{ad}$$

Mathematica [A] time = 0.910861, size = 99, normalized size = 0.58

$$\frac{-2i\cosh(2(c+dx))\left(2d^2(e+fx)^2+f^2\right)+8\sinh(c+dx)\left(2\left(d^2(e+fx)^2+2f^2\right)+idf(e+fx)\cosh(c+dx)\right)-32df(}{16ad^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (-32*d*f*(e + f*x)*Cosh[c + d*x] - (2*I)*(f^2 + 2*d^2*(e + f*x)^2)*Cosh[2*(
c + d*x)] + 8*(2*(2*f^2 + d^2*(e + f*x)^2) + I*d*f*(e + f*x)*Cosh[c + d*x])
*Sinh[c + d*x])/(16*a*d^3)
```

Maple [A] time = 0.138, size = 241, normalized size = 1.4

$$\frac{-\frac{i}{16}\left(2f^2x^2d^2+4d^2efx+2d^2e^2-2df^2x-2efd+f^2\right)e^{2dx+2c}}{ad^3}+\frac{\left(f^2x^2d^2+2d^2efx+d^2e^2-2df^2x-2efd+2f^2\right)}{2ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -1/16*I*(2*d^2*f^2*x^2+4*d^2*e*f*x+2*d^2*e^2-2*d*f^2*x-2*d*e*f+f^2)/a/d^3*e
xp(2*d*x+2*c)+1/2*(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2-2*d*f^2*x-2*d*e*f+2*f^2)
/a/d^3*exp(d*x+c)-1/2*(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2+2*d*f^2*x+2*d*e*f+2*
f^2)/a/d^3*exp(-d*x-c)-1/16*I*(2*d^2*f^2*x^2+4*d^2*e*f*x+2*d^2*e^2+2*d*f^2*
x+2*d*e*f+f^2)/a/d^3*exp(-2*d*x-2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.13009, size = 520, normalized size = 3.04

$$\frac{\left(-2i d^2 f^2 x^2 - 2i d^2 e^2 - 2i d e f - i f^2 + \left(-4i d^2 e f - 2i d f^2\right)x + \left(-2i d^2 f^2 x^2 - 2i d^2 e^2 + 2i d e f - i f^2 + \left(-4i d^2 e f + 2i d f^2\right)x \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/16*(-2*I*d^2*f^2*x^2 - 2*I*d^2*e^2 - 2*I*d*e*f - I*f^2 + (-4*I*d^2*e*f -
2*I*d*f^2)*x + (-2*I*d^2*f^2*x^2 - 2*I*d^2*e^2 + 2*I*d*e*f - I*f^2 + (-4*I*
d^2*e*f + 2*I*d*f^2)*x)*e^(4*d*x + 4*c) + 8*(d^2*f^2*x^2 + d^2*e^2 - 2*d*e*
f + 2*f^2 + 2*(d^2*e*f - d*f^2)*x)*e^(3*d*x + 3*c) - 8*(d^2*f^2*x^2 + d^2*e
^2 + 2*d*e*f + 2*f^2 + 2*(d^2*e*f + d*f^2)*x)*e^(d*x + c))*e^(-2*d*x - 2*c)
/(a*d^3)
```

Sympy [A] time = 3.36033, size = 644, normalized size = 3.77

$$\begin{cases} \frac{\left(\left(-512a^7d^{17}e^2e^{6c}-1024a^7d^{17}efxe^{6c}-512a^7d^{17}f^2x^2e^{6c}-1024a^7d^{16}efe^{6c}-1024a^7d^{16}f^2xe^{6c}-1024a^7d^{15}f^2e^{6c}\right)e^{-dx}+\left(512a^7d^{17}e^2e^{8c}+1024a^7d^{17}efxe^{8c}+512a^7d^{17}f\right.\right.}{} \\ -\frac{x^3\left(if^2e^{4c}-2f^2e^{3c}-2f^2e^{c}-if^2\right)e^{-2c}}{12a} - \frac{x^2\left(iefe^{4c}-2efe^{3c}-2efe^{c}-ief\right)e^{-2c}}{4a} - \frac{x\left(ie^2e^{4c}-2e^2e^{3c}-2e^2e^{c}-ie^2\right)e^{-2c}}{4a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise((((-512*a**7*d**17*e**2*exp(6*c) - 1024*a**7*d**17*e*f*x*exp(6*c)
 - 512*a**7*d**17*f**2*x**2*exp(6*c) - 1024*a**7*d**16*e*f*exp(6*c) - 1024*
a**7*d**16*f**2*x*exp(6*c) - 1024*a**7*d**15*f**2*exp(6*c))*exp(-d*x) + (51
2*a**7*d**17*e**2*exp(8*c) + 1024*a**7*d**17*e*f*x*exp(8*c) + 512*a**7*d**1
7*f**2*x**2*exp(8*c) - 1024*a**7*d**16*e*f*exp(8*c) - 1024*a**7*d**16*f**2*
x*exp(8*c) + 1024*a**7*d**15*f**2*exp(8*c))*exp(d*x) + (-128*I*a**7*d**17*e
**2*exp(5*c) - 256*I*a**7*d**17*e*f*x*exp(5*c) - 128*I*a**7*d**17*f**2*x**2
*exp(5*c) - 128*I*a**7*d**16*e*f*exp(5*c) - 128*I*a**7*d**16*f**2*x*exp(5*c
) - 64*I*a**7*d**15*f**2*exp(5*c))*exp(-2*d*x) + (-128*I*a**7*d**17*e**2*ex
p(9*c) - 256*I*a**7*d**17*e*f*x*exp(9*c) - 128*I*a**7*d**17*f**2*x**2*exp(9
*c) + 128*I*a**7*d**16*e*f*exp(9*c) + 128*I*a**7*d**16*f**2*x*exp(9*c) - 64
*I*a**7*d**15*f**2*exp(9*c))*exp(2*d*x))*exp(-7*c)/(1024*a**8*d**18), Ne(10
24*a**8*d**18*exp(7*c), 0)), (-x**3*(I*f**2*exp(4*c) - 2*f**2*exp(3*c) - 2*
f**2*exp(c) - I*f**2)*exp(-2*c)/(12*a) - x**2*(I*e*f*exp(4*c) - 2*e*f*exp(3
*c) - 2*e*f*exp(c) - I*e*f)*exp(-2*c)/(4*a) - x*(I*e**2*exp(4*c) - 2*e**2*e
xp(3*c) - 2*e**2*exp(c) - I*e**2)*exp(-2*c)/(4*a), True))
```

Giac [B] time = 1.22192, size = 764, normalized size = 4.47

$$\frac{-2i d^2 f^2 x^2 e^{(5dx+6c)} + 6d^2 f^2 x^2 e^{(4dx+5c)} - 8i d^2 f^2 x^2 e^{(3dx+4c)} - 8d^2 f^2 x^2 e^{(2dx+3c)} + 6i d^2 f^2 x^2 e^{(dx+2c)} - 2d^2 f^2 x^2 e^{c} - 4i d^2 f\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (-2*I*d^2*f^2*x^2*e^(5*d*x + 6*c) + 6*d^2*f^2*x^2*e^(4*d*x + 5*c) - 8*I*d^2
*f^2*x^2*e^(3*d*x + 4*c) - 8*d^2*f^2*x^2*e^(2*d*x + 3*c) + 6*I*d^2*f^2*x^2*
e^(d*x + 2*c) - 2*d^2*f^2*x^2*e^c - 4*I*d^2*f*x*e^(5*d*x + 6*c + 1) + 2*I*d
*f^2*x*e^(5*d*x + 6*c) + 12*d^2*f*x*e^(4*d*x + 5*c + 1) - 14*d*f^2*x*e^(4*d
*x + 5*c) - 16*I*d^2*f*x*e^(3*d*x + 4*c + 1) + 16*I*d*f^2*x*e^(3*d*x + 4*c)
 - 16*d^2*f*x*e^(2*d*x + 3*c + 1) - 16*d*f^2*x*e^(2*d*x + 3*c) + 12*I*d^2*f
*x*e^(d*x + 2*c + 1) + 14*I*d*f^2*x*e^(d*x + 2*c) - 4*d^2*f*x*e^(c + 1) - 2
*d*f^2*x*e^c - 2*I*d^2*e^(5*d*x + 6*c + 2) + 2*I*d*f*e^(5*d*x + 6*c + 1) -
I*f^2*e^(5*d*x + 6*c) + 6*d^2*e^(4*d*x + 5*c + 2) - 14*d*f*e^(4*d*x + 5*c +
 1) + 15*f^2*e^(4*d*x + 5*c) - 8*I*d^2*e^(3*d*x + 4*c + 2) + 16*I*d*f*e^(3*
d*x + 4*c + 1) - 16*I*f^2*e^(3*d*x + 4*c) - 8*d^2*e^(2*d*x + 3*c + 2) - 16*
d*f*e^(2*d*x + 3*c + 1) - 16*f^2*e^(2*d*x + 3*c) + 6*I*d^2*e^(d*x + 2*c + 2
) + 14*I*d*f*e^(d*x + 2*c + 1) + 15*I*f^2*e^(d*x + 2*c) - 2*d^2*e^(c + 2) -
 2*d*f*e^(c + 1) - f^2*e^c)/(16*a*d^3*e^(3*d*x + 4*c) - 16*I*a*d^3*e^(2*d*x
 + 3*c))
```

3.267 $\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=98

$$-\frac{f\cosh(c+dx)}{ad^2}+\frac{if\sinh(c+dx)\cosh(c+dx)}{4ad^2}-\frac{i(e+fx)\sinh^2(c+dx)}{2ad}+\frac{(e+fx)\sinh(c+dx)}{ad}-\frac{ifx}{4ad}$$

```
[Out] ((-I/4)*f*x)/(a*d) - (f*Cosh[c + d*x])/(a*d^2) + ((e + f*x)*Sinh[c + d*x])/
(a*d) + ((I/4)*f*Cosh[c + d*x]*Sinh[c + d*x])/(a*d^2) - ((I/2)*(e + f*x)*Si
nh[c + d*x]^2)/(a*d)
```

Rubi [A] time = 0.0995435, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {5563, 3296, 2638, 5446, 2635, 8}

$$-\frac{f\cosh(c+dx)}{ad^2}+\frac{if\sinh(c+dx)\cosh(c+dx)}{4ad^2}-\frac{i(e+fx)\sinh^2(c+dx)}{2ad}+\frac{(e+fx)\sinh(c+dx)}{ad}-\frac{ifx}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-I/4)*f*x)/(a*d) - (f*Cosh[c + d*x])/(a*d^2) + ((e + f*x)*Sinh[c + d*x])/
(a*d) + ((I/4)*f*Cosh[c + d*x]*Sinh[c + d*x])/(a*d^2) - ((I/2)*(e + f*x)*Si
nh[c + d*x]^2)/(a*d)
```

Rule 5563

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cosh[c
+ d*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Si
nh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ
[a^2 + b^2, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int (e+fx)\cosh(c+dx)\sinh(c+dx)\,dx}{a} + \frac{\int (e+fx)\cosh(c+dx)\,dx}{a}$$
$$= \frac{(e+fx)\sinh(c+dx)}{ad} - \frac{i(e+fx)\sinh^2(c+dx)}{2ad} + \frac{(if)\int \sinh^2(c+dx)\,dx}{2ad} - \frac{f\int \sinh(\ldots}{a\ldots}$$
$$= -\frac{f\cosh(c+dx)}{ad^2} + \frac{(e+fx)\sinh(c+dx)}{ad} + \frac{if\cosh(c+dx)\sinh(c+dx)}{4ad^2} - \frac{i(e+fx)s\ldots}{2\ldots}$$
$$= -\frac{ifx}{4ad} - \frac{f\cosh(c+dx)}{ad^2} + \frac{(e+fx)\sinh(c+dx)}{ad} + \frac{if\cosh(c+dx)\sinh(c+dx)}{4ad^2} - \frac{i(e\ldots}{\ldots}$$

Mathematica [A] time = 1.13317, size = 60, normalized size = 0.61

$$\frac{d(e+fx)(4\sinh(c+dx)-i\cosh(2(c+dx)))+if(\sinh(c+dx)+4i)\cosh(c+dx)}{4ad^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (I*f*Cosh[c + d*x]*(4*I + Sinh[c + d*x]) + d*(e + f*x)*((-I)*Cosh[2*(c + d*
x)] + 4*Sinh[c + d*x]))/(4*a*d^2)
```

Maple [A] time = 0.121, size = 113, normalized size = 1.2

$$\frac{-\frac{i}{16}\left(2dfx+2de-f\right)e^{2dx+2c}}{ad^2} + \frac{\left(dfx+de-f\right)e^{dx+c}}{2ad^2} - \frac{\left(dfx+de+f\right)e^{-dx-c}}{2ad^2} - \frac{\frac{i}{16}\left(2dfx+2de+f\right)e^{-2dx-2c}}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -1/16*I*(2*d*f*x+2*d*e-f)/a/d^2*exp(2*d*x+2*c)+1/2*(d*f*x+d*e-f)/a/d^2*exp(
d*x+c)-1/2*(d*f*x+d*e+f)/a/d^2*exp(-d*x-c)-1/16*I*(2*d*f*x+2*d*e+f)/a/d^2*e
xp(-2*d*x-2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.14459, size = 242, normalized size = 2.47

$$\frac{\left(-2 i d f x - 2 i d e + \left(-2 i d f x - 2 i d e + i f\right) e^{(4 d x + 4 c)} + 8 \left(d f x + d e - f\right) e^{(3 d x + 3 c)} - 8 \left(d f x + d e + f\right) e^{(d x + c)} - i f\right) e^{(-2 d x - 2 c)}}{16 a d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(-2*I*d*f*x - 2*I*d*e + (-2*I*d*f*x - 2*I*d*e + I*f)*e^(4*d*x + 4*c) +
 8*(d*f*x + d*e - f)*e^(3*d*x + 3*c) - 8*(d*f*x + d*e + f)*e^(d*x + c) - I*
f)*e^(-2*d*x - 2*c)/(a*d^2)
```

Sympy [A] time = 2.07587, size = 330, normalized size = 3.37

$$\begin{cases} \frac{\left(\left(-512 a^5 d^9 e e^{4c} - 512 a^5 d^9 f x e^{4c} - 512 a^5 d^8 f e^{4c}\right) e^{-dx} + \left(512 a^5 d^9 e e^{6c} + 512 a^5 d^9 f x e^{6c} - 512 a^5 d^8 f e^{6c}\right) e^{dx} + \left(-128 i a^5 d^9 e e^{3c} - 128 i a^5 d^9 f x e^{3c} - 64 i a^5 d^8 f e^{3c}\right) e^{-2dx} + \left(-128 i a \right.}{1024 a^6 d^{10}} \\ -\frac{x^2 \left(i f e^{4c} - 2 f e^{3c} - 2 f e^{c} - i f\right) e^{-2c}}{8a} - \frac{x \left(i e e^{4c} - 2 e e^{3c} - 2 e e^{c} - i e\right) e^{-2c}}{4a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise((((-512*a**5*d**9*e*exp(4*c) - 512*a**5*d**9*f*x*exp(4*c) - 512*a
**5*d**8*f*exp(4*c))*exp(-d*x) + (512*a**5*d**9*e*exp(6*c) + 512*a**5*d**9*
f*x*exp(6*c) - 512*a**5*d**8*f*exp(6*c))*exp(d*x) + (-128*I*a**5*d**9*e*exp
(3*c) - 128*I*a**5*d**9*f*x*exp(3*c) - 64*I*a**5*d**8*f*exp(3*c))*exp(-2*d*
x) + (-128*I*a**5*d**9*e*exp(7*c) - 128*I*a**5*d**9*f*x*exp(7*c) + 64*I*a**
5*d**8*f*exp(7*c))*exp(2*d*x))*exp(-5*c)/(1024*a**6*d**10), Ne(1024*a**6*d*
*10*exp(5*c), 0)), (-x**2*(I*f*exp(4*c) - 2*f*exp(3*c) - 2*f*exp(c) - I*f)*
exp(-2*c)/(8*a) - x*(I*e*exp(4*c) - 2*e*exp(3*c) - 2*e*exp(c) - I*e)*exp(-2
*c)/(4*a), True))
```

Giac [B] time = 1.19529, size = 332, normalized size = 3.39

$$\frac{-2 i d f x e^{(5 d x + 6 c)} + 6 d f x e^{(4 d x + 5 c)} - 8 i d f x e^{(3 d x + 4 c)} - 8 d f x e^{(2 d x + 3 c)} + 6 i d f x e^{(d x + 2 c)} - 2 d f x e^{c} - 2 i d e^{(5 d x + 6 c + 1)} + i f e^{(5 d x}}{16 a d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (-2*I*d*f*x*e^(5*d*x + 6*c) + 6*d*f*x*e^(4*d*x + 5*c) - 8*I*d*f*x*e^(3*d*x
+ 4*c) - 8*d*f*x*e^(2*d*x + 3*c) + 6*I*d*f*x*e^(d*x + 2*c) - 2*d*f*x*e^c -
2*I*d*e^(5*d*x + 6*c + 1) + I*f*e^(5*d*x + 6*c) + 6*d*e^(4*d*x + 5*c + 1) -
 7*f*e^(4*d*x + 5*c) - 8*I*d*e^(3*d*x + 4*c + 1) + 8*I*f*e^(3*d*x + 4*c) -
8*d*e^(2*d*x + 3*c + 1) - 8*f*e^(2*d*x + 3*c) + 6*I*d*e^(d*x + 2*c + 1) + 7
*I*f*e^(d*x + 2*c) - 2*d*e^(c + 1) - f*e^c)/(16*a*d^2*e^(3*d*x + 4*c) - 16*
I*a*d^2*e^(2*d*x + 3*c))
```

3.268 $\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{\sinh(c+dx)}{ad} - \frac{i\sinh^2(c+dx)}{2ad}$$

```
[Out] Sinh[c + d*x]/(a*d) - ((I/2)*Sinh[c + d*x]^2)/(a*d)
```

Rubi [A] time = 0.047608, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2667}

$$\frac{\sinh(c+dx)}{ad} - \frac{i\sinh^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] Sinh[c + d*x]/(a*d) - ((I/2)*Sinh[c + d*x]^2)/(a*d)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(c+dx)}{a+ia\sinh(c+dx)}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)\,dx, x, ia\sinh(c+dx)\right)}{a^3d}\\ &= \frac{\sinh(c+dx)}{ad} - \frac{i\sinh^2(c+dx)}{2ad}\end{aligned}$$

Mathematica [A] time = 0.0491364, size = 28, normalized size = 0.82

$$\frac{(2-i\sinh(c+dx))\sinh(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((2 - I*Sinh[c + d*x])*Sinh[c + d*x])/(2*a*d)
```

Maple [A] time = 0.015, size = 29, normalized size = 0.9

$$-\frac{\frac{i}{2}(\sinh(dx+c))^2 - \sinh(dx+c)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -1/d/a*(1/2*I*sinh(d*x+c)^2-sinh(d*x+c))
```

Maxima [A] time = 1.16329, size = 81, normalized size = 2.38

$$-\frac{i\left(4i\,e^{(-dx-c)}+1\right)e^{(2\,dx+2\,c)}}{8\,ad}-\frac{i\left(-4i\,e^{(-dx-c)}+e^{(-2\,dx-2\,c)}\right)}{8\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*I*(4*I*e^(-d*x - c) + 1)*e^(2*d*x + 2*c)/(a*d) - 1/8*I*(-4*I*e^(-d*x -
 c) + e^(-2*d*x - 2*c))/(a*d)
```

Fricas [A] time = 2.08925, size = 120, normalized size = 3.53

$$\frac{\left(-i\,e^{(4\,dx+4\,c)}+4\,e^{(3\,dx+3\,c)}-4\,e^{(dx+c)}-i\right)e^{(-2\,dx-2\,c)}}{8\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(-I*e^(4*d*x + 4*c) + 4*e^(3*d*x + 3*c) - 4*e^(d*x + c) - I)*e^(-2*d*x
- 2*c)/(a*d)
```

Sympy [A] time = 0.766, size = 136, normalized size = 4.

$$\begin{cases} \frac{\left(-32ia^3d^3e^{5c}e^{2dx}+128a^3d^3e^{4c}e^{dx}-128a^3d^3e^{2c}e^{-dx}-32ia^3d^3e^{c}e^{-2dx}\right)e^{-3c}}{256a^4d^4} & \text{for}\ 256a^4d^4e^{3c}\neq 0 \\ -\frac{x\left(ie^{4c}-2e^{3c}-2e^{c}-i\right)e^{-2c}}{4a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise(((-32*I*a**3*d**3*exp(5*c)*exp(2*d*x) + 128*a**3*d**3*exp(4*c)*ex
p(d*x) - 128*a**3*d**3*exp(2*c)*exp(-d*x) - 32*I*a**3*d**3*exp(c)*exp(-2*d*
x))*exp(-3*c)/(256*a**4*d**4), Ne(256*a**4*d**4*exp(3*c), 0)), (-x*(I*exp(4
*c) - 2*exp(3*c) - 2*exp(c) - I)*exp(-2*c)/(4*a), True))
```

Giac [A] time = 1.15882, size = 65, normalized size = 1.91

$$-\frac{\left(4\,e^{(dx+c)}+i\right)e^{(-2\,dx-2\,c)}+i\,e^{(2\,dx+2\,c)}-4\,e^{(dx+c)}}{8\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/8*((4*e^(d*x + c) + I)*e^(-2*d*x - 2*c) + I*e^(2*d*x + 2*c) - 4*e^(d*x +
 c))/(a*d)
```

3.269 $\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=131

$$-\frac{i\sinh\left(2c-\frac{2de}{f}\right)\text{Chi}\left(\frac{2de}{f}+2dx\right)}{2af}+\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{af}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{af}-\frac{i\cosh\left(2c-\frac{2de}{f}\right)\text{Shi}\left(\right.}{2af}$$

```
[Out] (Cosh[c - (d*e)/f]*CoshIntegral[(d*e)/f + d*x])/(a*f) - ((I/2)*CoshIntegral
[(2*d*e)/f + 2*d*x]*Sinh[2*c - (2*d*e)/f])/(a*f) + (Sinh[c - (d*e)/f]*SinhI
ntegral[(d*e)/f + d*x])/(a*f) - ((I/2)*Cosh[2*c - (2*d*e)/f]*SinhIntegral[(
2*d*e)/f + 2*d*x])/(a*f)
```

Rubi [A] time = 0.324126, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {5563, 3303, 3298, 3301, 5448, 12}

$$-\frac{i\sinh\left(2c-\frac{2de}{f}\right)\text{Chi}\left(\frac{2de}{f}+2dx\right)}{2af}+\frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{af}+\frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{af}-\frac{i\cosh\left(2c-\frac{2de}{f}\right)\text{Shi}\left(\right.}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] (Cosh[c - (d*e)/f]*CoshIntegral[(d*e)/f + d*x])/(a*f) - ((I/2)*CoshIntegral
[(2*d*e)/f + 2*d*x]*Sinh[2*c - (2*d*e)/f])/(a*f) + (Sinh[c - (d*e)/f]*SinhI
ntegral[(d*e)/f + d*x])/(a*f) - ((I/2)*Cosh[2*c - (2*d*e)/f]*SinhIntegral[(
2*d*e)/f + 2*d*x])/(a*f)
```

Rule 5563

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cosh[c
+ d*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Si
nh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ
[a^2 + b^2, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{e+fx}\,dx}{a} + \frac{\int \frac{\cosh(c+dx)}{e+fx}\,dx}{a}$$

$$= -\frac{i\int \frac{\sinh(2c+2dx)}{2(e+fx)}\,dx}{a} + \frac{\cosh\left(c-\frac{de}{f}\right)\int \frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx}\,dx}{a} + \frac{\sinh\left(c-\frac{de}{f}\right)\int \frac{\sinh\left(\frac{de}{f}+dx\right)}{e+fx}}{a}$$

$$= \frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{af} + \frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{af} - \frac{i\int \frac{\sinh(2c+2dx)}{e+fx}\,dx}{2a}$$

$$= \frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{af} + \frac{\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(\frac{de}{f}+dx\right)}{af} - \frac{\left(i\cosh\left(2c-\frac{2de}{f}\right)\right.}{2}$$

$$= \frac{\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{af} - \frac{i\text{Chi}\left(\frac{2de}{f}+2dx\right)\sinh\left(2c-\frac{2de}{f}\right)}{2af} + \frac{\sinh\left(c-\frac{de}{f}\right)}{}$$

Mathematica [A] time = 0.418634, size = 112, normalized size = 0.85

$$\frac{2\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(d\left(\frac{e}{f}+x\right)\right) - i\left(\sinh\left(2c-\frac{2de}{f}\right)\text{Chi}\left(\frac{2d(e+fx)}{f}\right) + 2i\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(d\left(\frac{e}{f}+x\right)\right) + \cosh\left(2c-\frac{2de}{f}\right)\right.}{2af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] (2*Cosh[c - (d*e)/f]*CoshIntegral[d*(e/f + x)] - I*(CoshIntegral[(2*d*(e +
f*x))/f]*Sinh[2*c - (2*d*e)/f] + (2*I)*Sinh[c - (d*e)/f]*SinhIntegral[d*(e/
f + x)] + Cosh[2*c - (2*d*e)/f]*SinhIntegral[(2*d*(e + f*x))/f]))/(2*a*f)
```

Maple [A] time = 0.114, size = 180, normalized size = 1.4

$$-\frac{1}{2af}e^{-\frac{cf-de}{f}}\text{Ei}\left(1,dx+c-\frac{cf-de}{f}\right) - \frac{1}{2af}e^{\frac{cf-de}{f}}\text{Ei}\left(1,-dx-c-\frac{-cf+de}{f}\right) + \frac{\frac{i}{4}}{af}e^{2\frac{cf-de}{f}}\text{Ei}\left(1,-2dx-2c-2\frac{-cf+}{f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -1/2/a/f*exp(-(c*f-d*e)/f)*Ei(1,d*x+c-(c*f-d*e)/f)-1/2/a/f*exp((c*f-d*e)/f)
*Ei(1,-d*x-c-(-c*f+d*e)/f)+1/4*I/a/f*exp(2*(c*f-d*e)/f)*Ei(1,-2*d*x-2*c-2*(
-c*f+d*e)/f)-1/4*I/a/f*exp(-2*(c*f-d*e)/f)*Ei(1,2*d*x+2*c-2*(c*f-d*e)/f)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.12151, size = 258, normalized size = 1.97

$$\frac{i\,\mathrm{Ei}\left(-\frac{2\left(dfx+de\right)}{f}\right)e^{\left(\frac{2\left(de-cf\right)}{f}\right)}+2\,\mathrm{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(\frac{de-cf}{f}\right)}+2\,\mathrm{Ei}\left(\frac{dfx+de}{f}\right)e^{\left(-\frac{de-cf}{f}\right)}-i\,\mathrm{Ei}\left(\frac{2\left(dfx+de\right)}{f}\right)e^{\left(-\frac{2\left(de-cf\right)}{f}\right)}}{4\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(I*Ei(-2*(d*f*x + d*e)/f)*e^(2*(d*e - c*f)/f) + 2*Ei(-(d*f*x + d*e)/f)*
e^((d*e - c*f)/f) + 2*Ei((d*f*x + d*e)/f)*e^(-(d*e - c*f)/f) - I*Ei(2*(d*f*
x + d*e)/f)*e^(-2*(d*e - c*f)/f))/(a*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18696, size = 208, normalized size = 1.59

$$-\frac{\left(i\,\mathrm{Ei}\left(\frac{2\left(dfx+de\right)}{f}\right)e^{\left(4\,c-\frac{2\,de}{f}\right)}-2\,\mathrm{Ei}\left(\frac{dfx+de}{f}\right)e^{\left(3\,c-\frac{de}{f}\right)}-2\,\mathrm{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(c+\frac{de}{f}\right)}-i\,\mathrm{Ei}\left(-\frac{2\left(dfx+de\right)}{f}\right)e^{\left(\frac{2\,de}{f}\right)}+3i\,e^{\left(2\,c\right)}\log\left(fx+e\right.}{4\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(I*Ei(2*(d*f*x + d*e)/f)*e^(4*c - 2*d*e/f) - 2*Ei((d*f*x + d*e)/f)*e^(
3*c - d*e/f) - 2*Ei(-(d*f*x + d*e)/f)*e^(c + d*e/f) - I*Ei(-2*(d*f*x + d*e)
/f)*e^(2*d*e/f) + 3*I*e^(2*c)*log(f*x + e) - 3*I*e^(2*c)*log(I*f*x + I*e))*
e^(-2*c)/(a*f)
```

3.270 $\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=180

$$\frac{d\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{af^2}-\frac{id\cosh\left(2c-\frac{2de}{f}\right)\text{Chi}\left(\frac{2de}{f}+2dx\right)}{af^2}-\frac{id\sinh\left(2c-\frac{2de}{f}\right)\text{Shi}\left(\frac{2de}{f}+2dx\right)}{af^2}+\frac{d\cosh\left(c-\frac{de}{f}\right.}{a}$$

```
[Out] -(Cosh[c + d*x]/(a*f*(e + f*x))) - (I*d*Cosh[2*c - (2*d*e)/f]*CoshIntegral[
(2*d*e)/f + 2*d*x])/(a*f^2) + (d*CoshIntegral[(d*e)/f + d*x]*Sinh[c - (d*e)
/f])/(a*f^2) + ((I/2)*Sinh[2*c + 2*d*x])/(a*f*(e + f*x)) + (d*Cosh[c - (d*e
)/f]*SinhIntegral[(d*e)/f + d*x])/(a*f^2) - (I*d*Sinh[2*c - (2*d*e)/f]*Sinh
Integral[(2*d*e)/f + 2*d*x])/(a*f^2)
```

Rubi [A] time = 0.391807, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {5563, 3297, 3303, 3298, 3301, 5448, 12}

$$\frac{d\sinh\left(c-\frac{de}{f}\right)\text{Chi}\left(\frac{de}{f}+dx\right)}{af^2}-\frac{id\cosh\left(2c-\frac{2de}{f}\right)\text{Chi}\left(\frac{2de}{f}+2dx\right)}{af^2}-\frac{id\sinh\left(2c-\frac{2de}{f}\right)\text{Shi}\left(\frac{2de}{f}+2dx\right)}{af^2}+\frac{d\cosh\left(c-\frac{de}{f}\right.}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```


```
[Out] -(Cosh[c + d*x]/(a*f*(e + f*x))) - (I*d*Cosh[2*c - (2*d*e)/f]*CoshIntegral[
(2*d*e)/f + 2*d*x])/(a*f^2) + (d*CoshIntegral[(d*e)/f + d*x]*Sinh[c - (d*e)
/f])/(a*f^2) + ((I/2)*Sinh[2*c + 2*d*x])/(a*f*(e + f*x)) + (d*Cosh[c - (d*e
)/f]*SinhIntegral[(d*e)/f + d*x])/(a*f^2) - (I*d*Sinh[2*c - (2*d*e)/f]*Sinh
Integral[(2*d*e)/f + 2*d*x])/(a*f^2)
```


Rule 5563

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Cosh[c
+ d*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Si
nh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ
[a^2 + b^2, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx = -\frac{i\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2}\,dx}{a}+\frac{\int \frac{\cosh(c+dx)}{(e+fx)^2}\,dx}{a}$$

$$= -\frac{\cosh(c+dx)}{af(e+fx)}-\frac{i\int \frac{\sinh(2c+2dx)}{2(e+fx)^2}\,dx}{a}+\frac{d\int \frac{\sinh(c+dx)}{e+fx}\,dx}{af}$$

$$= -\frac{\cosh(c+dx)}{af(e+fx)}-\frac{i\int \frac{\sinh(2c+2dx)}{(e+fx)^2}\,dx}{2a}+\frac{\left(d\cosh\left(c-\frac{de}{f}\right)\right)\int \frac{\sinh\left(\frac{de}{f}+dx\right)}{e+fx}\,dx}{af}+\frac{(d\,\mathrm{s}}{}$$

$$= -\frac{\cosh(c+dx)}{af(e+fx)}+\frac{d\mathrm{Chi}\left(\frac{de}{f}+dx\right)\sinh\left(c-\frac{de}{f}\right)}{af^2}+\frac{i\sinh(2c+2dx)}{2af(e+fx)}+\frac{d\cosh\left(c\right.}{}$$

$$= -\frac{\cosh(c+dx)}{af(e+fx)}+\frac{d\mathrm{Chi}\left(\frac{de}{f}+dx\right)\sinh\left(c-\frac{de}{f}\right)}{af^2}+\frac{i\sinh(2c+2dx)}{2af(e+fx)}+\frac{d\cosh\left(c\right.}{}$$

$$= -\frac{\cosh(c+dx)}{af(e+fx)}-\frac{id\cosh\left(2c-\frac{2de}{f}\right)\mathrm{Chi}\left(\frac{2de}{f}+2dx\right)}{af^2}+\frac{d\mathrm{Chi}\left(\frac{de}{f}+dx\right)\sinh\left(c\right.}{af^2}$$

Mathematica [A] time = 0.712969, size = 212, normalized size = 1.18

$$\frac{2d(e+fx)\sinh\left(c-\frac{de}{f}\right)\mathrm{Chi}\left(d\left(\frac{e}{f}+x\right)\right)-2id(e+fx)\cosh\left(2c-\frac{2de}{f}\right)\mathrm{Chi}\left(\frac{2d(e+fx)}{f}\right)-2ide\sinh\left(2c-\frac{2de}{f}\right)\mathrm{Shi}\left(\frac{2d(e+}{f}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] (-2*f*Cosh[c + d*x] - (2*I)*d*(e + f*x)*Cosh[2*c - (2*d*e)/f]*CoshIntegral[
(2*d*(e + f*x))/f] + 2*d*(e + f*x)*CoshIntegral[d*(e/f + x)]*Sinh[c - (d*e)
/f] + I*f*Sinh[2*(c + d*x)] + 2*d*e*Cosh[c - (d*e)/f]*SinhIntegral[d*(e/f +
```

```
 x)] + 2*d*f*x*Cosh[c - (d*e)/f]*SinhIntegral[d*(e/f + x)] - (2*I)*d*e*Sinh
[2*c - (2*d*e)/f]*SinhIntegral[(2*d*(e + f*x))/f] - (2*I)*d*f*x*Sinh[2*c -
(2*d*e)/f]*SinhIntegral[(2*d*(e + f*x))/f])/(2*a*f^2*(e + f*x))
```

Maple [A] time = 0.125, size = 299, normalized size = 1.7

$$-\frac{d\mathrm{e}^{-dx-c}}{2\,af\left(dfx+de\right)}+\frac{d}{2\,af^{2}}\mathrm{e}^{-\frac{cf-de}{f}}\mathrm{Ei}\left(1,dx+c-\frac{cf-de}{f}\right)-\frac{d\mathrm{e}^{dx+c}}{2\,af^{2}}\left(\frac{de}{f}+dx\right)^{-1}-\frac{d}{2\,af^{2}}\mathrm{e}^{\frac{cf-de}{f}}\mathrm{Ei}\left(1,-dx-c-\frac{-cf+de}{f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -1/2/a*d*exp(-d*x-c)/f/(d*f*x+d*e)+1/2/a*d/f^2*exp(-(c*f-d*e)/f)*Ei(1,d*x+c
-(c*f-d*e)/f)-1/2*d/a/f^2*exp(d*x+c)/(d*e/f+d*x)-1/2*d/a/f^2*exp((c*f-d*e)/
f)*Ei(1,-d*x-c-(-c*f+d*e)/f)+1/4*I*d/a/f^2*exp(2*d*x+2*c)/(d*e/f+d*x)+1/2*I
*d/a/f^2*exp(2*(c*f-d*e)/f)*Ei(1,-2*d*x-2*c-2*(-c*f+d*e)/f)-1/4*I/a*d*exp(-
2*d*x-2*c)/f/(d*f*x+d*e)+1/2*I/a*d/f^2*exp(-2*(c*f-d*e)/f)*Ei(1,2*d*x+2*c-2
*(c*f-d*e)/f)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.21271, size = 510, normalized size = 2.83

$$\frac{\left(i\,fe^{(4\,dx+4\,c)}-2\,fe^{(3\,dx+3\,c)}+\left(\left(-2i\,dfx-2i\,de\right)\mathrm{Ei}\left(-\frac{2\left(dfx+de\right)}{f}\right)e^{\left(\frac{2\left(de-cf\right)}{f}\right)}-2\left(dfx+de\right)\mathrm{Ei}\left(-\frac{dfx+de}{f}\right)e^{\left(\frac{de-cf}{f}\right)}+2\left(dfx+d\right.\right.\right.}{4\left(af^{3}x+aef^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/4*(I*f*e^(4*d*x + 4*c) - 2*f*e^(3*d*x + 3*c) + ((-2*I*d*f*x - 2*I*d*e)*Ei
(-2*(d*f*x + d*e)/f)*e^(2*(d*e - c*f)/f) - 2*(d*f*x + d*e)*Ei(-(d*f*x + d*e
)/f)*e^((d*e - c*f)/f) + 2*(d*f*x + d*e)*Ei((d*f*x + d*e)/f)*e^(-(d*e - c*f
)/f) + (-2*I*d*f*x - 2*I*d*e)*Ei(2*(d*f*x + d*e)/f)*e^(-2*(d*e - c*f)/f))*e
^(2*d*x + 2*c) - 2*f*e^(d*x + c) - I*f)*e^(-2*d*x - 2*c)/(a*f^3*x + a*e*f^2
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.271 $\int \frac{(e+fx)^3 \mathbf{sech}(c+dx)}{a+ia\sinh(c+dx)} dx$

Optimal. Leaf size=463

$$\frac{3if^2(e+fx)\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^3}-\frac{3if^2(e+fx)\text{PolyLog}\left(3,ie^{c+dx}\right)}{ad^3}-\frac{3if(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2ad^2}+\frac{3if(e+fx)^2}{}$$

```
[Out] (((-3*I)/2)*f*(e + f*x)^2)/(a*d^2) - (6*f^2*(e + f*x)*ArcTan[E^(c + d*x)])/
(a*d^3) + ((e + f*x)^3*ArcTan[E^(c + d*x)])/(a*d) + ((3*I)*f^2*(e + f*x)*Lo
g[1 + E^(2*(c + d*x))])/(a*d^3) + ((3*I)*f^3*PolyLog[2, (-I)*E^(c + d*x)])/
(a*d^4) - (((3*I)/2)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) -
((3*I)*f^3*PolyLog[2, I*E^(c + d*x)])/(a*d^4) + (((3*I)/2)*f*(e + f*x)^2*Po
lyLog[2, I*E^(c + d*x)])/(a*d^2) + (((3*I)/2)*f^3*PolyLog[2, -E^(2*(c + d*x
))])/(a*d^4) + ((3*I)*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^3) -
 ((3*I)*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a*d^3) - ((3*I)*f^3*PolyL
og[4, (-I)*E^(c + d*x)])/(a*d^4) + ((3*I)*f^3*PolyLog[4, I*E^(c + d*x)])/(a
*d^4) + (3*f*(e + f*x)^2*Sech[c + d*x])/(2*a*d^2) + ((I/2)*(e + f*x)^3*Sech
[c + d*x]^2)/(a*d) - (((3*I)/2)*f*(e + f*x)^2*Tanh[c + d*x])/(a*d^2) + ((e
+ f*x)^3*Sech[c + d*x]*Tanh[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.483055, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {5571, 4186, 4180, 2279, 2391, 2531, 6609, 2282, 6589, 5451, 4184, 3718, 2190}

$$\frac{3if^2(e+fx)\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^3}-\frac{3if^2(e+fx)\text{PolyLog}\left(3,ie^{c+dx}\right)}{ad^3}-\frac{3if(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2ad^2}+\frac{3if(e+fx)^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```


```
[Out] (((-3*I)/2)*f*(e + f*x)^2)/(a*d^2) - (6*f^2*(e + f*x)*ArcTan[E^(c + d*x)])/
(a*d^3) + ((e + f*x)^3*ArcTan[E^(c + d*x)])/(a*d) + ((3*I)*f^2*(e + f*x)*Lo
g[1 + E^(2*(c + d*x))])/(a*d^3) + ((3*I)*f^3*PolyLog[2, (-I)*E^(c + d*x)])/
(a*d^4) - (((3*I)/2)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) -
((3*I)*f^3*PolyLog[2, I*E^(c + d*x)])/(a*d^4) + (((3*I)/2)*f*(e + f*x)^2*Po
lyLog[2, I*E^(c + d*x)])/(a*d^2) + (((3*I)/2)*f^3*PolyLog[2, -E^(2*(c + d*x
))])/(a*d^4) + ((3*I)*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^3) -
 ((3*I)*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a*d^3) - ((3*I)*f^3*PolyL
og[4, (-I)*E^(c + d*x)])/(a*d^4) + ((3*I)*f^3*PolyLog[4, I*E^(c + d*x)])/(a
*d^4) + (3*f*(e + f*x)^2*Sech[c + d*x])/(2*a*d^2) + ((I/2)*(e + f*x)^3*Sech
[c + d*x]^2)/(a*d) - (((3*I)/2)*f*(e + f*x)^2*Tanh[c + d*x])/(a*d^2) + ((e
+ f*x)^3*Sech[c + d*x]*Tanh[c + d*x])/(2*a*d)
```


Rule 5571

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sech[c
 + d*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)*T
anh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && Eq
Q[a^2 + b^2, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
```

```
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
```

```
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int (e+fx)^3\operatorname{sech}^2(c+dx)\tanh(c+dx)\,dx}{a}+\frac{\int (e+fx)^3\operatorname{sech}^3(c+dx)\,dx}{a}$$
$$= \frac{3f(e+fx)^2\operatorname{sech}(c+dx)}{2ad^2}+\frac{i(e+fx)^3\operatorname{sech}^2(c+dx)}{2ad}+\frac{(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx}{2ad}$$
$$= -\frac{6f^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{ad}+\frac{3f(e+fx)^2\operatorname{sech}(c+dx)}{2ad^2}+\frac{i(e+}{}$$
$$= -\frac{3if(e+fx)^2}{2ad^2}-\frac{6f^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{ad}-\frac{3if(e+fx)^2\mathrm{Li}_2(}{2ad^2}$$
$$= -\frac{3if(e+fx)^2}{2ad^2}-\frac{6f^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{ad}+\frac{3if^2(e+fx)\log(}{ad^3}$$
$$= -\frac{3if(e+fx)^2}{2ad^2}-\frac{6f^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{ad}+\frac{3if^2(e+fx)\log(}{ad^3}$$
$$= -\frac{3if(e+fx)^2}{2ad^2}-\frac{6f^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^3}+\frac{(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{ad}+\frac{3if^2(e+fx)\log(}{ad^3}$$

Mathematica [A] time = 11.3799, size = 767, normalized size = 1.66

$$-\frac{12\left(1+ie^c\right)f^2\left(4f^2-d^2e^2\right)\operatorname{PolyLog}\left(2,ie^{-c-dx}\right)-12\left(1+ie^c\right)f^4\left(d^2x^2\operatorname{PolyLog}\left(2,ie^{-c-dx}\right)+2\left(dx\operatorname{PolyLog}\left(3,ie^{-c-dx}\right)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] -((e + f*x)^4/f + (4*(1 - I*E^c)*(e + f*x)^3*Log[1 + I*E^(-c - d*x)])/d + (
(12*I)*(I + E^c)*f*(d^2*(e + f*x)^2*PolyLog[2, (-I)*E^(-c - d*x)] + 2*f*(d*
(e + f*x)*PolyLog[3, (-I)*E^(-c - d*x)] + f*PolyLog[4, (-I)*E^(-c - d*x)]))
)/d^4)/(8*a*(I + E^c)) - ((-12*f^2 + d^2*(e + f*x)^2)^2 + 12*d*(1 + I*E^c)*
f^2*(d^2*e^2 - 4*f^2)*x*Log[1 - I*E^(-c - d*x)] + 12*d^3*e*(1 + I*E^c)*f^3*
x^2*Log[1 - I*E^(-c - d*x)] + 4*d^3*(1 + I*E^c)*f^4*x^3*Log[1 - I*E^(-c - d
*x)] - 4*d*e*(1 + I*E^c)*f*(d^2*e^2 - 12*f^2)*(d*x - Log[I - E^(c + d*x)])
+ 12*(1 + I*E^c)*f^2*(-(d^2*e^2) + 4*f^2)*PolyLog[2, I*E^(-c - d*x)] - 24*d
*e*(1 + I*E^c)*f^3*(d*x*PolyLog[2, I*E^(-c - d*x)] + PolyLog[3, I*E^(-c - d
*x)]) - 12*(1 + I*E^c)*f^4*(d^2*x^2*PolyLog[2, I*E^(-c - d*x)] + 2*(d*x*Pol
yLog[3, I*E^(-c - d*x)] + PolyLog[4, I*E^(-c - d*x)])))/(8*a*d^4*(-I + E^c)
*f) + (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(8*a*(Cosh[c/2] - I*S
inh[c/2])*(Cosh[c/2] + I*Sinh[c/2])) + ((I/2)*(e + f*x)^3)/(a*d*(Cosh[c/2 +
 (d*x)/2] + I*Sinh[c/2 + (d*x)/2])^2) - ((3*I)*(e^2*f*Sinh[(d*x)/2] + 2*e*f
^2*x*Sinh[(d*x)/2] + f^3*x^2*Sinh[(d*x)/2]))/(a*d^2*(Cosh[c/2] + I*Sinh[c/2
])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2]))
```

Maple [B] time = 0.244, size = 1152, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 3*I/a/d^2*polylog(2,I*exp(d*x+c))*e*f^2*x-3/2*I/a/d^3*e*f^2*c^2*ln(exp(d*x+
c)-I)+3/2*I/a/d^3*e*f^2*c^2*ln(exp(d*x+c)+I)+3/2*I/a/d^3*ln(1+I*exp(d*x+c))
*c^2*e*f^2-3/2*I/a/d^3*ln(1-I*exp(d*x+c))*c^2*e*f^2+3/2*I/a/d^2*e^2*f*c*ln(
exp(d*x+c)-I)-3/2*I/a/d^2*e^2*f*c*ln(exp(d*x+c)+I)+3/2*I/a/d*ln(1-I*exp(d*x
+c))*e^2*f*x+3/2*I/a/d^2*ln(1-I*exp(d*x+c))*c*e^2*f-3/2*I/a/d*ln(1+I*exp(d*
x+c))*e^2*f*x-3/2*I/a/d^2*ln(1+I*exp(d*x+c))*c*e^2*f-3/2*I/a/d*ln(1+I*exp(d
*x+c))*e*f^2*x^2-3*I/a/d^2*polylog(2,-I*exp(d*x+c))*e*f^2*x+3/2*I/a/d*ln(1-
I*exp(d*x+c))*e*f^2*x^2-3*I*f^3*polylog(4,-I*exp(d*x+c))/a/d^4+3*I*f^3*poly
log(4,I*exp(d*x+c))/a/d^4+1/2*I/a/d*e^3*ln(exp(d*x+c)+I)-3*I/a/d^2*f^3*x^2-
3*I/a/d^4*f^3*c^2+6*I/a/d^4*f^3*polylog(2,-I*exp(d*x+c))-1/2*I/a/d*e^3*ln(e
xp(d*x+c)-I)-3/2*I/a/d^2*f^3*polylog(2,-I*exp(d*x+c))*x^2+6*I/a/d^3*e*f^2*l
n(exp(d*x+c)-I)-6*I/a/d^3*e*f^2*ln(exp(d*x+c))+3*I/a/d^3*e*f^2*polylog(3,-I
*exp(d*x+c))-3*I/a/d^3*e*f^2*polylog(3,I*exp(d*x+c))-6*I/a/d^3*f^3*c*x-6*I/
a/d^4*f^3*c*ln(exp(d*x+c)-I)-3/2*I/a/d^2*e^2*f*polylog(2,-I*exp(d*x+c))+3/2
*I/a/d^2*e^2*f*polylog(2,I*exp(d*x+c))+1/2*I/a/d^4*f^3*c^3*ln(exp(d*x+c)-I)
-1/2*I/a/d^4*f^3*c^3*ln(exp(d*x+c)+I)+6*I/a/d^4*f^3*c*ln(exp(d*x+c))+6*I/a/
d^4*f^3*c*ln(1+I*exp(d*x+c))+1/2*I/a/d^4*f^3*c^3*ln(1-I*exp(d*x+c))-1/2*I/a
/d^4*f^3*c^3*ln(1+I*exp(d*x+c))-1/2*I/a/d*f^3*ln(1+I*exp(d*x+c))*x^3+6*I/a/
d^3*f^3*ln(1+I*exp(d*x+c))*x+1/2*I/a/d*f^3*ln(1-I*exp(d*x+c))*x^3+3/2*I/a/d
^2*f^3*polylog(2,I*exp(d*x+c))*x^2-3*I/a/d^3*f^3*polylog(3,I*exp(d*x+c))*x+
3*I/a/d^3*f^3*polylog(3,-I*exp(d*x+c))*x+(d*f^3*x^3*exp(d*x+c)+3*d*e*f^2*x^
2*exp(d*x+c)+3*d*e^2*f*x*exp(d*x+c)+d*e^3*exp(d*x+c)+3*f^3*x^2*exp(d*x+c)-3
*I*f^3*x^2+6*e*f^2*x*exp(d*x+c)-6*I*e*f^2*x+3*e^2*f*exp(d*x+c)-3*I*e^2*f)/(
exp(d*x+c)-I)^2/d^2/a
```

Maxima [A] time = 2.02883, size = 923, normalized size = 1.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*e^3*(4*e^(-d*x - c)/((4*I*a*e^(-d*x - c) + 2*a*e^(-2*d*x - 2*c) - 2*a)
*d) + I*log(e^(-d*x - c) + I)/(a*d) - I*log(I*e^(-d*x - c) + 1)/(a*d)) + 3/
2*I*(d*x*log(-I*e^(d*x + c) + 1) + dilog(I*e^(d*x + c)))*e^2*f/(a*d^2) - 6*
I*e*f^2*x/(a*d^2) + (-3*I*f^3*x^2 - 6*I*e*f^2*x - 3*I*e^2*f + (d*f^3*x^3*e^
c + 3*e^2*f*e^c + 3*(d*e*f^2 + f^3)*x^2*e^c + 3*(d*e^2*f + 2*e*f^2)*x*e^c)*
e^(d*x))/(a*d^2*e^(2*d*x + 2*c) - 2*I*a*d^2*e^(d*x + c) - a*d^2) - 3/2*I*(d
^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x + c)) - 2*polylog(3,
-I*e^(d*x + c)))*e*f^2/(a*d^3) + 3/2*I*(d^2*x^2*log(-I*e^(d*x + c) + 1) + 2
*d*x*dilog(I*e^(d*x + c)) - 2*polylog(3, I*e^(d*x + c)))*e*f^2/(a*d^3) + 6*
I*e*f^2*log(I*e^(d*x + c) + 1)/(a*d^3) - 1/2*I*(d^3*x^3*log(I*e^(d*x + c) +
 1) + 3*d^2*x^2*dilog(-I*e^(d*x + c)) - 6*d*x*polylog(3, -I*e^(d*x + c)) +
6*polylog(4, -I*e^(d*x + c)))*f^3/(a*d^4) + 1/2*I*(d^3*x^3*log(-I*e^(d*x +
c) + 1) + 3*d^2*x^2*dilog(I*e^(d*x + c)) - 6*d*x*polylog(3, I*e^(d*x + c))
+ 6*polylog(4, I*e^(d*x + c)))*f^3/(a*d^4) - 3/2*I*(d^2*e^2*f - 4*f^3)*(d*x
*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))/(a*d^4) - 1/8*(I*d^4*f^3*x
^4 + 4*I*d^4*e*f^2*x^3 + 6*I*d^4*e^2*f*x^2)/(a*d^4) + 1/8*(I*d^4*f^3*x^4 +
4*I*d^4*e*f^2*x^3 + (6*I*d^2*e^2*f - 24*I*f^3)*d^2*x^2)/(a*d^4)
```

Fricas [C] time = 2.45374, size = 3418, normalized size = 7.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (-6*I*d^2*e^2*f + 12*I*c*d*e*f^2 - 6*I*c^2*f^3 + (-3*I*d^2*f^3*x^2 - 6*I*d^
2*e*f^2*x - 3*I*d^2*e^2*f + (3*I*d^2*f^3*x^2 + 6*I*d^2*e*f^2*x + 3*I*d^2*e^
2*f)*e^(2*d*x + 2*c) + 6*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*e^(d*x +
 c))*dilog(I*e^(d*x + c)) + (3*I*d^2*f^3*x^2 + 6*I*d^2*e*f^2*x + 3*I*d^2*e^
2*f - 12*I*f^3 + (-3*I*d^2*f^3*x^2 - 6*I*d^2*e*f^2*x - 3*I*d^2*e^2*f + 12*I
*f^3)*e^(2*d*x + 2*c) - 6*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f - 4*f^3)
*e^(d*x + c))*dilog(-I*e^(d*x + c)) + (-6*I*d^2*f^3*x^2 - 12*I*d^2*e*f^2*x
- 12*I*c*d*e*f^2 + 6*I*c^2*f^3)*e^(2*d*x + 2*c) + 2*(d^3*f^3*x^3 + d^3*e^3
+ 3*d^2*e^2*f - 12*c*d*e*f^2 + 6*c^2*f^3 + 3*(d^3*e*f^2 - d^2*f^3)*x^2 + 3*
(d^3*e^2*f - 2*d^2*e*f^2)*x)*e^(d*x + c) + (-I*d^3*e^3 + 3*I*c*d^2*e^2*f -
3*I*c^2*d*e*f^2 + I*c^3*f^3 + (I*d^3*e^3 - 3*I*c*d^2*e^2*f + 3*I*c^2*d*e*f^
2 - I*c^3*f^3)*e^(2*d*x + 2*c) + 2*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2
 - c^3*f^3)*e^(d*x + c))*log(e^(d*x + c) + I) + (I*d^3*e^3 - 3*I*c*d^2*e^2*
f + (3*I*c^2 - 12*I)*d*e*f^2 + (-I*c^3 + 12*I*c)*f^3 + (-I*d^3*e^3 + 3*I*c*
d^2*e^2*f + (-3*I*c^2 + 12*I)*d*e*f^2 + (I*c^3 - 12*I*c)*f^3)*e^(2*d*x + 2*
c) - 2*(d^3*e^3 - 3*c*d^2*e^2*f + 3*(c^2 - 4)*d*e*f^2 - (c^3 - 12*c)*f^3)*e
^(d*x + c))*log(e^(d*x + c) - I) + (I*d^3*f^3*x^3 + 3*I*d^3*e*f^2*x^2 + 3*I
*c*d^2*e^2*f - 3*I*c^2*d*e*f^2 + (I*c^3 - 12*I*c)*f^3 + (3*I*d^3*e^2*f - 12
*I*d*f^3)*x + (-I*d^3*f^3*x^3 - 3*I*d^3*e*f^2*x^2 - 3*I*c*d^2*e^2*f + 3*I*c
^2*d*e*f^2 + (-I*c^3 + 12*I*c)*f^3 + (-3*I*d^3*e^2*f + 12*I*d*f^3)*x)*e^(2*
d*x + 2*c) - 2*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*c*d^2*e^2*f - 3*c^2*d*e*f
^2 + (c^3 - 12*c)*f^3 + 3*(d^3*e^2*f - 4*d*f^3)*x)*e^(d*x + c))*log(I*e^(d*
x + c) + 1) + (-I*d^3*f^3*x^3 - 3*I*d^3*e*f^2*x^2 - 3*I*d^3*e^2*f*x - 3*I*c
*d^2*e^2*f + 3*I*c^2*d*e*f^2 - I*c^3*f^3 + (I*d^3*f^3*x^3 + 3*I*d^3*e*f^2*x
^2 + 3*I*d^3*e^2*f*x + 3*I*c*d^2*e^2*f - 3*I*c^2*d*e*f^2 + I*c^3*f^3)*e^(2*
d*x + 2*c) + 2*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2
*f - 3*c^2*d*e*f^2 + c^3*f^3)*e^(d*x + c))*log(-I*e^(d*x + c) + 1) + (6*I*f
^3*e^(2*d*x + 2*c) + 12*f^3*e^(d*x + c) - 6*I*f^3)*polylog(4, I*e^(d*x + c)
) + (-6*I*f^3*e^(2*d*x + 2*c) - 12*f^3*e^(d*x + c) + 6*I*f^3)*polylog(4, -I
```

```
*e^(d*x + c)) + (6*I*d*f^3*x + 6*I*d*e*f^2 + (-6*I*d*f^3*x - 6*I*d*e*f^2)*e
^(2*d*x + 2*c) - 12*(d*f^3*x + d*e*f^2)*e^(d*x + c))*polylog(3, I*e^(d*x +
c)) + (-6*I*d*f^3*x - 6*I*d*e*f^2 + (6*I*d*f^3*x + 6*I*d*e*f^2)*e^(2*d*x +
2*c) + 12*(d*f^3*x + d*e*f^2)*e^(d*x + c))*polylog(3, -I*e^(d*x + c)))/(2*a
*d^4*e^(2*d*x + 2*c) - 4*I*a*d^4*e^(d*x + c) - 2*a*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \operatorname{sech}(dx+c)}{ia\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sech(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.272 $\int \frac{(e+fx)^2\mathbf{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=268

$$-\frac{if(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2}+\frac{if(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{ad^2}+\frac{if^2\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^3}-\frac{if^2\text{PolyLog}\left(3,ie^{c+dx}\right)}{ad^3}$$

```
[Out] ((e + f*x)^2*ArcTan[E^(c + d*x)])/(a*d) - (f^2*ArcTan[Sinh[c + d*x]])/(a*d^
3) + (I*f^2*Log[Cosh[c + d*x]])/(a*d^3) - (I*f*(e + f*x)*PolyLog[2, (-I)*E^
(c + d*x)])/(a*d^2) + (I*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + (
I*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^3) - (I*f^2*PolyLog[3, I*E^(c + d*
x)])/(a*d^3) + (f*(e + f*x)*Sech[c + d*x])/(a*d^2) + ((I/2)*(e + f*x)^2*Sec
h[c + d*x]^2)/(a*d) - (I*f*(e + f*x)*Tanh[c + d*x])/(a*d^2) + ((e + f*x)^2*
Sech[c + d*x]*Tanh[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.263905, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {5571, 4186, 3770, 4180, 2531, 2282, 6589, 5451, 4184, 3475}

$$-\frac{if(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2}+\frac{if(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{ad^2}+\frac{if^2\text{PolyLog}\left(3,-ie^{c+dx}\right)}{ad^3}-\frac{if^2\text{PolyLog}\left(3,ie^{c+dx}\right)}{ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((e + f*x)^2*ArcTan[E^(c + d*x)])/(a*d) - (f^2*ArcTan[Sinh[c + d*x]])/(a*d^
3) + (I*f^2*Log[Cosh[c + d*x]])/(a*d^3) - (I*f*(e + f*x)*PolyLog[2, (-I)*E^
(c + d*x)])/(a*d^2) + (I*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + (
I*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^3) - (I*f^2*PolyLog[3, I*E^(c + d*
x)])/(a*d^3) + (f*(e + f*x)*Sech[c + d*x])/(a*d^2) + ((I/2)*(e + f*x)^2*Sec
h[c + d*x]^2)/(a*d) - (I*f*(e + f*x)*Tanh[c + d*x])/(a*d^2) + ((e + f*x)^2*
Sech[c + d*x]*Tanh[c + d*x])/(2*a*d)
```

Rule 5571

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sech[c
 + d*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)*T
anh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && Eq
Q[a^2 + b^2, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int (e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)\,dx}{a} + \frac{\int (e+fx)^2\text{sech}^3(c+dx)\,dx}{a}$$

$$= \frac{f(e+fx)\text{sech}(c+dx)}{ad^2} + \frac{i(e+fx)^2\text{sech}^2(c+dx)}{2ad} + \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{2ad}$$

$$= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad} - \frac{f^2\tan^{-1}(\sinh(c+dx))}{ad^3} + \frac{f(e+fx)\text{sech}(c+dx)}{ad^2} + \frac{i(e+fx)^2\text{se}}{2}$$

$$= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad} - \frac{f^2\tan^{-1}(\sinh(c+dx))}{ad^3} + \frac{if^2\log(\cosh(c+dx))}{ad^3} - \frac{if(e+fx)\text{Li}}{ad}$$

$$= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad} - \frac{f^2\tan^{-1}(\sinh(c+dx))}{ad^3} + \frac{if^2\log(\cosh(c+dx))}{ad^3} - \frac{if(e+fx)\text{Li}}{ad}$$

$$= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad} - \frac{f^2\tan^{-1}(\sinh(c+dx))}{ad^3} + \frac{if^2\log(\cosh(c+dx))}{ad^3} - \frac{if(e+fx)\text{Li}}{ad}$$

Mathematica [A] time = 11.3621, size = 501, normalized size = 1.87

$$-\frac{\frac{-6(1+ie^c)ef\text{PolyLog}\left(2,ie^{-c-dx}\right)-6(1+ie^c)f^2\left(x\text{PolyLog}\left(2,ie^{-c-dx}\right)+\frac{\text{PolyLog}\left(3,ie^{-c-dx}\right)}{d}\right)-\frac{3(1+ie^c)\left(d^2e^2-4f^2\right)\left(dx-\log\left(-e^{c+dx}+i\right)\right)}{d}+6(1+ie^c)defx\log\left(1-ie^{-c-dx}\right)+3(1}{(e^c-i)d^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] -(((e + f*x)^3/f + (3*(1 - I*E^c)*(e + f*x)^2*Log[1 + I*E^(-c - d*x)])/d +
((6*I)*(I + E^c)*f*(d*(e + f*x)*PolyLog[2, (-I)*E^(-c - d*x)] + f*PolyLog[3
, (-I)*E^(-c - d*x)]))/d^3)/(I + E^c) + (3*(d^2*e^2 - 4*f^2)*x + 3*d^2*e*f*
x^2 + d^2*f^2*x^3 + 6*d*e*(1 + I*E^c)*f*x*Log[1 - I*E^(-c - d*x)] + 3*d*(1
+ I*E^c)*f^2*x^2*Log[1 - I*E^(-c - d*x)] - (3*(1 + I*E^c)*(d^2*e^2 - 4*f^2)
*(d*x - Log[I - E^(c + d*x)]))/d - 6*e*(1 + I*E^c)*f*PolyLog[2, I*E^(-c - d
*x)] - 6*(1 + I*E^c)*f^2*(x*PolyLog[2, I*E^(-c - d*x)] + PolyLog[3, I*E^(-c
 - d*x)]/d))/(d^2*(-I + E^c)) - x*(3*e^2 + 3*e*f*x + f^2*x^2)*Sech[c] - ((3
*I)*(e + f*x)^2)/(d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2) + ((12*I)*
f*(e + f*x)*Sinh[(d*x)/2])/(d^2*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2
] + I*Sinh[(c + d*x)/2])))/(6*a)
```

Maple [B] time = 0.132, size = 613, normalized size = 2.3

$$\frac{dx^2f^2e^{dx+c}+2defxe^{dx+c}+de^2e^{dx+c}-2if^2x+2f^2xe^{dx+c}-2ief+2efe^{dx+c}}{\left(e^{dx+c}-i\right)^2d^2a} - \frac{ief\text{polylog}\left(2,-ie^{dx+c}\right)}{ad^2} - \frac{i\ln\left(1+ie^{dx+c}\right.}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] (d*x^2*f^2*exp(d*x+c)+2*d*e*f*x*exp(d*x+c)+d*e^2*exp(d*x+c)-2*I*f^2*x+2*f^2
*x*exp(d*x+c)-2*I*e*f+2*e*f*exp(d*x+c))/(exp(d*x+c)-I)^2/d^2/a-I/a/d^2*e*f*
polylog(2,-I*exp(d*x+c))-I/a/d*ln(1+I*exp(d*x+c))*e*f*x+1/2*I/a/d*e^2*ln(ex
p(d*x+c)+I)-1/2*I/a/d*e^2*ln(exp(d*x+c)-I)-1/2*I/a/d^3*c^2*f^2*ln(exp(d*x+c
)-I)-I/a/d^2*e*f*c*ln(exp(d*x+c)+I)-1/2*I/a/d^3*ln(1-I*exp(d*x+c))*c^2*f^2+
```

```
I*f^2*polylog(3,-I*exp(d*x+c))/a/d^3-2*I/a/d^3*f^2*ln(exp(d*x+c))+1/2*I/a/d
^3*ln(1+I*exp(d*x+c))*c^2*f^2+I/a/d^2*ln(1-I*exp(d*x+c))*c*e*f+I/a/d^2*poly
log(2,I*exp(d*x+c))*f^2*x-1/2*I/a/d*ln(1+I*exp(d*x+c))*f^2*x^2-I/a/d^2*poly
log(2,-I*exp(d*x+c))*f^2*x+I/a/d*ln(1-I*exp(d*x+c))*e*f*x+1/2*I/a/d*ln(1-I*
exp(d*x+c))*f^2*x^2+1/2*I/a/d^3*c^2*f^2*ln(exp(d*x+c)+I)+2*I/a/d^3*f^2*ln(e
xp(d*x+c)-I)-I/a/d^2*ln(1+I*exp(d*x+c))*c*e*f+I/a/d^2*e*f*c*ln(exp(d*x+c)-I
)-I*f^2*polylog(3,I*exp(d*x+c))/a/d^3+I/a/d^2*e*f*polylog(2,I*exp(d*x+c))
```

Maxima [A] time = 1.99471, size = 522, normalized size = 1.95

$$-\frac{1}{2}e^2\left(\frac{4\,e^{(-dx-c)}}{\left(4i\,ae^{(-dx-c)}+2\,ae^{(-2\,dx-2\,c)}-2\,a\right)d}+\frac{i\,\log\left(e^{(-dx-c)}+i\right)}{ad}-\frac{i\,\log\left(i\,e^{(-dx-c)}+1\right)}{ad}\right)+\frac{-2i\,f^2x-2i\,ef+\left(df^2x^2e^c+\right.}{ad^2e^{(2\,dx+2\,c)}-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*e^2*(4*e^(-d*x - c)/((4*I*a*e^(-d*x - c) + 2*a*e^(-2*d*x - 2*c) - 2*a)
*d) + I*log(e^(-d*x - c) + I)/(a*d) - I*log(I*e^(-d*x - c) + 1)/(a*d)) + (-
2*I*f^2*x - 2*I*e*f + (d*f^2*x^2*e^c + 2*e*f*e^c + 2*(d*e*f + f^2)*x*e^c)*e
^(d*x))/(a*d^2*e^(2*d*x + 2*c) - 2*I*a*d^2*e^(d*x + c) - a*d^2) - I*(d*x*lo
g(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*e*f/(a*d^2) + I*(d*x*log(-I*e
^(d*x + c) + 1) + dilog(I*e^(d*x + c)))*e*f/(a*d^2) - 2*I*f^2*x/(a*d^2) - 1
/2*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x + c)) - 2*poly
log(3, -I*e^(d*x + c)))*f^2/(a*d^3) + 1/2*I*(d^2*x^2*log(-I*e^(d*x + c) + 1
) + 2*d*x*dilog(I*e^(d*x + c)) - 2*polylog(3, I*e^(d*x + c)))*f^2/(a*d^3) +
 2*I*f^2*log(I*e^(d*x + c) + 1)/(a*d^3)
```

Fricas [C] time = 2.38792, size = 1958, normalized size = 7.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (-4*I*d*e*f + 4*I*c*f^2 + (-2*I*d*f^2*x - 2*I*d*e*f + (2*I*d*f^2*x + 2*I*d*
e*f)*e^(2*d*x + 2*c) + 4*(d*f^2*x + d*e*f)*e^(d*x + c))*dilog(I*e^(d*x + c)
) + (2*I*d*f^2*x + 2*I*d*e*f + (-2*I*d*f^2*x - 2*I*d*e*f)*e^(2*d*x + 2*c) -
 4*(d*f^2*x + d*e*f)*e^(d*x + c))*dilog(-I*e^(d*x + c)) + (-4*I*d*f^2*x - 4
*I*c*f^2)*e^(2*d*x + 2*c) + 2*(d^2*f^2*x^2 + d^2*e^2 + 2*d*e*f - 4*c*f^2 +
2*(d^2*e*f - d*f^2)*x)*e^(d*x + c) + (-I*d^2*e^2 + 2*I*c*d*e*f - I*c^2*f^2
+ (I*d^2*e^2 - 2*I*c*d*e*f + I*c^2*f^2)*e^(2*d*x + 2*c) + 2*(d^2*e^2 - 2*c*
d*e*f + c^2*f^2)*e^(d*x + c))*log(e^(d*x + c) + I) + (I*d^2*e^2 - 2*I*c*d*e
*f + (I*c^2 - 4*I)*f^2 + (-I*d^2*e^2 + 2*I*c*d*e*f + (-I*c^2 + 4*I)*f^2)*e^
(2*d*x + 2*c) - 2*(d^2*e^2 - 2*c*d*e*f + (c^2 - 4)*f^2)*e^(d*x + c))*log(e^
(d*x + c) - I) + (I*d^2*f^2*x^2 + 2*I*d^2*e*f*x + 2*I*c*d*e*f - I*c^2*f^2 +
 (-I*d^2*f^2*x^2 - 2*I*d^2*e*f*x - 2*I*c*d*e*f + I*c^2*f^2)*e^(2*d*x + 2*c)
 - 2*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(d*x + c))*log(I*e
^(d*x + c) + 1) + (-I*d^2*f^2*x^2 - 2*I*d^2*e*f*x - 2*I*c*d*e*f + I*c^2*f^2
 + (I*d^2*f^2*x^2 + 2*I*d^2*e*f*x + 2*I*c*d*e*f - I*c^2*f^2)*e^(2*d*x + 2*c
) + 2*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(d*x + c))*log(-I
*e^(d*x + c) + 1) + (-2*I*f^2*e^(2*d*x + 2*c) - 4*f^2*e^(d*x + c) + 2*I*f^2
)*polylog(3, I*e^(d*x + c)) + (2*I*f^2*e^(2*d*x + 2*c) + 4*f^2*e^(d*x + c)
```

```
- 2*I*f^2)*polylog(3, -I*e^(d*x + c)))/(2*a*d^3*e^(2*d*x + 2*c) - 4*I*a*d^3
*e^(d*x + c) - 2*a*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c)}{i\,a\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sech(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.273 $\int \frac{(e+fx)\mathbf{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=161

$$-\frac{if\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{2ad^2}+\frac{if\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{2ad^2}-\frac{if\tanh(c+dx)}{2ad^2}+\frac{f\mathrm{sech}(c+dx)}{2ad^2}+\frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad}+\frac{i(e}{}$$

```
[Out] ((e + f*x)*ArcTan[E^(c + d*x)])/(a*d) - ((I/2)*f*PolyLog[2, (-I)*E^(c + d*x
)])/(a*d^2) + ((I/2)*f*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + (f*Sech[c + d*x
])/(2*a*d^2) + ((I/2)*(e + f*x)*Sech[c + d*x]^2)/(a*d) - ((I/2)*f*Tanh[c +
d*x])/(a*d^2) + ((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.142473, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5571, 4185, 4180, 2279, 2391, 5451, 3767, 8}

$$-\frac{if\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{2ad^2}+\frac{if\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{2ad^2}-\frac{if\tanh(c+dx)}{2ad^2}+\frac{f\mathrm{sech}(c+dx)}{2ad^2}+\frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad}+\frac{i(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```


```
[Out] ((e + f*x)*ArcTan[E^(c + d*x)])/(a*d) - ((I/2)*f*PolyLog[2, (-I)*E^(c + d*x
)])/(a*d^2) + ((I/2)*f*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + (f*Sech[c + d*x
])/(2*a*d^2) + ((I/2)*(e + f*x)*Sech[c + d*x]^2)/(a*d) - ((I/2)*f*Tanh[c +
d*x])/(a*d^2) + ((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*a*d)
```


Rule 5571

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sech[c
 + d*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)*T
anh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && Eq
Q[a^2 + b^2, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)\,dx}{a}+\frac{\int(e+fx)\text{sech}^3(c+dx)\,dx}{a}$$
$$=\frac{f\text{sech}(c+dx)}{2ad^2}+\frac{i(e+fx)\text{sech}^2(c+dx)}{2ad}+\frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{2ad}+\frac{\int(e+fx\ldots}{}$$
$$=\frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad}+\frac{f\text{sech}(c+dx)}{2ad^2}+\frac{i(e+fx)\text{sech}^2(c+dx)}{2ad}+\frac{(e+fx)\text{sech}(c+dx)\ldots}{2ad}$$
$$=\frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad}+\frac{f\text{sech}(c+dx)}{2ad^2}+\frac{i(e+fx)\text{sech}^2(c+dx)}{2ad}-\frac{if\tanh(c+dx)}{2ad^2}+\frac{(e\ldots}{}$$
$$=\frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad}-\frac{if\text{Li}_2\left(-ie^{c+dx}\right)}{2ad^2}+\frac{if\text{Li}_2\left(ie^{c+dx}\right)}{2ad^2}+\frac{f\text{sech}(c+dx)}{2ad^2}+\frac{i(e+fx)\text{sech}\ldots}{2ad\ldots}$$

Mathematica [B] time = 2.69126, size = 707, normalized size = 4.39

$$\frac{\sqrt{2}f\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2\left(2(-1)^{3/4}(c+dx)^2+\sqrt{2}\left(-4i\text{PolyLog}\left(2,-ie^{-c-dx}\right)+(4ic+4idx-2\pi)\log\ldots\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((8*I)*d*(e + f*x) - 4*(c + d*x)*(c*f - d*(2*e + f*x))*(Cosh[(c + d*x)/2] +
 I*Sinh[(c + d*x)/2])^2 - 4*d*e*(c + d*x - (2*I)*Log[Cosh[(c + d*x)/2] - I*
Sinh[(c + d*x)/2]])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 + 4*c*f*(c
+ d*x - (2*I)*Log[Cosh[(c + d*x)/2] - I*Sinh[(c + d*x)/2]])*(Cosh[(c + d*x)
/2] + I*Sinh[(c + d*x)/2])^2 - 4*d*e*(c + d*x + (2*I)*Log[Cosh[(c + d*x)/2]
 + I*Sinh[(c + d*x)/2]])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 + 4*c*
f*(c + d*x + (2*I)*Log[Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])*(Cosh[(c +
 d*x)/2] + I*Sinh[(c + d*x)/2])^2 + Sqrt[2]*f*(2*(-1)^(3/4)*(c + d*x)^2 + S
```

```
qrt[2]*(((4*I)*c - 2*Pi + (4*I)*d*x)*Log[1 + I*E^(-c - d*x)] + Pi*(-3*c - 3
*d*x + 4*Log[1 + E^(c + d*x)] - 4*Log[Cosh[(c + d*x)/2]] + 2*Log[-Sin[(Pi -
 (2*I)*(c + d*x))/4]]) - (4*I)*PolyLog[2, (-I)*E^(-c - d*x)]))*(Cosh[(c + d
*x)/2] + I*Sinh[(c + d*x)/2])^2 + Sqrt[2]*f*(-2*(-1)^(1/4)*(c + d*x)^2 + Sq
rt[2]*(-2*((2*I)*c + Pi + (2*I)*d*x)*Log[1 - I*E^(-c - d*x)] + Pi*(c + d*x
- 4*Log[1 + E^(c + d*x)] + 4*Log[Cosh[(c + d*x)/2]] + 2*Log[Sin[(Pi + (2*I)
*(c + d*x))/4]]) + (4*I)*PolyLog[2, I*E^(-c - d*x)]))*(Cosh[(c + d*x)/2] +
I*Sinh[(c + d*x)/2])^2 + 16*f*Sinh[(c + d*x)/2]*((-I)*Cosh[(c + d*x)/2] + S
inh[(c + d*x)/2]))/(16*d^2*(a + I*a*Sinh[c + d*x]))
```

Maple [A] time = 0.151, size = 268, normalized size = 1.7

$$\frac{dfx\mathrm{e}^{dx+c}+de\mathrm{e}^{dx+c}+f\mathrm{e}^{dx+c}-if}{\left(\mathrm{e}^{dx+c}-i\right)^{2}d^{2}a}-\frac{\frac{i}{2}e\ln\left(\mathrm{e}^{dx+c}-i\right)}{da}+\frac{\frac{i}{2}e\ln\left(\mathrm{e}^{dx+c}+i\right)}{da}-\frac{\frac{i}{2}f\ln\left(1+i\mathrm{e}^{dx+c}\right)x}{da}-\frac{\frac{i}{2}f\ln\left(1+i\mathrm{e}^{dx+c}\right)c}{ad^{2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] (d*f*x*exp(d*x+c)+d*e*exp(d*x+c)+f*exp(d*x+c)-I*f)/(exp(d*x+c)-I)^2/d^2/a-1
/2*I/a/d*e*ln(exp(d*x+c)-I)+1/2*I/a/d*e*ln(exp(d*x+c)+I)-1/2*I/a/d*f*ln(1+I
*exp(d*x+c))*x-1/2*I/a/d^2*f*ln(1+I*exp(d*x+c))*c-1/2*I*f*polylog(2,-I*exp(
d*x+c))/a/d^2+1/2*I/a/d*f*ln(1-I*exp(d*x+c))*x+1/2*I/a/d^2*f*ln(1-I*exp(d*x
+c))*c+1/2*I*f*polylog(2,I*exp(d*x+c))/a/d^2+1/2*I/a/d^2*f*c*ln(exp(d*x+c)-
I)-1/2*I/a/d^2*f*c*ln(exp(d*x+c)+I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2f\left(\frac{(dxe^{c}+e^{c})e^{(dx)}-i}{2\,ad^{2}e^{(2\,dx+2\,c)}-4i\,ad^{2}e^{(dx+c)}-2\,ad^{2}}+\int\frac{x}{4\left(ae^{(dx+c)}+i\,a\right)}\,dx+\int\frac{x}{4\left(ae^{(dx+c)}-i\,a\right)}\,dx\right)-\frac{1}{2}e\left(\frac{4\,e^{(-dx}}{\left(4i\,ae^{(-dx-c)}+2\,ae^{(}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*f*(((d*x*e^c + e^c)*e^(d*x) - I)/(2*a*d^2*e^(2*d*x + 2*c) - 4*I*a*d^2*e^(
d*x + c) - 2*a*d^2) + integrate(1/4*x/(a*e^(d*x + c) + I*a), x) + integrate
(1/4*x/(a*e^(d*x + c) - I*a), x)) - 1/2*e*(4*e^(-d*x - c)/((4*I*a*e^(-d*x -
 c) + 2*a*e^(-2*d*x - 2*c) - 2*a)*d) + I*log(e^(-d*x - c) + I)/(a*d) - I*lo
g(I*e^(-d*x - c) + 1)/(a*d))
```

Fricas [B] time = 2.28914, size = 902, normalized size = 5.6

$$\frac{\left(i\,fe^{(2\,dx+2\,c)}+2\,fe^{(dx+c)}-i\,f\right)\mathrm{Li}_2\left(i\,e^{(dx+c)}\right)+\left(-i\,fe^{(2\,dx+2\,c)}-2\,fe^{(dx+c)}+i\,f\right)\mathrm{Li}_2\left(-i\,e^{(dx+c)}\right)+2\left(dfx+de+f\right)e^{(dx+c)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((I*f*e^(2*d*x + 2*c) + 2*f*e^(d*x + c) - I*f)*dilog(I*e^(d*x + c)) + (-I*f
*e^(2*d*x + 2*c) - 2*f*e^(d*x + c) + I*f)*dilog(-I*e^(d*x + c)) + 2*(d*f*x
+ d*e + f)*e^(d*x + c) + (-I*d*e + I*c*f + (I*d*e - I*c*f)*e^(2*d*x + 2*c)
+ 2*(d*e - c*f)*e^(d*x + c))*log(e^(d*x + c) + I) + (I*d*e - I*c*f + (-I*d*
e + I*c*f)*e^(2*d*x + 2*c) - 2*(d*e - c*f)*e^(d*x + c))*log(e^(d*x + c) - I
) + (I*d*f*x + I*c*f + (-I*d*f*x - I*c*f)*e^(2*d*x + 2*c) - 2*(d*f*x + c*f)
*e^(d*x + c))*log(I*e^(d*x + c) + 1) + (-I*d*f*x - I*c*f + (I*d*f*x + I*c*f
)*e^(2*d*x + 2*c) + 2*(d*f*x + c*f)*e^(d*x + c))*log(-I*e^(d*x + c) + 1) -
2*I*f)/(2*a*d^2*e^(2*d*x + 2*c) - 4*I*a*d^2*e^(d*x + c) - 2*a*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e \operatorname{sech}(c+dx)}{i \sinh(c+dx)+1}\, dx + \int \frac{fx \operatorname{sech}(c+dx)}{i \sinh(c+dx)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] (Integral(e*sech(c + d*x)/(I*sinh(c + d*x) + 1), x) + Integral(f*x*sech(c +
 d*x)/(I*sinh(c + d*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\operatorname{sech}(dx+c)}{i\, a \sinh(dx+c)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sech(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

3.274 $\int \frac{\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}(\sinh(c+dx))}{2ad}+\frac{i}{2d(a+ia\sinh(c+dx))}$$

```
[Out] ArcTan[Sinh[c + d*x]]/(2*a*d) + (I/2)/(d*(a + I*a*Sinh[c + d*x]))
```

Rubi [A] time = 0.0566407, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2667, 44, 206}

$$\frac{\tan^{-1}(\sinh(c+dx))}{2ad}+\frac{i}{2d(a+ia\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ArcTan[Sinh[c + d*x]]/(2*a*d) + (I/2)/(d*(a + I*a*Sinh[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{(ia)\operatorname{Subst}\left(\int \frac{1}{(a-x)(a+x)^2}\,dx, x, ia\sinh(c+dx)\right)}{d}$$
$$= -\frac{(ia)\operatorname{Subst}\left(\int \left(\frac{1}{2a(a+x)^2}+\frac{1}{2a(a^2-x^2)}\right)dx, x, ia\sinh(c+dx)\right)}{d}$$
$$= \frac{i}{2d(a+ia\sinh(c+dx))} - \frac{i\operatorname{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, ia\sinh(c+dx)\right)}{2d}$$
$$= \frac{\tan^{-1}(\sinh(c+dx))}{2ad} + \frac{i}{2d(a+ia\sinh(c+dx))}$$

Mathematica [A] time = 0.0480522, size = 30, normalized size = 0.71

$$\frac{\tan^{-1}(\sinh(c+dx)) + \frac{1}{\sinh(c+dx)-i}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (ArcTan[Sinh[c + d*x]] + (-I + Sinh[c + d*x])^(-1))/(2*a*d)
```

Maple [B] time = 0.046, size = 91, normalized size = 2.2

$$\frac{\frac{i}{2}}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right) - \frac{i}{da}\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-2} - \frac{\frac{i}{2}}{da}\ln\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right) - \frac{1}{da}\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 1/2*I/d/a*ln(tanh(1/2*d*x+1/2*c)+I)-I/d/a/(-I+tanh(1/2*d*x+1/2*c))^2-1/2*I/
d/a*ln(-I+tanh(1/2*d*x+1/2*c))-1/d/a/(-I+tanh(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.16804, size = 117, normalized size = 2.79

$$-\frac{2\,e^{(-dx-c)}}{\left(4i\,ae^{(-dx-c)} + 2\,ae^{(-2\,dx-2\,c)} - 2\,a\right)d} - \frac{i\,\log\left(e^{(-dx-c)}+i\right)}{2\,ad} + \frac{i\,\log\left(i\,e^{(-dx-c)}+1\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*e^(-d*x - c)/((4*I*a*e^(-d*x - c) + 2*a*e^(-2*d*x - 2*c) - 2*a)*d) - 1/2
*I*log(e^(-d*x - c) + I)/(a*d) + 1/2*I*log(I*e^(-d*x - c) + 1)/(a*d)
```

Fricas [B] time = 2.12009, size = 267, normalized size = 6.36

$$\frac{\left(ie^{(2\,dx+2\,c)}+2\,e^{(dx+c)}-i\right)\log\left(e^{(dx+c)}+i\right)+\left(-ie^{(2\,dx+2\,c)}-2\,e^{(dx+c)}+i\right)\log\left(e^{(dx+c)}-i\right)+2\,e^{(dx+c)}}{2\,ade^{(2\,dx+2\,c)}-4i\,ade^{(dx+c)}-2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((I*e^(2*d*x + 2*c) + 2*e^(d*x + c) - I)*log(e^(d*x + c) + I) + (-I*e^(2*d*
x + 2*c) - 2*e^(d*x + c) + I)*log(e^(d*x + c) - I) + 2*e^(d*x + c))/(2*a*d*
e^(2*d*x + 2*c) - 4*I*a*d*e^(d*x + c) - 2*a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{sech}(c+dx)}{i\sinh(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Integral(sech(c + d*x)/(I*sinh(c + d*x) + 1), x)/a
```

Giac [B] time = 1.15309, size = 147, normalized size = 3.5

$$-\frac{i\log\left(e^{(dx+c)}-e^{(-dx-c)}-2i\right)}{4\,ad}+\frac{i\log\left(ie^{(dx+c)}-ie^{(-dx-c)}-2\right)}{4\,ad}-\frac{-ie^{(dx+c)}+ie^{(-dx-c)}-6}{4\,ad\left(e^{(dx+c)}-e^{(-dx-c)}-2i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*I*log(e^(d*x + c) - e^(-d*x - c) - 2*I)/(a*d) + 1/4*I*log(I*e^(d*x + c
) - I*e^(-d*x - c) - 2)/(a*d) - 1/4*(-I*e^(d*x + c) + I*e^(-d*x - c) - 6)/(
a*d*(e^(d*x + c) - e^(-d*x - c) - 2*I))
```

3.275 $\int \frac{\text{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{\text{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Sech[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0496738, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sech[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]

[Out] Defer[Int][Sech[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\text{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx = \int \frac{\text{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Mathematica [A] time = 65.4834, size = 0, normalized size = 0.

$$\int \frac{\text{sech}(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]

[Out] Integrate[Sech[c + d*x]/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Maple [A] time = 0.714, size = 0, normalized size = 0.

$$\int \frac{\text{sech}(dx+c)}{(fx+e)(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)

[Out] int(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(\left(dfxe^{c}+\left(de-f\right)e^{c}\right)e^{(dx)}+if\right)}{2\,ad^{2}f^{2}x^{2}+4\,ad^{2}efx+2\,ad^{2}e^{2}-2\left(ad^{2}f^{2}x^{2}e^{(2\,c)}+2\,ad^{2}efxe^{(2\,c)}+ad^{2}e^{2}e^{(2\,c)}\right)e^{(2\,dx)}-\left(-4i\,ad^{2}f^{2}x^{2}e^{c}-8i\,ad^{2}efxe^{c}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*((d*f*x*e^c + (d*e - f)*e^c)*e^(d*x) + I*f)/(2*a*d^2*f^2*x^2 + 4*a*d^2*e
*f*x + 2*a*d^2*e^2 - 2*(a*d^2*f^2*x^2*e^(2*c) + 2*a*d^2*e*f*x*e^(2*c) + a*d
^2*e^2*e^(2*c))*e^(2*d*x) - (-4*I*a*d^2*f^2*x^2*e^c - 8*I*a*d^2*e*f*x*e^c -
 4*I*a*d^2*e^2*e^c)*e^(d*x)) + 2*integrate((d^2*f^2*x^2 + 2*d^2*e*f*x + d^2
*e^2 - 4*f^2)/(-4*I*a*d^2*f^3*x^3 - 12*I*a*d^2*e*f^2*x^2 - 12*I*a*d^2*e^2*f
*x - 4*I*a*d^2*e^3 + 4*(a*d^2*f^3*x^3*e^c + 3*a*d^2*e*f^2*x^2*e^c + 3*a*d^2
*e^2*f*x*e^c + a*d^2*e^3*e^c)*e^(d*x)), x) + 2*integrate(1/(4*I*a*f*x + 4*I
*a*e + 4*(a*f*x*e^c + a*e*e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(dfx+de-f\right)e^{(dx+c)}-\left(ad^{2}f^{2}x^{2}+2\,ad^{2}efx+ad^{2}e^{2}-\left(ad^{2}f^{2}x^{2}+2\,ad^{2}efx+ad^{2}e^{2}\right)e^{(2\,dx+2\,c)}-\left(-2i\,ad^{2}f^{2}x^{2}-4i\,ac\right.\right.}{ad^{2}f^{2}x^{2}+2\,ad^{2}efx+ad^{2}e^{2}-\left(ad^{2}f^{2}x^{2}+2\,ad^{2}efx+ad^{2}e^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((d*f*x + d*e - f)*e^(d*x + c) - (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^
2 - (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*e^(2*d*x + 2*c) - (-2*I*a*d
^2*f^2*x^2 - 4*I*a*d^2*e*f*x - 2*I*a*d^2*e^2)*e^(d*x + c))*integral((-2*I*f
^2 + (d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2 - 2*f^2)*e^(d*x + c))/(a*d^2*f^3*
x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*
a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^(2*d*x + 2*c)), x) + I*f)/
(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2 - (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x
 + a*d^2*e^2)*e^(2*d*x + 2*c) - (-2*I*a*d^2*f^2*x^2 - 4*I*a*d^2*e*f*x - 2*I
*a*d^2*e^2)*e^(d*x + c))
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\operatorname{sech}\left(c+dx\right)}{ie\sinh\left(c+dx\right)+e+ifx\sinh\left(c+dx\right)+fx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Integral(sech(c + d*x)/(I*e*sinh(c + d*x) + e + I*f*x*sinh(c + d*x) + f*x),
 x)/a
```

<u>Giac</u> [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(dx+c)}{(fx+e)(ia\sinh(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sech(d*x + c)/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

3.276 $\int \frac{\text{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=31

$$\text{Unintegrable}\left(\frac{\text{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Sech[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0503592, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sech[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Sech[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\text{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\int \frac{\text{sech}(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.007, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sech[c + d*x]/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] $Aborted
```

Maple [A] time = 1.079, size = 0, normalized size = 0.

$$\int \frac{\text{sech}\,(dx+c)}{\left(fx+e\right)^2(a+ia\sinh\,(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(\left(dfxe^{c}+\left(de-2f\right)e^{c}\right)e^{(dx)}+2if\right)}{2\,ad^{2}f^{3}x^{3}+6\,ad^{2}ef^{2}x^{2}+6\,ad^{2}e^{2}fx+2\,ad^{2}e^{3}-2\left(ad^{2}f^{3}x^{3}e^{(2c)}+3\,ad^{2}ef^{2}x^{2}e^{(2c)}+3\,ad^{2}e^{2}fxe^{(2c)}+ad^{2}e^{3}e^{(2c)}\right)e^{(2dx)}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*((d*f*x*e^c + (d*e - 2*f)*e^c)*e^(d*x) + 2*I*f)/(2*a*d^2*f^3*x^3 + 6*a*d
^2*e*f^2*x^2 + 6*a*d^2*e^2*f*x + 2*a*d^2*e^3 - 2*(a*d^2*f^3*x^3*e^(2*c) + 3
*a*d^2*e*f^2*x^2*e^(2*c) + 3*a*d^2*e^2*f*x*e^(2*c) + a*d^2*e^3*e^(2*c))*e^(
2*d*x) - (-4*I*a*d^2*f^3*x^3*e^c - 12*I*a*d^2*e*f^2*x^2*e^c - 12*I*a*d^2*e^
2*f*x*e^c - 4*I*a*d^2*e^3*e^c)*e^(d*x)) + 2*integrate((d^2*f^2*x^2 + 2*d^2*
e*f*x + d^2*e^2 - 12*f^2)/(-4*I*a*d^2*f^4*x^4 - 16*I*a*d^2*e*f^3*x^3 - 24*I
*a*d^2*e^2*f^2*x^2 - 16*I*a*d^2*e^3*f*x - 4*I*a*d^2*e^4 + 4*(a*d^2*f^4*x^4*
e^c + 4*a*d^2*e*f^3*x^3*e^c + 6*a*d^2*e^2*f^2*x^2*e^c + 4*a*d^2*e^3*f*x*e^c
 + a*d^2*e^4*e^c)*e^(d*x)), x) + 2*integrate(1/(4*I*a*f^2*x^2 + 8*I*a*e*f*x
 + 4*I*a*e^2 + 4*(a*f^2*x^2*e^c + 2*a*e*f*x*e^c + a*e^2*e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(dfx+de-2f\right)e^{(dx+c)}-\left(ad^{2}f^{3}x^{3}+3\,ad^{2}ef^{2}x^{2}+3\,ad^{2}e^{2}fx+ad^{2}e^{3}-\left(ad^{2}f^{3}x^{3}+3\,ad^{2}ef^{2}x^{2}+3\,ad^{2}e^{2}fx+ad^{2}e^{3}\right)e^{(2c}}{ad^{2}f^{3}x^{3}+3\,ad^{2}ef^{2}x^{2}+3\,ad^{2}e^{2}fx+ad^{2}e^{3}-\left(ad^{2}f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((d*f*x + d*e - 2*f)*e^(d*x + c) - (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*
a*d^2*e^2*f*x + a*d^2*e^3 - (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^
2*f*x + a*d^2*e^3)*e^(2*d*x + 2*c) - (-2*I*a*d^2*f^3*x^3 - 6*I*a*d^2*e*f^2*
x^2 - 6*I*a*d^2*e^2*f*x - 2*I*a*d^2*e^3)*e^(d*x + c))*integral((-6*I*f^2 +
(d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2 - 6*f^2)*e^(d*x + c))/(a*d^2*f^4*x^4 +
 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4 + (a
*d^2*f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x +
a*d^2*e^4)*e^(2*d*x + 2*c)), x) + 2*I*f)/(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2
 + 3*a*d^2*e^2*f*x + a*d^2*e^3 - (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d
^2*e^2*f*x + a*d^2*e^3)*e^(2*d*x + 2*c) - (-2*I*a*d^2*f^3*x^3 - 6*I*a*d^2*e
*f^2*x^2 - 6*I*a*d^2*e^2*f*x - 2*I*a*d^2*e^3)*e^(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(dx+c)}{(fx+e)^2(ia\sinh(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sech(d*x + c)/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

3.277 $\int \frac{(e+fx)^3\mathbf{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=450

$$-\frac{f^2(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{f^2(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{ad^3}-\frac{2f^2(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{ad^3}+\frac{f^3\text{PolyLog}\left(\right.}{ad^4}$$

```
[Out] (2*(e + f*x)^3)/(3*a*d) - (I*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/(a*d^2) + (
I*f^3*ArcTan[Sinh[c + d*x]])/(a*d^4) - (2*f*(e + f*x)^2*Log[1 + E^(2*(c + d
*x))])/(a*d^2) + (f^3*Log[Cosh[c + d*x]])/(a*d^4) - (f^2*(e + f*x)*PolyLog[
2, (-I)*E^(c + d*x)])/(a*d^3) + (f^2*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(
a*d^3) - (2*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(a*d^3) + (f^3*Poly
Log[3, (-I)*E^(c + d*x)])/(a*d^4) - (f^3*PolyLog[3, I*E^(c + d*x)])/(a*d^4)
 + (f^3*PolyLog[3, -E^(2*(c + d*x))])/(a*d^4) - (I*f^2*(e + f*x)*Sech[c + d
*x])/(a*d^3) + (f*(e + f*x)^2*Sech[c + d*x]^2)/(2*a*d^2) + ((I/3)*(e + f*x)
^3*Sech[c + d*x]^3)/(a*d) - (f^2*(e + f*x)*Tanh[c + d*x])/(a*d^3) + (2*(e +
 f*x)^3*Tanh[c + d*x])/(3*a*d) - ((I/2)*f*(e + f*x)^2*Sech[c + d*x]*Tanh[c
+ d*x])/(a*d^2) + ((e + f*x)^3*Sech[c + d*x]^2*Tanh[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.589366, antiderivative size = 450, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {5571, 4186, 4184, 3475, 3718, 2190, 2531, 2282, 6589, 5451, 3770, 4180}

$$-\frac{f^2(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^3}+\frac{f^2(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{ad^3}-\frac{2f^2(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{ad^3}+\frac{f^3\text{PolyLog}\left(\right.}{ad^4}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```


```
[Out] (2*(e + f*x)^3)/(3*a*d) - (I*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/(a*d^2) + (
I*f^3*ArcTan[Sinh[c + d*x]])/(a*d^4) - (2*f*(e + f*x)^2*Log[1 + E^(2*(c + d
*x))])/(a*d^2) + (f^3*Log[Cosh[c + d*x]])/(a*d^4) - (f^2*(e + f*x)*PolyLog[
2, (-I)*E^(c + d*x)])/(a*d^3) + (f^2*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(
a*d^3) - (2*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(a*d^3) + (f^3*Poly
Log[3, (-I)*E^(c + d*x)])/(a*d^4) - (f^3*PolyLog[3, I*E^(c + d*x)])/(a*d^4)
 + (f^3*PolyLog[3, -E^(2*(c + d*x))])/(a*d^4) - (I*f^2*(e + f*x)*Sech[c + d
*x])/(a*d^3) + (f*(e + f*x)^2*Sech[c + d*x]^2)/(2*a*d^2) + ((I/3)*(e + f*x)
^3*Sech[c + d*x]^3)/(a*d) - (f^2*(e + f*x)*Tanh[c + d*x])/(a*d^3) + (2*(e +
 f*x)^3*Tanh[c + d*x])/(3*a*d) - ((I/2)*f*(e + f*x)^2*Sech[c + d*x]*Tanh[c
+ d*x])/(a*d^2) + ((e + f*x)^3*Sech[c + d*x]^2*Tanh[c + d*x])/(3*a*d)
```


Rule 5571

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sech[c
 + d*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)*T
anh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && Eq
Q[a^2 + b^2, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
```

```
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
```

```
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int (e+fx)^3\text{sech}^3(c+dx)\tanh(c+dx)\,dx}{a} + \frac{\int (e+fx)^3\text{sech}^4(c+dx)\,dx}{a}$$

$$= \frac{f(e+fx)^2\text{sech}^2(c+dx)}{2ad^2} + \frac{i(e+fx)^3\text{sech}^3(c+dx)}{3ad} + \frac{(e+fx)^3\text{sech}^2(c+dx)\tanh(c+\ldots}{3ad}$$

$$= -\frac{if^2(e+fx)\text{sech}(c+dx)}{ad^3} + \frac{f(e+fx)^2\text{sech}^2(c+dx)}{2ad^2} + \frac{i(e+fx)^3\text{sech}^3(c+dx)}{3ad} - \frac{f^2(e\ldots}{}$$

$$= \frac{2(e+fx)^3}{3ad} - \frac{if(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{if^3\tan^{-1}(\sinh(c+dx))}{ad^4} + \frac{f^3\log(\cosh(c+dx\ldots}{ad^4}$$

$$= \frac{2(e+fx)^3}{3ad} - \frac{if(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{if^3\tan^{-1}(\sinh(c+dx))}{ad^4} - \frac{2f(e+fx)^2\log\left(1+\ldots\right.}{ad^2}$$

$$= \frac{2(e+fx)^3}{3ad} - \frac{if(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{if^3\tan^{-1}(\sinh(c+dx))}{ad^4} - \frac{2f(e+fx)^2\log\left(1+\ldots\right.}{ad^2}$$

$$= \frac{2(e+fx)^3}{3ad} - \frac{if(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{if^3\tan^{-1}(\sinh(c+dx))}{ad^4} - \frac{2f(e+fx)^2\log\left(1+\ldots\right.}{ad^2}$$

$$= \frac{2(e+fx)^3}{3ad} - \frac{if(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{if^3\tan^{-1}(\sinh(c+dx))}{ad^4} - \frac{2f(e+fx)^2\log\left(1+\ldots\right.}{ad^2}$$

Mathematica [B] time = 12.2762, size = 1049, normalized size = 2.33

$$-\frac{if\left(\frac{(e+fx)^3}{f} + \frac{3(1-ie^c)\log\left(1+ie^{-c-dx}\right)(e+fx)^2}{d} + \frac{6i(i+e^c)f\left(d(e+fx)\text{PolyLog}\left(2,-ie^{-c-dx}\right)+f\text{PolyLog}\left(3,-ie^{-c-dx}\right)\right)}{d^3}\right)}{2ad\,(i+e^c)} + \frac{if\left(5d^2f^2x^3+15d^2efx^2+\ldots\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-I/2)*f*((e + f*x)^3/f + (3*(1 - I*E^c)*(e + f*x)^2*Log[1 + I*E^(-c - d*x
)])/d + ((6*I)*(I + E^c)*f*(d*(e + f*x)*PolyLog[2, (-I)*E^(-c - d*x)] + f*P
olyLog[3, (-I)*E^(-c - d*x)]))/d^3))/(a*d*(I + E^c)) + ((I/6)*f*(3*(5*d^2*e
^2 - 4*f^2)*x + 15*d^2*e*f*x^2 + 5*d^2*f^2*x^3 + 30*d*e*(1 + I*E^c)*f*x*Log
[1 - I*E^(-c - d*x)] + 15*d*(1 + I*E^c)*f^2*x^2*Log[1 - I*E^(-c - d*x)] - (
3*(1 + I*E^c)*(5*d^2*e^2 - 4*f^2)*(d*x - Log[I - E^(c + d*x)]))/d - 30*e*(1
 + I*E^c)*f*PolyLog[2, I*E^(-c - d*x)] - 30*(1 + I*E^c)*f^2*(x*PolyLog[2, I
```

```
*E^(-c - d*x)] + PolyLog[3, I*E^(-c - d*x)]/d)))/(a*d^3*(-I + E^c)) + (e^3*
Sinh[(d*x)/2] + 3*e^2*f*x*Sinh[(d*x)/2] + 3*e*f^2*x^2*Sinh[(d*x)/2] + f^3*x
^3*Sinh[(d*x)/2])/(2*a*d*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] - I
*Sinh[c/2 + (d*x)/2])) + (e^3*Sinh[(d*x)/2] + 3*e^2*f*x*Sinh[(d*x)/2] + 3*e
*f^2*x^2*Sinh[(d*x)/2] + f^3*x^3*Sinh[(d*x)/2])/(3*a*d*(Cosh[c/2] + I*Sinh[
c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2])^3) + (I*d*e^3*Cosh[c/2]
 + 3*e^2*f*Cosh[c/2] + (3*I)*d*e^2*f*x*Cosh[c/2] + 6*e*f^2*x*Cosh[c/2] + (3
*I)*d*e*f^2*x^2*Cosh[c/2] + 3*f^3*x^2*Cosh[c/2] + I*d*f^3*x^3*Cosh[c/2] + d
*e^3*Sinh[c/2] + (3*I)*e^2*f*Sinh[c/2] + 3*d*e^2*f*x*Sinh[c/2] + (6*I)*e*f^
2*x*Sinh[c/2] + 3*d*e*f^2*x^2*Sinh[c/2] + (3*I)*f^3*x^2*Sinh[c/2] + d*f^3*x
^3*Sinh[c/2])/(6*a*d^2*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] + I*S
inh[c/2 + (d*x)/2])^2) + (5*d^2*e^3*Sinh[(d*x)/2] - 12*e*f^2*Sinh[(d*x)/2]
+ 15*d^2*e^2*f*x*Sinh[(d*x)/2] - 12*f^3*x*Sinh[(d*x)/2] + 15*d^2*e*f^2*x^2*
Sinh[(d*x)/2] + 5*d^2*f^3*x^3*Sinh[(d*x)/2])/(6*a*d^3*(Cosh[c/2] + I*Sinh[c
/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2]))
```

Maple [B] time = 0.18, size = 1001, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -5*f^2/d^2/a*e*ln(1+I*exp(d*x+c))*x-5*f^2/d^3/a*e*ln(1+I*exp(d*x+c))*c-8*f^
2/d^3/a*e*c*ln(exp(d*x+c))+5*f^2/d^3/a*e*c*ln(exp(d*x+c)-I)+8*f^2/d^2/a*e*c
*x-8/3*f^3/d^4/a*c^3+4/3*f^3/d/a*x^3+3*f^3*polylog(3,I*exp(d*x+c))/a/d^4+5*
f^3*polylog(3,-I*exp(d*x+c))/a/d^4+1/3*I*(6*I*f^3*x*exp(2*d*x+2*c)+8*d^2*f^
3*x^3*exp(d*x+c)-3*d*f^3*x^2*exp(3*d*x+3*c)-4*I*d^2*f^3*x^3+24*d^2*e*f^2*x^
2*exp(d*x+c)-6*d*e*f^2*x*exp(3*d*x+3*c)+6*I*e*f^2*exp(2*d*x+2*c)+6*I*e*f^2+
24*d^2*e^2*f*x*exp(d*x+c)-3*d*e^2*f*exp(3*d*x+3*c)-3*d*f^3*x^2*exp(d*x+c)-6
*f^3*x*exp(3*d*x+3*c)+6*I*f^3*x-4*I*d^2*e^3+8*d^2*e^3*exp(d*x+c)-6*d*e*f^2*
x*exp(d*x+c)-6*e*f^2*exp(3*d*x+3*c)-12*I*d^2*e^2*f*x-3*d*e^2*f*exp(d*x+c)-6
*f^3*x*exp(d*x+c)-12*I*d^2*e*f^2*x^2-6*e*f^2*exp(d*x+c))/(exp(d*x+c)+I)/(ex
p(d*x+c)-I)^3/d^3/a-2*f^3/d^4/a*ln(exp(d*x+c))+2*f^3/d^4/a*ln(exp(d*x+c)-I)
-4*f^3/d^3/a*c^2*x+4*f/d^2/a*ln(exp(d*x+c))*e^2-5/2*f^3/d^2/a*ln(1+I*exp(d*
x+c))*x^2+5/2*f^3/d^4/a*ln(1+I*exp(d*x+c))*c^2+4*f^2/d^3/a*e*c^2-5/2*f/d^2/
a*ln(exp(d*x+c)-I)*e^2-5/2*f^3/d^4/a*c^2*ln(exp(d*x+c)-I)+4*f^2/d/a*e*x^2+4
*f^3/d^4/a*c^2*ln(exp(d*x+c))-5*f^3/d^3/a*polylog(2,-I*exp(d*x+c))*x-5*f^2/
d^3/a*e*polylog(2,-I*exp(d*x+c))-3/2*f^3/d^2/a*ln(1-I*exp(d*x+c))*x^2-3*f^3
/d^3/a*polylog(2,I*exp(d*x+c))*x+3/2*f^3/d^4/a*ln(1-I*exp(d*x+c))*c^2-3/2*f
/d^2/a*e^2*ln(exp(d*x+c)+I)-3/2*f^3/d^4/a*c^2*ln(exp(d*x+c)+I)-3*f^2/d^3/a*
e*polylog(2,I*exp(d*x+c))+3*f^2/d^3/a*e*c*ln(exp(d*x+c)+I)-3*f^2/d^2/a*ln(1
-I*exp(d*x+c))*e*x-3*f^2/d^3/a*ln(1-I*exp(d*x+c))*c*e
```

Maxima [A] time = 2.5021, size = 984, normalized size = 2.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/2*e^2*f*(24*(4*I*d*x*e^(4*d*x + 4*c) + (8*d*x*e^(3*c) + e^(3*c))*e^(3*d*x
) + e^(d*x + c))/(12*I*a*d^2*e^(4*d*x + 4*c) + 24*a*d^2*e^(3*d*x + 3*c) + 2
4*a*d^2*e^(d*x + c) - 12*I*a*d^2) - 3*log((e^(d*x + c) + I)*e^(-c))/(a*d^2)
 - 5*log(-I*(I*e^(d*x + c) + 1)*e^(-c))/(a*d^2)) + 4*e^3*(2*e^(-d*x - c)/((
6*a*e^(-d*x - c) + 6*a*e^(-3*d*x - 3*c) - 3*I*a*e^(-4*d*x - 4*c) + 3*I*a)*d
) + I/((6*a*e^(-d*x - c) + 6*a*e^(-3*d*x - 3*c) - 3*I*a*e^(-4*d*x - 4*c) +
3*I*a)*d)) + (4*I*d^2*f^3*x^3 + 12*I*d^2*e*f^2*x^2 - 6*I*f^3*x - 6*I*e*f^2
+ 3*(d*f^3*x^2*e^(3*c) + 2*e*f^2*e^(3*c) + 2*(d*e*f^2 + f^3)*x*e^(3*c))*e^(
3*d*x) + (-6*I*f^3*x*e^(2*c) - 6*I*e*f^2*e^(2*c))*e^(2*d*x) - (8*d^2*f^3*x^
3*e^c - 6*e*f^2*e^c + 3*(8*d^2*e*f^2 - d*f^3)*x^2*e^c - 6*(d*e*f^2 + f^3)*x
*e^c)*e^(d*x))/(3*I*a*d^3*e^(4*d*x + 4*c) + 6*a*d^3*e^(3*d*x + 3*c) + 6*a*d
^3*e^(d*x + c) - 3*I*a*d^3) - 5*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d
*x + c)))*e*f^2/(a*d^3) - 3*(d*x*log(-I*e^(d*x + c) + 1) + dilog(I*e^(d*x +
 c)))*e*f^2/(a*d^3) - 2*f^3*x/(a*d^3) - 5/2*(d^2*x^2*log(I*e^(d*x + c) + 1)
 + 2*d*x*dilog(-I*e^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a*d^4)
- 3/2*(d^2*x^2*log(-I*e^(d*x + c) + 1) + 2*d*x*dilog(I*e^(d*x + c)) - 2*pol
ylog(3, I*e^(d*x + c)))*f^3/(a*d^4) + 2*f^3*log(e^(d*x + c) - I)/(a*d^4) +
4/3*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2)/(a*d^4)
```

Fricas [C] time = 2.41309, size = 3393, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] (8*d^3*e^3 - 24*c*d^2*e^2*f + 12*(2*c^2 - 1)*d*e*f^2 - 4*(2*c^3 - 3*c)*f^3
+ (18*d*f^3*x + 18*d*e*f^2 - 18*(d*f^3*x + d*e*f^2)*e^(4*d*x + 4*c) + (36*I
*d*f^3*x + 36*I*d*e*f^2)*e^(3*d*x + 3*c) + (36*I*d*f^3*x + 36*I*d*e*f^2)*e^
(d*x + c))*dilog(I*e^(d*x + c)) + (30*d*f^3*x + 30*d*e*f^2 - 30*(d*f^3*x +
d*e*f^2)*e^(4*d*x + 4*c) + (60*I*d*f^3*x + 60*I*d*e*f^2)*e^(3*d*x + 3*c) +
(60*I*d*f^3*x + 60*I*d*e*f^2)*e^(d*x + c))*dilog(-I*e^(d*x + c)) + 4*(2*d^3
*f^3*x^3 + 6*d^3*e*f^2*x^2 + 6*c*d^2*e^2*f - 6*c^2*d*e*f^2 + (2*c^3 - 3*c)*
f^3 + 3*(2*d^3*e^2*f - d*f^3)*x)*e^(4*d*x + 4*c) + (-16*I*d^3*f^3*x^3 + (-4
8*I*c - 6*I)*d^2*e^2*f + (48*I*c^2 - 12*I)*d*e*f^2 + (-16*I*c^3 + 24*I*c)*f
^3 + (-48*I*d^3*e*f^2 - 6*I*d^2*f^3)*x^2 + (-48*I*d^3*e^2*f - 12*I*d^2*e*f^
2 + 12*I*d*f^3)*x)*e^(3*d*x + 3*c) - 12*(d*f^3*x + d*e*f^2)*e^(2*d*x + 2*c)
 + (-6*I*d^2*f^3*x^2 + 16*I*d^3*e^3 + (-48*I*c - 6*I)*d^2*e^2*f + (48*I*c^2
 - 12*I)*d*e*f^2 + (-16*I*c^3 + 24*I*c)*f^3 + (-12*I*d^2*e*f^2 + 12*I*d*f^3
)*x)*e^(d*x + c) + (9*d^2*e^2*f - 18*c*d*e*f^2 + 9*c^2*f^3 - 9*(d^2*e^2*f -
 2*c*d*e*f^2 + c^2*f^3)*e^(4*d*x + 4*c) + (18*I*d^2*e^2*f - 36*I*c*d*e*f^2
+ 18*I*c^2*f^3)*e^(3*d*x + 3*c) + (18*I*d^2*e^2*f - 36*I*c*d*e*f^2 + 18*I*c
^2*f^3)*e^(d*x + c))*log(e^(d*x + c) + I) + (15*d^2*e^2*f - 30*c*d*e*f^2 +
3*(5*c^2 - 4)*f^3 - 3*(5*d^2*e^2*f - 10*c*d*e*f^2 + (5*c^2 - 4)*f^3)*e^(4*d
*x + 4*c) + (30*I*d^2*e^2*f - 60*I*c*d*e*f^2 + (30*I*c^2 - 24*I)*f^3)*e^(3*
d*x + 3*c) + (30*I*d^2*e^2*f - 60*I*c*d*e*f^2 + (30*I*c^2 - 24*I)*f^3)*e^(d
*x + c))*log(e^(d*x + c) - I) + (15*d^2*f^3*x^2 + 30*d^2*e*f^2*x + 30*c*d*e
*f^2 - 15*c^2*f^3 - 15*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3
)*e^(4*d*x + 4*c) + (30*I*d^2*f^3*x^2 + 60*I*d^2*e*f^2*x + 60*I*c*d*e*f^2 -
 30*I*c^2*f^3)*e^(3*d*x + 3*c) + (30*I*d^2*f^3*x^2 + 60*I*d^2*e*f^2*x + 60*
I*c*d*e*f^2 - 30*I*c^2*f^3)*e^(d*x + c))*log(I*e^(d*x + c) + 1) + (9*d^2*f^
3*x^2 + 18*d^2*e*f^2*x + 18*c*d*e*f^2 - 9*c^2*f^3 - 9*(d^2*f^3*x^2 + 2*d^2*
e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*e^(4*d*x + 4*c) + (18*I*d^2*f^3*x^2 + 36*I
*d^2*e*f^2*x + 36*I*c*d*e*f^2 - 18*I*c^2*f^3)*e^(3*d*x + 3*c) + (18*I*d^2*f
^3*x^2 + 36*I*d^2*e*f^2*x + 36*I*c*d*e*f^2 - 18*I*c^2*f^3)*e^(d*x + c))*log
```

```
(-I*e^(d*x + c) + 1) + (18*f^3*e^(4*d*x + 4*c) - 36*I*f^3*e^(3*d*x + 3*c) -
 36*I*f^3*e^(d*x + c) - 18*f^3)*polylog(3, I*e^(d*x + c)) + (30*f^3*e^(4*d*
x + 4*c) - 60*I*f^3*e^(3*d*x + 3*c) - 60*I*f^3*e^(d*x + c) - 30*f^3)*polylo
g(3, -I*e^(d*x + c)))/(6*a*d^4*e^(4*d*x + 4*c) - 12*I*a*d^4*e^(3*d*x + 3*c)
 - 12*I*a*d^4*e^(d*x + c) - 6*a*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sech(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \operatorname{sech}(dx+c)^2}{i a \sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sech(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

3.278 $\int \frac{(e+fx)^2\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=325

$$-\frac{f^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{3ad^3}+\frac{f^2\text{PolyLog}\left(2,ie^{c+dx}\right)}{3ad^3}-\frac{2f^2\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{3ad^3}-\frac{4f(e+fx)\log\left(e^{2(c+dx)}+1\right)}{3ad^2}-\frac{2if(e+$$

```
[Out] (2*(e + f*x)^2)/(3*a*d) - (((2*I)/3)*f*(e + f*x)*ArcTan[E^(c + d*x)])/(a*d^
2) - (4*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(3*a*d^2) - (f^2*PolyLog[2, (
-I)*E^(c + d*x)])/(3*a*d^3) + (f^2*PolyLog[2, I*E^(c + d*x)])/(3*a*d^3) - (
2*f^2*PolyLog[2, -E^(2*(c + d*x))])/(3*a*d^3) - ((I/3)*f^2*Sech[c + d*x])/(
a*d^3) + (f*(e + f*x)*Sech[c + d*x]^2)/(3*a*d^2) + ((I/3)*(e + f*x)^2*Sech[
c + d*x]^3)/(a*d) - (f^2*Tanh[c + d*x])/(3*a*d^3) + (2*(e + f*x)^2*Tanh[c +
 d*x])/(3*a*d) - ((I/3)*f*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(a*d^2) +
((e + f*x)^2*Sech[c + d*x]^2*Tanh[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.386601, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {5571, 4186, 3767, 8, 4184, 3718, 2190, 2279, 2391, 5451, 4185, 4180}

$$-\frac{f^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{3ad^3}+\frac{f^2\text{PolyLog}\left(2,ie^{c+dx}\right)}{3ad^3}-\frac{2f^2\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{3ad^3}-\frac{4f(e+fx)\log\left(e^{2(c+dx)}+1\right)}{3ad^2}-\frac{2if(e+$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```


```
[Out] (2*(e + f*x)^2)/(3*a*d) - (((2*I)/3)*f*(e + f*x)*ArcTan[E^(c + d*x)])/(a*d^
2) - (4*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(3*a*d^2) - (f^2*PolyLog[2, (
-I)*E^(c + d*x)])/(3*a*d^3) + (f^2*PolyLog[2, I*E^(c + d*x)])/(3*a*d^3) - (
2*f^2*PolyLog[2, -E^(2*(c + d*x))])/(3*a*d^3) - ((I/3)*f^2*Sech[c + d*x])/(
a*d^3) + (f*(e + f*x)*Sech[c + d*x]^2)/(3*a*d^2) + ((I/3)*(e + f*x)^2*Sech[
c + d*x]^3)/(a*d) - (f^2*Tanh[c + d*x])/(3*a*d^3) + (2*(e + f*x)^2*Tanh[c +
 d*x])/(3*a*d) - ((I/3)*f*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(a*d^2) +
((e + f*x)^2*Sech[c + d*x]^2*Tanh[c + d*x])/(3*a*d)
```


Rule 5571

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sech[c
 + d*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)*T
anh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && Eq
Q[a^2 + b^2, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
```

```
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int (e+fx)^2\text{sech}^3(c+dx)\tanh(c+dx)\,dx}{a} + \frac{\int (e+fx)^2\text{sech}^4(c+dx)\,dx}{a}$$

$$= \frac{f(e+fx)\text{sech}^2(c+dx)}{3ad^2} + \frac{i(e+fx)^2\text{sech}^3(c+dx)}{3ad} + \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+d}{3ad}$$

$$= -\frac{if^2\text{sech}(c+dx)}{3ad^3} + \frac{f(e+fx)\text{sech}^2(c+dx)}{3ad^2} + \frac{i(e+fx)^2\text{sech}^3(c+dx)}{3ad} + \frac{2(e+fx)^2\tan}{3ad}$$

$$= \frac{2(e+fx)^2}{3ad} - \frac{2if(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{3ad^2} - \frac{if^2\text{sech}(c+dx)}{3ad^3} + \frac{f(e+fx)\text{sech}^2(c+dx)}{3ad^2} + \ldots$$

$$= \frac{2(e+fx)^2}{3ad} - \frac{2if(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{3ad^2} - \frac{4f(e+fx)\log\left(1+e^{2(c+dx)}\right)}{3ad^2} - \frac{if^2\text{sech}(c+dx}{3ad^3}$$

$$= \frac{2(e+fx)^2}{3ad} - \frac{2if(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{3ad^2} - \frac{4f(e+fx)\log\left(1+e^{2(c+dx)}\right)}{3ad^2} - \frac{f^2\text{Li}_2\left(-ie^{c+dx}\right)}{3ad^3}$$

$$= \frac{2(e+fx)^2}{3ad} - \frac{2if(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{3ad^2} - \frac{4f(e+fx)\log\left(1+e^{2(c+dx)}\right)}{3ad^2} - \frac{f^2\text{Li}_2\left(-ie^{c+dx}\right)}{3ad^3}$$

Mathematica [A] time = 8.26164, size = 564, normalized size = 1.74

$$12f^2\text{PolyLog}\left(2,-ie^{-c-dx}\right)+20f^2\text{PolyLog}\left(2,ie^{-c-dx}\right)+\frac{d^2e^2\sinh(2(c+dx))-2id^2e^2\cosh(c+dx)+4id^2e^2\cosh(c+2dx)+2d^2efx\sinh(2(c+dx))}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (((10*I)*d*(e + f*x)*(d*(e + f*x) + 2*(1 + I*E^c)*f*Log[1 - I*E^(-c - d*x)]
))/(-I + E^c) - ((6*I)*d*(e + f*x)*(d*(e + f*x) + 2*(1 - I*E^c)*f*Log[1 + I
*E^(-c - d*x)]))/(I + E^c) + 12*f^2*PolyLog[2, (-I)*E^(-c - d*x)] + 20*f^2*
PolyLog[2, I*E^(-c - d*x)] + ((-2*I)*f^2*Cosh[c] + 2*d*f*(e + f*x)*Cosh[d*x
] - (2*I)*d^2*e^2*Cosh[c + d*x] + (4*I)*f^2*Cosh[c + d*x] - (4*I)*d^2*e*f*x
*Cosh[c + d*x] - (2*I)*d^2*f^2*x^2*Cosh[c + d*x] + 2*d*e*f*Cosh[2*c + d*x]
+ 2*d*f^2*x*Cosh[2*c + d*x] + (4*I)*d^2*e^2*Cosh[c + 2*d*x] - (2*I)*f^2*Cos
h[c + 2*d*x] + (8*I)*d^2*e*f*x*Cosh[c + 2*d*x] + (4*I)*d^2*f^2*x^2*Cosh[c +
 2*d*x] + 8*d^2*e^2*Sinh[d*x] - 2*f^2*Sinh[d*x] + 16*d^2*e*f*x*Sinh[d*x] +
8*d^2*f^2*x^2*Sinh[d*x] + d^2*e^2*Sinh[2*(c + d*x)] - 2*f^2*Sinh[2*(c + d*x
)] + 2*d^2*e*f*x*Sinh[2*(c + d*x)] + d^2*f^2*x^2*Sinh[2*(c + d*x)] + 2*f^2*
Sinh[2*c + d*x])/((Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2] + I*Sinh[c/2])*(Cosh
[(c + d*x)/2] - I*Sinh[(c + d*x)/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/
2])^3))/(12*a*d^3)
```

Maple [A] time = 0.154, size = 509, normalized size = 1.6

$$\frac{\frac{2i}{3}\left(-2\,id^2f^2x^2+4\,d^2x^2f^2e^{dx+c}-df^2xe^{3\,dx+3\,c}-4\,id^2efx+8\,d^2efxe^{dx+c}-defe^{3\,dx+3\,c}-2\,id^2e^2+if^2e^{2\,dx+2\,c}+4\,d^2e^2e^{dx}\ldots\right.}{\left(e^{dx+c}+i\right)\left(e^{dx+c}-i\right)^3d^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 2/3*I*(-2*I*d^2*f^2*x^2+4*d^2*x^2*f^2*exp(d*x+c)-d*f^2*x*exp(3*d*x+3*c)-4*I
*d^2*e*f*x+8*d^2*e*f*x*exp(d*x+c)-d*e*f*exp(3*d*x+3*c)-2*I*d^2*e^2+I*f^2*ex
p(2*d*x+2*c)+4*d^2*e^2*exp(d*x+c)-d*f^2*x*exp(d*x+c)-f^2*exp(3*d*x+3*c)-d*e
*f*exp(d*x+c)+I*f^2-f^2*exp(d*x+c))/(exp(d*x+c)+I)/(exp(d*x+c)-I)^3/d^3/a-5
/3*f/d^2/a*ln(exp(d*x+c)-I)*e-f/d^2/a*e*ln(exp(d*x+c)+I)+8/3*f/d^2/a*ln(exp
(d*x+c))*e+5/3*f^2/d^3/a*c*ln(exp(d*x+c)-I)-8/3*f^2/d^3/a*c*ln(exp(d*x+c))+
f^2/d^3/a*c*ln(exp(d*x+c)+I)+4/3*f^2*x^2/a/d+8/3*f^2/d^2/a*c*x+4/3*f^2/d^3/
a*c^2-5/3*f^2/d^2/a*ln(1+I*exp(d*x+c))*x-5/3*f^2/d^3/a*ln(1+I*exp(d*x+c))*c
-5/3*f^2*polylog(2,-I*exp(d*x+c))/a/d^3-f^2/d^2/a*ln(1-I*exp(d*x+c))*x-f^2/
d^3/a*ln(1-I*exp(d*x+c))*c-f^2*polylog(2,I*exp(d*x+c))/a/d^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$4\,f^2\left(\frac{2i\,d^2x^2+\left(dxe^{(3\,c)}+e^{(3\,c)}\right)e^{(3\,dx)}-\left(4\,d^2x^2e^c-dxe^c-e^c\right)e^{(dx)}-i\,e^{(2\,dx+2\,c)}-i}{6i\,ad^3e^{(4\,dx+4\,c)}+12\,ad^3e^{(3\,dx+3\,c)}+12\,ad^3e^{(dx+c)}-6i\,ad^3}+i\int\frac{x}{4\left(ade^{(dx+c)}+i\,ad\right)}\,dx-5i\int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 4*f^2*((2*I*d^2*x^2 + (d*x*e^(3*c) + e^(3*c))*e^(3*d*x) - (4*d^2*x^2*e^c -
d*x*e^c - e^c)*e^(d*x) - I*e^(2*d*x + 2*c) - I)/(6*I*a*d^3*e^(4*d*x + 4*c)
+ 12*a*d^3*e^(3*d*x + 3*c) + 12*a*d^3*e^(d*x + c) - 6*I*a*d^3) + I*integrat
e(1/4*x/(a*d*e^(d*x + c) + I*a*d), x) - 5*I*integrate(1/12*x/(a*d*e^(d*x +
c) - I*a*d), x)) + 1/3*e*f*(24*(4*I*d*x*e^(4*d*x + 4*c) + (8*d*x*e^(3*c) +
e^(3*c))*e^(3*d*x) + e^(d*x + c))/(12*I*a*d^2*e^(4*d*x + 4*c) + 24*a*d^2*e^
(3*d*x + 3*c) + 24*a*d^2*e^(d*x + c) - 12*I*a*d^2) - 3*log((e^(d*x + c) + I
)*e^(-c))/(a*d^2) - 5*log(-I*(I*e^(d*x + c) + 1)*e^(-c))/(a*d^2)) + 4*e^2*(
2*e^(-d*x - c)/((6*a*e^(-d*x - c) + 6*a*e^(-3*d*x - 3*c) - 3*I*a*e^(-4*d*x
- 4*c) + 3*I*a)*d) + I/((6*a*e^(-d*x - c) + 6*a*e^(-3*d*x - 3*c) - 3*I*a*e^
(-4*d*x - 4*c) + 3*I*a)*d))
```

Fricas [B] time = 2.2461, size = 1760, normalized size = 5.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] (4*d^2*e^2 - 8*c*d*e*f + 2*(2*c^2 - 1)*f^2 - 2*f^2*e^(2*d*x + 2*c) - (3*f^2
*e^(4*d*x + 4*c) - 6*I*f^2*e^(3*d*x + 3*c) - 6*I*f^2*e^(d*x + c) - 3*f^2)*d
ilog(I*e^(d*x + c)) - (5*f^2*e^(4*d*x + 4*c) - 10*I*f^2*e^(3*d*x + 3*c) - 1
0*I*f^2*e^(d*x + c) - 5*f^2)*dilog(-I*e^(d*x + c)) + 4*(d^2*f^2*x^2 + 2*d^2
*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(4*d*x + 4*c) + (-8*I*d^2*f^2*x^2 + (-16*I*
c - 2*I)*d*e*f + (8*I*c^2 - 2*I)*f^2 + (-16*I*d^2*e*f - 2*I*d*f^2)*x)*e^(3*
d*x + 3*c) + (8*I*d^2*e^2 + (-16*I*c - 2*I)*d*e*f - 2*I*d*f^2*x + (8*I*c^2
- 2*I)*f^2)*e^(d*x + c) + (3*d*e*f - 3*c*f^2 - 3*(d*e*f - c*f^2)*e^(4*d*x +
```

```
4*c) + (6*I*d*e*f - 6*I*c*f^2)*e^(3*d*x + 3*c) + (6*I*d*e*f - 6*I*c*f^2)*e
^(d*x + c))*log(e^(d*x + c) + I) + (5*d*e*f - 5*c*f^2 - 5*(d*e*f - c*f^2)*e
^(4*d*x + 4*c) + (10*I*d*e*f - 10*I*c*f^2)*e^(3*d*x + 3*c) + (10*I*d*e*f -
10*I*c*f^2)*e^(d*x + c))*log(e^(d*x + c) - I) + (5*d*f^2*x + 5*c*f^2 - 5*(d
*f^2*x + c*f^2)*e^(4*d*x + 4*c) + (10*I*d*f^2*x + 10*I*c*f^2)*e^(3*d*x + 3*
c) + (10*I*d*f^2*x + 10*I*c*f^2)*e^(d*x + c))*log(I*e^(d*x + c) + 1) + (3*d
*f^2*x + 3*c*f^2 - 3*(d*f^2*x + c*f^2)*e^(4*d*x + 4*c) + (6*I*d*f^2*x + 6*I
*c*f^2)*e^(3*d*x + 3*c) + (6*I*d*f^2*x + 6*I*c*f^2)*e^(d*x + c))*log(-I*e^(
d*x + c) + 1))/(3*a*d^3*e^(4*d*x + 4*c) - 6*I*a*d^3*e^(3*d*x + 3*c) - 6*I*a
*d^3*e^(d*x + c) - 3*a*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sech(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \operatorname{sech}\left(dx+c\right)^{2}}{i\,a\sinh\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sech(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

3.279 $\int \frac{(e+fx)\,\textbf{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=158

$$\frac{f\,\text{sech}^2(c+dx)}{6ad^2} - \frac{if\tan^{-1}(\sinh(c+dx))}{6ad^2} - \frac{2f\log(\cosh(c+dx))}{3ad^2} - \frac{if\tanh(c+dx)\text{sech}(c+dx)}{6ad^2} + \frac{2(e+fx)\tanh(c}{3ad}$$

```
[Out] ((-I/6)*f*ArcTan[Sinh[c + d*x]])/(a*d^2) - (2*f*Log[Cosh[c + d*x]])/(3*a*d^
2) + (f*Sech[c + d*x]^2)/(6*a*d^2) + ((I/3)*(e + f*x)*Sech[c + d*x]^3)/(a*d
) + (2*(e + f*x)*Tanh[c + d*x])/(3*a*d) - ((I/6)*f*Sech[c + d*x]*Tanh[c + d
*x])/(a*d^2) + ((e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.157548, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {5571, 4185, 4184, 3475, 5451, 3768, 3770}

$$\frac{f\,\text{sech}^2(c+dx)}{6ad^2} - \frac{if\tan^{-1}(\sinh(c+dx))}{6ad^2} - \frac{2f\log(\cosh(c+dx))}{3ad^2} - \frac{if\tanh(c+dx)\text{sech}(c+dx)}{6ad^2} + \frac{2(e+fx)\tanh(c}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-I/6)*f*ArcTan[Sinh[c + d*x]])/(a*d^2) - (2*f*Log[Cosh[c + d*x]])/(3*a*d^
2) + (f*Sech[c + d*x]^2)/(6*a*d^2) + ((I/3)*(e + f*x)*Sech[c + d*x]^3)/(a*d
) + (2*(e + f*x)*Tanh[c + d*x])/(3*a*d) - ((I/6)*f*Sech[c + d*x]*Tanh[c + d
*x])/(a*d^2) + ((e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x])/(3*a*d)
```

Rule 5571

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sech[c
 + d*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)*T
anh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && Eq
Q[a^2 + b^2, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int (e+fx)\text{sech}^3(c+dx)\tanh(c+dx)\,dx}{a} + \frac{\int (e+fx)\text{sech}^4(c+dx)\,dx}{a}$$
$$= \frac{f\text{sech}^2(c+dx)}{6ad^2} + \frac{i(e+fx)\text{sech}^3(c+dx)}{3ad} + \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{3ad} + \frac{2\int (e$$
$$= \frac{f\text{sech}^2(c+dx)}{6ad^2} + \frac{i(e+fx)\text{sech}^3(c+dx)}{3ad} + \frac{2(e+fx)\tanh(c+dx)}{3ad} - \frac{if\text{sech}(c+dx)\tan}{6ad^2}$$
$$= -\frac{if\tan^{-1}(\sinh(c+dx))}{6ad^2} - \frac{2f\log(\cosh(c+dx))}{3ad^2} + \frac{f\text{sech}^2(c+dx)}{6ad^2} + \frac{i(e+fx)\text{sech}^3(c+}{3ad}$$

Mathematica [A] time = 1.13355, size = 194, normalized size = 1.23

$$\frac{2d(e+fx)(\cosh(2(c+dx))-2i\sinh(c+dx))+\cosh(c+dx)\left(-i\sinh(c+dx)\left(2f\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-4if\log(\text{co}\right.\right.}{6ad^2(\sinh(c+dx)-i)\left(\cosh\left(\frac{1}{2}(c+dx)\right)-i\sinh\left(\frac{1}{2}(c+dx)\right)\right)(\text{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sech[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (2*d*(e + f*x)*(Cosh[2*(c + d*x)] - (2*I)*Sinh[c + d*x]) + Cosh[c + d*x]*(-
(d*e) - I*f + c*f - 2*f*ArcTan[Tanh[(c + d*x)/2]] + (4*I)*f*Log[Cosh[c + d*
x]] - I*(d*e - c*f + 2*f*ArcTan[Tanh[(c + d*x)/2]] - (4*I)*f*Log[Cosh[c + d
*x]])*Sinh[c + d*x]))/(6*a*d^2*(Cosh[(c + d*x)/2] - I*Sinh[(c + d*x)/2])*(C
osh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(-I + Sinh[c + d*x]))
```

Maple [A] time = 0.164, size = 143, normalized size = 0.9

$$\frac{4\,fx}{3\,da} + \frac{4\,cf}{3\,ad^2} - \frac{\frac{i}{3}\left(-8\,dfx\mathrm{e}^{dx+c} + f\mathrm{e}^{3\,dx+3\,c} - 8\,de\mathrm{e}^{dx+c} + f\mathrm{e}^{dx+c} + 4\,idfx + 4\,ide\right)}{\left(\mathrm{e}^{dx+c}+i\right)\left(\mathrm{e}^{dx+c}-i\right)^3 d^2 a} - \frac{5\,f\ln\left(\mathrm{e}^{dx+c}-i\right)}{6\,ad^2} - \frac{f\ln\left(\mathrm{e}^{dx+c}+i\right)}{2\,ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 4/3*f*x/a/d+4/3*f/a/d^2*c-1/3*I*(-8*d*f*x*exp(d*x+c)+f*exp(3*d*x+3*c)-8*d*e
*exp(d*x+c)+f*exp(d*x+c)+4*I*d*f*x+4*I*d*e)/(exp(d*x+c)+I)/(exp(d*x+c)-I)^3
/d^2/a-5/6*f/a/d^2*ln(exp(d*x+c)-I)-1/2*f/a/d^2*ln(exp(d*x+c)+I)
```

Maxima [A] time = 1.33948, size = 339, normalized size = 2.15

$$\frac{1}{6}f\left(\frac{24\left(4i\,dxe^{(4\,dx+4\,c)}+\left(8\,dxe^{(3\,c)}+e^{(3\,c)}\right)e^{(3\,dx)}+e^{(dx+c)}\right)}{12i\,ad^2e^{(4\,dx+4\,c)}+24\,ad^2e^{(3\,dx+3\,c)}+24\,ad^2e^{(dx+c)}-12i\,ad^2}-\frac{3\,\log\left(\left(e^{(dx+c)}+i\right)e^{(-c)}\right)}{ad^2}-\frac{5\,\log\left(-i\left(i\,e^{(dx+c)}+1\right)e^{(-\cdots}\right.}{ad^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*f*(24*(4*I*d*x*e^(4*d*x + 4*c) + (8*d*x*e^(3*c) + e^(3*c))*e^(3*d*x) +
e^(d*x + c))/(12*I*a*d^2*e^(4*d*x + 4*c) + 24*a*d^2*e^(3*d*x + 3*c) + 24*a*
d^2*e^(d*x + c) - 12*I*a*d^2) - 3*log((e^(d*x + c) + I)*e^(-c))/(a*d^2) - 5
*log(-I*(I*e^(d*x + c) + 1)*e^(-c))/(a*d^2)) + 4*e*(2*e^(-d*x - c)/((6*a*e^
(-d*x - c) + 6*a*e^(-3*d*x - 3*c) - 3*I*a*e^(-4*d*x - 4*c) + 3*I*a)*d) + I/
((6*a*e^(-d*x - c) + 6*a*e^(-3*d*x - 3*c) - 3*I*a*e^(-4*d*x - 4*c) + 3*I*a)
*d))
```

Fricas [A] time = 2.29769, size = 527, normalized size = 3.34

$$\frac{8\,dfxe^{(4\,dx+4\,c)}+8\,de+\left(-16i\,dfx-2i\,f\right)e^{(3\,dx+3\,c)}+\left(16i\,de-2i\,f\right)e^{(dx+c)}-\left(3\,fe^{(4\,dx+4\,c)}-6i\,fe^{(3\,dx+3\,c)}-6i\,fe^{(dx+c)}-\cdots\right.}{6\,ad^2e^{(4\,dx+4\,c)}-12i\,ad^2e^{(3\,dx+3\,c)}-12i\,ad^2e^{(\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (8*d*f*x*e^(4*d*x + 4*c) + 8*d*e + (-16*I*d*f*x - 2*I*f)*e^(3*d*x + 3*c) +
(16*I*d*e - 2*I*f)*e^(d*x + c) - (3*f*e^(4*d*x + 4*c) - 6*I*f*e^(3*d*x + 3*
c) - 6*I*f*e^(d*x + c) - 3*f)*log(e^(d*x + c) + I) - (5*f*e^(4*d*x + 4*c) -
 10*I*f*e^(3*d*x + 3*c) - 10*I*f*e^(d*x + c) - 5*f)*log(e^(d*x + c) - I))/(
6*a*d^2*e^(4*d*x + 4*c) - 12*I*a*d^2*e^(3*d*x + 3*c) - 12*I*a*d^2*e^(d*x +
c) - 6*a*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21864, size = 352, normalized size = 2.23

$$\frac{8\,dfxe^{(4\,dx+4\,c)}-16i\,dfxe^{(3\,dx+3\,c)}-3\,fe^{(4\,dx+4\,c)}\log\left(e^{(dx+c)}+i\right)+6i\,fe^{(3\,dx+3\,c)}\log\left(e^{(dx+c)}+i\right)+6i\,fe^{(dx+c)}\log\left(e^{(dx+c)}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (8*d*f*x*e^(4*d*x + 4*c) - 16*I*d*f*x*e^(3*d*x + 3*c) - 3*f*e^(4*d*x + 4*c)
*log(e^(d*x + c) + I) + 6*I*f*e^(3*d*x + 3*c)*log(e^(d*x + c) + I) + 6*I*f*
e^(d*x + c)*log(e^(d*x + c) + I) - 5*f*e^(4*d*x + 4*c)*log(e^(d*x + c) - I)
 + 10*I*f*e^(3*d*x + 3*c)*log(e^(d*x + c) - I) + 10*I*f*e^(d*x + c)*log(e^(
d*x + c) - I) + 8*d*e - 2*I*f*e^(3*d*x + 3*c) + 16*I*d*e^(d*x + c + 1) - 2*
I*f*e^(d*x + c) + 3*f*log(e^(d*x + c) + I) + 5*f*log(e^(d*x + c) - I))/(6*a
*d^2*e^(4*d*x + 4*c) - 12*I*a*d^2*e^(3*d*x + 3*c) - 12*I*a*d^2*e^(d*x + c)
- 6*a*d^2)
```

3.280 $\int \frac{\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=47

$$\frac{2\tanh(c+dx)}{3ad}+\frac{i\text{sech}(c+dx)}{3d(a+ia\sinh(c+dx))}$$

```
[Out] ((I/3)*Sech[c + d*x])/(d*(a + I*a*Sinh[c + d*x])) + (2*Tanh[c + d*x])/(3*a*
d)
```

Rubi [A] time = 0.054996, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2672, 3767, 8}

$$\frac{2\tanh(c+dx)}{3ad}+\frac{i\text{sech}(c+dx)}{3d(a+ia\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((I/3)*Sech[c + d*x])/(d*(a + I*a*Sinh[c + d*x])) + (2*Tanh[c + d*x])/(3*a*
d)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{sech}^2(c+dx)}{a+ia\sinh(c+dx)}\,dx &= \frac{i\text{sech}(c+dx)}{3d(a+ia\sinh(c+dx))}+\frac{2\int \text{sech}^2(c+dx)\,dx}{3a}\\ &= \frac{i\text{sech}(c+dx)}{3d(a+ia\sinh(c+dx))}+\frac{(2i)\,\text{Subst}\left(\int 1\,dx,x,-i\tanh(c+dx)\right)}{3ad}\\ &= \frac{i\text{sech}(c+dx)}{3d(a+ia\sinh(c+dx))}+\frac{2\tanh(c+dx)}{3ad}\end{aligned}$$

Mathematica [A] time = 0.0461272, size = 47, normalized size = 1.

$$\frac{\operatorname{sech}(c+dx)(\cosh(2(c+dx))-2i\sinh(c+dx))}{3ad(\sinh(c+dx)-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (Sech[c + d*x]*(Cosh[2*(c + d*x)] - (2*I)*Sinh[c + d*x]))/(3*a*d*(-I + Sinh
[c + d*x]))
```

Maple [A] time = 0.049, size = 75, normalized size = 1.6

$$2\frac{1}{da}\left(1/4\,(\tanh(1/2\,dx+c/2)+i)^{-1}-1/3\,(-i+\tanh(1/2\,dx+c/2))^{-3}+\frac{i/2}{(-i+\tanh(1/2\,dx+c/2))^{2}}+3/4\,(-i+\tanh(1/$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 2/d/a*(1/4/(tanh(1/2*d*x+1/2*c)+I)-1/3/(-I+tanh(1/2*d*x+1/2*c))^3+1/2*I/(-I
+tanh(1/2*d*x+1/2*c))^2+3/4/(-I+tanh(1/2*d*x+1/2*c)))
```

Maxima [B] time = 1.18232, size = 140, normalized size = 2.98

$$\frac{8\,e^{(-dx-c)}}{\left(6\,ae^{(-dx-c)}+6\,ae^{(-3\,dx-3\,c)}-3i\,ae^{(-4\,dx-4\,c)}+3i\,a\right)d}+\frac{4i}{\left(6\,ae^{(-dx-c)}+6\,ae^{(-3\,dx-3\,c)}-3i\,ae^{(-4\,dx-4\,c)}+3i\,a\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 8*e^(-d*x - c)/((6*a*e^(-d*x - c) + 6*a*e^(-3*d*x - 3*c) - 3*I*a*e^(-4*d*x
- 4*c) + 3*I*a)*d) + 4*I/((6*a*e^(-d*x - c) + 6*a*e^(-3*d*x - 3*c) - 3*I*a*
e^(-4*d*x - 4*c) + 3*I*a)*d)
```

Fricas [A] time = 2.07062, size = 144, normalized size = 3.06

$$-\frac{4\left(-2i\,e^{(dx+c)}-1\right)}{3\,ade^{(4\,dx+4\,c)}-6i\,ade^{(3\,dx+3\,c)}-6i\,ade^{(dx+c)}-3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -4*(-2*I*e^(d*x + c) - 1)/(3*a*d*e^(4*d*x + 4*c) - 6*I*a*d*e^(3*d*x + 3*c)
- 6*I*a*d*e^(d*x + c) - 3*a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{sech}^2(c+dx)}{i\sinh(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Integral(sech(c + d*x)**2/(I*sinh(c + d*x) + 1), x)/a
```

Giac [A] time = 1.15949, size = 81, normalized size = 1.72

$$\frac{1}{2\,ad\left(i\,e^{(dx+c)}-1\right)}-\frac{-3i\,e^{(2\,dx+2\,c)}-12\,e^{(dx+c)}+5i}{6\,ad\left(e^{(dx+c)}-i\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2/(a*d*(I*e^(d*x + c) - 1)) - 1/6*(-3*I*e^(2*d*x + 2*c) - 12*e^(d*x + c)
+ 5*I)/(a*d*(e^(d*x + c) - I)^3)
```

3.281 $\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Sech[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0780557, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sech[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Sech[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Mathematica [A] time = 122.852, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Sech[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Maple [A] time = 0.248, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{sech}(dx+c))^2}{\left(fx+e\right)(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-4if\int \frac{1}{8i\,adf^2x^2 + 16i\,adefx + 8i\,ade^2 + 8\left(adf^2x^2e^c + 2\,adefxe^c + ade^2e^c\right)e^{(dx)}}\,dx - \frac{}{12\,ad^3f^3x^3 + 36\,ad^3ef^2x^2 + 36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -4*I*f*integrate(1/(8*I*a*d*f^2*x^2 + 16*I*a*d*e*f*x + 8*I*a*d*e^2 + 8*(a*d
*f^2*x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) - 4*(4*d^2*f^2*x
^2 + 8*d^2*e*f*x + 4*d^2*e^2 - 2*f^2*e^(2*d*x + 2*c) - 2*f^2 + (I*d*f^2*x*e
^(3*c) + (I*d*e*f - 2*I*f^2)*e^(3*c))*e^(3*d*x) + (8*I*d^2*f^2*x^2*e^c + (1
6*I*d^2*e*f + I*d*f^2)*x*e^c + (8*I*d^2*e^2 + I*d*e*f - 2*I*f^2)*e^c)*e^(d*
x))/(12*a*d^3*f^3*x^3 + 36*a*d^3*e*f^2*x^2 + 36*a*d^3*e^2*f*x + 12*a*d^3*e^
3 - 12*(a*d^3*f^3*x^3*e^(4*c) + 3*a*d^3*e*f^2*x^2*e^(4*c) + 3*a*d^3*e^2*f*x
*e^(4*c) + a*d^3*e^3*e^(4*c))*e^(4*d*x) - (-24*I*a*d^3*f^3*x^3*e^(3*c) - 72
*I*a*d^3*e*f^2*x^2*e^(3*c) - 72*I*a*d^3*e^2*f*x*e^(3*c) - 24*I*a*d^3*e^3*e^
(3*c))*e^(3*d*x) - (-24*I*a*d^3*f^3*x^3*e^c - 72*I*a*d^3*e*f^2*x^2*e^c - 72
*I*a*d^3*e^2*f*x*e^c - 24*I*a*d^3*e^3*e^c)*e^(d*x)) - 4*integrate((5*d^2*f^
3*x^2 + 10*d^2*e*f^2*x + 5*d^2*e^2*f - 12*f^3)/(24*a*d^3*f^4*x^4 + 96*a*d^3
*e*f^3*x^3 + 144*a*d^3*e^2*f^2*x^2 + 96*a*d^3*e^3*f*x + 24*a*d^3*e^4 + (24*
I*a*d^3*f^4*x^4*e^c + 96*I*a*d^3*e*f^3*x^3*e^c + 144*I*a*d^3*e^2*f^2*x^2*e^
c + 96*I*a*d^3*e^3*f*x*e^c + 24*I*a*d^3*e^4*e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(4*d^2*f^2*x^2 + 8*d^2*e*f*x + 4*d^2*e^2 - 2*f^2*e^(2*d*x + 2*c) - 2*f^2 +
 (I*d*f^2*x + I*d*e*f - 2*I*f^2)*e^(3*d*x + 3*c) + (8*I*d^2*f^2*x^2 + 8*I*d
^2*e^2 + I*d*e*f - 2*I*f^2 + (16*I*d^2*e*f + I*d*f^2)*x)*e^(d*x + c) - (3*a
*d^3*f^3*x^3 + 9*a*d^3*e*f^2*x^2 + 9*a*d^3*e^2*f*x + 3*a*d^3*e^3 - 3*(a*d^3
*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*e^(4*d*x + 4*c)
 - (-6*I*a*d^3*f^3*x^3 - 18*I*a*d^3*e*f^2*x^2 - 18*I*a*d^3*e^2*f*x - 6*I*a*
d^3*e^3)*e^(3*d*x + 3*c) - (-6*I*a*d^3*f^3*x^3 - 18*I*a*d^3*e*f^2*x^2 - 18*
I*a*d^3*e^2*f*x - 6*I*a*d^3*e^3)*e^(d*x + c))*integral(-1/3*(4*d^2*f^3*x^2
+ 8*d^2*e*f^2*x + 4*d^2*e^2*f - 6*f^3 - (I*d^2*f^3*x^2 + 2*I*d^2*e*f^2*x +
I*d^2*e^2*f - 6*I*f^3)*e^(d*x + c))/(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*
a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4 + (a*d^3*f^4*x^4 + 4*a*d^3*
e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*e^(2*d*x + 2
*c)), x))/(3*a*d^3*f^3*x^3 + 9*a*d^3*e*f^2*x^2 + 9*a*d^3*e^2*f*x + 3*a*d^3*
e^3 - 3*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*e
^(4*d*x + 4*c) - (-6*I*a*d^3*f^3*x^3 - 18*I*a*d^3*e*f^2*x^2 - 18*I*a*d^3*e^
2*f*x - 6*I*a*d^3*e^3)*e^(3*d*x + 3*c) - (-6*I*a*d^3*f^3*x^3 - 18*I*a*d^3*e
*f^2*x^2 - 18*I*a*d^3*e^2*f*x - 6*I*a*d^3*e^3)*e^(d*x + c))
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{sech}^2(c+dx)}{ie\sinh(c+dx)+e+ifx\sinh(c+dx)+fx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Integral(sech(c + d*x)**2/(I*e*sinh(c + d*x) + e + I*f*x*sinh(c + d*x) + f*
x), x)/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(dx+c)^2}{(fx+e)(i\,a\sinh(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sech(d*x + c)^2/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

3.282 $\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

[Out] Unintegrable[Sech[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]

Rubi [A] time = 0.0767881, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

[In] Int[Sech[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]

[Out] Defer[Int][Sech[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.02, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

[In] Integrate[Sech[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]

[Out] $Aborted

Maple [A] time = 1.753, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{sech}(dx+c))^2}{(fx+e)^2(a+ia\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)

[Out] int(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -4*I*f*integrate(1/(4*I*a*d*f^3*x^3 + 12*I*a*d*e*f^2*x^2 + 12*I*a*d*e^2*f*x
 + 4*I*a*d*e^3 + 4*(a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e
^c + a*d*e^3*e^c)*e^(d*x)), x) - 4*(2*d^2*f^2*x^2 + 4*d^2*e*f*x + 2*d^2*e^2
 - 3*f^2*e^(2*d*x + 2*c) - 3*f^2 + (I*d*f^2*x*e^(3*c) + (I*d*e*f - 3*I*f^2)
*e^(3*c))*e^(3*d*x) + (4*I*d^2*f^2*x^2*e^c + (8*I*d^2*e*f + I*d*f^2)*x*e^c
+ (4*I*d^2*e^2 + I*d*e*f - 3*I*f^2)*e^c)*e^(d*x))/(6*a*d^3*f^4*x^4 + 24*a*d
^3*e*f^3*x^3 + 36*a*d^3*e^2*f^2*x^2 + 24*a*d^3*e^3*f*x + 6*a*d^3*e^4 - 6*(a
*d^3*f^4*x^4*e^(4*c) + 4*a*d^3*e*f^3*x^3*e^(4*c) + 6*a*d^3*e^2*f^2*x^2*e^(4
*c) + 4*a*d^3*e^3*f*x*e^(4*c) + a*d^3*e^4*e^(4*c))*e^(4*d*x) - (-12*I*a*d^3
*f^4*x^4*e^(3*c) - 48*I*a*d^3*e*f^3*x^3*e^(3*c) - 72*I*a*d^3*e^2*f^2*x^2*e^
(3*c) - 48*I*a*d^3*e^3*f*x*e^(3*c) - 12*I*a*d^3*e^4*e^(3*c))*e^(3*d*x) - (-
12*I*a*d^3*f^4*x^4*e^c - 48*I*a*d^3*e*f^3*x^3*e^c - 72*I*a*d^3*e^2*f^2*x^2*
e^c - 48*I*a*d^3*e^3*f*x*e^c - 12*I*a*d^3*e^4*e^c)*e^(d*x)) - 4*integrate((
5*d^2*f^3*x^2 + 10*d^2*e*f^2*x + 5*d^2*e^2*f - 24*f^3)/(12*a*d^3*f^5*x^5 +
60*a*d^3*e*f^4*x^4 + 120*a*d^3*e^2*f^3*x^3 + 120*a*d^3*e^3*f^2*x^2 + 60*a*d
^3*e^4*f*x + 12*a*d^3*e^5 + (12*I*a*d^3*f^5*x^5*e^c + 60*I*a*d^3*e*f^4*x^4*
e^c + 120*I*a*d^3*e^2*f^3*x^3*e^c + 120*I*a*d^3*e^3*f^2*x^2*e^c + 60*I*a*d^
3*e^4*f*x*e^c + 12*I*a*d^3*e^5*e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -(4*d^2*f^2*x^2 + 8*d^2*e*f*x + 4*d^2*e^2 - 6*f^2*e^(2*d*x + 2*c) - 6*f^2 +
 (2*I*d*f^2*x + 2*I*d*e*f - 6*I*f^2)*e^(3*d*x + 3*c) + (8*I*d^2*f^2*x^2 + 8
*I*d^2*e^2 + 2*I*d*e*f - 6*I*f^2 + (16*I*d^2*e*f + 2*I*d*f^2)*x)*e^(d*x + c
) - (3*a*d^3*f^4*x^4 + 12*a*d^3*e*f^3*x^3 + 18*a*d^3*e^2*f^2*x^2 + 12*a*d^3
*e^3*f*x + 3*a*d^3*e^4 - 3*(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2
*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*e^(4*d*x + 4*c) - (-6*I*a*d^3*f^4*x
^4 - 24*I*a*d^3*e*f^3*x^3 - 36*I*a*d^3*e^2*f^2*x^2 - 24*I*a*d^3*e^3*f*x - 6
*I*a*d^3*e^4)*e^(3*d*x + 3*c) - (-6*I*a*d^3*f^4*x^4 - 24*I*a*d^3*e*f^3*x^3
- 36*I*a*d^3*e^2*f^2*x^2 - 24*I*a*d^3*e^3*f*x - 6*I*a*d^3*e^4)*e^(d*x + c))
*integral(-1/3*(8*d^2*f^3*x^2 + 16*d^2*e*f^2*x + 8*d^2*e^2*f - 24*f^3 - (2*
I*d^2*f^3*x^2 + 4*I*d^2*e*f^2*x + 2*I*d^2*e^2*f - 24*I*f^3)*e^(d*x + c))/(a
*d^3*f^5*x^5 + 5*a*d^3*e*f^4*x^4 + 10*a*d^3*e^2*f^3*x^3 + 10*a*d^3*e^3*f^2*
x^2 + 5*a*d^3*e^4*f*x + a*d^3*e^5 + (a*d^3*f^5*x^5 + 5*a*d^3*e*f^4*x^4 + 10
*a*d^3*e^2*f^3*x^3 + 10*a*d^3*e^3*f^2*x^2 + 5*a*d^3*e^4*f*x + a*d^3*e^5)*e^
(2*d*x + 2*c)), x))/(3*a*d^3*f^4*x^4 + 12*a*d^3*e*f^3*x^3 + 18*a*d^3*e^2*f^
2*x^2 + 12*a*d^3*e^3*f*x + 3*a*d^3*e^4 - 3*(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x
^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*e^(4*d*x + 4*c) - (
-6*I*a*d^3*f^4*x^4 - 24*I*a*d^3*e*f^3*x^3 - 36*I*a*d^3*e^2*f^2*x^2 - 24*I*a
```

```
*d^3*e^3*f*x - 6*I*a*d^3*e^4)*e^(3*d*x + 3*c) - (-6*I*a*d^3*f^4*x^4 - 24*I*
a*d^3*e*f^3*x^3 - 36*I*a*d^3*e^2*f^2*x^2 - 24*I*a*d^3*e^3*f*x - 6*I*a*d^3*e
^4)*e^(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.283 $\int \frac{(e+fx)^3 \mathbf{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=667

$$\frac{9if^2(e+fx)\text{PolyLog}\left(3,-ie^{c+dx}\right)}{4ad^3}-\frac{9if^2(e+fx)\text{PolyLog}\left(3,ie^{c+dx}\right)}{4ad^3}-\frac{9if(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{8ad^2}+\frac{9if(e+fx)^2}{}$$

```
[Out] ((-I/2)*f*(e + f*x)^2)/(a*d^2) - (5*f^2*(e + f*x)*ArcTan[E^(c + d*x)])/(a*d
^3) + (3*(e + f*x)^3*ArcTan[E^(c + d*x)])/(4*a*d) + (I*f^2*(e + f*x)*Log[1
+ E^(2*(c + d*x))])/(a*d^3) + (((5*I)/2)*f^3*PolyLog[2, (-I)*E^(c + d*x)])/
(a*d^4) - (((9*I)/8)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) -
(((5*I)/2)*f^3*PolyLog[2, I*E^(c + d*x)])/(a*d^4) + (((9*I)/8)*f*(e + f*x)^
2*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + ((I/2)*f^3*PolyLog[2, -E^(2*(c + d*x
))])/(a*d^4) + (((9*I)/4)*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^
3) - (((9*I)/4)*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a*d^3) - (((9*I)/
4)*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(a*d^4) + (((9*I)/4)*f^3*PolyLog[4, I*
E^(c + d*x)])/(a*d^4) - (f^3*Sech[c + d*x])/(4*a*d^4) + (9*f*(e + f*x)^2*Se
ch[c + d*x])/(8*a*d^2) - ((I/4)*f^2*(e + f*x)*Sech[c + d*x]^2)/(a*d^3) + (f
*(e + f*x)^2*Sech[c + d*x]^3)/(4*a*d^2) + ((I/4)*(e + f*x)^3*Sech[c + d*x]^
4)/(a*d) + ((I/4)*f^3*Tanh[c + d*x])/(a*d^4) - ((I/2)*f*(e + f*x)^2*Tanh[c
+ d*x])/(a*d^2) - (f^2*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(4*a*d^3) + (
3*(e + f*x)^3*Sech[c + d*x]*Tanh[c + d*x])/(8*a*d) - ((I/4)*f*(e + f*x)^2*S
ech[c + d*x]^2*Tanh[c + d*x])/(a*d^2) + ((e + f*x)^3*Sech[c + d*x]^3*Tanh[c
 + d*x])/(4*a*d)
```

Rubi [A] time = 0.71112, antiderivative size = 667, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 16, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.516, Rules used = {5571, 4186, 4185, 4180, 2279, 2391, 2531, 6609, 2282, 6589, 5451, 3767, 8, 4184, 3718, 2190}

$$\frac{9if^2(e+fx)\text{PolyLog}\left(3,-ie^{c+dx}\right)}{4ad^3}-\frac{9if^2(e+fx)\text{PolyLog}\left(3,ie^{c+dx}\right)}{4ad^3}-\frac{9if(e+fx)^2\text{PolyLog}\left(2,-ie^{c+dx}\right)}{8ad^2}+\frac{9if(e+fx)^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-I/2)*f*(e + f*x)^2)/(a*d^2) - (5*f^2*(e + f*x)*ArcTan[E^(c + d*x)])/(a*d
^3) + (3*(e + f*x)^3*ArcTan[E^(c + d*x)])/(4*a*d) + (I*f^2*(e + f*x)*Log[1
+ E^(2*(c + d*x))])/(a*d^3) + (((5*I)/2)*f^3*PolyLog[2, (-I)*E^(c + d*x)])/
(a*d^4) - (((9*I)/8)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) -
(((5*I)/2)*f^3*PolyLog[2, I*E^(c + d*x)])/(a*d^4) + (((9*I)/8)*f*(e + f*x)^
2*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + ((I/2)*f^3*PolyLog[2, -E^(2*(c + d*x
))])/(a*d^4) + (((9*I)/4)*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^
3) - (((9*I)/4)*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a*d^3) - (((9*I)/
4)*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(a*d^4) + (((9*I)/4)*f^3*PolyLog[4, I*
E^(c + d*x)])/(a*d^4) - (f^3*Sech[c + d*x])/(4*a*d^4) + (9*f*(e + f*x)^2*Se
ch[c + d*x])/(8*a*d^2) - ((I/4)*f^2*(e + f*x)*Sech[c + d*x]^2)/(a*d^3) + (f
*(e + f*x)^2*Sech[c + d*x]^3)/(4*a*d^2) + ((I/4)*(e + f*x)^3*Sech[c + d*x]^
4)/(a*d) + ((I/4)*f^3*Tanh[c + d*x])/(a*d^4) - ((I/2)*f*(e + f*x)^2*Tanh[c
+ d*x])/(a*d^2) - (f^2*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(4*a*d^3) + (
3*(e + f*x)^3*Sech[c + d*x]*Tanh[c + d*x])/(8*a*d) - ((I/4)*f*(e + f*x)^2*S
ech[c + d*x]^2*Tanh[c + d*x])/(a*d^2) + ((e + f*x)^3*Sech[c + d*x]^3*Tanh[c
 + d*x])/(4*a*d)
```

Rule 5571

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sech[c
 + d*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)*T
anh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && Eq
Q[a^2 + b^2, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int (e+fx)^3\operatorname{sech}^4(c+dx)\tanh(c+dx)\,dx}{a} + \frac{\int (e+fx)^3\operatorname{sech}^5(c+dx)\,dx}{a}$$

$$= \frac{f(e+fx)^2\operatorname{sech}^3(c+dx)}{4ad^2} + \frac{i(e+fx)^3\operatorname{sech}^4(c+dx)}{4ad} + \frac{(e+fx)^3\operatorname{sech}^3(c+dx)\tanh(c}{4ad}$$

$$= -\frac{f^3\operatorname{sech}(c+dx)}{4ad^4} + \frac{9f(e+fx)^2\operatorname{sech}(c+dx)}{8ad^2} - \frac{if^2(e+fx)\operatorname{sech}^2(c+dx)}{4ad^3} + \frac{f(e+fx}{}$$

$$= -\frac{5f^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{4ad} - \frac{f^3\operatorname{sech}(c+dx)}{4ad^4} + \frac{9f(e+f}{}$$

$$= -\frac{if(e+fx)^2}{2ad^2} - \frac{5f^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{4ad} - \frac{9if(e+fx)^2\mathrm{I}}{8ad}$$

$$= -\frac{if(e+fx)^2}{2ad^2} - \frac{5f^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{4ad} + \frac{if^2(e+fx)\,\mathrm{lo}}{a}$$

$$= -\frac{if(e+fx)^2}{2ad^2} - \frac{5f^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{4ad} + \frac{if^2(e+fx)\,\mathrm{lo}}{a}$$

$$= -\frac{if(e+fx)^2}{2ad^2} - \frac{5f^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^3} + \frac{3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{4ad} + \frac{if^2(e+fx)\,\mathrm{lo}}{a}$$

Mathematica [B] time = 13.394, size = 1804, normalized size = 2.7

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (-3*(4*d^2*e*(d^2*e^2 - 4*f^2)*x + 2*d^2*f*(3*d^2*e^2 - 4*f^2)*x^2 + 4*d^4*
e*f^2*x^3 + d^4*f^3*x^4 + 4*d*(1 - I*E^c)*f*(3*d^2*e^2 - 4*f^2)*x*Log[1 + I
*E^(-c - d*x)] + 12*d^3*e*(1 - I*E^c)*f^2*x^2*Log[1 + I*E^(-c - d*x)] + 4*d
^3*(1 - I*E^c)*f^3*x^3*Log[1 + I*E^(-c - d*x)] + (4*I)*d*e*(I + E^c)*(d^2*e
^2 - 4*f^2)*(d*x - Log[I + E^(c + d*x)]) + 4*(1 - I*E^c)*f*(-3*d^2*e^2 + 4*
f^2)*PolyLog[2, (-I)*E^(-c - d*x)] + (24*I)*d*e*(I + E^c)*f^2*(d*x*PolyLog[
2, (-I)*E^(-c - d*x)] + PolyLog[3, (-I)*E^(-c - d*x)]) + (12*I)*(I + E^c)*f
^3*(d^2*x^2*PolyLog[2, (-I)*E^(-c - d*x)] + 2*(d*x*PolyLog[3, (-I)*E^(-c -
d*x)] + PolyLog[4, (-I)*E^(-c - d*x)]))))/(32*a*d^4*(I + E^c)) - ((28*f^2 -
 3*d^2*(e + f*x)^2)^2 + 12*d*(1 + I*E^c)*f^2*(9*d^2*e^2 - 28*f^2)*x*Log[1 -
 I*E^(-c - d*x)] + 108*d^3*e*(1 + I*E^c)*f^3*x^2*Log[1 - I*E^(-c - d*x)] +
36*d^3*(1 + I*E^c)*f^4*x^3*Log[1 - I*E^(-c - d*x)] - 12*d*e*(1 + I*E^c)*f*(
3*d^2*e^2 - 28*f^2)*(d*x - Log[I - E^(c + d*x)]) + 12*(1 + I*E^c)*f^2*(-9*d
^2*e^2 + 28*f^2)*PolyLog[2, I*E^(-c - d*x)] - 216*d*e*(1 + I*E^c)*f^3*(d*x*
PolyLog[2, I*E^(-c - d*x)] + PolyLog[3, I*E^(-c - d*x)]) - 108*(1 + I*E^c)*
f^4*(d^2*x^2*PolyLog[2, I*E^(-c - d*x)] + 2*(d*x*PolyLog[3, I*E^(-c - d*x)]
 + PolyLog[4, I*E^(-c - d*x)])))/(96*a*d^4*(-I + E^c)*f) + ((3*e^3*x*Cosh[c
])/(4*a) + (3*e^3*x*Sinh[c])/(4*a))/(1 + Cosh[2*c] + Sinh[2*c]) + ((9*e^2*f
*x^2*Cosh[c])/(8*a) + (9*e^2*f*x^2*Sinh[c])/(8*a))/(1 + Cosh[2*c] + Sinh[2*
c]) + ((3*e*f^2*x^3*Cosh[c])/(4*a) + (3*e*f^2*x^3*Sinh[c])/(4*a))/(1 + Cosh
[2*c] + Sinh[2*c]) + ((3*f^3*x^4*Cosh[c])/(16*a) + (3*f^3*x^4*Sinh[c])/(16*
a))/(1 + Cosh[2*c] + Sinh[2*c]) - ((I/8)*(e^3 + 3*e^2*f*x + 3*e*f^2*x^2 + f
^3*x^3))/(a*d*(Cosh[c/2 + (d*x)/2] - I*Sinh[c/2 + (d*x)/2])^2) + (((3*I)/4)
*(e^2*f*Sinh[(d*x)/2] + 2*e*f^2*x*Sinh[(d*x)/2] + f^3*x^2*Sinh[(d*x)/2]))/(
a*d^2*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] - I*Sinh[c/2 + (d*x)/2
])) + ((I/8)*(e^3 + 3*e^2*f*x + 3*e*f^2*x^2 + f^3*x^3))/(a*d*(Cosh[c/2 + (d
*x)/2] + I*Sinh[c/2 + (d*x)/2])^4) - ((I/4)*(e^2*f*Sinh[(d*x)/2] + 2*e*f^2*
x*Sinh[(d*x)/2] + f^3*x^2*Sinh[(d*x)/2]))/(a*d^2*(Cosh[c/2] + I*Sinh[c/2])*
```

```
(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2])^3) + ((2*I)*d^2*e^3*Cosh[c/2]
 + d*e^2*f*Cosh[c/2] - (2*I)*e*f^2*Cosh[c/2] + (6*I)*d^2*e^2*f*x*Cosh[c/2]
+ 2*d*e*f^2*x*Cosh[c/2] - (2*I)*f^3*x*Cosh[c/2] + (6*I)*d^2*e*f^2*x^2*Cosh[
c/2] + d*f^3*x^2*Cosh[c/2] + (2*I)*d^2*f^3*x^3*Cosh[c/2] - 2*d^2*e^3*Sinh[c
/2] - I*d*e^2*f*Sinh[c/2] + 2*e*f^2*Sinh[c/2] - 6*d^2*e^2*f*x*Sinh[c/2] - (
2*I)*d*e*f^2*x*Sinh[c/2] + 2*f^3*x*Sinh[c/2] - 6*d^2*e*f^2*x^2*Sinh[c/2] -
I*d*f^3*x^2*Sinh[c/2] - 2*d^2*f^3*x^3*Sinh[c/2]])/(8*a*d^3*(Cosh[c/2] + I*Si
nh[c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2])^2) - ((I/4)*(7*d^2*e
^2*f*Sinh[(d*x)/2] - 2*f^3*Sinh[(d*x)/2] + 14*d^2*e*f^2*x*Sinh[(d*x)/2] + 7
*d^2*f^3*x^2*Sinh[(d*x)/2]))/(a*d^4*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c/2 + (
d*x)/2] + I*Sinh[c/2 + (d*x)/2]))
```

Maple [B] time = 0.259, size = 2026, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 9/8*I/a/d^2*ln(1-I*exp(d*x+c))*c*e^2*f-9/8*I/a/d*ln(1+I*exp(d*x+c))*e^2*f*x
-9/8*I/a/d^2*ln(1+I*exp(d*x+c))*c*e^2*f+9/8*I/a/d^2*e^2*f*c*ln(exp(d*x+c)-I
)+9/8*I/a/d^3*ln(1+I*exp(d*x+c))*c^2*e*f^2-9/8*I/a/d^3*e*f^2*c^2*ln(exp(d*x
+c)-I)+9/8*I/a/d*ln(1-I*exp(d*x+c))*e*f^2*x^2+9/4*I/a/d^2*polylog(2,I*exp(d
*x+c))*e*f^2*x+9/8*I/a/d*ln(1-I*exp(d*x+c))*e^2*f*x-9/4*I*f^3*polylog(4,-I*
exp(d*x+c))/a/d^4+9/4*I*f^3*polylog(4,I*exp(d*x+c))/a/d^4-I/a/d^4*f^3*c^2+3
/8*I/a/d*e^3*ln(exp(d*x+c)+I)+7/2*I/a/d^4*f^3*polylog(2,-I*exp(d*x+c))-3/2*
I/a/d^4*f^3*polylog(2,I*exp(d*x+c))-3/8*I/a/d*e^3*ln(exp(d*x+c)-I)-I/a/d^2*
f^3*x^2-2*I/a/d^3*e*f^2*ln(exp(d*x+c))-3/2*I/a/d^3*f^3*ln(1-I*exp(d*x+c))*x
-3/2*I/a/d^4*f^3*ln(1-I*exp(d*x+c))*c-3/8*I/a/d*f^3*ln(1+I*exp(d*x+c))*x^3-
3/8*I/a/d^4*f^3*ln(1+I*exp(d*x+c))*c^3-9/8*I/a/d^2*f^3*polylog(2,-I*exp(d*x
+c))*x^2+9/4*I/a/d^3*f^3*polylog(3,-I*exp(d*x+c))*x+3/8*I/a/d*f^3*ln(1-I*ex
p(d*x+c))*x^3+3/8*I/a/d^4*f^3*ln(1-I*exp(d*x+c))*c^3+9/8*I/a/d^2*f^3*polylo
g(2,I*exp(d*x+c))*x^2-9/4*I/a/d^3*f^3*polylog(3,I*exp(d*x+c))*x+7/2*I/a/d^3
*f^3*ln(1+I*exp(d*x+c))*x+7/2*I/a/d^4*f^3*ln(1+I*exp(d*x+c))*c+9/4*I/a/d^3*
e*f^2*polylog(3,-I*exp(d*x+c))-9/4*I/a/d^3*e*f^2*polylog(3,I*exp(d*x+c))+7/
2*I/a/d^3*e*f^2*ln(exp(d*x+c)-I)-3/2*I/a/d^3*e*f^2*ln(exp(d*x+c)+I)+2*I/a/d
^4*f^3*c*ln(exp(d*x+c))-9/8*I/a/d^2*e^2*f*polylog(2,-I*exp(d*x+c))+9/8*I/a/
d^2*e^2*f*polylog(2,I*exp(d*x+c))-7/2*I/a/d^4*f^3*c*ln(exp(d*x+c)-I)+3/2*I/
a/d^4*f^3*c*ln(exp(d*x+c)+I)+3/8*I/a/d^4*f^3*c^3*ln(exp(d*x+c)-I)-3/8*I/a/d
^4*f^3*c^3*ln(exp(d*x+c)+I)-2*I/a/d^3*f^3*c*x+9/8*I/a/d^3*e*f^2*c^2*ln(exp(
d*x+c)+I)-9/8*I/a/d^3*ln(1-I*exp(d*x+c))*c^2*e*f^2-9/8*I/a/d*ln(1+I*exp(d*x
+c))*e*f^2*x^2-9/4*I/a/d^2*polylog(2,-I*exp(d*x+c))*e*f^2*x-9/8*I/a/d^2*e^2
*f*c*ln(exp(d*x+c)+I)+1/4*(2*I*f^3-22*I*d^2*f^3*x^2*exp(2*d*x+2*c)-22*I*d^2
*e^2*f*exp(2*d*x+2*c)+9*d^3*e*f^2*x^2*exp(5*d*x+5*c)+9*d^3*e^2*f*x*exp(5*d*
x+5*c)+18*d^2*e*f^2*x*exp(5*d*x+5*c)+16*d^2*e*f^2*x*exp(3*d*x+3*c)-8*I*d^2*
e*f^2*x-44*I*d^2*e*f^2*x*exp(2*d*x+2*c)-d^2*f^3*x^2*exp(d*x+c)-d^2*e^2*f*ex
p(d*x+c)+3*d^3*f^3*x^3*exp(d*x+c)-2*d*f^3*x*exp(d*x+c)-2*d*e*f^2*exp(d*x+c)
+6*I*d^3*f^3*x^3*exp(2*d*x+2*c)-6*I*d^3*f^3*x^3*exp(4*d*x+4*c)-18*I*d^2*f^3
*x^2*exp(4*d*x+4*c)-18*I*d^2*e^2*f*exp(4*d*x+4*c)-18*I*d^3*e^2*f*x*exp(4*d*
x+4*c)+18*I*d^3*e*f^2*x^2*exp(2*d*x+2*c)-18*I*d^3*e*f^2*x^2*exp(4*d*x+4*c)+
18*I*d^3*e^2*f*x*exp(2*d*x+2*c)-36*I*d^2*e*f^2*x*exp(4*d*x+4*c)+3*d^3*e^3*e
xp(5*d*x+5*c)+2*I*f^3*exp(4*d*x+4*c)+4*I*f^3*exp(2*d*x+2*c)+2*d^3*e^3*exp(3
*d*x+3*c)+9*d^3*e*f^2*x^2*exp(d*x+c)+9*d^3*e^2*f*x*exp(d*x+c)-2*d^2*e*f^2*x
*exp(d*x+c)-2*f^3*exp(d*x+c)+3*d^3*e^3*exp(d*x+c)-4*I*d^2*e^2*f-4*I*d^2*f^3
*x^2-2*f^3*exp(5*d*x+5*c)-4*f^3*exp(3*d*x+3*c)+6*d^3*e*f^2*x^2*exp(3*d*x+3*
c)+6*d^3*e^2*f*x*exp(3*d*x+3*c)+2*d^3*f^3*x^3*exp(3*d*x+3*c)+6*I*d^3*e^3*ex
```

```
p(2*d*x+2*c)-6*I*d^3*e^3*exp(4*d*x+4*c)+9*d^2*e^2*f*exp(5*d*x+5*c)+8*d^2*e^
2*f*exp(3*d*x+3*c)-2*d*f^3*x*exp(5*d*x+5*c)-4*d*f^3*x*exp(3*d*x+3*c)-2*d*e*
f^2*exp(5*d*x+5*c)-4*d*e*f^2*exp(3*d*x+3*c)+3*d^3*f^3*x^3*exp(5*d*x+5*c)+9*
d^2*f^3*x^2*exp(5*d*x+5*c)+8*d^2*f^3*x^2*exp(3*d*x+3*c))/(exp(d*x+c)+I)^2/(
exp(d*x+c)-I)^4/d^4/a
```

Maxima [B] time = 6.25254, size = 1800, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/8*e^3*(64*(3*e^(-d*x - c) - 6*I*e^(-2*d*x - 2*c) + 2*e^(-3*d*x - 3*c) +
6*I*e^(-4*d*x - 4*c) + 3*e^(-5*d*x - 5*c))/((64*I*a*e^(-d*x - c) - 32*a*e^(
-2*d*x - 2*c) + 128*I*a*e^(-3*d*x - 3*c) + 32*a*e^(-4*d*x - 4*c) + 64*I*a*e
^(-5*d*x - 5*c) + 32*a*e^(-6*d*x - 6*c) - 32*a)*d) + 3*I*log(e^(-d*x - c) +
 I)/(a*d) - 3*I*log(e^(-d*x - c) - I)/(a*d)) - 2*I*e*f^2*x/(a*d^2) + (-4*I*
d^2*f^3*x^2 - 8*I*d^2*e*f^2*x - 4*I*d^2*e^2*f + 2*I*f^3 + (3*d^3*f^3*x^3*e^
(5*c) + 9*(d^3*e*f^2 + d^2*f^3)*x^2*e^(5*c) + (9*d^3*e^2*f + 18*d^2*e*f^2 -
 2*d*f^3)*x*e^(5*c) + (9*d^2*e^2*f - 2*d*e*f^2 - 2*f^3)*e^(5*c))*e^(5*d*x)
+ (-6*I*d^3*f^3*x^3*e^(4*c) + (-18*I*d^3*e*f^2 - 18*I*d^2*f^3)*x^2*e^(4*c)
+ (-18*I*d^3*e^2*f - 36*I*d^2*e*f^2)*x*e^(4*c) + (-18*I*d^2*e^2*f + 2*I*f^3
)*e^(4*c))*e^(4*d*x) + 2*(d^3*f^3*x^3*e^(3*c) + (3*d^3*e*f^2 + 4*d^2*f^3)*x
^2*e^(3*c) + (3*d^3*e^2*f + 8*d^2*e*f^2 - 2*d*f^3)*x*e^(3*c) + 2*(2*d^2*e^2
*f - d*e*f^2 - f^3)*e^(3*c))*e^(3*d*x) + (6*I*d^3*f^3*x^3*e^(2*c) + (18*I*d
^3*e*f^2 - 22*I*d^2*f^3)*x^2*e^(2*c) + (18*I*d^3*e^2*f - 44*I*d^2*e*f^2)*x*
e^(2*c) + (-22*I*d^2*e^2*f + 4*I*f^3)*e^(2*c))*e^(2*d*x) + (3*d^3*f^3*x^3*e
^c + (9*d^3*e*f^2 - d^2*f^3)*x^2*e^c + (9*d^3*e^2*f - 2*d^2*e*f^2 - 2*d*f^3
)*x*e^c - (d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*e^c)*e^(d*x))/(4*a*d^4*e^(6*d*x +
 6*c) - 8*I*a*d^4*e^(5*d*x + 5*c) + 4*a*d^4*e^(4*d*x + 4*c) - 16*I*a*d^4*e^
(3*d*x + 3*c) - 4*a*d^4*e^(2*d*x + 2*c) - 8*I*a*d^4*e^(d*x + c) - 4*a*d^4)
- 9/8*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x + c)) - 2*p
olylog(3, -I*e^(d*x + c)))*e*f^2/(a*d^3) + 9/8*I*(d^2*x^2*log(-I*e^(d*x + c
) + 1) + 2*d*x*dilog(I*e^(d*x + c)) - 2*polylog(3, I*e^(d*x + c)))*e*f^2/(a
*d^3) + 7/2*I*e*f^2*log(I*e^(d*x + c) + 1)/(a*d^3) - 3/2*I*e*f^2*log(I*e^(d
*x + c) - 1)/(a*d^3) - 3/8*I*(d^3*x^3*log(I*e^(d*x + c) + 1) + 3*d^2*x^2*di
log(-I*e^(d*x + c)) - 6*d*x*polylog(3, -I*e^(d*x + c)) + 6*polylog(4, -I*e^
(d*x + c)))*f^3/(a*d^4) + 3/8*I*(d^3*x^3*log(-I*e^(d*x + c) + 1) + 3*d^2*x^
2*dilog(I*e^(d*x + c)) - 6*d*x*polylog(3, I*e^(d*x + c)) + 6*polylog(4, I*e
^(d*x + c)))*f^3/(a*d^4) - 1/8*I*(9*d^2*e^2*f - 28*f^3)*(d*x*log(I*e^(d*x +
 c) + 1) + dilog(-I*e^(d*x + c)))/(a*d^4) + 3/8*I*(3*d^2*e^2*f - 4*f^3)*(d*
x*log(-I*e^(d*x + c) + 1) + dilog(I*e^(d*x + c)))/(a*d^4) - 1/32*(3*I*d^4*f
^3*x^4 + 12*I*d^4*e*f^2*x^3 + (18*I*d^2*e^2*f - 24*I*f^3)*d^2*x^2)/(a*d^4)
+ 1/32*(3*I*d^4*f^3*x^4 + 12*I*d^4*e*f^2*x^3 + (18*I*d^2*e^2*f - 56*I*f^3)*
d^2*x^2)/(a*d^4)
```

Fricas [C] time = 2.89648, size = 9095, normalized size = 13.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] (-8*I*d^2*e^2*f + 16*I*c*d*e*f^2 + (-8*I*c^2 + 4*I)*f^3 + (-9*I*d^2*f^3*x^2
 - 18*I*d^2*e*f^2*x - 9*I*d^2*e^2*f + 12*I*f^3 + (9*I*d^2*f^3*x^2 + 18*I*d^
2*e*f^2*x + 9*I*d^2*e^2*f - 12*I*f^3)*e^(6*d*x + 6*c) + 6*(3*d^2*f^3*x^2 +
6*d^2*e*f^2*x + 3*d^2*e^2*f - 4*f^3)*e^(5*d*x + 5*c) + (9*I*d^2*f^3*x^2 + 1
8*I*d^2*e*f^2*x + 9*I*d^2*e^2*f - 12*I*f^3)*e^(4*d*x + 4*c) + 12*(3*d^2*f^3
*x^2 + 6*d^2*e*f^2*x + 3*d^2*e^2*f - 4*f^3)*e^(3*d*x + 3*c) + (-9*I*d^2*f^3
*x^2 - 18*I*d^2*e*f^2*x - 9*I*d^2*e^2*f + 12*I*f^3)*e^(2*d*x + 2*c) + 6*(3*
d^2*f^3*x^2 + 6*d^2*e*f^2*x + 3*d^2*e^2*f - 4*f^3)*e^(d*x + c))*dilog(I*e^(
d*x + c)) + (9*I*d^2*f^3*x^2 + 18*I*d^2*e*f^2*x + 9*I*d^2*e^2*f - 28*I*f^3
+ (-9*I*d^2*f^3*x^2 - 18*I*d^2*e*f^2*x - 9*I*d^2*e^2*f + 28*I*f^3)*e^(6*d*x
 + 6*c) - 2*(9*d^2*f^3*x^2 + 18*d^2*e*f^2*x + 9*d^2*e^2*f - 28*f^3)*e^(5*d*
x + 5*c) + (-9*I*d^2*f^3*x^2 - 18*I*d^2*e*f^2*x - 9*I*d^2*e^2*f + 28*I*f^3)
*e^(4*d*x + 4*c) - 4*(9*d^2*f^3*x^2 + 18*d^2*e*f^2*x + 9*d^2*e^2*f - 28*f^3
)*e^(3*d*x + 3*c) + (9*I*d^2*f^3*x^2 + 18*I*d^2*e*f^2*x + 9*I*d^2*e^2*f - 2
8*I*f^3)*e^(2*d*x + 2*c) - 2*(9*d^2*f^3*x^2 + 18*d^2*e*f^2*x + 9*d^2*e^2*f
 - 28*f^3)*e^(d*x + c))*dilog(-I*e^(d*x + c)) + (-8*I*d^2*f^3*x^2 - 16*I*d^2
*e*f^2*x - 16*I*c*d*e*f^2 + 8*I*c^2*f^3)*e^(6*d*x + 6*c) + 2*(3*d^3*f^3*x^3
 + 3*d^3*e^3 + 9*d^2*e^2*f - 2*(8*c + 1)*d*e*f^2 + 2*(4*c^2 - 1)*f^3 + (9*d
^3*e*f^2 + d^2*f^3)*x^2 + (9*d^3*e^2*f + 2*d^2*e*f^2 - 2*d*f^3)*x)*e^(5*d*x
 + 5*c) + (-12*I*d^3*f^3*x^3 - 12*I*d^3*e^3 - 36*I*d^2*e^2*f - 16*I*c*d*e*f
^2 + (8*I*c^2 + 4*I)*f^3 + (-36*I*d^3*e*f^2 - 44*I*d^2*f^3)*x^2 + (-36*I*d^
3*e^2*f - 88*I*d^2*e*f^2)*x)*e^(4*d*x + 4*c) + 4*(d^3*f^3*x^3 + d^3*e^3 + 4
*d^2*e^2*f - 2*(8*c + 1)*d*e*f^2 + 2*(4*c^2 - 1)*f^3 + (3*d^3*e*f^2 - 4*d^2
*f^3)*x^2 + (3*d^3*e^2*f - 8*d^2*e*f^2 - 2*d*f^3)*x)*e^(3*d*x + 3*c) + (12*
I*d^3*f^3*x^3 + 12*I*d^3*e^3 - 44*I*d^2*e^2*f + 16*I*c*d*e*f^2 + (-8*I*c^2
+ 8*I)*f^3 + (36*I*d^3*e*f^2 - 36*I*d^2*f^3)*x^2 + (36*I*d^3*e^2*f - 72*I*d
^2*e*f^2)*x)*e^(2*d*x + 2*c) + 2*(3*d^3*f^3*x^3 + 3*d^3*e^3 - d^2*e^2*f - 2
*(8*c + 1)*d*e*f^2 + 2*(4*c^2 - 1)*f^3 + 9*(d^3*e*f^2 - d^2*f^3)*x^2 + (9*d
^3*e^2*f - 18*d^2*e*f^2 - 2*d*f^3)*x)*e^(d*x + c) + (-3*I*d^3*e^3 + 9*I*c*d
^2*e^2*f + (-9*I*c^2 + 12*I)*d*e*f^2 + (3*I*c^3 - 12*I*c)*f^3 + (3*I*d^3*e^
3 - 9*I*c*d^2*e^2*f + (9*I*c^2 - 12*I)*d*e*f^2 + (-3*I*c^3 + 12*I*c)*f^3)*e
^(6*d*x + 6*c) + 6*(d^3*e^3 - 3*c*d^2*e^2*f + (3*c^2 - 4)*d*e*f^2 - (c^3 -
4*c)*f^3)*e^(5*d*x + 5*c) + (3*I*d^3*e^3 - 9*I*c*d^2*e^2*f + (9*I*c^2 - 12*
I)*d*e*f^2 + (-3*I*c^3 + 12*I*c)*f^3)*e^(4*d*x + 4*c) + 12*(d^3*e^3 - 3*c*d
^2*e^2*f + (3*c^2 - 4)*d*e*f^2 - (c^3 - 4*c)*f^3)*e^(3*d*x + 3*c) + (-3*I*d
^3*e^3 + 9*I*c*d^2*e^2*f + (-9*I*c^2 + 12*I)*d*e*f^2 + (3*I*c^3 - 12*I*c)*f
^3)*e^(2*d*x + 2*c) + 6*(d^3*e^3 - 3*c*d^2*e^2*f + (3*c^2 - 4)*d*e*f^2 - (c
^3 - 4*c)*f^3)*e^(d*x + c))*log(e^(d*x + c) + I) + (3*I*d^3*e^3 - 9*I*c*d^2
*e^2*f + (9*I*c^2 - 28*I)*d*e*f^2 + (-3*I*c^3 + 28*I*c)*f^3 + (-3*I*d^3*e^3
 + 9*I*c*d^2*e^2*f + (-9*I*c^2 + 28*I)*d*e*f^2 + (3*I*c^3 - 28*I*c)*f^3)*e^
(6*d*x + 6*c) - 2*(3*d^3*e^3 - 9*c*d^2*e^2*f + (9*c^2 - 28)*d*e*f^2 - (3*c^
3 - 28*c)*f^3)*e^(5*d*x + 5*c) + (-3*I*d^3*e^3 + 9*I*c*d^2*e^2*f + (-9*I*c^
2 + 28*I)*d*e*f^2 + (3*I*c^3 - 28*I*c)*f^3)*e^(4*d*x + 4*c) - 4*(3*d^3*e^3
 - 9*c*d^2*e^2*f + (9*c^2 - 28)*d*e*f^2 - (3*c^3 - 28*c)*f^3)*e^(3*d*x + 3*c
) + (3*I*d^3*e^3 - 9*I*c*d^2*e^2*f + (9*I*c^2 - 28*I)*d*e*f^2 + (-3*I*c^3 +
 28*I*c)*f^3)*e^(2*d*x + 2*c) - 2*(3*d^3*e^3 - 9*c*d^2*e^2*f + (9*c^2 - 28)
*d*e*f^2 - (3*c^3 - 28*c)*f^3)*e^(d*x + c))*log(e^(d*x + c) - I) + (3*I*d^3
*f^3*x^3 + 9*I*d^3*e*f^2*x^2 + 9*I*c*d^2*e^2*f - 9*I*c^2*d*e*f^2 + (3*I*c^3
 - 28*I*c)*f^3 + (9*I*d^3*e^2*f - 28*I*d*f^3)*x + (-3*I*d^3*f^3*x^3 - 9*I*d
^3*e*f^2*x^2 - 9*I*c*d^2*e^2*f + 9*I*c^2*d*e*f^2 + (-3*I*c^3 + 28*I*c)*f^3
+ (-9*I*d^3*e^2*f + 28*I*d*f^3)*x)*e^(6*d*x + 6*c) - 2*(3*d^3*f^3*x^3 + 9*d
^3*e*f^2*x^2 + 9*c*d^2*e^2*f - 9*c^2*d*e*f^2 + (3*c^3 - 28*c)*f^3 + (9*d^3*
e^2*f - 28*d*f^3)*x)*e^(5*d*x + 5*c) + (-3*I*d^3*f^3*x^3 - 9*I*d^3*e*f^2*x^
2 - 9*I*c*d^2*e^2*f + 9*I*c^2*d*e*f^2 + (-3*I*c^3 + 28*I*c)*f^3 + (-9*I*d^3
*e^2*f + 28*I*d*f^3)*x)*e^(4*d*x + 4*c) - 4*(3*d^3*f^3*x^3 + 9*d^3*e*f^2*x^
2 + 9*c*d^2*e^2*f - 9*c^2*d*e*f^2 + (3*c^3 - 28*c)*f^3 + (9*d^3*e^2*f - 28*
```

```
d*f^3)*x)*e^(3*d*x + 3*c) + (3*I*d^3*f^3*x^3 + 9*I*d^3*e*f^2*x^2 + 9*I*c*d^
2*e^2*f - 9*I*c^2*d*e*f^2 + (3*I*c^3 - 28*I*c)*f^3 + (9*I*d^3*e^2*f - 28*I*
d*f^3)*x)*e^(2*d*x + 2*c) - 2*(3*d^3*f^3*x^3 + 9*d^3*e*f^2*x^2 + 9*c*d^2*e^
2*f - 9*c^2*d*e*f^2 + (3*c^3 - 28*c)*f^3 + (9*d^3*e^2*f - 28*d*f^3)*x)*e^(d
*x + c))*log(I*e^(d*x + c) + 1) + (-3*I*d^3*f^3*x^3 - 9*I*d^3*e*f^2*x^2 - 9
*I*c*d^2*e^2*f + 9*I*c^2*d*e*f^2 + (-3*I*c^3 + 12*I*c)*f^3 + (-9*I*d^3*e^2*
f + 12*I*d*f^3)*x + (3*I*d^3*f^3*x^3 + 9*I*d^3*e*f^2*x^2 + 9*I*c*d^2*e^2*f
- 9*I*c^2*d*e*f^2 + (3*I*c^3 - 12*I*c)*f^3 + (9*I*d^3*e^2*f - 12*I*d*f^3)*x
)*e^(6*d*x + 6*c) + 6*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*c*d^2*e^2*f - 3*c^
2*d*e*f^2 + (c^3 - 4*c)*f^3 + (3*d^3*e^2*f - 4*d*f^3)*x)*e^(5*d*x + 5*c) +
(3*I*d^3*f^3*x^3 + 9*I*d^3*e*f^2*x^2 + 9*I*c*d^2*e^2*f - 9*I*c^2*d*e*f^2 +
(3*I*c^3 - 12*I*c)*f^3 + (9*I*d^3*e^2*f - 12*I*d*f^3)*x)*e^(4*d*x + 4*c) +
12*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + (c^3 -
4*c)*f^3 + (3*d^3*e^2*f - 4*d*f^3)*x)*e^(3*d*x + 3*c) + (-3*I*d^3*f^3*x^3 -
 9*I*d^3*e*f^2*x^2 - 9*I*c*d^2*e^2*f + 9*I*c^2*d*e*f^2 + (-3*I*c^3 + 12*I*c
)*f^3 + (-9*I*d^3*e^2*f + 12*I*d*f^3)*x)*e^(2*d*x + 2*c) + 6*(d^3*f^3*x^3 +
 3*d^3*e*f^2*x^2 + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + (c^3 - 4*c)*f^3 + (3*d^3
*e^2*f - 4*d*f^3)*x)*e^(d*x + c))*log(-I*e^(d*x + c) + 1) + (18*I*f^3*e^(6*
d*x + 6*c) + 36*f^3*e^(5*d*x + 5*c) + 18*I*f^3*e^(4*d*x + 4*c) + 72*f^3*e^(
3*d*x + 3*c) - 18*I*f^3*e^(2*d*x + 2*c) + 36*f^3*e^(d*x + c) - 18*I*f^3)*po
lylog(4, I*e^(d*x + c)) + (-18*I*f^3*e^(6*d*x + 6*c) - 36*f^3*e^(5*d*x + 5*
c) - 18*I*f^3*e^(4*d*x + 4*c) - 72*f^3*e^(3*d*x + 3*c) + 18*I*f^3*e^(2*d*x
+ 2*c) - 36*f^3*e^(d*x + c) + 18*I*f^3)*polylog(4, -I*e^(d*x + c)) + (18*I*
d*f^3*x + 18*I*d*e*f^2 + (-18*I*d*f^3*x - 18*I*d*e*f^2)*e^(6*d*x + 6*c) - 3
6*(d*f^3*x + d*e*f^2)*e^(5*d*x + 5*c) + (-18*I*d*f^3*x - 18*I*d*e*f^2)*e^(4
*d*x + 4*c) - 72*(d*f^3*x + d*e*f^2)*e^(3*d*x + 3*c) + (18*I*d*f^3*x + 18*I
*d*e*f^2)*e^(2*d*x + 2*c) - 36*(d*f^3*x + d*e*f^2)*e^(d*x + c))*polylog(3,
I*e^(d*x + c)) + (-18*I*d*f^3*x - 18*I*d*e*f^2 + (18*I*d*f^3*x + 18*I*d*e*f
^2)*e^(6*d*x + 6*c) + 36*(d*f^3*x + d*e*f^2)*e^(5*d*x + 5*c) + (18*I*d*f^3*
x + 18*I*d*e*f^2)*e^(4*d*x + 4*c) + 72*(d*f^3*x + d*e*f^2)*e^(3*d*x + 3*c)
+ (-18*I*d*f^3*x - 18*I*d*e*f^2)*e^(2*d*x + 2*c) + 36*(d*f^3*x + d*e*f^2)*e
^(d*x + c))*polylog(3, -I*e^(d*x + c)))/(8*a*d^4*e^(6*d*x + 6*c) - 16*I*a*d
^4*e^(5*d*x + 5*c) + 8*a*d^4*e^(4*d*x + 4*c) - 32*I*a*d^4*e^(3*d*x + 3*c) -
 8*a*d^4*e^(2*d*x + 2*c) - 16*I*a*d^4*e^(d*x + c) - 8*a*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sech(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \operatorname{sech}(dx+c)^3}{i a \sinh(dx+c)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sech(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

3.284 $\int \frac{(e+fx)^2\mathbf{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=423

$$-\frac{3if(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{4ad^2}+\frac{3if(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{4ad^2}+\frac{3if^2\text{PolyLog}\left(3,-ie^{c+dx}\right)}{4ad^3}-\frac{3if^2\text{PolyLog}\left(3,ie^{c-}\right.}{4ad^3}$$

```
[Out] (3*(e + f*x)^2*ArcTan[E^(c + d*x)])/(4*a*d) - (5*f^2*ArcTan[Sinh[c + d*x]])
/(6*a*d^3) + ((I/3)*f^2*Log[Cosh[c + d*x]])/(a*d^3) - (((3*I)/4)*f*(e + f*x
)*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) + (((3*I)/4)*f*(e + f*x)*PolyLog[2,
 I*E^(c + d*x)])/(a*d^2) + (((3*I)/4)*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(a*
d^3) - (((3*I)/4)*f^2*PolyLog[3, I*E^(c + d*x)])/(a*d^3) + (3*f*(e + f*x)*S
ech[c + d*x])/(4*a*d^2) - ((I/12)*f^2*Sech[c + d*x]^2)/(a*d^3) + (f*(e + f*
x)*Sech[c + d*x]^3)/(6*a*d^2) + ((I/4)*(e + f*x)^2*Sech[c + d*x]^4)/(a*d) -
 ((I/3)*f*(e + f*x)*Tanh[c + d*x])/(a*d^2) - (f^2*Sech[c + d*x]*Tanh[c + d*
x])/(12*a*d^3) + (3*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(8*a*d) - ((I/
6)*f*(e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x])/(a*d^2) + ((e + f*x)^2*Sech[c
 + d*x]^3*Tanh[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.39833, antiderivative size = 423, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {5571, 4186, 3768, 3770, 4180, 2531, 2282, 6589, 5451, 4185, 4184, 3475}

$$-\frac{3if(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{4ad^2}+\frac{3if(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{4ad^2}+\frac{3if^2\text{PolyLog}\left(3,-ie^{c+dx}\right)}{4ad^3}-\frac{3if^2\text{PolyLog}\left(3,ie^{c-}\right.}{4ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```


```
[Out] (3*(e + f*x)^2*ArcTan[E^(c + d*x)])/(4*a*d) - (5*f^2*ArcTan[Sinh[c + d*x]])
/(6*a*d^3) + ((I/3)*f^2*Log[Cosh[c + d*x]])/(a*d^3) - (((3*I)/4)*f*(e + f*x
)*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) + (((3*I)/4)*f*(e + f*x)*PolyLog[2,
 I*E^(c + d*x)])/(a*d^2) + (((3*I)/4)*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(a*
d^3) - (((3*I)/4)*f^2*PolyLog[3, I*E^(c + d*x)])/(a*d^3) + (3*f*(e + f*x)*S
ech[c + d*x])/(4*a*d^2) - ((I/12)*f^2*Sech[c + d*x]^2)/(a*d^3) + (f*(e + f*
x)*Sech[c + d*x]^3)/(6*a*d^2) + ((I/4)*(e + f*x)^2*Sech[c + d*x]^4)/(a*d) -
 ((I/3)*f*(e + f*x)*Tanh[c + d*x])/(a*d^2) - (f^2*Sech[c + d*x]*Tanh[c + d*
x])/(12*a*d^3) + (3*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(8*a*d) - ((I/
6)*f*(e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x])/(a*d^2) + ((e + f*x)^2*Sech[c
 + d*x]^3*Tanh[c + d*x])/(4*a*d)
```


Rule 5571

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sech[c
 + d*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)*T
anh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && Eq
Q[a^2 + b^2, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
```

```
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
```

```
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int (e+fx)^2\text{sech}^4(c+dx)\tanh(c+dx)\,dx}{a} + \frac{\int (e+fx)^2\text{sech}^5(c+dx)\,dx}{a}$$
$$= \frac{f(e+fx)\text{sech}^3(c+dx)}{6ad^2} + \frac{i(e+fx)^2\text{sech}^4(c+dx)}{4ad} + \frac{(e+fx)^2\text{sech}^3(c+dx)\tanh(c+d}{4ad}$$
$$= \frac{3f(e+fx)\text{sech}(c+dx)}{4ad^2} - \frac{if^2\text{sech}^2(c+dx)}{12ad^3} + \frac{f(e+fx)\text{sech}^3(c+dx)}{6ad^2} + \frac{i(e+fx)^2\text{sech}}{4ad}$$
$$= \frac{3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{4ad} - \frac{5f^2\tan^{-1}(\sinh(c+dx))}{6ad^3} + \frac{3f(e+fx)\text{sech}(c+dx)}{4ad^2} - \frac{if^2\text{sec}}{1}$$
$$= \frac{3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{4ad} - \frac{5f^2\tan^{-1}(\sinh(c+dx))}{6ad^3} + \frac{if^2\log(\cosh(c+dx))}{3ad^3} - \frac{3if(e+f}{}$$
$$= \frac{3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{4ad} - \frac{5f^2\tan^{-1}(\sinh(c+dx))}{6ad^3} + \frac{if^2\log(\cosh(c+dx))}{3ad^3} - \frac{3if(e+f}{}$$
$$= \frac{3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{4ad} - \frac{5f^2\tan^{-1}(\sinh(c+dx))}{6ad^3} + \frac{if^2\log(\cosh(c+dx))}{3ad^3} - \frac{3if(e+f}{}$$

Mathematica [B] time = 12.9861, size = 1284, normalized size = 3.04

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] -((9*d^2*e^2 - 28*f^2)*x + 9*d^2*e*f*x^2 + 3*d^2*f^2*x^3 + 18*d*e*(1 + I*E^
c)*f*x*Log[1 - I*E^(-c - d*x)] + 9*d*(1 + I*E^c)*f^2*x^2*Log[1 - I*E^(-c -
d*x)] - ((1 + I*E^c)*(9*d^2*e^2 - 28*f^2)*(d*x - Log[I - E^(c + d*x)]))/d -
 18*e*(1 + I*E^c)*f*PolyLog[2, I*E^(-c - d*x)] - 18*(1 + I*E^c)*f^2*(x*Poly
Log[2, I*E^(-c - d*x)] + PolyLog[3, I*E^(-c - d*x)]/d))/(24*a*d^2*(-I + E^c
)) - (3*d^2*e^2*x - 4*f^2*x + 3*d^2*e*f*x^2 + d^2*f^2*x^3 - (6*I)*d*e*f*x*L
og[1 + I*Cosh[c + d*x] - I*Sinh[c + d*x]]*(I + Cosh[c] + Sinh[c]) - (3*I)*d
*f^2*x^2*Log[1 + I*Cosh[c + d*x] - I*Sinh[c + d*x]]*(I + Cosh[c] + Sinh[c])
 + (I*(3*d^2*e^2 - 4*f^2)*(d*x - Log[I + Cosh[c + d*x] + Sinh[c + d*x]])*(I
 + Cosh[c] + Sinh[c]))/d + (6*I)*e*f*PolyLog[2, (-I)*(Cosh[c + d*x] - Sinh[
c + d*x])]*(I + Cosh[c] + Sinh[c]) + ((6*I)*f^2*(d*x*PolyLog[2, (-I)*(Cosh[
c + d*x] - Sinh[c + d*x])] + PolyLog[3, (-I)*(Cosh[c + d*x] - Sinh[c + d*x]
)])*(I + Cosh[c] + Sinh[c]))/d)/(8*a*d^2*(I + Cosh[c] + Sinh[c])) + ((3*e^2
*x*Cosh[c])/(4*a) + (3*e^2*x*Sinh[c])/(4*a))/(1 + Cosh[2*c] + Sinh[2*c]) +
```

```
((3*e*f*x^2*Cosh[c])/(4*a) + (3*e*f*x^2*Sinh[c])/(4*a))/(1 + Cosh[2*c] + Si
nh[2*c]) + ((f^2*x^3*Cosh[c])/(4*a) + (f^2*x^3*Sinh[c])/(4*a))/(1 + Cosh[2*
c] + Sinh[2*c]) - ((I/8)*(e^2 + 2*e*f*x + f^2*x^2))/(a*d*(Cosh[c/2 + (d*x)/
2] - I*Sinh[c/2 + (d*x)/2])^2) + ((I/2)*(e*f*Sinh[(d*x)/2] + f^2*x*Sinh[(d*
x)/2]))/(a*d^2*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] - I*Sinh[c/2
+ (d*x)/2])) + ((I/8)*(e^2 + 2*e*f*x + f^2*x^2))/(a*d*(Cosh[c/2 + (d*x)/2]
+ I*Sinh[c/2 + (d*x)/2])^4) - ((I/6)*(e*f*Sinh[(d*x)/2] + f^2*x*Sinh[(d*x)/
2]))/(a*d^2*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (
d*x)/2])^3) + ((3*I)*d^2*e^2*Cosh[c/2] + d*e*f*Cosh[c/2] - I*f^2*Cosh[c/2]
+ (6*I)*d^2*e*f*x*Cosh[c/2] + d*f^2*x*Cosh[c/2] + (3*I)*d^2*f^2*x^2*Cosh[c/
2] - 3*d^2*e^2*Sinh[c/2] - I*d*e*f*Sinh[c/2] + f^2*Sinh[c/2] - 6*d^2*e*f*x*
Sinh[c/2] - I*d*f^2*x*Sinh[c/2] - 3*d^2*f^2*x^2*Sinh[c/2])/(12*a*d^3*(Cosh[
c/2] + I*Sinh[c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2])^2) - (((7
*I)/6)*(e*f*Sinh[(d*x)/2] + f^2*x*Sinh[(d*x)/2]))/(a*d^2*(Cosh[c/2] + I*Sin
h[c/2])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2]))
```

Maple [B] time = 0.208, size = 1044, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 1/12*(-2*f^2*exp(5*d*x+5*c)-4*f^2*exp(3*d*x+3*c)-2*f^2*exp(d*x+c)-44*I*d*e*
f*exp(2*d*x+2*c)+12*d^2*e*f*x*exp(3*d*x+3*c)+18*d^2*e*f*x*exp(5*d*x+5*c)+6*
d^2*f^2*x^2*exp(3*d*x+3*c)+9*d^2*f^2*x^2*exp(5*d*x+5*c)+18*d*f^2*x*exp(5*d*
x+5*c)+9*d^2*x^2*f^2*exp(d*x+c)-2*d*f^2*x*exp(d*x+c)-2*d*e*f*exp(d*x+c)+16*
d*f^2*x*exp(3*d*x+3*c)+16*d*e*f*exp(3*d*x+3*c)-36*I*d*e*f*exp(4*d*x+4*c)-18
*I*d^2*f^2*x^2*exp(4*d*x+4*c)+18*I*d^2*f^2*x^2*exp(2*d*x+2*c)-44*I*d*f^2*x*
exp(2*d*x+2*c)+36*I*d^2*e*f*x*exp(2*d*x+2*c)-36*I*d^2*e*f*x*exp(4*d*x+4*c)-
8*I*d*f*e-8*I*d*f^2*x+6*d^2*e^2*exp(3*d*x+3*c)+9*d^2*e^2*exp(5*d*x+5*c)+18*
d^2*e*f*x*exp(d*x+c)-36*I*d*f^2*x*exp(4*d*x+4*c)+9*d^2*e^2*exp(d*x+c)+18*d*
e*f*exp(5*d*x+5*c)-18*I*d^2*e^2*exp(4*d*x+4*c)+18*I*d^2*e^2*exp(2*d*x+2*c))
/(exp(d*x+c)+I)^2/(exp(d*x+c)-I)^4/d^3/a-3/4*I/a/d^2*e*f*c*ln(exp(d*x+c)+I)
-3/8*I/a/d*ln(1+I*exp(d*x+c))*f^2*x^2-3/4*I/a/d^2*polylog(2,-I*exp(d*x+c))*
f^2*x+3/4*I/a/d^2*e*f*c*ln(exp(d*x+c)-I)+3/8*I/a/d^3*ln(1+I*exp(d*x+c))*c^2
*f^2+3/8*I/a/d*ln(1-I*exp(d*x+c))*f^2*x^2-3/8*I/a/d*e^2*ln(exp(d*x+c)-I)-3/
4*I/a/d*ln(1+I*exp(d*x+c))*e*f*x+3/4*I/a/d^2*polylog(2,I*exp(d*x+c))*f^2*x-
3/8*I/a/d^3*c^2*f^2*ln(exp(d*x+c)-I)-3/4*I*f^2*polylog(3,I*exp(d*x+c))/a/d^
3+3/4*I/a/d^2*ln(1-I*exp(d*x+c))*c*e*f-3/4*I/a/d^2*ln(1+I*exp(d*x+c))*c*e*f
+3/4*I*f^2*polylog(3,-I*exp(d*x+c))/a/d^3+3/4*I/a/d^2*e*f*polylog(2,I*exp(d
*x+c))-2/3*I/a/d^3*f^2*ln(exp(d*x+c))-3/4*I/a/d^2*e*f*polylog(2,-I*exp(d*x+
c))-3/8*I/a/d^3*ln(1-I*exp(d*x+c))*c^2*f^2+3/8*I/a/d*e^2*ln(exp(d*x+c)+I)+7
/6*I/a/d^3*f^2*ln(exp(d*x+c)-I)+3/8*I/a/d^3*c^2*f^2*ln(exp(d*x+c)+I)-1/2*I/
a/d^3*f^2*ln(exp(d*x+c)+I)+3/4*I/a/d*ln(1-I*exp(d*x+c))*e*f*x
```

Maxima [B] time = 3.73354, size = 1081, normalized size = 2.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/8*e^2*(64*(3*e^(-d*x - c) - 6*I*e^(-2*d*x - 2*c) + 2*e^(-3*d*x - 3*c) +
6*I*e^(-4*d*x - 4*c) + 3*e^(-5*d*x - 5*c))/((64*I*a*e^(-d*x - c) - 32*a*e^(
-2*d*x - 2*c) + 128*I*a*e^(-3*d*x - 3*c) + 32*a*e^(-4*d*x - 4*c) + 64*I*a*e
^(-5*d*x - 5*c) + 32*a*e^(-6*d*x - 6*c) - 32*a)*d) + 3*I*log(e^(-d*x - c) +
 I)/(a*d) - 3*I*log(e^(-d*x - c) - I)/(a*d)) + (-8*I*d*f^2*x - 8*I*d*e*f +
(9*d^2*f^2*x^2*e^(5*c) + 18*(d^2*e*f + d*f^2)*x*e^(5*c) + 2*(9*d*e*f - f^2)
*e^(5*c))*e^(5*d*x) + (-18*I*d^2*f^2*x^2*e^(4*c) - 36*I*d*e*f*e^(4*c) + (-3
6*I*d^2*e*f - 36*I*d*f^2)*x*e^(4*c))*e^(4*d*x) + 2*(3*d^2*f^2*x^2*e^(3*c) +
 2*(3*d^2*e*f + 4*d*f^2)*x*e^(3*c) + 2*(4*d*e*f - f^2)*e^(3*c))*e^(3*d*x) +
 (18*I*d^2*f^2*x^2*e^(2*c) - 44*I*d*e*f*e^(2*c) + (36*I*d^2*e*f - 44*I*d*f^
2)*x*e^(2*c))*e^(2*d*x) + (9*d^2*f^2*x^2*e^c + 2*(9*d^2*e*f - d*f^2)*x*e^c
- 2*(d*e*f + f^2)*e^c)*e^(d*x))/(12*a*d^3*e^(6*d*x + 6*c) - 24*I*a*d^3*e^(5
*d*x + 5*c) + 12*a*d^3*e^(4*d*x + 4*c) - 48*I*a*d^3*e^(3*d*x + 3*c) - 12*a*
d^3*e^(2*d*x + 2*c) - 24*I*a*d^3*e^(d*x + c) - 12*a*d^3) - 3/4*I*(d*x*log(I
*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*e*f/(a*d^2) + 3/4*I*(d*x*log(-I*
e^(d*x + c) + 1) + dilog(I*e^(d*x + c)))*e*f/(a*d^2) - 2/3*I*f^2*x/(a*d^2)
- 3/8*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x + c)) - 2*p
olylog(3, -I*e^(d*x + c)))*f^2/(a*d^3) + 3/8*I*(d^2*x^2*log(-I*e^(d*x + c)
+ 1) + 2*d*x*dilog(I*e^(d*x + c)) - 2*polylog(3, I*e^(d*x + c)))*f^2/(a*d^3
) + 7/6*I*f^2*log(I*e^(d*x + c) + 1)/(a*d^3) - 1/2*I*f^2*log(I*e^(d*x + c)
- 1)/(a*d^3)
```

Fricas [C] time = 2.77009, size = 5145, normalized size = 12.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] (-16*I*d*e*f + 16*I*c*f^2 + (-18*I*d*f^2*x - 18*I*d*e*f + (18*I*d*f^2*x + 1
8*I*d*e*f)*e^(6*d*x + 6*c) + 36*(d*f^2*x + d*e*f)*e^(5*d*x + 5*c) + (18*I*d
*f^2*x + 18*I*d*e*f)*e^(4*d*x + 4*c) + 72*(d*f^2*x + d*e*f)*e^(3*d*x + 3*c)
 + (-18*I*d*f^2*x - 18*I*d*e*f)*e^(2*d*x + 2*c) + 36*(d*f^2*x + d*e*f)*e^(d
*x + c))*dilog(I*e^(d*x + c)) + (18*I*d*f^2*x + 18*I*d*e*f + (-18*I*d*f^2*x
 - 18*I*d*e*f)*e^(6*d*x + 6*c) - 36*(d*f^2*x + d*e*f)*e^(5*d*x + 5*c) + (-1
8*I*d*f^2*x - 18*I*d*e*f)*e^(4*d*x + 4*c) - 72*(d*f^2*x + d*e*f)*e^(3*d*x +
 3*c) + (18*I*d*f^2*x + 18*I*d*e*f)*e^(2*d*x + 2*c) - 36*(d*f^2*x + d*e*f)*
e^(d*x + c))*dilog(-I*e^(d*x + c)) + (-16*I*d*f^2*x - 16*I*c*f^2)*e^(6*d*x
+ 6*c) + 2*(9*d^2*f^2*x^2 + 9*d^2*e^2 + 18*d*e*f - 2*(8*c + 1)*f^2 + 2*(9*d
^2*e*f + d*f^2)*x)*e^(5*d*x + 5*c) + (-36*I*d^2*f^2*x^2 - 36*I*d^2*e^2 - 72
*I*d*e*f - 16*I*c*f^2 + (-72*I*d^2*e*f - 88*I*d*f^2)*x)*e^(4*d*x + 4*c) + 4
*(3*d^2*f^2*x^2 + 3*d^2*e^2 + 8*d*e*f - 2*(8*c + 1)*f^2 + 2*(3*d^2*e*f - 4*
d*f^2)*x)*e^(3*d*x + 3*c) + (36*I*d^2*f^2*x^2 + 36*I*d^2*e^2 - 88*I*d*e*f +
 16*I*c*f^2 + (72*I*d^2*e*f - 72*I*d*f^2)*x)*e^(2*d*x + 2*c) + 2*(9*d^2*f^2
*x^2 + 9*d^2*e^2 - 2*d*e*f - 2*(8*c + 1)*f^2 + 18*(d^2*e*f - d*f^2)*x)*e^(d
*x + c) + (-9*I*d^2*e^2 + 18*I*c*d*e*f + (-9*I*c^2 + 12*I)*f^2 + (9*I*d^2*e
^2 - 18*I*c*d*e*f + (9*I*c^2 - 12*I)*f^2)*e^(6*d*x + 6*c) + 6*(3*d^2*e^2 -
6*c*d*e*f + (3*c^2 - 4)*f^2)*e^(5*d*x + 5*c) + (9*I*d^2*e^2 - 18*I*c*d*e*f
+ (9*I*c^2 - 12*I)*f^2)*e^(4*d*x + 4*c) + 12*(3*d^2*e^2 - 6*c*d*e*f + (3*c^
2 - 4)*f^2)*e^(3*d*x + 3*c) + (-9*I*d^2*e^2 + 18*I*c*d*e*f + (-9*I*c^2 + 12
*I)*f^2)*e^(2*d*x + 2*c) + 6*(3*d^2*e^2 - 6*c*d*e*f + (3*c^2 - 4)*f^2)*e^(d
*x + c))*log(e^(d*x + c) + I) + (9*I*d^2*e^2 - 18*I*c*d*e*f + (9*I*c^2 - 28
*I)*f^2 + (-9*I*d^2*e^2 + 18*I*c*d*e*f + (-9*I*c^2 + 28*I)*f^2)*e^(6*d*x +
6*c) - 2*(9*d^2*e^2 - 18*c*d*e*f + (9*c^2 - 28)*f^2)*e^(5*d*x + 5*c) + (-9*
I*d^2*e^2 + 18*I*c*d*e*f + (-9*I*c^2 + 28*I)*f^2)*e^(4*d*x + 4*c) - 4*(9*d^
```

```
2*e^2 - 18*c*d*e*f + (9*c^2 - 28)*f^2)*e^(3*d*x + 3*c) + (9*I*d^2*e^2 - 18*
I*c*d*e*f + (9*I*c^2 - 28*I)*f^2)*e^(2*d*x + 2*c) - 2*(9*d^2*e^2 - 18*c*d*e
*f + (9*c^2 - 28)*f^2)*e^(d*x + c))*log(e^(d*x + c) - I) + (9*I*d^2*f^2*x^2
 + 18*I*d^2*e*f*x + 18*I*c*d*e*f - 9*I*c^2*f^2 + (-9*I*d^2*f^2*x^2 - 18*I*d
^2*e*f*x - 18*I*c*d*e*f + 9*I*c^2*f^2)*e^(6*d*x + 6*c) - 18*(d^2*f^2*x^2 +
2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(5*d*x + 5*c) + (-9*I*d^2*f^2*x^2 - 18
*I*d^2*e*f*x - 18*I*c*d*e*f + 9*I*c^2*f^2)*e^(4*d*x + 4*c) - 36*(d^2*f^2*x^
2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(3*d*x + 3*c) + (9*I*d^2*f^2*x^2 +
 18*I*d^2*e*f*x + 18*I*c*d*e*f - 9*I*c^2*f^2)*e^(2*d*x + 2*c) - 18*(d^2*f^2
*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(d*x + c))*log(I*e^(d*x + c) +
1) + (-9*I*d^2*f^2*x^2 - 18*I*d^2*e*f*x - 18*I*c*d*e*f + 9*I*c^2*f^2 + (9*I
*d^2*f^2*x^2 + 18*I*d^2*e*f*x + 18*I*c*d*e*f - 9*I*c^2*f^2)*e^(6*d*x + 6*c)
 + 18*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(5*d*x + 5*c) + (
9*I*d^2*f^2*x^2 + 18*I*d^2*e*f*x + 18*I*c*d*e*f - 9*I*c^2*f^2)*e^(4*d*x + 4
*c) + 36*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(3*d*x + 3*c)
+ (-9*I*d^2*f^2*x^2 - 18*I*d^2*e*f*x - 18*I*c*d*e*f + 9*I*c^2*f^2)*e^(2*d*x
 + 2*c) + 18*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(d*x + c))
*log(-I*e^(d*x + c) + 1) + (-18*I*f^2*e^(6*d*x + 6*c) - 36*f^2*e^(5*d*x + 5
*c) - 18*I*f^2*e^(4*d*x + 4*c) - 72*f^2*e^(3*d*x + 3*c) + 18*I*f^2*e^(2*d*x
 + 2*c) - 36*f^2*e^(d*x + c) + 18*I*f^2)*polylog(3, I*e^(d*x + c)) + (18*I*
f^2*e^(6*d*x + 6*c) + 36*f^2*e^(5*d*x + 5*c) + 18*I*f^2*e^(4*d*x + 4*c) + 7
2*f^2*e^(3*d*x + 3*c) - 18*I*f^2*e^(2*d*x + 2*c) + 36*f^2*e^(d*x + c) - 18*
I*f^2)*polylog(3, -I*e^(d*x + c)))/(24*a*d^3*e^(6*d*x + 6*c) - 48*I*a*d^3*e
^(5*d*x + 5*c) + 24*a*d^3*e^(4*d*x + 4*c) - 96*I*a*d^3*e^(3*d*x + 3*c) - 24
*a*d^3*e^(2*d*x + 2*c) - 48*I*a*d^3*e^(d*x + c) - 24*a*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sech(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c)^3}{i\,a\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sech(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

3.285 $\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=233

$$-\frac{3ifPolyLog\left(2,-ie^{c+dx}\right)}{8ad^2}+\frac{3ifPolyLog\left(2,ie^{c+dx}\right)}{8ad^2}+\frac{if\tanh^3(c+dx)}{12ad^2}-\frac{if\tanh(c+dx)}{4ad^2}+\frac{f\text{sech}^3(c+dx)}{12ad^2}+\frac{3f\text{sech}}{8a}$$

```
[Out] (3*(e + f*x)*ArcTan[E^(c + d*x)])/(4*a*d) - (((3*I)/8)*f*PolyLog[2, (-I)*E^
(c + d*x)])/(a*d^2) + (((3*I)/8)*f*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + (3*
f*Sech[c + d*x])/(8*a*d^2) + (f*Sech[c + d*x]^3)/(12*a*d^2) + ((I/4)*(e + f
*x)*Sech[c + d*x]^4)/(a*d) - ((I/4)*f*Tanh[c + d*x])/(a*d^2) + (3*(e + f*x)
*Sech[c + d*x]*Tanh[c + d*x])/(8*a*d) + ((e + f*x)*Sech[c + d*x]^3*Tanh[c +
 d*x])/(4*a*d) + ((I/12)*f*Tanh[c + d*x]^3)/(a*d^2)
```

Rubi [A] time = 0.189825, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {5571, 4185, 4180, 2279, 2391, 5451, 3767}

$$-\frac{3ifPolyLog\left(2,-ie^{c+dx}\right)}{8ad^2}+\frac{3ifPolyLog\left(2,ie^{c+dx}\right)}{8ad^2}+\frac{if\tanh^3(c+dx)}{12ad^2}-\frac{if\tanh(c+dx)}{4ad^2}+\frac{f\text{sech}^3(c+dx)}{12ad^2}+\frac{3f\text{sech}}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (3*(e + f*x)*ArcTan[E^(c + d*x)])/(4*a*d) - (((3*I)/8)*f*PolyLog[2, (-I)*E^
(c + d*x)])/(a*d^2) + (((3*I)/8)*f*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + (3*
f*Sech[c + d*x])/(8*a*d^2) + (f*Sech[c + d*x]^3)/(12*a*d^2) + ((I/4)*(e + f
*x)*Sech[c + d*x]^4)/(a*d) - ((I/4)*f*Tanh[c + d*x])/(a*d^2) + (3*(e + f*x)
*Sech[c + d*x]*Tanh[c + d*x])/(8*a*d) + ((e + f*x)*Sech[c + d*x]^3*Tanh[c +
 d*x])/(4*a*d) + ((I/12)*f*Tanh[c + d*x]^3)/(a*d^2)
```

Rule 5571

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sech[c
 + d*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)*T
anh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && Eq
Q[a^2 + b^2, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{i\int (e+fx)\text{sech}^4(c+dx)\tanh(c+dx)\,dx}{a}+\frac{\int (e+fx)\text{sech}^5(c+dx)\,dx}{a}$$
$$= \frac{f\text{sech}^3(c+dx)}{12ad^2}+\frac{i(e+fx)\text{sech}^4(c+dx)}{4ad}+\frac{(e+fx)\text{sech}^3(c+dx)\tanh(c+dx)}{4ad}+\frac{3\int$$
$$= \frac{3f\text{sech}(c+dx)}{8ad^2}+\frac{f\text{sech}^3(c+dx)}{12ad^2}+\frac{i(e+fx)\text{sech}^4(c+dx)}{4ad}+\frac{3(e+fx)\text{sech}(c+dx)}{8ad}$$
$$= \frac{3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{4ad}+\frac{3f\text{sech}(c+dx)}{8ad^2}+\frac{f\text{sech}^3(c+dx)}{12ad^2}+\frac{i(e+fx)\text{sech}^4(c+dx}{4ad}$$
$$= \frac{3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{4ad}+\frac{3f\text{sech}(c+dx)}{8ad^2}+\frac{f\text{sech}^3(c+dx)}{12ad^2}+\frac{i(e+fx)\text{sech}^4(c+dx}{4ad}$$
$$= \frac{3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{4ad}-\frac{3if\text{Li}_2\left(-ie^{c+dx}\right)}{8ad^2}+\frac{3if\text{Li}_2\left(ie^{c+dx}\right)}{8ad^2}+\frac{3f\text{sech}(c+dx)}{8ad^2}+\frac{f\text{se}}{}$$

Mathematica [B] time = 6.66753, size = 1290, normalized size = 5.54

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((I/24)*(6*d*e - I*f - 6*c*f + 6*f*(c + d*x)))/(d^2*(a + I*a*Sinh[c + d*x])
) + ((I/8)*(d*e - c*f + f*(c + d*x)))/(d^2*(Cosh[(c + d*x)/2] + I*Sinh[(c +
 d*x)/2])^2*(a + I*a*Sinh[c + d*x])) + (3*(c + d*x)*(2*d*e - 2*c*f + f*(c +
 d*x))*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2)/(16*d^2*(a + I*a*Sinh[c
 + d*x])) + (((3*I)/8)*e*((I/2)*(c + d*x) + Log[Cosh[(c + d*x)/2] - I*Sinh[
```

```
(c + d*x)/2]])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2)/(d*(a + I*a*Sin
h[c + d*x])) - (((3*I)/8)*c*f*((I/2)*(c + d*x) + Log[Cosh[(c + d*x)/2] - I*
Sinh[(c + d*x)/2]])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2)/(d^2*(a +
I*a*Sinh[c + d*x])) - (((3*I)/8)*e*((-I/2)*(c + d*x) + Log[Cosh[(c + d*x)/2
] + I*Sinh[(c + d*x)/2]])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2)/(d*(
a + I*a*Sinh[c + d*x])) + (((3*I)/8)*c*f*((-I/2)*(c + d*x) + Log[Cosh[(c +
d*x)/2] + I*Sinh[(c + d*x)/2]])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2
)/(d^2*(a + I*a*Sinh[c + d*x])) + (3*f*(-(c + d*x)^2/(4*E^((I/4)*Pi)) - ((3
*Pi*(c + d*x))/4 - Pi*Log[1 + E^(c + d*x)] - 2*(-Pi/4 + (I/2)*(c + d*x))*Lo
g[1 - E^((2*I)*(-Pi/4 + (I/2)*(c + d*x)))] + Pi*Log[Cosh[(c + d*x)/2]] - (P
i*Log[-Sin[Pi/4 - (I/2)*(c + d*x)]])/2 + I*PolyLog[2, E^((2*I)*(-Pi/4 + (I/
2)*(c + d*x)))])/Sqrt[2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2)/(4*S
qrt[2]*d^2*(a + I*a*Sinh[c + d*x])) + (3*f*(-(E^((I/4)*Pi)*(c + d*x)^2)/4 +
 ((Pi*(c + d*x))/4 - Pi*Log[1 + E^(c + d*x)] - 2*(Pi/4 + (I/2)*(c + d*x))*L
og[1 - E^((2*I)*(Pi/4 + (I/2)*(c + d*x)))] + Pi*Log[Cosh[(c + d*x)/2]] + (P
i*Log[Sin[Pi/4 + (I/2)*(c + d*x)]])/2 + I*PolyLog[2, E^((2*I)*(Pi/4 + (I/2)
*(c + d*x)))])/Sqrt[2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2)/(4*Sqr
t[2]*d^2*(a + I*a*Sinh[c + d*x])) - ((I/8)*(d*e - c*f + f*(c + d*x))*(Cosh[
(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2)/(d^2*(Cosh[(c + d*x)/2] - I*Sinh[(c
+ d*x)/2])^2*(a + I*a*Sinh[c + d*x])) - ((I/12)*f*Sinh[(c + d*x)/2])/(d^2*(
Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(a + I*a*Sinh[c + d*x])) - (((7*I)
/12)*f*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*Sinh[(c + d*x)/2])/(d^2*(a
 + I*a*Sinh[c + d*x])) + ((I/4)*f*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])
^2*Sinh[(c + d*x)/2])/(d^2*(Cosh[(c + d*x)/2] - I*Sinh[(c + d*x)/2])*(a + I
*a*Sinh[c + d*x]))
```

Maple [B] time = 0.213, size = 445, normalized size = 1.9

$$\frac{9\,f e^{5\,dx+5\,c}+8\,f e^{3\,dx+3\,c}-f e^{dx+c}-18\,i f e^{4\,dx+4\,c}-18\,i d f x e^{4\,dx+4\,c}-18\,i d e e^{4\,dx+4\,c}-22\,i f e^{2\,dx+2\,c}+6\,d e e^{3\,dx+3\,c}+9\,d f x e^{5}}{12\,\left(e^{dx+c}+i\right)^{2}\left(e^{dx+c}-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 1/12*(9*f*exp(5*d*x+5*c)+8*f*exp(3*d*x+3*c)-f*exp(d*x+c)-18*I*f*exp(4*d*x+4
*c)-18*I*d*f*x*exp(4*d*x+4*c)-18*I*d*e*exp(4*d*x+4*c)-22*I*f*exp(2*d*x+2*c)
+6*d*e*exp(3*d*x+3*c)+9*d*f*x*exp(5*d*x+5*c)+6*d*f*x*exp(3*d*x+3*c)+18*I*d*
f*x*exp(2*d*x+2*c)-4*I*f+9*d*f*x*exp(d*x+c)+18*I*d*e*exp(2*d*x+2*c)+9*d*e*e
xp(5*d*x+5*c)+9*d*e*exp(d*x+c))/(exp(d*x+c)+I)^2/(exp(d*x+c)-I)^4/d^2/a-3/8
*I/a/d*e*ln(exp(d*x+c)-I)+3/8*I/a/d*e*ln(exp(d*x+c)+I)-3/8*I/a/d*f*ln(1+I*e
xp(d*x+c))*x-3/8*I/a/d^2*f*ln(1+I*exp(d*x+c))*c-3/8*I*f*polylog(2,-I*exp(d*
x+c))/a/d^2+3/8*I/a/d*f*ln(1-I*exp(d*x+c))*x+3/8*I/a/d^2*f*ln(1-I*exp(d*x+c
))*c+3/8*I*f*polylog(2,I*exp(d*x+c))/a/d^2+3/8*I/a/d^2*f*c*ln(exp(d*x+c)-I)
-3/8*I/a/d^2*f*c*ln(exp(d*x+c)+I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$8\,f\left(\frac{9\left(dxe^{(5\,c)}+e^{(5\,c)}\right)e^{(5\,dx)}+\left(-18i\,dxe^{(4\,c)}-18i\,e^{(4\,c)}\right)e^{(4\,dx)}+2\left(3\,dxe^{(3\,c)}+4\,e^{(3\,c)}\right)e^{(3\,dx)}+\left(18i\,dxe^{(2\,c)}-22i\,e^{(2\,c)}\right)e^{(2\,dx)}+}{96\,ad^{2}e^{(6\,dx+6\,c)}-192i\,ad^{2}e^{(5\,dx+5\,c)}+96\,ad^{2}e^{(4\,dx+4\,c)}-384i\,ad^{2}e^{(3\,dx+3\,c)}-96\,ad^{2}e^{(2\,dx+2\,c)}-192i\,ad^{2}e^{(dx+c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 8*f*((9*(d*x*e^(5*c) + e^(5*c))*e^(5*d*x) + (-18*I*d*x*e^(4*c) - 18*I*e^(4*
c))*e^(4*d*x) + 2*(3*d*x*e^(3*c) + 4*e^(3*c))*e^(3*d*x) + (18*I*d*x*e^(2*c)
 - 22*I*e^(2*c))*e^(2*d*x) + (9*d*x*e^c - e^c)*e^(d*x) - 4*I)/(96*a*d^2*e^(
6*d*x + 6*c) - 192*I*a*d^2*e^(5*d*x + 5*c) + 96*a*d^2*e^(4*d*x + 4*c) - 384
*I*a*d^2*e^(3*d*x + 3*c) - 96*a*d^2*e^(2*d*x + 2*c) - 192*I*a*d^2*e^(d*x +
c) - 96*a*d^2) + 3*integrate(1/64*x/(a*e^(d*x + c) + I*a), x) + 3*integrate
(1/64*x/(a*e^(d*x + c) - I*a), x)) - 1/8*e*(64*(3*e^(-d*x - c) - 6*I*e^(-2*
d*x - 2*c) + 2*e^(-3*d*x - 3*c) + 6*I*e^(-4*d*x - 4*c) + 3*e^(-5*d*x - 5*c)
)/((64*I*a*e^(-d*x - c) - 32*a*e^(-2*d*x - 2*c) + 128*I*a*e^(-3*d*x - 3*c)
+ 32*a*e^(-4*d*x - 4*c) + 64*I*a*e^(-5*d*x - 5*c) + 32*a*e^(-6*d*x - 6*c) -
 32*a)*d) + 3*I*log(e^(-d*x - c) + I)/(a*d) - 3*I*log(e^(-d*x - c) - I)/(a*
d))
```

Fricas [B] time = 2.44546, size = 2423, normalized size = 10.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((9*I*f*e^(6*d*x + 6*c) + 18*f*e^(5*d*x + 5*c) + 9*I*f*e^(4*d*x + 4*c) + 36
*f*e^(3*d*x + 3*c) - 9*I*f*e^(2*d*x + 2*c) + 18*f*e^(d*x + c) - 9*I*f)*dilo
g(I*e^(d*x + c)) + (-9*I*f*e^(6*d*x + 6*c) - 18*f*e^(5*d*x + 5*c) - 9*I*f*e
^(4*d*x + 4*c) - 36*f*e^(3*d*x + 3*c) + 9*I*f*e^(2*d*x + 2*c) - 18*f*e^(d*x
 + c) + 9*I*f)*dilog(-I*e^(d*x + c)) + 18*(d*f*x + d*e + f)*e^(5*d*x + 5*c)
 + (-36*I*d*f*x - 36*I*d*e - 36*I*f)*e^(4*d*x + 4*c) + 4*(3*d*f*x + 3*d*e +
 4*f)*e^(3*d*x + 3*c) + (36*I*d*f*x + 36*I*d*e - 44*I*f)*e^(2*d*x + 2*c) +
2*(9*d*f*x + 9*d*e - f)*e^(d*x + c) + (-9*I*d*e + 9*I*c*f + (9*I*d*e - 9*I*
c*f)*e^(6*d*x + 6*c) + 18*(d*e - c*f)*e^(5*d*x + 5*c) + (9*I*d*e - 9*I*c*f)
*e^(4*d*x + 4*c) + 36*(d*e - c*f)*e^(3*d*x + 3*c) + (-9*I*d*e + 9*I*c*f)*e^
(2*d*x + 2*c) + 18*(d*e - c*f)*e^(d*x + c))*log(e^(d*x + c) + I) + (9*I*d*e
 - 9*I*c*f + (-9*I*d*e + 9*I*c*f)*e^(6*d*x + 6*c) - 18*(d*e - c*f)*e^(5*d*x
 + 5*c) + (-9*I*d*e + 9*I*c*f)*e^(4*d*x + 4*c) - 36*(d*e - c*f)*e^(3*d*x +
3*c) + (9*I*d*e - 9*I*c*f)*e^(2*d*x + 2*c) - 18*(d*e - c*f)*e^(d*x + c))*lo
g(e^(d*x + c) - I) + (9*I*d*f*x + 9*I*c*f + (-9*I*d*f*x - 9*I*c*f)*e^(6*d*x
 + 6*c) - 18*(d*f*x + c*f)*e^(5*d*x + 5*c) + (-9*I*d*f*x - 9*I*c*f)*e^(4*d*
x + 4*c) - 36*(d*f*x + c*f)*e^(3*d*x + 3*c) + (9*I*d*f*x + 9*I*c*f)*e^(2*d*
x + 2*c) - 18*(d*f*x + c*f)*e^(d*x + c))*log(I*e^(d*x + c) + 1) + (-9*I*d*f
*x - 9*I*c*f + (9*I*d*f*x + 9*I*c*f)*e^(6*d*x + 6*c) + 18*(d*f*x + c*f)*e^(
5*d*x + 5*c) + (9*I*d*f*x + 9*I*c*f)*e^(4*d*x + 4*c) + 36*(d*f*x + c*f)*e^(
3*d*x + 3*c) + (-9*I*d*f*x - 9*I*c*f)*e^(2*d*x + 2*c) + 18*(d*f*x + c*f)*e^
(d*x + c))*log(-I*e^(d*x + c) + 1) - 8*I*f)/(24*a*d^2*e^(6*d*x + 6*c) - 48*
I*a*d^2*e^(5*d*x + 5*c) + 24*a*d^2*e^(4*d*x + 4*c) - 96*I*a*d^2*e^(3*d*x +
3*c) - 24*a*d^2*e^(2*d*x + 2*c) - 48*I*a*d^2*e^(d*x + c) - 24*a*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\operatorname{sech}(dx+c)^3}{i\,a\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sech(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

3.286 $\int \frac{\text{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{ia}{8d(a+ia\sinh(c+dx))^2}-\frac{i}{8d(a-ia\sinh(c+dx))}+\frac{i}{4d(a+ia\sinh(c+dx))}+\frac{3\tan^{-1}(\sinh(c+dx))}{8ad}$$

```
[Out] (3*ArcTan[Sinh[c + d*x]])/(8*a*d) - (I/8)/(d*(a - I*a*Sinh[c + d*x])) + ((I
/8)*a)/(d*(a + I*a*Sinh[c + d*x])^2) + (I/4)/(d*(a + I*a*Sinh[c + d*x]))
```

Rubi [A] time = 0.0811465, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2667, 44, 206}

$$\frac{ia}{8d(a+ia\sinh(c+dx))^2}-\frac{i}{8d(a-ia\sinh(c+dx))}+\frac{i}{4d(a+ia\sinh(c+dx))}+\frac{3\tan^{-1}(\sinh(c+dx))}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (3*ArcTan[Sinh[c + d*x]])/(8*a*d) - (I/8)/(d*(a - I*a*Sinh[c + d*x])) + ((I
/8)*a)/(d*(a + I*a*Sinh[c + d*x])^2) + (I/4)/(d*(a + I*a*Sinh[c + d*x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)}\,dx = -\frac{(ia^3)\operatorname{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^3}\,dx, x, ia\sinh(c+dx)\right)}{d}$$

$$= -\frac{(ia^3)\operatorname{Subst}\left(\int\left(\frac{1}{8a^3(a-x)^2}+\frac{1}{4a^2(a+x)^3}+\frac{1}{4a^3(a+x)^2}+\frac{3}{8a^3(a^2-x^2)}\right)dx, x, ia\sinh(c+dx)\right)}{d}$$

$$= -\frac{i}{8d(a-ia\sinh(c+dx))}+\frac{ia}{8d(a+ia\sinh(c+dx))^2}+\frac{i}{4d(a+ia\sinh(c+dx))}-\frac{(3i)\operatorname{Subst}}{}$$

$$= \frac{3\tan^{-1}(\sinh(c+dx))}{8ad}-\frac{i}{8d(a-ia\sinh(c+dx))}+\frac{ia}{8d(a+ia\sinh(c+dx))^2}+\frac{i}{4d(a+ia\sinh}$$

Mathematica [A] time = 0.10488, size = 101, normalized size = 1.11

$$\frac{\operatorname{sech}^2(c+dx)\left(3\sinh^3(c+dx)\tan^{-1}(\sinh(c+dx))+\sinh^2(c+dx)\left(3-3i\tan^{-1}(\sinh(c+dx))\right)+3\sinh(c+dx)\left(\tan^{-1}\right.\right.}{8ad(\sinh(c+dx)-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (Sech[c + d*x]^2*(2 - (3*I)*ArcTan[Sinh[c + d*x]] + 3*(-I + ArcTan[Sinh[c +
 d*x]])*Sinh[c + d*x] + (3 - (3*I)*ArcTan[Sinh[c + d*x]])*Sinh[c + d*x]^2 +
 3*ArcTan[Sinh[c + d*x]]*Sinh[c + d*x]^3))/(8*a*d*(-I + Sinh[c + d*x]))
```

Maple [B] time = 0.056, size = 180, normalized size = 2.

$$\frac{\frac{i}{4}}{da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{-2}+\frac{\frac{3i}{8}}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)-\frac{1}{4da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{-1}+\frac{\frac{i}{2}}{da}\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 1/4*I/d/a/(tanh(1/2*d*x+1/2*c)+I)^2+3/8*I/d/a*ln(tanh(1/2*d*x+1/2*c)+I)-1/4
/d/a/(tanh(1/2*d*x+1/2*c)+I)+1/2*I/d/a/(-I+tanh(1/2*d*x+1/2*c))^4-3/8*I/d/a
*ln(-I+tanh(1/2*d*x+1/2*c))-3/2*I/d/a/(-I+tanh(1/2*d*x+1/2*c))^2+1/d/a/(-I+
tanh(1/2*d*x+1/2*c))^3-1/d/a/(-I+tanh(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.14857, size = 243, normalized size = 2.67

$$-\frac{8\left(3e^{(-dx-c)}-6ie^{(-2dx-2c)}+2e^{(-3dx-3c)}+6ie^{(-4dx-4c)}+3e^{(-5dx-5c)}\right)}{\left(64iae^{(-dx-c)}-32ae^{(-2dx-2c)}+128iae^{(-3dx-3c)}+32ae^{(-4dx-4c)}+64iae^{(-5dx-5c)}+32ae^{(-6dx-6c)}-32a\right)d}-\frac{3i\log\left(e^{(-}\right.}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -8*(3*e^(-d*x - c) - 6*I*e^(-2*d*x - 2*c) + 2*e^(-3*d*x - 3*c) + 6*I*e^(-4*
d*x - 4*c) + 3*e^(-5*d*x - 5*c))/((64*I*a*e^(-d*x - c) - 32*a*e^(-2*d*x - 2
```

```
*c) + 128*I*a*e^(-3*d*x - 3*c) + 32*a*e^(-4*d*x - 4*c) + 64*I*a*e^(-5*d*x -
 5*c) + 32*a*e^(-6*d*x - 6*c) - 32*a)*d) - 3/8*I*log(e^(-d*x - c) + I)/(a*d
) + 3/8*I*log(e^(-d*x - c) - I)/(a*d)
```

Fricas [B] time = 2.10379, size = 763, normalized size = 8.38

$$\frac{\left(3i\,e^{(6\,dx+6\,c)}+6\,e^{(5\,dx+5\,c)}+3i\,e^{(4\,dx+4\,c)}+12\,e^{(3\,dx+3\,c)}-3i\,e^{(2\,dx+2\,c)}+6\,e^{(dx+c)}-3i\right)\log\left(e^{(dx+c)}+i\right)+\left(-3i\,e^{(6\,dx+6\,c)}-6\right.}{8\,ade^{(6\,dx+6\,c)}-16i\,ade^{(5\,dx+5\,c)}+8\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((3*I*e^(6*d*x + 6*c) + 6*e^(5*d*x + 5*c) + 3*I*e^(4*d*x + 4*c) + 12*e^(3*d
*x + 3*c) - 3*I*e^(2*d*x + 2*c) + 6*e^(d*x + c) - 3*I)*log(e^(d*x + c) + I)
 + (-3*I*e^(6*d*x + 6*c) - 6*e^(5*d*x + 5*c) - 3*I*e^(4*d*x + 4*c) - 12*e^(
3*d*x + 3*c) + 3*I*e^(2*d*x + 2*c) - 6*e^(d*x + c) + 3*I)*log(e^(d*x + c) -
 I) + 6*e^(5*d*x + 5*c) - 12*I*e^(4*d*x + 4*c) + 4*e^(3*d*x + 3*c) + 12*I*e
^(2*d*x + 2*c) + 6*e^(d*x + c))/(8*a*d*e^(6*d*x + 6*c) - 16*I*a*d*e^(5*d*x
+ 5*c) + 8*a*d*e^(4*d*x + 4*c) - 32*I*a*d*e^(3*d*x + 3*c) - 8*a*d*e^(2*d*x
+ 2*c) - 16*I*a*d*e^(d*x + c) - 8*a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{sech}^3(c+dx)}{i\sinh(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Integral(sech(c + d*x)**3/(I*sinh(c + d*x) + 1), x)/a
```

Giac [B] time = 1.19123, size = 250, normalized size = 2.75

$$\frac{3i\log\left(-i\,e^{(dx+c)}+i\,e^{(-dx-c)}+2\right)}{16\,ad}-\frac{3i\log\left(-i\,e^{(dx+c)}+i\,e^{(-dx-c)}-2\right)}{16\,ad}+\frac{3\,e^{(dx+c)}-3\,e^{(-dx-c)}+10i}{16\,ad\left(i\,e^{(dx+c)}-i\,e^{(-dx-c)}-2\right)}-\frac{-9i\left(e^{(dx+c)}-e\right.}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/16*I*log(-I*e^(d*x + c) + I*e^(-d*x - c) + 2)/(a*d) - 3/16*I*log(-I*e^(d*
x + c) + I*e^(-d*x - c) - 2)/(a*d) + 1/16*(3*e^(d*x + c) - 3*e^(-d*x - c) +
 10*I)/(a*d*(I*e^(d*x + c) - I*e^(-d*x - c) - 2)) - 1/32*(-9*I*(e^(d*x + c)
 - e^(-d*x - c))^2 - 52*e^(d*x + c) + 52*e^(-d*x - c) + 84*I)/(a*d*(e^(d*x
+ c) - e^(-d*x - c) - 2*I)^2)
```

3.287 $\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))} dx$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Sech[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0796316, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

```
[In] Int[Sech[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Sech[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))} dx = \int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))} dx$$

Mathematica [A] time = 115.857, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Sech[c + d*x]^3/((e + f*x)*(a + I*a*Sinh[c + d*x])), x]
```

Maple [A] time = 1.718, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{sech}(dx+c))^3}{(fx+e)(a+ia\sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -8*(4*I*d^2*f^3*x^2 + 8*I*d^2*e*f^2*x + 4*I*d^2*e^2*f - 6*I*f^3 + (9*d^3*f^
3*x^3*e^(5*c) + 9*(3*d^3*e*f^2 - d^2*f^3)*x^2*e^(5*c) + (27*d^3*e^2*f - 18*
d^2*e*f^2 - 2*d*f^3)*x*e^(5*c) + (9*d^3*e^3 - 9*d^2*e^2*f - 2*d*e*f^2 + 6*f
^3)*e^(5*c))*e^(5*d*x) + (-18*I*d^3*f^3*x^3*e^(4*c) + (-54*I*d^3*e*f^2 + 18
*I*d^2*f^3)*x^2*e^(4*c) + (-54*I*d^3*e^2*f + 36*I*d^2*e*f^2)*x*e^(4*c) + (-
18*I*d^3*e^3 + 18*I*d^2*e^2*f - 6*I*f^3)*e^(4*c))*e^(4*d*x) + 2*(3*d^3*f^3*
x^3*e^(3*c) + (9*d^3*e*f^2 - 4*d^2*f^3)*x^2*e^(3*c) + (9*d^3*e^2*f - 8*d^2*
e*f^2 - 2*d*f^3)*x*e^(3*c) + (3*d^3*e^3 - 4*d^2*e^2*f - 2*d*e*f^2 + 6*f^3)*
e^(3*c))*e^(3*d*x) + (18*I*d^3*f^3*x^3*e^(2*c) + (54*I*d^3*e*f^2 + 22*I*d^2
*f^3)*x^2*e^(2*c) + (54*I*d^3*e^2*f + 44*I*d^2*e*f^2)*x*e^(2*c) + (18*I*d^3
*e^3 + 22*I*d^2*e^2*f - 12*I*f^3)*e^(2*c))*e^(2*d*x) + (9*d^3*f^3*x^3*e^c +
 (27*d^3*e*f^2 + d^2*f^3)*x^2*e^c + (27*d^3*e^2*f + 2*d^2*e*f^2 - 2*d*f^3)*
x*e^c + (9*d^3*e^3 + d^2*e^2*f - 2*d*e*f^2 + 6*f^3)*e^c)*e^(d*x))/(96*a*d^4
*f^4*x^4 + 384*a*d^4*e*f^3*x^3 + 576*a*d^4*e^2*f^2*x^2 + 384*a*d^4*e^3*f*x
+ 96*a*d^4*e^4 - 96*(a*d^4*f^4*x^4*e^(6*c) + 4*a*d^4*e*f^3*x^3*e^(6*c) + 6*
a*d^4*e^2*f^2*x^2*e^(6*c) + 4*a*d^4*e^3*f*x*e^(6*c) + a*d^4*e^4*e^(6*c))*e^
(6*d*x) - (-192*I*a*d^4*f^4*x^4*e^(5*c) - 768*I*a*d^4*e*f^3*x^3*e^(5*c) - 1
152*I*a*d^4*e^2*f^2*x^2*e^(5*c) - 768*I*a*d^4*e^3*f*x*e^(5*c) - 192*I*a*d^4
*e^4*e^(5*c))*e^(5*d*x) - 96*(a*d^4*f^4*x^4*e^(4*c) + 4*a*d^4*e*f^3*x^3*e^(
4*c) + 6*a*d^4*e^2*f^2*x^2*e^(4*c) + 4*a*d^4*e^3*f*x*e^(4*c) + a*d^4*e^4*e^
(4*c))*e^(4*d*x) - (-384*I*a*d^4*f^4*x^4*e^(3*c) - 1536*I*a*d^4*e*f^3*x^3*e
^(3*c) - 2304*I*a*d^4*e^2*f^2*x^2*e^(3*c) - 1536*I*a*d^4*e^3*f*x*e^(3*c) -
384*I*a*d^4*e^4*e^(3*c))*e^(3*d*x) + 96*(a*d^4*f^4*x^4*e^(2*c) + 4*a*d^4*e*
f^3*x^3*e^(2*c) + 6*a*d^4*e^2*f^2*x^2*e^(2*c) + 4*a*d^4*e^3*f*x*e^(2*c) + a
*d^4*e^4*e^(2*c))*e^(2*d*x) - (-192*I*a*d^4*f^4*x^4*e^c - 768*I*a*d^4*e*f^3
*x^3*e^c - 1152*I*a*d^4*e^2*f^2*x^2*e^c - 768*I*a*d^4*e^3*f*x*e^c - 192*I*a
*d^4*e^4*e^c)*e^(d*x)) + 8*integrate((9*d^4*f^4*x^4 + 36*d^4*e*f^3*x^3 + 9*
d^4*e^4 - 28*d^2*e^2*f^2 + 48*f^4 + 2*(27*d^4*e^2*f^2 - 14*d^2*f^4)*x^2 + 4
*(9*d^4*e^3*f - 14*d^2*e*f^3)*x)/(-192*I*a*d^4*f^5*x^5 - 960*I*a*d^4*e*f^4*
x^4 - 1920*I*a*d^4*e^2*f^3*x^3 - 1920*I*a*d^4*e^3*f^2*x^2 - 960*I*a*d^4*e^4
*f*x - 192*I*a*d^4*e^5 + 192*(a*d^4*f^5*x^5*e^c + 5*a*d^4*e*f^4*x^4*e^c + 1
0*a*d^4*e^2*f^3*x^3*e^c + 10*a*d^4*e^3*f^2*x^2*e^c + 5*a*d^4*e^4*f*x*e^c +
a*d^4*e^5*e^c)*e^(d*x)), x) + 8*integrate((3*d^2*f^2*x^2 + 6*d^2*e*f*x + 3*
d^2*e^2 - 4*f^2)/(64*I*a*d^2*f^3*x^3 + 192*I*a*d^2*e*f^2*x^2 + 192*I*a*d^2*
e^2*f*x + 64*I*a*d^2*e^3 + 64*(a*d^2*f^3*x^3*e^c + 3*a*d^2*e*f^2*x^2*e^c +
3*a*d^2*e^2*f*x*e^c + a*d^2*e^3*e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(4*I*d^2*f^3*x^2 + 8*I*d^2*e*f^2*x + 4*I*d^2*e^2*f - 6*I*f^3 + (9*d^3*f^3*
x^3 + 9*d^3*e^3 - 9*d^2*e^2*f - 2*d*e*f^2 + 6*f^3 + 9*(3*d^3*e*f^2 - d^2*f^
```

```
3)*x^2 + (27*d^3*e^2*f - 18*d^2*e*f^2 - 2*d*f^3)*x)*e^(5*d*x + 5*c) + (-18*
I*d^3*f^3*x^3 - 18*I*d^3*e^3 + 18*I*d^2*e^2*f - 6*I*f^3 + (-54*I*d^3*e*f^2
+ 18*I*d^2*f^3)*x^2 + (-54*I*d^3*e^2*f + 36*I*d^2*e*f^2)*x)*e^(4*d*x + 4*c)
 + 2*(3*d^3*f^3*x^3 + 3*d^3*e^3 - 4*d^2*e^2*f - 2*d*e*f^2 + 6*f^3 + (9*d^3*
e*f^2 - 4*d^2*f^3)*x^2 + (9*d^3*e^2*f - 8*d^2*e*f^2 - 2*d*f^3)*x)*e^(3*d*x
+ 3*c) + (18*I*d^3*f^3*x^3 + 18*I*d^3*e^3 + 22*I*d^2*e^2*f - 12*I*f^3 + (54
*I*d^3*e*f^2 + 22*I*d^2*f^3)*x^2 + (54*I*d^3*e^2*f + 44*I*d^2*e*f^2)*x)*e^(
2*d*x + 2*c) + (9*d^3*f^3*x^3 + 9*d^3*e^3 + d^2*e^2*f - 2*d*e*f^2 + 6*f^3 +
 (27*d^3*e*f^2 + d^2*f^3)*x^2 + (27*d^3*e^2*f + 2*d^2*e*f^2 - 2*d*f^3)*x)*e
^(d*x + c) - (12*a*d^4*f^4*x^4 + 48*a*d^4*e*f^3*x^3 + 72*a*d^4*e^2*f^2*x^2
+ 48*a*d^4*e^3*f*x + 12*a*d^4*e^4 - 12*(a*d^4*f^4*x^4 + 4*a*d^4*e*f^3*x^3 +
 6*a*d^4*e^2*f^2*x^2 + 4*a*d^4*e^3*f*x + a*d^4*e^4)*e^(6*d*x + 6*c) - (-24*
I*a*d^4*f^4*x^4 - 96*I*a*d^4*e*f^3*x^3 - 144*I*a*d^4*e^2*f^2*x^2 - 96*I*a*d
^4*e^3*f*x - 24*I*a*d^4*e^4)*e^(5*d*x + 5*c) - 12*(a*d^4*f^4*x^4 + 4*a*d^4*
e*f^3*x^3 + 6*a*d^4*e^2*f^2*x^2 + 4*a*d^4*e^3*f*x + a*d^4*e^4)*e^(4*d*x + 4
*c) - (-48*I*a*d^4*f^4*x^4 - 192*I*a*d^4*e*f^3*x^3 - 288*I*a*d^4*e^2*f^2*x^
2 - 192*I*a*d^4*e^3*f*x - 48*I*a*d^4*e^4)*e^(3*d*x + 3*c) + 12*(a*d^4*f^4*x
^4 + 4*a*d^4*e*f^3*x^3 + 6*a*d^4*e^2*f^2*x^2 + 4*a*d^4*e^3*f*x + a*d^4*e^4)
*e^(2*d*x + 2*c) - (-24*I*a*d^4*f^4*x^4 - 96*I*a*d^4*e*f^3*x^3 - 144*I*a*d^
4*e^2*f^2*x^2 - 96*I*a*d^4*e^3*f*x - 24*I*a*d^4*e^4)*e^(d*x + c))*integral(
1/12*(-8*I*d^2*f^4*x^2 - 16*I*d^2*e*f^3*x - 8*I*d^2*e^2*f^2 + 24*I*f^4 + (9
*d^4*f^4*x^4 + 36*d^4*e*f^3*x^3 + 9*d^4*e^4 - 20*d^2*e^2*f^2 + 24*f^4 + 2*(
27*d^4*e^2*f^2 - 10*d^2*f^4)*x^2 + 4*(9*d^4*e^3*f - 10*d^2*e*f^3)*x)*e^(d*x
 + c))/(a*d^4*f^5*x^5 + 5*a*d^4*e*f^4*x^4 + 10*a*d^4*e^2*f^3*x^3 + 10*a*d^4
*e^3*f^2*x^2 + 5*a*d^4*e^4*f*x + a*d^4*e^5 + (a*d^4*f^5*x^5 + 5*a*d^4*e*f^4
*x^4 + 10*a*d^4*e^2*f^3*x^3 + 10*a*d^4*e^3*f^2*x^2 + 5*a*d^4*e^4*f*x + a*d^
4*e^5)*e^(2*d*x + 2*c)), x))/(12*a*d^4*f^4*x^4 + 48*a*d^4*e*f^3*x^3 + 72*a*
d^4*e^2*f^2*x^2 + 48*a*d^4*e^3*f*x + 12*a*d^4*e^4 - 12*(a*d^4*f^4*x^4 + 4*a
*d^4*e*f^3*x^3 + 6*a*d^4*e^2*f^2*x^2 + 4*a*d^4*e^3*f*x + a*d^4*e^4)*e^(6*d*
x + 6*c) - (-24*I*a*d^4*f^4*x^4 - 96*I*a*d^4*e*f^3*x^3 - 144*I*a*d^4*e^2*f^
2*x^2 - 96*I*a*d^4*e^3*f*x - 24*I*a*d^4*e^4)*e^(5*d*x + 5*c) - 12*(a*d^4*f^
4*x^4 + 4*a*d^4*e*f^3*x^3 + 6*a*d^4*e^2*f^2*x^2 + 4*a*d^4*e^3*f*x + a*d^4*e
^4)*e^(4*d*x + 4*c) - (-48*I*a*d^4*f^4*x^4 - 192*I*a*d^4*e*f^3*x^3 - 288*I*
a*d^4*e^2*f^2*x^2 - 192*I*a*d^4*e^3*f*x - 48*I*a*d^4*e^4)*e^(3*d*x + 3*c) +
 12*(a*d^4*f^4*x^4 + 4*a*d^4*e*f^3*x^3 + 6*a*d^4*e^2*f^2*x^2 + 4*a*d^4*e^3*
f*x + a*d^4*e^4)*e^(2*d*x + 2*c) - (-24*I*a*d^4*f^4*x^4 - 96*I*a*d^4*e*f^3*
x^3 - 144*I*a*d^4*e^2*f^2*x^2 - 96*I*a*d^4*e^3*f*x - 24*I*a*d^4*e^4)*e^(d*x
 + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")

[Out] Timed out
```

3.288 $\int \frac{\text{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{\text{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Sech[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0811407, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sech[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Sech[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\text{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx=\int \frac{\text{sech}^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.034, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sech[c + d*x]^3/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] $Aborted
```

Maple [A] time = 2.361, size = 0, normalized size = 0.

$$\int \frac{(\text{sech}\,(dx+c))^3}{(fx+e)^2\,(a+ia\sinh\,(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] int(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -8*(8*I*d^2*f^3*x^2 + 16*I*d^2*e*f^2*x + 8*I*d^2*e^2*f - 24*I*f^3 + 3*(3*d^
3*f^3*x^3*e^(5*c) + 3*(3*d^3*e*f^2 - 2*d^2*f^3)*x^2*e^(5*c) + (9*d^3*e^2*f
- 12*d^2*e*f^2 - 2*d*f^3)*x*e^(5*c) + (3*d^3*e^3 - 6*d^2*e^2*f - 2*d*e*f^2
+ 8*f^3)*e^(5*c))*e^(5*d*x) + (-18*I*d^3*f^3*x^3*e^(4*c) + (-54*I*d^3*e*f^2
 + 36*I*d^2*f^3)*x^2*e^(4*c) + (-54*I*d^3*e^2*f + 72*I*d^2*e*f^2)*x*e^(4*c)
 + (-18*I*d^3*e^3 + 36*I*d^2*e^2*f - 24*I*f^3)*e^(4*c))*e^(4*d*x) + 2*(3*d^
3*f^3*x^3*e^(3*c) + (9*d^3*e*f^2 - 8*d^2*f^3)*x^2*e^(3*c) + (9*d^3*e^2*f -
16*d^2*e*f^2 - 6*d*f^3)*x*e^(3*c) + (3*d^3*e^3 - 8*d^2*e^2*f - 6*d*e*f^2 +
24*f^3)*e^(3*c))*e^(3*d*x) + (18*I*d^3*f^3*x^3*e^(2*c) + (54*I*d^3*e*f^2 +
44*I*d^2*f^3)*x^2*e^(2*c) + (54*I*d^3*e^2*f + 88*I*d^2*e*f^2)*x*e^(2*c) + (
18*I*d^3*e^3 + 44*I*d^2*e^2*f - 48*I*f^3)*e^(2*c))*e^(2*d*x) + (9*d^3*f^3*x
^3*e^c + (27*d^3*e*f^2 + 2*d^2*f^3)*x^2*e^c + (27*d^3*e^2*f + 4*d^2*e*f^2 -
 6*d*f^3)*x*e^c + (9*d^3*e^3 + 2*d^2*e^2*f - 6*d*e*f^2 + 24*f^3)*e^c)*e^(d*
x))/(96*a*d^4*f^5*x^5 + 480*a*d^4*e*f^4*x^4 + 960*a*d^4*e^2*f^3*x^3 + 960*a
*d^4*e^3*f^2*x^2 + 480*a*d^4*e^4*f*x + 96*a*d^4*e^5 - 96*(a*d^4*f^5*x^5*e^(
6*c) + 5*a*d^4*e*f^4*x^4*e^(6*c) + 10*a*d^4*e^2*f^3*x^3*e^(6*c) + 10*a*d^4*
e^3*f^2*x^2*e^(6*c) + 5*a*d^4*e^4*f*x*e^(6*c) + a*d^4*e^5*e^(6*c))*e^(6*d*x
) - (-192*I*a*d^4*f^5*x^5*e^(5*c) - 960*I*a*d^4*e*f^4*x^4*e^(5*c) - 1920*I*
a*d^4*e^2*f^3*x^3*e^(5*c) - 1920*I*a*d^4*e^3*f^2*x^2*e^(5*c) - 960*I*a*d^4*
e^4*f*x*e^(5*c) - 192*I*a*d^4*e^5*e^(5*c))*e^(5*d*x) - 96*(a*d^4*f^5*x^5*e^
(4*c) + 5*a*d^4*e*f^4*x^4*e^(4*c) + 10*a*d^4*e^2*f^3*x^3*e^(4*c) + 10*a*d^4
*e^3*f^2*x^2*e^(4*c) + 5*a*d^4*e^4*f*x*e^(4*c) + a*d^4*e^5*e^(4*c))*e^(4*d*
x) - (-384*I*a*d^4*f^5*x^5*e^(3*c) - 1920*I*a*d^4*e*f^4*x^4*e^(3*c) - 3840*
I*a*d^4*e^2*f^3*x^3*e^(3*c) - 3840*I*a*d^4*e^3*f^2*x^2*e^(3*c) - 1920*I*a*d
^4*e^4*f*x*e^(3*c) - 384*I*a*d^4*e^5*e^(3*c))*e^(3*d*x) + 96*(a*d^4*f^5*x^5
*e^(2*c) + 5*a*d^4*e*f^4*x^4*e^(2*c) + 10*a*d^4*e^2*f^3*x^3*e^(2*c) + 10*a*
d^4*e^3*f^2*x^2*e^(2*c) + 5*a*d^4*e^4*f*x*e^(2*c) + a*d^4*e^5*e^(2*c))*e^(2
*d*x) - (-192*I*a*d^4*f^5*x^5*e^c - 960*I*a*d^4*e*f^4*x^4*e^c - 1920*I*a*d^
4*e^2*f^3*x^3*e^c - 1920*I*a*d^4*e^3*f^2*x^2*e^c - 960*I*a*d^4*e^4*f*x*e^c
- 192*I*a*d^4*e^5*e^c)*e^(d*x)) + 8*integrate((3*d^4*f^4*x^4 + 12*d^4*e*f^3
*x^3 + 3*d^4*e^4 - 28*d^2*e^2*f^2 + 80*f^4 + 2*(9*d^4*e^2*f^2 - 14*d^2*f^4)
*x^2 + 4*(3*d^4*e^3*f - 14*d^2*e*f^3)*x)/(-64*I*a*d^4*f^6*x^6 - 384*I*a*d^4
*e*f^5*x^5 - 960*I*a*d^4*e^2*f^4*x^4 - 1280*I*a*d^4*e^3*f^3*x^3 - 960*I*a*d
^4*e^4*f^2*x^2 - 384*I*a*d^4*e^5*f*x - 64*I*a*d^4*e^6 + 64*(a*d^4*f^6*x^6*e
^c + 6*a*d^4*e*f^5*x^5*e^c + 15*a*d^4*e^2*f^4*x^4*e^c + 20*a*d^4*e^3*f^3*x^
3*e^c + 15*a*d^4*e^4*f^2*x^2*e^c + 6*a*d^4*e^5*f*x*e^c + a*d^4*e^6*e^c)*e^(
d*x)), x) + 8*integrate(3*(d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2 - 4*f^2)/(64
*I*a*d^2*f^4*x^4 + 256*I*a*d^2*e*f^3*x^3 + 384*I*a*d^2*e^2*f^2*x^2 + 256*I*
a*d^2*e^3*f*x + 64*I*a*d^2*e^4 + 64*(a*d^2*f^4*x^4*e^c + 4*a*d^2*e*f^3*x^3*
e^c + 6*a*d^2*e^2*f^2*x^2*e^c + 4*a*d^2*e^3*f*x*e^c + a*d^2*e^4*e^c)*e^(d*x
)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -(8*I*d^2*f^3*x^2 + 16*I*d^2*e*f^2*x + 8*I*d^2*e^2*f - 24*I*f^3 + 3*(3*d^3*
f^3*x^3 + 3*d^3*e^3 - 6*d^2*e^2*f - 2*d*e*f^2 + 8*f^3 + 3*(3*d^3*e*f^2 - 2*
d^2*f^3)*x^2 + (9*d^3*e^2*f - 12*d^2*e*f^2 - 2*d*f^3)*x)*e^(5*d*x + 5*c) +
(-18*I*d^3*f^3*x^3 - 18*I*d^3*e^3 + 36*I*d^2*e^2*f - 24*I*f^3 + (-54*I*d^3*
e*f^2 + 36*I*d^2*f^3)*x^2 + (-54*I*d^3*e^2*f + 72*I*d^2*e*f^2)*x)*e^(4*d*x
+ 4*c) + 2*(3*d^3*f^3*x^3 + 3*d^3*e^3 - 8*d^2*e^2*f - 6*d*e*f^2 + 24*f^3 +
(9*d^3*e*f^2 - 8*d^2*f^3)*x^2 + (9*d^3*e^2*f - 16*d^2*e*f^2 - 6*d*f^3)*x)*e
^(3*d*x + 3*c) + (18*I*d^3*f^3*x^3 + 18*I*d^3*e^3 + 44*I*d^2*e^2*f - 48*I*f
^3 + (54*I*d^3*e*f^2 + 44*I*d^2*f^3)*x^2 + (54*I*d^3*e^2*f + 88*I*d^2*e*f^2
)*x)*e^(2*d*x + 2*c) + (9*d^3*f^3*x^3 + 9*d^3*e^3 + 2*d^2*e^2*f - 6*d*e*f^2
 + 24*f^3 + (27*d^3*e*f^2 + 2*d^2*f^3)*x^2 + (27*d^3*e^2*f + 4*d^2*e*f^2 -
6*d*f^3)*x)*e^(d*x + c) - (12*a*d^4*f^5*x^5 + 60*a*d^4*e*f^4*x^4 + 120*a*d^
4*e^2*f^3*x^3 + 120*a*d^4*e^3*f^2*x^2 + 60*a*d^4*e^4*f*x + 12*a*d^4*e^5 - 1
2*(a*d^4*f^5*x^5 + 5*a*d^4*e*f^4*x^4 + 10*a*d^4*e^2*f^3*x^3 + 10*a*d^4*e^3*
f^2*x^2 + 5*a*d^4*e^4*f*x + a*d^4*e^5)*e^(6*d*x + 6*c) - (-24*I*a*d^4*f^5*x
^5 - 120*I*a*d^4*e*f^4*x^4 - 240*I*a*d^4*e^2*f^3*x^3 - 240*I*a*d^4*e^3*f^2*
x^2 - 120*I*a*d^4*e^4*f*x - 24*I*a*d^4*e^5)*e^(5*d*x + 5*c) - 12*(a*d^4*f^5
*x^5 + 5*a*d^4*e*f^4*x^4 + 10*a*d^4*e^2*f^3*x^3 + 10*a*d^4*e^3*f^2*x^2 + 5*
a*d^4*e^4*f*x + a*d^4*e^5)*e^(4*d*x + 4*c) - (-48*I*a*d^4*f^5*x^5 - 240*I*a
*d^4*e*f^4*x^4 - 480*I*a*d^4*e^2*f^3*x^3 - 480*I*a*d^4*e^3*f^2*x^2 - 240*I*
a*d^4*e^4*f*x - 48*I*a*d^4*e^5)*e^(3*d*x + 3*c) + 12*(a*d^4*f^5*x^5 + 5*a*d
^4*e*f^4*x^4 + 10*a*d^4*e^2*f^3*x^3 + 10*a*d^4*e^3*f^2*x^2 + 5*a*d^4*e^4*f*
x + a*d^4*e^5)*e^(2*d*x + 2*c) - (-24*I*a*d^4*f^5*x^5 - 120*I*a*d^4*e*f^4*x
^4 - 240*I*a*d^4*e^2*f^3*x^3 - 240*I*a*d^4*e^3*f^2*x^2 - 120*I*a*d^4*e^4*f*
x - 24*I*a*d^4*e^5)*e^(d*x + c))*integral(1/4*(-8*I*d^2*f^4*x^2 - 16*I*d^2*
e*f^3*x - 8*I*d^2*e^2*f^2 + 40*I*f^4 + (3*d^4*f^4*x^4 + 12*d^4*e*f^3*x^3 +
3*d^4*e^4 - 20*d^2*e^2*f^2 + 40*f^4 + 2*(9*d^4*e^2*f^2 - 10*d^2*f^4)*x^2 +
4*(3*d^4*e^3*f - 10*d^2*e*f^3)*x)*e^(d*x + c))/(a*d^4*f^6*x^6 + 6*a*d^4*e*f
^5*x^5 + 15*a*d^4*e^2*f^4*x^4 + 20*a*d^4*e^3*f^3*x^3 + 15*a*d^4*e^4*f^2*x^2
 + 6*a*d^4*e^5*f*x + a*d^4*e^6 + (a*d^4*f^6*x^6 + 6*a*d^4*e*f^5*x^5 + 15*a*
d^4*e^2*f^4*x^4 + 20*a*d^4*e^3*f^3*x^3 + 15*a*d^4*e^4*f^2*x^2 + 6*a*d^4*e^5
*f*x + a*d^4*e^6)*e^(2*d*x + 2*c)), x))/(12*a*d^4*f^5*x^5 + 60*a*d^4*e*f^4*
x^4 + 120*a*d^4*e^2*f^3*x^3 + 120*a*d^4*e^3*f^2*x^2 + 60*a*d^4*e^4*f*x + 12
*a*d^4*e^5 - 12*(a*d^4*f^5*x^5 + 5*a*d^4*e*f^4*x^4 + 10*a*d^4*e^2*f^3*x^3 +
 10*a*d^4*e^3*f^2*x^2 + 5*a*d^4*e^4*f*x + a*d^4*e^5)*e^(6*d*x + 6*c) - (-24
*I*a*d^4*f^5*x^5 - 120*I*a*d^4*e*f^4*x^4 - 240*I*a*d^4*e^2*f^3*x^3 - 240*I*
a*d^4*e^3*f^2*x^2 - 120*I*a*d^4*e^4*f*x - 24*I*a*d^4*e^5)*e^(5*d*x + 5*c) -
 12*(a*d^4*f^5*x^5 + 5*a*d^4*e*f^4*x^4 + 10*a*d^4*e^2*f^3*x^3 + 10*a*d^4*e^
3*f^2*x^2 + 5*a*d^4*e^4*f*x + a*d^4*e^5)*e^(4*d*x + 4*c) - (-48*I*a*d^4*f^5
*x^5 - 240*I*a*d^4*e*f^4*x^4 - 480*I*a*d^4*e^2*f^3*x^3 - 480*I*a*d^4*e^3*f^
2*x^2 - 240*I*a*d^4*e^4*f*x - 48*I*a*d^4*e^5)*e^(3*d*x + 3*c) + 12*(a*d^4*f
^5*x^5 + 5*a*d^4*e*f^4*x^4 + 10*a*d^4*e^2*f^3*x^3 + 10*a*d^4*e^3*f^2*x^2 +
5*a*d^4*e^4*f*x + a*d^4*e^5)*e^(2*d*x + 2*c) - (-24*I*a*d^4*f^5*x^5 - 120*I
*a*d^4*e*f^4*x^4 - 240*I*a*d^4*e^2*f^3*x^3 - 240*I*a*d^4*e^3*f^2*x^2 - 120*
I*a*d^4*e^4*f*x - 24*I*a*d^4*e^5)*e^(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.289 $\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=356

$$-\frac{6f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3}-\frac{6f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3}+\frac{3f(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{3}{}$$

```
[Out] -(e + f*x)^4/(4*b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
])/(b*d) + (3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2]))])/(b*d^2) + (3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/(b*d^2) - (6*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - S
qrt[a^2 + b^2]))])/(b*d^3) - (6*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/(b*d^3) + (6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(b*d^4) + (6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/(b*d^4)
```

Rubi [A] time = 0.482998, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5561, 2190, 2531, 6609, 2282, 6589}

$$-\frac{6f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3}-\frac{6f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3}+\frac{3f(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -(e + f*x)^4/(4*b*f) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/(b*d) + ((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
])/(b*d) + (3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2]))])/(b*d^2) + (3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/(b*d^2) - (6*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - S
qrt[a^2 + b^2]))])/(b*d^3) - (6*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/(b*d^3) + (6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(b*d^4) + (6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/(b*d^4)
```


Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(e+fx)^4}{4bf} + \int \frac{e^{c+dx}(e+fx)^3}{a-\sqrt{a^2+b^2}+be^{c+dx}}\,dx + \int \frac{e^{c+dx}(e+fx)^3}{a+\sqrt{a^2+b^2}+be^{c+dx}}\,dx$$

$$= -\frac{(e+fx)^4}{4bf} + \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd} - \frac{(3f)\int(e\ \cdots}{\cdots}$$

$$= -\frac{(e+fx)^4}{4bf} + \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd} + \frac{3f(e+f\cdots}{\cdots}$$

$$= -\frac{(e+fx)^4}{4bf} + \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd} + \frac{3f(e+f\cdots}{\cdots}$$

$$= -\frac{(e+fx)^4}{4bf} + \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd} + \frac{3f(e+f\cdots}{\cdots}$$

$$= -\frac{(e+fx)^4}{4bf} + \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd} + \frac{3f(e+f\cdots}{\cdots}$$

Mathematica [A] time = 0.149016, size = 329, normalized size = 0.92

$$\frac{\frac{12f\left(d^2(e+fx)^2\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)-2df(e+fx)\text{PolyLog}\left(3,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+2f^2\text{PolyLog}\left(4,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)\right)}{d^4}+\frac{12f\left(d^2(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)-2df(e+f\cdots\right.}{\cdots}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-((e + f*x)^4/f) + (4*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2])])/d + (4*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/
d + (12*f*(d^2*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]
)] - 2*d*f*(e + f*x)*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2
*f^2*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]))/d^4 + (12*f*(d^2*
(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*d*f*(e
 + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 2*f^2*PolyLo
g[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/d^4)/(4*b)
```

Maple [F] time = 0.259, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3}\cosh\left(dx+c\right)}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{e^{3}\log\left(b\sinh\left(dx+c\right)+a\right)}{bd}+\frac{f^{3}x^{4}+4\,ef^{2}x^{3}+6\,e^{2}fx^{2}}{4\,b}-\int -\frac{2\left(bf^{3}x^{3}+3\,bef^{2}x^{2}+3\,be^{2}fx-\left(af^{3}x^{3}e^{c}+3\,aef^{2}x^{2}e^{c}+3\right.\right.}{b^{2}e^{(2\,dx+2\,c)}+2\,abe^{(dx+c)}-b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^3*log(b*sinh(d*x + c) + a)/(b*d) + 1/4*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x
^2)/b - integrate(-2*(b*f^3*x^3 + 3*b*e*f^2*x^2 + 3*b*e^2*f*x - (a*f^3*x^3*
e^c + 3*a*e*f^2*x^2*e^c + 3*a*e^2*f*x*e^c)*e^(d*x))/(b^2*e^(2*d*x + 2*c) +
2*a*b*e^(d*x + c) - b^2), x)
```

Fricas [C] time = 2.47518, size = 2128, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(d^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*d^4*e^2*f*x^2 + 4*d^4*e^3*x - 24*f^
3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 24*f^3*polylog(4, (a*cosh(d*x + c) +
 a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
))/b) - 12*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*dilog((a*cosh(d*x + c)
 + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2) - b)/b + 1) - 12*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*dilog((a*cos
```

```
h(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b + 1) - 4*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^
3*f^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2
) + 2*a) - 4*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*log(2*b*co
sh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 4*(d^3
*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2
+ c^3*f^3)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*s
inh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 4*(d^3*f^3*x^3 + 3*d^3*e*f^2*
x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*log(-(a*cosh
(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b) + 24*(d*f^3*x + d*e*f^2)*polylog(3, (a*cosh(d*x + c) +
 a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
))/b) + 24*(d*f^3*x + d*e*f^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c
) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b))/(b*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \cosh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.290 $\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=264

$$\frac{2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2}-\frac{2f^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3}-\frac{2f^2\text{PolyLog}}{b}$$

```
[Out] -(e + f*x)^3/(3*b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
])/(b*d) + (2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
))])/(b*d^2) + (2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))])/(b*d^2) - (2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
))])/(b*d^3) - (2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])
/(b*d^3)
```

Rubi [A] time = 0.411303, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5561, 2190, 2531, 2282, 6589}

$$\frac{2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2}-\frac{2f^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3}-\frac{2f^2\text{PolyLog}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -(e + f*x)^3/(3*b*f) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/(b*d) + ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
])/(b*d) + (2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
))])/(b*d^2) + (2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))])/(b*d^2) - (2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
))])/(b*d^3) - (2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])
/(b*d^3)
```


Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(e+fx)^3}{3bf}+\int \frac{e^{c+dx}(e+fx)^2}{a-\sqrt{a^2+b^2}+be^{c+dx}}\,dx+\int \frac{e^{c+dx}(e+fx)^2}{a+\sqrt{a^2+b^2}+be^{c+dx}}\,dx$$

$$= -\frac{(e+fx)^3}{3bf}+\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd}+\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd}-\frac{(2f)\int(e\ \cdots}{\cdots}$$

$$= -\frac{(e+fx)^3}{3bf}+\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd}+\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd}+\frac{2f(e+f\cdots}{\cdots}$$

$$= -\frac{(e+fx)^3}{3bf}+\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd}+\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd}+\frac{2f(e+f\cdots}{\cdots}$$

$$= -\frac{(e+fx)^3}{3bf}+\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd}+\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd}+\frac{2f(e+f\cdots}{\cdots}$$

Mathematica [A] time = 0.145766, size = 244, normalized size = 0.92

$$\frac{\frac{6f\left(d(e+fx)\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)-f\text{PolyLog}\left(3,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)\right)}{d^3}+\frac{6f\left(d(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)-f\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)\right)}{d^3}+\frac{3(e+fx)^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\cdots\right)}{d}\cdots}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-((e + f*x)^3/f) + (3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2])])/d + (3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/
d + (6*f*(d*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] -
f*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]))/d^3 + (6*f*(d*(e + f
*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - f*PolyLog[3, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/d^3)/(3*b)
```

Maple [F] time = 0.204, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \cosh(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{e^2\log(b\sinh(dx+c)+a)}{bd} + \frac{f^2x^3+3\,efx^2}{3\,b} - \int -\frac{2\left(bf^2x^2+2\,befx-\left(af^2x^2e^c+2\,aefxe^c\right)e^{(dx)}\right)}{b^2e^{(2\,dx+2\,c)}+2\,abe^{(dx+c)}-b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^2*log(b*sinh(d*x + c) + a)/(b*d) + 1/3*(f^2*x^3 + 3*e*f*x^2)/b - integrat
e(-2*(b*f^2*x^2 + 2*b*e*f*x - (a*f^2*x^2*e^c + 2*a*e*f*x*e^c)*e^(d*x))/(b^2
*e^(2*d*x + 2*c) + 2*a*b*e^(d*x + c) - b^2), x)
```

Fricas [C] time = 2.31073, size = 1504, normalized size = 5.7

$$-\frac{d^3f^2x^3+3\,d^3efx^2+3\,d^3e^2x+6\,f^2\mathrm{polylog}\left(3,\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}}{b}\right)+6\,f^2\mathrm{polylog}\left(3,-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d^3*e^2*x + 6*f^2*polylog(3, (a*cosh(
d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2))/b) + 6*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*(d*f^2*x + d
*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 6*(d*f^2*x + d*e*f)*dilog((a*
cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2) - b)/b + 1) - 3*(d^2*e^2 - 2*c*d*e*f + c^2*f^2)*log(2*b*co
sh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(d^2
*e^2 - 2*c*d*e*f + c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b
*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f -
c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 3*(d^2*f^2*x^2 + 2*d^2*e*f*x +
2*c*d*e*f - c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b))/(b*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \cosh(dx+c)}{b \sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.291 $\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=170

$$\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e\cdots}{}$$

```
[Out] -(e + f*x)^2/(2*b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(
b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (
f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)
```

Rubi [A] time = 0.234474, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5561, 2190, 2279, 2391}

$$\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2}+\frac{f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd}+\frac{(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bd}-\frac{(e\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(e + f*x)^2/(2*b*f) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])])/(b*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(
b*d) + (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (
f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2)
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{(e+fx)^2}{2bf} + \int \frac{e^{c+dx}(e+fx)}{a-\sqrt{a^2+b^2}+be^{c+dx}}\,dx + \int \frac{e^{c+dx}(e+fx)}{a+\sqrt{a^2+b^2}+be^{c+dx}}\,dx$$

$$= -\frac{(e+fx)^2}{2bf} + \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd} - \frac{f\int\log\left(1+\right.}{b}$$

$$= -\frac{(e+fx)^2}{2bf} + \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd} - \frac{f\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{(e+fx)^2}{2bf} + \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd} + \frac{f\mathrm{Li}_2\left(-\frac{be^{c+}}{a-\sqrt{a}}\right.}{bd^2}$$

Mathematica [A] time = 0.0348309, size = 157, normalized size = 0.92

$$\frac{2f^2\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+2f^2\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)-d(e+fx)\left(-2f\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)-2f\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)\right.}{2bd^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-(d*(e + f*x)*(d*e + d*f*x - 2*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])] - 2*f*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])) + 2*f^2*PolyLo
g[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[2, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))])/(2*b*d^2*f)
```

Maple [B] time = 0.077, size = 412, normalized size = 2.4

$$-\frac{fx^2}{2b}+\frac{ex}{b}-\frac{cf\ln\left(b e^{2dx+2c}+2ae^{dx+c}-b\right)}{d^2b}+2\frac{cf\ln\left(e^{dx+c}\right)}{d^2b}+\frac{e\ln\left(be^{2dx+2c}+2ae^{dx+c}-b\right)}{bd}-2\frac{e\ln\left(e^{dx+c}\right)}{bd}+\frac{fx}{bd}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/2*f*x^2/b+e*x/b-1/d^2/b*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d^2/
b*f*c*ln(exp(d*x+c))+1/d/b*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d/b*e*
ln(exp(d*x+c))+1/d/b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(
1/2)))*x+1/d^2/b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)
))*c+1/d/b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d
^2/b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d^2/b*f
*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/d^2/b*f*dilo
g((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2/d/b*f*c*x-1/d^2
/b*f*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}f\left(\frac{x^2}{b}-\int\frac{4\left(axe^{(dx+c)}-bx\right)}{b^2e^{(2\,dx+2\,c)}+2\,abe^{(dx+c)}-b^2}\,dx\right)+\frac{e\log\left(b\sinh\left(dx+c\right)+a\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*f*(x^2/b - integrate(4*(a*x*e^(d*x + c) - b*x)/(b^2*e^(2*d*x + 2*c) + 2
*a*b*e^(d*x + c) - b^2), x)) + e*log(b*sinh(d*x + c) + a)/(b*d)
```

Fricas [B] time = 2.16192, size = 964, normalized size = 5.67

$$-\frac{d^2fx^2+2d^2ex-2f\,\mathrm{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}+1\right)-2f\,\mathrm{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\,\mathrm{c}\ldots}{\ldots}\right.\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*f*x^2 + 2*d^2*e*x - 2*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
+ (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2
*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(d*e - c*f)*log(2*b*cosh(d*x +
 c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(d*e - c*f)*
log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a
) - 2*(d*f*x + c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
 c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(d*f*x + c*f)*log(
-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 + b^2)/b^2) - b)/b))/(b*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*cosh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\cosh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.292 $\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(a+b\sinh(c+dx))}{bd}$$

```
[Out] Log[a + b*Sinh[c + d*x]]/(b*d)
```

Rubi [A] time = 0.0273205, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2668, 31}

$$\frac{\log(a+b\sinh(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Log[a + b*Sinh[c + d*x]]/(b*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(c+dx)\right)}{bd}\\ &= \frac{\log(a+b\sinh(c+dx))}{bd}\end{aligned}$$

Mathematica [A] time = 0.0069378, size = 18, normalized size = 1.

$$\frac{\log(a+b\sinh(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Log[a + b*Sinh[c + d*x]]/(b*d)
```

Maple [A] time = 0.002, size = 19, normalized size = 1.1

$$\frac{\ln\left(a+b\sinh\left(dx+c\right)\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] ln(a+b*sinh(d*x+c))/b/d
```

Maxima [A] time = 1.15693, size = 24, normalized size = 1.33

$$\frac{\log\left(b\sinh\left(dx+c\right)+a\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] log(b*sinh(d*x + c) + a)/(b*d)
```

Fricas [B] time = 2.04264, size = 104, normalized size = 5.78

$$-\frac{dx-\log\left(\frac{2\left(b\sinh(dx+c)+a\right)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(d*x - log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))))/(b*d)
```

Sympy [A] time = 1.09728, size = 41, normalized size = 2.28

$$\begin{cases} \frac{x\cosh\left(c\right)}{a} & \text{for}\ b=0\wedge d=0\\ \frac{x\cosh\left(c\right)}{a+b\sinh\left(c\right)} & \text{for}\ d=0\\ \frac{\sinh\left(c+dx\right)}{ad} & \text{for}\ b=0\\ \frac{\log\left(\frac{a}{b}+\sinh\left(c+dx\right)\right)}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Piecewise((x*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (x*cosh(c)/(a + b*sinh(c)), E
q(d, 0)), (sinh(c + d*x)/(a*d), Eq(b, 0)), (log(a/b + sinh(c + d*x))/(b*d),
 True))
```

Giac [A] time = 1.1611, size = 45, normalized size = 2.5

$$\frac{\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(b*d)
```

3.293 $\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Cosh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0471313, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cosh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Cosh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A] time = 12.8653, size = 0, normalized size = 0.

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cosh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Cosh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A] time = 0.197, size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{\left(fx+e\right)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\log\left(fx+e\right)}{bf}-\frac{1}{2}\int-\frac{4\left(ae^{(dx+c)}-b\right)}{b^2fx+b^2e-\left(b^2fxe^{(2c)}+b^2ee^{(2c)}\right)e^{(2dx)}-2\left(abfxe^c+abee^c\right)e^{(dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] log(f*x + e)/(b*f) - 1/2*integrate(-4*(a*e^(d*x + c) - b)/(b^2*f*x + b^2*e
- (b^2*f*x*e^(2*c) + b^2*e*e^(2*c))*e^(2*d*x) - 2*(a*b*f*x*e^c + a*b*e*e^c)
*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh\left(dx+c\right)}{afx+ae+\left(bfx+be\right)\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh\left(dx+c\right)}{\left(fx+e\right)\left(b\sinh\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

3.294 $\int \frac{(e+fx)^3 \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=527

$$-\frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3}+\frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^3}+\frac{3f\sqrt{a^2+b^2}(e+fx)^2\text{P}}{b^2}$$

```
[Out] -(a*(e + f*x)^4)/(4*b^2*f) + (6*f^2*(e + f*x)*Cosh[c + d*x])/(b*d^3) + ((e
+ f*x)^3*Cosh[c + d*x])/(b*d) + (Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)^3*Lo
g[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*d) + (3*Sqrt[a^2 + b^2]*
f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^
2) - (3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/(b^2*d^2) - (6*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^3) + (6*Sqrt[a^2 + b^2]*f^2
*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^3)
+ (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
)])/(b^2*d^4) - (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/(b^2*d^4) - (6*f^3*Sinh[c + d*x])/(b*d^4) - (3*f*(e + f*x
)^2*Sinh[c + d*x])/(b*d^2)
```

Rubi [A] time = 0.908079, antiderivative size = 527, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {5565, 32, 3296, 2637, 3322, 2264, 2190, 2531, 6609, 2282, 6589}

$$-\frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3}+\frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^3}+\frac{3f\sqrt{a^2+b^2}(e+fx)^2\text{P}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -(a*(e + f*x)^4)/(4*b^2*f) + (6*f^2*(e + f*x)*Cosh[c + d*x])/(b*d^3) + ((e
+ f*x)^3*Cosh[c + d*x])/(b*d) + (Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)^3*Lo
g[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*d) + (3*Sqrt[a^2 + b^2]*
f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^
2) - (3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/(b^2*d^2) - (6*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^3) + (6*Sqrt[a^2 + b^2]*f^2
*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^3)
+ (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
)])/(b^2*d^4) - (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/(b^2*d^4) - (6*f^3*Sinh[c + d*x])/(b*d^4) - (3*f*(e + f*x
)^2*Sinh[c + d*x])/(b*d^2)
```


Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\int (e+fx)^3\,dx}{b^2}+\frac{\int (e+fx)^3\sinh(c+dx)\,dx}{b}+\frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}\,dx}{b^2}$$

$$= -\frac{a(e+fx)^4}{4b^2f}+\frac{(e+fx)^3\cosh(c+dx)}{bd}+\frac{\left(2\left(a^2+b^2\right)\right)\int \frac{e^{c+dx}(e+fx)^3}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{b^2}-\frac{(3f)\int (e \ldots}{\ldots}$$

$$= -\frac{a(e+fx)^4}{4b^2f}+\frac{(e+fx)^3\cosh(c+dx)}{bd}-\frac{3f(e+fx)^2\sinh(c+dx)}{bd^2}+\frac{\left(2\sqrt{a^2+b^2}\right)\int \frac{\ldots}{2a-2\ldots}}{b}$$

$$= -\frac{a(e+fx)^4}{4b^2f}+\frac{6f^2(e+fx)\cosh(c+dx)}{bd^3}+\frac{(e+fx)^3\cosh(c+dx)}{bd}+\frac{\sqrt{a^2+b^2}(e+fx)^3\ldots}{b^2\ldots}$$

$$= -\frac{a(e+fx)^4}{4b^2f}+\frac{6f^2(e+fx)\cosh(c+dx)}{bd^3}+\frac{(e+fx)^3\cosh(c+dx)}{bd}+\frac{\sqrt{a^2+b^2}(e+fx)^3\ldots}{b^2\ldots}$$

$$= -\frac{a(e+fx)^4}{4b^2f}+\frac{6f^2(e+fx)\cosh(c+dx)}{bd^3}+\frac{(e+fx)^3\cosh(c+dx)}{bd}+\frac{\sqrt{a^2+b^2}(e+fx)^3\ldots}{b^2\ldots}$$

$$= -\frac{a(e+fx)^4}{4b^2f}+\frac{6f^2(e+fx)\cosh(c+dx)}{bd^3}+\frac{(e+fx)^3\cosh(c+dx)}{bd}+\frac{\sqrt{a^2+b^2}(e+fx)^3\ldots}{b^2\ldots}$$

$$= -\frac{a(e+fx)^4}{4b^2f}+\frac{6f^2(e+fx)\cosh(c+dx)}{bd^3}+\frac{(e+fx)^3\cosh(c+dx)}{bd}+\frac{\sqrt{a^2+b^2}(e+fx)^3\ldots}{b^2\ldots}$$

Mathematica [A] time = 3.28391, size = 933, normalized size = 1.77

$$-\frac{af^3x^4d^4+4aef^2x^3d^4+6ae^2fx^2d^4+4ae^3xd^4+8\sqrt{a^2+b^2}e^3\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)d^3-4be^3\cosh(c+dx)d^3-4bf^3x^3\cosh(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(4*a*d^4*e^3*x + 6*a*d^4*e^2*f*x^2 + 4*a*d^4*e*f^2*x^3 + a*d^4*f^3*x^4 + 8
*Sqrt[a^2 + b^2]*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 4*b
*d^3*e^3*Cosh[c + d*x] - 24*b*d*e*f^2*Cosh[c + d*x] - 12*b*d^3*e^2*f*x*Cosh
[c + d*x] - 24*b*d*f^3*x*Cosh[c + d*x] - 12*b*d^3*e*f^2*x^2*Cosh[c + d*x] -
 4*b*d^3*f^3*x^3*Cosh[c + d*x] - 12*Sqrt[a^2 + b^2]*d^3*e^2*f*x*Log[1 + (b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 12*Sqrt[a^2 + b^2]*d^3*e*f^2*x^2*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 4*Sqrt[a^2 + b^2]*d^3*f^3*x^3*
Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 12*Sqrt[a^2 + b^2]*d^3*e^2
*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 12*Sqrt[a^2 + b^2]*d^
3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 4*Sqrt[a^2 + b
^2]*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 12*Sqrt[a^
```

```
2 + b^2]*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]
)] + 12*Sqrt[a^2 + b^2]*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))] + 24*Sqrt[a^2 + b^2]*d*e*f^2*PolyLog[3, (b*E^(c + d*x))
/(-a + Sqrt[a^2 + b^2])] + 24*Sqrt[a^2 + b^2]*d*f^3*x*PolyLog[3, (b*E^(c +
d*x))/(-a + Sqrt[a^2 + b^2])] - 24*Sqrt[a^2 + b^2]*d*e*f^2*PolyLog[3, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 24*Sqrt[a^2 + b^2]*d*f^3*x*PolyLog[3
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 24*Sqrt[a^2 + b^2]*f^3*PolyLo
g[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 24*Sqrt[a^2 + b^2]*f^3*PolyL
og[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 12*b*d^2*e^2*f*Sinh[c + d
*x] + 24*b*f^3*Sinh[c + d*x] + 24*b*d^2*e*f^2*x*Sinh[c + d*x] + 12*b*d^2*f^
3*x^2*Sinh[c + d*x])/(4*b^2*d^4)
```

Maple [F] time = 0.269, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 (\cosh(dx+c))^2}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)

[Out] int((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [C] time = 3.06394, size = 4691, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")

[Out] 1/4*(2*b*d^3*f^3*x^3 + 2*b*d^3*e^3 + 6*b*d^2*e^2*f + 12*b*d*e*f^2 + 12*b*f^
3 + 6*(b*d^3*e*f^2 + b*d^2*f^3)*x^2 + 2*(b*d^3*f^3*x^3 + b*d^3*e^3 - 3*b*d^
2*e^2*f + 6*b*d*e*f^2 - 6*b*f^3 + 3*(b*d^3*e*f^2 - b*d^2*f^3)*x^2 + 3*(b*d^
3*e^2*f - 2*b*d^2*e*f^2 + 2*b*d*f^3)*x)*cosh(d*x + c)^2 + 2*(b*d^3*f^3*x^3
+ b*d^3*e^3 - 3*b*d^2*e^2*f + 6*b*d*e*f^2 - 6*b*f^3 + 3*(b*d^3*e*f^2 - b*d^
2*f^3)*x^2 + 3*(b*d^3*e^2*f - 2*b*d^2*e*f^2 + 2*b*d*f^3)*x)*sinh(d*x + c)^2
 + 12*((b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c) + (b*d
^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 12*((b*d^2*f^3*x^2 + 2*b*d^2*
```

```
e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c) + (b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b
*d^2*e^2*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a
*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b + 1) - 4*((b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3
)*cosh(d*x + c) + (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^
3)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*
x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 4*((b*d^3*e^3 - 3*b*c*d^2*e^2*f
 + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*cosh(d*x + c) + (b*d^3*e^3 - 3*b*c*d^2*e^2*
f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2
*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 4
*((b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f -
3*b*c^2*d*e*f^2 + b*c^3*f^3)*cosh(d*x + c) + (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2
*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) +
 (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 4*((b*
d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c
^2*d*e*f^2 + b*c^3*f^3)*cosh(d*x + c) + (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2
+ 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 24*(b*f^3*c
osh(d*x + c) + b*f^3*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2))/b) - 24*(b*f^3*cosh(d*x + c) + b*f^3*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
 c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 24*((b*d*f^3*x + b*d*e*f
^2)*cosh(d*x + c) + (b*d*f^3*x + b*d*e*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*((b*d*f^3*x + b*d*e*f^2)*cosh
(d*x + c) + (b*d*f^3*x + b*d*e*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*po
lylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*(b*d^3*e^2*f + 2*b*d^2*e*f^2 + 2*b*d*f
^3)*x - (a*d^4*f^3*x^4 + 4*a*d^4*e*f^2*x^3 + 6*a*d^4*e^2*f*x^2 + 4*a*d^4*e^
3*x)*cosh(d*x + c) - (a*d^4*f^3*x^4 + 4*a*d^4*e*f^2*x^3 + 6*a*d^4*e^2*f*x^2
 + 4*a*d^4*e^3*x - 4*(b*d^3*f^3*x^3 + b*d^3*e^3 - 3*b*d^2*e^2*f + 6*b*d*e*f
^2 - 6*b*f^3 + 3*(b*d^3*e*f^2 - b*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f - 2*b*d^2*e
*f^2 + 2*b*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c))/(b^2*d^4*cosh(d*x + c) +
 b^2*d^4*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^3\cosh\left(dx+c\right)^2}{b\sinh\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

3.295 $\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=389

$$\frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}-\frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^2}-\frac{2f^2\sqrt{a^2+b^2}\text{PolyLog}\left(3,-\right.}{b^2d^3}$$

```
[Out] -(a*(e + f*x)^3)/(3*b^2*f) + (2*f^2*Cosh[c + d*x])/(b*d^3) + ((e + f*x)^2*C
osh[c + d*x])/(b*d) + (Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])])/(b^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*d) + (2*Sqrt[a^2 + b^2]*f*(e + f*x
)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^2) - (2*Sqrt
[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/(b^2*d^2) - (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2]))])/(b^2*d^3) + (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^3) - (2*f*(e + f*x)*Sinh[c + d*x])/(b
*d^2)
```

Rubi [A] time = 0.785279, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5565, 32, 3296, 2638, 3322, 2264, 2190, 2531, 2282, 6589}

$$\frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}-\frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^2}-\frac{2f^2\sqrt{a^2+b^2}\text{PolyLog}\left(3,-\right.}{b^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -(a*(e + f*x)^3)/(3*b^2*f) + (2*f^2*Cosh[c + d*x])/(b*d^3) + ((e + f*x)^2*C
osh[c + d*x])/(b*d) + (Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])])/(b^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*d) + (2*Sqrt[a^2 + b^2]*f*(e + f*x
)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^2) - (2*Sqrt
[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/(b^2*d^2) - (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2]))])/(b^2*d^3) + (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^3) - (2*f*(e + f*x)*Sinh[c + d*x])/(b
*d^2)
```


Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\int (e+fx)^2\,dx}{b^2} + \frac{\int (e+fx)^2\sinh(c+dx)\,dx}{b} + \frac{\left(a^2+b^2\right)\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}\,dx}{b^2}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{(e+fx)^2\cosh(c+dx)}{bd} + \frac{\left(2\left(a^2+b^2\right)\right)\int \frac{e^{c+dx}(e+fx)^2}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{b^2} - \frac{(2f)\int (e}{}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{(e+fx)^2\cosh(c+dx)}{bd} - \frac{2f(e+fx)\sinh(c+dx)}{bd^2} + \frac{\left(2\sqrt{a^2+b^2}\right)\int \frac{e^c}{2a-2\sqrt{}}}{b}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cosh(c+dx)}{bd^3} + \frac{(e+fx)^2\cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\right.}{b^2d}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cosh(c+dx)}{bd^3} + \frac{(e+fx)^2\cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\right.}{b^2d}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cosh(c+dx)}{bd^3} + \frac{(e+fx)^2\cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\right.}{b^2d}$$

$$= -\frac{a(e+fx)^3}{3b^2f} + \frac{2f^2\cosh(c+dx)}{bd^3} + \frac{(e+fx)^2\cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\right.}{b^2d}$$

Mathematica [A] time = 2.91874, size = 447, normalized size = 1.15

$$-\frac{3\sqrt{a^2+b^2}\left(-2df(e+fx)\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+2df(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+2f^2\text{PolyLog}\left(3,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(a*d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2) + 3*Sqrt[a^2 + b^2]*(2*d^2*e^2*ArcTan
h[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2])] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d
^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)
*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyL
og[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 2*f^2*PolyLog[3, (b*E^(c
+ d*x))/(-a + Sqrt[a^2 + b^2])] - 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))]) - 3*b*Cosh[d*x]*((2*f^2 + d^2*(e + f*x)^2)*Cosh[c] - 2*d
*f*(e + f*x)*Sinh[c]) + 3*b*(2*d*f*(e + f*x)*Cosh[c] - (2*f^2 + d^2*(e + f*
x)^2)*Sinh[c])*Sinh[d*x])/(3*b^2*d^3)
```

Maple [F] time = 0.23, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^2(\cosh(dx+c))^2}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.75005, size = 3163, normalized size = 8.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*b*d^2*f^2*x^2 + 3*b*d^2*e^2 + 6*b*d*e*f + 6*b*f^2 + 3*(b*d^2*f^2*x^2
 + b*d^2*e^2 - 2*b*d*e*f + 2*b*f^2 + 2*(b*d^2*e*f - b*d*f^2)*x)*cosh(d*x +
c)^2 + 3*(b*d^2*f^2*x^2 + b*d^2*e^2 - 2*b*d*e*f + 2*b*f^2 + 2*(b*d^2*e*f -
b*d*f^2)*x)*sinh(d*x + c)^2 + 12*((b*d*f^2*x + b*d*e*f)*cosh(d*x + c) + (b*
d*f^2*x + b*d*e*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x +
 c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2) - b)/b + 1) - 12*((b*d*f^2*x + b*d*e*f)*cosh(d*x + c) + (b*d*f^2*x +
 b*d*e*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*s
inh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
b)/b + 1) - 6*((b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cosh(d*x + c) + (b*d^2
*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*
b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 6*
((b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cosh(d*x + c) + (b*d^2*e^2 - 2*b*c*d
*e*f + b*c^2*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c
) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 6*((b*d^2*f^2*x^
2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*cosh(d*x + c) + (b*d^2*f^2*x^2
 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 6*((b*d^2*f^2*x^2 + 2*b*d^2*e*f*x
+ 2*b*c*d*e*f - b*c^2*f^2)*cosh(d*x + c) + (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x +
 2*b*c*d*e*f - b*c^2*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh
(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b) - 12*(b*f^2*cosh(d*x + c) + b*f^2*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*
x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 12*(b*f^2*cosh(d*x +
c) + b*f^2*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c)
 + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2))/b) + 6*(b*d^2*e*f + b*d*f^2)*x - 2*(a*d^3*f^2*x^3 + 3*a*d^3*e*f*x^2 +
3*a*d^3*e^2*x)*cosh(d*x + c) - 2*(a*d^3*f^2*x^3 + 3*a*d^3*e*f*x^2 + 3*a*d^3
*e^2*x - 3*(b*d^2*f^2*x^2 + b*d^2*e^2 - 2*b*d*e*f + 2*b*f^2 + 2*(b*d^2*e*f
- b*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))/(b^2*d^3*cosh(d*x + c) + b^2*d^
3*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^2 \cosh\left(dx+c\right)^2}{b\sinh\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

3.296 $\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=252

$$\frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}-\frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^2}+\frac{\sqrt{a^2+b^2}(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^2d}-\frac{\sqrt{a}}{}$$

```
[Out] -((a*e*x)/b^2) - (a*f*x^2)/(2*b^2) + ((e + f*x)*Cosh[c + d*x])/(b*d) + (Sqr
t[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2
*d) - (Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(b^2*d) + (Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(b^2*d^2) - (Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(b^2*d^2) - (f*Sinh[c + d*x])/(b*d^2)
```

Rubi [A] time = 0.442924, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5565, 3296, 2637, 3322, 2264, 2190, 2279, 2391}

$$\frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}-\frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^2}+\frac{\sqrt{a^2+b^2}(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^2d}-\frac{\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -((a*e*x)/b^2) - (a*f*x^2)/(2*b^2) + ((e + f*x)*Cosh[c + d*x])/(b*d) + (Sqr
t[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2
*d) - (Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(b^2*d) + (Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(b^2*d^2) - (Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(b^2*d^2) - (f*Sinh[c + d*x])/(b*d^2)
```


Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\int(e+fx)\,dx}{b^2}+\frac{\int(e+fx)\sinh(c+dx)\,dx}{b}+\frac{\left(a^2+b^2\right)\int\frac{e+fx}{a+b\sinh(c+dx)}\,dx}{b^2}$$

$$= -\frac{aex}{b^2}-\frac{afx^2}{2b^2}+\frac{(e+fx)\cosh(c+dx)}{bd}+\frac{\left(2\left(a^2+b^2\right)\right)\int\frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{b^2}-\frac{f\int\cosh(\ldots}{b\ldots}$$

$$= -\frac{aex}{b^2}-\frac{afx^2}{2b^2}+\frac{(e+fx)\cosh(c+dx)}{bd}-\frac{f\sinh(c+dx)}{bd^2}+\frac{\left(2\sqrt{a^2+b^2}\right)\int\frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+\ldots}}}{b}$$

$$= -\frac{aex}{b^2}-\frac{afx^2}{2b^2}+\frac{(e+fx)\cosh(c+dx)}{bd}+\frac{\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{\sqrt{a^2+b^2}\ldots}{\ldots}$$

$$= -\frac{aex}{b^2}-\frac{afx^2}{2b^2}+\frac{(e+fx)\cosh(c+dx)}{bd}+\frac{\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{\sqrt{a^2+b^2}\ldots}{\ldots}$$

$$= -\frac{aex}{b^2}-\frac{afx^2}{2b^2}+\frac{(e+fx)\cosh(c+dx)}{bd}+\frac{\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{\sqrt{a^2+b^2}\ldots}{\ldots}$$

Mathematica [A] time = 1.95755, size = 258, normalized size = 1.02

$$\frac{2\sqrt{a^2+b^2}\left(f\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)-f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)-2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+f(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)-\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(c + d*x)*(c*f - d*(2*e + f*x)) + 2*b*d*(e + f*x)*Cosh[c + d*x] + 2*Sqrt
[a^2 + b^2]*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*Ar
cTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - f*P
olyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) - 2*b*f*Sinh[c + d*x])
/(2*b^2*d^2)
```

Maple [B] time = 0.089, size = 901, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/2*a*f*x^2/b^2-a*e*x/b^2+1/2*(d*f*x+d*e-f)/d^2/b*exp(d*x+c)+1/2*(d*f*x+d*
e+f)/d^2/b*exp(-d*x-c)-2*a^2/b^2/d*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d
*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+
c)+2*a)/(a^2+b^2)^(1/2))+a^2/b^2/d*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2
+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+a^2/b^2/d^2*f/(a^2+b^2)^(1/2)*ln((-b
*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-a^2/b^2/d*f/(a^2+b^2
)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-a^2/b^2/
d^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2
)))*c+a^2/b^2/d^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)
/(-a+(a^2+b^2)^(1/2)))-a^2/b^2/d^2*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a
^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/d*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+
c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d^2*f/(a^2+b^2)^(1/2)*ln((-
b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d*f/(a^2+b^2)^(1/
2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*f/(a^2+
b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d^2
*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1
/2)))-1/d^2*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^
2+b^2)^(1/2)))+2*a^2/b^2/d^2*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c
)+2*a)/(a^2+b^2)^(1/2))+2/d^2*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+
c)+2*a)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.32499, size = 1872, normalized size = 7.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*f*x + b*d*e + (b*d*f*x + b*d*e - b*f)*cosh(d*x + c)^2 + (b*d*f*x +
 b*d*e - b*f)*sinh(d*x + c)^2 + 2*(b*f*cosh(d*x + c) + b*f*sinh(d*x + c))*s
qrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b*f*cosh(d*
x + c) + b*f*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) +
a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
 - b)/b + 1) - 2*((b*d*e - b*c*f)*cosh(d*x + c) + (b*d*e - b*c*f)*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b
*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((b*d*e - b*c*f)*cosh(d*x + c) + (b*d*e -
 b*c*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*si
nh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((b*d*f*x + b*c*f)*cosh(
d*x + c) + (b*d*f*x + b*c*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*c
osh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((
a^2 + b^2)/b^2) - b)/b) - 2*((b*d*f*x + b*c*f)*cosh(d*x + c) + (b*d*f*x + b
*c*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d
*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b)
 + b*f - (a*d^2*f*x^2 + 2*a*d^2*e*x)*cosh(d*x + c) - (a*d^2*f*x^2 + 2*a*d^2
*e*x - 2*(b*d*f*x + b*d*e - b*f)*cosh(d*x + c))*sinh(d*x + c))/(b^2*d^2*cos
h(d*x + c) + b^2*d^2*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\cosh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

3.297 $\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)} dx$

Optimal. Leaf size=68

$$-\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{ax}{b^2}+\frac{\cosh(c+dx)}{bd}$$

```
[Out] -((a*x)/b^2) - (2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^
2 + b^2]])/(b^2*d) + Cosh[c + d*x]/(b*d)
```

Rubi [A] time = 0.116962, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2695, 2735, 2660, 618, 204}

$$-\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{ax}{b^2}+\frac{\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a*x)/b^2) - (2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^
2 + b^2]])/(b^2*d) + Cosh[c + d*x]/(b*d)
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{\cosh(c+dx)}{bd} + \frac{i\int \frac{-ib+ia\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}\\
&= -\frac{ax}{b^2} + \frac{\cosh(c+dx)}{bd} + \frac{\left(a^2+b^2\right)\int \frac{1}{a+b\sinh(c+dx)}\,dx}{b^2}\\
&= -\frac{ax}{b^2} + \frac{\cosh(c+dx)}{bd} - \frac{\left(2i\left(a^2+b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{b^2d}\\
&= -\frac{ax}{b^2} + \frac{\cosh(c+dx)}{bd} + \frac{\left(4i\left(a^2+b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^2+b^2\right)-x^2}\,dx, x, -2ib+2a\tan\left(\frac{1}{2}(ic+idx)\right)\right.}{b^2d}\\
&= -\frac{ax}{b^2} - \frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^2d} + \frac{\cosh(c+dx)}{bd}
\end{aligned}$$

Mathematica [C] time = 2.08619, size = 492, normalized size = 7.24

$$\frac{\cosh(c+dx)\left(2\sqrt{b^2}(a-ib)\sqrt{1+i\sinh(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a-ib}\sqrt{-\frac{b(\sinh(c+dx)+i)}{a-ib}}}{\sqrt{a+ib}\sqrt{-\frac{b(\sinh(c+dx)-i)}{a+ib}}}\right)+\sqrt{a+ib}\left(\sqrt{b^2}\sqrt{\frac{b(1+i\sinh(c+dx))}{b-ia}}\left(\sqrt{a-ib}\sqrt{1\right.\right.\right.}{b\sqrt{b^2}d\sqrt{a-ib}\sqrt{a+ib}\sqrt{1+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (Cosh[c + d*x]*(2*(a - I*b)*Sqrt[b^2]*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I +
 Sinh[c + d*x]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[c + d*x])
)/(a + I*b))])]*Sqrt[1 + I*Sinh[c + d*x]] + Sqrt[a + I*b]*((-2*I)*Sqrt[a -
I*b]*b*ArcTan[(Sqrt[(-I)*b]*Sqrt[-((b*(I + Sinh[c + d*x]))/(a - I*b))])/(Sq
rt[I*b]*Sqrt[-((b*(-I + Sinh[c + d*x]))/(a + I*b))])]*Sqrt[1 + I*Sinh[c + d
*x]] + Sqrt[b^2]*Sqrt[(b*(1 + I*Sinh[c + d*x]))/((-I)*a + b)]*(-2*(-1)^(3/4
)*Sqrt[b]*ArcSin[((-1)^(1/4)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[c + d*x]))/(
a - I*b))])/(Sqrt[2]*Sqrt[b])] + Sqrt[a - I*b]*Sqrt[1 + I*Sinh[c + d*x]]*Sq
rt[-((b*(I + Sinh[c + d*x]))/(a - I*b))]))))/(Sqrt[a - I*b]*Sqrt[a + I*b]*b
*Sqrt[b^2]*d*Sqrt[1 + I*Sinh[c + d*x]]*Sqrt[-((b*(-I + Sinh[c + d*x]))/(a +
 I*b))]*Sqrt[-((b*(I + Sinh[c + d*x]))/(a - I*b))])
```

Maple [B] time = 0.039, size = 174, normalized size = 2.6

$$\frac{1}{bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} - \frac{a}{db^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) - \frac{1}{bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} + \frac{a}{db^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/d/b/(tanh(1/2*d*x+1/2*c)+1)-1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/b/(ta
nh(1/2*d*x+1/2*c)-1)+1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)+2/d*a^2/b^2/(a^2+b
^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))+2/d/(a
^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.16567, size = 684, normalized size = 10.06

$$-\frac{2\,adx\cosh\left(dx+c\right)-b\cosh\left(dx+c\right)^{2}-b\sinh\left(dx+c\right)^{2}-2\,\sqrt{a^{2}+b^{2}}\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)\log\left(\frac{b^{2}\cosh(dx+c)\cdots}{\cdots}\right.\cdots}{2\left(b^{2}d\cos\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*x*cosh(d*x + c) - b*cosh(d*x + c)^2 - b*sinh(d*x + c)^2 - 2*sqr
t(a^2 + b^2)*(cosh(d*x + c) + sinh(d*x + c))*log((b^2*cosh(d*x + c)^2 + b^2
*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c)
 + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c
) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*c
osh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(a*d*x - b*cosh(d*x + c))*sinh(d*
x + c) - b)/(b^2*d*cosh(d*x + c) + b^2*d*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18774, size = 157, normalized size = 2.31

$$-\frac{(dx+c)a}{b^{2}d}+\frac{e^{(dx+c)}}{2\,bd}+\frac{e^{(-dx-c)}}{2\,bd}+\frac{\sqrt{a^{2}+b^{2}}\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -(d*x + c)*a/(b^2*d) + 1/2*e^(d*x + c)/(b*d) + 1/2*e^(-d*x - c)/(b*d) + sqr
t(a^2 + b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(
d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(b^2*d)
```

3.298 $\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Cosh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0748446, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cosh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Cosh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A] time = 25.0941, size = 0, normalized size = 0.

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cosh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Cosh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A] time = 0.111, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx+c))^2}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$2\left(a^2e^c + b^2e^c\right)\int -\frac{e^{(dx)}}{b^3fx + b^3e - \left(b^3fxe^{(2c)} + b^3ee^{(2c)}\right)e^{(2dx)} - 2\left(ab^2fxe^c + ab^2ee^c\right)e^{(dx)}}\,dx + \frac{e^{\left(-c+\frac{de}{f}\right)}E_1\left(\frac{(fx+e)d}{f}\right)}{2bf} - \frac{e^{\left(c-\frac{de}{f}\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*(a^2*e^c + b^2*e^c)*integrate(-e^(d*x)/(b^3*f*x + b^3*e - (b^3*f*x*e^(2*c
) + b^3*e*e^(2*c))*e^(2*d*x) - 2*(a*b^2*f*x*e^c + a*b^2*e*e^c)*e^(d*x)), x)
 + 1/2*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b*f) - 1/2*e^(c - d
*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b*f) - a*log(f*x + e)/(b^2*f)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx+c)^2}{afx + ae + \left(bfx + be\right)\sinh(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^2}{\left(fx+e\right)(b\sinh(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

3.299 $\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=642

$$-\frac{6f^2\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}-\frac{6f^2\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^3}+\frac{3f\left(a^2+b^2\right)(e+$$

```
[Out] (3*f^3*x)/(8*b*d^3) + (e + f*x)^3/(4*b*d) - ((a^2 + b^2)*(e + f*x)^4)/(4*b^
3*f) + (6*a*f^3*Cosh[c + d*x])/(b^2*d^4) + (3*a*f*(e + f*x)^2*Cosh[c + d*x]
)/(b^2*d^2) + ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2])])/(b^3*d) + ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2])])/(b^3*d) + (3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^2) + (3*(a^2 + b^2)*f*(e + f*x
)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^2) - (6*(a
^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))
])/(b^3*d^3) - (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2]))])/(b^3*d^3) + (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^4) + (6*(a^2 + b^2)*f^3*PolyLog[4,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^4) - (6*a*f^2*(e + f*x)*S
inh[c + d*x])/(b^2*d^3) - (a*(e + f*x)^3*Sinh[c + d*x])/(b^2*d) - (3*f^3*Co
sh[c + d*x]*Sinh[c + d*x])/(8*b*d^4) - (3*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[
c + d*x])/(4*b*d^2) + (3*f^2*(e + f*x)*Sinh[c + d*x]^2)/(4*b*d^3) + ((e + f
*x)^3*Sinh[c + d*x]^2)/(2*b*d)
```

Rubi [A] time = 0.784937, antiderivative size = 642, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5565, 3296, 2638, 5446, 3311, 32, 2635, 8, 5561, 2190, 2531, 6609, 2282, 6589}

$$-\frac{6f^2\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}-\frac{6f^2\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^3}+\frac{3f\left(a^2+b^2\right)(e+$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (3*f^3*x)/(8*b*d^3) + (e + f*x)^3/(4*b*d) - ((a^2 + b^2)*(e + f*x)^4)/(4*b^
3*f) + (6*a*f^3*Cosh[c + d*x])/(b^2*d^4) + (3*a*f*(e + f*x)^2*Cosh[c + d*x]
)/(b^2*d^2) + ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2])])/(b^3*d) + ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2])])/(b^3*d) + (3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^2) + (3*(a^2 + b^2)*f*(e + f*x
)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^2) - (6*(a
^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))
])/(b^3*d^3) - (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2]))])/(b^3*d^3) + (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^4) + (6*(a^2 + b^2)*f^3*PolyLog[4,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^4) - (6*a*f^2*(e + f*x)*S
inh[c + d*x])/(b^2*d^3) - (a*(e + f*x)^3*Sinh[c + d*x])/(b^2*d) - (3*f^3*Co
sh[c + d*x]*Sinh[c + d*x])/(8*b*d^4) - (3*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[
c + d*x])/(4*b*d^2) + (3*f^2*(e + f*x)*Sinh[c + d*x]^2)/(4*b*d^3) + ((e + f
*x)^3*Sinh[c + d*x]^2)/(2*b*d)
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

<u>Rule 3296</u>

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

<u>Rule 2638</u>

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

<u>Rule 5446</u>

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

<u>Rule 3311</u>

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

<u>Rule 32</u>

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

<u>Rule 2635</u>

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

<u>Rule 8</u>

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

<u>Rule 5561</u>

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

<u>Rule 2190</u>

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\int (e+fx)^3\cosh(c+dx)\,dx}{b^2} + \frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)\,dx}{b} + \frac{\left(a^2+b^2\right)\int}{}$$

$$= -\frac{\left(a^2+b^2\right)(e+fx)^4}{4b^3f} - \frac{a(e+fx)^3\sinh(c+dx)}{b^2d} + \frac{(e+fx)^3\sinh^2(c+dx)}{2bd} + \frac{\left(a^2+b^2\right)\int}{}$$

$$= -\frac{\left(a^2+b^2\right)(e+fx)^4}{4b^3f} + \frac{3af(e+fx)^2\cosh(c+dx)}{b^2d^2} + \frac{\left(a^2+b^2\right)(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b}}\right.}{b^3d}$$

$$= \frac{(e+fx)^3}{4bd} - \frac{\left(a^2+b^2\right)(e+fx)^4}{4b^3f} + \frac{3af(e+fx)^2\cosh(c+dx)}{b^2d^2} + \frac{\left(a^2+b^2\right)(e+fx)^3\log\left(\right.}{b^3d}$$

$$= \frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} - \frac{\left(a^2+b^2\right)(e+fx)^4}{4b^3f} + \frac{6af^3\cosh(c+dx)}{b^2d^4} + \frac{3af(e+fx)^2\cosh(c+d}{b^2d^2}$$

$$= \frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} - \frac{\left(a^2+b^2\right)(e+fx)^4}{4b^3f} + \frac{6af^3\cosh(c+dx)}{b^2d^4} + \frac{3af(e+fx)^2\cosh(c+d}{b^2d^2}$$

$$= \frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} - \frac{\left(a^2+b^2\right)(e+fx)^4}{4b^3f} + \frac{6af^3\cosh(c+dx)}{b^2d^4} + \frac{3af(e+fx)^2\cosh(c+d}{b^2d^2}$$

Mathematica [B] time = 23.9018, size = 10263, normalized size = 15.99

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.28, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^3 (\cosh(dx+c))^3}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*e^3*((4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) - 8*(a^2 + b^2)*(d
*x + c)/(b^3*d) - (4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d) - 8*(a^2
+ b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d)) + 1/32*(8*(
a^2*d^4*f^3*e^(2*c) + b^2*d^4*f^3*e^(2*c))*x^4 + 32*(a^2*d^4*e*f^2*e^(2*c)
+ b^2*d^4*e*f^2*e^(2*c))*x^3 + 48*(a^2*d^4*e^2*f*e^(2*c) + b^2*d^4*e^2*f*e^
(2*c))*x^2 + (4*b^2*d^3*f^3*x^3*e^(4*c) + 6*(2*d^3*e*f^2 - d^2*f^3)*b^2*x^2
*e^(4*c) + 6*(2*d^3*e^2*f - 2*d^2*e*f^2 + d*f^3)*b^2*x*e^(4*c) - 3*(2*d^2*e
^2*f - 2*d*e*f^2 + f^3)*b^2*e^(4*c))*e^(2*d*x) - 16*(a*b*d^3*f^3*x^3*e^(3*c
) + 3*(d^3*e*f^2 - d^2*f^3)*a*b*x^2*e^(3*c) + 3*(d^3*e^2*f - 2*d^2*e*f^2 +
2*d*f^3)*a*b*x*e^(3*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a*b*e^(3*c))*e^(
d*x) + 16*(a*b*d^3*f^3*x^3*e^c + 3*(d^3*e*f^2 + d^2*f^3)*a*b*x^2*e^c + 3*(d
^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*a*b*x*e^c + 3*(d^2*e^2*f + 2*d*e*f^2 + 2*
f^3)*a*b*e^c)*e^(-d*x) + (4*b^2*d^3*f^3*x^3 + 6*(2*d^3*e*f^2 + d^2*f^3)*b^2
*x^2 + 6*(2*d^3*e^2*f + 2*d^2*e*f^2 + d*f^3)*b^2*x + 3*(2*d^2*e^2*f + 2*d*e
*f^2 + f^3)*b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^4) - integrate(-2*((a^2*b*f^3
+ b^3*f^3)*x^3 + 3*(a^2*b*e*f^2 + b^3*e*f^2)*x^2 + 3*(a^2*b*e^2*f + b^3*e^2
*f)*x - ((a^3*f^3*e^c + a*b^2*f^3*e^c)*x^3 + 3*(a^3*e*f^2*e^c + a*b^2*e*f^2
*e^c)*x^2 + 3*(a^3*e^2*f*e^c + a*b^2*e^2*f*e^c)*x)*e^(d*x))/(b^4*e^(2*d*x +
 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x)
```

Fricas [C] time = 3.06476, size = 9852, normalized size = 15.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/32*(4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 + 6*b^2*d^2*e^2*f + 6*b^2*d*e*f^2 +
 3*b^2*f^3 + (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f + 6*b^2*d
*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^2*d^3*e
^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*cosh(d*x + c)^4 + (4*b^2*d^3*f^3*x^3
 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f + 6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d
^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^2*d^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*
f^3)*x)*sinh(d*x + c)^4 - 16*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3 - 3*a*b*d^2*e^2
*f + 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(a
*b*d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*x + c)^3 - 4*(4*a*b
*d^3*f^3*x^3 + 4*a*b*d^3*e^3 - 12*a*b*d^2*e^2*f + 24*a*b*d*e*f^2 - 24*a*b*f
^3 + 12*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 12*(a*b*d^3*e^2*f - 2*a*b*d^2*e
*f^2 + 2*a*b*d*f^3)*x - (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*
f + 6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(
2*b^2*d^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*cosh(d*x + c))*sinh(d*x +
 c)^3 + 6*(2*b^2*d^3*e*f^2 + b^2*d^2*f^3)*x^2 - 8*((a^2 + b^2)*d^4*f^3*x^4
+ 4*(a^2 + b^2)*d^4*e*f^2*x^3 + 6*(a^2 + b^2)*d^4*e^2*f*x^2 + 4*(a^2 + b^2)
*d^4*e^3*x + 8*(a^2 + b^2)*c*d^3*e^3 - 12*(a^2 + b^2)*c^2*d^2*e^2*f + 8*(a^
2 + b^2)*c^3*d*e*f^2 - 2*(a^2 + b^2)*c^4*f^3)*cosh(d*x + c)^2 - 2*(4*(a^2 +
 b^2)*d^4*f^3*x^4 + 16*(a^2 + b^2)*d^4*e*f^2*x^3 + 24*(a^2 + b^2)*d^4*e^2*f
*x^2 + 16*(a^2 + b^2)*d^4*e^3*x + 32*(a^2 + b^2)*c*d^3*e^3 - 48*(a^2 + b^2)
*c^2*d^2*e^2*f + 32*(a^2 + b^2)*c^3*d*e*f^2 - 8*(a^2 + b^2)*c^4*f^3 - 3*(4*
b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f + 6*b^2*d*e*f^2 - 3*b^2*f
^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^2*d^3*e^2*f - 2*b^2*d^2
*e*f^2 + b^2*d*f^3)*x)*cosh(d*x + c)^2 + 24*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3
- 3*a*b*d^2*e^2*f + 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b*d^3*e*f^2 - a*b*d^2*
f^3)*x^2 + 3*(a*b*d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*x +
c))*sinh(d*x + c)^2 + 6*(2*b^2*d^3*e^2*f + 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x +
 16*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3 + 3*a*b*d^2*e^2*f + 6*a*b*d*e*f^2 + 6*a*
b*f^3 + 3*(a*b*d^3*e*f^2 + a*b*d^2*f^3)*x^2 + 3*(a*b*d^3*e^2*f + 2*a*b*d^2*
```

```
e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*x + c) + 96*(((a^2 + b^2)*d^2*f^3*x^2 + 2*(a
^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*cosh(d*x + c)^2 + 2*((a^2 +
b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*cosh(
d*x + c)*sinh(d*x + c) + ((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2
*x + (a^2 + b^2)*d^2*e^2*f)*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sin
h(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)
/b + 1) + 96*(((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 +
 b^2)*d^2*e^2*f)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^
2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) + ((a^2
 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*si
nh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 32*(((a^2 + b^2)*d^3
*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*
c^3*f^3)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2
*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*cosh(d*x + c)*sinh(d*
x + c) + ((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c
^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) +
2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 32*(((a^2 + b^2)*d^3
*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*
c^3*f^3)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2
*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*cosh(d*x + c)*sinh(d*
x + c) + ((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c
^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) +
2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 32*(((a^2 + b^2)*d^3
*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + 3*(a^2
 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*f^3)*cosh
(d*x + c)^2 + 2*((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*
(a^2 + b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e
*f^2 + (a^2 + b^2)*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 + b^2)*d^3*
f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + 3*(a^2
+ b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*f^3)*sinh(
d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 32*(((a^2 + b^2)*d^3*f^3*x^3
 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*
c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*f^3)*cosh(d*x + c
)^2 + 2*((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b
^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (
a^2 + b^2)*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 + b^2)*d^3*f^3*x^3
+ 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c
*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*f^3)*sinh(d*x + c)
^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 192*((a^2 + b^2)*f^3*cosh(d*x + c)^2
 + 2*(a^2 + b^2)*f^3*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*f^3*sinh(d*x
 + c)^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 192*((a^2 + b^2)*f^3*cosh(d*x
 + c)^2 + 2*(a^2 + b^2)*f^3*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*f^3*s
inh(d*x + c)^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 192*(((a^2 + b^2)*d*f^
3*x + (a^2 + b^2)*d*e*f^2)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*d*f^3*x + (a^2
+ b^2)*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 + b^2)*d*f^3*x + (a^2 +
 b^2)*d*e*f^2)*sinh(d*x + c)^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x +
c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 192*((
(a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*cosh(d*x + c)^2 + 2*((a^2 + b^2)
*d*f^3*x + (a^2 + b^2)*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 + b^2)*
d*f^3*x + (a^2 + b^2)*d*e*f^2)*sinh(d*x + c)^2)*polylog(3, (a*cosh(d*x + c)
 + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2))/b) + 4*(4*a*b*d^3*f^3*x^3 + 4*a*b*d^3*e^3 + 12*a*b*d^2*e^2*f + 24*a*b*
d*e*f^2 + 24*a*b*f^3 + (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f
 + 6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2
```

```
*b^2*d^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*cosh(d*x + c)^3 + 12*(a*b*
d^3*e*f^2 + a*b*d^2*f^3)*x^2 - 12*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3 - 3*a*b*d^
2*e^2*f + 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x^2 +
 3*(a*b*d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*x + c)^2 + 12*
(a*b*d^3*e^2*f + 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x - 4*((a^2 + b^2)*d^4*f^3*
x^4 + 4*(a^2 + b^2)*d^4*e*f^2*x^3 + 6*(a^2 + b^2)*d^4*e^2*f*x^2 + 4*(a^2 +
b^2)*d^4*e^3*x + 8*(a^2 + b^2)*c*d^3*e^3 - 12*(a^2 + b^2)*c^2*d^2*e^2*f + 8
*(a^2 + b^2)*c^3*d*e*f^2 - 2*(a^2 + b^2)*c^4*f^3)*cosh(d*x + c))*sinh(d*x +
 c))/(b^3*d^4*cosh(d*x + c)^2 + 2*b^3*d^4*cosh(d*x + c)*sinh(d*x + c) + b^3
*d^4*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3\cosh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

3.300 $\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=477

$$\frac{2f\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{2f\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2}-\frac{2f^2\left(a^2+b^2\right)\text{PolyLog}\left(\right.}{b^3d^3}$$

```
[Out] (e*f*x)/(2*b*d) + (f^2*x^2)/(4*b*d) - ((a^2 + b^2)*(e + f*x)^3)/(3*b^3*f) +
 (2*a*f*(e + f*x)*Cosh[c + d*x])/(b^2*d^2) + ((a^2 + b^2)*(e + f*x)^2*Log[1
 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) + ((a^2 + b^2)*(e + f*x)
^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*d) + (2*(a^2 + b^2)
*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^2
) + (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/(b^3*d^2) - (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2]))])/(b^3*d^3) - (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^3) - (2*a*f^2*Sinh[c + d*x])/(b^2*d^3)
 - (a*(e + f*x)^2*Sinh[c + d*x])/(b^2*d) - (f*(e + f*x)*Cosh[c + d*x]*Sinh[
c + d*x])/(2*b*d^2) + (f^2*Sinh[c + d*x]^2)/(4*b*d^3) + ((e + f*x)^2*Sinh[c
 + d*x]^2)/(2*b*d)
```

Rubi [A] time = 0.642494, antiderivative size = 477, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5565, 3296, 2637, 5446, 3310, 5561, 2190, 2531, 2282, 6589}

$$\frac{2f\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{2f\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2}-\frac{2f^2\left(a^2+b^2\right)\text{PolyLog}\left(\right.}{b^3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (e*f*x)/(2*b*d) + (f^2*x^2)/(4*b*d) - ((a^2 + b^2)*(e + f*x)^3)/(3*b^3*f) +
 (2*a*f*(e + f*x)*Cosh[c + d*x])/(b^2*d^2) + ((a^2 + b^2)*(e + f*x)^2*Log[1
 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) + ((a^2 + b^2)*(e + f*x)
^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*d) + (2*(a^2 + b^2)
*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^2
) + (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/(b^3*d^2) - (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2]))])/(b^3*d^3) - (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^3) - (2*a*f^2*Sinh[c + d*x])/(b^2*d^3)
 - (a*(e + f*x)^2*Sinh[c + d*x])/(b^2*d) - (f*(e + f*x)*Cosh[c + d*x]*Sinh[
c + d*x])/(2*b*d^2) + (f^2*Sinh[c + d*x]^2)/(4*b*d^3) + ((e + f*x)^2*Sinh[c
 + d*x]^2)/(2*b*d)
```


Rule *5565*

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule *3296*

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
```

```
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\int (e+fx)^2\cosh(c+dx)\,dx}{b^2}+\frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)\,dx}{b}+\frac{\left(a^2+b^2\right)\int}{}$$

$$= -\frac{\left(a^2+b^2\right)(e+fx)^3}{3b^3f}-\frac{a(e+fx)^2\sinh(c+dx)}{b^2d}+\frac{(e+fx)^2\sinh^2(c+dx)}{2bd}+\frac{\left(a^2+b^2\right)\int}{}$$

$$= -\frac{\left(a^2+b^2\right)(e+fx)^3}{3b^3f}+\frac{2af(e+fx)\cosh(c+dx)}{b^2d^2}+\frac{\left(a^2+b^2\right)(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{b^3d}$$

$$= \frac{efx}{2bd}+\frac{f^2x^2}{4bd}-\frac{\left(a^2+b^2\right)(e+fx)^3}{3b^3f}+\frac{2af(e+fx)\cosh(c+dx)}{b^2d^2}+\frac{\left(a^2+b^2\right)(e+fx)^2\log}{b^3d}$$

$$= \frac{efx}{2bd}+\frac{f^2x^2}{4bd}-\frac{\left(a^2+b^2\right)(e+fx)^3}{3b^3f}+\frac{2af(e+fx)\cosh(c+dx)}{b^2d^2}+\frac{\left(a^2+b^2\right)(e+fx)^2\log}{b^3d}$$

$$= \frac{efx}{2bd}+\frac{f^2x^2}{4bd}-\frac{\left(a^2+b^2\right)(e+fx)^3}{3b^3f}+\frac{2af(e+fx)\cosh(c+dx)}{b^2d^2}+\frac{\left(a^2+b^2\right)(e+fx)^2\log}{b^3d}$$

Mathematica [B] time = 17.6291, size = 3021, normalized size = 6.33

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a^2 + b^2)*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6
*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[
-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^
(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2
)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*
d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt
[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 +
 E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(
c + d*x)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^
2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt
[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c -
Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x
))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d
*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*
E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 +
 (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2
*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6
*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a
^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*
E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*f^2*PolyLog[
3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2
*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))
])/d^3 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^
(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqr
```

```
t[(a^2 + b^2)*E^(2*c)]))])/d^3))/(3*b^3*(-1 + E^(2*c))) + Csch[c]*(Cosh[2*c
 + 2*d*x]/(96*b^3*d^3) - Sinh[2*c + 2*d*x]/(96*b^3*d^3))*(-24*a*b*d^2*e^2*C
osh[d*x] - 48*a*b*d*e*f*Cosh[d*x] - 48*a*b*f^2*Cosh[d*x] - 48*a*b*d^2*e*f*x
*Cosh[d*x] - 48*a*b*d*f^2*x*Cosh[d*x] - 24*a*b*d^2*f^2*x^2*Cosh[d*x] + 24*a
*b*d^2*e^2*Cosh[2*c + d*x] + 48*a*b*d*e*f*Cosh[2*c + d*x] + 48*a*b*f^2*Cosh
[2*c + d*x] + 48*a*b*d^2*e*f*x*Cosh[2*c + d*x] + 48*a*b*d*f^2*x*Cosh[2*c +
d*x] + 24*a*b*d^2*f^2*x^2*Cosh[2*c + d*x] + 48*a^2*d^3*e^2*x*Cosh[c + 2*d*x
] + 48*b^2*d^3*e^2*x*Cosh[c + 2*d*x] + 48*a^2*d^3*e*f*x^2*Cosh[c + 2*d*x] +
 48*b^2*d^3*e*f*x^2*Cosh[c + 2*d*x] + 16*a^2*d^3*f^2*x^3*Cosh[c + 2*d*x] +
16*b^2*d^3*f^2*x^3*Cosh[c + 2*d*x] + 48*a^2*d^3*e^2*x*Cosh[3*c + 2*d*x] + 4
8*b^2*d^3*e^2*x*Cosh[3*c + 2*d*x] + 48*a^2*d^3*e*f*x^2*Cosh[3*c + 2*d*x] +
48*b^2*d^3*e*f*x^2*Cosh[3*c + 2*d*x] + 16*a^2*d^3*f^2*x^3*Cosh[3*c + 2*d*x]
 + 16*b^2*d^3*f^2*x^3*Cosh[3*c + 2*d*x] + 24*a*b*d^2*e^2*Cosh[2*c + 3*d*x]
- 48*a*b*d*e*f*Cosh[2*c + 3*d*x] + 48*a*b*f^2*Cosh[2*c + 3*d*x] + 48*a*b*d^
2*e*f*x*Cosh[2*c + 3*d*x] - 48*a*b*d*f^2*x*Cosh[2*c + 3*d*x] + 24*a*b*d^2*f
^2*x^2*Cosh[2*c + 3*d*x] - 24*a*b*d^2*e^2*Cosh[4*c + 3*d*x] + 48*a*b*d*e*f*
Cosh[4*c + 3*d*x] - 48*a*b*f^2*Cosh[4*c + 3*d*x] - 48*a*b*d^2*e*f*x*Cosh[4*
c + 3*d*x] + 48*a*b*d*f^2*x*Cosh[4*c + 3*d*x] - 24*a*b*d^2*f^2*x^2*Cosh[4*c
 + 3*d*x] - 6*b^2*d^2*e^2*Cosh[3*c + 4*d*x] + 6*b^2*d*e*f*Cosh[3*c + 4*d*x]
 - 3*b^2*f^2*Cosh[3*c + 4*d*x] - 12*b^2*d^2*e*f*x*Cosh[3*c + 4*d*x] + 6*b^2
*d*f^2*x*Cosh[3*c + 4*d*x] - 6*b^2*d^2*f^2*x^2*Cosh[3*c + 4*d*x] + 6*b^2*d^
2*e^2*Cosh[5*c + 4*d*x] - 6*b^2*d*e*f*Cosh[5*c + 4*d*x] + 3*b^2*f^2*Cosh[5*
c + 4*d*x] + 12*b^2*d^2*e*f*x*Cosh[5*c + 4*d*x] - 6*b^2*d*f^2*x*Cosh[5*c +
4*d*x] + 6*b^2*d^2*f^2*x^2*Cosh[5*c + 4*d*x] + 12*b^2*d^2*e^2*Sinh[c] + 12*
b^2*d*e*f*Sinh[c] + 6*b^2*f^2*Sinh[c] + 24*b^2*d^2*e*f*x*Sinh[c] + 12*b^2*d
*f^2*x*Sinh[c] + 12*b^2*d^2*f^2*x^2*Sinh[c] - 24*a*b*d^2*e^2*Sinh[d*x] - 48
*a*b*d*e*f*Sinh[d*x] - 48*a*b*f^2*Sinh[d*x] - 48*a*b*d^2*e*f*x*Sinh[d*x] -
48*a*b*d*f^2*x*Sinh[d*x] - 24*a*b*d^2*f^2*x^2*Sinh[d*x] + 24*a*b*d^2*e^2*Si
nh[2*c + d*x] + 48*a*b*d*e*f*Sinh[2*c + d*x] + 48*a*b*f^2*Sinh[2*c + d*x] +
 48*a*b*d^2*e*f*x*Sinh[2*c + d*x] + 48*a*b*d*f^2*x*Sinh[2*c + d*x] + 24*a*b
*d^2*f^2*x^2*Sinh[2*c + d*x] + 48*a^2*d^3*e^2*x*Sinh[c + 2*d*x] + 48*b^2*d^
3*e^2*x*Sinh[c + 2*d*x] + 48*a^2*d^3*e*f*x^2*Sinh[c + 2*d*x] + 48*b^2*d^3*e
*f*x^2*Sinh[c + 2*d*x] + 16*a^2*d^3*f^2*x^3*Sinh[c + 2*d*x] + 16*b^2*d^3*f^
2*x^3*Sinh[c + 2*d*x] + 48*a^2*d^3*e^2*x*Sinh[3*c + 2*d*x] + 48*b^2*d^3*e^2
*x*Sinh[3*c + 2*d*x] + 48*a^2*d^3*e*f*x^2*Sinh[3*c + 2*d*x] + 48*b^2*d^3*e*
f*x^2*Sinh[3*c + 2*d*x] + 16*a^2*d^3*f^2*x^3*Sinh[3*c + 2*d*x] + 16*b^2*d^3
*f^2*x^3*Sinh[3*c + 2*d*x] + 24*a*b*d^2*e^2*Sinh[2*c + 3*d*x] - 48*a*b*d*e*
f*Sinh[2*c + 3*d*x] + 48*a*b*f^2*Sinh[2*c + 3*d*x] + 48*a*b*d^2*e*f*x*Sinh[
2*c + 3*d*x] - 48*a*b*d*f^2*x*Sinh[2*c + 3*d*x] + 24*a*b*d^2*f^2*x^2*Sinh[2
*c + 3*d*x] - 24*a*b*d^2*e^2*Sinh[4*c + 3*d*x] + 48*a*b*d*e*f*Sinh[4*c + 3*
d*x] - 48*a*b*f^2*Sinh[4*c + 3*d*x] - 48*a*b*d^2*e*f*x*Sinh[4*c + 3*d*x] +
48*a*b*d*f^2*x*Sinh[4*c + 3*d*x] - 24*a*b*d^2*f^2*x^2*Sinh[4*c + 3*d*x] - 6
*b^2*d^2*e^2*Sinh[3*c + 4*d*x] + 6*b^2*d*e*f*Sinh[3*c + 4*d*x] - 3*b^2*f^2*
Sinh[3*c + 4*d*x] - 12*b^2*d^2*e*f*x*Sinh[3*c + 4*d*x] + 6*b^2*d*f^2*x*Sinh
[3*c + 4*d*x] - 6*b^2*d^2*f^2*x^2*Sinh[3*c + 4*d*x] + 6*b^2*d^2*e^2*Sinh[5*
c + 4*d*x] - 6*b^2*d*e*f*Sinh[5*c + 4*d*x] + 3*b^2*f^2*Sinh[5*c + 4*d*x] +
12*b^2*d^2*e*f*x*Sinh[5*c + 4*d*x] - 6*b^2*d*f^2*x*Sinh[5*c + 4*d*x] + 6*b^
2*d^2*f^2*x^2*Sinh[5*c + 4*d*x])
```

Maple [F] time = 0.234, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 (\cosh(dx+c))^3}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}e^{2}\left(\frac{\left(4\,ae^{(-dx-c)}-b\right)e^{(2\,dx+2\,c)}}{b^{2}d}-\frac{8\left(a^{2}+b^{2}\right)(dx+c)}{b^{3}d}-\frac{4\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}}{b^{2}d}-\frac{8\left(a^{2}+b^{2}\right)\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-}\right.}{b^{3}d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*e^2*((4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) - 8*(a^2 + b^2)*(d
*x + c)/(b^3*d) - (4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d) - 8*(a^2
+ b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d)) + 1/48*(16*
(a^2*d^3*f^2*e^(2*c) + b^2*d^3*f^2*e^(2*c))*x^3 + 48*(a^2*d^3*e*f*e^(2*c) +
 b^2*d^3*e*f*e^(2*c))*x^2 + 3*(2*b^2*d^2*f^2*x^2*e^(4*c) + 2*(2*d^2*e*f - d
*f^2)*b^2*x*e^(4*c) - (2*d*e*f - f^2)*b^2*e^(4*c))*e^(2*d*x) - 24*(a*b*d^2*
f^2*x^2*e^(3*c) + 2*(d^2*e*f - d*f^2)*a*b*x*e^(3*c) - 2*(d*e*f - f^2)*a*b*e
^(3*c))*e^(d*x) + 24*(a*b*d^2*f^2*x^2*e^c + 2*(d^2*e*f + d*f^2)*a*b*x*e^c +
 2*(d*e*f + f^2)*a*b*e^c)*e^(-d*x) + 3*(2*b^2*d^2*f^2*x^2 + 2*(2*d^2*e*f +
d*f^2)*b^2*x + (2*d*e*f + f^2)*b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^3) - integr
ate(-2*((a^2*b*f^2 + b^3*f^2)*x^2 + 2*(a^2*b*e*f + b^3*e*f)*x - ((a^3*f^2*e
^c + a*b^2*f^2*e^c)*x^2 + 2*(a^3*e*f*e^c + a*b^2*e*f*e^c)*x)*e^(d*x))/(b^4*
e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x)
```

Fricas [C] time = 2.64666, size = 6356, normalized size = 13.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(6*b^2*d^2*f^2*x^2 + 6*b^2*d^2*e^2 + 6*b^2*d*e*f + 3*(2*b^2*d^2*f^2*x^
2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x
)*cosh(d*x + c)^4 + 3*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^
2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*sinh(d*x + c)^4 + 3*b^2*f^2 - 24*(
a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f -
a*b*d*f^2)*x)*cosh(d*x + c)^3 - 12*(2*a*b*d^2*f^2*x^2 + 2*a*b*d^2*e^2 - 4*a
*b*d*e*f + 4*a*b*f^2 + 4*(a*b*d^2*e*f - a*b*d*f^2)*x - (2*b^2*d^2*f^2*x^2 +
 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*c
osh(d*x + c))*sinh(d*x + c)^3 - 16*((a^2 + b^2)*d^3*f^2*x^3 + 3*(a^2 + b^2)
*d^3*e*f*x^2 + 3*(a^2 + b^2)*d^3*e^2*x + 6*(a^2 + b^2)*c*d^2*e^2 - 6*(a^2 +
 b^2)*c^2*d*e*f + 2*(a^2 + b^2)*c^3*f^2)*cosh(d*x + c)^2 - 2*(8*(a^2 + b^2)
*d^3*f^2*x^3 + 24*(a^2 + b^2)*d^3*e*f*x^2 + 24*(a^2 + b^2)*d^3*e^2*x + 48*(
a^2 + b^2)*c*d^2*e^2 - 48*(a^2 + b^2)*c^2*d*e*f + 16*(a^2 + b^2)*c^3*f^2 -
9*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2
*e*f - b^2*d*f^2)*x)*cosh(d*x + c)^2 + 36*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 -
2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*x)*cosh(d*x + c))*sin
h(d*x + c)^2 + 6*(2*b^2*d^2*e*f + b^2*d*f^2)*x + 24*(a*b*d^2*f^2*x^2 + a*b*
d^2*e^2 + 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f + a*b*d*f^2)*x)*cosh(d*x
 + c) + 96*(((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c)^2 + 2*(
(a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + ((a^
2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*sinh(d*x + c)^2)*dilog((a*cosh(d*x +
```

```
c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2) - b)/b + 1) + 96*(((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x
+ c)^2 + 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c)*sinh(d*x
 + c) + ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*sinh(d*x + c)^2)*dilog((a
*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b + 1) + 48*(((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*
d*e*f + (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*d^2*e^2 - 2*(
a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + ((a
^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*sinh(d*x +
 c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2
) + 2*a) + 48*(((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c
^2*f^2)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f +
(a^2 + b^2)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 + b^2)*d^2*e^2 - 2
*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*sinh(d*x + c)^2)*log(2*b*cosh(d
*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 48*(((a^2
+ b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2
 + b^2)*c^2*f^2)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^
2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)*s
inh(d*x + c) + ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2
+ b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c
) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2) - b)/b) + 48*(((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(
a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*
d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)
*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 +
 b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*sinh(d*x + c
)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 96*((a^2 + b^2)*f^2*cosh(d*x + c)^2
 + 2*(a^2 + b^2)*f^2*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*f^2*sinh(d*x
 + c)^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 96*((a^2 + b^2)*f^2*cosh(d*x
+ c)^2 + 2*(a^2 + b^2)*f^2*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*f^2*si
nh(d*x + c)^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 4*(6*a*b*d^2*f^2*x^2 +
6*a*b*d^2*e^2 + 12*a*b*d*e*f + 12*a*b*f^2 + 3*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^
2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*cosh(d*x +
 c)^3 - 18*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*
b*d^2*e*f - a*b*d*f^2)*x)*cosh(d*x + c)^2 + 12*(a*b*d^2*e*f + a*b*d*f^2)*x
- 8*((a^2 + b^2)*d^3*f^2*x^3 + 3*(a^2 + b^2)*d^3*e*f*x^2 + 3*(a^2 + b^2)*d^
3*e^2*x + 6*(a^2 + b^2)*c*d^2*e^2 - 6*(a^2 + b^2)*c^2*d*e*f + 2*(a^2 + b^2)
*c^3*f^2)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d^3*cosh(d*x + c)^2 + 2*b^3*d^
3*cosh(d*x + c)*sinh(d*x + c) + b^3*d^3*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \cosh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

3.301 $\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=298

$$\frac{f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2}+\frac{\left(a^2+b^2\right)(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3d}+$$

```
[Out] (f*x)/(4*b*d) - ((a^2 + b^2)*(e + f*x)^2)/(2*b^3*f) + (a*f*Cosh[c + d*x])/(
b^2*d^2) + ((a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])])/(b^3*d) + ((a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/(b^3*d) + ((a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2]))])/(b^3*d^2) + ((a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2]))])/(b^3*d^2) - (a*(e + f*x)*Sinh[c + d*x])/(b^2*d) - (f
*Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^2) + ((e + f*x)*Sinh[c + d*x]^2)/(2*b*
d)
```

Rubi [A] time = 0.357755, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5565, 3296, 2638, 5446, 2635, 8, 5561, 2190, 2279, 2391}

$$\frac{f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2}+\frac{\left(a^2+b^2\right)(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3d}+$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (f*x)/(4*b*d) - ((a^2 + b^2)*(e + f*x)^2)/(2*b^3*f) + (a*f*Cosh[c + d*x])/(
b^2*d^2) + ((a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])])/(b^3*d) + ((a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/(b^3*d) + ((a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2]))])/(b^3*d^2) + ((a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2]))])/(b^3*d^2) - (a*(e + f*x)*Sinh[c + d*x])/(b^2*d) - (f
*Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^2) + ((e + f*x)*Sinh[c + d*x]^2)/(2*b*
d)
```


Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{a\int(e+fx)\cosh(c+dx)\,dx}{b^2} + \frac{\int(e+fx)\cosh(c+dx)\sinh(c+dx)\,dx}{b} + \frac{\left(a^2+b^2\right)\int}{}$$

$$= -\frac{\left(a^2+b^2\right)(e+fx)^2}{2b^3f} - \frac{a(e+fx)\sinh(c+dx)}{b^2d} + \frac{(e+fx)\sinh^2(c+dx)}{2bd} + \frac{\left(a^2+b^2\right)\int}{}$$

$$= -\frac{\left(a^2+b^2\right)(e+fx)^2}{2b^3f} + \frac{af\cosh(c+dx)}{b^2d^2} + \frac{\left(a^2+b^2\right)(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d} + \frac{\left(a^2\right.}{}$$

$$= \frac{fx}{4bd} - \frac{\left(a^2+b^2\right)(e+fx)^2}{2b^3f} + \frac{af\cosh(c+dx)}{b^2d^2} + \frac{\left(a^2+b^2\right)(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d}$$

$$= \frac{fx}{4bd} - \frac{\left(a^2+b^2\right)(e+fx)^2}{2b^3f} + \frac{af\cosh(c+dx)}{b^2d^2} + \frac{\left(a^2+b^2\right)(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d}$$

Mathematica [A] time = 1.3663, size = 251, normalized size = 0.84

$$\frac{8\left(a^2+b^2\right)\left(f\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+f(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)+f(c+dx)\log\left(\frac{be^{c}}{\sqrt{a^2+}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (8*a*b*f*Cosh[c + d*x] + 2*b^2*d*(e + f*x)*Cosh[2*(c + d*x)] + 8*(a^2 + b^2
)*(-(f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d*e*
Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLog[2, (b*E
^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))]) - 8*a*b*d*(e + f*x)*Sinh[c + d*x] - b^2*f*Sinh[2*(c + d*
x)])/(8*b^3*d^2)
```

Maple [B] time = 0.11, size = 975, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/2*f*x^2/b+1/16*(2*d*f*x+2*d*e-f)/d^2/b*exp(2*d*x+2*c)+1/16*(2*d*f*x+2*d*
e+f)/d^2/b*exp(-2*d*x-2*c)-1/d^2/b*f*c^2+1/d/b*e*ln(b*exp(2*d*x+2*c)+2*a*ex
p(d*x+c)-b)-2/d/b*e*ln(exp(d*x+c))+1/d^2/b*f*dilog((b*exp(d*x+c)+(a^2+b^2)^
(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/d^2/b*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2
)-a)/(-a+(a^2+b^2)^(1/2)))-1/2*a^2*f*x^2/b^3-1/b^3/d^2*a^2*f*c*ln(b*exp(2*d
*x+2*c)+2*a*exp(d*x+c)-b)+2/b^3/d^2*a^2*f*c*ln(exp(d*x+c))+1/b^3/d*a^2*f*ln
((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/b^3/d^2*a^2*f*ln
((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/b^3/d*a^2*f*ln((
-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/b^3/d^2*a^2*f*ln
((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/b^3/d*a^2*f*c*
x+a^2*e*x/b^3+e*x/b-1/b^3/d^2*a^2*f*c^2+1/b^3/d^2*a^2*f*dilog((b*exp(d*x+c)
+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/b^3/d^2*a^2*f*dilog((-b*exp(d*x+
```

```
c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/b^3/d*a^2*e*ln(b*exp(2*d*x+2*
c)+2*a*exp(d*x+c)-b)-2/b^3/d*a^2*e*ln(exp(d*x+c))-2/d/b*f*c*x-1/d^2/b*f*c*l
n(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d^2/b*f*c*ln(exp(d*x+c))+1/d/b*f*ln(
(-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d^2/b*f*ln((-b*
exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d/b*f*ln((b*exp(d*x
+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2/b*f*ln((b*exp(d*x+c)+(a
^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/2*a*(d*f*x+d*e+f)/b^2/d^2*exp(-d*
x-c)-1/2*a*(d*f*x+d*e-f)/b^2/d^2*exp(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}e\left(\frac{\left(4\,ae^{(-dx-c)}-b\right)e^{(2\,dx+2\,c)}}{b^2d}-\frac{8\left(a^2+b^2\right)(dx+c)}{b^3d}-\frac{4\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}}{b^2d}-\frac{8\left(a^2+b^2\right)\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2}\right.}{b^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*e*((4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) - 8*(a^2 + b^2)*(d*x
 + c)/(b^3*d) - (4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d) - 8*(a^2 +
b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d)) + 1/16*f*((8*
(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*x^2 + (2*b^2*d*x*e^(4*c) - b^2*e^(4*c))
*e^(2*d*x) - 8*(a*b*d*x*e^(3*c) - a*b*e^(3*c))*e^(d*x) + 8*(a*b*d*x*e^c + a
*b*e^c)*e^(-d*x) + (2*b^2*d*x + b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^2) - 2*int
egrate(16*((a^3*e^c + a*b^2*e^c)*x*e^(d*x) - (a^2*b + b^3)*x)/(b^4*e^(2*d*x
 + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x))
```

Fricas [B] time = 2.36882, size = 3507, normalized size = 11.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(2*b^2*d*f*x + (2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^4 + (2*
b^2*d*f*x + 2*b^2*d*e - b^2*f)*sinh(d*x + c)^4 + 2*b^2*d*e - 8*(a*b*d*f*x +
 a*b*d*e - a*b*f)*cosh(d*x + c)^3 - 4*(2*a*b*d*f*x + 2*a*b*d*e - 2*a*b*f -
(2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c))*sinh(d*x + c)^3 + b^2*f -
8*((a^2 + b^2)*d^2*f*x^2 + 2*(a^2 + b^2)*d^2*e*x + 4*(a^2 + b^2)*c*d*e - 2*
(a^2 + b^2)*c^2*f)*cosh(d*x + c)^2 - 2*(4*(a^2 + b^2)*d^2*f*x^2 + 8*(a^2 +
b^2)*d^2*e*x + 16*(a^2 + b^2)*c*d*e - 8*(a^2 + b^2)*c^2*f - 3*(2*b^2*d*f*x
+ 2*b^2*d*e - b^2*f)*cosh(d*x + c)^2 + 12*(a*b*d*f*x + a*b*d*e - a*b*f)*cos
h(d*x + c))*sinh(d*x + c)^2 + 8*(a*b*d*f*x + a*b*d*e + a*b*f)*cosh(d*x + c)
 + 16*((a^2 + b^2)*f*cosh(d*x + c)^2 + 2*(a^2 + b^2)*f*cosh(d*x + c)*sinh(d
*x + c) + (a^2 + b^2)*f*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*
x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b +
 1) + 16*((a^2 + b^2)*f*cosh(d*x + c)^2 + 2*(a^2 + b^2)*f*cosh(d*x + c)*sin
h(d*x + c) + (a^2 + b^2)*f*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh
(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/
b + 1) + 16*(((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)^2 + 2*((a^2
+ b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 + b^2)*d*
e - (a^2 + b^2)*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x
+ c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 16*(((a^2 + b^2)*d*e - (a^2 + b^2
```

```
)*c*f)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c
)*sinh(d*x + c) + ((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*sinh(d*x + c)^2)*log(
2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) +
16*(((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*
d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 + b^2)*d*f*x +
 (a^2 + b^2)*c*f)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
+ (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 16*((
(a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*d*f*x
 + (a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 + b^2)*d*f*x + (a^2
 + b^2)*c*f)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*(2*a*b*d
*f*x + 2*a*b*d*e + (2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^3 + 2*a*
b*f - 6*(a*b*d*f*x + a*b*d*e - a*b*f)*cosh(d*x + c)^2 - 4*((a^2 + b^2)*d^2*
f*x^2 + 2*(a^2 + b^2)*d^2*e*x + 4*(a^2 + b^2)*c*d*e - 2*(a^2 + b^2)*c^2*f)*
cosh(d*x + c))*sinh(d*x + c))/(b^3*d^2*cosh(d*x + c)^2 + 2*b^3*d^2*cosh(d*x
 + c)*sinh(d*x + c) + b^3*d^2*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**3/(a+b*sinh(d*x+c)),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\cosh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")

[Out] integrate((f*x + e)*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

3.302 $\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{b^3d}-\frac{a\sinh(c+dx)}{b^2d}+\frac{\sinh^2(c+dx)}{2bd}$$

```
[Out] ((a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/(b^3*d) - (a*Sinh[c + d*x])/(b^2*d)
+ Sinh[c + d*x]^2/(2*b*d)
```

Rubi [A] time = 0.0692951, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{b^3d}-\frac{a\sinh(c+dx)}{b^2d}+\frac{\sinh^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/(b^3*d) - (a*Sinh[c + d*x])/(b^2*d)
+ Sinh[c + d*x]^2/(2*b*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{-b^2-x^2}{a+x}\,dx, x, b\sinh(c+dx)\right)}{b^3d}\\
&= -\frac{\text{Subst}\left(\int \left(a-x+\frac{-a^2-b^2}{a+x}\right)dx, x, b\sinh(c+dx)\right)}{b^3d}\\
&= \frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{b^3d}-\frac{a\sinh(c+dx)}{b^2d}+\frac{\sinh^2(c+dx)}{2bd}
\end{aligned}$$

Mathematica [A] time = 0.0505462, size = 53, normalized size = 0.9

$$-\frac{-(a^2+b^2)\log(a+b\sinh(c+dx))+ab\sinh(c+dx)-\frac{1}{2}b^2\sinh^2(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((-((a^2 + b^2)*Log[a + b*Sinh[c + d*x]]) + a*b*Sinh[c + d*x] - (b^2*Sinh[
c + d*x]^2)/2)/(b^3*d))
```

Maple [B] time = 0.043, size = 291, normalized size = 4.9

$$\frac{1}{2\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}-\frac{1}{2\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+\frac{a}{db^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{a^2}{db^3}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2-1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)+1/d/b^2/(
tanh(1/2*d*x+1/2*c)+1)*a-1/d/b^3*ln(tanh(1/2*d*x+1/2*c)+1)*a^2-1/d/b*ln(tan
h(1/2*d*x+1/2*c)+1)+1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/b/(tanh(1/2*d*x
+1/2*c)-1)+1/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*a-1/d/b^3*ln(tanh(1/2*d*x+1/2*c)
-1)*a^2-1/d/b*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/b^3*ln(tanh(1/2*d*x+1/2*c)^2*a-
2*tanh(1/2*d*x+1/2*c)*b-a)*a^2+1/d/b*ln(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*
d*x+1/2*c)*b-a)
```

Maxima [B] time = 1.16017, size = 171, normalized size = 2.9

$$-\frac{\left(4\,ae^{(-dx-c)}-b\right)e^{(2\,dx+2\,c)}}{8\,b^2d}+\frac{\left(a^2+b^2\right)(dx+c)}{b^3d}+\frac{4\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}}{8\,b^2d}+\frac{\left(a^2+b^2\right)\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-\right.}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) + (a^2 + b^2)*(d*x + c)
/(b^3*d) + 1/8*(4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d) + (a^2 + b^2
)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d)
```

Fricas [B] time = 2.27073, size = 848, normalized size = 14.37

$$\frac{b^2\cosh(dx+c)^4+b^2\sinh(dx+c)^4-8\left(a^2+b^2\right)dx\cosh(dx+c)^2-4\,ab\cosh(dx+c)^3+4\left(b^2\cosh(dx+c)-ab\right)\sinh\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*cosh(d*x + c)^4 + b^2*sinh(d*x + c)^4 - 8*(a^2 + b^2)*d*x*cosh(d*x
 + c)^2 - 4*a*b*cosh(d*x + c)^3 + 4*(b^2*cosh(d*x + c) - a*b)*sinh(d*x + c)
^3 + 4*a*b*cosh(d*x + c) + 2*(3*b^2*cosh(d*x + c)^2 - 4*(a^2 + b^2)*d*x - 6
*a*b*cosh(d*x + c))*sinh(d*x + c)^2 + b^2 + 8*((a^2 + b^2)*cosh(d*x + c)^2
```

```
+ 2*(a^2 + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*sinh(d*x + c)^2)*
log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(b^2*cosh(
d*x + c)^3 - 4*(a^2 + b^2)*d*x*cosh(d*x + c) - 3*a*b*cosh(d*x + c)^2 + a*b)
*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^2 + 2*b^3*d*cosh(d*x + c)*sinh(d*x + c
) + b^3*d*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16287, size = 128, normalized size = 2.17

$$\frac{\left(a^2+b^2\right)\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{b^3 d}+\frac{bd\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2-4\,ad\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{8\,b^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (a^2 + b^2)*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(b^3*d) + 1/8*(b
*d*(e^(d*x + c) - e^(-d*x - c))^2 - 4*a*d*(e^(d*x + c) - e^(-d*x - c)))/(b^
2*d^2)
```

3.303 $\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Cosh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0768147, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

```
[In] Int[Cosh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Cosh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Mathematica [A] time = 81.9985, size = 0, normalized size = 0.

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cosh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Cosh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A] time = 0.234, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx+c))^3}{(fx+e)(a+b\sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{e^{\left(-2c+\frac{2de}{f}\right)}E_1\left(\frac{2(fx+e)d}{f}\right)}{4bf}+\frac{ae^{\left(-c+\frac{de}{f}\right)}E_1\left(\frac{(fx+e)d}{f}\right)}{2b^2f}+\frac{ae^{\left(c-\frac{de}{f}\right)}E_1\left(-\frac{(fx+e)d}{f}\right)}{2b^2f}-\frac{e^{\left(2c-\frac{2de}{f}\right)}E_1\left(-\frac{2(fx+e)d}{f}\right)}{4bf}+\frac{(a^2+b^2)\log(fx+}{b^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b*f) + 1/2*a*e^(
-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^2*f) + 1/2*a*e^(c - d*e/f)*
exp_integral_e(1, -(f*x + e)*d/f)/(b^2*f) - 1/4*e^(2*c - 2*d*e/f)*exp_integ
ral_e(1, -2*(f*x + e)*d/f)/(b*f) + (a^2 + b^2)*log(f*x + e)/(b^3*f) - 1/8*i
ntegrate(16*(a^2*b + b^3 - (a^3*e^c + a*b^2*e^c)*e^(d*x))/(b^4*f*x + b^4*e
- (b^4*f*x*e^(2*c) + b^4*e*e^(2*c))*e^(2*d*x) - 2*(a*b^3*f*x*e^c + a*b^3*e*
e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx+c)^3}{afx+ae+(bfx+be)\sinh(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.304 $\int \frac{(e+fx)^3\mathbf{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=786

$$\frac{6iaf^2(e+fx)\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d^3\left(a^2+b^2\right)}-\frac{6iaf^2(e+fx)\text{PolyLog}\left(3,ie^{c+dx}\right)}{d^3\left(a^2+b^2\right)}-\frac{6bf^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^3\left(a^2+b^2\right)}-\frac{6b}{}$$

```
[Out] (2*a*(e + f*x)^3*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) + (b*(e + f*x)^3*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) + (b*(e + f*x)^
3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) - (b*(e +
 f*x)^3*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)*d) - ((3*I)*a*f*(e + f*x)^2*
PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) + ((3*I)*a*f*(e + f*x)^2*Po
lyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2,
-((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2) + (3*b*f*(e +
f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d
^2) - (3*b*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)*d^2)
+ ((6*I)*a*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) -
((6*I)*a*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)*d^3) - (6*b*
f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 +
 b^2)*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))])/((a^2 + b^2)*d^3) + (3*b*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x
))])/(2*(a^2 + b^2)*d^3) - ((6*I)*a*f^3*PolyLog[4, (-I)*E^(c + d*x)])/((a^2
 + b^2)*d^4) + ((6*I)*a*f^3*PolyLog[4, I*E^(c + d*x)])/((a^2 + b^2)*d^4) +
(6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)
*d^4) + (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^
2 + b^2)*d^4) - (3*b*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*(a^2 + b^2)*d^4)
```

Rubi [A] time = 1.45704, antiderivative size = 786, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5573, 5561, 2190, 2531, 6609, 2282, 6589, 6742, 4180, 3718}

$$\frac{6iaf^2(e+fx)\text{PolyLog}\left(3,-ie^{c+dx}\right)}{d^3\left(a^2+b^2\right)}-\frac{6iaf^2(e+fx)\text{PolyLog}\left(3,ie^{c+dx}\right)}{d^3\left(a^2+b^2\right)}-\frac{6bf^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^3\left(a^2+b^2\right)}-\frac{6b}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a*(e + f*x)^3*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) + (b*(e + f*x)^3*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) + (b*(e + f*x)^
3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) - (b*(e +
 f*x)^3*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)*d) - ((3*I)*a*f*(e + f*x)^2*
PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) + ((3*I)*a*f*(e + f*x)^2*Po
lyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2,
-((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2) + (3*b*f*(e +
f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d
^2) - (3*b*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)*d^2)
+ ((6*I)*a*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) -
((6*I)*a*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)*d^3) - (6*b*
f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 +
 b^2)*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))])/((a^2 + b^2)*d^3) + (3*b*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x
))])/(2*(a^2 + b^2)*d^3) - ((6*I)*a*f^3*PolyLog[4, (-I)*E^(c + d*x)])/((a^2
 + b^2)*d^4) + ((6*I)*a*f^3*PolyLog[4, I*E^(c + d*x)])/((a^2 + b^2)*d^4) +
```

```
(6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)
*d^4) + (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^
2 + b^2)*d^4) - (3*b*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*(a^2 + b^2)*d^4)
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\operatorname{sech}(c+dx)(a-b\sinh(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}$$

$$= -\frac{b(e+fx)^4}{4\left(a^2+b^2\right)f} + \frac{\int \left(a(e+fx)^3\operatorname{sech}(c+dx) - b(e+fx)^3\tanh(c+dx)\right)dx}{a^2+b^2} + \frac{b^2\int \frac{e^{\cdots}}{a-\sqrt{\cdots}}}{a\cdots}$$

$$= -\frac{b(e+fx)^4}{4\left(a^2+b^2\right)f} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{a\int(\cdots}{\cdots}$$

$$= \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+d\cdots}}{a+\sqrt{a^2\cdots}}\right)}{\left(a^2+b^2\right)d}$$

$$= \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+d\cdots}}{a+\sqrt{a^2\cdots}}\right)}{\left(a^2+b^2\right)d}$$

$$= \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+d\cdots}}{a+\sqrt{a^2\cdots}}\right)}{\left(a^2+b^2\right)d}$$

$$= \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+d\cdots}}{a+\sqrt{a^2\cdots}}\right)}{\left(a^2+b^2\right)d}$$

$$= \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+d\cdots}}{a+\sqrt{a^2\cdots}}\right)}{\left(a^2+b^2\right)d}$$

$$= \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^3\log\left(1+\frac{be^{c+d\cdots}}{a+\sqrt{a^2\cdots}}\right)}{\left(a^2+b^2\right)d}$$

Mathematica [B] time = 28.6787, size = 10644, normalized size = 13.54

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.351, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-e^3\left(\frac{2\,a\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d} - \frac{b\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^2+b^2)d} + \frac{b\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d}\right) + \int \frac{4\,f^3x^3}{\left(b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -e^3*(2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - b*log(-2*a*e^(-d*x - c) +
b*e^(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^2
 + b^2)*d)) + integrate(4*f^3*x^3/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(
e^(d*x + c) + e^(-d*x - c))) + 12*e*f^2*x^2/((b*(e^(d*x + c) - e^(-d*x - c)
) + 2*a)*(e^(d*x + c) + e^(-d*x - c))) + 12*e^2*f*x/((b*(e^(d*x + c) - e^(-
d*x - c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))), x)
```

Fricas [C] time = 3.03533, size = 4177, normalized size = 5.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (6*b*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*b*f^3*polylog(4, (a*cosh(d*
x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2))/b) + 3*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*dilog((a*
cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2) - b)/b + 1) + 3*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e
^2*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (3*I*a*d^2*f^3*x^2 - 3*b*d^2*
f^3*x^2 + 6*I*a*d^2*e*f^2*x - 6*b*d^2*e*f^2*x + 3*I*a*d^2*e^2*f - 3*b*d^2*e
^2*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + (-3*I*a*d^2*f^3*x^2 - 3*b*
d^2*f^3*x^2 - 6*I*a*d^2*e*f^2*x - 6*b*d^2*e*f^2*x - 3*I*a*d^2*e^2*f - 3*b*d
```

```
^2*e^2*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + (b*d^3*e^3 - 3*b*c*d^
2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x
 + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3
*b*c^2*d*e*f^2 - b*c^3*f^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b
*sqrt((a^2 + b^2)/b^2) + 2*a) + (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^
3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*log(-(a*cosh(d*x
 + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2) - b)/b) + (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3
*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*log(-(a*cosh(d*x + c) + a*sin
h(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)
/b) + (I*a*d^3*e^3 - b*d^3*e^3 - 3*I*a*c*d^2*e^2*f + 3*b*c*d^2*e^2*f + 3*I*
a*c^2*d*e*f^2 - 3*b*c^2*d*e*f^2 - I*a*c^3*f^3 + b*c^3*f^3)*log(cosh(d*x + c
) + sinh(d*x + c) + I) + (-I*a*d^3*e^3 - b*d^3*e^3 + 3*I*a*c*d^2*e^2*f + 3*
b*c*d^2*e^2*f - 3*I*a*c^2*d*e*f^2 - 3*b*c^2*d*e*f^2 + I*a*c^3*f^3 + b*c^3*f
^3)*log(cosh(d*x + c) + sinh(d*x + c) - I) + (-I*a*d^3*f^3*x^3 - b*d^3*f^3*
x^3 - 3*I*a*d^3*e*f^2*x^2 - 3*b*d^3*e*f^2*x^2 - 3*I*a*d^3*e^2*f*x - 3*b*d^3
*e^2*f*x - 3*I*a*c*d^2*e^2*f - 3*b*c*d^2*e^2*f + 3*I*a*c^2*d*e*f^2 + 3*b*c^
2*d*e*f^2 - I*a*c^3*f^3 - b*c^3*f^3)*log(I*cosh(d*x + c) + I*sinh(d*x + c)
+ 1) + (I*a*d^3*f^3*x^3 - b*d^3*f^3*x^3 + 3*I*a*d^3*e*f^2*x^2 - 3*b*d^3*e*f
^2*x^2 + 3*I*a*d^3*e^2*f*x - 3*b*d^3*e^2*f*x + 3*I*a*c*d^2*e^2*f - 3*b*c*d^
2*e^2*f - 3*I*a*c^2*d*e*f^2 + 3*b*c^2*d*e*f^2 + I*a*c^3*f^3 - b*c^3*f^3)*lo
g(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1) - 6*(-I*a*f^3 + b*f^3)*polylog(4,
 I*cosh(d*x + c) + I*sinh(d*x + c)) - 6*(I*a*f^3 + b*f^3)*polylog(4, -I*cos
h(d*x + c) - I*sinh(d*x + c)) - 6*(b*d*f^3*x + b*d*e*f^2)*polylog(3, (a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2))/b) - 6*(b*d*f^3*x + b*d*e*f^2)*polylog(3, (a*cosh(d*x + c) +
 a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
))/b) + (-6*I*a*d*f^3*x + 6*b*d*f^3*x - 6*I*a*d*e*f^2 + 6*b*d*e*f^2)*polylo
g(3, I*cosh(d*x + c) + I*sinh(d*x + c)) + (6*I*a*d*f^3*x + 6*b*d*f^3*x + 6*
I*a*d*e*f^2 + 6*b*d*e*f^2)*polylog(3, -I*cosh(d*x + c) - I*sinh(d*x + c)))/
((a^2 + b^2)*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e+fx\right)^3 \operatorname{sech}\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^3 \operatorname{sech}\left(dx+c\right)}{b\sinh\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sech(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.305 $\int \frac{(e+fx)^2 \mathbf{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=558

$$-\frac{2iaf(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)}+\frac{2iaf(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)}+\frac{2bf(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)}+\frac{2bf(e+$$

```
[Out] (2*a*(e + f*x)^2*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) + (b*(e + f*x)^2*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) + (b*(e + f*x)^
2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) - (b*(e +
 f*x)^2*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)*d) - ((2*I)*a*f*(e + f*x)*Po
lyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) + ((2*I)*a*f*(e + f*x)*PolyLo
g[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) + (2*b*f*(e + f*x)*PolyLog[2, -((b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2) + (2*b*f*(e + f*x)*P
olyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2) - (b
*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2)*d^2) + ((2*I)*a*f^2
*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) - ((2*I)*a*f^2*PolyLog[3,
I*E^(c + d*x)])/((a^2 + b^2)*d^3) - (2*b*f^2*PolyLog[3, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^3) - (2*b*f^2*PolyLog[3, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^3) + (b*f^2*PolyLog[3, -E^(2
*(c + d*x))])/(2*(a^2 + b^2)*d^3)
```

Rubi [A] time = 1.03445, antiderivative size = 558, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5573, 5561, 2190, 2531, 2282, 6589, 6742, 4180, 3718}

$$-\frac{2iaf(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)}+\frac{2iaf(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)}+\frac{2bf(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)}+\frac{2bf(e+$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (2*a*(e + f*x)^2*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) + (b*(e + f*x)^2*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) + (b*(e + f*x)^
2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) - (b*(e +
 f*x)^2*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)*d) - ((2*I)*a*f*(e + f*x)*Po
lyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) + ((2*I)*a*f*(e + f*x)*PolyLo
g[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) + (2*b*f*(e + f*x)*PolyLog[2, -((b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2) + (2*b*f*(e + f*x)*P
olyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2) - (b
*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2)*d^2) + ((2*I)*a*f^2
*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) - ((2*I)*a*f^2*PolyLog[3,
I*E^(c + d*x)])/((a^2 + b^2)*d^3) - (2*b*f^2*PolyLog[3, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^3) - (2*b*f^2*PolyLog[3, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^3) + (b*f^2*PolyLog[3, -E^(2
*(c + d*x))])/(2*(a^2 + b^2)*d^3)
```


Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
```

```
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
```

```
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}$$

$$= -\frac{b(e+fx)^3}{3\left(a^2+b^2\right)f} + \frac{\int \left(a(e+fx)^2\text{sech}(c+dx)-b(e+fx)^2\tanh(c+dx)\right)dx}{a^2+b^2} + \frac{b^2\int \frac{e^{c+dx}(\ldots}{a-\sqrt{a^2+\ldots}}}{a^2+\ldots}$$

$$= -\frac{b(e+fx)^3}{3\left(a^2+b^2\right)f} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{a\int (e+\ldots}{\ldots}$$

$$= \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d}\ldots$$

$$= \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d}\ldots$$

$$= \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d}\ldots$$

$$= \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d}\ldots$$

$$= \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d}\ldots$$

Mathematica [B] time = 21.5492, size = 1639, normalized size = 2.94

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (12*b*d^3*e^2*E^(2*c)*x - 12*b*d^3*e^2*(1 + E^(2*c))*x - 12*b*d^3*e*f*x^2 -
 4*b*d^3*f^2*x^3 + 12*a*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] + 6*b*d^2
*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*a*d*e*(1 + E
^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - PolyLog[
2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) + 6*b*d*e*(1 + E^(2*c))*f
*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]) +
(6*I)*a*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log[1 +
 I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2, I*E
^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*x)])
 + b*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))]) - 6*
d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))]))/(6*(a^2
 + b^2)*d^3*(1 + E^(2*c))) - (b*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^
(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a
^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c
)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*
a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((
-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b
```

```
*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(
c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x
) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^
c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d
*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c
 + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1
 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log
[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*
c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d +
(3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/
d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*
E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x)
)/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x
)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2
 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]
))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2
 + b^2)*E^(2*c)]))])/d^3 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + S
qrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c +
d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3))/(3*(a^2 + b^2)*(-1 + E^(
2*c))) + (b*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Csch[c/2]*Sech[c/2]*Sech[c])/(6*(
a^2 + b^2))
```

Maple [F] time = 0.267, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-e^2\left(\frac{2\,a\arctan\left(e^{(-dx-c)}\right)}{\left(a^2+b^2\right)d}-\frac{b\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{\left(a^2+b^2\right)d}+\frac{b\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\left(a^2+b^2\right)d}\right)+\int\frac{4\,f^2}{\left(b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -e^2*(2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - b*log(-2*a*e^(-d*x - c) +
b*e^(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^2
 + b^2)*d)) + integrate(4*f^2*x^2/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(
e^(d*x + c) + e^(-d*x - c))) + 8*e*f*x/((b*(e^(d*x + c) - e^(-d*x - c)) + 2
*a)*(e^(d*x + c) + e^(-d*x - c))), x)
```

Fricas [C] time = 2.61712, size = 2745, normalized size = 4.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*b*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*b*f^2*polylog(3, (a*cosh(d
*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2))/b) - 2*(b*d*f^2*x + b*d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*
x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b +
 1) - 2*(b*d*f^2*x + b*d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (2*I*
a*d*f^2*x - 2*b*d*f^2*x + 2*I*a*d*e*f - 2*b*d*e*f)*dilog(I*cosh(d*x + c) +
I*sinh(d*x + c)) - (-2*I*a*d*f^2*x - 2*b*d*f^2*x - 2*I*a*d*e*f - 2*b*d*e*f)
*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c
^2*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^
2) + 2*a) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(2*b*cosh(d*x + c) + 2
*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b*d^2*f^2*x^2 + 2*b*
d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c
) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b*
d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*log(-(a*cosh(d*x + c
) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2) - b)/b) - (I*a*d^2*e^2 - b*d^2*e^2 - 2*I*a*c*d*e*f + 2*b*c*d*e*f + I*a
*c^2*f^2 - b*c^2*f^2)*log(cosh(d*x + c) + sinh(d*x + c) + I) - (-I*a*d^2*e^
2 - b*d^2*e^2 + 2*I*a*c*d*e*f + 2*b*c*d*e*f - I*a*c^2*f^2 - b*c^2*f^2)*log(
cosh(d*x + c) + sinh(d*x + c) - I) - (-I*a*d^2*f^2*x^2 - b*d^2*f^2*x^2 - 2*
I*a*d^2*e*f*x - 2*b*d^2*e*f*x - 2*I*a*c*d*e*f - 2*b*c*d*e*f + I*a*c^2*f^2 +
 b*c^2*f^2)*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) - (I*a*d^2*f^2*x^2 -
 b*d^2*f^2*x^2 + 2*I*a*d^2*e*f*x - 2*b*d^2*e*f*x + 2*I*a*c*d*e*f - 2*b*c*d*
e*f - I*a*c^2*f^2 + b*c^2*f^2)*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1)
+ 2*(I*a*f^2 - b*f^2)*polylog(3, I*cosh(d*x + c) + I*sinh(d*x + c)) + 2*(-I
*a*f^2 - b*f^2)*polylog(3, -I*cosh(d*x + c) - I*sinh(d*x + c)))/((a^2 + b^2
)*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sech(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.306 $\int \frac{(e+fx)\textbf{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=334

$$-\frac{iaf\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)}+\frac{iaf\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)}+\frac{bf\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)}+\frac{bf\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2\left(a^2+b^2\right)}-\frac{bf}{}$$

```
[Out] (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) + (b*(e + f*x)*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) + (b*(e + f*x)*Log[
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) - (b*(e + f*x)*
Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)*d) - (I*a*f*PolyLog[2, (-I)*E^(c + d
*x)])/((a^2 + b^2)*d^2) + (I*a*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^
2) + (b*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2
)*d^2) + (b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 +
 b^2)*d^2) - (b*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)*d^2)
```

Rubi [A] time = 0.59629, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5573, 5561, 2190, 2279, 2391, 6742, 4180, 3718}

$$-\frac{iaf\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)}+\frac{iaf\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)}+\frac{bf\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)}+\frac{bf\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2\left(a^2+b^2\right)}-\frac{bf}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) + (b*(e + f*x)*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) + (b*(e + f*x)*Log[
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) - (b*(e + f*x)*
Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)*d) - (I*a*f*PolyLog[2, (-I)*E^(c + d
*x)])/((a^2 + b^2)*d^2) + (I*a*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^
2) + (b*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2
)*d^2) + (b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 +
 b^2)*d^2) - (b*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)*d^2)
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}dx}{a^2+b^2}$$

$$= -\frac{b(e+fx)^2}{2\left(a^2+b^2\right)f} + \frac{\int (a(e+fx)\text{sech}(c+dx) - b(e+fx)\tanh(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{e^{c+dx}(\ldots}{a-\sqrt{a^2+b^2}\ldots}}{a^2+\ldots}$$

$$= -\frac{b(e+fx)^2}{2\left(a^2+b^2\right)f} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{a\int(e+\ldots}{\ldots}$$

$$= \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d}$$

$$= \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d}$$

$$= \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d}$$

$$= \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d}$$

Mathematica [A] time = 2.73318, size = 439, normalized size = 1.31

$$\frac{2bf\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) + 2bf\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) - 2iaf\text{PolyLog}(2,-i(\sinh(c+dx)+\cosh(c+dx))) + 2iaf\ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*b*c*d*e - 2*b*c^2*f + 2*b*d^2*e*x - 2*b*c*d*f*x + 4*a*d*e*ArcTan[Cosh[c
+ d*x] + Sinh[c + d*x]] + 4*a*d*f*x*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] +
 2*b*c*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*b*d*f*x*Log[1 +
 (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*b*c*f*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])] + 2*b*d*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])] + 2*b*d*e*Log[a + b*Sinh[c + d*x]] - 2*b*c*f*Log[a + b*Sinh[c + d*x]]
 - 2*b*d*e*Log[1 + Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]] - 2*b*d*f*x*Log[1
 + Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]] + 2*b*f*PolyLog[2, (b*E^(c + d*x)
)/(-a + Sqrt[a^2 + b^2])] + 2*b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))] - (2*I)*a*f*PolyLog[2, (-I)*(Cosh[c + d*x] + Sinh[c + d*x])] +
(2*I)*a*f*PolyLog[2, I*(Cosh[c + d*x] + Sinh[c + d*x])] - b*f*PolyLog[2, -C
osh[2*(c + d*x)] - Sinh[2*(c + d*x)]])/(2*(a^2 + b^2)*d^2)
```

Maple [B] time = 0.128, size = 954, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] -2/d*e/(2*a^2+2*b^2)*b*ln(1+exp(2*d*x+2*c))+4/d*e/(2*a^2+2*b^2)*a*arctan(ex
p(d*x+c))+2/d*e*b/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d*f
/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*b*x-2/d^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+
c))*b*c+2*I/d^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*c-2*I/d^2*f/(2*a^2+2*b
^2)*dilog(1+I*exp(d*x+c))*a-2/d*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*b*x-2/d^
2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*b*c+2*I/d*f/(2*a^2+2*b^2)*ln(1-I*exp(d
*x+c))*a*x+2*I/d^2*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))*a-2/d^2*f/(2*a^2+2
*b^2)*dilog(1+I*exp(d*x+c))*b-2*I/d*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*a*x-
2/d^2*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))*b-2*I/d^2*f/(2*a^2+2*b^2)*ln(1+
I*exp(d*x+c))*a*c+2/d*f*b/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a
)/(-a+(a^2+b^2)^(1/2)))*x+2/d^2*f*b/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^
2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+2/d*f*b/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+
(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+2/d^2*f*b/(2*a^2+2*b^2)*ln((b*exp
(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/d^2*f*b/(2*a^2+2*b^2)*d
ilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+2/d^2*f*b/(2*a
^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/d^2
*f*c/(2*a^2+2*b^2)*b*ln(1+exp(2*d*x+2*c))-4/d^2*f*c/(2*a^2+2*b^2)*a*arctan(
exp(d*x+c))-2/d^2*f*c*b/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-e\left(\frac{2\,a\arctan\left(e^{(-dx-c)}\right)}{\left(a^2+b^2\right)d}-\frac{b\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{\left(a^2+b^2\right)d}+\frac{b\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\left(a^2+b^2\right)d}\right)+2\,f\int\frac{2\,x}{\left(b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -e*(2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - b*log(-2*a*e^(-d*x - c) + b*
e^(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^2 +
 b^2)*d)) + 2*f*integrate(2*x/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(e^(d
*x + c) + e^(-d*x - c))), x)
```

Fricas [A] time = 2.41935, size = 1547, normalized size = 4.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (b*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + b*f*dilog((a*cosh(d*x + c) + a
*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b + 1) + (I*a*f - b*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + (-I*
a*f - b*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + (b*d*e - b*c*f)*log(
2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) +
(b*d*e - b*c*f)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 +
 b^2)/b^2) + 2*a) + (b*d*f*x + b*c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x +
c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b
*d*f*x + b*c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (I*a*d*e - b*d*e - I*a*c
*f + b*c*f)*log(cosh(d*x + c) + sinh(d*x + c) + I) + (-I*a*d*e - b*d*e + I*
a*c*f + b*c*f)*log(cosh(d*x + c) + sinh(d*x + c) - I) + (-I*a*d*f*x - b*d*f
```

```
*x - I*a*c*f - b*c*f)*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) + (I*a*d*f
*x - b*d*f*x + I*a*c*f - b*c*f)*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1)
)/((a^2 + b^2)*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\operatorname{sech}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sech(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.307 $\int \frac{\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=69

$$\frac{b\log(a+b\sinh(c+dx))}{d(a^2+b^2)}+\frac{a\tan^{-1}(\sinh(c+dx))}{d(a^2+b^2)}-\frac{b\log(\cosh(c+dx))}{d(a^2+b^2)}$$

```
[Out] (a*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d) - (b*Log[Cosh[c + d*x]])/((a^2 +
b^2)*d) + (b*Log[a + b*Sinh[c + d*x]])/((a^2 + b^2)*d)
```

Rubi [A] time = 0.0698349, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2668, 706, 31, 635, 204, 260}

$$\frac{b\log(a+b\sinh(c+dx))}{d(a^2+b^2)}+\frac{a\tan^{-1}(\sinh(c+dx))}{d(a^2+b^2)}-\frac{b\log(\cosh(c+dx))}{d(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d) - (b*Log[Cosh[c + d*x]])/((a^2 +
b^2)*d) + (b*Log[a + b*Sinh[c + d*x]])/((a^2 + b^2)*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b\,\text{Subst}\left(\int \frac{1}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(c+dx)\right)}{d}$$
$$= \frac{b\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(c+dx)\right)}{\left(a^2+b^2\right)d} + \frac{b\,\text{Subst}\left(\int \frac{-a+x}{-b^2-x^2}\,dx, x, b\sinh(c+dx)\right)}{\left(a^2+b^2\right)d}$$
$$= \frac{b\log(a+b\sinh(c+dx))}{\left(a^2+b^2\right)d} + \frac{b\,\text{Subst}\left(\int \frac{x}{-b^2-x^2}\,dx, x, b\sinh(c+dx)\right)}{\left(a^2+b^2\right)d} - \frac{(ab)\,\text{Subst}\left(\int \frac{1}{-b^2-x}\right.}{\left(a^2\right.}$$
$$= \frac{a\tan^{-1}(\sinh(c+dx))}{\left(a^2+b^2\right)d} - \frac{b\log(\cosh(c+dx))}{\left(a^2+b^2\right)d} + \frac{b\log(a+b\sinh(c+dx))}{\left(a^2+b^2\right)d}$$

Mathematica [A] time = 0.0963634, size = 114, normalized size = 1.65

$$-\frac{b\left(\left(\sqrt{-b^2}-a\right)\log\left(\sqrt{-b^2}-b\sinh(c+dx)\right)-2\sqrt{-b^2}\log(a+b\sinh(c+dx))+\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\sinh(c+d\right.\right.}{2\sqrt{-b^2}d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(b*((-a + Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[c + d*x]] - 2*Sqrt[-b^2]*Log
[a + b*Sinh[c + d*x]] + (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sinh[c + d*x]])
)/(2*Sqrt[-b^2]*(a^2 + b^2)*d)
```

Maple [A] time = 0.004, size = 100, normalized size = 1.5

$$\frac{b}{d\left(a^2+b^2\right)}\ln\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 a-2\tanh(1/2\,dx+c/2)\,b-a\right)-\frac{b}{d\left(a^2+b^2\right)}\ln\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+1\right)+2\,\frac{a\,\text{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/d*b/(a^2+b^2)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)*b-a)-1/d/(
a^2+b^2)*b*ln(tanh(1/2*d*x+1/2*c)^2+1)+2/d/(a^2+b^2)*a*arctan(tanh(1/2*d*x+
1/2*c))
```

Maxima [A] time = 1.6696, size = 128, normalized size = 1.86

$$-\frac{2\,a\arctan\left(e^{(-dx-c)}\right)}{\left(a^2+b^2\right)d}+\frac{b\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{\left(a^2+b^2\right)d}-\frac{b\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\left(a^2+b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(-2*a*e^(-d*x - c) + b*e^(
-2*d*x - 2*c) - b)/((a^2 + b^2)*d) - b*log(e^(-2*d*x - 2*c) + 1)/((a^2 + b^
2)*d)
```

Fricas [A] time = 2.15555, size = 247, normalized size = 3.58

$$\frac{2\,a\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)+b\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)-b\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{\left(a^2+b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*a*arctan(cosh(d*x + c) + sinh(d*x + c)) + b*log(2*(b*sinh(d*x + c) + a)/
(cosh(d*x + c) - sinh(d*x + c))) - b*log(2*cosh(d*x + c)/(cosh(d*x + c) - s
inh(d*x + c))))/((a^2 + b^2)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A] time = 1.14886, size = 171, normalized size = 2.48

$$\frac{b^2\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^2bd+b^3d}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)a}{2\left(a^2d+b^2d\right)}-\frac{b\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{2\left(a^2d+b^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^2*b*d + b^3*d) + 1/2*
(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*a/(a^2*d + b^2*d) -
 1/2*b*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2*d + b^2*d)
```

3.308 $\int \frac{\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Sech[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0489457, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sech[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Sech[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A] time = 18.4726, size = 0, normalized size = 0.

$$\int \frac{\text{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Sech[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A] time = 0.074, size = 0, normalized size = 0.

$$\int \frac{\text{sech}\,(dx+c)}{(fx+e)\,(a+b\sinh\,(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sech(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{sech}(dx+c)}{afx+ae+(bfx+be)\sinh(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sech(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sech(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sech(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

3.309 $\int \frac{(e+fx)^3 \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=780

$$\frac{6ibf^2(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^3\left(a^2+b^2\right)}-\frac{6ibf^2(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^3\left(a^2+b^2\right)}-\frac{3af^2(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{d^3\left(a^2+b^2\right)}-\frac{6b^2f}{}$$

```
[Out] (a*(e + f*x)^3)/((a^2 + b^2)*d) - (6*b*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/(
(a^2 + b^2)*d^2) + (b^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/((a^2 + b^2)^(3/2)*d) - (b^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (3*a*f*(e + f*x)^2*Log[1 + E
^(2*(c + d*x))])/((a^2 + b^2)*d^2) + ((6*I)*b*f^2*(e + f*x)*PolyLog[2, (-I)
*E^(c + d*x)])/((a^2 + b^2)*d^3) - ((6*I)*b*f^2*(e + f*x)*PolyLog[2, I*E^(c
 + d*x)])/((a^2 + b^2)*d^3) + (3*b^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (3*b^2*f*(e + f*x)^
2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*
d^2) - (3*a*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2)*d^3) -
 ((6*I)*b*f^3*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^4) + ((6*I)*b*f^
3*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)*d^4) - (6*b^2*f^2*(e + f*x)*PolyL
og[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) +
(6*b^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/
((a^2 + b^2)^(3/2)*d^3) + (3*a*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*(a^2 +
b^2)*d^4) + (6*b^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/((a^2 + b^2)^(3/2)*d^4) - (6*b^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^4) + (b*(e + f*x)^3*Sech[c + d*x])/(
(a^2 + b^2)*d) + (a*(e + f*x)^3*Tanh[c + d*x])/((a^2 + b^2)*d)
```

Rubi [A] time = 1.68822, antiderivative size = 780, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {5573, 3322, 2264, 2190, 2531, 6609, 2282, 6589, 6742, 4184, 3718, 5451, 4180}

$$\frac{6ibf^2(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^3\left(a^2+b^2\right)}-\frac{6ibf^2(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^3\left(a^2+b^2\right)}-\frac{3af^2(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{d^3\left(a^2+b^2\right)}-\frac{6b^2f}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(e + f*x)^3)/((a^2 + b^2)*d) - (6*b*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/(
(a^2 + b^2)*d^2) + (b^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/((a^2 + b^2)^(3/2)*d) - (b^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (3*a*f*(e + f*x)^2*Log[1 + E
^(2*(c + d*x))])/((a^2 + b^2)*d^2) + ((6*I)*b*f^2*(e + f*x)*PolyLog[2, (-I)
*E^(c + d*x)])/((a^2 + b^2)*d^3) - ((6*I)*b*f^2*(e + f*x)*PolyLog[2, I*E^(c
 + d*x)])/((a^2 + b^2)*d^3) + (3*b^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (3*b^2*f*(e + f*x)^
2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*
d^2) - (3*a*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2)*d^3) -
 ((6*I)*b*f^3*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^4) + ((6*I)*b*f^
3*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)*d^4) - (6*b^2*f^2*(e + f*x)*PolyL
og[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) +
(6*b^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/
((a^2 + b^2)^(3/2)*d^3) + (3*a*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*(a^2 +
```

```
b^2)*d^4) + (6*b^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/((a^2 + b^2)^(3/2)*d^4) - (6*b^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^4) + (b*(e + f*x)^3*Sech[c + d*x])/(
(a^2 + b^2)*d) + (a*(e + f*x)^3*Tanh[c + d*x])/((a^2 + b^2)*d)
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{\int \left(a(e+fx)^3\operatorname{sech}^2(c+dx) - b(e+fx)^3\operatorname{sech}(c+dx)\tanh(c+dx)\right)dx}{a^2+b^2} + \frac{\left(2b^2\right)\int \frac{e^{c+\cdots}}{-b+2ae\cdots}}{a^2\cdots}$$

$$= \frac{\left(2b^3\right)\int \frac{e^{c+dx}(e+fx)^3}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\left(a^2+b^2\right)^{3/2}} - \frac{\left(2b^3\right)\int \frac{e^{c+dx}(e+fx)^3}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\left(a^2+b^2\right)^{3/2}} + \frac{a\int (e+fx)^3\operatorname{sech}^2(c+dx\cdots}{a^2+b^2}$$

$$= \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} - \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} + \frac{b(e+fx)^3\operatorname{sech}(c+dx\cdots}{\left(a^2+b^2\right)d}$$

$$= \frac{a(e+fx)^3}{\left(a^2+b^2\right)d} - \frac{6bf(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d^2} + \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} - \frac{b^2(e+fx)^3\cdots}{\left(a^2\cdots\right.}$$

$$= \frac{a(e+fx)^3}{\left(a^2+b^2\right)d} - \frac{6bf(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d^2} + \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} - \frac{b^2(e+fx)^3\cdots}{\left(a^2\cdots\right.}$$

$$= \frac{a(e+fx)^3}{\left(a^2+b^2\right)d} - \frac{6bf(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d^2} + \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} - \frac{b^2(e+fx)^3\cdots}{\left(a^2\cdots\right.}$$

$$= \frac{a(e+fx)^3}{\left(a^2+b^2\right)d} - \frac{6bf(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d^2} + \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} - \frac{b^2(e+fx)^3\cdots}{\left(a^2\cdots\right.}$$

$$= \frac{a(e+fx)^3}{\left(a^2+b^2\right)d} - \frac{6bf(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d^2} + \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} - \frac{b^2(e+fx)^3\cdots}{\left(a^2\cdots\right.}$$

Mathematica [A] time = 13.8837, size = 1143, normalized size = 1.47

$$\frac{\left(-2e^3\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)d^3 + f^3x^3\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^3 + 3ef^2x^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^3 + 3e^2fx\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^3 - f^3\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(f*(-12*a*d^3*e^2*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*d^3*e*f*
x^2 + 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] - 6*
a*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*b*d*e*(
1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - Pol
yLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*e*(1 + E^(2*
c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))
]) + (6*I)*b*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Lo
g[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2
, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d
*x)]) - a*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))])
 - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))]))))/(
2*(a^2 + b^2)*d^4*(1 + E^(2*c))) + (b^2*(-2*d^3*e^3*ArcTanh[(a + b*E^(c + d
*x))/Sqrt[a^2 + b^2]] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]
+ d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 3*d^3*e^2*f*
```

```
x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 3*d^3*e*f^2*x^2*Log[1 +
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^3*f^3*x^3*Log[1 + (b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(
-a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))] - 6*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^
2 + b^2])] - 6*d*f^3*x*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] +
 6*d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*d*f^3*x
*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*f^3*PolyLog[4, (b
*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*f^3*PolyLog[4, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))]]))/((a^2 + b^2)^(3/2)*d^4) + (Sech[c]*Sech[c + d*x]*
(b*e^3*Cosh[c] + 3*b*e^2*f*x*Cosh[c] + 3*b*e*f^2*x^2*Cosh[c] + b*f^3*x^3*Co
sh[c] + a*e^3*Sinh[d*x] + 3*a*e^2*f*x*Sinh[d*x] + 3*a*e*f^2*x^2*Sinh[d*x] +
 a*f^3*x^3*Sinh[d*x]))/((a^2 + b^2)*d)
```

Maple [F] time = 0.705, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 (\operatorname{sech}(dx+c))^2}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)

[Out] int((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [C] time = 4.21643, size = 14519, normalized size = 18.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")

[Out] -1/2*(4*(a^3 + a*b^2)*d^3*e^3 - 12*(a^3 + a*b^2)*c*d^2*e^2*f + 12*(a^3 + a*
b^2)*c^2*d*e*f^2 - 4*(a^3 + a*b^2)*c^3*f^3 - 4*((a^3 + a*b^2)*d^3*f^3*x^3 +
 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b
^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*cosh
(d*x + c)^2 - 4*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2
+ 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^
2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*sinh(d*x + c)^2 - 6*(b^3*d^2*f^3*x^
2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f + (b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*
```

```
x + b^3*d^2*e^2*f)*cosh(d*x + c)^2 + 2*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x
 + b^3*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d^2*f^3*x^2 + 2*b^3*d^
2*e*f^2*x + b^3*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*
cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2) - b)/b + 1) + 6*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3
*d^2*e^2*f + (b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*cosh(d*x
 + c)^2 + 2*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*cosh(d*x
+ c)*sinh(d*x + c) + (b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*
sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x
+ c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1
) + 2*(b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3 +
(b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*cosh(d*
x + c)^2 + 2*(b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3
*f^3)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^
3*c^2*d*e*f^2 - b^3*c^3*f^3)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b
*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(
b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3 + (b^3*d^
3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*cosh(d*x + c)^
2 + 2*(b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*c
osh(d*x + c)*sinh(d*x + c) + (b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d
*e*f^2 - b^3*c^3*f^3)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d
*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(b^3*d^3
*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*
b^3*c^2*d*e*f^2 + b^3*c^3*f^3 + (b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*
b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3)*cosh
(d*x + c)^2 + 2*(b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x
+ 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3)*cosh(d*x + c)*sinh(d
*x + c) + (b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^
3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3)*sinh(d*x + c)^2)*sqrt((a^2
 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(b^3*d^3*f^3*x^3 + 3*b^3*
d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 +
 b^3*c^3*f^3 + (b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x +
 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3)*cosh(d*x + c)^2 + 2*(
b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2
*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d^
3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3
*b^3*c^2*d*e*f^2 + b^3*c^3*f^3)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(
-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 + b^2)/b^2) - b)/b) - 12*(b^3*f^3*cosh(d*x + c)^2 + 2*b^3*f^3*cosh
(d*x + c)*sinh(d*x + c) + b^3*f^3*sinh(d*x + c)^2 + b^3*f^3)*sqrt((a^2 + b^
2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 12*(b^3*f^3*cosh(d*x + c)^2 +
2*b^3*f^3*cosh(d*x + c)*sinh(d*x + c) + b^3*f^3*sinh(d*x + c)^2 + b^3*f^3)*
sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*co
sh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 12*(b^3*d*f^3*x
+ b^3*d*e*f^2 + (b^3*d*f^3*x + b^3*d*e*f^2)*cosh(d*x + c)^2 + 2*(b^3*d*f^3*
x + b^3*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d*f^3*x + b^3*d*e*f^2)*
sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh
(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) -
 12*(b^3*d*f^3*x + b^3*d*e*f^2 + (b^3*d*f^3*x + b^3*d*e*f^2)*cosh(d*x + c)^
2 + 2*(b^3*d*f^3*x + b^3*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d*f^3*
x + b^3*d*e*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(
d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2))/b) - 4*((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3)*d^3*e*f^2*
x^2 + 3*(a^2*b + b^3)*d^3*e^2*f*x + (a^2*b + b^3)*d^3*e^3)*cosh(d*x + c) +
(12*(a^3 + a*b^2)*d*f^3*x + 12*I*(a^2*b + b^3)*d*f^3*x + 12*(a^3 + a*b^2)*d
*e*f^2 + 12*I*(a^2*b + b^3)*d*e*f^2 + (12*(a^3 + a*b^2)*d*f^3*x + 12*I*(a^2
*b + b^3)*d*f^3*x + 12*(a^3 + a*b^2)*d*e*f^2 + 12*I*(a^2*b + b^3)*d*e*f^2)*
```

```
cosh(d*x + c)^2 + (24*(a^3 + a*b^2)*d*f^3*x + 24*I*(a^2*b + b^3)*d*f^3*x +
24*(a^3 + a*b^2)*d*e*f^2 + 24*I*(a^2*b + b^3)*d*e*f^2)*cosh(d*x + c)*sinh(d
*x + c) + (12*(a^3 + a*b^2)*d*f^3*x + 12*I*(a^2*b + b^3)*d*f^3*x + 12*(a^3
+ a*b^2)*d*e*f^2 + 12*I*(a^2*b + b^3)*d*e*f^2)*sinh(d*x + c)^2)*dilog(I*cos
h(d*x + c) + I*sinh(d*x + c)) + (12*(a^3 + a*b^2)*d*f^3*x - 12*I*(a^2*b + b
^3)*d*f^3*x + 12*(a^3 + a*b^2)*d*e*f^2 - 12*I*(a^2*b + b^3)*d*e*f^2 + (12*(
a^3 + a*b^2)*d*f^3*x - 12*I*(a^2*b + b^3)*d*f^3*x + 12*(a^3 + a*b^2)*d*e*f^
2 - 12*I*(a^2*b + b^3)*d*e*f^2)*cosh(d*x + c)^2 + (24*(a^3 + a*b^2)*d*f^3*x
 - 24*I*(a^2*b + b^3)*d*f^3*x + 24*(a^3 + a*b^2)*d*e*f^2 - 24*I*(a^2*b + b^
3)*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) + (12*(a^3 + a*b^2)*d*f^3*x - 12*I*
(a^2*b + b^3)*d*f^3*x + 12*(a^3 + a*b^2)*d*e*f^2 - 12*I*(a^2*b + b^3)*d*e*f
^2)*sinh(d*x + c)^2)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + (6*(a^3 +
a*b^2)*d^2*e^2*f + 6*I*(a^2*b + b^3)*d^2*e^2*f - 12*(a^3 + a*b^2)*c*d*e*f^2
 - 12*I*(a^2*b + b^3)*c*d*e*f^2 + 6*(a^3 + a*b^2)*c^2*f^3 + 6*I*(a^2*b + b^
3)*c^2*f^3 + (6*(a^3 + a*b^2)*d^2*e^2*f + 6*I*(a^2*b + b^3)*d^2*e^2*f - 12*
(a^3 + a*b^2)*c*d*e*f^2 - 12*I*(a^2*b + b^3)*c*d*e*f^2 + 6*(a^3 + a*b^2)*c^
2*f^3 + 6*I*(a^2*b + b^3)*c^2*f^3)*cosh(d*x + c)^2 + (12*(a^3 + a*b^2)*d^2*
e^2*f + 12*I*(a^2*b + b^3)*d^2*e^2*f - 24*(a^3 + a*b^2)*c*d*e*f^2 - 24*I*(a
^2*b + b^3)*c*d*e*f^2 + 12*(a^3 + a*b^2)*c^2*f^3 + 12*I*(a^2*b + b^3)*c^2*f
^3)*cosh(d*x + c)*sinh(d*x + c) + (6*(a^3 + a*b^2)*d^2*e^2*f + 6*I*(a^2*b +
 b^3)*d^2*e^2*f - 12*(a^3 + a*b^2)*c*d*e*f^2 - 12*I*(a^2*b + b^3)*c*d*e*f^2
 + 6*(a^3 + a*b^2)*c^2*f^3 + 6*I*(a^2*b + b^3)*c^2*f^3)*sinh(d*x + c)^2)*lo
g(cosh(d*x + c) + sinh(d*x + c) + I) + (6*(a^3 + a*b^2)*d^2*e^2*f - 6*I*(a^
2*b + b^3)*d^2*e^2*f - 12*(a^3 + a*b^2)*c*d*e*f^2 + 12*I*(a^2*b + b^3)*c*d*
e*f^2 + 6*(a^3 + a*b^2)*c^2*f^3 - 6*I*(a^2*b + b^3)*c^2*f^3 + (6*(a^3 + a*b
^2)*d^2*e^2*f - 6*I*(a^2*b + b^3)*d^2*e^2*f - 12*(a^3 + a*b^2)*c*d*e*f^2 +
12*I*(a^2*b + b^3)*c*d*e*f^2 + 6*(a^3 + a*b^2)*c^2*f^3 - 6*I*(a^2*b + b^3)*
c^2*f^3)*cosh(d*x + c)^2 + (12*(a^3 + a*b^2)*d^2*e^2*f - 12*I*(a^2*b + b^3)
*d^2*e^2*f - 24*(a^3 + a*b^2)*c*d*e*f^2 + 24*I*(a^2*b + b^3)*c*d*e*f^2 + 12
*(a^3 + a*b^2)*c^2*f^3 - 12*I*(a^2*b + b^3)*c^2*f^3)*cosh(d*x + c)*sinh(d*x
 + c) + (6*(a^3 + a*b^2)*d^2*e^2*f - 6*I*(a^2*b + b^3)*d^2*e^2*f - 12*(a^3
+ a*b^2)*c*d*e*f^2 + 12*I*(a^2*b + b^3)*c*d*e*f^2 + 6*(a^3 + a*b^2)*c^2*f^3
 - 6*I*(a^2*b + b^3)*c^2*f^3)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x
 + c) - I) + (6*(a^3 + a*b^2)*d^2*f^3*x^2 - 6*I*(a^2*b + b^3)*d^2*f^3*x^2 +
 12*(a^3 + a*b^2)*d^2*e*f^2*x - 12*I*(a^2*b + b^3)*d^2*e*f^2*x + 12*(a^3 +
a*b^2)*c*d*e*f^2 - 12*I*(a^2*b + b^3)*c*d*e*f^2 - 6*(a^3 + a*b^2)*c^2*f^3 +
 6*I*(a^2*b + b^3)*c^2*f^3 + (6*(a^3 + a*b^2)*d^2*f^3*x^2 - 6*I*(a^2*b + b^
3)*d^2*f^3*x^2 + 12*(a^3 + a*b^2)*d^2*e*f^2*x - 12*I*(a^2*b + b^3)*d^2*e*f^
2*x + 12*(a^3 + a*b^2)*c*d*e*f^2 - 12*I*(a^2*b + b^3)*c*d*e*f^2 - 6*(a^3 +
a*b^2)*c^2*f^3 + 6*I*(a^2*b + b^3)*c^2*f^3)*cosh(d*x + c)^2 + (12*(a^3 + a*
b^2)*d^2*f^3*x^2 - 12*I*(a^2*b + b^3)*d^2*f^3*x^2 + 24*(a^3 + a*b^2)*d^2*e*
f^2*x - 24*I*(a^2*b + b^3)*d^2*e*f^2*x + 24*(a^3 + a*b^2)*c*d*e*f^2 - 24*I*
(a^2*b + b^3)*c*d*e*f^2 - 12*(a^3 + a*b^2)*c^2*f^3 + 12*I*(a^2*b + b^3)*c^2
*f^3)*cosh(d*x + c)*sinh(d*x + c) + (6*(a^3 + a*b^2)*d^2*f^3*x^2 - 6*I*(a^2
*b + b^3)*d^2*f^3*x^2 + 12*(a^3 + a*b^2)*d^2*e*f^2*x - 12*I*(a^2*b + b^3)*d
^2*e*f^2*x + 12*(a^3 + a*b^2)*c*d*e*f^2 - 12*I*(a^2*b + b^3)*c*d*e*f^2 - 6*
(a^3 + a*b^2)*c^2*f^3 + 6*I*(a^2*b + b^3)*c^2*f^3)*sinh(d*x + c)^2)*log(I*c
osh(d*x + c) + I*sinh(d*x + c) + 1) + (6*(a^3 + a*b^2)*d^2*f^3*x^2 + 6*I*(a
^2*b + b^3)*d^2*f^3*x^2 + 12*(a^3 + a*b^2)*d^2*e*f^2*x + 12*I*(a^2*b + b^3)
*d^2*e*f^2*x + 12*(a^3 + a*b^2)*c*d*e*f^2 + 12*I*(a^2*b + b^3)*c*d*e*f^2 -
6*(a^3 + a*b^2)*c^2*f^3 - 6*I*(a^2*b + b^3)*c^2*f^3 + (6*(a^3 + a*b^2)*d^2*
f^3*x^2 + 6*I*(a^2*b + b^3)*d^2*f^3*x^2 + 12*(a^3 + a*b^2)*d^2*e*f^2*x + 12
*I*(a^2*b + b^3)*d^2*e*f^2*x + 12*(a^3 + a*b^2)*c*d*e*f^2 + 12*I*(a^2*b + b
^3)*c*d*e*f^2 - 6*(a^3 + a*b^2)*c^2*f^3 - 6*I*(a^2*b + b^3)*c^2*f^3)*cosh(d
*x + c)^2 + (12*(a^3 + a*b^2)*d^2*f^3*x^2 + 12*I*(a^2*b + b^3)*d^2*f^3*x^2
+ 24*(a^3 + a*b^2)*d^2*e*f^2*x + 24*I*(a^2*b + b^3)*d^2*e*f^2*x + 24*(a^3 +
 a*b^2)*c*d*e*f^2 + 24*I*(a^2*b + b^3)*c*d*e*f^2 - 12*(a^3 + a*b^2)*c^2*f^3
 - 12*I*(a^2*b + b^3)*c^2*f^3)*cosh(d*x + c)*sinh(d*x + c) + (6*(a^3 + a*b^
2)*d^2*f^3*x^2 + 6*I*(a^2*b + b^3)*d^2*f^3*x^2 + 12*(a^3 + a*b^2)*d^2*e*f^2
```

```
*x + 12*I*(a^2*b + b^3)*d^2*e*f^2*x + 12*(a^3 + a*b^2)*c*d*e*f^2 + 12*I*(a^
2*b + b^3)*c*d*e*f^2 - 6*(a^3 + a*b^2)*c^2*f^3 - 6*I*(a^2*b + b^3)*c^2*f^3)
*sinh(d*x + c)^2)*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1) - 12*((a^3 +
a*b^2)*f^3 + I*(a^2*b + b^3)*f^3 + ((a^3 + a*b^2)*f^3 + I*(a^2*b + b^3)*f^3
)*cosh(d*x + c)^2 + 2*((a^3 + a*b^2)*f^3 + I*(a^2*b + b^3)*f^3)*cosh(d*x +
c)*sinh(d*x + c) + ((a^3 + a*b^2)*f^3 + I*(a^2*b + b^3)*f^3)*sinh(d*x + c)^
2)*polylog(3, I*cosh(d*x + c) + I*sinh(d*x + c)) - (12*(a^3 + a*b^2)*f^3 -
12*I*(a^2*b + b^3)*f^3 + 12*((a^3 + a*b^2)*f^3 - I*(a^2*b + b^3)*f^3)*cosh(
d*x + c)^2 + 24*((a^3 + a*b^2)*f^3 - I*(a^2*b + b^3)*f^3)*cosh(d*x + c)*sin
h(d*x + c) + 12*((a^3 + a*b^2)*f^3 - I*(a^2*b + b^3)*f^3)*sinh(d*x + c)^2)*
polylog(3, -I*cosh(d*x + c) - I*sinh(d*x + c)) - 4*((a^2*b + b^3)*d^3*f^3*x
^3 + 3*(a^2*b + b^3)*d^3*e*f^2*x^2 + 3*(a^2*b + b^3)*d^3*e^2*f*x + (a^2*b +
 b^3)*d^3*e^3 + 2*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^
2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*
b^2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*cosh(d*x + c))*sinh(d*x + c))/((a
^4 + 2*a^2*b^2 + b^4)*d^4*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^4*c
osh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*d^4*sinh(d*x + c)^2 +
(a^4 + 2*a^2*b^2 + b^4)*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.310 $\int \frac{(e+fx)^2 \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=548

$$\frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^{3/2}}-\frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2\left(a^2+b^2\right)^{3/2}}-\frac{2b^2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^3\left(a^2+b^2\right)^{3/2}}+2b^2$$

```
[Out] (a*(e + f*x)^2)/((a^2 + b^2)*d) - (4*b*f*(e + f*x)*ArcTan[E^(c + d*x)])/((a
^2 + b^2)*d^2) + (b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])])/((a^2 + b^2)^(3/2)*d) - (b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (2*a*f*(e + f*x)*Log[1 + E^(2*
(c + d*x))])/((a^2 + b^2)*d^2) + ((2*I)*b*f^2*PolyLog[2, (-I)*E^(c + d*x)])
/((a^2 + b^2)*d^3) - ((2*I)*b*f^2*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d
^3) + (2*b^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
)])/((a^2 + b^2)^(3/2)*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (a*f^2*PolyLog[2, -E^
(2*(c + d*x))])/((a^2 + b^2)*d^3) - (2*b^2*f^2*PolyLog[3, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) + (2*b^2*f^2*PolyLog[3, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) + (b*(e +
 f*x)^2*Sech[c + d*x])/((a^2 + b^2)*d) + (a*(e + f*x)^2*Tanh[c + d*x])/((a^
2 + b^2)*d)
```

Rubi [A] time = 1.29548, antiderivative size = 548, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5573, 3322, 2264, 2190, 2531, 2282, 6589, 6742, 4184, 3718, 2279, 2391, 5451, 4180}

$$\frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^{3/2}}-\frac{2b^2f(e+fx)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2\left(a^2+b^2\right)^{3/2}}-\frac{2b^2f^2\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^3\left(a^2+b^2\right)^{3/2}}+2b^2$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(e + f*x)^2)/((a^2 + b^2)*d) - (4*b*f*(e + f*x)*ArcTan[E^(c + d*x)])/((a
^2 + b^2)*d^2) + (b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])])/((a^2 + b^2)^(3/2)*d) - (b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (2*a*f*(e + f*x)*Log[1 + E^(2*
(c + d*x))])/((a^2 + b^2)*d^2) + ((2*I)*b*f^2*PolyLog[2, (-I)*E^(c + d*x)])
/((a^2 + b^2)*d^3) - ((2*I)*b*f^2*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d
^3) + (2*b^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
)])/((a^2 + b^2)^(3/2)*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (a*f^2*PolyLog[2, -E^
(2*(c + d*x))])/((a^2 + b^2)*d^3) - (2*b^2*f^2*PolyLog[3, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) + (2*b^2*f^2*PolyLog[3, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) + (b*(e +
 f*x)^2*Sech[c + d*x])/((a^2 + b^2)*d) + (a*(e + f*x)^2*Tanh[c + d*x])/((a^
2 + b^2)*d)
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
```

```
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
```

```
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{\int \left(a(e+fx)^2\text{sech}^2(c+dx) - b(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)\right)\,dx}{a^2+b^2} + \frac{\left(2b^2\right)\int \frac{e^{c+}}{-b+2ae}}{a^2 -}$$

$$= \frac{\left(2b^3\right)\int \frac{e^{c+dx}(e+fx)^2}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\left(a^2+b^2\right)^{3/2}} - \frac{\left(2b^3\right)\int \frac{e^{c+dx}(e+fx)^2}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\left(a^2+b^2\right)^{3/2}} + \frac{a\int (e+fx)^2\text{sech}^2(c+dx}{a^2+b^2}$$

$$= \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} - \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} + \frac{b(e+fx)^2\text{sech}(c+dx}{\left(a^2+b^2\right)d}$$

$$= \frac{a(e+fx)^2}{\left(a^2+b^2\right)d} - \frac{4bf(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d^2} + \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} - \frac{b^2(e+fx)^2\,\text{l}}{\left(a^2\right.}$$

$$= \frac{a(e+fx)^2}{\left(a^2+b^2\right)d} - \frac{4bf(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d^2} + \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} - \frac{b^2(e+fx)^2\,\text{l}}{\left(a^2\right.}$$

$$= \frac{a(e+fx)^2}{\left(a^2+b^2\right)d} - \frac{4bf(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d^2} + \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} - \frac{b^2(e+fx)^2\,\text{l}}{\left(a^2\right.}$$

$$= \frac{a(e+fx)^2}{\left(a^2+b^2\right)d} - \frac{4bf(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d^2} + \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} - \frac{b^2(e+fx)^2\,\text{l}}{\left(a^2\right.}$$

Mathematica [A] time = 8.35974, size = 905, normalized size = 1.65

$$\frac{\left(-2e^2\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)d^2 + f^2x^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^2 + 2efx\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^2 - f^2x^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)d^2 - 2efx}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b^2*(-2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*x
*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])] + 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
- 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*
f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[3, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/((a^2 + b^2)^(3/2)*d^3) - (2*a*e
*f*Sech[c]*(Cosh[c]*Log[Cosh[c]*Cosh[d*x] + Sinh[c]*Sinh[d*x]] - d*x*Sinh[c
]))/((a^2 + b^2)*d^2*(Cosh[c]^2 - Sinh[c]^2)) - (4*b*e*f*ArcTan[(Sinh[c] +
Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]])/((a^2 + b^2)*d^2*Sqrt[
Cosh[c]^2 - Sinh[c]^2]) + (a*f^2*Csch[c]*(-((d^2*x^2)/E^ArcTanh[Coth[c]]) +
 (I*Coth[c]*(-(d*x*(-Pi + (2*I)*ArcTanh[Coth[c]])) - Pi*Log[1 + E^(2*d*x)]
- 2*(I*d*x + I*ArcTanh[Coth[c]])*Log[1 - E^((2*I)*(I*d*x + I*ArcTanh[Coth[c
]]))] + Pi*Log[Cosh[d*x]] + (2*I)*ArcTanh[Coth[c]]*Log[I*Sinh[d*x + ArcTanh
```

```
[Coth[c]]]] + I*PolyLog[2, E^((2*I)*(I*d*x + I*ArcTanh[Coth[c]]))]]))/Sqrt[1
 - Coth[c]^2])*Sech[c])/((a^2 + b^2)*d^3*Sqrt[Csch[c]^2*(-Cosh[c]^2 + Sinh[
c]^2)]) - (2*b*f^2*(((-I)*Csch[c]*(I*(d*x + ArcTanh[Coth[c]])*(Log[1 - E^(-
(d*x) - ArcTanh[Coth[c]])] - Log[1 + E^(-(d*x) - ArcTanh[Coth[c]])]) + I*(P
olyLog[2, -E^(-(d*x) - ArcTanh[Coth[c]])] - PolyLog[2, E^(-(d*x) - ArcTanh[
Coth[c]])])))/Sqrt[1 - Coth[c]^2] - (2*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)
/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]]*ArcTanh[Coth[c]])/Sqrt[Cosh[c]^2 - Sinh[c
]^2]))/((a^2 + b^2)*d^3) + (Sech[c]*Sech[c + d*x]*(b*e^2*Cosh[c] + 2*b*e*f*
x*Cosh[c] + b*f^2*x^2*Cosh[c] + a*e^2*Sinh[d*x] + 2*a*e*f*x*Sinh[d*x] + a*f
^2*x^2*Sinh[d*x]))/((a^2 + b^2)*d)
```

Maple [F] time = 0.539, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \left(\operatorname{sech}\left(dx+c\right)\right)^{2}}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.29049, size = 8370, normalized size = 15.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(a^3 + a*b^2)*d^2*e^2 - 8*(a^3 + a*b^2)*c*d*e*f + 4*(a^3 + a*b^2)*c
^2*f^2 - 4*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3
+ a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*cosh(d*x + c)^2 - 4*((a^3 + a*b^2
)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3
+ a*b^2)*c^2*f^2)*sinh(d*x + c)^2 - 4*(b^3*d*f^2*x + b^3*d*e*f + (b^3*d*f^2
*x + b^3*d*e*f)*cosh(d*x + c)^2 + 2*(b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)
*sinh(d*x + c) + (b^3*d*f^2*x + b^3*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2
)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 4*(b^3*d*f^2*x + b^3*d*e*f +
 (b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)^2 + 2*(b^3*d*f^2*x + b^3*d*e*f)*co
sh(d*x + c)*sinh(d*x + c) + (b^3*d*f^2*x + b^3*d*e*f)*sinh(d*x + c)^2)*sqrt
```

```
((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^3*d^2*e^2 -
2*b^3*c*d*e*f + b^3*c^2*f^2 + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*c
osh(d*x + c)^2 + 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c
)*sinh(d*x + c) + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*sinh(d*x + c)
^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*s
qrt((a^2 + b^2)/b^2) + 2*a) - 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2
+ (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)^2 + 2*(b^3*d^2*
e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d^2*e
^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*lo
g(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
- 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2 + (b^3
*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x + c)
^2 + 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*co
sh(d*x + c)*sinh(d*x + c) + (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*
e*f - b^3*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x
+ c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2) - b)/b) + 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^
3*c^2*f^2 + (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^
2)*cosh(d*x + c)^2 + 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f -
 b^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*
f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*l
og(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*(b^3*f^2*cosh(d*x + c)^2 + 2*b^3*f^2*co
sh(d*x + c)*sinh(d*x + c) + b^3*f^2*sinh(d*x + c)^2 + b^3*f^2)*sqrt((a^2 +
b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 4*(b^3*f^2*cosh(d*x + c)^2 +
 2*b^3*f^2*cosh(d*x + c)*sinh(d*x + c) + b^3*f^2*sinh(d*x + c)^2 + b^3*f^2)
*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 4*((a^2*b + b^3
)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*d^2*e*f*x + (a^2*b + b^3)*d^2*e^2)*cosh(d*x
 + c) + 4*((a^3 + a*b^2)*f^2 + I*(a^2*b + b^3)*f^2 + ((a^3 + a*b^2)*f^2 + I
*(a^2*b + b^3)*f^2)*cosh(d*x + c)^2 + 2*((a^3 + a*b^2)*f^2 + I*(a^2*b + b^3
)*f^2)*cosh(d*x + c)*sinh(d*x + c) + ((a^3 + a*b^2)*f^2 + I*(a^2*b + b^3)*f
^2)*sinh(d*x + c)^2)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + (4*(a^3 + a
*b^2)*f^2 - 4*I*(a^2*b + b^3)*f^2 + 4*((a^3 + a*b^2)*f^2 - I*(a^2*b + b^3)*
f^2)*cosh(d*x + c)^2 + 8*((a^3 + a*b^2)*f^2 - I*(a^2*b + b^3)*f^2)*cosh(d*x
 + c)*sinh(d*x + c) + 4*((a^3 + a*b^2)*f^2 - I*(a^2*b + b^3)*f^2)*sinh(d*x
+ c)^2)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + (4*(a^3 + a*b^2)*d*e*f
+ 4*I*(a^2*b + b^3)*d*e*f - 4*(a^3 + a*b^2)*c*f^2 - 4*I*(a^2*b + b^3)*c*f^2
 + (4*(a^3 + a*b^2)*d*e*f + 4*I*(a^2*b + b^3)*d*e*f - 4*(a^3 + a*b^2)*c*f^2
 - 4*I*(a^2*b + b^3)*c*f^2)*cosh(d*x + c)^2 + (8*(a^3 + a*b^2)*d*e*f + 8*I*
(a^2*b + b^3)*d*e*f - 8*(a^3 + a*b^2)*c*f^2 - 8*I*(a^2*b + b^3)*c*f^2)*cosh
(d*x + c)*sinh(d*x + c) + (4*(a^3 + a*b^2)*d*e*f + 4*I*(a^2*b + b^3)*d*e*f
- 4*(a^3 + a*b^2)*c*f^2 - 4*I*(a^2*b + b^3)*c*f^2)*sinh(d*x + c)^2)*log(cos
h(d*x + c) + sinh(d*x + c) + I) + (4*(a^3 + a*b^2)*d*e*f - 4*I*(a^2*b + b^3
)*d*e*f - 4*(a^3 + a*b^2)*c*f^2 + 4*I*(a^2*b + b^3)*c*f^2 + (4*(a^3 + a*b^2
)*d*e*f - 4*I*(a^2*b + b^3)*d*e*f - 4*(a^3 + a*b^2)*c*f^2 + 4*I*(a^2*b + b^
3)*c*f^2)*cosh(d*x + c)^2 + (8*(a^3 + a*b^2)*d*e*f - 8*I*(a^2*b + b^3)*d*e*
f - 8*(a^3 + a*b^2)*c*f^2 + 8*I*(a^2*b + b^3)*c*f^2)*cosh(d*x + c)*sinh(d*x
 + c) + (4*(a^3 + a*b^2)*d*e*f - 4*I*(a^2*b + b^3)*d*e*f - 4*(a^3 + a*b^2)*
c*f^2 + 4*I*(a^2*b + b^3)*c*f^2)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(
d*x + c) - I) + (4*(a^3 + a*b^2)*d*f^2*x - 4*I*(a^2*b + b^3)*d*f^2*x + 4*(a
^3 + a*b^2)*c*f^2 - 4*I*(a^2*b + b^3)*c*f^2 + (4*(a^3 + a*b^2)*d*f^2*x - 4*
I*(a^2*b + b^3)*d*f^2*x + 4*(a^3 + a*b^2)*c*f^2 - 4*I*(a^2*b + b^3)*c*f^2)*
cosh(d*x + c)^2 + (8*(a^3 + a*b^2)*d*f^2*x - 8*I*(a^2*b + b^3)*d*f^2*x + 8*
(a^3 + a*b^2)*c*f^2 - 8*I*(a^2*b + b^3)*c*f^2)*cosh(d*x + c)*sinh(d*x + c)
+ (4*(a^3 + a*b^2)*d*f^2*x - 4*I*(a^2*b + b^3)*d*f^2*x + 4*(a^3 + a*b^2)*c*
f^2 - 4*I*(a^2*b + b^3)*c*f^2)*sinh(d*x + c)^2)*log(I*cosh(d*x + c) + I*sin
```

```
h(d*x + c) + 1) + (4*(a^3 + a*b^2)*d*f^2*x + 4*I*(a^2*b + b^3)*d*f^2*x + 4*
(a^3 + a*b^2)*c*f^2 + 4*I*(a^2*b + b^3)*c*f^2 + (4*(a^3 + a*b^2)*d*f^2*x +
4*I*(a^2*b + b^3)*d*f^2*x + 4*(a^3 + a*b^2)*c*f^2 + 4*I*(a^2*b + b^3)*c*f^2
)*cosh(d*x + c)^2 + (8*(a^3 + a*b^2)*d*f^2*x + 8*I*(a^2*b + b^3)*d*f^2*x +
8*(a^3 + a*b^2)*c*f^2 + 8*I*(a^2*b + b^3)*c*f^2)*cosh(d*x + c)*sinh(d*x + c
) + (4*(a^3 + a*b^2)*d*f^2*x + 4*I*(a^2*b + b^3)*d*f^2*x + 4*(a^3 + a*b^2)*
c*f^2 + 4*I*(a^2*b + b^3)*c*f^2)*sinh(d*x + c)^2)*log(-I*cosh(d*x + c) - I*
sinh(d*x + c) + 1) - 4*((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*d^2*e*f
*x + (a^2*b + b^3)*d^2*e^2 + 2*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)
*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*cosh(d*x + c)
)*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c)^2 + 2*(a^4 + 2*
a^2*b^2 + b^4)*d^3*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*d^
3*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^2 \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.311 $\int \frac{(e+fx)\mathbf{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=295

$$\frac{b^2 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^{3/2}}-\frac{b^2 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2\left(a^2+b^2\right)^{3/2}}-\frac{bf\tan^{-1}(\sinh(c+dx))}{d^2\left(a^2+b^2\right)}-\frac{af\log(\cosh(c+dx))}{d^2\left(a^2+b^2\right)}+\frac{b^2(e+f}{}$$

```
[Out] -((b*f*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d^2)) + (b^2*(e + f*x)*Log[1 + (
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (b^2*(e + f*
x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) -
(a*f*Log[Cosh[c + d*x]])/((a^2 + b^2)*d^2) + (b^2*f*PolyLog[2, -((b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (b^2*f*PolyLog[2,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) + (b*(e
+ f*x)*Sech[c + d*x])/((a^2 + b^2)*d) + (a*(e + f*x)*Tanh[c + d*x])/((a^2 +
 b^2)*d)
```

Rubi [A] time = 0.71703, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5573, 3322, 2264, 2190, 2279, 2391, 6742, 4184, 3475, 5451, 3770}

$$\frac{b^2 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^{3/2}}-\frac{b^2 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2\left(a^2+b^2\right)^{3/2}}-\frac{bf\tan^{-1}(\sinh(c+dx))}{d^2\left(a^2+b^2\right)}-\frac{af\log(\cosh(c+dx))}{d^2\left(a^2+b^2\right)}+\frac{b^2(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((b*f*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d^2)) + (b^2*(e + f*x)*Log[1 + (
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (b^2*(e + f*
x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) -
(a*f*Log[Cosh[c + d*x]])/((a^2 + b^2)*d^2) + (b^2*f*PolyLog[2, -((b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (b^2*f*PolyLog[2,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) + (b*(e
+ f*x)*Sech[c + d*x])/((a^2 + b^2)*d) + (a*(e + f*x)*Tanh[c + d*x])/((a^2 +
 b^2)*d)
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e+fx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{sech}^2(c+dx)(a-b\sinh(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{e+fx}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{\int \left(a(e+fx)\text{sech}^2(c+dx) - b(e+fx)\text{sech}(c+dx)\tanh(c+dx)\right)\,dx}{a^2+b^2} + \frac{(2b^2)\int \frac{e^{c+dx}(e}{-b+2ae^{c+dx}}}{a^2+b^2}$$

$$= \frac{(2b^3)\int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{(a^2+b^2)^{3/2}} - \frac{(2b^3)\int \frac{e^{c+dx}(e+fx)}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{(a^2+b^2)^{3/2}} + \frac{a\int (e+fx)\text{sech}^2(c+dx)\,d}{a^2+b^2}$$

$$= \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} - \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} + \frac{b(e+fx)\text{sech}(c+dx)}{(a^2+b^2)\,d} +$$

$$= -\frac{bf\tan^{-1}(\sinh(c+dx))}{(a^2+b^2)\,d^2} + \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} - \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}$$

$$= -\frac{bf\tan^{-1}(\sinh(c+dx))}{(a^2+b^2)\,d^2} + \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} - \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}$$

Mathematica [A] time = 3.16199, size = 284, normalized size = 0.96

$$\frac{\frac{b^2\left(f\,\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)-f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)-2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+f(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)-f(c+dx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)+2cf\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)\right)}{(a^2+b^2)^{3/2}}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((-2*b*f*ArcTan[Tanh[(c + d*x)/2]])/(a^2 + b^2) - (a*f*Log[Cosh[c + d*x]])/
(a^2 + b^2) + (b^2*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2
*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])
] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^2 + b^2)^(3
/2) + (d*(e + f*x)*Sech[c + d*x]*(b + a*Sinh[c + d*x]))/(a^2 + b^2))/d^2
```

Maple [B] time = 0.169, size = 1928, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] -2/d^2/(a^2+b^2)^(3/2)*b^2*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)
+a)/(a+(a^2+b^2)^(1/2)))*a^2*c+2/d^2/(a^2+b^2)^(3/2)*b^2*f*c/(2*a^2+2*b^2)*
arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2+2/d/(a^2+b^2)^(3/2)*b
^2*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)
))*a^2*x-2/d/(a^2+b^2)^(3/2)*b^2*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)
^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^2*x+2/d^2/(a^2+b^2)^(3/2)*b^2*f/(2*a^2+2*b
```

```
^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^2*c+2/d^2/
(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-4/d^2/(
a^2+b^2)*b^3*f/(2*a^2+2*b^2)*arctan(exp(d*x+c))+2/d^2/(a^2+b^2)^(5/2)*a^4*f
*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/2/d^2/(a^2+b^2)^2*a*b^
2*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^
2)*ln(1+exp(2*d*x+2*c))+2/d^2/(a^2+b^2)^(3/2)*b^4*f/(2*a^2+2*b^2)*arctanh(1
/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-4/d^2/(a^2+b^2)*a^2*b*f/(2*a^2+2*b
^2)*arctan(exp(d*x+c))+2/d^2/(a^2+b^2)^(5/2)*a^2*b^2*f*arctanh(1/2*(2*b*exp
(d*x+c)+2*a)/(a^2+b^2)^(1/2))-4/d^2/(a^2+b^2)^(1/2)*a^2*f/(2*a^2+2*b^2)*arc
tanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d/(a^2+b^2)^(3/2)*b^4*e/(2
*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d/(a^2+b^2)
^(1/2)*b^2*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2)
)-2/d^2/(a^2+b^2)*b^2*f/(2*a^2+2*b^2)*a*ln(1+exp(2*d*x+2*c))+1/d^2/(a^2+b^2
)*b^2*f/(2*a^2+2*b^2)*a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d^2/(a^2+b^
2)^(1/2)*b^2*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/
2))-2/d^2/(a^2+b^2)^(3/2)*b^4*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)
^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/d^2/(a^2+b^2)^(3/2)*b^4*f/(2*a^2+2*b^2)*di
log((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+2/d^2/(a^2+b^2)
^(3/2)*b^2*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2
+b^2)^(1/2)))*a^2+2/d^2/(a^2+b^2)^(3/2)*b^4*f*c/(2*a^2+2*b^2)*arctanh(1/2*(
2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d/(a^2+b^2)^(3/2)*b^4*f/(2*a^2+2*b^2
)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-2/d/(a^2+b^2
)^(3/2)*b^4*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2
)^(1/2)))*x-2/d/(a^2+b^2)^(3/2)*b^2*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*
x+c)+2*a)/(a^2+b^2)^(1/2))*a^2+2/d^2/(a^2+b^2)^(3/2)*b^4*f/(2*a^2+2*b^2)*ln
((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/d^2/(a^2+b^2)^
(3/2)*b^4*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^
(1/2)))*c+2/d^2/(a^2+b^2)^(1/2)*b^2*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(
d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d^2/(a^2+b^2)^(3/2)*b^2*f/(2*a^2+2*b^2)*arct
anh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2-2/d^2/(a^2+b^2)^(3/2)*b^2
*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))
)*a^2-1/d^2/(a^2+b^2)^2*a^3*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d^2/(
a^2+b^2)*a*f*ln(exp(d*x+c))-2*(f*x+e)*(-b*exp(d*x+c)+a)/d/(a^2+b^2)/(1+exp(
2*d*x+2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.48013, size = 3164, normalized size = 10.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")

[Out] (2*(a^3 + a*b^2)*d*f*x*cosh(d*x + c)^2 + 2*(a^3 + a*b^2)*d*f*x*sinh(d*x + c
)^2 - 2*(a^3 + a*b^2)*d*e + (b^3*f*cosh(d*x + c)^2 + 2*b^3*f*cosh(d*x + c)*
```

```
sinh(d*x + c) + b^3*f*sinh(d*x + c)^2 + b^3*f)*sqrt((a^2 + b^2)/b^2)*dilog(
(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sq
rt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*f*cosh(d*x + c)^2 + 2*b^3*f*cosh(d*x
 + c)*sinh(d*x + c) + b^3*f*sinh(d*x + c)^2 + b^3*f)*sqrt((a^2 + b^2)/b^2)*
dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*d*e - b^3*c*f + (b^3*d*e - b^3
*c*f)*cosh(d*x + c)^2 + 2*(b^3*d*e - b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) +
 (b^3*d*e - b^3*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*
x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*d*e -
b^3*c*f + (b^3*d*e - b^3*c*f)*cosh(d*x + c)^2 + 2*(b^3*d*e - b^3*c*f)*cosh(
d*x + c)*sinh(d*x + c) + (b^3*d*e - b^3*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b
^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b
^2) + 2*a) + (b^3*d*f*x + b^3*c*f + (b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^2 +
 2*(b^3*d*f*x + b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d*f*x + b^3*c*f
)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x
 + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) -
 (b^3*d*f*x + b^3*c*f + (b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^2 + 2*(b^3*d*f*
x + b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d*f*x + b^3*c*f)*sinh(d*x +
 c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*((a^2*b +
 b^3)*f*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*f*cosh(d*x + c)*sinh(d*x + c) + (
a^2*b + b^3)*f*sinh(d*x + c)^2 + (a^2*b + b^3)*f)*arctan(cosh(d*x + c) + si
nh(d*x + c)) + 2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e)*cosh(d*x + c) -
((a^3 + a*b^2)*f*cosh(d*x + c)^2 + 2*(a^3 + a*b^2)*f*cosh(d*x + c)*sinh(d*x
 + c) + (a^3 + a*b^2)*f*sinh(d*x + c)^2 + (a^3 + a*b^2)*f)*log(2*cosh(d*x +
 c)/(cosh(d*x + c) - sinh(d*x + c))) + 2*(2*(a^3 + a*b^2)*d*f*x*cosh(d*x +
c) + (a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e)*sinh(d*x + c))/((a^4 + 2*a^2*
b^2 + b^4)*d^2*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(d*x + c
)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*d^2*sinh(d*x + c)^2 + (a^4 + 2*a^
2*b^2 + b^4)*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)\operatorname{sech}^2(c + dx)}{a + b\sinh(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)\operatorname{sech}(dx + c)^2}{b\sinh(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sech(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

3.312 $\int \frac{\mathrm{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{\mathrm{sech}(c+dx)(a\sinh(c+dx)+b)}{d\left(a^2+b^2\right)}-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

```
[Out] (-2*b^2*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3
/2)*d) + (Sech[c + d*x]*(b + a*Sinh[c + d*x]))/((a^2 + b^2)*d)
```

Rubi [A] time = 0.0996727, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2696, 12, 2660, 618, 204}

$$\frac{\mathrm{sech}(c+dx)(a\sinh(c+dx)+b)}{d\left(a^2+b^2\right)}-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*b^2*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3
/2)*d) + (Sech[c + d*x]*(b + a*Sinh[c + d*x]))/((a^2 + b^2)*d)
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{(a^2+b^2)\,d}+\frac{\int \frac{b^2}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}\\
&= \frac{\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{(a^2+b^2)\,d}+\frac{b^2\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}\\
&= \frac{\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{(a^2+b^2)\,d}-\frac{(2ib^2)\operatorname{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{(a^2+b^2)\,d}\\
&= \frac{\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{(a^2+b^2)\,d}+\frac{(4ib^2)\operatorname{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx,x,-2ib+2a\tan\left(\frac{1}{2}(ic+i\right.\right.}{(a^2+b^2)\,d}\\
&= -\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}+\frac{\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{(a^2+b^2)\,d}
\end{aligned}$$

Mathematica [A] time = 0.172899, size = 104, normalized size = 1.35

$$-\frac{a\sqrt{-a^2-b^2}\tanh(c+dx)+b\sqrt{-a^2-b^2}\operatorname{sech}(c+dx)+2b^2\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{d\left(-a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((2*b^2*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] + b*Sqrt[-a^2 -
 b^2]*Sech[c + d*x] + a*Sqrt[-a^2 - b^2]*Tanh[c + d*x])/((-a^2 - b^2)^(3/2)
*d))
```

Maple [A] time = 0.003, size = 90, normalized size = 1.2

$$\frac{1}{d}\left(2\frac{b^2}{(a^2+b^2)^{3/2}}\operatorname{Artanh}\left(1/2\frac{2a\tanh(1/2\,dx+c/2)-2b}{\sqrt{a^2+b^2}}\right)-2\frac{-a\tanh(1/2\,dx+c/2)-b}{(a^2+b^2)\left((\tanh(1/2\,dx+c/2))^2+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/d*(2*b^2/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b
^2)^(1/2))-2/(a^2+b^2)*(-a*tanh(1/2*d*x+1/2*c)-b)/(tanh(1/2*d*x+1/2*c)^2+1)
```

)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15788, size = 884, normalized size = 11.48

$$-\frac{2\,a^{3}+2\,ab^{2}-\left(b^{2}\cosh\left(dx+c\right)^{2}+2\,b^{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b^{2}\sinh\left(dx+c\right)^{2}+b^{2}\right)\sqrt{a^{2}+b^{2}}\log\left(\frac{b^{2}\cosh(dx+c}{\cdots}\right.}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)d\cosh\left(dx+c\right)^{2}+2\left(a^{4}+2\,a^{2}b^{2}+b\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*a^3 + 2*a*b^2 - (b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c
) + b^2*sinh(d*x + c)^2 + b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b
^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x +
c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x +
 c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b
*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*(a^2*b + b^3)*cosh(d*x + c) - 2
*(a^2*b + b^3)*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 +
2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2
+ b^4)*d*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^{2}\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [A] time = 1.23397, size = 165, normalized size = 2.14

$$-\frac{b^{2}\log\left(\frac{\left|-2\,be^{(dx+c)}-2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|-2\,be^{(dx+c)}-2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{2}d+b^{2}d\right)\sqrt{a^{2}+b^{2}}}+\frac{2\left(be^{(dx+c)}-a\right)}{\left(a^{2}d+b^{2}d\right)\left(e^{(2\,dx+2\,c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -b^2*log(abs(-2*b*e^(d*x + c) - 2*a - 2*sqrt(a^2 + b^2))/abs(-2*b*e^(d*x +
c) - 2*a + 2*sqrt(a^2 + b^2)))/((a^2*d + b^2*d)*sqrt(a^2 + b^2)) + 2*(b*e^(
d*x + c) - a)/((a^2*d + b^2*d)*(e^(2*d*x + 2*c) + 1))
```

3.313 $\int \frac{\text{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\text{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Sech[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0769621, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sech[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Sech[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\text{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\text{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A] time = 68.3938, size = 0, normalized size = 0.

$$\int \frac{\text{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Sech[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A] time = 0.567, size = 0, normalized size = 0.

$$\int \frac{(\text{sech}\,(dx+c))^2}{\left(fx+e\right)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$4\,b^2 \int -\frac{e^{(dx+c)}}{2\left(a^2be + b^3e + \left(a^2bf + b^3f\right)x - \left(a^2bee^{(2\,c)} + b^3ee^{(2\,c)} + \left(a^2bfe^{(2\,c)} + b^3fe^{(2\,c)}\right)x\right)e^{(2\,dx)} - 2\left(a^3ee^c + ab^2ee^c + \left(a^3fe\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 4*b^2*integrate(-1/2*e^(d*x + c)/(a^2*b*e + b^3*e + (a^2*b*f + b^3*f)*x - (
a^2*b*e*e^(2*c) + b^3*e*e^(2*c) + (a^2*b*f*e^(2*c) + b^3*f*e^(2*c))*x)*e^(2
*d*x) - 2*(a^3*e*e^c + a*b^2*e*e^c + (a^3*f*e^c + a*b^2*f*e^c)*x)*e^(d*x)),
 x) + 2*(b*e^(d*x + c) - a)/(a^2*d*e + b^2*d*e + (a^2*d*f + b^2*d*f)*x + (a
^2*d*e*e^(2*c) + b^2*d*e*e^(2*c) + (a^2*d*f*e^(2*c) + b^2*d*f*e^(2*c))*x)*e
^(2*d*x)) + 4*integrate(1/2*(b*f*e^(d*x + c) - a*f)/(a^2*d*e^2 + b^2*d*e^2
+ (a^2*d*f^2 + b^2*d*f^2)*x^2 + 2*(a^2*d*e*f + b^2*d*e*f)*x + (a^2*d*e^2*e^
(2*c) + b^2*d*e^2*e^(2*c) + (a^2*d*f^2*e^(2*c) + b^2*d*f^2*e^(2*c))*x^2 + 2
*(a^2*d*e*f*e^(2*c) + b^2*d*e*f*e^(2*c))*x)*e^(2*d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{sech}\left(dx+c\right)^2}{afx+ae+\left(bfx+be\right)\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sech(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sech(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.314 $\int \frac{(e+fx)^2 \mathbf{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx$

Optimal. Leaf size=928

result too large to display

```
[Out] (2*a*b^2*(e + f*x)^2*ArcTan[E^(c + d*x)])/((a^2 + b^2)^2*d) + (a*(e + f*x)^
2*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (a*f^2*ArcTan[Sinh[c + d*x]])/((a^
2 + b^2)*d^3) + (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])])/((a^2 + b^2)^2*d) + (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])])/((a^2 + b^2)^2*d) - (b^3*(e + f*x)^2*Log[1 + E^(2*(c + d*x)
)])/((a^2 + b^2)^2*d) + (b*f^2*Log[Cosh[c + d*x]])/((a^2 + b^2)*d^3) - ((2*
I)*a*b^2*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - (I
*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) + ((2*I)*a*b
^2*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) + (I*a*f*(e +
 f*x)*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) + (2*b^3*f*(e + f*x)*Pol
yLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) + (2
*b^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^
2 + b^2)^2*d^2) - (b^3*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^
2)^2*d^2) + ((2*I)*a*b^2*f^2*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d
^3) + (I*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) - ((2*I)*a*b
^2*f^2*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)^2*d^3) - (I*a*f^2*PolyLog[3,
 I*E^(c + d*x)])/((a^2 + b^2)*d^3) - (2*b^3*f^2*PolyLog[3, -((b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) - (2*b^3*f^2*PolyLog[3, -((b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) + (b^3*f^2*PolyL
og[3, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^3) + (a*f*(e + f*x)*Sech[c + d*
x])/((a^2 + b^2)*d^2) + (b*(e + f*x)^2*Sech[c + d*x]^2)/(2*(a^2 + b^2)*d) -
 (b*f*(e + f*x)*Tanh[c + d*x])/((a^2 + b^2)*d^2) + (a*(e + f*x)^2*Sech[c +
d*x]*Tanh[c + d*x])/(2*(a^2 + b^2)*d)
```

Rubi [A] time = 1.77192, antiderivative size = 928, normalized size of antiderivative = 1., number of steps used = 39, number of rules used = 14, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5573, 5561, 2190, 2531, 2282, 6589, 6742, 4180, 3718, 4186, 3770, 5451, 4184, 3475}

$$\frac{(e+fx)^2 \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)b^3}{\left(a^2+b^2\right)^2 d}+\frac{(e+fx)^2 \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)b^3}{\left(a^2+b^2\right)^2 d}-\frac{(e+fx)^2 \log\left(1+e^{2(c+dx)}\right)b^3}{\left(a^2+b^2\right)^2 d}+\frac{2f(e+fx)\text{PolyLo}}{\left(a^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a*b^2*(e + f*x)^2*ArcTan[E^(c + d*x)])/((a^2 + b^2)^2*d) + (a*(e + f*x)^
2*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (a*f^2*ArcTan[Sinh[c + d*x]])/((a^
2 + b^2)*d^3) + (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])])/((a^2 + b^2)^2*d) + (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])])/((a^2 + b^2)^2*d) - (b^3*(e + f*x)^2*Log[1 + E^(2*(c + d*x)
)])/((a^2 + b^2)^2*d) + (b*f^2*Log[Cosh[c + d*x]])/((a^2 + b^2)*d^3) - ((2*
I)*a*b^2*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - (I
*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) + ((2*I)*a*b
^2*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) + (I*a*f*(e +
 f*x)*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) + (2*b^3*f*(e + f*x)*Pol
yLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) + (2
*b^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^
2 + b^2)^2*d^2) - (b^3*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^
```

```
2)^2*d^2) + ((2*I)*a*b^2*f^2*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d
^3) + (I*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) - ((2*I)*a*b
^2*f^2*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)^2*d^3) - (I*a*f^2*PolyLog[3,
 I*E^(c + d*x)])/((a^2 + b^2)*d^3) - (2*b^3*f^2*PolyLog[3, -((b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) - (2*b^3*f^2*PolyLog[3, -((b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) + (b^3*f^2*PolyL
og[3, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^3) + (a*f*(e + f*x)*Sech[c + d*
x])/((a^2 + b^2)*d^2) + (b*(e + f*x)^2*Sech[c + d*x]^2)/(2*(a^2 + b^2)*d) -
 (b*f*(e + f*x)*Tanh[c + d*x])/((a^2 + b^2)*d^2) + (a*(e + f*x)^2*Sech[c +
d*x]*Tanh[c + d*x])/(2*(a^2 + b^2)*d)
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e+fx)^2 \text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2 \text{sech}^3(c+dx)(a-b\sinh(c+dx))\,dx}{a^2+b^2} + \frac{b^2 \int \frac{(e+fx)^2 \text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{b^2 \int (e+fx)^2 \text{sech}(c+dx)(a-b\sinh(c+dx))\,dx}{(a^2+b^2)^2} + \frac{b^4 \int \frac{(e+fx)^2 \cosh(c+dx)}{a+b\sinh(c+dx)}\,dx}{(a^2+b^2)^2} + \frac{\int \big(a(e+ \ldots}{}$$

$$= -\frac{b^3(e+fx)^3}{3(a^2+b^2)^2 f} + \frac{b^2 \int \left(a(e+fx)^2 \text{sech}(c+dx) - b(e+fx)^2 \tanh(c+dx)\right)dx}{(a^2+b^2)^2} + \frac{b^4 \int \ldots}{\ldots}$$

$$= -\frac{b^3(e+fx)^3}{3(a^2+b^2)^2 f} + \frac{b^3(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} + \frac{b^3(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} + \frac{af(\ldots}{\ldots}$$

$$= \frac{2ab^2(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)^2 d} + \frac{a(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2) d} - \frac{af^2 \tan^{-1}(\sinh(c+dx))}{(a^2+b^2) d^3} + \frac{b^3(\ldots}{\ldots}$$

$$= \frac{2ab^2(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)^2 d} + \frac{a(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2) d} - \frac{af^2 \tan^{-1}(\sinh(c+dx))}{(a^2+b^2) d^3} + \frac{b^3(\ldots}{\ldots}$$

$$= \frac{2ab^2(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)^2 d} + \frac{a(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2) d} - \frac{af^2 \tan^{-1}(\sinh(c+dx))}{(a^2+b^2) d^3} + \frac{b^3(\ldots}{\ldots}$$

$$= \frac{2ab^2(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)^2 d} + \frac{a(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2) d} - \frac{af^2 \tan^{-1}(\sinh(c+dx))}{(a^2+b^2) d^3} + \frac{b^3(\ldots}{\ldots}$$

$$= \frac{2ab^2(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)^2 d} + \frac{a(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2) d} - \frac{af^2 \tan^{-1}(\sinh(c+dx))}{(a^2+b^2) d^3} + \frac{b^3(\ldots}{\ldots}$$

Mathematica [B] time = 31.5229, size = 3368, normalized size = 3.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(-12*b^3*d^3*e^2*E^(2*c)*x + 12*a^2*b*d*E^(2*c)*f^2*x + 12*b^3*d*E^(2*c)*f
^2*x - 12*b^3*d^3*e*E^(2*c)*f*x^2 - 4*b^3*d^3*E^(2*c)*f^2*x^3 - 6*a^3*d^2*e
^2*ArcTan[E^(c + d*x)] - 18*a*b^2*d^2*e^2*ArcTan[E^(c + d*x)] - 6*a^3*d^2*e
^2*E^(2*c)*ArcTan[E^(c + d*x)] - 18*a*b^2*d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x
)] + 12*a^3*f^2*ArcTan[E^(c + d*x)] + 12*a*b^2*f^2*ArcTan[E^(c + d*x)] + 12
*a^3*E^(2*c)*f^2*ArcTan[E^(c + d*x)] + 12*a*b^2*E^(2*c)*f^2*ArcTan[E^(c + d
*x)] - (6*I)*a^3*d^2*e*f*x*Log[1 - I*E^(c + d*x)] - (18*I)*a*b^2*d^2*e*f*x*
Log[1 - I*E^(c + d*x)] - (6*I)*a^3*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)]
 - (18*I)*a*b^2*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] - (3*I)*a^3*d^2*f^
2*x^2*Log[1 - I*E^(c + d*x)] - (9*I)*a*b^2*d^2*f^2*x^2*Log[1 - I*E^(c + d*x
)] - (3*I)*a^3*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] - (9*I)*a*b^2*d^2
*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (6*I)*a^3*d^2*e*f*x*Log[1 + I*E^(
c + d*x)] + (18*I)*a*b^2*d^2*e*f*x*Log[1 + I*E^(c + d*x)] + (6*I)*a^3*d^2*e
*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] + (18*I)*a*b^2*d^2*e*E^(2*c)*f*x*Log[1
+ I*E^(c + d*x)] + (3*I)*a^3*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] + (9*I)*a*b
^2*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] + (3*I)*a^3*d^2*E^(2*c)*f^2*x^2*Log[1
```

+ I*E^(c + d*x)] + (9*I)*a*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 + I*E^(c + d*x)] + 6*b^3*d^2*e^2*Log[1 + E^(2*(c + d*x))] + 6*b^3*d^2*e^2*E^(2*c)*Log[1 + E^(2*(c + d*x))] - 6*a^2*b*f^2*Log[1 + E^(2*(c + d*x))] - 6*b^3*f^2*Log[1 + E^(2*(c + d*x))] - 6*a^2*b*E^(2*c)*f^2*Log[1 + E^(2*(c + d*x))] - 6*b^3*E^(2*c)*f^2*Log[1 + E^(2*(c + d*x))] + 12*b^3*d^2*e*f*x*Log[1 + E^(2*(c + d*x))] + 12*b^3*d^2*e*E^(2*c)*f*x*Log[1 + E^(2*(c + d*x))] + 6*b^3*d^2*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 6*b^3*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(2*(c + d*x))] + (6*I)*a*(a^2 + 3*b^2)*d*(1 + E^(2*c))*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)] - (6*I)*a*(a^2 + 3*b^2)*d*(1 + E^(2*c))*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)] + 6*b^3*d*e*f*PolyLog[2, -E^(2*(c + d*x))] + 6*b^3*d*e*E^(2*c)*f*PolyLog[2, -E^(2*(c + d*x))] + 6*b^3*d*f^2*x*PolyLog[2, -E^(2*(c + d*x))] + 6*b^3*d*E^(2*c)*f^2*x*PolyLog[2, -E^(2*(c + d*x))] - (6*I)*a^3*f^2*PolyLog[3, (-I)*E^(c + d*x)] - (18*I)*a*b^2*f^2*PolyLog[3, (-I)*E^(c + d*x)] - (6*I)*a^3*E^(2*c)*f^2*PolyLog[3, (-I)*E^(c + d*x)] - (18*I)*a*b^2*E^(2*c)*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (6*I)*a^3*f^2*PolyLog[3, I*E^(c + d*x)] + (18*I)*a*b^2*f^2*PolyLog[3, I*E^(c + d*x)] + (6*I)*a^3*E^(2*c)*f^2*PolyLog[3, I*E^(c + d*x)] + (18*I)*a*b^2*E^(2*c)*f^2*PolyLog[3, I*E^(c + d*x)] - 3*b^3*f^2*PolyLog[3, -E^(2*(c + d*x))] - 3*b^3*E^(2*c)*f^2*PolyLog[3, -E^(2*(c + d*x))]))/(6*(a^2 + b^2)^2*d^3*(1 + E^(2*c))) - (b^3*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3))/(3*(a^2 + b^2)^2*(-1 + E^(2*c))) + (Csch[c]*Sech[c]*Sech[c + d*x]^2*(-6*a^2*b*e*f - 6*b^3*e*f + 12*b^3*d^2*e^2*x - 6*a^2*b*f^2*x - 6*b^3*f^2*x + 12*b^3*d^2*e*f*x^2 + 4*b^3*d^2*f^2*x^3 + 6*a^2*b*e*f*Cosh[2*c] + 6*b^3*e*f*Cosh[2*c] + 6*a^2*b*f^2*x*Cosh[2*c] + 6*b^3*f^2*x*Cosh[2*c] + 6*a^2*b*e*f*Cosh[2*d*x] + 6*b^3*e*f*Cosh[2*d*x] + 6*a^2*b*f^2*x*Cosh[2*d*x] + 6*b^3*f^2*x*Cosh[2*d*x] - 3*a^3*d*e^2*Cosh[c - d*x] - 3*a*b^2*d*e^2*Cosh[c - d*x] - 6*a^3*d*e*f*x*Cosh[c - d*x] - 6*a*b^2*d*e*f*x*Cosh[c - d*x] - 3*a^3*d*f^2*x^2*Cosh[c - d*x] - 3*a*b^2*d*f^2*x^2*Cosh[c - d*x] + 3*a^3*d*e^2*Cosh[3*c + d*x] + 3*a*b^2*d*e^2*Cosh[3*c + d*x] + 6*a^3*d*e*f*x*Cosh[3*c + d*x] + 6*a*b^2*d*e*f*x*Cosh[3*c + d*x] + 3*a^3*d*f^2*x^2*Cosh[3*c + d*x] + 3*a*b^2*d*f^2*x^2*Cosh[3*c + d*x] - 6*a^2*b*e*f*Cosh[2*c + 2*d*x] - 6*b^3*e*f*Cosh[2*c + 2*d*x] + 12*b^3*d^2*e^2*x*Cosh[2*c + 2*d*x] - 6*a^2*b*f^2*x*Cosh[2*c + 2*d*x] - 6*b^3*f^2*x*Cosh[2*c + 2*d*x] + 12*b^3*d^2*e*f*x^2*Cosh[2*c + 2*d*x] + 4*b^3*d^2*f^2*x^3*Cosh[2*c + 2*d*x] + 6*a^2*b*d*e^2*Sinh[2*c] + 6*b^3*d*e^2*Sinh[2*c] + 12*a^2*b*d*e*f*x*Sinh[2*c] + 12*b^3*d*e*f*x*Sinh[2*c] + 6*a^2*b*d*f^2*x^2*Sinh[2*c] + 6*b^3*d*f^2*x^2*Sinh[2*c] + 6*a^3*e*f*Sinh[c - d*x] +

```
6*a*b^2*e*f*Sinh[c - d*x] + 6*a^3*f^2*x*Sinh[c - d*x] + 6*a*b^2*f^2*x*Sinh[
c - d*x] + 6*a^3*e*f*Sinh[3*c + d*x] + 6*a*b^2*e*f*Sinh[3*c + d*x] + 6*a^3*
f^2*x*Sinh[3*c + d*x] + 6*a*b^2*f^2*x*Sinh[3*c + d*x]))/(24*(a^2 + b^2)^2*d
^2)
```

Maple [F] time = 0.477, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \left(\operatorname{sech}\left(dx+c\right)\right)^{3}}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] a^3*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*
d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b
^4*d^2), x) + 3*a*b^2*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x +
 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 +
 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*b^3*d^2*f^2*integrate(x^2/(a^4*d^2*e^(2*d
*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d
^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*a^3*d^2*e*f*integrate(x*e^(d*x + c)/(
a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x
+ 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 6*a*b^2*d^2*e*f*integrate
(x*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b
^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 4*b^3*d^2
*e*f*integrate(x/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) +
 b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - a^2*b*f
^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/((
a^4 + 2*a^2*b^2 + b^4)*d^3)) - b^3*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4
)*d^3) - log(e^(2*d*x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) + (b^3*log
(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b^4)*d) -
b^3*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^3 + 3*a*b^2)
*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (a*e^(-d*x - c) + 2*b*e
^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*e^(-2*d*x
 - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d))*e^2 - 2*a^3*f^2*arctan(e^(d*x +
 c))/((a^4 + 2*a^2*b^2 + b^4)*d^3) - 2*a*b^2*f^2*arctan(e^(d*x + c))/((a^4
+ 2*a^2*b^2 + b^4)*d^3) + (2*b*f^2*x + 2*b*e*f + (a*d*f^2*x^2*e^(3*c) + 2*a
*e*f*e^(3*c) + 2*(d*e*f + f^2)*a*x*e^(3*c))*e^(3*d*x) + 2*(b*d*f^2*x^2*e^(2
*c) + b*e*f*e^(2*c) + (2*d*e*f + f^2)*b*x*e^(2*c))*e^(2*d*x) - (a*d*f^2*x^2
*e^c - 2*a*e*f*e^c + 2*(d*e*f - f^2)*a*x*e^c)*e^(d*x))/(a^2*d^2 + b^2*d^2 +
 (a^2*d^2*e^(4*c) + b^2*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d
^2*e^(2*c))*e^(2*d*x)) - integrate(2*(b^4*f^2*x^2 + 2*b^4*e*f*x - (a*b^3*f^
2*x^2*e^c + 2*a*b^3*e*f*x*e^c)*e^(d*x))/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b*e
^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c + 2*a^3*b^
```

```
2*e^c + a*b^4*e^c)*e^(d*x)), x)
```

Fricas [C] time = 5.2455, size = 24035, normalized size = 25.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*cosh(d*x + c)^4 + 4*(
(a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*sinh(d*x + c)^4 - 4*(a^2*b + b
^3)*d*e*f + 4*(a^2*b + b^3)*c*f^2 - 2*((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 + a
*b^2)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^3 + a*b^2)*d^2*e*f + (a^3 + a
*b^2)*d*f^2)*x)*cosh(d*x + c)^3 - 2*((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 + a*b
^2)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^3 + a*b^2)*d^2*e*f + (a^3 + a*b
^2)*d*f^2)*x - 8*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*cosh(d*x + c
))*sinh(d*x + c)^3 - 4*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 +
 (a^2*b + b^3)*d*e*f - 2*(a^2*b + b^3)*c*f^2 + (2*(a^2*b + b^3)*d^2*e*f - (
a^2*b + b^3)*d*f^2)*x)*cosh(d*x + c)^2 - 2*(2*(a^2*b + b^3)*d^2*f^2*x^2 + 2
*(a^2*b + b^3)*d^2*e^2 + 2*(a^2*b + b^3)*d*e*f - 4*(a^2*b + b^3)*c*f^2 - 12
*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*cosh(d*x + c)^2 + 2*(2*(a^2*
b + b^3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x + 3*((a^3 + a*b^2)*d^2*f^2*x^2 +
(a^3 + a*b^2)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^3 + a*b^2)*d^2*e*f +
(a^3 + a*b^2)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*((a^3 + a*b^2)*d
^2*f^2*x^2 + (a^3 + a*b^2)*d^2*e^2 - 2*(a^3 + a*b^2)*d*e*f + 2*((a^3 + a*b^
2)*d^2*e*f - (a^3 + a*b^2)*d*f^2)*x)*cosh(d*x + c) - 4*(b^3*d*f^2*x + b^3*d
*e*f + (b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)^4 + 4*(b^3*d*f^2*x + b^3*d*e
*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^3*d*f^2*x + b^3*d*e*f)*sinh(d*x + c)
^4 + 2*(b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)^2 + 2*(b^3*d*f^2*x + b^3*d*e
*f + 3*(b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^3
*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)^3 + (b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x
+ c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 4*(b^3*d*f^2*x
 + b^3*d*e*f + (b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)^4 + 4*(b^3*d*f^2*x +
 b^3*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^3*d*f^2*x + b^3*d*e*f)*sinh(
d*x + c)^4 + 2*(b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)^2 + 2*(b^3*d*f^2*x +
 b^3*d*e*f + 3*(b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
 4*((b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)^3 + (b^3*d*f^2*x + b^3*d*e*f)*c
osh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (4*b^3
*d*f^2*x + 4*b^3*d*e*f - 2*I*(a^3 + 3*a*b^2)*d*f^2*x + (4*b^3*d*f^2*x + 4*b
^3*d*e*f - 2*I*(a^3 + 3*a*b^2)*d*f^2*x - 2*I*(a^3 + 3*a*b^2)*d*e*f)*cosh(d*
x + c)^4 + (16*b^3*d*f^2*x + 16*b^3*d*e*f - 8*I*(a^3 + 3*a*b^2)*d*f^2*x - 8
*I*(a^3 + 3*a*b^2)*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (4*b^3*d*f^2*x +
4*b^3*d*e*f - 2*I*(a^3 + 3*a*b^2)*d*f^2*x - 2*I*(a^3 + 3*a*b^2)*d*e*f)*sinh
(d*x + c)^4 - 2*I*(a^3 + 3*a*b^2)*d*e*f + (8*b^3*d*f^2*x + 8*b^3*d*e*f - 4*
I*(a^3 + 3*a*b^2)*d*f^2*x - 4*I*(a^3 + 3*a*b^2)*d*e*f)*cosh(d*x + c)^2 + (8
*b^3*d*f^2*x + 8*b^3*d*e*f - 4*I*(a^3 + 3*a*b^2)*d*f^2*x - 4*I*(a^3 + 3*a*b
^2)*d*e*f + (24*b^3*d*f^2*x + 24*b^3*d*e*f - 12*I*(a^3 + 3*a*b^2)*d*f^2*x -
 12*I*(a^3 + 3*a*b^2)*d*e*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((16*b^3*d*
f^2*x + 16*b^3*d*e*f - 8*I*(a^3 + 3*a*b^2)*d*f^2*x - 8*I*(a^3 + 3*a*b^2)*d*
e*f)*cosh(d*x + c)^3 + (16*b^3*d*f^2*x + 16*b^3*d*e*f - 8*I*(a^3 + 3*a*b^2)
*d*f^2*x - 8*I*(a^3 + 3*a*b^2)*d*e*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(I
*cosh(d*x + c) + I*sinh(d*x + c)) + (4*b^3*d*f^2*x + 4*b^3*d*e*f + 2*I*(a^3
 + 3*a*b^2)*d*f^2*x + (4*b^3*d*f^2*x + 4*b^3*d*e*f + 2*I*(a^3 + 3*a*b^2)*d*
f^2*x + 2*I*(a^3 + 3*a*b^2)*d*e*f)*cosh(d*x + c)^4 + (16*b^3*d*f^2*x + 16*b
```

^3*d*e*f + 8*I*(a^3 + 3*a*b^2)*d*f^2*x + 8*I*(a^3 + 3*a*b^2)*d*e*f)*cosh(d*
x + c)*sinh(d*x + c)^3 + (4*b^3*d*f^2*x + 4*b^3*d*e*f + 2*I*(a^3 + 3*a*b^2)
*d*f^2*x + 2*I*(a^3 + 3*a*b^2)*d*e*f)*sinh(d*x + c)^4 + 2*I*(a^3 + 3*a*b^2)
*d*e*f + (8*b^3*d*f^2*x + 8*b^3*d*e*f + 4*I*(a^3 + 3*a*b^2)*d*f^2*x + 4*I*(
a^3 + 3*a*b^2)*d*e*f)*cosh(d*x + c)^2 + (8*b^3*d*f^2*x + 8*b^3*d*e*f + 4*I*
(a^3 + 3*a*b^2)*d*f^2*x + 4*I*(a^3 + 3*a*b^2)*d*e*f + (24*b^3*d*f^2*x + 24*
b^3*d*e*f + 12*I*(a^3 + 3*a*b^2)*d*f^2*x + 12*I*(a^3 + 3*a*b^2)*d*e*f)*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + ((16*b^3*d*f^2*x + 16*b^3*d*e*f + 8*I*(a^3 +
 3*a*b^2)*d*f^2*x + 8*I*(a^3 + 3*a*b^2)*d*e*f)*cosh(d*x + c)^3 + (16*b^3*d*
f^2*x + 16*b^3*d*e*f + 8*I*(a^3 + 3*a*b^2)*d*f^2*x + 8*I*(a^3 + 3*a*b^2)*d*
e*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)
) - 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2 + (b^3*d^2*e^2 - 2*b^3*c*d
*e*f + b^3*c^2*f^2)*cosh(d*x + c)^4 + 4*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*
c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3
*c^2*f^2)*sinh(d*x + c)^4 + 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*c
osh(d*x + c)^2 + 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2 + 3*(b^3*d^2*
e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b
^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)^3 + (b^3*d^2*e^2 -
2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x
 + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(b^3*d^2*e
^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2 + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f
^2)*cosh(d*x + c)^4 + 4*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*
x + c)*sinh(d*x + c)^3 + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*sinh(d
x + c)^4 + 2(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)^2 +
 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2 + 3*(b^3*d^2*e^2 - 2*b^3*c*d*
e*f + b^3*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^3*d^2*e^2 - 2*b
^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)^3 + (b^3*d^2*e^2 - 2*b^3*c*d*e*f +
b^3*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh
(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(b^3*d^2*f^2*x^2 + 2*b^3*d
^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2 + (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x
 + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x + c)^4 + 4*(b^3*d^2*f^2*x^2 + 2*b^
3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 +
(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*sinh(d*x
+ c)^4 + 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2
)*cosh(d*x + c)^2 + 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f -
b^3*c^2*f^2 + 3*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^
2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^3*d^2*f^2*x^2 + 2*b^3*d^2*e
*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x + c)^3 + (b^3*d^2*f^2*x^2 + 2*
b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*
log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2
*b^3*c*d*e*f - b^3*c^2*f^2 + (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d
*e*f - b^3*c^2*f^2)*cosh(d*x + c)^4 + 4*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x
+ 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^3*d^2*f^2
*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*sinh(d*x + c)^4 + 2*(
b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x +
 c)^2 + 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2
+ 3*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(
d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3
*c*d*e*f - b^3*c^2*f^2)*cosh(d*x + c)^3 + (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*
x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(-(a*cosh
(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b) + (2*b^3*d^2*e^2 - 4*b^3*c*d*e*f - I*(a^3 + 3*a*b^2)*d
^2*e^2 + 2*I*(a^3 + 3*a*b^2)*c*d*e*f + (2*b^3*d^2*e^2 - 4*b^3*c*d*e*f - I*(
a^3 + 3*a*b^2)*d^2*e^2 + 2*I*(a^3 + 3*a*b^2)*c*d*e*f + 2*(b^3*c^2 - a^2*b -
 b^3)*f^2 + I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2)*f^2)*cosh(d*x + c)^4
+ (8*b^3*d^2*e^2 - 16*b^3*c*d*e*f - 4*I*(a^3 + 3*a*b^2)*d^2*e^2 + 8*I*(a^3
+ 3*a*b^2)*c*d*e*f + 8*(b^3*c^2 - a^2*b - b^3)*f^2 + 4*I*(2*a^3 + 2*a*b^2 -
 (a^3 + 3*a*b^2)*c^2)*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*b^3*d^2*e^2 -

4*b^3*c*d*e*f - I*(a^3 + 3*a*b^2)*d^2*e^2 + 2*I*(a^3 + 3*a*b^2)*c*d*e*f +
2*(b^3*c^2 - a^2*b - b^3)*f^2 + I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2)*f
^2)*sinh(d*x + c)^4 + 2*(b^3*c^2 - a^2*b - b^3)*f^2 + I*(2*a^3 + 2*a*b^2 -
(a^3 + 3*a*b^2)*c^2)*f^2 + (4*b^3*d^2*e^2 - 8*b^3*c*d*e*f - 2*I*(a^3 + 3*a*
b^2)*d^2*e^2 + 4*I*(a^3 + 3*a*b^2)*c*d*e*f + 4*(b^3*c^2 - a^2*b - b^3)*f^2
+ 2*I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2)*f^2)*cosh(d*x + c)^2 + (4*b^3
*d^2*e^2 - 8*b^3*c*d*e*f - 2*I*(a^3 + 3*a*b^2)*d^2*e^2 + 4*I*(a^3 + 3*a*b^2
)*c*d*e*f + 4*(b^3*c^2 - a^2*b - b^3)*f^2 + 2*I*(2*a^3 + 2*a*b^2 - (a^3 + 3
*a*b^2)*c^2)*f^2 + (12*b^3*d^2*e^2 - 24*b^3*c*d*e*f - 6*I*(a^3 + 3*a*b^2)*d
^2*e^2 + 12*I*(a^3 + 3*a*b^2)*c*d*e*f + 12*(b^3*c^2 - a^2*b - b^3)*f^2 + 6*
I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2)*f^2)*cosh(d*x + c)^2)*sinh(d*x +
c)^2 + ((8*b^3*d^2*e^2 - 16*b^3*c*d*e*f - 4*I*(a^3 + 3*a*b^2)*d^2*e^2 + 8*I
*(a^3 + 3*a*b^2)*c*d*e*f + 8*(b^3*c^2 - a^2*b - b^3)*f^2 + 4*I*(2*a^3 + 2*a
*b^2 - (a^3 + 3*a*b^2)*c^2)*f^2)*cosh(d*x + c)^3 + (8*b^3*d^2*e^2 - 16*b^3*
c*d*e*f - 4*I*(a^3 + 3*a*b^2)*d^2*e^2 + 8*I*(a^3 + 3*a*b^2)*c*d*e*f + 8*(b^
3*c^2 - a^2*b - b^3)*f^2 + 4*I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2)*f^2)
*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + I) + (2*
b^3*d^2*e^2 - 4*b^3*c*d*e*f + I*(a^3 + 3*a*b^2)*d^2*e^2 - 2*I*(a^3 + 3*a*b^
2)*c*d*e*f + (2*b^3*d^2*e^2 - 4*b^3*c*d*e*f + I*(a^3 + 3*a*b^2)*d^2*e^2 - 2
I(a^3 + 3*a*b^2)*c*d*e*f + 2*(b^3*c^2 - a^2*b - b^3)*f^2 - I*(2*a^3 + 2*a
*b^2 - (a^3 + 3*a*b^2)*c^2)*f^2)*cosh(d*x + c)^4 + (8*b^3*d^2*e^2 - 16*b^3*
c*d*e*f + 4*I*(a^3 + 3*a*b^2)*d^2*e^2 - 8*I*(a^3 + 3*a*b^2)*c*d*e*f + 8*(b^
3*c^2 - a^2*b - b^3)*f^2 - 4*I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2)*f^2)
*cosh(d*x + c)*sinh(d*x + c)^3 + (2*b^3*d^2*e^2 - 4*b^3*c*d*e*f + I*(a^3 +
3*a*b^2)*d^2*e^2 - 2*I*(a^3 + 3*a*b^2)*c*d*e*f + 2*(b^3*c^2 - a^2*b - b^3)*
f^2 - I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2)*f^2)*sinh(d*x + c)^4 + 2*(b
^3*c^2 - a^2*b - b^3)*f^2 - I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2)*f^2 +
(4*b^3*d^2*e^2 - 8*b^3*c*d*e*f + 2*I*(a^3 + 3*a*b^2)*d^2*e^2 - 4*I*(a^3 +
3*a*b^2)*c*d*e*f + 4*(b^3*c^2 - a^2*b - b^3)*f^2 - 2*I*(2*a^3 + 2*a*b^2 - (
a^3 + 3*a*b^2)*c^2)*f^2)*cosh(d*x + c)^2 + (4*b^3*d^2*e^2 - 8*b^3*c*d*e*f +
2*I*(a^3 + 3*a*b^2)*d^2*e^2 - 4*I*(a^3 + 3*a*b^2)*c*d*e*f + 4*(b^3*c^2 - a
^2*b - b^3)*f^2 - 2*I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2)*f^2 + (12*b^3
*d^2*e^2 - 24*b^3*c*d*e*f + 6*I*(a^3 + 3*a*b^2)*d^2*e^2 - 12*I*(a^3 + 3*a*b
^2)*c*d*e*f + 12*(b^3*c^2 - a^2*b - b^3)*f^2 - 6*I*(2*a^3 + 2*a*b^2 - (a^3
+ 3*a*b^2)*c^2)*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*b^3*d^2*e^2 - 1
6*b^3*c*d*e*f + 4*I*(a^3 + 3*a*b^2)*d^2*e^2 - 8*I*(a^3 + 3*a*b^2)*c*d*e*f +
8*(b^3*c^2 - a^2*b - b^3)*f^2 - 4*I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2
)*f^2)*cosh(d*x + c)^3 + (8*b^3*d^2*e^2 - 16*b^3*c*d*e*f + 4*I*(a^3 + 3*a*b
^2)*d^2*e^2 - 8*I*(a^3 + 3*a*b^2)*c*d*e*f + 8*(b^3*c^2 - a^2*b - b^3)*f^2 -
4*I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2)*f^2)*cosh(d*x + c))*sinh(d*x +
c))*log(cosh(d*x + c) + sinh(d*x + c) - I) + (2*b^3*d^2*f^2*x^2 + 4*b^3*d^
2*e*f*x + 4*b^3*c*d*e*f - 2*b^3*c^2*f^2 + I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 + 2
I(a^3 + 3*a*b^2)*d^2*e*f*x + 2*I*(a^3 + 3*a*b^2)*c*d*e*f - I*(a^3 + 3*a*b
^2)*c^2*f^2 + (2*b^3*d^2*f^2*x^2 + 4*b^3*d^2*e*f*x + 4*b^3*c*d*e*f - 2*b^3*
c^2*f^2 + I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 + 2*I*(a^3 + 3*a*b^2)*d^2*e*f*x + 2
I(a^3 + 3*a*b^2)*c*d*e*f - I*(a^3 + 3*a*b^2)*c^2*f^2)*cosh(d*x + c)^4 + (
8*b^3*d^2*f^2*x^2 + 16*b^3*d^2*e*f*x + 16*b^3*c*d*e*f - 8*b^3*c^2*f^2 + 4*I
*(a^3 + 3*a*b^2)*d^2*f^2*x^2 + 8*I*(a^3 + 3*a*b^2)*d^2*e*f*x + 8*I*(a^3 + 3
*a*b^2)*c*d*e*f - 4*I*(a^3 + 3*a*b^2)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^
3 + (2*b^3*d^2*f^2*x^2 + 4*b^3*d^2*e*f*x + 4*b^3*c*d*e*f - 2*b^3*c^2*f^2 +
I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 + 2*I*(a^3 + 3*a*b^2)*d^2*e*f*x + 2*I*(a^3 +
3*a*b^2)*c*d*e*f - I*(a^3 + 3*a*b^2)*c^2*f^2)*sinh(d*x + c)^4 + (4*b^3*d^2*
f^2*x^2 + 8*b^3*d^2*e*f*x + 8*b^3*c*d*e*f - 4*b^3*c^2*f^2 + 2*I*(a^3 + 3*a*
b^2)*d^2*f^2*x^2 + 4*I*(a^3 + 3*a*b^2)*d^2*e*f*x + 4*I*(a^3 + 3*a*b^2)*c*d*
e*f - 2*I*(a^3 + 3*a*b^2)*c^2*f^2)*cosh(d*x + c)^2 + (4*b^3*d^2*f^2*x^2 + 8
*b^3*d^2*e*f*x + 8*b^3*c*d*e*f - 4*b^3*c^2*f^2 + 2*I*(a^3 + 3*a*b^2)*d^2*f^
2*x^2 + 4*I*(a^3 + 3*a*b^2)*d^2*e*f*x + 4*I*(a^3 + 3*a*b^2)*c*d*e*f - 2*I*(
a^3 + 3*a*b^2)*c^2*f^2 + (12*b^3*d^2*f^2*x^2 + 24*b^3*d^2*e*f*x + 24*b^3*c*
d*e*f - 12*b^3*c^2*f^2 + 6*I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 + 12*I*(a^3 + 3*a*

```
b^2)*d^2*e*f*x + 12*I*(a^3 + 3*a*b^2)*c*d*e*f - 6*I*(a^3 + 3*a*b^2)*c^2*f^2
)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*b^3*d^2*f^2*x^2 + 16*b^3*d^2*e*f*x
 + 16*b^3*c*d*e*f - 8*b^3*c^2*f^2 + 4*I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 + 8*I*(
a^3 + 3*a*b^2)*d^2*e*f*x + 8*I*(a^3 + 3*a*b^2)*c*d*e*f - 4*I*(a^3 + 3*a*b^2
)*c^2*f^2)*cosh(d*x + c)^3 + (8*b^3*d^2*f^2*x^2 + 16*b^3*d^2*e*f*x + 16*b^3
*c*d*e*f - 8*b^3*c^2*f^2 + 4*I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 + 8*I*(a^3 + 3*a
*b^2)*d^2*e*f*x + 8*I*(a^3 + 3*a*b^2)*c*d*e*f - 4*I*(a^3 + 3*a*b^2)*c^2*f^2
)*cosh(d*x + c))*sinh(d*x + c))*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1)
 + (2*b^3*d^2*f^2*x^2 + 4*b^3*d^2*e*f*x + 4*b^3*c*d*e*f - 2*b^3*c^2*f^2 - I*
(a^3 + 3*a*b^2)*d^2*f^2*x^2 - 2*I*(a^3 + 3*a*b^2)*d^2*e*f*x - 2*I*(a^3 + 3*
a*b^2)*c*d*e*f + I*(a^3 + 3*a*b^2)*c^2*f^2 + (2*b^3*d^2*f^2*x^2 + 4*b^3*d^2
*e*f*x + 4*b^3*c*d*e*f - 2*b^3*c^2*f^2 - I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 - 2*
I*(a^3 + 3*a*b^2)*d^2*e*f*x - 2*I*(a^3 + 3*a*b^2)*c*d*e*f + I*(a^3 + 3*a*b^
2)*c^2*f^2)*cosh(d*x + c)^4 + (8*b^3*d^2*f^2*x^2 + 16*b^3*d^2*e*f*x + 16*b^
3*c*d*e*f - 8*b^3*c^2*f^2 - 4*I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 - 8*I*(a^3 + 3*
a*b^2)*d^2*e*f*x - 8*I*(a^3 + 3*a*b^2)*c*d*e*f + 4*I*(a^3 + 3*a*b^2)*c^2*f^
2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*b^3*d^2*f^2*x^2 + 4*b^3*d^2*e*f*x + 4
*b^3*c*d*e*f - 2*b^3*c^2*f^2 - I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 - 2*I*(a^3 + 3
*a*b^2)*d^2*e*f*x - 2*I*(a^3 + 3*a*b^2)*c*d*e*f + I*(a^3 + 3*a*b^2)*c^2*f^2
)*sinh(d*x + c)^4 + (4*b^3*d^2*f^2*x^2 + 8*b^3*d^2*e*f*x + 8*b^3*c*d*e*f -
4*b^3*c^2*f^2 - 2*I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 - 4*I*(a^3 + 3*a*b^2)*d^2*e
*f*x - 4*I*(a^3 + 3*a*b^2)*c*d*e*f + 2*I*(a^3 + 3*a*b^2)*c^2*f^2)*cosh(d*x
 + c)^2 + (4*b^3*d^2*f^2*x^2 + 8*b^3*d^2*e*f*x + 8*b^3*c*d*e*f - 4*b^3*c^2*f
^2 - 2*I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 - 4*I*(a^3 + 3*a*b^2)*d^2*e*f*x - 4*I*
(a^3 + 3*a*b^2)*c*d*e*f + 2*I*(a^3 + 3*a*b^2)*c^2*f^2 + (12*b^3*d^2*f^2*x^2
 + 24*b^3*d^2*e*f*x + 24*b^3*c*d*e*f - 12*b^3*c^2*f^2 - 6*I*(a^3 + 3*a*b^2)
*d^2*f^2*x^2 - 12*I*(a^3 + 3*a*b^2)*d^2*e*f*x - 12*I*(a^3 + 3*a*b^2)*c*d*e*
f + 6*I*(a^3 + 3*a*b^2)*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*b^3
*d^2*f^2*x^2 + 16*b^3*d^2*e*f*x + 16*b^3*c*d*e*f - 8*b^3*c^2*f^2 - 4*I*(a^3
 + 3*a*b^2)*d^2*f^2*x^2 - 8*I*(a^3 + 3*a*b^2)*d^2*e*f*x - 8*I*(a^3 + 3*a*b^
2)*c*d*e*f + 4*I*(a^3 + 3*a*b^2)*c^2*f^2)*cosh(d*x + c)^3 + (8*b^3*d^2*f^2*
x^2 + 16*b^3*d^2*e*f*x + 16*b^3*c*d*e*f - 8*b^3*c^2*f^2 - 4*I*(a^3 + 3*a*b^
2)*d^2*f^2*x^2 - 8*I*(a^3 + 3*a*b^2)*d^2*e*f*x - 8*I*(a^3 + 3*a*b^2)*c*d*e*
f + 4*I*(a^3 + 3*a*b^2)*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(-I*cosh(
d*x + c) - I*sinh(d*x + c) + 1) + 4*(b^3*f^2*cosh(d*x + c)^4 + 4*b^3*f^2*co
sh(d*x + c)*sinh(d*x + c)^3 + b^3*f^2*sinh(d*x + c)^4 + 2*b^3*f^2*cosh(d*x
 + c)^2 + b^3*f^2 + 2*(3*b^3*f^2*cosh(d*x + c)^2 + b^3*f^2)*sinh(d*x + c)^2
 + 4*(b^3*f^2*cosh(d*x + c)^3 + b^3*f^2*cosh(d*x + c))*sinh(d*x + c))*polylo
g(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2))/b) + 4*(b^3*f^2*cosh(d*x + c)^4 + 4*b^3*f^2*cosh(
d*x + c)*sinh(d*x + c)^3 + b^3*f^2*sinh(d*x + c)^4 + 2*b^3*f^2*cosh(d*x + c
)^2 + b^3*f^2 + 2*(3*b^3*f^2*cosh(d*x + c)^2 + b^3*f^2)*sinh(d*x + c)^2 + 4
*(b^3*f^2*cosh(d*x + c)^3 + b^3*f^2*cosh(d*x + c))*sinh(d*x + c))*polylog(3
, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2))/b) - (4*b^3*f^2 + (4*b^3*f^2 - 2*I*(a^3 + 3*a*b^2)*f
^2)*cosh(d*x + c)^4 + (16*b^3*f^2 - 8*I*(a^3 + 3*a*b^2)*f^2)*cosh(d*x + c)*
sinh(d*x + c)^3 + (4*b^3*f^2 - 2*I*(a^3 + 3*a*b^2)*f^2)*sinh(d*x + c)^4 - 2
*I*(a^3 + 3*a*b^2)*f^2 + (8*b^3*f^2 - 4*I*(a^3 + 3*a*b^2)*f^2)*cosh(d*x + c
)^2 + (8*b^3*f^2 - 4*I*(a^3 + 3*a*b^2)*f^2 + (24*b^3*f^2 - 12*I*(a^3 + 3*a*
b^2)*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((16*b^3*f^2 - 8*I*(a^3 + 3*a*
b^2)*f^2)*cosh(d*x + c)^3 + (16*b^3*f^2 - 8*I*(a^3 + 3*a*b^2)*f^2)*cosh(d*x
 + c))*sinh(d*x + c))*polylog(3, I*cosh(d*x + c) + I*sinh(d*x + c)) - (4*b^
3*f^2 + (4*b^3*f^2 + 2*I*(a^3 + 3*a*b^2)*f^2)*cosh(d*x + c)^4 + (16*b^3*f^2
 + 8*I*(a^3 + 3*a*b^2)*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (4*b^3*f^2 + 2*
I*(a^3 + 3*a*b^2)*f^2)*sinh(d*x + c)^4 + 2*I*(a^3 + 3*a*b^2)*f^2 + (8*b^3*f
^2 + 4*I*(a^3 + 3*a*b^2)*f^2)*cosh(d*x + c)^2 + (8*b^3*f^2 + 4*I*(a^3 + 3*a
*b^2)*f^2 + (24*b^3*f^2 + 12*I*(a^3 + 3*a*b^2)*f^2)*cosh(d*x + c)^2)*sinh(d
*x + c)^2 + ((16*b^3*f^2 + 8*I*(a^3 + 3*a*b^2)*f^2)*cosh(d*x + c)^3 + (16*b
^3*f^2 + 8*I*(a^3 + 3*a*b^2)*f^2)*cosh(d*x + c))*sinh(d*x + c))*polylog(3,
```

```
-I*cosh(d*x + c) - I*sinh(d*x + c)) + 2*((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 +
 a*b^2)*d^2*e^2 - 2*(a^3 + a*b^2)*d*e*f + 8*((a^2*b + b^3)*d*f^2*x + (a^2*b
 + b^3)*c*f^2)*cosh(d*x + c)^3 - 3*((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 + a*b^
2)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^3 + a*b^2)*d^2*e*f + (a^3 + a*b^
2)*d*f^2)*x)*cosh(d*x + c)^2 + 2*((a^3 + a*b^2)*d^2*e*f - (a^3 + a*b^2)*d*f
^2)*x - 4*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 + (a^2*b + b^3
)*d*e*f - 2*(a^2*b + b^3)*c*f^2 + (2*(a^2*b + b^3)*d^2*e*f - (a^2*b + b^3)*
d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d
*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c)*sinh(d*x + c)^3 + (
a^4 + 2*a^2*b^2 + b^4)*d^3*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*
cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^3 + 2*(3*(a^4 + 2*a^2*b^2 + b^4
)*d^3*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^3)*sinh(d*x + c)^2 + 4*((
a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d^3*co
sh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \operatorname{sech}(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sech(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

3.315 $\int \frac{(e+fx)\mathbf{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=560

$$\frac{b^3 f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^2}+\frac{b^3 f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2\left(a^2+b^2\right)^2}-\frac{b^3 f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d^2\left(a^2+b^2\right)^2}-\frac{iab^2 f\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)^2}+$$

```
[Out] (2*a*b^2*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)^2*d) + (a*(e + f*x)*Ar
cTan[E^(c + d*x)])/((a^2 + b^2)*d) + (b^3*(e + f*x)*Log[1 + (b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) + (b^3*(e + f*x)*Log[1 + (b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) - (b^3*(e + f*x)*Log[1 +
 E^(2*(c + d*x))])/((a^2 + b^2)^2*d) - (I*a*b^2*f*PolyLog[2, (-I)*E^(c + d*
x)])/((a^2 + b^2)^2*d^2) - ((I/2)*a*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 +
 b^2)*d^2) + (I*a*b^2*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) + ((
I/2)*a*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) + (b^3*f*PolyLog[2, -
((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) + (b^3*f*Poly
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) - (b^
3*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^2) + (a*f*Sech[c + d*x
])/(2*(a^2 + b^2)*d^2) + (b*(e + f*x)*Sech[c + d*x]^2)/(2*(a^2 + b^2)*d) -
(b*f*Tanh[c + d*x])/(2*(a^2 + b^2)*d^2) + (a*(e + f*x)*Sech[c + d*x]*Tanh[c
 + d*x])/(2*(a^2 + b^2)*d)
```

Rubi [A] time = 0.938346, antiderivative size = 560, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {5573, 5561, 2190, 2279, 2391, 6742, 4180, 3718, 4185, 5451, 3767, 8}

$$\frac{b^3 f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^2}+\frac{b^3 f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2\left(a^2+b^2\right)^2}-\frac{b^3 f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2d^2\left(a^2+b^2\right)^2}-\frac{iab^2 f\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a*b^2*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)^2*d) + (a*(e + f*x)*Ar
cTan[E^(c + d*x)])/((a^2 + b^2)*d) + (b^3*(e + f*x)*Log[1 + (b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) + (b^3*(e + f*x)*Log[1 + (b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) - (b^3*(e + f*x)*Log[1 +
 E^(2*(c + d*x))])/((a^2 + b^2)^2*d) - (I*a*b^2*f*PolyLog[2, (-I)*E^(c + d*
x)])/((a^2 + b^2)^2*d^2) - ((I/2)*a*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 +
 b^2)*d^2) + (I*a*b^2*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) + ((
I/2)*a*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) + (b^3*f*PolyLog[2, -
((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) + (b^3*f*Poly
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) - (b^
3*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^2) + (a*f*Sech[c + d*x
])/(2*(a^2 + b^2)*d^2) + (b*(e + f*x)*Sech[c + d*x]^2)/(2*(a^2 + b^2)*d) -
(b*f*Tanh[c + d*x])/(2*(a^2 + b^2)*d^2) + (a*(e + f*x)*Sech[c + d*x]*Tanh[c
 + d*x])/(2*(a^2 + b^2)*d)
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
```

```
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{sech}^3(c+dx)(a-b\sinh(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{b^2\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))\,dx}{\left(a^2+b^2\right)^2} + \frac{b^4\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx}{\left(a^2+b^2\right)^2} + \frac{\int \left(a(e+fx)\right.}{}$$

$$= -\frac{b^3(e+fx)^2}{2\left(a^2+b^2\right)^2 f} + \frac{b^2\int (a(e+fx)\text{sech}(c+dx) - b(e+fx)\tanh(c+dx))\,dx}{\left(a^2+b^2\right)^2} + \frac{b^4\int \frac{e^{c+dx}}{a-\sqrt{a^2+}}}{\left(a^2+\right.}$$

$$= -\frac{b^3(e+fx)^2}{2\left(a^2+b^2\right)^2 f} + \frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2 d} + \frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2 d} + \frac{af\text{sec}}{2\left(a^2\right.}$$

$$= \frac{2ab^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} + \frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right) d} + \frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2 d} + \frac{b}{}$$

$$= \frac{2ab^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} + \frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right) d} + \frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2 d} + \frac{b}{}$$

$$= \frac{2ab^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} + \frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right) d} + \frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2 d} + \frac{b}{}$$

$$= \frac{2ab^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} + \frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right) d} + \frac{b^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2 d} + \frac{b}{}$$

Mathematica [A] time = 7.07565, size = 588, normalized size = 1.05

$$\frac{-iaf\left(a^2+3b^2\right)\text{PolyLog}\left(2,-ie^{c+dx}\right) + iaf\left(a^2+3b^2\right)\text{PolyLog}\left(2,ie^{c+dx}\right) + 2b^3f\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) + 2b^3f\text{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*b^3*d*e*(c + d*x) - 2*b^3*c*f*(c + d*x) + 2*a^3*d*e*ArcTan[E^(c + d*x)]
+ 6*a*b^2*d*e*ArcTan[E^(c + d*x)] - 2*a^3*c*f*ArcTan[E^(c + d*x)] - 6*a*b^2
*c*f*ArcTan[E^(c + d*x)] + I*a^3*f*(c + d*x)*Log[1 - I*E^(c + d*x)] + (3*I)
*a*b^2*f*(c + d*x)*Log[1 - I*E^(c + d*x)] - I*a^3*f*(c + d*x)*Log[1 + I*E^(
c + d*x)] - (3*I)*a*b^2*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + 2*b^3*f*(c + d
*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*b^3*f*(c + d*x)*Log[
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*b^3*d*e*Log[1 + E^(2*(c + d*
x))] + 2*b^3*c*f*Log[1 + E^(2*(c + d*x))] - 2*b^3*f*(c + d*x)*Log[1 + E^(2*
(c + d*x))] + 2*b^3*d*e*Log[a + b*Sinh[c + d*x]] - 2*b^3*c*f*Log[a + b*Sinh
[c + d*x]] - I*a*(a^2 + 3*b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] + I*a*(a^2 +
3*b^2)*f*PolyLog[2, I*E^(c + d*x)] + 2*b^3*f*PolyLog[2, (b*E^(c + d*x))/(-a
 + Sqrt[a^2 + b^2])] + 2*b^3*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))] - b^3*f*PolyLog[2, -E^(2*(c + d*x))] + (a^2 + b^2)*d*(e + f*x)*Sec
h[c + d*x]^2*(b + a*Sinh[c + d*x]) + (a^2 + b^2)*f*Sech[c + d*x]*(a - b*Sin
h[c + d*x]))/(2*(a^2 + b^2)^2*d^2)
```

Maple [B] time = 0.176, size = 2051, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] 2/(a^2+b^2)/d^2*b^3*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/
(a+(a^2+b^2)^(1/2)))-2/(a^2+b^2)/d*b^3*e/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))
+2/(a^2+b^2)/d*b^3*e/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/
(a^2+b^2)/d*a^3*e/(2*a^2+2*b^2)*arctan(exp(d*x+c))-2/(a^2+b^2)/d^2*b^3*f/(2
*a^2+2*b^2)*dilog(1+I*exp(d*x+c))-2/(a^2+b^2)/d^2*b^3*f/(2*a^2+2*b^2)*dilog
(1-I*exp(d*x+c))+2/(a^2+b^2)/d^2*b^3*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(
a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2/(a^2+b^2)/d*b^3*f/(2*a^2+2*b^2)*l
n(1+I*exp(d*x+c))*x-2/(a^2+b^2)/d^2*b^3*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*
c-2/(a^2+b^2)/d*b^3*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x-2/(a^2+b^2)/d^2*b^
3*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c+2/(a^2+b^2)/d^2*b^3*f*c/(2*a^2+2*b^2
)*ln(1+exp(2*d*x+2*c))-2/(a^2+b^2)/d^2*b^3*f*c/(2*a^2+2*b^2)*ln(b*exp(2*d*x
+2*c)+2*a*exp(d*x+c)-b)+I/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*dilog(1-I*exp(d
*x+c))-2/(a^2+b^2)/d^2*a^3*f*c/(2*a^2+2*b^2)*arctan(exp(d*x+c))-I/(a^2+b^2)
/d^2*a^3*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))+1/(a^2+b^2)^(3/2)/d^2*a*b^3*
f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+
b^2)^(3/2)/d^2*a^3*b*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^
2+b^2)^(1/2))-1/(a^2+b^2)^(1/2)/d^2*a*b*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*
exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+3*I/(a^2+b^2)/d^2*a*b^2*f/(2*a^2+2*b^2)*ln
(1-I*exp(d*x+c))*c-3*I/(a^2+b^2)/d*a*b^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))
*x-3*I/(a^2+b^2)/d^2*a*b^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c+3*I/(a^2+b^
2)/d*a*b^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x-3*I/(a^2+b^2)/d^2*a*b^2*f/(
2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))+3*I/(a^2+b^2)/d^2*a*b^2*f/(2*a^2+2*b^2)*
dilog(1-I*exp(d*x+c))-I/(a^2+b^2)/d*a^3*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*
x-I/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c-1/(a^2+b^2)^(3/2
)/d*a*b^3*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
-1/(a^2+b^2)^(3/2)/d*a^3*b*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)
/(a^2+b^2)^(1/2))-6/(a^2+b^2)/d^2*a*b^2*f*c/(2*a^2+2*b^2)*arctan(exp(d*x+c)
)+I/(a^2+b^2)/d*a^3*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x+I/(a^2+b^2)/d^2*a^
3*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c+1/(a^2+b^2)^(1/2)/d*a*b*e/(2*a^2+2*b
^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+6/(a^2+b^2)/d*a*b^2*e
/(2*a^2+2*b^2)*arctan(exp(d*x+c))+2/(a^2+b^2)/d*b^3*f/(2*a^2+2*b^2)*ln((-b*
exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+2/(a^2+b^2)/d^2*b^3*f
/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c
```

```
+2/(a^2+b^2)/d*b^3*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(
a^2+b^2)^(1/2)))*x+2/(a^2+b^2)/d^2*b^3*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^
2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+(a*d*f*x*exp(3*d*x+3*c)+a*d*e*exp(3*
d*x+3*c)+2*b*d*f*x*exp(2*d*x+2*c)-a*d*f*x*exp(d*x+c)+a*f*exp(3*d*x+3*c)+2*b
*d*e*exp(2*d*x+2*c)-a*d*e*exp(d*x+c)+b*f*exp(2*d*x+2*c)+a*f*exp(d*x+c)+b*f)
/d^2/(a^2+b^2)/(1+exp(2*d*x+2*c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(\frac{b^3 \log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{\left(a^4 + 2\,a^2b^2 + b^4\right)d} - \frac{b^3 \log\left(e^{(-2\,dx-2\,c)} + 1\right)}{\left(a^4 + 2\,a^2b^2 + b^4\right)d} - \frac{\left(a^3 + 3\,ab^2\right)\arctan\left(e^{(-dx-c)}\right)}{\left(a^4 + 2\,a^2b^2 + b^4\right)d} + \frac{ae^{(-dx-c)} + 2\,b}{\left(a^2 + b^2 + 2\left(a^2 + b^2\right)e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b^
4)*d) - b^3*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^3 +
3*a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (a*e^(-d*x - c)
 + 2*b*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*e
^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d))*e + f*(((a*d*x*e^(3*c)
+ a*e^(3*c))*e^(3*d*x) + (2*b*d*x*e^(2*c) + b*e^(2*c))*e^(2*d*x) - (a*d*x*e
^c - a*e^c)*e^(d*x) + b)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) + b^2*d^2*e^
(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*x)) - 8*int
egrate(-1/4*(a*b^3*x*e^(d*x + c) - b^4*x)/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b
*e^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c + 2*a^3*
b^2*e^c + a*b^4*e^c)*e^(d*x)), x) + 8*integrate(1/8*(2*b^3*x + (a^3*e^c + 3
*a*b^2*e^c)*x*e^(d*x))/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(2*c) + 2*a^2*b^2*e^
(2*c) + b^4*e^(2*c))*e^(2*d*x)), x))
```

Fricas [B] time = 3.51499, size = 11385, normalized size = 20.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3 + a*b^2)*f)*cosh(d*x
 + c)^3 + 2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3 + a*b^2)*f)*sin
h(d*x + c)^3 + 2*(2*(a^2*b + b^3)*d*f*x + 2*(a^2*b + b^3)*d*e + (a^2*b + b^
3)*f)*cosh(d*x + c)^2 + 2*(2*(a^2*b + b^3)*d*f*x + 2*(a^2*b + b^3)*d*e + (a
^2*b + b^3)*f + 3*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3 + a*b^2)*
f)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(a^2*b + b^3)*f - 2*((a^3 + a*b^2)*d*
f*x + (a^3 + a*b^2)*d*e - (a^3 + a*b^2)*f)*cosh(d*x + c) + 2*(b^3*f*cosh(d*
x + c)^4 + 4*b^3*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*f*sinh(d*x + c)^4 +
2*b^3*f*cosh(d*x + c)^2 + b^3*f + 2*(3*b^3*f*cosh(d*x + c)^2 + b^3*f)*sinh(
d*x + c)^2 + 4*(b^3*f*cosh(d*x + c)^3 + b^3*f*cosh(d*x + c))*sinh(d*x + c))
*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x +
 c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^3*f*cosh(d*x + c)^4 + 4*b^3*f
*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*f*sinh(d*x + c)^4 + 2*b^3*f*cosh(d*x +
 c)^2 + b^3*f + 2*(3*b^3*f*cosh(d*x + c)^2 + b^3*f)*sinh(d*x + c)^2 + 4*(b^
3*f*cosh(d*x + c)^3 + b^3*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x
 + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
```

```
^2)/b^2) - b)/b + 1) - ((2*b^3*f - I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)^4 + (
8*b^3*f - 4*I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*b^3*f -
 I*(a^3 + 3*a*b^2)*f)*sinh(d*x + c)^4 + 2*b^3*f + (4*b^3*f - 2*I*(a^3 + 3*a
*b^2)*f)*cosh(d*x + c)^2 + (4*b^3*f + (12*b^3*f - 6*I*(a^3 + 3*a*b^2)*f)*co
sh(d*x + c)^2 - 2*I*(a^3 + 3*a*b^2)*f)*sinh(d*x + c)^2 - I*(a^3 + 3*a*b^2)*
f + ((8*b^3*f - 4*I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)^3 + (8*b^3*f - 4*I*(a^
3 + 3*a*b^2)*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(I*cosh(d*x + c) + I*sin
h(d*x + c)) - ((2*b^3*f + I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)^4 + (8*b^3*f +
 4*I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*b^3*f + I*(a^3 +
 3*a*b^2)*f)*sinh(d*x + c)^4 + 2*b^3*f + (4*b^3*f + 2*I*(a^3 + 3*a*b^2)*f)*
cosh(d*x + c)^2 + (4*b^3*f + (12*b^3*f + 6*I*(a^3 + 3*a*b^2)*f)*cosh(d*x +
c)^2 + 2*I*(a^3 + 3*a*b^2)*f)*sinh(d*x + c)^2 + I*(a^3 + 3*a*b^2)*f + ((8*b
^3*f + 4*I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)^3 + (8*b^3*f + 4*I*(a^3 + 3*a*b
^2)*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(-I*cosh(d*x + c) - I*sinh(d*x +
c)) + 2*(b^3*d*e - b^3*c*f + (b^3*d*e - b^3*c*f)*cosh(d*x + c)^4 + 4*(b^3*d
*e - b^3*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^3*d*e - b^3*c*f)*sinh(d*x
+ c)^4 + 2*(b^3*d*e - b^3*c*f)*cosh(d*x + c)^2 + 2*(b^3*d*e - b^3*c*f + 3*(
b^3*d*e - b^3*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^3*d*e - b^3*c*f
)*cosh(d*x + c)^3 + (b^3*d*e - b^3*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(2
*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2
*(b^3*d*e - b^3*c*f + (b^3*d*e - b^3*c*f)*cosh(d*x + c)^4 + 4*(b^3*d*e - b^
3*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^3*d*e - b^3*c*f)*sinh(d*x + c)^4
+ 2*(b^3*d*e - b^3*c*f)*cosh(d*x + c)^2 + 2*(b^3*d*e - b^3*c*f + 3*(b^3*d*e
 - b^3*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^3*d*e - b^3*c*f)*cosh(
d*x + c)^3 + (b^3*d*e - b^3*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh
(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(b^3*d
*f*x + b^3*c*f + (b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^4 + 4*(b^3*d*f*x + b^3
*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^3*d*f*x + b^3*c*f)*sinh(d*x + c)^4
 + 2*(b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^2 + 2*(b^3*d*f*x + b^3*c*f + 3*(b^
3*d*f*x + b^3*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^3*d*f*x + b^3*c
*f)*cosh(d*x + c)^3 + (b^3*d*f*x + b^3*c*f)*cosh(d*x + c))*sinh(d*x + c))*l
og(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(b^3*d*f*x + b^3*c*f + (b^3*d*f*x + b^3
*c*f)*cosh(d*x + c)^4 + 4*(b^3*d*f*x + b^3*c*f)*cosh(d*x + c)*sinh(d*x + c)
^3 + (b^3*d*f*x + b^3*c*f)*sinh(d*x + c)^4 + 2*(b^3*d*f*x + b^3*c*f)*cosh(d
*x + c)^2 + 2*(b^3*d*f*x + b^3*c*f + 3*(b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + 4*((b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^3 + (b^3*d*f*x
+ b^3*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x
 + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) -
 (2*b^3*d*e - 2*b^3*c*f + (2*b^3*d*e - 2*b^3*c*f - I*(a^3 + 3*a*b^2)*d*e +
I*(a^3 + 3*a*b^2)*c*f)*cosh(d*x + c)^4 + (8*b^3*d*e - 8*b^3*c*f - 4*I*(a^3
+ 3*a*b^2)*d*e + 4*I*(a^3 + 3*a*b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (
2*b^3*d*e - 2*b^3*c*f - I*(a^3 + 3*a*b^2)*d*e + I*(a^3 + 3*a*b^2)*c*f)*sinh
(d*x + c)^4 - I*(a^3 + 3*a*b^2)*d*e + I*(a^3 + 3*a*b^2)*c*f + (4*b^3*d*e -
4*b^3*c*f - 2*I*(a^3 + 3*a*b^2)*d*e + 2*I*(a^3 + 3*a*b^2)*c*f)*cosh(d*x + c
)^2 + (4*b^3*d*e - 4*b^3*c*f - 2*I*(a^3 + 3*a*b^2)*d*e + 2*I*(a^3 + 3*a*b^2
)*c*f + (12*b^3*d*e - 12*b^3*c*f - 6*I*(a^3 + 3*a*b^2)*d*e + 6*I*(a^3 + 3*a
*b^2)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*b^3*d*e - 8*b^3*c*f - 4*I
*(a^3 + 3*a*b^2)*d*e + 4*I*(a^3 + 3*a*b^2)*c*f)*cosh(d*x + c)^3 + (8*b^3*d*
e - 8*b^3*c*f - 4*I*(a^3 + 3*a*b^2)*d*e + 4*I*(a^3 + 3*a*b^2)*c*f)*cosh(d*x
 + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + I) - (2*b^3*d*e -
 2*b^3*c*f + (2*b^3*d*e - 2*b^3*c*f + I*(a^3 + 3*a*b^2)*d*e - I*(a^3 + 3*a*
b^2)*c*f)*cosh(d*x + c)^4 + (8*b^3*d*e - 8*b^3*c*f + 4*I*(a^3 + 3*a*b^2)*d*
e - 4*I*(a^3 + 3*a*b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*b^3*d*e - 2
*b^3*c*f + I*(a^3 + 3*a*b^2)*d*e - I*(a^3 + 3*a*b^2)*c*f)*sinh(d*x + c)^4 +
 I*(a^3 + 3*a*b^2)*d*e - I*(a^3 + 3*a*b^2)*c*f + (4*b^3*d*e - 4*b^3*c*f + 2
*I*(a^3 + 3*a*b^2)*d*e - 2*I*(a^3 + 3*a*b^2)*c*f)*cosh(d*x + c)^2 + (4*b^3*
d*e - 4*b^3*c*f + 2*I*(a^3 + 3*a*b^2)*d*e - 2*I*(a^3 + 3*a*b^2)*c*f + (12*b
^3*d*e - 12*b^3*c*f + 6*I*(a^3 + 3*a*b^2)*d*e - 6*I*(a^3 + 3*a*b^2)*c*f)*co
```

```
sh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*b^3*d*e - 8*b^3*c*f + 4*I*(a^3 + 3*a*b
^2)*d*e - 4*I*(a^3 + 3*a*b^2)*c*f)*cosh(d*x + c)^3 + (8*b^3*d*e - 8*b^3*c*f
 + 4*I*(a^3 + 3*a*b^2)*d*e - 4*I*(a^3 + 3*a*b^2)*c*f)*cosh(d*x + c))*sinh(d
*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - I) - (2*b^3*d*f*x + 2*b^3*c*f
+ (2*b^3*d*f*x + 2*b^3*c*f + I*(a^3 + 3*a*b^2)*d*f*x + I*(a^3 + 3*a*b^2)*c*
f)*cosh(d*x + c)^4 + (8*b^3*d*f*x + 8*b^3*c*f + 4*I*(a^3 + 3*a*b^2)*d*f*x +
 4*I*(a^3 + 3*a*b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*b^3*d*f*x + 2*
b^3*c*f + I*(a^3 + 3*a*b^2)*d*f*x + I*(a^3 + 3*a*b^2)*c*f)*sinh(d*x + c)^4
+ I*(a^3 + 3*a*b^2)*d*f*x + I*(a^3 + 3*a*b^2)*c*f + (4*b^3*d*f*x + 4*b^3*c*
f + 2*I*(a^3 + 3*a*b^2)*d*f*x + 2*I*(a^3 + 3*a*b^2)*c*f)*cosh(d*x + c)^2 +
(4*b^3*d*f*x + 4*b^3*c*f + 2*I*(a^3 + 3*a*b^2)*d*f*x + 2*I*(a^3 + 3*a*b^2)*
c*f + (12*b^3*d*f*x + 12*b^3*c*f + 6*I*(a^3 + 3*a*b^2)*d*f*x + 6*I*(a^3 + 3
*a*b^2)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*b^3*d*f*x + 8*b^3*c*f +
 4*I*(a^3 + 3*a*b^2)*d*f*x + 4*I*(a^3 + 3*a*b^2)*c*f)*cosh(d*x + c)^3 + (8*
b^3*d*f*x + 8*b^3*c*f + 4*I*(a^3 + 3*a*b^2)*d*f*x + 4*I*(a^3 + 3*a*b^2)*c*f
)*cosh(d*x + c))*sinh(d*x + c))*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1)
- (2*b^3*d*f*x + 2*b^3*c*f + (2*b^3*d*f*x + 2*b^3*c*f - I*(a^3 + 3*a*b^2)*d
*f*x - I*(a^3 + 3*a*b^2)*c*f)*cosh(d*x + c)^4 + (8*b^3*d*f*x + 8*b^3*c*f -
4*I*(a^3 + 3*a*b^2)*d*f*x - 4*I*(a^3 + 3*a*b^2)*c*f)*cosh(d*x + c)*sinh(d*x
 + c)^3 + (2*b^3*d*f*x + 2*b^3*c*f - I*(a^3 + 3*a*b^2)*d*f*x - I*(a^3 + 3*a
*b^2)*c*f)*sinh(d*x + c)^4 - I*(a^3 + 3*a*b^2)*d*f*x - I*(a^3 + 3*a*b^2)*c*
f + (4*b^3*d*f*x + 4*b^3*c*f - 2*I*(a^3 + 3*a*b^2)*d*f*x - 2*I*(a^3 + 3*a*b
^2)*c*f)*cosh(d*x + c)^2 + (4*b^3*d*f*x + 4*b^3*c*f - 2*I*(a^3 + 3*a*b^2)*d
*f*x - 2*I*(a^3 + 3*a*b^2)*c*f + (12*b^3*d*f*x + 12*b^3*c*f - 6*I*(a^3 + 3*
a*b^2)*d*f*x - 6*I*(a^3 + 3*a*b^2)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
((8*b^3*d*f*x + 8*b^3*c*f - 4*I*(a^3 + 3*a*b^2)*d*f*x - 4*I*(a^3 + 3*a*b^2)
*c*f)*cosh(d*x + c)^3 + (8*b^3*d*f*x + 8*b^3*c*f - 4*I*(a^3 + 3*a*b^2)*d*f*
x - 4*I*(a^3 + 3*a*b^2)*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-I*cosh(d*x
+ c) - I*sinh(d*x + c) + 1) - 2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e -
3*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3 + a*b^2)*f)*cosh(d*x + c)
^2 - (a^3 + a*b^2)*f - 2*(2*(a^2*b + b^3)*d*f*x + 2*(a^2*b + b^3)*d*e + (a^
2*b + b^3)*f)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^2*co
sh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(d*x + c)*sinh(d*x + c)^3
 + (a^4 + 2*a^2*b^2 + b^4)*d^2*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*
d^2*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2 + 2*(3*(a^4 + 2*a^2*b^2 +
 b^4)*d^2*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)*sinh(d*x + c)^2 +
4*((a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d^
2*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)\operatorname{sech}^3(c + dx)}{a + b\sinh(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*sech(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)\operatorname{sech}(dx + c)^3}{b\sinh(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sech(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

3.316 $\int \frac{\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=119

$$\frac{b^3\log(a+b\sinh(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{a\left(a^2+3b^2\right)\tan^{-1}(\sinh(c+dx))}{2d\left(a^2+b^2\right)^2}-\frac{b^3\log(\cosh(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{\text{sech}^2(c+dx)(a\sinh(c+dx)+}{2d\left(a^2+b^2\right)}$$

```
[Out] (a*(a^2 + 3*b^2)*ArcTan[Sinh[c + d*x]])/(2*(a^2 + b^2)^2*d) - (b^3*Log[Cosh
[c + d*x]])/((a^2 + b^2)^2*d) + (b^3*Log[a + b*Sinh[c + d*x]])/((a^2 + b^2)
^2*d) + (Sech[c + d*x]^2*(b + a*Sinh[c + d*x]))/(2*(a^2 + b^2)*d)
```

Rubi [A] time = 0.143604, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2668, 741, 801, 635, 203, 260}

$$\frac{b^3\log(a+b\sinh(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{a\left(a^2+3b^2\right)\tan^{-1}(\sinh(c+dx))}{2d\left(a^2+b^2\right)^2}-\frac{b^3\log(\cosh(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{\text{sech}^2(c+dx)(a\sinh(c+dx)+}{2d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(a^2 + 3*b^2)*ArcTan[Sinh[c + d*x]])/(2*(a^2 + b^2)^2*d) - (b^3*Log[Cosh
[c + d*x]])/((a^2 + b^2)^2*d) + (b^3*Log[a + b*Sinh[c + d*x]])/((a^2 + b^2)
^2*d) + (Sech[c + d*x]^2*(b + a*Sinh[c + d*x]))/(2*(a^2 + b^2)*d)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b^3\operatorname{Subst}\left(\int \frac{1}{(a+x)\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2\left(a^2+b^2\right)d} - \frac{b\operatorname{Subst}\left(\int \frac{a^2+2b^2+ax}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(c+dx)\right)}{2\left(a^2+b^2\right)d}$$

$$= \frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2\left(a^2+b^2\right)d} - \frac{b\operatorname{Subst}\left(\int \left(-\frac{2b^2}{\left(a^2+b^2\right)(a+x)} + \frac{-a^3-3ab^2+2b^2x}{\left(a^2+b^2\right)\left(b^2+x^2\right)}\right) dx, x, b\sin}{2\left(a^2+b^2\right)d}$$

$$= \frac{b^3\log(a+b\sinh(c+dx))}{\left(a^2+b^2\right)^2 d} + \frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2\left(a^2+b^2\right)d} - \frac{b\operatorname{Subst}\left(\int \frac{-a^3-3ab^2+2b^2x}{b^2+x^2}\right.}{2\left(a^2+\right.}$$

$$= \frac{b^3\log(a+b\sinh(c+dx))}{\left(a^2+b^2\right)^2 d} + \frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2\left(a^2+b^2\right)d} - \frac{b^3\operatorname{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x,\right.}{\left(a^2+b^2\right)^2}$$

$$= \frac{a\left(a^2+3b^2\right)\tan^{-1}(\sinh(c+dx))}{2\left(a^2+b^2\right)^2 d} - \frac{b^3\log(\cosh(c+dx))}{\left(a^2+b^2\right)^2 d} + \frac{b^3\log(a+b\sinh(c+dx))}{\left(a^2+b^2\right)^2 d} + \ldots$$

Mathematica [A] time = 0.188339, size = 104, normalized size = 0.87

$$\frac{b\left(a^2+b^2\right)\operatorname{sech}^2(c+dx) + 2a\left(a^2+3b^2\right)\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + a\left(a^2+b^2\right)\tanh(c+dx)\operatorname{sech}(c+dx) + 2b^3(\log}{2d\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a*(a^2 + 3*b^2)*ArcTan[Tanh[(c + d*x)/2]] + 2*b^3*(-Log[Cosh[c + d*x]] +
 Log[a + b*Sinh[c + d*x]]) + b*(a^2 + b^2)*Sech[c + d*x]^2 + a*(a^2 + b^2)*
Sech[c + d*x]*Tanh[c + d*x])/(2*(a^2 + b^2)^2*d)
```

Maple [B] time = 0.003, size = 468, normalized size = 3.9

$$\frac{b^3}{d\left(a^4+2a^2b^2+b^4\right)}\ln\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 a - 2\tanh(1/2\,dx+c/2)\,b - a\right) - \frac{a^3}{d\left(a^4+2a^2b^2+b^4\right)}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/d*b^3/(a^4+2*a^2*b^2+b^4)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c
)*b-a)-1/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2
*c)^3*a^3-1/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+
1/2*c)^3*a*b^2-2/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2
*d*x+1/2*c)^2*a^2*b-2/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^2+1)^2*tan
h(1/2*d*x+1/2*c)^2*b^3+1/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^2+1)^2*
tanh(1/2*d*x+1/2*c)*a^3+1/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^2+1)^2
*tanh(1/2*d*x+1/2*c)*a*b^2-1/d/(a^4+2*a^2*b^2+b^4)*b^3*ln(tanh(1/2*d*x+1/2*
c)^2+1)+1/d/(a^4+2*a^2*b^2+b^4)*arctan(tanh(1/2*d*x+1/2*c))*a^3+3/d/(a^4+2*
a^2*b^2+b^4)*arctan(tanh(1/2*d*x+1/2*c))*a*b^2
```

Maxima [A] time = 1.96289, size = 292, normalized size = 2.45

$$\frac{b^3 \log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{\left(a^4+2\,a^2b^2+b^4\right)d} - \frac{b^3 \log\left(e^{(-2\,dx-2\,c)}+1\right)}{\left(a^4+2\,a^2b^2+b^4\right)d} - \frac{\left(a^3+3\,ab^2\right)\arctan\left(e^{(-dx-c)}\right)}{\left(a^4+2\,a^2b^2+b^4\right)d} + \frac{ae^{(-dx-c)} + 2\,b\ldots}{\left(a^2+b^2+2\left(a^2+b^2\right)e^{\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b^4
)*d) - b^3*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^3 + 3
*a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (a*e^(-d*x - c)
+ 2*b*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*e^
(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d)
```

Fricas [B] time = 2.43699, size = 2215, normalized size = 18.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((a^3 + a*b^2)*cosh(d*x + c)^3 + (a^3 + a*b^2)*sinh(d*x + c)^3 + 2*(a^2*b +
 b^3)*cosh(d*x + c)^2 + (2*a^2*b + 2*b^3 + 3*(a^3 + a*b^2)*cosh(d*x + c))*s
inh(d*x + c)^2 + ((a^3 + 3*a*b^2)*cosh(d*x + c)^4 + 4*(a^3 + 3*a*b^2)*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a*b^2)*sinh(d*x + c)^4 + a^3 + 3*a*b^2
+ 2*(a^3 + 3*a*b^2)*cosh(d*x + c)^2 + 2*(a^3 + 3*a*b^2 + 3*(a^3 + 3*a*b^2)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3 + 3*a*b^2)*cosh(d*x + c)^3 + (a^
3 + 3*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x
+ c)) - (a^3 + a*b^2)*cosh(d*x + c) + (b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x
 + c)*sinh(d*x + c)^3 + b^3*sinh(d*x + c)^4 + 2*b^3*cosh(d*x + c)^2 + b^3 +
 2*(3*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 +
 b^3*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x +
c) - sinh(d*x + c))) - (b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x
+ c)^3 + b^3*sinh(d*x + c)^4 + 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(
d*x + c)^2 + b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 + b^3*cosh(d*x +
 c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) -
(a^3 + a*b^2 - 3*(a^3 + a*b^2)*cosh(d*x + c)^2 - 4*(a^2*b + b^3)*cosh(d*x +
 c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^4 + 4*(a^4 + 2
*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d
```

```
*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4
+ 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d)*sinh(d*x
+ c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d + 4*((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x
+ c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sech(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Giac [B] time = 1.36663, size = 400, normalized size = 3.36

$$\frac{b^4\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^4bd+2\,a^2b^3d+b^5d}-\frac{b^3\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{2\left(a^4d+2\,a^2b^2d+b^4d\right)}+\frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(a^3\right.}{4\left(a^4d+2\,a^2b^2d+b^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^4*b*d + 2*a^2*b^3*d +
 b^5*d) - 1/2*b^3*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^4*d + 2*a^2*b^
2*d + b^4*d) + 1/4*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*
(a^3 + 3*a*b^2)/(a^4*d + 2*a^2*b^2*d + b^4*d) + 1/2*(b^3*(e^(d*x + c) - e^(
-d*x - c))^2 + 2*a^3*(e^(d*x + c) - e^(-d*x - c)) + 2*a*b^2*(e^(d*x + c) -
e^(-d*x - c)) + 4*a^2*b + 8*b^3)/((a^4*d + 2*a^2*b^2*d + b^4*d)*((e^(d*x +
c) - e^(-d*x - c))^2 + 4))
```

3.317 $\int \frac{\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Sech[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0768256, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

```
[In] Int[Sech[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]

[Out] Defer[Int][Sech[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Mathematica [A] time = 133.342, size = 0, normalized size = 0.

$$\int \frac{\text{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]

[Out] Integrate[Sech[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A] time = 1.058, size = 0, normalized size = 0.

$$\int \frac{(\text{sech}(dx+c))^3}{(fx+e)(a+b\sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)

[Out] int(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b*f - (a*d*f*x*e^(3*c) + (d*e - f)*a*e^(3*c))*e^(3*d*x) - (2*b*d*f*x*e^(2
*c) + (2*d*e - f)*b*e^(2*c))*e^(2*d*x) + (a*d*f*x*e^c + (d*e + f)*a*e^c)*e^
(d*x))/(a^2*d^2*e^2 + b^2*d^2*e^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*(a^
2*d^2*e*f + b^2*d^2*e*f)*x + (a^2*d^2*e^2*e^(4*c) + b^2*d^2*e^2*e^(4*c) + (
a^2*d^2*f^2*e^(4*c) + b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^2*d^2*e*f*e^(4*c) + b
^2*d^2*e*f*e^(4*c))*x)*e^(4*d*x) + 2*(a^2*d^2*e^2*e^(2*c) + b^2*d^2*e^2*e^(
2*c) + (a^2*d^2*f^2*e^(2*c) + b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^2*d^2*e*f*e^(
2*c) + b^2*d^2*e*f*e^(2*c))*x)*e^(2*d*x)) + 8*integrate(1/8*(2*b^3*d^2*f^2*
x^2 + 4*b^3*d^2*e*f*x - 2*a^2*b*f^2 + 2*(d^2*e^2 - f^2)*b^3 + ((d^2*e^2 - 2
*f^2)*a^3*e^c + (3*d^2*e^2 - 2*f^2)*a*b^2*e^c + (a^3*d^2*f^2*e^c + 3*a*b^2*
d^2*f^2*e^c)*x^2 + 2*(a^3*d^2*e*f*e^c + 3*a*b^2*d^2*e*f*e^c)*x)*e^(d*x))/(a
^4*d^2*e^3 + 2*a^2*b^2*d^2*e^3 + b^4*d^2*e^3 + (a^4*d^2*f^3 + 2*a^2*b^2*d^2
*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*e*f^2 + 2*a^2*b^2*d^2*e*f^2 + b^4*d^2*
e*f^2)*x^2 + 3*(a^4*d^2*e^2*f + 2*a^2*b^2*d^2*e^2*f + b^4*d^2*e^2*f)*x + (a
^4*d^2*e^3*e^(2*c) + 2*a^2*b^2*d^2*e^3*e^(2*c) + b^4*d^2*e^3*e^(2*c) + (a^4
*d^2*f^3*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(2*c) + b^4*d^2*f^3*e^(2*c))*x^3 + 3
*(a^4*d^2*e*f^2*e^(2*c) + 2*a^2*b^2*d^2*e*f^2*e^(2*c) + b^4*d^2*e*f^2*e^(2*
c))*x^2 + 3*(a^4*d^2*e^2*f*e^(2*c) + 2*a^2*b^2*d^2*e^2*f*e^(2*c) + b^4*d^2*
e^2*f*e^(2*c))*x)*e^(2*d*x)), x) - 8*integrate(-1/4*(a*b^3*e^(d*x + c) - b^
4)/(a^4*b*e + 2*a^2*b^3*e + b^5*e + (a^4*b*f + 2*a^2*b^3*f + b^5*f)*x - (a^
4*b*e*e^(2*c) + 2*a^2*b^3*e*e^(2*c) + b^5*e*e^(2*c) + (a^4*b*f*e^(2*c) + 2*
a^2*b^3*f*e^(2*c) + b^5*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^5*e*e^c + 2*a^3*b^2*
e*e^c + a*b^4*e*e^c + (a^5*f*e^c + 2*a^3*b^2*f*e^c + a*b^4*f*e^c)*x)*e^(d*x
)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{sech}(dx+c)^3}{afx+ae+(bfx+be)\sinh(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sech(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sech(c + d*x)**3/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.318 $\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)} dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)},x\right)$$

```
[Out] Unintegrable[(x^m*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]), x]
```

Rubi [A] time = 0.0616135, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Defer[Int][(x^m*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]), x]
```

Rubi steps

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)} dx = \int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)} dx$$

Mathematica [A] time = 11.3477, size = 0, normalized size = 0.

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Integrate[(x^m*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]), x]
```

Maple [A] time = 0.072, size = 0, normalized size = 0.

$$\int \frac{x^m (\cosh(dx+c))^3}{a+b\sinh(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \cosh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(x^m*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m \cosh(dx+c)^3}{b\sinh(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(x^m*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(x**m*cosh(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \cosh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

3.319 $\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)},x\right)$$

```
[Out] Unintegrable[(x^m*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]), x]
```

Rubi [A] time = 0.0617755, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Defer[Int][(x^m*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]), x]
```

Rubi steps

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Mathematica [A] time = 8.57976, size = 0, normalized size = 0.

$$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Integrate[(x^m*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]), x]
```

Maple [A] time = 0.115, size = 0, normalized size = 0.

$$\int \frac{x^m (\cosh(dx+c))^2}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \cosh (dx + c)^2}{b \sinh (dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(x^m*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m \cosh (dx + c)^2}{b \sinh (dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(x^m*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \cosh^2 (c + dx)}{a + b \sinh (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(x**m*cosh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \cosh (dx + c)^2}{b \sinh (dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

3.320 $\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)} \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)}, x\right)$$

```
[Out] Unintegrable[(x^m*Cosh[c + d*x])/(a + b*Sinh[c + d*x]), x]
```

Rubi [A] time = 0.0382973, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Defer[Int][(x^m*Cosh[c + d*x])/(a + b*Sinh[c + d*x]), x]
```

Rubi steps

$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)} \, dx = \int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)} \, dx$$

Mathematica [A] time = 5.52251, size = 0, normalized size = 0.

$$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Integrate[(x^m*Cosh[c + d*x])/(a + b*Sinh[c + d*x]), x]
```

Maple [A] time = 0.068, size = 0, normalized size = 0.

$$\int \frac{x^m \cosh(dx+c)}{a+b\sinh(dx+c)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{xe^{(2\,dx+m\log(x)+2\,c)}}{b(m+1)e^{(2\,dx+2\,c)}+2\,a(m+1)e^{(dx+c)}-b(m+1)}-\frac{1}{2}\int\frac{2\left(2\,adxe^{(3\,dx+3\,c)}-2\,a(m+1)e^{(dx+c)}+b(m+1)-\right.}{b^2(m+1)e^{(4\,dx+4\,c)}+4\,ab(m+1)e^{(3\,dx+3\,c)}-4\,ab(m+1)e^{(dx+c)}+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] x*e^(2*d*x + m*log(x) + 2*c)/(b*(m + 1)*e^(2*d*x + 2*c) + 2*a*(m + 1)*e^(d*
x + c) - b*(m + 1)) - 1/2*integrate(2*(2*a*d*x*e^(3*d*x + 3*c) - 2*a*(m + 1
)*e^(d*x + c) + b*(m + 1) - (2*b*d*x*e^(2*c) + b*(m + 1)*e^(2*c))*e^(2*d*x)
)*x^m/(b^2*(m + 1)*e^(4*d*x + 4*c) + 4*a*b*(m + 1)*e^(3*d*x + 3*c) - 4*a*b*
(m + 1)*e^(d*x + c) + b^2*(m + 1) + 2*(2*a^2*(m + 1)*e^(2*c) - b^2*(m + 1)*
e^(2*c))*e^(2*d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m\cosh\left(dx+c\right)}{b\sinh\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(x^m*cosh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^m\cosh\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(x**m*cosh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^m\cosh\left(dx+c\right)}{b\sinh\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.321 $\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2} dx$

Optimal. Leaf size=74

$$-\frac{2f\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}-\frac{e+fx}{bd(a+b\sinh(c+dx))}$$

```
[Out] (-2*f*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b*Sqrt[a^2 + b^2
]*d^2) - (e + f*x)/(b*d*(a + b*Sinh[c + d*x]))
```

Rubi [A] time = 0.0729192, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5464, 2660, 618, 204}

$$-\frac{2f\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}-\frac{e+fx}{bd(a+b\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] (-2*f*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b*Sqrt[a^2 + b^2
]*d^2) - (e + f*x)/(b*d*(a + b*Sinh[c + d*x]))
```

Rule 5464

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sinh[c +
d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = -\frac{e+fx}{bd(a+b\sinh(c+dx))} + \frac{f\int \frac{1}{a+b\sinh(c+dx)}\,dx}{bd}$$
$$= -\frac{e+fx}{bd(a+b\sinh(c+dx))} - \frac{(2if)\,\text{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{bd^2}$$
$$= -\frac{e+fx}{bd(a+b\sinh(c+dx))} + \frac{(4if)\,\text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx, x, -2ib+2a\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{bd^2}$$
$$= -\frac{2f\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{e+fx}{bd(a+b\sinh(c+dx))}$$

Mathematica [A] time = 0.503442, size = 78, normalized size = 1.05

$$\frac{\frac{2f\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - \frac{d(e+fx)}{a+b\sinh(c+dx)}}{bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] ((2*f*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
- (d*(e + f*x))/(a + b*Sinh[c + d*x]))/(b*d^2)
```

Maple [B] time = 0.214, size = 164, normalized size = 2.2

$$-2\,\frac{(fx+e)\,e^{dx+c}}{bd\left(be^{2\,dx+2\,c}+2\,ae^{dx+c}-b\right)} + \frac{f}{d^2b}\ln\left(e^{dx+c}+\frac{1}{b}\left(a\sqrt{a^2+b^2}-a^2-b^2\right)\frac{1}{\sqrt{a^2+b^2}}\right)\frac{1}{\sqrt{a^2+b^2}} - \frac{f}{d^2b}\ln\left(e^{dx+c}+\frac{1}{b}\left(a\sqrt{a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

```
[Out] -2*(f*x+e)/b/d*exp(d*x+c)/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/(a^2+b^2)^(
1/2)*f/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(1/2)-a^2-b^2)/(a^2+b^2)^(1/2)/b)-1
/(a^2+b^2)^(1/2)*f/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(1/2)+a^2+b^2)/(a^2+b^2
)^(1/2)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.1182, size = 1015, normalized size = 13.72

$$\frac{\left(bf\cosh\left(dx+c\right)^{2}+bf\sinh\left(dx+c\right)^{2}+2\,af\cosh\left(dx+c\right)-bf+2\left(bf\cosh\left(dx+c\right)+af\right)\sinh\left(dx+c\right)\right)\sqrt{a^{2}+b^{2}}\,\mathrm{lo}}{\left(a^{2}b^{2}+b^{4}\right)d^{2}\cosh\left(dx+c\right)^{2}+\left(a^{2}b^{2}+b^{4}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((b*f*cosh(d*x + c)^2 + b*f*sinh(d*x + c)^2 + 2*a*f*cosh(d*x + c) - b*f + 2
*(b*f*cosh(d*x + c) + a*f)*sinh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*cosh(d*x
 + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*
cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b
*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x
+ c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*((a^2 + b^2)*d*f*x +
 (a^2 + b^2)*d*e)*cosh(d*x + c) - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*d*e)*s
inh(d*x + c))/((a^2*b^2 + b^4)*d^2*cosh(d*x + c)^2 + (a^2*b^2 + b^4)*d^2*si
nh(d*x + c)^2 + 2*(a^3*b + a*b^3)*d^2*cosh(d*x + c) - (a^2*b^2 + b^4)*d^2 +
 2*((a^2*b^2 + b^4)*d^2*cosh(d*x + c) + (a^3*b + a*b^3)*d^2)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(fx+e\right)\cosh\left(dx+c\right)}{\left(b\sinh\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

3.322 $\int \frac{(e+fx)^2 \cosh(c+dx)}{(a+b\sinh(c+dx))^2} dx$

Optimal. Leaf size=234

$$\frac{2f^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{2f^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}}+\frac{2f(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd^2\sqrt{a^2+b^2}}-\frac{2f(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right.}{bd^2\sqrt{a^2+b^2}}$$

```
[Out] (2*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*Sqrt[a^2
+ b^2]*d^2) - (2*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
)/(b*Sqrt[a^2 + b^2]*d^2) + (2*f^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (2*f^2*PolyLog[2, -((b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (e + f*x)^2/(b*d*(a + b
*Sinh[c + d*x]))
```

Rubi [A] time = 0.440379, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5464, 3322, 2264, 2190, 2279, 2391}

$$\frac{2f^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{2f^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}}+\frac{2f(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd^2\sqrt{a^2+b^2}}-\frac{2f(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right.}{bd^2\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] (2*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*Sqrt[a^2
+ b^2]*d^2) - (2*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
)/(b*Sqrt[a^2 + b^2]*d^2) + (2*f^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (2*f^2*PolyLog[2, -((b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (e + f*x)^2/(b*d*(a + b
*Sinh[c + d*x]))
```

Rule 5464

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sinh[c +
d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = -\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}+\frac{(2f)\int \frac{e+fx}{a+b\sinh(c+dx)}\,dx}{bd}$$

$$= -\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}+\frac{(4f)\int \frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{bd}$$

$$= -\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}+\frac{(4f)\int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\sqrt{a^2+b^2}d}-\frac{(4f)\int \frac{e^{c+dx}(e+fx)}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\sqrt{a^2+b^2}d}$$

$$= \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx$$

$$= \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx$$

$$= \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}+\frac{2f^2\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3}$$

Mathematica [A] time = 1.37844, size = 175, normalized size = 0.75

$$\frac{2f\left(f\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)-f\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+d(e+fx)\left(\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)-\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)\right)\right)}{bd^3\sqrt{a^2+b^2}}-\frac{}{bd(}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] (2*f*(d*(e + f*x)*(Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - Log[1 +
 (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]) + f*PolyLog[2, (b*E^(c + d*x))/(-a
 + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)]))/(b*Sqrt[a^2 + b^2]*d^3) - (e + f*x)^2/(b*d*(a + b*Sinh[c + d*x]))
```

Maple [B] time = 0.19, size = 491, normalized size = 2.1

$$-2\frac{(x^2f^2+2efx+e^2)\,\mathrm{e}^{dx+c}}{bd\left(b\mathrm{e}^{2dx+2c}+2a\mathrm{e}^{dx+c}-b\right)}-4\frac{ef}{d^2b\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\frac{2b\mathrm{e}^{dx+c}+2a}{\sqrt{a^2+b^2}}\right)+2\frac{f^2x}{d^2b\sqrt{a^2+b^2}}\ln\left(\frac{-b\mathrm{e}^{dx+c}+\sqrt{a^2+b^2}-}{-a+\sqrt{a^2+b^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

```
[Out] -2*(f^2*x^2+2*e*f*x+e^2)/b/d*exp(d*x+c)/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)
-4*f/b/d^2*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/
2))+2*f^2/b/d^2/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a
^2+b^2)^(1/2)))*x+2*f^2/b/d^3/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(
1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2*f^2/b/d^2/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c
)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-2*f^2/b/d^3/(a^2+b^2)^(1/2)*ln(
(b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2*f^2/b/d^3/(a^2+b^
2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2*f^
2/b/d^3/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)
^(1/2)))+4*f^2/b/d^3*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^
2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.40127, size = 3195, normalized size = 13.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 2*((b^2*f^2*cosh(d*x + c)^2 + b^2*f^2*sinh(d*x + c)^2 + 2*a*b*f^2*cosh(d*x
+ c) - b^2*f^2 + 2*(b^2*f^2*cosh(d*x + c) + a*b*f^2)*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^2*f^2*cosh(d*x +
c)^2 + b^2*f^2*sinh(d*x + c)^2 + 2*a*b*f^2*cosh(d*x + c) - b^2*f^2 + 2*(b^2
*f^2*cosh(d*x + c) + a*b*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a
*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b + 1) + (b^2*d*e*f - b^2*c*f^2 - (b^2*d*e*f - b^2*c
*f^2)*cosh(d*x + c)^2 - (b^2*d*e*f - b^2*c*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*
e*f - a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*e*f - a*b*c*f^2 + (b^2*d*e*f - b^
2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d
*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*d*e*f
 - b^2*c*f^2 - (b^2*d*e*f - b^2*c*f^2)*cosh(d*x + c)^2 - (b^2*d*e*f - b^2*c
*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*e*f - a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*
```

```
e*f - a*b*c*f^2 + (b^2*d*e*f - b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqr
t((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^
2 + b^2)/b^2) + 2*a) - (b^2*d*f^2*x + b^2*c*f^2 - (b^2*d*f^2*x + b^2*c*f^2)
*cosh(d*x + c)^2 - (b^2*d*f^2*x + b^2*c*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*f^2
*x + a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*f^2*x + a*b*c*f^2 + (b^2*d*f^2*x +
 b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b) + (b^2*d*f^2*x + b^2*c*f^2 - (b^2*d*f^2*x + b^2*c*f^2
)*cosh(d*x + c)^2 - (b^2*d*f^2*x + b^2*c*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*f^
2*x + a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*f^2*x + a*b*c*f^2 + (b^2*d*f^2*x
+ b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*co
sh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b) - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x
 + (a^2 + b^2)*d^2*e^2)*cosh(d*x + c) - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 +
 b^2)*d^2*e*f*x + (a^2 + b^2)*d^2*e^2)*sinh(d*x + c))/((a^2*b^2 + b^4)*d^3*
cosh(d*x + c)^2 + (a^2*b^2 + b^4)*d^3*sinh(d*x + c)^2 + 2*(a^3*b + a*b^3)*d
^3*cosh(d*x + c) - (a^2*b^2 + b^4)*d^3 + 2*((a^2*b^2 + b^4)*d^3*cosh(d*x +
c) + (a^3*b + a*b^3)*d^3)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \cosh(dx+c)}{(b\sinh(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

3.323 $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=348

$$\frac{6f^2(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{6f^2(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{6f^3\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4\sqrt{a^2+b^2}}+\frac{6f^3\text{PolyLo}}{bd^4}$$

```
[Out] (3*f*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*Sqrt[a^
2 + b^2]*d^2) - (3*f*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(b*Sqrt[a^2 + b^2]*d^2) + (6*f^2*(e + f*x)*PolyLog[2, -((b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (6*f^2*(e + f*x)*Poly
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) -
 (6*f^3*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 +
 b^2]*d^4) + (6*f^3*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
b*Sqrt[a^2 + b^2]*d^4) - (e + f*x)^3/(b*d*(a + b*Sinh[c + d*x]))
```

Rubi [A] time = 0.739712, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5464, 3322, 2264, 2190, 2531, 2282, 6589}

$$\frac{6f^2(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{6f^2(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{6f^3\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4\sqrt{a^2+b^2}}+\frac{6f^3\text{PolyLo}}{bd^4}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```


```
[Out] (3*f*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*Sqrt[a^
2 + b^2]*d^2) - (3*f*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(b*Sqrt[a^2 + b^2]*d^2) + (6*f^2*(e + f*x)*PolyLog[2, -((b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (6*f^2*(e + f*x)*Poly
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) -
 (6*f^3*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 +
 b^2]*d^4) + (6*f^3*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
b*Sqrt[a^2 + b^2]*d^4) - (e + f*x)^3/(b*d*(a + b*Sinh[c + d*x]))
```


Rule 5464

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sinh[c +
d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
```

```
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = -\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}+\frac{(3f)\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}\,dx}{bd}$$

$$= -\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}+\frac{(6f)\int\frac{e^{c+dx}(e+fx)^2}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{bd}$$

$$= -\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}+\frac{(6f)\int\frac{e^{c+dx}(e+fx)^2}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\sqrt{a^2+b^2}d}-\frac{(6f)\int\frac{e^{c+dx}(e+fx)^2}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\sqrt{a^2+b^2}d}$$

$$= \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{(e+fx)^3}{bd(a+b\sinh(c+\ldots}$$

$$= \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}+\frac{6f^2(e+fx)\text{Li}_2\left(-\ldots\right.}{b\sqrt{a^2+b^2}\ldots}$$

$$= \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}+\frac{6f^2(e+fx)\text{Li}_2\left(-\ldots\right.}{b\sqrt{a^2+b^2}\ldots}$$

$$= \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}+\frac{6f^2(e+fx)\text{Li}_2\left(-\ldots\right.}{b\sqrt{a^2+b^2}\ldots}$$

Mathematica [A] time = 2.56684, size = 368, normalized size = 1.06

$$3f\left(2df(e+fx)\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)-2df(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)-2f^2\text{PolyLog}\left(3,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+2f^2\text{PolyLo}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] (3*f*(-2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*x
*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])] + 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
- 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*
f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[3, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(b*Sqrt[a^2 + b^2]*d^4) - (e + f
*x)^3/(b*d*(a + b*Sinh[c + d*x]))
```

Maple [F] time = 0.32, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3\cosh(dx+c)}{(a+b\sinh(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.78683, size = 5462, normalized size = 15.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(6*(b^2*d*f^3*x + b^2*d*e*f^2 - (b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c)^
2 - (b^2*d*f^3*x + b^2*d*e*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*f^3*x + a*b*d*e*
```

```
f^2)*cosh(d*x + c) - 2*(a*b*d*f^3*x + a*b*d*e*f^2 + (b^2*d*f^3*x + b^2*d*e*
f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x
+ c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2) - b)/b + 1) - 6*(b^2*d*f^3*x + b^2*d*e*f^2 - (b^2*d*f^3*x + b^2*d*e
*f^2)*cosh(d*x + c)^2 - (b^2*d*f^3*x + b^2*d*e*f^2)*sinh(d*x + c)^2 - 2*(a*
b*d*f^3*x + a*b*d*e*f^2)*cosh(d*x + c) - 2*(a*b*d*f^3*x + a*b*d*e*f^2 + (b^
2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(b^2*d^2*e^2*f - 2*b^2*c*d*e*f^
2 + b^2*c^2*f^3 - (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*cosh(d*x
+ c)^2 - (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*sinh(d*x + c)^2 -
2*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3)*cosh(d*x + c) - 2*(a*b*d^
2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3 + (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2
+ b^2*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*
cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 3*(b
^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3 - (b^2*d^2*e^2*f - 2*b^2*c*d*e
*f^2 + b^2*c^2*f^3)*cosh(d*x + c)^2 - (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^
2*c^2*f^3)*sinh(d*x + c)^2 - 2*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f
^3)*cosh(d*x + c) - 2*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3 + (b^2
*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c))*s
qrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((
a^2 + b^2)/b^2) + 2*a) + 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d
*e*f^2 - b^2*c^2*f^3 - (b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f
^2 - b^2*c^2*f^3)*cosh(d*x + c)^2 - (b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x +
2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*sinh(d*x + c)^2 - 2*(a*b*d^2*f^3*x^2 + 2*a*b
*d^2*e*f^2*x + 2*a*b*c*d*e*f^2 - a*b*c^2*f^3)*cosh(d*x + c) - 2*(a*b*d^2*f^
3*x^2 + 2*a*b*d^2*e*f^2*x + 2*a*b*c*d*e*f^2 - a*b*c^2*f^3 + (b^2*d^2*f^3*x^
2 + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*cosh(d*x + c))*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 3*(b^2*d
^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3 - (b^2*d^2*f
^3*x^2 + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*cosh(d*x + c)^2
 - (b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*si
nh(d*x + c)^2 - 2*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2*x + 2*a*b*c*d*e*f^2 -
a*b*c^2*f^3)*cosh(d*x + c) - 2*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2*x + 2*a*b
*c*d*e*f^2 - a*b*c^2*f^3 + (b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d
*e*f^2 - b^2*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*l
og(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2) - b)/b) + 6*(b^2*f^3*cosh(d*x + c)^2 + b^2*f^3*sinh
(d*x + c)^2 + 2*a*b*f^3*cosh(d*x + c) - b^2*f^3 + 2*(b^2*f^3*cosh(d*x + c)
+ a*b*f^3)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c)
 + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2))/b) - 6*(b^2*f^3*cosh(d*x + c)^2 + b^2*f^3*sinh(d*x + c)^2 + 2*a*b*f^3*
cosh(d*x + c) - b^2*f^3 + 2*(b^2*f^3*cosh(d*x + c) + a*b*f^3)*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*((a^2 + b^2)
*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + (a
^2 + b^2)*d^3*e^3)*cosh(d*x + c) + 2*((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^
2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + (a^2 + b^2)*d^3*e^3)*sinh(d*
x + c))/((a^2*b^2 + b^4)*d^4*cosh(d*x + c)^2 + (a^2*b^2 + b^4)*d^4*sinh(d*x
 + c)^2 + 2*(a^3*b + a*b^3)*d^4*cosh(d*x + c) - (a^2*b^2 + b^4)*d^4 + 2*((a
^2*b^2 + b^4)*d^4*cosh(d*x + c) + (a^3*b + a*b^3)*d^4)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \cosh(dx+c)}{(b\sinh(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

3.324 $\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=74

$$-\frac{2f\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}-\frac{e+fx}{bd(a+b\sinh(c+dx))}$$

```
[Out] (-2*f*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b*Sqrt[a^2 + b^2
]*d^2) - (e + f*x)/(b*d*(a + b*Sinh[c + d*x]))
```

Rubi [A] time = 0.0700089, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5464, 2660, 618, 204}

$$-\frac{2f\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}}-\frac{e+fx}{bd(a+b\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] (-2*f*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b*Sqrt[a^2 + b^2
]*d^2) - (e + f*x)/(b*d*(a + b*Sinh[c + d*x]))
```

Rule 5464

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sinh[c +
d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = -\frac{e+fx}{bd(a+b\sinh(c+dx))}+\frac{f\int \frac{1}{a+b\sinh(c+dx)}\,dx}{bd}$$

$$= -\frac{e+fx}{bd(a+b\sinh(c+dx))}-\frac{(2if)\,\text{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{bd^2}$$

$$= -\frac{e+fx}{bd(a+b\sinh(c+dx))}+\frac{(4if)\,\text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx,x,-2ib+2a\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{bd^2}$$

$$= -\frac{2f\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{e+fx}{bd(a+b\sinh(c+dx))}$$

Mathematica [A] time = 0.396371, size = 78, normalized size = 1.05

$$\frac{\frac{2f\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-\frac{d(e+fx)}{a+b\sinh(c+dx)}}{bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] ((2*f*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
- (d*(e + f*x))/(a + b*Sinh[c + d*x]))/(b*d^2)
```

Maple [B] time = 0., size = 164, normalized size = 2.2

$$-2\,\frac{\left(fx+e\right)e^{dx+c}}{bd\left(be^{2\,dx+2\,c}+2\,ae^{dx+c}-b\right)}+\frac{f}{d^2b}\ln\left(e^{dx+c}+\frac{1}{b}\left(a\sqrt{a^2+b^2}-a^2-b^2\right)\frac{1}{\sqrt{a^2+b^2}}\right)\frac{1}{\sqrt{a^2+b^2}}-\frac{f}{d^2b}\ln\left(e^{dx+c}+\frac{1}{b}\left(a\sqrt{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

```
[Out] -2*(f*x+e)/b/d*exp(d*x+c)/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/(a^2+b^2)^(
1/2)*f/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(1/2)-a^2-b^2)/(a^2+b^2)^(1/2)/b)-1
/(a^2+b^2)^(1/2)*f/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(1/2)+a^2+b^2)/(a^2+b^2
)^(1/2)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.60907, size = 1015, normalized size = 13.72

$$\frac{\left(bf\cosh\left(dx+c\right)^{2}+bf\sinh\left(dx+c\right)^{2}+2\,af\cosh\left(dx+c\right)-bf+2\left(bf\cosh\left(dx+c\right)+af\right)\sinh\left(dx+c\right)\right)\sqrt{a^{2}+b^{2}}\,\mathrm{lo}}{\left(a^{2}b^{2}+b^{4}\right)d^{2}\cosh\left(dx+c\right)^{2}+\left(a^{2}b^{2}+b^{4}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((b*f*cosh(d*x + c)^2 + b*f*sinh(d*x + c)^2 + 2*a*f*cosh(d*x + c) - b*f + 2
*(b*f*cosh(d*x + c) + a*f)*sinh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*cosh(d*x
 + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*
cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b
*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x
+ c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*((a^2 + b^2)*d*f*x +
 (a^2 + b^2)*d*e)*cosh(d*x + c) - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*d*e)*s
inh(d*x + c))/((a^2*b^2 + b^4)*d^2*cosh(d*x + c)^2 + (a^2*b^2 + b^4)*d^2*si
nh(d*x + c)^2 + 2*(a^3*b + a*b^3)*d^2*cosh(d*x + c) - (a^2*b^2 + b^4)*d^2 +
 2*((a^2*b^2 + b^4)*d^2*cosh(d*x + c) + (a^3*b + a*b^3)*d^2)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)\cosh\left(dx+c\right)}{\left(b\sinh\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

3.325 $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=234

$$\frac{2f^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{2f^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}}+\frac{2f(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd^2\sqrt{a^2+b^2}}-\frac{2f(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right.}{bd^2\sqrt{a^2+b^2}}$$

```
[Out] (2*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*Sqrt[a^2
+ b^2]*d^2) - (2*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
)/(b*Sqrt[a^2 + b^2]*d^2) + (2*f^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (2*f^2*PolyLog[2, -((b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (e + f*x)^2/(b*d*(a + b
*Sinh[c + d*x]))
```

Rubi [A] time = 0.439988, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5464, 3322, 2264, 2190, 2279, 2391}

$$\frac{2f^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{2f^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}}+\frac{2f(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd^2\sqrt{a^2+b^2}}-\frac{2f(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right.}{bd^2\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] (2*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*Sqrt[a^2
+ b^2]*d^2) - (2*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
)/(b*Sqrt[a^2 + b^2]*d^2) + (2*f^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (2*f^2*PolyLog[2, -((b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (e + f*x)^2/(b*d*(a + b
*Sinh[c + d*x]))
```

Rule 5464

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sinh[c +
d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx &= -\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}+\frac{(2f)\int \frac{e+fx}{a+b\sinh(c+dx)}\,dx}{bd}\\
&= -\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}+\frac{(4f)\int \frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{bd}\\
&= -\frac{(e+fx)^2}{bd(a+b\sinh(c+dx))}+\frac{(4f)\int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\sqrt{a^2+b^2}d}-\frac{(4f)\int \frac{e^{c+dx}(e+fx)}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\sqrt{a^2+b^2}d}\\
&= \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx}\\
&= \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{(e+fx)^2}{bd(a+b\sinh(c+dx}\\
&= \frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{2f(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}+\frac{2f^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3}
\end{aligned}
$$

Mathematica [A] time = 0.550565, size = 175, normalized size = 0.75

$$
\frac{2f\left(f\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)-f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+d(e+fx)\left(\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)-\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)\right)\right)}{bd^3\sqrt{a^2+b^2}}-\frac{}{bd(}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] (2*f*(d*(e + f*x)*(Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - Log[1 +
 (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]) + f*PolyLog[2, (b*E^(c + d*x))/(-a
 + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)]))/(b*Sqrt[a^2 + b^2]*d^3) - (e + f*x)^2/(b*d*(a + b*Sinh[c + d*x]))
```

Maple [B] time = 0.001, size = 491, normalized size = 2.1

$$-2\frac{\left(x^2f^2+2efx+e^2\right)e^{dx+c}}{bd\left(be^{2dx+2c}+2ae^{dx+c}-b\right)}-4\frac{ef}{d^2b\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\frac{2be^{dx+c}+2a}{\sqrt{a^2+b^2}}\right)+2\frac{f^2x}{d^2b\sqrt{a^2+b^2}}\ln\left(\frac{-be^{dx+c}+\sqrt{a^2+b^2}-}{-a+\sqrt{a^2+b^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

```
[Out] -2*(f^2*x^2+2*e*f*x+e^2)/b/d*exp(d*x+c)/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)
-4*f/b/d^2*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/
2))+2*f^2/b/d^2/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a
^2+b^2)^(1/2)))*x+2*f^2/b/d^3/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(
1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2*f^2/b/d^2/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c
)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-2*f^2/b/d^3/(a^2+b^2)^(1/2)*ln(
(b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2*f^2/b/d^3/(a^2+b^
2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2*f^
2/b/d^3/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)
^(1/2)))+4*f^2/b/d^3*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^
2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.93246, size = 3195, normalized size = 13.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 2*((b^2*f^2*cosh(d*x + c)^2 + b^2*f^2*sinh(d*x + c)^2 + 2*a*b*f^2*cosh(d*x
+ c) - b^2*f^2 + 2*(b^2*f^2*cosh(d*x + c) + a*b*f^2)*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^2*f^2*cosh(d*x +
c)^2 + b^2*f^2*sinh(d*x + c)^2 + 2*a*b*f^2*cosh(d*x + c) - b^2*f^2 + 2*(b^2
*f^2*cosh(d*x + c) + a*b*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a
*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b + 1) + (b^2*d*e*f - b^2*c*f^2 - (b^2*d*e*f - b^2*c
*f^2)*cosh(d*x + c)^2 - (b^2*d*e*f - b^2*c*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*
e*f - a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*e*f - a*b*c*f^2 + (b^2*d*e*f - b^
2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d
*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*d*e*f
 - b^2*c*f^2 - (b^2*d*e*f - b^2*c*f^2)*cosh(d*x + c)^2 - (b^2*d*e*f - b^2*c
*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*e*f - a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*
```

```
e*f - a*b*c*f^2 + (b^2*d*e*f - b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqr
t((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^
2 + b^2)/b^2) + 2*a) - (b^2*d*f^2*x + b^2*c*f^2 - (b^2*d*f^2*x + b^2*c*f^2)
*cosh(d*x + c)^2 - (b^2*d*f^2*x + b^2*c*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*f^2
*x + a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*f^2*x + a*b*c*f^2 + (b^2*d*f^2*x +
 b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b) + (b^2*d*f^2*x + b^2*c*f^2 - (b^2*d*f^2*x + b^2*c*f^2
)*cosh(d*x + c)^2 - (b^2*d*f^2*x + b^2*c*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*f^
2*x + a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*f^2*x + a*b*c*f^2 + (b^2*d*f^2*x
+ b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*co
sh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b) - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x
 + (a^2 + b^2)*d^2*e^2)*cosh(d*x + c) - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 +
 b^2)*d^2*e*f*x + (a^2 + b^2)*d^2*e^2)*sinh(d*x + c))/((a^2*b^2 + b^4)*d^3*
cosh(d*x + c)^2 + (a^2*b^2 + b^4)*d^3*sinh(d*x + c)^2 + 2*(a^3*b + a*b^3)*d
^3*cosh(d*x + c) - (a^2*b^2 + b^4)*d^3 + 2*((a^2*b^2 + b^4)*d^3*cosh(d*x +
c) + (a^3*b + a*b^3)*d^3)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2\cosh(dx+c)}{(b\sinh(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

3.326 $\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=348

$$\frac{6f^2(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{6f^2(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{6f^3\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4\sqrt{a^2+b^2}}+\frac{6f^3\text{PolyLo}}{bd^4}$$

```
[Out] (3*f*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*Sqrt[a^
2 + b^2]*d^2) - (3*f*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(b*Sqrt[a^2 + b^2]*d^2) + (6*f^2*(e + f*x)*PolyLog[2, -((b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (6*f^2*(e + f*x)*Poly
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) -
 (6*f^3*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 +
 b^2]*d^4) + (6*f^3*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
b*Sqrt[a^2 + b^2]*d^4) - (e + f*x)^3/(b*d*(a + b*Sinh[c + d*x]))
```

Rubi [A] time = 0.73091, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5464, 3322, 2264, 2190, 2531, 2282, 6589}

$$\frac{6f^2(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{6f^2(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}}-\frac{6f^3\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4\sqrt{a^2+b^2}}+\frac{6f^3\text{PolyLo}}{bd^4}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```


```
[Out] (3*f*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*Sqrt[a^
2 + b^2]*d^2) - (3*f*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(b*Sqrt[a^2 + b^2]*d^2) + (6*f^2*(e + f*x)*PolyLog[2, -((b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (6*f^2*(e + f*x)*Poly
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) -
 (6*f^3*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 +
 b^2]*d^4) + (6*f^3*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
b*Sqrt[a^2 + b^2]*d^4) - (e + f*x)^3/(b*d*(a + b*Sinh[c + d*x]))
```


Rule 5464

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sinh[c +
d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
```

```
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx = -\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}+\frac{(3f)\int\frac{(e+fx)^2}{a+b\sinh(c+dx)}\,dx}{bd}$$

$$= -\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}+\frac{(6f)\int\frac{e^{c+dx}(e+fx)^2}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{bd}$$

$$= -\frac{(e+fx)^3}{bd(a+b\sinh(c+dx))}+\frac{(6f)\int\frac{e^{c+dx}(e+fx)^2}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\sqrt{a^2+b^2}d}-\frac{(6f)\int\frac{e^{c+dx}(e+fx)^2}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{\sqrt{a^2+b^2}d}$$

$$= \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{(e+fx)^3}{bd(a+b\sinh(c+\ldots}$$

$$= \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}+\frac{6f^2(e+fx)\mathrm{Li}_2\left(-\ldots\right.}{b\sqrt{a^2+b^2}\ldots}$$

$$= \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}+\frac{6f^2(e+fx)\mathrm{Li}_2\left(-\ldots\right.}{b\sqrt{a^2+b^2}\ldots}$$

$$= \frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}-\frac{3f(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2}+\frac{6f^2(e+fx)\mathrm{Li}_2\left(-\ldots\right.}{b\sqrt{a^2+b^2}\ldots}$$

Mathematica [A] time = 0.388116, size = 368, normalized size = 1.06

$$3f\left(2df(e+fx)\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)-2df(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)-2f^2\text{PolyLog}\left(3,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+2f^2\text{PolyLo}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] (3*f*(-2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*x
*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])] + 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
- 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*
f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[3, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(b*Sqrt[a^2 + b^2]*d^4) - (e + f
*x)^3/(b*d*(a + b*Sinh[c + d*x]))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \cosh(dx+c)}{(a+b\sinh(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.89884, size = 5462, normalized size = 15.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(6*(b^2*d*f^3*x + b^2*d*e*f^2 - (b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c)^
2 - (b^2*d*f^3*x + b^2*d*e*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*f^3*x + a*b*d*e*
```

```
f^2)*cosh(d*x + c) - 2*(a*b*d*f^3*x + a*b*d*e*f^2 + (b^2*d*f^3*x + b^2*d*e*
f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x
+ c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2) - b)/b + 1) - 6*(b^2*d*f^3*x + b^2*d*e*f^2 - (b^2*d*f^3*x + b^2*d*e
*f^2)*cosh(d*x + c)^2 - (b^2*d*f^3*x + b^2*d*e*f^2)*sinh(d*x + c)^2 - 2*(a*
b*d*f^3*x + a*b*d*e*f^2)*cosh(d*x + c) - 2*(a*b*d*f^3*x + a*b*d*e*f^2 + (b^
2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(b^2*d^2*e^2*f - 2*b^2*c*d*e*f^
2 + b^2*c^2*f^3 - (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*cosh(d*x
+ c)^2 - (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*sinh(d*x + c)^2 -
2*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3)*cosh(d*x + c) - 2*(a*b*d^
2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3 + (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2
+ b^2*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*
cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 3*(b
^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3 - (b^2*d^2*e^2*f - 2*b^2*c*d*e
*f^2 + b^2*c^2*f^3)*cosh(d*x + c)^2 - (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^
2*c^2*f^3)*sinh(d*x + c)^2 - 2*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f
^3)*cosh(d*x + c) - 2*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3 + (b^2
*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c))*s
qrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((
a^2 + b^2)/b^2) + 2*a) + 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d
*e*f^2 - b^2*c^2*f^3 - (b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f
^2 - b^2*c^2*f^3)*cosh(d*x + c)^2 - (b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x +
2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*sinh(d*x + c)^2 - 2*(a*b*d^2*f^3*x^2 + 2*a*b
*d^2*e*f^2*x + 2*a*b*c*d*e*f^2 - a*b*c^2*f^3)*cosh(d*x + c) - 2*(a*b*d^2*f^
3*x^2 + 2*a*b*d^2*e*f^2*x + 2*a*b*c*d*e*f^2 - a*b*c^2*f^3 + (b^2*d^2*f^3*x^
2 + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*cosh(d*x + c))*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 3*(b^2*d
^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3 - (b^2*d^2*f
^3*x^2 + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*cosh(d*x + c)^2
 - (b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*si
nh(d*x + c)^2 - 2*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2*x + 2*a*b*c*d*e*f^2 -
a*b*c^2*f^3)*cosh(d*x + c) - 2*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2*x + 2*a*b
*c*d*e*f^2 - a*b*c^2*f^3 + (b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d
*e*f^2 - b^2*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*l
og(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2) - b)/b) + 6*(b^2*f^3*cosh(d*x + c)^2 + b^2*f^3*sinh
(d*x + c)^2 + 2*a*b*f^3*cosh(d*x + c) - b^2*f^3 + 2*(b^2*f^3*cosh(d*x + c)
+ a*b*f^3)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c)
 + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2))/b) - 6*(b^2*f^3*cosh(d*x + c)^2 + b^2*f^3*sinh(d*x + c)^2 + 2*a*b*f^3*
cosh(d*x + c) - b^2*f^3 + 2*(b^2*f^3*cosh(d*x + c) + a*b*f^3)*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*((a^2 + b^2)
*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + (a
^2 + b^2)*d^3*e^3)*cosh(d*x + c) + 2*((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^
2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + (a^2 + b^2)*d^3*e^3)*sinh(d*
x + c))/((a^2*b^2 + b^4)*d^4*cosh(d*x + c)^2 + (a^2*b^2 + b^4)*d^4*sinh(d*x
 + c)^2 + 2*(a^3*b + a*b^3)*d^4*cosh(d*x + c) - (a^2*b^2 + b^4)*d^4 + 2*((a
^2*b^2 + b^4)*d^4*cosh(d*x + c) + (a^3*b + a*b^3)*d^4)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^3\cosh\left(dx+c\right)}{\left(b\sinh\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

3.327 $\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=112

$$-\frac{af\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd^2\left(a^2+b^2\right)^{3/2}}-\frac{f\cosh(c+dx)}{2d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

```
[Out] -((a*f*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b*(a^2 + b^2)^(
3/2)*d^2)) - (e + f*x)/(2*b*d*(a + b*Sinh[c + d*x])^2) - (f*Cosh[c + d*x])/
(2*(a^2 + b^2)*d^2*(a + b*Sinh[c + d*x]))
```

Rubi [A] time = 0.0983786, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5464, 2664, 12, 2660, 618, 204}

$$-\frac{af\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd^2\left(a^2+b^2\right)^{3/2}}-\frac{f\cosh(c+dx)}{2d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```



```
[Out] -((a*f*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b*(a^2 + b^2)^(
3/2)*d^2)) - (e + f*x)/(2*b*d*(a + b*Sinh[c + d*x])^2) - (f*Cosh[c + d*x])/
(2*(a^2 + b^2)*d^2*(a + b*Sinh[c + d*x]))
```


Rule 5464

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sinh[c +
d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}+\frac{f\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx}{2bd}$$

$$= -\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}-\frac{f\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))}+\frac{f\int \frac{a}{a+b\sinh(c+dx)}\,dx}{2b\left(a^2+b^2\right)d}$$

$$= -\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}-\frac{f\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))}+\frac{(af)\int \frac{1}{a+b\sinh(c+dx)}\,dx}{2b\left(a^2+b^2\right)d}$$

$$= -\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}-\frac{f\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))}-\frac{(iaf)\,\text{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\right.}{b\left(a^2+\right.}$$

$$= -\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}-\frac{f\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))}+\frac{(2iaf)\,\text{Subst}\left(\int \frac{1}{-4\left(a^2+b^2\right.}\right.}{}$$

$$= -\frac{af\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^2}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}-\frac{f\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+\right.}$$

Mathematica [A] time = 1.16218, size = 112, normalized size = 1.

$$-\frac{\frac{\frac{2af\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}}+\frac{d(e+fx)}{(a+b\sinh(c+dx))^2}}{b}+\frac{f\cosh(c+dx)}{\left(a^2+b^2\right)(a+b\sinh(c+dx))}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] -((f*Cosh[c + d*x])/((a^2 + b^2)*(a + b*Sinh[c + d*x])) + ((2*a*f*ArcTan[(b
 - a*Tanh[(c + d*x)/2]])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) + (d*(e + f*x
))/(a + b*Sinh[c + d*x])^2)/b)/(2*d^2)
```

Maple [B] time = 0.319, size = 308, normalized size = 2.8

$$-\frac{2 a^{2} d f x e^{2 d x+2 c}+2 b^{2} d f x e^{2 d x+2 c}+2 a^{2} d e e^{2 d x+2 c}-a b f e^{3 d x+3 c}+2 b^{2} d e e^{2 d x+2 c}-2 a^{2} f e^{2 d x+2 c}+b^{2} f e^{2 d x+2 c}+3 f a}{d^{2} b \left(b e^{2 d x+2 c}+2 a e^{d x+c}-b\right)^{2} \left(a^{2}+b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

```
[Out] -1/b*(2*a^2*d*f*x*exp(2*d*x+2*c)+2*b^2*d*f*x*exp(2*d*x+2*c)+2*a^2*d*e*exp(2
*d*x+2*c)-a*b*f*exp(3*d*x+3*c)+2*b^2*d*e*exp(2*d*x+2*c)-2*a^2*f*exp(2*d*x+2
*c)+b^2*f*exp(2*d*x+2*c)+3*f*a*exp(d*x+c)*b-f*b^2)/d^2/(b*exp(2*d*x+2*c)+2*
a*exp(d*x+c)-b)^2/(a^2+b^2)+1/2/(a^2+b^2)^(3/2)*f*a/d^2/b*ln(exp(d*x+c)+(a*
(a^2+b^2)^(3/2)-a^4-2*a^2*b^2-b^4)/(a^2+b^2)^(3/2)/b)-1/2/(a^2+b^2)^(3/2)*f
*a/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(3/2)+a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(3/2
)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.3144, size = 2811, normalized size = 25.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a^3*b + a*b^3)*f*cosh(d*x + c)^3 + 2*(a^3*b + a*b^3)*f*sinh(d*x + c
)^3 - 6*(a^3*b + a*b^3)*f*cosh(d*x + c) - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*f*
x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*e - (2*a^4 + a^2*b^2 - b^4)*f)*cosh(d*x + c
)^2 - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*e -
3*(a^3*b + a*b^3)*f*cosh(d*x + c) - (2*a^4 + a^2*b^2 - b^4)*f)*sinh(d*x + c
)^2 + (a*b^2*f*cosh(d*x + c)^4 + a*b^2*f*sinh(d*x + c)^4 + 4*a^2*b*f*cosh(d
*x + c)^3 - 4*a^2*b*f*cosh(d*x + c) + a*b^2*f + 2*(2*a^3 - a*b^2)*f*cosh(d*
x + c)^2 + 4*(a*b^2*f*cosh(d*x + c) + a^2*b*f)*sinh(d*x + c)^3 + 2*(3*a*b^2
*f*cosh(d*x + c)^2 + 6*a^2*b*f*cosh(d*x + c) + (2*a^3 - a*b^2)*f)*sinh(d*x
+ c)^2 + 4*(a*b^2*f*cosh(d*x + c)^3 + 3*a^2*b*f*cosh(d*x + c)^2 - a^2*b*f +
 (2*a^3 - a*b^2)*f*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*c
osh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 +
2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x +
 c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*co
sh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(a^2*b^2 + b^
4)*f + 2*(3*(a^3*b + a*b^3)*f*cosh(d*x + c)^2 - 3*(a^3*b + a*b^3)*f - 2*(2*
(a^4 + 2*a^2*b^2 + b^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*e - (2*a^4 + a^
2*b^2 - b^4)*f)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b^3 + 2*a^2*b^5 + b^7)*
```

```
d^2*cosh(d*x + c)^4 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^2*sinh(d*x + c)^4 + 4*(
a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2*cosh(d*x + c)^3 + 2*(2*a^6*b + 3*a^4*b^3 -
 b^7)*d^2*cosh(d*x + c)^2 - 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2*cosh(d*x +
c) + 4*((a^4*b^3 + 2*a^2*b^5 + b^7)*d^2*cosh(d*x + c) + (a^5*b^2 + 2*a^3*b^
4 + a*b^6)*d^2)*sinh(d*x + c)^3 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^2 + 2*(3*(a
^4*b^3 + 2*a^2*b^5 + b^7)*d^2*cosh(d*x + c)^2 + 6*(a^5*b^2 + 2*a^3*b^4 + a*
b^6)*d^2*cosh(d*x + c) + (2*a^6*b + 3*a^4*b^3 - b^7)*d^2)*sinh(d*x + c)^2 +
 4*((a^4*b^3 + 2*a^2*b^5 + b^7)*d^2*cosh(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^
4 + a*b^6)*d^2*cosh(d*x + c)^2 + (2*a^6*b + 3*a^4*b^3 - b^7)*d^2*cosh(d*x +
 c) - (a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

3.328 $\int \frac{(e+fx)^2 \cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=306

$$\frac{af^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}-\frac{af^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}+\frac{af(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd^2\left(a^2+b^2\right)^{3/2}}-\frac{af(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right.}{bd^2\left(a^2+b^2\right)^{3/2}}$$

```
[Out] (a*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^
2)^(3/2)*d^2) - (a*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)])/(b*(a^2 + b^2)^(3/2)*d^2) + (f^2*Log[a + b*Sinh[c + d*x]])/(b*(a^2 + b^
2)*d^3) + (a*f^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(
a^2 + b^2)^(3/2)*d^3) - (a*f^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) - (e + f*x)^2/(2*b*d*(a + b*Sinh[c + d*
x])^2) - (f*(e + f*x)*Cosh[c + d*x])/((a^2 + b^2)*d^2*(a + b*Sinh[c + d*x])
)
```

Rubi [A] time = 0.519948, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5464, 3324, 3322, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{af^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}-\frac{af^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}+\frac{af(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd^2\left(a^2+b^2\right)^{3/2}}-\frac{af(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right.}{bd^2\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```


```
[Out] (a*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^
2)^(3/2)*d^2) - (a*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)])/(b*(a^2 + b^2)^(3/2)*d^2) + (f^2*Log[a + b*Sinh[c + d*x]])/(b*(a^2 + b^
2)*d^3) + (a*f^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(
a^2 + b^2)^(3/2)*d^3) - (a*f^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) - (e + f*x)^2/(2*b*d*(a + b*Sinh[c + d*
x])^2) - (f*(e + f*x)*Cosh[c + d*x])/((a^2 + b^2)*d^2*(a + b*Sinh[c + d*x])
)
```


Rule 5464

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sinh[c +
d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2} + \frac{f\int \frac{e+fx}{(a+b\sinh(c+dx))^2}\,dx}{bd}$$

$$= -\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2} - \frac{f(e+fx)\cosh(c+dx)}{(a^2+b^2)\,d^2(a+b\sinh(c+dx))} + \frac{(af)\int \frac{e+fx}{a+b\sinh(c+dx)}\,dx}{b(a^2+b^2)\,d}$$

$$= -\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2} - \frac{f(e+fx)\cosh(c+dx)}{(a^2+b^2)\,d^2(a+b\sinh(c+dx))} + \frac{(2af)\int \frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{b(a^2+b^2)\,d}$$

$$= \frac{f^2\log(a+b\sinh(c+dx))}{b(a^2+b^2)\,d^3} - \frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2} - \frac{f(e+fx)\cosh(c+dx)}{(a^2+b^2)\,d^2(a+b\sinh(c+d$$

$$= \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^2} - \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^2} + \frac{f^2\log(a+b\sinh(c}{b(a^2+b^2)\,d^3}$$

$$= \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^2} - \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^2} + \frac{f^2\log(a+b\sinh(c}{b(a^2+b^2)\,d^3}$$

$$= \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^2} - \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^2} + \frac{f^2\log(a+b\sinh(c}{b(a^2+b^2)\,d^3}$$

Mathematica [B] time = 16.4123, size = 623, normalized size = 2.04

$$\frac{2e^cf\left(\frac{a(e^{2c}-1)f\left(\text{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c-\sqrt{e^{2c}(a^2+b^2)}}\right)-\text{PolyLog}\left(2,-\frac{be^{2c+dx}}{\sqrt{e^{2c}(a^2+b^2)}+ae^c}\right)+dx\left(\log\left(\frac{be^{2c+dx}}{ae^c-\sqrt{e^{2c}(a^2+b^2)}}+1\right)-\log\left(\frac{be^{2c+dx}}{\sqrt{e^{2c}(a^2+b^2)}+ae^c}+1\right)\right)\right)}{2d\sqrt{e^{2c}(a^2+b^2)}} - \frac{ae^{-c}(e^{2c}-1)e}{\sqrt{}}\right.}{b(e}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] (f^2*x*Coth[c])/(b*(a^2 + b^2)*d^2) + (2*E^c*f*(-(E^c*f*x) + ((-1 + E^(2*c)
)*f*x)/E^c - (a*e*(-1 + E^(2*c))*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2
]])/(Sqrt[a^2 + b^2]*E^c) + (a*(-1 + E^(2*c))*f*ArcTanh[(a + b*E^(c + d*x))
/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d*E^c) + ((-1 + E^(2*c))*f*(-2*x + (2*a
*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-a^2 - b^2]*d) + Log[2
*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))]/d))/(2*E^c) + (a*(-1 + E^(2*c))*
f*(d*x*(Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - Lo
g[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])]) + PolyLog[2,
-((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - PolyLog[2, -((b
*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(2*d*Sqrt[(a^2 + b^
2)*E^(2*c)])))/(b*(a^2 + b^2)*d^2*(-1 + E^(2*c))) - (f^2*x*Cosh[c]*Csch[c/2
]*Sech[c/2])/(2*b*(a^2 + b^2)*d^2) - (e + f*x)^2/(2*b*d*(a + b*Sinh[c + d*x
])^2) + (Csch[c/2]*Sech[c/2]*(a*e*f*Cosh[c] + a*f^2*x*Cosh[c] + b*e*f*Sinh[
d*x] + b*f^2*x*Sinh[d*x]))/(2*b*(a^2 + b^2)*d^2*(a + b*Sinh[c + d*x]))
```

Maple [B] time = 0.3, size = 805, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

```
[Out] -2/b*(a^2*d*f^2*x^2*exp(2*d*x+2*c)+b^2*d*f^2*x^2*exp(2*d*x+2*c)+2*a^2*d*e*f
*x*exp(2*d*x+2*c)-a*b*f^2*x*exp(3*d*x+3*c)+2*b^2*d*e*f*x*exp(2*d*x+2*c)+a^2
*d*e^2*exp(2*d*x+2*c)-2*a^2*f^2*x*exp(2*d*x+2*c)-a*b*e*f*exp(3*d*x+3*c)+b^2
*d*e^2*exp(2*d*x+2*c)+b^2*f^2*x*exp(2*d*x+2*c)-2*a^2*e*f*exp(2*d*x+2*c)+3*a
*b*f^2*x*exp(d*x+c)+b^2*e*f*exp(2*d*x+2*c)+3*a*b*e*f*exp(d*x+c)-b^2*f^2*x-b
^2*e*f)/d^2/(a^2+b^2)/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)^2+1/b*f^2/d^3/(a^
2+b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/b*f^2/d^3/(a^2+b^2)*ln(exp(d
*x+c))-2/b*f/d^2/(a^2+b^2)^(3/2)*a*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+
b^2)^(1/2))+1/b*f^2/d^2/(a^2+b^2)^(3/2)*a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))*x+1/b*f^2/d^3/(a^2+b^2)^(3/2)*a*ln((-b*exp(d*x+c)
+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/b*f^2/d^2/(a^2+b^2)^(3/2)*a*l
n((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/b*f^2/d^3/(a^2+
b^2)^(3/2)*a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/b
*f^2/d^3/(a^2+b^2)^(3/2)*a*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2
+b^2)^(1/2)))-1/b*f^2/d^3/(a^2+b^2)^(3/2)*a*dilog((b*exp(d*x+c)+(a^2+b^2)^(
1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/b*f^2/d^3/(a^2+b^2)^(3/2)*a*c*arctanh(1/2*(2
*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.17122, size = 11416, normalized size = 37.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(2*((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*cosh(d*x + c)^4 + 2*(
(a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*sinh(d*x + c)^4 - 2*(a^2*b
^2 + b^4)*d*e*f + 2*(a^2*b^2 + b^4)*c*f^2 + 2*(3*(a^3*b + a*b^3)*d*f^2*x -
(a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^3)*c*f^2)*cosh(d*x + c)^3 + 2*(3*(a^
3*b + a*b^3)*d*f^2*x - (a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^3)*c*f^2 + 4*
((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*cosh(d*x + c))*sinh(d*x +
 c)^3 + 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + (a^4 + 2*a^2*b^2 + b^4)*d^
2*e^2 - (2*a^4 + a^2*b^2 - b^4)*d*e*f + 2*(2*a^4 + a^2*b^2 - b^4)*c*f^2 + (
2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f + (2*a^4 + a^2*b^2 - b^4)*d*f^2)*x)*cosh(
d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + (a^4 + 2*a^2*b^2 + b^
4)*d^2*e^2 - (2*a^4 + a^2*b^2 - b^4)*d*e*f + 2*(2*a^4 + a^2*b^2 - b^4)*c*f^
2 + 6*((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*cosh(d*x + c)^2 + (
2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f + (2*a^4 + a^2*b^2 - b^4)*d*f^2)*x + 3*(3
*(a^3*b + a*b^3)*d*f^2*x - (a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^3)*c*f^2)
```

```
*cosh(d*x + c))*sinh(d*x + c)^2 - (a*b^3*f^2*cosh(d*x + c)^4 + a*b^3*f^2*si
nh(d*x + c)^4 + 4*a^2*b^2*f^2*cosh(d*x + c)^3 - 4*a^2*b^2*f^2*cosh(d*x + c)
 + a*b^3*f^2 + 2*(2*a^3*b - a*b^3)*f^2*cosh(d*x + c)^2 + 4*(a*b^3*f^2*cosh(
d*x + c) + a^2*b^2*f^2)*sinh(d*x + c)^3 + 2*(3*a*b^3*f^2*cosh(d*x + c)^2 +
6*a^2*b^2*f^2*cosh(d*x + c) + (2*a^3*b - a*b^3)*f^2)*sinh(d*x + c)^2 + 4*(a
*b^3*f^2*cosh(d*x + c)^3 + 3*a^2*b^2*f^2*cosh(d*x + c)^2 - a^2*b^2*f^2 + (2
*a^3*b - a*b^3)*f^2*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dil
og((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (a*b^3*f^2*cosh(d*x + c)^4 + a*b^3*f^2
*sinh(d*x + c)^4 + 4*a^2*b^2*f^2*cosh(d*x + c)^3 - 4*a^2*b^2*f^2*cosh(d*x +
 c) + a*b^3*f^2 + 2*(2*a^3*b - a*b^3)*f^2*cosh(d*x + c)^2 + 4*(a*b^3*f^2*co
sh(d*x + c) + a^2*b^2*f^2)*sinh(d*x + c)^3 + 2*(3*a*b^3*f^2*cosh(d*x + c)^2
 + 6*a^2*b^2*f^2*cosh(d*x + c) + (2*a^3*b - a*b^3)*f^2)*sinh(d*x + c)^2 + 4
*(a*b^3*f^2*cosh(d*x + c)^3 + 3*a^2*b^2*f^2*cosh(d*x + c)^2 - a^2*b^2*f^2 +
 (2*a^3*b - a*b^3)*f^2*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*
dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (a*b^3*d*f^2*x + a*b^3*c*f^2 + (a*b
^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x + c)^4 + (a*b^3*d*f^2*x + a*b^3*c*f^2)*s
inh(d*x + c)^4 + 4*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*cosh(d*x + c)^3 + 4*(a
^2*b^2*d*f^2*x + a^2*b^2*c*f^2 + (a*b^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x + c
))*sinh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d*f^2*x + (2*a^3*b - a*b^3)*c*f^2
)*cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d*f^2*x + (2*a^3*b - a*b^3)*c*f^2
+ 3*(a*b^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x + c)^2 + 6*(a^2*b^2*d*f^2*x + a^
2*b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*(a^2*b^2*d*f^2*x + a^2*b^2*
c*f^2)*cosh(d*x + c) - 4*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2 - (a*b^3*d*f^2*x
+ a*b^3*c*f^2)*cosh(d*x + c)^3 - 3*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*cosh(d
*x + c)^2 - ((2*a^3*b - a*b^3)*d*f^2*x + (2*a^3*b - a*b^3)*c*f^2)*cosh(d*x
+ c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d
*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b)
 + (a*b^3*d*f^2*x + a*b^3*c*f^2 + (a*b^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x +
c)^4 + (a*b^3*d*f^2*x + a*b^3*c*f^2)*sinh(d*x + c)^4 + 4*(a^2*b^2*d*f^2*x +
 a^2*b^2*c*f^2)*cosh(d*x + c)^3 + 4*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2 + (a*b
^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*((2*a^3*b - a*
b^3)*d*f^2*x + (2*a^3*b - a*b^3)*c*f^2)*cosh(d*x + c)^2 + 2*((2*a^3*b - a*b
^3)*d*f^2*x + (2*a^3*b - a*b^3)*c*f^2 + 3*(a*b^3*d*f^2*x + a*b^3*c*f^2)*cos
h(d*x + c)^2 + 6*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*cosh(d*x + c))*sinh(d*x
+ c)^2 - 4*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*cosh(d*x + c) - 4*(a^2*b^2*d*f
^2*x + a^2*b^2*c*f^2 - (a*b^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x + c)^3 - 3*(a
^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*cosh(d*x + c)^2 - ((2*a^3*b - a*b^3)*d*f^2*
x + (2*a^3*b - a*b^3)*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*((a^3*b + a*b^3)*d*f^2*x - 3*(a^
3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^3)*c*f^2)*cosh(d*x + c) - ((a^2*b^2 + b
^4)*f^2*cosh(d*x + c)^4 + (a^2*b^2 + b^4)*f^2*sinh(d*x + c)^4 + 4*(a^3*b +
a*b^3)*f^2*cosh(d*x + c)^3 + 2*(2*a^4 + a^2*b^2 - b^4)*f^2*cosh(d*x + c)^2
- 4*(a^3*b + a*b^3)*f^2*cosh(d*x + c) + 4*((a^2*b^2 + b^4)*f^2*cosh(d*x + c
) + (a^3*b + a*b^3)*f^2)*sinh(d*x + c)^3 + (a^2*b^2 + b^4)*f^2 + 2*(3*(a^2*
b^2 + b^4)*f^2*cosh(d*x + c)^2 + 6*(a^3*b + a*b^3)*f^2*cosh(d*x + c) + (2*a
^4 + a^2*b^2 - b^4)*f^2)*sinh(d*x + c)^2 + 4*((a^2*b^2 + b^4)*f^2*cosh(d*x
+ c)^3 + 3*(a^3*b + a*b^3)*f^2*cosh(d*x + c)^2 + (2*a^4 + a^2*b^2 - b^4)*f^
2*cosh(d*x + c) - (a^3*b + a*b^3)*f^2)*sinh(d*x + c) - (a*b^3*d*e*f - a*b^3
*c*f^2 + (a*b^3*d*e*f - a*b^3*c*f^2)*cosh(d*x + c)^4 + (a*b^3*d*e*f - a*b^3
*c*f^2)*sinh(d*x + c)^4 + 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*cosh(d*x + c)^3
 + 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2 + (a*b^3*d*e*f - a*b^3*c*f^2)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^
2)*cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^2 +
 3*(a*b^3*d*e*f - a*b^3*c*f^2)*cosh(d*x + c)^2 + 6*(a^2*b^2*d*e*f - a^2*b^2
*c*f^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*
cosh(d*x + c) - 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2 - (a*b^3*d*e*f - a*b^3*c*f
```

```
^2)*cosh(d*x + c)^3 - 3*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*cosh(d*x + c)^2 - (
(2*a^3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^2)*cosh(d*x + c))*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*
b*sqrt((a^2 + b^2)/b^2) + 2*a) - ((a^2*b^2 + b^4)*f^2*cosh(d*x + c)^4 + (a^
2*b^2 + b^4)*f^2*sinh(d*x + c)^4 + 4*(a^3*b + a*b^3)*f^2*cosh(d*x + c)^3 +
2*(2*a^4 + a^2*b^2 - b^4)*f^2*cosh(d*x + c)^2 - 4*(a^3*b + a*b^3)*f^2*cosh(
d*x + c) + 4*((a^2*b^2 + b^4)*f^2*cosh(d*x + c) + (a^3*b + a*b^3)*f^2)*sinh
(d*x + c)^3 + (a^2*b^2 + b^4)*f^2 + 2*(3*(a^2*b^2 + b^4)*f^2*cosh(d*x + c)^
2 + 6*(a^3*b + a*b^3)*f^2*cosh(d*x + c) + (2*a^4 + a^2*b^2 - b^4)*f^2)*sinh
(d*x + c)^2 + 4*((a^2*b^2 + b^4)*f^2*cosh(d*x + c)^3 + 3*(a^3*b + a*b^3)*f^
2*cosh(d*x + c)^2 + (2*a^4 + a^2*b^2 - b^4)*f^2*cosh(d*x + c) - (a^3*b + a*
b^3)*f^2)*sinh(d*x + c) + (a*b^3*d*e*f - a*b^3*c*f^2 + (a*b^3*d*e*f - a*b^3
*c*f^2)*cosh(d*x + c)^4 + (a*b^3*d*e*f - a*b^3*c*f^2)*sinh(d*x + c)^4 + 4*(
a^2*b^2*d*e*f - a^2*b^2*c*f^2)*cosh(d*x + c)^3 + 4*(a^2*b^2*d*e*f - a^2*b^2
*c*f^2 + (a*b^3*d*e*f - a*b^3*c*f^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*((2
*a^3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^2)*cosh(d*x + c)^2 + 2*((2*a^
3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^2 + 3*(a*b^3*d*e*f - a*b^3*c*f^2
)*cosh(d*x + c)^2 + 6*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*cosh(d*x + c))*sinh(d
*x + c)^2 - 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*cosh(d*x + c) - 4*(a^2*b^2*d*
e*f - a^2*b^2*c*f^2 - (a*b^3*d*e*f - a*b^3*c*f^2)*cosh(d*x + c)^3 - 3*(a^2*
b^2*d*e*f - a^2*b^2*c*f^2)*cosh(d*x + c)^2 - ((2*a^3*b - a*b^3)*d*e*f - (2*
a^3*b - a*b^3)*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))*
log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a
) - 2*((a^3*b + a*b^3)*d*f^2*x - 3*(a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^3
)*c*f^2 - 4*((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*cosh(d*x + c)
^3 - 3*(3*(a^3*b + a*b^3)*d*f^2*x - (a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^
3)*c*f^2)*cosh(d*x + c)^2 - 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + (a^4 +
 2*a^2*b^2 + b^4)*d^2*e^2 - (2*a^4 + a^2*b^2 - b^4)*d*e*f + 2*(2*a^4 + a^2*
b^2 - b^4)*c*f^2 + (2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f + (2*a^4 + a^2*b^2 -
b^4)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b^3 + 2*a^2*b^5 + b^7)*d
^3*cosh(d*x + c)^4 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^3*sinh(d*x + c)^4 + 4*(a
^5*b^2 + 2*a^3*b^4 + a*b^6)*d^3*cosh(d*x + c)^3 + 2*(2*a^6*b + 3*a^4*b^3 -
b^7)*d^3*cosh(d*x + c)^2 - 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^3*cosh(d*x + c
) + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^3 + 4*((a^4*b^3 + 2*a^2*b^5 + b^7)*d^3*co
sh(d*x + c) + (a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^3)*sinh(d*x + c)^3 + 2*(3*(a^
4*b^3 + 2*a^2*b^5 + b^7)*d^3*cosh(d*x + c)^2 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b
^6)*d^3*cosh(d*x + c) + (2*a^6*b + 3*a^4*b^3 - b^7)*d^3)*sinh(d*x + c)^2 +
4*((a^4*b^3 + 2*a^2*b^5 + b^7)*d^3*cosh(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4
 + a*b^6)*d^3*cosh(d*x + c)^2 + (2*a^6*b + 3*a^4*b^3 - b^7)*d^3*cosh(d*x +
c) - (a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^3)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

3.329 $\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b\sinh(c+dx))^3} dx$

Optimal. Leaf size=631

$$\frac{3af^2(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}-\frac{3af^2(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}+\frac{3f^3\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4\left(a^2+b^2\right)}+\frac{3f^3\text{Poly}}{b}$$

```
[Out] (-3*f*(e + f*x)^2)/(2*b*(a^2 + b^2)*d^2) + (3*f^2*(e + f*x)*Log[1 + (b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d^3) + (3*a*f*(e + f*x)^2*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(2*b*(a^2 + b^2)^(3/2)*d^2)
+ (3*f^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2
+ b^2)*d^3) - (3*a*f*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(2*b*(a^2 + b^2)^(3/2)*d^2) + (3*f^3*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^4) + (3*a*f^2*(e + f*x)*PolyLog[2, -
((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) + (3*f^
3*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^4)
 - (3*a*f^2*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])
/(b*(a^2 + b^2)^(3/2)*d^3) - (3*a*f^3*PolyLog[3, -((b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^4) + (3*a*f^3*PolyLog[3, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^4) - (e + f*x)^3/(2
*b*d*(a + b*Sinh[c + d*x])^2) - (3*f*(e + f*x)^2*Cosh[c + d*x])/(2*(a^2 + b
^2)*d^2*(a + b*Sinh[c + d*x]))
```

Rubi [A] time = 1.09479, antiderivative size = 631, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5464, 3324, 3322, 2264, 2190, 2531, 2282, 6589, 5561, 2279, 2391}

$$\frac{3af^2(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}-\frac{3af^2(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}+\frac{3f^3\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4\left(a^2+b^2\right)}+\frac{3f^3\text{Poly}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```


```
[Out] (-3*f*(e + f*x)^2)/(2*b*(a^2 + b^2)*d^2) + (3*f^2*(e + f*x)*Log[1 + (b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d^3) + (3*a*f*(e + f*x)^2*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(2*b*(a^2 + b^2)^(3/2)*d^2)
+ (3*f^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2
+ b^2)*d^3) - (3*a*f*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(2*b*(a^2 + b^2)^(3/2)*d^2) + (3*f^3*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^4) + (3*a*f^2*(e + f*x)*PolyLog[2, -
((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) + (3*f^
3*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^4)
 - (3*a*f^2*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])
/(b*(a^2 + b^2)^(3/2)*d^3) - (3*a*f^3*PolyLog[3, -((b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^4) + (3*a*f^3*PolyLog[3, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^4) - (e + f*x)^3/(2
*b*d*(a + b*Sinh[c + d*x])^2) - (3*f*(e + f*x)^2*Cosh[c + d*x])/(2*(a^2 + b
^2)*d^2*(a + b*Sinh[c + d*x]))
```


Rule 5464

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sinh[c +
```

```
d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}+\frac{(3f)\int \frac{(e+fx)^2}{(a+b\sinh(c+dx))^2}\,dx}{2bd}$$

$$= -\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}-\frac{3f(e+fx)^2\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))}+\frac{(3af)\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}\,dx}{2b\left(a^2+b^2\right)d}$$

$$= -\frac{3f(e+fx)^2}{2b\left(a^2+b^2\right)d^2}-\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2}-\frac{3f(e+fx)^2\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))}+\frac{(3af)}{}$$

$$= -\frac{3f(e+fx)^2}{2b\left(a^2+b^2\right)d^2}+\frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3}+\frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3}-$$

$$= -\frac{3f(e+fx)^2}{2b\left(a^2+b^2\right)d^2}+\frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3}+\frac{3af(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2b\left(a^2+b^2\right)^{3/2}d^2}+$$

$$= -\frac{3f(e+fx)^2}{2b\left(a^2+b^2\right)d^2}+\frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3}+\frac{3af(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2b\left(a^2+b^2\right)^{3/2}d^2}+$$

$$= -\frac{3f(e+fx)^2}{2b\left(a^2+b^2\right)d^2}+\frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3}+\frac{3af(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2b\left(a^2+b^2\right)^{3/2}d^2}+$$

$$= -\frac{3f(e+fx)^2}{2b\left(a^2+b^2\right)d^2}+\frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3}+\frac{3af(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2b\left(a^2+b^2\right)^{3/2}d^2}+$$

Mathematica [B] time = 24.6796, size = 5753, normalized size = 9.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

[Out] Result too large to show

Maple [F] time = 0.43, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \cosh(dx+c)}{(a+b\sinh(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 4.17084, size = 24868, normalized size = 39.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*(a^2*b^2 + b^4)*d^2*e^2*f - 12*(a^2*b^2 + b^4)*c*d*e*f^2 + 6*(a^2*b^
2 + b^4)*c^2*f^3 - 6*((a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^2*b^2 + b^4)*d^2*e
*f^2*x + 2*(a^2*b^2 + b^4)*c*d*e*f^2 - (a^2*b^2 + b^4)*c^2*f^3)*cosh(d*x +
c)^4 - 6*((a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^2*b^2 + b^4)*d^2*e*f^2*x + 2*(
a^2*b^2 + b^4)*c*d*e*f^2 - (a^2*b^2 + b^4)*c^2*f^3)*sinh(d*x + c)^4 - 6*(3*
(a^3*b + a*b^3)*d^2*f^3*x^2 + 6*(a^3*b + a*b^3)*d^2*e*f^2*x - (a^3*b + a*b^
3)*d^2*e^2*f + 8*(a^3*b + a*b^3)*c*d*e*f^2 - 4*(a^3*b + a*b^3)*c^2*f^3)*cos
h(d*x + c)^3 - 6*(3*(a^3*b + a*b^3)*d^2*f^3*x^2 + 6*(a^3*b + a*b^3)*d^2*e*f
^2*x - (a^3*b + a*b^3)*d^2*e^2*f + 8*(a^3*b + a*b^3)*c*d*e*f^2 - 4*(a^3*b +
 a*b^3)*c^2*f^3 + 4*((a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^2*b^2 + b^4)*d^2*e*
f^2*x + 2*(a^2*b^2 + b^4)*c*d*e*f^2 - (a^2*b^2 + b^4)*c^2*f^3)*cosh(d*x + c
))*sinh(d*x + c)^3 - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 2*(a^4 + 2*
a^2*b^2 + b^4)*d^3*e^3 - 3*(2*a^4 + a^2*b^2 - b^4)*d^2*e^2*f + 12*(2*a^4 +
a^2*b^2 - b^4)*c*d*e*f^2 - 6*(2*a^4 + a^2*b^2 - b^4)*c^2*f^3 + 3*(2*(a^4 +
2*a^2*b^2 + b^4)*d^3*e*f^2 + (2*a^4 + a^2*b^2 - b^4)*d^2*f^3)*x^2 + 6*((a^4
 + 2*a^2*b^2 + b^4)*d^3*e^2*f + (2*a^4 + a^2*b^2 - b^4)*d^2*e*f^2)*x)*cosh(
d*x + c)^2 - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 2*(a^4 + 2*a^2*b^2
+ b^4)*d^3*e^3 - 3*(2*a^4 + a^2*b^2 - b^4)*d^2*e^2*f + 12*(2*a^4 + a^2*b^2
- b^4)*c*d*e*f^2 - 6*(2*a^4 + a^2*b^2 - b^4)*c^2*f^3 + 3*(2*(a^4 + 2*a^2*b^
2 + b^4)*d^3*e*f^2 + (2*a^4 + a^2*b^2 - b^4)*d^2*f^3)*x^2 + 18*((a^2*b^2 +
```

b^4)*d^2*f^3*x^2 + 2*(a^2*b^2 + b^4)*d^2*e*f^2*x + 2*(a^2*b^2 + b^4)*c*d*e*
f^2 - (a^2*b^2 + b^4)*c^2*f^3)*cosh(d*x + c)^2 + 6*((a^4 + 2*a^2*b^2 + b^4)
*d^3*e^2*f + (2*a^4 + a^2*b^2 - b^4)*d^2*e*f^2)*x + 9*(3*(a^3*b + a*b^3)*d^
2*f^3*x^2 + 6*(a^3*b + a*b^3)*d^2*e*f^2*x - (a^3*b + a*b^3)*d^2*e^2*f + 8*(
a^3*b + a*b^3)*c*d*e*f^2 - 4*(a^3*b + a*b^3)*c^2*f^3)*cosh(d*x + c))*sinh(d
x + c)^2 - 6(a*b^3*f^3*cosh(d*x + c)^4 + a*b^3*f^3*sinh(d*x + c)^4 + 4*a^
2*b^2*f^3*cosh(d*x + c)^3 - 4*a^2*b^2*f^3*cosh(d*x + c) + a*b^3*f^3 + 2*(2*
a^3*b - a*b^3)*f^3*cosh(d*x + c)^2 + 4*(a*b^3*f^3*cosh(d*x + c) + a^2*b^2*f
^3)*sinh(d*x + c)^3 + 2*(3*a*b^3*f^3*cosh(d*x + c)^2 + 6*a^2*b^2*f^3*cosh(d
*x + c) + (2*a^3*b - a*b^3)*f^3)*sinh(d*x + c)^2 + 4*(a*b^3*f^3*cosh(d*x +
c)^3 + 3*a^2*b^2*f^3*cosh(d*x + c)^2 - a^2*b^2*f^3 + (2*a^3*b - a*b^3)*f^3*
cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x
+ c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2))/b) + 6*(a*b^3*f^3*cosh(d*x + c)^4 + a*b^3*f^3*sinh(d*x + c)^4 + 4*
a^2*b^2*f^3*cosh(d*x + c)^3 - 4*a^2*b^2*f^3*cosh(d*x + c) + a*b^3*f^3 + 2*(
2*a^3*b - a*b^3)*f^3*cosh(d*x + c)^2 + 4*(a*b^3*f^3*cosh(d*x + c) + a^2*b^2
*f^3)*sinh(d*x + c)^3 + 2*(3*a*b^3*f^3*cosh(d*x + c)^2 + 6*a^2*b^2*f^3*cosh
(d*x + c) + (2*a^3*b - a*b^3)*f^3)*sinh(d*x + c)^2 + 4*(a*b^3*f^3*cosh(d*x
+ c)^3 + 3*a^2*b^2*f^3*cosh(d*x + c)^2 - a^2*b^2*f^3 + (2*a^3*b - a*b^3)*f^
3*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*
x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2))/b) + 6*((a^3*b + a*b^3)*d^2*f^3*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f^
2*x - 3*(a^3*b + a*b^3)*d^2*e^2*f + 8*(a^3*b + a*b^3)*c*d*e*f^2 - 4*(a^3*b
+ a*b^3)*c^2*f^3)*cosh(d*x + c) + 6*((a^2*b^2 + b^4)*f^3*cosh(d*x + c)^4 +
(a^2*b^2 + b^4)*f^3*sinh(d*x + c)^4 + 4*(a^3*b + a*b^3)*f^3*cosh(d*x + c)^3
 + 2*(2*a^4 + a^2*b^2 - b^4)*f^3*cosh(d*x + c)^2 - 4*(a^3*b + a*b^3)*f^3*co
sh(d*x + c) + (a^2*b^2 + b^4)*f^3 + 4*((a^2*b^2 + b^4)*f^3*cosh(d*x + c) +
(a^3*b + a*b^3)*f^3)*sinh(d*x + c)^3 + 2*(3*(a^2*b^2 + b^4)*f^3*cosh(d*x +
c)^2 + 6*(a^3*b + a*b^3)*f^3*cosh(d*x + c) + (2*a^4 + a^2*b^2 - b^4)*f^3)*s
inh(d*x + c)^2 + 4*((a^2*b^2 + b^4)*f^3*cosh(d*x + c)^3 + 3*(a^3*b + a*b^3)
*f^3*cosh(d*x + c)^2 + (2*a^4 + a^2*b^2 - b^4)*f^3*cosh(d*x + c) - (a^3*b +
 a*b^3)*f^3)*sinh(d*x + c) + (a*b^3*d*f^3*x + a*b^3*d*e*f^2 + (a*b^3*d*f^3*
x + a*b^3*d*e*f^2)*cosh(d*x + c)^4 + (a*b^3*d*f^3*x + a*b^3*d*e*f^2)*sinh(d
x + c)^4 + 4(a^2*b^2*d*f^3*x + a^2*b^2*d*e*f^2)*cosh(d*x + c)^3 + 4*(a^2*
b^2*d*f^3*x + a^2*b^2*d*e*f^2 + (a*b^3*d*f^3*x + a*b^3*d*e*f^2)*cosh(d*x +
c))*sinh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d*f^3*x + (2*a^3*b - a*b^3)*d*e*
f^2)*cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d*f^3*x + (2*a^3*b - a*b^3)*d*e
f^2 + 3(a*b^3*d*f^3*x + a*b^3*d*e*f^2)*cosh(d*x + c)^2 + 6*(a^2*b^2*d*f^3
*x + a^2*b^2*d*e*f^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*(a^2*b^2*d*f^3*x +
 a^2*b^2*d*e*f^2)*cosh(d*x + c) - 4*(a^2*b^2*d*f^3*x + a^2*b^2*d*e*f^2 - (a
*b^3*d*f^3*x + a*b^3*d*e*f^2)*cosh(d*x + c)^3 - 3*(a^2*b^2*d*f^3*x + a^2*b^
2*d*e*f^2)*cosh(d*x + c)^2 - ((2*a^3*b - a*b^3)*d*f^3*x + (2*a^3*b - a*b^3)
*d*e*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))*dilog((a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b + 1) + 6*((a^2*b^2 + b^4)*f^3*cosh(d*x + c)^4 + (a^2*b
^2 + b^4)*f^3*sinh(d*x + c)^4 + 4*(a^3*b + a*b^3)*f^3*cosh(d*x + c)^3 + 2*(
2*a^4 + a^2*b^2 - b^4)*f^3*cosh(d*x + c)^2 - 4*(a^3*b + a*b^3)*f^3*cosh(d*x
 + c) + (a^2*b^2 + b^4)*f^3 + 4*((a^2*b^2 + b^4)*f^3*cosh(d*x + c) + (a^3*b
 + a*b^3)*f^3)*sinh(d*x + c)^3 + 2*(3*(a^2*b^2 + b^4)*f^3*cosh(d*x + c)^2 +
 6*(a^3*b + a*b^3)*f^3*cosh(d*x + c) + (2*a^4 + a^2*b^2 - b^4)*f^3)*sinh(d*
x + c)^2 + 4*((a^2*b^2 + b^4)*f^3*cosh(d*x + c)^3 + 3*(a^3*b + a*b^3)*f^3*c
osh(d*x + c)^2 + (2*a^4 + a^2*b^2 - b^4)*f^3*cosh(d*x + c) - (a^3*b + a*b^3
)*f^3)*sinh(d*x + c) - (a*b^3*d*f^3*x + a*b^3*d*e*f^2 + (a*b^3*d*f^3*x + a*
b^3*d*e*f^2)*cosh(d*x + c)^4 + (a*b^3*d*f^3*x + a*b^3*d*e*f^2)*sinh(d*x + c
)^4 + 4*(a^2*b^2*d*f^3*x + a^2*b^2*d*e*f^2)*cosh(d*x + c)^3 + 4*(a^2*b^2*d*
f^3*x + a^2*b^2*d*e*f^2 + (a*b^3*d*f^3*x + a*b^3*d*e*f^2)*cosh(d*x + c))*si
nh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d*f^3*x + (2*a^3*b - a*b^3)*d*e*f^2)*c
osh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d*f^3*x + (2*a^3*b - a*b^3)*d*e*f^2 +
 3*(a*b^3*d*f^3*x + a*b^3*d*e*f^2)*cosh(d*x + c)^2 + 6*(a^2*b^2*d*f^3*x + a

```
^2*b^2*d*e*f^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*(a^2*b^2*d*f^3*x + a^2*b
^2*d*e*f^2)*cosh(d*x + c) - 4*(a^2*b^2*d*f^3*x + a^2*b^2*d*e*f^2 - (a*b^3*d
*f^3*x + a*b^3*d*e*f^2)*cosh(d*x + c)^3 - 3*(a^2*b^2*d*f^3*x + a^2*b^2*d*e*
f^2)*cosh(d*x + c)^2 - ((2*a^3*b - a*b^3)*d*f^3*x + (2*a^3*b - a*b^3)*d*e*f
^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))*dilog((a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2) - b)/b + 1) + 3*(2*(a^2*b^2 + b^4)*d*e*f^2 - 2*(a^2*b^2 + b^4)*c*f^
3 + 2*((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3)*cosh(d*x + c)^4 + 2
*((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3)*sinh(d*x + c)^4 + 8*((a^
3*b + a*b^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c)^3 + 8*((a^3*b +
 a*b^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3 + ((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b
^2 + b^4)*c*f^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*((2*a^4 + a^2*b^2 - b^4
)*d*e*f^2 - (2*a^4 + a^2*b^2 - b^4)*c*f^3)*cosh(d*x + c)^2 + 4*((2*a^4 + a^
2*b^2 - b^4)*d*e*f^2 - (2*a^4 + a^2*b^2 - b^4)*c*f^3 + 3*((a^2*b^2 + b^4)*d
*e*f^2 - (a^2*b^2 + b^4)*c*f^3)*cosh(d*x + c)^2 + 6*((a^3*b + a*b^3)*d*e*f^
2 - (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 8*((a^3*b + a*b
^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c) - 8*((a^3*b + a*b^3)*d*e
*f^2 - (a^3*b + a*b^3)*c*f^3 - ((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c
*f^3)*cosh(d*x + c)^3 - 3*((a^3*b + a*b^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3)
*cosh(d*x + c)^2 - ((2*a^4 + a^2*b^2 - b^4)*d*e*f^2 - (2*a^4 + a^2*b^2 - b^
4)*c*f^3)*cosh(d*x + c))*sinh(d*x + c) - (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f
^2 + a*b^3*c^2*f^3 + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*
cosh(d*x + c)^4 + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*sin
h(d*x + c)^4 + 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3
)*cosh(d*x + c)^3 + 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^
2*f^3 + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cosh(d*x + c)
)*sinh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*
d*e*f^2 + (2*a^3*b - a*b^3)*c^2*f^3)*cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)
*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*d*e*f^2 + (2*a^3*b - a*b^3)*c^2*f^3 + 3*
(a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cosh(d*x + c)^2 + 6*(
a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*cosh(d*x + c))*s
inh(d*x + c)^2 - 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f
^3)*cosh(d*x + c) - 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^
2*f^3 - (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cosh(d*x + c)
^3 - 3*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*cosh(d*x
 + c)^2 - ((2*a^3*b - a*b^3)*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*d*e*f^2 + (2
*a^3*b - a*b^3)*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) +
2*a) + 3*(2*(a^2*b^2 + b^4)*d*e*f^2 - 2*(a^2*b^2 + b^4)*c*f^3 + 2*((a^2*b^2
 + b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3)*cosh(d*x + c)^4 + 2*((a^2*b^2 + b^
4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3)*sinh(d*x + c)^4 + 8*((a^3*b + a*b^3)*d*
e*f^2 - (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c)^3 + 8*((a^3*b + a*b^3)*d*e*f^2
 - (a^3*b + a*b^3)*c*f^3 + ((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3
)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*((2*a^4 + a^2*b^2 - b^4)*d*e*f^2 - (2*
a^4 + a^2*b^2 - b^4)*c*f^3)*cosh(d*x + c)^2 + 4*((2*a^4 + a^2*b^2 - b^4)*d*
e*f^2 - (2*a^4 + a^2*b^2 - b^4)*c*f^3 + 3*((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b
^2 + b^4)*c*f^3)*cosh(d*x + c)^2 + 6*((a^3*b + a*b^3)*d*e*f^2 - (a^3*b + a*
b^3)*c*f^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 8*((a^3*b + a*b^3)*d*e*f^2 - (
a^3*b + a*b^3)*c*f^3)*cosh(d*x + c) - 8*((a^3*b + a*b^3)*d*e*f^2 - (a^3*b +
 a*b^3)*c*f^3 - ((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3)*cosh(d*x
+ c)^3 - 3*((a^3*b + a*b^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c)^
2 - ((2*a^4 + a^2*b^2 - b^4)*d*e*f^2 - (2*a^4 + a^2*b^2 - b^4)*c*f^3)*cosh(
d*x + c))*sinh(d*x + c) + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*
f^3 + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cosh(d*x + c)^4
 + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*sinh(d*x + c)^4 +
4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*cosh(d*x + c)
^3 + 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3 + (a*b^3*
d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c)
^3 + 2*((2*a^3*b - a*b^3)*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*d*e*f^2 + (2*a^
```

3*b - a*b^3)*c^2*f^3)*cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*d*e*f^2 + (2*a^3*b - a*b^3)*c^2*f^3 + 3*(a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cosh(d*x + c)^2 + 6*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*cosh(d*x + c) - 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3 - (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cosh(d*x + c)^3 - 3*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*cosh(d*x + c)^2 - ((2*a^3*b - a*b^3)*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*d*e*f^2 + (2*a^3*b - a*b^3)*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 3*(2*(a^2*b^2 + b^4)*d*f^3*x + 2*(a^2*b^2 + b^4)*c*f^3 + 2*((a^2*b^2 + b^4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3)*cosh(d*x + c)^4 + 2*((a^2*b^2 + b^4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3)*sinh(d*x + c)^4 + 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c)^3 + 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3 + ((a^2*b^2 + b^4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*((2*a^4 + a^2*b^2 - b^4)*d*f^3*x + (2*a^4 + a^2*b^2 - b^4)*c*f^3)*cosh(d*x + c)^2 + 4*((2*a^4 + a^2*b^2 - b^4)*d*f^3*x + (2*a^4 + a^2*b^2 - b^4)*c*f^3 + 3*((a^2*b^2 + b^4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3)*cosh(d*x + c)^2 + 6*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c) - 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3 - ((a^2*b^2 + b^4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3)*cosh(d*x + c)^3 - 3*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c)^2 - ((2*a^4 + a^2*b^2 - b^4)*d*f^3*x + (2*a^4 + a^2*b^2 - b^4)*c*f^3)*cosh(d*x + c))*sinh(d*x + c) + (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^3 + (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^3)*cosh(d*x + c)^4 + (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^3)*sinh(d*x + c)^4 + 4*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*cosh(d*x + c)^3 + 4*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3 + (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d^2*f^3*x^2 + 2*(2*a^3*b - a*b^3)*d^2*e*f^2*x + 2*(2*a^3*b - a*b^3)*c*d*e*f^2 - (2*a^3*b - a*b^3)*c^2*f^3)*cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d^2*f^3*x^2 + 2*(2*a^3*b - a*b^3)*d^2*e*f^2*x + 2*(2*a^3*b - a*b^3)*c*d*e*f^2 - (2*a^3*b - a*b^3)*c^2*f^3 + 3*(a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^3)*cosh(d*x + c)^2 + 6*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*cosh(d*x + c) - 4*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3 - (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^3)*cosh(d*x + c)^3 - 3*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*cosh(d*x + c)^2 - ((2*a^3*b - a*b^3)*d^2*f^3*x^2 + 2*(2*a^3*b - a*b^3)*d^2*e*f^2*x + 2*(2*a^3*b - a*b^3)*c*d*e*f^2 - (2*a^3*b - a*b^3)*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 3*(2*(a^2*b^2 + b^4)*d*f^3*x + 2*(a^2*b^2 + b^4)*c*f^3 + 2*((a^2*b^2 + b^4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3)*cosh(d*x + c)^4 + 2*((a^2*b^2 + b^4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3)*sinh(d*x + c)^4 + 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c)^3 + 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3 + ((a^2*b^2 + b^4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*((2*a^4 + a^2*b^2 - b^4)*d*f^3*x + (2*a^4 + a^2*b^2 - b^4)*c*f^3)*cosh(d*x + c)^2 + 4*((2*a^4 + a^2*b^2 - b^4)*d*f^3*x + (2*a^4 + a^2*b^2 - b^4)*c*f^3 + 3*((a^2*b^2 + b^4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3)*cosh(d*x + c)^2 + 6*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c) - 8*(

```
(a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3 - ((a^2*b^2 + b^4)*d*f^3*x
+ (a^2*b^2 + b^4)*c*f^3)*cosh(d*x + c)^3 - 3*((a^3*b + a*b^3)*d*f^3*x + (a^
3*b + a*b^3)*c*f^3)*cosh(d*x + c)^2 - ((2*a^4 + a^2*b^2 - b^4)*d*f^3*x + (2
*a^4 + a^2*b^2 - b^4)*c*f^3)*cosh(d*x + c))*sinh(d*x + c) - (a*b^3*d^2*f^3*
x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^3 + (a*b^3*d^2*
f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^3)*cosh(d*x
 + c)^4 + (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a*
b^3*c^2*f^3)*sinh(d*x + c)^4 + 4*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2
*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*cosh(d*x + c)^3 + 4*(a^2*b^2*d^
2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3 +
 (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f
^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d^2*f^3*x^2 + 2*(
2*a^3*b - a*b^3)*d^2*e*f^2*x + 2*(2*a^3*b - a*b^3)*c*d*e*f^2 - (2*a^3*b - a
*b^3)*c^2*f^3)*cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d^2*f^3*x^2 + 2*(2*a^
3*b - a*b^3)*d^2*e*f^2*x + 2*(2*a^3*b - a*b^3)*c*d*e*f^2 - (2*a^3*b - a*b^3
)*c^2*f^3 + 3*(a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2
- a*b^3*c^2*f^3)*cosh(d*x + c)^2 + 6*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e
*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c
)^2 - 4*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2
- a^2*b^2*c^2*f^3)*cosh(d*x + c) - 4*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e
*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3 - (a*b^3*d^2*f^3*x^2 + 2*a*b
^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^3)*cosh(d*x + c)^3 - 3*(a^
2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c
^2*f^3)*cosh(d*x + c)^2 - ((2*a^3*b - a*b^3)*d^2*f^3*x^2 + 2*(2*a^3*b - a*b
^3)*d^2*e*f^2*x + 2*(2*a^3*b - a*b^3)*c*d*e*f^2 - (2*a^3*b - a*b^3)*c^2*f^3
)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))*log(-(a*cosh(d*x + c
) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2) - b)/b) + 2*(3*(a^3*b + a*b^3)*d^2*f^3*x^2 + 6*(a^3*b + a*b^3)*d^2*e*f
^2*x - 9*(a^3*b + a*b^3)*d^2*e^2*f + 24*(a^3*b + a*b^3)*c*d*e*f^2 - 12*(a^3
*b + a*b^3)*c^2*f^3 - 12*((a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^2*b^2 + b^4)*d
^2*e*f^2*x + 2*(a^2*b^2 + b^4)*c*d*e*f^2 - (a^2*b^2 + b^4)*c^2*f^3)*cosh(d*
x + c)^3 - 9*(3*(a^3*b + a*b^3)*d^2*f^3*x^2 + 6*(a^3*b + a*b^3)*d^2*e*f^2*x
 - (a^3*b + a*b^3)*d^2*e^2*f + 8*(a^3*b + a*b^3)*c*d*e*f^2 - 4*(a^3*b + a*b
^3)*c^2*f^3)*cosh(d*x + c)^2 - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 2
*(a^4 + 2*a^2*b^2 + b^4)*d^3*e^3 - 3*(2*a^4 + a^2*b^2 - b^4)*d^2*e^2*f + 12
*(2*a^4 + a^2*b^2 - b^4)*c*d*e*f^2 - 6*(2*a^4 + a^2*b^2 - b^4)*c^2*f^3 + 3*
(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*e*f^2 + (2*a^4 + a^2*b^2 - b^4)*d^2*f^3)*x^2
 + 6*((a^4 + 2*a^2*b^2 + b^4)*d^3*e^2*f + (2*a^4 + a^2*b^2 - b^4)*d^2*e*f^2
)*x)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b^3 + 2*a^2*b^5 + b^7)*d^4*cosh(d*
x + c)^4 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^4*sinh(d*x + c)^4 + 4*(a^5*b^2 + 2
*a^3*b^4 + a*b^6)*d^4*cosh(d*x + c)^3 + 2*(2*a^6*b + 3*a^4*b^3 - b^7)*d^4*c
osh(d*x + c)^2 - 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^4*cosh(d*x + c) + (a^4*b
^3 + 2*a^2*b^5 + b^7)*d^4 + 4*((a^4*b^3 + 2*a^2*b^5 + b^7)*d^4*cosh(d*x + c
) + (a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^4)*sinh(d*x + c)^3 + 2*(3*(a^4*b^3 + 2*
a^2*b^5 + b^7)*d^4*cosh(d*x + c)^2 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^4*co
sh(d*x + c) + (2*a^6*b + 3*a^4*b^3 - b^7)*d^4)*sinh(d*x + c)^2 + 4*((a^4*b^
3 + 2*a^2*b^5 + b^7)*d^4*cosh(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*
d^4*cosh(d*x + c)^2 + (2*a^6*b + 3*a^4*b^3 - b^7)*d^4*cosh(d*x + c) - (a^5*
b^2 + 2*a^3*b^4 + a*b^6)*d^4)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^3\cosh\left(dx+c\right)}{\left(b\sinh\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

3.330 $\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=112

$$-\frac{af\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd^2\left(a^2+b^2\right)^{3/2}}-\frac{f\cosh(c+dx)}{2d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

```
[Out] -((a*f*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b*(a^2 + b^2)^(
3/2)*d^2)) - (e + f*x)/(2*b*d*(a + b*Sinh[c + d*x])^2) - (f*Cosh[c + d*x])/
(2*(a^2 + b^2)*d^2*(a + b*Sinh[c + d*x]))
```

Rubi [A] time = 0.0951033, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5464, 2664, 12, 2660, 618, 204}

$$-\frac{af\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd^2\left(a^2+b^2\right)^{3/2}}-\frac{f\cosh(c+dx)}{2d^2\left(a^2+b^2\right)(a+b\sinh(c+dx))}-\frac{e+fx}{2bd(a+b\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] -((a*f*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b*(a^2 + b^2)^(
3/2)*d^2)) - (e + f*x)/(2*b*d*(a + b*Sinh[c + d*x])^2) - (f*Cosh[c + d*x])/
(2*(a^2 + b^2)*d^2*(a + b*Sinh[c + d*x]))
```

Rule 5464

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sinh[c +
d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(e+fx)\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{e+fx}{2bd(a+b\sinh(c+dx))^2} + \frac{f\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx}{2bd}$$

$$= -\frac{e+fx}{2bd(a+b\sinh(c+dx))^2} - \frac{f\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))} + \frac{f\int \frac{a}{a+b\sinh(c+dx)}\,dx}{2b\left(a^2+b^2\right)d}$$

$$= -\frac{e+fx}{2bd(a+b\sinh(c+dx))^2} - \frac{f\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))} + \frac{(af)\int \frac{1}{a+b\sinh(c+dx)}\,dx}{2b\left(a^2+b^2\right)d}$$

$$= -\frac{e+fx}{2bd(a+b\sinh(c+dx))^2} - \frac{f\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))} - \frac{(iaf)\operatorname{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\right.}{b\left(a^2+\right.}$$

$$= -\frac{e+fx}{2bd(a+b\sinh(c+dx))^2} - \frac{f\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))} + \frac{(2iaf)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^2+b^2\right.}\right.}{}$$

$$= -\frac{af\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d^2} - \frac{e+fx}{2bd(a+b\sinh(c+dx))^2} - \frac{f\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+\right.}$$

Mathematica [A] time = 0.373767, size = 112, normalized size = 1.

$$-\frac{\frac{\frac{2af\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}}+\frac{d(e+fx)}{(a+b\sinh(c+dx))^2}}{b}+\frac{f\cosh(c+dx)}{\left(a^2+b^2\right)(a+b\sinh(c+dx))}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] -((f*Cosh[c + d*x])/((a^2 + b^2)*(a + b*Sinh[c + d*x])) + ((2*a*f*ArcTan[(b
 - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) + (d*(e + f*x
))/(a + b*Sinh[c + d*x])^2)/b)/(2*d^2)
```

Maple [B] time = 0., size = 308, normalized size = 2.8

$$-\frac{2\,a^{2}dfx\mathrm{e}^{2\,dx+2\,c}+2\,b^{2}dfx\mathrm{e}^{2\,dx+2\,c}+2\,a^{2}de\mathrm{e}^{2\,dx+2\,c}-abf\mathrm{e}^{3\,dx+3\,c}+2\,b^{2}de\mathrm{e}^{2\,dx+2\,c}-2\,a^{2}f\mathrm{e}^{2\,dx+2\,c}+b^{2}f\mathrm{e}^{2\,dx+2\,c}+3\,fa}{d^{2}b\left(b\mathrm{e}^{2\,dx+2\,c}+2\,a\mathrm{e}^{dx+c}-b\right)^{2}\left(a^{2}+b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

```
[Out] -1/b*(2*a^2*d*f*x*exp(2*d*x+2*c)+2*b^2*d*f*x*exp(2*d*x+2*c)+2*a^2*d*e*exp(2
*d*x+2*c)-a*b*f*exp(3*d*x+3*c)+2*b^2*d*e*exp(2*d*x+2*c)-2*a^2*f*exp(2*d*x+2
*c)+b^2*f*exp(2*d*x+2*c)+3*f*a*exp(d*x+c)*b-f*b^2)/d^2/(b*exp(2*d*x+2*c)+2*
a*exp(d*x+c)-b)^2/(a^2+b^2)+1/2/(a^2+b^2)^(3/2)*f*a/d^2/b*ln(exp(d*x+c)+(a*
(a^2+b^2)^(3/2)-a^4-2*a^2*b^2-b^4)/(a^2+b^2)^(3/2)/b)-1/2/(a^2+b^2)^(3/2)*f
*a/d^2/b*ln(exp(d*x+c)+(a*(a^2+b^2)^(3/2)+a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(3/2
)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.21918, size = 2811, normalized size = 25.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a^3*b + a*b^3)*f*cosh(d*x + c)^3 + 2*(a^3*b + a*b^3)*f*sinh(d*x + c
)^3 - 6*(a^3*b + a*b^3)*f*cosh(d*x + c) - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*f*
x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*e - (2*a^4 + a^2*b^2 - b^4)*f)*cosh(d*x + c
)^2 - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*e -
3*(a^3*b + a*b^3)*f*cosh(d*x + c) - (2*a^4 + a^2*b^2 - b^4)*f)*sinh(d*x + c
)^2 + (a*b^2*f*cosh(d*x + c)^4 + a*b^2*f*sinh(d*x + c)^4 + 4*a^2*b*f*cosh(d
*x + c)^3 - 4*a^2*b*f*cosh(d*x + c) + a*b^2*f + 2*(2*a^3 - a*b^2)*f*cosh(d*
x + c)^2 + 4*(a*b^2*f*cosh(d*x + c) + a^2*b*f)*sinh(d*x + c)^3 + 2*(3*a*b^2
*f*cosh(d*x + c)^2 + 6*a^2*b*f*cosh(d*x + c) + (2*a^3 - a*b^2)*f)*sinh(d*x
+ c)^2 + 4*(a*b^2*f*cosh(d*x + c)^3 + 3*a^2*b*f*cosh(d*x + c)^2 - a^2*b*f +
 (2*a^3 - a*b^2)*f*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*c
osh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 +
2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x +
 c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*co
sh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(a^2*b^2 + b^
4)*f + 2*(3*(a^3*b + a*b^3)*f*cosh(d*x + c)^2 - 3*(a^3*b + a*b^3)*f - 2*(2*
(a^4 + 2*a^2*b^2 + b^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*e - (2*a^4 + a^
2*b^2 - b^4)*f)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b^3 + 2*a^2*b^5 + b^7)*
```

```
d^2*cosh(d*x + c)^4 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^2*sinh(d*x + c)^4 + 4*(
a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2*cosh(d*x + c)^3 + 2*(2*a^6*b + 3*a^4*b^3 -
 b^7)*d^2*cosh(d*x + c)^2 - 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2*cosh(d*x +
c) + 4*((a^4*b^3 + 2*a^2*b^5 + b^7)*d^2*cosh(d*x + c) + (a^5*b^2 + 2*a^3*b^
4 + a*b^6)*d^2)*sinh(d*x + c)^3 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^2 + 2*(3*(a
^4*b^3 + 2*a^2*b^5 + b^7)*d^2*cosh(d*x + c)^2 + 6*(a^5*b^2 + 2*a^3*b^4 + a*
b^6)*d^2*cosh(d*x + c) + (2*a^6*b + 3*a^4*b^3 - b^7)*d^2)*sinh(d*x + c)^2 +
 4*((a^4*b^3 + 2*a^2*b^5 + b^7)*d^2*cosh(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^
4 + a*b^6)*d^2*cosh(d*x + c)^2 + (2*a^6*b + 3*a^4*b^3 - b^7)*d^2*cosh(d*x +
 c) - (a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

3.331 $\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=306

$$\frac{af^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}-\frac{af^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}+\frac{af(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd^2\left(a^2+b^2\right)^{3/2}}-\frac{af(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+\cdots}\right.}{bd^2\left(a^2+b^2\right)^{3/2}}$$

```
[Out] (a*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^
2)^(3/2)*d^2) - (a*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)])/(b*(a^2 + b^2)^(3/2)*d^2) + (f^2*Log[a + b*Sinh[c + d*x]])/(b*(a^2 + b^
2)*d^3) + (a*f^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(
a^2 + b^2)^(3/2)*d^3) - (a*f^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) - (e + f*x)^2/(2*b*d*(a + b*Sinh[c + d*
x])^2) - (f*(e + f*x)*Cosh[c + d*x])/((a^2 + b^2)*d^2*(a + b*Sinh[c + d*x])
)
```

Rubi [A] time = 0.521416, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5464, 3324, 3322, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{af^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}-\frac{af^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}+\frac{af(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bd^2\left(a^2+b^2\right)^{3/2}}-\frac{af(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+\cdots}\right.}{bd^2\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] (a*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^
2)^(3/2)*d^2) - (a*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)])/(b*(a^2 + b^2)^(3/2)*d^2) + (f^2*Log[a + b*Sinh[c + d*x]])/(b*(a^2 + b^
2)*d^3) + (a*f^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(
a^2 + b^2)^(3/2)*d^3) - (a*f^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) - (e + f*x)^2/(2*b*d*(a + b*Sinh[c + d*
x])^2) - (f*(e + f*x)*Cosh[c + d*x])/((a^2 + b^2)*d^2*(a + b*Sinh[c + d*x])
)
```

Rule 5464

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sinh[c +
d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2} + \frac{f\int \frac{e+fx}{(a+b\sinh(c+dx))^2}\,dx}{bd}$$

$$= -\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2} - \frac{f(e+fx)\cosh(c+dx)}{(a^2+b^2)\,d^2(a+b\sinh(c+dx))} + \frac{(af)\int \frac{e+fx}{a+b\sinh(c+dx)}\,dx}{b(a^2+b^2)\,d}$$

$$= -\frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2} - \frac{f(e+fx)\cosh(c+dx)}{(a^2+b^2)\,d^2(a+b\sinh(c+dx))} + \frac{(2af)\int \frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c}}}{b(a^2+b^2)\,d}$$

$$= \frac{f^2\log(a+b\sinh(c+dx))}{b(a^2+b^2)\,d^3} - \frac{(e+fx)^2}{2bd(a+b\sinh(c+dx))^2} - \frac{f(e+fx)\cosh(c+dx)}{(a^2+b^2)\,d^2(a+b\sinh(c+d}$$

$$= \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^2} - \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^2} + \frac{f^2\log(a+b\sinh(c}{b(a^2+b^2)\,d^3}$$

$$= \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^2} - \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^2} + \frac{f^2\log(a+b\sinh(c}{b(a^2+b^2)\,d^3}$$

$$= \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^2} - \frac{af(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d^2} + \frac{f^2\log(a+b\sinh(c}{b(a^2+b^2)\,d^3}$$

Mathematica [B] time = 7.10572, size = 623, normalized size = 2.04

$$\frac{2e^c f\left(\frac{a(e^{2c}-1)f\left(\text{PolyLog}\left(2,-\frac{be^{2c+dx}}{ae^c-\sqrt{e^{2c}(a^2+b^2)}}\right)-\text{PolyLog}\left(2,-\frac{be^{2c+dx}}{\sqrt{e^{2c}(a^2+b^2)}+ae^c}\right)+dx\left(\log\left(\frac{be^{2c+dx}}{ae^c-\sqrt{e^{2c}(a^2+b^2)}}+1\right)-\log\left(\frac{be^{2c+dx}}{\sqrt{e^{2c}(a^2+b^2)}+ae^c}+1\right)\right)\right)}{2d\sqrt{e^{2c}(a^2+b^2)}} - \frac{ae^{-c}(e^{2c}-1)e}{\sqrt{}}\right.}{b(e}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] (f^2*x*Coth[c])/(b*(a^2 + b^2)*d^2) + (2*E^c*f*(-(E^c*f*x) + ((-1 + E^(2*c)
)*f*x)/E^c - (a*e*(-1 + E^(2*c))*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2
]])/(Sqrt[a^2 + b^2]*E^c) + (a*(-1 + E^(2*c))*f*ArcTanh[(a + b*E^(c + d*x))
/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d*E^c) + ((-1 + E^(2*c))*f*(-2*x + (2*a
*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-a^2 - b^2]*d) + Log[2
*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))]/d))/(2*E^c) + (a*(-1 + E^(2*c))*
f*(d*x*(Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - Lo
g[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])]) + PolyLog[2,
-((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - PolyLog[2, -((b
*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/(2*d*Sqrt[(a^2 + b^
2)*E^(2*c)])))/(b*(a^2 + b^2)*d^2*(-1 + E^(2*c))) - (f^2*x*Cosh[c]*Csch[c/2
]*Sech[c/2])/(2*b*(a^2 + b^2)*d^2) - (e + f*x)^2/(2*b*d*(a + b*Sinh[c + d*x
])^2) + (Csch[c/2]*Sech[c/2]*(a*e*f*Cosh[c] + a*f^2*x*Cosh[c] + b*e*f*Sinh[
d*x] + b*f^2*x*Sinh[d*x]))/(2*b*(a^2 + b^2)*d^2*(a + b*Sinh[c + d*x]))
```

Maple [B] time = 0., size = 805, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

```
[Out] -2/b*(a^2*d*f^2*x^2*exp(2*d*x+2*c)+b^2*d*f^2*x^2*exp(2*d*x+2*c)+2*a^2*d*e*f
*x*exp(2*d*x+2*c)-a*b*f^2*x*exp(3*d*x+3*c)+2*b^2*d*e*f*x*exp(2*d*x+2*c)+a^2
*d*e^2*exp(2*d*x+2*c)-2*a^2*f^2*x*exp(2*d*x+2*c)-a*b*e*f*exp(3*d*x+3*c)+b^2
*d*e^2*exp(2*d*x+2*c)+b^2*f^2*x*exp(2*d*x+2*c)-2*a^2*e*f*exp(2*d*x+2*c)+3*a
*b*f^2*x*exp(d*x+c)+b^2*e*f*exp(2*d*x+2*c)+3*a*b*e*f*exp(d*x+c)-b^2*f^2*x-b
^2*e*f)/d^2/(a^2+b^2)/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)^2+1/b*f^2/d^3/(a^
2+b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/b*f^2/d^3/(a^2+b^2)*ln(exp(d
*x+c))-2/b*f/d^2/(a^2+b^2)^(3/2)*a*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+
b^2)^(1/2))+1/b*f^2/d^2/(a^2+b^2)^(3/2)*a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))*x+1/b*f^2/d^3/(a^2+b^2)^(3/2)*a*ln((-b*exp(d*x+c)
+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/b*f^2/d^2/(a^2+b^2)^(3/2)*a*l
n((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/b*f^2/d^3/(a^2+
b^2)^(3/2)*a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/b
*f^2/d^3/(a^2+b^2)^(3/2)*a*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2
+b^2)^(1/2)))-1/b*f^2/d^3/(a^2+b^2)^(3/2)*a*dilog((b*exp(d*x+c)+(a^2+b^2)^(
1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/b*f^2/d^3/(a^2+b^2)^(3/2)*a*c*arctanh(1/2*(2
*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.04368, size = 11416, normalized size = 37.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(2*((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*cosh(d*x + c)^4 + 2*(
(a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*sinh(d*x + c)^4 - 2*(a^2*b
^2 + b^4)*d*e*f + 2*(a^2*b^2 + b^4)*c*f^2 + 2*(3*(a^3*b + a*b^3)*d*f^2*x -
(a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^3)*c*f^2)*cosh(d*x + c)^3 + 2*(3*(a^
3*b + a*b^3)*d*f^2*x - (a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^3)*c*f^2 + 4*
((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*cosh(d*x + c))*sinh(d*x +
 c)^3 + 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + (a^4 + 2*a^2*b^2 + b^4)*d^
2*e^2 - (2*a^4 + a^2*b^2 - b^4)*d*e*f + 2*(2*a^4 + a^2*b^2 - b^4)*c*f^2 + (
2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f + (2*a^4 + a^2*b^2 - b^4)*d*f^2)*x)*cosh(
d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + (a^4 + 2*a^2*b^2 + b^
4)*d^2*e^2 - (2*a^4 + a^2*b^2 - b^4)*d*e*f + 2*(2*a^4 + a^2*b^2 - b^4)*c*f^
2 + 6*((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*cosh(d*x + c)^2 + (
2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f + (2*a^4 + a^2*b^2 - b^4)*d*f^2)*x + 3*(3
*(a^3*b + a*b^3)*d*f^2*x - (a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^3)*c*f^2)
```

```
*cosh(d*x + c))*sinh(d*x + c)^2 - (a*b^3*f^2*cosh(d*x + c)^4 + a*b^3*f^2*si
nh(d*x + c)^4 + 4*a^2*b^2*f^2*cosh(d*x + c)^3 - 4*a^2*b^2*f^2*cosh(d*x + c)
 + a*b^3*f^2 + 2*(2*a^3*b - a*b^3)*f^2*cosh(d*x + c)^2 + 4*(a*b^3*f^2*cosh(
d*x + c) + a^2*b^2*f^2)*sinh(d*x + c)^3 + 2*(3*a*b^3*f^2*cosh(d*x + c)^2 +
6*a^2*b^2*f^2*cosh(d*x + c) + (2*a^3*b - a*b^3)*f^2)*sinh(d*x + c)^2 + 4*(a
*b^3*f^2*cosh(d*x + c)^3 + 3*a^2*b^2*f^2*cosh(d*x + c)^2 - a^2*b^2*f^2 + (2
*a^3*b - a*b^3)*f^2*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dil
og((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (a*b^3*f^2*cosh(d*x + c)^4 + a*b^3*f^2
*sinh(d*x + c)^4 + 4*a^2*b^2*f^2*cosh(d*x + c)^3 - 4*a^2*b^2*f^2*cosh(d*x +
 c) + a*b^3*f^2 + 2*(2*a^3*b - a*b^3)*f^2*cosh(d*x + c)^2 + 4*(a*b^3*f^2*co
sh(d*x + c) + a^2*b^2*f^2)*sinh(d*x + c)^3 + 2*(3*a*b^3*f^2*cosh(d*x + c)^2
 + 6*a^2*b^2*f^2*cosh(d*x + c) + (2*a^3*b - a*b^3)*f^2)*sinh(d*x + c)^2 + 4
*(a*b^3*f^2*cosh(d*x + c)^3 + 3*a^2*b^2*f^2*cosh(d*x + c)^2 - a^2*b^2*f^2 +
 (2*a^3*b - a*b^3)*f^2*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*
dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (a*b^3*d*f^2*x + a*b^3*c*f^2 + (a*b
^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x + c)^4 + (a*b^3*d*f^2*x + a*b^3*c*f^2)*s
inh(d*x + c)^4 + 4*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*cosh(d*x + c)^3 + 4*(a
^2*b^2*d*f^2*x + a^2*b^2*c*f^2 + (a*b^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x + c
))*sinh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d*f^2*x + (2*a^3*b - a*b^3)*c*f^2
)*cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d*f^2*x + (2*a^3*b - a*b^3)*c*f^2
+ 3*(a*b^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x + c)^2 + 6*(a^2*b^2*d*f^2*x + a^
2*b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*(a^2*b^2*d*f^2*x + a^2*b^2*
c*f^2)*cosh(d*x + c) - 4*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2 - (a*b^3*d*f^2*x
+ a*b^3*c*f^2)*cosh(d*x + c)^3 - 3*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*cosh(d
*x + c)^2 - ((2*a^3*b - a*b^3)*d*f^2*x + (2*a^3*b - a*b^3)*c*f^2)*cosh(d*x
+ c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d
*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b)
 + (a*b^3*d*f^2*x + a*b^3*c*f^2 + (a*b^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x +
c)^4 + (a*b^3*d*f^2*x + a*b^3*c*f^2)*sinh(d*x + c)^4 + 4*(a^2*b^2*d*f^2*x +
 a^2*b^2*c*f^2)*cosh(d*x + c)^3 + 4*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2 + (a*b
^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*((2*a^3*b - a*
b^3)*d*f^2*x + (2*a^3*b - a*b^3)*c*f^2)*cosh(d*x + c)^2 + 2*((2*a^3*b - a*b
^3)*d*f^2*x + (2*a^3*b - a*b^3)*c*f^2 + 3*(a*b^3*d*f^2*x + a*b^3*c*f^2)*cos
h(d*x + c)^2 + 6*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*cosh(d*x + c))*sinh(d*x
+ c)^2 - 4*(a^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*cosh(d*x + c) - 4*(a^2*b^2*d*f
^2*x + a^2*b^2*c*f^2 - (a*b^3*d*f^2*x + a*b^3*c*f^2)*cosh(d*x + c)^3 - 3*(a
^2*b^2*d*f^2*x + a^2*b^2*c*f^2)*cosh(d*x + c)^2 - ((2*a^3*b - a*b^3)*d*f^2*
x + (2*a^3*b - a*b^3)*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*((a^3*b + a*b^3)*d*f^2*x - 3*(a^
3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^3)*c*f^2)*cosh(d*x + c) - ((a^2*b^2 + b
^4)*f^2*cosh(d*x + c)^4 + (a^2*b^2 + b^4)*f^2*sinh(d*x + c)^4 + 4*(a^3*b +
a*b^3)*f^2*cosh(d*x + c)^3 + 2*(2*a^4 + a^2*b^2 - b^4)*f^2*cosh(d*x + c)^2
- 4*(a^3*b + a*b^3)*f^2*cosh(d*x + c) + 4*((a^2*b^2 + b^4)*f^2*cosh(d*x + c
) + (a^3*b + a*b^3)*f^2)*sinh(d*x + c)^3 + (a^2*b^2 + b^4)*f^2 + 2*(3*(a^2*
b^2 + b^4)*f^2*cosh(d*x + c)^2 + 6*(a^3*b + a*b^3)*f^2*cosh(d*x + c) + (2*a
^4 + a^2*b^2 - b^4)*f^2)*sinh(d*x + c)^2 + 4*((a^2*b^2 + b^4)*f^2*cosh(d*x
+ c)^3 + 3*(a^3*b + a*b^3)*f^2*cosh(d*x + c)^2 + (2*a^4 + a^2*b^2 - b^4)*f^
2*cosh(d*x + c) - (a^3*b + a*b^3)*f^2)*sinh(d*x + c) - (a*b^3*d*e*f - a*b^3
*c*f^2 + (a*b^3*d*e*f - a*b^3*c*f^2)*cosh(d*x + c)^4 + (a*b^3*d*e*f - a*b^3
*c*f^2)*sinh(d*x + c)^4 + 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*cosh(d*x + c)^3
 + 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2 + (a*b^3*d*e*f - a*b^3*c*f^2)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^
2)*cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^2 +
 3*(a*b^3*d*e*f - a*b^3*c*f^2)*cosh(d*x + c)^2 + 6*(a^2*b^2*d*e*f - a^2*b^2
*c*f^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*
cosh(d*x + c) - 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2 - (a*b^3*d*e*f - a*b^3*c*f
```

```
^2)*cosh(d*x + c)^3 - 3*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*cosh(d*x + c)^2 - (
(2*a^3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^2)*cosh(d*x + c))*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*
b*sqrt((a^2 + b^2)/b^2) + 2*a) - ((a^2*b^2 + b^4)*f^2*cosh(d*x + c)^4 + (a^
2*b^2 + b^4)*f^2*sinh(d*x + c)^4 + 4*(a^3*b + a*b^3)*f^2*cosh(d*x + c)^3 +
2*(2*a^4 + a^2*b^2 - b^4)*f^2*cosh(d*x + c)^2 - 4*(a^3*b + a*b^3)*f^2*cosh(
d*x + c) + 4*((a^2*b^2 + b^4)*f^2*cosh(d*x + c) + (a^3*b + a*b^3)*f^2)*sinh
(d*x + c)^3 + (a^2*b^2 + b^4)*f^2 + 2*(3*(a^2*b^2 + b^4)*f^2*cosh(d*x + c)^
2 + 6*(a^3*b + a*b^3)*f^2*cosh(d*x + c) + (2*a^4 + a^2*b^2 - b^4)*f^2)*sinh
(d*x + c)^2 + 4*((a^2*b^2 + b^4)*f^2*cosh(d*x + c)^3 + 3*(a^3*b + a*b^3)*f^
2*cosh(d*x + c)^2 + (2*a^4 + a^2*b^2 - b^4)*f^2*cosh(d*x + c) - (a^3*b + a*
b^3)*f^2)*sinh(d*x + c) + (a*b^3*d*e*f - a*b^3*c*f^2 + (a*b^3*d*e*f - a*b^3
*c*f^2)*cosh(d*x + c)^4 + (a*b^3*d*e*f - a*b^3*c*f^2)*sinh(d*x + c)^4 + 4*(
a^2*b^2*d*e*f - a^2*b^2*c*f^2)*cosh(d*x + c)^3 + 4*(a^2*b^2*d*e*f - a^2*b^2
*c*f^2 + (a*b^3*d*e*f - a*b^3*c*f^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*((2
*a^3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^2)*cosh(d*x + c)^2 + 2*((2*a^
3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^2 + 3*(a*b^3*d*e*f - a*b^3*c*f^2
)*cosh(d*x + c)^2 + 6*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*cosh(d*x + c))*sinh(d
*x + c)^2 - 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*cosh(d*x + c) - 4*(a^2*b^2*d*
e*f - a^2*b^2*c*f^2 - (a*b^3*d*e*f - a*b^3*c*f^2)*cosh(d*x + c)^3 - 3*(a^2*
b^2*d*e*f - a^2*b^2*c*f^2)*cosh(d*x + c)^2 - ((2*a^3*b - a*b^3)*d*e*f - (2*
a^3*b - a*b^3)*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))*
log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a
) - 2*((a^3*b + a*b^3)*d*f^2*x - 3*(a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^3
)*c*f^2 - 4*((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*cosh(d*x + c)
^3 - 3*(3*(a^3*b + a*b^3)*d*f^2*x - (a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^
3)*c*f^2)*cosh(d*x + c)^2 - 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + (a^4 +
 2*a^2*b^2 + b^4)*d^2*e^2 - (2*a^4 + a^2*b^2 - b^4)*d*e*f + 2*(2*a^4 + a^2*
b^2 - b^4)*c*f^2 + (2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f + (2*a^4 + a^2*b^2 -
b^4)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b^3 + 2*a^2*b^5 + b^7)*d
^3*cosh(d*x + c)^4 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^3*sinh(d*x + c)^4 + 4*(a
^5*b^2 + 2*a^3*b^4 + a*b^6)*d^3*cosh(d*x + c)^3 + 2*(2*a^6*b + 3*a^4*b^3 -
b^7)*d^3*cosh(d*x + c)^2 - 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^3*cosh(d*x + c
) + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^3 + 4*((a^4*b^3 + 2*a^2*b^5 + b^7)*d^3*co
sh(d*x + c) + (a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^3)*sinh(d*x + c)^3 + 2*(3*(a^
4*b^3 + 2*a^2*b^5 + b^7)*d^3*cosh(d*x + c)^2 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b
^6)*d^3*cosh(d*x + c) + (2*a^6*b + 3*a^4*b^3 - b^7)*d^3)*sinh(d*x + c)^2 +
4*((a^4*b^3 + 2*a^2*b^5 + b^7)*d^3*cosh(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4
 + a*b^6)*d^3*cosh(d*x + c)^2 + (2*a^6*b + 3*a^4*b^3 - b^7)*d^3*cosh(d*x +
c) - (a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^3)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

3.332 $\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b\sinh(c+dx))^3} \, dx$

Optimal. Leaf size=631

$$\frac{3af^2(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}-\frac{3af^2(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}+\frac{3f^3\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4\left(a^2+b^2\right)}+\frac{3f^3\text{Poly}}{b}$$

```
[Out] (-3*f*(e + f*x)^2)/(2*b*(a^2 + b^2)*d^2) + (3*f^2*(e + f*x)*Log[1 + (b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d^3) + (3*a*f*(e + f*x)^2*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(2*b*(a^2 + b^2)^(3/2)*d^2)
+ (3*f^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2
+ b^2)*d^3) - (3*a*f*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(2*b*(a^2 + b^2)^(3/2)*d^2) + (3*f^3*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^4) + (3*a*f^2*(e + f*x)*PolyLog[2, -
((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) + (3*f^
3*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^4)
 - (3*a*f^2*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])
/(b*(a^2 + b^2)^(3/2)*d^3) - (3*a*f^3*PolyLog[3, -((b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^4) + (3*a*f^3*PolyLog[3, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^4) - (e + f*x)^3/(2
*b*d*(a + b*Sinh[c + d*x])^2) - (3*f*(e + f*x)^2*Cosh[c + d*x])/(2*(a^2 + b
^2)*d^2*(a + b*Sinh[c + d*x]))
```

Rubi [A] time = 1.08746, antiderivative size = 631, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5464, 3324, 3322, 2264, 2190, 2531, 2282, 6589, 5561, 2279, 2391}

$$\frac{3af^2(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}-\frac{3af^2(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\left(a^2+b^2\right)^{3/2}}+\frac{3f^3\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4\left(a^2+b^2\right)}+\frac{3f^3\text{Poly}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] (-3*f*(e + f*x)^2)/(2*b*(a^2 + b^2)*d^2) + (3*f^2*(e + f*x)*Log[1 + (b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d^3) + (3*a*f*(e + f*x)^2*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(2*b*(a^2 + b^2)^(3/2)*d^2)
+ (3*f^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2
+ b^2)*d^3) - (3*a*f*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(2*b*(a^2 + b^2)^(3/2)*d^2) + (3*f^3*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^4) + (3*a*f^2*(e + f*x)*PolyLog[2, -
((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) + (3*f^
3*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^4)
 - (3*a*f^2*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])
/(b*(a^2 + b^2)^(3/2)*d^3) - (3*a*f^3*PolyLog[3, -((b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^4) + (3*a*f^3*PolyLog[3, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^4) - (e + f*x)^3/(2
*b*d*(a + b*Sinh[c + d*x])^2) - (3*f*(e + f*x)^2*Cosh[c + d*x])/(2*(a^2 + b
^2)*d^2*(a + b*Sinh[c + d*x]))
```

Rule 5464

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sinh[c +
```

```
d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx = -\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2} + \frac{(3f)\int \frac{(e+fx)^2}{(a+b\sinh(c+dx))^2}\,dx}{2bd}$$

$$= -\frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2} - \frac{3f(e+fx)^2\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))} + \frac{(3af)\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}\,dx}{2b\left(a^2+b^2\right)d}$$

$$= -\frac{3f(e+fx)^2}{2b\left(a^2+b^2\right)d^2} - \frac{(e+fx)^3}{2bd(a+b\sinh(c+dx))^2} - \frac{3f(e+fx)^2\cosh(c+dx)}{2\left(a^2+b^2\right)d^2(a+b\sinh(c+dx))} + \frac{(3af)}{}$$

$$= -\frac{3f(e+fx)^2}{2b\left(a^2+b^2\right)d^2} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3} - \frac{}{2b}$$

$$= -\frac{3f(e+fx)^2}{2b\left(a^2+b^2\right)d^2} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3} + \frac{3af(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2b\left(a^2+b^2\right)^{3/2}d^2} + \cdots$$

$$= -\frac{3f(e+fx)^2}{2b\left(a^2+b^2\right)d^2} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3} + \frac{3af(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2b\left(a^2+b^2\right)^{3/2}d^2} + \cdots$$

$$= -\frac{3f(e+fx)^2}{2b\left(a^2+b^2\right)d^2} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3} + \frac{3af(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2b\left(a^2+b^2\right)^{3/2}d^2} + \cdots$$

$$= -\frac{3f(e+fx)^2}{2b\left(a^2+b^2\right)d^2} + \frac{3f^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)d^3} + \frac{3af(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2b\left(a^2+b^2\right)^{3/2}d^2} + \cdots$$

Mathematica [B] time = 7.56222, size = 5753, normalized size = 9.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \cosh(dx+c)}{(a+b\sinh(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 4.70474, size = 24868, normalized size = 39.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*(a^2*b^2 + b^4)*d^2*e^2*f - 12*(a^2*b^2 + b^4)*c*d*e*f^2 + 6*(a^2*b^
2 + b^4)*c^2*f^3 - 6*((a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^2*b^2 + b^4)*d^2*e
*f^2*x + 2*(a^2*b^2 + b^4)*c*d*e*f^2 - (a^2*b^2 + b^4)*c^2*f^3)*cosh(d*x +
c)^4 - 6*((a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^2*b^2 + b^4)*d^2*e*f^2*x + 2*(
a^2*b^2 + b^4)*c*d*e*f^2 - (a^2*b^2 + b^4)*c^2*f^3)*sinh(d*x + c)^4 - 6*(3*
(a^3*b + a*b^3)*d^2*f^3*x^2 + 6*(a^3*b + a*b^3)*d^2*e*f^2*x - (a^3*b + a*b^
3)*d^2*e^2*f + 8*(a^3*b + a*b^3)*c*d*e*f^2 - 4*(a^3*b + a*b^3)*c^2*f^3)*cos
h(d*x + c)^3 - 6*(3*(a^3*b + a*b^3)*d^2*f^3*x^2 + 6*(a^3*b + a*b^3)*d^2*e*f
^2*x - (a^3*b + a*b^3)*d^2*e^2*f + 8*(a^3*b + a*b^3)*c*d*e*f^2 - 4*(a^3*b +
 a*b^3)*c^2*f^3 + 4*((a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^2*b^2 + b^4)*d^2*e*
f^2*x + 2*(a^2*b^2 + b^4)*c*d*e*f^2 - (a^2*b^2 + b^4)*c^2*f^3)*cosh(d*x + c
))*sinh(d*x + c)^3 - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 2*(a^4 + 2*
a^2*b^2 + b^4)*d^3*e^3 - 3*(2*a^4 + a^2*b^2 - b^4)*d^2*e^2*f + 12*(2*a^4 +
a^2*b^2 - b^4)*c*d*e*f^2 - 6*(2*a^4 + a^2*b^2 - b^4)*c^2*f^3 + 3*(2*(a^4 +
2*a^2*b^2 + b^4)*d^3*e*f^2 + (2*a^4 + a^2*b^2 - b^4)*d^2*f^3)*x^2 + 6*((a^4
 + 2*a^2*b^2 + b^4)*d^3*e^2*f + (2*a^4 + a^2*b^2 - b^4)*d^2*e*f^2)*x)*cosh(
d*x + c)^2 - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 2*(a^4 + 2*a^2*b^2
+ b^4)*d^3*e^3 - 3*(2*a^4 + a^2*b^2 - b^4)*d^2*e^2*f + 12*(2*a^4 + a^2*b^2
- b^4)*c*d*e*f^2 - 6*(2*a^4 + a^2*b^2 - b^4)*c^2*f^3 + 3*(2*(a^4 + 2*a^2*b^
2 + b^4)*d^3*e*f^2 + (2*a^4 + a^2*b^2 - b^4)*d^2*f^3)*x^2 + 18*((a^2*b^2 +
```

b^4)*d^2*f^3*x^2 + 2*(a^2*b^2 + b^4)*d^2*e*f^2*x + 2*(a^2*b^2 + b^4)*c*d*e*
f^2 - (a^2*b^2 + b^4)*c^2*f^3)*cosh(d*x + c)^2 + 6*((a^4 + 2*a^2*b^2 + b^4)
*d^3*e^2*f + (2*a^4 + a^2*b^2 - b^4)*d^2*e*f^2)*x + 9*(3*(a^3*b + a*b^3)*d^
2*f^3*x^2 + 6*(a^3*b + a*b^3)*d^2*e*f^2*x - (a^3*b + a*b^3)*d^2*e^2*f + 8*(
a^3*b + a*b^3)*c*d*e*f^2 - 4*(a^3*b + a*b^3)*c^2*f^3)*cosh(d*x + c))*sinh(d
x + c)^2 - 6(a*b^3*f^3*cosh(d*x + c)^4 + a*b^3*f^3*sinh(d*x + c)^4 + 4*a^
2*b^2*f^3*cosh(d*x + c)^3 - 4*a^2*b^2*f^3*cosh(d*x + c) + a*b^3*f^3 + 2*(2*
a^3*b - a*b^3)*f^3*cosh(d*x + c)^2 + 4*(a*b^3*f^3*cosh(d*x + c) + a^2*b^2*f
^3)*sinh(d*x + c)^3 + 2*(3*a*b^3*f^3*cosh(d*x + c)^2 + 6*a^2*b^2*f^3*cosh(d
*x + c) + (2*a^3*b - a*b^3)*f^3)*sinh(d*x + c)^2 + 4*(a*b^3*f^3*cosh(d*x +
c)^3 + 3*a^2*b^2*f^3*cosh(d*x + c)^2 - a^2*b^2*f^3 + (2*a^3*b - a*b^3)*f^3*
cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x
+ c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2))/b) + 6*(a*b^3*f^3*cosh(d*x + c)^4 + a*b^3*f^3*sinh(d*x + c)^4 + 4*
a^2*b^2*f^3*cosh(d*x + c)^3 - 4*a^2*b^2*f^3*cosh(d*x + c) + a*b^3*f^3 + 2*(
2*a^3*b - a*b^3)*f^3*cosh(d*x + c)^2 + 4*(a*b^3*f^3*cosh(d*x + c) + a^2*b^2
*f^3)*sinh(d*x + c)^3 + 2*(3*a*b^3*f^3*cosh(d*x + c)^2 + 6*a^2*b^2*f^3*cosh
(d*x + c) + (2*a^3*b - a*b^3)*f^3)*sinh(d*x + c)^2 + 4*(a*b^3*f^3*cosh(d*x
+ c)^3 + 3*a^2*b^2*f^3*cosh(d*x + c)^2 - a^2*b^2*f^3 + (2*a^3*b - a*b^3)*f^
3*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*
x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2))/b) + 6*((a^3*b + a*b^3)*d^2*f^3*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f^
2*x - 3*(a^3*b + a*b^3)*d^2*e^2*f + 8*(a^3*b + a*b^3)*c*d*e*f^2 - 4*(a^3*b
+ a*b^3)*c^2*f^3)*cosh(d*x + c) + 6*((a^2*b^2 + b^4)*f^3*cosh(d*x + c)^4 +
(a^2*b^2 + b^4)*f^3*sinh(d*x + c)^4 + 4*(a^3*b + a*b^3)*f^3*cosh(d*x + c)^3
 + 2*(2*a^4 + a^2*b^2 - b^4)*f^3*cosh(d*x + c)^2 - 4*(a^3*b + a*b^3)*f^3*co
sh(d*x + c) + (a^2*b^2 + b^4)*f^3 + 4*((a^2*b^2 + b^4)*f^3*cosh(d*x + c) +
(a^3*b + a*b^3)*f^3)*sinh(d*x + c)^3 + 2*(3*(a^2*b^2 + b^4)*f^3*cosh(d*x +
c)^2 + 6*(a^3*b + a*b^3)*f^3*cosh(d*x + c) + (2*a^4 + a^2*b^2 - b^4)*f^3)*s
inh(d*x + c)^2 + 4*((a^2*b^2 + b^4)*f^3*cosh(d*x + c)^3 + 3*(a^3*b + a*b^3)
*f^3*cosh(d*x + c)^2 + (2*a^4 + a^2*b^2 - b^4)*f^3*cosh(d*x + c) - (a^3*b +
 a*b^3)*f^3)*sinh(d*x + c) + (a*b^3*d*f^3*x + a*b^3*d*e*f^2 + (a*b^3*d*f^3*
x + a*b^3*d*e*f^2)*cosh(d*x + c)^4 + (a*b^3*d*f^3*x + a*b^3*d*e*f^2)*sinh(d
x + c)^4 + 4(a^2*b^2*d*f^3*x + a^2*b^2*d*e*f^2)*cosh(d*x + c)^3 + 4*(a^2*
b^2*d*f^3*x + a^2*b^2*d*e*f^2 + (a*b^3*d*f^3*x + a*b^3*d*e*f^2)*cosh(d*x +
c))*sinh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d*f^3*x + (2*a^3*b - a*b^3)*d*e*
f^2)*cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d*f^3*x + (2*a^3*b - a*b^3)*d*e
f^2 + 3(a*b^3*d*f^3*x + a*b^3*d*e*f^2)*cosh(d*x + c)^2 + 6*(a^2*b^2*d*f^3
*x + a^2*b^2*d*e*f^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*(a^2*b^2*d*f^3*x +
 a^2*b^2*d*e*f^2)*cosh(d*x + c) - 4*(a^2*b^2*d*f^3*x + a^2*b^2*d*e*f^2 - (a
*b^3*d*f^3*x + a*b^3*d*e*f^2)*cosh(d*x + c)^3 - 3*(a^2*b^2*d*f^3*x + a^2*b^
2*d*e*f^2)*cosh(d*x + c)^2 - ((2*a^3*b - a*b^3)*d*f^3*x + (2*a^3*b - a*b^3)
*d*e*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))*dilog((a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b + 1) + 6*((a^2*b^2 + b^4)*f^3*cosh(d*x + c)^4 + (a^2*b
^2 + b^4)*f^3*sinh(d*x + c)^4 + 4*(a^3*b + a*b^3)*f^3*cosh(d*x + c)^3 + 2*(
2*a^4 + a^2*b^2 - b^4)*f^3*cosh(d*x + c)^2 - 4*(a^3*b + a*b^3)*f^3*cosh(d*x
 + c) + (a^2*b^2 + b^4)*f^3 + 4*((a^2*b^2 + b^4)*f^3*cosh(d*x + c) + (a^3*b
 + a*b^3)*f^3)*sinh(d*x + c)^3 + 2*(3*(a^2*b^2 + b^4)*f^3*cosh(d*x + c)^2 +
 6*(a^3*b + a*b^3)*f^3*cosh(d*x + c) + (2*a^4 + a^2*b^2 - b^4)*f^3)*sinh(d*
x + c)^2 + 4*((a^2*b^2 + b^4)*f^3*cosh(d*x + c)^3 + 3*(a^3*b + a*b^3)*f^3*c
osh(d*x + c)^2 + (2*a^4 + a^2*b^2 - b^4)*f^3*cosh(d*x + c) - (a^3*b + a*b^3
)*f^3)*sinh(d*x + c) - (a*b^3*d*f^3*x + a*b^3*d*e*f^2 + (a*b^3*d*f^3*x + a*
b^3*d*e*f^2)*cosh(d*x + c)^4 + (a*b^3*d*f^3*x + a*b^3*d*e*f^2)*sinh(d*x + c
)^4 + 4*(a^2*b^2*d*f^3*x + a^2*b^2*d*e*f^2)*cosh(d*x + c)^3 + 4*(a^2*b^2*d*
f^3*x + a^2*b^2*d*e*f^2 + (a*b^3*d*f^3*x + a*b^3*d*e*f^2)*cosh(d*x + c))*si
nh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d*f^3*x + (2*a^3*b - a*b^3)*d*e*f^2)*c
osh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d*f^3*x + (2*a^3*b - a*b^3)*d*e*f^2 +
 3*(a*b^3*d*f^3*x + a*b^3*d*e*f^2)*cosh(d*x + c)^2 + 6*(a^2*b^2*d*f^3*x + a

^2*b^2*d*e*f^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*(a^2*b^2*d*f^3*x + a^2*b
^2*d*e*f^2)*cosh(d*x + c) - 4*(a^2*b^2*d*f^3*x + a^2*b^2*d*e*f^2 - (a*b^3*d
*f^3*x + a*b^3*d*e*f^2)*cosh(d*x + c)^3 - 3*(a^2*b^2*d*f^3*x + a^2*b^2*d*e*
f^2)*cosh(d*x + c)^2 - ((2*a^3*b - a*b^3)*d*f^3*x + (2*a^3*b - a*b^3)*d*e*f
^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))*dilog((a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2) - b)/b + 1) + 3*(2*(a^2*b^2 + b^4)*d*e*f^2 - 2*(a^2*b^2 + b^4)*c*f^
3 + 2*((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3)*cosh(d*x + c)^4 + 2
*((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3)*sinh(d*x + c)^4 + 8*((a^
3*b + a*b^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c)^3 + 8*((a^3*b +
a*b^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3 + ((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b
^2 + b^4)*c*f^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*((2*a^4 + a^2*b^2 - b^4
)*d*e*f^2 - (2*a^4 + a^2*b^2 - b^4)*c*f^3)*cosh(d*x + c)^2 + 4*((2*a^4 + a^
2*b^2 - b^4)*d*e*f^2 - (2*a^4 + a^2*b^2 - b^4)*c*f^3 + 3*((a^2*b^2 + b^4)*d
*e*f^2 - (a^2*b^2 + b^4)*c*f^3)*cosh(d*x + c)^2 + 6*((a^3*b + a*b^3)*d*e*f^
2 - (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 8*((a^3*b + a*b
^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c) - 8*((a^3*b + a*b^3)*d*e
*f^2 - (a^3*b + a*b^3)*c*f^3 - ((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c
*f^3)*cosh(d*x + c)^3 - 3*((a^3*b + a*b^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3)
*cosh(d*x + c)^2 - ((2*a^4 + a^2*b^2 - b^4)*d*e*f^2 - (2*a^4 + a^2*b^2 - b^
4)*c*f^3)*cosh(d*x + c))*sinh(d*x + c) - (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f
^2 + a*b^3*c^2*f^3 + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*
cosh(d*x + c)^4 + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*sin
h(d*x + c)^4 + 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3
)*cosh(d*x + c)^3 + 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^
2*f^3 + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cosh(d*x + c)
)*sinh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*
d*e*f^2 + (2*a^3*b - a*b^3)*c^2*f^3)*cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)
*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*d*e*f^2 + (2*a^3*b - a*b^3)*c^2*f^3 + 3*
(a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cosh(d*x + c)^2 + 6*(
a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*cosh(d*x + c))*s
inh(d*x + c)^2 - 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f
^3)*cosh(d*x + c) - 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^
2*f^3 - (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cosh(d*x + c)
^3 - 3*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*cosh(d*x
+ c)^2 - ((2*a^3*b - a*b^3)*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*d*e*f^2 + (2
*a^3*b - a*b^3)*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) +
2*a) + 3*(2*(a^2*b^2 + b^4)*d*e*f^2 - 2*(a^2*b^2 + b^4)*c*f^3 + 2*((a^2*b^2
+ b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3)*cosh(d*x + c)^4 + 2*((a^2*b^2 + b^
4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3)*sinh(d*x + c)^4 + 8*((a^3*b + a*b^3)*d*
e*f^2 - (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c)^3 + 8*((a^3*b + a*b^3)*d*e*f^2
- (a^3*b + a*b^3)*c*f^3 + ((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3
)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*((2*a^4 + a^2*b^2 - b^4)*d*e*f^2 - (2*
a^4 + a^2*b^2 - b^4)*c*f^3)*cosh(d*x + c)^2 + 4*((2*a^4 + a^2*b^2 - b^4)*d*
e*f^2 - (2*a^4 + a^2*b^2 - b^4)*c*f^3 + 3*((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b
^2 + b^4)*c*f^3)*cosh(d*x + c)^2 + 6*((a^3*b + a*b^3)*d*e*f^2 - (a^3*b + a*
b^3)*c*f^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 8*((a^3*b + a*b^3)*d*e*f^2 - (
a^3*b + a*b^3)*c*f^3)*cosh(d*x + c) - 8*((a^3*b + a*b^3)*d*e*f^2 - (a^3*b +
a*b^3)*c*f^3 - ((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3)*cosh(d*x
+ c)^3 - 3*((a^3*b + a*b^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c)^
2 - ((2*a^4 + a^2*b^2 - b^4)*d*e*f^2 - (2*a^4 + a^2*b^2 - b^4)*c*f^3)*cosh(
d*x + c))*sinh(d*x + c) + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*
f^3 + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cosh(d*x + c)^4
+ (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*sinh(d*x + c)^4 +
4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*cosh(d*x + c)
^3 + 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3 + (a*b^3*
d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c)
^3 + 2*((2*a^3*b - a*b^3)*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*d*e*f^2 + (2*a^

```
3*b - a*b^3)*c^2*f^3)*cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d^2*e^2*f - 2*
(2*a^3*b - a*b^3)*c*d*e*f^2 + (2*a^3*b - a*b^3)*c^2*f^3 + 3*(a*b^3*d^2*e^2*
f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cosh(d*x + c)^2 + 6*(a^2*b^2*d^2*e^2
*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c)^2
 - 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*cosh(d*x +
c) - 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3 - (a*b^3*
d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cosh(d*x + c)^3 - 3*(a^2*b^2
*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*cosh(d*x + c)^2 - ((2*a
^3*b - a*b^3)*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*d*e*f^2 + (2*a^3*b - a*b^3)
*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))*log(2*b*cosh
(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 3*(2*(a^
2*b^2 + b^4)*d*f^3*x + 2*(a^2*b^2 + b^4)*c*f^3 + 2*((a^2*b^2 + b^4)*d*f^3*x
 + (a^2*b^2 + b^4)*c*f^3)*cosh(d*x + c)^4 + 2*((a^2*b^2 + b^4)*d*f^3*x + (a
^2*b^2 + b^4)*c*f^3)*sinh(d*x + c)^4 + 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b
+ a*b^3)*c*f^3)*cosh(d*x + c)^3 + 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b
^3)*c*f^3 + ((a^2*b^2 + b^4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3)*cosh(d*x + c)
)*sinh(d*x + c)^3 + 4*((2*a^4 + a^2*b^2 - b^4)*d*f^3*x + (2*a^4 + a^2*b^2 -
 b^4)*c*f^3)*cosh(d*x + c)^2 + 4*((2*a^4 + a^2*b^2 - b^4)*d*f^3*x + (2*a^4
+ a^2*b^2 - b^4)*c*f^3 + 3*((a^2*b^2 + b^4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3
)*cosh(d*x + c)^2 + 6*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3)*cos
h(d*x + c))*sinh(d*x + c)^2 - 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*
c*f^3)*cosh(d*x + c) - 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3 -
 ((a^2*b^2 + b^4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3)*cosh(d*x + c)^3 - 3*((a^
3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c)^2 - ((2*a^4 + a
^2*b^2 - b^4)*d*f^3*x + (2*a^4 + a^2*b^2 - b^4)*c*f^3)*cosh(d*x + c))*sinh(
d*x + c) + (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a
*b^3*c^2*f^3 + (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2
 - a*b^3*c^2*f^3)*cosh(d*x + c)^4 + (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*
x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^3)*sinh(d*x + c)^4 + 4*(a^2*b^2*d^2*f^3
*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*cosh(
d*x + c)^3 + 4*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d
*e*f^2 - a^2*b^2*c^2*f^3 + (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b
^3*c*d*e*f^2 - a*b^3*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*((2*a^3*b
 - a*b^3)*d^2*f^3*x^2 + 2*(2*a^3*b - a*b^3)*d^2*e*f^2*x + 2*(2*a^3*b - a*b^3
)*c*d*e*f^2 - (2*a^3*b - a*b^3)*c^2*f^3)*cosh(d*x + c)^2 + 2*((2*a^3*b - a*
b^3)*d^2*f^3*x^2 + 2*(2*a^3*b - a*b^3)*d^2*e*f^2*x + 2*(2*a^3*b - a*b^3)*c*
d*e*f^2 - (2*a^3*b - a*b^3)*c^2*f^3 + 3*(a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*
f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^3)*cosh(d*x + c)^2 + 6*(a^2*b^2*d^2
*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*c
osh(d*x + c))*sinh(d*x + c)^2 - 4*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^
2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*cosh(d*x + c) - 4*(a^2*b^2*d^2
*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3 -
(a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^
3)*cosh(d*x + c)^3 - 3*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2
*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*cosh(d*x + c)^2 - ((2*a^3*b - a*b^3)*d^2*
f^3*x^2 + 2*(2*a^3*b - a*b^3)*d^2*e*f^2*x + 2*(2*a^3*b - a*b^3)*c*d*e*f^2 -
 (2*a^3*b - a*b^3)*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 3*(2*(a^2*b^2 + b^4)*d*f^3*x + 2*(
a^2*b^2 + b^4)*c*f^3 + 2*((a^2*b^2 + b^4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3)*
cosh(d*x + c)^4 + 2*((a^2*b^2 + b^4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3)*sinh(
d*x + c)^4 + 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3)*cosh(d*x +
 c)^3 + 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3 + ((a^2*b^2 + b^
4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*((2*
a^4 + a^2*b^2 - b^4)*d*f^3*x + (2*a^4 + a^2*b^2 - b^4)*c*f^3)*cosh(d*x + c)
^2 + 4*((2*a^4 + a^2*b^2 - b^4)*d*f^3*x + (2*a^4 + a^2*b^2 - b^4)*c*f^3 + 3
*((a^2*b^2 + b^4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3)*cosh(d*x + c)^2 + 6*((a^
3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c))*sinh(d*x + c)^
2 - 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c) - 8*(
```

```
(a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3 - ((a^2*b^2 + b^4)*d*f^3*x
+ (a^2*b^2 + b^4)*c*f^3)*cosh(d*x + c)^3 - 3*((a^3*b + a*b^3)*d*f^3*x + (a^
3*b + a*b^3)*c*f^3)*cosh(d*x + c)^2 - ((2*a^4 + a^2*b^2 - b^4)*d*f^3*x + (2
*a^4 + a^2*b^2 - b^4)*c*f^3)*cosh(d*x + c))*sinh(d*x + c) - (a*b^3*d^2*f^3*
x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^3 + (a*b^3*d^2*
f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^3)*cosh(d*x
 + c)^4 + (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a*
b^3*c^2*f^3)*sinh(d*x + c)^4 + 4*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2
*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*cosh(d*x + c)^3 + 4*(a^2*b^2*d^
2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3 +
 (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f
^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d^2*f^3*x^2 + 2*(
2*a^3*b - a*b^3)*d^2*e*f^2*x + 2*(2*a^3*b - a*b^3)*c*d*e*f^2 - (2*a^3*b - a
*b^3)*c^2*f^3)*cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d^2*f^3*x^2 + 2*(2*a^
3*b - a*b^3)*d^2*e*f^2*x + 2*(2*a^3*b - a*b^3)*c*d*e*f^2 - (2*a^3*b - a*b^3
)*c^2*f^3 + 3*(a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2
- a*b^3*c^2*f^3)*cosh(d*x + c)^2 + 6*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e
*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c
)^2 - 4*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2
- a^2*b^2*c^2*f^3)*cosh(d*x + c) - 4*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e
*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3 - (a*b^3*d^2*f^3*x^2 + 2*a*b
^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^3)*cosh(d*x + c)^3 - 3*(a^
2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c
^2*f^3)*cosh(d*x + c)^2 - ((2*a^3*b - a*b^3)*d^2*f^3*x^2 + 2*(2*a^3*b - a*b
^3)*d^2*e*f^2*x + 2*(2*a^3*b - a*b^3)*c*d*e*f^2 - (2*a^3*b - a*b^3)*c^2*f^3
)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))*log(-(a*cosh(d*x + c
) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2) - b)/b) + 2*(3*(a^3*b + a*b^3)*d^2*f^3*x^2 + 6*(a^3*b + a*b^3)*d^2*e*f
^2*x - 9*(a^3*b + a*b^3)*d^2*e^2*f + 24*(a^3*b + a*b^3)*c*d*e*f^2 - 12*(a^3
*b + a*b^3)*c^2*f^3 - 12*((a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^2*b^2 + b^4)*d
^2*e*f^2*x + 2*(a^2*b^2 + b^4)*c*d*e*f^2 - (a^2*b^2 + b^4)*c^2*f^3)*cosh(d*
x + c)^3 - 9*(3*(a^3*b + a*b^3)*d^2*f^3*x^2 + 6*(a^3*b + a*b^3)*d^2*e*f^2*x
 - (a^3*b + a*b^3)*d^2*e^2*f + 8*(a^3*b + a*b^3)*c*d*e*f^2 - 4*(a^3*b + a*b
^3)*c^2*f^3)*cosh(d*x + c)^2 - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 2
*(a^4 + 2*a^2*b^2 + b^4)*d^3*e^3 - 3*(2*a^4 + a^2*b^2 - b^4)*d^2*e^2*f + 12
*(2*a^4 + a^2*b^2 - b^4)*c*d*e*f^2 - 6*(2*a^4 + a^2*b^2 - b^4)*c^2*f^3 + 3*
(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*e*f^2 + (2*a^4 + a^2*b^2 - b^4)*d^2*f^3)*x^2
 + 6*((a^4 + 2*a^2*b^2 + b^4)*d^3*e^2*f + (2*a^4 + a^2*b^2 - b^4)*d^2*e*f^2
)*x)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b^3 + 2*a^2*b^5 + b^7)*d^4*cosh(d*
x + c)^4 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^4*sinh(d*x + c)^4 + 4*(a^5*b^2 + 2
*a^3*b^4 + a*b^6)*d^4*cosh(d*x + c)^3 + 2*(2*a^6*b + 3*a^4*b^3 - b^7)*d^4*c
osh(d*x + c)^2 - 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^4*cosh(d*x + c) + (a^4*b
^3 + 2*a^2*b^5 + b^7)*d^4 + 4*((a^4*b^3 + 2*a^2*b^5 + b^7)*d^4*cosh(d*x + c
) + (a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^4)*sinh(d*x + c)^3 + 2*(3*(a^4*b^3 + 2*
a^2*b^5 + b^7)*d^4*cosh(d*x + c)^2 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^4*co
sh(d*x + c) + (2*a^6*b + 3*a^4*b^3 - b^7)*d^4)*sinh(d*x + c)^2 + 4*((a^4*b^
3 + 2*a^2*b^5 + b^7)*d^4*cosh(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*
d^4*cosh(d*x + c)^2 + (2*a^6*b + 3*a^4*b^3 - b^7)*d^4*cosh(d*x + c) - (a^5*
b^2 + 2*a^3*b^4 + a*b^6)*d^4)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)/(a+b*sinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \cosh(dx+c)}{(b\sinh(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)/(b*sinh(d*x + c) + a)^3, x)
```

3.333 $\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=448

$$\frac{6af^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3}+\frac{6af^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^3}-\frac{3af(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}$$

```
[Out] (a*(e + f*x)^4)/(4*b^2*f) - (6*f^3*Cosh[c + d*x])/(b*d^4) - (3*f*(e + f*x)^
2*Cosh[c + d*x])/(b*d^2) - (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2])])/(b^2*d) - (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/(b^2*d) - (3*a*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(b^2*d^2) - (3*a*f*(e + f*x)^2*PolyLog[2, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^2) + (6*a*f^2*(e + f*x)*PolyLog[3, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^3) + (6*a*f^2*(e + f*x)*Pol
yLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^3) - (6*a*f^3*Pol
yLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^4) - (6*a*f^3*Pol
yLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^4) + (6*f^2*(e +
f*x)*Sinh[c + d*x])/(b*d^3) + ((e + f*x)^3*Sinh[c + d*x])/(b*d)
```

Rubi [A] time = 0.646703, antiderivative size = 448, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {5579, 3296, 2638, 5561, 2190, 2531, 6609, 2282, 6589}

$$\frac{6af^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3}+\frac{6af^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^3}-\frac{3af(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (a*(e + f*x)^4)/(4*b^2*f) - (6*f^3*Cosh[c + d*x])/(b*d^4) - (3*f*(e + f*x)^
2*Cosh[c + d*x])/(b*d^2) - (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2])])/(b^2*d) - (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/(b^2*d) - (3*a*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(b^2*d^2) - (3*a*f*(e + f*x)^2*PolyLog[2, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^2) + (6*a*f^2*(e + f*x)*PolyLog[3, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^3) + (6*a*f^2*(e + f*x)*Pol
yLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^3) - (6*a*f^3*Pol
yLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^4) - (6*a*f^3*Pol
yLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^4) + (6*f^2*(e +
f*x)*Sinh[c + d*x])/(b*d^3) + ((e + f*x)^3*Sinh[c + d*x])/(b*d)
```


Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\cosh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{(e+fx)^3\sinh(c+dx)}{bd} - \frac{a\int \frac{e^{c+dx}(e+fx)^3}{a-\sqrt{a^2+b^2}+be^{c+dx}}\,dx}{b} - \frac{a\int \frac{e^{c+dx}}{a+\sqrt{a^2}}}{}$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{3f(e+fx)^2\cosh(c+dx)}{bd^2} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d} -$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{3f(e+fx)^2\cosh(c+dx)}{bd^2} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d} -$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{6f^3\cosh(c+dx)}{bd^4} - \frac{3f(e+fx)^2\cosh(c+dx)}{bd^2} - \frac{a(e+fx)^3}{}$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{6f^3\cosh(c+dx)}{bd^4} - \frac{3f(e+fx)^2\cosh(c+dx)}{bd^2} - \frac{a(e+fx)^3}{}$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{6f^3\cosh(c+dx)}{bd^4} - \frac{3f(e+fx)^2\cosh(c+dx)}{bd^2} - \frac{a(e+fx)^3}{}$$

Mathematica [B] time = 20.9593, size = 10378, normalized size = 23.17

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.198, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3\cosh(dx+c)\sinh(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}e^3\left(\frac{2(dx+c)a}{b^2d} - \frac{e^{(dx+c)}}{bd} + \frac{e^{(-dx-c)}}{bd} + \frac{2a\log\left(-2ae^{(-dx-c)}+be^{(-2dx-2c)}-b\right)}{b^2d}\right) - \frac{\left(ad^4f^3x^4e^c+4ad^4ef^2x^3e^c+6ad^4e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/2*e^3*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) +
2*a*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^2*d)) - 1/4*(a*d^4*f
^3*x^4*e^c + 4*a*d^4*e*f^2*x^3*e^c + 6*a*d^4*e^2*f*x^2*e^c - 2*(b*d^3*f^3*x
^3*e^(2*c) + 3*(d^3*e*f^2 - d^2*f^3)*b*x^2*e^(2*c) + 3*(d^3*e^2*f - 2*d^2*e
*f^2 + 2*d*f^3)*b*x*e^(2*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*b*e^(2*c))*
e^(d*x) + 2*(b*d^3*f^3*x^3 + 3*(d^3*e*f^2 + d^2*f^3)*b*x^2 + 3*(d^3*e^2*f +
 2*d^2*e*f^2 + 2*d*f^3)*b*x + 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*b)*e^(-d*x)
)*e^(-c)/(b^2*d^4) + integrate(-2*(a*b*f^3*x^3 + 3*a*b*e*f^2*x^2 + 3*a*b*e^
2*f*x - (a^2*f^3*x^3*e^c + 3*a^2*e*f^2*x^2*e^c + 3*a^2*e^2*f*x*e^c)*e^(d*x)
)/(b^3*e^(2*d*x + 2*c) + 2*a*b^2*e^(d*x + c) - b^3), x)
```

Fricas [C] time = 2.77144, size = 4582, normalized size = 10.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -1/4*(2*b*d^3*f^3*x^3 + 2*b*d^3*e^3 + 6*b*d^2*e^2*f + 12*b*d*e*f^2 + 12*b*f
^3 + 6*(b*d^3*e*f^2 + b*d^2*f^3)*x^2 - 2*(b*d^3*f^3*x^3 + b*d^3*e^3 - 3*b*d
^2*e^2*f + 6*b*d*e*f^2 - 6*b*f^3 + 3*(b*d^3*e*f^2 - b*d^2*f^3)*x^2 + 3*(b*d
^3*e^2*f - 2*b*d^2*e*f^2 + 2*b*d*f^3)*x)*cosh(d*x + c)^2 - 2*(b*d^3*f^3*x^3
 + b*d^3*e^3 - 3*b*d^2*e^2*f + 6*b*d*e*f^2 - 6*b*f^3 + 3*(b*d^3*e*f^2 - b*d
^2*f^3)*x^2 + 3*(b*d^3*e^2*f - 2*b*d^2*e*f^2 + 2*b*d*f^3)*x)*sinh(d*x + c)^
2 + 6*(b*d^3*e^2*f + 2*b*d^2*e*f^2 + 2*b*d*f^3)*x - (a*d^4*f^3*x^4 + 4*a*d^
4*e*f^2*x^3 + 6*a*d^4*e^2*f*x^2 + 4*a*d^4*e^3*x + 8*a*c*d^3*e^3 - 12*a*c^2*
d^2*e^2*f + 8*a*c^3*d*e*f^2 - 2*a*c^4*f^3)*cosh(d*x + c) + 12*((a*d^2*f^3*x
^2 + 2*a*d^2*e*f^2*x + a*d^2*e^2*f)*cosh(d*x + c) + (a*d^2*f^3*x^2 + 2*a*d^
2*e*f^2*x + a*d^2*e^2*f)*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x
 + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b +
1) + 12*((a*d^2*f^3*x^2 + 2*a*d^2*e*f^2*x + a*d^2*e^2*f)*cosh(d*x + c) + (a
*d^2*f^3*x^2 + 2*a*d^2*e*f^2*x + a*d^2*e^2*f)*sinh(d*x + c))*dilog((a*cosh(
d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2) - b)/b + 1) + 4*((a*d^3*e^3 - 3*a*c*d^2*e^2*f + 3*a*c^2*d*e*f^2
 - a*c^3*f^3)*cosh(d*x + c) + (a*d^3*e^3 - 3*a*c*d^2*e^2*f + 3*a*c^2*d*e*f^
2 - a*c^3*f^3)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2
*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 4*((a*d^3*e^3 - 3*a*c*d^2*e^2*f + 3*a*c^2
*d*e*f^2 - a*c^3*f^3)*cosh(d*x + c) + (a*d^3*e^3 - 3*a*c*d^2*e^2*f + 3*a*c^
2*d*e*f^2 - a*c^3*f^3)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x
+ c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 4*((a*d^3*f^3*x^3 + 3*a*d^3*e*f^2
*x^2 + 3*a*d^3*e^2*f*x + 3*a*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 + a*c^3*f^3)*cos
h(d*x + c) + (a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + 3*a*c*d
^2*e^2*f - 3*a*c^2*d*e*f^2 + a*c^3*f^3)*sinh(d*x + c))*log(-(a*cosh(d*x + c
) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2) - b)/b) + 4*((a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + 3*
a*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 + a*c^3*f^3)*cosh(d*x + c) + (a*d^3*f^3*x^3
 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + 3*a*c*d^2*e^2*f - 3*a*c^2*d*e*f^2
+ a*c^3*f^3)*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*co
sh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 24*(a*f^3*co
sh(d*x + c) + a*f^3*sinh(d*x + c))*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x
 + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*
(a*f^3*cosh(d*x + c) + a*f^3*sinh(d*x + c))*polylog(4, (a*cosh(d*x + c) + a
```

```
*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))
/b) - 24*((a*d*f^3*x + a*d*e*f^2)*cosh(d*x + c) + (a*d*f^3*x + a*d*e*f^2)*s
inh(d*x + c))*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
 c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 24*((a*d*f^3*x + a*d*e*f
^2)*cosh(d*x + c) + (a*d*f^3*x + a*d*e*f^2)*sinh(d*x + c))*polylog(3, (a*co
sh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2))/b) - (a*d^4*f^3*x^4 + 4*a*d^4*e*f^2*x^3 + 6*a*d^4*e^2*f*x^2
 + 4*a*d^4*e^3*x + 8*a*c*d^3*e^3 - 12*a*c^2*d^2*e^2*f + 8*a*c^3*d*e*f^2 - 2
*a*c^4*f^3 + 4*(b*d^3*f^3*x^3 + b*d^3*e^3 - 3*b*d^2*e^2*f + 6*b*d*e*f^2 - 6
*b*f^3 + 3*(b*d^3*e*f^2 - b*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f - 2*b*d^2*e*f^2 +
 2*b*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c))/(b^2*d^4*cosh(d*x + c) + b^2*d
^4*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \cosh(dx+c)\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.334 $\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=330

$$-\frac{2af(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}-\frac{2af(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^2}+\frac{2af^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3}+\frac{2af^2\text{Pol}}{}$$

```
[Out] (a*(e + f*x)^3)/(3*b^2*f) - (2*f*(e + f*x)*Cosh[c + d*x])/(b*d^2) - (a*(e +
 f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*d) - (a*(e + f
*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*d) - (2*a*f*(e +
 f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^2) - (2*
a*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^
2) + (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^
3) + (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^
3) + (2*f^2*Sinh[c + d*x])/(b*d^3) + ((e + f*x)^2*Sinh[c + d*x])/(b*d)
```

Rubi [A] time = 0.549545, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5579, 3296, 2637, 5561, 2190, 2531, 2282, 6589}

$$-\frac{2af(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}-\frac{2af(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^2}+\frac{2af^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3}+\frac{2af^2\text{Pol}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(e + f*x)^3)/(3*b^2*f) - (2*f*(e + f*x)*Cosh[c + d*x])/(b*d^2) - (a*(e +
 f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*d) - (a*(e + f
*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*d) - (2*a*f*(e +
 f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^2) - (2*
a*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^
2) + (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^
3) + (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^
3) + (2*f^2*Sinh[c + d*x])/(b*d^3) + ((e + f*x)^2*Sinh[c + d*x])/(b*d)
```

Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{a(e+fx)^3}{3b^2f} + \frac{(e+fx)^2\sinh(c+dx)}{bd} - \frac{a\int \frac{e^{c+dx}(e+fx)^2}{a-\sqrt{a^2+b^2}+be^{c+dx}}\,dx}{b} - \frac{a\int \frac{e^{c+dx}(e+\cdots}{a+\sqrt{a^2+b^2}+\cdots}}{b}$$

$$= \frac{a(e+fx)^3}{3b^2f} - \frac{2f(e+fx)\cosh(c+dx)}{bd^2} - \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d} - \frac{a(e\cdots}{\cdots}$$

$$= \frac{a(e+fx)^3}{3b^2f} - \frac{2f(e+fx)\cosh(c+dx)}{bd^2} - \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d} - \frac{a(e\cdots}{\cdots}$$

$$= \frac{a(e+fx)^3}{3b^2f} - \frac{2f(e+fx)\cosh(c+dx)}{bd^2} - \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d} - \frac{a(e\cdots}{\cdots}$$

$$= \frac{a(e+fx)^3}{3b^2f} - \frac{2f(e+fx)\cosh(c+dx)}{bd^2} - \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d} - \frac{a(e\cdots}{\cdots}$$

Mathematica [B] time = 14.2168, size = 1301, normalized size = 3.94

$$\frac{1}{2}\left(\frac{2a\left(2e^{2c}f^2x^3+6ee^{2c}fx^2-\frac{3e^{2c}f^2\log\left(\frac{e^{2c+dx}b}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}+1\right)x^2}{d}+\frac{3f^2\log\left(\frac{e^{2c+dx}b}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}+1\right)x^2}{d}-\frac{3e^{2c}f^2\log\left(\frac{e^{2c+dx}b}{e^ca+\sqrt{(a^2+b^2)e^{2c}}}+1\right)x^2}{d}+\frac{3f^2\log(\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] ((2*a*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[
a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 +
b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x
))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2
*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*
a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b
^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c
 + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)
))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*
c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 +
b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^
2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^
c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*
E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c +
 d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2
*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log
[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(-1 + E
^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2
)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c +
 d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*f^2*PolyLog[3, -((b*
E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*
```

```
PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 -
 (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))
])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 +
 b^2)*E^(2*c)]))])/d^3))/(3*b^2*(-1 + E^(2*c))) - (a*x*(3*e^2 + 3*e*f*x + f
^2*x^2)*Cosh[c]*Csch[c/2]*Sech[c/2])/(3*b^2) + (2*Cosh[d*x]*(-2*d*e*f*Cosh[
c] - 2*d*f^2*x*Cosh[c] + d^2*e^2*Sinh[c] + 2*f^2*Sinh[c] + 2*d^2*e*f*x*Sinh
[c] + d^2*f^2*x^2*Sinh[c]))/(b*d^3) + (2*(d^2*e^2*Cosh[c] + 2*f^2*Cosh[c] +
 2*d^2*e*f*x*Cosh[c] + d^2*f^2*x^2*Cosh[c] - 2*d*e*f*Sinh[c] - 2*d*f^2*x*Si
nh[c])*Sinh[d*x])/(b*d^3))/2
```

Maple [F] time = 0.126, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}e^{2}\left(\frac{2\left(dx+c\right)a}{b^{2}d}-\frac{e^{\left(dx+c\right)}}{bd}+\frac{e^{\left(-dx-c\right)}}{bd}+\frac{2\,a\log\left(-2\,ae^{\left(-dx-c\right)}+be^{\left(-2\,dx-2\,c\right)}-b\right)}{b^{2}d}\right)-\frac{\left(2\,ad^{3}f^{2}x^{3}e^{c}+6\,ad^{3}efx^{2}e^{c}-3\left(bd^{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/2*e^2*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) +
2*a*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^2*d)) - 1/6*(2*a*d^3
*f^2*x^3*e^c + 6*a*d^3*e*f*x^2*e^c - 3*(b*d^2*f^2*x^2*e^(2*c) + 2*(d^2*e*f
- d*f^2)*b*x*e^(2*c) - 2*(d*e*f - f^2)*b*e^(2*c))*e^(d*x) + 3*(b*d^2*f^2*x^
2 + 2*(d^2*e*f + d*f^2)*b*x + 2*(d*e*f + f^2)*b)*e^(-d*x))*e^(-c)/(b^2*d^3)
 + integrate(-2*(a*b*f^2*x^2 + 2*a*b*e*f*x - (a^2*f^2*x^2*e^c + 2*a^2*e*f*x
*e^c)*e^(d*x))/(b^3*e^(2*d*x + 2*c) + 2*a*b^2*e^(d*x + c) - b^3), x)
```

Fricas [C] time = 2.47519, size = 3051, normalized size = 9.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -1/6*(3*b*d^2*f^2*x^2 + 3*b*d^2*e^2 + 6*b*d*e*f + 6*b*f^2 - 3*(b*d^2*f^2*x^
2 + b*d^2*e^2 - 2*b*d*e*f + 2*b*f^2 + 2*(b*d^2*e*f - b*d*f^2)*x)*cosh(d*x +
 c)^2 - 3*(b*d^2*f^2*x^2 + b*d^2*e^2 - 2*b*d*e*f + 2*b*f^2 + 2*(b*d^2*e*f -
```

```
b*d*f^2)*x)*sinh(d*x + c)^2 + 6*(b*d^2*e*f + b*d*f^2)*x - 2*(a*d^3*f^2*x^3
 + 3*a*d^3*e*f*x^2 + 3*a*d^3*e^2*x + 6*a*c*d^2*e^2 - 6*a*c^2*d*e*f + 2*a*c^
3*f^2)*cosh(d*x + c) + 12*((a*d*f^2*x + a*d*e*f)*cosh(d*x + c) + (a*d*f^2*x
 + a*d*e*f)*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*co
sh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*((a*d
*f^2*x + a*d*e*f)*cosh(d*x + c) + (a*d*f^2*x + a*d*e*f)*sinh(d*x + c))*dilo
g((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 6*((a*d^2*e^2 - 2*a*c*d*e*f + a*c^2*f^2
)*cosh(d*x + c) + (a*d^2*e^2 - 2*a*c*d*e*f + a*c^2*f^2)*sinh(d*x + c))*log(
2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) +
6*((a*d^2*e^2 - 2*a*c*d*e*f + a*c^2*f^2)*cosh(d*x + c) + (a*d^2*e^2 - 2*a*c
*d*e*f + a*c^2*f^2)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c
) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 6*((a*d^2*f^2*x^2 + 2*a*d^2*e*f*x +
2*a*c*d*e*f - a*c^2*f^2)*cosh(d*x + c) + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + 2
*a*c*d*e*f - a*c^2*f^2)*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x +
 c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 6
*((a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*cosh(d*x + c) +
 (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*sinh(d*x + c))*l
og(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2) - b)/b) - 12*(a*f^2*cosh(d*x + c) + a*f^2*sinh(d*x
+ c))*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 12*(a*f^2*cosh(d*x + c) + a*f^2*
sinh(d*x + c))*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*(a*d^3*f^2*x^3 + 3*a*
d^3*e*f*x^2 + 3*a*d^3*e^2*x + 6*a*c*d^2*e^2 - 6*a*c^2*d*e*f + 2*a*c^3*f^2 +
 3*(b*d^2*f^2*x^2 + b*d^2*e^2 - 2*b*d*e*f + 2*b*f^2 + 2*(b*d^2*e*f - b*d*f^
2)*x)*cosh(d*x + c))*sinh(d*x + c))/(b^2*d^3*cosh(d*x + c) + b^2*d^3*sinh(d
*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \cosh(dx+c)\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.335 $\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=212

$$-\frac{af\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}-\frac{af\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^2}-\frac{a(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^2d}-\frac{a(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+\right.}{b^2d}$$

```
[Out] (a*(e + f*x)^2)/(2*b^2*f) - (f*Cosh[c + d*x])/(b*d^2) - (a*(e + f*x)*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*d) - (a*(e + f*x)*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*d) - (a*f*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^2) - (a*f*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^2) + ((e + f*x)*Sinh[c + d*x])/(b*d)
```

Rubi [A] time = 0.312137, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {5579, 3296, 2638, 5561, 2190, 2279, 2391}

$$-\frac{af\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}-\frac{af\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^2}-\frac{a(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^2d}-\frac{a(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+\right.}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(e + f*x)^2)/(2*b^2*f) - (f*Cosh[c + d*x])/(b*d^2) - (a*(e + f*x)*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*d) - (a*(e + f*x)*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*d) - (a*f*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^2) - (a*f*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^2) + ((e + f*x)*Sinh[c + d*x])/(b*d)
```

Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
```

```
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{a(e+fx)^2}{2b^2f} + \frac{(e+fx)\sinh(c+dx)}{bd} - \frac{a\int \frac{e^{c+dx}(e+fx)}{a-\sqrt{a^2+b^2}+be^{c+dx}}\,dx}{b} - \frac{a\int \frac{e^{c+dx}(e+fx)}{a+\sqrt{a^2+b^2}+be}}{b}$$

$$= \frac{a(e+fx)^2}{2b^2f} - \frac{f\cosh(c+dx)}{bd^2} - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d} - \frac{a(e+fx)\log\Big(}{b^2}$$

$$= \frac{a(e+fx)^2}{2b^2f} - \frac{f\cosh(c+dx)}{bd^2} - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d} - \frac{a(e+fx)\log\Big(}{b^2}$$

$$= \frac{a(e+fx)^2}{2b^2f} - \frac{f\cosh(c+dx)}{bd^2} - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d} - \frac{a(e+fx)\log\Big(}{b^2}$$

Mathematica [A] time = 1.05742, size = 206, normalized size = 0.97

$$\frac{-a\left(f\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+f(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)+f(c+dx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)+\right.}{b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-(b*f*Cosh[c + d*x]) - a*(-(f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2])] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d
*x]] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) + b*d*(e + f*x)*Sinh[c + d*x])/(
b^2*d^2)
```

Maple [B] time = 0.064, size = 483, normalized size = 2.3

$$\frac{afx^2}{2b^2}-\frac{aex}{b^2}+\frac{\left(dfx+de-f\right)e^{dx+c}}{2d^2b}-\frac{\left(dfx+de+f\right)e^{-dx-c}}{2d^2b}+\frac{afc\ln\left(be^{2dx+2c}+2ae^{dx+c}-b\right)}{b^2d^2}-2\frac{afc\ln\left(e^{dx+c}\right)}{b^2d^2}-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/2*a*f*x^2/b^2-a*e*x/b^2+1/2*(d*f*x+d*e-f)/d^2/b*exp(d*x+c)-1/2*(d*f*x+d*e
+f)/d^2/b*exp(-d*x-c)+a/b^2/d^2*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2
*a/b^2/d^2*f*c*ln(exp(d*x+c))-a/b^2/d*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-
b)+2*a/b^2/d*e*ln(exp(d*x+c))-a/b^2/d*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a
)/(-a+(a^2+b^2)^(1/2)))*x-a/b^2/d^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
(-a+(a^2+b^2)^(1/2)))*c-a/b^2/d*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a
^2+b^2)^(1/2)))*x-a/b^2/d^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b
^2)^(1/2)))*c-a/b^2/d^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^
2)^(1/2)))-a/b^2/d^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2
)^(1/2)))+2*a/b^2/d*f*c*x+a/b^2/d^2*f*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}e\left(\frac{2(dx+c)a}{b^2d}-\frac{e^{(dx+c)}}{bd}+\frac{e^{(-dx-c)}}{bd}+\frac{2a\log\left(-2ae^{(-dx-c)}+be^{(-2dx-2c)}-b\right)}{b^2d}\right)-\frac{1}{4}f\left(\frac{2\left(ad^2x^2e^c-\left(bdxe^{(2c)}-be^{(2c)}\right)e\ldots}{b^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] -1/2*e*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) + 2*
a*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^2*d)) - 1/4*f*(2*(a*d^
2*x^2*e^c - (b*d*x*e^(2*c) - b*e^(2*c))*e^(d*x) + (b*d*x + b)*e^(-d*x))*e^(
-c)/(b^2*d^2) - integrate(8*(a^2*x*e^(d*x + c) - a*b*x)/(b^3*e^(2*d*x + 2*c
) + 2*a*b^2*e^(d*x + c) - b^3), x))
```

Fricas [B] time = 2.27905, size = 1760, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] -1/2*(b*d*f*x + b*d*e - (b*d*f*x + b*d*e - b*f)*cosh(d*x + c)^2 - (b*d*f*x
+ b*d*e - b*f)*sinh(d*x + c)^2 + b*f - (a*d^2*f*x^2 + 2*a*d^2*e*x + 4*a*c*d
*e - 2*a*c^2*f)*cosh(d*x + c) + 2*(a*f*cosh(d*x + c) + a*f*sinh(d*x + c))*d
ilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a*f*cosh(d*x + c) + a*f*sinh(d*x
+ c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*((a*d*e - a*c*f)*cosh(d*x +
```

```
 c) + (a*d*e - a*c*f)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
 c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((a*d*e - a*c*f)*cosh(d*x + c) +
 (a*d*e - a*c*f)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) -
 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((a*d*f*x + a*c*f)*cosh(d*x + c) + (a
*d*f*x + a*c*f)*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*((a*d*f
*x + a*c*f)*cosh(d*x + c) + (a*d*f*x + a*c*f)*sinh(d*x + c))*log(-(a*cosh(d
*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2) - b)/b) - (a*d^2*f*x^2 + 2*a*d^2*e*x + 4*a*c*d*e - 2*a*c^2*f + 2
*(b*d*f*x + b*d*e - b*f)*cosh(d*x + c))*sinh(d*x + c))/(b^2*d^2*cosh(d*x +
c) + b^2*d^2*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\cosh(dx+c)\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.336 $\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{\sinh(c+dx)}{bd}-\frac{a\log(a+b\sinh(c+dx))}{b^2d}$$

```
[Out] -((a*Log[a + b*Sinh[c + d*x]])/(b^2*d)) + Sinh[c + d*x]/(b*d)
```

Rubi [A] time = 0.0569536, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 43}

$$\frac{\sinh(c+dx)}{bd}-\frac{a\log(a+b\sinh(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a*Log[a + b*Sinh[c + d*x]])/(b^2*d)) + Sinh[c + d*x]/(b*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{b(a+x)}\,dx, x, b\sinh(c+dx)\right)}{bd}\\
&= \frac{\text{Subst}\left(\int \frac{x}{a+x}\,dx, x, b\sinh(c+dx)\right)}{b^2d}\\
&= \frac{\text{Subst}\left(\int \left(1-\frac{a}{a+x}\right)dx, x, b\sinh(c+dx)\right)}{b^2d}\\
&= -\frac{a\log(a+b\sinh(c+dx))}{b^2d}+\frac{\sinh(c+dx)}{bd}
\end{aligned}$$

Mathematica [A] time = 0.0386967, size = 33, normalized size = 0.97

$$-\frac{\frac{a\log(a+b\sinh(c+dx))}{b^2}-\frac{\sinh(c+dx)}{b}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(((a*Log[a + b*Sinh[c + d*x]])/b^2 - Sinh[c + d*x]/b)/d)
```

Maple [A] time = 0.013, size = 35, normalized size = 1.

$$-\frac{a\ln\left(a+b\sinh\left(dx+c\right)\right)}{b^2d}+\frac{\sinh\left(dx+c\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] -a*ln(a+b*sinh(d*x+c))/b^2/d+sinh(d*x+c)/b/d
```

Maxima [B] time = 1.07533, size = 112, normalized size = 3.29

$$-\frac{(dx+c)a}{b^2d}+\frac{e^{(dx+c)}}{2\,bd}-\frac{e^{(-dx-c)}}{2\,bd}-\frac{a\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(d*x + c)*a/(b^2*d) + 1/2*e^(d*x + c)/(b*d) - 1/2*e^(-d*x - c)/(b*d) - a*l
og(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^2*d)
```

Fricas [B] time = 2.12315, size = 354, normalized size = 10.41

$$\frac{2\,adx\cosh\left(dx+c\right)+b\cosh\left(dx+c\right)^2+b\sinh\left(dx+c\right)^2-2\left(a\cosh\left(dx+c\right)+a\sinh\left(dx+c\right)\right)\log\left(\frac{2\left(b\sinh(dx+c)+a\right)}{\cosh(dx+c)-\sinh(dx+c)}\right)\cdots}{2\left(b^2d\cosh\left(dx+c\right)+b^2d\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*x*cosh(d*x + c) + b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 - 2*(a*c
osh(d*x + c) + a*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c)
- sinh(d*x + c))) + 2*(a*d*x + b*cosh(d*x + c))*sinh(d*x + c) - b)/(b^2*d*c
osh(d*x + c) + b^2*d*sinh(d*x + c))
```

Sympy [A] time = 1.47607, size = 65, normalized size = 1.91

$$\begin{cases} \frac{x\sinh(c)\cosh(c)}{a} & \text{for } b = 0 \wedge d = 0 \\ \frac{x\sinh(c)\cosh(c)}{a+b\sinh(c)} & \text{for } d = 0 \\ \frac{\sinh^2(c+dx)}{2ad} & \text{for } b = 0 \\ -\frac{a\log\left(\frac{a}{b}+\sinh(c+dx)\right)}{b^2d} + \frac{\sinh(c+dx)}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Piecewise((x*sinh(c)*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (x*sinh(c)*cosh(c)/(a
 + b*sinh(c)), Eq(d, 0)), (sinh(c + d*x)**2/(2*a*d), Eq(b, 0)), (-a*log(a/b
 + sinh(c + d*x))/(b**2*d) + sinh(c + d*x)/(b*d), True))
```

Giac [A] time = 1.14879, size = 84, normalized size = 2.47

$$\frac{e^{(dx+c)} - e^{(-dx-c)}}{2\,bd} - \frac{a\log\left(\left|b\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 2\,a\right|\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(e^(d*x + c) - e^(-d*x - c))/(b*d) - a*log(abs(b*(e^(d*x + c) - e^(-d*x
 - c)) + 2*a))/(b^2*d)
```

3.337 $\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=34

$$\text{Unintegrable}\left(\frac{\sinh(c+dx)\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Cosh[c + d*x]*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [A] time = 0.0571966, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[c + d*x]*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Cosh[c + d*x]*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

Rubi steps

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A] time = 73.8462, size = 0, normalized size = 0.

$$\int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[c + d*x]*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x
]
```

```
[Out] Integrate[(Cosh[c + d*x]*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

Maple [A] time = 0.118, size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)\sinh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{e^{\left(-c+\frac{de}{f}\right)}E_1\left(\frac{(fx+e)d}{f}\right)}{2\,bf}-\frac{e^{\left(c-\frac{de}{f}\right)}E_1\left(-\frac{(fx+e)d}{f}\right)}{2\,bf}-\frac{a\log\left(fx+e\right)}{b^2f}+\frac{1}{4}\int-\frac{8\left(a^2e^{(dx+c)}-ab\right)}{b^3fx+b^3e-\left(b^3fxe^{(2\,c)}+b^3ee^{(2\,c)}\right)e^{(2\,dx)}-2\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] -1/2*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b*f) - 1/2*e^(c - d*e
/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b*f) - a*log(f*x + e)/(b^2*f) + 1/4*
integrate(-8*(a^2*e^(d*x + c) - a*b)/(b^3*f*x + b^3*e - (b^3*f*x*e^(2*c) +
b^3*e*e^(2*c))*e^(2*d*x) - 2*(a*b^2*f*x*e^c + a*b^2*e*e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh\left(dx+c\right)\sinh\left(dx+c\right)}{afx+ae+\left(bfx+be\right)\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] integral(cosh(d*x + c)*sinh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
+ c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh\left(dx+c\right)\sinh\left(dx+c\right)}{\left(fx+e\right)\left(b\sinh\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] integrate(cosh(d*x + c)*sinh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

3.338 $\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal. Leaf size=696

$$\frac{6af^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}-\frac{6af^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^3}-\frac{3af\sqrt{a^2+b^2}(e+f}{}$$

```
[Out] (3*e*f^2*x)/(4*b*d^2) + (3*f^3*x^2)/(8*b*d^2) + (a^2*(e + f*x)^4)/(4*b^3*f)
 + (e + f*x)^4/(8*b*f) - (6*a*f^2*(e + f*x)*Cosh[c + d*x])/(b^2*d^3) - (a*(
e + f*x)^3*Cosh[c + d*x])/(b^2*d) - (3*f^3*Cosh[c + d*x]^2)/(8*b*d^4) - (3*
f*(e + f*x)^2*Cosh[c + d*x]^2)/(4*b*d^2) - (a*Sqrt[a^2 + b^2]*(e + f*x)^3*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) + (a*Sqrt[a^2 + b^2]
*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*d) - (3*a
*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/(b^3*d^2) + (3*a*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^2) + (6*a*Sqrt[a^2 + b^2]*f^2*(e
 + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^3) - (
6*a*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/(b^3*d^3) - (6*a*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^4) + (6*a*Sqrt[a^2 + b^2]*f^3*PolyLog[4
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^4) + (6*a*f^3*Sinh[c +
d*x])/(b^2*d^4) + (3*a*f*(e + f*x)^2*Sinh[c + d*x])/(b^2*d^2) + (3*f^2*(e +
 f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^3) + ((e + f*x)^3*Cosh[c + d*x]*S
inh[c + d*x])/(2*b*d)
```

Rubi [A] time = 1.12957, antiderivative size = 696, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 14, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5579, 3311, 32, 3310, 5565, 3296, 2637, 3322, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{6af^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}-\frac{6af^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^3}-\frac{3af\sqrt{a^2+b^2}(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (3*e*f^2*x)/(4*b*d^2) + (3*f^3*x^2)/(8*b*d^2) + (a^2*(e + f*x)^4)/(4*b^3*f)
 + (e + f*x)^4/(8*b*f) - (6*a*f^2*(e + f*x)*Cosh[c + d*x])/(b^2*d^3) - (a*(
e + f*x)^3*Cosh[c + d*x])/(b^2*d) - (3*f^3*Cosh[c + d*x]^2)/(8*b*d^4) - (3*
f*(e + f*x)^2*Cosh[c + d*x]^2)/(4*b*d^2) - (a*Sqrt[a^2 + b^2]*(e + f*x)^3*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) + (a*Sqrt[a^2 + b^2]
*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*d) - (3*a
*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/(b^3*d^2) + (3*a*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^2) + (6*a*Sqrt[a^2 + b^2]*f^2*(e
 + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^3) - (
6*a*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/(b^3*d^3) - (6*a*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^4) + (6*a*Sqrt[a^2 + b^2]*f^3*PolyLog[4
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^4) + (6*a*f^3*Sinh[c +
d*x])/(b^2*d^4) + (3*a*f*(e + f*x)^2*Sinh[c + d*x])/(b^2*d^2) + (3*f^2*(e +
 f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^3) + ((e + f*x)^3*Cosh[c + d*x]*S
inh[c + d*x])/(2*b*d)
```

Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\cosh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{3f(e+fx)^2\cosh^2(c+dx)}{4bd^2} + \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{2bd} + \frac{a^2\int(\ldots}{}$$

$$= \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{a(e+fx)^3\cosh(c+dx)}{b^2d} - \frac{3f^3\cosh^2(c+dx)}{8bd^4} - \ldots$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{a(e+fx)^3\cosh(c+dx)}{b^2d} - 3f\ldots$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3} - \ldots$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3} - \ldots$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3} - \ldots$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3} - \ldots$$

$$= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx)\cosh(c+dx)}{b^2d^3} - \ldots$$

Mathematica [C] time = 14.6937, size = 2961, normalized size = 4.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (e^3*(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(S
qrt[-a^2 - b^2]*d)))/(4*b) + (3*e^2*f*(x^2 + (2*a*((I*Pi*ArcTanh[(-b + a*Ta
nh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (2*((-I)*c + ArcCos[((
-I)*a)/b])*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2
- b^2]] + ((-2*I)*c + Pi - (2*I)*d*x)*ArcTanh[((a - I*b)*Tan[((2*I)*c + Pi
+ (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] - (ArcCos[((-I)*a)/b] + (2*I)*ArcTanh[((
a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[((I*a +
b)*(a + I*(b + Sqrt[-a^2 - b^2]))*(-I + Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))
/(b*(I*a + b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))] - (A
rcCos[((-I)*a)/b] - (2*I)*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)
/4])/Sqrt[-a^2 - b^2]])*Log[((I*a + b)*(I*a - b + Sqrt[-a^2 - b^2])*(I + Co
t[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(a - I*b + Sqrt[-a^2 - b^2]*Cot[((2*I)
*c + Pi + (2*I)*d*x)/4]))] + (ArcCos[((-I)*a)/b] - (2*I)*ArcTanh[((a + I*b)
*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] - (2*I)*ArcTanh[((a -
 I*b)*Tan[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[-(((-1)^(3/
4)*Sqrt[-a^2 - b^2]*E^(-c/2 - (d*x)/2))/(Sqrt[2]*Sqrt[(-I)*b]*Sqrt[a + b*Si
nh[c + d*x]]))] + (ArcCos[((-I)*a)/b] + (2*I)*(ArcTanh[((a + I*b)*Cot[((2*I
)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] + ArcTanh[((a - I*b)*Tan[((2*I)
*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]]))*Log[((-1)^(1/4)*Sqrt[-a^2 - b^
```

```
2]*E^((c + d*x)/2))/(Sqrt[2]*Sqrt[(-I)*b]*Sqrt[a + b*Sinh[c + d*x]])] + I*(
PolyLog[2, ((I*a + Sqrt[-a^2 - b^2])*(I*a + b - I*Sqrt[-a^2 - b^2]*Cot[((2*
I)*c + Pi + (2*I)*d*x)/4]))/(b*(I*a + b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c +
 Pi + (2*I)*d*x)/4]))] - PolyLog[2, ((a + I*Sqrt[-a^2 - b^2])*(-a + I*b + S
qrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(I*a + b + I*Sqrt[-a
^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))]))/Sqrt[-a^2 - b^2]))/d^2))/(
8*b) + (e*f^2*(x^3 - (3*a*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2])] - d^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*Pol
yLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*x*PolyLog[2, -((b*E^(
c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt
[a^2 + b^2])] + 2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(S
qrt[a^2 + b^2]*d^3)))/(4*b) + (f^3*(x^4 - (4*a*(d^3*x^3*Log[1 + (b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2])] - d^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2])] + 3*d^2*x^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] -
3*d^2*x^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*x*Poly
Log[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*d*x*PolyLog[3, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[
a^2 + b^2])] - 6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sq
rt[a^2 + b^2]*d^4)))/(16*b) + (e*f^2*(2*(4*a^2 + b^2)*x^3 - (6*a*(4*a^2 + 3
*b^2)*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2*x^2*Log
[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, (b*E^(c + d*
x))/(-a + Sqrt[a^2 + b^2])] - 2*d*x*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2]))] - 2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*Po
lyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^2 + b^2]*d^3)
- (24*a*b*Cosh[d*x]*((2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[c]))/d^3 + (3*b^2*C
osh[2*d*x]*(-2*d*x*Cosh[2*c] + (1 + 2*d^2*x^2)*Sinh[2*c]))/d^3 - (24*a*b*(-
2*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c])*Sinh[d*x])/d^3 + (3*b^2*((1 + 2*d^2*
x^2)*Cosh[2*c] - 2*d*x*Sinh[2*c])*Sinh[2*d*x])/d^3))/(8*b^3) + (f^3*((4*a^2
 + b^2)*x^4 - (4*a*(4*a^2 + 3*b^2)*(d^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2])] - d^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] +
3*d^2*x^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 3*d^2*x^2*Po
lyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*x*PolyLog[3, (b*E^
(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*d*x*PolyLog[3, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))] + 6*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
 - 6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^2 + b^2
]*d^4) - (16*a*b*Cosh[d*x]*(d*x*(6 + d^2*x^2)*Cosh[c] - 3*(2 + d^2*x^2)*Sin
h[c]))/d^4 + (b^2*Cosh[2*d*x]*(-3*(1 + 2*d^2*x^2)*Cosh[2*c] + 2*d*x*(3 + 2*
d^2*x^2)*Sinh[2*c]))/d^4 - (16*a*b*(-3*(2 + d^2*x^2)*Cosh[c] + d*x*(6 + d^2
*x^2)*Sinh[c])*Sinh[d*x])/d^4 + (b^2*(2*d*x*(3 + 2*d^2*x^2)*Cosh[2*c] - 3*(
1 + 2*d^2*x^2)*Sinh[2*c])*Sinh[2*d*x])/d^4))/(16*b^3) + (e^3*((4*a^2 + b^2)
*(c + d*x) - (2*a*(4*a^2 + 3*b^2)*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^
2 - b^2]])/Sqrt[-a^2 - b^2] - 4*a*b*Cosh[c + d*x] + b^2*Sinh[2*(c + d*x)]))
/(4*b^3*d) + (3*e^2*f*((4*a^2 + b^2)*(-c + d*x)*(c + d*x) - 8*a*b*d*x*Cosh[
c + d*x] - b^2*Cosh[2*(c + d*x)] - (2*a*(4*a^2 + 3*b^2)*(2*c*ArcTanh[(a + b
*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + (c + d*x)*Log[1 + (b*(
Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] - (c + d*x)*Log[1 +
(b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])] + PolyLog[2, (b*
(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])] - PolyLog[2, -((b*
(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2]
+ 8*a*b*Sinh[c + d*x] + 2*b^2*d*x*Sinh[2*(c + d*x)]))/(8*b^3*d^2)
```

Maple [F] time = 0.191, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3}\left(\cosh\left(dx+c\right)\right)^{2}\sinh\left(dx+c\right)}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.10445, size = 8699, normalized size = 12.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/32*(4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 + 6*b^2*d^2*e^2*f + 6*b^2*d*e*f^2
+ 3*b^2*f^3 - (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f + 6*b^2*
d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^2*d^3*
e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*cosh(d*x + c)^4 - (4*b^2*d^3*f^3*x^
3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f + 6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*
d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^2*d^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d
*f^3)*x)*sinh(d*x + c)^4 + 16*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3 - 3*a*b*d^2*e^
2*f + 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(
a*b*d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*x + c)^3 + 4*(4*a*
b*d^3*f^3*x^3 + 4*a*b*d^3*e^3 - 12*a*b*d^2*e^2*f + 24*a*b*d*e*f^2 - 24*a*b*
f^3 + 12*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 12*(a*b*d^3*e^2*f - 2*a*b*d^2*
e*f^2 + 2*a*b*d*f^3)*x - (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2
*f + 6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*
(2*b^2*d^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 6*(2*b^2*d^3*e*f^2 + b^2*d^2*f^3)*x^2 - 4*((2*a^2 + b^2)*d^4*f^3*x
^4 + 4*(2*a^2 + b^2)*d^4*e*f^2*x^3 + 6*(2*a^2 + b^2)*d^4*e^2*f*x^2 + 4*(2*a
^2 + b^2)*d^4*e^3*x)*cosh(d*x + c)^2 - 2*(2*(2*a^2 + b^2)*d^4*f^3*x^4 + 8*(
2*a^2 + b^2)*d^4*e*f^2*x^3 + 12*(2*a^2 + b^2)*d^4*e^2*f*x^2 + 8*(2*a^2 + b^
2)*d^4*e^3*x + 3*(4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f + 6*b
^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^2*d
^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*cosh(d*x + c)^2 - 24*(a*b*d^3*f^
3*x^3 + a*b*d^3*e^3 - 3*a*b*d^2*e^2*f + 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b*
d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(a*b*d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d
*f^3)*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 96*((a*b*d^2*f^3*x^2 + 2*a*b*d^2*
e*f^2*x + a*b*d^2*e^2*f)*cosh(d*x + c)^2 + 2*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e
*f^2*x + a*b*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d^2*f^3*x^2 + 2*
a*b*d^2*e*f^2*x + a*b*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dil
og((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 96*((a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2
*x + a*b*d^2*e^2*f)*cosh(d*x + c)^2 + 2*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2*
```

```
x + a*b*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d^2*f^3*x^2 + 2*a*b*d
^2*e*f^2*x + a*b*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a
*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b + 1) - 32*((a*b*d^3*e^3 - 3*a*b*c*d^2*e^2*f + 3*a*
b*c^2*d*e*f^2 - a*b*c^3*f^3)*cosh(d*x + c)^2 + 2*(a*b*d^3*e^3 - 3*a*b*c*d^2
*e^2*f + 3*a*b*c^2*d*e*f^2 - a*b*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) + (a*
b*d^3*e^3 - 3*a*b*c*d^2*e^2*f + 3*a*b*c^2*d*e*f^2 - a*b*c^3*f^3)*sinh(d*x +
 c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*
b*sqrt((a^2 + b^2)/b^2) + 2*a) + 32*((a*b*d^3*e^3 - 3*a*b*c*d^2*e^2*f + 3*a
*b*c^2*d*e*f^2 - a*b*c^3*f^3)*cosh(d*x + c)^2 + 2*(a*b*d^3*e^3 - 3*a*b*c*d^
2*e^2*f + 3*a*b*c^2*d*e*f^2 - a*b*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) + (a
*b*d^3*e^3 - 3*a*b*c*d^2*e^2*f + 3*a*b*c^2*d*e*f^2 - a*b*c^3*f^3)*sinh(d*x
+ c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2
*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 32*((a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^
2 + 3*a*b*d^3*e^2*f*x + 3*a*b*c*d^2*e^2*f - 3*a*b*c^2*d*e*f^2 + a*b*c^3*f^3
)*cosh(d*x + c)^2 + 2*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*e^
2*f*x + 3*a*b*c*d^2*e^2*f - 3*a*b*c^2*d*e*f^2 + a*b*c^3*f^3)*cosh(d*x + c)*
sinh(d*x + c) + (a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*e^2*f*x
+ 3*a*b*c*d^2*e^2*f - 3*a*b*c^2*d*e*f^2 + a*b*c^3*f^3)*sinh(d*x + c)^2)*sqr
t((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 32*((a*b*d^3*f^3*x^3
+ 3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*e^2*f*x + 3*a*b*c*d^2*e^2*f - 3*a*b*c^2*d
*e*f^2 + a*b*c^3*f^3)*cosh(d*x + c)^2 + 2*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^
2*x^2 + 3*a*b*d^3*e^2*f*x + 3*a*b*c*d^2*e^2*f - 3*a*b*c^2*d*e*f^2 + a*b*c^3
*f^3)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2
+ 3*a*b*d^3*e^2*f*x + 3*a*b*c*d^2*e^2*f - 3*a*b*c^2*d*e*f^2 + a*b*c^3*f^3)*
sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x +
 c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 1
92*(a*b*f^3*cosh(d*x + c)^2 + 2*a*b*f^3*cosh(d*x + c)*sinh(d*x + c) + a*b*f
^3*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*s
inh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b
) - 192*(a*b*f^3*cosh(d*x + c)^2 + 2*a*b*f^3*cosh(d*x + c)*sinh(d*x + c) +
a*b*f^3*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c)
+ a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2))/b) - 192*((a*b*d*f^3*x + a*b*d*e*f^2)*cosh(d*x + c)^2 + 2*(a*b*d*f^3*x
+ a*b*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d*f^3*x + a*b*d*e*f^2)*si
nh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d
*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 1
92*((a*b*d*f^3*x + a*b*d*e*f^2)*cosh(d*x + c)^2 + 2*(a*b*d*f^3*x + a*b*d*e*
f^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d*f^3*x + a*b*d*e*f^2)*sinh(d*x + c
)^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) -
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*(2*b^2*d^
3*e^2*f + 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x + 16*(a*b*d^3*f^3*x^3 + a*b*d^3*e^
3 + 3*a*b*d^2*e^2*f + 6*a*b*d*e*f^2 + 6*a*b*f^3 + 3*(a*b*d^3*e*f^2 + a*b*d^
2*f^3)*x^2 + 3*(a*b*d^3*e^2*f + 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*x
+ c) + 4*(4*a*b*d^3*f^3*x^3 + 4*a*b*d^3*e^3 + 12*a*b*d^2*e^2*f + 24*a*b*d*e
*f^2 + 24*a*b*f^3 - (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f +
6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^
2*d^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*cosh(d*x + c)^3 + 12*(a*b*d^3
*e*f^2 + a*b*d^2*f^3)*x^2 + 12*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3 - 3*a*b*d^2*e
^2*f + 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*
(a*b*d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*x + c)^2 + 12*(a*
b*d^3*e^2*f + 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x - 2*((2*a^2 + b^2)*d^4*f^3*x
^4 + 4*(2*a^2 + b^2)*d^4*e*f^2*x^3 + 6*(2*a^2 + b^2)*d^4*e^2*f*x^2 + 4*(2*a
^2 + b^2)*d^4*e^3*x)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d^4*cosh(d*x + c)^2
 + 2*b^3*d^4*cosh(d*x + c)*sinh(d*x + c) + b^3*d^4*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \cosh(dx+c)^2 \sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)^2*sinh(d*x + c)/(b*sinh(d*x + c) + a),
x)
```

3.339 $\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=510

$$-\frac{2af\sqrt{a^2+b^2}(e+fx)\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{2af\sqrt{a^2+b^2}(e+fx)\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2}+\frac{2af^2\sqrt{a^2+b^2}\mathrm{Poly}}{b^3}$$

```
[Out] (f^2*x)/(4*b*d^2) + (a^2*(e + f*x)^3)/(3*b^3*f) + (e + f*x)^3/(6*b*f) - (2*
a*f^2*Cosh[c + d*x])/(b^2*d^3) - (a*(e + f*x)^2*Cosh[c + d*x])/(b^2*d) - (f
*(e + f*x)*Cosh[c + d*x]^2)/(2*b*d^2) - (a*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) + (a*Sqrt[a^2 + b^2]*(e
 + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*d) - (2*a*Sq
rt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
))])/(b^3*d^2) + (2*a*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^2) + (2*a*Sqrt[a^2 + b^2]*f^2*PolyLog[3
, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^3) - (2*a*Sqrt[a^2 + b^
2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^3) + (2
*a*f*(e + f*x)*Sinh[c + d*x])/(b^2*d^2) + (f^2*Cosh[c + d*x]*Sinh[c + d*x])
/(4*b*d^3) + ((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.962293, antiderivative size = 510, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 14, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5579, 3311, 32, 2635, 8, 5565, 3296, 2638, 3322, 2264, 2190, 2531, 2282, 6589}

$$-\frac{2af\sqrt{a^2+b^2}(e+fx)\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{2af\sqrt{a^2+b^2}(e+fx)\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2}+\frac{2af^2\sqrt{a^2+b^2}\mathrm{Poly}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (f^2*x)/(4*b*d^2) + (a^2*(e + f*x)^3)/(3*b^3*f) + (e + f*x)^3/(6*b*f) - (2*
a*f^2*Cosh[c + d*x])/(b^2*d^3) - (a*(e + f*x)^2*Cosh[c + d*x])/(b^2*d) - (f
*(e + f*x)*Cosh[c + d*x]^2)/(2*b*d^2) - (a*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) + (a*Sqrt[a^2 + b^2]*(e
 + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*d) - (2*a*Sq
rt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
))])/(b^3*d^2) + (2*a*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^2) + (2*a*Sqrt[a^2 + b^2]*f^2*PolyLog[3
, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^3) - (2*a*Sqrt[a^2 + b^
2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^3) + (2
*a*f*(e + f*x)*Sinh[c + d*x])/(b^2*d^2) + (f^2*Cosh[c + d*x]*Sinh[c + d*x])
/(4*b*d^3) + ((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)
```


Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{f(e+fx)\cosh^2(c+dx)}{2bd^2} + \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{2bd} + \frac{a^2\int$$

$$= \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{a(e+fx)^2\cosh(c+dx)}{b^2d} - \frac{f(e+fx)\cosh^2(c}{2bd^2}$$

$$= \frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{a(e+fx)^2\cosh(c+dx)}{b^2d} - \frac{f(e+fx)}{2}$$

$$= \frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2\cosh(c+dx)}{b^2d^3} - \frac{a(e+fx)^2\cosh}{b^2d}$$

$$= \frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2\cosh(c+dx)}{b^2d^3} - \frac{a(e+fx)^2\cosh}{b^2d}$$

$$= \frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2\cosh(c+dx)}{b^2d^3} - \frac{a(e+fx)^2\cosh}{b^2d}$$

$$= \frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2\cosh(c+dx)}{b^2d^3} - \frac{a(e+fx)^2\cosh}{b^2d}$$

Mathematica [C] time = 10.6121, size = 2170, normalized size = 4.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (e^2*(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(S
qrt[-a^2 - b^2]*d)))/(4*b) + (e*f*(x^2 + (2*a*((I*Pi*ArcTanh[(-b + a*Tanh[(
c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (2*((-I)*c + ArcCos[((-I)*
a)/b])*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^
2]] + ((-2*I)*c + Pi - (2*I)*d*x)*ArcTanh[((a - I*b)*Tan[((2*I)*c + Pi + (2
*I)*d*x)/4])/Sqrt[-a^2 - b^2]] - (ArcCos[((-I)*a)/b] + (2*I)*ArcTanh[((a +
I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[((I*a + b)*(
a + I*(b + Sqrt[-a^2 - b^2]))*(-I + Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*
(I*a + b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))] - (ArcCo
s[((-I)*a)/b] - (2*I)*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])
/Sqrt[-a^2 - b^2]])*Log[((I*a + b)*(I*a - b + Sqrt[-a^2 - b^2])*(I + Cot[((
2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(a - I*b + Sqrt[-a^2 - b^2]*Cot[((2*I)*c +
 Pi + (2*I)*d*x)/4]))] + (ArcCos[((-I)*a)/b] - (2*I)*ArcTanh[((a + I*b)*Cot
[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] - (2*I)*ArcTanh[((a - I*b
)*Tan[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[-(((-1)^(3/4)*S
qrt[-a^2 - b^2]*E^(-c/2 - (d*x)/2))/(Sqrt[2]*Sqrt[(-I)*b]*Sqrt[a + b*Sinh[c
 + d*x]]))] + (ArcCos[((-I)*a)/b] + (2*I)*(ArcTanh[((a + I*b)*Cot[((2*I)*c
+ Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] + ArcTanh[((a - I*b)*Tan[((2*I)*c +
 Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]]))*Log[((-1)^(1/4)*Sqrt[-a^2 - b^2]*E
^((c + d*x)/2))/(Sqrt[2]*Sqrt[(-I)*b]*Sqrt[a + b*Sinh[c + d*x]])] + I*(Poly
Log[2, ((I*a + Sqrt[-a^2 - b^2])*(I*a + b - I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c
 + Pi + (2*I)*d*x)/4]))/(b*(I*a + b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi
+ (2*I)*d*x)/4]))] - PolyLog[2, ((a + I*Sqrt[-a^2 - b^2])*(-a + I*b + Sqrt[
-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(I*a + b + I*Sqrt[-a^2 -
 b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))]))/Sqrt[-a^2 - b^2]))/d^2))/(4*b)
 + (f^2*(x^3 - (3*a*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]
 - d^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2
, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*x*PolyLog[2, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))] - 2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 +
 b^2])] + 2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^
2 + b^2]*d^3)))/(12*b) + (f^2*(2*(4*a^2 + b^2)*x^3 - (6*a*(4*a^2 + 3*b^2)*(
d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2*x^2*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, (b*E^(c + d*x))/(-a
 + Sqrt[a^2 + b^2])] - 2*d*x*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2]))] - 2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*PolyLog[3
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(Sqrt[a^2 + b^2]*d^3) - (24*a
*b*Cosh[d*x]*((2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[c]))/d^3 + (3*b^2*Cosh[2*d
*x]*(-2*d*x*Cosh[2*c] + (1 + 2*d^2*x^2)*Sinh[2*c]))/d^3 - (24*a*b*(-2*d*x*C
osh[c] + (2 + d^2*x^2)*Sinh[c])*Sinh[d*x])/d^3 + (3*b^2*((1 + 2*d^2*x^2)*Co
sh[2*c] - 2*d*x*Sinh[2*c])*Sinh[2*d*x])/d^3))/(24*b^3) + (e^2*((4*a^2 + b^2
)*(c + d*x) - (2*a*(4*a^2 + 3*b^2)*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a
^2 - b^2]])/Sqrt[-a^2 - b^2] - 4*a*b*Cosh[c + d*x] + b^2*Sinh[2*(c + d*x)])
)/(4*b^3*d) + (e*f*((4*a^2 + b^2)*(-c + d*x)*(c + d*x) - 8*a*b*d*x*Cosh[c +
 d*x] - b^2*Cosh[2*(c + d*x)] - (2*a*(4*a^2 + 3*b^2)*(2*c*ArcTanh[(a + b*Co
sh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + (c + d*x)*Log[1 + (b*(Cos
h[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] - (c + d*x)*Log[1 + (b*
(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])] + PolyLog[2, (b*(Co
sh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])] - PolyLog[2, -((b*(Co
sh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2] + 8
*a*b*Sinh[c + d*x] + 2*b^2*d*x*Sinh[2*(c + d*x)]))/(4*b^3*d^2)
```

Maple [F] time = 0.145, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \left(\cosh \left(dx+c\right)\right)^{2} \sinh \left(dx+c\right)}{a+b\sinh \left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.71597, size = 5658, normalized size = 11.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/48*(6*b^2*d^2*f^2*x^2 + 6*b^2*d^2*e^2 + 6*b^2*d*e*f - 3*(2*b^2*d^2*f^2*x
^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*
x)*cosh(d*x + c)^4 - 3*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b
^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*sinh(d*x + c)^4 + 3*b^2*f^2 + 24*
(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f -
 a*b*d*f^2)*x)*cosh(d*x + c)^3 + 12*(2*a*b*d^2*f^2*x^2 + 2*a*b*d^2*e^2 - 4*
a*b*d*e*f + 4*a*b*f^2 + 4*(a*b*d^2*e*f - a*b*d*f^2)*x - (2*b^2*d^2*f^2*x^2
+ 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*
cosh(d*x + c))*sinh(d*x + c)^3 - 8*((2*a^2 + b^2)*d^3*f^2*x^3 + 3*(2*a^2 +
b^2)*d^3*e*f*x^2 + 3*(2*a^2 + b^2)*d^3*e^2*x)*cosh(d*x + c)^2 - 2*(4*(2*a^2
 + b^2)*d^3*f^2*x^3 + 12*(2*a^2 + b^2)*d^3*e*f*x^2 + 12*(2*a^2 + b^2)*d^3*e
^2*x + 9*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*
b^2*d^2*e*f - b^2*d*f^2)*x)*cosh(d*x + c)^2 - 36*(a*b*d^2*f^2*x^2 + a*b*d^2
*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*x)*cosh(d*x +
c))*sinh(d*x + c)^2 + 96*((a*b*d*f^2*x + a*b*d*e*f)*cosh(d*x + c)^2 + 2*(a*
b*d*f^2*x + a*b*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d*f^2*x + a*b*d*e
*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(
d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b
 + 1) - 96*((a*b*d*f^2*x + a*b*d*e*f)*cosh(d*x + c)^2 + 2*(a*b*d*f^2*x + a*
b*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d*f^2*x + a*b*d*e*f)*sinh(d*x +
 c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
```

```
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 48*((a
*b*d^2*e^2 - 2*a*b*c*d*e*f + a*b*c^2*f^2)*cosh(d*x + c)^2 + 2*(a*b*d^2*e^2
- 2*a*b*c*d*e*f + a*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d^2*e^2 -
 2*a*b*c*d*e*f + a*b*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*
b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 48
*((a*b*d^2*e^2 - 2*a*b*c*d*e*f + a*b*c^2*f^2)*cosh(d*x + c)^2 + 2*(a*b*d^2*
e^2 - 2*a*b*c*d*e*f + a*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d^2*e
^2 - 2*a*b*c*d*e*f + a*b*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*lo
g(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
+ 48*((a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*f^2)*cos
h(d*x + c)^2 + 2*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c
^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x +
2*a*b*c*d*e*f - a*b*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a
*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b) - 48*((a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*
c*d*e*f - a*b*c^2*f^2)*cosh(d*x + c)^2 + 2*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f
*x + 2*a*b*c*d*e*f - a*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d^2*f^
2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*f^2)*sinh(d*x + c)^2)*sqr
t((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 96*(a*b*f^2*cosh(d*x
+ c)^2 + 2*a*b*f^2*cosh(d*x + c)*sinh(d*x + c) + a*b*f^2*sinh(d*x + c)^2)*s
qrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 96*(a*b*f^2*cosh(
d*x + c)^2 + 2*a*b*f^2*cosh(d*x + c)*sinh(d*x + c) + a*b*f^2*sinh(d*x + c)^
2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*(2*b^2*d^2*
e*f + b^2*d*f^2)*x + 24*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 + 2*a*b*d*e*f + 2*a*
b*f^2 + 2*(a*b*d^2*e*f + a*b*d*f^2)*x)*cosh(d*x + c) + 4*(6*a*b*d^2*f^2*x^2
 + 6*a*b*d^2*e^2 + 12*a*b*d*e*f + 12*a*b*f^2 - 3*(2*b^2*d^2*f^2*x^2 + 2*b^2
*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*cosh(d*
x + c)^3 + 18*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*
(a*b*d^2*e*f - a*b*d*f^2)*x)*cosh(d*x + c)^2 + 12*(a*b*d^2*e*f + a*b*d*f^2)
*x - 4*((2*a^2 + b^2)*d^3*f^2*x^3 + 3*(2*a^2 + b^2)*d^3*e*f*x^2 + 3*(2*a^2
+ b^2)*d^3*e^2*x)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d^3*cosh(d*x + c)^2 +
2*b^3*d^3*cosh(d*x + c)*sinh(d*x + c) + b^3*d^3*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \cosh(dx+c)^2 \sinh(dx+c)}{b \sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)^2*sinh(d*x + c)/(b*sinh(d*x + c) + a),
x)
```

3.340 $\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=327

$$-\frac{af\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{af\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2}-\frac{a\sqrt{a^2+b^2}(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3d}+\cdots$$

```
[Out] (a^2*e*x)/b^3 + (e*x)/(2*b) + (a^2*f*x^2)/(2*b^3) + (f*x^2)/(4*b) - (a*(e +
 f*x)*Cosh[c + d*x])/(b^2*d) - (f*Cosh[c + d*x]^2)/(4*b*d^2) - (a*Sqrt[a^2
+ b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) +
(a*Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
)/(b^3*d) - (a*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2]))])/(b^3*d^2) + (a*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(b^3*d^2) + (a*f*Sinh[c + d*x])/(b^2*d^2) + ((e + f
*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.550004, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5579, 3310, 5565, 3296, 2637, 3322, 2264, 2190, 2279, 2391}

$$-\frac{af\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{af\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2}-\frac{a\sqrt{a^2+b^2}(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3d}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (a^2*e*x)/b^3 + (e*x)/(2*b) + (a^2*f*x^2)/(2*b^3) + (f*x^2)/(4*b) - (a*(e +
 f*x)*Cosh[c + d*x])/(b^2*d) - (f*Cosh[c + d*x]^2)/(4*b*d^2) - (a*Sqrt[a^2
+ b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) +
(a*Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
)/(b^3*d) - (a*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2]))])/(b^3*d^2) + (a*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(b^3*d^2) + (a*f*Sinh[c + d*x])/(b^2*d^2) + ((e + f
*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)
```


Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{f\cosh^2(c+dx)}{4bd^2} + \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{2bd} + \frac{a^2\int (e+fx)\,dx}{b^3} -$$

$$= \frac{a^2ex}{b^3} + \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} + \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} - \frac{f\cosh^2(c+dx)}{4bd^2} + (e$$

$$= \frac{a^2ex}{b^3} + \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} + \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} - \frac{f\cosh^2(c+dx)}{4bd^2} + a$$

$$= \frac{a^2ex}{b^3} + \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} + \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} - \frac{f\cosh^2(c+dx)}{4bd^2} - a$$

$$= \frac{a^2ex}{b^3} + \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} + \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} - \frac{f\cosh^2(c+dx)}{4bd^2} - a$$

$$= \frac{a^2ex}{b^3} + \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} + \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} - \frac{f\cosh^2(c+dx)}{4bd^2} - a$$

Mathematica [C] time = 4.61866, size = 1549, normalized size = 4.74

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (2*b^2*e*(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]]
)/(Sqrt[-a^2 - b^2]*d)) + b^2*f*(x^2 + (2*a*((I*Pi*ArcTanh[(-b + a*Tanh[(c
+ d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (2*((-I)*c + ArcCos[((-I)*a)
/b])*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]
] + ((-2*I)*c + Pi - (2*I)*d*x)*ArcTanh[((a - I*b)*Tan[((2*I)*c + Pi + (2*I
)*d*x)/4])/Sqrt[-a^2 - b^2]] - (ArcCos[((-I)*a)/b] + (2*I)*ArcTanh[((a + I*
b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[((I*a + b)*(a
+ I*(b + Sqrt[-a^2 - b^2]))*(-I + Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(I
*a + b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))] - (ArcCos[
((-I)*a)/b] - (2*I)*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/S
qrt[-a^2 - b^2]])*Log[((I*a + b)*(I*a - b + Sqrt[-a^2 - b^2])*(I + Cot[((2*
I)*c + Pi + (2*I)*d*x)/4]))/(b*(a - I*b + Sqrt[-a^2 - b^2]*Cot[((2*I)*c + P
i + (2*I)*d*x)/4]))] + (ArcCos[((-I)*a)/b] - (2*I)*ArcTanh[((a + I*b)*Cot[(
(2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] - (2*I)*ArcTanh[((a - I*b)*
Tan[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[-(((-1)^(3/4)*Sqr
t[-a^2 - b^2]*E^(-c/2 - (d*x)/2))/(Sqrt[2]*Sqrt[(-I)*b]*Sqrt[a + b*Sinh[c +
 d*x]]))] + (ArcCos[((-I)*a)/b] + (2*I)*(ArcTanh[((a + I*b)*Cot[((2*I)*c +
Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] + ArcTanh[((a - I*b)*Tan[((2*I)*c + P
i + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]]))*Log[((-1)^(1/4)*Sqrt[-a^2 - b^2]*E^(
(c + d*x)/2))/(Sqrt[2]*Sqrt[(-I)*b]*Sqrt[a + b*Sinh[c + d*x]])] + I*(PolyLo
g[2, ((I*a + Sqrt[-a^2 - b^2])*(I*a + b - I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c +
 Pi + (2*I)*d*x)/4]))/(b*(I*a + b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi +
(2*I)*d*x)/4]))] - PolyLog[2, ((a + I*Sqrt[-a^2 - b^2])*(-a + I*b + Sqrt[-a
^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(I*a + b + I*Sqrt[-a^2 - b
^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))]))/Sqrt[-a^2 - b^2]))/d^2) + (2*e*(
(4*a^2 + b^2)*(c + d*x) - (2*a*(4*a^2 + 3*b^2)*ArcTan[(b - a*Tanh[(c + d*x)
```

```
/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 4*a*b*Cosh[c + d*x] + b^2*Sinh[2
*(c + d*x)]))/d + (f*((4*a^2 + b^2)*(-c + d*x)*(c + d*x) - 8*a*b*d*x*Cosh[c
 + d*x] - b^2*Cosh[2*(c + d*x)] - (2*a*(4*a^2 + 3*b^2)*(2*c*ArcTanh[(a + b*
Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + (c + d*x)*Log[1 + (b*(C
osh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] - (c + d*x)*Log[1 + (
b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])] + PolyLog[2, (b*(
Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])] - PolyLog[2, -((b*(
Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2] +
 8*a*b*Sinh[c + d*x] + 2*b^2*d*x*Sinh[2*(c + d*x)]))/d^2)/(8*b^3)
```

Maple [B] time = 0.082, size = 1012, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/2*a^2*f*x^2/b^3+1/4*f*x^2/b+a^2*e*x/b^3+1/2*e*x/b+1/16*(2*d*f*x+2*d*e-f)/
d^2/b*exp(2*d*x+2*c)-1/2*a*(d*f*x+d*e-f)/b^2/d^2*exp(d*x+c)-1/2*a*(d*f*x+d*
e+f)/b^2/d^2*exp(-d*x-c)-1/16*(2*d*f*x+2*d*e+f)/d^2/b*exp(-2*d*x-2*c)+2*a^3
/b^3/d*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+
2*a/b/d*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
-a^3/b^3/d*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+
b^2)^(1/2)))*x-a^3/b^3/d^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1
/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+a^3/b^3/d*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)
+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+a^3/b^3/d^2*f/(a^2+b^2)^(1/2)*ln
((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-a^3/b^3/d^2*f/(a^2
+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+a
^3/b^3/d^2*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2
+b^2)^(1/2)))-a/b/d*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
(-a+(a^2+b^2)^(1/2)))*x-a/b/d^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^
2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+a/b/d*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c
)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+a/b/d^2*f/(a^2+b^2)^(1/2)*ln((b
*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-a/b/d^2*f/(a^2+b^2)^(
1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+a/b/d^2*
f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)
))-2*a^3/b^3/d^2*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+
b^2)^(1/2))-2*a/b/d^2*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/
(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.38246, size = 3200, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -1/16*(2*b^2*d*f*x - (2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^4 - (2
*b^2*d*f*x + 2*b^2*d*e - b^2*f)*sinh(d*x + c)^4 + 2*b^2*d*e + 8*(a*b*d*f*x
+ a*b*d*e - a*b*f)*cosh(d*x + c)^3 + 4*(2*a*b*d*f*x + 2*a*b*d*e - 2*a*b*f -
 (2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c))*sinh(d*x + c)^3 + b^2*f -
 4*((2*a^2 + b^2)*d^2*f*x^2 + 2*(2*a^2 + b^2)*d^2*e*x)*cosh(d*x + c)^2 - 2*
(2*(2*a^2 + b^2)*d^2*f*x^2 + 4*(2*a^2 + b^2)*d^2*e*x + 3*(2*b^2*d*f*x + 2*b
^2*d*e - b^2*f)*cosh(d*x + c)^2 - 12*(a*b*d*f*x + a*b*d*e - a*b*f)*cosh(d*x
 + c))*sinh(d*x + c)^2 + 16*(a*b*f*cosh(d*x + c)^2 + 2*a*b*f*cosh(d*x + c)*
sinh(d*x + c) + a*b*f*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(
d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2) - b)/b + 1) - 16*(a*b*f*cosh(d*x + c)^2 + 2*a*b*f*cosh(d*x + c)
*sinh(d*x + c) + a*b*f*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh
(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b + 1) - 16*((a*b*d*e - a*b*c*f)*cosh(d*x + c)^2 + 2*(a*b
*d*e - a*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d*e - a*b*c*f)*sinh(d*x
+ c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2
*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 16*((a*b*d*e - a*b*c*f)*cosh(d*x + c)^2 +
 2*(a*b*d*e - a*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d*e - a*b*c*f)*si
nh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
 c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 16*((a*b*d*f*x + a*b*c*f)*cosh(d*x
 + c)^2 + 2*(a*b*d*f*x + a*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d*f*x
+ a*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a
*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b) - 16*((a*b*d*f*x + a*b*c*f)*cosh(d*x + c)^2 + 2*(a*b*d*f*x + a*b*c*
f)*cosh(d*x + c)*sinh(d*x + c) + (a*b*d*f*x + a*b*c*f)*sinh(d*x + c)^2)*sqr
t((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 8*(a*b*d*f*x + a*b*d*
e + a*b*f)*cosh(d*x + c) + 4*(2*a*b*d*f*x + 2*a*b*d*e - (2*b^2*d*f*x + 2*b^
2*d*e - b^2*f)*cosh(d*x + c)^3 + 2*a*b*f + 6*(a*b*d*f*x + a*b*d*e - a*b*f)*
cosh(d*x + c)^2 - 2*((2*a^2 + b^2)*d^2*f*x^2 + 2*(2*a^2 + b^2)*d^2*e*x)*cos
h(d*x + c))*sinh(d*x + c))/(b^3*d^2*cosh(d*x + c)^2 + 2*b^3*d^2*cosh(d*x +
c)*sinh(d*x + c) + b^3*d^2*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\cosh(dx+c)^2\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)^2*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.341 $\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{2a\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3d}+\frac{x\left(2a^2+b^2\right)}{2b^3}-\frac{\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}$$

```
[Out] ((2*a^2 + b^2)*x)/(2*b^3) + (2*a*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[(c + d
*x)/2])/Sqrt[a^2 + b^2]])/(b^3*d) - (Cosh[c + d*x]*(2*a - b*Sinh[c + d*x]))
/(2*b^2*d)
```

Rubi [A] time = 0.180593, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2865, 2735, 2660, 618, 204}

$$\frac{2a\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3d}+\frac{x\left(2a^2+b^2\right)}{2b^3}-\frac{\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((2*a^2 + b^2)*x)/(2*b^3) + (2*a*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[(c + d
*x)/2])/Sqrt[a^2 + b^2]])/(b^3*d) - (Cosh[c + d*x]*(2*a - b*Sinh[c + d*x]))
/(2*b^2*d)
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}+\frac{i\int \frac{iab-i\left(2a^2+b^2\right)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx}{2b^2}$$

$$=\frac{\left(2a^2+b^2\right)x}{2b^3}-\frac{\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}-\frac{\left(a\left(a^2+b^2\right)\right)\int \frac{1}{a+b\sinh(c+dx)}\,d}{b^3}$$

$$=\frac{\left(2a^2+b^2\right)x}{2b^3}-\frac{\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}+\frac{\left(2ia\left(a^2+b^2\right)\right)\text{Subst}\left(\int \frac{}{a-2i}\right.}{b}$$

$$=\frac{\left(2a^2+b^2\right)x}{2b^3}-\frac{\cosh(c+dx)(2a-b\sinh(c+dx))}{2b^2d}-\frac{\left(4ia\left(a^2+b^2\right)\right)\text{Subst}\left(\int \frac{}{-4(a}\right.}{}$$

$$=\frac{\left(2a^2+b^2\right)x}{2b^3}+\frac{2a\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3d}-\frac{\cosh(c+dx)(2a-b\sinh}{2b^2d}$$

Mathematica [A] time = 0.427519, size = 109, normalized size = 1.15

$$\frac{8a\sqrt{-a^2-b^2}\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)+4a^2c+4a^2dx-4ab\cosh(c+dx)+b^2\sinh(2(c+dx))+2b^2c+2b^2dx}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (4*a^2*c + 2*b^2*c + 4*a^2*d*x + 2*b^2*d*x + 8*a*Sqrt[-a^2 - b^2]*ArcTan[(b
 - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] - 4*a*b*Cosh[c + d*x] + b^2*Sinh[
2*(c + d*x)])/(4*b^3*d)
```

Maple [B] time = 0.035, size = 260, normalized size = 2.7

$$-\frac{1}{2\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+\frac{1}{2\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{a}{db^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+\frac{a^2}{db^3}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)-1/d/b^2/
(tanh(1/2*d*x+1/2*c)+1)*a+1/d/b^3*ln(tanh(1/2*d*x+1/2*c)+1)*a^2+1/2/d/b*ln(
```

```
tanh(1/2*d*x+1/2*c)+1)+1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/b/(tanh(1/2*
d*x+1/2*c)-1)+1/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*a-1/d/b^3*ln(tanh(1/2*d*x+1/2
*c)-1)*a^2-1/2/d/b*ln(tanh(1/2*d*x+1/2*c)-1)-2/d*a*(a^2+b^2)^(1/2)/b^3*arct
anh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.27919, size = 1150, normalized size = 12.11

$$b^2\cosh(dx+c)^4 + b^2\sinh(dx+c)^4 + 4\left(2a^2+b^2\right)dx\cosh(dx+c)^2 - 4ab\cosh(dx+c)^3 + 4\left(b^2\cosh(dx+c)-ab\right)\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/8*(b^2*cosh(d*x + c)^4 + b^2*sinh(d*x + c)^4 + 4*(2*a^2 + b^2)*d*x*cosh(d
*x + c)^2 - 4*a*b*cosh(d*x + c)^3 + 4*(b^2*cosh(d*x + c) - a*b)*sinh(d*x +
c)^3 - 4*a*b*cosh(d*x + c) + 2*(3*b^2*cosh(d*x + c)^2 + 2*(2*a^2 + b^2)*d*x
 - 6*a*b*cosh(d*x + c))*sinh(d*x + c)^2 + 8*(a*cosh(d*x + c)^2 + 2*a*cosh(d
*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2)*sqrt(a^2 + b^2)*log((b^2*cosh(d*
x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2
*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) +
b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x
 + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - b^2 + 4*(b^2*cosh(d*x
 + c)^3 + 2*(2*a^2 + b^2)*d*x*cosh(d*x + c) - 3*a*b*cosh(d*x + c)^2 - a*b)*
sinh(d*x + c))/(b^3*d*cosh(d*x + c)^2 + 2*b^3*d*cosh(d*x + c)*sinh(d*x + c)
 + b^3*d*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15216, size = 223, normalized size = 2.35

$$\frac{\left(2\,a^2+b^2\right)(dx+c)}{2\,b^3d}-\frac{\left(4\,abe^{(dx+c)}+b^2\right)e^{(-2\,dx-2\,c)}}{8\,b^3d}-\frac{\left(a^3+ab^2\right)\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^3d}+\frac{bde^{(2\,dx+2\,c)}-4\,ade^{(dx+c)}}{8\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2 + b^2)*(d*x + c)/(b^3*d) - 1/8*(4*a*b*e^(d*x + c) + b^2)*e^(-2*d
*x - 2*c)/(b^3*d) - (a^3 + a*b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^
2 + b^2))/abs(2*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*
b^3*d) + 1/8*(b*d*e^(2*d*x + 2*c) - 4*a*d*e^(d*x + c))/(b^2*d^2)
```

3.342 $\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{\sinh(c+dx)\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Cosh[c + d*x]^2*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [A] time = 0.0815182, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[c + d*x]^2*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Cosh[c + d*x]^2*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^2(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.109, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx+c))^2\sinh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-2\left(a^3e^c + ab^2e^c\right)\int -\frac{e^{(dx)}}{b^4fx + b^4e - \left(b^4fxe^{(2c)} + b^4ee^{(2c)}\right)e^{(2dx)} - 2\left(ab^3fxe^c + ab^3ee^c\right)e^{(dx)}}\,dx - \frac{e^{\left(-2c+\frac{2de}{f}\right)}E_1\left(\frac{2(fx+e)d}{f}\right.}{4bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -2*(a^3*e^c + a*b^2*e^c)*integrate(-e^(d*x)/(b^4*f*x + b^4*e - (b^4*f*x*e^(
2*c) + b^4*e*e^(2*c))*e^(2*d*x) - 2*(a*b^3*f*x*e^c + a*b^3*e*e^c)*e^(d*x)),
 x) - 1/4*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b*f) - 1/2
*a*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^2*f) + 1/2*a*e^(c - d
*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^2*f) - 1/4*e^(2*c - 2*d*e/f)*exp
_integral_e(1, -2*(f*x + e)*d/f)/(b*f) + 1/2*(2*a^2 + b^2)*log(f*x + e)/(b^
3*f)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx+c)^2\sinh(dx+c)}{afx + ae + \left(bfx + be\right)\sinh(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(cosh(d*x + c)^2*sinh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^2\sinh(dx+c)}{\left(fx+e\right)\left(b\sinh(dx+c)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate(cosh(d*x + c)^2*sinh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)),
x)
```

3.343 $\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=864

$$\frac{a\left(a^2+b^2\right)(e+fx)^4}{4b^4f}-\frac{a\sinh^2(c+dx)(e+fx)^3}{2b^2d}-\frac{a\left(a^2+b^2\right)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)(e+fx)^3}{b^4d}-\frac{a\left(a^2+b^2\right)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}\right.}{b^4d}$$

```
[Out] (-3*a*f^3*x)/(8*b^2*d^3) - (a*(e + f*x)^3)/(4*b^2*d) + (a*(a^2 + b^2)*(e +
f*x)^4)/(4*b^4*f) - (6*a^2*f^3*Cosh[c + d*x])/(b^3*d^4) - (40*f^3*Cosh[c +
d*x])/(9*b*d^4) - (3*a^2*f*(e + f*x)^2*Cosh[c + d*x])/(b^3*d^2) - (2*f*(e +
 f*x)^2*Cosh[c + d*x])/(b*d^2) - (2*f^3*Cosh[c + d*x]^3)/(27*b*d^4) - (f*(e
 + f*x)^2*Cosh[c + d*x]^3)/(3*b*d^2) - (a*(a^2 + b^2)*(e + f*x)^3*Log[1 + (
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a*(a^2 + b^2)*(e + f*x)^3
*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^4*d) - (3*a*(a^2 + b^2)
*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d
^2) - (3*a*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2]))])/(b^4*d^2) + (6*a*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^3) + (6*a*(a^2 + b^2)*f^2*(e +
 f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^3) - (6*
a*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^
4*d^4) - (6*a*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))])/(b^4*d^4) + (6*a^2*f^2*(e + f*x)*Sinh[c + d*x])/(b^3*d^3) + (40*f^
2*(e + f*x)*Sinh[c + d*x])/(9*b*d^3) + (a^2*(e + f*x)^3*Sinh[c + d*x])/(b^3
*d) + (2*(e + f*x)^3*Sinh[c + d*x])/(3*b*d) + (3*a*f^3*Cosh[c + d*x]*Sinh[c
 + d*x])/(8*b^2*d^4) + (3*a*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b
^2*d^2) + (2*f^2*(e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(9*b*d^3) + ((e +
 f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b*d) - (3*a*f^2*(e + f*x)*Sinh[c
+ d*x]^2)/(4*b^2*d^3) - (a*(e + f*x)^3*Sinh[c + d*x]^2)/(2*b^2*d)
```

Rubi [A] time = 1.11732, antiderivative size = 864, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 16, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {5579, 3311, 3296, 2638, 3310, 5565, 5446, 32, 2635, 8, 5561, 2190, 2531, 6609, 2282, 6589}

$$\frac{a\left(a^2+b^2\right)(e+fx)^4}{4b^4f}-\frac{a\sinh^2(c+dx)(e+fx)^3}{2b^2d}-\frac{a\left(a^2+b^2\right)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)(e+fx)^3}{b^4d}-\frac{a\left(a^2+b^2\right)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}\right.}{b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (-3*a*f^3*x)/(8*b^2*d^3) - (a*(e + f*x)^3)/(4*b^2*d) + (a*(a^2 + b^2)*(e +
f*x)^4)/(4*b^4*f) - (6*a^2*f^3*Cosh[c + d*x])/(b^3*d^4) - (40*f^3*Cosh[c +
d*x])/(9*b*d^4) - (3*a^2*f*(e + f*x)^2*Cosh[c + d*x])/(b^3*d^2) - (2*f*(e +
 f*x)^2*Cosh[c + d*x])/(b*d^2) - (2*f^3*Cosh[c + d*x]^3)/(27*b*d^4) - (f*(e
 + f*x)^2*Cosh[c + d*x]^3)/(3*b*d^2) - (a*(a^2 + b^2)*(e + f*x)^3*Log[1 + (
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a*(a^2 + b^2)*(e + f*x)^3
*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^4*d) - (3*a*(a^2 + b^2)
*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d
^2) - (3*a*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2]))])/(b^4*d^2) + (6*a*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^3) + (6*a*(a^2 + b^2)*f^2*(e +
 f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^3) - (6*
a*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^
4*d^4) - (6*a*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
```

```
b^2]))])/(b^4*d^4) + (6*a^2*f^2*(e + f*x)*Sinh[c + d*x])/(b^3*d^3) + (40*f^
2*(e + f*x)*Sinh[c + d*x])/(9*b*d^3) + (a^2*(e + f*x)^3*Sinh[c + d*x])/(b^3
*d) + (2*(e + f*x)^3*Sinh[c + d*x])/(3*b*d) + (3*a*f^3*Cosh[c + d*x]*Sinh[c
 + d*x])/(8*b^2*d^4) + (3*a*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b
^2*d^2) + (2*f^2*(e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(9*b*d^3) + ((e +
 f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b*d) - (3*a*f^2*(e + f*x)*Sinh[c
+ d*x]^2)/(4*b^2*d^3) - (a*(e + f*x)^3*Sinh[c + d*x]^2)/(2*b^2*d)
```

Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
```

```
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\cosh^3(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{f(e+fx)^2\cosh^3(c+dx)}{3bd^2} + \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{3bd} + \frac{a^2\int(\cdots}{}$$

$$= \frac{a\left(a^2+b^2\right)(e+fx)^4}{4b^4f} - \frac{2f^3\cosh^3(c+dx)}{27bd^4} - \frac{f(e+fx)^2\cosh^3(c+dx)}{3bd^2} + \frac{a^2}{}$$

$$= \frac{a\left(a^2+b^2\right)(e+fx)^4}{4b^4f} - \frac{3a^2f(e+fx)^2\cosh(c+dx)}{b^3d^2} - \frac{2f(e+fx)^2\cosh(c+}{bd^2}$$

$$= -\frac{a(e+fx)^3}{4b^2d} + \frac{a\left(a^2+b^2\right)(e+fx)^4}{4b^4f} - \frac{4f^3\cosh(c+dx)}{9bd^4} - \frac{3a^2f(e+fx)^2\,\mathrm{co}}{b^3d^2}$$

$$= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a\left(a^2+b^2\right)(e+fx)^4}{4b^4f} - \frac{6a^2f^3\cosh(c+dx)}{b^3d^4} - \frac{40f^3}{}$$

$$= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a\left(a^2+b^2\right)(e+fx)^4}{4b^4f} - \frac{6a^2f^3\cosh(c+dx)}{b^3d^4} - \frac{40f^3}{}$$

$$= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a\left(a^2+b^2\right)(e+fx)^4}{4b^4f} - \frac{6a^2f^3\cosh(c+dx)}{b^3d^4} - \frac{40f^3}{}$$

Mathematica [B] time = 51.0765, size = 7460, normalized size = 8.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.24, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3(\cosh(dx+c))^3\sinh(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/24*e^3*((3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 + 3*b^2)*e^(-2*d*x - 2*c))*
e^(3*d*x + 3*c)/(b^3*d) + 24*(a^3 + a*b^2)*(d*x + c)/(b^4*d) + (3*a*b*e^(-2
*d*x - 2*c) + b^2*e^(-3*d*x - 3*c) + 3*(4*a^2 + 3*b^2)*e^(-d*x - c))/(b^3*d
) + 24*(a^3 + a*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^4*d
)) - 1/864*(216*(a^3*d^4*f^3*e^(3*c) + a*b^2*d^4*f^3*e^(3*c))*x^4 + 864*(a^
3*d^4*e*f^2*e^(3*c) + a*b^2*d^4*e*f^2*e^(3*c))*x^3 + 1296*(a^3*d^4*e^2*f*e^
(3*c) + a*b^2*d^4*e^2*f*e^(3*c))*x^2 - 4*(9*b^3*d^3*f^3*x^3*e^(6*c) + 9*(3*
d^3*e*f^2 - d^2*f^3)*b^3*x^2*e^(6*c) + 3*(9*d^3*e^2*f - 6*d^2*e*f^2 + 2*d*f
^3)*b^3*x*e^(6*c) - (9*d^2*e^2*f - 6*d*e*f^2 + 2*f^3)*b^3*e^(6*c))*e^(3*d*x
) + 27*(4*a*b^2*d^3*f^3*x^3*e^(5*c) + 6*(2*d^3*e*f^2 - d^2*f^3)*a*b^2*x^2*e
^(5*c) + 6*(2*d^3*e^2*f - 2*d^2*e*f^2 + d*f^3)*a*b^2*x*e^(5*c) - 3*(2*d^2*e
^2*f - 2*d*e*f^2 + f^3)*a*b^2*e^(5*c))*e^(2*d*x) + 108*(12*(d^2*e^2*f - 2*d
*e*f^2 + 2*f^3)*a^2*b*e^(4*c) + 9*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*b^3*e^(4*
c) - (4*a^2*b*d^3*f^3*e^(4*c) + 3*b^3*d^3*f^3*e^(4*c))*x^3 - 3*(4*(d^3*e*f^
2 - d^2*f^3)*a^2*b*e^(4*c) + 3*(d^3*e*f^2 - d^2*f^3)*b^3*e^(4*c))*x^2 - 3*(
4*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*a^2*b*e^(4*c) + 3*(d^3*e^2*f - 2*d^2*
e*f^2 + 2*d*f^3)*b^3*e^(4*c))*x)*e^(d*x) + 108*(12*(d^2*e^2*f + 2*d*e*f^2 +
 2*f^3)*a^2*b*e^(2*c) + 9*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*b^3*e^(2*c) + (4*
a^2*b*d^3*f^3*e^(2*c) + 3*b^3*d^3*f^3*e^(2*c))*x^3 + 3*(4*(d^3*e*f^2 + d^2*
f^3)*a^2*b*e^(2*c) + 3*(d^3*e*f^2 + d^2*f^3)*b^3*e^(2*c))*x^2 + 3*(4*(d^3*e
^2*f + 2*d^2*e*f^2 + 2*d*f^3)*a^2*b*e^(2*c) + 3*(d^3*e^2*f + 2*d^2*e*f^2 +
2*d*f^3)*b^3*e^(2*c))*x)*e^(-d*x) + 27*(4*a*b^2*d^3*f^3*x^3*e^c + 6*(2*d^3*
e*f^2 + d^2*f^3)*a*b^2*x^2*e^c + 6*(2*d^3*e^2*f + 2*d^2*e*f^2 + d*f^3)*a*b^
2*x*e^c + 3*(2*d^2*e^2*f + 2*d*e*f^2 + f^3)*a*b^2*e^c)*e^(-2*d*x) + 4*(9*b^
3*d^3*f^3*x^3 + 9*(3*d^3*e*f^2 + d^2*f^3)*b^3*x^2 + 3*(9*d^3*e^2*f + 6*d^2*
e*f^2 + 2*d*f^3)*b^3*x + (9*d^2*e^2*f + 6*d*e*f^2 + 2*f^3)*b^3)*e^(-3*d*x))
*e^(-3*c)/(b^4*d^4) + integrate(-2*((a^3*b*f^3 + a*b^3*f^3)*x^3 + 3*(a^3*b*
e*f^2 + a*b^3*e*f^2)*x^2 + 3*(a^3*b*e^2*f + a*b^3*e^2*f)*x - ((a^4*f^3*e^c
+ a^2*b^2*f^3*e^c)*x^3 + 3*(a^4*e*f^2*e^c + a^2*b^2*e*f^2*e^c)*x^2 + 3*(a^4
*e^2*f*e^c + a^2*b^2*e^2*f*e^c)*x)*e^(d*x))/(b^5*e^(2*d*x + 2*c) + 2*a*b^4*
e^(d*x + c) - b^5), x)
```

Fricas [C] time = 3.58887, size = 17256, normalized size = 19.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/864*(36*b^3*d^3*f^3*x^3 + 36*b^3*d^3*e^3 + 36*b^3*d^2*e^2*f + 24*b^3*d*e
*f^2 - 4*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f
^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f
 - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*cosh(d*x + c)^6 - 4*(9*b^3*d^3*f^3*x^3
 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d
```

^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*
d*f^3)*x)*sinh(d*x + c)^6 + 8*b^3*f^3 + 27*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d
^3*e^3 - 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3
*e*f^2 - a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*
b^2*d*f^3)*x)*cosh(d*x + c)^5 + 3*(36*a*b^2*d^3*f^3*x^3 + 36*a*b^2*d^3*e^3
- 54*a*b^2*d^2*e^2*f + 54*a*b^2*d*e*f^2 - 27*a*b^2*f^3 + 54*(2*a*b^2*d^3*e*
f^2 - a*b^2*d^2*f^3)*x^2 + 54*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^
2*d*f^3)*x - 8*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3
*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3
*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c)^5 -
108*((4*a^2*b + 3*b^3)*d^3*f^3*x^3 + (4*a^2*b + 3*b^3)*d^3*e^3 - 3*(4*a^2*
b + 3*b^3)*d^2*e^2*f + 6*(4*a^2*b + 3*b^3)*d*e*f^2 - 6*(4*a^2*b + 3*b^3)*f^
3 + 3*((4*a^2*b + 3*b^3)*d^3*e*f^2 - (4*a^2*b + 3*b^3)*d^2*f^3)*x^2 + 3*((4
*a^2*b + 3*b^3)*d^3*e^2*f - 2*(4*a^2*b + 3*b^3)*d^2*e*f^2 + 2*(4*a^2*b + 3*
b^3)*d*f^3)*x)*cosh(d*x + c)^4 - 3*(36*(4*a^2*b + 3*b^3)*d^3*f^3*x^3 + 36*(
4*a^2*b + 3*b^3)*d^3*e^3 - 108*(4*a^2*b + 3*b^3)*d^2*e^2*f + 216*(4*a^2*b +
3*b^3)*d*e*f^2 - 216*(4*a^2*b + 3*b^3)*f^3 + 108*((4*a^2*b + 3*b^3)*d^3*e*
f^2 - (4*a^2*b + 3*b^3)*d^2*f^3)*x^2 + 20*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^
3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*
d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*cosh(
d*x + c)^2 + 108*((4*a^2*b + 3*b^3)*d^3*e^2*f - 2*(4*a^2*b + 3*b^3)*d^2*e*f
^2 + 2*(4*a^2*b + 3*b^3)*d*f^3)*x - 45*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e
^3 - 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f
^2 - a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^2*
d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c)^4 - 216*((a^3 + a*b^2)*d^4*f^3*x^4 +
4*(a^3 + a*b^2)*d^4*e*f^2*x^3 + 6*(a^3 + a*b^2)*d^4*e^2*f*x^2 + 4*(a^3 + a
*b^2)*d^4*e^3*x + 8*(a^3 + a*b^2)*c*d^3*e^3 - 12*(a^3 + a*b^2)*c^2*d^2*e^2*
f + 8*(a^3 + a*b^2)*c^3*d*e*f^2 - 2*(a^3 + a*b^2)*c^4*f^3)*cosh(d*x + c)^3
- 2*(108*(a^3 + a*b^2)*d^4*f^3*x^4 + 432*(a^3 + a*b^2)*d^4*e*f^2*x^3 + 648*
(a^3 + a*b^2)*d^4*e^2*f*x^2 + 432*(a^3 + a*b^2)*d^4*e^3*x + 864*(a^3 + a*b^
2)*c*d^3*e^3 - 1296*(a^3 + a*b^2)*c^2*d^2*e^2*f + 864*(a^3 + a*b^2)*c^3*d*e
f^2 - 216(a^3 + a*b^2)*c^4*f^3 + 40*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 -
9*b^3*d^2*e^2*f + 6*b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*
f^3)*x^2 + 3*(9*b^3*d^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*cosh(d*x
+ c)^3 - 135*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e^3 - 6*a*b^2*d^2*e^2*f + 6
*a*b^2*d*e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f^2 - a*b^2*d^2*f^3)*x^2 +
6*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x)*cosh(d*x + c)^2
+ 216*((4*a^2*b + 3*b^3)*d^3*f^3*x^3 + (4*a^2*b + 3*b^3)*d^3*e^3 - 3*(4*a^2
*b + 3*b^3)*d^2*e^2*f + 6*(4*a^2*b + 3*b^3)*d*e*f^2 - 6*(4*a^2*b + 3*b^3)*f
^3 + 3*((4*a^2*b + 3*b^3)*d^3*e*f^2 - (4*a^2*b + 3*b^3)*d^2*f^3)*x^2 + 3*((
4*a^2*b + 3*b^3)*d^3*e^2*f - 2*(4*a^2*b + 3*b^3)*d^2*e*f^2 + 2*(4*a^2*b + 3
*b^3)*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 36*(3*b^3*d^3*e*f^2 + b^3*
d^2*f^3)*x^2 + 108*((4*a^2*b + 3*b^3)*d^3*f^3*x^3 + (4*a^2*b + 3*b^3)*d^3*e
^3 + 3*(4*a^2*b + 3*b^3)*d^2*e^2*f + 6*(4*a^2*b + 3*b^3)*d*e*f^2 + 6*(4*a^2
*b + 3*b^3)*f^3 + 3*((4*a^2*b + 3*b^3)*d^3*e*f^2 + (4*a^2*b + 3*b^3)*d^2*f^
3)*x^2 + 3*((4*a^2*b + 3*b^3)*d^3*e^2*f + 2*(4*a^2*b + 3*b^3)*d^2*e*f^2 + 2
*(4*a^2*b + 3*b^3)*d*f^3)*x)*cosh(d*x + c)^2 + 6*(18*(4*a^2*b + 3*b^3)*d^3*
f^3*x^3 + 18*(4*a^2*b + 3*b^3)*d^3*e^3 + 54*(4*a^2*b + 3*b^3)*d^2*e^2*f + 1
08*(4*a^2*b + 3*b^3)*d*e*f^2 - 10*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^
3*d^2*e^2*f + 6*b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)
x^2 + 3(9*b^3*d^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*cosh(d*x + c)
^4 + 108*(4*a^2*b + 3*b^3)*f^3 + 45*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e^3
- 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f^2
- a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^2*d*f
^3)*x)*cosh(d*x + c)^3 + 54*((4*a^2*b + 3*b^3)*d^3*e*f^2 + (4*a^2*b + 3*b^3
)*d^2*f^3)*x^2 - 108*((4*a^2*b + 3*b^3)*d^3*f^3*x^3 + (4*a^2*b + 3*b^3)*d^3
e^3 - 3(4*a^2*b + 3*b^3)*d^2*e^2*f + 6*(4*a^2*b + 3*b^3)*d*e*f^2 - 6*(4*a
^2*b + 3*b^3)*f^3 + 3*((4*a^2*b + 3*b^3)*d^3*e*f^2 - (4*a^2*b + 3*b^3)*d^2*
f^3)*x^2 + 3*((4*a^2*b + 3*b^3)*d^3*e^2*f - 2*(4*a^2*b + 3*b^3)*d^2*e*f^2 +

2*(4*a^2*b + 3*b^3)*d*f^3)*x)*cosh(d*x + c)^2 + 54*((4*a^2*b + 3*b^3)*d^3*
e^2*f + 2*(4*a^2*b + 3*b^3)*d^2*e*f^2 + 2*(4*a^2*b + 3*b^3)*d*f^3)*x - 108*
((a^3 + a*b^2)*d^4*f^3*x^4 + 4*(a^3 + a*b^2)*d^4*e*f^2*x^3 + 6*(a^3 + a*b^2
)*d^4*e^2*f*x^2 + 4*(a^3 + a*b^2)*d^4*e^3*x + 8*(a^3 + a*b^2)*c*d^3*e^3 - 1
2*(a^3 + a*b^2)*c^2*d^2*e^2*f + 8*(a^3 + a*b^2)*c^3*d*e*f^2 - 2*(a^3 + a*b^
2)*c^4*f^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 12*(9*b^3*d^3*e^2*f + 6*b^3*d^
2*e*f^2 + 2*b^3*d*f^3)*x + 27*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e^3 + 6*a*
b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 + 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f^2 + a*b^
2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*e^2*f + 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x)
*cosh(d*x + c) + 2592*(((a^3 + a*b^2)*d^2*f^3*x^2 + 2*(a^3 + a*b^2)*d^2*e*f
^2*x + (a^3 + a*b^2)*d^2*e^2*f)*cosh(d*x + c)^3 + 3*((a^3 + a*b^2)*d^2*f^3*
x^2 + 2*(a^3 + a*b^2)*d^2*e*f^2*x + (a^3 + a*b^2)*d^2*e^2*f)*cosh(d*x + c)^
2*sinh(d*x + c) + 3*((a^3 + a*b^2)*d^2*f^3*x^2 + 2*(a^3 + a*b^2)*d^2*e*f^2*
x + (a^3 + a*b^2)*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c)^2 + ((a^3 + a*b^2)
*d^2*f^3*x^2 + 2*(a^3 + a*b^2)*d^2*e*f^2*x + (a^3 + a*b^2)*d^2*e^2*f)*sinh(
d*x + c)^3)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2592*(((a^3 + a*b^2)*d^
2*f^3*x^2 + 2*(a^3 + a*b^2)*d^2*e*f^2*x + (a^3 + a*b^2)*d^2*e^2*f)*cosh(d*x
 + c)^3 + 3*((a^3 + a*b^2)*d^2*f^3*x^2 + 2*(a^3 + a*b^2)*d^2*e*f^2*x + (a^3
 + a*b^2)*d^2*e^2*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3*((a^3 + a*b^2)*d^2*f
^3*x^2 + 2*(a^3 + a*b^2)*d^2*e*f^2*x + (a^3 + a*b^2)*d^2*e^2*f)*cosh(d*x +
c)*sinh(d*x + c)^2 + ((a^3 + a*b^2)*d^2*f^3*x^2 + 2*(a^3 + a*b^2)*d^2*e*f^2
*x + (a^3 + a*b^2)*d^2*e^2*f)*sinh(d*x + c)^3)*dilog((a*cosh(d*x + c) + a*s
inh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
b)/b + 1) + 864*(((a^3 + a*b^2)*d^3*e^3 - 3*(a^3 + a*b^2)*c*d^2*e^2*f + 3*(
a^3 + a*b^2)*c^2*d*e*f^2 - (a^3 + a*b^2)*c^3*f^3)*cosh(d*x + c)^3 + 3*((a^3
 + a*b^2)*d^3*e^3 - 3*(a^3 + a*b^2)*c*d^2*e^2*f + 3*(a^3 + a*b^2)*c^2*d*e*f
^2 - (a^3 + a*b^2)*c^3*f^3)*cosh(d*x + c)^2*sinh(d*x + c) + 3*((a^3 + a*b^2
)*d^3*e^3 - 3*(a^3 + a*b^2)*c*d^2*e^2*f + 3*(a^3 + a*b^2)*c^2*d*e*f^2 - (a^
3 + a*b^2)*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c)^2 + ((a^3 + a*b^2)*d^3*e^3
- 3*(a^3 + a*b^2)*c*d^2*e^2*f + 3*(a^3 + a*b^2)*c^2*d*e*f^2 - (a^3 + a*b^2)
*c^3*f^3)*sinh(d*x + c)^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*
sqrt((a^2 + b^2)/b^2) + 2*a) + 864*(((a^3 + a*b^2)*d^3*e^3 - 3*(a^3 + a*b^2
)*c*d^2*e^2*f + 3*(a^3 + a*b^2)*c^2*d*e*f^2 - (a^3 + a*b^2)*c^3*f^3)*cosh(d
x + c)^3 + 3((a^3 + a*b^2)*d^3*e^3 - 3*(a^3 + a*b^2)*c*d^2*e^2*f + 3*(a^3
 + a*b^2)*c^2*d*e*f^2 - (a^3 + a*b^2)*c^3*f^3)*cosh(d*x + c)^2*sinh(d*x + c
) + 3*((a^3 + a*b^2)*d^3*e^3 - 3*(a^3 + a*b^2)*c*d^2*e^2*f + 3*(a^3 + a*b^2
)*c^2*d*e*f^2 - (a^3 + a*b^2)*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c)^2 + ((a^
3 + a*b^2)*d^3*e^3 - 3*(a^3 + a*b^2)*c*d^2*e^2*f + 3*(a^3 + a*b^2)*c^2*d*e*
f^2 - (a^3 + a*b^2)*c^3*f^3)*sinh(d*x + c)^3)*log(2*b*cosh(d*x + c) + 2*b*s
inh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 864*(((a^3 + a*b^2)*d^3*f
^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a
^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f
^3)*cosh(d*x + c)^3 + 3*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*
f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^
3 + a*b^2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*cosh(d*x + c)^2*sinh(d*x +
c) + 3*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3
+ a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*
e*f^2 + (a^3 + a*b^2)*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c)^2 + ((a^3 + a*b^
2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*
x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 + (a^3 + a*b^
2)*c^3*f^3)*sinh(d*x + c)^3)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 864*(((a^3
+ a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*
e^2*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 + (a^3
+ a*b^2)*c^3*f^3)*cosh(d*x + c)^3 + 3*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 +
 a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*d^2
*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*cosh(d*x + c)
^2*sinh(d*x + c) + 3*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2

```
*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 +
 a*b^2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c)^2
+ ((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b
^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2
 + (a^3 + a*b^2)*c^3*f^3)*sinh(d*x + c)^3)*log(-(a*cosh(d*x + c) + a*sinh(d
*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b)
 + 5184*((a^3 + a*b^2)*f^3*cosh(d*x + c)^3 + 3*(a^3 + a*b^2)*f^3*cosh(d*x +
 c)^2*sinh(d*x + c) + 3*(a^3 + a*b^2)*f^3*cosh(d*x + c)*sinh(d*x + c)^2 + (
a^3 + a*b^2)*f^3*sinh(d*x + c)^3)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x
+ c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 5184
*((a^3 + a*b^2)*f^3*cosh(d*x + c)^3 + 3*(a^3 + a*b^2)*f^3*cosh(d*x + c)^2*s
inh(d*x + c) + 3*(a^3 + a*b^2)*f^3*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3 + a
*b^2)*f^3*sinh(d*x + c)^3)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) -
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 5184*(((a^3
 + a*b^2)*d*f^3*x + (a^3 + a*b^2)*d*e*f^2)*cosh(d*x + c)^3 + 3*((a^3 + a*b^
2)*d*f^3*x + (a^3 + a*b^2)*d*e*f^2)*cosh(d*x + c)^2*sinh(d*x + c) + 3*((a^3
 + a*b^2)*d*f^3*x + (a^3 + a*b^2)*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c)^2 +
((a^3 + a*b^2)*d*f^3*x + (a^3 + a*b^2)*d*e*f^2)*sinh(d*x + c)^3)*polylog(3,
 (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 + b^2)/b^2))/b) - 5184*(((a^3 + a*b^2)*d*f^3*x + (a^3 + a*b^2)*d*e
*f^2)*cosh(d*x + c)^3 + 3*((a^3 + a*b^2)*d*f^3*x + (a^3 + a*b^2)*d*e*f^2)*c
osh(d*x + c)^2*sinh(d*x + c) + 3*((a^3 + a*b^2)*d*f^3*x + (a^3 + a*b^2)*d*e
*f^2)*cosh(d*x + c)*sinh(d*x + c)^2 + ((a^3 + a*b^2)*d*f^3*x + (a^3 + a*b^2
)*d*e*f^2)*sinh(d*x + c)^3)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) -
 (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 3*(36*a*b^
2*d^3*f^3*x^3 + 36*a*b^2*d^3*e^3 + 54*a*b^2*d^2*e^2*f + 54*a*b^2*d*e*f^2 +
27*a*b^2*f^3 - 8*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b
^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d
^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*cosh(d*x + c)^5 + 45*(4*a*b^2*
d^3*f^3*x^3 + 4*a*b^2*d^3*e^3 - 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 - 3*a*b
^2*f^3 + 6*(2*a*b^2*d^3*e*f^2 - a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*e^2*f -
 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x)*cosh(d*x + c)^4 - 144*((4*a^2*b + 3*b^
3)*d^3*f^3*x^3 + (4*a^2*b + 3*b^3)*d^3*e^3 - 3*(4*a^2*b + 3*b^3)*d^2*e^2*f
+ 6*(4*a^2*b + 3*b^3)*d*e*f^2 - 6*(4*a^2*b + 3*b^3)*f^3 + 3*((4*a^2*b + 3*b
^3)*d^3*e*f^2 - (4*a^2*b + 3*b^3)*d^2*f^3)*x^2 + 3*((4*a^2*b + 3*b^3)*d^3*e
^2*f - 2*(4*a^2*b + 3*b^3)*d^2*e*f^2 + 2*(4*a^2*b + 3*b^3)*d*f^3)*x)*cosh(d
*x + c)^3 + 54*(2*a*b^2*d^3*e*f^2 + a*b^2*d^2*f^3)*x^2 - 216*((a^3 + a*b^2)
*d^4*f^3*x^4 + 4*(a^3 + a*b^2)*d^4*e*f^2*x^3 + 6*(a^3 + a*b^2)*d^4*e^2*f*x^
2 + 4*(a^3 + a*b^2)*d^4*e^3*x + 8*(a^3 + a*b^2)*c*d^3*e^3 - 12*(a^3 + a*b^2
)*c^2*d^2*e^2*f + 8*(a^3 + a*b^2)*c^3*d*e*f^2 - 2*(a^3 + a*b^2)*c^4*f^3)*co
sh(d*x + c)^2 + 54*(2*a*b^2*d^3*e^2*f + 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x
+ 72*((4*a^2*b + 3*b^3)*d^3*f^3*x^3 + (4*a^2*b + 3*b^3)*d^3*e^3 + 3*(4*a^2*
b + 3*b^3)*d^2*e^2*f + 6*(4*a^2*b + 3*b^3)*d*e*f^2 + 6*(4*a^2*b + 3*b^3)*f^
3 + 3*((4*a^2*b + 3*b^3)*d^3*e*f^2 + (4*a^2*b + 3*b^3)*d^2*f^3)*x^2 + 3*((4
*a^2*b + 3*b^3)*d^3*e^2*f + 2*(4*a^2*b + 3*b^3)*d^2*e*f^2 + 2*(4*a^2*b + 3*
b^3)*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c))/(b^4*d^4*cosh(d*x + c)^3 + 3*b
^4*d^4*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^4*d^4*cosh(d*x + c)*sinh(d*x + c
)^2 + b^4*d^4*sinh(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \cosh(dx+c)^3 \sinh(dx+c)}{b \sinh(dx+c)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)^3*sinh(d*x + c)/(b*sinh(d*x + c) + a),
x)
```

3.344 $\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=636

$$-\frac{2af\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{2af\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4d^2}+\frac{2af^2\left(a^2+b^2\right)\text{PolyI}}{b^4d}$$

```
[Out] -(a*e*f*x)/(2*b^2*d) - (a*f^2*x^2)/(4*b^2*d) + (a*(a^2 + b^2)*(e + f*x)^3)/
(3*b^4*f) - (2*a^2*f*(e + f*x)*Cosh[c + d*x])/(b^3*d^2) - (4*f*(e + f*x)*Co
sh[c + d*x])/(3*b*d^2) - (2*f*(e + f*x)*Cosh[c + d*x]^3)/(9*b*d^2) - (a*(a^
2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4*d
) - (a*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)])/(b^4*d) - (2*a*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/(b^4*d^2) - (2*a*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^2) + (2*a*(a^2 + b^2)*f^2*
PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^3) + (2*a*(a^2
 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^3)
 + (2*a^2*f^2*Sinh[c + d*x])/(b^3*d^3) + (14*f^2*Sinh[c + d*x])/(9*b*d^3) +
 (a^2*(e + f*x)^2*Sinh[c + d*x])/(b^3*d) + (2*(e + f*x)^2*Sinh[c + d*x])/(3
*b*d) + (a*f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^2*d^2) + ((e + f*x
)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b*d) - (a*f^2*Sinh[c + d*x]^2)/(4*b^2
*d^3) - (a*(e + f*x)^2*Sinh[c + d*x]^2)/(2*b^2*d) + (2*f^2*Sinh[c + d*x]^3)
/(27*b*d^3)
```

Rubi [A] time = 0.875195, antiderivative size = 636, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 13, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.382, Rules used = {5579, 3311, 3296, 2637, 2633, 5565, 5446, 3310, 5561, 2190, 2531, 2282, 6589}

$$-\frac{2af\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{2af\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4d^2}+\frac{2af^2\left(a^2+b^2\right)\text{PolyI}}{b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -(a*e*f*x)/(2*b^2*d) - (a*f^2*x^2)/(4*b^2*d) + (a*(a^2 + b^2)*(e + f*x)^3)/
(3*b^4*f) - (2*a^2*f*(e + f*x)*Cosh[c + d*x])/(b^3*d^2) - (4*f*(e + f*x)*Co
sh[c + d*x])/(3*b*d^2) - (2*f*(e + f*x)*Cosh[c + d*x]^3)/(9*b*d^2) - (a*(a^
2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4*d
) - (a*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)])/(b^4*d) - (2*a*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/(b^4*d^2) - (2*a*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^2) + (2*a*(a^2 + b^2)*f^2*
PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^3) + (2*a*(a^2
 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^3)
 + (2*a^2*f^2*Sinh[c + d*x])/(b^3*d^3) + (14*f^2*Sinh[c + d*x])/(9*b*d^3) +
 (a^2*(e + f*x)^2*Sinh[c + d*x])/(b^3*d) + (2*(e + f*x)^2*Sinh[c + d*x])/(3
*b*d) + (a*f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^2*d^2) + ((e + f*x
)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b*d) - (a*f^2*Sinh[c + d*x]^2)/(4*b^2
*d^3) - (a*(e + f*x)^2*Sinh[c + d*x]^2)/(2*b^2*d) + (2*f^2*Sinh[c + d*x]^3)
/(27*b*d^3)
```


Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh^3(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{2f(e+fx)\cosh^3(c+dx)}{9bd^2} + \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+dx)}{3bd} + \frac{a^2}{}$$

$$= \frac{a\left(a^2+b^2\right)(e+fx)^3}{3b^4f} - \frac{2f(e+fx)\cosh^3(c+dx)}{9bd^2} + \frac{a^2(e+fx)^2\sinh(c+}{b^3d}$$

$$= \frac{a\left(a^2+b^2\right)(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx)\cosh(c+dx)}{b^3d^2} - \frac{4f(e+fx)\cosh(c}{3bd^2}$$

$$= -\frac{aefx}{2b^2d} - \frac{af^2x^2}{4b^2d} + \frac{a\left(a^2+b^2\right)(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx)\cosh(c+dx)}{b^3d^2} -$$

$$= -\frac{aefx}{2b^2d} - \frac{af^2x^2}{4b^2d} + \frac{a\left(a^2+b^2\right)(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx)\cosh(c+dx)}{b^3d^2} -$$

$$= -\frac{aefx}{2b^2d} - \frac{af^2x^2}{4b^2d} + \frac{a\left(a^2+b^2\right)(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx)\cosh(c+dx)}{b^3d^2} -$$

Mathematica [B] time = 16.7223, size = 3509, normalized size = 5.52

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (f^2*(2*a*x^3*(-1 + Coth[c]) - 2*a*x^3*Coth[c] - (6*a*b^2*(d^2*x^2*Log[1 +
((a - Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 2*d*x*PolyLog[
2, ((-a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 2*PolyLog[
3, ((-a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b]))/(Sqrt[a^2
+ b^2]*(-a + Sqrt[a^2 + b^2])*d^3) - (6*a*b^2*(d^2*x^2*Log[1 + ((a + Sqrt[a
^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 2*d*x*PolyLog[2, ((a + Sqr
t[a^2 + b^2])*(-Cosh[c + d*x] + Sinh[c + d*x]))/b] - 2*PolyLog[3, ((a + Sqr
t[a^2 + b^2])*(-Cosh[c + d*x] + Sinh[c + d*x]))/b]))/(Sqrt[a^2 + b^2]*(a +
Sqrt[a^2 + b^2])*d^3) + (6*a^2*(d^2*x^2*Log[1 + (b*(Cosh[c + d*x] + Sinh[c
+ d*x]))/(a - Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, (b*(Cosh[c + d*x] + Sinh
[c + d*x]))/(-a + Sqrt[a^2 + b^2])] - 2*PolyLog[3, (b*(Cosh[c + d*x] + Sinh
[c + d*x]))/(-a + Sqrt[a^2 + b^2])]))/(Sqrt[a^2 + b^2]*d^3) - (6*a^2*(d^2*x
^2*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])] + 2*d
*x*PolyLog[2, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))]
 - 2*PolyLog[3, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])
)]))/(Sqrt[a^2 + b^2]*d^3) + (6*b*Cosh[d*x]*(-2*d*x*Cosh[c] + (2 + d^2*x^2)
*Sinh[c]))/d^3 + (6*b*((2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[c])*Sinh[d*x])/d^
3))/(12*b^2) - (e^2*((a*Log[a + b*Sinh[c + d*x]])/b^2 - Sinh[c + d*x]/b))/(
2*d) + (e*f*(-(b*Cosh[c + d*x]) - a*(-(c + d*x)^2/2 + (c + d*x)*Log[1 + (b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])] - c*Log[a + b*Sinh[c + d*x]] + PolyLog[2, (b*E^(c + d*x
))/(-a + Sqrt[a^2 + b^2])] + PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2]))]) + b*d*x*Sinh[c + d*x]))/(b^2*d^2) + (e^2*(-3*a*(2*a^2 + b^2)*Log[a
+ b*Sinh[c + d*x]] + 3*b*(2*a^2 + b^2)*Sinh[c + d*x] - 3*a*b^2*Sinh[c + d*x
]^2 + 2*b^3*Sinh[c + d*x]^3))/(6*b^4*d) + (e*f*(-18*b*(4*a^2 + b^2)*Cosh[c
```

+ d*x] - 18*a*b^2*d*x*Cosh[2*(c + d*x)] - 2*b^3*Cosh[3*(c + d*x)] - 36*a*(2
a^2 + b^2)(-(c + d*x)^2/2 + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2])] + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - c*
Log[a + b*Sinh[c + d*x]] + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]
)] + PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) + 18*b*(4*a^2 +
b^2)*d*x*Sinh[c + d*x] + 9*a*b^2*Sinh[2*(c + d*x)] + 6*b^3*d*x*Sinh[3*(c +
d*x)]))/(36*b^4*d^2) + (f^2*((2*a*(2*a^2 + b^2)*(-1 + Coth[c])*(2*x^3 + (6*
a*(d^2*x^2*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2]
)] + 2*d*x*PolyLog[2, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 +
b^2])] - 2*PolyLog[3, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 +
b^2])])*Sinh[c]*(Cosh[c] + Sinh[c]))/(Sqrt[a^2 + b^2]*d^3) - (3*b^2*(d^2*x^
2*Log[1 + ((a - Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 2*d*
x*PolyLog[2, ((-a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] -
2*PolyLog[3, ((-a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b])*(
-1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 + b^2]*(-a + Sqrt[a^2 + b^2])*d^3) -
 (3*b^2*(d^2*x^2*Log[1 + ((a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d
*x]))/b] - 2*d*x*PolyLog[2, ((a + Sqrt[a^2 + b^2])*(-Cosh[c + d*x] + Sinh[c
 + d*x]))/b] - 2*PolyLog[3, ((a + Sqrt[a^2 + b^2])*(-Cosh[c + d*x] + Sinh[c
 + d*x]))/b])*(-1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 + b^2]*(a + Sqrt[a^2
+ b^2])*d^3) - (3*a*(d^2*x^2*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a
 + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, -((b*(Cosh[c + d*x] + Sinh[c + d*x]
))/(a + Sqrt[a^2 + b^2]))] - 2*PolyLog[3, -((b*(Cosh[c + d*x] + Sinh[c + d*
x]))/(a + Sqrt[a^2 + b^2]))])*(-1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 + b^2
]*d^3)))/(3*b^4) + Csch[c]*(Cosh[3*c + 3*d*x]/(108*b^4*d^3) - Sinh[3*c + 3*
d*x]/(108*b^4*d^3))*(27*a*b^2*Cosh[d*x] + 54*a*b^2*d*x*Cosh[d*x] + 54*a*b^2
*d^2*x^2*Cosh[d*x] - 27*a*b^2*Cosh[2*c + d*x] - 54*a*b^2*d*x*Cosh[2*c + d*x
] - 54*a*b^2*d^2*x^2*Cosh[2*c + d*x] + 432*a^2*b*Cosh[c + 2*d*x] + 108*b^3*
Cosh[c + 2*d*x] + 432*a^2*b*d*x*Cosh[c + 2*d*x] + 108*b^3*d*x*Cosh[c + 2*d*
x] + 216*a^2*b*d^2*x^2*Cosh[c + 2*d*x] + 54*b^3*d^2*x^2*Cosh[c + 2*d*x] - 4
32*a^2*b*Cosh[3*c + 2*d*x] - 108*b^3*Cosh[3*c + 2*d*x] - 432*a^2*b*d*x*Cosh
[3*c + 2*d*x] - 108*b^3*d*x*Cosh[3*c + 2*d*x] - 216*a^2*b*d^2*x^2*Cosh[3*c
+ 2*d*x] - 54*b^3*d^2*x^2*Cosh[3*c + 2*d*x] - 144*a^3*d^3*x^3*Cosh[2*c + 3*
d*x] - 72*a*b^2*d^3*x^3*Cosh[2*c + 3*d*x] - 144*a^3*d^3*x^3*Cosh[4*c + 3*d*
x] - 72*a*b^2*d^3*x^3*Cosh[4*c + 3*d*x] - 432*a^2*b*Cosh[3*c + 4*d*x] - 108
*b^3*Cosh[3*c + 4*d*x] + 432*a^2*b*d*x*Cosh[3*c + 4*d*x] + 108*b^3*d*x*Cosh
[3*c + 4*d*x] - 216*a^2*b*d^2*x^2*Cosh[3*c + 4*d*x] - 54*b^3*d^2*x^2*Cosh[3
*c + 4*d*x] + 432*a^2*b*Cosh[5*c + 4*d*x] + 108*b^3*Cosh[5*c + 4*d*x] - 432
*a^2*b*d*x*Cosh[5*c + 4*d*x] - 108*b^3*d*x*Cosh[5*c + 4*d*x] + 216*a^2*b*d^
2*x^2*Cosh[5*c + 4*d*x] + 54*b^3*d^2*x^2*Cosh[5*c + 4*d*x] + 27*a*b^2*Cosh[
4*c + 5*d*x] - 54*a*b^2*d*x*Cosh[4*c + 5*d*x] + 54*a*b^2*d^2*x^2*Cosh[4*c +
 5*d*x] - 27*a*b^2*Cosh[6*c + 5*d*x] + 54*a*b^2*d*x*Cosh[6*c + 5*d*x] - 54*
a*b^2*d^2*x^2*Cosh[6*c + 5*d*x] - 4*b^3*Cosh[5*c + 6*d*x] + 12*b^3*d*x*Cosh
[5*c + 6*d*x] - 18*b^3*d^2*x^2*Cosh[5*c + 6*d*x] + 4*b^3*Cosh[7*c + 6*d*x]
- 12*b^3*d*x*Cosh[7*c + 6*d*x] + 18*b^3*d^2*x^2*Cosh[7*c + 6*d*x] - 8*b^3*S
inh[c] - 24*b^3*d*x*Sinh[c] - 36*b^3*d^2*x^2*Sinh[c] + 27*a*b^2*Sinh[d*x] +
 54*a*b^2*d*x*Sinh[d*x] + 54*a*b^2*d^2*x^2*Sinh[d*x] - 27*a*b^2*Sinh[2*c +
d*x] - 54*a*b^2*d*x*Sinh[2*c + d*x] - 54*a*b^2*d^2*x^2*Sinh[2*c + d*x] + 43
2*a^2*b*Sinh[c + 2*d*x] + 108*b^3*Sinh[c + 2*d*x] + 432*a^2*b*d*x*Sinh[c +
2*d*x] + 108*b^3*d*x*Sinh[c + 2*d*x] + 216*a^2*b*d^2*x^2*Sinh[c + 2*d*x] +
54*b^3*d^2*x^2*Sinh[c + 2*d*x] - 432*a^2*b*Sinh[3*c + 2*d*x] - 108*b^3*Sinh
[3*c + 2*d*x] - 432*a^2*b*d*x*Sinh[3*c + 2*d*x] - 108*b^3*d*x*Sinh[3*c + 2*
d*x] - 216*a^2*b*d^2*x^2*Sinh[3*c + 2*d*x] - 54*b^3*d^2*x^2*Sinh[3*c + 2*d*
x] - 144*a^3*d^3*x^3*Sinh[2*c + 3*d*x] - 72*a*b^2*d^3*x^3*Sinh[2*c + 3*d*x]
 - 144*a^3*d^3*x^3*Sinh[4*c + 3*d*x] - 72*a*b^2*d^3*x^3*Sinh[4*c + 3*d*x] -
 432*a^2*b*Sinh[3*c + 4*d*x] - 108*b^3*Sinh[3*c + 4*d*x] + 432*a^2*b*d*x*Si
nh[3*c + 4*d*x] + 108*b^3*d*x*Sinh[3*c + 4*d*x] - 216*a^2*b*d^2*x^2*Sinh[3*
c + 4*d*x] - 54*b^3*d^2*x^2*Sinh[3*c + 4*d*x] + 432*a^2*b*Sinh[5*c + 4*d*x]
 + 108*b^3*Sinh[5*c + 4*d*x] - 432*a^2*b*d*x*Sinh[5*c + 4*d*x] - 108*b^3*d*
x*Sinh[5*c + 4*d*x] + 216*a^2*b*d^2*x^2*Sinh[5*c + 4*d*x] + 54*b^3*d^2*x^2*

```
Sinh[5*c + 4*d*x] + 27*a*b^2*Sinh[4*c + 5*d*x] - 54*a*b^2*d*x*Sinh[4*c + 5*
d*x] + 54*a*b^2*d^2*x^2*Sinh[4*c + 5*d*x] - 27*a*b^2*Sinh[6*c + 5*d*x] + 54
*a*b^2*d*x*Sinh[6*c + 5*d*x] - 54*a*b^2*d^2*x^2*Sinh[6*c + 5*d*x] - 4*b^3*S
inh[5*c + 6*d*x] + 12*b^3*d*x*Sinh[5*c + 6*d*x] - 18*b^3*d^2*x^2*Sinh[5*c +
 6*d*x] + 4*b^3*Sinh[7*c + 6*d*x] - 12*b^3*d*x*Sinh[7*c + 6*d*x] + 18*b^3*d
^2*x^2*Sinh[7*c + 6*d*x])))/8
```

Maple [F] time = 0.191, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \left(\cosh \left(dx+c\right)\right)^{3} \sinh \left(dx+c\right)}{a+b \sinh \left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/24*e^2*((3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 + 3*b^2)*e^(-2*d*x - 2*c))*
e^(3*d*x + 3*c)/(b^3*d) + 24*(a^3 + a*b^2)*(d*x + c)/(b^4*d) + (3*a*b*e^(-2
*d*x - 2*c) + b^2*e^(-3*d*x - 3*c) + 3*(4*a^2 + 3*b^2)*e^(-d*x - c))/(b^3*d
) + 24*(a^3 + a*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^4*d
)) - 1/432*(144*(a^3*d^3*f^2*e^(3*c) + a*b^2*d^3*f^2*e^(3*c))*x^3 + 432*(a^
3*d^3*e*f*e^(3*c) + a*b^2*d^3*e*f*e^(3*c))*x^2 - 2*(9*b^3*d^2*f^2*x^2*e^(6*
c) + 6*(3*d^2*e*f - d*f^2)*b^3*x*e^(6*c) - 2*(3*d*e*f - f^2)*b^3*e^(6*c))*e
^(3*d*x) + 27*(2*a*b^2*d^2*f^2*x^2*e^(5*c) + 2*(2*d^2*e*f - d*f^2)*a*b^2*x*
e^(5*c) - (2*d*e*f - f^2)*a*b^2*e^(5*c))*e^(2*d*x) + 54*(8*(d*e*f - f^2)*a^
2*b*e^(4*c) + 6*(d*e*f - f^2)*b^3*e^(4*c) - (4*a^2*b*d^2*f^2*e^(4*c) + 3*b^
3*d^2*f^2*e^(4*c))*x^2 - 2*(4*(d^2*e*f - d*f^2)*a^2*b*e^(4*c) + 3*(d^2*e*f
- d*f^2)*b^3*e^(4*c))*x)*e^(d*x) + 54*(8*(d*e*f + f^2)*a^2*b*e^(2*c) + 6*(d
*e*f + f^2)*b^3*e^(2*c) + (4*a^2*b*d^2*f^2*e^(2*c) + 3*b^3*d^2*f^2*e^(2*c))
*x^2 + 2*(4*(d^2*e*f + d*f^2)*a^2*b*e^(2*c) + 3*(d^2*e*f + d*f^2)*b^3*e^(2*
c))*x)*e^(-d*x) + 27*(2*a*b^2*d^2*f^2*x^2*e^c + 2*(2*d^2*e*f + d*f^2)*a*b^2
*x*e^c + (2*d*e*f + f^2)*a*b^2*e^c)*e^(-2*d*x) + 2*(9*b^3*d^2*f^2*x^2 + 6*(
3*d^2*e*f + d*f^2)*b^3*x + 2*(3*d*e*f + f^2)*b^3)*e^(-3*d*x))*e^(-3*c)/(b^4
*d^3) + integrate(-2*((a^3*b*f^2 + a*b^3*f^2)*x^2 + 2*(a^3*b*e*f + a*b^3*e*
f)*x - ((a^4*f^2*e^c + a^2*b^2*f^2*e^c)*x^2 + 2*(a^4*e*f*e^c + a^2*b^2*e*f*
e^c)*x)*e^(d*x))/(b^5*e^(2*d*x + 2*c) + 2*a*b^4*e^(d*x + c) - b^5), x)
```

Fricas [C] time = 3.04855, size = 10886, normalized size = 17.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")

[Out] -1/432*(18*b^3*d^2*f^2*x^2 + 18*b^3*d^2*e^2 - 2*(9*b^3*d^2*f^2*x^2 + 9*b^3*
d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d
*x + c)^6 - 2*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2
+ 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*sinh(d*x + c)^6 + 12*b^3*d*e*f + 27*(2*a
*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2
*d^2*e*f - a*b^2*d*f^2)*x)*cosh(d*x + c)^5 + 3*(18*a*b^2*d^2*f^2*x^2 + 18*a
*b^2*d^2*e^2 - 18*a*b^2*d*e*f + 9*a*b^2*f^2 + 18*(2*a*b^2*d^2*e*f - a*b^2*d
*f^2)*x - 4*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 +
6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*b^3*f^2
 - 54*((4*a^2*b + 3*b^3)*d^2*f^2*x^2 + (4*a^2*b + 3*b^3)*d^2*e^2 - 2*(4*a^2
*b + 3*b^3)*d*e*f + 2*(4*a^2*b + 3*b^3)*f^2 + 2*((4*a^2*b + 3*b^3)*d^2*e*f
- (4*a^2*b + 3*b^3)*d*f^2)*x)*cosh(d*x + c)^4 - 3*(18*(4*a^2*b + 3*b^3)*d^2
*f^2*x^2 + 18*(4*a^2*b + 3*b^3)*d^2*e^2 - 36*(4*a^2*b + 3*b^3)*d*e*f + 36*(
4*a^2*b + 3*b^3)*f^2 + 10*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f
+ 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*x + c)^2 + 36*((4*a^2
*b + 3*b^3)*d^2*e*f - (4*a^2*b + 3*b^3)*d*f^2)*x - 45*(2*a*b^2*d^2*f^2*x^2
+ 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*
d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^4 - 144*((a^3 + a*b^2)*d^3*f^2*x^3 +
 3*(a^3 + a*b^2)*d^3*e*f*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*x + 6*(a^3 + a*b^2)*
c*d^2*e^2 - 6*(a^3 + a*b^2)*c^2*d*e*f + 2*(a^3 + a*b^2)*c^3*f^2)*cosh(d*x +
 c)^3 - 2*(72*(a^3 + a*b^2)*d^3*f^2*x^3 + 216*(a^3 + a*b^2)*d^3*e*f*x^2 + 2
16*(a^3 + a*b^2)*d^3*e^2*x + 432*(a^3 + a*b^2)*c*d^2*e^2 - 432*(a^3 + a*b^2
)*c^2*d*e*f + 144*(a^3 + a*b^2)*c^3*f^2 + 20*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2
*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*x
+ c)^3 - 135*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2
*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*cosh(d*x + c)^2 + 108*((4*a^2*b
 + 3*b^3)*d^2*f^2*x^2 + (4*a^2*b + 3*b^3)*d^2*e^2 - 2*(4*a^2*b + 3*b^3)*d*e
*f + 2*(4*a^2*b + 3*b^3)*f^2 + 2*((4*a^2*b + 3*b^3)*d^2*e*f - (4*a^2*b + 3*
b^3)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 54*((4*a^2*b + 3*b^3)*d^2*f
^2*x^2 + (4*a^2*b + 3*b^3)*d^2*e^2 + 2*(4*a^2*b + 3*b^3)*d*e*f + 2*(4*a^2*b
 + 3*b^3)*f^2 + 2*((4*a^2*b + 3*b^3)*d^2*e*f + (4*a^2*b + 3*b^3)*d*f^2)*x)*
cosh(d*x + c)^2 + 6*(9*(4*a^2*b + 3*b^3)*d^2*f^2*x^2 + 9*(4*a^2*b + 3*b^3)*
d^2*e^2 - 5*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 +
6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*x + c)^4 + 18*(4*a^2*b + 3*b^3)*d*e
*f + 45*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2
+ 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*cosh(d*x + c)^3 + 18*(4*a^2*b + 3*b^
3)*f^2 - 54*((4*a^2*b + 3*b^3)*d^2*f^2*x^2 + (4*a^2*b + 3*b^3)*d^2*e^2 - 2*
(4*a^2*b + 3*b^3)*d*e*f + 2*(4*a^2*b + 3*b^3)*f^2 + 2*((4*a^2*b + 3*b^3)*d^
2*e*f - (4*a^2*b + 3*b^3)*d*f^2)*x)*cosh(d*x + c)^2 + 18*((4*a^2*b + 3*b^3)
*d^2*e*f + (4*a^2*b + 3*b^3)*d*f^2)*x - 72*((a^3 + a*b^2)*d^3*f^2*x^3 + 3*(
a^3 + a*b^2)*d^3*e*f*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*x + 6*(a^3 + a*b^2)*c*d^
2*e^2 - 6*(a^3 + a*b^2)*c^2*d*e*f + 2*(a^3 + a*b^2)*c^3*f^2)*cosh(d*x + c))
*sinh(d*x + c)^2 + 12*(3*b^3*d^2*e*f + b^3*d*f^2)*x + 27*(2*a*b^2*d^2*f^2*x
^2 + 2*a*b^2*d^2*e^2 + 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f + a*b
^2*d*f^2)*x)*cosh(d*x + c) + 864*(((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*d*
e*f)*cosh(d*x + c)^3 + 3*((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*d*e*f)*cosh
(d*x + c)^2*sinh(d*x + c) + 3*((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*d*e*f)
*cosh(d*x + c)*sinh(d*x + c)^2 + ((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*d*e
*f)*sinh(d*x + c)^3)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 864*(((a^3 + a
*b^2)*d*f^2*x + (a^3 + a*b^2)*d*e*f)*cosh(d*x + c)^3 + 3*((a^3 + a*b^2)*d*f
^2*x + (a^3 + a*b^2)*d*e*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3*((a^3 + a*b^2
)*d*f^2*x + (a^3 + a*b^2)*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^2 + ((a^3 + a*
b^2)*d*f^2*x + (a^3 + a*b^2)*d*e*f)*sinh(d*x + c)^3)*dilog((a*cosh(d*x + c)
 + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2) - b)/b + 1) + 432*(((a^3 + a*b^2)*d^2*e^2 - 2*(a^3 + a*b^2)*c*d*e*f + (
```

```
a^3 + a*b^2)*c^2*f^2)*cosh(d*x + c)^3 + 3*((a^3 + a*b^2)*d^2*e^2 - 2*(a^3 +
 a*b^2)*c*d*e*f + (a^3 + a*b^2)*c^2*f^2)*cosh(d*x + c)^2*sinh(d*x + c) + 3*
((a^3 + a*b^2)*d^2*e^2 - 2*(a^3 + a*b^2)*c*d*e*f + (a^3 + a*b^2)*c^2*f^2)*c
osh(d*x + c)*sinh(d*x + c)^2 + ((a^3 + a*b^2)*d^2*e^2 - 2*(a^3 + a*b^2)*c*d
*e*f + (a^3 + a*b^2)*c^2*f^2)*sinh(d*x + c)^3)*log(2*b*cosh(d*x + c) + 2*b*
sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 432*(((a^3 + a*b^2)*d^2*
e^2 - 2*(a^3 + a*b^2)*c*d*e*f + (a^3 + a*b^2)*c^2*f^2)*cosh(d*x + c)^3 + 3*
((a^3 + a*b^2)*d^2*e^2 - 2*(a^3 + a*b^2)*c*d*e*f + (a^3 + a*b^2)*c^2*f^2)*c
osh(d*x + c)^2*sinh(d*x + c) + 3*((a^3 + a*b^2)*d^2*e^2 - 2*(a^3 + a*b^2)*c
*d*e*f + (a^3 + a*b^2)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^2 + ((a^3 + a*b
^2)*d^2*e^2 - 2*(a^3 + a*b^2)*c*d*e*f + (a^3 + a*b^2)*c^2*f^2)*sinh(d*x + c
)^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2)
+ 2*a) + 432*(((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a
^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*cosh(d*x + c)^3 + 3*((a^3 + a*
b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a
^3 + a*b^2)*c^2*f^2)*cosh(d*x + c)^2*sinh(d*x + c) + 3*((a^3 + a*b^2)*d^2*f
^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2
)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^2 + ((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(
a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*s
inh(d*x + c)^3)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 432*(((a^3 + a*b^2)*d^2*
f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^
2)*c^2*f^2)*cosh(d*x + c)^3 + 3*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2
)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*cosh(d*x + c
)^2*sinh(d*x + c) + 3*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*
x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*cosh(d*x + c)*sinh(d*x
 + c)^2 + ((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 +
 a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*sinh(d*x + c)^3)*log(-(a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2) - b)/b) - 864*((a^3 + a*b^2)*f^2*cosh(d*x + c)^3 + 3*(a^3 + a*b^2)*
f^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3 + a*b^2)*f^2*cosh(d*x + c)*sinh(
d*x + c)^2 + (a^3 + a*b^2)*f^2*sinh(d*x + c)^3)*polylog(3, (a*cosh(d*x + c)
 + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2))/b) - 864*((a^3 + a*b^2)*f^2*cosh(d*x + c)^3 + 3*(a^3 + a*b^2)*f^2*cosh
(d*x + c)^2*sinh(d*x + c) + 3*(a^3 + a*b^2)*f^2*cosh(d*x + c)*sinh(d*x + c)
^2 + (a^3 + a*b^2)*f^2*sinh(d*x + c)^3)*polylog(3, (a*cosh(d*x + c) + a*sin
h(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b)
+ 3*(18*a*b^2*d^2*f^2*x^2 + 18*a*b^2*d^2*e^2 + 18*a*b^2*d*e*f - 4*(9*b^3*d^
2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^
3*d*f^2)*x)*cosh(d*x + c)^5 + 9*a*b^2*f^2 + 45*(2*a*b^2*d^2*f^2*x^2 + 2*a*b
^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*
x)*cosh(d*x + c)^4 - 72*((4*a^2*b + 3*b^3)*d^2*f^2*x^2 + (4*a^2*b + 3*b^3)*
d^2*e^2 - 2*(4*a^2*b + 3*b^3)*d*e*f + 2*(4*a^2*b + 3*b^3)*f^2 + 2*((4*a^2*b
 + 3*b^3)*d^2*e*f - (4*a^2*b + 3*b^3)*d*f^2)*x)*cosh(d*x + c)^3 - 144*((a^3
 + a*b^2)*d^3*f^2*x^3 + 3*(a^3 + a*b^2)*d^3*e*f*x^2 + 3*(a^3 + a*b^2)*d^3*e
^2*x + 6*(a^3 + a*b^2)*c*d^2*e^2 - 6*(a^3 + a*b^2)*c^2*d*e*f + 2*(a^3 + a*b
^2)*c^3*f^2)*cosh(d*x + c)^2 + 18*(2*a*b^2*d^2*e*f + a*b^2*d*f^2)*x + 36*((
4*a^2*b + 3*b^3)*d^2*f^2*x^2 + (4*a^2*b + 3*b^3)*d^2*e^2 + 2*(4*a^2*b + 3*b
^3)*d*e*f + 2*(4*a^2*b + 3*b^3)*f^2 + 2*((4*a^2*b + 3*b^3)*d^2*e*f + (4*a^2
*b + 3*b^3)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))/(b^4*d^3*cosh(d*x + c)^
3 + 3*b^4*d^3*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^4*d^3*cosh(d*x + c)*sinh(
d*x + c)^2 + b^4*d^3*sinh(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \cosh(dx+c)^3 \sinh(dx+c)}{b \sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)^3*sinh(d*x + c)/(b*sinh(d*x + c) + a),
x)
```

3.345 $\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=400

$$-\frac{af\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{af\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4d^2}-\frac{a^2f\cosh(c+dx)}{b^3d^2}-\frac{a\left(a^2+b^2\right)(e+\ldots}{\ldots}$$

```
[Out] -(a*f*x)/(4*b^2*d) + (a*(a^2 + b^2)*(e + f*x)^2)/(2*b^4*f) - (a^2*f*Cosh[c
+ d*x])/(b^3*d^2) - (2*f*Cosh[c + d*x])/(3*b*d^2) - (f*Cosh[c + d*x]^3)/(9*
b*d^2) - (a*(a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])])/(b^4*d) - (a*(a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])])/(b^4*d) - (a*(a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(b^4*d^2) - (a*(a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^2) + (a^2*(e + f*x)*Sinh[c + d*x])/(b^3
*d) + (2*(e + f*x)*Sinh[c + d*x])/(3*b*d) + (a*f*Cosh[c + d*x]*Sinh[c + d*x
])/(4*b^2*d^2) + ((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b*d) - (a*(e
+ f*x)*Sinh[c + d*x]^2)/(2*b^2*d)
```

Rubi [A] time = 0.484925, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5579, 3310, 3296, 2638, 5565, 5446, 2635, 8, 5561, 2190, 2279, 2391}

$$-\frac{af\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{af\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4d^2}-\frac{a^2f\cosh(c+dx)}{b^3d^2}-\frac{a\left(a^2+b^2\right)(e+\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -(a*f*x)/(4*b^2*d) + (a*(a^2 + b^2)*(e + f*x)^2)/(2*b^4*f) - (a^2*f*Cosh[c
+ d*x])/(b^3*d^2) - (2*f*Cosh[c + d*x])/(3*b*d^2) - (f*Cosh[c + d*x]^3)/(9*
b*d^2) - (a*(a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2])])/(b^4*d) - (a*(a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])])/(b^4*d) - (a*(a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(b^4*d^2) - (a*(a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^2) + (a^2*(e + f*x)*Sinh[c + d*x])/(b^3
*d) + (2*(e + f*x)*Sinh[c + d*x])/(3*b*d) + (a*f*Cosh[c + d*x]*Sinh[c + d*x
])/(4*b^2*d^2) + ((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b*d) - (a*(e
+ f*x)*Sinh[c + d*x]^2)/(2*b^2*d)
```


Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
```

```
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh^3(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{f\cosh^3(c+dx)}{9bd^2} + \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{3bd} + \frac{a^2\int(e+fx)\,\cdots}{\cdots}$$

$$= \frac{a\left(a^2+b^2\right)(e+fx)^2}{2b^4f} - \frac{f\cosh^3(c+dx)}{9bd^2} + \frac{a^2(e+fx)\sinh(c+dx)}{b^3d} + \frac{2(e\cdots}{\cdots}$$

$$= \frac{a\left(a^2+b^2\right)(e+fx)^2}{2b^4f} - \frac{a^2f\cosh(c+dx)}{b^3d^2} - \frac{2f\cosh(c+dx)}{3bd^2} - \frac{f\cosh^3(c\cdots}{9bd^2\cdots}$$

$$= -\frac{afx}{4b^2d} + \frac{a\left(a^2+b^2\right)(e+fx)^2}{2b^4f} - \frac{a^2f\cosh(c+dx)}{b^3d^2} - \frac{2f\cosh(c+dx)}{3bd^2} - \cdots$$

$$= -\frac{afx}{4b^2d} + \frac{a\left(a^2+b^2\right)(e+fx)^2}{2b^4f} - \frac{a^2f\cosh(c+dx)}{b^3d^2} - \frac{2f\cosh(c+dx)}{3bd^2} - \cdots$$

Mathematica [A] time = 3.43536, size = 551, normalized size = 1.38

$$-\frac{36b^2f\left(a\left(\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)+(c+dx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] -(-36*b^2*d*e*(-(a*Log[a + b*Sinh[c + d*x]]) + b*Sinh[c + d*x]) + 36*b^2*f*
(b*Cosh[c + d*x] + a*(-(c + d*x)^2/2 + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])] + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])] - c*Log[a + b*Sinh[c + d*x]] + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a
^2 + b^2])] + PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) - b*d*x
*Sinh[c + d*x]) + 12*d*e*(3*a*(2*a^2 + b^2)*Log[a + b*Sinh[c + d*x]] - 3*b*
(2*a^2 + b^2)*Sinh[c + d*x] + 3*a*b^2*Sinh[c + d*x]^2 - 2*b^3*Sinh[c + d*x]
^3) + f*(18*b*(4*a^2 + b^2)*Cosh[c + d*x] + 18*a*b^2*d*x*Cosh[2*(c + d*x)]
+ 2*b^3*Cosh[3*(c + d*x)] + 36*a*(2*a^2 + b^2)*(-(c + d*x)^2/2 + (c + d*x)*
Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + (c + d*x)*Log[1 + (b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2])] - c*Log[a + b*Sinh[c + d*x]] + PolyLog[2, (b
*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + PolyLog[2, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))]]) - 18*b*(4*a^2 + b^2)*d*x*Sinh[c + d*x] - 9*a*b^2*Sinh[2*
(c + d*x)] - 6*b^3*d*x*Sinh[3*(c + d*x)]))/(72*b^4*d^2)
```

Maple [B] time = 0.099, size = 1102, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/16*a*(2*d*f*x+2*d*e-f)/b^2/d^2*exp(2*d*x+2*c)-1/8*(4*a^2+3*b^2)*(d*f*x+d
*e+f)/b^3/d^2*exp(-d*x-c)-1/16*a*(2*d*f*x+2*d*e+f)/b^2/d^2*exp(-2*d*x-2*c)+
1/2*a^3*f*x^2/b^4+1/2*a*f*x^2/b^2-a^3/b^4/d*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(
1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-a^3/b^4/d^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(
1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-a^3/b^4/d*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2
)+a)/(a+(a^2+b^2)^(1/2)))*x-a^3/b^4/d^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+
a)/(a+(a^2+b^2)^(1/2)))*c+a^3/b^4/d^2*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c
)-b)-2*a^3/b^4/d^2*f*c*ln(exp(d*x+c))+2*a^3/b^4/d*f*c*x-a^3*e*x/b^4-a*e*x/b
^2+a/b^2/d^2*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2*a/b^2/d^2*f*c*ln(e
xp(d*x+c))-a/b^2/d*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/
2)))*x-a/b^2/d^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)
))*c-a/b^2/d*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-a
/b^2/d^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2*a/b
^2/d*f*c*x-1/72*(3*d*f*x+3*d*e+f)/d^2/b*exp(-3*d*x-3*c)+1/72*(3*d*f*x+3*d*e
-f)/d^2/b*exp(3*d*x+3*c)+1/8*(4*a^2*d*f*x+3*b^2*d*f*x+4*a^2*d*e+3*b^2*d*e-4
*a^2*f-3*b^2*f)/b^3/d^2*exp(d*x+c)+a/b^2/d^2*f*c^2-a/b^2/d*e*ln(b*exp(2*d*x
+2*c)+2*a*exp(d*x+c)-b)+2*a/b^2/d*e*ln(exp(d*x+c))-a/b^2/d^2*f*dilog((b*exp
(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-a/b^2/d^2*f*dilog((-b*exp(d
*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+a^3/b^4/d^2*f*c^2-a^3/b^4/d^
2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-a^3/b^4/d^2
*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-a^3/b^4/d*
e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2*a^3/b^4/d*e*ln(exp(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{24}e\left(\frac{\left(3\,abe^{(-dx-c)}-b^2-3\left(4\,a^2+3\,b^2\right)e^{(-2\,dx-2\,c)}\right)e^{(3\,dx+3\,c)}}{b^3d}+\frac{24\left(a^3+ab^2\right)(dx+c)}{b^4d}+\frac{3\,abe^{(-2\,dx-2\,c)}+b^2e^{(-3\,dx-3\,c)}+3\,(}{b^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/24*e*((3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 + 3*b^2)*e^(-2*d*x - 2*c))*e^
(3*d*x + 3*c)/(b^3*d) + 24*(a^3 + a*b^2)*(d*x + c)/(b^4*d) + (3*a*b*e^(-2*d
*x - 2*c) + b^2*e^(-3*d*x - 3*c) + 3*(4*a^2 + 3*b^2)*e^(-d*x - c))/(b^3*d)
+ 24*(a^3 + a*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^4*d))
 - 1/144*f*((72*(a^3*d^2*e^(3*c) + a*b^2*d^2*e^(3*c))*x^2 - 2*(3*b^3*d*x*e^
(6*c) - b^3*e^(6*c))*e^(3*d*x) + 9*(2*a*b^2*d*x*e^(5*c) - a*b^2*e^(5*c))*e^
(2*d*x) + 18*(4*a^2*b*e^(4*c) + 3*b^3*e^(4*c) - (4*a^2*b*d*e^(4*c) + 3*b^3*
d*e^(4*c))*x)*e^(d*x) + 18*(4*a^2*b*e^(2*c) + 3*b^3*e^(2*c) + (4*a^2*b*d*e^
(2*c) + 3*b^3*d*e^(2*c))*x)*e^(-d*x) + 9*(2*a*b^2*d*x*e^c + a*b^2*e^c)*e^(-
2*d*x) + 2*(3*b^3*d*x + b^3)*e^(-3*d*x))*e^(-3*c)/(b^4*d^2) - 9*integrate(3
2*((a^4*e^c + a^2*b^2*e^c)*x*e^(d*x) - (a^3*b + a*b^3)*x)/(b^5*e^(2*d*x + 2
*c) + 2*a*b^4*e^(d*x + c) - b^5), x))
```

Fricas [B] time = 2.56898, size = 5828, normalized size = 14.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/144*(2*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^6 + 2*(3*b^3*d*f*x
 + 3*b^3*d*e - b^3*f)*sinh(d*x + c)^6 - 6*b^3*d*f*x - 9*(2*a*b^2*d*f*x + 2*
a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^5 - 3*(6*a*b^2*d*f*x + 6*a*b^2*d*e - 3*a
*b^2*f - 4*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c))*sinh(d*x + c)^5
 - 6*b^3*d*e + 18*((4*a^2*b + 3*b^3)*d*f*x + (4*a^2*b + 3*b^3)*d*e - (4*a^2
*b + 3*b^3)*f)*cosh(d*x + c)^4 + 3*(6*(4*a^2*b + 3*b^3)*d*f*x + 6*(4*a^2*b
+ 3*b^3)*d*e + 10*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^2 - 6*(4*
a^2*b + 3*b^3)*f - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)
)*sinh(d*x + c)^4 - 2*b^3*f + 72*((a^3 + a*b^2)*d^2*f*x^2 + 2*(a^3 + a*b^2)
*d^2*e*x + 4*(a^3 + a*b^2)*c*d*e - 2*(a^3 + a*b^2)*c^2*f)*cosh(d*x + c)^3 +
 2*(36*(a^3 + a*b^2)*d^2*f*x^2 + 72*(a^3 + a*b^2)*d^2*e*x + 144*(a^3 + a*b^
2)*c*d*e - 72*(a^3 + a*b^2)*c^2*f + 20*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*co
sh(d*x + c)^3 - 45*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^2
+ 36*((4*a^2*b + 3*b^3)*d*f*x + (4*a^2*b + 3*b^3)*d*e - (4*a^2*b + 3*b^3)*f
)*cosh(d*x + c))*sinh(d*x + c)^3 - 18*((4*a^2*b + 3*b^3)*d*f*x + (4*a^2*b +
 3*b^3)*d*e + (4*a^2*b + 3*b^3)*f)*cosh(d*x + c)^2 + 6*(5*(3*b^3*d*f*x + 3*
b^3*d*e - b^3*f)*cosh(d*x + c)^4 - 3*(4*a^2*b + 3*b^3)*d*f*x - 15*(2*a*b^2*
d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^3 - 3*(4*a^2*b + 3*b^3)*d*e +
18*((4*a^2*b + 3*b^3)*d*f*x + (4*a^2*b + 3*b^3)*d*e - (4*a^2*b + 3*b^3)*f)*
cosh(d*x + c)^2 - 3*(4*a^2*b + 3*b^3)*f + 36*((a^3 + a*b^2)*d^2*f*x^2 + 2*(
a^3 + a*b^2)*d^2*e*x + 4*(a^3 + a*b^2)*c*d*e - 2*(a^3 + a*b^2)*c^2*f)*cosh(
d*x + c))*sinh(d*x + c)^2 - 9*(2*a*b^2*d*f*x + 2*a*b^2*d*e + a*b^2*f)*cosh(
d*x + c) - 144*((a^3 + a*b^2)*f*cosh(d*x + c)^3 + 3*(a^3 + a*b^2)*f*cosh(d*
x + c)^2*sinh(d*x + c) + 3*(a^3 + a*b^2)*f*cosh(d*x + c)*sinh(d*x + c)^2 +
(a^3 + a*b^2)*f*sinh(d*x + c)^3)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) +
 (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 14
4*((a^3 + a*b^2)*f*cosh(d*x + c)^3 + 3*(a^3 + a*b^2)*f*cosh(d*x + c)^2*sinh
(d*x + c) + 3*(a^3 + a*b^2)*f*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3 + a*b^2)
*f*sinh(d*x + c)^3)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 144*(((a^3 + a*
b^2)*d*e - (a^3 + a*b^2)*c*f)*cosh(d*x + c)^3 + 3*((a^3 + a*b^2)*d*e - (a^3
 + a*b^2)*c*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3*((a^3 + a*b^2)*d*e - (a^3
+ a*b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)^2 + ((a^3 + a*b^2)*d*e - (a^3 + a
*b^2)*c*f)*sinh(d*x + c)^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b
*sqrt((a^2 + b^2)/b^2) + 2*a) - 144*(((a^3 + a*b^2)*d*e - (a^3 + a*b^2)*c*f
)*cosh(d*x + c)^3 + 3*((a^3 + a*b^2)*d*e - (a^3 + a*b^2)*c*f)*cosh(d*x + c)
^2*sinh(d*x + c) + 3*((a^3 + a*b^2)*d*e - (a^3 + a*b^2)*c*f)*cosh(d*x + c)*
sinh(d*x + c)^2 + ((a^3 + a*b^2)*d*e - (a^3 + a*b^2)*c*f)*sinh(d*x + c)^3)*
log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a
) - 144*(((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*cosh(d*x + c)^3 + 3*((a^
3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3*((a
^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)^2 + ((a^
3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*sinh(d*x + c)^3)*log(-(a*cosh(d*x + c
) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2) - b)/b) - 144*(((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*cosh(d*x + c)
^3 + 3*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*cosh(d*x + c)^2*sinh(d*x +
 c) + 3*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*cosh(d*x + c)*sinh(d*x +
c)^2 + ((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*sinh(d*x + c)^3)*log(-(a*c
osh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((
```

```
a^2 + b^2)/b^2) - b)/b) - 3*(6*a*b^2*d*f*x - 4*(3*b^3*d*f*x + 3*b^3*d*e - b
^3*f)*cosh(d*x + c)^5 + 6*a*b^2*d*e + 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b
^2*f)*cosh(d*x + c)^4 + 3*a*b^2*f - 24*((4*a^2*b + 3*b^3)*d*f*x + (4*a^2*b
+ 3*b^3)*d*e - (4*a^2*b + 3*b^3)*f)*cosh(d*x + c)^3 - 72*((a^3 + a*b^2)*d^2
*f*x^2 + 2*(a^3 + a*b^2)*d^2*e*x + 4*(a^3 + a*b^2)*c*d*e - 2*(a^3 + a*b^2)*
c^2*f)*cosh(d*x + c)^2 + 12*((4*a^2*b + 3*b^3)*d*f*x + (4*a^2*b + 3*b^3)*d*
e + (4*a^2*b + 3*b^3)*f)*cosh(d*x + c))*sinh(d*x + c))/(b^4*d^2*cosh(d*x +
c)^3 + 3*b^4*d^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^4*d^2*cosh(d*x + c)*si
nh(d*x + c)^2 + b^4*d^2*sinh(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\cosh(dx+c)^3\sinh(dx+c)}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)^3*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.346 $\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{(a^2+b^2)\sinh(c+dx)}{b^3d}-\frac{a(a^2+b^2)\log(a+b\sinh(c+dx))}{b^4d}-\frac{a\sinh^2(c+dx)}{2b^2d}+\frac{\sinh^3(c+dx)}{3bd}$$

```
[Out] -((a*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/(b^4*d)) + ((a^2 + b^2)*Sinh[c +
 d*x])/(b^3*d) - (a*Sinh[c + d*x]^2)/(2*b^2*d) + Sinh[c + d*x]^3/(3*b*d)
```

Rubi [A] time = 0.12166, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 772}

$$\frac{(a^2+b^2)\sinh(c+dx)}{b^3d}-\frac{a(a^2+b^2)\log(a+b\sinh(c+dx))}{b^4d}-\frac{a\sinh^2(c+dx)}{2b^2d}+\frac{\sinh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/(b^4*d)) + ((a^2 + b^2)*Sinh[c +
 d*x])/(b^3*d) - (a*Sinh[c + d*x]^2)/(2*b^2*d) + Sinh[c + d*x]^3/(3*b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x(-b^2-x^2)}{b(a+x)}\,dx, x, b\sinh(c+dx)\right)}{b^3d}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{x(-b^2-x^2)}{a+x}\,dx, x, b\sinh(c+dx)\right)}{b^4d}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(-a^2\left(1+\frac{b^2}{a^2}\right)+ax-x^2+\frac{a(a^2+b^2)}{a+x}\right)dx, x, b\sinh(c+dx)\right)}{b^4d}$$
$$= -\frac{a(a^2+b^2)\log(a+b\sinh(c+dx))}{b^4d} + \frac{(a^2+b^2)\sinh(c+dx)}{b^3d} - \frac{a\sinh^2(c+dx)}{2b^2d} + \frac{\text{s}}{}$$

Mathematica [A] time = 0.164234, size = 75, normalized size = 0.88

$$\frac{6b(a^2+b^2)\sinh(c+dx) - 6a(a^2+b^2)\log(a+b\sinh(c+dx)) - 3ab^2\sinh^2(c+dx) + 2b^3\sinh^3(c+dx)}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-6*a*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]] + 6*b*(a^2 + b^2)*Sinh[c + d*x]
- 3*a*b^2*Sinh[c + d*x]^2 + 2*b^3*Sinh[c + d*x]^3)/(6*b^4*d)
```

Maple [B] time = 0.039, size = 428, normalized size = 5.

$$-\frac{1}{3\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3} + \frac{1}{2\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2} - \frac{a}{2\,db^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2} - \frac{a^2}{db^3}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/3/d/b/(tanh(1/2*d*x+1/2*c)+1)^3+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2-1/2/d/
b^2/(tanh(1/2*d*x+1/2*c)+1)^2*a-1/d/b^3/(tanh(1/2*d*x+1/2*c)+1)*a^2+1/2/d/b
^2/(tanh(1/2*d*x+1/2*c)+1)*a-1/d/b/(tanh(1/2*d*x+1/2*c)+1)+1/d*a^3/b^4*ln(t
anh(1/2*d*x+1/2*c)+1)+1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/3/d/b/(tanh(1/2
*d*x+1/2*c)-1)^3-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^2*a-1/2/d/b/(tanh(1/2*d*
x+1/2*c)-1)^2-1/d/b^3/(tanh(1/2*d*x+1/2*c)-1)*a^2-1/2/d/b^2/(tanh(1/2*d*x+1
/2*c)-1)*a-1/d/b/(tanh(1/2*d*x+1/2*c)-1)+1/d*a^3/b^4*ln(tanh(1/2*d*x+1/2*c)
-1)+1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)-1/d*a^3/b^4*ln(tanh(1/2*d*x+1/2*c)^
2*a-2*tanh(1/2*d*x+1/2*c)*b-a)-1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(
1/2*d*x+1/2*c)*b-a)
```

Maxima [B] time = 1.06732, size = 247, normalized size = 2.91

$$-\frac{\left(3\,abe^{(-dx-c)} - b^2 - 3\left(4\,a^2+3\,b^2\right)e^{(-2\,dx-2\,c)}\right)e^{(3\,dx+3\,c)}}{24\,b^3d} - \frac{\left(a^3+ab^2\right)(dx+c)}{b^4d} - \frac{3\,abe^{(-2\,dx-2\,c)} + b^2e^{(-3\,dx-3\,c)} + 3\left(4\,a^2+3\,b\right.}{24\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/24*(3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 + 3*b^2)*e^(-2*d*x - 2*c))*e^(3*
d*x + 3*c)/(b^3*d) - (a^3 + a*b^2)*(d*x + c)/(b^4*d) - 1/24*(3*a*b*e^(-2*d*
x - 2*c) + b^2*e^(-3*d*x - 3*c) + 3*(4*a^2 + 3*b^2)*e^(-d*x - c))/(b^3*d) -
 (a^3 + a*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^4*d)
```

Fricas [B] time = 2.20706, size = 1613, normalized size = 18.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/24*(b^3*cosh(d*x + c)^6 + b^3*sinh(d*x + c)^6 - 3*a*b^2*cosh(d*x + c)^5 +
 24*(a^3 + a*b^2)*d*x*cosh(d*x + c)^3 + 3*(2*b^3*cosh(d*x + c) - a*b^2)*sin
h(d*x + c)^5 + 3*(4*a^2*b + 3*b^3)*cosh(d*x + c)^4 + 3*(5*b^3*cosh(d*x + c)
^2 - 5*a*b^2*cosh(d*x + c) + 4*a^2*b + 3*b^3)*sinh(d*x + c)^4 - 3*a*b^2*cos
h(d*x + c) + 2*(10*b^3*cosh(d*x + c)^3 - 15*a*b^2*cosh(d*x + c)^2 + 12*(a^3
 + a*b^2)*d*x + 6*(4*a^2*b + 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - b^3 -
3*(4*a^2*b + 3*b^3)*cosh(d*x + c)^2 + 3*(5*b^3*cosh(d*x + c)^4 - 10*a*b^2*c
osh(d*x + c)^3 + 24*(a^3 + a*b^2)*d*x*cosh(d*x + c) - 4*a^2*b - 3*b^3 + 6*(
4*a^2*b + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 24*((a^3 + a*b^2)*cosh(
d*x + c)^3 + 3*(a^3 + a*b^2)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3 + a*b^2
)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3 + a*b^2)*sinh(d*x + c)^3)*log(2*(b*s
inh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 3*(2*b^3*cosh(d*x + c)
^5 - 5*a*b^2*cosh(d*x + c)^4 + 24*(a^3 + a*b^2)*d*x*cosh(d*x + c)^2 + 4*(4*
a^2*b + 3*b^3)*cosh(d*x + c)^3 - a*b^2 - 2*(4*a^2*b + 3*b^3)*cosh(d*x + c))
*sinh(d*x + c))/(b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c)^2*sinh(d*x +
 c) + 3*b^4*d*cosh(d*x + c)*sinh(d*x + c)^2 + b^4*d*sinh(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15208, size = 215, normalized size = 2.53

$$-\frac{\left(a^3+ab^2\right)\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{b^4 d}+\frac{b^2d^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3-3\,abd^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+12\,a^2d^2\left(e^{(dx+c)}-\right.}{24\,b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -(a^3 + a*b^2)*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(b^4*d) + 1/2
4*(b^2*d^2*(e^(d*x + c) - e^(-d*x - c))^3 - 3*a*b*d^2*(e^(d*x + c) - e^(-d*
x - c))^2 + 12*a^2*d^2*(e^(d*x + c) - e^(-d*x - c)) + 12*b^2*d^2*(e^(d*x +
c) - e^(-d*x - c)))/(b^3*d^3)
```

3.347 $\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{\sinh(c+dx)\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Cosh[c + d*x]^3*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [A] time = 0.0842915, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[c + d*x]^3*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Cosh[c + d*x]^3*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^3(c+dx)\sinh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.155, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx+c))^3\sinh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{e^{\left(-3\,c+\frac{3\,de}{f}\right)}E_1\left(\frac{3\left(fx+e\right)d}{f}\right)}{8\,bf}-\frac{ae^{\left(-2\,c+\frac{2\,de}{f}\right)}E_1\left(\frac{2\left(fx+e\right)d}{f}\right)}{4\,b^2f}+\frac{ae^{\left(2\,c-\frac{2\,de}{f}\right)}E_1\left(-\frac{2\left(fx+e\right)d}{f}\right)}{4\,b^2f}-\frac{e^{\left(3\,c-\frac{3\,de}{f}\right)}E_1\left(-\frac{3\left(fx+e\right)d}{f}\right)}{8\,bf}-\frac{\left(4\,a^2+3\,b^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/8*e^(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b*f) - 1/4*a*e^
(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b^2*f) + 1/4*a*e^(2*c
- 2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b^2*f) - 1/8*e^(3*c - 3*d*e
/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b*f) - 1/8*(4*a^2 + 3*b^2)*e^(-c +
 d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^3*f) - 1/8*(4*a^2*e^c + 3*b^2*e
^c)*e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^3*f) - (a^3 + a*b^2)*lo
g(f*x + e)/(b^4*f) + 1/16*integrate(32*(a^3*b + a*b^3 - (a^4*e^c + a^2*b^2*
e^c)*e^(d*x))/(b^5*f*x + b^5*e - (b^5*f*x*e^(2*c) + b^5*e*e^(2*c))*e^(2*d*x
) - 2*(a*b^4*f*x*e^c + a*b^4*e*e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)}{afx+ae+\left(bfx+be\right)\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(cosh(d*x + c)^3*sinh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)}{\left(fx+e\right)\left(b\sinh\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate(cosh(d*x + c)^3*sinh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)),
x)
```

3.348 $\int \frac{(e+fx)^3 \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1021

result too large to display

```
[Out] (2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*(e + f*x)^3*ArcTan[E^(c
+ d*x)])/(b*(a^2 + b^2)*d) - (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2])])/((a^2 + b^2)*d) - (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) + (a*(e + f*x)^3*Log[1 + E^(2*(c + d
*x))])/((a^2 + b^2)*d) - ((3*I)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])
/(b*d^2) + ((3*I)*a^2*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(b*(a^2 +
 b^2)*d^2) + ((3*I)*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - ((3*
I)*a^2*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) - (3*a*
f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 +
 b^2)*d^2) - (3*a*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))])/((a^2 + b^2)*d^2) + (3*a*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x
))])/(2*(a^2 + b^2)*d^2) + ((6*I)*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)
])/(b*d^3) - ((6*I)*a^2*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(b*(a^2
 + b^2)*d^3) - ((6*I)*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(b*d^3) + ((
6*I)*a^2*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^3) + (6*
a*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2
 + b^2)*d^3) + (6*a*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/((a^2 + b^2)*d^3) - (3*a*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d
*x))])/(2*(a^2 + b^2)*d^3) - ((6*I)*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(b*d^
4) + ((6*I)*a^2*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^4) + ((6
*I)*f^3*PolyLog[4, I*E^(c + d*x)])/(b*d^4) - ((6*I)*a^2*f^3*PolyLog[4, I*E^
(c + d*x)])/(b*(a^2 + b^2)*d^4) - (6*a*f^3*PolyLog[4, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^4) - (6*a*f^3*PolyLog[4, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^4) + (3*a*f^3*PolyLog[4, -E^(2
*(c + d*x))])/(4*(a^2 + b^2)*d^4)
```

Rubi [A] time = 1.32959, antiderivative size = 1021, normalized size of antiderivative = 1., number of steps used = 39, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {5567, 4180, 2531, 6609, 2282, 6589, 5573, 5561, 2190, 6742, 3718}

$$-\frac{6i\text{PolyLog}\left(4,-ie^{c+dx}\right)f^3}{bd^4}+\frac{6ia^2\text{PolyLog}\left(4,-ie^{c+dx}\right)f^3}{b\left(a^2+b^2\right)d^4}+\frac{6i\text{PolyLog}\left(4,ie^{c+dx}\right)f^3}{bd^4}-\frac{6ia^2\text{PolyLog}\left(4,ie^{c+dx}\right)f^3}{b\left(a^2+b^2\right)d^4}-\frac{6}{-}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*(e + f*x)^3*ArcTan[E^(c
+ d*x)])/(b*(a^2 + b^2)*d) - (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2])])/((a^2 + b^2)*d) - (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) + (a*(e + f*x)^3*Log[1 + E^(2*(c + d
*x))])/((a^2 + b^2)*d) - ((3*I)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])
/(b*d^2) + ((3*I)*a^2*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(b*(a^2 +
 b^2)*d^2) + ((3*I)*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - ((3*
I)*a^2*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) - (3*a*
f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 +
 b^2)*d^2) - (3*a*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))])/((a^2 + b^2)*d^2) + (3*a*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x
))])/(2*(a^2 + b^2)*d^2) + ((6*I)*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)
```

```
])/(b*d^3) - ((6*I)*a^2*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(b*(a^2
 + b^2)*d^3) - ((6*I)*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(b*d^3) + ((
6*I)*a^2*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^3) + (6*
a*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2
 + b^2)*d^3) + (6*a*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/((a^2 + b^2)*d^3) - (3*a*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d
*x))])/(2*(a^2 + b^2)*d^3) - ((6*I)*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(b*d^
4) + ((6*I)*a^2*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^4) + ((6
*I)*f^3*PolyLog[4, I*E^(c + d*x)])/(b*d^4) - ((6*I)*a^2*f^3*PolyLog[4, I*E^
(c + d*x)])/(b*(a^2 + b^2)*d^4) - (6*a*f^3*PolyLog[4, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^4) - (6*a*f^3*PolyLog[4, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^4) + (3*a*f^3*PolyLog[4, -E^(2
*(c + d*x))])/(4*(a^2 + b^2)*d^4)
```

Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c +
 d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\operatorname{sech}(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a\int (e+fx)^3\operatorname{sech}(c+dx)(a-b\sinh(c+dx))\,dx}{b\left(a^2+b^2\right)} - \frac{(ab)\int \cdots}{\cdots}$$

$$= \frac{a(e+fx)^4}{4\left(a^2+b^2\right)f} + \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{3if(e+fx)^2\operatorname{Li}_2\left(-ie^{c+dx}\right)}{bd^2} + \frac{3if(e+fx)^2\mathrm{L}\cdots}{bd^2}$$

$$= \frac{a(e+fx)^4}{4\left(a^2+b^2\right)f} + \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} - \frac{a(e+fx)^3\cdots}{(\cdots}$$

$$= \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d}\cdots$$

$$= \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d}\cdots$$

$$= \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d}\cdots$$

$$= \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d}\cdots$$

$$= \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d}\cdots$$

$$= \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d}\cdots$$

Mathematica [B] time = 25.5273, size = 2333, normalized size = 2.29

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (8*b*Sqrt[-(a^2 + b^2)^2]*d^3*e^3*ArcTan[E^(c + d*x)] - 8*a^2*Sqrt[a^2 + b^
2]*d^3*e^3*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]] - 8*a^2*Sqrt[-a^2 -
 b^2]*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + (12*I)*b*Sqrt[
-(a^2 + b^2)^2]*d^3*e^2*f*x*Log[1 - I*E^(c + d*x)] + (12*I)*b*Sqrt[-(a^2 +
b^2)^2]*d^3*e*f^2*x^2*Log[1 - I*E^(c + d*x)] + (4*I)*b*Sqrt[-(a^2 + b^2)^2]
*d^3*f^3*x^3*Log[1 - I*E^(c + d*x)] - (12*I)*b*Sqrt[-(a^2 + b^2)^2]*d^3*e^2
*f*x*Log[1 + I*E^(c + d*x)] - (12*I)*b*Sqrt[-(a^2 + b^2)^2]*d^3*e*f^2*x^2*L
og[1 + I*E^(c + d*x)] - (4*I)*b*Sqrt[-(a^2 + b^2)^2]*d^3*f^3*x^3*Log[1 + I*
E^(c + d*x)] + 4*a*Sqrt[-(a^2 + b^2)^2]*d^3*e^3*Log[1 + E^(2*(c + d*x))] +
12*a*Sqrt[-(a^2 + b^2)^2]*d^3*e^2*f*x*Log[1 + E^(2*(c + d*x))] + 12*a*Sqrt[
-(a^2 + b^2)^2]*d^3*e*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 4*a*Sqrt[-(a^2 + b
^2)^2]*d^3*f^3*x^3*Log[1 + E^(2*(c + d*x))] - 4*a*Sqrt[-(a^2 + b^2)^2]*d^3*
e^3*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] - 12*a*Sqrt[-(a^2 + b^2
)^2]*d^3*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c
)])] - 12*a*Sqrt[-(a^2 + b^2)^2]*d^3*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a
```

```
*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 4*a*Sqrt[-(a^2 + b^2)^2]*d^3*f^3*x^3*L
og[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 12*a*Sqrt[-
(a^2 + b^2)^2]*d^3*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b
^2)*E^(2*c)])] - 12*a*Sqrt[-(a^2 + b^2)^2]*d^3*e*f^2*x^2*Log[1 + (b*E^(2*c
+ d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 4*a*Sqrt[-(a^2 + b^2)^2]*d^3
*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - (
12*I)*b*Sqrt[-(a^2 + b^2)^2]*d^2*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)]
 + (12*I)*b*Sqrt[-(a^2 + b^2)^2]*d^2*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)
] + 6*a*Sqrt[-(a^2 + b^2)^2]*d^2*e^2*f*PolyLog[2, -E^(2*(c + d*x))] + 12*a*
Sqrt[-(a^2 + b^2)^2]*d^2*e*f^2*x*PolyLog[2, -E^(2*(c + d*x))] + 6*a*Sqrt[-(
a^2 + b^2)^2]*d^2*f^3*x^2*PolyLog[2, -E^(2*(c + d*x))] - 12*a*Sqrt[-(a^2 +
b^2)^2]*d^2*e^2*f*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*
E^(2*c)]))] - 24*a*Sqrt[-(a^2 + b^2)^2]*d^2*e*f^2*x*PolyLog[2, -((b*E^(2*c
+ d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 12*a*Sqrt[-(a^2 + b^2)^2]*d
^2*f^3*x^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)
]))] - 12*a*Sqrt[-(a^2 + b^2)^2]*d^2*e^2*f*PolyLog[2, -((b*E^(2*c + d*x))/(
a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 24*a*Sqrt[-(a^2 + b^2)^2]*d^2*e*f^2*
x*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 12
*a*Sqrt[-(a^2 + b^2)^2]*d^2*f^3*x^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c +
 Sqrt[(a^2 + b^2)*E^(2*c)]))] + (24*I)*b*Sqrt[-(a^2 + b^2)^2]*d*e*f^2*PolyL
og[3, (-I)*E^(c + d*x)] + (24*I)*b*Sqrt[-(a^2 + b^2)^2]*d*f^3*x*PolyLog[3,
(-I)*E^(c + d*x)] - (24*I)*b*Sqrt[-(a^2 + b^2)^2]*d*e*f^2*PolyLog[3, I*E^(c
 + d*x)] - (24*I)*b*Sqrt[-(a^2 + b^2)^2]*d*f^3*x*PolyLog[3, I*E^(c + d*x)]
- 6*a*Sqrt[-(a^2 + b^2)^2]*d*e*f^2*PolyLog[3, -E^(2*(c + d*x))] - 6*a*Sqrt[
-(a^2 + b^2)^2]*d*f^3*x*PolyLog[3, -E^(2*(c + d*x))] + 24*a*Sqrt[-(a^2 + b^
2)^2]*d*e*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)]))] + 24*a*Sqrt[-(a^2 + b^2)^2]*d*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/
(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 24*a*Sqrt[-(a^2 + b^2)^2]*d*e*f^2*P
olyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 24*a*
Sqrt[-(a^2 + b^2)^2]*d*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(
a^2 + b^2)*E^(2*c)]))] - (24*I)*b*Sqrt[-(a^2 + b^2)^2]*f^3*PolyLog[4, (-I)*
E^(c + d*x)] + (24*I)*b*Sqrt[-(a^2 + b^2)^2]*f^3*PolyLog[4, I*E^(c + d*x)]
+ 3*a*Sqrt[-(a^2 + b^2)^2]*f^3*PolyLog[4, -E^(2*(c + d*x))] - 24*a*Sqrt[-(a
^2 + b^2)^2]*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E
^(2*c)]))] - 24*a*Sqrt[-(a^2 + b^2)^2]*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(
a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/(4*Sqrt[-a^2 - b^2]*(a^2 + b^2)^(3/2)
*d^4)
```

Maple [F] time = 0.353, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3}\tanh\left(dx+c\right)}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-e^{3}\left(\frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{\left(a^{2}+b^{2}\right)d}+\frac{a\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{\left(a^{2}+b^{2}\right)d}-\frac{a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\left(a^{2}+b^{2}\right)d}\right)+\int\frac{2\,f^{3}x^{3}\left(e^{(dx+c)}-\cdots\right.}{\left(b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -e^3*(2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(-2*a*e^(-d*x - c) +
b*e^(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a^2
 + b^2)*d)) + integrate(2*f^3*x^3*(e^(d*x + c) - e^(-d*x - c))/((b*(e^(d*x
+ c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))) + 6*e*f^2*x^2*(e^
(d*x + c) - e^(-d*x - c))/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(e^(d*x +
 c) + e^(-d*x - c))) + 6*e^2*f*x*(e^(d*x + c) - e^(-d*x - c))/((b*(e^(d*x +
 c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))), x)
```

Fricas [C] time = 3.12044, size = 4177, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(6*a*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*a*f^3*polylog(4, (a*cosh(d
*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2))/b) + 3*(a*d^2*f^3*x^2 + 2*a*d^2*e*f^2*x + a*d^2*e^2*f)*dilog((a
*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b + 1) + 3*(a*d^2*f^3*x^2 + 2*a*d^2*e*f^2*x + a*d^2*
e^2*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (3*a*d^2*f^3*x^2 + 3*I*b*d^2
*f^3*x^2 + 6*a*d^2*e*f^2*x + 6*I*b*d^2*e*f^2*x + 3*a*d^2*e^2*f + 3*I*b*d^2*
e^2*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) - (3*a*d^2*f^3*x^2 - 3*I*b*
d^2*f^3*x^2 + 6*a*d^2*e*f^2*x - 6*I*b*d^2*e*f^2*x + 3*a*d^2*e^2*f - 3*I*b*d
^2*e^2*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + (a*d^3*e^3 - 3*a*c*d^
2*e^2*f + 3*a*c^2*d*e*f^2 - a*c^3*f^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x
 + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (a*d^3*e^3 - 3*a*c*d^2*e^2*f + 3
*a*c^2*d*e*f^2 - a*c^3*f^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b
*sqrt((a^2 + b^2)/b^2) + 2*a) + (a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^
3*e^2*f*x + 3*a*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 + a*c^3*f^3)*log(-(a*cosh(d*x
 + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2) - b)/b) + (a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + 3
*a*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 + a*c^3*f^3)*log(-(a*cosh(d*x + c) + a*sin
h(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)
/b) - (a*d^3*e^3 + I*b*d^3*e^3 - 3*a*c*d^2*e^2*f - 3*I*b*c*d^2*e^2*f + 3*a*
c^2*d*e*f^2 + 3*I*b*c^2*d*e*f^2 - a*c^3*f^3 - I*b*c^3*f^3)*log(cosh(d*x + c
) + sinh(d*x + c) + I) - (a*d^3*e^3 - I*b*d^3*e^3 - 3*a*c*d^2*e^2*f + 3*I*b
*c*d^2*e^2*f + 3*a*c^2*d*e*f^2 - 3*I*b*c^2*d*e*f^2 - a*c^3*f^3 + I*b*c^3*f^
3)*log(cosh(d*x + c) + sinh(d*x + c) - I) - (a*d^3*f^3*x^3 - I*b*d^3*f^3*x^
3 + 3*a*d^3*e*f^2*x^2 - 3*I*b*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x - 3*I*b*d^3*e
^2*f*x + 3*a*c*d^2*e^2*f - 3*I*b*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 + 3*I*b*c^2*
d*e*f^2 + a*c^3*f^3 - I*b*c^3*f^3)*log(I*cosh(d*x + c) + I*sinh(d*x + c) +
1) - (a*d^3*f^3*x^3 + I*b*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*I*b*d^3*e*f^2
*x^2 + 3*a*d^3*e^2*f*x + 3*I*b*d^3*e^2*f*x + 3*a*c*d^2*e^2*f + 3*I*b*c*d^2*
e^2*f - 3*a*c^2*d*e*f^2 - 3*I*b*c^2*d*e*f^2 + a*c^3*f^3 + I*b*c^3*f^3)*log(
-I*cosh(d*x + c) - I*sinh(d*x + c) + 1) - (6*a*f^3 + 6*I*b*f^3)*polylog(4,
I*cosh(d*x + c) + I*sinh(d*x + c)) - (6*a*f^3 - 6*I*b*f^3)*polylog(4, -I*co
sh(d*x + c) - I*sinh(d*x + c)) - 6*(a*d*f^3*x + a*d*e*f^2)*polylog(3, (a*co
sh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2))/b) - 6*(a*d*f^3*x + a*d*e*f^2)*polylog(3, (a*cosh(d*x + c)
+ a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2))/b) + (6*a*d*f^3*x + 6*I*b*d*f^3*x + 6*a*d*e*f^2 + 6*I*b*d*e*f^2)*polylo
```

```
g(3, I*cosh(d*x + c) + I*sinh(d*x + c)) + (6*a*d*f^3*x - 6*I*b*d*f^3*x + 6*
a*d*e*f^2 - 6*I*b*d*e*f^2)*polylog(3, -I*cosh(d*x + c) - I*sinh(d*x + c)))/
((a^2 + b^2)*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^3 \tanh (c+dx)}{a+b \sinh (c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.349 $\int \frac{(e+fx)^2 \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=716

$$\frac{2ia^2f(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{bd^2\left(a^2+b^2\right)}-\frac{2ia^2f(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{bd^2\left(a^2+b^2\right)}-\frac{2af(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)}-\frac{2af}{}$$

```
[Out] (2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*(e + f*x)^2*ArcTan[E^(c
+ d*x)])/(b*(a^2 + b^2)*d) - (a*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2])])/((a^2 + b^2)*d) - (a*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) + (a*(e + f*x)^2*Log[1 + E^(2*(c + d
*x))])/((a^2 + b^2)*d) - ((2*I)*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(
b*d^2) + ((2*I)*a^2*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b*(a^2 + b^2
)*d^2) + ((2*I)*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - ((2*I)*a^2
*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) - (2*a*f*(e + f
*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2)
 - (2*a*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
(a^2 + b^2)*d^2) + (a*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2
)*d^2) + ((2*I)*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b*d^3) - ((2*I)*a^2*f^2*
PolyLog[3, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^3) - ((2*I)*f^2*PolyLog[3, I
*E^(c + d*x)])/(b*d^3) + ((2*I)*a^2*f^2*PolyLog[3, I*E^(c + d*x)])/(b*(a^2
+ b^2)*d^3) + (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/((a^2 + b^2)*d^3) + (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/((a^2 + b^2)*d^3) - (a*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*(a^2
+ b^2)*d^3)
```

Rubi [A] time = 1.06746, antiderivative size = 716, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5567, 4180, 2531, 2282, 6589, 5573, 5561, 2190, 6742, 3718}

$$\frac{2ia^2f(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{bd^2\left(a^2+b^2\right)}-\frac{2ia^2f(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)}{bd^2\left(a^2+b^2\right)}-\frac{2af(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)}-\frac{2af}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*(e + f*x)^2*ArcTan[E^(c
+ d*x)])/(b*(a^2 + b^2)*d) - (a*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2])])/((a^2 + b^2)*d) - (a*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) + (a*(e + f*x)^2*Log[1 + E^(2*(c + d
*x))])/((a^2 + b^2)*d) - ((2*I)*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(
b*d^2) + ((2*I)*a^2*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b*(a^2 + b^2
)*d^2) + ((2*I)*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - ((2*I)*a^2
*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) - (2*a*f*(e + f
*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2)
 - (2*a*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
(a^2 + b^2)*d^2) + (a*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2
)*d^2) + ((2*I)*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b*d^3) - ((2*I)*a^2*f^2*
PolyLog[3, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^3) - ((2*I)*f^2*PolyLog[3, I
*E^(c + d*x)])/(b*d^3) + ((2*I)*a^2*f^2*PolyLog[3, I*E^(c + d*x)])/(b*(a^2
+ b^2)*d^3) + (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/((a^2 + b^2)*d^3) + (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/((a^2 + b^2)*d^3) - (a*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*(a^2
```

```
+ b^2)*d^3)
```

Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c +
 d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\mathbf{sech}(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\mathrm{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a\int (e+fx)^2\mathbf{sech}(c+dx)(a-b\sinh(c+dx))\,dx}{b\left(a^2+b^2\right)} - \frac{(ab)\int \cdots}{\cdots}$$

$$= \frac{a(e+fx)^3}{3\left(a^2+b^2\right)f} + \frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2if(e+fx)\mathrm{Li}_2\left(-ie^{c+dx}\right)}{bd^2} + \frac{2if(e+fx)\mathrm{Li}_2 \cdots}{bd^2}$$

$$= \frac{a(e+fx)^3}{3\left(a^2+b^2\right)f} + \frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} - \frac{a(e+fx)^2 \cdots}{\cdots}$$

$$= \frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} \cdots$$

$$= \frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} \cdots$$

$$= \frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} \cdots$$

$$= \frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} \cdots$$

$$= \frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} \cdots$$

Mathematica [B] time = 19.14, size = 1640, normalized size = 2.29

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-12*a*d^3*e^2*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*d^3*e*f*x^2
+ 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] - 6*a*d^
2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*b*d*e*(1 +
E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - PolyLog
[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*e*(1 + E^(2*c))*
f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]) +
 (6*I)*b*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log[1
+ I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2, I*
E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*x)]
) - a*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))]) - 6
*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))]))/(6*(a^
2 + b^2)*d^3*(1 + E^(2*c))) + (a*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E
^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-
a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*
c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6
*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(
(-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a +
b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^
(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*
x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E
^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c +
d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*
c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[
1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Lo
g[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2
*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d +
 (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])
/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)
*E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x
))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*
x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^
2 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)
]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^
2 + b^2)*E^(2*c)]))])/d^3 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c +
Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c +
 d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3))/(3*(a^2 + b^2)*(-1 + E^
(2*c))) - (a*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Csch[c/2]*Sech[c/2]*Sech[c])/(6*
(a^2 + b^2))
```

Maple [F] time = 0.285, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \tanh \left(dx+c\right)}{a+b\sinh \left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-e^2\left(\frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d}+\frac{a\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{(a^2+b^2)d}-\frac{a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d}\right)+\int\frac{2\,f^2x^2\left(e^{(dx+c}\right.}{\left(b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -e^2*(2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(-2*a*e^(-d*x - c) +
b*e^(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a^2
 + b^2)*d)) + integrate(2*f^2*x^2*(e^(d*x + c) - e^(-d*x - c))/((b*(e^(d*x
+ c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))) + 4*e*f*x*(e^(d*x
 + c) - e^(-d*x - c))/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c)
+ e^(-d*x - c))), x)
```

Fricas [C] time = 2.70121, size = 2743, normalized size = 3.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*a*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*a*f^2*polylog(3, (a*cosh(d*
x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2))/b) - 2*(a*d*f^2*x + a*d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x
 + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b +
1) - 2*(a*d*f^2*x + a*d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (2*a*d
*f^2*x + 2*I*b*d*f^2*x + 2*a*d*e*f + 2*I*b*d*e*f)*dilog(I*cosh(d*x + c) + I
*sinh(d*x + c)) + (2*a*d*f^2*x - 2*I*b*d*f^2*x + 2*a*d*e*f - 2*I*b*d*e*f)*d
ilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) - (a*d^2*e^2 - 2*a*c*d*e*f + a*c^2
*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2)
 + 2*a) - (a*d^2*e^2 - 2*a*c*d*e*f + a*c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b
*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (a*d^2*f^2*x^2 + 2*a*d^
2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
+ (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (a*d^
2*f^2*x^2 + 2*a*d^2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*log(-(a*cosh(d*x + c)
+ a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2) - b)/b) + (a*d^2*e^2 + I*b*d^2*e^2 - 2*a*c*d*e*f - 2*I*b*c*d*e*f + a*c^2
*f^2 + I*b*c^2*f^2)*log(cosh(d*x + c) + sinh(d*x + c) + I) + (a*d^2*e^2 - I
*b*d^2*e^2 - 2*a*c*d*e*f + 2*I*b*c*d*e*f + a*c^2*f^2 - I*b*c^2*f^2)*log(cos
h(d*x + c) + sinh(d*x + c) - I) + (a*d^2*f^2*x^2 - I*b*d^2*f^2*x^2 + 2*a*d^
2*e*f*x - 2*I*b*d^2*e*f*x + 2*a*c*d*e*f - 2*I*b*c*d*e*f - a*c^2*f^2 + I*b*c
^2*f^2)*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) + (a*d^2*f^2*x^2 + I*b*d
^2*f^2*x^2 + 2*a*d^2*e*f*x + 2*I*b*d^2*e*f*x + 2*a*c*d*e*f + 2*I*b*c*d*e*f
- a*c^2*f^2 - I*b*c^2*f^2)*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1) - (2
*a*f^2 + 2*I*b*f^2)*polylog(3, I*cosh(d*x + c) + I*sinh(d*x + c)) - (2*a*f^
2 - 2*I*b*f^2)*polylog(3, -I*cosh(d*x + c) - I*sinh(d*x + c)))/((a^2 + b^2)
*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^2 \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.350 $\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=421

$$\frac{ia^2 f\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{bd^2\left(a^2+b^2\right)}-\frac{ia^2 f\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{bd^2\left(a^2+b^2\right)}-\frac{af\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)}-\frac{af\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2\left(a^2+b^2\right)}+ af$$

```
[Out] (2*(e + f*x)*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*(e + f*x)*ArcTan[E^(c + d*
x)])/(b*(a^2 + b^2)*d) - (a*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2])])/((a^2 + b^2)*d) - (a*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])])/((a^2 + b^2)*d) + (a*(e + f*x)*Log[1 + E^(2*(c + d*x))])/((a
^2 + b^2)*d) - (I*f*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + (I*a^2*f*PolyLo
g[2, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) + (I*f*PolyLog[2, I*E^(c + d*x)
])/(b*d^2) - (I*a^2*f*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) - (a*f
*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2) -
(a*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2
) + (a*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)*d^2)
```

Rubi [A] time = 0.596636, antiderivative size = 421, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5567, 4180, 2279, 2391, 5573, 5561, 2190, 6742, 3718}

$$\frac{ia^2 f\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{bd^2\left(a^2+b^2\right)}-\frac{ia^2 f\,\text{PolyLog}\left(2,ie^{c+dx}\right)}{bd^2\left(a^2+b^2\right)}-\frac{af\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)}-\frac{af\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2\left(a^2+b^2\right)}+ af$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*(e + f*x)*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*(e + f*x)*ArcTan[E^(c + d*
x)])/(b*(a^2 + b^2)*d) - (a*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
 + b^2])])/((a^2 + b^2)*d) - (a*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])])/((a^2 + b^2)*d) + (a*(e + f*x)*Log[1 + E^(2*(c + d*x))])/((a
^2 + b^2)*d) - (I*f*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + (I*a^2*f*PolyLo
g[2, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) + (I*f*PolyLog[2, I*E^(c + d*x)
])/(b*d^2) - (I*a^2*f*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) - (a*f
*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2) -
(a*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2
) + (a*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)*d^2)
```

Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c +
 d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
```

```
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{sech}(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a\int (e+fx)\text{sech}(c+dx)(a-b\sinh(c+dx))\,dx}{b\left(a^2+b^2\right)} - \frac{(ab)\int \frac{(e+f}{a+}}{a^2}$$

$$= \frac{a(e+fx)^2}{2\left(a^2+b^2\right)f} + \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a\int (a(e+fx)\text{sech}(c+dx) - b(e+fx)\tanh(}{b\left(a^2+b^2\right)}$$

$$= \frac{a(e+fx)^2}{2\left(a^2+b^2\right)f} + \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} - \frac{a(e+fx)\log}{\left(a^2+\right.}$$

$$= \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} - \frac{a}{}$$

$$= \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} - \frac{a}{}$$

$$= \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} - \frac{a}{}$$

$$= \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d} - \frac{a(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)d} - \frac{a}{}$$

Mathematica [A] time = 2.63113, size = 438, normalized size = 1.04

$$-2af\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) - 2af\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) + af\text{PolyLog}(2,-\sinh(2(c+dx))-\cosh(2(c+dx))) - 2$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*a*c*d*e + 2*a*c^2*f - 2*a*d^2*e*x + 2*a*c*d*f*x + 4*b*d*e*ArcTan[Cosh[c
 + d*x] + Sinh[c + d*x]] + 4*b*d*f*x*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]]
- 2*a*c*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*a*d*f*x*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*a*c*f*Log[1 + (b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2])] - 2*a*d*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2])] - 2*a*d*e*Log[a + b*Sinh[c + d*x]] + 2*a*c*f*Log[a + b*Sinh[c + d*x]
] + 2*a*d*e*Log[1 + Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]] + 2*a*d*f*x*Log[
1 + Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]] - 2*a*f*PolyLog[2, (b*E^(c + d*x
))/(-a + Sqrt[a^2 + b^2])] - 2*a*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2]))] - (2*I)*b*f*PolyLog[2, (-I)*(Cosh[c + d*x] + Sinh[c + d*x])] +
 (2*I)*b*f*PolyLog[2, I*(Cosh[c + d*x] + Sinh[c + d*x])] + a*f*PolyLog[2, -
Cosh[2*(c + d*x)] - Sinh[2*(c + d*x)]])/(2*(a^2 + b^2)*d^2)
```

Maple [B] time = 0.154, size = 1287, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 2/d^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*c-2/d*f/(2*a^2+2*b^2)*ln((-b*exp
(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a*x-2/d^2*f/(2*a^2+2*b^2)*
ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a*c-2/d*f/(2*a^2
+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a*x-2/d^2*
f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a*
c+2/d*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*a*x-4/d^2*f*c/(2*a^2+2*b^2)*b*arct
an(exp(d*x+c))+2/d^2*f*c/(2*a^2+2*b^2)*a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)
-b)-2/d^2*f*c/(2*a^2+2*b^2)*(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a
)/(a^2+b^2)^(1/2))-2/d^2*f*c/(2*a^2+2*b^2)*a*ln(1+exp(2*d*x+2*c))-2/d*e/(2*
a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2)
)*a^2+2*I/d^2*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))*b-2*I/d^2*f/(2*a^2+2*b^
2)*dilog(1+I*exp(d*x+c))*b+2/d^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*a*c+2/d
*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*x+2/d^2*f*c/(2*a^2+2*b^2)/(a^2+b^2)^(
1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2+2*I/d*f/(2*a^2+2
*b^2)*ln(1-I*exp(d*x+c))*b*x+2*I/d^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*b*c
-2*I/d*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*b*x-2*I/d^2*f/(2*a^2+2*b^2)*ln(1+
I*exp(d*x+c))*b*c-2/d/(a^2+b^2)^(1/2)*b^2*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*
exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d^2/(a^2+b^2)^(1/2)*b^2*f*c/(2*a^2+2*b^2
)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d^2*f/(2*a^2+2*b^2)*d
ilog(1-I*exp(d*x+c))*a-2/d^2*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^
(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a-2/d^2*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+
(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a+2/d*e/(2*a^2+2*b^2)*a*ln(1+exp(2
*d*x+2*c))+4/d*e/(2*a^2+2*b^2)*b*arctan(exp(d*x+c))-2/d*e/(2*a^2+2*b^2)*a*l
n(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d*e/(2*a^2+2*b^2)*(a^2+b^2)^(1/2)*ar
ctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d^2*f/(2*a^2+2*b^2)*dilog
(1+I*exp(d*x+c))*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-e\left(\frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{\left(a^2+b^2\right)d}+\frac{a\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{\left(a^2+b^2\right)d}-\frac{a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\left(a^2+b^2\right)d}\right)+f\int\frac{2\,x\left(e^{(dx+c)}-\cdots\right.}{\left(b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -e*(2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(-2*a*e^(-d*x - c) + b*
e^(-2*d*x - 2*c) - b)/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a^2 +
 b^2)*d)) + f*integrate(2*x*(e^(d*x + c) - e^(-d*x - c))/((b*(e^(d*x + c) -
 e^(-d*x - c)) + 2*a)*(e^(d*x + c) + e^(-d*x - c))), x)
```

Fricas [A] time = 2.46716, size = 1544, normalized size = 3.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(a*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + a*f*dilog((a*cosh(d*x + c) +
a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
```

```
 - b)/b + 1) - (a*f + I*b*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) - (a*
f - I*b*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + (a*d*e - a*c*f)*log(
2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) +
(a*d*e - a*c*f)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 +
 b^2)/b^2) + 2*a) + (a*d*f*x + a*c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x +
c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (a
*d*f*x + a*c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (a*d*e + I*b*d*e - a*c*f
 - I*b*c*f)*log(cosh(d*x + c) + sinh(d*x + c) + I) - (a*d*e - I*b*d*e - a*c
*f + I*b*c*f)*log(cosh(d*x + c) + sinh(d*x + c) - I) - (a*d*f*x - I*b*d*f*x
 + a*c*f - I*b*c*f)*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) - (a*d*f*x +
 I*b*d*f*x + a*c*f + I*b*c*f)*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1))/
((a^2 + b^2)*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.351 $\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=69

$$-\frac{a\log(a+b\sinh(c+dx))}{d\left(a^2+b^2\right)}+\frac{b\tan^{-1}(\sinh(c+dx))}{d\left(a^2+b^2\right)}+\frac{a\log(\cosh(c+dx))}{d\left(a^2+b^2\right)}$$

```
[Out] (b*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d) + (a*Log[Cosh[c + d*x]])/((a^2 +
b^2)*d) - (a*Log[a + b*Sinh[c + d*x]])/((a^2 + b^2)*d)
```

Rubi [A] time = 0.0769974, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2721, 801, 635, 203, 260}

$$-\frac{a\log(a+b\sinh(c+dx))}{d\left(a^2+b^2\right)}+\frac{b\tan^{-1}(\sinh(c+dx))}{d\left(a^2+b^2\right)}+\frac{a\log(\cosh(c+dx))}{d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d) + (a*Log[Cosh[c + d*x]])/((a^2 +
b^2)*d) - (a*Log[a + b*Sinh[c + d*x]])/((a^2 + b^2)*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x}{(a+x)(-b^2-x^2)}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a}{(a^2+b^2)(a+x)} + \frac{-b^2-ax}{(a^2+b^2)(b^2+x^2)}\right)dx, x, b\sinh(c+dx)\right)}{d}$$

$$= -\frac{a\log(a+b\sinh(c+dx))}{(a^2+b^2)\,d} - \frac{\text{Subst}\left(\int \frac{-b^2-ax}{b^2+x^2}\,dx, x, b\sinh(c+dx)\right)}{(a^2+b^2)\,d}$$

$$= -\frac{a\log(a+b\sinh(c+dx))}{(a^2+b^2)\,d} + \frac{a\,\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(c+dx)\right)}{(a^2+b^2)\,d} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{b^2+x^2}\,d\right.}{(a^2+}$$

$$= \frac{b\tan^{-1}(\sinh(c+dx))}{(a^2+b^2)\,d} + \frac{a\log(\cosh(c+dx))}{(a^2+b^2)\,d} - \frac{a\log(a+b\sinh(c+dx))}{(a^2+b^2)\,d}$$

Mathematica [A] time = 0.0767782, size = 51, normalized size = 0.74

$$\frac{a(\log(\cosh(c+dx)) - \log(a+b\sinh(c+dx))) + 2b\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*b*ArcTan[Tanh[(c + d*x)/2]] + a*(Log[Cosh[c + d*x]] - Log[a + b*Sinh[c +
 d*x]]))/((a^2 + b^2)*d)
```

Maple [A] time = 0.002, size = 113, normalized size = 1.6

$$-2\,\frac{a\ln\left((\tanh(1/2\,dx+c/2))^2 a - 2\tanh(1/2\,dx+c/2)\,b - a\right)}{d\left(2\,a^2+2\,b^2\right)} + 2\,\frac{a\ln\left((\tanh(1/2\,dx+c/2))^2+1\right)}{d\left(2\,a^2+2\,b^2\right)} + 4\,\frac{b\arctan(\tan}{d\left(2\,a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] -2/d*a/(2*a^2+2*b^2)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)*b-a)+
2/d/(2*a^2+2*b^2)*a*ln(tanh(1/2*d*x+1/2*c)^2+1)+4/d/(2*a^2+2*b^2)*b*arctan(
tanh(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.60528, size = 128, normalized size = 1.86

$$-\frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{(a^2+b^2)d} - \frac{a\log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{(a^2+b^2)d} + \frac{a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{(a^2+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(-2*a*e^(-d*x - c) + b*e^(
-2*d*x - 2*c) - b)/((a^2 + b^2)*d) + a*log(e^(-2*d*x - 2*c) + 1)/((a^2 + b^
2)*d)
```

Fricas [A] time = 2.13869, size = 247, normalized size = 3.58

$$\frac{2\,b\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)-a\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)+a\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{\left(a^2+b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*b*arctan(cosh(d*x + c) + sinh(d*x + c)) - a*log(2*(b*sinh(d*x + c) + a)/
(cosh(d*x + c) - sinh(d*x + c))) + a*log(2*cosh(d*x + c)/(cosh(d*x + c) - s
inh(d*x + c))))/((a^2 + b^2)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A] time = 1.44589, size = 115, normalized size = 1.67

$$\frac{\frac{2\,b\arctan\left(e^{(dx+c)}\right)}{a^2+b^2}+\frac{a\log\left(e^{(2\,dx+2\,c)}+1\right)}{a^2+b^2}-\frac{a\log\left(\left|be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}-b\right|\right)}{a^2+b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (2*b*arctan(e^(d*x + c))/(a^2 + b^2) + a*log(e^(2*d*x + 2*c) + 1)/(a^2 + b^
2) - a*log(abs(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - b))/(a^2 + b^2))/d
```

3.352 $\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Tanh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0476321, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Tanh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A] time = 16.8269, size = 0, normalized size = 0.

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Tanh[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A] time = 0.244, size = 0, normalized size = 0.

$$\int \frac{\tanh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh (dx + c)}{(fx + e)(b \sinh (dx + c) + a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tanh (dx + c)}{afx + ae + (bfx + be) \sinh (dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh (c + dx)}{(a + b \sinh (c + dx)) (e + fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.353 $\int \frac{(e+fx)^3 \mathbf{sech}(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$

Optimal. Leaf size=917

result too large to display

```
[Out] (e + f*x)^3/(b*d) - (a^2*(e + f*x)^3)/(b*(a^2 + b^2)*d) + (6*a*f*(e + f*x)^
2*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d^2) - (a*b*(e + f*x)^3*Log[1 + (b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) + (a*b*(e + f*x)^3*L
og[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (3*f
*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b*d^2) + (3*a^2*f*(e + f*x)^2*Log[1
 + E^(2*(c + d*x))])/(b*(a^2 + b^2)*d^2) - ((6*I)*a*f^2*(e + f*x)*PolyLog[2
, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) + ((6*I)*a*f^2*(e + f*x)*PolyLog[2,
I*E^(c + d*x)])/((a^2 + b^2)*d^3) - (3*a*b*f*(e + f*x)^2*PolyLog[2, -((b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) + (3*a*b*f*(e +
 f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^
(3/2)*d^2) - (3*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(b*d^3) + (3*a^
2*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(b*(a^2 + b^2)*d^3) + ((6*I)*
a*f^3*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^4) - ((6*I)*a*f^3*PolyLo
g[3, I*E^(c + d*x)])/((a^2 + b^2)*d^4) + (6*a*b*f^2*(e + f*x)*PolyLog[3, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) - (6*a*b*f
^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 +
b^2)^(3/2)*d^3) + (3*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*b*d^4) - (3*a^2*f
^3*PolyLog[3, -E^(2*(c + d*x))])/(2*b*(a^2 + b^2)*d^4) - (6*a*b*f^3*PolyLog
[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^4) + (6
*a*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)
^(3/2)*d^4) - (a*(e + f*x)^3*Sech[c + d*x])/((a^2 + b^2)*d) + ((e + f*x)^3*
Tanh[c + d*x])/(b*d) - (a^2*(e + f*x)^3*Tanh[c + d*x])/(b*(a^2 + b^2)*d)
```

Rubi [A] time = 1.71208, antiderivative size = 917, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 14, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5583, 4184, 3718, 2190, 2531, 2282, 6589, 5573, 3322, 2264, 6609, 6742, 5451, 4180}

$$\frac{6ia\text{PolyLog}\left(3,-ie^{c+dx}\right)f^3}{\left(a^2+b^2\right)d^4}-\frac{6ia\text{PolyLog}\left(3,ie^{c+dx}\right)f^3}{\left(a^2+b^2\right)d^4}+\frac{3\text{PolyLog}\left(3,-e^{2(c+dx)}\right)f^3}{2bd^4}-\frac{3a^2\text{PolyLog}\left(3,-e^{2(c+dx)}\right)f^3}{2b\left(a^2+b^2\right)d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (e + f*x)^3/(b*d) - (a^2*(e + f*x)^3)/(b*(a^2 + b^2)*d) + (6*a*f*(e + f*x)^
2*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d^2) - (a*b*(e + f*x)^3*Log[1 + (b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) + (a*b*(e + f*x)^3*L
og[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (3*f
*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b*d^2) + (3*a^2*f*(e + f*x)^2*Log[1
 + E^(2*(c + d*x))])/(b*(a^2 + b^2)*d^2) - ((6*I)*a*f^2*(e + f*x)*PolyLog[2
, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) + ((6*I)*a*f^2*(e + f*x)*PolyLog[2,
I*E^(c + d*x)])/((a^2 + b^2)*d^3) - (3*a*b*f*(e + f*x)^2*PolyLog[2, -((b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) + (3*a*b*f*(e +
 f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^
(3/2)*d^2) - (3*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(b*d^3) + (3*a^
2*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(b*(a^2 + b^2)*d^3) + ((6*I)*
a*f^3*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^4) - ((6*I)*a*f^3*PolyLo
g[3, I*E^(c + d*x)])/((a^2 + b^2)*d^4) + (6*a*b*f^2*(e + f*x)*PolyLog[3, -(
```

```
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) - (6*a*b*f
^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 +
b^2)^(3/2)*d^3) + (3*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*b*d^4) - (3*a^2*f
^3*PolyLog[3, -E^(2*(c + d*x))])/(2*b*(a^2 + b^2)*d^4) - (6*a*b*f^3*PolyLog
[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^4) + (6
*a*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)
^(3/2)*d^4) - (a*(e + f*x)^3*Sech[c + d*x])/((a^2 + b^2)*d) + ((e + f*x)^3*
Tanh[c + d*x])/(b*d) - (a^2*(e + f*x)^3*Tanh[c + d*x])/(b*(a^2 + b^2)*d)
```

Rule 5583

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\text{sech}^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)^3\tanh(c+dx)}{bd} - \frac{a\int (e+fx)^3\text{sech}^2(c+dx)(a-b\sinh(c+dx))\,dx}{b\left(a^2+b^2\right)}$$

$$= \frac{(e+fx)^3}{bd} + \frac{(e+fx)^3\tanh(c+dx)}{bd} - \frac{a\int \left(a(e+fx)^3\text{sech}^2(c+dx) - b(e+\right.}{b\left(a^2+\right.}$$

$$= \frac{(e+fx)^3}{bd} - \frac{3f(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{bd^2} + \frac{(e+fx)^3\tanh(c+dx)}{bd} - \frac{\left(2ab^2\right.}{}$$

$$= \frac{(e+fx)^3}{bd} - \frac{ab(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} + \frac{ab(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d}$$

$$= \frac{(e+fx)^3}{bd} - \frac{a^2(e+fx)^3}{b\left(a^2+b^2\right)d} + \frac{6af(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d^2} - \frac{ab(e+fx)^3\log\left(1\right.}{\left(a^2+b^2\right)^{3}}$$

$$= \frac{(e+fx)^3}{bd} - \frac{a^2(e+fx)^3}{b\left(a^2+b^2\right)d} + \frac{6af(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d^2} - \frac{ab(e+fx)^3\log\left(1\right.}{\left(a^2+b^2\right)^{3}}$$

$$= \frac{(e+fx)^3}{bd} - \frac{a^2(e+fx)^3}{b\left(a^2+b^2\right)d} + \frac{6af(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d^2} - \frac{ab(e+fx)^3\log\left(1\right.}{\left(a^2+b^2\right)^{3}}$$

$$= \frac{(e+fx)^3}{bd} - \frac{a^2(e+fx)^3}{b\left(a^2+b^2\right)d} + \frac{6af(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d^2} - \frac{ab(e+fx)^3\log\left(1\right.}{\left(a^2+b^2\right)^{3}}$$

$$= \frac{(e+fx)^3}{bd} - \frac{a^2(e+fx)^3}{b\left(a^2+b^2\right)d} + \frac{6af(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d^2} - \frac{ab(e+fx)^3\log\left(1\right.}{\left(a^2+b^2\right)^{3}}$$

Mathematica [A] time = 13.2951, size = 1143, normalized size = 1.25

$$\frac{f\left(-4bf^2x^3d^3 - 12befx^2d^3 + 12be^2e^{2c}xd^3 - 12be^2\left(1+e^{2c}\right)xd^3 + 12ae^2\left(1+e^{2c}\right)\tan^{-1}\left(e^{c+dx}\right)d^2 + 6be^2\left(1+e^{2c}\right)\left(2dx - \text{l}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (f*(12*b*d^3*e^2*E^(2*c)*x - 12*b*d^3*e^2*(1 + E^(2*c))*x - 12*b*d^3*e*f*x^
2 - 4*b*d^3*f^2*x^3 + 12*a*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] + 6*b*
d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*a*d*e*(1
+ E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - PolyL
og[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) + 6*b*d*e*(1 + E^(2*c)
)*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))])
 + (6*I)*a*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log[
1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2,
I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*x
)]) + b*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))]) -
```

```
 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))]))))/(2*
(a^2 + b^2)*d^4*(1 + E^(2*c))) + (a*b*(2*d^3*e^3*ArcTanh[(a + b*E^(c + d*x)
)/Sqrt[a^2 + b^2]] - 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2])] - 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d
^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 3*d^3*e^2*f*x*L
og[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a
+ Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))] + 6*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 +
 b^2])] + 6*d*f^3*x*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*
d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*f^3*x*Po
lyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*f^3*PolyLog[4, (b*E^
(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))]))/((a^2 + b^2)^(3/2)*d^4) + (Sech[c]*Sech[c + d*x]*(-(
a*e^3*Cosh[c]) - 3*a*e^2*f*x*Cosh[c] - 3*a*e*f^2*x^2*Cosh[c] - a*f^3*x^3*Co
sh[c] + b*e^3*Sinh[d*x] + 3*b*e^2*f*x*Sinh[d*x] + 3*b*e*f^2*x^2*Sinh[d*x] +
 b*f^3*x^3*Sinh[d*x]))/((a^2 + b^2)*d)
```

Maple [F] time = 0.728, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3} \operatorname{sech}\left(dx+c\right) \tanh\left(dx+c\right)}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 4.40787, size = 14924, normalized size = 16.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -1/2*(4*(a^2*b + b^3)*d^3*e^3 - 12*(a^2*b + b^3)*c*d^2*e^2*f + 12*(a^2*b +
b^3)*c^2*d*e*f^2 - 4*(a^2*b + b^3)*c^3*f^3 - 4*((a^2*b + b^3)*d^3*f^3*x^3 +
```

3*(a^2*b + b^3)*d^3*e*f^2*x^2 + 3*(a^2*b + b^3)*d^3*e^2*f*x + 3*(a^2*b + b
^3)*c*d^2*e^2*f - 3*(a^2*b + b^3)*c^2*d*e*f^2 + (a^2*b + b^3)*c^3*f^3)*cosh
(d*x + c)^2 - 4*((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3)*d^3*e*f^2*x^2
+ 3*(a^2*b + b^3)*d^3*e^2*f*x + 3*(a^2*b + b^3)*c*d^2*e^2*f - 3*(a^2*b + b^
3)*c^2*d*e*f^2 + (a^2*b + b^3)*c^3*f^3)*sinh(d*x + c)^2 + 6*(a*b^2*d^2*f^3*
x^2 + 2*a*b^2*d^2*e*f^2*x + a*b^2*d^2*e^2*f + (a*b^2*d^2*f^3*x^2 + 2*a*b^2*
d^2*e*f^2*x + a*b^2*d^2*e^2*f)*cosh(d*x + c)^2 + 2*(a*b^2*d^2*f^3*x^2 + 2*a
*b^2*d^2*e*f^2*x + a*b^2*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d^
2*f^3*x^2 + 2*a*b^2*d^2*e*f^2*x + a*b^2*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt((a
^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 6*(a*b^2*d^2*f^3*x^2
+ 2*a*b^2*d^2*e*f^2*x + a*b^2*d^2*e^2*f + (a*b^2*d^2*f^3*x^2 + 2*a*b^2*d^2
*e*f^2*x + a*b^2*d^2*e^2*f)*cosh(d*x + c)^2 + 2*(a*b^2*d^2*f^3*x^2 + 2*a*b^
2*d^2*e*f^2*x + a*b^2*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d^2*f
^3*x^2 + 2*a*b^2*d^2*e*f^2*x + a*b^2*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt((a^2
+ b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(a*b^2*d^3*e^3 - 3*a*
b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 - a*b^2*c^3*f^3 + (a*b^2*d^3*e^3 - 3*
a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 - a*b^2*c^3*f^3)*cosh(d*x + c)^2 +
2*(a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 - a*b^2*c^3*f^
3)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a
*b^2*c^2*d*e*f^2 - a*b^2*c^3*f^3)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*lo
g(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
+ 2*(a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 - a*b^2*c^3*
f^3 + (a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 - a*b^2*c^
3*f^3)*cosh(d*x + c)^2 + 2*(a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c
^2*d*e*f^2 - a*b^2*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d^3*e^3 -
3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 - a*b^2*c^3*f^3)*sinh(d*x + c)^2)
*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt
((a^2 + b^2)/b^2) + 2*a) + 2*(a*b^2*d^3*f^3*x^3 + 3*a*b^2*d^3*e*f^2*x^2 + 3
*a*b^2*d^3*e^2*f*x + 3*a*b^2*c*d^2*e^2*f - 3*a*b^2*c^2*d*e*f^2 + a*b^2*c^3*
f^3 + (a*b^2*d^3*f^3*x^3 + 3*a*b^2*d^3*e*f^2*x^2 + 3*a*b^2*d^3*e^2*f*x + 3*
a*b^2*c*d^2*e^2*f - 3*a*b^2*c^2*d*e*f^2 + a*b^2*c^3*f^3)*cosh(d*x + c)^2 +
2*(a*b^2*d^3*f^3*x^3 + 3*a*b^2*d^3*e*f^2*x^2 + 3*a*b^2*d^3*e^2*f*x + 3*a*b^
2*c*d^2*e^2*f - 3*a*b^2*c^2*d*e*f^2 + a*b^2*c^3*f^3)*cosh(d*x + c)*sinh(d*x
+ c) + (a*b^2*d^3*f^3*x^3 + 3*a*b^2*d^3*e*f^2*x^2 + 3*a*b^2*d^3*e^2*f*x +
3*a*b^2*c*d^2*e^2*f - 3*a*b^2*c^2*d*e*f^2 + a*b^2*c^3*f^3)*sinh(d*x + c)^2)
*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*
x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(a*b^2*d^3*f^3*
x^3 + 3*a*b^2*d^3*e*f^2*x^2 + 3*a*b^2*d^3*e^2*f*x + 3*a*b^2*c*d^2*e^2*f - 3
*a*b^2*c^2*d*e*f^2 + a*b^2*c^3*f^3 + (a*b^2*d^3*f^3*x^3 + 3*a*b^2*d^3*e*f^2
*x^2 + 3*a*b^2*d^3*e^2*f*x + 3*a*b^2*c*d^2*e^2*f - 3*a*b^2*c^2*d*e*f^2 + a*
b^2*c^3*f^3)*cosh(d*x + c)^2 + 2*(a*b^2*d^3*f^3*x^3 + 3*a*b^2*d^3*e*f^2*x^2
+ 3*a*b^2*d^3*e^2*f*x + 3*a*b^2*c*d^2*e^2*f - 3*a*b^2*c^2*d*e*f^2 + a*b^2*
c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d^3*f^3*x^3 + 3*a*b^2*d^3*e*f
^2*x^2 + 3*a*b^2*d^3*e^2*f*x + 3*a*b^2*c*d^2*e^2*f - 3*a*b^2*c^2*d*e*f^2 +
a*b^2*c^3*f^3)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c)
+ a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2) - b)/b) + 12*(a*b^2*f^3*cosh(d*x + c)^2 + 2*a*b^2*f^3*cosh(d*x + c)*sin
h(d*x + c) + a*b^2*f^3*sinh(d*x + c)^2 + a*b^2*f^3)*sqrt((a^2 + b^2)/b^2)*p
olylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2))/b) - 12*(a*b^2*f^3*cosh(d*x + c)^2 + 2*a*b^2
*f^3*cosh(d*x + c)*sinh(d*x + c) + a*b^2*f^3*sinh(d*x + c)^2 + a*b^2*f^3)*s
qrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 12*(a*b^2*d*f^3*x
+ a*b^2*d*e*f^2 + (a*b^2*d*f^3*x + a*b^2*d*e*f^2)*cosh(d*x + c)^2 + 2*(a*b
^2*d*f^3*x + a*b^2*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d*f^3*x +
a*b^2*d*e*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*
x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +

b^2)/b^2))/b) + 12*(a*b^2*d*f^3*x + a*b^2*d*e*f^2 + (a*b^2*d*f^3*x + a*b^2*
d*e*f^2)*cosh(d*x + c)^2 + 2*(a*b^2*d*f^3*x + a*b^2*d*e*f^2)*cosh(d*x + c)*
sinh(d*x + c) + (a*b^2*d*f^3*x + a*b^2*d*e*f^2)*sinh(d*x + c)^2)*sqrt((a^2
+ b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 4*((a^3 + a*b^2)*d^3*f^3*x
^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + (a^3 + a
*b^2)*d^3*e^3)*cosh(d*x + c) - (12*I*(a^3 + a*b^2)*d*f^3*x - 12*(a^2*b + b^
3)*d*f^3*x + 12*I*(a^3 + a*b^2)*d*e*f^2 - 12*(a^2*b + b^3)*d*e*f^2 + (12*I*
(a^3 + a*b^2)*d*f^3*x - 12*(a^2*b + b^3)*d*f^3*x + 12*I*(a^3 + a*b^2)*d*e*f
^2 - 12*(a^2*b + b^3)*d*e*f^2)*cosh(d*x + c)^2 + (24*I*(a^3 + a*b^2)*d*f^3*
x - 24*(a^2*b + b^3)*d*f^3*x + 24*I*(a^3 + a*b^2)*d*e*f^2 - 24*(a^2*b + b^3
)*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) + (12*I*(a^3 + a*b^2)*d*f^3*x - 12*(
a^2*b + b^3)*d*f^3*x + 12*I*(a^3 + a*b^2)*d*e*f^2 - 12*(a^2*b + b^3)*d*e*f^
2)*sinh(d*x + c)^2)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) - (-12*I*(a^3
+ a*b^2)*d*f^3*x - 12*(a^2*b + b^3)*d*f^3*x - 12*I*(a^3 + a*b^2)*d*e*f^2 -
12*(a^2*b + b^3)*d*e*f^2 + (-12*I*(a^3 + a*b^2)*d*f^3*x - 12*(a^2*b + b^3)*
d*f^3*x - 12*I*(a^3 + a*b^2)*d*e*f^2 - 12*(a^2*b + b^3)*d*e*f^2)*cosh(d*x +
c)^2 + (-24*I*(a^3 + a*b^2)*d*f^3*x - 24*(a^2*b + b^3)*d*f^3*x - 24*I*(a^3
+ a*b^2)*d*e*f^2 - 24*(a^2*b + b^3)*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) +
(-12*I*(a^3 + a*b^2)*d*f^3*x - 12*(a^2*b + b^3)*d*f^3*x - 12*I*(a^3 + a*b^
2)*d*e*f^2 - 12*(a^2*b + b^3)*d*e*f^2)*sinh(d*x + c)^2)*dilog(-I*cosh(d*x +
c) - I*sinh(d*x + c)) - (6*I*(a^3 + a*b^2)*d^2*e^2*f - 6*(a^2*b + b^3)*d^2
*e^2*f - 12*I*(a^3 + a*b^2)*c*d*e*f^2 + 12*(a^2*b + b^3)*c*d*e*f^2 + 6*I*(a
^3 + a*b^2)*c^2*f^3 - 6*(a^2*b + b^3)*c^2*f^3 + (6*I*(a^3 + a*b^2)*d^2*e^2*
f - 6*(a^2*b + b^3)*d^2*e^2*f - 12*I*(a^3 + a*b^2)*c*d*e*f^2 + 12*(a^2*b +
b^3)*c*d*e*f^2 + 6*I*(a^3 + a*b^2)*c^2*f^3 - 6*(a^2*b + b^3)*c^2*f^3)*cosh(
d*x + c)^2 + (12*I*(a^3 + a*b^2)*d^2*e^2*f - 12*(a^2*b + b^3)*d^2*e^2*f - 2
4*I*(a^3 + a*b^2)*c*d*e*f^2 + 24*(a^2*b + b^3)*c*d*e*f^2 + 12*I*(a^3 + a*b^
2)*c^2*f^3 - 12*(a^2*b + b^3)*c^2*f^3)*cosh(d*x + c)*sinh(d*x + c) + (6*I*(
a^3 + a*b^2)*d^2*e^2*f - 6*(a^2*b + b^3)*d^2*e^2*f - 12*I*(a^3 + a*b^2)*c*d
*e*f^2 + 12*(a^2*b + b^3)*c*d*e*f^2 + 6*I*(a^3 + a*b^2)*c^2*f^3 - 6*(a^2*b
+ b^3)*c^2*f^3)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) + I) - (
-6*I*(a^3 + a*b^2)*d^2*e^2*f - 6*(a^2*b + b^3)*d^2*e^2*f + 12*I*(a^3 + a*b^
2)*c*d*e*f^2 + 12*(a^2*b + b^3)*c*d*e*f^2 - 6*I*(a^3 + a*b^2)*c^2*f^3 - 6*(
a^2*b + b^3)*c^2*f^3 + (-6*I*(a^3 + a*b^2)*d^2*e^2*f - 6*(a^2*b + b^3)*d^2*
e^2*f + 12*I*(a^3 + a*b^2)*c*d*e*f^2 + 12*(a^2*b + b^3)*c*d*e*f^2 - 6*I*(a^
3 + a*b^2)*c^2*f^3 - 6*(a^2*b + b^3)*c^2*f^3)*cosh(d*x + c)^2 + (-12*I*(a^3
+ a*b^2)*d^2*e^2*f - 12*(a^2*b + b^3)*d^2*e^2*f + 24*I*(a^3 + a*b^2)*c*d*e
f^2 + 24(a^2*b + b^3)*c*d*e*f^2 - 12*I*(a^3 + a*b^2)*c^2*f^3 - 12*(a^2*b
+ b^3)*c^2*f^3)*cosh(d*x + c)*sinh(d*x + c) + (-6*I*(a^3 + a*b^2)*d^2*e^2*f
- 6*(a^2*b + b^3)*d^2*e^2*f + 12*I*(a^3 + a*b^2)*c*d*e*f^2 + 12*(a^2*b + b
^3)*c*d*e*f^2 - 6*I*(a^3 + a*b^2)*c^2*f^3 - 6*(a^2*b + b^3)*c^2*f^3)*sinh(d
*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) - I) - (-6*I*(a^3 + a*b^2)*d^2
*f^3*x^2 - 6*(a^2*b + b^3)*d^2*f^3*x^2 - 12*I*(a^3 + a*b^2)*d^2*e*f^2*x - 1
2*(a^2*b + b^3)*d^2*e*f^2*x - 12*I*(a^3 + a*b^2)*c*d*e*f^2 - 12*(a^2*b + b^
3)*c*d*e*f^2 + 6*I*(a^3 + a*b^2)*c^2*f^3 + 6*(a^2*b + b^3)*c^2*f^3 + (-6*I*
(a^3 + a*b^2)*d^2*f^3*x^2 - 6*(a^2*b + b^3)*d^2*f^3*x^2 - 12*I*(a^3 + a*b^2
)*d^2*e*f^2*x - 12*(a^2*b + b^3)*d^2*e*f^2*x - 12*I*(a^3 + a*b^2)*c*d*e*f^2
- 12*(a^2*b + b^3)*c*d*e*f^2 + 6*I*(a^3 + a*b^2)*c^2*f^3 + 6*(a^2*b + b^3)
*c^2*f^3)*cosh(d*x + c)^2 + (-12*I*(a^3 + a*b^2)*d^2*f^3*x^2 - 12*(a^2*b +
b^3)*d^2*f^3*x^2 - 24*I*(a^3 + a*b^2)*d^2*e*f^2*x - 24*(a^2*b + b^3)*d^2*e*
f^2*x - 24*I*(a^3 + a*b^2)*c*d*e*f^2 - 24*(a^2*b + b^3)*c*d*e*f^2 + 12*I*(a
^3 + a*b^2)*c^2*f^3 + 12*(a^2*b + b^3)*c^2*f^3)*cosh(d*x + c)*sinh(d*x + c)
+ (-6*I*(a^3 + a*b^2)*d^2*f^3*x^2 - 6*(a^2*b + b^3)*d^2*f^3*x^2 - 12*I*(a^
3 + a*b^2)*d^2*e*f^2*x - 12*(a^2*b + b^3)*d^2*e*f^2*x - 12*I*(a^3 + a*b^2)*
c*d*e*f^2 - 12*(a^2*b + b^3)*c*d*e*f^2 + 6*I*(a^3 + a*b^2)*c^2*f^3 + 6*(a^2
*b + b^3)*c^2*f^3)*sinh(d*x + c)^2)*log(I*cosh(d*x + c) + I*sinh(d*x + c) +
1) - (6*I*(a^3 + a*b^2)*d^2*f^3*x^2 - 6*(a^2*b + b^3)*d^2*f^3*x^2 + 12*I*(
a^3 + a*b^2)*d^2*e*f^2*x - 12*(a^2*b + b^3)*d^2*e*f^2*x + 12*I*(a^3 + a*b^2

```
)*c*d*e*f^2 - 12*(a^2*b + b^3)*c*d*e*f^2 - 6*I*(a^3 + a*b^2)*c^2*f^3 + 6*(a
^2*b + b^3)*c^2*f^3 + (6*I*(a^3 + a*b^2)*d^2*f^3*x^2 - 6*(a^2*b + b^3)*d^2*
f^3*x^2 + 12*I*(a^3 + a*b^2)*d^2*e*f^2*x - 12*(a^2*b + b^3)*d^2*e*f^2*x + 1
2*I*(a^3 + a*b^2)*c*d*e*f^2 - 12*(a^2*b + b^3)*c*d*e*f^2 - 6*I*(a^3 + a*b^2
)*c^2*f^3 + 6*(a^2*b + b^3)*c^2*f^3)*cosh(d*x + c)^2 + (12*I*(a^3 + a*b^2)*
d^2*f^3*x^2 - 12*(a^2*b + b^3)*d^2*f^3*x^2 + 24*I*(a^3 + a*b^2)*d^2*e*f^2*x
 - 24*(a^2*b + b^3)*d^2*e*f^2*x + 24*I*(a^3 + a*b^2)*c*d*e*f^2 - 24*(a^2*b
+ b^3)*c*d*e*f^2 - 12*I*(a^3 + a*b^2)*c^2*f^3 + 12*(a^2*b + b^3)*c^2*f^3)*c
osh(d*x + c)*sinh(d*x + c) + (6*I*(a^3 + a*b^2)*d^2*f^3*x^2 - 6*(a^2*b + b^
3)*d^2*f^3*x^2 + 12*I*(a^3 + a*b^2)*d^2*e*f^2*x - 12*(a^2*b + b^3)*d^2*e*f^
2*x + 12*I*(a^3 + a*b^2)*c*d*e*f^2 - 12*(a^2*b + b^3)*c*d*e*f^2 - 6*I*(a^3
+ a*b^2)*c^2*f^3 + 6*(a^2*b + b^3)*c^2*f^3)*sinh(d*x + c)^2)*log(-I*cosh(d*
x + c) - I*sinh(d*x + c) + 1) - (-12*I*(a^3 + a*b^2)*f^3 + 12*(a^2*b + b^3)
*f^3 + (-12*I*(a^3 + a*b^2)*f^3 + 12*(a^2*b + b^3)*f^3)*cosh(d*x + c)^2 + (
-24*I*(a^3 + a*b^2)*f^3 + 24*(a^2*b + b^3)*f^3)*cosh(d*x + c)*sinh(d*x + c)
 + (-12*I*(a^3 + a*b^2)*f^3 + 12*(a^2*b + b^3)*f^3)*sinh(d*x + c)^2)*polylo
g(3, I*cosh(d*x + c) + I*sinh(d*x + c)) - (12*I*(a^3 + a*b^2)*f^3 + 12*(a^2
*b + b^3)*f^3 + (12*I*(a^3 + a*b^2)*f^3 + 12*(a^2*b + b^3)*f^3)*cosh(d*x +
c)^2 + (24*I*(a^3 + a*b^2)*f^3 + 24*(a^2*b + b^3)*f^3)*cosh(d*x + c)*sinh(d
*x + c) + (12*I*(a^3 + a*b^2)*f^3 + 12*(a^2*b + b^3)*f^3)*sinh(d*x + c)^2)*
polylog(3, -I*cosh(d*x + c) - I*sinh(d*x + c)) + 4*((a^3 + a*b^2)*d^3*f^3*x
^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + (a^3 + a
*b^2)*d^3*e^3 - 2*((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3)*d^3*e*f^2*x^
2 + 3*(a^2*b + b^3)*d^3*e^2*f*x + 3*(a^2*b + b^3)*c*d^2*e^2*f - 3*(a^2*b +
b^3)*c^2*d*e*f^2 + (a^2*b + b^3)*c^3*f^3)*cosh(d*x + c))*sinh(d*x + c))/((a
^4 + 2*a^2*b^2 + b^4)*d^4*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^4*c
osh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*d^4*sinh(d*x + c)^2 +
(a^4 + 2*a^2*b^2 + b^4)*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.354 $\int \frac{(e+fx)^2\mathbf{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=648

$$-\frac{2abf(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^{3/2}}+\frac{2abf(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2\left(a^2+b^2\right)^{3/2}}+\frac{a^2f^2\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{bd^3\left(a^2+b^2\right)}-\frac{2iaf}{}$$

```
[Out] (e + f*x)^2/(b*d) - (a^2*(e + f*x)^2)/(b*(a^2 + b^2)*d) + (4*a*f*(e + f*x)*
ArcTan[E^(c + d*x)])/((a^2 + b^2)*d^2) - (a*b*(e + f*x)^2*Log[1 + (b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) + (a*b*(e + f*x)^2*Log
[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (2*f*(
e + f*x)*Log[1 + E^(2*(c + d*x))])/(b*d^2) + (2*a^2*f*(e + f*x)*Log[1 + E^(
2*(c + d*x))])/(b*(a^2 + b^2)*d^2) - ((2*I)*a*f^2*PolyLog[2, (-I)*E^(c + d*
x)])/((a^2 + b^2)*d^3) + ((2*I)*a*f^2*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^
2)*d^3) - (2*a*b*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/((a^2 + b^2)^(3/2)*d^2) + (2*a*b*f*(e + f*x)*PolyLog[2, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (f^2*PolyLog[2, -
E^(2*(c + d*x))])/(b*d^3) + (a^2*f^2*PolyLog[2, -E^(2*(c + d*x))])/(b*(a^2
+ b^2)*d^3) + (2*a*b*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
)])/((a^2 + b^2)^(3/2)*d^3) - (2*a*b*f^2*PolyLog[3, -((b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) - (a*(e + f*x)^2*Sech[c + d*x])
/((a^2 + b^2)*d) + ((e + f*x)^2*Tanh[c + d*x])/(b*d) - (a^2*(e + f*x)^2*Tan
h[c + d*x])/(b*(a^2 + b^2)*d)
```

Rubi [A] time = 1.32148, antiderivative size = 648, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 15, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.469, Rules used = {5583, 4184, 3718, 2190, 2279, 2391, 5573, 3322, 2264, 2531, 2282, 6589, 6742, 5451, 4180}

$$-\frac{2abf(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^{3/2}}+\frac{2abf(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2\left(a^2+b^2\right)^{3/2}}+\frac{a^2f^2\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{bd^3\left(a^2+b^2\right)}-\frac{2iaf}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e + f*x)^2/(b*d) - (a^2*(e + f*x)^2)/(b*(a^2 + b^2)*d) + (4*a*f*(e + f*x)*
ArcTan[E^(c + d*x)])/((a^2 + b^2)*d^2) - (a*b*(e + f*x)^2*Log[1 + (b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) + (a*b*(e + f*x)^2*Log
[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (2*f*(
e + f*x)*Log[1 + E^(2*(c + d*x))])/(b*d^2) + (2*a^2*f*(e + f*x)*Log[1 + E^(
2*(c + d*x))])/(b*(a^2 + b^2)*d^2) - ((2*I)*a*f^2*PolyLog[2, (-I)*E^(c + d*
x)])/((a^2 + b^2)*d^3) + ((2*I)*a*f^2*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^
2)*d^3) - (2*a*b*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/((a^2 + b^2)^(3/2)*d^2) + (2*a*b*f*(e + f*x)*PolyLog[2, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (f^2*PolyLog[2, -
E^(2*(c + d*x))])/(b*d^3) + (a^2*f^2*PolyLog[2, -E^(2*(c + d*x))])/(b*(a^2
+ b^2)*d^3) + (2*a*b*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
)])/((a^2 + b^2)^(3/2)*d^3) - (2*a*b*f^2*PolyLog[3, -((b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) - (a*(e + f*x)^2*Sech[c + d*x])
/((a^2 + b^2)*d) + ((e + f*x)^2*Tanh[c + d*x])/(b*d) - (a^2*(e + f*x)^2*Tan
h[c + d*x])/(b*(a^2 + b^2)*d)
```

Rule 5583

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
```

```
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{sech}^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)^2\tanh(c+dx)}{bd} - \frac{a\int (e+fx)^2\text{sech}^2(c+dx)(a-b\sinh(c+dx))\,dx}{b\left(a^2+b^2\right)}$$

$$= \frac{(e+fx)^2}{bd} + \frac{(e+fx)^2\tanh(c+dx)}{bd} - \frac{a\int \left(a(e+fx)^2\text{sech}^2(c+dx) - b(e+\right.}{b\left(a^2+\right.}$$

$$= \frac{(e+fx)^2}{bd} - \frac{2f(e+fx)\log\left(1+e^{2(c+dx)}\right)}{bd^2} + \frac{(e+fx)^2\tanh(c+dx)}{bd} - \frac{\left(2ab^2\right)}{}$$

$$= \frac{(e+fx)^2}{bd} - \frac{ab(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} + \frac{ab(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d}$$

$$= \frac{(e+fx)^2}{bd} - \frac{a^2(e+fx)^2}{b\left(a^2+b^2\right)d} + \frac{4af(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d^2} - \frac{ab(e+fx)^2\log\left(1\right.}{\left(a^2+b^2\right)^{3/}}$$

$$= \frac{(e+fx)^2}{bd} - \frac{a^2(e+fx)^2}{b\left(a^2+b^2\right)d} + \frac{4af(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d^2} - \frac{ab(e+fx)^2\log\left(1\right.}{\left(a^2+b^2\right)^{3/}}$$

$$= \frac{(e+fx)^2}{bd} - \frac{a^2(e+fx)^2}{b\left(a^2+b^2\right)d} + \frac{4af(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d^2} - \frac{ab(e+fx)^2\log\left(1\right.}{\left(a^2+b^2\right)^{3/}}$$

$$= \frac{(e+fx)^2}{bd} - \frac{a^2(e+fx)^2}{b\left(a^2+b^2\right)d} + \frac{4af(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d^2} - \frac{ab(e+fx)^2\log\left(1\right.}{\left(a^2+b^2\right)^{3/}}$$

Mathematica [A] time = 8.18767, size = 906, normalized size = 1.4

$$\frac{2a\left(-\frac{2\tan^{-1}\left(\frac{\sinh(c)+\cosh(c)\tanh\left(\frac{dx}{2}\right)}{\sqrt{\cosh^2(c)-\sinh^2(c)}}\right)\tanh^{-1}(\coth(c))}{\sqrt{\cosh^2(c)-\sinh^2(c)}} - \frac{i\text{csch}(c)\left(i\left(dx+\tanh^{-1}(\coth(c))\right)\left(\log\left(1-e^{-dx-\tanh^{-1}(\coth(c))}\right)-\log\left(1+e^{-dx-\tanh^{-1}(\coth(c))}\right)\right)+i\left(\right.}{\sqrt{1-\coth^2(c)}}\right.}{\left(a^2+b^2\right)d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (a*b*(2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x*
Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])] - 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] +
 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 2*f
^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*f^2*PolyLog[3, -(
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/((a^2 + b^2)^(3/2)*d^3) - (2*b*e*
f*Sech[c]*(Cosh[c]*Log[Cosh[c]*Cosh[d*x] + Sinh[c]*Sinh[d*x]] - d*x*Sinh[c]
))/((a^2 + b^2)*d^2*(Cosh[c]^2 - Sinh[c]^2)) + (4*a*e*f*ArcTan[(Sinh[c] + C
osh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]])/((a^2 + b^2)*d^2*Sqrt[C
```

```
osh[c]^2 - Sinh[c]^2]) + (b*f^2*Csch[c]*(-((d^2*x^2)/E^ArcTanh[Coth[c]]) +
(I*Coth[c]*(-(d*x*(-Pi + (2*I)*ArcTanh[Coth[c]])) - Pi*Log[1 + E^(2*d*x)] -
 2*(I*d*x + I*ArcTanh[Coth[c]])*Log[1 - E^((2*I)*(I*d*x + I*ArcTanh[Coth[c]
]))] + Pi*Log[Cosh[d*x]] + (2*I)*ArcTanh[Coth[c]]*Log[I*Sinh[d*x + ArcTanh[
Coth[c]]]] + I*PolyLog[2, E^((2*I)*(I*d*x + I*ArcTanh[Coth[c]]))]))/Sqrt[1
- Coth[c]^2])*Sech[c])/((a^2 + b^2)*d^3*Sqrt[Csch[c]^2*(-Cosh[c]^2 + Sinh[c
]^2)]) + (2*a*f^2*(((-I)*Csch[c]*(I*(d*x + ArcTanh[Coth[c]])*(Log[1 - E^(-(
d*x) - ArcTanh[Coth[c]])] - Log[1 + E^(-(d*x) - ArcTanh[Coth[c]])]) + I*(Po
lyLog[2, -E^(-(d*x) - ArcTanh[Coth[c]])] - PolyLog[2, E^(-(d*x) - ArcTanh[C
oth[c]])])))/Sqrt[1 - Coth[c]^2] - (2*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)/
2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]]*ArcTanh[Coth[c]])/Sqrt[Cosh[c]^2 - Sinh[c]
^2]))/((a^2 + b^2)*d^3) + (Sech[c]*Sech[c + d*x]*(-(a*e^2*Cosh[c]) - 2*a*e*
f*x*Cosh[c] - a*f^2*x^2*Cosh[c] + b*e^2*Sinh[d*x] + 2*b*e*f*x*Sinh[d*x] + b
*f^2*x^2*Sinh[d*x]))/((a^2 + b^2)*d)
```

Maple [F] time = 0.931, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \operatorname{sech}\left(dx+c\right) \tanh\left(dx+c\right)}{a+b\sinh\left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.44481, size = 8627, normalized size = 13.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -1/2*(4*(a^2*b + b^3)*d^2*e^2 - 8*(a^2*b + b^3)*c*d*e*f + 4*(a^2*b + b^3)*c
^2*f^2 - 4*((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*d^2*e*f*x + 2*(a^2*
b + b^3)*c*d*e*f - (a^2*b + b^3)*c^2*f^2)*cosh(d*x + c)^2 - 4*((a^2*b + b^3
)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*d^2*e*f*x + 2*(a^2*b + b^3)*c*d*e*f - (a^2*
b + b^3)*c^2*f^2)*sinh(d*x + c)^2 + 4*(a*b^2*d*f^2*x + a*b^2*d*e*f + (a*b^2
```

```
*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*f^2*x + a*b^2*d*e*f)*c
osh(d*x + c)*sinh(d*x + c) + (a*b^2*d*f^2*x + a*b^2*d*e*f)*sinh(d*x + c)^2)
*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 4*(a*b^2*d*f
^2*x + a*b^2*d*e*f + (a*b^2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)^2 + 2*(a*b
^2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d*f^2*x + a*
b^2*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) +
a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
 - b)/b + 1) - 2*(a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2 + (a*b^2*
d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(d*x + c)^2 + 2*(a*b^2*d^2*e
^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*
d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2
)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2
) + 2*a) + 2*(a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2 + (a*b^2*d^2*
e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(d*x + c)^2 + 2*(a*b^2*d^2*e^2 -
 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d^2*
e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^
2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) +
2*a) + 2*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c
^2*f^2 + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c
^2*f^2)*cosh(d*x + c)^2 + 2*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^
2*c*d*e*f - a*b^2*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d^2*f^2*x^2
 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*sinh(d*x + c)^2)*sq
rt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
 c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(a*b^2*d^2*f^2*x^2
 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2 + (a*b^2*d^2*f^2*x^2
 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*cosh(d*x + c)^2 + 2
*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*
cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*
b^2*c*d*e*f - a*b^2*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a
*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b) - 4*(a*b^2*f^2*cosh(d*x + c)^2 + 2*a*b^2*f^2*cosh
(d*x + c)*sinh(d*x + c) + a*b^2*f^2*sinh(d*x + c)^2 + a*b^2*f^2)*sqrt((a^2
+ b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 4*(a*b^2*f^2*cosh(d*x + c)
^2 + 2*a*b^2*f^2*cosh(d*x + c)*sinh(d*x + c) + a*b^2*f^2*sinh(d*x + c)^2 +
a*b^2*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x +
 c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 4*((a
^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + (a^3 + a*b^2)*d^2*e^2
)*cosh(d*x + c) - (4*I*(a^3 + a*b^2)*f^2 - 4*(a^2*b + b^3)*f^2 + (4*I*(a^3
+ a*b^2)*f^2 - 4*(a^2*b + b^3)*f^2)*cosh(d*x + c)^2 + (8*I*(a^3 + a*b^2)*f^
2 - 8*(a^2*b + b^3)*f^2)*cosh(d*x + c)*sinh(d*x + c) + (4*I*(a^3 + a*b^2)*f
^2 - 4*(a^2*b + b^3)*f^2)*sinh(d*x + c)^2)*dilog(I*cosh(d*x + c) + I*sinh(d
*x + c)) - (-4*I*(a^3 + a*b^2)*f^2 - 4*(a^2*b + b^3)*f^2 + (-4*I*(a^3 + a*b
^2)*f^2 - 4*(a^2*b + b^3)*f^2)*cosh(d*x + c)^2 + (-8*I*(a^3 + a*b^2)*f^2 -
8*(a^2*b + b^3)*f^2)*cosh(d*x + c)*sinh(d*x + c) + (-4*I*(a^3 + a*b^2)*f^2
- 4*(a^2*b + b^3)*f^2)*sinh(d*x + c)^2)*dilog(-I*cosh(d*x + c) - I*sinh(d*x
 + c)) - (4*I*(a^3 + a*b^2)*d*e*f - 4*(a^2*b + b^3)*d*e*f - 4*I*(a^3 + a*b^
2)*c*f^2 + 4*(a^2*b + b^3)*c*f^2 + (4*I*(a^3 + a*b^2)*d*e*f - 4*(a^2*b + b^
3)*d*e*f - 4*I*(a^3 + a*b^2)*c*f^2 + 4*(a^2*b + b^3)*c*f^2)*cosh(d*x + c)^2
 + (8*I*(a^3 + a*b^2)*d*e*f - 8*(a^2*b + b^3)*d*e*f - 8*I*(a^3 + a*b^2)*c*f
^2 + 8*(a^2*b + b^3)*c*f^2)*cosh(d*x + c)*sinh(d*x + c) + (4*I*(a^3 + a*b^2
)*d*e*f - 4*(a^2*b + b^3)*d*e*f - 4*I*(a^3 + a*b^2)*c*f^2 + 4*(a^2*b + b^3)
*c*f^2)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) + I) - (-4*I*(a^
3 + a*b^2)*d*e*f - 4*(a^2*b + b^3)*d*e*f + 4*I*(a^3 + a*b^2)*c*f^2 + 4*(a^2
*b + b^3)*c*f^2 + (-4*I*(a^3 + a*b^2)*d*e*f - 4*(a^2*b + b^3)*d*e*f + 4*I*(
a^3 + a*b^2)*c*f^2 + 4*(a^2*b + b^3)*c*f^2)*cosh(d*x + c)^2 + (-8*I*(a^3 +
a*b^2)*d*e*f - 8*(a^2*b + b^3)*d*e*f + 8*I*(a^3 + a*b^2)*c*f^2 + 8*(a^2*b +
 b^3)*c*f^2)*cosh(d*x + c)*sinh(d*x + c) + (-4*I*(a^3 + a*b^2)*d*e*f - 4*(a
```

```
^2*b + b^3)*d*e*f + 4*I*(a^3 + a*b^2)*c*f^2 + 4*(a^2*b + b^3)*c*f^2)*sinh(d
*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) - I) - (-4*I*(a^3 + a*b^2)*d*f
^2*x - 4*(a^2*b + b^3)*d*f^2*x - 4*I*(a^3 + a*b^2)*c*f^2 - 4*(a^2*b + b^3)*
c*f^2 + (-4*I*(a^3 + a*b^2)*d*f^2*x - 4*(a^2*b + b^3)*d*f^2*x - 4*I*(a^3 +
a*b^2)*c*f^2 - 4*(a^2*b + b^3)*c*f^2)*cosh(d*x + c)^2 + (-8*I*(a^3 + a*b^2)
*d*f^2*x - 8*(a^2*b + b^3)*d*f^2*x - 8*I*(a^3 + a*b^2)*c*f^2 - 8*(a^2*b + b
^3)*c*f^2)*cosh(d*x + c)*sinh(d*x + c) + (-4*I*(a^3 + a*b^2)*d*f^2*x - 4*(a
^2*b + b^3)*d*f^2*x - 4*I*(a^3 + a*b^2)*c*f^2 - 4*(a^2*b + b^3)*c*f^2)*sinh
(d*x + c)^2)*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) - (4*I*(a^3 + a*b^2
)*d*f^2*x - 4*(a^2*b + b^3)*d*f^2*x + 4*I*(a^3 + a*b^2)*c*f^2 - 4*(a^2*b +
b^3)*c*f^2 + (4*I*(a^3 + a*b^2)*d*f^2*x - 4*(a^2*b + b^3)*d*f^2*x + 4*I*(a^
3 + a*b^2)*c*f^2 - 4*(a^2*b + b^3)*c*f^2)*cosh(d*x + c)^2 + (8*I*(a^3 + a*b
^2)*d*f^2*x - 8*(a^2*b + b^3)*d*f^2*x + 8*I*(a^3 + a*b^2)*c*f^2 - 8*(a^2*b
+ b^3)*c*f^2)*cosh(d*x + c)*sinh(d*x + c) + (4*I*(a^3 + a*b^2)*d*f^2*x - 4*
(a^2*b + b^3)*d*f^2*x + 4*I*(a^3 + a*b^2)*c*f^2 - 4*(a^2*b + b^3)*c*f^2)*si
nh(d*x + c)^2)*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1) + 4*((a^3 + a*b^
2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + (a^3 + a*b^2)*d^2*e^2 - 2*((a^
2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*d^2*e*f*x + 2*(a^2*b + b^3)*c*d*e*
f - (a^2*b + b^3)*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2
+ b^4)*d^3*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c)*si
nh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*d^3*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^
2 + b^4)*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^2 \tanh(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*tanh(c + d*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.355 $\int \frac{(e+fx)\mathbf{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=335

$$-\frac{abf\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^{3/2}}+\frac{abf\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2\left(a^2+b^2\right)^{3/2}}+\frac{af\tan^{-1}(\sinh(c+dx))}{d^2\left(a^2+b^2\right)}+\frac{a^2f\log(\cosh(c+dx))}{bd^2\left(a^2+b^2\right)}-\frac{ab(e\cdots}{}$$

```
[Out] (a*f*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d^2) - (a*b*(e + f*x)*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) + (a*b*(e + f*x)*
Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (f*
Log[Cosh[c + d*x]])/(b*d^2) + (a^2*f*Log[Cosh[c + d*x]])/(b*(a^2 + b^2)*d^2
) - (a*b*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^
2)^(3/2)*d^2) + (a*b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/((a^2 + b^2)^(3/2)*d^2) - (a*(e + f*x)*Sech[c + d*x])/((a^2 + b^2)*d) + (
(e + f*x)*Tanh[c + d*x])/(b*d) - (a^2*(e + f*x)*Tanh[c + d*x])/(b*(a^2 + b^
2)*d)
```

Rubi [A] time = 0.662796, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5583, 4184, 3475, 5573, 3322, 2264, 2190, 2279, 2391, 6742, 5451, 3770}

$$-\frac{abf\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^{3/2}}+\frac{abf\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2\left(a^2+b^2\right)^{3/2}}+\frac{af\tan^{-1}(\sinh(c+dx))}{d^2\left(a^2+b^2\right)}+\frac{a^2f\log(\cosh(c+dx))}{bd^2\left(a^2+b^2\right)}-\frac{ab(e\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*f*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d^2) - (a*b*(e + f*x)*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) + (a*b*(e + f*x)*
Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (f*
Log[Cosh[c + d*x]])/(b*d^2) + (a^2*f*Log[Cosh[c + d*x]])/(b*(a^2 + b^2)*d^2
) - (a*b*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^
2)^(3/2)*d^2) + (a*b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/((a^2 + b^2)^(3/2)*d^2) - (a*(e + f*x)*Sech[c + d*x])/((a^2 + b^2)*d) + (
(e + f*x)*Tanh[c + d*x])/(b*d) - (a^2*(e + f*x)*Tanh[c + d*x])/(b*(a^2 + b^
2)*d)
```

Rule 5583

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\operatorname{sech}^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)\tanh(c+dx)}{bd} - \frac{a\int (e+fx)\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))\,dx}{b\left(a^2+b^2\right)} - \cdots$$

$$= -\frac{f\log(\cosh(c+dx))}{bd^2} + \frac{(e+fx)\tanh(c+dx)}{bd} - \frac{a\int \left(a(e+fx)\operatorname{sech}^2(c+dx\cdots\right.}{\cdots}$$

$$= -\frac{f\log(\cosh(c+dx))}{bd^2} + \frac{(e+fx)\tanh(c+dx)}{bd} - \frac{\left(2ab^2\right)\int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\cdots}{\left(a^2+b^2\right)^{3/2}}$$

$$= -\frac{ab(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} + \frac{ab(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} - \frac{f\log(\cosh\cdots}{bd\cdots}$$

$$= \frac{af\tan^{-1}(\sinh(c+dx))}{\left(a^2+b^2\right)d^2} - \frac{ab(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} + \frac{ab(e+fx)\log\left(1\cdots\right.}{\left(a^2+b^2\right)^{3\cdots}}$$

$$= \frac{af\tan^{-1}(\sinh(c+dx))}{\left(a^2+b^2\right)d^2} - \frac{ab(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} + \frac{ab(e+fx)\log\left(1\cdots\right.}{\left(a^2+b^2\right)^{3\cdots}}$$

Mathematica [A] time = 3.10648, size = 285, normalized size = 0.85

$$\frac{\frac{ab\left(-f\operatorname{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-f(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)+f(c+dx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)-2cf\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\cdots\right.}{\left(a^2+b^2\right)^{3/2}}\cdots}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((2*a*f*ArcTan[Tanh[(c + d*x)/2]])/(a^2 + b^2) - (b*f*Log[Cosh[c + d*x]])/(
a^2 + b^2) + (a*b*(2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*c
*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - f*(c + d*x)*Log[1 + (b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])] - f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
+ f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^2 + b^2)^(3/2
) + (d*(e + f*x)*Sech[c + d*x]*(-a + b*Sinh[c + d*x]))/(a^2 + b^2))/d^2
```

Maple [B] time = 0.168, size = 1858, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/(a^2+b^2)^2/d^2*f*b*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)*a^2-2/(a^2+b^2
)/d^2*f*b^3/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))+1/(a^2+b^2)/d^2*f*b^3/(2*a^2
+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+4/(a^2+b^2)/d^2*a^3*f/(2*a^2+
2*b^2)*arctan(exp(d*x+c))+2/(a^2+b^2)^(5/2)/d^2*f*b*arctanh(1/2*(2*b*exp(d*
x+c)+2*a)/(a^2+b^2)^(1/2))*a^3+2/(a^2+b^2)^(5/2)/d^2*f*b^3*arctanh(1/2*(2*b
*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a+4/(a^2+b^2)/d^2*f*b^2/(2*a^2+2*b^2)*a*a
rctan(exp(d*x+c))-2/(a^2+b^2)/d^2*a^2*b*f/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c)
)+2/(a^2+b^2)/d^2*a^2*b*f/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-
b)-2/(a^2+b^2)^(3/2)/d^2*a*b^3*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c
)+2*a)/(a^2+b^2)^(1/2))-2/(a^2+b^2)^(3/2)/d^2*a^3*b*f*c/(2*a^2+2*b^2)*arcta
nh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/(a^2+b^2)^(1/2)/d^2*a*b*f*c/
(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)
^(3/2)/d^2*a*b^3*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^
2+b^2)^(1/2)))*c-2/(a^2+b^2)^(3/2)/d^2*a^3*b*f/(2*a^2+2*b^2)*ln((-b*exp(d*x
+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/(a^2+b^2)^(3/2)/d^2*a*b^3*
f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*
c+2/(a^2+b^2)^(3/2)/d^2*a^3*b*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1
/2)+a)/(a+(a^2+b^2)^(1/2)))*c-2/(a^2+b^2)^(3/2)/d*a*b^3*f/(2*a^2+2*b^2)*ln(
(-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+2/(a^2+b^2)^(3/2)
/d*a^3*b*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(
1/2)))*x+2/(a^2+b^2)^(3/2)/d*a*b^3*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^
2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-2/(a^2+b^2)^(3/2)/d*a^3*b*f/(2*a^2+2*b^2
)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+2/(a^2+b^2)^
(3/2)/d*a*b^3*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1
/2))+2/(a^2+b^2)^(3/2)/d*a^3*b*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+
2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)^(1/2)/d*a*b*e/(2*a^2+2*b^2)*arctanh(1/2*(
2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/(a^2+b^2)^(3/2)/d^2*a*b^3*f/(2*a^2+2
*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+2/(a^2+
b^2)^(3/2)/d^2*a^3*b*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)
/(a+(a^2+b^2)^(1/2)))+2/(a^2+b^2)^(3/2)/d^2*a*b^3*f/(2*a^2+2*b^2)*dilog((b*
exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2/(a^2+b^2)^(3/2)/d^2*a^
3*b*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(
1/2)))-2/(a^2+b^2)^(1/2)/d^2*a*b*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c
)+2*a)/(a^2+b^2)^(1/2))-2/(a^2+b^2)^(3/2)/d^2*a*b^3*f/(2*a^2+2*b^2)*arctanh
(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/(a^2+b^2)^(3/2)/d^2*a^3*b*f/(2
*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)/d
^2*b*f*ln(exp(d*x+c))-1/2/(a^2+b^2)^2/d^2*f*b^3*ln(b*exp(2*d*x+2*c)+2*a*exp
(d*x+c)-b)-2*(f*x+e)*(a*exp(d*x+c)+b)/d/(a^2+b^2)/(1+exp(2*d*x+2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.50921, size = 3272, normalized size = 9.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] (2*(a^2*b + b^3)*d*f*x*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*d*f*x*sinh(d*x + c
)^2 - 2*(a^2*b + b^3)*d*e - (a*b^2*f*cosh(d*x + c)^2 + 2*a*b^2*f*cosh(d*x +
 c)*sinh(d*x + c) + a*b^2*f*sinh(d*x + c)^2 + a*b^2*f)*sqrt((a^2 + b^2)/b^2
)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (a*b^2*f*cosh(d*x + c)^2 + 2*a*b^
2*f*cosh(d*x + c)*sinh(d*x + c) + a*b^2*f*sinh(d*x + c)^2 + a*b^2*f)*sqrt((
a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (a*b^2*d*e - a*b^2*
c*f + (a*b^2*d*e - a*b^2*c*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*e - a*b^2*c*f)*c
osh(d*x + c)*sinh(d*x + c) + (a*b^2*d*e - a*b^2*c*f)*sinh(d*x + c)^2)*sqrt(
(a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2
+ b^2)/b^2) + 2*a) - (a*b^2*d*e - a*b^2*c*f + (a*b^2*d*e - a*b^2*c*f)*cosh(
d*x + c)^2 + 2*(a*b^2*d*e - a*b^2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2
*d*e - a*b^2*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x +
 c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (a*b^2*d*f*x +
 a*b^2*c*f + (a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*f*x + a
*b^2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d*f*x + a*b^2*c*f)*sinh(d*x
+ c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (a*b^2*d*f
*x + a*b^2*c*f + (a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*f*x
 + a*b^2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d*f*x + a*b^2*c*f)*sinh(
d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) -
 (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*((a^
3 + a*b^2)*f*cosh(d*x + c)^2 + 2*(a^3 + a*b^2)*f*cosh(d*x + c)*sinh(d*x + c
) + (a^3 + a*b^2)*f*sinh(d*x + c)^2 + (a^3 + a*b^2)*f)*arctan(cosh(d*x + c)
 + sinh(d*x + c)) - 2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e)*cosh(d*x +
c) - ((a^2*b + b^3)*f*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*f*cosh(d*x + c)*sin
h(d*x + c) + (a^2*b + b^3)*f*sinh(d*x + c)^2 + (a^2*b + b^3)*f)*log(2*cosh(
d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 2*(2*(a^2*b + b^3)*d*f*x*cosh(d
*x + c) - (a^3 + a*b^2)*d*f*x - (a^3 + a*b^2)*d*e)*sinh(d*x + c))/((a^4 + 2
*a^2*b^2 + b^4)*d^2*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(d*
x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*d^2*sinh(d*x + c)^2 + (a^4 +
 2*a^2*b^2 + b^4)*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)\tanh(c + dx)\operatorname{sech}(c + dx)}{a + b\sinh(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*tanh(c + d*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.356 $\int \frac{\text{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=78

$$\frac{2ab\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}-\frac{\text{sech}(c+dx)(a-b\sinh(c+dx))}{d\left(a^2+b^2\right)}$$

```
[Out] (2*a*b*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/
2)*d) - (Sech[c + d*x]*(a - b*Sinh[c + d*x]))/((a^2 + b^2)*d)
```

Rubi [A] time = 0.112229, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2866, 12, 2660, 618, 204}

$$\frac{2ab\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}-\frac{\text{sech}(c+dx)(a-b\sinh(c+dx))}{d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a*b*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/
2)*d) - (Sech[c + d*x]*(a - b*Sinh[c + d*x]))/((a^2 + b^2)*d)
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\operatorname{sech}(c+dx)(a-b\sinh(c+dx))}{\left(a^2+b^2\right)d} - \frac{\int \frac{ab}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}$$

$$= -\frac{\operatorname{sech}(c+dx)(a-b\sinh(c+dx))}{\left(a^2+b^2\right)d} - \frac{(ab)\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}$$

$$= -\frac{\operatorname{sech}(c+dx)(a-b\sinh(c+dx))}{\left(a^2+b^2\right)d} + \frac{(2iab)\operatorname{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(ic+\right.\right.}{\left(a^2+b^2\right)d}$$

$$= -\frac{\operatorname{sech}(c+dx)(a-b\sinh(c+dx))}{\left(a^2+b^2\right)d} - \frac{(4iab)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^2+b^2\right)-x^2}\,dx, x, -2ib+2a\right.}{\left(a^2+b^2\right)d}$$

$$= \frac{2ab\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} - \frac{\operatorname{sech}(c+dx)(a-b\sinh(c+dx))}{\left(a^2+b^2\right)d}$$

Mathematica [A] time = 0.192775, size = 104, normalized size = 1.33

$$-\frac{b\sqrt{-a^2-b^2}\tanh(c+dx) - a\sqrt{-a^2-b^2}\operatorname{sech}(c+dx) - 2ab\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{d\left(-a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((-2*a*b*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] - a*Sqrt[-a^2
- b^2]*Sech[c + d*x] + b*Sqrt[-a^2 - b^2]*Tanh[c + d*x])/((-a^2 - b^2)^(3/2
)*d))
```

Maple [A] time = 0.001, size = 100, normalized size = 1.3

$$\frac{1}{d}\left(-4\frac{ab}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}\operatorname{Artanh}\left(1/2\frac{2a\tanh(1/2dx+c/2)-2b}{\sqrt{a^2+b^2}}\right) - 2\frac{-\tanh(1/2dx+c/2)b+a}{\left(a^2+b^2\right)\left((\tanh(1/2dx+c/2))^2+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/d*(-4*a*b/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2
*c)-2*b)/(a^2+b^2)^(1/2))-2/(a^2+b^2)*(-tanh(1/2*d*x+1/2*c)*b+a)/(tanh(1/2*
```

```
d*x+1/2*c)^2+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15745, size = 884, normalized size = 11.33

$$-\frac{2\,a^2b+2\,b^3-\left(ab\cosh\left(dx+c\right)^2+2\,ab\cosh\left(dx+c\right)\sinh\left(dx+c\right)+ab\sinh\left(dx+c\right)^2+ab\right)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(dx+c)^2+}{}\right.}{\left(a^4+2\,a^2b^2+b^4\right)d\cosh\left(dx+c\right)^2+2\left(a^4+2\,a^2b^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*a^2*b + 2*b^3 - (a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c
) + a*b*sinh(d*x + c)^2 + a*b)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b
^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x +
c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x +
 c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b
*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(a^3 + a*b^2)*cosh(d*x + c) + 2
*(a^3 + a*b^2)*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 +
2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2
+ b^4)*d*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tanh\left(c+dx\right)\operatorname{sech}\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A] time = 1.27738, size = 158, normalized size = 2.03

$$\frac{\frac{ab\log\left(\frac{\left|-2\,be^{(dx+2\,c)}-2\,ae^{c}-2\,\sqrt{a^2+b^2}e^{c}\right|}{\left|-2\,be^{(dx+2\,c)}-2\,ae^{c}+2\,\sqrt{a^2+b^2}e^{c}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(ae^{(dx+c)}+b\right)}{\left(a^2+b^2\right)\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*b*log(abs(-2*b*e^(d*x + 2*c) - 2*a*e^c - 2*sqrt(a^2 + b^2)*e^c)/abs(-2*b
*e^(d*x + 2*c) - 2*a*e^c + 2*sqrt(a^2 + b^2)*e^c))/(a^2 + b^2)^(3/2) - 2*(a
*e^(d*x + c) + b)/((a^2 + b^2)*(e^(2*d*x + 2*c) + 1)))/d
```

3.357 $\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=34

$$\text{Unintegrable}\left(\frac{\tanh(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Sech[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [A] time = 0.0586461, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sech[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Sech[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A] time = 60.4988, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sech[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x
]
```

```
[Out] Integrate[(Sech[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

Maple [A] time = 0.559, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-2\,ab\int-\frac{e^{(dx+c)}}{a^2be+b^3e+\left(a^2bf+b^3f\right)x-\left(a^2bee^{(2\,c)}+b^3ee^{(2\,c)}+\left(a^2bfe^{(2\,c)}+b^3fe^{(2\,c)}\right)x\right)e^{(2\,dx)}-2\left(a^3ee^c+ab^2ee^c+\left(a^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] -2*a*b*integrate(-e^(d*x + c)/(a^2*b*e + b^3*e + (a^2*b*f + b^3*f)*x - (a^2
*b*e*e^(2*c) + b^3*e*e^(2*c) + (a^2*b*f*e^(2*c) + b^3*f*e^(2*c))*x)*e^(2*d*
x) - 2*(a^3*e*e^c + a*b^2*e*e^c + (a^3*f*e^c + a*b^2*f*e^c)*x)*e^(d*x)), x)
 - 2*(a*e^(d*x + c) + b)/(a^2*d*e + b^2*d*e + (a^2*d*f + b^2*d*f)*x + (a^2*
d*e*e^(2*c) + b^2*d*e*e^(2*c) + (a^2*d*f*e^(2*c) + b^2*d*f*e^(2*c))*x)*e^(2
*d*x)) - 2*integrate((a*f*e^(d*x + c) + b*f)/(a^2*d*e^2 + b^2*d*e^2 + (a^2*
d*f^2 + b^2*d*f^2)*x^2 + 2*(a^2*d*e*f + b^2*d*e*f)*x + (a^2*d*e^2*e^(2*c) +
 b^2*d*e^2*e^(2*c) + (a^2*d*f^2*e^(2*c) + b^2*d*f^2*e^(2*c))*x^2 + 2*(a^2*d
*e*f*e^(2*c) + b^2*d*e*f*e^(2*c))*x)*e^(2*d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{sech}\left(dx+c\right)\tanh\left(dx+c\right)}{afx+ae+\left(bfx+be\right)\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] integral(sech(d*x + c)*tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
+ c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tanh\left(c+dx\right)\operatorname{sech}\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)*sech(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.358 $\int \frac{(e+fx)^2\mathbf{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1176

result too large to display

```
[Out] ((e + f*x)^2*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*b*(e + f*x)^2*ArcTan[E^(c
+ d*x)])/((a^2 + b^2)^2*d) - (a^2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b*(a^2
+ b^2)*d) - (f^2*ArcTan[Sinh[c + d*x]])/(b*d^3) + (a^2*f^2*ArcTan[Sinh[c +
d*x]])/(b*(a^2 + b^2)*d^3) - (a*b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) - (a*b^2*(e + f*x)^2*Log[1 + (b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) + (a*b^2*(e + f*x)^2*Log
[1 + E^(2*(c + d*x))])/((a^2 + b^2)^2*d) - (a*f^2*Log[Cosh[c + d*x]])/((a^2
 + b^2)*d^3) - (I*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + ((2*I
)*a^2*b*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) + (I*
a^2*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) + (I*f*(e
 + f*x)*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - ((2*I)*a^2*b*f*(e + f*x)*PolyL
og[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - (I*a^2*f*(e + f*x)*PolyLog[2, I
*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) - (2*a*b^2*f*(e + f*x)*PolyLog[2, -((b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) - (2*a*b^2*f*(e +
f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d
^2) + (a*b^2*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2)^2*d^2)
+ (I*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b*d^3) - ((2*I)*a^2*b*f^2*PolyLog[3
, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^3) - (I*a^2*f^2*PolyLog[3, (-I)*E^(c
+ d*x)])/(b*(a^2 + b^2)*d^3) - (I*f^2*PolyLog[3, I*E^(c + d*x)])/(b*d^3) +
((2*I)*a^2*b*f^2*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)^2*d^3) + (I*a^2*f^
2*PolyLog[3, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^3) + (2*a*b^2*f^2*PolyLog[3,
-((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) + (2*a*b^2*f
^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3
) - (a*b^2*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^3) + (f*(e
+ f*x)*Sech[c + d*x])/(b*d^2) - (a^2*f*(e + f*x)*Sech[c + d*x])/(b*(a^2 + b
^2)*d^2) - (a*(e + f*x)^2*Sech[c + d*x]^2)/(2*(a^2 + b^2)*d) + (a*f*(e + f*
x)*Tanh[c + d*x])/((a^2 + b^2)*d^2) + ((e + f*x)^2*Sech[c + d*x]*Tanh[c + d
*x])/(2*b*d) - (a^2*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*b*(a^2 + b^
2)*d)
```

Rubi [A] time = 1.69576, antiderivative size = 1176, normalized size of antiderivative = 1., number of steps used = 49, number of rules used = 15, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.441, Rules used = {5583, 4186, 3770, 4180, 2531, 2282, 6589, 5573, 5561, 2190, 6742, 3718, 5451, 4184, 3475}

$$-\frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)a^2}{b\left(a^2+b^2\right)d}-\frac{2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)a^2}{\left(a^2+b^2\right)^2 d}+\frac{f^2\tan^{-1}(\sinh(c+dx))a^2}{b\left(a^2+b^2\right)d^3}+\frac{if(e+fx)\text{PolyLog}\left(2,-ie^{c+}\right.}{b\left(a^2+b^2\right)d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] ((e + f*x)^2*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*b*(e + f*x)^2*ArcTan[E^(c
+ d*x)])/((a^2 + b^2)^2*d) - (a^2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b*(a^2
+ b^2)*d) - (f^2*ArcTan[Sinh[c + d*x]])/(b*d^3) + (a^2*f^2*ArcTan[Sinh[c +
d*x]])/(b*(a^2 + b^2)*d^3) - (a*b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) - (a*b^2*(e + f*x)^2*Log[1 + (b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) + (a*b^2*(e + f*x)^2*Log
[1 + E^(2*(c + d*x))])/((a^2 + b^2)^2*d) - (a*f^2*Log[Cosh[c + d*x]])/((a^2
```

```
 + b^2)*d^3) - (I*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + ((2*I
)*a^2*b*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) + (I*
a^2*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) + (I*f*(e
 + f*x)*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - ((2*I)*a^2*b*f*(e + f*x)*PolyL
og[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - (I*a^2*f*(e + f*x)*PolyLog[2, I
*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) - (2*a*b^2*f*(e + f*x)*PolyLog[2, -((b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) - (2*a*b^2*f*(e +
f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d
^2) + (a*b^2*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2)^2*d^2)
+ (I*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b*d^3) - ((2*I)*a^2*b*f^2*PolyLog[3
, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^3) - (I*a^2*f^2*PolyLog[3, (-I)*E^(c
+ d*x)])/(b*(a^2 + b^2)*d^3) - (I*f^2*PolyLog[3, I*E^(c + d*x)])/(b*d^3) +
((2*I)*a^2*b*f^2*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)^2*d^3) + (I*a^2*f^
2*PolyLog[3, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^3) + (2*a*b^2*f^2*PolyLog[3,
-((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) + (2*a*b^2*f
^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3
) - (a*b^2*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^3) + (f*(e
+ f*x)*Sech[c + d*x])/(b*d^2) - (a^2*f*(e + f*x)*Sech[c + d*x])/(b*(a^2 + b
^2)*d^2) - (a*(e + f*x)^2*Sech[c + d*x]^2)/(2*(a^2 + b^2)*d) + (a*f*(e + f*
x)*Tanh[c + d*x])/((a^2 + b^2)*d^2) + ((e + f*x)^2*Sech[c + d*x]*Tanh[c + d
*x])/(2*b*d) - (a^2*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*b*(a^2 + b^
2)*d)
```

Rule 5583

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
```

```
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
```

```
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{sech}^3(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{f(e+fx)\text{sech}(c+dx)}{bd^2} + \frac{(e+fx)^2\text{sech}(c+dx)\tanh(c+dx)}{2bd} + \frac{\int (e+fx)}{}$$

$$= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{f^2\tan^{-1}(\sinh(c+dx))}{bd^3} + \frac{f(e+fx)\text{sech}(c+dx)}{bd^2}$$

$$= \frac{ab^2(e+fx)^3}{3\left(a^2+b^2\right)^2 f} + \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{f^2\tan^{-1}(\sinh(c+dx))}{bd^3} - \frac{if(e+}{}$$

$$= \frac{ab^2(e+fx)^3}{3\left(a^2+b^2\right)^2 f} + \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{f^2\tan^{-1}(\sinh(c+dx))}{bd^3} - \frac{ab^2(e}{}$$

$$= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{a^2(e+fx)^2\tan^{-1}}{b\left(a^2+b^2\right)d}$$

$$= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{a^2(e+fx)^2\tan^{-1}}{b\left(a^2+b^2\right)d}$$

$$= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{a^2(e+fx)^2\tan^{-1}}{b\left(a^2+b^2\right)d}$$

$$= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{a^2(e+fx)^2\tan^{-1}}{b\left(a^2+b^2\right)d}$$

$$= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{a^2(e+fx)^2\tan^{-1}}{b\left(a^2+b^2\right)d}$$

Mathematica [B] time = 31.4769, size = 3390, normalized size = 2.88

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (-12*a*b^2*d^3*e^2*E^(2*c)*x + 12*a^3*d*E^(2*c)*f^2*x + 12*a*b^2*d*E^(2*c)*
f^2*x - 12*a*b^2*d^3*e*E^(2*c)*f*x^2 - 4*a*b^2*d^3*E^(2*c)*f^2*x^3 - 6*a^2*
b*d^2*e^2*ArcTan[E^(c + d*x)] + 6*b^3*d^2*e^2*ArcTan[E^(c + d*x)] - 6*a^2*b
*d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)] + 6*b^3*d^2*e^2*E^(2*c)*ArcTan[E^(c +
d*x)] - 12*a^2*b*f^2*ArcTan[E^(c + d*x)] - 12*b^3*f^2*ArcTan[E^(c + d*x)] -
 12*a^2*b*E^(2*c)*f^2*ArcTan[E^(c + d*x)] - 12*b^3*E^(2*c)*f^2*ArcTan[E^(c
+ d*x)] - (6*I)*a^2*b*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (6*I)*b^3*d^2*e*f*
x*Log[1 - I*E^(c + d*x)] - (6*I)*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d
*x)] + (6*I)*b^3*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] - (3*I)*a^2*b*d^2
*f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*b^3*d^2*f^2*x^2*Log[1 - I*E^(c + d*
x)] - (3*I)*a^2*b*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*b^3*d^
2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (6*I)*a^2*b*d^2*e*f*x*Log[1 + I*
E^(c + d*x)] - (6*I)*b^3*d^2*e*f*x*Log[1 + I*E^(c + d*x)] + (6*I)*a^2*b*d^2
*e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (6*I)*b^3*d^2*e*E^(2*c)*f*x*Log[1 +
 I*E^(c + d*x)] + (3*I)*a^2*b*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - (3*I)*b^
3*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] + (3*I)*a^2*b*d^2*E^(2*c)*f^2*x^2*Log[
1 + I*E^(c + d*x)] - (3*I)*b^3*d^2*E^(2*c)*f^2*x^2*Log[1 + I*E^(c + d*x)] +
 6*a*b^2*d^2*e^2*Log[1 + E^(2*(c + d*x))] + 6*a*b^2*d^2*e^2*E^(2*c)*Log[1 +
 E^(2*(c + d*x))] - 6*a^3*f^2*Log[1 + E^(2*(c + d*x))] - 6*a*b^2*f^2*Log[1
+ E^(2*(c + d*x))] - 6*a^3*E^(2*c)*f^2*Log[1 + E^(2*(c + d*x))] - 6*a*b^2*E
^(2*c)*f^2*Log[1 + E^(2*(c + d*x))] + 12*a*b^2*d^2*e*f*x*Log[1 + E^(2*(c +
d*x))] + 12*a*b^2*d^2*e*E^(2*c)*f*x*Log[1 + E^(2*(c + d*x))] + 6*a*b^2*d^2*
f^2*x^2*Log[1 + E^(2*(c + d*x))] + 6*a*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(2
*(c + d*x))] + (6*I)*b*(a^2 - b^2)*d*(1 + E^(2*c))*f*(e + f*x)*PolyLog[2, (
-I)*E^(c + d*x)] + (6*I)*b*(-a^2 + b^2)*d*(1 + E^(2*c))*f*(e + f*x)*PolyLog
[2, I*E^(c + d*x)] + 6*a*b^2*d*e*f*PolyLog[2, -E^(2*(c + d*x))] + 6*a*b^2*d
*e*E^(2*c)*f*PolyLog[2, -E^(2*(c + d*x))] + 6*a*b^2*d*f^2*x*PolyLog[2, -E^(
2*(c + d*x))] + 6*a*b^2*d*E^(2*c)*f^2*x*PolyLog[2, -E^(2*(c + d*x))] - (6*I
)*a^2*b*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (6*I)*b^3*f^2*PolyLog[3, (-I)*E^
(c + d*x)] - (6*I)*a^2*b*E^(2*c)*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (6*I)*b
^3*E^(2*c)*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (6*I)*a^2*b*f^2*PolyLog[3, I*
E^(c + d*x)] - (6*I)*b^3*f^2*PolyLog[3, I*E^(c + d*x)] + (6*I)*a^2*b*E^(2*c
)*f^2*PolyLog[3, I*E^(c + d*x)] - (6*I)*b^3*E^(2*c)*f^2*PolyLog[3, I*E^(c +
 d*x)] - 3*a*b^2*f^2*PolyLog[3, -E^(2*(c + d*x))] - 3*a*b^2*E^(2*c)*f^2*Pol
yLog[3, -E^(2*(c + d*x))])/(6*(a^2 + b^2)^2*d^3*(1 + E^(2*c))) + (a*b^2*(6*
e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2
]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d
) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-
a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[
(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(
a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-
a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))
])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d +
(6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d
- (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*
E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[
(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqr
t[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a
*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x)
)/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E
^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(-1 + E^(2*c))*f
*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)
]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a
*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*f^2*PolyLog[3, -((b*E^(2*c +
d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3
```

```
, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (6*f^2*P
olyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 +
(6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(
2*c)]))])/d^3))/(3*(a^2 + b^2)^2*(-1 + E^(2*c))) + (Csch[c]*Sech[c]*Sech[c
+ d*x]^2*(6*a^3*e*f + 6*a*b^2*e*f - 12*a*b^2*d^2*e^2*x + 6*a^3*f^2*x + 6*a*
b^2*f^2*x - 12*a*b^2*d^2*e*f*x^2 - 4*a*b^2*d^2*f^2*x^3 - 6*a^3*e*f*Cosh[2*c
] - 6*a*b^2*e*f*Cosh[2*c] - 6*a^3*f^2*x*Cosh[2*c] - 6*a*b^2*f^2*x*Cosh[2*c]
 - 6*a^3*e*f*Cosh[2*d*x] - 6*a*b^2*e*f*Cosh[2*d*x] - 6*a^3*f^2*x*Cosh[2*d*x
] - 6*a*b^2*f^2*x*Cosh[2*d*x] - 3*a^2*b*d*e^2*Cosh[c - d*x] - 3*b^3*d*e^2*C
osh[c - d*x] - 6*a^2*b*d*e*f*x*Cosh[c - d*x] - 6*b^3*d*e*f*x*Cosh[c - d*x]
- 3*a^2*b*d*f^2*x^2*Cosh[c - d*x] - 3*b^3*d*f^2*x^2*Cosh[c - d*x] + 3*a^2*b
*d*e^2*Cosh[3*c + d*x] + 3*b^3*d*e^2*Cosh[3*c + d*x] + 6*a^2*b*d*e*f*x*Cosh
[3*c + d*x] + 6*b^3*d*e*f*x*Cosh[3*c + d*x] + 3*a^2*b*d*f^2*x^2*Cosh[3*c +
d*x] + 3*b^3*d*f^2*x^2*Cosh[3*c + d*x] + 6*a^3*e*f*Cosh[2*c + 2*d*x] + 6*a*
b^2*e*f*Cosh[2*c + 2*d*x] - 12*a*b^2*d^2*e^2*x*Cosh[2*c + 2*d*x] + 6*a^3*f^
2*x*Cosh[2*c + 2*d*x] + 6*a*b^2*f^2*x*Cosh[2*c + 2*d*x] - 12*a*b^2*d^2*e*f*
x^2*Cosh[2*c + 2*d*x] - 4*a*b^2*d^2*f^2*x^3*Cosh[2*c + 2*d*x] - 6*a^3*d*e^2
*Sinh[2*c] - 6*a*b^2*d*e^2*Sinh[2*c] - 12*a^3*d*e*f*x*Sinh[2*c] - 12*a*b^2*
d*e*f*x*Sinh[2*c] - 6*a^3*d*f^2*x^2*Sinh[2*c] - 6*a*b^2*d*f^2*x^2*Sinh[2*c]
 + 6*a^2*b*e*f*Sinh[c - d*x] + 6*b^3*e*f*Sinh[c - d*x] + 6*a^2*b*f^2*x*Sinh
[c - d*x] + 6*b^3*f^2*x*Sinh[c - d*x] + 6*a^2*b*e*f*Sinh[3*c + d*x] + 6*b^3
*e*f*Sinh[3*c + d*x] + 6*a^2*b*f^2*x*Sinh[3*c + d*x] + 6*b^3*f^2*x*Sinh[3*c
 + d*x]))/(24*(a^2 + b^2)^2*d^2)
```

Maple [F] time = 0.454, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \left(\operatorname{sech}\left(dx+c\right)\right)^{2} \tanh\left(dx+c\right)}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -a^2*b*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b
^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2
+ b^4*d^2), x) + b^3*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x +
2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 +
2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a*b^2*d^2*f^2*integrate(x^2/(a^4*d^2*e^(2*
d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*
d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a^2*b*d^2*e*f*integrate(x*e^(d*x + c
)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d
*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*b^3*d^2*e*f*integrat
e(x*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) +
```

```
b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 4*a*b^2*
d^2*e*f*integrate(x/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c
) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + a^3*
f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/(
(a^4 + 2*a^2*b^2 + b^4)*d^3)) + a*b^2*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 +
b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) - (a*b^
2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b^4)*
d) - a*b^2*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^2*b -
 b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) - (b*e^(-d*x - c) -
2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*e^(-
2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d))*e^2 - 2*a^2*b*f^2*arctan(e
^(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3) - 2*b^3*f^2*arctan(e^(d*x + c))/(
(a^4 + 2*a^2*b^2 + b^4)*d^3) - (2*a*f^2*x + 2*a*e*f - (b*d*f^2*x^2*e^(3*c)
+ 2*b*e*f*e^(3*c) + 2*(d*e*f + f^2)*b*x*e^(3*c))*e^(3*d*x) + 2*(a*d*f^2*x^2
*e^(2*c) + a*e*f*e^(2*c) + (2*d*e*f + f^2)*a*x*e^(2*c))*e^(2*d*x) + (b*d*f^
2*x^2*e^c - 2*b*e*f*e^c + 2*(d*e*f - f^2)*b*x*e^c)*e^(d*x))/(a^2*d^2 + b^2*
d^2 + (a^2*d^2*e^(4*c) + b^2*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) +
b^2*d^2*e^(2*c))*e^(2*d*x)) + integrate(2*(a*b^3*f^2*x^2 + 2*a*b^3*e*f*x -
(a^2*b^2*f^2*x^2*e^c + 2*a^2*b^2*e*f*x*e^c)*e^(d*x))/(a^4*b + 2*a^2*b^3 + b
^5 - (a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*e
^c + 2*a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x)
```

Fricas [C] time = 5.5478, size = 24520, normalized size = 20.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/2*(4*((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*cosh(d*x + c)^4 + 4*((
a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*sinh(d*x + c)^4 - 4*(a^3 + a*b^
2)*d*e*f + 4*(a^3 + a*b^2)*c*f^2 + 2*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b +
b^3)*d^2*e^2 + 2*(a^2*b + b^3)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + (a^2*b +
b^3)*d*f^2)*x)*cosh(d*x + c)^3 + 2*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^
3)*d^2*e^2 + 2*(a^2*b + b^3)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + (a^2*b + b^
3)*d*f^2)*x + 8*((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*cosh(d*x + c)
)*sinh(d*x + c)^3 - 4*((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 + a*b^2)*d^2*e^2 +
(a^3 + a*b^2)*d*e*f - 2*(a^3 + a*b^2)*c*f^2 + (2*(a^3 + a*b^2)*d^2*e*f - (a
^3 + a*b^2)*d*f^2)*x)*cosh(d*x + c)^2 - 2*(2*(a^3 + a*b^2)*d^2*f^2*x^2 + 2*
(a^3 + a*b^2)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f - 4*(a^3 + a*b^2)*c*f^2 - 12*
((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*cosh(d*x + c)^2 + 2*(2*(a^3 +
 a*b^2)*d^2*e*f - (a^3 + a*b^2)*d*f^2)*x - 3*((a^2*b + b^3)*d^2*f^2*x^2 + (
a^2*b + b^3)*d^2*e^2 + 2*(a^2*b + b^3)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + (
a^2*b + b^3)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*((a^2*b + b^3)*d^
2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 - 2*(a^2*b + b^3)*d*e*f + 2*((a^2*b + b^3
)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x)*cosh(d*x + c) - 4*(a*b^2*d*f^2*x + a*b^
2*d*e*f + (a*b^2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)^4 + 4*(a*b^2*d*f^2*x
+ a*b^2*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b^2*d*f^2*x + a*b^2*d*e*f
)*sinh(d*x + c)^4 + 2*(a*b^2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)^2 + 2*(a*
b^2*d*f^2*x + a*b^2*d*e*f + 3*(a*b^2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 4*((a*b^2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)^3 + (a*b
^2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x +
 c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2) - b)/b + 1) - 4*(a*b^2*d*f^2*x + a*b^2*d*e*f + (a*b^2*d*f^2*x + a*b^
2*d*e*f)*cosh(d*x + c)^4 + 4*(a*b^2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)*si
```

nh(d*x + c)^3 + (a*b^2*d*f^2*x + a*b^2*d*e*f)*sinh(d*x + c)^4 + 2*(a*b^2*d*
f^2*x + a*b^2*d*e*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*f^2*x + a*b^2*d*e*f + 3*(
a*b^2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a*b^2*d
*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)^3 + (a*b^2*d*f^2*x + a*b^2*d*e*f)*cosh(
d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (4*a*b^2*d
*f^2*x + 4*a*b^2*d*e*f - 2*I*(a^2*b - b^3)*d*f^2*x + (4*a*b^2*d*f^2*x + 4*a
*b^2*d*e*f - 2*I*(a^2*b - b^3)*d*f^2*x - 2*I*(a^2*b - b^3)*d*e*f)*cosh(d*x
+ c)^4 + (16*a*b^2*d*f^2*x + 16*a*b^2*d*e*f - 8*I*(a^2*b - b^3)*d*f^2*x - 8
I(a^2*b - b^3)*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (4*a*b^2*d*f^2*x +
4*a*b^2*d*e*f - 2*I*(a^2*b - b^3)*d*f^2*x - 2*I*(a^2*b - b^3)*d*e*f)*sinh(d
*x + c)^4 - 2*I*(a^2*b - b^3)*d*e*f + (8*a*b^2*d*f^2*x + 8*a*b^2*d*e*f - 4*
I*(a^2*b - b^3)*d*f^2*x - 4*I*(a^2*b - b^3)*d*e*f)*cosh(d*x + c)^2 + (8*a*b
^2*d*f^2*x + 8*a*b^2*d*e*f - 4*I*(a^2*b - b^3)*d*f^2*x - 4*I*(a^2*b - b^3)*
d*e*f + (24*a*b^2*d*f^2*x + 24*a*b^2*d*e*f - 12*I*(a^2*b - b^3)*d*f^2*x - 1
2*I*(a^2*b - b^3)*d*e*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((16*a*b^2*d*f^
2*x + 16*a*b^2*d*e*f - 8*I*(a^2*b - b^3)*d*f^2*x - 8*I*(a^2*b - b^3)*d*e*f)
*cosh(d*x + c)^3 + (16*a*b^2*d*f^2*x + 16*a*b^2*d*e*f - 8*I*(a^2*b - b^3)*d
*f^2*x - 8*I*(a^2*b - b^3)*d*e*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(I*cos
h(d*x + c) + I*sinh(d*x + c)) + (4*a*b^2*d*f^2*x + 4*a*b^2*d*e*f + 2*I*(a^2
*b - b^3)*d*f^2*x + (4*a*b^2*d*f^2*x + 4*a*b^2*d*e*f + 2*I*(a^2*b - b^3)*d*
f^2*x + 2*I*(a^2*b - b^3)*d*e*f)*cosh(d*x + c)^4 + (16*a*b^2*d*f^2*x + 16*a
*b^2*d*e*f + 8*I*(a^2*b - b^3)*d*f^2*x + 8*I*(a^2*b - b^3)*d*e*f)*cosh(d*x
+ c)*sinh(d*x + c)^3 + (4*a*b^2*d*f^2*x + 4*a*b^2*d*e*f + 2*I*(a^2*b - b^3)
*d*f^2*x + 2*I*(a^2*b - b^3)*d*e*f)*sinh(d*x + c)^4 + 2*I*(a^2*b - b^3)*d*e
*f + (8*a*b^2*d*f^2*x + 8*a*b^2*d*e*f + 4*I*(a^2*b - b^3)*d*f^2*x + 4*I*(a^
2*b - b^3)*d*e*f)*cosh(d*x + c)^2 + (8*a*b^2*d*f^2*x + 8*a*b^2*d*e*f + 4*I*
(a^2*b - b^3)*d*f^2*x + 4*I*(a^2*b - b^3)*d*e*f + (24*a*b^2*d*f^2*x + 24*a*
b^2*d*e*f + 12*I*(a^2*b - b^3)*d*f^2*x + 12*I*(a^2*b - b^3)*d*e*f)*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 + ((16*a*b^2*d*f^2*x + 16*a*b^2*d*e*f + 8*I*(a^2*b
 - b^3)*d*f^2*x + 8*I*(a^2*b - b^3)*d*e*f)*cosh(d*x + c)^3 + (16*a*b^2*d*f^
2*x + 16*a*b^2*d*e*f + 8*I*(a^2*b - b^3)*d*f^2*x + 8*I*(a^2*b - b^3)*d*e*f)
*cosh(d*x + c))*sinh(d*x + c))*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) -
2*(a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2 + (a*b^2*d^2*e^2 - 2*a*b
^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(d*x + c)^4 + 4*(a*b^2*d^2*e^2 - 2*a*b^2*c*
d*e*f + a*b^2*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b^2*d^2*e^2 - 2*a
*b^2*c*d*e*f + a*b^2*c^2*f^2)*sinh(d*x + c)^4 + 2*(a*b^2*d^2*e^2 - 2*a*b^2*
c*d*e*f + a*b^2*c^2*f^2)*cosh(d*x + c)^2 + 2*(a*b^2*d^2*e^2 - 2*a*b^2*c*d*e
*f + a*b^2*c^2*f^2 + 3*(a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*co
sh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^
2*c^2*f^2)*cosh(d*x + c)^3 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f
^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c)
 + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f +
a*b^2*c^2*f^2 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(d*x
+ c)^4 + 4*(a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(d*x + c)*
sinh(d*x + c)^3 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*sinh(d*
x + c)^4 + 2*(a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(d*x + c
)^2 + 2*(a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2 + 3*(a*b^2*d^2*e^2
 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((
a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(d*x + c)^3 + (a*b^2*d
^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log
(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) -
 2*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2
 + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2
)*cosh(d*x + c)^4 + 4*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*
e*f - a*b^2*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b^2*d^2*f^2*x^2 + 2
*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*sinh(d*x + c)^4 + 2*(a*
b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*cosh
(d*x + c)^2 + 2*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f -

a*b^2*c^2*f^2 + 3*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*cosh(d*x + c)^3 + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2 + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*cosh(d*x + c)^4 + 4*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*sinh(d*x + c)^4 + 2*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*cosh(d*x + c)^2 + 2*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2 + 3*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*cosh(d*x + c)^3 + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (2*a*b^2*d^2*e^2 - 4*a*b^2*c*d*e*f - I*(a^2*b - b^3)*d^2*e^2 + 2*I*(a^2*b - b^3)*c*d*e*f + (2*a*b^2*d^2*e^2 - 4*a*b^2*c*d*e*f - I*(a^2*b - b^3)*d^2*e^2 + 2*I*(a^2*b - b^3)*c*d*e*f + 2*(a*b^2*c^2 - a^3 - a*b^2)*f^2 - I*(2*a^2*b + 2*b^3 + (a^2*b - b^3)*c^2)*f^2)*cosh(d*x + c)^4 + (8*a*b^2*d^2*e^2 - 16*a*b^2*c*d*e*f - 4*I*(a^2*b - b^3)*d^2*e^2 + 8*I*(a^2*b - b^3)*c*d*e*f + 8*(a*b^2*c^2 - a^3 - a*b^2)*f^2 - 4*I*(2*a^2*b + 2*b^3 + (a^2*b - b^3)*c^2)*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b^2*d^2*e^2 - 4*a*b^2*c*d*e*f - I*(a^2*b - b^3)*d^2*e^2 + 2*I*(a^2*b - b^3)*c*d*e*f + 2*(a*b^2*c^2 - a^3 - a*b^2)*f^2 - I*(2*a^2*b + 2*b^3 + (a^2*b - b^3)*c^2)*f^2)*sinh(d*x + c)^4 + 2*(a*b^2*c^2 - a^3 - a*b^2)*f^2 - I*(2*a^2*b + 2*b^3 + (a^2*b - b^3)*c^2)*f^2 + (4*a*b^2*d^2*e^2 - 8*a*b^2*c*d*e*f - 2*I*(a^2*b - b^3)*d^2*e^2 + 4*I*(a^2*b - b^3)*c*d*e*f + 4*(a*b^2*c^2 - a^3 - a*b^2)*f^2 - 2*I*(2*a^2*b + 2*b^3 + (a^2*b - b^3)*c^2)*f^2)*cosh(d*x + c)^2 + (4*a*b^2*d^2*e^2 - 8*a*b^2*c*d*e*f - 2*I*(a^2*b - b^3)*d^2*e^2 + 4*I*(a^2*b - b^3)*c*d*e*f + 4*(a*b^2*c^2 - a^3 - a*b^2)*f^2 - 2*I*(2*a^2*b + 2*b^3 + (a^2*b - b^3)*c^2)*f^2 + (12*a*b^2*d^2*e^2 - 24*a*b^2*c*d*e*f - 6*I*(a^2*b - b^3)*d^2*e^2 + 12*I*(a^2*b - b^3)*c*d*e*f + 12*(a*b^2*c^2 - a^3 - a*b^2)*f^2 - 6*I*(2*a^2*b + 2*b^3 + (a^2*b - b^3)*c^2)*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a*b^2*d^2*e^2 - 16*a*b^2*c*d*e*f - 4*I*(a^2*b - b^3)*d^2*e^2 + 8*I*(a^2*b - b^3)*c*d*e*f + 8*(a*b^2*c^2 - a^3 - a*b^2)*f^2 - 4*I*(2*a^2*b + 2*b^3 + (a^2*b - b^3)*c^2)*f^2)*cosh(d*x + c)^3 + (8*a*b^2*d^2*e^2 - 16*a*b^2*c*d*e*f - 4*I*(a^2*b - b^3)*d^2*e^2 + 8*I*(a^2*b - b^3)*c*d*e*f + 8*(a*b^2*c^2 - a^3 - a*b^2)*f^2 - 4*I*(2*a^2*b + 2*b^3 + (a^2*b - b^3)*c^2)*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + I) + (2*a*b^2*d^2*e^2 - 4*a*b^2*c*d*e*f + I*(a^2*b - b^3)*d^2*e^2 - 2*I*(a^2*b - b^3)*c*d*e*f + (2*a*b^2*d^2*e^2 - 4*a*b^2*c*d*e*f + I*(a^2*b - b^3)*d^2*e^2 - 2*I*(a^2*b - b^3)*c*d*e*f + 2*(a*b^2*c^2 - a^3 - a*b^2)*f^2 + I*(2*a^2*b + 2*b^3 + (a^2*b - b^3)*c^2)*f^2)*cosh(d*x + c)^4 + (8*a*b^2*d^2*e^2 - 16*a*b^2*c*d*e*f + 4*I*(a^2*b - b^3)*d^2*e^2 - 8*I*(a^2*b - b^3)*c*d*e*f + 8*(a*b^2*c^2 - a^3 - a*b^2)*f^2 + 4*I*(2*a^2*b + 2*b^3 + (a^2*b - b^3)*c^2)*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b^2*d^2*e^2 - 4*a*b^2*c*d*e*f + I*(a^2*b - b^3)*d^2*e^2 - 2*I*(a^2*b - b^3)*c*d*e*f + 2*(a*b^2*c^2 - a^3 - a*b^2)*f^2 + I*(2*a^2*b + 2*b^3 + (a^2*b - b^3)*c^2)*f^2)*sinh(d*x + c)^4 + 2*(a*b^2*c^2 - a^3 - a*b^2)*f^2 + I*(2*a^2*b + 2*b^3 + (a^2*b - b^3)*c^2)*f^2 + (4*a*b^2*d^2*e^2 - 8*a*b^2*c*d*e*f + 2*I*(a^2*b - b^3)*d^2*e^2 - 4*I*(a^2*b - b^3)*c*d*e*f + 4*(a*b^2*c^2 - a^3 - a*b^2)*f^2 + 2*I*(2*a^2*b + 2*b^3 + (a^2*b - b^3)*c^2)*f^2)*cosh(d*x + c)^2 + (4*a*b^2*d^2*e^2 - 8*a*b^2*c*d*e*f + 2*I*(a^2*b - b^3)*d^2*e^2 - 4*I*(a^2*b - b^3)*c*d*e*f + 4*(a*b^2*c^2 - a^3 - a*b^2)*f^2 + 2*I*(2*a^2*b + 2*b^3 + (a^2*b - b^3)*c^2)*f^2 + (12*a*b^2*d^2*e^2 - 24*a*b^2*c*d*e*f + 6*I*(a^2*b - b^3)*d^2*e^2 - 12*I*(a^2*b - b^3)*c*d*e*f + 12*(a*b^2*c^2 - a^3 - a*b^2)*f^2 + 6*I

*(2*a^2*b + 2*b^3 + (a^2*b - b^3)*c^2)*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^
2 + ((8*a*b^2*d^2*e^2 - 16*a*b^2*c*d*e*f + 4*I*(a^2*b - b^3)*d^2*e^2 - 8*I*
(a^2*b - b^3)*c*d*e*f + 8*(a*b^2*c^2 - a^3 - a*b^2)*f^2 + 4*I*(2*a^2*b + 2*
b^3 + (a^2*b - b^3)*c^2)*f^2)*cosh(d*x + c)^3 + (8*a*b^2*d^2*e^2 - 16*a*b^2
*c*d*e*f + 4*I*(a^2*b - b^3)*d^2*e^2 - 8*I*(a^2*b - b^3)*c*d*e*f + 8*(a*b^2
*c^2 - a^3 - a*b^2)*f^2 + 4*I*(2*a^2*b + 2*b^3 + (a^2*b - b^3)*c^2)*f^2)*co
sh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - I) + (2*a*b
^2*d^2*f^2*x^2 + 4*a*b^2*d^2*e*f*x + 4*a*b^2*c*d*e*f - 2*a*b^2*c^2*f^2 + I*
(a^2*b - b^3)*d^2*f^2*x^2 + 2*I*(a^2*b - b^3)*d^2*e*f*x + 2*I*(a^2*b - b^3)
*c*d*e*f - I*(a^2*b - b^3)*c^2*f^2 + (2*a*b^2*d^2*f^2*x^2 + 4*a*b^2*d^2*e*f
*x + 4*a*b^2*c*d*e*f - 2*a*b^2*c^2*f^2 + I*(a^2*b - b^3)*d^2*f^2*x^2 + 2*I*
(a^2*b - b^3)*d^2*e*f*x + 2*I*(a^2*b - b^3)*c*d*e*f - I*(a^2*b - b^3)*c^2*f
^2)*cosh(d*x + c)^4 + (8*a*b^2*d^2*f^2*x^2 + 16*a*b^2*d^2*e*f*x + 16*a*b^2*
c*d*e*f - 8*a*b^2*c^2*f^2 + 4*I*(a^2*b - b^3)*d^2*f^2*x^2 + 8*I*(a^2*b - b^
3)*d^2*e*f*x + 8*I*(a^2*b - b^3)*c*d*e*f - 4*I*(a^2*b - b^3)*c^2*f^2)*cosh(
d*x + c)*sinh(d*x + c)^3 + (2*a*b^2*d^2*f^2*x^2 + 4*a*b^2*d^2*e*f*x + 4*a*b
^2*c*d*e*f - 2*a*b^2*c^2*f^2 + I*(a^2*b - b^3)*d^2*f^2*x^2 + 2*I*(a^2*b - b
^3)*d^2*e*f*x + 2*I*(a^2*b - b^3)*c*d*e*f - I*(a^2*b - b^3)*c^2*f^2)*sinh(d
*x + c)^4 + (4*a*b^2*d^2*f^2*x^2 + 8*a*b^2*d^2*e*f*x + 8*a*b^2*c*d*e*f - 4*
a*b^2*c^2*f^2 + 2*I*(a^2*b - b^3)*d^2*f^2*x^2 + 4*I*(a^2*b - b^3)*d^2*e*f*x
 + 4*I*(a^2*b - b^3)*c*d*e*f - 2*I*(a^2*b - b^3)*c^2*f^2)*cosh(d*x + c)^2 +
 (4*a*b^2*d^2*f^2*x^2 + 8*a*b^2*d^2*e*f*x + 8*a*b^2*c*d*e*f - 4*a*b^2*c^2*f
^2 + 2*I*(a^2*b - b^3)*d^2*f^2*x^2 + 4*I*(a^2*b - b^3)*d^2*e*f*x + 4*I*(a^2
*b - b^3)*c*d*e*f - 2*I*(a^2*b - b^3)*c^2*f^2 + (12*a*b^2*d^2*f^2*x^2 + 24*
a*b^2*d^2*e*f*x + 24*a*b^2*c*d*e*f - 12*a*b^2*c^2*f^2 + 6*I*(a^2*b - b^3)*d
^2*f^2*x^2 + 12*I*(a^2*b - b^3)*d^2*e*f*x + 12*I*(a^2*b - b^3)*c*d*e*f - 6*
I*(a^2*b - b^3)*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a*b^2*d^2*f
^2*x^2 + 16*a*b^2*d^2*e*f*x + 16*a*b^2*c*d*e*f - 8*a*b^2*c^2*f^2 + 4*I*(a^2
*b - b^3)*d^2*f^2*x^2 + 8*I*(a^2*b - b^3)*d^2*e*f*x + 8*I*(a^2*b - b^3)*c*d
*e*f - 4*I*(a^2*b - b^3)*c^2*f^2)*cosh(d*x + c)^3 + (8*a*b^2*d^2*f^2*x^2 +
16*a*b^2*d^2*e*f*x + 16*a*b^2*c*d*e*f - 8*a*b^2*c^2*f^2 + 4*I*(a^2*b - b^3)
*d^2*f^2*x^2 + 8*I*(a^2*b - b^3)*d^2*e*f*x + 8*I*(a^2*b - b^3)*c*d*e*f - 4*
I*(a^2*b - b^3)*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(I*cosh(d*x + c)
+ I*sinh(d*x + c) + 1) + (2*a*b^2*d^2*f^2*x^2 + 4*a*b^2*d^2*e*f*x + 4*a*b^2
*c*d*e*f - 2*a*b^2*c^2*f^2 - I*(a^2*b - b^3)*d^2*f^2*x^2 - 2*I*(a^2*b - b^3
)*d^2*e*f*x - 2*I*(a^2*b - b^3)*c*d*e*f + I*(a^2*b - b^3)*c^2*f^2 + (2*a*b^
2*d^2*f^2*x^2 + 4*a*b^2*d^2*e*f*x + 4*a*b^2*c*d*e*f - 2*a*b^2*c^2*f^2 - I*(
a^2*b - b^3)*d^2*f^2*x^2 - 2*I*(a^2*b - b^3)*d^2*e*f*x - 2*I*(a^2*b - b^3)*
c*d*e*f + I*(a^2*b - b^3)*c^2*f^2)*cosh(d*x + c)^4 + (8*a*b^2*d^2*f^2*x^2 +
 16*a*b^2*d^2*e*f*x + 16*a*b^2*c*d*e*f - 8*a*b^2*c^2*f^2 - 4*I*(a^2*b - b^3
)*d^2*f^2*x^2 - 8*I*(a^2*b - b^3)*d^2*e*f*x - 8*I*(a^2*b - b^3)*c*d*e*f + 4
I(a^2*b - b^3)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b^2*d^2*f^2*
x^2 + 4*a*b^2*d^2*e*f*x + 4*a*b^2*c*d*e*f - 2*a*b^2*c^2*f^2 - I*(a^2*b - b^
3)*d^2*f^2*x^2 - 2*I*(a^2*b - b^3)*d^2*e*f*x - 2*I*(a^2*b - b^3)*c*d*e*f +
I*(a^2*b - b^3)*c^2*f^2)*sinh(d*x + c)^4 + (4*a*b^2*d^2*f^2*x^2 + 8*a*b^2*d
^2*e*f*x + 8*a*b^2*c*d*e*f - 4*a*b^2*c^2*f^2 - 2*I*(a^2*b - b^3)*d^2*f^2*x^
2 - 4*I*(a^2*b - b^3)*d^2*e*f*x - 4*I*(a^2*b - b^3)*c*d*e*f + 2*I*(a^2*b -
b^3)*c^2*f^2)*cosh(d*x + c)^2 + (4*a*b^2*d^2*f^2*x^2 + 8*a*b^2*d^2*e*f*x +
8*a*b^2*c*d*e*f - 4*a*b^2*c^2*f^2 - 2*I*(a^2*b - b^3)*d^2*f^2*x^2 - 4*I*(a^
2*b - b^3)*d^2*e*f*x - 4*I*(a^2*b - b^3)*c*d*e*f + 2*I*(a^2*b - b^3)*c^2*f^
2 + (12*a*b^2*d^2*f^2*x^2 + 24*a*b^2*d^2*e*f*x + 24*a*b^2*c*d*e*f - 12*a*b^
2*c^2*f^2 - 6*I*(a^2*b - b^3)*d^2*f^2*x^2 - 12*I*(a^2*b - b^3)*d^2*e*f*x -
12*I*(a^2*b - b^3)*c*d*e*f + 6*I*(a^2*b - b^3)*c^2*f^2)*cosh(d*x + c)^2)*si
nh(d*x + c)^2 + ((8*a*b^2*d^2*f^2*x^2 + 16*a*b^2*d^2*e*f*x + 16*a*b^2*c*d*e
*f - 8*a*b^2*c^2*f^2 - 4*I*(a^2*b - b^3)*d^2*f^2*x^2 - 8*I*(a^2*b - b^3)*d^
2*e*f*x - 8*I*(a^2*b - b^3)*c*d*e*f + 4*I*(a^2*b - b^3)*c^2*f^2)*cosh(d*x +
 c)^3 + (8*a*b^2*d^2*f^2*x^2 + 16*a*b^2*d^2*e*f*x + 16*a*b^2*c*d*e*f - 8*a*
b^2*c^2*f^2 - 4*I*(a^2*b - b^3)*d^2*f^2*x^2 - 8*I*(a^2*b - b^3)*d^2*e*f*x -
 8*I*(a^2*b - b^3)*c*d*e*f + 4*I*(a^2*b - b^3)*c^2*f^2)*cosh(d*x + c))*sinh

```
(d*x + c))*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1) + 4*(a*b^2*f^2*cosh(
d*x + c)^4 + 4*a*b^2*f^2*cosh(d*x + c)*sinh(d*x + c)^3 + a*b^2*f^2*sinh(d*x
 + c)^4 + 2*a*b^2*f^2*cosh(d*x + c)^2 + a*b^2*f^2 + 2*(3*a*b^2*f^2*cosh(d*x
 + c)^2 + a*b^2*f^2)*sinh(d*x + c)^2 + 4*(a*b^2*f^2*cosh(d*x + c)^3 + a*b^2
*f^2*cosh(d*x + c))*sinh(d*x + c))*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x
 + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 4*(
a*b^2*f^2*cosh(d*x + c)^4 + 4*a*b^2*f^2*cosh(d*x + c)*sinh(d*x + c)^3 + a*b
^2*f^2*sinh(d*x + c)^4 + 2*a*b^2*f^2*cosh(d*x + c)^2 + a*b^2*f^2 + 2*(3*a*b
^2*f^2*cosh(d*x + c)^2 + a*b^2*f^2)*sinh(d*x + c)^2 + 4*(a*b^2*f^2*cosh(d*x
 + c)^3 + a*b^2*f^2*cosh(d*x + c))*sinh(d*x + c))*polylog(3, (a*cosh(d*x +
c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2))/b) - (4*a*b^2*f^2 + (4*a*b^2*f^2 - 2*I*(a^2*b - b^3)*f^2)*cosh(d*x +
 c)^4 + (16*a*b^2*f^2 - 8*I*(a^2*b - b^3)*f^2)*cosh(d*x + c)*sinh(d*x + c)^
3 + (4*a*b^2*f^2 - 2*I*(a^2*b - b^3)*f^2)*sinh(d*x + c)^4 - 2*I*(a^2*b - b^
3)*f^2 + (8*a*b^2*f^2 - 4*I*(a^2*b - b^3)*f^2)*cosh(d*x + c)^2 + (8*a*b^2*f
^2 - 4*I*(a^2*b - b^3)*f^2 + (24*a*b^2*f^2 - 12*I*(a^2*b - b^3)*f^2)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 + ((16*a*b^2*f^2 - 8*I*(a^2*b - b^3)*f^2)*cosh(d
*x + c)^3 + (16*a*b^2*f^2 - 8*I*(a^2*b - b^3)*f^2)*cosh(d*x + c))*sinh(d*x
+ c))*polylog(3, I*cosh(d*x + c) + I*sinh(d*x + c)) - (4*a*b^2*f^2 + (4*a*b
^2*f^2 + 2*I*(a^2*b - b^3)*f^2)*cosh(d*x + c)^4 + (16*a*b^2*f^2 + 8*I*(a^2*
b - b^3)*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (4*a*b^2*f^2 + 2*I*(a^2*b - b
^3)*f^2)*sinh(d*x + c)^4 + 2*I*(a^2*b - b^3)*f^2 + (8*a*b^2*f^2 + 4*I*(a^2*
b - b^3)*f^2)*cosh(d*x + c)^2 + (8*a*b^2*f^2 + 4*I*(a^2*b - b^3)*f^2 + (24*
a*b^2*f^2 + 12*I*(a^2*b - b^3)*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((16
*a*b^2*f^2 + 8*I*(a^2*b - b^3)*f^2)*cosh(d*x + c)^3 + (16*a*b^2*f^2 + 8*I*(
a^2*b - b^3)*f^2)*cosh(d*x + c))*sinh(d*x + c))*polylog(3, -I*cosh(d*x + c)
 - I*sinh(d*x + c)) - 2*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2
- 2*(a^2*b + b^3)*d*e*f - 8*((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*c
osh(d*x + c)^3 - 3*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 + 2*(
a^2*b + b^3)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + (a^2*b + b^3)*d*f^2)*x)*cos
h(d*x + c)^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x + 4*((a^3
+ a*b^2)*d^2*f^2*x^2 + (a^3 + a*b^2)*d^2*e^2 + (a^3 + a*b^2)*d*e*f - 2*(a^3
 + a*b^2)*c*f^2 + (2*(a^3 + a*b^2)*d^2*e*f - (a^3 + a*b^2)*d*f^2)*x)*cosh(d
*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c)^4 + 4*(a
^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2
+ b^4)*d^3*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c)^2
+ (a^4 + 2*a^2*b^2 + b^4)*d^3 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x +
 c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^3)*sinh(d*x + c)^2 + 4*((a^4 + 2*a^2*b^2
+ b^4)*d^3*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c))*sin
h(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sech(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.359 $\int \frac{(e+fx)\mathbf{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=711

$$\frac{ia^2f\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{2bd^2\left(a^2+b^2\right)}+\frac{ia^2bf\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)^2}-\frac{ia^2f\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{2bd^2\left(a^2+b^2\right)}-\frac{ia^2bf\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)^2}-\frac{ab^2 j}{}$$

```
[Out] ((e + f*x)*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*b*(e + f*x)*ArcTan[E^(c + d*
x)])/((a^2 + b^2)^2*d) - (a^2*(e + f*x)*ArcTan[E^(c + d*x)])/(b*(a^2 + b^2)
*d) - (a*b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^
2 + b^2)^2*d) - (a*b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/((a^2 + b^2)^2*d) + (a*b^2*(e + f*x)*Log[1 + E^(2*(c + d*x))])/((a^2
+ b^2)^2*d) - ((I/2)*f*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + (I*a^2*b*f*P
olyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) + ((I/2)*a^2*f*PolyLog[2,
(-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) + ((I/2)*f*PolyLog[2, I*E^(c + d*x)]
)/(b*d^2) - (I*a^2*b*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - ((I
/2)*a^2*f*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) - (a*b^2*f*PolyLog
[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) - (a*b^2
*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2
) + (a*b^2*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^2) + (f*Sech[
c + d*x])/(2*b*d^2) - (a^2*f*Sech[c + d*x])/(2*b*(a^2 + b^2)*d^2) - (a*(e +
 f*x)*Sech[c + d*x]^2)/(2*(a^2 + b^2)*d) + (a*f*Tanh[c + d*x])/(2*(a^2 + b^
2)*d^2) + ((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*b*d) - (a^2*(e + f*x)*
Sech[c + d*x]*Tanh[c + d*x])/(2*b*(a^2 + b^2)*d)
```

Rubi [A] time = 0.993033, antiderivative size = 711, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.406, Rules used = {5583, 4185, 4180, 2279, 2391, 5573, 5561, 2190, 6742, 3718, 5451, 3767, 8}

$$\frac{ia^2f\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{2bd^2\left(a^2+b^2\right)}+\frac{ia^2bf\mathrm{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)^2}-\frac{ia^2f\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{2bd^2\left(a^2+b^2\right)}-\frac{ia^2bf\mathrm{PolyLog}\left(2,ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)^2}-\frac{ab^2 j}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] ((e + f*x)*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*b*(e + f*x)*ArcTan[E^(c + d*
x)])/((a^2 + b^2)^2*d) - (a^2*(e + f*x)*ArcTan[E^(c + d*x)])/(b*(a^2 + b^2)
*d) - (a*b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^
2 + b^2)^2*d) - (a*b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/((a^2 + b^2)^2*d) + (a*b^2*(e + f*x)*Log[1 + E^(2*(c + d*x))])/((a^2
+ b^2)^2*d) - ((I/2)*f*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + (I*a^2*b*f*P
olyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) + ((I/2)*a^2*f*PolyLog[2,
(-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) + ((I/2)*f*PolyLog[2, I*E^(c + d*x)]
)/(b*d^2) - (I*a^2*b*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - ((I
/2)*a^2*f*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) - (a*b^2*f*PolyLog
[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) - (a*b^2
*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2
) + (a*b^2*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^2) + (f*Sech[
c + d*x])/(2*b*d^2) - (a^2*f*Sech[c + d*x])/(2*b*(a^2 + b^2)*d^2) - (a*(e +
 f*x)*Sech[c + d*x]^2)/(2*(a^2 + b^2)*d) + (a*f*Tanh[c + d*x])/(2*(a^2 + b^
2)*d^2) + ((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*b*d) - (a^2*(e + f*x)*
```

```
Sech[c + d*x]*Tanh[c + d*x])/(2*b*(a^2 + b^2)*d)
```

Rule 5583

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{sech}^3(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{f\text{sech}(c+dx)}{2bd^2} + \frac{(e+fx)\text{sech}(c+dx)\tanh(c+dx)}{2bd} + \frac{\int (e+fx)\text{sech}(c+ \cdots}{2b}$$

$$= \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} + \frac{f\text{sech}(c+dx)}{2bd^2} + \frac{(e+fx)\text{sech}(c+dx)\tanh(c+d\cdots}{2bd}$$

$$= \frac{ab^2(e+fx)^2}{2\left(a^2+b^2\right)^2 f} + \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} + \frac{f\text{sech}(c+dx)}{2bd^2} + \frac{(e+fx)\text{sech}(c \cdots}{2\cdots}$$

$$= \frac{ab^2(e+fx)^2}{2\left(a^2+b^2\right)^2 f} + \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{ab^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2 d} - \cdots$$

$$= \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{a^2(e+fx)\tan^{-1}\left(e^{c+\cdots}\right)}{b\left(a^2+b^2\right)d}$$

$$= \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{a^2(e+fx)\tan^{-1}\left(e^{c+\cdots}\right)}{b\left(a^2+b^2\right)d}$$

$$= \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{a^2(e+fx)\tan^{-1}\left(e^{c+\cdots}\right)}{b\left(a^2+b^2\right)d}$$

$$= \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^2b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{a^2(e+fx)\tan^{-1}\left(e^{c+\cdots}\right)}{b\left(a^2+b^2\right)d}$$

Mathematica [A] time = 8.54467, size = 587, normalized size = 0.83

$$\frac{b\left(if\left(a^2-b^2\right)\text{PolyLog}\left(2,-ie^{c+dx}\right)-if\left(a^2-b^2\right)\text{PolyLog}\left(2,ie^{c+dx}\right)+abf\text{PolyLog}\left(2,-e^{2(c+dx)}\right)-2a^2de\tan^{-1}\left(e^{c+dx}\right)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (-2*a*b^2*(-(f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLog
[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))]) + b*(-2*a*b*d*e*(c + d*x) + 2*a*b*c*f*(c + d*x)
- a*b*f*(c + d*x)^2 - 2*a^2*d*e*ArcTan[E^(c + d*x)] + 2*b^2*d*e*ArcTan[E^(c
 + d*x)] + 2*a^2*c*f*ArcTan[E^(c + d*x)] - 2*b^2*c*f*ArcTan[E^(c + d*x)] -
I*a^2*f*(c + d*x)*Log[1 - I*E^(c + d*x)] + I*b^2*f*(c + d*x)*Log[1 - I*E^(c
 + d*x)] + I*a^2*f*(c + d*x)*Log[1 + I*E^(c + d*x)] - I*b^2*f*(c + d*x)*Log
[1 + I*E^(c + d*x)] + 2*a*b*d*e*Log[1 + E^(2*(c + d*x))] - 2*a*b*c*f*Log[1
+ E^(2*(c + d*x))] + 2*a*b*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] + I*(a^2 -
b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] - I*(a^2 - b^2)*f*PolyLog[2, I*E^(c + d
*x)] + a*b*f*PolyLog[2, -E^(2*(c + d*x))]) + (a^2 + b^2)*f*Sech[c + d*x]*(b
```

```
 + a*Sinh[c + d*x]) + (a^2 + b^2)*d*(e + f*x)*Sech[c + d*x]^2*(-a + b*Sinh[
c + d*x]))/(2*(a^2 + b^2)^2*d^2)
```

Maple [B] time = 0.19, size = 2074, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/d^2/(a^2+b^2)^(3/2)*b^2*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a
)/(a^2+b^2)^(1/2))*a^2-1/d/(a^2+b^2)^(3/2)*b^4*e/(2*a^2+2*b^2)*arctanh(1/2*
(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d/(a^2+b^2)^(1/2)*b^2*e/(2*a^2+2*b^
2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+I*b/(a^2+b^2)/d^2*a^2*
f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c+I*b/(a^2+b^2)/d*a^2*f/(2*a^2+2*b^2)*ln
(1+I*exp(d*x+c))*x-I*b/(a^2+b^2)/d*a^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x
-I*b/(a^2+b^2)/d^2*a^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c+I*b/(a^2+b^2)/d
^2*a^2*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))+2*b^2/(a^2+b^2)/d*f/(2*a^2+2*b
^2)*ln(1-I*exp(d*x+c))*a*x+2*b^2/(a^2+b^2)/d^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d
*x+c))*a*c-2*b^2/(a^2+b^2)/d*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1
/2)-a)/(-a+(a^2+b^2)^(1/2)))*a*x-2*b^2/(a^2+b^2)/d^2*f/(2*a^2+2*b^2)*ln((-b
*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a*c-2*b^2/(a^2+b^2)/d*
f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a*
x-I*b^3/(a^2+b^2)/d*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x-I*b^3/(a^2+b^2)/d^
2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c-I*b/(a^2+b^2)/d^2*a^2*f/(2*a^2+2*b^2
)*dilog(1-I*exp(d*x+c))+2*b^2/(a^2+b^2)/d*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c)
)*a*x+2*b^2/(a^2+b^2)/d^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*a*c+I*b^3/(a^2
+b^2)/d*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x+I*b^3/(a^2+b^2)/d^2*f/(2*a^2+2
*b^2)*ln(1-I*exp(d*x+c))*c+2*b/(a^2+b^2)/d^2*a^2*f*c/(2*a^2+2*b^2)*arctan(e
xp(d*x+c))-2*b^2/(a^2+b^2)/d^2*f*c/(2*a^2+2*b^2)*a*ln(1+exp(2*d*x+2*c))+2*b
^2/(a^2+b^2)/d^2*f*c/(2*a^2+2*b^2)*a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-
2*b^2/(a^2+b^2)/d^2*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+
(a^2+b^2)^(1/2)))*a*c-2*b^2/(a^2+b^2)/d*e/(2*a^2+2*b^2)*a*ln(b*exp(2*d*x+2*
c)+2*a*exp(d*x+c)-b)+2*b^2/(a^2+b^2)/d*e/(2*a^2+2*b^2)*a*ln(1+exp(2*d*x+2*c
))-2*b^3/(a^2+b^2)/d^2*f*c/(2*a^2+2*b^2)*arctan(exp(d*x+c))+2*b^2/(a^2+b^2)
/d^2*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))*a+2*b^2/(a^2+b^2)/d^2*f/(2*a^2+2
*b^2)*dilog(1-I*exp(d*x+c))*a-2*b^2/(a^2+b^2)/d^2*f/(2*a^2+2*b^2)*dilog((-b
*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a+I*b^3/(a^2+b^2)/d^2*
f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))-2*b^2/(a^2+b^2)/d^2*f/(2*a^2+2*b^2)*d
ilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a-2*b/(a^2+b^2)/
d*a^2*e/(2*a^2+2*b^2)*arctan(exp(d*x+c))-I*b^3/(a^2+b^2)/d^2*f/(2*a^2+2*b^2
)*dilog(1+I*exp(d*x+c))+1/d^2/(a^2+b^2)^(3/2)*b^4*f*c/(2*a^2+2*b^2)*arctanh
(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d/(a^2+b^2)^(3/2)*b^2*e/(2*a^2
+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2-1/d^2/(a^2+b^
2)^(1/2)*b^2*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(
1/2))-(-b*d*f*x*exp(3*d*x+3*c)+2*a*d*f*x*exp(2*d*x+2*c)-b*d*e*exp(3*d*x+3*c
)+2*a*d*e*exp(2*d*x+2*c)+b*d*f*x*exp(d*x+c)-b*f*exp(3*d*x+3*c)+a*f*exp(2*d*
x+2*c)+b*d*e*exp(d*x+c)-f*b*exp(d*x+c)+a*f)/d^2/(a^2+b^2)/(1+exp(2*d*x+2*c)
)^2+2*b^3/(a^2+b^2)/d*e/(2*a^2+2*b^2)*arctan(exp(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\left(\frac{ab^2\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{\left(a^4+2\,a^2b^2+b^4\right)d}-\frac{ab^2\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\left(a^4+2\,a^2b^2+b^4\right)d}-\frac{\left(a^2b-b^3\right)\arctan\left(e^{(-dx-c)}\right)}{\left(a^4+2\,a^2b^2+b^4\right)d}-\frac{be^{(-dx-c)}}{\left(a^2+b^2+2\left(a^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -(a*b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 +
 b^4)*d) - a*b^2*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a
^2*b - b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) - (b*e^(-d*x -
 c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2
)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d))*e + f*(((b*d*x*e^(3*
c) + b*e^(3*c))*e^(3*d*x) - (2*a*d*x*e^(2*c) + a*e^(2*c))*e^(2*d*x) - (b*d*
x*e^c - b*e^c)*e^(d*x) - a)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) + b^2*d^2
*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*x)) + 4*
integrate(-1/2*(a^2*b^2*x*e^(d*x + c) - a*b^3*x)/(a^4*b + 2*a^2*b^3 + b^5 -
 (a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c +
 2*a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) - 4*integrate(1/4*(2*a*b^2*x + (a^
2*b*e^c - b^3*e^c)*x*e^(d*x))/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(2*c) + 2*a^2
*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x))
```

Fricas [B] time = 3.69688, size = 11628, normalized size = 16.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/2*(2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^2*b + b^3)*f)*cosh(d*x
 + c)^3 + 2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^2*b + b^3)*f)*sin
h(d*x + c)^3 - 2*(2*(a^3 + a*b^2)*d*f*x + 2*(a^3 + a*b^2)*d*e + (a^3 + a*b^
2)*f)*cosh(d*x + c)^2 - 2*(2*(a^3 + a*b^2)*d*f*x + 2*(a^3 + a*b^2)*d*e + (a
^3 + a*b^2)*f - 3*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^2*b + b^3)*
f)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(a^3 + a*b^2)*f - 2*((a^2*b + b^3)*d*
f*x + (a^2*b + b^3)*d*e - (a^2*b + b^3)*f)*cosh(d*x + c) - 2*(a*b^2*f*cosh(
d*x + c)^4 + 4*a*b^2*f*cosh(d*x + c)*sinh(d*x + c)^3 + a*b^2*f*sinh(d*x + c
)^4 + 2*a*b^2*f*cosh(d*x + c)^2 + a*b^2*f + 2*(3*a*b^2*f*cosh(d*x + c)^2 +
a*b^2*f)*sinh(d*x + c)^2 + 4*(a*b^2*f*cosh(d*x + c)^3 + a*b^2*f*cosh(d*x +
c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
 c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(a*b^2*f*cosh(
d*x + c)^4 + 4*a*b^2*f*cosh(d*x + c)*sinh(d*x + c)^3 + a*b^2*f*sinh(d*x + c
)^4 + 2*a*b^2*f*cosh(d*x + c)^2 + a*b^2*f + 2*(3*a*b^2*f*cosh(d*x + c)^2 +
a*b^2*f)*sinh(d*x + c)^2 + 4*(a*b^2*f*cosh(d*x + c)^3 + a*b^2*f*cosh(d*x +
c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
 c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + ((2*a*b^2*f - I*
(a^2*b - b^3)*f)*cosh(d*x + c)^4 + (8*a*b^2*f - 4*I*(a^2*b - b^3)*f)*cosh(d
*x + c)*sinh(d*x + c)^3 + (2*a*b^2*f - I*(a^2*b - b^3)*f)*sinh(d*x + c)^4 +
 2*a*b^2*f + (4*a*b^2*f - 2*I*(a^2*b - b^3)*f)*cosh(d*x + c)^2 + (4*a*b^2*f
 + (12*a*b^2*f - 6*I*(a^2*b - b^3)*f)*cosh(d*x + c)^2 - 2*I*(a^2*b - b^3)*f
)*sinh(d*x + c)^2 - I*(a^2*b - b^3)*f + ((8*a*b^2*f - 4*I*(a^2*b - b^3)*f)*
cosh(d*x + c)^3 + (8*a*b^2*f - 4*I*(a^2*b - b^3)*f)*cosh(d*x + c))*sinh(d*x
 + c))*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + ((2*a*b^2*f + I*(a^2*b -
b^3)*f)*cosh(d*x + c)^4 + (8*a*b^2*f + 4*I*(a^2*b - b^3)*f)*cosh(d*x + c)*s
inh(d*x + c)^3 + (2*a*b^2*f + I*(a^2*b - b^3)*f)*sinh(d*x + c)^4 + 2*a*b^2*
f + (4*a*b^2*f + 2*I*(a^2*b - b^3)*f)*cosh(d*x + c)^2 + (4*a*b^2*f + (12*a*
b^2*f + 6*I*(a^2*b - b^3)*f)*cosh(d*x + c)^2 + 2*I*(a^2*b - b^3)*f)*sinh(d*
x + c)^2 + I*(a^2*b - b^3)*f + ((8*a*b^2*f + 4*I*(a^2*b - b^3)*f)*cosh(d*x
```

```
+ c)^3 + (8*a*b^2*f + 4*I*(a^2*b - b^3)*f)*cosh(d*x + c))*sinh(d*x + c))*di
log(-I*cosh(d*x + c) - I*sinh(d*x + c)) - 2*(a*b^2*d*e - a*b^2*c*f + (a*b^2
*d*e - a*b^2*c*f)*cosh(d*x + c)^4 + 4*(a*b^2*d*e - a*b^2*c*f)*cosh(d*x + c)
*sinh(d*x + c)^3 + (a*b^2*d*e - a*b^2*c*f)*sinh(d*x + c)^4 + 2*(a*b^2*d*e -
 a*b^2*c*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*e - a*b^2*c*f + 3*(a*b^2*d*e - a*b
^2*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a*b^2*d*e - a*b^2*c*f)*cosh(
d*x + c)^3 + (a*b^2*d*e - a*b^2*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(2*b*
cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(a
*b^2*d*e - a*b^2*c*f + (a*b^2*d*e - a*b^2*c*f)*cosh(d*x + c)^4 + 4*(a*b^2*d
*e - a*b^2*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b^2*d*e - a*b^2*c*f)*sin
h(d*x + c)^4 + 2*(a*b^2*d*e - a*b^2*c*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*e - a
*b^2*c*f + 3*(a*b^2*d*e - a*b^2*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(
(a*b^2*d*e - a*b^2*c*f)*cosh(d*x + c)^3 + (a*b^2*d*e - a*b^2*c*f)*cosh(d*x
+ c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((
a^2 + b^2)/b^2) + 2*a) - 2*(a*b^2*d*f*x + a*b^2*c*f + (a*b^2*d*f*x + a*b^2*
c*f)*cosh(d*x + c)^4 + 4*(a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c)*sinh(d*x +
 c)^3 + (a*b^2*d*f*x + a*b^2*c*f)*sinh(d*x + c)^4 + 2*(a*b^2*d*f*x + a*b^2*
c*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*f*x + a*b^2*c*f + 3*(a*b^2*d*f*x + a*b^2*
c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a*b^2*d*f*x + a*b^2*c*f)*cosh(d
*x + c)^3 + (a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-(a
*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b) - 2*(a*b^2*d*f*x + a*b^2*c*f + (a*b^2*d*f*x + a*b
^2*c*f)*cosh(d*x + c)^4 + 4*(a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c)*sinh(d*
x + c)^3 + (a*b^2*d*f*x + a*b^2*c*f)*sinh(d*x + c)^4 + 2*(a*b^2*d*f*x + a*b
^2*c*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*f*x + a*b^2*c*f + 3*(a*b^2*d*f*x + a*b
^2*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a*b^2*d*f*x + a*b^2*c*f)*cos
h(d*x + c)^3 + (a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(
-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 + b^2)/b^2) - b)/b) + (2*a*b^2*d*e - 2*a*b^2*c*f + (2*a*b^2*d*e -
2*a*b^2*c*f - I*(a^2*b - b^3)*d*e + I*(a^2*b - b^3)*c*f)*cosh(d*x + c)^4 +
(8*a*b^2*d*e - 8*a*b^2*c*f - 4*I*(a^2*b - b^3)*d*e + 4*I*(a^2*b - b^3)*c*f)
*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b^2*d*e - 2*a*b^2*c*f - I*(a^2*b - b^
3)*d*e + I*(a^2*b - b^3)*c*f)*sinh(d*x + c)^4 - I*(a^2*b - b^3)*d*e + I*(a^
2*b - b^3)*c*f + (4*a*b^2*d*e - 4*a*b^2*c*f - 2*I*(a^2*b - b^3)*d*e + 2*I*(
a^2*b - b^3)*c*f)*cosh(d*x + c)^2 + (4*a*b^2*d*e - 4*a*b^2*c*f - 2*I*(a^2*b
 - b^3)*d*e + 2*I*(a^2*b - b^3)*c*f + (12*a*b^2*d*e - 12*a*b^2*c*f - 6*I*(a
^2*b - b^3)*d*e + 6*I*(a^2*b - b^3)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
 ((8*a*b^2*d*e - 8*a*b^2*c*f - 4*I*(a^2*b - b^3)*d*e + 4*I*(a^2*b - b^3)*c*
f)*cosh(d*x + c)^3 + (8*a*b^2*d*e - 8*a*b^2*c*f - 4*I*(a^2*b - b^3)*d*e + 4
*I*(a^2*b - b^3)*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sin
h(d*x + c) + I) + (2*a*b^2*d*e - 2*a*b^2*c*f + (2*a*b^2*d*e - 2*a*b^2*c*f +
 I*(a^2*b - b^3)*d*e - I*(a^2*b - b^3)*c*f)*cosh(d*x + c)^4 + (8*a*b^2*d*e
- 8*a*b^2*c*f + 4*I*(a^2*b - b^3)*d*e - 4*I*(a^2*b - b^3)*c*f)*cosh(d*x + c
)*sinh(d*x + c)^3 + (2*a*b^2*d*e - 2*a*b^2*c*f + I*(a^2*b - b^3)*d*e - I*(a
^2*b - b^3)*c*f)*sinh(d*x + c)^4 + I*(a^2*b - b^3)*d*e - I*(a^2*b - b^3)*c*
f + (4*a*b^2*d*e - 4*a*b^2*c*f + 2*I*(a^2*b - b^3)*d*e - 2*I*(a^2*b - b^3)*
c*f)*cosh(d*x + c)^2 + (4*a*b^2*d*e - 4*a*b^2*c*f + 2*I*(a^2*b - b^3)*d*e -
 2*I*(a^2*b - b^3)*c*f + (12*a*b^2*d*e - 12*a*b^2*c*f + 6*I*(a^2*b - b^3)*d
*e - 6*I*(a^2*b - b^3)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a*b^2*d*
e - 8*a*b^2*c*f + 4*I*(a^2*b - b^3)*d*e - 4*I*(a^2*b - b^3)*c*f)*cosh(d*x +
 c)^3 + (8*a*b^2*d*e - 8*a*b^2*c*f + 4*I*(a^2*b - b^3)*d*e - 4*I*(a^2*b - b
^3)*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) -
I) + (2*a*b^2*d*f*x + 2*a*b^2*c*f + (2*a*b^2*d*f*x + 2*a*b^2*c*f + I*(a^2*b
 - b^3)*d*f*x + I*(a^2*b - b^3)*c*f)*cosh(d*x + c)^4 + (8*a*b^2*d*f*x + 8*a
*b^2*c*f + 4*I*(a^2*b - b^3)*d*f*x + 4*I*(a^2*b - b^3)*c*f)*cosh(d*x + c)*s
inh(d*x + c)^3 + (2*a*b^2*d*f*x + 2*a*b^2*c*f + I*(a^2*b - b^3)*d*f*x + I*(
a^2*b - b^3)*c*f)*sinh(d*x + c)^4 + I*(a^2*b - b^3)*d*f*x + I*(a^2*b - b^3)
*c*f + (4*a*b^2*d*f*x + 4*a*b^2*c*f + 2*I*(a^2*b - b^3)*d*f*x + 2*I*(a^2*b
- b^3)*c*f)*cosh(d*x + c)^2 + (4*a*b^2*d*f*x + 4*a*b^2*c*f + 2*I*(a^2*b - b
```

```
^3)*d*f*x + 2*I*(a^2*b - b^3)*c*f + (12*a*b^2*d*f*x + 12*a*b^2*c*f + 6*I*(a
^2*b - b^3)*d*f*x + 6*I*(a^2*b - b^3)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2
 + ((8*a*b^2*d*f*x + 8*a*b^2*c*f + 4*I*(a^2*b - b^3)*d*f*x + 4*I*(a^2*b - b
^3)*c*f)*cosh(d*x + c)^3 + (8*a*b^2*d*f*x + 8*a*b^2*c*f + 4*I*(a^2*b - b^3)
*d*f*x + 4*I*(a^2*b - b^3)*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(I*cosh(d*
x + c) + I*sinh(d*x + c) + 1) + (2*a*b^2*d*f*x + 2*a*b^2*c*f + (2*a*b^2*d*f
*x + 2*a*b^2*c*f - I*(a^2*b - b^3)*d*f*x - I*(a^2*b - b^3)*c*f)*cosh(d*x +
c)^4 + (8*a*b^2*d*f*x + 8*a*b^2*c*f - 4*I*(a^2*b - b^3)*d*f*x - 4*I*(a^2*b
- b^3)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b^2*d*f*x + 2*a*b^2*c*f -
I*(a^2*b - b^3)*d*f*x - I*(a^2*b - b^3)*c*f)*sinh(d*x + c)^4 - I*(a^2*b - b
^3)*d*f*x - I*(a^2*b - b^3)*c*f + (4*a*b^2*d*f*x + 4*a*b^2*c*f - 2*I*(a^2*b
 - b^3)*d*f*x - 2*I*(a^2*b - b^3)*c*f)*cosh(d*x + c)^2 + (4*a*b^2*d*f*x + 4
*a*b^2*c*f - 2*I*(a^2*b - b^3)*d*f*x - 2*I*(a^2*b - b^3)*c*f + (12*a*b^2*d*
f*x + 12*a*b^2*c*f - 6*I*(a^2*b - b^3)*d*f*x - 6*I*(a^2*b - b^3)*c*f)*cosh(
d*x + c)^2)*sinh(d*x + c)^2 + ((8*a*b^2*d*f*x + 8*a*b^2*c*f - 4*I*(a^2*b -
b^3)*d*f*x - 4*I*(a^2*b - b^3)*c*f)*cosh(d*x + c)^3 + (8*a*b^2*d*f*x + 8*a*
b^2*c*f - 4*I*(a^2*b - b^3)*d*f*x - 4*I*(a^2*b - b^3)*c*f)*cosh(d*x + c))*s
inh(d*x + c))*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1) - 2*((a^2*b + b^3
)*d*f*x + (a^2*b + b^3)*d*e - 3*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e +
(a^2*b + b^3)*f)*cosh(d*x + c)^2 - (a^2*b + b^3)*f + 2*(2*(a^3 + a*b^2)*d*f
*x + 2*(a^3 + a*b^2)*d*e + (a^3 + a*b^2)*f)*cosh(d*x + c))*sinh(d*x + c))/(
(a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d^2
*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d^2*sinh(d*x + c)^
4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)
*d^2 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2
+ b^4)*d^2)*sinh(d*x + c)^2 + 4*((a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(d*x + c)^
3 + (a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx) \tanh(c + dx) \operatorname{sech}^2(c + dx)}{a + b \sinh(c + dx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*tanh(c + d*x)*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \operatorname{sech}(dx + c)^2 \tanh(dx + c)}{b \sinh(dx + c) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate((f*x + e)*sech(d*x + c)^2*tanh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.360 $\int \frac{\mathrm{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=122

$$-\frac{ab^2\log(a+b\sinh(c+dx))}{d\left(a^2+b^2\right)^2}-\frac{b\left(a^2-b^2\right)\tan^{-1}(\sinh(c+dx))}{2d\left(a^2+b^2\right)^2}+\frac{ab^2\log(\cosh(c+dx))}{d\left(a^2+b^2\right)^2}-\frac{\mathrm{sech}^2(c+dx)(a-b\sinh(c}{2d\left(a^2+b^2\right)}$$

```
[Out] -(b*(a^2 - b^2)*ArcTan[Sinh[c + d*x]])/(2*(a^2 + b^2)^2*d) + (a*b^2*Log[Cos
h[c + d*x]])/((a^2 + b^2)^2*d) - (a*b^2*Log[a + b*Sinh[c + d*x]])/((a^2 + b
^2)^2*d) - (Sech[c + d*x]^2*(a - b*Sinh[c + d*x]))/(2*(a^2 + b^2)*d)
```

Rubi [A] time = 0.199132, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2837, 12, 823, 801, 635, 203, 260}

$$-\frac{ab^2\log(a+b\sinh(c+dx))}{d\left(a^2+b^2\right)^2}-\frac{b\left(a^2-b^2\right)\tan^{-1}(\sinh(c+dx))}{2d\left(a^2+b^2\right)^2}+\frac{ab^2\log(\cosh(c+dx))}{d\left(a^2+b^2\right)^2}-\frac{\mathrm{sech}^2(c+dx)(a-b\sinh(c}{2d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(b*(a^2 - b^2)*ArcTan[Sinh[c + d*x]])/(2*(a^2 + b^2)^2*d) + (a*b^2*Log[Cos
h[c + d*x]])/((a^2 + b^2)^2*d) - (a*b^2*Log[a + b*Sinh[c + d*x]])/((a^2 + b
^2)^2*d) - (Sech[c + d*x]^2*(a - b*Sinh[c + d*x]))/(2*(a^2 + b^2)*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b^3\operatorname{Subst}\left(\int \frac{x}{b(a+x)\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= \frac{b^2\operatorname{Subst}\left(\int \frac{x}{(a+x)\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2\left(a^2+b^2\right)d} + \frac{\operatorname{Subst}\left(\int \frac{ab^2-b^2x}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(c+dx)\right)}{2\left(a^2+b^2\right)d}$$

$$= -\frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2\left(a^2+b^2\right)d} + \frac{\operatorname{Subst}\left(\int \left(-\frac{2ab^2}{\left(a^2+b^2\right)(a+x)} + \frac{b^2\left(-a^2+b^2+2ax\right)}{\left(a^2+b^2\right)\left(b^2+x^2\right)}\right) dx, \ldots}{2\left(a^2+b^2\right)d}$$

$$= -\frac{ab^2\log(a+b\sinh(c+dx))}{\left(a^2+b^2\right)^2 d} - \frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2\left(a^2+b^2\right)d} + \frac{b^2\operatorname{Subst}\left(\int \frac{-a^2 \ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{ab^2\log(a+b\sinh(c+dx))}{\left(a^2+b^2\right)^2 d} - \frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2\left(a^2+b^2\right)d} + \frac{\left(ab^2\right)\operatorname{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{b\left(a^2-b^2\right)\tan^{-1}(\sinh(c+dx))}{2\left(a^2+b^2\right)^2 d} + \frac{ab^2\log(\cosh(c+dx))}{\left(a^2+b^2\right)^2 d} - \frac{ab^2\log(a+b\sinh(c+\ldots}{\left(a^2+b^2\right)^2 d}$$

Mathematica [A] time = 0.208201, size = 105, normalized size = 0.86

$$\frac{-a\left(a^2+b^2\right)\operatorname{sech}^2(c+dx) + b\left(a^2+b^2\right)\tanh(c+dx)\operatorname{sech}(c+dx) + 2b\left(\left(b^2-a^2\right)\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right) + ab(\log(\mathrm{co}\ldots}{2d\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*b*((-a^2 + b^2)*ArcTan[Tanh[(c + d*x)/2]] + a*b*(Log[Cosh[c + d*x]] - Lo
g[a + b*Sinh[c + d*x]])) - a*(a^2 + b^2)*Sech[c + d*x]^2 + b*(a^2 + b^2)*Se
ch[c + d*x]*Tanh[c + d*x])/(2*(a^2 + b^2)^2*d)
```

Maple [B] time = 0.003, size = 474, normalized size = 3.9

$$-2\frac{ab^2\ln\left((\tanh(1/2\,dx+c/2))^2a-2\tanh(1/2\,dx+c/2)\,b-a\right)}{d\left(2\,a^4+4\,a^2b^2+2\,b^4\right)}-\frac{a^2b}{d\left(a^4+2\,a^2b^2+b^4\right)}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tanh\left(\frac{dx}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] -2/d*a*b^2/(2*a^4+4*a^2*b^2+2*b^4)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*
x+1/2*c)*b-a)-1/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*
d*x+1/2*c)^3*a^2*b-1/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh
(1/2*d*x+1/2*c)^3*b^3+2/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^2+1)^2*t
anh(1/2*d*x+1/2*c)^2*a^3+2/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^2+1)^
2*tanh(1/2*d*x+1/2*c)^2*a*b^2+1/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^
2+1)^2*tanh(1/2*d*x+1/2*c)*a^2*b+1/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*
c)^2+1)^2*tanh(1/2*d*x+1/2*c)*b^3+1/d/(a^4+2*a^2*b^2+b^4)*ln(tanh(1/2*d*x+1
/2*c)^2+1)*a*b^2-1/d/(a^4+2*a^2*b^2+b^4)*arctan(tanh(1/2*d*x+1/2*c))*a^2*b+
1/d/(a^4+2*a^2*b^2+b^4)*arctan(tanh(1/2*d*x+1/2*c))*b^3
```

Maxima [A] time = 1.58781, size = 294, normalized size = 2.41

$$-\frac{ab^2\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{\left(a^4+2\,a^2b^2+b^4\right)d}+\frac{ab^2\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\left(a^4+2\,a^2b^2+b^4\right)d}+\frac{\left(a^2b-b^3\right)\arctan\left(e^{(-dx-c)}\right)}{\left(a^4+2\,a^2b^2+b^4\right)d}+\frac{be^{(-dx-c)}}{\left(a^2+b^2+2\left(a^2+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -a*b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 +
b^4)*d) + a*b^2*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) + (a^
2*b - b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (b*e^(-d*x -
c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)
*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d)
```

Fricas [B] time = 2.46903, size = 2240, normalized size = 18.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] ((a^2*b + b^3)*cosh(d*x + c)^3 + (a^2*b + b^3)*sinh(d*x + c)^3 - 2*(a^3 + a
*b^2)*cosh(d*x + c)^2 - (2*a^3 + 2*a*b^2 - 3*(a^2*b + b^3)*cosh(d*x + c))*s
```

```
inh(d*x + c)^2 - ((a^2*b - b^3)*cosh(d*x + c)^4 + 4*(a^2*b - b^3)*cosh(d*x
+ c)*sinh(d*x + c)^3 + (a^2*b - b^3)*sinh(d*x + c)^4 + a^2*b - b^3 + 2*(a^2
*b - b^3)*cosh(d*x + c)^2 + 2*(a^2*b - b^3 + 3*(a^2*b - b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + 4*((a^2*b - b^3)*cosh(d*x + c)^3 + (a^2*b - b^3)*cosh(
d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - (a^2*b + b
^3)*cosh(d*x + c) - (a*b^2*cosh(d*x + c)^4 + 4*a*b^2*cosh(d*x + c)*sinh(d*x
 + c)^3 + a*b^2*sinh(d*x + c)^4 + 2*a*b^2*cosh(d*x + c)^2 + a*b^2 + 2*(3*a*
b^2*cosh(d*x + c)^2 + a*b^2)*sinh(d*x + c)^2 + 4*(a*b^2*cosh(d*x + c)^3 + a
*b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x +
c) - sinh(d*x + c))) + (a*b^2*cosh(d*x + c)^4 + 4*a*b^2*cosh(d*x + c)*sinh(
d*x + c)^3 + a*b^2*sinh(d*x + c)^4 + 2*a*b^2*cosh(d*x + c)^2 + a*b^2 + 2*(3
*a*b^2*cosh(d*x + c)^2 + a*b^2)*sinh(d*x + c)^2 + 4*(a*b^2*cosh(d*x + c)^3
+ a*b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) -
sinh(d*x + c))) - (a^2*b + b^3 - 3*(a^2*b + b^3)*cosh(d*x + c)^2 + 4*(a^3 +
 a*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x +
 c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 +
2*a^2*b^2 + b^4)*d*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x +
 c)^2 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 +
 b^4)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d + 4*((a^4 + 2*a^2*b^2
+ b^4)*d*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c))*sinh(d*
x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [A] time = 1.35872, size = 304, normalized size = 2.49

$$\frac{\frac{ab^2\log\left(e^{(2dx+2c)}+1\right)}{a^4+2a^2b^2+b^4}-\frac{ab^2\log\left(\left|-be^{(2dx+2c)}-2ae^{(dx+c)}+b\right|\right)}{a^4+2a^2b^2+b^4}-\frac{\left(a^2be^c-b^3e^c\right)\arctan\left(e^{(dx+c)}\right)e^{(-c)}}{a^4+2a^2b^2+b^4}+\frac{\left(a^2be^{(3c)}+b^3e^{(3c)}\right)e^{(3dx)}-2\left(a^3e^{(2c)}+ab^2e^{(2c)}\right)e^{(2dx)}-\left(a^2be\right.}{\left(a^2+b^2\right)^2\left(e^{(2dx+2c)}+1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*b^2*log(e^(2*d*x + 2*c) + 1)/(a^4 + 2*a^2*b^2 + b^4) - a*b^2*log(abs(-b*
e^(2*d*x + 2*c) - 2*a*e^(d*x + c) + b))/(a^4 + 2*a^2*b^2 + b^4) - (a^2*b*e^
c - b^3*e^c)*arctan(e^(d*x + c))*e^(-c)/(a^4 + 2*a^2*b^2 + b^4) + ((a^2*b*e
^(3*c) + b^3*e^(3*c))*e^(3*d*x) - 2*(a^3*e^(2*c) + a*b^2*e^(2*c))*e^(2*d*x)
 - (a^2*b*e^c + b^3*e^c)*e^(d*x))/((a^2 + b^2)^2*(e^(2*d*x + 2*c) + 1)^2))/
d
```

3.361 $\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{\tanh(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Sech[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [A] time = 0.0856575, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sech[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Sech[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A] time = 142.055, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sech[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] Integrate[(Sech[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
, x]
```

Maple [A] time = 1.159, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{sech}(dx+c))^2\tanh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] (a*f + (b*d*f*x*e^(3*c) + (d*e - f)*b*e^(3*c))*e^(3*d*x) - (2*a*d*f*x*e^(2*
c) + (2*d*e - f)*a*e^(2*c))*e^(2*d*x) - (b*d*f*x*e^c + (d*e + f)*b*e^c)*e^(
d*x))/(a^2*d^2*e^2 + b^2*d^2*e^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*(a^2
*d^2*e*f + b^2*d^2*e*f)*x + (a^2*d^2*e^2*e^(4*c) + b^2*d^2*e^2*e^(4*c) + (a
^2*d^2*f^2*e^(4*c) + b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^2*d^2*e*f*e^(4*c) + b^
2*d^2*e*f*e^(4*c))*x)*e^(4*d*x) + 2*(a^2*d^2*e^2*e^(2*c) + b^2*d^2*e^2*e^(2
*c) + (a^2*d^2*f^2*e^(2*c) + b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^2*d^2*e*f*e^(2
*c) + b^2*d^2*e*f*e^(2*c))*x)*e^(2*d*x)) - 4*integrate(1/4*(2*a*b^2*d^2*f^2
*x^2 + 4*a*b^2*d^2*e*f*x - 2*a^3*f^2 + 2*(d^2*e^2 - f^2)*a*b^2 + ((d^2*e^2
+ 2*f^2)*a^2*b*e^c - (d^2*e^2 - 2*f^2)*b^3*e^c + (a^2*b*d^2*f^2*e^c - b^3*d
^2*f^2*e^c)*x^2 + 2*(a^2*b*d^2*e*f*e^c - b^3*d^2*e*f*e^c)*x)*e^(d*x))/(a^4*
d^2*e^3 + 2*a^2*b^2*d^2*e^3 + b^4*d^2*e^3 + (a^4*d^2*f^3 + 2*a^2*b^2*d^2*f^
3 + b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*e*f^2 + 2*a^2*b^2*d^2*e*f^2 + b^4*d^2*e*f
^2)*x^2 + 3*(a^4*d^2*e^2*f + 2*a^2*b^2*d^2*e^2*f + b^4*d^2*e^2*f)*x + (a^4*
d^2*e^3*e^(2*c) + 2*a^2*b^2*d^2*e^3*e^(2*c) + b^4*d^2*e^3*e^(2*c) + (a^4*d^
2*f^3*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(2*c) + b^4*d^2*f^3*e^(2*c))*x^3 + 3*(a
^4*d^2*e*f^2*e^(2*c) + 2*a^2*b^2*d^2*e*f^2*e^(2*c) + b^4*d^2*e*f^2*e^(2*c))
*x^2 + 3*(a^4*d^2*e^2*f*e^(2*c) + 2*a^2*b^2*d^2*e^2*f*e^(2*c) + b^4*d^2*e^2
*f*e^(2*c))*x)*e^(2*d*x)), x) + 4*integrate(-1/2*(a^2*b^2*e^(d*x + c) - a*b
^3)/(a^4*b*e + 2*a^2*b^3*e + b^5*e + (a^4*b*f + 2*a^2*b^3*f + b^5*f)*x - (a
^4*b*e*e^(2*c) + 2*a^2*b^3*e*e^(2*c) + b^5*e*e^(2*c) + (a^4*b*f*e^(2*c) + 2
*a^2*b^3*f*e^(2*c) + b^5*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^5*e*e^c + 2*a^3*b^2
*e*e^c + a*b^4*e*e^c + (a^5*f*e^c + 2*a^3*b^2*f*e^c + a*b^4*f*e^c)*x)*e^(d*
x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{sech}\,(dx+c)^2\tanh\,(dx+c)}{afx+ae+\left(bfx+be\right)\sinh\,(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(sech(d*x + c)^2*tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(c+dx)\operatorname{sech}^2(c+dx)}{(a+b\sinh(c+dx))\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)*sech(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)),
x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.362 $\int \frac{(e+fx)^3 \cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=606

$$-\frac{6a^2f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}-\frac{6a^2f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^3}+\frac{3a^2f(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{b^3d^2}$$

```
[Out] (3*f^3*x)/(8*b*d^3) + (e + f*x)^3/(4*b*d) - (a^2*(e + f*x)^4)/(4*b^3*f) + (
6*a*f^3*Cosh[c + d*x])/(b^2*d^4) + (3*a*f*(e + f*x)^2*Cosh[c + d*x])/(b^2*d
^2) + (a^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3
*d) + (a^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3
*d) + (3*a^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/(b^3*d^2) + (3*a^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/(b^3*d^2) - (6*a^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^3) - (6*a^2*f^2*(e + f*x)*PolyLog[3, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^3) + (6*a^2*f^3*PolyLog[4,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^4) + (6*a^2*f^3*PolyLog[
4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^4) - (6*a*f^2*(e + f*x
)*Sinh[c + d*x])/(b^2*d^3) - (a*(e + f*x)^3*Sinh[c + d*x])/(b^2*d) - (3*f^3
*Cosh[c + d*x]*Sinh[c + d*x])/(8*b*d^4) - (3*f*(e + f*x)^2*Cosh[c + d*x]*Si
nh[c + d*x])/(4*b*d^2) + (3*f^2*(e + f*x)*Sinh[c + d*x]^2)/(4*b*d^3) + ((e
+ f*x)^3*Sinh[c + d*x]^2)/(2*b*d)
```

Rubi [A] time = 0.879494, antiderivative size = 606, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 14, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5579, 5446, 3311, 32, 2635, 8, 3296, 2638, 5561, 2190, 2531, 6609, 2282, 6589}

$$-\frac{6a^2f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}-\frac{6a^2f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^3}+\frac{3a^2f(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{b^3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (3*f^3*x)/(8*b*d^3) + (e + f*x)^3/(4*b*d) - (a^2*(e + f*x)^4)/(4*b^3*f) + (
6*a*f^3*Cosh[c + d*x])/(b^2*d^4) + (3*a*f*(e + f*x)^2*Cosh[c + d*x])/(b^2*d
^2) + (a^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3
*d) + (a^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3
*d) + (3*a^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/(b^3*d^2) + (3*a^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/(b^3*d^2) - (6*a^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^3) - (6*a^2*f^2*(e + f*x)*PolyLog[3, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^3) + (6*a^2*f^3*PolyLog[4,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^4) + (6*a^2*f^3*PolyLog[
4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^4) - (6*a*f^2*(e + f*x
)*Sinh[c + d*x])/(b^2*d^3) - (a*(e + f*x)^3*Sinh[c + d*x])/(b^2*d) - (3*f^3
*Cosh[c + d*x]*Sinh[c + d*x])/(8*b*d^4) - (3*f*(e + f*x)^2*Cosh[c + d*x]*Si
nh[c + d*x])/(4*b*d^2) + (3*f^2*(e + f*x)*Sinh[c + d*x]^2)/(4*b*d^3) + ((e
+ f*x)^3*Sinh[c + d*x]^2)/(2*b*d)
```


Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
```

```
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}}{b}$$

$$= \frac{(e+fx)^3\sinh^2(c+dx)}{2bd} - \frac{a\int (e+fx)^3\cosh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^3\cosh}{a+b\sinh}}{b^2}$$

$$= -\frac{a^2(e+fx)^4}{4b^3f} - \frac{a(e+fx)^3\sinh(c+dx)}{b^2d} - \frac{3f(e+fx)^2\cosh(c+dx)\sinh}{4bd^2}$$

$$= \frac{(e+fx)^3}{4bd} - \frac{a^2(e+fx)^4}{4b^3f} + \frac{3af(e+fx)^2\cosh(c+dx)}{b^2d^2} + \frac{a^2(e+fx)^3\log}{b^3}$$

$$= \frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} - \frac{a^2(e+fx)^4}{4b^3f} + \frac{3af(e+fx)^2\cosh(c+dx)}{b^2d^2} + \frac{a^2(e+}{}$$

$$= \frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} - \frac{a^2(e+fx)^4}{4b^3f} + \frac{6af^3\cosh(c+dx)}{b^2d^4} + \frac{3af(e+fx)^2\,\mathrm{c}}{b^2d^2}$$

$$= \frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} - \frac{a^2(e+fx)^4}{4b^3f} + \frac{6af^3\cosh(c+dx)}{b^2d^4} + \frac{3af(e+fx)^2\,\mathrm{c}}{b^2d^2}$$

$$= \frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} - \frac{a^2(e+fx)^4}{4b^3f} + \frac{6af^3\cosh(c+dx)}{b^2d^4} + \frac{3af(e+fx)^2\,\mathrm{c}}{b^2d^2}$$

Mathematica [B] time = 28.4807, size = 2872, normalized size = 4.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] -(e^3*Log[a + b*Sinh[c + d*x]])/(4*b*d) - (3*e^2*f*(-x^2/(2*b) + (x*Log[1 +
 (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (x*Log[1 + (b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2])])/(b*d) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2
+ b^2])]/(b*d^2) + PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(b*
d^2)))/4 - (3*e*f^2*(-x^3/(3*b) + (x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2])])/(b*d) + (x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b
*d) + (2*x*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) +
(2*x*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2) - (2*Pol
yLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])])/(b*d^3) - (2*PolyLog[3, -(
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^3)))/4 - (f^3*(-x^4/(4*b) + (
x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (x^3*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (3*x^2*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (3*x^2*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))])/(b*d^2) - (6*x*PolyLog[3, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/(b*d^3) - (6*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2]))])/(b*d^3) + (6*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^
2])])/(b*d^4) + (6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b
*d^4)))/4 + (f^3*(((4*a^2 + b^2)*x^4*Cosh[c]*Csch[c/2]*Sech[c/2])/(8*b^3) -
 (4*a*Cosh[d*x]*(-6*Cosh[c] - 3*d^2*x^2*Cosh[c] + 6*d*x*Sinh[c] + d^3*x^3*S
inh[c]))/(b^2*d^4) + (Cosh[2*d*x]*(6*d*x*Cosh[2*c] + 4*d^3*x^3*Cosh[2*c] -
3*Sinh[2*c] - 6*d^2*x^2*Sinh[2*c]))/(4*b*d^4) - ((4*a^2 + b^2)*(x^4 - (2*b^
2*(d^3*x^3*Log[1 + ((a - Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/
```

```
b] - 3*d^2*x^2*PolyLog[2, ((-a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c +
 d*x]))/b] - 6*d*x*PolyLog[3, ((-a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh
[c + d*x]))/b] - 6*PolyLog[4, ((-a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh
[c + d*x]))/b])*(-1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 + b^2]*(-a + Sqrt[a
^2 + b^2])*d^4) - (2*b^2*(d^3*x^3*Log[1 + ((a + Sqrt[a^2 + b^2])*(Cosh[c +
d*x] - Sinh[c + d*x]))/b] - 3*d^2*x^2*PolyLog[2, -(((a + Sqrt[a^2 + b^2])*(
Cosh[c + d*x] - Sinh[c + d*x]))/b)] - 6*d*x*PolyLog[3, -(((a + Sqrt[a^2 + b
^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b)] - 6*PolyLog[4, -(((a + Sqrt[a^2 +
 b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b)])*(-1 + Cosh[2*c] + Sinh[2*c]))/
(Sqrt[a^2 + b^2]*(a + Sqrt[a^2 + b^2])*d^4) + (2*a*(d^3*x^3*Log[1 + (b*(Cos
h[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] + 3*d^2*x^2*PolyLog[2,
(b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])] - 6*d*x*PolyLog
[3, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])] + 6*PolyLog
[4, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])])*(-1 + Cosh
[2*c] + Sinh[2*c]))/(Sqrt[a^2 + b^2]*d^4) - (2*a*(d^3*x^3*Log[1 + (b*(Cosh[
c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])] + 3*d^2*x^2*PolyLog[2, -(
(b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))] - 6*d*x*PolyLog
[3, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))] + 6*PolyL
og[4, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))])*(-1 +
Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 + b^2]*d^4)))/(2*b^3*(-1 + Cosh[2*c] + Si
nh[2*c])) - (4*a*(6*d*x*Cosh[c] + d^3*x^3*Cosh[c] - 6*Sinh[c] - 3*d^2*x^2*S
inh[c])*Sinh[d*x])/(b^2*d^4) + ((-3*Cosh[2*c] - 6*d^2*x^2*Cosh[2*c] + 6*d*x
*Sinh[2*c] + 4*d^3*x^3*Sinh[2*c])*Sinh[2*d*x])/(4*b*d^4)))/4 + (e*f^2*(2*(4
*a^2 + b^2)*x^3*Coth[c] - (24*a*b*Cosh[d*x]*(-2*d*x*Cosh[c] + (2 + d^2*x^2)
*Sinh[c]))/d^3 + (3*b^2*Cosh[2*d*x]*((1 + 2*d^2*x^2)*Cosh[2*c] - 2*d*x*Sinh
[2*c]))/d^3 - (4*a^2 + b^2)*(-1 + Coth[c])*(2*x^3 + (6*a*(d^2*x^2*Log[1 + (
b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2
, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])] - 2*PolyLog[3
, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])])*Sinh[c]*(Cos
h[c] + Sinh[c]))/(Sqrt[a^2 + b^2]*d^3) - (3*b^2*(d^2*x^2*Log[1 + ((a - Sqrt
[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 2*d*x*PolyLog[2, ((-a +
Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 2*PolyLog[3, ((-a +
Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b])*(-1 + Cosh[2*c] + Sin
h[2*c]))/(Sqrt[a^2 + b^2]*(-a + Sqrt[a^2 + b^2])*d^3) - (3*b^2*(d^2*x^2*Log
[1 + ((a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 2*d*x*Pol
yLog[2, ((a + Sqrt[a^2 + b^2])*(-Cosh[c + d*x] + Sinh[c + d*x]))/b] - 2*Pol
yLog[3, ((a + Sqrt[a^2 + b^2])*(-Cosh[c + d*x] + Sinh[c + d*x]))/b])*(-1 +
Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 + b^2]*(a + Sqrt[a^2 + b^2])*d^3) - (3*a*
(d^2*x^2*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])]
 + 2*d*x*PolyLog[2, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b
^2]))] - 2*PolyLog[3, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 +
 b^2]))])*(-1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 + b^2]*d^3)) - (24*a*b*((
2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[c])*Sinh[d*x])/d^3 + (3*b^2*(-2*d*x*Cosh[
2*c] + (1 + 2*d^2*x^2)*Sinh[2*c])*Sinh[2*d*x])/d^3))/(8*b^3) + (e^3*((4*a^2
 + b^2)*Log[a + b*Sinh[c + d*x]] - 4*a*b*Sinh[c + d*x] + 2*b^2*Sinh[c + d*x
]^2))/(4*b^3*d) + (3*e^2*f*(8*a*b*Cosh[c + d*x] + 2*b^2*d*x*Cosh[2*(c + d*x
)] + 2*(4*a^2 + b^2)*(-(c + d*x)^2/2 + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])] + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])] - c*Log[a + b*Sinh[c + d*x]] + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a
^2 + b^2])] + PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) - 8*a*b
*d*x*Sinh[c + d*x] - b^2*Sinh[2*(c + d*x)]))/(8*b^3*d^2)
```

Maple [F] time = 0.216, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3}\cosh\left(dx+c\right)\left(\sinh\left(dx+c\right)\right)^{2}}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)

[Out] int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")

[Out] 1/8*e^3*(8*(d*x + c)*a^2/(b^3*d) - (4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(
b^2*d) + 8*a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d) + (4
*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d)) + 1/32*(8*a^2*d^4*f^3*x^4*e^
(2*c) + 32*a^2*d^4*e*f^2*x^3*e^(2*c) + 48*a^2*d^4*e^2*f*x^2*e^(2*c) + (4*b^
2*d^3*f^3*x^3*e^(4*c) + 6*(2*d^3*e*f^2 - d^2*f^3)*b^2*x^2*e^(4*c) + 6*(2*d^
3*e^2*f - 2*d^2*e*f^2 + d*f^3)*b^2*x*e^(4*c) - 3*(2*d^2*e^2*f - 2*d*e*f^2 +
 f^3)*b^2*e^(4*c))*e^(2*d*x) - 16*(a*b*d^3*f^3*x^3*e^(3*c) + 3*(d^3*e*f^2 -
 d^2*f^3)*a*b*x^2*e^(3*c) + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*a*b*x*e^(
3*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a*b*e^(3*c))*e^(d*x) + 16*(a*b*d^3
*f^3*x^3*e^c + 3*(d^3*e*f^2 + d^2*f^3)*a*b*x^2*e^c + 3*(d^3*e^2*f + 2*d^2*e
*f^2 + 2*d*f^3)*a*b*x*e^c + 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*a*b*e^c)*e^(-
d*x) + (4*b^2*d^3*f^3*x^3 + 6*(2*d^3*e*f^2 + d^2*f^3)*b^2*x^2 + 6*(2*d^3*e^
2*f + 2*d^2*e*f^2 + d*f^3)*b^2*x + 3*(2*d^2*e^2*f + 2*d*e*f^2 + f^3)*b^2)*e
^(-2*d*x))*e^(-2*c)/(b^3*d^4) - integrate(-2*(a^2*b*f^3*x^3 + 3*a^2*b*e*f^2
*x^2 + 3*a^2*b*e^2*f*x - (a^3*f^3*x^3*e^c + 3*a^3*e*f^2*x^2*e^c + 3*a^3*e^2
*f*x*e^c)*e^(d*x))/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x)
```

Fricas [C] time = 2.986, size = 8556, normalized size = 14.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")

[Out] 1/32*(4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 + 6*b^2*d^2*e^2*f + 6*b^2*d*e*f^2 +
 3*b^2*f^3 + (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f + 6*b^2*d
*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^2*d^3*e
^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*cosh(d*x + c)^4 + (4*b^2*d^3*f^3*x^3
 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f + 6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d
^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^2*d^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*
f^3)*x)*sinh(d*x + c)^4 - 16*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3 - 3*a*b*d^2*e^2
*f + 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(a
*b*d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*x + c)^3 - 4*(4*a*b
*d^3*f^3*x^3 + 4*a*b*d^3*e^3 - 12*a*b*d^2*e^2*f + 24*a*b*d*e*f^2 - 24*a*b*f
^3 + 12*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 12*(a*b*d^3*e^2*f - 2*a*b*d^2*e
*f^2 + 2*a*b*d*f^3)*x - (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*
f + 6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(
2*b^2*d^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*cosh(d*x + c))*sinh(d*x +
 c)^3 + 6*(2*b^2*d^3*e*f^2 + b^2*d^2*f^3)*x^2 - 8*(a^2*d^4*f^3*x^4 + 4*a^2*
```

d^4*e*f^2*x^3 + 6*a^2*d^4*e^2*f*x^2 + 4*a^2*d^4*e^3*x + 8*a^2*c*d^3*e^3 - 1
2*a^2*c^2*d^2*e^2*f + 8*a^2*c^3*d*e*f^2 - 2*a^2*c^4*f^3)*cosh(d*x + c)^2 -
2*(4*a^2*d^4*f^3*x^4 + 16*a^2*d^4*e*f^2*x^3 + 24*a^2*d^4*e^2*f*x^2 + 16*a^2
*d^4*e^3*x + 32*a^2*c*d^3*e^3 - 48*a^2*c^2*d^2*e^2*f + 32*a^2*c^3*d*e*f^2 -
 8*a^2*c^4*f^3 - 3*(4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f + 6
*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^2
*d^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*cosh(d*x + c)^2 + 24*(a*b*d^3*
f^3*x^3 + a*b*d^3*e^3 - 3*a*b*d^2*e^2*f + 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*
b*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(a*b*d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b
*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 6*(2*b^2*d^3*e^2*f + 2*b^2*d^2*
e*f^2 + b^2*d*f^3)*x + 16*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3 + 3*a*b*d^2*e^2*f
+ 6*a*b*d*e*f^2 + 6*a*b*f^3 + 3*(a*b*d^3*e*f^2 + a*b*d^2*f^3)*x^2 + 3*(a*b*
d^3*e^2*f + 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*x + c) + 96*((a^2*d^2*
f^3*x^2 + 2*a^2*d^2*e*f^2*x + a^2*d^2*e^2*f)*cosh(d*x + c)^2 + 2*(a^2*d^2*f
^3*x^2 + 2*a^2*d^2*e*f^2*x + a^2*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) + (
a^2*d^2*f^3*x^2 + 2*a^2*d^2*e*f^2*x + a^2*d^2*e^2*f)*sinh(d*x + c)^2)*dilog
((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 + b^2)/b^2) - b)/b + 1) + 96*((a^2*d^2*f^3*x^2 + 2*a^2*d^2*e*f^2*x
 + a^2*d^2*e^2*f)*cosh(d*x + c)^2 + 2*(a^2*d^2*f^3*x^2 + 2*a^2*d^2*e*f^2*x
+ a^2*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d^2*f^3*x^2 + 2*a^2*d^2
*e*f^2*x + a^2*d^2*e^2*f)*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(
d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b
 + 1) + 32*((a^2*d^3*e^3 - 3*a^2*c*d^2*e^2*f + 3*a^2*c^2*d*e*f^2 - a^2*c^3*
f^3)*cosh(d*x + c)^2 + 2*(a^2*d^3*e^3 - 3*a^2*c*d^2*e^2*f + 3*a^2*c^2*d*e*f
^2 - a^2*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d^3*e^3 - 3*a^2*c*d^2*
e^2*f + 3*a^2*c^2*d*e*f^2 - a^2*c^3*f^3)*sinh(d*x + c)^2)*log(2*b*cosh(d*x
+ c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 32*((a^2*d^3*
e^3 - 3*a^2*c*d^2*e^2*f + 3*a^2*c^2*d*e*f^2 - a^2*c^3*f^3)*cosh(d*x + c)^2
+ 2*(a^2*d^3*e^3 - 3*a^2*c*d^2*e^2*f + 3*a^2*c^2*d*e*f^2 - a^2*c^3*f^3)*cos
h(d*x + c)*sinh(d*x + c) + (a^2*d^3*e^3 - 3*a^2*c*d^2*e^2*f + 3*a^2*c^2*d*e
*f^2 - a^2*c^3*f^3)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
 c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 32*((a^2*d^3*f^3*x^3 + 3*a^2*d^3*e
*f^2*x^2 + 3*a^2*d^3*e^2*f*x + 3*a^2*c*d^2*e^2*f - 3*a^2*c^2*d*e*f^2 + a^2*
c^3*f^3)*cosh(d*x + c)^2 + 2*(a^2*d^3*f^3*x^3 + 3*a^2*d^3*e*f^2*x^2 + 3*a^2
*d^3*e^2*f*x + 3*a^2*c*d^2*e^2*f - 3*a^2*c^2*d*e*f^2 + a^2*c^3*f^3)*cosh(d*
x + c)*sinh(d*x + c) + (a^2*d^3*f^3*x^3 + 3*a^2*d^3*e*f^2*x^2 + 3*a^2*d^3*e
^2*f*x + 3*a^2*c*d^2*e^2*f - 3*a^2*c^2*d*e*f^2 + a^2*c^3*f^3)*sinh(d*x + c)
^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 32*((a^2*d^3*f^3*x^3 + 3*a^2*d^3*e*f
^2*x^2 + 3*a^2*d^3*e^2*f*x + 3*a^2*c*d^2*e^2*f - 3*a^2*c^2*d*e*f^2 + a^2*c^
3*f^3)*cosh(d*x + c)^2 + 2*(a^2*d^3*f^3*x^3 + 3*a^2*d^3*e*f^2*x^2 + 3*a^2*d
^3*e^2*f*x + 3*a^2*c*d^2*e^2*f - 3*a^2*c^2*d*e*f^2 + a^2*c^3*f^3)*cosh(d*x
+ c)*sinh(d*x + c) + (a^2*d^3*f^3*x^3 + 3*a^2*d^3*e*f^2*x^2 + 3*a^2*d^3*e^2
*f*x + 3*a^2*c*d^2*e^2*f - 3*a^2*c^2*d*e*f^2 + a^2*c^3*f^3)*sinh(d*x + c)^2
)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
 c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 192*(a^2*f^3*cosh(d*x + c)^2 + 2*a^2*f
^3*cosh(d*x + c)*sinh(d*x + c) + a^2*f^3*sinh(d*x + c)^2)*polylog(4, (a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2))/b) + 192*(a^2*f^3*cosh(d*x + c)^2 + 2*a^2*f^3*cosh(d*x + c)*
sinh(d*x + c) + a^2*f^3*sinh(d*x + c)^2)*polylog(4, (a*cosh(d*x + c) + a*si
nh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b)
 - 192*((a^2*d*f^3*x + a^2*d*e*f^2)*cosh(d*x + c)^2 + 2*(a^2*d*f^3*x + a^2*
d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d*f^3*x + a^2*d*e*f^2)*sinh(d*x
 + c)^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 192*((a^2*d*f^3*x + a^2*d*e*f
^2)*cosh(d*x + c)^2 + 2*(a^2*d*f^3*x + a^2*d*e*f^2)*cosh(d*x + c)*sinh(d*x
+ c) + (a^2*d*f^3*x + a^2*d*e*f^2)*sinh(d*x + c)^2)*polylog(3, (a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2))/b) + 4*(4*a*b*d^3*f^3*x^3 + 4*a*b*d^3*e^3 + 12*a*b*d^2*e^2*f + 24*

```
a*b*d*e*f^2 + 24*a*b*f^3 + (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e
^2*f + 6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 +
6*(2*b^2*d^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*cosh(d*x + c)^3 + 12*(
a*b*d^3*e*f^2 + a*b*d^2*f^3)*x^2 - 12*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3 - 3*a*
b*d^2*e^2*f + 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x
^2 + 3*(a*b*d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*x + c)^2 +
 12*(a*b*d^3*e^2*f + 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x - 4*(a^2*d^4*f^3*x^4
+ 4*a^2*d^4*e*f^2*x^3 + 6*a^2*d^4*e^2*f*x^2 + 4*a^2*d^4*e^3*x + 8*a^2*c*d^3
*e^3 - 12*a^2*c^2*d^2*e^2*f + 8*a^2*c^3*d*e*f^2 - 2*a^2*c^4*f^3)*cosh(d*x +
 c))*sinh(d*x + c))/(b^3*d^4*cosh(d*x + c)^2 + 2*b^3*d^4*cosh(d*x + c)*sinh
(d*x + c) + b^3*d^4*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3\cosh(dx+c)\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)*sinh(d*x + c)^2/(b*sinh(d*x + c) + a),
x)
```

3.363 $\int \frac{(e+fx)^2 \cosh(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal. Leaf size=449

$$\frac{2a^2 f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{2a^2 f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2}-\frac{2a^2 f^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}-\frac{2a^2f^2\text{I}}{}$$

```
[Out] (e*f*x)/(2*b*d) + (f^2*x^2)/(4*b*d) - (a^2*(e + f*x)^3)/(3*b^3*f) + (2*a*f*
(e + f*x)*Cosh[c + d*x])/(b^2*d^2) + (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2])])/(b^3*d) + (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2])])/(b^3*d) + (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^2) + (2*a^2*f*(e + f*x)*PolyLog[2
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^2) - (2*a^2*f^2*PolyLog
[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^3) - (2*a^2*f^2*PolyL
og[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^3) - (2*a*f^2*Sinh[
c + d*x])/(b^2*d^3) - (a*(e + f*x)^2*Sinh[c + d*x])/(b^2*d) - (f*(e + f*x)*
Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d^2) + (f^2*Sinh[c + d*x]^2)/(4*b*d^3) +
((e + f*x)^2*Sinh[c + d*x]^2)/(2*b*d)
```

Rubi [A] time = 0.716799, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5579, 5446, 3310, 3296, 2637, 5561, 2190, 2531, 2282, 6589}

$$\frac{2a^2 f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{2a^2 f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2}-\frac{2a^2 f^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}-\frac{2a^2f^2\text{I}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (e*f*x)/(2*b*d) + (f^2*x^2)/(4*b*d) - (a^2*(e + f*x)^3)/(3*b^3*f) + (2*a*f*
(e + f*x)*Cosh[c + d*x])/(b^2*d^2) + (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2])])/(b^3*d) + (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2])])/(b^3*d) + (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^2) + (2*a^2*f*(e + f*x)*PolyLog[2
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^2) - (2*a^2*f^2*PolyLog
[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^3) - (2*a^2*f^2*PolyL
og[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^3) - (2*a*f^2*Sinh[
c + d*x])/(b^2*d^3) - (a*(e + f*x)^2*Sinh[c + d*x])/(b^2*d) - (f*(e + f*x)*
Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d^2) + (f^2*Sinh[c + d*x]^2)/(4*b*d^3) +
((e + f*x)^2*Sinh[c + d*x]^2)/(2*b*d)
```


Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
```

```
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)^2\sinh^2(c+dx)}{2bd} - \frac{a\int (e+fx)^2\cosh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^2\cosh(c}{a+b\sinh(c+}}{b^2}$$

$$= -\frac{a^2(e+fx)^3}{3b^3f} - \frac{a(e+fx)^2\sinh(c+dx)}{b^2d} - \frac{f(e+fx)\cosh(c+dx)\sinh(c+}{2bd^2}$$

$$= \frac{efx}{2bd} + \frac{f^2x^2}{4bd} - \frac{a^2(e+fx)^3}{3b^3f} + \frac{2af(e+fx)\cosh(c+dx)}{b^2d^2} + \frac{a^2(e+fx)^2\log\Big(}{b^3d}$$

$$= \frac{efx}{2bd} + \frac{f^2x^2}{4bd} - \frac{a^2(e+fx)^3}{3b^3f} + \frac{2af(e+fx)\cosh(c+dx)}{b^2d^2} + \frac{a^2(e+fx)^2\log\Big(}{b^3d}$$

$$= \frac{efx}{2bd} + \frac{f^2x^2}{4bd} - \frac{a^2(e+fx)^3}{3b^3f} + \frac{2af(e+fx)\cosh(c+dx)}{b^2d^2} + \frac{a^2(e+fx)^2\log\Big(}{b^3d}$$

$$= \frac{efx}{2bd} + \frac{f^2x^2}{4bd} - \frac{a^2(e+fx)^3}{3b^3f} + \frac{2af(e+fx)\cosh(c+dx)}{b^2d^2} + \frac{a^2(e+fx)^2\log\Big(}{b^3d}$$

Mathematica [B] time = 11.766, size = 1496, normalized size = 3.33

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] -(e^2*Log[a + b*Sinh[c + d*x]])/(4*b*d) - (e*f*(-x^2/(2*b) + (x*Log[1 + (b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d) + (x*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])])/(b*d) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^
2])]/(b*d^2) + PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(b*d^2)
))/2 - (f^2*(-x^3/(3*b) + (x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
)])/(b*d) + (x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d) + (2
*x*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*d^2) + (2*x*Pol
yLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^2) - (2*PolyLog[3,
(b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])])/(b*d^3) - (2*PolyLog[3, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(b*d^3)))/4 + (f^2*(2*(4*a^2 + b^2)*x^3*Co
th[c] - (24*a*b*Cosh[d*x]*(-2*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c]))/d^3 + (
3*b^2*Cosh[2*d*x]*((1 + 2*d^2*x^2)*Cosh[2*c] - 2*d*x*Sinh[2*c]))/d^3 - (4*a
^2 + b^2)*(-1 + Coth[c])*(2*x^3 + (6*a*(d^2*x^2*Log[1 + (b*(Cosh[c + d*x] +
 Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, (b*(Cosh[c + d*x
] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])] - 2*PolyLog[3, (b*(Cosh[c + d*x
] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])])*Sinh[c]*(Cosh[c] + Sinh[c]))/(
Sqrt[a^2 + b^2]*d^3) - (3*b^2*(d^2*x^2*Log[1 + ((a - Sqrt[a^2 + b^2])*(Cosh
[c + d*x] - Sinh[c + d*x]))/b] - 2*d*x*PolyLog[2, ((-a + Sqrt[a^2 + b^2])*(
Cosh[c + d*x] - Sinh[c + d*x]))/b] - 2*PolyLog[3, ((-a + Sqrt[a^2 + b^2])*(
Cosh[c + d*x] - Sinh[c + d*x]))/b])*(-1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2
 + b^2]*(-a + Sqrt[a^2 + b^2])*d^3) - (3*b^2*(d^2*x^2*Log[1 + ((a + Sqrt[a^
```

```
2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 2*d*x*PolyLog[2, ((a + Sqrt
[a^2 + b^2])*(-Cosh[c + d*x] + Sinh[c + d*x]))/b] - 2*PolyLog[3, ((a + Sqrt
[a^2 + b^2])*(-Cosh[c + d*x] + Sinh[c + d*x]))/b])*(-1 + Cosh[2*c] + Sinh[2
*c]))/(Sqrt[a^2 + b^2]*(a + Sqrt[a^2 + b^2])*d^3) - (3*a*(d^2*x^2*Log[1 + (
b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2
, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))] - 2*PolyLog
[3, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))])*(-1 + Co
sh[2*c] + Sinh[2*c]))/(Sqrt[a^2 + b^2]*d^3)) - (24*a*b*((2 + d^2*x^2)*Cosh[
c] - 2*d*x*Sinh[c])*Sinh[d*x])/d^3 + (3*b^2*(-2*d*x*Cosh[2*c] + (1 + 2*d^2*
x^2)*Sinh[2*c])*Sinh[2*d*x])/d^3))/(24*b^3) + (e^2*((4*a^2 + b^2)*Log[a + b
*Sinh[c + d*x]] - 4*a*b*Sinh[c + d*x] + 2*b^2*Sinh[c + d*x]^2))/(4*b^3*d) +
 (e*f*(8*a*b*Cosh[c + d*x] + 2*b^2*d*x*Cosh[2*(c + d*x)] + 2*(4*a^2 + b^2)*
(-(c + d*x)^2/2 + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]
+ (c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - c*Log[a + b*Si
nh[c + d*x]] + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + PolyLog
[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) - 8*a*b*d*x*Sinh[c + d*x] -
b^2*Sinh[2*(c + d*x)]))/(4*b^3*d^2)
```

Maple [F] time = 0.302, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \cosh\left(dx+c\right) \left(\sinh\left(dx+c\right)\right)^{2}}{a+b\sinh\left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}e^{2}\left(\frac{8\,(dx+c)a^{2}}{b^{3}d}-\frac{\left(4\,ae^{(-dx-c)}-b\right)e^{(2\,dx+2\,c)}}{b^{2}d}+\frac{8\,a^{2}\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{b^{3}d}+\frac{4\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}}{b^{2}d}\right)+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] 1/8*e^2*(8*(d*x + c)*a^2/(b^3*d) - (4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(
b^2*d) + 8*a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d) + (4
*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d)) + 1/48*(16*a^2*d^3*f^2*x^3*e
^(2*c) + 48*a^2*d^3*e*f*x^2*e^(2*c) + 3*(2*b^2*d^2*f^2*x^2*e^(4*c) + 2*(2*d
^2*e*f - d*f^2)*b^2*x*e^(4*c) - (2*d*e*f - f^2)*b^2*e^(4*c))*e^(2*d*x) - 24
*(a*b*d^2*f^2*x^2*e^(3*c) + 2*(d^2*e*f - d*f^2)*a*b*x*e^(3*c) - 2*(d*e*f -
f^2)*a*b*e^(3*c))*e^(d*x) + 24*(a*b*d^2*f^2*x^2*e^c + 2*(d^2*e*f + d*f^2)*a
*b*x*e^c + 2*(d*e*f + f^2)*a*b*e^c)*e^(-d*x) + 3*(2*b^2*d^2*f^2*x^2 + 2*(2*
d^2*e*f + d*f^2)*b^2*x + (2*d*e*f + f^2)*b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^3
) - integrate(-2*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x - (a^3*f^2*x^2*e^c + 2*a^3*
e*f*x*e^c)*e^(d*x))/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x)
```

Fricas [C] time = 2.6464, size = 5513, normalized size = 12.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/48*(6*b^2*d^2*f^2*x^2 + 6*b^2*d^2*e^2 + 6*b^2*d*e*f + 3*(2*b^2*d^2*f^2*x^
2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x
)*cosh(d*x + c)^4 + 3*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^
2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*sinh(d*x + c)^4 + 3*b^2*f^2 - 24*(
a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f -
a*b*d*f^2)*x)*cosh(d*x + c)^3 - 12*(2*a*b*d^2*f^2*x^2 + 2*a*b*d^2*e^2 - 4*a
*b*d*e*f + 4*a*b*f^2 + 4*(a*b*d^2*e*f - a*b*d*f^2)*x - (2*b^2*d^2*f^2*x^2 +
 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*c
osh(d*x + c))*sinh(d*x + c)^3 - 16*(a^2*d^3*f^2*x^3 + 3*a^2*d^3*e*f*x^2 + 3
*a^2*d^3*e^2*x + 6*a^2*c*d^2*e^2 - 6*a^2*c^2*d*e*f + 2*a^2*c^3*f^2)*cosh(d*
x + c)^2 - 2*(8*a^2*d^3*f^2*x^3 + 24*a^2*d^3*e*f*x^2 + 24*a^2*d^3*e^2*x + 4
8*a^2*c*d^2*e^2 - 48*a^2*c^2*d*e*f + 16*a^2*c^3*f^2 - 9*(2*b^2*d^2*f^2*x^2
+ 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*
cosh(d*x + c)^2 + 36*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f
^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 6*(2*b
^2*d^2*e*f + b^2*d*f^2)*x + 24*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 + 2*a*b*d*e*f
 + 2*a*b*f^2 + 2*(a*b*d^2*e*f + a*b*d*f^2)*x)*cosh(d*x + c) + 96*((a^2*d*f^
2*x + a^2*d*e*f)*cosh(d*x + c)^2 + 2*(a^2*d*f^2*x + a^2*d*e*f)*cosh(d*x + c
)*sinh(d*x + c) + (a^2*d*f^2*x + a^2*d*e*f)*sinh(d*x + c)^2)*dilog((a*cosh(
d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b + 1) + 96*((a^2*d*f^2*x + a^2*d*e*f)*cosh(d*x + c)^2 + 2
*(a^2*d*f^2*x + a^2*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d*f^2*x + a^2
*d*e*f)*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 48*((a^2*d
^2*e^2 - 2*a^2*c*d*e*f + a^2*c^2*f^2)*cosh(d*x + c)^2 + 2*(a^2*d^2*e^2 - 2*
a^2*c*d*e*f + a^2*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d^2*e^2 - 2*a
^2*c*d*e*f + a^2*c^2*f^2)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh
(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 48*((a^2*d^2*e^2 - 2*a^2*c*d
*e*f + a^2*c^2*f^2)*cosh(d*x + c)^2 + 2*(a^2*d^2*e^2 - 2*a^2*c*d*e*f + a^2*
c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d^2*e^2 - 2*a^2*c*d*e*f + a^2*c
^2*f^2)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sq
rt((a^2 + b^2)/b^2) + 2*a) + 48*((a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + 2*a^2
*c*d*e*f - a^2*c^2*f^2)*cosh(d*x + c)^2 + 2*(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*
f*x + 2*a^2*c*d*e*f - a^2*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d^2*f
^2*x^2 + 2*a^2*d^2*e*f*x + 2*a^2*c*d*e*f - a^2*c^2*f^2)*sinh(d*x + c)^2)*lo
g(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2) - b)/b) + 48*((a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + 2
*a^2*c*d*e*f - a^2*c^2*f^2)*cosh(d*x + c)^2 + 2*(a^2*d^2*f^2*x^2 + 2*a^2*d^
2*e*f*x + 2*a^2*c*d*e*f - a^2*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d
^2*f^2*x^2 + 2*a^2*d^2*e*f*x + 2*a^2*c*d*e*f - a^2*c^2*f^2)*sinh(d*x + c)^2
)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
 c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 96*(a^2*f^2*cosh(d*x + c)^2 + 2*a^2*f^
2*cosh(d*x + c)*sinh(d*x + c) + a^2*f^2*sinh(d*x + c)^2)*polylog(3, (a*cosh
(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2))/b) - 96*(a^2*f^2*cosh(d*x + c)^2 + 2*a^2*f^2*cosh(d*x + c)*si
nh(d*x + c) + a^2*f^2*sinh(d*x + c)^2)*polylog(3, (a*cosh(d*x + c) + a*sinh
(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) +
 4*(6*a*b*d^2*f^2*x^2 + 6*a*b*d^2*e^2 + 12*a*b*d*e*f + 12*a*b*f^2 + 3*(2*b^
2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f -
b^2*d*f^2)*x)*cosh(d*x + c)^3 - 18*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d
```

```
*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*x)*cosh(d*x + c)^2 + 12*(a*b
*d^2*e*f + a*b*d*f^2)*x - 8*(a^2*d^3*f^2*x^3 + 3*a^2*d^3*e*f*x^2 + 3*a^2*d^
3*e^2*x + 6*a^2*c*d^2*e^2 - 6*a^2*c^2*d*e*f + 2*a^2*c^3*f^2)*cosh(d*x + c))
*sinh(d*x + c))/(b^3*d^3*cosh(d*x + c)^2 + 2*b^3*d^3*cosh(d*x + c)*sinh(d*x
 + c) + b^3*d^3*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \cosh(dx+c)\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)*sinh(d*x + c)^2/(b*sinh(d*x + c) + a),
x)
```

3.364 $\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=278

$$\frac{a^2f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{a^2f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2}+\frac{a^2(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3d}+\frac{a^2(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+\right.}{b^3d}$$

```
[Out] (f*x)/(4*b*d) - (a^2*(e + f*x)^2)/(2*b^3*f) + (a*f*Cosh[c + d*x])/(b^2*d^2)
 + (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) +
 (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*d) + (
a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^2) + (a^
2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^2) - (a*(e
 + f*x)*Sinh[c + d*x])/(b^2*d) - (f*Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^2)
+ ((e + f*x)*Sinh[c + d*x]^2)/(2*b*d)
```

Rubi [A] time = 0.418473, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5579, 5446, 2635, 8, 3296, 2638, 5561, 2190, 2279, 2391}

$$\frac{a^2f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2}+\frac{a^2f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2}+\frac{a^2(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3d}+\frac{a^2(e+fx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+\right.}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (f*x)/(4*b*d) - (a^2*(e + f*x)^2)/(2*b^3*f) + (a*f*Cosh[c + d*x])/(b^2*d^2)
 + (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) +
 (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*d) + (
a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^2) + (a^
2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^2) - (a*(e
 + f*x)*Sinh[c + d*x])/(b^2*d) - (f*Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^2)
+ ((e + f*x)*Sinh[c + d*x]^2)/(2*b*d)
```

Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh(c+dx)\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)\sinh^2(c+dx)}{2bd} - \frac{a\int (e+fx)\cosh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}}{b^2}$$

$$= -\frac{a^2(e+fx)^2}{2b^3f} - \frac{a(e+fx)\sinh(c+dx)}{b^2d} - \frac{f\cosh(c+dx)\sinh(c+dx)}{4bd^2} + (e+$$

$$= \frac{fx}{4bd} - \frac{a^2(e+fx)^2}{2b^3f} + \frac{af\cosh(c+dx)}{b^2d^2} + \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d} + a^2($$

$$= \frac{fx}{4bd} - \frac{a^2(e+fx)^2}{2b^3f} + \frac{af\cosh(c+dx)}{b^2d^2} + \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d} + a^2($$

$$= \frac{fx}{4bd} - \frac{a^2(e+fx)^2}{2b^3f} + \frac{af\cosh(c+dx)}{b^2d^2} + \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d} + a^2($$

Mathematica [A] time = 1.08718, size = 423, normalized size = 1.52

$$b^2f\left(-2\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)-2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+dx\left(-2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)-2\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)+dx\right)\right)+f$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (-2*b^2*d*e*Log[a + b*Sinh[c + d*x]] + b^2*f*(d*x*(d*x - 2*Log[1 + (b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2])] - 2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2])]) - 2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*PolyLog[
2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) + 2*d*e*((4*a^2 + b^2)*Log[a
+ b*Sinh[c + d*x]] - 4*a*b*Sinh[c + d*x] + 2*b^2*Sinh[c + d*x]^2) + f*(8*a*
b*Cosh[c + d*x] + 2*b^2*d*x*Cosh[2*(c + d*x)] + 2*(4*a^2 + b^2)*(-(c + d*x)
^2/2 + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + (c + d*x)
*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - c*Log[a + b*Sinh[c + d*x]
] + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))]) - 8*a*b*d*x*Sinh[c + d*x] - b^2*Sinh[2*
(c + d*x)]))/(8*b^3*d^2)
```

Maple [B] time = 0.116, size = 565, normalized size = 2.

$$-\frac{a^2fx^2}{2b^3}+\frac{a^2ex}{b^3}+\frac{\left(2dfx+2de-f\right)e^{2dx+2c}}{16d^2b}-\frac{a\left(dfx+de-f\right)e^{dx+c}}{2b^2d^2}+\frac{a\left(dfx+de+f\right)e^{-dx-c}}{2b^2d^2}+\frac{\left(2dfx+2de+f\right)e^{-}}{16d^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/2*a^2*f*x^2/b^3+a^2*e*x/b^3+1/16*(2*d*f*x+2*d*e-f)/d^2/b*exp(2*d*x+2*c)-
1/2*a*(d*f*x+d*e-f)/b^2/d^2*exp(d*x+c)+1/2*a*(d*f*x+d*e+f)/b^2/d^2*exp(-d*x
-c)+1/16*(2*d*f*x+2*d*e+f)/d^2/b*exp(-2*d*x-2*c)-1/b^3/d^2*a^2*f*c*ln(b*exp
```

```
(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/b^3/d^2*a^2*f*c*ln(exp(d*x+c))+1/b^3/d*a^2*
e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/b^3/d*a^2*e*ln(exp(d*x+c))+1/b^3/
d*a^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/b^3/
d^2*a^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/b^
3/d*a^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/b^3/
d^2*a^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/b^3/
d^2*a^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/b^3
/d^2*a^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2/
b^3/d*a^2*f*c*x-1/b^3/d^2*a^2*f*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}e\left(\frac{8\,(dx+c)a^2}{b^3d}-\frac{\left(4\,ae^{(-dx-c)}-b\right)e^{(2\,dx+2\,c)}}{b^2d}+\frac{8\,a^2\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{b^3d}+\frac{4\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}}{b^2d}\right)+\frac{1}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] 1/8*e*(8*(d*x + c)*a^2/(b^3*d) - (4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^
2*d) + 8*a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d) + (4*a
*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d)) + 1/16*f*((8*a^2*d^2*x^2*e^(2*
c) + (2*b^2*d*x*e^(4*c) - b^2*e^(4*c))*e^(2*d*x) - 8*(a*b*d*x*e^(3*c) - a*b
*e^(3*c))*e^(d*x) + 8*(a*b*d*x*e^c + a*b*e^c)*e^(-d*x) + (2*b^2*d*x + b^2)*
e^(-2*d*x))*e^(-2*c)/(b^3*d^2) - 2*integrate(16*(a^3*x*e^(d*x + c) - a^2*b*
x)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x))
```

Fricas [B] time = 2.38726, size = 3054, normalized size = 10.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/16*(2*b^2*d*f*x + (2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^4 + (2*
b^2*d*f*x + 2*b^2*d*e - b^2*f)*sinh(d*x + c)^4 + 2*b^2*d*e - 8*(a*b*d*f*x +
 a*b*d*e - a*b*f)*cosh(d*x + c)^3 - 4*(2*a*b*d*f*x + 2*a*b*d*e - 2*a*b*f -
(2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c))*sinh(d*x + c)^3 + b^2*f -
8*(a^2*d^2*f*x^2 + 2*a^2*d^2*e*x + 4*a^2*c*d*e - 2*a^2*c^2*f)*cosh(d*x + c)
^2 - 2*(4*a^2*d^2*f*x^2 + 8*a^2*d^2*e*x + 16*a^2*c*d*e - 8*a^2*c^2*f - 3*(2
*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^2 + 12*(a*b*d*f*x + a*b*d*e -
 a*b*f)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*(a*b*d*f*x + a*b*d*e + a*b*f)*co
sh(d*x + c) + 16*(a^2*f*cosh(d*x + c)^2 + 2*a^2*f*cosh(d*x + c)*sinh(d*x +
c) + a^2*f*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 16*(a^2
*f*cosh(d*x + c)^2 + 2*a^2*f*cosh(d*x + c)*sinh(d*x + c) + a^2*f*sinh(d*x +
 c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 16*((a^2*d*e - a^2*c*f)*cosh
(d*x + c)^2 + 2*(a^2*d*e - a^2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d*e
- a^2*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b
*sqrt((a^2 + b^2)/b^2) + 2*a) + 16*((a^2*d*e - a^2*c*f)*cosh(d*x + c)^2 + 2
```

```
*(a^2*d*e - a^2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d*e - a^2*c*f)*sinh
(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^
2)/b^2) + 2*a) + 16*((a^2*d*f*x + a^2*c*f)*cosh(d*x + c)^2 + 2*(a^2*d*f*x +
 a^2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d*f*x + a^2*c*f)*sinh(d*x + c)
^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 16*((a^2*d*f*x + a^2*c*f)*cosh(d*x +
 c)^2 + 2*(a^2*d*f*x + a^2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d*f*x +
a^2*c*f)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*(2*a*b*d*f*x
 + 2*a*b*d*e + (2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^3 + 2*a*b*f
- 6*(a*b*d*f*x + a*b*d*e - a*b*f)*cosh(d*x + c)^2 - 4*(a^2*d^2*f*x^2 + 2*a^
2*d^2*e*x + 4*a^2*c*d*e - 2*a^2*c^2*f)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d
^2*cosh(d*x + c)^2 + 2*b^3*d^2*cosh(d*x + c)*sinh(d*x + c) + b^3*d^2*sinh(d
*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\cosh(dx+c)\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"giac")

[Out] integrate((f*x + e)*cosh(d*x + c)*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

3.365 $\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{a^2\log(a+b\sinh(c+dx))}{b^3d}-\frac{a\sinh(c+dx)}{b^2d}+\frac{\sinh^2(c+dx)}{2bd}$$

```
[Out] (a^2*Log[a + b*Sinh[c + d*x]])/(b^3*d) - (a*Sinh[c + d*x])/(b^2*d) + Sinh[c
 + d*x]^2/(2*b*d)
```

Rubi [A] time = 0.0808121, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{a^2\log(a+b\sinh(c+dx))}{b^3d}-\frac{a\sinh(c+dx)}{b^2d}+\frac{\sinh^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*Log[a + b*Sinh[c + d*x]])/(b^3*d) - (a*Sinh[c + d*x])/(b^2*d) + Sinh[c
 + d*x]^2/(2*b*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{b^2(a+x)}\,dx, x, b\sinh(c+dx)\right)}{bd}\\
&= \frac{\text{Subst}\left(\int \frac{x^2}{a+x}\,dx, x, b\sinh(c+dx)\right)}{b^3d}\\
&= \frac{\text{Subst}\left(\int \left(-a+x+\frac{a^2}{a+x}\right)dx, x, b\sinh(c+dx)\right)}{b^3d}\\
&= \frac{a^2\log(a+b\sinh(c+dx))}{b^3d}-\frac{a\sinh(c+dx)}{b^2d}+\frac{\sinh^2(c+dx)}{2bd}
\end{aligned}$$

Mathematica [A] time = 0.0711854, size = 49, normalized size = 0.89

$$\frac{2a^2\log(a+b\sinh(c+dx))-2ab\sinh(c+dx)+b^2\sinh^2(c+dx)}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a^2*Log[a + b*Sinh[c + d*x]] - 2*a*b*Sinh[c + d*x] + b^2*Sinh[c + d*x]^2
)/(2*b^3*d)
```

Maple [A] time = 0.017, size = 54, normalized size = 1.

$$\frac{a^2\ln(a+b\sinh(dx+c))}{b^3d}-\frac{a\sinh(dx+c)}{b^2d}+\frac{(\sinh(dx+c))^2}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] a^2*ln(a+b*sinh(d*x+c))/b^3/d-a*sinh(d*x+c)/b^2/d+1/2*sinh(d*x+c)^2/b/d
```

Maxima [B] time = 1.05146, size = 161, normalized size = 2.93

$$\frac{(dx+c)a^2}{b^3d}-\frac{\left(4\,ae^{(-dx-c)}-b\right)e^{(2\,dx+2\,c)}}{8\,b^2d}+\frac{a^2\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{b^3d}+\frac{4\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}}{8\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] (d*x + c)*a^2/(b^3*d) - 1/8*(4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d)
+ a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d) + 1/8*(4*a*e^
(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d)
```

Fricas [B] time = 2.21071, size = 784, normalized size = 14.25

$$-\frac{8\,a^2dx\cosh(dx+c)^2-b^2\cosh(dx+c)^4-b^2\sinh(dx+c)^4+4\,ab\cosh(dx+c)^3-4\left(b^2\cosh(dx+c)-ab\right)\sinh(dx+\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/8*(8*a^2*d*x*cosh(d*x + c)^2 - b^2*cosh(d*x + c)^4 - b^2*sinh(d*x + c)^4
 + 4*a*b*cosh(d*x + c)^3 - 4*(b^2*cosh(d*x + c) - a*b)*sinh(d*x + c)^3 - 4*
a*b*cosh(d*x + c) + 2*(4*a^2*d*x - 3*b^2*cosh(d*x + c)^2 + 6*a*b*cosh(d*x +
 c))*sinh(d*x + c)^2 - b^2 - 8*(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*s
```

```
inh(d*x + c) + a^2*sinh(d*x + c)^2)*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x +
 c) - sinh(d*x + c))) + 4*(4*a^2*d*x*cosh(d*x + c) - b^2*cosh(d*x + c)^3 +
3*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^2 + 2*b^3*
d*cosh(d*x + c)*sinh(d*x + c) + b^3*d*sinh(d*x + c)^2)
```

Sympy [A] time = 3.59674, size = 87, normalized size = 1.58

$$\begin{cases} \frac{x\sinh^2(c)\cosh(c)}{a} & \text{for } b = 0 \wedge d = 0 \\ \frac{\sinh^3(c+dx)}{3ad} & \text{for } b = 0 \\ \frac{x\sinh^2(c)\cosh(c)}{a+b\sinh(c)} & \text{for } d = 0 \\ \frac{a^2\log\left(\frac{a}{b}+\sinh(c+dx)\right)}{b^3d} - \frac{a\sinh(c+dx)}{b^2d} + \frac{\sinh^2(c+dx)}{2bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Piecewise((x*sinh(c)**2*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)**3/
(3*a*d), Eq(b, 0)), (x*sinh(c)**2*cosh(c)/(a + b*sinh(c)), Eq(d, 0)), (a**2
*log(a/b + sinh(c + d*x))/(b**3*d) - a*sinh(c + d*x)/(b**2*d) + sinh(c + d*
x)**2/(2*b*d), True))
```

Giac [A] time = 1.29886, size = 123, normalized size = 2.24

$$\frac{a^2\log\left(\left|b\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 2a\right|\right)}{b^3d} + \frac{bd\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 - 4ad\left(e^{(dx+c)} - e^{(-dx-c)}\right)}{8b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] a^2*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(b^3*d) + 1/8*(b*d*(e^(d
*x + c) - e^(-d*x - c))^2 - 4*a*d*(e^(d*x + c) - e^(-d*x - c)))/(b^2*d^2)
```

3.366 $\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{\sinh^2(c+dx)\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Cosh[c + d*x]*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [A] time = 0.0869597, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[c + d*x]*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Cosh[c + d*x]*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.002, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[c + d*x]*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.26, size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)(\sinh(dx+c))^2}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{e^{\left(-2c+\frac{2de}{f}\right)}E_1\left(\frac{2(fx+e)d}{f}\right)}{4bf}+\frac{ae^{\left(-c+\frac{de}{f}\right)}E_1\left(\frac{(fx+e)d}{f}\right)}{2b^2f}+\frac{ae^{\left(c-\frac{de}{f}\right)}E_1\left(-\frac{(fx+e)d}{f}\right)}{2b^2f}-\frac{e^{\left(2c-\frac{2de}{f}\right)}E_1\left(-\frac{2(fx+e)d}{f}\right)}{4bf}+\frac{a^2\log\left(fx+e\right)}{b^3f}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] 1/4*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b*f) + 1/2*a*e^(
-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^2*f) + 1/2*a*e^(c - d*e/f)*
exp_integral_e(1, -(f*x + e)*d/f)/(b^2*f) - 1/4*e^(2*c - 2*d*e/f)*exp_integ
ral_e(1, -2*(f*x + e)*d/f)/(b*f) + a^2*log(f*x + e)/(b^3*f) - 1/8*integrate
(-16*(a^3*e^(d*x + c) - a^2*b)/(b^4*f*x + b^4*e - (b^4*f*x*e^(2*c) + b^4*e*
e^(2*c))*e^(2*d*x) - 2*(a*b^3*f*x*e^c + a*b^3*e*e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2}{afx+ae+\left(bfx+be\right)\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(cosh(d*x + c)*sinh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2}{\left(fx+e\right)\left(b\sinh\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate(cosh(d*x + c)*sinh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a)),
x)
```

3.367 $\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal. Leaf size=897

$$-\frac{a(e+fx)^4}{8b^2f} - \frac{a^3(e+fx)^4}{4b^4f} + \frac{\cosh^3(c+dx)(e+fx)^3}{3bd} + \frac{a^2\cosh(c+dx)(e+fx)^3}{b^3d} + \frac{a^2\sqrt{a^2+b^2}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)(e+}{b^4d}$$

```
[Out] (-3*a*e*f^2*x)/(4*b^2*d^2) - (3*a*f^3*x^2)/(8*b^2*d^2) - (a^3*(e + f*x)^4)/
(4*b^4*f) - (a*(e + f*x)^4)/(8*b^2*f) + (6*a^2*f^2*(e + f*x)*Cosh[c + d*x])
/(b^3*d^3) + (4*f^2*(e + f*x)*Cosh[c + d*x])/(3*b*d^3) + (a^2*(e + f*x)^3*C
osh[c + d*x])/(b^3*d) + (3*a*f^3*Cosh[c + d*x]^2)/(8*b^2*d^4) + (3*a*f*(e +
 f*x)^2*Cosh[c + d*x]^2)/(4*b^2*d^2) + (2*f^2*(e + f*x)*Cosh[c + d*x]^3)/(9
*b*d^3) + ((e + f*x)^3*Cosh[c + d*x]^3)/(3*b*d) + (a^2*Sqrt[a^2 + b^2]*(e +
 f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a^2*Sqrt
[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^
4*d) + (3*a^2*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2]))])/(b^4*d^2) - (3*a^2*Sqrt[a^2 + b^2]*f*(e + f*x)^2*Pol
yLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^2) - (6*a^2*Sqrt[
a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
)])/(b^4*d^3) + (6*a^2*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^3) + (6*a^2*Sqrt[a^2 + b^2]*f^3*Poly
Log[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^4) - (6*a^2*Sqrt[a
^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^
4) - (6*a^2*f^3*Sinh[c + d*x])/(b^3*d^4) - (14*f^3*Sinh[c + d*x])/(9*b*d^4)
 - (3*a^2*f*(e + f*x)^2*Sinh[c + d*x])/(b^3*d^2) - (2*f*(e + f*x)^2*Sinh[c
+ d*x])/(3*b*d^2) - (3*a*f^2*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^2*
d^3) - (a*(e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^2*d) - (f*(e + f*x)
^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b*d^2) - (2*f^3*Sinh[c + d*x]^3)/(27*b
*d^4)
```

Rubi [A] time = 1.47301, antiderivative size = 897, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 16, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5579, 5447, 3311, 3296, 2637, 2633, 32, 3310, 5565, 3322, 2264, 2190, 2531, 6609, 2282, 6589}

$$-\frac{a(e+fx)^4}{8b^2f} - \frac{a^3(e+fx)^4}{4b^4f} + \frac{\cosh^3(c+dx)(e+fx)^3}{3bd} + \frac{a^2\cosh(c+dx)(e+fx)^3}{b^3d} + \frac{a^2\sqrt{a^2+b^2}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)(e+}{b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-3*a*e*f^2*x)/(4*b^2*d^2) - (3*a*f^3*x^2)/(8*b^2*d^2) - (a^3*(e + f*x)^4)/
(4*b^4*f) - (a*(e + f*x)^4)/(8*b^2*f) + (6*a^2*f^2*(e + f*x)*Cosh[c + d*x])
/(b^3*d^3) + (4*f^2*(e + f*x)*Cosh[c + d*x])/(3*b*d^3) + (a^2*(e + f*x)^3*C
osh[c + d*x])/(b^3*d) + (3*a*f^3*Cosh[c + d*x]^2)/(8*b^2*d^4) + (3*a*f*(e +
 f*x)^2*Cosh[c + d*x]^2)/(4*b^2*d^2) + (2*f^2*(e + f*x)*Cosh[c + d*x]^3)/(9
*b*d^3) + ((e + f*x)^3*Cosh[c + d*x]^3)/(3*b*d) + (a^2*Sqrt[a^2 + b^2]*(e +
 f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a^2*Sqrt
[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^
4*d) + (3*a^2*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2]))])/(b^4*d^2) - (3*a^2*Sqrt[a^2 + b^2]*f*(e + f*x)^2*Pol
yLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^2) - (6*a^2*Sqrt[
a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
)])/(b^4*d^3) + (6*a^2*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c +
```

```
 d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^3) + (6*a^2*Sqrt[a^2 + b^2]*f^3*Poly
Log[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^4) - (6*a^2*Sqrt[a
^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^
4) - (6*a^2*f^3*Sinh[c + d*x])/(b^3*d^4) - (14*f^3*Sinh[c + d*x])/(9*b*d^4)
 - (3*a^2*f*(e + f*x)^2*Sinh[c + d*x])/(b^3*d^2) - (2*f*(e + f*x)^2*Sinh[c
+ d*x])/(3*b*d^2) - (3*a*f^2*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^2*
d^3) - (a*(e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^2*d) - (f*(e + f*x)
^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b*d^2) - (2*f^3*Sinh[c + d*x]^3)/(27*b
*d^4)
```

Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\cosh^2(c+dx)\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}}{b}$$

$$= \frac{(e+fx)^3\cosh^3(c+dx)}{3bd} - \frac{a\int (e+fx)^3\cosh^2(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^3\cos}{a+b\sinh(}}{b^2}$$

$$= \frac{3af(e+fx)^2\cosh^2(c+dx)}{4b^2d^2} + \frac{2f^2(e+fx)\cosh^3(c+dx)}{9bd^3} + \frac{(e+fx)^3\cosh}{3bd}$$

$$= -\frac{a^3(e+fx)^4}{4b^4f} - \frac{a(e+fx)^4}{8b^2f} + \frac{a^2(e+fx)^3\cosh(c+dx)}{b^3d} + \frac{3af^3\cosh^2(c+}{8b^2d^4}$$

$$= -\frac{3aef^2x}{4b^2d^2} - \frac{3af^3x^2}{8b^2d^2} - \frac{a^3(e+fx)^4}{4b^4f} - \frac{a(e+fx)^4}{8b^2f} + \frac{4f^2(e+fx)\cosh(c+dx}{3bd^3}$$

$$= -\frac{3aef^2x}{4b^2d^2} - \frac{3af^3x^2}{8b^2d^2} - \frac{a^3(e+fx)^4}{4b^4f} - \frac{a(e+fx)^4}{8b^2f} + \frac{6a^2f^2(e+fx)\cosh(c+}{b^3d^3}$$

$$= -\frac{3aef^2x}{4b^2d^2} - \frac{3af^3x^2}{8b^2d^2} - \frac{a^3(e+fx)^4}{4b^4f} - \frac{a(e+fx)^4}{8b^2f} + \frac{6a^2f^2(e+fx)\cosh(c+}{b^3d^3}$$

$$= -\frac{3aef^2x}{4b^2d^2} - \frac{3af^3x^2}{8b^2d^2} - \frac{a^3(e+fx)^4}{4b^4f} - \frac{a(e+fx)^4}{8b^2f} + \frac{6a^2f^2(e+fx)\cosh(c+}{b^3d^3}$$

$$= -\frac{3aef^2x}{4b^2d^2} - \frac{3af^3x^2}{8b^2d^2} - \frac{a^3(e+fx)^4}{4b^4f} - \frac{a(e+fx)^4}{8b^2f} + \frac{6a^2f^2(e+fx)\cosh(c+}{b^3d^3}$$

$$= -\frac{3aef^2x}{4b^2d^2} - \frac{3af^3x^2}{8b^2d^2} - \frac{a^3(e+fx)^4}{4b^4f} - \frac{a(e+fx)^4}{8b^2f} + \frac{6a^2f^2(e+fx)\cosh(c+}{b^3d^3}$$

Mathematica [A] time = 7.79542, size = 1667, normalized size = 1.86

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x
]),x]
```

```
[Out] -(432*a^3*d^4*e^3*x + 216*a*b^2*d^4*e^3*x + 648*a^3*d^4*e^2*f*x^2 + 324*a*b
^2*d^4*e^2*f*x^2 + 432*a^3*d^4*e*f^2*x^3 + 216*a*b^2*d^4*e*f^2*x^3 + 108*a^
3*d^4*f^3*x^4 + 54*a*b^2*d^4*f^3*x^4 + 864*a^2*Sqrt[a^2 + b^2]*d^3*e^3*ArcT
anh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 432*a^2*b*d^3*e^3*Cosh[c + d*x]
- 108*b^3*d^3*e^3*Cosh[c + d*x] - 2592*a^2*b*d*e*f^2*Cosh[c + d*x] - 648*b^
3*d*e*f^2*Cosh[c + d*x] - 1296*a^2*b*d^3*e^2*f*x*Cosh[c + d*x] - 324*b^3*d^
```

```
3*e^2*f*x*Cosh[c + d*x] - 2592*a^2*b*d*f^3*x*Cosh[c + d*x] - 648*b^3*d*f^3*
x*Cosh[c + d*x] - 1296*a^2*b*d^3*e*f^2*x^2*Cosh[c + d*x] - 324*b^3*d^3*e*f^
2*x^2*Cosh[c + d*x] - 432*a^2*b*d^3*f^3*x^3*Cosh[c + d*x] - 108*b^3*d^3*f^3
*x^3*Cosh[c + d*x] - 162*a*b^2*d^2*e^2*f*Cosh[2*(c + d*x)] - 81*a*b^2*f^3*C
osh[2*(c + d*x)] - 324*a*b^2*d^2*e*f^2*x*Cosh[2*(c + d*x)] - 162*a*b^2*d^2*
f^3*x^2*Cosh[2*(c + d*x)] - 36*b^3*d^3*e^3*Cosh[3*(c + d*x)] - 24*b^3*d*e*f
^2*Cosh[3*(c + d*x)] - 108*b^3*d^3*e^2*f*x*Cosh[3*(c + d*x)] - 24*b^3*d*f^3
*x*Cosh[3*(c + d*x)] - 108*b^3*d^3*e*f^2*x^2*Cosh[3*(c + d*x)] - 36*b^3*d^3
*f^3*x^3*Cosh[3*(c + d*x)] - 1296*a^2*Sqrt[a^2 + b^2]*d^3*e^2*f*x*Log[1 + (
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 1296*a^2*Sqrt[a^2 + b^2]*d^3*e*f^2*
x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 432*a^2*Sqrt[a^2 + b^2
]*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 1296*a^2*Sqr
t[a^2 + b^2]*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 1
296*a^2*Sqrt[a^2 + b^2]*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2])] + 432*a^2*Sqrt[a^2 + b^2]*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])] - 1296*a^2*Sqrt[a^2 + b^2]*d^2*f*(e + f*x)^2*PolyLog[2,
 (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 1296*a^2*Sqrt[a^2 + b^2]*d^2*f*(
e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 2592*a^2*
Sqrt[a^2 + b^2]*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
 + 2592*a^2*Sqrt[a^2 + b^2]*d*f^3*x*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^
2 + b^2])] - 2592*a^2*Sqrt[a^2 + b^2]*d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))] - 2592*a^2*Sqrt[a^2 + b^2]*d*f^3*x*PolyLog[3, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2592*a^2*Sqrt[a^2 + b^2]*f^3*PolyLog
[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2592*a^2*Sqrt[a^2 + b^2]*f^3*
PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 1296*a^2*b*d^2*e^2*f
*Sinh[c + d*x] + 324*b^3*d^2*e^2*f*Sinh[c + d*x] + 2592*a^2*b*f^3*Sinh[c +
d*x] + 648*b^3*f^3*Sinh[c + d*x] + 2592*a^2*b*d^2*e*f^2*x*Sinh[c + d*x] + 6
48*b^3*d^2*e*f^2*x*Sinh[c + d*x] + 1296*a^2*b*d^2*f^3*x^2*Sinh[c + d*x] + 3
24*b^3*d^2*f^3*x^2*Sinh[c + d*x] + 108*a*b^2*d^3*e^3*Sinh[2*(c + d*x)] + 16
2*a*b^2*d*e*f^2*Sinh[2*(c + d*x)] + 324*a*b^2*d^3*e^2*f*x*Sinh[2*(c + d*x)]
 + 162*a*b^2*d*f^3*x*Sinh[2*(c + d*x)] + 324*a*b^2*d^3*e*f^2*x^2*Sinh[2*(c
+ d*x)] + 108*a*b^2*d^3*f^3*x^3*Sinh[2*(c + d*x)] + 36*b^3*d^2*e^2*f*Sinh[3
*(c + d*x)] + 8*b^3*f^3*Sinh[3*(c + d*x)] + 72*b^3*d^2*e*f^2*x*Sinh[3*(c +
d*x)] + 36*b^3*d^2*f^3*x^2*Sinh[3*(c + d*x)])/(432*b^4*d^4)
```

Maple [F] time = 0.217, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3}\left(\cosh\left(dx+c\right)\right)^{2}\left(\sinh\left(dx+c\right)\right)^{2}}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.59105, size = 15351, normalized size = 17.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="fricas")
```

```
[Out] 1/864*(36*b^3*d^3*f^3*x^3 + 36*b^3*d^3*e^3 + 36*b^3*d^2*e^2*f + 24*b^3*d*e*
f^2 + 4*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f^
2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f
- 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*cosh(d*x + c)^6 + 4*(9*b^3*d^3*f^3*x^3
+ 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^
3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d
*f^3)*x)*sinh(d*x + c)^6 + 8*b^3*f^3 - 27*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^
3*e^3 - 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*
e*f^2 - a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b
^2*d*f^3)*x)*cosh(d*x + c)^5 - 3*(36*a*b^2*d^3*f^3*x^3 + 36*a*b^2*d^3*e^3 -
 54*a*b^2*d^2*e^2*f + 54*a*b^2*d*e*f^2 - 27*a*b^2*f^3 + 54*(2*a*b^2*d^3*e*f
^2 - a*b^2*d^2*f^3)*x^2 + 54*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^2
*d*f^3)*x - 8*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*
d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*
e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c)^5 +
108*((4*a^2*b + b^3)*d^3*f^3*x^3 + (4*a^2*b + b^3)*d^3*e^3 - 3*(4*a^2*b + b
^3)*d^2*e^2*f + 6*(4*a^2*b + b^3)*d*e*f^2 - 6*(4*a^2*b + b^3)*f^3 + 3*((4*a
^2*b + b^3)*d^3*e*f^2 - (4*a^2*b + b^3)*d^2*f^3)*x^2 + 3*((4*a^2*b + b^3)*d
^3*e^2*f - 2*(4*a^2*b + b^3)*d^2*e*f^2 + 2*(4*a^2*b + b^3)*d*f^3)*x)*cosh(d
*x + c)^4 + 3*(36*(4*a^2*b + b^3)*d^3*f^3*x^3 + 36*(4*a^2*b + b^3)*d^3*e^3
- 108*(4*a^2*b + b^3)*d^2*e^2*f + 216*(4*a^2*b + b^3)*d*e*f^2 - 216*(4*a^2*
b + b^3)*f^3 + 108*((4*a^2*b + b^3)*d^3*e*f^2 - (4*a^2*b + b^3)*d^2*f^3)*x^
2 + 20*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f^2
 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f -
 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*cosh(d*x + c)^2 + 108*((4*a^2*b + b^3)*d
^3*e^2*f - 2*(4*a^2*b + b^3)*d^2*e*f^2 + 2*(4*a^2*b + b^3)*d*f^3)*x - 45*(4
*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e^3 - 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2
- 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f^2 - a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*
e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c)^4
- 108*((2*a^3 + a*b^2)*d^4*f^3*x^4 + 4*(2*a^3 + a*b^2)*d^4*e*f^2*x^3 + 6*(2
*a^3 + a*b^2)*d^4*e^2*f*x^2 + 4*(2*a^3 + a*b^2)*d^4*e^3*x)*cosh(d*x + c)^3
- 2*(54*(2*a^3 + a*b^2)*d^4*f^3*x^4 + 216*(2*a^3 + a*b^2)*d^4*e*f^2*x^3 + 3
24*(2*a^3 + a*b^2)*d^4*e^2*f*x^2 + 216*(2*a^3 + a*b^2)*d^4*e^3*x - 40*(9*b^
3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f^2 - 2*b^3*f^3
 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f - 6*b^3*d^2*e
*f^2 + 2*b^3*d*f^3)*x)*cosh(d*x + c)^3 + 135*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2
*d^3*e^3 - 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d
^3*e*f^2 - a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 +
a*b^2*d*f^3)*x)*cosh(d*x + c)^2 - 216*((4*a^2*b + b^3)*d^3*f^3*x^3 + (4*a^2
*b + b^3)*d^3*e^3 - 3*(4*a^2*b + b^3)*d^2*e^2*f + 6*(4*a^2*b + b^3)*d*e*f^2
 - 6*(4*a^2*b + b^3)*f^3 + 3*((4*a^2*b + b^3)*d^3*e*f^2 - (4*a^2*b + b^3)*d
^2*f^3)*x^2 + 3*((4*a^2*b + b^3)*d^3*e^2*f - 2*(4*a^2*b + b^3)*d^2*e*f^2 +
2*(4*a^2*b + b^3)*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 36*(3*b^3*d^3*
e*f^2 + b^3*d^2*f^3)*x^2 + 108*((4*a^2*b + b^3)*d^3*f^3*x^3 + (4*a^2*b + b^
3)*d^3*e^3 + 3*(4*a^2*b + b^3)*d^2*e^2*f + 6*(4*a^2*b + b^3)*d*e*f^2 + 6*(4
*a^2*b + b^3)*f^3 + 3*((4*a^2*b + b^3)*d^3*e*f^2 + (4*a^2*b + b^3)*d^2*f^3)
```

x^2 + 3((4*a^2*b + b^3)*d^3*e^2*f + 2*(4*a^2*b + b^3)*d^2*e*f^2 + 2*(4*a^
2*b + b^3)*d*f^3)*x)*cosh(d*x + c)^2 + 6*(18*(4*a^2*b + b^3)*d^3*f^3*x^3 +
18*(4*a^2*b + b^3)*d^3*e^3 + 54*(4*a^2*b + b^3)*d^2*e^2*f + 108*(4*a^2*b +
b^3)*d*e*f^2 + 10*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*
b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*
d^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*cosh(d*x + c)^4 + 108*(4*a^2*
b + b^3)*f^3 - 45*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e^3 - 6*a*b^2*d^2*e^2*
f + 6*a*b^2*d*e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f^2 - a*b^2*d^2*f^3)*x
^2 + 6*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x)*cosh(d*x +
c)^3 + 54*((4*a^2*b + b^3)*d^3*e*f^2 + (4*a^2*b + b^3)*d^2*f^3)*x^2 + 108*(
(4*a^2*b + b^3)*d^3*f^3*x^3 + (4*a^2*b + b^3)*d^3*e^3 - 3*(4*a^2*b + b^3)*d
^2*e^2*f + 6*(4*a^2*b + b^3)*d*e*f^2 - 6*(4*a^2*b + b^3)*f^3 + 3*((4*a^2*b
+ b^3)*d^3*e*f^2 - (4*a^2*b + b^3)*d^2*f^3)*x^2 + 3*((4*a^2*b + b^3)*d^3*e^
2*f - 2*(4*a^2*b + b^3)*d^2*e*f^2 + 2*(4*a^2*b + b^3)*d*f^3)*x)*cosh(d*x +
c)^2 + 54*((4*a^2*b + b^3)*d^3*e^2*f + 2*(4*a^2*b + b^3)*d^2*e*f^2 + 2*(4*a
^2*b + b^3)*d*f^3)*x - 54*((2*a^3 + a*b^2)*d^4*f^3*x^4 + 4*(2*a^3 + a*b^2)*
d^4*e*f^2*x^3 + 6*(2*a^3 + a*b^2)*d^4*e^2*f*x^2 + 4*(2*a^3 + a*b^2)*d^4*e^3
*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 2592*((a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2
*e*f^2*x + a^2*b*d^2*e^2*f)*cosh(d*x + c)^3 + 3*(a^2*b*d^2*f^3*x^2 + 2*a^2*
b*d^2*e*f^2*x + a^2*b*d^2*e^2*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^2*b*d
^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2*b*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x
+ c)^2 + (a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2*b*d^2*e^2*f)*sinh(d
*x + c)^3)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) +
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 25
92*((a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2*b*d^2*e^2*f)*cosh(d*x +
c)^3 + 3*(a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2*b*d^2*e^2*f)*cosh(d
*x + c)^2*sinh(d*x + c) + 3*(a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2*
b*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2*b*d^2*f^3*x^2 + 2*a^2*b*d
^2*e*f^2*x + a^2*b*d^2*e^2*f)*sinh(d*x + c)^3)*sqrt((a^2 + b^2)/b^2)*dilog(
(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sq
rt((a^2 + b^2)/b^2) - b)/b + 1) - 864*((a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f
+ 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*cosh(d*x + c)^3 + 3*(a^2*b*d^3*e^3
- 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*cosh(d*x + c)^
2*sinh(d*x + c) + 3*(a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*
f^2 - a^2*b*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2*b*d^3*e^3 - 3*a^2
*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*sinh(d*x + c)^3)*sqrt
((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2
+ b^2)/b^2) + 2*a) + 864*((a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c
^2*d*e*f^2 - a^2*b*c^3*f^3)*cosh(d*x + c)^3 + 3*(a^2*b*d^3*e^3 - 3*a^2*b*c*
d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*cosh(d*x + c)^2*sinh(d*x +
c) + 3*(a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*
c^3*f^3)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2
*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*sinh(d*x + c)^3)*sqrt((a^2 + b^2)
/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2)
+ 2*a) + 864*((a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2
*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*cosh(d*x
+ c)^3 + 3*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x
+ 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*cosh(d*x + c)
^2*sinh(d*x + c) + 3*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d
^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*cos
h(d*x + c)*sinh(d*x + c)^2 + (a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3
*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*
f^3)*sinh(d*x + c)^3)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(
d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b
) - 864*((a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x +
3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*cosh(d*x + c)^3
+ 3*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a
^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*cosh(d*x + c)^2*sin
h(d*x + c) + 3*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2

```
*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*cosh(d*x
+ c)*sinh(d*x + c)^2 + (a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b
*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*s
inh(d*x + c)^3)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x +
c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 51
84*(a^2*b*f^3*cosh(d*x + c)^3 + 3*a^2*b*f^3*cosh(d*x + c)^2*sinh(d*x + c) +
 3*a^2*b*f^3*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*b*f^3*sinh(d*x + c)^3)*sqr
t((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 5184*(a^2*b*f^3*cos
h(d*x + c)^3 + 3*a^2*b*f^3*cosh(d*x + c)^2*sinh(d*x + c) + 3*a^2*b*f^3*cosh
(d*x + c)*sinh(d*x + c)^2 + a^2*b*f^3*sinh(d*x + c)^3)*sqrt((a^2 + b^2)/b^2
)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 5184*((a^2*b*d*f^3*x + a^2*b*d*e*f^2
)*cosh(d*x + c)^3 + 3*(a^2*b*d*f^3*x + a^2*b*d*e*f^2)*cosh(d*x + c)^2*sinh(
d*x + c) + 3*(a^2*b*d*f^3*x + a^2*b*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c)^2
+ (a^2*b*d*f^3*x + a^2*b*d*e*f^2)*sinh(d*x + c)^3)*sqrt((a^2 + b^2)/b^2)*po
lylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2))/b) + 5184*((a^2*b*d*f^3*x + a^2*b*d*e*f^2)*co
sh(d*x + c)^3 + 3*(a^2*b*d*f^3*x + a^2*b*d*e*f^2)*cosh(d*x + c)^2*sinh(d*x
+ c) + 3*(a^2*b*d*f^3*x + a^2*b*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (a
^2*b*d*f^3*x + a^2*b*d*e*f^2)*sinh(d*x + c)^3)*sqrt((a^2 + b^2)/b^2)*polylo
g(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2))/b) + 12*(9*b^3*d^3*e^2*f + 6*b^3*d^2*e*f^2 + 2*b^
3*d*f^3)*x + 27*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e^3 + 6*a*b^2*d^2*e^2*f
+ 6*a*b^2*d*e*f^2 + 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f^2 + a*b^2*d^2*f^3)*x^2
 + 6*(2*a*b^2*d^3*e^2*f + 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x)*cosh(d*x + c)
 + 3*(36*a*b^2*d^3*f^3*x^3 + 36*a*b^2*d^3*e^3 + 54*a*b^2*d^2*e^2*f + 54*a*b
^2*d*e*f^2 + 27*a*b^2*f^3 + 8*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^
2*e^2*f + 6*b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2
 + 3*(9*b^3*d^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*cosh(d*x + c)^5 -
 45*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e^3 - 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*
e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f^2 - a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^
2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x)*cosh(d*x + c)^4 + 144*((4
*a^2*b + b^3)*d^3*f^3*x^3 + (4*a^2*b + b^3)*d^3*e^3 - 3*(4*a^2*b + b^3)*d^2
*e^2*f + 6*(4*a^2*b + b^3)*d*e*f^2 - 6*(4*a^2*b + b^3)*f^3 + 3*((4*a^2*b +
b^3)*d^3*e*f^2 - (4*a^2*b + b^3)*d^2*f^3)*x^2 + 3*((4*a^2*b + b^3)*d^3*e^2*
f - 2*(4*a^2*b + b^3)*d^2*e*f^2 + 2*(4*a^2*b + b^3)*d*f^3)*x)*cosh(d*x + c)
^3 + 54*(2*a*b^2*d^3*e*f^2 + a*b^2*d^2*f^3)*x^2 - 108*((2*a^3 + a*b^2)*d^4*
f^3*x^4 + 4*(2*a^3 + a*b^2)*d^4*e*f^2*x^3 + 6*(2*a^3 + a*b^2)*d^4*e^2*f*x^2
 + 4*(2*a^3 + a*b^2)*d^4*e^3*x)*cosh(d*x + c)^2 + 54*(2*a*b^2*d^3*e^2*f + 2
*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x + 72*((4*a^2*b + b^3)*d^3*f^3*x^3 + (4*a^
2*b + b^3)*d^3*e^3 + 3*(4*a^2*b + b^3)*d^2*e^2*f + 6*(4*a^2*b + b^3)*d*e*f^
2 + 6*(4*a^2*b + b^3)*f^3 + 3*((4*a^2*b + b^3)*d^3*e*f^2 + (4*a^2*b + b^3)*
d^2*f^3)*x^2 + 3*((4*a^2*b + b^3)*d^3*e^2*f + 2*(4*a^2*b + b^3)*d^2*e*f^2 +
 2*(4*a^2*b + b^3)*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c))/(b^4*d^4*cosh(d*
x + c)^3 + 3*b^4*d^4*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^4*d^4*cosh(d*x + c
)*sinh(d*x + c)^2 + b^4*d^4*sinh(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**2*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3\cosh(dx+c)^2\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)^2*sinh(d*x + c)^2/(b*sinh(d*x + c) + a)
, x)
```

3.368 $\int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal. Leaf size=649

$$\frac{2a^2 f\sqrt{a^2+b^2}(e+fx)\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{2a^2 f\sqrt{a^2+b^2}(e+fx)\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4d^2}-\frac{2a^2f^2\sqrt{a^2+b^2}\mathrm{PolyL}}{b^4d^3}$$

```
[Out] -(a*f^2*x)/(4*b^2*d^2) - (a^3*(e + f*x)^3)/(3*b^4*f) - (a*(e + f*x)^3)/(6*b
^2*f) + (2*a^2*f^2*Cosh[c + d*x])/(b^3*d^3) + (4*f^2*Cosh[c + d*x])/(9*b*d^
3) + (a^2*(e + f*x)^2*Cosh[c + d*x])/(b^3*d) + (a*f*(e + f*x)*Cosh[c + d*x]
^2)/(2*b^2*d^2) + (2*f^2*Cosh[c + d*x]^3)/(27*b*d^3) + ((e + f*x)^2*Cosh[c
+ d*x]^3)/(3*b*d) + (a^2*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a^2*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1
+ (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^4*d) + (2*a^2*Sqrt[a^2 + b^2]*
f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^2)
 - (2*a^2*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/(b^4*d^2) - (2*a^2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^3) + (2*a^2*Sqrt[a^2 + b^2]*f^2*P
olyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^3) - (2*a^2*f*(
e + f*x)*Sinh[c + d*x])/(b^3*d^2) - (4*f*(e + f*x)*Sinh[c + d*x])/(9*b*d^2)
 - (a*f^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^2*d^3) - (a*(e + f*x)^2*Cosh[c
+ d*x]*Sinh[c + d*x])/(2*b^2*d) - (2*f*(e + f*x)*Cosh[c + d*x]^2*Sinh[c + d
*x])/(9*b*d^2)
```

Rubi [A] time = 1.20186, antiderivative size = 649, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 16, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5579, 5447, 3310, 3296, 2638, 3311, 32, 2635, 8, 5565, 3322, 2264, 2190, 2531, 2282, 6589}

$$\frac{2a^2 f\sqrt{a^2+b^2}(e+fx)\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{2a^2 f\sqrt{a^2+b^2}(e+fx)\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4d^2}-\frac{2a^2f^2\sqrt{a^2+b^2}\mathrm{PolyL}}{b^4d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(a*f^2*x)/(4*b^2*d^2) - (a^3*(e + f*x)^3)/(3*b^4*f) - (a*(e + f*x)^3)/(6*b
^2*f) + (2*a^2*f^2*Cosh[c + d*x])/(b^3*d^3) + (4*f^2*Cosh[c + d*x])/(9*b*d^
3) + (a^2*(e + f*x)^2*Cosh[c + d*x])/(b^3*d) + (a*f*(e + f*x)*Cosh[c + d*x]
^2)/(2*b^2*d^2) + (2*f^2*Cosh[c + d*x]^3)/(27*b*d^3) + ((e + f*x)^2*Cosh[c
+ d*x]^3)/(3*b*d) + (a^2*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a^2*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1
+ (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^4*d) + (2*a^2*Sqrt[a^2 + b^2]*
f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^2)
 - (2*a^2*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/(b^4*d^2) - (2*a^2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^3) + (2*a^2*Sqrt[a^2 + b^2]*f^2*P
olyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^3) - (2*a^2*f*(
e + f*x)*Sinh[c + d*x])/(b^3*d^2) - (4*f*(e + f*x)*Sinh[c + d*x])/(9*b*d^2)
 - (a*f^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^2*d^3) - (a*(e + f*x)^2*Cosh[c
+ d*x]*Sinh[c + d*x])/(2*b^2*d) - (2*f*(e + f*x)*Cosh[c + d*x]^2*Sinh[c + d
*x])/(9*b*d^2)
```

Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh^2(c+dx)\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh(c+}{a+b\sinh(c+dx)}}{b}$$

$$= \frac{(e+fx)^2\cosh^3(c+dx)}{3bd} - \frac{a\int (e+fx)^2\cosh^2(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^2}{a+b\,\mathrm{s}}}{}$$

$$= \frac{af(e+fx)\cosh^2(c+dx)}{2b^2d^2} + \frac{2f^2\cosh^3(c+dx)}{27bd^3} + \frac{(e+fx)^2\cosh^3(c+}{3bd}$$

$$= -\frac{a^3(e+fx)^3}{3b^4f} - \frac{a(e+fx)^3}{6b^2f} + \frac{a^2(e+fx)^2\cosh(c+dx)}{b^3d} + \frac{af(e+fx)\,\mathrm{co}}{2b^2}$$

$$= -\frac{af^2x}{4b^2d^2} - \frac{a^3(e+fx)^3}{3b^4f} - \frac{a(e+fx)^3}{6b^2f} + \frac{4f^2\cosh(c+dx)}{9bd^3} + \frac{a^2(e+fx)^2}{b}$$

$$= -\frac{af^2x}{4b^2d^2} - \frac{a^3(e+fx)^3}{3b^4f} - \frac{a(e+fx)^3}{6b^2f} + \frac{2a^2f^2\cosh(c+dx)}{b^3d^3} + \frac{4f^2\cosh(}{9bd}$$

$$= -\frac{af^2x}{4b^2d^2} - \frac{a^3(e+fx)^3}{3b^4f} - \frac{a(e+fx)^3}{6b^2f} + \frac{2a^2f^2\cosh(c+dx)}{b^3d^3} + \frac{4f^2\cosh(}{9bd}$$

$$= -\frac{af^2x}{4b^2d^2} - \frac{a^3(e+fx)^3}{3b^4f} - \frac{a(e+fx)^3}{6b^2f} + \frac{2a^2f^2\cosh(c+dx)}{b^3d^3} + \frac{4f^2\cosh(}{9bd}$$

$$= -\frac{af^2x}{4b^2d^2} - \frac{a^3(e+fx)^3}{3b^4f} - \frac{a(e+fx)^3}{6b^2f} + \frac{2a^2f^2\cosh(c+dx)}{b^3d^3} + \frac{4f^2\cosh(}{9bd}$$

Mathematica [A] time = 4.98927, size = 966, normalized size = 1.49

$$-\frac{-54d^2e^2\cosh(c+dx)b^3 - 108f^2\cosh(c+dx)b^3 - 54d^2f^2x^2\cosh(c+dx)b^3 - 108d^2efx\cosh(c+dx)b^3 - 18d^2e^2\,\mathrm{cos}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x
]),x]
```

```
[Out] -(216*a^3*d^3*e^2*x + 108*a*b^2*d^3*e^2*x + 216*a^3*d^3*e*f*x^2 + 108*a*b^2
*d^3*e*f*x^2 + 72*a^3*d^3*f^2*x^3 + 36*a*b^2*d^3*f^2*x^3 + 432*a^2*Sqrt[a^2
 + b^2]*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 216*a^2*b*d^
2*e^2*Cosh[c + d*x] - 54*b^3*d^2*e^2*Cosh[c + d*x] - 432*a^2*b*f^2*Cosh[c +
 d*x] - 108*b^3*f^2*Cosh[c + d*x] - 432*a^2*b*d^2*e*f*x*Cosh[c + d*x] - 108
*b^3*d^2*e*f*x*Cosh[c + d*x] - 216*a^2*b*d^2*f^2*x^2*Cosh[c + d*x] - 54*b^3
*d^2*f^2*x^2*Cosh[c + d*x] - 54*a*b^2*d*e*f*Cosh[2*(c + d*x)] - 54*a*b^2*d*
f^2*x*Cosh[2*(c + d*x)] - 18*b^3*d^2*e^2*Cosh[3*(c + d*x)] - 4*b^3*f^2*Cosh
[3*(c + d*x)] - 36*b^3*d^2*e*f*x*Cosh[3*(c + d*x)] - 18*b^3*d^2*f^2*x^2*Cos
h[3*(c + d*x)] - 432*a^2*Sqrt[a^2 + b^2]*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])] - 216*a^2*Sqrt[a^2 + b^2]*d^2*f^2*x^2*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])] + 432*a^2*Sqrt[a^2 + b^2]*d^2*e*f*x*Log[1
+ (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 216*a^2*Sqrt[a^2 + b^2]*d^2*f^2*
x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 432*a^2*Sqrt[a^2 + b^2
]*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 432*a^
2*Sqrt[a^2 + b^2]*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))] + 432*a^2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sq
rt[a^2 + b^2])] - 432*a^2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/
```

```
(a + Sqrt[a^2 + b^2]))] + 432*a^2*b*d*e*f*Sinh[c + d*x] + 108*b^3*d*e*f*Sin
h[c + d*x] + 432*a^2*b*d*f^2*x*Sinh[c + d*x] + 108*b^3*d*f^2*x*Sinh[c + d*x
] + 54*a*b^2*d^2*e^2*Sinh[2*(c + d*x)] + 27*a*b^2*f^2*Sinh[2*(c + d*x)] + 1
08*a*b^2*d^2*e*f*x*Sinh[2*(c + d*x)] + 54*a*b^2*d^2*f^2*x^2*Sinh[2*(c + d*x
)] + 12*b^3*d*e*f*Sinh[3*(c + d*x)] + 12*b^3*d*f^2*x*Sinh[3*(c + d*x)])/(21
6*b^4*d^3)
```

Maple [F] time = 0.228, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^2 (\cosh(dx+c))^2 (\sinh(dx+c))^2}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)

[Out] int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="maxima")

[Out] Exception raised: ValueError
```

Fricas [C] time = 3.49251, size = 9708, normalized size = 14.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="fricas")

[Out] 1/432*(18*b^3*d^2*f^2*x^2 + 18*b^3*d^2*e^2 + 2*(9*b^3*d^2*f^2*x^2 + 9*b^3*d
^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*
x + c)^6 + 2*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 +
 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*sinh(d*x + c)^6 + 12*b^3*d*e*f - 27*(2*a*
b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*
d^2*e*f - a*b^2*d*f^2)*x)*cosh(d*x + c)^5 - 3*(18*a*b^2*d^2*f^2*x^2 + 18*a*
b^2*d^2*e^2 - 18*a*b^2*d*e*f + 9*a*b^2*f^2 + 18*(2*a*b^2*d^2*e*f - a*b^2*d*
f^2)*x - 4*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6
*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*b^3*f^2
+ 54*((4*a^2*b + b^3)*d^2*f^2*x^2 + (4*a^2*b + b^3)*d^2*e^2 - 2*(4*a^2*b +
b^3)*d*e*f + 2*(4*a^2*b + b^3)*f^2 + 2*((4*a^2*b + b^3)*d^2*e*f - (4*a^2*b
+ b^3)*d*f^2)*x)*cosh(d*x + c)^4 + 3*(18*(4*a^2*b + b^3)*d^2*f^2*x^2 + 18*(
4*a^2*b + b^3)*d^2*e^2 - 36*(4*a^2*b + b^3)*d*e*f + 36*(4*a^2*b + b^3)*f^2
```

+ 10*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*x + c)^2 + 36*((4*a^2*b + b^3)*d^2*e*f - (4*a^2*b + b^3)*d*f^2)*x - 45*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^4 - 72*((2*a^3 + a*b^2)*d^3*f^2*x^3 + 3*(2*a^3 + a*b^2)*d^3*e*f*x^2 + 3*(2*a^3 + a*b^2)*d^3*e^2*x)*cosh(d*x + c)^3 - 2*(36*(2*a^3 + a*b^2)*d^3*f^2*x^3 + 108*(2*a^3 + a*b^2)*d^3*e*f*x^2 + 108*(2*a^3 + a*b^2)*d^3*e^2*x - 20*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*x + c)^3 + 135*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*cosh(d*x + c)^2 - 108*((4*a^2*b + b^3)*d^2*f^2*x^2 + (4*a^2*b + b^3)*d^2*e^2 - 2*(4*a^2*b + b^3)*d*e*f + 2*(4*a^2*b + b^3)*f^2 + 2*((4*a^2*b + b^3)*d^2*e*f - (4*a^2*b + b^3)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 54*((4*a^2*b + b^3)*d^2*f^2*x^2 + (4*a^2*b + b^3)*d^2*e^2 + 2*(4*a^2*b + b^3)*d*e*f + 2*(4*a^2*b + b^3)*f^2 + 2*((4*a^2*b + b^3)*d^2*e*f + (4*a^2*b + b^3)*d*f^2)*x)*cosh(d*x + c)^2 + 6*(9*(4*a^2*b + b^3)*d^2*f^2*x^2 + 9*(4*a^2*b + b^3)*d^2*e^2 + 5*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*x + c)^4 + 18*(4*a^2*b + b^3)*d*e*f - 45*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*cosh(d*x + c)^3 + 18*(4*a^2*b + b^3)*f^2 + 54*((4*a^2*b + b^3)*d^2*f^2*x^2 + (4*a^2*b + b^3)*d^2*e^2 - 2*(4*a^2*b + b^3)*d*e*f + 2*(4*a^2*b + b^3)*f^2 + 2*((4*a^2*b + b^3)*d^2*e*f - (4*a^2*b + b^3)*d*f^2)*x)*cosh(d*x + c)^2 + 18*((4*a^2*b + b^3)*d^2*e*f + (4*a^2*b + b^3)*d*f^2)*x - 36*((2*a^3 + a*b^2)*d^3*f^2*x^3 + 3*(2*a^3 + a*b^2)*d^3*e*f*x^2 + 3*(2*a^3 + a*b^2)*d^3*e^2*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 864*((a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)^3 + 3*(a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2*b*d*f^2*x + a^2*b*d*e*f)*sinh(d*x + c)^3)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 864*((a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)^3 + 3*(a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2*b*d*f^2*x + a^2*b*d*e*f)*sinh(d*x + c)^3)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 432*((a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c)^3 + 3*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*sinh(d*x + c)^3)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 432*((a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c)^3 + 3*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*sinh(d*x + c)^3)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 432*((a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*cosh(d*x + c)^3 + 3*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*sinh(d*x + c)^3)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 432*((a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*cosh(d*x + c)^3 + 3*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*sinh(d*x + c)^3)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sin

```
h(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)
/b) - 864*(a^2*b*f^2*cosh(d*x + c)^3 + 3*a^2*b*f^2*cosh(d*x + c)^2*sinh(d*x
 + c) + 3*a^2*b*f^2*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*b*f^2*sinh(d*x + c)
^3)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 864*(a^2*b*f
^2*cosh(d*x + c)^3 + 3*a^2*b*f^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*a^2*b*f^
2*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*b*f^2*sinh(d*x + c)^3)*sqrt((a^2 + b^
2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 12*(3*b^3*d^2*e*f + b^3*d*f^2)
*x + 27*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 + 2*a*b^2*d*e*f + a*b^2*f^2
+ 2*(2*a*b^2*d^2*e*f + a*b^2*d*f^2)*x)*cosh(d*x + c) + 3*(18*a*b^2*d^2*f^2*
x^2 + 18*a*b^2*d^2*e^2 + 18*a*b^2*d*e*f + 4*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*
e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*x +
 c)^5 + 9*a*b^2*f^2 - 45*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d
*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*cosh(d*x + c)^4 + 7
2*((4*a^2*b + b^3)*d^2*f^2*x^2 + (4*a^2*b + b^3)*d^2*e^2 - 2*(4*a^2*b + b^3
)*d*e*f + 2*(4*a^2*b + b^3)*f^2 + 2*((4*a^2*b + b^3)*d^2*e*f - (4*a^2*b + b
^3)*d*f^2)*x)*cosh(d*x + c)^3 - 72*((2*a^3 + a*b^2)*d^3*f^2*x^3 + 3*(2*a^3
+ a*b^2)*d^3*e*f*x^2 + 3*(2*a^3 + a*b^2)*d^3*e^2*x)*cosh(d*x + c)^2 + 18*(2
*a*b^2*d^2*e*f + a*b^2*d*f^2)*x + 36*((4*a^2*b + b^3)*d^2*f^2*x^2 + (4*a^2*
b + b^3)*d^2*e^2 + 2*(4*a^2*b + b^3)*d*e*f + 2*(4*a^2*b + b^3)*f^2 + 2*((4*
a^2*b + b^3)*d^2*e*f + (4*a^2*b + b^3)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x +
c))/(b^4*d^3*cosh(d*x + c)^3 + 3*b^4*d^3*cosh(d*x + c)^2*sinh(d*x + c) + 3*
b^4*d^3*cosh(d*x + c)*sinh(d*x + c)^2 + b^4*d^3*sinh(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**2*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \cosh(dx+c)^2 \sinh(dx+c)^2}{b \sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)^2*sinh(d*x + c)^2/(b*sinh(d*x + c) + a)
, x)
```

3.369 $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=403

$$\frac{a^2 f\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{a^2 f\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4d^2}-\frac{a^2 f\sinh(c+dx)}{b^3d^2}+\frac{a^2\sqrt{a^2+b^2}(e+f}{}$$

```
[Out] -((a^3*e*x)/b^4) - (a*e*x)/(2*b^2) - (a^3*f*x^2)/(2*b^4) - (a*f*x^2)/(4*b^2
) + (a^2*(e + f*x)*Cosh[c + d*x])/(b^3*d) + (a*f*Cosh[c + d*x]^2)/(4*b^2*d^
2) + ((e + f*x)*Cosh[c + d*x]^3)/(3*b*d) + (a^2*Sqrt[a^2 + b^2]*(e + f*x)*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a^2*Sqrt[a^2 + b^
2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^4*d) + (a^2
*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b
^4*d^2) - (a^2*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2]))])/(b^4*d^2) - (a^2*f*Sinh[c + d*x])/(b^3*d^2) - (f*Sinh[c + d*x])
/(3*b*d^2) - (a*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^2*d) - (f*Sinh[
c + d*x]^3)/(9*b*d^2)
```

Rubi [A] time = 0.690013, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 12, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5579, 5447, 2633, 3310, 5565, 3296, 2637, 3322, 2264, 2190, 2279, 2391}

$$\frac{a^2 f\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{a^2 f\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4d^2}-\frac{a^2 f\sinh(c+dx)}{b^3d^2}+\frac{a^2\sqrt{a^2+b^2}(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a^3*e*x)/b^4) - (a*e*x)/(2*b^2) - (a^3*f*x^2)/(2*b^4) - (a*f*x^2)/(4*b^2
) + (a^2*(e + f*x)*Cosh[c + d*x])/(b^3*d) + (a*f*Cosh[c + d*x]^2)/(4*b^2*d^
2) + ((e + f*x)*Cosh[c + d*x]^3)/(3*b*d) + (a^2*Sqrt[a^2 + b^2]*(e + f*x)*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a^2*Sqrt[a^2 + b^
2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^4*d) + (a^2
*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b
^4*d^2) - (a^2*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2]))])/(b^4*d^2) - (a^2*f*Sinh[c + d*x])/(b^3*d^2) - (f*Sinh[c + d*x])
/(3*b*d^2) - (a*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^2*d) - (f*Sinh[
c + d*x]^3)/(9*b*d^2)
```

Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
```

```
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh^2(c+dx)\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}}{b}$$

$$= \frac{(e+fx)\cosh^3(c+dx)}{3bd} - \frac{a\int (e+fx)\cosh^2(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)\cosh}{a+b\sinh(}}{b^2}$$

$$= \frac{af\cosh^2(c+dx)}{4b^2d^2} + \frac{(e+fx)\cosh^3(c+dx)}{3bd} - \frac{a(e+fx)\cosh(c+dx)\sin}{2b^2d}$$

$$= -\frac{a^3ex}{b^4} - \frac{aex}{2b^2} - \frac{a^3fx^2}{2b^4} - \frac{afx^2}{4b^2} + \frac{a^2(e+fx)\cosh(c+dx)}{b^3d} + \frac{af\cosh^2(c}{4b^2d^2}$$

$$= -\frac{a^3ex}{b^4} - \frac{aex}{2b^2} - \frac{a^3fx^2}{2b^4} - \frac{afx^2}{4b^2} + \frac{a^2(e+fx)\cosh(c+dx)}{b^3d} + \frac{af\cosh^2(c}{4b^2d^2}$$

$$= -\frac{a^3ex}{b^4} - \frac{aex}{2b^2} - \frac{a^3fx^2}{2b^4} - \frac{afx^2}{4b^2} + \frac{a^2(e+fx)\cosh(c+dx)}{b^3d} + \frac{af\cosh^2(c}{4b^2d^2}$$

$$= -\frac{a^3ex}{b^4} - \frac{aex}{2b^2} - \frac{a^3fx^2}{2b^4} - \frac{afx^2}{4b^2} + \frac{a^2(e+fx)\cosh(c+dx)}{b^3d} + \frac{af\cosh^2(c}{4b^2d^2}$$

$$= -\frac{a^3ex}{b^4} - \frac{aex}{2b^2} - \frac{a^3fx^2}{2b^4} - \frac{afx^2}{4b^2} + \frac{a^2(e+fx)\cosh(c+dx)}{b^3d} + \frac{af\cosh^2(c}{4b^2d^2}$$

Mathematica [A] time = 3.05828, size = 676, normalized size = 1.68

$$-\frac{-72a^2f\sqrt{a^2+b^2}\text{PolyLog}\left(2,\frac{b(\sinh(c+dx)+\cosh(c+dx))}{\sqrt{a^2+b^2}-a}\right)+72a^2f\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{b(\sinh(c+dx)+\cosh(c+dx))}{\sqrt{a^2+b^2}+a}\right)+144a^2c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] -(72*a^3*c*d*e + 36*a*b^2*c*d*e - 36*a^3*c^2*f - 18*a*b^2*c^2*f + 72*a^3*d^
2*e*x + 36*a*b^2*d^2*e*x + 36*a^3*d^2*f*x^2 + 18*a*b^2*d^2*f*x^2 + 144*a^2*
Sqrt[a^2 + b^2]*d*e*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^
2 + b^2]] - 144*a^2*Sqrt[a^2 + b^2]*c*f*ArcTanh[(a + b*Cosh[c + d*x] + b*Si
nh[c + d*x])/Sqrt[a^2 + b^2]] - 72*a^2*b*d*e*Cosh[c + d*x] - 18*b^3*d*e*Cos
h[c + d*x] - 72*a^2*b*d*f*x*Cosh[c + d*x] - 18*b^3*d*f*x*Cosh[c + d*x] - 9*
a*b^2*f*Cosh[2*(c + d*x)] - 6*b^3*d*e*Cosh[3*(c + d*x)] - 6*b^3*d*f*x*Cosh[
3*(c + d*x)] - 72*a^2*Sqrt[a^2 + b^2]*c*f*Log[1 + (b*(Cosh[c + d*x] + Sinh[
c + d*x]))/(a - Sqrt[a^2 + b^2])] - 72*a^2*Sqrt[a^2 + b^2]*d*f*x*Log[1 + (b
*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] + 72*a^2*Sqrt[a^2
```

```
+ b^2]*c*f*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]
)] + 72*a^2*Sqrt[a^2 + b^2]*d*f*x*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]
))/(a + Sqrt[a^2 + b^2])] - 72*a^2*Sqrt[a^2 + b^2]*f*PolyLog[2, (b*(Cosh[c
+ d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])] + 72*a^2*Sqrt[a^2 + b^2]*f
*PolyLog[2, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))] +
 72*a^2*b*f*Sinh[c + d*x] + 18*b^3*f*Sinh[c + d*x] + 18*a*b^2*d*e*Sinh[2*(c
 + d*x)] + 18*a*b^2*d*f*x*Sinh[2*(c + d*x)] + 2*b^3*f*Sinh[3*(c + d*x)])/(7
2*b^4*d^2)
```

Maple [B] time = 0.106, size = 1128, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/16*a*(2*d*f*x+2*d*e-f)/b^2/d^2*exp(2*d*x+2*c)+1/16*a*(2*d*f*x+2*d*e+f)/b
^2/d^2*exp(-2*d*x-2*c)-1/2*a^3*f*x^2/b^4-1/4*a*f*x^2/b^2-a^4/b^4/d^2*f/(a^2
+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2*a
^4/b^4/d*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2)
)+a^4/b^4/d^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a
+(a^2+b^2)^(1/2)))+1/8*(4*a^2+b^2)*(d*f*x+d*e+f)/b^3/d^2*exp(-d*x-c)-a^3*e*
x/b^4-1/2*a*e*x/b^2-2*a^2/b^2/d*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+
c)+2*a)/(a^2+b^2)^(1/2))+a^2/b^2/d^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)
+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-a^2/b^2/d^2*f/(a^2+b^2)^(1/2)*dil
og((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+a^4/b^4/d*f/(a^2+b
^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+a^4/
b^4/d^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2
)^(1/2)))*c-a^4/b^4/d*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)
/(a+(a^2+b^2)^(1/2)))*x-a^4/b^4/d^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2
+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2*a^4/b^4/d^2*f*c/(a^2+b^2)^(1/2)*arc
tanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+a^2/b^2/d*f/(a^2+b^2)^(1/2)*
ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+a^2/b^2/d^2*f/
(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*
c-a^2/b^2/d*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b
^2)^(1/2)))*x-a^2/b^2/d^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2
)+a)/(a+(a^2+b^2)^(1/2)))*c+2*a^2/b^2/d^2*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(
2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/72*(3*d*f*x+3*d*e+f)/d^2/b*exp(-3*d*
x-3*c)+1/72*(3*d*f*x+3*d*e-f)/d^2/b*exp(3*d*x+3*c)+1/8*(4*a^2*d*f*x+b^2*d*f
*x+4*a^2*d*e+b^2*d*e-4*a^2*f-b^2*f)/b^3/d^2*exp(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.80926, size = 5268, normalized size = 13.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/144*(2*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^6 + 2*(3*b^3*d*f*x
 + 3*b^3*d*e - b^3*f)*sinh(d*x + c)^6 + 6*b^3*d*f*x - 9*(2*a*b^2*d*f*x + 2*
a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^5 - 3*(6*a*b^2*d*f*x + 6*a*b^2*d*e - 3*a
*b^2*f - 4*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c))*sinh(d*x + c)^5
 + 6*b^3*d*e + 18*((4*a^2*b + b^3)*d*f*x + (4*a^2*b + b^3)*d*e - (4*a^2*b +
 b^3)*f)*cosh(d*x + c)^4 + 3*(6*(4*a^2*b + b^3)*d*f*x + 6*(4*a^2*b + b^3)*d
*e + 10*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^2 - 6*(4*a^2*b + b^
3)*f - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c))*sinh(d*x +
 c)^4 + 2*b^3*f - 36*((2*a^3 + a*b^2)*d^2*f*x^2 + 2*(2*a^3 + a*b^2)*d^2*e*x
)*cosh(d*x + c)^3 - 2*(18*(2*a^3 + a*b^2)*d^2*f*x^2 + 36*(2*a^3 + a*b^2)*d^
2*e*x - 20*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^3 + 45*(2*a*b^2*
d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^2 - 36*((4*a^2*b + b^3)*d*f*x
+ (4*a^2*b + b^3)*d*e - (4*a^2*b + b^3)*f)*cosh(d*x + c))*sinh(d*x + c)^3 +
 18*((4*a^2*b + b^3)*d*f*x + (4*a^2*b + b^3)*d*e + (4*a^2*b + b^3)*f)*cosh(
d*x + c)^2 + 6*(5*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^4 + 3*(4*
a^2*b + b^3)*d*f*x - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x +
c)^3 + 3*(4*a^2*b + b^3)*d*e + 18*((4*a^2*b + b^3)*d*f*x + (4*a^2*b + b^3)*
d*e - (4*a^2*b + b^3)*f)*cosh(d*x + c)^2 + 3*(4*a^2*b + b^3)*f - 18*((2*a^3
 + a*b^2)*d^2*f*x^2 + 2*(2*a^3 + a*b^2)*d^2*e*x)*cosh(d*x + c))*sinh(d*x +
c)^2 + 144*(a^2*b*f*cosh(d*x + c)^3 + 3*a^2*b*f*cosh(d*x + c)^2*sinh(d*x +
c) + 3*a^2*b*f*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*b*f*sinh(d*x + c)^3)*sqr
t((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
 c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 144*(a^2*b*f*cos
h(d*x + c)^3 + 3*a^2*b*f*cosh(d*x + c)^2*sinh(d*x + c) + 3*a^2*b*f*cosh(d*x
 + c)*sinh(d*x + c)^2 + a^2*b*f*sinh(d*x + c)^3)*sqrt((a^2 + b^2)/b^2)*dilo
g((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 144*((a^2*b*d*e - a^2*b*c*f)*cosh(d*x +
 c)^3 + 3*(a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^2*b*
d*e - a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2*b*d*e - a^2*b*c*f)*si
nh(d*x + c)^3)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
 c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 144*((a^2*b*d*e - a^2*b*c*f)*cosh(
d*x + c)^3 + 3*(a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a
^2*b*d*e - a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2*b*d*e - a^2*b*c*
f)*sinh(d*x + c)^3)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(
d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 144*((a^2*b*d*f*x + a^2*b*c*f
)*cosh(d*x + c)^3 + 3*(a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^2*sinh(d*x +
c) + 3*(a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2*b*d*f
*x + a^2*b*c*f)*sinh(d*x + c)^3)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c
) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2) - b)/b) - 144*((a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^3 + 3*(a^2*b*d*
f*x + a^2*b*c*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^2*b*d*f*x + a^2*b*c*f
)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2*b*d*f*x + a^2*b*c*f)*sinh(d*x + c)^3
)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 9*(2*a*b^2*d*f*x
 + 2*a*b^2*d*e + a*b^2*f)*cosh(d*x + c) + 3*(6*a*b^2*d*f*x + 4*(3*b^3*d*f*x
 + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^5 + 6*a*b^2*d*e - 15*(2*a*b^2*d*f*x + 2
*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^4 + 3*a*b^2*f + 24*((4*a^2*b + b^3)*d*f
*x + (4*a^2*b + b^3)*d*e - (4*a^2*b + b^3)*f)*cosh(d*x + c)^3 - 36*((2*a^3
+ a*b^2)*d^2*f*x^2 + 2*(2*a^3 + a*b^2)*d^2*e*x)*cosh(d*x + c)^2 + 12*((4*a^
2*b + b^3)*d*f*x + (4*a^2*b + b^3)*d*e + (4*a^2*b + b^3)*f)*cosh(d*x + c))*
```

```
sinh(d*x + c))/(b^4*d^2*cosh(d*x + c)^3 + 3*b^4*d^2*cosh(d*x + c)^2*sinh(d*
x + c) + 3*b^4*d^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^4*d^2*sinh(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**2*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\cosh(dx+c)^2\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)^2*sinh(d*x + c)^2/(b*sinh(d*x + c) + a),
x)
```

3.370 $\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=141

$$\frac{(3a^2+b^2)\cosh(c+dx)}{3b^3d}-\frac{2a^2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{ax(2a^2+b^2)}{2b^4}-\frac{a\sinh(c+dx)\cosh(c+dx)}{2b^2d}+\text{si}$$

```
[Out] -(a*(2*a^2 + b^2)*x)/(2*b^4) - (2*a^2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[(
c + d*x)/2])/Sqrt[a^2 + b^2]])/(b^4*d) + ((3*a^2 + b^2)*Cosh[c + d*x])/(3*b
^3*d) - (a*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^2*d) + (Cosh[c + d*x]*Sinh[c +
 d*x]^2)/(3*b*d)
```

Rubi [A] time = 0.498199, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3050, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{(3a^2+b^2)\cosh(c+dx)}{3b^3d}-\frac{2a^2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{ax(2a^2+b^2)}{2b^4}-\frac{a\sinh(c+dx)\cosh(c+dx)}{2b^2d}+\text{si}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(a*(2*a^2 + b^2)*x)/(2*b^4) - (2*a^2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[(
c + d*x)/2])/Sqrt[a^2 + b^2]])/(b^4*d) + ((3*a^2 + b^2)*Cosh[c + d*x])/(3*b
^3*d) - (a*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^2*d) + (Cosh[c + d*x]*Sinh[c +
 d*x]^2)/(3*b*d)
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
```

```
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh^2(c+dx)\left(1+\sinh^2(c+dx)\right)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{\cosh(c+dx)\sinh^2(c+dx)}{3bd} + \frac{\int \frac{\sinh(c+dx)\left(-2a+b\sinh(c+dx)-3a\sinh^2(c+dx)\right)}{a+b\sinh(c+dx)}dx}{3b}$$

$$= -\frac{a\cosh(c+dx)\sinh(c+dx)}{2b^2d} + \frac{\cosh(c+dx)\sinh^2(c+dx)}{3bd} + \frac{\int \frac{3a^2-ab\sinh(c+dx)+}{a+b\sinh}}{}$$

$$= \frac{\left(3a^2+b^2\right)\cosh(c+dx)}{3b^3d} - \frac{a\cosh(c+dx)\sinh(c+dx)}{2b^2d} + \frac{\cosh(c+dx)\sinh^2(c+}{3bd}$$

$$= -\frac{a\left(2a^2+b^2\right)x}{2b^4} + \frac{\left(3a^2+b^2\right)\cosh(c+dx)}{3b^3d} - \frac{a\cosh(c+dx)\sinh(c+dx)}{2b^2d} + \cosh$$

$$= -\frac{a\left(2a^2+b^2\right)x}{2b^4} + \frac{\left(3a^2+b^2\right)\cosh(c+dx)}{3b^3d} - \frac{a\cosh(c+dx)\sinh(c+dx)}{2b^2d} + \cosh$$

$$= -\frac{a\left(2a^2+b^2\right)x}{2b^4} + \frac{\left(3a^2+b^2\right)\cosh(c+dx)}{3b^3d} - \frac{a\cosh(c+dx)\sinh(c+dx)}{2b^2d} + \cosh$$

$$= -\frac{a\left(2a^2+b^2\right)x}{2b^4} - \frac{2a^2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^4d} + \frac{\left(3a^2+b^2\right)\cosh(c+}{3b^3d}$$

Mathematica [A] time = 0.391655, size = 123, normalized size = 0.87

$$\frac{3b\left(4a^2+b^2\right)\cosh(c+dx) - 3a\left(2\left(2a^2+b^2\right)(c+dx) + 8a\sqrt{-a^2-b^2}\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right) + b^2\sinh(2(c+dx))\right) +}{12b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (3*b*(4*a^2 + b^2)*Cosh[c + d*x] + b^3*Cosh[3*(c + d*x)] - 3*a*(2*(2*a^2 +
b^2)*(c + d*x) + 8*a*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt
[-a^2 - b^2]] + b^2*Sinh[2*(c + d*x)]))/(12*b^4*d)
```

Maple [B] time = 0.037, size = 398, normalized size = 2.8

$$\frac{1}{3bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3} - \frac{1}{2bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2} + \frac{a}{2db^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2} + \frac{a^2}{db^3}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/3/d/b/(tanh(1/2*d*x+1/2*c)+1)^3-1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/b
^2/(tanh(1/2*d*x+1/2*c)+1)^2*a+1/d/b^3/(tanh(1/2*d*x+1/2*c)+1)*a^2-1/2/d/b^
2/(tanh(1/2*d*x+1/2*c)+1)*a+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)-1/d*a^3/b^4*ln(
tanh(1/2*d*x+1/2*c)+1)-1/2/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/3/d/b/(tanh(
1/2*d*x+1/2*c)-1)^3-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^2*a-1/2/d/b/(tanh(1/2
*d*x+1/2*c)-1)^2-1/d/b^3/(tanh(1/2*d*x+1/2*c)-1)*a^2-1/2/d/b^2/(tanh(1/2*d*
```

```
x+1/2*c)-1)*a-1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)+1/d*a^3/b^4*ln(tanh(1/2*d*x+1
/2*c)-1)+1/2/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)+2/d*a^2*(a^2+b^2)^(1/2)/b^4*
arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.13802, size = 1882, normalized size = 13.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="frica
s")
```

```
[Out] 1/24*(b^3*cosh(d*x + c)^6 + b^3*sinh(d*x + c)^6 - 3*a*b^2*cosh(d*x + c)^5 -
 12*(2*a^3 + a*b^2)*d*x*cosh(d*x + c)^3 + 3*(2*b^3*cosh(d*x + c) - a*b^2)*s
inh(d*x + c)^5 + 3*(4*a^2*b + b^3)*cosh(d*x + c)^4 + 3*(5*b^3*cosh(d*x + c)
^2 - 5*a*b^2*cosh(d*x + c) + 4*a^2*b + b^3)*sinh(d*x + c)^4 + 3*a*b^2*cosh(
d*x + c) + 2*(10*b^3*cosh(d*x + c)^3 - 15*a*b^2*cosh(d*x + c)^2 - 6*(2*a^3
+ a*b^2)*d*x + 6*(4*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + b^3 + 3*(
4*a^2*b + b^3)*cosh(d*x + c)^2 + 3*(5*b^3*cosh(d*x + c)^4 - 10*a*b^2*cosh(d
*x + c)^3 - 12*(2*a^3 + a*b^2)*d*x*cosh(d*x + c) + 4*a^2*b + b^3 + 6*(4*a^2
*b + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 24*(a^2*cosh(d*x + c)^3 + 3*a^
2*cosh(d*x + c)^2*sinh(d*x + c) + 3*a^2*cosh(d*x + c)*sinh(d*x + c)^2 + a^2
*sinh(d*x + c)^3)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x +
 c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sin
h(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*
cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c
) + a)*sinh(d*x + c) - b)) + 3*(2*b^3*cosh(d*x + c)^5 - 5*a*b^2*cosh(d*x +
c)^4 - 12*(2*a^3 + a*b^2)*d*x*cosh(d*x + c)^2 + 4*(4*a^2*b + b^3)*cosh(d*x
+ c)^3 + a*b^2 + 2*(4*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c))/(b^4*d*cos
h(d*x + c)^3 + 3*b^4*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^4*d*cosh(d*x + c
)*sinh(d*x + c)^2 + b^4*d*sinh(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 1.16917, size = 309, normalized size = 2.19

$$-\frac{(2a^3+ab^2)(dx+c)}{2b^4d}+\frac{\left(3ab^2e^{(dx+c)}+b^3+3\left(4a^2b+b^3\right)e^{(2dx+2c)}\right)e^{(-3dx-3c)}}{24b^4d}+\frac{\left(a^4+a^2b^2\right)\log\left(\frac{\left|2be^{(dx+c)}+2a-2\sqrt{a^2+b^2}\right|}{\left|2be^{(dx+c)}+2a+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac"
)
```

```
[Out] -1/2*(2*a^3 + a*b^2)*(d*x + c)/(b^4*d) + 1/24*(3*a*b^2*e^(d*x + c) + b^3 +
3*(4*a^2*b + b^3)*e^(2*d*x + 2*c))*e^(-3*d*x - 3*c)/(b^4*d) + (a^4 + a^2*b^
2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c) +
 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4*d) + 1/24*(b^2*d^2*e^(3*d*x
 + 3*c) - 3*a*b*d^2*e^(2*d*x + 2*c) + 12*a^2*d^2*e^(d*x + c) + 3*b^2*d^2*e^
(d*x + c))/(b^3*d^3)
```

3.371 $\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=38

$$\text{Unintegrable}\left(\frac{\sinh^2(c+dx)\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Cosh[c + d*x]^2*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d
*x])), x]
```

Rubi [A] time = 0.125769, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[c + d*x]^2*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Cosh[c + d*x]^2*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^2(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]
)),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.135, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx+c))^2(\sinh(dx+c))^2}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$2\left(a^4e^c + a^2b^2e^c\right)\int -\frac{e^{(dx)}}{b^5fx + b^5e - \left(b^5fxe^{(2c)} + b^5ee^{(2c)}\right)e^{(2dx)} - 2\left(ab^4fxe^c + ab^4ee^c\right)e^{(dx)}}\,dx + \frac{e^{\left(-3c+\frac{3de}{f}\right)}E_1\left(\frac{3(fx+e)d}{f}\right)}{8bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] 2*(a^4*e^c + a^2*b^2*e^c)*integrate(-e^(d*x)/(b^5*f*x + b^5*e - (b^5*f*x*e^
(2*c) + b^5*e*e^(2*c))*e^(2*d*x) - 2*(a*b^4*f*x*e^c + a*b^4*e*e^c)*e^(d*x))
, x) + 1/8*e^(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b*f) + 1/
4*a*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b^2*f) + 1/4*a*e
^(2*c - 2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b^2*f) - 1/8*e^(3*c -
 3*d*e/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b*f) + 1/8*(4*a^2 + b^2)*e^(
-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^3*f) - 1/8*(4*a^2*e^c + b^2
*e^c)*e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^3*f) - 1/2*(2*a^3 + a
*b^2)*log(f*x + e)/(b^4*f)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx+c)^2\sinh(dx+c)^2}{afx + ae + (bfx + be)\sinh(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] integral(cosh(d*x + c)^2*sinh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(
d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*sinh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] sage0*x
```

3.372 $\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1123

result too large to display

```
[Out] (3*a^2*f^3*x)/(8*b^3*d^3) - (45*f^3*x)/(256*b*d^3) + (a^2*(e + f*x)^3)/(4*b
^3*d) - (3*(e + f*x)^3)/(32*b*d) - (a^2*(a^2 + b^2)*(e + f*x)^4)/(4*b^5*f)
+ (6*a^3*f^3*Cosh[c + d*x])/(b^4*d^4) + (40*a*f^3*Cosh[c + d*x])/(9*b^2*d^4
) + (3*a^3*f*(e + f*x)^2*Cosh[c + d*x])/(b^4*d^2) + (2*a*f*(e + f*x)^2*Cosh
[c + d*x])/(b^2*d^2) + (9*f^2*(e + f*x)*Cosh[c + d*x]^2)/(32*b*d^3) + (2*a*
f^3*Cosh[c + d*x]^3)/(27*b^2*d^4) + (a*f*(e + f*x)^2*Cosh[c + d*x]^3)/(3*b^
2*d^2) + (3*f^2*(e + f*x)*Cosh[c + d*x]^4)/(32*b*d^3) + ((e + f*x)^3*Cosh[c
 + d*x]^4)/(4*b*d) + (a^2*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])])/(b^5*d) + (a^2*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^5*d) + (3*a^2*(a^2 + b^2)*f*(e + f*x)
^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^2) + (3*a^2
*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
]))])/(b^5*d^2) - (6*a^2*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^3) - (6*a^2*(a^2 + b^2)*f^2*(e + f*x)*
PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^3) + (6*a^2*(a
^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^
4) + (6*a^2*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2]))])/(b^5*d^4) - (6*a^3*f^2*(e + f*x)*Sinh[c + d*x])/(b^4*d^3) - (40*a*f^
2*(e + f*x)*Sinh[c + d*x])/(9*b^2*d^3) - (a^3*(e + f*x)^3*Sinh[c + d*x])/(b
^4*d) - (2*a*(e + f*x)^3*Sinh[c + d*x])/(3*b^2*d) - (3*a^2*f^3*Cosh[c + d*x
]*Sinh[c + d*x])/(8*b^3*d^4) - (45*f^3*Cosh[c + d*x]*Sinh[c + d*x])/(256*b*
d^4) - (3*a^2*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^3*d^2) - (9*f
*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(32*b*d^2) - (2*a*f^2*(e + f*x)*C
osh[c + d*x]^2*Sinh[c + d*x])/(9*b^2*d^3) - (a*(e + f*x)^3*Cosh[c + d*x]^2*
Sinh[c + d*x])/(3*b^2*d) - (3*f^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(128*b*d^4
) - (3*f*(e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(16*b*d^2) + (3*a^2*f^2
*(e + f*x)*Sinh[c + d*x]^2)/(4*b^3*d^3) + (a^2*(e + f*x)^3*Sinh[c + d*x]^2)
/(2*b^3*d)
```

Rubi [A] time = 1.52371, antiderivative size = 1123, normalized size of antiderivative = 1., number of steps used = 40, number of rules used = 17, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.472, Rules used = {5579, 5447, 3311, 32, 2635, 8, 3296, 2638, 3310, 5565, 5446, 5561, 2190, 2531, 6609, 2282, 6589}

$$-\frac{a^2\left(a^2+b^2\right)(e+fx)^4}{4b^5f}+\frac{\cosh^4(c+dx)(e+fx)^3}{4bd}+\frac{a^2\sinh^2(c+dx)(e+fx)^3}{2b^3d}+\frac{a^2\left(a^2+b^2\right)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)(e+f}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (3*a^2*f^3*x)/(8*b^3*d^3) - (45*f^3*x)/(256*b*d^3) + (a^2*(e + f*x)^3)/(4*b
^3*d) - (3*(e + f*x)^3)/(32*b*d) - (a^2*(a^2 + b^2)*(e + f*x)^4)/(4*b^5*f)
+ (6*a^3*f^3*Cosh[c + d*x])/(b^4*d^4) + (40*a*f^3*Cosh[c + d*x])/(9*b^2*d^4
) + (3*a^3*f*(e + f*x)^2*Cosh[c + d*x])/(b^4*d^2) + (2*a*f*(e + f*x)^2*Cosh
[c + d*x])/(b^2*d^2) + (9*f^2*(e + f*x)*Cosh[c + d*x]^2)/(32*b*d^3) + (2*a*
f^3*Cosh[c + d*x]^3)/(27*b^2*d^4) + (a*f*(e + f*x)^2*Cosh[c + d*x]^3)/(3*b^
2*d^2) + (3*f^2*(e + f*x)*Cosh[c + d*x]^4)/(32*b*d^3) + ((e + f*x)^3*Cosh[c
 + d*x]^4)/(4*b*d) + (a^2*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(
```

```
a - Sqrt[a^2 + b^2])])/(b^5*d) + (a^2*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^5*d) + (3*a^2*(a^2 + b^2)*f*(e + f*x)
^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^2) + (3*a^2
*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
]))])/(b^5*d^2) - (6*a^2*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^3) - (6*a^2*(a^2 + b^2)*f^2*(e + f*x)*
PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^3) + (6*a^2*(a
^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^
4) + (6*a^2*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2]))])/(b^5*d^4) - (6*a^3*f^2*(e + f*x)*Sinh[c + d*x])/(b^4*d^3) - (40*a*f^
2*(e + f*x)*Sinh[c + d*x])/(9*b^2*d^3) - (a^3*(e + f*x)^3*Sinh[c + d*x])/(b
^4*d) - (2*a*(e + f*x)^3*Sinh[c + d*x])/(3*b^2*d) - (3*a^2*f^3*Cosh[c + d*x
]*Sinh[c + d*x])/(8*b^3*d^4) - (45*f^3*Cosh[c + d*x]*Sinh[c + d*x])/(256*b*
d^4) - (3*a^2*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^3*d^2) - (9*f
*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(32*b*d^2) - (2*a*f^2*(e + f*x)*C
osh[c + d*x]^2*Sinh[c + d*x])/(9*b^2*d^3) - (a*(e + f*x)^3*Cosh[c + d*x]^2*
Sinh[c + d*x])/(3*b^2*d) - (3*f^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(128*b*d^4
) - (3*f*(e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(16*b*d^2) + (3*a^2*f^2
*(e + f*x)*Sinh[c + d*x]^2)/(4*b^3*d^3) + (a^2*(e + f*x)^3*Sinh[c + d*x]^2)
/(2*b^3*d)
```

Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\cosh^3(c+dx)\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}}{b}$$

$$= \frac{(e+fx)^3\cosh^4(c+dx)}{4bd} - \frac{a\int (e+fx)^3\cosh^3(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^3\cos}{a+b\sinh(}}{b^2}$$

$$= \frac{af(e+fx)^2\cosh^3(c+dx)}{3b^2d^2} + \frac{3f^2(e+fx)\cosh^4(c+dx)}{32bd^3} + \frac{(e+fx)^3\cosh^4}{4bd}$$

$$= -\frac{a^2\left(a^2+b^2\right)(e+fx)^4}{4b^5f} + \frac{9f^2(e+fx)\cosh^2(c+dx)}{32bd^3} + \frac{2af^3\cosh^3(c+dx}{27b^2d^4}$$

$$= -\frac{3(e+fx)^3}{32bd} - \frac{a^2\left(a^2+b^2\right)(e+fx)^4}{4b^5f} + \frac{3a^3f(e+fx)^2\cosh(c+dx)}{b^4d^2} + \frac{2af}{}$$

$$= -\frac{45f^3x}{256bd^3} + \frac{a^2(e+fx)^3}{4b^3d} - \frac{3(e+fx)^3}{32bd} - \frac{a^2\left(a^2+b^2\right)(e+fx)^4}{4b^5f} + \frac{4af^3\cos}{9b^2}$$

$$= \frac{3a^2f^3x}{8b^3d^3} - \frac{45f^3x}{256bd^3} + \frac{a^2(e+fx)^3}{4b^3d} - \frac{3(e+fx)^3}{32bd} - \frac{a^2\left(a^2+b^2\right)(e+fx)^4}{4b^5f} +$$

$$= \frac{3a^2f^3x}{8b^3d^3} - \frac{45f^3x}{256bd^3} + \frac{a^2(e+fx)^3}{4b^3d} - \frac{3(e+fx)^3}{32bd} - \frac{a^2\left(a^2+b^2\right)(e+fx)^4}{4b^5f} +$$

$$= \frac{3a^2f^3x}{8b^3d^3} - \frac{45f^3x}{256bd^3} + \frac{a^2(e+fx)^3}{4b^3d} - \frac{3(e+fx)^3}{32bd} - \frac{a^2\left(a^2+b^2\right)(e+fx)^4}{4b^5f} +$$

Mathematica [B] time = 42.1848, size = 8734, normalized size = 7.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x
]),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.273, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 (\cosh(dx+c))^3 (\sinh(dx+c))^2}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="maxima")
```

```
[Out] -1/192*e^3*((8*a*b^2*e^(-d*x - c) - 3*b^3 - 12*(2*a^2*b + b^3)*e^(-2*d*x -
2*c) + 24*(4*a^3 + 3*a*b^2)*e^(-3*d*x - 3*c))*e^(4*d*x + 4*c)/(b^4*d) - 192
*(a^4 + a^2*b^2)*(d*x + c)/(b^5*d) - (8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(-
4*d*x - 4*c) + 24*(4*a^3 + 3*a*b^2)*e^(-d*x - c) + 12*(2*a^2*b + b^3)*e^(-2
*d*x - 2*c))/(b^4*d) - 192*(a^4 + a^2*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*
d*x - 2*c) - b)/(b^5*d)) + 1/55296*(13824*(a^4*d^4*f^3*e^(4*c) + a^2*b^2*d^
4*f^3*e^(4*c))*x^4 + 55296*(a^4*d^4*e*f^2*e^(4*c) + a^2*b^2*d^4*e*f^2*e^(4*
c))*x^3 + 82944*(a^4*d^4*e^2*f*e^(4*c) + a^2*b^2*d^4*e^2*f*e^(4*c))*x^2 + 2
7*(32*b^4*d^3*f^3*x^3*e^(8*c) + 24*(4*d^3*e*f^2 - d^2*f^3)*b^4*x^2*e^(8*c)
+ 12*(8*d^3*e^2*f - 4*d^2*e*f^2 + d*f^3)*b^4*x*e^(8*c) - 3*(8*d^2*e^2*f - 4
*d*e*f^2 + f^3)*b^4*e^(8*c))*e^(4*d*x) - 256*(9*a*b^3*d^3*f^3*x^3*e^(7*c) +
 9*(3*d^3*e*f^2 - d^2*f^3)*a*b^3*x^2*e^(7*c) + 3*(9*d^3*e^2*f - 6*d^2*e*f^2
 + 2*d*f^3)*a*b^3*x*e^(7*c) - (9*d^2*e^2*f - 6*d*e*f^2 + 2*f^3)*a*b^3*e^(7*
c))*e^(3*d*x) - 864*(6*(2*d^2*e^2*f - 2*d*e*f^2 + f^3)*a^2*b^2*e^(6*c) + 3*
(2*d^2*e^2*f - 2*d*e*f^2 + f^3)*b^4*e^(6*c) - 4*(2*a^2*b^2*d^3*f^3*e^(6*c)
+ b^4*d^3*f^3*e^(6*c))*x^3 - 6*(2*(2*d^3*e*f^2 - d^2*f^3)*a^2*b^2*e^(6*c) +
 (2*d^3*e*f^2 - d^2*f^3)*b^4*e^(6*c))*x^2 - 6*(2*(2*d^3*e^2*f - 2*d^2*e*f^2
 + d*f^3)*a^2*b^2*e^(6*c) + (2*d^3*e^2*f - 2*d^2*e*f^2 + d*f^3)*b^4*e^(6*c)
)*x)*e^(2*d*x) + 6912*(12*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a^3*b*e^(5*c) + 9
*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a*b^3*e^(5*c) - (4*a^3*b*d^3*f^3*e^(5*c) +
 3*a*b^3*d^3*f^3*e^(5*c))*x^3 - 3*(4*(d^3*e*f^2 - d^2*f^3)*a^3*b*e^(5*c) +
3*(d^3*e*f^2 - d^2*f^3)*a*b^3*e^(5*c))*x^2 - 3*(4*(d^3*e^2*f - 2*d^2*e*f^2
+ 2*d*f^3)*a^3*b*e^(5*c) + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*a*b^3*e^(5
*c))*x)*e^(d*x) + 6912*(12*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*a^3*b*e^(3*c) +
9*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*a*b^3*e^(3*c) + (4*a^3*b*d^3*f^3*e^(3*c)
+ 3*a*b^3*d^3*f^3*e^(3*c))*x^3 + 3*(4*(d^3*e*f^2 + d^2*f^3)*a^3*b*e^(3*c) +
 3*(d^3*e*f^2 + d^2*f^3)*a*b^3*e^(3*c))*x^2 + 3*(4*(d^3*e^2*f + 2*d^2*e*f^2
```

```
 + 2*d*f^3)*a^3*b*e^(3*c) + 3*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*a*b^3*e^(
3*c))*x)*e^(-d*x) + 864*(6*(2*d^2*e^2*f + 2*d*e*f^2 + f^3)*a^2*b^2*e^(2*c)
+ 3*(2*d^2*e^2*f + 2*d*e*f^2 + f^3)*b^4*e^(2*c) + 4*(2*a^2*b^2*d^3*f^3*e^(2
*c) + b^4*d^3*f^3*e^(2*c))*x^3 + 6*(2*(2*d^3*e*f^2 + d^2*f^3)*a^2*b^2*e^(2*
c) + (2*d^3*e*f^2 + d^2*f^3)*b^4*e^(2*c))*x^2 + 6*(2*(2*d^3*e^2*f + 2*d^2*e
*f^2 + d*f^3)*a^2*b^2*e^(2*c) + (2*d^3*e^2*f + 2*d^2*e*f^2 + d*f^3)*b^4*e^(
2*c))*x)*e^(-2*d*x) + 256*(9*a*b^3*d^3*f^3*x^3*e^c + 9*(3*d^3*e*f^2 + d^2*f
^3)*a*b^3*x^2*e^c + 3*(9*d^3*e^2*f + 6*d^2*e*f^2 + 2*d*f^3)*a*b^3*x*e^c + (
9*d^2*e^2*f + 6*d*e*f^2 + 2*f^3)*a*b^3*e^c)*e^(-3*d*x) + 27*(32*b^4*d^3*f^3
*x^3 + 24*(4*d^3*e*f^2 + d^2*f^3)*b^4*x^2 + 12*(8*d^3*e^2*f + 4*d^2*e*f^2 +
 d*f^3)*b^4*x + 3*(8*d^2*e^2*f + 4*d*e*f^2 + f^3)*b^4)*e^(-4*d*x))*e^(-4*c)
/(b^5*d^4) - integrate(-2*((a^4*b*f^3 + a^2*b^3*f^3)*x^3 + 3*(a^4*b*e*f^2 +
 a^2*b^3*e*f^2)*x^2 + 3*(a^4*b*e^2*f + a^2*b^3*e^2*f)*x - ((a^5*f^3*e^c + a
^3*b^2*f^3*e^c)*x^3 + 3*(a^5*e*f^2*e^c + a^3*b^2*e*f^2*e^c)*x^2 + 3*(a^5*e^
2*f*e^c + a^3*b^2*e^2*f*e^c)*x)*e^(d*x))/(b^6*e^(2*d*x + 2*c) + 2*a*b^5*e^(
d*x + c) - b^6), x)
```

Fricas [C] time = 4.35762, size = 26953, normalized size = 24.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="fricas")
```

```
[Out] 1/55296*(864*b^4*d^3*f^3*x^3 + 864*b^4*d^3*e^3 + 648*b^4*d^2*e^2*f + 27*(32
*b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4*d^2*e^2*f + 12*b^4*d*e*f^2 - 3*b
^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3)*x^2 + 12*(8*b^4*d^3*e^2*f - 4*b
^4*d^2*e*f^2 + b^4*d*f^3)*x)*cosh(d*x + c)^8 + 27*(32*b^4*d^3*f^3*x^3 + 32*
b^4*d^3*e^3 - 24*b^4*d^2*e^2*f + 12*b^4*d*e*f^2 - 3*b^4*f^3 + 24*(4*b^4*d^3
*e*f^2 - b^4*d^2*f^3)*x^2 + 12*(8*b^4*d^3*e^2*f - 4*b^4*d^2*e*f^2 + b^4*d*f
^3)*x)*sinh(d*x + c)^8 + 324*b^4*d*e*f^2 - 256*(9*a*b^3*d^3*f^3*x^3 + 9*a*b
^3*d^3*e^3 - 9*a*b^3*d^2*e^2*f + 6*a*b^3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3
*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2 + 3*(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2
+ 2*a*b^3*d*f^3)*x)*cosh(d*x + c)^7 - 8*(288*a*b^3*d^3*f^3*x^3 + 288*a*b^3*
d^3*e^3 - 288*a*b^3*d^2*e^2*f + 192*a*b^3*d*e*f^2 - 64*a*b^3*f^3 + 288*(3*a
*b^3*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2 + 96*(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e
*f^2 + 2*a*b^3*d*f^3)*x - 27*(32*b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4*
d^2*e^2*f + 12*b^4*d*e*f^2 - 3*b^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3)
*x^2 + 12*(8*b^4*d^3*e^2*f - 4*b^4*d^2*e*f^2 + b^4*d*f^3)*x)*cosh(d*x + c))
*sinh(d*x + c)^7 + 81*b^4*f^3 + 864*(4*(2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 4*(2
*a^2*b^2 + b^4)*d^3*e^3 - 6*(2*a^2*b^2 + b^4)*d^2*e^2*f + 6*(2*a^2*b^2 + b^
4)*d*e*f^2 - 3*(2*a^2*b^2 + b^4)*f^3 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e*f^2 - (
2*a^2*b^2 + b^4)*d^2*f^3)*x^2 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e^2*f - 2*(2*a^2
*b^2 + b^4)*d^2*e*f^2 + (2*a^2*b^2 + b^4)*d*f^3)*x)*cosh(d*x + c)^6 + 4*(86
4*(2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 864*(2*a^2*b^2 + b^4)*d^3*e^3 - 1296*(2*a
^2*b^2 + b^4)*d^2*e^2*f + 1296*(2*a^2*b^2 + b^4)*d*e*f^2 - 648*(2*a^2*b^2 +
 b^4)*f^3 + 1296*(2*(2*a^2*b^2 + b^4)*d^3*e*f^2 - (2*a^2*b^2 + b^4)*d^2*f^3
)*x^2 + 189*(32*b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4*d^2*e^2*f + 12*b^
4*d*e*f^2 - 3*b^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3)*x^2 + 12*(8*b^4*
d^3*e^2*f - 4*b^4*d^2*e*f^2 + b^4*d*f^3)*x)*cosh(d*x + c)^2 + 1296*(2*(2*a^
2*b^2 + b^4)*d^3*e^2*f - 2*(2*a^2*b^2 + b^4)*d^2*e*f^2 + (2*a^2*b^2 + b^4)*
d*f^3)*x - 448*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 - 9*a*b^3*d^2*e^2*f +
 6*a*b^3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2
+ 3*(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x)*cosh(d*x + c
))*sinh(d*x + c)^6 - 6912*((4*a^3*b + 3*a*b^3)*d^3*f^3*x^3 + (4*a^3*b + 3*a
```

*b^3)*d^3*e^3 - 3*(4*a^3*b + 3*a*b^3)*d^2*e^2*f + 6*(4*a^3*b + 3*a*b^3)*d*e
f^2 - 6(4*a^3*b + 3*a*b^3)*f^3 + 3*((4*a^3*b + 3*a*b^3)*d^3*e*f^2 - (4*a^
3*b + 3*a*b^3)*d^2*f^3)*x^2 + 3*((4*a^3*b + 3*a*b^3)*d^3*e^2*f - 2*(4*a^3*b
 + 3*a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + 3*a*b^3)*d*f^3)*x)*cosh(d*x + c)^5 - 2
4*(288*(4*a^3*b + 3*a*b^3)*d^3*f^3*x^3 + 288*(4*a^3*b + 3*a*b^3)*d^3*e^3 -
864*(4*a^3*b + 3*a*b^3)*d^2*e^2*f + 1728*(4*a^3*b + 3*a*b^3)*d*e*f^2 - 1728
*(4*a^3*b + 3*a*b^3)*f^3 - 63*(32*b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4
*d^2*e^2*f + 12*b^4*d*e*f^2 - 3*b^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3
)*x^2 + 12*(8*b^4*d^3*e^2*f - 4*b^4*d^2*e*f^2 + b^4*d*f^3)*x)*cosh(d*x + c)
^3 + 864*((4*a^3*b + 3*a*b^3)*d^3*e*f^2 - (4*a^3*b + 3*a*b^3)*d^2*f^3)*x^2
+ 224*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 - 9*a*b^3*d^2*e^2*f + 6*a*b^3*
d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2 + 3*(9*a*
b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x)*cosh(d*x + c)^2 + 864
*((4*a^3*b + 3*a*b^3)*d^3*e^2*f - 2*(4*a^3*b + 3*a*b^3)*d^2*e*f^2 + 2*(4*a^
3*b + 3*a*b^3)*d*f^3)*x - 216*(4*(2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 4*(2*a^2*b
^2 + b^4)*d^3*e^3 - 6*(2*a^2*b^2 + b^4)*d^2*e^2*f + 6*(2*a^2*b^2 + b^4)*d*e
f^2 - 3(2*a^2*b^2 + b^4)*f^3 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e*f^2 - (2*a^2*
b^2 + b^4)*d^2*f^3)*x^2 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e^2*f - 2*(2*a^2*b^2 +
 b^4)*d^2*e*f^2 + (2*a^2*b^2 + b^4)*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c)^
5 - 13824*((a^4 + a^2*b^2)*d^4*f^3*x^4 + 4*(a^4 + a^2*b^2)*d^4*e*f^2*x^3 +
6*(a^4 + a^2*b^2)*d^4*e^2*f*x^2 + 4*(a^4 + a^2*b^2)*d^4*e^3*x + 8*(a^4 + a^
2*b^2)*c*d^3*e^3 - 12*(a^4 + a^2*b^2)*c^2*d^2*e^2*f + 8*(a^4 + a^2*b^2)*c^3
*d*e*f^2 - 2*(a^4 + a^2*b^2)*c^4*f^3)*cosh(d*x + c)^4 - 2*(6912*(a^4 + a^2*
b^2)*d^4*f^3*x^4 + 27648*(a^4 + a^2*b^2)*d^4*e*f^2*x^3 + 41472*(a^4 + a^2*b
^2)*d^4*e^2*f*x^2 + 27648*(a^4 + a^2*b^2)*d^4*e^3*x + 55296*(a^4 + a^2*b^2)
*c*d^3*e^3 - 82944*(a^4 + a^2*b^2)*c^2*d^2*e^2*f + 55296*(a^4 + a^2*b^2)*c^
3*d*e*f^2 - 13824*(a^4 + a^2*b^2)*c^4*f^3 - 945*(32*b^4*d^3*f^3*x^3 + 32*b^
4*d^3*e^3 - 24*b^4*d^2*e^2*f + 12*b^4*d*e*f^2 - 3*b^4*f^3 + 24*(4*b^4*d^3*e
*f^2 - b^4*d^2*f^3)*x^2 + 12*(8*b^4*d^3*e^2*f - 4*b^4*d^2*e*f^2 + b^4*d*f^3
)*x)*cosh(d*x + c)^4 + 4480*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 - 9*a*b^
3*d^2*e^2*f + 6*a*b^3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 - a*b^3*
d^2*f^3)*x^2 + 3*(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x)
*cosh(d*x + c)^3 - 6480*(4*(2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 4*(2*a^2*b^2 + b
^4)*d^3*e^3 - 6*(2*a^2*b^2 + b^4)*d^2*e^2*f + 6*(2*a^2*b^2 + b^4)*d*e*f^2 -
 3*(2*a^2*b^2 + b^4)*f^3 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e*f^2 - (2*a^2*b^2 +
b^4)*d^2*f^3)*x^2 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e^2*f - 2*(2*a^2*b^2 + b^4)*
d^2*e*f^2 + (2*a^2*b^2 + b^4)*d*f^3)*x)*cosh(d*x + c)^2 + 17280*((4*a^3*b +
 3*a*b^3)*d^3*f^3*x^3 + (4*a^3*b + 3*a*b^3)*d^3*e^3 - 3*(4*a^3*b + 3*a*b^3)
*d^2*e^2*f + 6*(4*a^3*b + 3*a*b^3)*d*e*f^2 - 6*(4*a^3*b + 3*a*b^3)*f^3 + 3*
((4*a^3*b + 3*a*b^3)*d^3*e*f^2 - (4*a^3*b + 3*a*b^3)*d^2*f^3)*x^2 + 3*((4*a
^3*b + 3*a*b^3)*d^3*e^2*f - 2*(4*a^3*b + 3*a*b^3)*d^2*e*f^2 + 2*(4*a^3*b +
3*a*b^3)*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c)^4 + 6912*((4*a^3*b + 3*a*b^
3)*d^3*f^3*x^3 + (4*a^3*b + 3*a*b^3)*d^3*e^3 + 3*(4*a^3*b + 3*a*b^3)*d^2*e^
2*f + 6*(4*a^3*b + 3*a*b^3)*d*e*f^2 + 6*(4*a^3*b + 3*a*b^3)*f^3 + 3*((4*a^3
*b + 3*a*b^3)*d^3*e*f^2 + (4*a^3*b + 3*a*b^3)*d^2*f^3)*x^2 + 3*((4*a^3*b +
3*a*b^3)*d^3*e^2*f + 2*(4*a^3*b + 3*a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + 3*a*b^3
)*d*f^3)*x)*cosh(d*x + c)^3 + 8*(864*(4*a^3*b + 3*a*b^3)*d^3*f^3*x^3 + 864*
(4*a^3*b + 3*a*b^3)*d^3*e^3 + 2592*(4*a^3*b + 3*a*b^3)*d^2*e^2*f + 189*(32*
b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4*d^2*e^2*f + 12*b^4*d*e*f^2 - 3*b^
4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3)*x^2 + 12*(8*b^4*d^3*e^2*f - 4*b^
4*d^2*e*f^2 + b^4*d*f^3)*x)*cosh(d*x + c)^5 + 5184*(4*a^3*b + 3*a*b^3)*d*e*
f^2 - 1120*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 - 9*a*b^3*d^2*e^2*f + 6*a
*b^3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2 + 3*
(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x)*cosh(d*x + c)^4
+ 5184*(4*a^3*b + 3*a*b^3)*f^3 + 2160*(4*(2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 4*
(2*a^2*b^2 + b^4)*d^3*e^3 - 6*(2*a^2*b^2 + b^4)*d^2*e^2*f + 6*(2*a^2*b^2 +
b^4)*d*e*f^2 - 3*(2*a^2*b^2 + b^4)*f^3 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e*f^2 -
 (2*a^2*b^2 + b^4)*d^2*f^3)*x^2 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e^2*f - 2*(2*a
^2*b^2 + b^4)*d^2*e*f^2 + (2*a^2*b^2 + b^4)*d*f^3)*x)*cosh(d*x + c)^3 + 259

```
2*((4*a^3*b + 3*a*b^3)*d^3*e*f^2 + (4*a^3*b + 3*a*b^3)*d^2*f^3)*x^2 - 8640*
((4*a^3*b + 3*a*b^3)*d^3*f^3*x^3 + (4*a^3*b + 3*a*b^3)*d^3*e^3 - 3*(4*a^3*b
 + 3*a*b^3)*d^2*e^2*f + 6*(4*a^3*b + 3*a*b^3)*d*e*f^2 - 6*(4*a^3*b + 3*a*b^
3)*f^3 + 3*((4*a^3*b + 3*a*b^3)*d^3*e*f^2 - (4*a^3*b + 3*a*b^3)*d^2*f^3)*x^
2 + 3*((4*a^3*b + 3*a*b^3)*d^3*e^2*f - 2*(4*a^3*b + 3*a*b^3)*d^2*e*f^2 + 2*
(4*a^3*b + 3*a*b^3)*d*f^3)*x)*cosh(d*x + c)^2 + 2592*((4*a^3*b + 3*a*b^3)*d
^3*e^2*f + 2*(4*a^3*b + 3*a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + 3*a*b^3)*d*f^3)*x
 - 6912*((a^4 + a^2*b^2)*d^4*f^3*x^4 + 4*(a^4 + a^2*b^2)*d^4*e*f^2*x^3 + 6*
(a^4 + a^2*b^2)*d^4*e^2*f*x^2 + 4*(a^4 + a^2*b^2)*d^4*e^3*x + 8*(a^4 + a^2*
b^2)*c*d^3*e^3 - 12*(a^4 + a^2*b^2)*c^2*d^2*e^2*f + 8*(a^4 + a^2*b^2)*c^3*d
*e*f^2 - 2*(a^4 + a^2*b^2)*c^4*f^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 648*(4
*b^4*d^3*e*f^2 + b^4*d^2*f^3)*x^2 + 864*(4*(2*a^2*b^2 + b^4)*d^3*f^3*x^3 +
4*(2*a^2*b^2 + b^4)*d^3*e^3 + 6*(2*a^2*b^2 + b^4)*d^2*e^2*f + 6*(2*a^2*b^2
+ b^4)*d*e*f^2 + 3*(2*a^2*b^2 + b^4)*f^3 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e*f^2
 + (2*a^2*b^2 + b^4)*d^2*f^3)*x^2 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e^2*f + 2*(2
*a^2*b^2 + b^4)*d^2*e*f^2 + (2*a^2*b^2 + b^4)*d*f^3)*x)*cosh(d*x + c)^2 + 1
2*(288*(2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 288*(2*a^2*b^2 + b^4)*d^3*e^3 + 63*(
32*b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4*d^2*e^2*f + 12*b^4*d*e*f^2 - 3
*b^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3)*x^2 + 12*(8*b^4*d^3*e^2*f - 4
*b^4*d^2*e*f^2 + b^4*d*f^3)*x)*cosh(d*x + c)^6 + 432*(2*a^2*b^2 + b^4)*d^2*
e^2*f - 448*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 - 9*a*b^3*d^2*e^2*f + 6*
a*b^3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2 + 3
*(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x)*cosh(d*x + c)^5
 + 432*(2*a^2*b^2 + b^4)*d*e*f^2 + 1080*(4*(2*a^2*b^2 + b^4)*d^3*f^3*x^3 +
4*(2*a^2*b^2 + b^4)*d^3*e^3 - 6*(2*a^2*b^2 + b^4)*d^2*e^2*f + 6*(2*a^2*b^2
+ b^4)*d*e*f^2 - 3*(2*a^2*b^2 + b^4)*f^3 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e*f^2
 - (2*a^2*b^2 + b^4)*d^2*f^3)*x^2 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e^2*f - 2*(2
*a^2*b^2 + b^4)*d^2*e*f^2 + (2*a^2*b^2 + b^4)*d*f^3)*x)*cosh(d*x + c)^4 + 2
16*(2*a^2*b^2 + b^4)*f^3 - 5760*((4*a^3*b + 3*a*b^3)*d^3*f^3*x^3 + (4*a^3*b
 + 3*a*b^3)*d^3*e^3 - 3*(4*a^3*b + 3*a*b^3)*d^2*e^2*f + 6*(4*a^3*b + 3*a*b^
3)*d*e*f^2 - 6*(4*a^3*b + 3*a*b^3)*f^3 + 3*((4*a^3*b + 3*a*b^3)*d^3*e*f^2 -
 (4*a^3*b + 3*a*b^3)*d^2*f^3)*x^2 + 3*((4*a^3*b + 3*a*b^3)*d^3*e^2*f - 2*(4
*a^3*b + 3*a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + 3*a*b^3)*d*f^3)*x)*cosh(d*x + c)
^3 + 432*(2*(2*a^2*b^2 + b^4)*d^3*e*f^2 + (2*a^2*b^2 + b^4)*d^2*f^3)*x^2 -
6912*((a^4 + a^2*b^2)*d^4*f^3*x^4 + 4*(a^4 + a^2*b^2)*d^4*e*f^2*x^3 + 6*(a^
4 + a^2*b^2)*d^4*e^2*f*x^2 + 4*(a^4 + a^2*b^2)*d^4*e^3*x + 8*(a^4 + a^2*b^2
)*c*d^3*e^3 - 12*(a^4 + a^2*b^2)*c^2*d^2*e^2*f + 8*(a^4 + a^2*b^2)*c^3*d*e*
f^2 - 2*(a^4 + a^2*b^2)*c^4*f^3)*cosh(d*x + c)^2 + 432*(2*(2*a^2*b^2 + b^4)
*d^3*e^2*f + 2*(2*a^2*b^2 + b^4)*d^2*e*f^2 + (2*a^2*b^2 + b^4)*d*f^3)*x + 1
728*((4*a^3*b + 3*a*b^3)*d^3*f^3*x^3 + (4*a^3*b + 3*a*b^3)*d^3*e^3 + 3*(4*a
^3*b + 3*a*b^3)*d^2*e^2*f + 6*(4*a^3*b + 3*a*b^3)*d*e*f^2 + 6*(4*a^3*b + 3*
a*b^3)*f^3 + 3*((4*a^3*b + 3*a*b^3)*d^3*e*f^2 + (4*a^3*b + 3*a*b^3)*d^2*f^3
)*x^2 + 3*((4*a^3*b + 3*a*b^3)*d^3*e^2*f + 2*(4*a^3*b + 3*a*b^3)*d^2*e*f^2
+ 2*(4*a^3*b + 3*a*b^3)*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 324*(8*b
^4*d^3*e^2*f + 4*b^4*d^2*e*f^2 + b^4*d*f^3)*x + 256*(9*a*b^3*d^3*f^3*x^3 +
9*a*b^3*d^3*e^3 + 9*a*b^3*d^2*e^2*f + 6*a*b^3*d*e*f^2 + 2*a*b^3*f^3 + 9*(3*
a*b^3*d^3*e*f^2 + a*b^3*d^2*f^3)*x^2 + 3*(9*a*b^3*d^3*e^2*f + 6*a*b^3*d^2*e
*f^2 + 2*a*b^3*d*f^3)*x)*cosh(d*x + c) + 165888*(((a^4 + a^2*b^2)*d^2*f^3*x
^2 + 2*(a^4 + a^2*b^2)*d^2*e*f^2*x + (a^4 + a^2*b^2)*d^2*e^2*f)*cosh(d*x +
c)^4 + 4*((a^4 + a^2*b^2)*d^2*f^3*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f^2*x + (a^
4 + a^2*b^2)*d^2*e^2*f)*cosh(d*x + c)^3*sinh(d*x + c) + 6*((a^4 + a^2*b^2)*
d^2*f^3*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f^2*x + (a^4 + a^2*b^2)*d^2*e^2*f)*co
sh(d*x + c)^2*sinh(d*x + c)^2 + 4*((a^4 + a^2*b^2)*d^2*f^3*x^2 + 2*(a^4 + a
^2*b^2)*d^2*e*f^2*x + (a^4 + a^2*b^2)*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c
)^3 + ((a^4 + a^2*b^2)*d^2*f^3*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f^2*x + (a^4 +
 a^2*b^2)*d^2*e^2*f)*sinh(d*x + c)^4)*dilog((a*cosh(d*x + c) + a*sinh(d*x +
 c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1)
 + 165888*(((a^4 + a^2*b^2)*d^2*f^3*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f^2*x + (
a^4 + a^2*b^2)*d^2*e^2*f)*cosh(d*x + c)^4 + 4*((a^4 + a^2*b^2)*d^2*f^3*x^2
```

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+ 2*(a^4 + a^2*b^2)*d^2*e*f^2*x + (a^4 + a^2*b^2)*d^2*e^2*f)*cosh(d*x + c)^
3*sinh(d*x + c) + 6*((a^4 + a^2*b^2)*d^2*f^3*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*
f^2*x + (a^4 + a^2*b^2)*d^2*e^2*f)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*((a^
4 + a^2*b^2)*d^2*f^3*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f^2*x + (a^4 + a^2*b^2)*
d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 + a^2*b^2)*d^2*f^3*x^2 + 2
*(a^4 + a^2*b^2)*d^2*e*f^2*x + (a^4 + a^2*b^2)*d^2*e^2*f)*sinh(d*x + c)^4)*
dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 55296*(((a^4 + a^2*b^2)*d^3*e^3 - 3
*(a^4 + a^2*b^2)*c*d^2*e^2*f + 3*(a^4 + a^2*b^2)*c^2*d*e*f^2 - (a^4 + a^2*b
^2)*c^3*f^3)*cosh(d*x + c)^4 + 4*((a^4 + a^2*b^2)*d^3*e^3 - 3*(a^4 + a^2*b^
2)*c*d^2*e^2*f + 3*(a^4 + a^2*b^2)*c^2*d*e*f^2 - (a^4 + a^2*b^2)*c^3*f^3)*c
osh(d*x + c)^3*sinh(d*x + c) + 6*((a^4 + a^2*b^2)*d^3*e^3 - 3*(a^4 + a^2*b^
2)*c*d^2*e^2*f + 3*(a^4 + a^2*b^2)*c^2*d*e*f^2 - (a^4 + a^2*b^2)*c^3*f^3)*c
osh(d*x + c)^2*sinh(d*x + c)^2 + 4*((a^4 + a^2*b^2)*d^3*e^3 - 3*(a^4 + a^2*
b^2)*c*d^2*e^2*f + 3*(a^4 + a^2*b^2)*c^2*d*e*f^2 - (a^4 + a^2*b^2)*c^3*f^3)
*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 + a^2*b^2)*d^3*e^3 - 3*(a^4 + a^2*b^
2)*c*d^2*e^2*f + 3*(a^4 + a^2*b^2)*c^2*d*e*f^2 - (a^4 + a^2*b^2)*c^3*f^3)*s
inh(d*x + c)^4)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 +
 b^2)/b^2) + 2*a) + 55296*(((a^4 + a^2*b^2)*d^3*e^3 - 3*(a^4 + a^2*b^2)*c*d
^2*e^2*f + 3*(a^4 + a^2*b^2)*c^2*d*e*f^2 - (a^4 + a^2*b^2)*c^3*f^3)*cosh(d*
x + c)^4 + 4*((a^4 + a^2*b^2)*d^3*e^3 - 3*(a^4 + a^2*b^2)*c*d^2*e^2*f + 3*(
a^4 + a^2*b^2)*c^2*d*e*f^2 - (a^4 + a^2*b^2)*c^3*f^3)*cosh(d*x + c)^3*sinh(
d*x + c) + 6*((a^4 + a^2*b^2)*d^3*e^3 - 3*(a^4 + a^2*b^2)*c*d^2*e^2*f + 3*(
a^4 + a^2*b^2)*c^2*d*e*f^2 - (a^4 + a^2*b^2)*c^3*f^3)*cosh(d*x + c)^2*sinh(
d*x + c)^2 + 4*((a^4 + a^2*b^2)*d^3*e^3 - 3*(a^4 + a^2*b^2)*c*d^2*e^2*f + 3
*(a^4 + a^2*b^2)*c^2*d*e*f^2 - (a^4 + a^2*b^2)*c^3*f^3)*cosh(d*x + c)*sinh(
d*x + c)^3 + ((a^4 + a^2*b^2)*d^3*e^3 - 3*(a^4 + a^2*b^2)*c*d^2*e^2*f + 3*(
a^4 + a^2*b^2)*c^2*d*e*f^2 - (a^4 + a^2*b^2)*c^3*f^3)*sinh(d*x + c)^4)*log(
2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) +
55296*(((a^4 + a^2*b^2)*d^3*f^3*x^3 + 3*(a^4 + a^2*b^2)*d^3*e*f^2*x^2 + 3*(
a^4 + a^2*b^2)*d^3*e^2*f*x + 3*(a^4 + a^2*b^2)*c*d^2*e^2*f - 3*(a^4 + a^2*b
^2)*c^2*d*e*f^2 + (a^4 + a^2*b^2)*c^3*f^3)*cosh(d*x + c)^4 + 4*((a^4 + a^2*
b^2)*d^3*f^3*x^3 + 3*(a^4 + a^2*b^2)*d^3*e*f^2*x^2 + 3*(a^4 + a^2*b^2)*d^3*
e^2*f*x + 3*(a^4 + a^2*b^2)*c*d^2*e^2*f - 3*(a^4 + a^2*b^2)*c^2*d*e*f^2 + (
a^4 + a^2*b^2)*c^3*f^3)*cosh(d*x + c)^3*sinh(d*x + c) + 6*((a^4 + a^2*b^2)*
d^3*f^3*x^3 + 3*(a^4 + a^2*b^2)*d^3*e*f^2*x^2 + 3*(a^4 + a^2*b^2)*d^3*e^2*f
*x + 3*(a^4 + a^2*b^2)*c*d^2*e^2*f - 3*(a^4 + a^2*b^2)*c^2*d*e*f^2 + (a^4 +
 a^2*b^2)*c^3*f^3)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*((a^4 + a^2*b^2)*d^3
*f^3*x^3 + 3*(a^4 + a^2*b^2)*d^3*e*f^2*x^2 + 3*(a^4 + a^2*b^2)*d^3*e^2*f*x
+ 3*(a^4 + a^2*b^2)*c*d^2*e^2*f - 3*(a^4 + a^2*b^2)*c^2*d*e*f^2 + (a^4 + a^
2*b^2)*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 + a^2*b^2)*d^3*f^3*x^
3 + 3*(a^4 + a^2*b^2)*d^3*e*f^2*x^2 + 3*(a^4 + a^2*b^2)*d^3*e^2*f*x + 3*(a^
4 + a^2*b^2)*c*d^2*e^2*f - 3*(a^4 + a^2*b^2)*c^2*d*e*f^2 + (a^4 + a^2*b^2)*
c^3*f^3)*sinh(d*x + c)^4)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 55296*(((a^4 +
 a^2*b^2)*d^3*f^3*x^3 + 3*(a^4 + a^2*b^2)*d^3*e*f^2*x^2 + 3*(a^4 + a^2*b^2)
*d^3*e^2*f*x + 3*(a^4 + a^2*b^2)*c*d^2*e^2*f - 3*(a^4 + a^2*b^2)*c^2*d*e*f^
2 + (a^4 + a^2*b^2)*c^3*f^3)*cosh(d*x + c)^4 + 4*((a^4 + a^2*b^2)*d^3*f^3*x
^3 + 3*(a^4 + a^2*b^2)*d^3*e*f^2*x^2 + 3*(a^4 + a^2*b^2)*d^3*e^2*f*x + 3*(a
^4 + a^2*b^2)*c*d^2*e^2*f - 3*(a^4 + a^2*b^2)*c^2*d*e*f^2 + (a^4 + a^2*b^2)
*c^3*f^3)*cosh(d*x + c)^3*sinh(d*x + c) + 6*((a^4 + a^2*b^2)*d^3*f^3*x^3 +
3*(a^4 + a^2*b^2)*d^3*e*f^2*x^2 + 3*(a^4 + a^2*b^2)*d^3*e^2*f*x + 3*(a^4 +
a^2*b^2)*c*d^2*e^2*f - 3*(a^4 + a^2*b^2)*c^2*d*e*f^2 + (a^4 + a^2*b^2)*c^3*
f^3)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*((a^4 + a^2*b^2)*d^3*f^3*x^3 + 3*(
a^4 + a^2*b^2)*d^3*e*f^2*x^2 + 3*(a^4 + a^2*b^2)*d^3*e^2*f*x + 3*(a^4 + a^2
*b^2)*c*d^2*e^2*f - 3*(a^4 + a^2*b^2)*c^2*d*e*f^2 + (a^4 + a^2*b^2)*c^3*f^3
)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 + a^2*b^2)*d^3*f^3*x^3 + 3*(a^4 + a
^2*b^2)*d^3*e*f^2*x^2 + 3*(a^4 + a^2*b^2)*d^3*e^2*f*x + 3*(a^4 + a^2*b^2)*c
*d^2*e^2*f - 3*(a^4 + a^2*b^2)*c^2*d*e*f^2 + (a^4 + a^2*b^2)*c^3*f^3)*sinh(
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d*x + c)^4)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 331776*((a^4 + a^2*b^2)*f^3*
cosh(d*x + c)^4 + 4*(a^4 + a^2*b^2)*f^3*cosh(d*x + c)^3*sinh(d*x + c) + 6*(
a^4 + a^2*b^2)*f^3*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^4 + a^2*b^2)*f^3*
cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + a^2*b^2)*f^3*sinh(d*x + c)^4)*polylo
g(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2))/b) + 331776*((a^4 + a^2*b^2)*f^3*cosh(d*x + c)^4
 + 4*(a^4 + a^2*b^2)*f^3*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^4 + a^2*b^2)*f
^3*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^4 + a^2*b^2)*f^3*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^4 + a^2*b^2)*f^3*sinh(d*x + c)^4)*polylog(4, (a*cosh(d*x
 + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2))/b) - 331776*(((a^4 + a^2*b^2)*d*f^3*x + (a^4 + a^2*b^2)*d*e*f^2)*
cosh(d*x + c)^4 + 4*((a^4 + a^2*b^2)*d*f^3*x + (a^4 + a^2*b^2)*d*e*f^2)*cos
h(d*x + c)^3*sinh(d*x + c) + 6*((a^4 + a^2*b^2)*d*f^3*x + (a^4 + a^2*b^2)*d
*e*f^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*((a^4 + a^2*b^2)*d*f^3*x + (a^4
 + a^2*b^2)*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 + a^2*b^2)*d*f^3
*x + (a^4 + a^2*b^2)*d*e*f^2)*sinh(d*x + c)^4)*polylog(3, (a*cosh(d*x + c)
 + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2))/b) - 331776*(((a^4 + a^2*b^2)*d*f^3*x + (a^4 + a^2*b^2)*d*e*f^2)*cosh(d
*x + c)^4 + 4*((a^4 + a^2*b^2)*d*f^3*x + (a^4 + a^2*b^2)*d*e*f^2)*cosh(d*x
 + c)^3*sinh(d*x + c) + 6*((a^4 + a^2*b^2)*d*f^3*x + (a^4 + a^2*b^2)*d*e*f^2
)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*((a^4 + a^2*b^2)*d*f^3*x + (a^4 + a^2
*b^2)*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 + a^2*b^2)*d*f^3*x + (
a^4 + a^2*b^2)*d*e*f^2)*sinh(d*x + c)^4)*polylog(3, (a*cosh(d*x + c) + a*si
nh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b)
 + 8*(288*a*b^3*d^3*f^3*x^3 + 288*a*b^3*d^3*e^3 + 288*a*b^3*d^2*e^2*f + 192
*a*b^3*d*e*f^2 + 27*(32*b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4*d^2*e^2*f
 + 12*b^4*d*e*f^2 - 3*b^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3)*x^2 + 12
*(8*b^4*d^3*e^2*f - 4*b^4*d^2*e*f^2 + b^4*d*f^3)*x)*cosh(d*x + c)^7 + 64*a*
b^3*f^3 - 224*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 - 9*a*b^3*d^2*e^2*f +
6*a*b^3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2 +
 3*(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x)*cosh(d*x + c)
^6 + 648*(4*(2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 4*(2*a^2*b^2 + b^4)*d^3*e^3 - 6
*(2*a^2*b^2 + b^4)*d^2*e^2*f + 6*(2*a^2*b^2 + b^4)*d*e*f^2 - 3*(2*a^2*b^2 +
 b^4)*f^3 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e*f^2 - (2*a^2*b^2 + b^4)*d^2*f^3)*x
^2 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e^2*f - 2*(2*a^2*b^2 + b^4)*d^2*e*f^2 + (2*
a^2*b^2 + b^4)*d*f^3)*x)*cosh(d*x + c)^5 - 4320*((4*a^3*b + 3*a*b^3)*d^3*f^
3*x^3 + (4*a^3*b + 3*a*b^3)*d^3*e^3 - 3*(4*a^3*b + 3*a*b^3)*d^2*e^2*f + 6*(
4*a^3*b + 3*a*b^3)*d*e*f^2 - 6*(4*a^3*b + 3*a*b^3)*f^3 + 3*((4*a^3*b + 3*a*
b^3)*d^3*e*f^2 - (4*a^3*b + 3*a*b^3)*d^2*f^3)*x^2 + 3*((4*a^3*b + 3*a*b^3)*
d^3*e^2*f - 2*(4*a^3*b + 3*a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + 3*a*b^3)*d*f^3)*
x)*cosh(d*x + c)^4 - 6912*((a^4 + a^2*b^2)*d^4*f^3*x^4 + 4*(a^4 + a^2*b^2)*
d^4*e*f^2*x^3 + 6*(a^4 + a^2*b^2)*d^4*e^2*f*x^2 + 4*(a^4 + a^2*b^2)*d^4*e^3
*x + 8*(a^4 + a^2*b^2)*c*d^3*e^3 - 12*(a^4 + a^2*b^2)*c^2*d^2*e^2*f + 8*(a^
4 + a^2*b^2)*c^3*d*e*f^2 - 2*(a^4 + a^2*b^2)*c^4*f^3)*cosh(d*x + c)^3 + 288
*(3*a*b^3*d^3*e*f^2 + a*b^3*d^2*f^3)*x^2 + 2592*((4*a^3*b + 3*a*b^3)*d^3*f^
3*x^3 + (4*a^3*b + 3*a*b^3)*d^3*e^3 + 3*(4*a^3*b + 3*a*b^3)*d^2*e^2*f + 6*(
4*a^3*b + 3*a*b^3)*d*e*f^2 + 6*(4*a^3*b + 3*a*b^3)*f^3 + 3*((4*a^3*b + 3*a*
b^3)*d^3*e*f^2 + (4*a^3*b + 3*a*b^3)*d^2*f^3)*x^2 + 3*((4*a^3*b + 3*a*b^3)*
d^3*e^2*f + 2*(4*a^3*b + 3*a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + 3*a*b^3)*d*f^3)*
x)*cosh(d*x + c)^2 + 96*(9*a*b^3*d^3*e^2*f + 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*
f^3)*x + 216*(4*(2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 4*(2*a^2*b^2 + b^4)*d^3*e^3
 + 6*(2*a^2*b^2 + b^4)*d^2*e^2*f + 6*(2*a^2*b^2 + b^4)*d*e*f^2 + 3*(2*a^2*b
^2 + b^4)*f^3 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e*f^2 + (2*a^2*b^2 + b^4)*d^2*f^
3)*x^2 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e^2*f + 2*(2*a^2*b^2 + b^4)*d^2*e*f^2 +
 (2*a^2*b^2 + b^4)*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c))/(b^5*d^4*cosh(d*
x + c)^4 + 4*b^5*d^4*cosh(d*x + c)^3*sinh(d*x + c) + 6*b^5*d^4*cosh(d*x + c
)^2*sinh(d*x + c)^2 + 4*b^5*d^4*cosh(d*x + c)*sinh(d*x + c)^3 + b^5*d^4*sin
h(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \cosh(dx+c)^3 \sinh(dx+c)^2}{b \sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)^3*sinh(d*x + c)^2/(b*sinh(d*x + c) + a)
, x)
```

3.373 $\int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh^2(c+dx)}{a+b\sinh(c+dx)} \, dx$

Optimal. Leaf size=819

$$\frac{f^2 \cosh^4(c+dx)}{32bd^3} + \frac{(e+fx)^2 \cosh^4(c+dx)}{4bd} + \frac{2af(e+fx)\cosh^3(c+dx)}{9b^2d^2} - \frac{f(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{8bd^2} + 3f^2$$

```
[Out] (a^2*e*f*x)/(2*b^3*d) - (3*e*f*x)/(16*b*d) + (a^2*f^2*x^2)/(4*b^3*d) - (3*f
^2*x^2)/(32*b*d) - (a^2*(a^2 + b^2)*(e + f*x)^3)/(3*b^5*f) + (2*a^3*f*(e +
f*x)*Cosh[c + d*x])/(b^4*d^2) + (4*a*f*(e + f*x)*Cosh[c + d*x])/(3*b^2*d^2)
 + (3*f^2*Cosh[c + d*x]^2)/(32*b*d^3) + (2*a*f*(e + f*x)*Cosh[c + d*x]^3)/(
9*b^2*d^2) + (f^2*Cosh[c + d*x]^4)/(32*b*d^3) + ((e + f*x)^2*Cosh[c + d*x]^
4)/(4*b*d) + (a^2*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2])])/(b^5*d) + (a^2*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2])])/(b^5*d) + (2*a^2*(a^2 + b^2)*f*(e + f*x)*PolyLog
[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^2) + (2*a^2*(a^2 + b^
2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d
^2) - (2*a^2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/(b^5*d^3) - (2*a^2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))])/(b^5*d^3) - (2*a^3*f^2*Sinh[c + d*x])/(b^4*d^3) - (14
*a*f^2*Sinh[c + d*x])/(9*b^2*d^3) - (a^3*(e + f*x)^2*Sinh[c + d*x])/(b^4*d)
 - (2*a*(e + f*x)^2*Sinh[c + d*x])/(3*b^2*d) - (a^2*f*(e + f*x)*Cosh[c + d*
x]*Sinh[c + d*x])/(2*b^3*d^2) - (3*f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])
/(16*b*d^2) - (a*(e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b^2*d) - (f*
(e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x])/(8*b*d^2) + (a^2*f^2*Sinh[c + d*x]
^2)/(4*b^3*d^3) + (a^2*(e + f*x)^2*Sinh[c + d*x]^2)/(2*b^3*d) - (2*a*f^2*Si
nh[c + d*x]^3)/(27*b^2*d^3)
```

Rubi [A] time = 1.17102, antiderivative size = 819, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 14, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5579, 5447, 3310, 3311, 3296, 2637, 2633, 5565, 5446, 5561, 2190, 2531, 2282, 6589}

$$\frac{f^2 \cosh^4(c+dx)}{32bd^3} + \frac{(e+fx)^2 \cosh^4(c+dx)}{4bd} + \frac{2af(e+fx)\cosh^3(c+dx)}{9b^2d^2} - \frac{f(e+fx)\sinh(c+dx)\cosh^3(c+dx)}{8bd^2} + 3f^2$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*e*f*x)/(2*b^3*d) - (3*e*f*x)/(16*b*d) + (a^2*f^2*x^2)/(4*b^3*d) - (3*f
^2*x^2)/(32*b*d) - (a^2*(a^2 + b^2)*(e + f*x)^3)/(3*b^5*f) + (2*a^3*f*(e +
f*x)*Cosh[c + d*x])/(b^4*d^2) + (4*a*f*(e + f*x)*Cosh[c + d*x])/(3*b^2*d^2)
 + (3*f^2*Cosh[c + d*x]^2)/(32*b*d^3) + (2*a*f*(e + f*x)*Cosh[c + d*x]^3)/(
9*b^2*d^2) + (f^2*Cosh[c + d*x]^4)/(32*b*d^3) + ((e + f*x)^2*Cosh[c + d*x]^
4)/(4*b*d) + (a^2*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2])])/(b^5*d) + (a^2*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2])])/(b^5*d) + (2*a^2*(a^2 + b^2)*f*(e + f*x)*PolyLog
[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^2) + (2*a^2*(a^2 + b^
2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d
^2) - (2*a^2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/(b^5*d^3) - (2*a^2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))])/(b^5*d^3) - (2*a^3*f^2*Sinh[c + d*x])/(b^4*d^3) - (14
*a*f^2*Sinh[c + d*x])/(9*b^2*d^3) - (a^3*(e + f*x)^2*Sinh[c + d*x])/(b^4*d)
 - (2*a*(e + f*x)^2*Sinh[c + d*x])/(3*b^2*d) - (a^2*f*(e + f*x)*Cosh[c + d*
```

```
x]*Sinh[c + d*x])/(2*b^3*d^2) - (3*f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])
/(16*b*d^2) - (a*(e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b^2*d) - (f*
(e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x])/(8*b*d^2) + (a^2*f^2*Sinh[c + d*x]
^2)/(4*b^3*d^3) + (a^2*(e + f*x)^2*Sinh[c + d*x]^2)/(2*b^3*d) - (2*a*f^2*Si
nh[c + d*x]^3)/(27*b^2*d^3)
```

Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
```

```
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh^3(c+dx)\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c+}{a+b\sinh(c+dx)}}{b}$$

$$= \frac{(e+fx)^2\cosh^4(c+dx)}{4bd} - \frac{a\int (e+fx)^2\cosh^3(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^2}{a+b\,\mathrm{s}}}{}$$

$$= \frac{2af(e+fx)\cosh^3(c+dx)}{9b^2d^2} + \frac{f^2\cosh^4(c+dx)}{32bd^3} + \frac{(e+fx)^2\cosh^4(c+c}{4bd}$$

$$= -\frac{a^2\left(a^2+b^2\right)(e+fx)^3}{3b^5f} + \frac{3f^2\cosh^2(c+dx)}{32bd^3} + \frac{2af(e+fx)\cosh^3(c+}{9b^2d^2}$$

$$= -\frac{3efx}{16bd} - \frac{3f^2x^2}{32bd} - \frac{a^2\left(a^2+b^2\right)(e+fx)^3}{3b^5f} + \frac{2a^3f(e+fx)\cosh(c+dx)}{b^4d^2}$$

$$= \frac{a^2efx}{2b^3d} - \frac{3efx}{16bd} + \frac{a^2f^2x^2}{4b^3d} - \frac{3f^2x^2}{32bd} - \frac{a^2\left(a^2+b^2\right)(e+fx)^3}{3b^5f} + \frac{2a^3f(e+}{}$$

$$= \frac{a^2efx}{2b^3d} - \frac{3efx}{16bd} + \frac{a^2f^2x^2}{4b^3d} - \frac{3f^2x^2}{32bd} - \frac{a^2\left(a^2+b^2\right)(e+fx)^3}{3b^5f} + \frac{2a^3f(e+}{}$$

$$= \frac{a^2efx}{2b^3d} - \frac{3efx}{16bd} + \frac{a^2f^2x^2}{4b^3d} - \frac{3f^2x^2}{32bd} - \frac{a^2\left(a^2+b^2\right)(e+fx)^3}{3b^5f} + \frac{2a^3f(e+}{}$$

Mathematica [B] time = 19.3254, size = 5198, normalized size = 6.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x
]),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.243, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^2(\cosh(dx+c))^3(\sinh(dx+c))^2}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="maxima")
```

```
[Out] -1/192*e^2*((8*a*b^2*e^(-d*x - c) - 3*b^3 - 12*(2*a^2*b + b^3)*e^(-2*d*x -
2*c) + 24*(4*a^3 + 3*a*b^2)*e^(-3*d*x - 3*c))*e^(4*d*x + 4*c)/(b^4*d) - 192
*(a^4 + a^2*b^2)*(d*x + c)/(b^5*d) - (8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(-
4*d*x - 4*c) + 24*(4*a^3 + 3*a*b^2)*e^(-d*x - c) + 12*(2*a^2*b + b^3)*e^(-2
*d*x - 2*c))/(b^4*d) - 192*(a^4 + a^2*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*
d*x - 2*c) - b)/(b^5*d)) + 1/13824*(4608*(a^4*d^3*f^2*e^(4*c) + a^2*b^2*d^3
*f^2*e^(4*c))*x^3 + 13824*(a^4*d^3*e*f*e^(4*c) + a^2*b^2*d^3*e*f*e^(4*c))*x
^2 + 27*(8*b^4*d^2*f^2*x^2*e^(8*c) + 4*(4*d^2*e*f - d*f^2)*b^4*x*e^(8*c) -
(4*d*e*f - f^2)*b^4*e^(8*c))*e^(4*d*x) - 64*(9*a*b^3*d^2*f^2*x^2*e^(7*c) +
6*(3*d^2*e*f - d*f^2)*a*b^3*x*e^(7*c) - 2*(3*d*e*f - f^2)*a*b^3*e^(7*c))*e^
(3*d*x) - 432*(2*(2*d*e*f - f^2)*a^2*b^2*e^(6*c) + (2*d*e*f - f^2)*b^4*e^(6
*c) - 2*(2*a^2*b^2*d^2*f^2*e^(6*c) + b^4*d^2*f^2*e^(6*c))*x^2 - 2*(2*(2*d^2
*e*f - d*f^2)*a^2*b^2*e^(6*c) + (2*d^2*e*f - d*f^2)*b^4*e^(6*c))*x)*e^(2*d*
x) + 1728*(8*(d*e*f - f^2)*a^3*b*e^(5*c) + 6*(d*e*f - f^2)*a*b^3*e^(5*c) -
(4*a^3*b*d^2*f^2*e^(5*c) + 3*a*b^3*d^2*f^2*e^(5*c))*x^2 - 2*(4*(d^2*e*f - d
*f^2)*a^3*b*e^(5*c) + 3*(d^2*e*f - d*f^2)*a*b^3*e^(5*c))*x)*e^(d*x) + 1728*
(8*(d*e*f + f^2)*a^3*b*e^(3*c) + 6*(d*e*f + f^2)*a*b^3*e^(3*c) + (4*a^3*b*d
^2*f^2*e^(3*c) + 3*a*b^3*d^2*f^2*e^(3*c))*x^2 + 2*(4*(d^2*e*f + d*f^2)*a^3*
b*e^(3*c) + 3*(d^2*e*f + d*f^2)*a*b^3*e^(3*c))*x)*e^(-d*x) + 432*(2*(2*d*e*
f + f^2)*a^2*b^2*e^(2*c) + (2*d*e*f + f^2)*b^4*e^(2*c) + 2*(2*a^2*b^2*d^2*f
^2*e^(2*c) + b^4*d^2*f^2*e^(2*c))*x^2 + 2*(2*(2*d^2*e*f + d*f^2)*a^2*b^2*e^
(2*c) + (2*d^2*e*f + d*f^2)*b^4*e^(2*c))*x)*e^(-2*d*x) + 64*(9*a*b^3*d^2*f^
2*x^2*e^c + 6*(3*d^2*e*f + d*f^2)*a*b^3*x*e^c + 2*(3*d*e*f + f^2)*a*b^3*e^c
)*e^(-3*d*x) + 27*(8*b^4*d^2*f^2*x^2 + 4*(4*d^2*e*f + d*f^2)*b^4*x + (4*d*e
*f + f^2)*b^4)*e^(-4*d*x))*e^(-4*c)/(b^5*d^3) - integrate(-2*((a^4*b*f^2 +
a^2*b^3*f^2)*x^2 + 2*(a^4*b*e*f + a^2*b^3*e*f)*x - ((a^5*f^2*e^c + a^3*b^2*
f^2*e^c)*x^2 + 2*(a^5*e*f*e^c + a^3*b^2*e*f*e^c)*x)*e^(d*x))/(b^6*e^(2*d*x
+ 2*c) + 2*a*b^5*e^(d*x + c) - b^6), x)
```

Fricas [C] time = 3.44704, size = 16764, normalized size = 20.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="fricas")
```

```
[Out] 1/13824*(216*b^4*d^2*f^2*x^2 + 27*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b^
4*d*e*f + b^4*f^2 + 4*(4*b^4*d^2*e*f - b^4*d*f^2)*x)*cosh(d*x + c)^8 + 27*(
8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b^4*d*e*f + b^4*f^2 + 4*(4*b^4*d^2*e*
f - b^4*d*f^2)*x)*sinh(d*x + c)^8 + 216*b^4*d^2*e^2 - 64*(9*a*b^3*d^2*f^2*x
^2 + 9*a*b^3*d^2*e^2 - 6*a*b^3*d*e*f + 2*a*b^3*f^2 + 6*(3*a*b^3*d^2*e*f - a
*b^3*d*f^2)*x)*cosh(d*x + c)^7 - 8*(72*a*b^3*d^2*f^2*x^2 + 72*a*b^3*d^2*e^2
 - 48*a*b^3*d*e*f + 16*a*b^3*f^2 + 48*(3*a*b^3*d^2*e*f - a*b^3*d*f^2)*x - 2
7*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b^4*d*e*f + b^4*f^2 + 4*(4*b^4*d^2
*e*f - b^4*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^7 + 108*b^4*d*e*f + 432*(
2*(2*a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(2*a^2*b^2 + b^4)*d^2*e^2 - 2*(2*a^2*b^
2 + b^4)*d*e*f + (2*a^2*b^2 + b^4)*f^2 + 2*(2*(2*a^2*b^2 + b^4)*d^2*e*f - (
2*a^2*b^2 + b^4)*d*f^2)*x)*cosh(d*x + c)^6 + 4*(216*(2*a^2*b^2 + b^4)*d^2*f
^2*x^2 + 216*(2*a^2*b^2 + b^4)*d^2*e^2 - 216*(2*a^2*b^2 + b^4)*d*e*f + 108*
(2*a^2*b^2 + b^4)*f^2 + 189*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b^4*d*e*
f + b^4*f^2 + 4*(4*b^4*d^2*e*f - b^4*d*f^2)*x)*cosh(d*x + c)^2 + 216*(2*(2*
a^2*b^2 + b^4)*d^2*e*f - (2*a^2*b^2 + b^4)*d*f^2)*x - 112*(9*a*b^3*d^2*f^2*
```

x^2 + 9*a*b^3*d^2*e^2 - 6*a*b^3*d*e*f + 2*a*b^3*f^2 + 6*(3*a*b^3*d^2*e*f - a*b^3*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^6 + 27*b^4*f^2 - 1728*((4*a^3*b + 3*a*b^3)*d^2*f^2*x^2 + (4*a^3*b + 3*a*b^3)*d^2*e^2 - 2*(4*a^3*b + 3*a*b^3)*d*e*f + 2*(4*a^3*b + 3*a*b^3)*f^2 + 2*((4*a^3*b + 3*a*b^3)*d^2*e*f - (4*a^3*b + 3*a*b^3)*d*f^2)*x)*cosh(d*x + c)^5 - 24*(72*(4*a^3*b + 3*a*b^3)*d^2*f^2*x^2 + 72*(4*a^3*b + 3*a*b^3)*d^2*e^2 - 144*(4*a^3*b + 3*a*b^3)*d*e*f - 63*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b^4*d*e*f + b^4*f^2 + 4*(4*b^4*d^2*e*f - b^4*d*f^2)*x)*cosh(d*x + c)^3 + 144*(4*a^3*b + 3*a*b^3)*f^2 + 56*(9*a*b^3*d^2*f^2*x^2 + 9*a*b^3*d^2*e^2 - 6*a*b^3*d*e*f + 2*a*b^3*f^2 + 6*(3*a*b^3*d^2*e*f - a*b^3*d*f^2)*x)*cosh(d*x + c)^2 + 144*((4*a^3*b + 3*a*b^3)*d^2*e*f - (4*a^3*b + 3*a*b^3)*d*f^2)*x - 108*(2*(2*a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(2*a^2*b^2 + b^4)*d^2*e^2 - 2*(2*a^2*b^2 + b^4)*d*e*f + (2*a^2*b^2 + b^4)*f^2 + 2*(2*(2*a^2*b^2 + b^4)*d^2*e*f - (2*a^2*b^2 + b^4)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 4608*((a^4 + a^2*b^2)*d^3*f^2*x^3 + 3*(a^4 + a^2*b^2)*d^3*e*f*x^2 + 3*(a^4 + a^2*b^2)*d^3*e^2*x + 6*(a^4 + a^2*b^2)*c*d^2*e^2 - 6*(a^4 + a^2*b^2)*c^2*d*e*f + 2*(a^4 + a^2*b^2)*c^3*f^2)*cosh(d*x + c)^4 - 2*(2304*(a^4 + a^2*b^2)*d^3*f^2*x^3 + 6912*(a^4 + a^2*b^2)*d^3*e*f*x^2 + 6912*(a^4 + a^2*b^2)*d^3*e^2*x + 13824*(a^4 + a^2*b^2)*c*d^2*e^2 - 13824*(a^4 + a^2*b^2)*c^2*d*e*f + 4608*(a^4 + a^2*b^2)*c^3*f^2 - 945*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b^4*d*e*f + b^4*f^2 + 4*(4*b^4*d^2*e*f - b^4*d*f^2)*x)*cosh(d*x + c)^4 + 1120*(9*a*b^3*d^2*f^2*x^2 + 9*a*b^3*d^2*e^2 - 6*a*b^3*d*e*f + 2*a*b^3*f^2 + 6*(3*a*b^3*d^2*e*f - a*b^3*d*f^2)*x)*cosh(d*x + c)^3 - 3240*(2*(2*a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(2*a^2*b^2 + b^4)*d^2*e^2 - 2*(2*a^2*b^2 + b^4)*d*e*f + (2*a^2*b^2 + b^4)*f^2 + 2*(2*(2*a^2*b^2 + b^4)*d^2*e*f - (2*a^2*b^2 + b^4)*d*f^2)*x)*cosh(d*x + c)^2 + 4320*((4*a^3*b + 3*a*b^3)*d^2*f^2*x^2 + (4*a^3*b + 3*a*b^3)*d^2*e^2 - 2*(4*a^3*b + 3*a*b^3)*d*e*f + 2*(4*a^3*b + 3*a*b^3)*f^2 + 2*((4*a^3*b + 3*a*b^3)*d^2*e*f - (4*a^3*b + 3*a*b^3)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^4 + 1728*((4*a^3*b + 3*a*b^3)*d^2*f^2*x^2 + (4*a^3*b + 3*a*b^3)*d^2*e^2 + 2*(4*a^3*b + 3*a*b^3)*d*e*f + 2*(4*a^3*b + 3*a*b^3)*f^2 + 2*((4*a^3*b + 3*a*b^3)*d^2*e*f + (4*a^3*b + 3*a*b^3)*d*f^2)*x)*cosh(d*x + c)^3 + 8*(216*(4*a^3*b + 3*a*b^3)*d^2*f^2*x^2 + 189*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b^4*d*e*f + b^4*f^2 + 4*(4*b^4*d^2*e*f - b^4*d*f^2)*x)*cosh(d*x + c)^5 + 216*(4*a^3*b + 3*a*b^3)*d^2*e^2 - 280*(9*a*b^3*d^2*f^2*x^2 + 9*a*b^3*d^2*e^2 - 6*a*b^3*d*e*f + 2*a*b^3*f^2 + 6*(3*a*b^3*d^2*e*f - a*b^3*d*f^2)*x)*cosh(d*x + c)^4 + 432*(4*a^3*b + 3*a*b^3)*d*e*f + 1080*(2*(2*a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(2*a^2*b^2 + b^4)*d^2*e^2 - 2*(2*a^2*b^2 + b^4)*d*e*f + (2*a^2*b^2 + b^4)*f^2 + 2*(2*(2*a^2*b^2 + b^4)*d^2*e*f - (2*a^2*b^2 + b^4)*d*f^2)*x)*cosh(d*x + c)^3 + 432*(4*a^3*b + 3*a*b^3)*f^2 - 2160*((4*a^3*b + 3*a*b^3)*d^2*f^2*x^2 + (4*a^3*b + 3*a*b^3)*d^2*e^2 - 2*(4*a^3*b + 3*a*b^3)*d*e*f + 2*(4*a^3*b + 3*a*b^3)*f^2 + 2*((4*a^3*b + 3*a*b^3)*d^2*e*f - (4*a^3*b + 3*a*b^3)*d*f^2)*x)*cosh(d*x + c)^2 + 432*((4*a^3*b + 3*a*b^3)*d^2*e*f + (4*a^3*b + 3*a*b^3)*d*f^2)*x - 2304*((a^4 + a^2*b^2)*d^3*f^2*x^3 + 3*(a^4 + a^2*b^2)*d^3*e*f*x^2 + 3*(a^4 + a^2*b^2)*d^3*e^2*x + 6*(a^4 + a^2*b^2)*c*d^2*e^2 - 6*(a^4 + a^2*b^2)*c^2*d*e*f + 2*(a^4 + a^2*b^2)*c^3*f^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 432*(2*(2*a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(2*a^2*b^2 + b^4)*d^2*e^2 + 2*(2*a^2*b^2 + b^4)*d*e*f + (2*a^2*b^2 + b^4)*f^2 + 2*(2*(2*a^2*b^2 + b^4)*d^2*e*f + (2*a^2*b^2 + b^4)*d*f^2)*x)*cosh(d*x + c)^2 + 12*(72*(2*a^2*b^2 + b^4)*d^2*f^2*x^2 + 63*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b^4*d*e*f + b^4*f^2 + 4*(4*b^4*d^2*e*f - b^4*d*f^2)*x)*cosh(d*x + c)^6 - 112*(9*a*b^3*d^2*f^2*x^2 + 9*a*b^3*d^2*e^2 - 6*a*b^3*d*e*f + 2*a*b^3*f^2 + 6*(3*a*b^3*d^2*e*f - a*b^3*d*f^2)*x)*cosh(d*x + c)^5 + 72*(2*a^2*b^2 + b^4)*d^2*e^2 + 540*(2*(2*a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(2*a^2*b^2 + b^4)*d^2*e^2 - 2*(2*a^2*b^2 + b^4)*d*e*f + (2*a^2*b^2 + b^4)*f^2 + 2*(2*(2*a^2*b^2 + b^4)*d^2*e*f - (2*a^2*b^2 + b^4)*d*f^2)*x)*cosh(d*x + c)^4 + 72*(2*a^2*b^2 + b^4)*d*e*f - 1440*((4*a^3*b + 3*a*b^3)*d^2*f^2*x^2 + (4*a^3*b + 3*a*b^3)*d^2*e^2 - 2*(4*a^3*b + 3*a*b^3)*d*e*f + 2*(4*a^3*b + 3*a*b^3)*f^2 + 2*((4*a^3*b + 3*a*b^3)*d^2*e*f - (4*a^3*b + 3*a*b^3)*d*f^2)*x)*cosh(d*x + c)^3 + 36*(2*a^2*b^2 + b^4)*f^2 - 2304*((a^4 + a^2*b^2)*d^3*f^2*x^3 + 3*(a^4 + a^2*b^2)*d^3*e*f*x^2 + 3*(a

^4 + a^2*b^2)*d^3*e^2*x + 6*(a^4 + a^2*b^2)*c*d^2*e^2 - 6*(a^4 + a^2*b^2)*c ^2*d*e*f + 2*(a^4 + a^2*b^2)*c^3*f^2)*cosh(d*x + c)^2 + 72*(2*(2*a^2*b^2 + b^4)*d^2*e*f + (2*a^2*b^2 + b^4)*d*f^2)*x + 432*((4*a^3*b + 3*a*b^3)*d^2*f^ 2*x^2 + (4*a^3*b + 3*a*b^3)*d^2*e^2 + 2*(4*a^3*b + 3*a*b^3)*d*e*f + 2*(4*a^ 3*b + 3*a*b^3)*f^2 + 2*((4*a^3*b + 3*a*b^3)*d^2*e*f + (4*a^3*b + 3*a*b^3)*d *f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 108*(4*b^4*d^2*e*f + b^4*d*f^2)*x + 64*(9*a*b^3*d^2*f^2*x^2 + 9*a*b^3*d^2*e^2 + 6*a*b^3*d*e*f + 2*a*b^3*f^2 + 6*(3*a*b^3*d^2*e*f + a*b^3*d*f^2)*x)*cosh(d*x + c) + 27648*(((a^4 + a^2*b ^2)*d*f^2*x + (a^4 + a^2*b^2)*d*e*f)*cosh(d*x + c)^4 + 4*((a^4 + a^2*b^2)*d *f^2*x + (a^4 + a^2*b^2)*d*e*f)*cosh(d*x + c)^3*sinh(d*x + c) + 6*((a^4 + a ^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*d*e*f)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*((a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*d*e*f)*sinh(d*x + c)^4) *dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 27648*(((a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*d*e*f)*cosh(d*x + c)^4 + 4*((a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*d*e*f)*cosh(d*x + c)^3*sinh(d*x + c) + 6*((a^4 + a^2*b^2)*d*f^2* x + (a^4 + a^2*b^2)*d*e*f)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*((a^4 + a^2* b^2)*d*f^2*x + (a^4 + a^2*b^2)*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*d*e*f)*sinh(d*x + c)^4)*dilog((a*cosh (d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 13824*(((a^4 + a^2*b^2)*d^2*e^2 - 2*(a^4 + a^2*b ^2)*c*d*e*f + (a^4 + a^2*b^2)*c^2*f^2)*cosh(d*x + c)^4 + 4*((a^4 + a^2*b^2) *d^2*e^2 - 2*(a^4 + a^2*b^2)*c*d*e*f + (a^4 + a^2*b^2)*c^2*f^2)*cosh(d*x + c)^3*sinh(d*x + c) + 6*((a^4 + a^2*b^2)*d^2*e^2 - 2*(a^4 + a^2*b^2)*c*d*e*f + (a^4 + a^2*b^2)*c^2*f^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*((a^4 + a^2 *b^2)*d^2*e^2 - 2*(a^4 + a^2*b^2)*c*d*e*f + (a^4 + a^2*b^2)*c^2*f^2)*cosh(d *x + c)*sinh(d*x + c)^3 + ((a^4 + a^2*b^2)*d^2*e^2 - 2*(a^4 + a^2*b^2)*c*d* e*f + (a^4 + a^2*b^2)*c^2*f^2)*sinh(d*x + c)^4)*log(2*b*cosh(d*x + c) + 2*b *sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 13824*(((a^4 + a^2*b^2) *d^2*e^2 - 2*(a^4 + a^2*b^2)*c*d*e*f + (a^4 + a^2*b^2)*c^2*f^2)*cosh(d*x + c)^4 + 4*((a^4 + a^2*b^2)*d^2*e^2 - 2*(a^4 + a^2*b^2)*c*d*e*f + (a^4 + a^2* b^2)*c^2*f^2)*cosh(d*x + c)^3*sinh(d*x + c) + 6*((a^4 + a^2*b^2)*d^2*e^2 - 2*(a^4 + a^2*b^2)*c*d*e*f + (a^4 + a^2*b^2)*c^2*f^2)*cosh(d*x + c)^2*sinh(d *x + c)^2 + 4*((a^4 + a^2*b^2)*d^2*e^2 - 2*(a^4 + a^2*b^2)*c*d*e*f + (a^4 + a^2*b^2)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 + a^2*b^2)*d^2*e^2 - 2*(a^4 + a^2*b^2)*c*d*e*f + (a^4 + a^2*b^2)*c^2*f^2)*sinh(d*x + c)^4)*lo g(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 13824*(((a^4 + a^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + 2*(a^ 4 + a^2*b^2)*c*d*e*f - (a^4 + a^2*b^2)*c^2*f^2)*cosh(d*x + c)^4 + 4*((a^4 + a^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + 2*(a^4 + a^2*b^2)*c*d *e*f - (a^4 + a^2*b^2)*c^2*f^2)*cosh(d*x + c)^3*sinh(d*x + c) + 6*((a^4 + a ^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + 2*(a^4 + a^2*b^2)*c*d*e *f - (a^4 + a^2*b^2)*c^2*f^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*((a^4 + a ^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + 2*(a^4 + a^2*b^2)*c*d*e *f - (a^4 + a^2*b^2)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 + a^2*b ^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + 2*(a^4 + a^2*b^2)*c*d*e*f - (a^4 + a^2*b^2)*c^2*f^2)*sinh(d*x + c)^4)*log(-(a*cosh(d*x + c) + a*sinh(d *x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 13824*(((a^4 + a^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + 2*(a ^4 + a^2*b^2)*c*d*e*f - (a^4 + a^2*b^2)*c^2*f^2)*cosh(d*x + c)^4 + 4*((a^4 + a^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + 2*(a^4 + a^2*b^2)*c* d*e*f - (a^4 + a^2*b^2)*c^2*f^2)*cosh(d*x + c)^3*sinh(d*x + c) + 6*((a^4 + a^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + 2*(a^4 + a^2*b^2)*c*d* e*f - (a^4 + a^2*b^2)*c^2*f^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*((a^4 + a^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + 2*(a^4 + a^2*b^2)*c*d* e*f - (a^4 + a^2*b^2)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 + a^2* b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + 2*(a^4 + a^2*b^2)*c*d*e*f - (a^4 + a^2*b^2)*c^2*f^2)*sinh(d*x + c)^4)*log(-(a*cosh(d*x + c) + a*sinh(

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d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b
) - 27648*((a^4 + a^2*b^2)*f^2*cosh(d*x + c)^4 + 4*(a^4 + a^2*b^2)*f^2*cosh
(d*x + c)^3*sinh(d*x + c) + 6*(a^4 + a^2*b^2)*f^2*cosh(d*x + c)^2*sinh(d*x
+ c)^2 + 4*(a^4 + a^2*b^2)*f^2*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + a^2*b
^2)*f^2*sinh(d*x + c)^4)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 27648*((a^4 +
 a^2*b^2)*f^2*cosh(d*x + c)^4 + 4*(a^4 + a^2*b^2)*f^2*cosh(d*x + c)^3*sinh(
d*x + c) + 6*(a^4 + a^2*b^2)*f^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^4 +
 a^2*b^2)*f^2*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + a^2*b^2)*f^2*sinh(d*x
+ c)^4)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 8*(72*a*b^3*d^2*f^2*x^2 + 72*a
*b^3*d^2*e^2 + 27*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b^4*d*e*f + b^4*f^
2 + 4*(4*b^4*d^2*e*f - b^4*d*f^2)*x)*cosh(d*x + c)^7 + 48*a*b^3*d*e*f - 56*
(9*a*b^3*d^2*f^2*x^2 + 9*a*b^3*d^2*e^2 - 6*a*b^3*d*e*f + 2*a*b^3*f^2 + 6*(3
*a*b^3*d^2*e*f - a*b^3*d*f^2)*x)*cosh(d*x + c)^6 + 16*a*b^3*f^2 + 324*(2*(2
*a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(2*a^2*b^2 + b^4)*d^2*e^2 - 2*(2*a^2*b^2 +
b^4)*d*e*f + (2*a^2*b^2 + b^4)*f^2 + 2*(2*(2*a^2*b^2 + b^4)*d^2*e*f - (2*a^
2*b^2 + b^4)*d*f^2)*x)*cosh(d*x + c)^5 - 1080*((4*a^3*b + 3*a*b^3)*d^2*f^2*
x^2 + (4*a^3*b + 3*a*b^3)*d^2*e^2 - 2*(4*a^3*b + 3*a*b^3)*d*e*f + 2*(4*a^3*
b + 3*a*b^3)*f^2 + 2*((4*a^3*b + 3*a*b^3)*d^2*e*f - (4*a^3*b + 3*a*b^3)*d*f
^2)*x)*cosh(d*x + c)^4 - 2304*((a^4 + a^2*b^2)*d^3*f^2*x^3 + 3*(a^4 + a^2*b
^2)*d^3*e*f*x^2 + 3*(a^4 + a^2*b^2)*d^3*e^2*x + 6*(a^4 + a^2*b^2)*c*d^2*e^2
 - 6*(a^4 + a^2*b^2)*c^2*d*e*f + 2*(a^4 + a^2*b^2)*c^3*f^2)*cosh(d*x + c)^3
 + 648*((4*a^3*b + 3*a*b^3)*d^2*f^2*x^2 + (4*a^3*b + 3*a*b^3)*d^2*e^2 + 2*(
4*a^3*b + 3*a*b^3)*d*e*f + 2*(4*a^3*b + 3*a*b^3)*f^2 + 2*((4*a^3*b + 3*a*b^
3)*d^2*e*f + (4*a^3*b + 3*a*b^3)*d*f^2)*x)*cosh(d*x + c)^2 + 48*(3*a*b^3*d^
2*e*f + a*b^3*d*f^2)*x + 108*(2*(2*a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(2*a^2*b^
2 + b^4)*d^2*e^2 + 2*(2*a^2*b^2 + b^4)*d*e*f + (2*a^2*b^2 + b^4)*f^2 + 2*(2
*(2*a^2*b^2 + b^4)*d^2*e*f + (2*a^2*b^2 + b^4)*d*f^2)*x)*cosh(d*x + c))*sin
h(d*x + c))/(b^5*d^3*cosh(d*x + c)^4 + 4*b^5*d^3*cosh(d*x + c)^3*sinh(d*x +
 c) + 6*b^5*d^3*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^5*d^3*cosh(d*x + c)*s
inh(d*x + c)^3 + b^5*d^3*sinh(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \cosh(dx+c)^3 \sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)^3*sinh(d*x + c)^2/(b*sinh(d*x + c) + a)
, x)
```

3.374 $\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=499

$$\frac{a^2f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^2}+\frac{a^2f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^5d^2}+\frac{a^3f\cosh(c+dx)}{b^4d^2}-\frac{a^2f\sinh(c+dx)\,\mathrm{c}}{4b^3d^2}$$

```
[Out] (a^2*f*x)/(4*b^3*d) - (3*f*x)/(32*b*d) - (a^2*(a^2 + b^2)*(e + f*x)^2)/(2*b
^5*f) + (a^3*f*Cosh[c + d*x])/(b^4*d^2) + (2*a*f*Cosh[c + d*x])/(3*b^2*d^2)
 + (a*f*Cosh[c + d*x]^3)/(9*b^2*d^2) + ((e + f*x)*Cosh[c + d*x]^4)/(4*b*d)
+ (a^2*(a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]
)/(b^5*d) + (a^2*(a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2])])/(b^5*d) + (a^2*(a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2]))])/(b^5*d^2) + (a^2*(a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^2) - (a^3*(e + f*x)*Sinh[c + d*x])/(b^
4*d) - (2*a*(e + f*x)*Sinh[c + d*x])/(3*b^2*d) - (a^2*f*Cosh[c + d*x]*Sinh[
c + d*x])/(4*b^3*d^2) - (3*f*Cosh[c + d*x]*Sinh[c + d*x])/(32*b*d^2) - (a*(
e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b^2*d) - (f*Cosh[c + d*x]^3*Sinh
[c + d*x])/(16*b*d^2) + (a^2*(e + f*x)*Sinh[c + d*x]^2)/(2*b^3*d)
```

Rubi [A] time = 0.675401, antiderivative size = 499, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 13, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.382, Rules used = {5579, 5447, 2635, 8, 3310, 3296, 2638, 5565, 5446, 5561, 2190, 2279, 2391}

$$\frac{a^2f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^2}+\frac{a^2f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^5d^2}+\frac{a^3f\cosh(c+dx)}{b^4d^2}-\frac{a^2f\sinh(c+dx)\,\mathrm{c}}{4b^3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (a^2*f*x)/(4*b^3*d) - (3*f*x)/(32*b*d) - (a^2*(a^2 + b^2)*(e + f*x)^2)/(2*b
^5*f) + (a^3*f*Cosh[c + d*x])/(b^4*d^2) + (2*a*f*Cosh[c + d*x])/(3*b^2*d^2)
 + (a*f*Cosh[c + d*x]^3)/(9*b^2*d^2) + ((e + f*x)*Cosh[c + d*x]^4)/(4*b*d)
+ (a^2*(a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]
)/(b^5*d) + (a^2*(a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2])])/(b^5*d) + (a^2*(a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2]))])/(b^5*d^2) + (a^2*(a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^2) - (a^3*(e + f*x)*Sinh[c + d*x])/(b^
4*d) - (2*a*(e + f*x)*Sinh[c + d*x])/(3*b^2*d) - (a^2*f*Cosh[c + d*x]*Sinh[
c + d*x])/(4*b^3*d^2) - (3*f*Cosh[c + d*x]*Sinh[c + d*x])/(32*b*d^2) - (a*(
e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b^2*d) - (f*Cosh[c + d*x]^3*Sinh
[c + d*x])/(16*b*d^2) + (a^2*(e + f*x)*Sinh[c + d*x]^2)/(2*b^3*d)
```


Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
```

```
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh^3(c+dx)\sinh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)\cosh^4(c+dx)}{4bd} - \frac{a\int (e+fx)\cosh^3(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)\cosh^3(c+}{a+b\sinh(c+d}}{b^2}$$

$$= \frac{af\cosh^3(c+dx)}{9b^2d^2} + \frac{(e+fx)\cosh^4(c+dx)}{4bd} - \frac{a(e+fx)\cosh^2(c+dx)\sinh}{3b^2d}$$

$$= -\frac{a^2\left(a^2+b^2\right)(e+fx)^2}{2b^5f} + \frac{af\cosh^3(c+dx)}{9b^2d^2} + \frac{(e+fx)\cosh^4(c+dx)}{4bd} - \frac{a^3(}{}$$

$$= -\frac{3fx}{32bd} - \frac{a^2\left(a^2+b^2\right)(e+fx)^2}{2b^5f} + \frac{a^3f\cosh(c+dx)}{b^4d^2} + \frac{2af\cosh(c+dx)}{3b^2d^2} +$$

$$= \frac{a^2fx}{4b^3d} - \frac{3fx}{32bd} - \frac{a^2\left(a^2+b^2\right)(e+fx)^2}{2b^5f} + \frac{a^3f\cosh(c+dx)}{b^4d^2} + \frac{2af\cosh(c+}{3b^2d^2}$$

$$= \frac{a^2fx}{4b^3d} - \frac{3fx}{32bd} - \frac{a^2\left(a^2+b^2\right)(e+fx)^2}{2b^5f} + \frac{a^3f\cosh(c+dx)}{b^4d^2} + \frac{2af\cosh(c+}{3b^2d^2}$$

Mathematica [A] time = 3.20973, size = 853, normalized size = 1.71

$$-144de\log(a+b\sinh(c+dx))b^4 + 72f\left(dx\left(dx - 2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right) - 2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)\right) - 2\text{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (-144*b^4*d*e*Log[a + b*Sinh[c + d*x]] + 72*b^4*f*(d*x*(d*x - 2*Log[1 + (b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])]) - 2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*Pol
yLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) + 72*b^2*d*e*((4*a^2 + b
^2)*Log[a + b*Sinh[c + d*x]] - 4*a*b*Sinh[c + d*x] + 2*b^2*Sinh[c + d*x]^2)
 + 24*d*e*(3*(16*a^4 + 12*a^2*b^2 + b^4)*Log[a + b*Sinh[c + d*x]] - 12*a*b*
(4*a^2 + 3*b^2)*Sinh[c + d*x] + 6*b^2*(4*a^2 + 3*b^2)*Sinh[c + d*x]^2 - 16*
a*b^3*Sinh[c + d*x]^3 + 12*b^4*Sinh[c + d*x]^4) + 36*b^2*f*(8*a*b*Cosh[c +
d*x] + 2*b^2*d*x*Cosh[2*(c + d*x)] + 2*(4*a^2 + b^2)*(-(c + d*x)^2/2 + (c +
 d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + (c + d*x)*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - c*Log[a + b*Sinh[c + d*x]] + PolyLog
[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + PolyLog[2, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))]) - 8*a*b*d*x*Sinh[c + d*x] - b^2*Sinh[2*(c + d*x)])
 + f*(576*a*b*(2*a^2 + b^2)*Cosh[c + d*x] + 72*b^2*(4*a^2 + b^2)*d*x*Cosh[2
*(c + d*x)] + 32*a*b^3*Cosh[3*(c + d*x)] + 36*b^4*d*x*Cosh[4*(c + d*x)] + 7
2*(16*a^4 + 12*a^2*b^2 + b^4)*(-(c + d*x)^2/2 + (c + d*x)*Log[1 + (b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2])] + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])] - c*Log[a + b*Sinh[c + d*x]] + PolyLog[2, (b*E^(c + d*x))/(-a
 + Sqrt[a^2 + b^2])] + PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
) - 576*a*b*(2*a^2 + b^2)*d*x*Sinh[c + d*x] - 36*b^2*(4*a^2 + b^2)*Sinh[2*(
c + d*x)] - 96*a*b^3*d*x*Sinh[3*(c + d*x)] - 9*b^4*Sinh[4*(c + d*x)]))/(115
2*b^5*d^2)
```

Maple [B] time = 0.118, size = 1217, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] a^4/b^5/d^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+
a^4/b^5/d*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+a^4/
b^5/d^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+a^4/b^
5/d*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-a^4/b^5/
d^2*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2*a^4/b^5/d^2*f*c*ln(exp(d*x+
c))-2*a^4/b^5/d*f*c*x-1/2*a^4*f*x^2/b^5-1/2*a^2*f*x^2/b^3+1/8*a*(4*a^2+3*b^
2)*(d*f*x+d*e+f)/b^4/d^2*exp(-d*x-c)-1/b^3/d^2*a^2*f*c*ln(b*exp(2*d*x+2*c)+
2*a*exp(d*x+c)-b)+2/b^3/d^2*a^2*f*c*ln(exp(d*x+c))+1/b^3/d*a^2*f*ln((b*exp(
d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/b^3/d^2*a^2*f*ln((b*exp(
d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/b^3/d*a^2*f*ln((-b*exp(d
*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/b^3/d^2*a^2*f*ln((-b*exp
(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/b^3/d*a^2*f*c*x+a^4*e*
x/b^5+a^2*e*x/b^3+1/256*(4*d*f*x+4*d*e-f)/d^2/b*exp(4*d*x+4*c)+1/32*(4*a^2*
d*f*x+2*b^2*d*f*x+4*a^2*d*e+2*b^2*d*e-2*a^2*f-b^2*f)/b^3/d^2*exp(2*d*x+2*c)
-1/72*a*(3*d*f*x+3*d*e-f)/b^2/d^2*exp(3*d*x+3*c)-1/b^3/d^2*a^2*f*c^2+1/b^3/
d^2*a^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/b^3
/d^2*a^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/
b^3/d*a^2*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/b^3/d*a^2*e*ln(exp(d*x+
c))+1/256*(4*d*f*x+4*d*e+f)/d^2/b*exp(-4*d*x-4*c)+1/32*(2*a^2+b^2)*(2*d*f*x
+2*d*e+f)/b^3/d^2*exp(-2*d*x-2*c)+1/72*a*(3*d*f*x+3*d*e+f)/b^2/d^2*exp(-3*d
*x-3*c)-1/8*a*(4*a^2*d*f*x+3*b^2*d*f*x+4*a^2*d*e+3*b^2*d*e-4*a^2*f-3*b^2*f)
/b^4/d^2*exp(d*x+c)-a^4/b^5/d^2*f*c^2+a^4/b^5/d*e*ln(b*exp(2*d*x+2*c)+2*a*e
xp(d*x+c)-b)-2*a^4/b^5/d*e*ln(exp(d*x+c))+a^4/b^5/d^2*f*dilog((b*exp(d*x+c)
+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+a^4/b^5/d^2*f*dilog((-b*exp(d*x+c)
+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/192*e*((8*a*b^2*e^(-d*x - c) - 3*b^3 - 12*(2*a^2*b + b^3)*e^(-2*d*x - 2*
c) + 24*(4*a^3 + 3*a*b^2)*e^(-3*d*x - 3*c))*e^(4*d*x + 4*c)/(b^4*d) - 192*(
a^4 + a^2*b^2)*(d*x + c)/(b^5*d) - (8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(-4*
d*x - 4*c) + 24*(4*a^3 + 3*a*b^2)*e^(-d*x - c) + 12*(2*a^2*b + b^3)*e^(-2*d
*x - 2*c))/(b^4*d) - 192*(a^4 + a^2*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*
x - 2*c) - b)/(b^5*d)) + 1/2304*f*((1152*(a^4*d^2*e^(4*c) + a^2*b^2*d^2*e^(
4*c))*x^2 + 9*(4*b^4*d*x*e^(8*c) - b^4*e^(8*c))*e^(4*d*x) - 32*(3*a*b^3*d*x
*e^(7*c) - a*b^3*e^(7*c))*e^(3*d*x) - 72*(2*a^2*b^2*e^(6*c) + b^4*e^(6*c) -
 2*(2*a^2*b^2*d*e^(6*c) + b^4*d*e^(6*c))*x)*e^(2*d*x) + 288*(4*a^3*b*e^(5*c
) + 3*a*b^3*e^(5*c) - (4*a^3*b*d*e^(5*c) + 3*a*b^3*d*e^(5*c))*x)*e^(d*x) +
288*(4*a^3*b*e^(3*c) + 3*a*b^3*e^(3*c) + (4*a^3*b*d*e^(3*c) + 3*a*b^3*d*e^(
3*c))*x)*e^(-d*x) + 72*(2*a^2*b^2*e^(2*c) + b^4*e^(2*c) + 2*(2*a^2*b^2*d*e^
(2*c) + b^4*d*e^(2*c))*x)*e^(-2*d*x) + 32*(3*a*b^3*d*x*e^c + a*b^3*e^c)*e^(
-3*d*x) + 9*(4*b^4*d*x + b^4)*e^(-4*d*x))*e^(-4*c)/(b^5*d^2) - 72*integrate
(64*((a^5*e^c + a^3*b^2*e^c)*x*e^(d*x) - (a^4*b + a^2*b^3)*x)/(b^6*e^(2*d*x
 + 2*c) + 2*a*b^5*e^(d*x + c) - b^6), x))
```

Fricas [B] time = 2.75812, size = 8818, normalized size = 17.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/2304*(9*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*cosh(d*x + c)^8 + 9*(4*b^4*d*f*
x + 4*b^4*d*e - b^4*f)*sinh(d*x + c)^8 - 32*(3*a*b^3*d*f*x + 3*a*b^3*d*e -
a*b^3*f)*cosh(d*x + c)^7 - 8*(12*a*b^3*d*f*x + 12*a*b^3*d*e - 4*a*b^3*f - 9
*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*cosh(d*x + c))*sinh(d*x + c)^7 + 36*b^4*
d*f*x + 72*(2*(2*a^2*b^2 + b^4)*d*f*x + 2*(2*a^2*b^2 + b^4)*d*e - (2*a^2*b^
2 + b^4)*f)*cosh(d*x + c)^6 + 4*(36*(2*a^2*b^2 + b^4)*d*f*x + 36*(2*a^2*b^2
 + b^4)*d*e + 63*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*cosh(d*x + c)^2 - 18*(2*
a^2*b^2 + b^4)*f - 56*(3*a*b^3*d*f*x + 3*a*b^3*d*e - a*b^3*f)*cosh(d*x + c)
)*sinh(d*x + c)^6 + 36*b^4*d*e - 288*((4*a^3*b + 3*a*b^3)*d*f*x + (4*a^3*b
+ 3*a*b^3)*d*e - (4*a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^5 - 24*(12*(4*a^3*b +
 3*a*b^3)*d*f*x - 21*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*cosh(d*x + c)^3 + 12
*(4*a^3*b + 3*a*b^3)*d*e + 28*(3*a*b^3*d*f*x + 3*a*b^3*d*e - a*b^3*f)*cosh(
d*x + c)^2 - 12*(4*a^3*b + 3*a*b^3)*f - 18*(2*(2*a^2*b^2 + b^4)*d*f*x + 2*(
2*a^2*b^2 + b^4)*d*e - (2*a^2*b^2 + b^4)*f)*cosh(d*x + c))*sinh(d*x + c)^5
+ 9*b^4*f - 1152*((a^4 + a^2*b^2)*d^2*f*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*x + 4
*(a^4 + a^2*b^2)*c*d*e - 2*(a^4 + a^2*b^2)*c^2*f)*cosh(d*x + c)^4 - 2*(576*
(a^4 + a^2*b^2)*d^2*f*x^2 + 1152*(a^4 + a^2*b^2)*d^2*e*x - 315*(4*b^4*d*f*x
 + 4*b^4*d*e - b^4*f)*cosh(d*x + c)^4 + 2304*(a^4 + a^2*b^2)*c*d*e - 1152*(
a^4 + a^2*b^2)*c^2*f + 560*(3*a*b^3*d*f*x + 3*a*b^3*d*e - a*b^3*f)*cosh(d*x
 + c)^3 - 540*(2*(2*a^2*b^2 + b^4)*d*f*x + 2*(2*a^2*b^2 + b^4)*d*e - (2*a^2
```

```
*b^2 + b^4)*f)*cosh(d*x + c)^2 + 720*((4*a^3*b + 3*a*b^3)*d*f*x + (4*a^3*b
+ 3*a*b^3)*d*e - (4*a^3*b + 3*a*b^3)*f)*cosh(d*x + c))*sinh(d*x + c)^4 + 28
8*((4*a^3*b + 3*a*b^3)*d*f*x + (4*a^3*b + 3*a*b^3)*d*e + (4*a^3*b + 3*a*b^3
)*f)*cosh(d*x + c)^3 + 8*(63*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*cosh(d*x + c
)^5 - 140*(3*a*b^3*d*f*x + 3*a*b^3*d*e - a*b^3*f)*cosh(d*x + c)^4 + 36*(4*a
^3*b + 3*a*b^3)*d*f*x + 180*(2*(2*a^2*b^2 + b^4)*d*f*x + 2*(2*a^2*b^2 + b^4
)*d*e - (2*a^2*b^2 + b^4)*f)*cosh(d*x + c)^3 + 36*(4*a^3*b + 3*a*b^3)*d*e -
 360*((4*a^3*b + 3*a*b^3)*d*f*x + (4*a^3*b + 3*a*b^3)*d*e - (4*a^3*b + 3*a*
b^3)*f)*cosh(d*x + c)^2 + 36*(4*a^3*b + 3*a*b^3)*f - 576*((a^4 + a^2*b^2)*d
^2*f*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*x + 4*(a^4 + a^2*b^2)*c*d*e - 2*(a^4 + a
^2*b^2)*c^2*f)*cosh(d*x + c))*sinh(d*x + c)^3 + 72*(2*(2*a^2*b^2 + b^4)*d*f
*x + 2*(2*a^2*b^2 + b^4)*d*e + (2*a^2*b^2 + b^4)*f)*cosh(d*x + c)^2 + 12*(2
1*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*cosh(d*x + c)^6 - 56*(3*a*b^3*d*f*x + 3
*a*b^3*d*e - a*b^3*f)*cosh(d*x + c)^5 + 90*(2*(2*a^2*b^2 + b^4)*d*f*x + 2*(
2*a^2*b^2 + b^4)*d*e - (2*a^2*b^2 + b^4)*f)*cosh(d*x + c)^4 + 12*(2*a^2*b^2
 + b^4)*d*f*x - 240*((4*a^3*b + 3*a*b^3)*d*f*x + (4*a^3*b + 3*a*b^3)*d*e -
(4*a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^3 + 12*(2*a^2*b^2 + b^4)*d*e - 576*((a
^4 + a^2*b^2)*d^2*f*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*x + 4*(a^4 + a^2*b^2)*c*d
*e - 2*(a^4 + a^2*b^2)*c^2*f)*cosh(d*x + c)^2 + 6*(2*a^2*b^2 + b^4)*f + 72*
((4*a^3*b + 3*a*b^3)*d*f*x + (4*a^3*b + 3*a*b^3)*d*e + (4*a^3*b + 3*a*b^3)*
f)*cosh(d*x + c))*sinh(d*x + c)^2 + 32*(3*a*b^3*d*f*x + 3*a*b^3*d*e + a*b^3
*f)*cosh(d*x + c) + 2304*((a^4 + a^2*b^2)*f*cosh(d*x + c)^4 + 4*(a^4 + a^2*
b^2)*f*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^4 + a^2*b^2)*f*cosh(d*x + c)^2*
sinh(d*x + c)^2 + 4*(a^4 + a^2*b^2)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4
+ a^2*b^2)*f*sinh(d*x + c)^4)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2304*
((a^4 + a^2*b^2)*f*cosh(d*x + c)^4 + 4*(a^4 + a^2*b^2)*f*cosh(d*x + c)^3*si
nh(d*x + c) + 6*(a^4 + a^2*b^2)*f*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^4
+ a^2*b^2)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + a^2*b^2)*f*sinh(d*x + c
)^4)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2304*(((a^4 + a^2*b^2)*d*e - (
a^4 + a^2*b^2)*c*f)*cosh(d*x + c)^4 + 4*((a^4 + a^2*b^2)*d*e - (a^4 + a^2*b
^2)*c*f)*cosh(d*x + c)^3*sinh(d*x + c) + 6*((a^4 + a^2*b^2)*d*e - (a^4 + a^
2*b^2)*c*f)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*((a^4 + a^2*b^2)*d*e - (a^4
 + a^2*b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 + a^2*b^2)*d*e - (a^
4 + a^2*b^2)*c*f)*sinh(d*x + c)^4)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c
) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2304*(((a^4 + a^2*b^2)*d*e - (a^4 +
a^2*b^2)*c*f)*cosh(d*x + c)^4 + 4*((a^4 + a^2*b^2)*d*e - (a^4 + a^2*b^2)*c*
f)*cosh(d*x + c)^3*sinh(d*x + c) + 6*((a^4 + a^2*b^2)*d*e - (a^4 + a^2*b^2)
*c*f)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*((a^4 + a^2*b^2)*d*e - (a^4 + a^2
*b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 + a^2*b^2)*d*e - (a^4 + a^
2*b^2)*c*f)*sinh(d*x + c)^4)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*
b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2304*(((a^4 + a^2*b^2)*d*f*x + (a^4 + a^2*
b^2)*c*f)*cosh(d*x + c)^4 + 4*((a^4 + a^2*b^2)*d*f*x + (a^4 + a^2*b^2)*c*f)
*cosh(d*x + c)^3*sinh(d*x + c) + 6*((a^4 + a^2*b^2)*d*f*x + (a^4 + a^2*b^2)
*c*f)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*((a^4 + a^2*b^2)*d*f*x + (a^4 + a
^2*b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 + a^2*b^2)*d*f*x + (a^4
+ a^2*b^2)*c*f)*sinh(d*x + c)^4)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) +
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2304*((
(a^4 + a^2*b^2)*d*f*x + (a^4 + a^2*b^2)*c*f)*cosh(d*x + c)^4 + 4*((a^4 + a^
2*b^2)*d*f*x + (a^4 + a^2*b^2)*c*f)*cosh(d*x + c)^3*sinh(d*x + c) + 6*((a^4
 + a^2*b^2)*d*f*x + (a^4 + a^2*b^2)*c*f)*cosh(d*x + c)^2*sinh(d*x + c)^2 +
4*((a^4 + a^2*b^2)*d*f*x + (a^4 + a^2*b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)
^3 + ((a^4 + a^2*b^2)*d*f*x + (a^4 + a^2*b^2)*c*f)*sinh(d*x + c)^4)*log(-(a
*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b) + 8*(9*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*cosh(d*x
 + c)^7 + 12*a*b^3*d*f*x - 28*(3*a*b^3*d*f*x + 3*a*b^3*d*e - a*b^3*f)*cosh(
d*x + c)^6 + 12*a*b^3*d*e + 54*(2*(2*a^2*b^2 + b^4)*d*f*x + 2*(2*a^2*b^2 +
b^4)*d*e - (2*a^2*b^2 + b^4)*f)*cosh(d*x + c)^5 + 4*a*b^3*f - 180*((4*a^3*b
```

```
 + 3*a*b^3)*d*f*x + (4*a^3*b + 3*a*b^3)*d*e - (4*a^3*b + 3*a*b^3)*f)*cosh(d
*x + c)^4 - 576*((a^4 + a^2*b^2)*d^2*f*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*x + 4*
(a^4 + a^2*b^2)*c*d*e - 2*(a^4 + a^2*b^2)*c^2*f)*cosh(d*x + c)^3 + 108*((4*
a^3*b + 3*a*b^3)*d*f*x + (4*a^3*b + 3*a*b^3)*d*e + (4*a^3*b + 3*a*b^3)*f)*c
osh(d*x + c)^2 + 18*(2*(2*a^2*b^2 + b^4)*d*f*x + 2*(2*a^2*b^2 + b^4)*d*e +
(2*a^2*b^2 + b^4)*f)*cosh(d*x + c))*sinh(d*x + c))/(b^5*d^2*cosh(d*x + c)^4
 + 4*b^5*d^2*cosh(d*x + c)^3*sinh(d*x + c) + 6*b^5*d^2*cosh(d*x + c)^2*sinh
(d*x + c)^2 + 4*b^5*d^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^5*d^2*sinh(d*x +
c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\cosh(dx+c)^3\sinh(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)^3*sinh(d*x + c)^2/(b*sinh(d*x + c) + a),
x)
```

3.375 $\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=113

$$\frac{(a^2+b^2)\sinh^2(c+dx)}{2b^3d}-\frac{a(a^2+b^2)\sinh(c+dx)}{b^4d}+\frac{a^2(a^2+b^2)\log(a+b\sinh(c+dx))}{b^5d}-\frac{a\sinh^3(c+dx)}{3b^2d}+\frac{\sinh^4(c+dx)}{4b d}$$

```
[Out] (a^2*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/(b^5*d) - (a*(a^2 + b^2)*Sinh[c
+ d*x])/(b^4*d) + ((a^2 + b^2)*Sinh[c + d*x]^2)/(2*b^3*d) - (a*Sinh[c + d*x
]^3)/(3*b^2*d) + Sinh[c + d*x]^4/(4*b*d)
```

Rubi [A] time = 0.162831, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$\frac{(a^2+b^2)\sinh^2(c+dx)}{2b^3d}-\frac{a(a^2+b^2)\sinh(c+dx)}{b^4d}+\frac{a^2(a^2+b^2)\log(a+b\sinh(c+dx))}{b^5d}-\frac{a\sinh^3(c+dx)}{3b^2d}+\frac{\sinh^4(c+dx)}{4b d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/(b^5*d) - (a*(a^2 + b^2)*Sinh[c
+ d*x])/(b^4*d) + ((a^2 + b^2)*Sinh[c + d*x]^2)/(2*b^3*d) - (a*Sinh[c + d*x
]^3)/(3*b^2*d) + Sinh[c + d*x]^4/(4*b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^2(-b^2-x^2)}{b^2(a+x)}\,dx, x, b\sinh(c+dx)\right)}{b^3d}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{x^2(-b^2-x^2)}{a+x}\,dx, x, b\sinh(c+dx)\right)}{b^5d}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(a\left(a^2+b^2\right)-\left(a^2+b^2\right)x+ax^2-x^3-\frac{a^2(a^2+b^2)}{a+x}\right)dx, x, b\sinh(c+dx)\right)}{b^5d}$$
$$= \frac{a^2\left(a^2+b^2\right)\log(a+b\sinh(c+dx))}{b^5d} - \frac{a\left(a^2+b^2\right)\sinh(c+dx)}{b^4d} + \frac{\left(a^2+b^2\right)\sinh^2(c}{2b^3d}$$

Mathematica [A] time = 0.302434, size = 98, normalized size = 0.87

$$\frac{6b^2\left(a^2+b^2\right)\sinh^2(c+dx)-12ab\left(a^2+b^2\right)\sinh(c+dx)+12a^2\left(a^2+b^2\right)\log(a+b\sinh(c+dx))-4ab^3\sinh^3(c+dx)+}{12b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (12*a^2*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]] - 12*a*b*(a^2 + b^2)*Sinh[c +
d*x] + 6*b^2*(a^2 + b^2)*Sinh[c + d*x]^2 - 4*a*b^3*Sinh[c + d*x]^3 + 3*b^4*
Sinh[c + d*x]^4)/(12*b^5*d)
```

Maple [B] time = 0.046, size = 614, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] 5/8/d/b/(tanh(1/2*d*x+1/2*c)+1)^2+5/8/d/b/(tanh(1/2*d*x+1/2*c)-1)^2-1/d*a^4
/b^5*ln(tanh(1/2*d*x+1/2*c)-1)+1/d*a^4/b^5*ln(tanh(1/2*d*x+1/2*c)^2*a-2*tan
h(1/2*d*x+1/2*c)*b-a)+1/2/d/b^3/(tanh(1/2*d*x+1/2*c)+1)^2*a^2+1/d/b^4/(tanh
(1/2*d*x+1/2*c)+1)*a^3-1/d*a^4/b^5*ln(tanh(1/2*d*x+1/2*c)+1)+1/3/d/b^2/(tan
h(1/2*d*x+1/2*c)+1)^3*a+1/3/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^3*a+1/2/d/b^3/(ta
nh(1/2*d*x+1/2*c)-1)^2*a^2+1/d/b^4/(tanh(1/2*d*x+1/2*c)-1)*a^3-1/2/d/b^2/(t
anh(1/2*d*x+1/2*c)+1)^2*a-1/2/d/b^3/(tanh(1/2*d*x+1/2*c)+1)*a^2+1/2/d/b^2/(
tanh(1/2*d*x+1/2*c)-1)^2*a+1/2/d/b^3/(tanh(1/2*d*x+1/2*c)-1)*a^2+1/d/b^3*ln
(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)*b-a)*a^2+1/d/b^2/(tanh(1/2*d
*x+1/2*c)+1)*a-1/d/b^3*ln(tanh(1/2*d*x+1/2*c)+1)*a^2+1/d/b^2/(tanh(1/2*d*x+
1/2*c)-1)*a-1/d/b^3*ln(tanh(1/2*d*x+1/2*c)-1)*a^2+1/4/d/b/(tanh(1/2*d*x+1/2
*c)-1)^4+1/4/d/b/(tanh(1/2*d*x+1/2*c)+1)^4-1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^
3+1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^3-3/8/d/b/(tanh(1/2*d*x+1/2*c)+1)+3/8/d/b
/(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.04974, size = 316, normalized size = 2.8

$$-\frac{\left(8\,ab^2e^{(-dx-c)}-3\,b^3-12\left(2\,a^2b+b^3\right)e^{(-2\,dx-2\,c)}+24\left(4\,a^3+3\,ab^2\right)e^{(-3\,dx-3\,c)}\right)e^{(4\,dx+4\,c)}}{192\,b^4d}+\frac{\left(a^4+a^2b^2\right)(dx+c)}{b^5d}+\frac{8\,ab^2e^{(-3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxim
a")
```

```
[Out] -1/192*(8*a*b^2*e^(-d*x - c) - 3*b^3 - 12*(2*a^2*b + b^3)*e^(-2*d*x - 2*c)
+ 24*(4*a^3 + 3*a*b^2)*e^(-3*d*x - 3*c))*e^(4*d*x + 4*c)/(b^4*d) + (a^4 + a
^2*b^2)*(d*x + c)/(b^5*d) + 1/192*(8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(-4*d
*x - 4*c) + 24*(4*a^3 + 3*a*b^2)*e^(-d*x - c) + 12*(2*a^2*b + b^3)*e^(-2*d*
x - 2*c))/(b^4*d) + (a^4 + a^2*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2
*c) - b)/(b^5*d)
```

Fricas [B] time = 2.23817, size = 2631, normalized size = 23.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="frica
s")
```

```
[Out] 1/192*(3*b^4*cosh(d*x + c)^8 + 3*b^4*sinh(d*x + c)^8 - 8*a*b^3*cosh(d*x + c
)^7 + 8*(3*b^4*cosh(d*x + c) - a*b^3)*sinh(d*x + c)^7 - 192*(a^4 + a^2*b^2)
*d*x*cosh(d*x + c)^4 + 12*(2*a^2*b^2 + b^4)*cosh(d*x + c)^6 + 4*(21*b^4*cos
h(d*x + c)^2 - 14*a*b^3*cosh(d*x + c) + 6*a^2*b^2 + 3*b^4)*sinh(d*x + c)^6
- 24*(4*a^3*b + 3*a*b^3)*cosh(d*x + c)^5 + 24*(7*b^4*cosh(d*x + c)^3 - 7*a*
b^3*cosh(d*x + c)^2 - 4*a^3*b - 3*a*b^3 + 3*(2*a^2*b^2 + b^4)*cosh(d*x + c)
)*sinh(d*x + c)^5 + 8*a*b^3*cosh(d*x + c) + 2*(105*b^4*cosh(d*x + c)^4 - 14
0*a*b^3*cosh(d*x + c)^3 - 96*(a^4 + a^2*b^2)*d*x + 90*(2*a^2*b^2 + b^4)*cos
h(d*x + c)^2 - 60*(4*a^3*b + 3*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 3*b^
4 + 24*(4*a^3*b + 3*a*b^3)*cosh(d*x + c)^3 + 8*(21*b^4*cosh(d*x + c)^5 - 35
*a*b^3*cosh(d*x + c)^4 + 12*a^3*b + 9*a*b^3 - 96*(a^4 + a^2*b^2)*d*x*cosh(d
*x + c) + 30*(2*a^2*b^2 + b^4)*cosh(d*x + c)^3 - 30*(4*a^3*b + 3*a*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^3 + 12*(2*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 12*(
7*b^4*cosh(d*x + c)^6 - 14*a*b^3*cosh(d*x + c)^5 - 96*(a^4 + a^2*b^2)*d*x*c
osh(d*x + c)^2 + 15*(2*a^2*b^2 + b^4)*cosh(d*x + c)^4 + 2*a^2*b^2 + b^4 - 2
0*(4*a^3*b + 3*a*b^3)*cosh(d*x + c)^3 + 6*(4*a^3*b + 3*a*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^2 + 192*((a^4 + a^2*b^2)*cosh(d*x + c)^4 + 4*(a^4 + a^2*b^2
)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^4 + a^2*b^2)*cosh(d*x + c)^2*sinh(d*
x + c)^2 + 4*(a^4 + a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + a^2*b^2
)*sinh(d*x + c)^4)*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x +
c))) + 8*(3*b^4*cosh(d*x + c)^7 - 7*a*b^3*cosh(d*x + c)^6 - 96*(a^4 + a^2*b
^2)*d*x*cosh(d*x + c)^3 + 9*(2*a^2*b^2 + b^4)*cosh(d*x + c)^5 - 15*(4*a^3*b
 + 3*a*b^3)*cosh(d*x + c)^4 + a*b^3 + 9*(4*a^3*b + 3*a*b^3)*cosh(d*x + c)^2
 + 3*(2*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c))/(b^5*d*cosh(d*x + c)^4
 + 4*b^5*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*b^5*d*cosh(d*x + c)^2*sinh(d*x
 + c)^2 + 4*b^5*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^5*d*sinh(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20187, size = 298, normalized size = 2.64

$$\frac{\left(a^4+a^2b^2\right)\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{b^5d}+\frac{3\,b^3d^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^4-8\,ab^2d^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+24\,a^2bd^3\left(e^{(dx+c)}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac"
)
```

```
[Out] (a^4 + a^2*b^2)*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(b^5*d) + 1/
192*(3*b^3*d^3*(e^(d*x + c) - e^(-d*x - c))^4 - 8*a*b^2*d^3*(e^(d*x + c) -
e^(-d*x - c))^3 + 24*a^2*b*d^3*(e^(d*x + c) - e^(-d*x - c))^2 + 24*b^3*d^3*
(e^(d*x + c) - e^(-d*x - c))^2 - 96*a^3*d^3*(e^(d*x + c) - e^(-d*x - c)) -
96*a*b^2*d^3*(e^(d*x + c) - e^(-d*x - c)))/(b^4*d^4)
```

3.376 $\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=38

$$\text{Unintegrable}\left(\frac{\sinh^2(c+dx)\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d
*x])), x]
```

Rubi [A] time = 0.13164, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^3(c+dx)\sinh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]
)),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.194, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx+c))^3(\sinh(dx+c))^2}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{e^{\left(-4c+\frac{4de}{f}\right)}E_1\left(\frac{4(fx+e)d}{f}\right)}{16bf}+\frac{ae^{\left(-3c+\frac{3de}{f}\right)}E_1\left(\frac{3(fx+e)d}{f}\right)}{8b^2f}+\frac{ae^{\left(3c-\frac{3de}{f}\right)}E_1\left(-\frac{3(fx+e)d}{f}\right)}{8b^2f}-\frac{e^{\left(4c-\frac{4de}{f}\right)}E_1\left(-\frac{4(fx+e)d}{f}\right)}{16bf}+\frac{(2a^2+b^2)e^{(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] 1/16*e^(-4*c + 4*d*e/f)*exp_integral_e(1, 4*(f*x + e)*d/f)/(b*f) + 1/8*a*e^
(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b^2*f) + 1/8*a*e^(3*c
- 3*d*e/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b^2*f) - 1/16*e^(4*c - 4*d*
e/f)*exp_integral_e(1, -4*(f*x + e)*d/f)/(b*f) + 1/8*(2*a^2 + b^2)*e^(-2*c
+ 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b^3*f) - 1/8*(2*a^2*e^(2*c)
+ b^2*e^(2*c))*e^(-2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b^3*f) + 1
/8*(4*a^3 + 3*a*b^2)*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^4*f
) + 1/8*(4*a^3*e^c + 3*a*b^2*e^c)*e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d
/f)/(b^4*f) + (a^4 + a^2*b^2)*log(f*x + e)/(b^5*f) - 1/32*integrate(64*(a^4
*b + a^2*b^3 - (a^5*e^c + a^3*b^2*e^c)*e^(d*x))/(b^6*f*x + b^6*e - (b^6*f*x
*e^(2*c) + b^6*e*e^(2*c))*e^(2*d*x) - 2*(a*b^5*f*x*e^c + a*b^5*e*e^c)*e^(d*
x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx+c)^3\sinh(dx+c)^2}{afx+ae+(bfx+be)\sinh(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] integral(cosh(d*x + c)^3*sinh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(
d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*sinh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^3 \sinh(dx+c)^2}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate(cosh(d*x + c)^3*sinh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a))
, x)
```

3.377 $\int \frac{(e+fx)^3 \sinh(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1218

result too large to display

```
[Out] -(e + f*x)^4/(4*b*f) - (2*a*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b^2*d) + (2*a
^3*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b^2*(a^2 + b^2)*d) + (a^2*(e + f*x)^3*
Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d) + (a^2*(e
 + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d)
 + ((e + f*x)^3*Log[1 + E^(2*(c + d*x))])/(b*d) - (a^2*(e + f*x)^3*Log[1 +
E^(2*(c + d*x))])/(b*(a^2 + b^2)*d) + ((3*I)*a*f*(e + f*x)^2*PolyLog[2, (-I
)*E^(c + d*x)])/(b^2*d^2) - ((3*I)*a^3*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c +
 d*x)])/(b^2*(a^2 + b^2)*d^2) - ((3*I)*a*f*(e + f*x)^2*PolyLog[2, I*E^(c +
d*x)])/(b^2*d^2) + ((3*I)*a^3*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b^2
*(a^2 + b^2)*d^2) + (3*a^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^2) + (3*a^2*f*(e + f*x)^2*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^2) + (3*f*(e + f
*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*b*d^2) - (3*a^2*f*(e + f*x)^2*PolyLo
g[2, -E^(2*(c + d*x))])/(2*b*(a^2 + b^2)*d^2) - ((6*I)*a*f^2*(e + f*x)*Poly
Log[3, (-I)*E^(c + d*x)])/(b^2*d^3) + ((6*I)*a^3*f^2*(e + f*x)*PolyLog[3, (
-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) + ((6*I)*a*f^2*(e + f*x)*PolyLog[3,
 I*E^(c + d*x)])/(b^2*d^3) - ((6*I)*a^3*f^2*(e + f*x)*PolyLog[3, I*E^(c + d
*x)])/(b^2*(a^2 + b^2)*d^3) - (6*a^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^3) - (6*a^2*f^2*(e + f*x)*Po
lyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^3) - (
3*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*b*d^3) + (3*a^2*f^2*(e + f
*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*b*(a^2 + b^2)*d^3) + ((6*I)*a*f^3*Poly
Log[4, (-I)*E^(c + d*x)])/(b^2*d^4) - ((6*I)*a^3*f^3*PolyLog[4, (-I)*E^(c +
 d*x)])/(b^2*(a^2 + b^2)*d^4) - ((6*I)*a*f^3*PolyLog[4, I*E^(c + d*x)])/(b^
2*d^4) + ((6*I)*a^3*f^3*PolyLog[4, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^4) +
(6*a^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 +
b^2)*d^4) + (6*a^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/(b*(a^2 + b^2)*d^4) + (3*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*b*d^4) - (3
*a^2*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*b*(a^2 + b^2)*d^4)
```

Rubi [A] time = 1.69431, antiderivative size = 1218, normalized size of antiderivative = 1., number of steps used = 46, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5581, 3718, 2190, 2531, 6609, 2282, 6589, 5567, 4180, 5573, 5561, 6742}

$$-\frac{(e+fx)^4}{4bf}+\frac{2a^3\tan^{-1}\left(e^{c+dx}\right)(e+fx)^3}{b^2\left(a^2+b^2\right)d}-\frac{2a\tan^{-1}\left(e^{c+dx}\right)(e+fx)^3}{b^2d}+\frac{a^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)(e+fx)^3}{b\left(a^2+b^2\right)d}+\frac{a^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+}}\right.}{b\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(e + f*x)^4/(4*b*f) - (2*a*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b^2*d) + (2*a
^3*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b^2*(a^2 + b^2)*d) + (a^2*(e + f*x)^3*
Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d) + (a^2*(e
 + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d)
 + ((e + f*x)^3*Log[1 + E^(2*(c + d*x))])/(b*d) - (a^2*(e + f*x)^3*Log[1 +
E^(2*(c + d*x))])/(b*(a^2 + b^2)*d) + ((3*I)*a*f*(e + f*x)^2*PolyLog[2, (-I
)*E^(c + d*x)])/(b^2*d^2) - ((3*I)*a^3*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c +
```

```
d*x)])/(b^2*(a^2 + b^2)*d^2) - ((3*I)*a*f*(e + f*x)^2*PolyLog[2, I*E^(c +
d*x)])/(b^2*d^2) + ((3*I)*a^3*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b^2
*(a^2 + b^2)*d^2) + (3*a^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^2) + (3*a^2*f*(e + f*x)^2*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^2) + (3*f*(e + f
*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*b*d^2) - (3*a^2*f*(e + f*x)^2*PolyLo
g[2, -E^(2*(c + d*x))])/(2*b*(a^2 + b^2)*d^2) - ((6*I)*a*f^2*(e + f*x)*Poly
Log[3, (-I)*E^(c + d*x)])/(b^2*d^3) + ((6*I)*a^3*f^2*(e + f*x)*PolyLog[3, (
-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) + ((6*I)*a*f^2*(e + f*x)*PolyLog[3,
 I*E^(c + d*x)])/(b^2*d^3) - ((6*I)*a^3*f^2*(e + f*x)*PolyLog[3, I*E^(c + d
*x)])/(b^2*(a^2 + b^2)*d^3) - (6*a^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^3) - (6*a^2*f^2*(e + f*x)*Po
lyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^3) - (
3*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*b*d^3) + (3*a^2*f^2*(e + f
*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*b*(a^2 + b^2)*d^3) + ((6*I)*a*f^3*Poly
Log[4, (-I)*E^(c + d*x)])/(b^2*d^4) - ((6*I)*a^3*f^3*PolyLog[4, (-I)*E^(c +
 d*x)])/(b^2*(a^2 + b^2)*d^4) - ((6*I)*a*f^3*PolyLog[4, I*E^(c + d*x)])/(b^
2*d^4) + ((6*I)*a^3*f^3*PolyLog[4, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^4) +
(6*a^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 +
b^2)*d^4) + (6*a^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/(b*(a^2 + b^2)*d^4) + (3*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*b*d^4) - (3
*a^2*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*b*(a^2 + b^2)*d^4)
```

Rule 5581

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - Di
st[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n)/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c +
 d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{a\int (e+fx)^3\text{sech}(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{b^2} + \cdots$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{(e+fx)^3\log\left(1+e^{2(c+dx)}\right)}{bd} + \cdots$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{a^2(e+fx)^4}{4b\left(a^2+b^2\right)f} - \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{(e+fx)^3\log \cdots}{b \cdots}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{a^2(e+fx)^4}{4b\left(a^2+b^2\right)f} - \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{a^2(e+fx)^3 \cdots}{b\left(a^2 \cdots\right)}$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d} + \cdots$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d} + \cdots$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d} + \cdots$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d} + \cdots$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d} + \cdots$$

$$= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d} + \cdots$$

Mathematica [B] time = 26.8379, size = 3387, normalized size = 2.78

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (-8*b*d^4*e^3*E^(2*c)*x - 12*b*d^4*e^2*E^(2*c)*f*x^2 - 8*b*d^4*e*E^(2*c)*f^
2*x^3 - 2*b*d^4*E^(2*c)*f^3*x^4 - 8*a*d^3*e^3*ArcTan[E^(c + d*x)] - 8*a*d^3
*e^3*E^(2*c)*ArcTan[E^(c + d*x)] - (12*I)*a*d^3*e^2*f*x*Log[1 - I*E^(c + d*
x)] - (12*I)*a*d^3*e^2*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] - (12*I)*a*d^3*e*
f^2*x^2*Log[1 - I*E^(c + d*x)] - (12*I)*a*d^3*e*E^(2*c)*f^2*x^2*Log[1 - I*E
^(c + d*x)] - (4*I)*a*d^3*f^3*x^3*Log[1 - I*E^(c + d*x)] - (4*I)*a*d^3*E^(2
*c)*f^3*x^3*Log[1 - I*E^(c + d*x)] + (12*I)*a*d^3*e^2*f*x*Log[1 + I*E^(c +
d*x)] + (12*I)*a*d^3*e^2*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] + (12*I)*a*d^3*
e*f^2*x^2*Log[1 + I*E^(c + d*x)] + (12*I)*a*d^3*e*E^(2*c)*f^2*x^2*Log[1 + I
*E^(c + d*x)] + (4*I)*a*d^3*f^3*x^3*Log[1 + I*E^(c + d*x)] + (4*I)*a*d^3*E^
(2*c)*f^3*x^3*Log[1 + I*E^(c + d*x)] + 4*b*d^3*e^3*Log[1 + E^(2*(c + d*x))]
```

+ 4*b*d^3*e^3*E^(2*c)*Log[1 + E^(2*(c + d*x))] + 12*b*d^3*e^2*f*x*Log[1 + E^(2*(c + d*x))] + 12*b*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(2*(c + d*x))] + 12*b*d^3*e*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 12*b*d^3*e*E^(2*c)*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 4*b*d^3*f^3*x^3*Log[1 + E^(2*(c + d*x))] + 4*b*d^3*E^(2*c)*f^3*x^3*Log[1 + E^(2*(c + d*x))] + (12*I)*a*d^2*(1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)] - (12*I)*a*d^2*(1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)] + 6*b*d^2*e^2*f*PolyLog[2, -E^(2*(c + d*x))] + 6*b*d^2*e^2*E^(2*c)*f*PolyLog[2, -E^(2*(c + d*x))] + 12*b*d^2*e*f^2*x*PolyLog[2, -E^(2*(c + d*x))] + 12*b*d^2*e*E^(2*c)*f^2*x*PolyLog[2, -E^(2*(c + d*x))] + 6*b*d^2*f^3*x^2*PolyLog[2, -E^(2*(c + d*x))] + 6*b*d^2*E^(2*c)*f^3*x^2*PolyLog[2, -E^(2*(c + d*x))] - (24*I)*a*d*e*f^2*PolyLog[3, (-I)*E^(c + d*x)] - (24*I)*a*d*e*E^(2*c)*f^2*PolyLog[3, (-I)*E^(c + d*x)] - (24*I)*a*d*f^3*x*PolyLog[3, (-I)*E^(c + d*x)] - (24*I)*a*d*E^(2*c)*f^3*x*PolyLog[3, (-I)*E^(c + d*x)] + (24*I)*a*d*e*f^2*PolyLog[3, I*E^(c + d*x)] + (24*I)*a*d*e*E^(2*c)*f^2*PolyLog[3, I*E^(c + d*x)] + (24*I)*a*d*f^3*x*PolyLog[3, I*E^(c + d*x)] + (24*I)*a*d*E^(2*c)*f^3*x*PolyLog[3, I*E^(c + d*x)] - 6*b*d*e*f^2*PolyLog[3, -E^(2*(c + d*x))] - 6*b*d*e*E^(2*c)*f^2*PolyLog[3, -E^(2*(c + d*x))] - 6*b*d*f^3*x*PolyLog[3, -E^(2*(c + d*x))] - 6*b*d*E^(2*c)*f^3*x*PolyLog[3, -E^(2*(c + d*x))] + (24*I)*a*f^3*PolyLog[4, (-I)*E^(c + d*x)] + (24*I)*a*E^(2*c)*f^3*PolyLog[4, (-I)*E^(c + d*x)] - (24*I)*a*f^3*PolyLog[4, I*E^(c + d*x)] - (24*I)*a*E^(2*c)*f^3*PolyLog[4, I*E^(c + d*x)] + 3*b*f^3*PolyLog[4, -E^(2*(c + d*x))] + 3*b*E^(2*c)*f^3*PolyLog[4, -E^(2*(c + d*x))]])/(4*(a^2 + b^2)*d^4*(1 + E^(2*c))) - (a^2*(4*e^3*E^(2*c)*x + 6*e^2*E^(2*c)*f*x^2 + 4*e*E^(2*c)*f^2*x^3 + E^(2*c)*f^3*x^4 + (4*a*Sqrt[a^2 + b^2]*e^3*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (2*e^3*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (2*e^3*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (2*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (2*E^(2*c)*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (2*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (2*E^(2*c)*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (12*e*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*e*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (12*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*E^(2*c)*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (12*e*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*e*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (12*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*E^(2*c)*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^4 - (12*E^(2*c)*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d

```
^4 + (12*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*
c)]))])/d^4 - (12*E^(2*c)*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[
(a^2 + b^2)*E^(2*c)]))])/d^4))/(2*b*(a^2 + b^2)*(-1 + E^(2*c))) + ((4*a^2*e
^3*x - 4*b^2*e^3*x + 6*a^2*e^2*f*x^2 - 6*b^2*e^2*f*x^2 + 4*a^2*e*f^2*x^3 -
4*b^2*e*f^2*x^3 + a^2*f^3*x^4 - b^2*f^3*x^4 + 4*a^2*e^3*x*Cosh[2*c] + 4*b^2
*e^3*x*Cosh[2*c] + 6*a^2*e^2*f*x^2*Cosh[2*c] + 6*b^2*e^2*f*x^2*Cosh[2*c] +
4*a^2*e*f^2*x^3*Cosh[2*c] + 4*b^2*e*f^2*x^3*Cosh[2*c] + a^2*f^3*x^4*Cosh[2*
c] + b^2*f^3*x^4*Cosh[2*c])*Csch[c]*Sech[c])/(8*b*(a^2 + b^2))
```

Maple [F] time = 0.652, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3}\sinh\left(dx+c\right)\tanh\left(dx+c\right)}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$e^{3}\left(\frac{a^{2}\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{\left(a^{2}b+b^{3}\right)d}+\frac{2\,a\arctan\left(e^{(-dx-c)}\right)}{\left(a^{2}+b^{2}\right)d}+\frac{b\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\left(a^{2}+b^{2}\right)d}+\frac{dx+c}{bd}\right)+\frac{f^{3}x^{4}+4\,ef^{2}x^{3}-}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] e^3*(a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b + b^3)*d)
+ 2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((
a^2 + b^2)*d) + (d*x + c)/(b*d)) + 1/4*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2
)/b - integrate(2*(a^2*b*f^3*x^3 + 3*a^2*b*e*f^2*x^2 + 3*a^2*b*e^2*f*x - (a
^3*f^3*x^3*e^c + 3*a^3*e*f^2*x^2*e^c + 3*a^3*e^2*f*x*e^c)*e^(d*x))/(a^2*b^2
 + b^4 - (a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x) - 2*(a^3*b*e^c + a*b^3*e
^c)*e^(d*x)), x) - integrate(2*(b*f^3*x^3 + 3*b*e*f^2*x^2 + 3*b*e^2*f*x + (
a*f^3*x^3*e^c + 3*a*e*f^2*x^2*e^c + 3*a*e^2*f*x*e^c)*e^(d*x))/(a^2 + b^2 +
(a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [C] time = 3.27461, size = 4721, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -1/4*((a^2 + b^2)*d^4*f^3*x^4 + 4*(a^2 + b^2)*d^4*e*f^2*x^3 + 6*(a^2 + b^2)
*d^4*e^2*f*x^2 + 4*(a^2 + b^2)*d^4*e^3*x - 24*a^2*f^3*polylog(4, (a*cosh(d*
```

```
x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2))/b) - 24*a^2*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) -
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 12*(a^2*d^2
*f^3*x^2 + 2*a^2*d^2*e*f^2*x + a^2*d^2*e^2*f)*dilog((a*cosh(d*x + c) + a*si
nh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b
)/b + 1) - 12*(a^2*d^2*f^3*x^2 + 2*a^2*d^2*e*f^2*x + a^2*d^2*e^2*f)*dilog((
a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqr
t((a^2 + b^2)/b^2) - b)/b + 1) - (-12*I*a*b*d^2*f^3*x^2 + 12*b^2*d^2*f^3*x^
2 - 24*I*a*b*d^2*e*f^2*x + 24*b^2*d^2*e*f^2*x - 12*I*a*b*d^2*e^2*f + 12*b^2
*d^2*e^2*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) - (12*I*a*b*d^2*f^3*x^
2 + 12*b^2*d^2*f^3*x^2 + 24*I*a*b*d^2*e*f^2*x + 24*b^2*d^2*e*f^2*x + 12*I*a
*b*d^2*e^2*f + 12*b^2*d^2*e^2*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c))
- 4*(a^2*d^3*e^3 - 3*a^2*c*d^2*e^2*f + 3*a^2*c^2*d*e*f^2 - a^2*c^3*f^3)*log
(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) -
 4*(a^2*d^3*e^3 - 3*a^2*c*d^2*e^2*f + 3*a^2*c^2*d*e*f^2 - a^2*c^3*f^3)*log(
2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) -
4*(a^2*d^3*f^3*x^3 + 3*a^2*d^3*e*f^2*x^2 + 3*a^2*d^3*e^2*f*x + 3*a^2*c*d^2*
e^2*f - 3*a^2*c^2*d*e*f^2 + a^2*c^3*f^3)*log(-(a*cosh(d*x + c) + a*sinh(d*x
 + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) -
 4*(a^2*d^3*f^3*x^3 + 3*a^2*d^3*e*f^2*x^2 + 3*a^2*d^3*e^2*f*x + 3*a^2*c*d^2
*e^2*f - 3*a^2*c^2*d*e*f^2 + a^2*c^3*f^3)*log(-(a*cosh(d*x + c) + a*sinh(d*
x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b)
- (-4*I*a*b*d^3*e^3 + 4*b^2*d^3*e^3 + 12*I*a*b*c*d^2*e^2*f - 12*b^2*c*d^2*e
^2*f - 12*I*a*b*c^2*d*e*f^2 + 12*b^2*c^2*d*e*f^2 + 4*I*a*b*c^3*f^3 - 4*b^2*
c^3*f^3)*log(cosh(d*x + c) + sinh(d*x + c) + I) - (4*I*a*b*d^3*e^3 + 4*b^2*
d^3*e^3 - 12*I*a*b*c*d^2*e^2*f - 12*b^2*c*d^2*e^2*f + 12*I*a*b*c^2*d*e*f^2
+ 12*b^2*c^2*d*e*f^2 - 4*I*a*b*c^3*f^3 - 4*b^2*c^3*f^3)*log(cosh(d*x + c) +
 sinh(d*x + c) - I) - (4*I*a*b*d^3*f^3*x^3 + 4*b^2*d^3*f^3*x^3 + 12*I*a*b*d
^3*e*f^2*x^2 + 12*b^2*d^3*e*f^2*x^2 + 12*I*a*b*d^3*e^2*f*x + 12*b^2*d^3*e^2
*f*x + 12*I*a*b*c*d^2*e^2*f + 12*b^2*c*d^2*e^2*f - 12*I*a*b*c^2*d*e*f^2 - 1
2*b^2*c^2*d*e*f^2 + 4*I*a*b*c^3*f^3 + 4*b^2*c^3*f^3)*log(I*cosh(d*x + c) +
I*sinh(d*x + c) + 1) - (-4*I*a*b*d^3*f^3*x^3 + 4*b^2*d^3*f^3*x^3 - 12*I*a*b
*d^3*e*f^2*x^2 + 12*b^2*d^3*e*f^2*x^2 - 12*I*a*b*d^3*e^2*f*x + 12*b^2*d^3*e
^2*f*x - 12*I*a*b*c*d^2*e^2*f + 12*b^2*c*d^2*e^2*f + 12*I*a*b*c^2*d*e*f^2 -
 12*b^2*c^2*d*e*f^2 - 4*I*a*b*c^3*f^3 + 4*b^2*c^3*f^3)*log(-I*cosh(d*x + c)
 - I*sinh(d*x + c) + 1) + 24*(I*a*b*f^3 - b^2*f^3)*polylog(4, I*cosh(d*x +
c) + I*sinh(d*x + c)) + 24*(-I*a*b*f^3 - b^2*f^3)*polylog(4, -I*cosh(d*x +
c) - I*sinh(d*x + c)) + 24*(a^2*d*f^3*x + a^2*d*e*f^2)*polylog(3, (a*cosh(d
*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2))/b) + 24*(a^2*d*f^3*x + a^2*d*e*f^2)*polylog(3, (a*cosh(d*x + c)
 + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2))/b) - (24*I*a*b*d*f^3*x - 24*b^2*d*f^3*x + 24*I*a*b*d*e*f^2 - 24*b^2*d*
e*f^2)*polylog(3, I*cosh(d*x + c) + I*sinh(d*x + c)) - (-24*I*a*b*d*f^3*x -
 24*b^2*d*f^3*x - 24*I*a*b*d*e*f^2 - 24*b^2*d*e*f^2)*polylog(3, -I*cosh(d*x
 + c) - I*sinh(d*x + c)))/((a^2*b + b^3)*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^3 \sinh(c + dx) \tanh(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*sinh(c + d*x)*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.378 $\int \frac{(e+fx)^2 \sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=861

$$\frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)a^3}{b^2\left(a^2+b^2\right)d}-\frac{2if(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)a^3}{b^2\left(a^2+b^2\right)d^2}+\frac{2if(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)a^3}{b^2\left(a^2+b^2\right)d^2}+\frac{2if^2\text{PolyLog}\left(\right.}{b^2\left(a^2+\right.}$$

```
[Out] -(e + f*x)^3/(3*b*f) - (2*a*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^2*d) + (2*a
^3*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^2*(a^2 + b^2)*d) + (a^2*(e + f*x)^2*
Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d) + (a^2*(e
 + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d)
 + ((e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b*d) - (a^2*(e + f*x)^2*Log[1 +
E^(2*(c + d*x))])/(b*(a^2 + b^2)*d) + ((2*I)*a*f*(e + f*x)*PolyLog[2, (-I)*
E^(c + d*x)])/(b^2*d^2) - ((2*I)*a^3*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x
)])/(b^2*(a^2 + b^2)*d^2) - ((2*I)*a*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])
/(b^2*d^2) + ((2*I)*a^3*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b^2*(a^2 +
b^2)*d^2) + (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/(b*(a^2 + b^2)*d^2) + (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^2) + (f*(e + f*x)*PolyLog[2,
 -E^(2*(c + d*x))])/(b*d^2) - (a^2*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))]
)/(b*(a^2 + b^2)*d^2) - ((2*I)*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b^2*d^3
) + ((2*I)*a^3*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) + ((
2*I)*a*f^2*PolyLog[3, I*E^(c + d*x)])/(b^2*d^3) - ((2*I)*a^3*f^2*PolyLog[3,
 I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) - (2*a^2*f^2*PolyLog[3, -((b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^3) - (2*a^2*f^2*PolyLog[3,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^3) - (f^2*PolyL
og[3, -E^(2*(c + d*x))])/(2*b*d^3) + (a^2*f^2*PolyLog[3, -E^(2*(c + d*x))])
/(2*b*(a^2 + b^2)*d^3)
```

Rubi [A] time = 1.36649, antiderivative size = 861, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {5581, 3718, 2190, 2531, 2282, 6589, 5567, 4180, 5573, 5561, 6742}

$$\frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)a^3}{b^2\left(a^2+b^2\right)d}-\frac{2if(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)a^3}{b^2\left(a^2+b^2\right)d^2}+\frac{2if(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right)a^3}{b^2\left(a^2+b^2\right)d^2}+\frac{2if^2\text{PolyLog}\left(\right.}{b^2\left(a^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```



```
[Out] -(e + f*x)^3/(3*b*f) - (2*a*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^2*d) + (2*a
^3*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^2*(a^2 + b^2)*d) + (a^2*(e + f*x)^2*
Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d) + (a^2*(e
 + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d)
 + ((e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b*d) - (a^2*(e + f*x)^2*Log[1 +
E^(2*(c + d*x))])/(b*(a^2 + b^2)*d) + ((2*I)*a*f*(e + f*x)*PolyLog[2, (-I)*
E^(c + d*x)])/(b^2*d^2) - ((2*I)*a^3*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x
)])/(b^2*(a^2 + b^2)*d^2) - ((2*I)*a*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])
/(b^2*d^2) + ((2*I)*a^3*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b^2*(a^2 +
b^2)*d^2) + (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/(b*(a^2 + b^2)*d^2) + (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^2) + (f*(e + f*x)*PolyLog[2,
 -E^(2*(c + d*x))])/(b*d^2) - (a^2*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))]
)/(b*(a^2 + b^2)*d^2) - ((2*I)*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b^2*d^3
```

```
) + ((2*I)*a^3*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) + ((
2*I)*a*f^2*PolyLog[3, I*E^(c + d*x)])/(b^2*d^3) - ((2*I)*a^3*f^2*PolyLog[3,
 I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) - (2*a^2*f^2*PolyLog[3, -((b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^3) - (2*a^2*f^2*PolyLog[3,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^3) - (f^2*PolyL
og[3, -E^(2*(c + d*x))])/(2*b*d^3) + (a^2*f^2*PolyLog[3, -E^(2*(c + d*x))])
/(2*b*(a^2 + b^2)*d^3)
```

Rule 5581

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - Di
st[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n)/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c +
```

```
 d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{a\int (e+fx)^2\text{sech}(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{b^2} + \cdots$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{bd} + \cdots$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{a^2(e+fx)^3}{3b\left(a^2+b^2\right)f} - \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{(e+fx)^2\log \cdots}{b \cdots}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{a^2(e+fx)^3}{3b\left(a^2+b^2\right)f} - \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{a^2(e+fx)^2\,\text{lo}\cdots}{b\left(a^2 \cdots\right.}$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d} + \cdots$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d} + \cdots$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d} + \cdots$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d} + \cdots$$

$$= -\frac{(e+fx)^3}{3bf} - \frac{2a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d} + \cdots$$

Mathematica [B] time = 19.7431, size = 1759, normalized size = 2.04

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] -(12*b*d^3*e^2*E^(2*c)*x - 12*b*d^3*e^2*(1 + E^(2*c))*x - 12*b*d^3*e*f*x^2
- 4*b*d^3*f^2*x^3 + 12*a*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] + 6*b*d^
2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*a*d*e*(1 +
E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - PolyLog
[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) + 6*b*d*e*(1 + E^(2*c))*
f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]) +
 (6*I)*a*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log[1
+ I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2, I*
E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*x)]
) + b*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))]) - 6
*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))]))/(6*(a^
2 + b^2)*d^3*(1 + E^(2*c))) - (a^2*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2
*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt
[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(
```

```
2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) -
(6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])
/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a
+ b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*
E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c +
d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a
*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c
+ d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(
2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Lo
g[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*
Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^
(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d
 + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])
])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^
2)*E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d
*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e +
f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/
d^2 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*
c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(
a^2 + b^2)*E^(2*c)]))])/d^3 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c
+ Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c
 + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3))/(3*b*(a^2 + b^2)*(-1
+ E^(2*c))) + ((3*a^2*e^2*x - 3*b^2*e^2*x + 3*a^2*e*f*x^2 - 3*b^2*e*f*x^2 +
 a^2*f^2*x^3 - b^2*f^2*x^3 + 3*a^2*e^2*x*Cosh[2*c] + 3*b^2*e^2*x*Cosh[2*c]
+ 3*a^2*e*f*x^2*Cosh[2*c] + 3*b^2*e*f*x^2*Cosh[2*c] + a^2*f^2*x^3*Cosh[2*c]
 + b^2*f^2*x^3*Cosh[2*c])*Csch[c]*Sech[c])/(6*b*(a^2 + b^2))
```

Maple [F] time = 0.49, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \sinh \left(dx+c\right) \tanh \left(dx+c\right)}{a+b \sinh \left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$e^{2}\left(\frac{a^{2} \log\left(-2\, ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{\left(a^{2}b+b^{3}\right)d}+\frac{2\,a \arctan\left(e^{(-dx-c)}\right)}{\left(a^{2}+b^{2}\right)d}+\frac{b \log\left(e^{(-2\,dx-2\,c)}+1\right)}{\left(a^{2}+b^{2}\right)d}+\frac{dx+c}{bd}\right)+\frac{f^{2}x^{3}+3\,efx^{2}}{3\,b}-\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] e^2*(a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b + b^3)*d)
+ 2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((
a^2 + b^2)*d) + (d*x + c)/(b*d)) + 1/3*(f^2*x^3 + 3*e*f*x^2)/b - integrate(
2*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x - (a^3*f^2*x^2*e^c + 2*a^3*e*f*x*e^c)*e^(d
*x))/(a^2*b^2 + b^4 - (a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x) - 2*(a^3*b*
e^c + a*b^3*e^c)*e^(d*x)), x) - integrate(2*(b*f^2*x^2 + 2*b*e*f*x + (a*f^2
```

```
*x^2*e^c + 2*a*e*f*x*e^c)*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))
*e^(2*d*x)), x)
```

Fricas [C] time = 2.84092, size = 3077, normalized size = 3.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -1/3*((a^2 + b^2)*d^3*f^2*x^3 + 3*(a^2 + b^2)*d^3*e*f*x^2 + 3*(a^2 + b^2)*d
^3*e^2*x + 6*a^2*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*a^2*f^2*polylog
(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2))/b) - 6*(a^2*d*f^2*x + a^2*d*e*f)*dilog((a*cosh(d*x
 + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2) - b)/b + 1) - 6*(a^2*d*f^2*x + a^2*d*e*f)*dilog((a*cosh(d*x + c) +
 a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
) - b)/b + 1) - (-6*I*a*b*d*f^2*x + 6*b^2*d*f^2*x - 6*I*a*b*d*e*f + 6*b^2*d
*e*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) - (6*I*a*b*d*f^2*x + 6*b^2*d
*f^2*x + 6*I*a*b*d*e*f + 6*b^2*d*e*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x +
 c)) - 3*(a^2*d^2*e^2 - 2*a^2*c*d*e*f + a^2*c^2*f^2)*log(2*b*cosh(d*x + c)
+ 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(a^2*d^2*e^2 - 2
*a^2*c*d*e*f + a^2*c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b
*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + 2*a^
2*c*d*e*f - a^2*c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 3*(a^2*d^2*f^2*
x^2 + 2*a^2*d^2*e*f*x + 2*a^2*c*d*e*f - a^2*c^2*f^2)*log(-(a*cosh(d*x + c)
+ a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2) - b)/b) - (-3*I*a*b*d^2*e^2 + 3*b^2*d^2*e^2 + 6*I*a*b*c*d*e*f - 6*b^2*c*
d*e*f - 3*I*a*b*c^2*f^2 + 3*b^2*c^2*f^2)*log(cosh(d*x + c) + sinh(d*x + c)
+ I) - (3*I*a*b*d^2*e^2 + 3*b^2*d^2*e^2 - 6*I*a*b*c*d*e*f - 6*b^2*c*d*e*f +
 3*I*a*b*c^2*f^2 + 3*b^2*c^2*f^2)*log(cosh(d*x + c) + sinh(d*x + c) - I) -
(3*I*a*b*d^2*f^2*x^2 + 3*b^2*d^2*f^2*x^2 + 6*I*a*b*d^2*e*f*x + 6*b^2*d^2*e*
f*x + 6*I*a*b*c*d*e*f + 6*b^2*c*d*e*f - 3*I*a*b*c^2*f^2 - 3*b^2*c^2*f^2)*lo
g(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) - (-3*I*a*b*d^2*f^2*x^2 + 3*b^2*d^
2*f^2*x^2 - 6*I*a*b*d^2*e*f*x + 6*b^2*d^2*e*f*x - 6*I*a*b*c*d*e*f + 6*b^2*c
*d*e*f + 3*I*a*b*c^2*f^2 - 3*b^2*c^2*f^2)*log(-I*cosh(d*x + c) - I*sinh(d*x
 + c) + 1) + 6*(-I*a*b*f^2 + b^2*f^2)*polylog(3, I*cosh(d*x + c) + I*sinh(d
*x + c)) + 6*(I*a*b*f^2 + b^2*f^2)*polylog(3, -I*cosh(d*x + c) - I*sinh(d*x
 + c)))/((a^2*b + b^3)*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^2 \sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*sinh(c + d*x)*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.379 $\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=516

$$-\frac{ia^3 f\text{PolyLog}\left(2,-ie^{c+dx}\right)}{b^2d^2\left(a^2+b^2\right)}+\frac{ia^3 f\text{PolyLog}\left(2,ie^{c+dx}\right)}{b^2d^2\left(a^2+b^2\right)}+\frac{a^2 f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2\left(a^2+b^2\right)}+\frac{a^2 f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2\left(a^2+b^2\right)}$$

```
[Out] -(e + f*x)^2/(2*b*f) - (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/(b^2*d) + (2*a^3
*(e + f*x)*ArcTan[E^(c + d*x)])/(b^2*(a^2 + b^2)*d) + (a^2*(e + f*x)*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d) + (a^2*(e + f*x
)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d) + ((e +
 f*x)*Log[1 + E^(2*(c + d*x))])/(b*d) - (a^2*(e + f*x)*Log[1 + E^(2*(c + d*
x))])/(b*(a^2 + b^2)*d) + (I*a*f*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*d^2) -
(I*a^3*f*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) - (I*a*f*PolyL
og[2, I*E^(c + d*x)])/(b^2*d^2) + (I*a^3*f*PolyLog[2, I*E^(c + d*x)])/(b^2*
(a^2 + b^2)*d^2) + (a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
))])/(b*(a^2 + b^2)*d^2) + (a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/(b*(a^2 + b^2)*d^2) + (f*PolyLog[2, -E^(2*(c + d*x))])/(2*b*d^
2) - (a^2*f*PolyLog[2, -E^(2*(c + d*x))])/(2*b*(a^2 + b^2)*d^2)
```

Rubi [A] time = 0.775852, antiderivative size = 516, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5581, 3718, 2190, 2279, 2391, 5567, 4180, 5573, 5561, 6742}

$$-\frac{ia^3 f\text{PolyLog}\left(2,-ie^{c+dx}\right)}{b^2d^2\left(a^2+b^2\right)}+\frac{ia^3 f\text{PolyLog}\left(2,ie^{c+dx}\right)}{b^2d^2\left(a^2+b^2\right)}+\frac{a^2 f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2\left(a^2+b^2\right)}+\frac{a^2 f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(e + f*x)^2/(2*b*f) - (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/(b^2*d) + (2*a^3
*(e + f*x)*ArcTan[E^(c + d*x)])/(b^2*(a^2 + b^2)*d) + (a^2*(e + f*x)*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d) + (a^2*(e + f*x
)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d) + ((e +
 f*x)*Log[1 + E^(2*(c + d*x))])/(b*d) - (a^2*(e + f*x)*Log[1 + E^(2*(c + d*
x))])/(b*(a^2 + b^2)*d) + (I*a*f*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*d^2) -
(I*a^3*f*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) - (I*a*f*PolyL
og[2, I*E^(c + d*x)])/(b^2*d^2) + (I*a^3*f*PolyLog[2, I*E^(c + d*x)])/(b^2*
(a^2 + b^2)*d^2) + (a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
))])/(b*(a^2 + b^2)*d^2) + (a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/(b*(a^2 + b^2)*d^2) + (f*PolyLog[2, -E^(2*(c + d*x))])/(2*b*d^
2) - (a^2*f*PolyLog[2, -E^(2*(c + d*x))])/(2*b*(a^2 + b^2)*d^2)
```

Rule 5581

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - Di
st[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n)/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c +
 d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx = \frac{\int (e+fx)\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}dx}{b}$$

$$= -\frac{(e+fx)^2}{2bf} - \frac{a\int (e+fx)\text{sech}(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}dx}{b^2} + \frac{2\int}{}$$

$$= -\frac{(e+fx)^2}{2bf} - \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{(e+fx)\log\left(1+e^{2(c+dx)}\right)}{bd} + \frac{a^2\int}{}$$

$$= -\frac{(e+fx)^2}{2bf} - \frac{a^2(e+fx)^2}{2b\left(a^2+b^2\right)f} - \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{(e+fx)\log\left(1\right.}{bd}$$

$$= -\frac{(e+fx)^2}{2bf} - \frac{a^2(e+fx)^2}{2b\left(a^2+b^2\right)f} - \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{a^2(e+fx)\log\left(\right.}{b\left(a^2+\right.}$$

$$= -\frac{(e+fx)^2}{2bf} - \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d} + \frac{a^2(e}{}$$

$$= -\frac{(e+fx)^2}{2bf} - \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d} + \frac{a^2(e}{}$$

$$= -\frac{(e+fx)^2}{2bf} - \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d} + \frac{a^2(e}{}$$

$$= -\frac{(e+fx)^2}{2bf} - \frac{2a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2\left(a^2+b^2\right)d} + \frac{a^2(e}{}$$

Mathematica [A] time = 2.87536, size = 438, normalized size = 0.85

$$\frac{a^2\left(f\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+f(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)+f(c+dx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)+de\log(a+b\sinh(c+dx))-cf\log(a+b\sinh(c\cdots\right.}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-(b*d*e*(c + d*x)) + b*c*f*(c + d*x) - (b*f*(c + d*x)^2)/2 - 2*a*d*e*ArcTa
n[Cosh[c + d*x] + Sinh[c + d*x]] + 2*a*c*f*ArcTan[Cosh[c + d*x] + Sinh[c +
d*x]] - 2*a*f*(c + d*x)*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] + b*d*e*Log[1
 + Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]] - b*c*f*Log[1 + Cosh[2*(c + d*x)]
 + Sinh[2*(c + d*x)]] + b*f*(c + d*x)*Log[1 + Cosh[2*(c + d*x)] + Sinh[2*(c
 + d*x)]] + (a^2*(-(f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2])] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*
PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))]))/b + I*a*f*PolyLog[2, (-I)*(Cosh[c + d*x]
 + Sinh[c + d*x])] - I*a*f*PolyLog[2, I*(Cosh[c + d*x] + Sinh[c + d*x])] +
```

```
(b*f*PolyLog[2, -Cosh[2*(c + d*x)] - Sinh[2*(c + d*x)]])/2)/((a^2 + b^2)*d^
2)
```

Maple [B] time = 0.225, size = 3882, normalized size = 7.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/2*f*x^2/b-1/d^2/b*f*c^2-2/d/b*e*ln(exp(d*x+c))+2/d^2*f/(2*a^2+2*b^2)*ln(
1+I*exp(d*x+c))*b*c+2/d*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*b*x+2/d^2*f/(2*a
^2+2*b^2)*ln(1-I*exp(d*x+c))*b*c-1/d*f*b/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a
^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d^2*f*b/(2*a^2+2*b^2)*ln((-b*exp
(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d*f*b/(2*a^2+2*b^2)*ln
((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*f*b/(2*a^2+2
*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-2/d^2*f*c/
(2*a^2+2*b^2)*b*ln(1+exp(2*d*x+2*c))+4/d^2*f*c/(2*a^2+2*b^2)*a*arctan(exp(d
*x+c))+1/d^2*f*c*b/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d*
f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*b*x+e*x/b-2/(a^2+b^2)^(1/2)/d^2*a*b*f*c/
(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2*b/d^2*a*f
/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+
b^2)^(1/2)))*c-2/b/d^2*a^3*f*c/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2
*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/b/d^2*a*f*c/(2*a^2+2*b^2)*(a^2+b^2)^(
1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/b/d*a^3*f/(2*a^2+2
*b^2)/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2
)))*x+2/b/d^2*a^3*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2
)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-2/b/d*a^3*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)
*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-2/b/d^2*a^3*f
/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^
2+b^2)^(1/2)))*c-2*b/d*a*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+
(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-2*b/d^2*a*f/(2*a^2+2*b^2)/(a^2+b
^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+2*b/
d*a*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+
(a^2+b^2)^(1/2)))*x+2/(a^2+b^2)^(1/2)/d*a*b*e/(2*a^2+2*b^2)*arctanh(1/2*(2*
b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d/b*f*c*x+2/d^2/b*f*c*ln(exp(d*x+c))+1
/2*b/d^2*f/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)
))*c-b/d^2*f/(a^2+b^2)^(3/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2
+b^2)^(1/2)))*a+b/d^2*f/(a^2+b^2)^(3/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2
)-a)/(-a+(a^2+b^2)^(1/2)))*a-1/2*b/d^2*f*c/(a^2+b^2)*ln(b*exp(2*d*x+2*c)+2*
a*exp(d*x+c)-b)+1/2*b/d*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-
a+(a^2+b^2)^(1/2)))*x+1/2*b/d^2*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/
2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/b/d*e/(a^2+b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp
(d*x+c)-b)*a^2-2*I/d^2*a*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))+2*I/d^2*a*f/
(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))-2/b/d*e/(a^2+b^2)^(3/2)*arctanh(1/2*(2*
b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^3-2*b/d*e/(a^2+b^2)^(3/2)*arctanh(1/2*
(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a+1/b/d^2*f/(a^2+b^2)*dilog((b*exp(d*
x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^2+1/b/d^2*f/(a^2+b^2)*dilog(
(-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^2-1/b/d^2*f/(a^2+
b^2)^(3/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^3+
1/b/d^2*f/(a^2+b^2)^(3/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+
b^2)^(1/2)))*a^3+1/2*b/d*f/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a
+(a^2+b^2)^(1/2)))*x+2/d*e/(2*a^2+2*b^2)*b*ln(1+exp(2*d*x+2*c))-4/d*e/(2*a^
2+2*b^2)*a*arctan(exp(d*x+c))-1/d*e*b/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a
*exp(d*x+c)-b)+2/d^2*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))*b+2/d^2*f/(2*a^2
+2*b^2)*dilog(1-I*exp(d*x+c))*b-1/d^2*f*b/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c
```

```
)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*f*b/(2*a^2+2*b^2)*dilog((b
*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/2*b/d*e/(a^2+b^2)*ln(
b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/2*b/d^2*f/(a^2+b^2)*dilog((b*exp(d*x+c
)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/2*b/d^2*f/(a^2+b^2)*dilog((-b*e
xp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+2*b/d^2*a*f/(2*a^2+2*b^2
)/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)
))-2*b/d^2*a*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)
^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+2*b/d^2*f*c/(a^2+b^2)^(3/2)*arctanh(1/2*(2*
b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a-2/b/d^2*a^3*f/(2*a^2+2*b^2)/(a^2+b^2)^
(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+2/b/d^2
*f*c/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^3-
1/b/d^2*f*c/(a^2+b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)*a^2+2/b/d^2*a^3
*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+
(a^2+b^2)^(1/2)))+1/b/d^2*f/(a^2+b^2)^(3/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/
2)-a)/(-a+(a^2+b^2)^(1/2)))*a^3*c+1/b/d*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+
b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^2*x+1/b/d^2*f/(a^2+b^2)*ln((-b*exp(d*
x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^2*c+1/b/d*f/(a^2+b^2)*ln((b
*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^2*x+1/b/d^2*f/(a^2+b^
2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^2*c-1/b/d*f/(
a^2+b^2)^(3/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^3
*x-1/b/d^2*f/(a^2+b^2)^(3/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^
2)^(1/2)))*a^3*c+b/d*f/(a^2+b^2)^(3/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)
/(-a+(a^2+b^2)^(1/2)))*a*x+b/d^2*f/(a^2+b^2)^(3/2)*ln((-b*exp(d*x+c)+(a^2+b
^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a*c-b/d*f/(a^2+b^2)^(3/2)*ln((b*exp(d*x+
c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a*x-b/d^2*f/(a^2+b^2)^(3/2)*ln((
b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a*c+2/b/d*a*e/(2*a^2+2
*b^2)*(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/b
/d*a^3*e/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^
2+b^2)^(1/2))+1/b/d*f/(a^2+b^2)^(3/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
(-a+(a^2+b^2)^(1/2)))*a^3*x-2*I/d^2*a*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c+
2*I/d^2*a*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c-2*I/d*a*f/(2*a^2+2*b^2)*ln(1
-I*exp(d*x+c))*x+2*I/d*a*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}f\left(\frac{x^2}{b}-\int-\frac{4\left(a^3xe^{(dx+c)}-a^2bx\right)}{a^2b^2+b^4-\left(a^2b^2e^{(2c)}+b^4e^{(2c)}\right)e^{(2dx)}-2\left(a^3be^c+ab^3e^c\right)e^{(dx)}}\,dx-\int\frac{4\left(axe^{(dx+c)}+bx\right)}{a^2+b^2+\left(a^2e^{(2c)}+b^2e^{(2c)}\right)e^{(2dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] 1/2*f*(x^2/b - integrate(-4*(a^3*x*e^(d*x + c) - a^2*b*x)/(a^2*b^2 + b^4 -
(a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x) - 2*(a^3*b*e^c + a*b^3*e^c)*e^(d*
x)), x) - integrate(4*(a*x*e^(d*x + c) + b*x)/(a^2 + b^2 + (a^2*e^(2*c) + b
^2*e^(2*c))*e^(2*d*x)), x)) + e*(a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x -
2*c) - b)/((a^2*b + b^3)*d) + 2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*
log(e^(-2*d*x - 2*c) + 1)/((a^2 + b^2)*d) + (d*x + c)/(b*d))
```

Fricas [A] time = 2.5035, size = 1767, normalized size = 3.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] -1/2*((a^2 + b^2)*d^2*f*x^2 + 2*(a^2 + b^2)*d^2*e*x - 2*a^2*f*dilog((a*cosh
(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b + 1) - 2*a^2*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
 - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) +
2*(I*a*b*f - b^2*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + 2*(-I*a*b*f
- b^2*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) - 2*(a^2*d*e - a^2*c*f)*
log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a
) - 2*(a^2*d*e - a^2*c*f)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*s
qrt((a^2 + b^2)/b^2) + 2*a) - 2*(a^2*d*f*x + a^2*c*f)*log(-(a*cosh(d*x + c)
 + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2) - b)/b) - 2*(a^2*d*f*x + a^2*c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x +
c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (-
2*I*a*b*d*e + 2*b^2*d*e + 2*I*a*b*c*f - 2*b^2*c*f)*log(cosh(d*x + c) + sinh
(d*x + c) + I) - (2*I*a*b*d*e + 2*b^2*d*e - 2*I*a*b*c*f - 2*b^2*c*f)*log(co
sh(d*x + c) + sinh(d*x + c) - I) - (2*I*a*b*d*f*x + 2*b^2*d*f*x + 2*I*a*b*c
*f + 2*b^2*c*f)*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) - (-2*I*a*b*d*f*
x + 2*b^2*d*f*x - 2*I*a*b*c*f + 2*b^2*c*f)*log(-I*cosh(d*x + c) - I*sinh(d*
x + c) + 1))/((a^2*b + b^3)*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*sinh(c + d*x)*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.380 $\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{a^2\log(a+b\sinh(c+dx))}{bd\left(a^2+b^2\right)}-\frac{a\tan^{-1}(\sinh(c+dx))}{d\left(a^2+b^2\right)}+\frac{b\log(\cosh(c+dx))}{d\left(a^2+b^2\right)}$$

```
[Out] -((a*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d)) + (b*Log[Cosh[c + d*x]])/((a^2
 + b^2)*d) + (a^2*Log[a + b*Sinh[c + d*x]])/(b*(a^2 + b^2)*d)
```

Rubi [A] time = 0.159761, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2837, 12, 1629, 635, 203, 260}

$$\frac{a^2\log(a+b\sinh(c+dx))}{bd\left(a^2+b^2\right)}-\frac{a\tan^{-1}(\sinh(c+dx))}{d\left(a^2+b^2\right)}+\frac{b\log(\cosh(c+dx))}{d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d)) + (b*Log[Cosh[c + d*x]])/((a^2
 + b^2)*d) + (a^2*Log[a + b*Sinh[c + d*x]])/(b*(a^2 + b^2)*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{b\,\mathrm{Subst}\left(\int \frac{x^2}{b^2(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(c+dx)\right)}{d}\\
&= -\frac{\mathrm{Subst}\left(\int \frac{x^2}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(c+dx)\right)}{bd}\\
&= -\frac{\mathrm{Subst}\left(\int \left(-\frac{a^2}{\left(a^2+b^2\right)(a+x)}+\frac{b^2(a-x)}{\left(a^2+b^2\right)\left(b^2+x^2\right)}\right)dx, x, b\sinh(c+dx)\right)}{bd}\\
&= \frac{a^2\log(a+b\sinh(c+dx))}{b\left(a^2+b^2\right)d}-\frac{b\,\mathrm{Subst}\left(\int \frac{a-x}{b^2+x^2}\,dx, x, b\sinh(c+dx)\right)}{\left(a^2+b^2\right)d}\\
&= \frac{a^2\log(a+b\sinh(c+dx))}{b\left(a^2+b^2\right)d}+\frac{b\,\mathrm{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(c+dx)\right)}{\left(a^2+b^2\right)d}-\frac{(ab)\,\mathrm{Subst}\left(\int\right.}{}\\
&= -\frac{a\tan^{-1}(\sinh(c+dx))}{\left(a^2+b^2\right)d}+\frac{b\log(\cosh(c+dx))}{\left(a^2+b^2\right)d}+\frac{a^2\log(a+b\sinh(c+dx))}{b\left(a^2+b^2\right)d}
\end{aligned}
$$

Mathematica [C] time = 0.085892, size = 78, normalized size = 1.05

$$
\frac{2a^2\log(a+b\sinh(c+dx))+b(b+ia)\log(-\sinh(c+dx)+i)+b(b-ia)\log(\sinh(c+dx)+i)}{2bd\left(a^2+b^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*(I*a + b)*Log[I - Sinh[c + d*x]] + b*((-I)*a + b)*Log[I + Sinh[c + d*x]]
 + 2*a^2*Log[a + b*Sinh[c + d*x]])/(2*b*(a^2 + b^2)*d)
```

Maple [B] time = 0.001, size = 153, normalized size = 2.1

$$
-\frac{1}{bd}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{1}{bd}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{a^2}{bd\left(a^2+b^2\right)}\ln\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 a-2\tanh(1/2\,dx+c/2
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/d/b*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/b*ln(tanh(1/2*d*x+1/2*c)-1)+1/d*a^2/b/
(a^2+b^2)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)*b-a)+4/d/(4*a^2+
4*b^2)*b*ln(tanh(1/2*d*x+1/2*c)^2+1)-8/d/(4*a^2+4*b^2)*a*arctan(tanh(1/2*d*
x+1/2*c))
```

Maxima [A] time = 1.62767, size = 149, normalized size = 2.01

$$\frac{a^2 \log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{\left(a^2b + b^3\right)d} + \frac{2\,a \arctan\left(e^{(-dx-c)}\right)}{\left(a^2 + b^2\right)d} + \frac{b \log\left(e^{(-2\,dx-2\,c)} + 1\right)}{\left(a^2 + b^2\right)d} + \frac{dx + c}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b + b^3)*d) + 2*a
*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^2 +
 b^2)*d) + (d*x + c)/(b*d)
```

Fricas [A] time = 2.30851, size = 284, normalized size = 3.84

$$-\frac{\left(a^2 + b^2\right)dx + 2\,ab \arctan\left(\cosh\left(dx + c\right) + \sinh\left(dx + c\right)\right) - a^2 \log\left(\frac{2\left(b \sinh(dx+c)+a\right)}{\cosh(dx+c)-\sinh(dx+c)}\right) - b^2 \log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{\left(a^2b + b^3\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((a^2 + b^2)*d*x + 2*a*b*arctan(cosh(d*x + c) + sinh(d*x + c)) - a^2*log(2
*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) - b^2*log(2*cosh(d*
x + c)/(cosh(d*x + c) - sinh(d*x + c))))/((a^2*b + b^3)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(c + dx\right) \tanh\left(c + dx\right)}{a + b \sinh\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sinh(c + d*x)*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A] time = 1.52435, size = 128, normalized size = 1.73

$$\frac{\frac{a^2 \log\left(\left|be^{(2\,dx+2\,c)} + 2\,ae^{(dx+c)} - b\right|\right)}{a^2b + b^3} - \frac{dx}{b} - \frac{2\,a \arctan\left(e^{(dx+c)}\right)}{a^2 + b^2} + \frac{b \log\left(e^{(2\,dx+2\,c)} + 1\right)}{a^2 + b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (a^2*log(abs(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - b))/(a^2*b + b^3) - d*x/
b - 2*a*arctan(e^(d*x + c))/(a^2 + b^2) + b*log(e^(2*d*x + 2*c) + 1)/(a^2 +
 b^2))/d
```

3.381 $\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=34

$$\text{Unintegrable}\left(\frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Sinh[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [A] time = 0.0598635, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sinh[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Sinh[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

Rubi steps

$$\int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Sinh[c + d*x]*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x
]
```

```
[Out] $Aborted
```

Maple [A] time = 0.421, size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)\tanh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\log\left(fx+e\right)}{bf} - \frac{1}{2}\int -\frac{4\left(a^3e^{(dx+c)} - a^2b\right)}{a^2b^2e + b^4e + \left(a^2b^2f + b^4f\right)x - \left(a^2b^2ee^{(2\,c)} + b^4ee^{(2\,c)} + \left(a^2b^2fe^{(2\,c)} + b^4fe^{(2\,c)}\right)x\right)e^{(2\,dx)} - 2\left(a^3be\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] log(f*x + e)/(b*f) - 1/2*integrate(-4*(a^3*e^(d*x + c) - a^2*b)/(a^2*b^2*e
+ b^4*e + (a^2*b^2*f + b^4*f)*x - (a^2*b^2*e*e^(2*c) + b^4*e*e^(2*c) + (a^2
*b^2*f*e^(2*c) + b^4*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^3*b*e*e^c + a*b^3*e*e^c
 + (a^3*b*f*e^c + a*b^3*f*e^c)*x)*e^(d*x)), x) - 1/2*integrate(4*(a*e^(d*x
+ c) + b)/(a^2*e + b^2*e + (a^2*f + b^2*f)*x + (a^2*e*e^(2*c) + b^2*e*e^(2*
c) + (a^2*f*e^(2*c) + b^2*f*e^(2*c))*x)*e^(2*d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh\left(dx+c\right)\tanh\left(dx+c\right)}{afx + ae + \left(bfx + be\right)\sinh\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] integral(sinh(d*x + c)*tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
+ c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(c+dx\right)\tanh\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sinh(c + d*x)*tanh(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.382 $\int \frac{(e+fx)^3 \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1118

result too large to display

```
[Out] -((a*(e + f*x)^3)/(b^2*d)) + (a^3*(e + f*x)^3)/(b^2*(a^2 + b^2)*d) + (6*f*(
e + f*x)^2*ArcTan[E^(c + d*x)])/(b*d^2) - (6*a^2*f*(e + f*x)^2*ArcTan[E^(c
+ d*x)])/(b*(a^2 + b^2)*d^2) + (a^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (a^2*(e + f*x)^3*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) + (3*a*f*(e + f*x)
^2*Log[1 + E^(2*(c + d*x))])/(b^2*d^2) - (3*a^3*f*(e + f*x)^2*Log[1 + E^(2*
(c + d*x))])/(b^2*(a^2 + b^2)*d^2) - ((6*I)*f^2*(e + f*x)*PolyLog[2, (-I)*E
^(c + d*x)])/(b*d^3) + ((6*I)*a^2*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)
])/(b*(a^2 + b^2)*d^3) + ((6*I)*f^2*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b
*d^3) - ((6*I)*a^2*f^2*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)*
d^3) + (3*a^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2]))])/((a^2 + b^2)^(3/2)*d^2) - (3*a^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) + (3*a*f^2*(e + f*
x)*PolyLog[2, -E^(2*(c + d*x))])/(b^2*d^3) - (3*a^3*f^2*(e + f*x)*PolyLog[2
, -E^(2*(c + d*x))])/(b^2*(a^2 + b^2)*d^3) + ((6*I)*f^3*PolyLog[3, (-I)*E^(
c + d*x)])/(b*d^4) - ((6*I)*a^2*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(b*(a^2 +
 b^2)*d^4) - ((6*I)*f^3*PolyLog[3, I*E^(c + d*x)])/(b*d^4) + ((6*I)*a^2*f^3
*PolyLog[3, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^4) - (6*a^2*f^2*(e + f*x)*Poly
Log[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) +
 (6*a^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])
/((a^2 + b^2)^(3/2)*d^3) - (3*a*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*d^
4) + (3*a^3*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*(a^2 + b^2)*d^4) + (6*
a^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^
(3/2)*d^4) - (6*a^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/((a^2 + b^2)^(3/2)*d^4) - ((e + f*x)^3*Sech[c + d*x])/(b*d) + (a^2*(e +
f*x)^3*Sech[c + d*x])/(b*(a^2 + b^2)*d) - (a*(e + f*x)^3*Tanh[c + d*x])/(b^
2*d) + (a^3*(e + f*x)^3*Tanh[c + d*x])/(b^2*(a^2 + b^2)*d)
```

Rubi [A] time = 1.98044, antiderivative size = 1118, normalized size of antiderivative = 1., number of steps used = 45, number of rules used = 15, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.536, Rules used = {5567, 5451, 4180, 2531, 2282, 6589, 5583, 4184, 3718, 2190, 5573, 3322, 2264, 6609, 6742}

$$\frac{(e+fx)^3a^3}{b^2\left(a^2+b^2\right)d}-\frac{3f(e+fx)^2\log\left(1+e^{2(c+dx)}\right)a^3}{b^2\left(a^2+b^2\right)d^2}-\frac{3f^2(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)a^3}{b^2\left(a^2+b^2\right)d^3}+\frac{3f^3\text{PolyLog}\left(3,-e^{2(c+dx)}\right)}{2b^2\left(a^2+b^2\right)d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a*(e + f*x)^3)/(b^2*d)) + (a^3*(e + f*x)^3)/(b^2*(a^2 + b^2)*d) + (6*f*(
e + f*x)^2*ArcTan[E^(c + d*x)])/(b*d^2) - (6*a^2*f*(e + f*x)^2*ArcTan[E^(c
+ d*x)])/(b*(a^2 + b^2)*d^2) + (a^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (a^2*(e + f*x)^3*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) + (3*a*f*(e + f*x)
^2*Log[1 + E^(2*(c + d*x))])/(b^2*d^2) - (3*a^3*f*(e + f*x)^2*Log[1 + E^(2*
(c + d*x))])/(b^2*(a^2 + b^2)*d^2) - ((6*I)*f^2*(e + f*x)*PolyLog[2, (-I)*E
^(c + d*x)])/(b*d^3) + ((6*I)*a^2*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)
])/(b*(a^2 + b^2)*d^3) + ((6*I)*f^2*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b
```

```
*d^3) - ((6*I)*a^2*f^2*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)*
d^3) + (3*a^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2]))])/((a^2 + b^2)^(3/2)*d^2) - (3*a^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) + (3*a*f^2*(e + f*
x)*PolyLog[2, -E^(2*(c + d*x))])/(b^2*d^3) - (3*a^3*f^2*(e + f*x)*PolyLog[2
, -E^(2*(c + d*x))])/(b^2*(a^2 + b^2)*d^3) + ((6*I)*f^3*PolyLog[3, (-I)*E^(
c + d*x)])/(b*d^4) - ((6*I)*a^2*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(b*(a^2 +
 b^2)*d^4) - ((6*I)*f^3*PolyLog[3, I*E^(c + d*x)])/(b*d^4) + ((6*I)*a^2*f^3
*PolyLog[3, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^4) - (6*a^2*f^2*(e + f*x)*Poly
Log[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) +
 (6*a^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])
/((a^2 + b^2)^(3/2)*d^3) - (3*a*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*d^
4) + (3*a^3*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*(a^2 + b^2)*d^4) + (6*
a^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^
(3/2)*d^4) - (6*a^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/((a^2 + b^2)^(3/2)*d^4) - ((e + f*x)^3*Sech[c + d*x])/(b*d) + (a^2*(e +
f*x)^3*Sech[c + d*x])/(b*(a^2 + b^2)*d) - (a*(e + f*x)^3*Tanh[c + d*x])/(b^
2*d) + (a^3*(e + f*x)^3*Tanh[c + d*x])/(b^2*(a^2 + b^2)*d)
```

Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c +
 d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5583

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\mathrm{sech}(c+dx)\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\mathrm{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)^3\mathrm{sech}(c+dx)}{bd} - \frac{a\int (e+fx)^3\mathrm{sech}^2(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^3\mathrm{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b^2} + \ldots$$

$$= \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{(e+fx)^3\mathrm{sech}(c+dx)}{bd} - \frac{a(e+fx)^3\tanh(c+dx)}{b^2d} + \ldots$$

$$= -\frac{a(e+fx)^3}{b^2d} + \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{6if^2(e+fx)\mathrm{Li}_2\left(-ie^{c+dx}\right)}{bd^3} + \frac{6if^2(e+fx)\mathrm{Li}_2\ldots}{bd^3}$$

$$= -\frac{a(e+fx)^3}{b^2d} + \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd^2} + \frac{3af(e+fx)^2\log\left(1+e^{2(c+dx)}\right)}{b^2d^2} - \ldots$$

$$= -\frac{a(e+fx)^3}{b^2d} + \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd^2} + \frac{a^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} - \ldots$$

$$= -\frac{a(e+fx)^3}{b^2d} + \frac{a^3(e+fx)^3}{b^2\left(a^2+b^2\right)d} + \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{6a^2f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d^2} \ldots$$

$$= -\frac{a(e+fx)^3}{b^2d} + \frac{a^3(e+fx)^3}{b^2\left(a^2+b^2\right)d} + \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{6a^2f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d^2} \ldots$$

$$= -\frac{a(e+fx)^3}{b^2d} + \frac{a^3(e+fx)^3}{b^2\left(a^2+b^2\right)d} + \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{6a^2f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d^2} \ldots$$

$$= -\frac{a(e+fx)^3}{b^2d} + \frac{a^3(e+fx)^3}{b^2\left(a^2+b^2\right)d} + \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{6a^2f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d^2} \ldots$$

$$= -\frac{a(e+fx)^3}{b^2d} + \frac{a^3(e+fx)^3}{b^2\left(a^2+b^2\right)d} + \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{6a^2f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d^2} \ldots$$

Mathematica [A] time = 13.669, size = 1147, normalized size = 1.03

$$\left(-2e^3\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)d^3 + f^3x^3\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^3 + 3ef^2x^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^3 + 3e^2fx\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^3 - f^3\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (f*(-12*a*d^3*e^2*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*d^3*e*f*x
^2 + 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] - 6*a
*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*b*d*e*(1
 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - Poly
Log[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*e*(1 + E^(2*c
))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]
) + (6*I)*b*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log
[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2,
 I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*
x)]) - a*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))])
- 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))])))/(2
*(a^2 + b^2)*d^4*(1 + E^(2*c))) + (a^2*(-2*d^3*e^3*ArcTanh[(a + b*E^(c + d*
x))/Sqrt[a^2 + b^2]] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] +
 d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 3*d^3*e^2*f*x
*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 3*d^3*e*f^2*x^2*Log[1 + (
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-
a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2]))] - 6*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2
 + b^2])] - 6*d*f^3*x*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] +
6*d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*d*f^3*x*
PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*f^3*PolyLog[4, (b*
E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*f^3*PolyLog[4, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))]))/((a^2 + b^2)^(3/2)*d^4) + (Sech[c]*Sech[c + d*x]*(
-(b*e^3*Cosh[c]) - 3*b*e^2*f*x*Cosh[c] - 3*b*e*f^2*x^2*Cosh[c] - b*f^3*x^3*
Cosh[c] - a*e^3*Sinh[d*x] - 3*a*e^2*f*x*Sinh[d*x] - 3*a*e*f^2*x^2*Sinh[d*x]
 - a*f^3*x^3*Sinh[d*x]))/((a^2 + b^2)*d)
```

Maple [F] time = 0.732, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 (\tanh (dx+c))^2}{a+b\sinh (dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 4.22311, size = 14864, normalized size = 13.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*(a^3 + a*b^2)*d^3*e^3 - 12*(a^3 + a*b^2)*c*d^2*e^2*f + 12*(a^3 + a*b
^2)*c^2*d*e*f^2 - 4*(a^3 + a*b^2)*c^3*f^3 - 4*((a^3 + a*b^2)*d^3*f^3*x^3 +
3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b^
2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*cosh(
d*x + c)^2 - 4*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 +
 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2
)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*sinh(d*x + c)^2 + 6*(a^2*b*d^2*f^3*x
^2 + 2*a^2*b*d^2*e*f^2*x + a^2*b*d^2*e^2*f + (a^2*b*d^2*f^3*x^2 + 2*a^2*b*d
^2*e*f^2*x + a^2*b*d^2*e^2*f)*cosh(d*x + c)^2 + 2*(a^2*b*d^2*f^3*x^2 + 2*a^
2*b*d^2*e*f^2*x + a^2*b*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d^2
*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2*b*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt((a^
2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 6*(a^2*b*d^2*f^3*x^2
+ 2*a^2*b*d^2*e*f^2*x + a^2*b*d^2*e^2*f + (a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*
e*f^2*x + a^2*b*d^2*e^2*f)*cosh(d*x + c)^2 + 2*(a^2*b*d^2*f^3*x^2 + 2*a^2*b
*d^2*e*f^2*x + a^2*b*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d^2*f^
3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2*b*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt((a^2 +
 b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(a^2*b*d^3*e^3 - 3*a^2
*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3 + (a^2*b*d^3*e^3 - 3*a
^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*cosh(d*x + c)^2 + 2
*(a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3
)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^
2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log
(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) +
 2*(a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f
^3 + (a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3
*f^3)*cosh(d*x + c)^2 + 2*(a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^
2*d*e*f^2 - a^2*b*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d^3*e^3 - 3
*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*sinh(d*x + c)^2)*
sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt(
(a^2 + b^2)/b^2) + 2*a) + 2*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*
a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f
^3 + (a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a
^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*cosh(d*x + c)^2 + 2
*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b
*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*cosh(d*x + c)*sinh(d*x
+ c) + (a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3
*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*sinh(d*x + c)^2)*
sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(a^2*b*d^3*f^3*x
^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*
a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3 + (a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*
x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2
*b*c^3*f^3)*cosh(d*x + c)^2 + 2*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2
+ 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c
^3*f^3)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^
2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a
^2*b*c^3*f^3)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c)
+ a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
```

2) - b)/b) + 12*(a^2*b*f^3*cosh(d*x + c)^2 + 2*a^2*b*f^3*cosh(d*x + c)*sinh (d*x + c) + a^2*b*f^3*sinh(d*x + c)^2 + a^2*b*f^3)*sqrt((a^2 + b^2)/b^2)*po lylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 12*(a^2*b*f^3*cosh(d*x + c)^2 + 2*a^2*b* f^3*cosh(d*x + c)*sinh(d*x + c) + a^2*b*f^3*sinh(d*x + c)^2 + a^2*b*f^3)*sq rt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh (d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 12*(a^2*b*d*f^3*x + a^2*b*d*e*f^2 + (a^2*b*d*f^3*x + a^2*b*d*e*f^2)*cosh(d*x + c)^2 + 2*(a^2* b*d*f^3*x + a^2*b*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d*f^3*x + a ^2*b*d*e*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b ^2)/b^2))/b) + 12*(a^2*b*d*f^3*x + a^2*b*d*e*f^2 + (a^2*b*d*f^3*x + a^2*b*d *e*f^2)*cosh(d*x + c)^2 + 2*(a^2*b*d*f^3*x + a^2*b*d*e*f^2)*cosh(d*x + c)*s inh(d*x + c) + (a^2*b*d*f^3*x + a^2*b*d*e*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 4*((a^2*b + b^3)*d^3*f^3*x^ 3 + 3*(a^2*b + b^3)*d^3*e*f^2*x^2 + 3*(a^2*b + b^3)*d^3*e^2*f*x + (a^2*b + b^3)*d^3*e^3)*cosh(d*x + c) + (12*(a^3 + a*b^2)*d*f^3*x + 12*I*(a^2*b + b^3)*d*f^3*x + 12*(a^3 + a*b^2)*d*e*f^2 + 12*I*(a^2*b + b^3)*d*e*f^2 + (12*(a^ 3 + a*b^2)*d*f^3*x + 12*I*(a^2*b + b^3)*d*f^3*x + 12*(a^3 + a*b^2)*d*e*f^2 + 12*I*(a^2*b + b^3)*d*e*f^2)*cosh(d*x + c)^2 + (24*(a^3 + a*b^2)*d*f^3*x + 24*I*(a^2*b + b^3)*d*f^3*x + 24*(a^3 + a*b^2)*d*e*f^2 + 24*I*(a^2*b + b^3) *d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) + (12*(a^3 + a*b^2)*d*f^3*x + 12*I*(a ^2*b + b^3)*d*f^3*x + 12*(a^3 + a*b^2)*d*e*f^2 + 12*I*(a^2*b + b^3)*d*e*f^2)*sinh(d*x + c)^2)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + (12*(a^3 + a* b^2)*d*f^3*x - 12*I*(a^2*b + b^3)*d*f^3*x + 12*(a^3 + a*b^2)*d*e*f^2 - 12*I *(a^2*b + b^3)*d*e*f^2 + (12*(a^3 + a*b^2)*d*f^3*x - 12*I*(a^2*b + b^3)*d*f ^3*x + 12*(a^3 + a*b^2)*d*e*f^2 - 12*I*(a^2*b + b^3)*d*e*f^2)*cosh(d*x + c) ^2 + (24*(a^3 + a*b^2)*d*f^3*x - 24*I*(a^2*b + b^3)*d*f^3*x + 24*(a^3 + a*b ^2)*d*e*f^2 - 24*I*(a^2*b + b^3)*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) + (12 *(a^3 + a*b^2)*d*f^3*x - 12*I*(a^2*b + b^3)*d*f^3*x + 12*(a^3 + a*b^2)*d*e* f^2 - 12*I*(a^2*b + b^3)*d*e*f^2)*sinh(d*x + c)^2)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + (6*(a^3 + a*b^2)*d^2*e^2*f + 6*I*(a^2*b + b^3)*d^2*e^2* f - 12*(a^3 + a*b^2)*c*d*e*f^2 - 12*I*(a^2*b + b^3)*c*d*e*f^2 + 6*(a^3 + a* b^2)*c^2*f^3 + 6*I*(a^2*b + b^3)*c^2*f^3 + (6*(a^3 + a*b^2)*d^2*e^2*f + 6*I *(a^2*b + b^3)*d^2*e^2*f - 12*(a^3 + a*b^2)*c*d*e*f^2 - 12*I*(a^2*b + b^3)* c*d*e*f^2 + 6*(a^3 + a*b^2)*c^2*f^3 + 6*I*(a^2*b + b^3)*c^2*f^3)*cosh(d*x + c)^2 + (12*(a^3 + a*b^2)*d^2*e^2*f + 12*I*(a^2*b + b^3)*d^2*e^2*f - 24*(a^ 3 + a*b^2)*c*d*e*f^2 - 24*I*(a^2*b + b^3)*c*d*e*f^2 + 12*(a^3 + a*b^2)*c^2* f^3 + 12*I*(a^2*b + b^3)*c^2*f^3)*cosh(d*x + c)*sinh(d*x + c) + (6*(a^3 + a *b^2)*d^2*e^2*f + 6*I*(a^2*b + b^3)*d^2*e^2*f - 12*(a^3 + a*b^2)*c*d*e*f^2 - 12*I*(a^2*b + b^3)*c*d*e*f^2 + 6*(a^3 + a*b^2)*c^2*f^3 + 6*I*(a^2*b + b^3)*c^2*f^3)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) + I) + (6*(a^ 3 + a*b^2)*d^2*e^2*f - 6*I*(a^2*b + b^3)*d^2*e^2*f - 12*(a^3 + a*b^2)*c*d*e *f^2 + 12*I*(a^2*b + b^3)*c*d*e*f^2 + 6*(a^3 + a*b^2)*c^2*f^3 - 6*I*(a^2*b + b^3)*c^2*f^3 + (6*(a^3 + a*b^2)*d^2*e^2*f - 6*I*(a^2*b + b^3)*d^2*e^2*f - 12*(a^3 + a*b^2)*c*d*e*f^2 + 12*I*(a^2*b + b^3)*c*d*e*f^2 + 6*(a^3 + a*b^2)*c^2*f^3 - 6*I*(a^2*b + b^3)*c^2*f^3)*cosh(d*x + c)^2 + (12*(a^3 + a*b^2)* d^2*e^2*f - 12*I*(a^2*b + b^3)*d^2*e^2*f - 24*(a^3 + a*b^2)*c*d*e*f^2 + 24* I*(a^2*b + b^3)*c*d*e*f^2 + 12*(a^3 + a*b^2)*c^2*f^3 - 12*I*(a^2*b + b^3)*c ^2*f^3)*cosh(d*x + c)*sinh(d*x + c) + (6*(a^3 + a*b^2)*d^2*e^2*f - 6*I*(a^2 *b + b^3)*d^2*e^2*f - 12*(a^3 + a*b^2)*c*d*e*f^2 + 12*I*(a^2*b + b^3)*c*d*e *f^2 + 6*(a^3 + a*b^2)*c^2*f^3 - 6*I*(a^2*b + b^3)*c^2*f^3)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) - I) + (6*(a^3 + a*b^2)*d^2*f^3*x^2 - 6 *I*(a^2*b + b^3)*d^2*f^3*x^2 + 12*(a^3 + a*b^2)*d^2*e*f^2*x - 12*I*(a^2*b + b^3)*d^2*e*f^2*x + 12*(a^3 + a*b^2)*c*d*e*f^2 - 12*I*(a^2*b + b^3)*c*d*e*f ^2 - 6*(a^3 + a*b^2)*c^2*f^3 + 6*I*(a^2*b + b^3)*c^2*f^3 + (6*(a^3 + a*b^2) *d^2*f^3*x^2 - 6*I*(a^2*b + b^3)*d^2*f^3*x^2 + 12*(a^3 + a*b^2)*d^2*e*f^2*x - 12*I*(a^2*b + b^3)*d^2*e*f^2*x + 12*(a^3 + a*b^2)*c*d*e*f^2 - 12*I*(a^2*

```
b + b^3)*c*d*e*f^2 - 6*(a^3 + a*b^2)*c^2*f^3 + 6*I*(a^2*b + b^3)*c^2*f^3)*c
osh(d*x + c)^2 + (12*(a^3 + a*b^2)*d^2*f^3*x^2 - 12*I*(a^2*b + b^3)*d^2*f^3
*x^2 + 24*(a^3 + a*b^2)*d^2*e*f^2*x - 24*I*(a^2*b + b^3)*d^2*e*f^2*x + 24*(
a^3 + a*b^2)*c*d*e*f^2 - 24*I*(a^2*b + b^3)*c*d*e*f^2 - 12*(a^3 + a*b^2)*c^
2*f^3 + 12*I*(a^2*b + b^3)*c^2*f^3)*cosh(d*x + c)*sinh(d*x + c) + (6*(a^3 +
 a*b^2)*d^2*f^3*x^2 - 6*I*(a^2*b + b^3)*d^2*f^3*x^2 + 12*(a^3 + a*b^2)*d^2*
e*f^2*x - 12*I*(a^2*b + b^3)*d^2*e*f^2*x + 12*(a^3 + a*b^2)*c*d*e*f^2 - 12*
I*(a^2*b + b^3)*c*d*e*f^2 - 6*(a^3 + a*b^2)*c^2*f^3 + 6*I*(a^2*b + b^3)*c^2
*f^3)*sinh(d*x + c)^2)*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) + (6*(a^3
 + a*b^2)*d^2*f^3*x^2 + 6*I*(a^2*b + b^3)*d^2*f^3*x^2 + 12*(a^3 + a*b^2)*d^
2*e*f^2*x + 12*I*(a^2*b + b^3)*d^2*e*f^2*x + 12*(a^3 + a*b^2)*c*d*e*f^2 + 1
2*I*(a^2*b + b^3)*c*d*e*f^2 - 6*(a^3 + a*b^2)*c^2*f^3 - 6*I*(a^2*b + b^3)*c
^2*f^3 + (6*(a^3 + a*b^2)*d^2*f^3*x^2 + 6*I*(a^2*b + b^3)*d^2*f^3*x^2 + 12*
(a^3 + a*b^2)*d^2*e*f^2*x + 12*I*(a^2*b + b^3)*d^2*e*f^2*x + 12*(a^3 + a*b^
2)*c*d*e*f^2 + 12*I*(a^2*b + b^3)*c*d*e*f^2 - 6*(a^3 + a*b^2)*c^2*f^3 - 6*I
*(a^2*b + b^3)*c^2*f^3)*cosh(d*x + c)^2 + (12*(a^3 + a*b^2)*d^2*f^3*x^2 + 1
2*I*(a^2*b + b^3)*d^2*f^3*x^2 + 24*(a^3 + a*b^2)*d^2*e*f^2*x + 24*I*(a^2*b
+ b^3)*d^2*e*f^2*x + 24*(a^3 + a*b^2)*c*d*e*f^2 + 24*I*(a^2*b + b^3)*c*d*e*
f^2 - 12*(a^3 + a*b^2)*c^2*f^3 - 12*I*(a^2*b + b^3)*c^2*f^3)*cosh(d*x + c)*
sinh(d*x + c) + (6*(a^3 + a*b^2)*d^2*f^3*x^2 + 6*I*(a^2*b + b^3)*d^2*f^3*x^
2 + 12*(a^3 + a*b^2)*d^2*e*f^2*x + 12*I*(a^2*b + b^3)*d^2*e*f^2*x + 12*(a^3
 + a*b^2)*c*d*e*f^2 + 12*I*(a^2*b + b^3)*c*d*e*f^2 - 6*(a^3 + a*b^2)*c^2*f^
3 - 6*I*(a^2*b + b^3)*c^2*f^3)*sinh(d*x + c)^2)*log(-I*cosh(d*x + c) - I*si
nh(d*x + c) + 1) - 12*((a^3 + a*b^2)*f^3 + I*(a^2*b + b^3)*f^3 + ((a^3 + a*
b^2)*f^3 + I*(a^2*b + b^3)*f^3)*cosh(d*x + c)^2 + 2*((a^3 + a*b^2)*f^3 + I*
(a^2*b + b^3)*f^3)*cosh(d*x + c)*sinh(d*x + c) + ((a^3 + a*b^2)*f^3 + I*(a^
2*b + b^3)*f^3)*sinh(d*x + c)^2)*polylog(3, I*cosh(d*x + c) + I*sinh(d*x +
c)) - (12*(a^3 + a*b^2)*f^3 - 12*I*(a^2*b + b^3)*f^3 + 12*((a^3 + a*b^2)*f^
3 - I*(a^2*b + b^3)*f^3)*cosh(d*x + c)^2 + 24*((a^3 + a*b^2)*f^3 - I*(a^2*b
 + b^3)*f^3)*cosh(d*x + c)*sinh(d*x + c) + 12*((a^3 + a*b^2)*f^3 - I*(a^2*b
 + b^3)*f^3)*sinh(d*x + c)^2)*polylog(3, -I*cosh(d*x + c) - I*sinh(d*x + c)
) - 4*((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3)*d^3*e*f^2*x^2 + 3*(a^2*b
 + b^3)*d^3*e^2*f*x + (a^2*b + b^3)*d^3*e^3 + 2*((a^3 + a*b^2)*d^3*f^3*x^3
+ 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*
b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*cos
h(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^4*cosh(d*x + c)^2 + 2
*(a^4 + 2*a^2*b^2 + b^4)*d^4*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2
 + b^4)*d^4*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e + fx\right)^{3} \tanh^{2}\left(c + dx\right)}{a + b \sinh\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")

[Out] Timed out
```

3.383 $\int \frac{(e+fx)^2 \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=772

$$\frac{2a^2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^{3/2}}-\frac{2a^2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2\left(a^2+b^2\right)^{3/2}}-\frac{a^3f^2\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{b^2d^3\left(a^2+b^2\right)}+\frac{2ia^2f^2\text{Po}}{bd}$$

```
[Out] -((a*(e + f*x)^2)/(b^2*d)) + (a^3*(e + f*x)^2)/(b^2*(a^2 + b^2)*d) + (4*f*(
e + f*x)*ArcTan[E^(c + d*x)])/(b*d^2) - (4*a^2*f*(e + f*x)*ArcTan[E^(c + d*
x)])/(b*(a^2 + b^2)*d^2) + (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (a^2*(e + f*x)^2*Log[1 + (b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) + (2*a*f*(e + f*x)*Log
[1 + E^(2*(c + d*x))])/(b^2*d^2) - (2*a^3*f*(e + f*x)*Log[1 + E^(2*(c + d*x
))])/(b^2*(a^2 + b^2)*d^2) - ((2*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^
3) + ((2*I)*a^2*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^3) + ((2
*I)*f^2*PolyLog[2, I*E^(c + d*x)])/(b*d^3) - ((2*I)*a^2*f^2*PolyLog[2, I*E^
(c + d*x)])/(b*(a^2 + b^2)*d^3) + (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (2*a^2*f*(e + f*x
)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*
d^2) + (a*f^2*PolyLog[2, -E^(2*(c + d*x))])/(b^2*d^3) - (a^3*f^2*PolyLog[2,
 -E^(2*(c + d*x))])/(b^2*(a^2 + b^2)*d^3) - (2*a^2*f^2*PolyLog[3, -((b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) + (2*a^2*f^2*Poly
Log[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) -
 ((e + f*x)^2*Sech[c + d*x])/(b*d) + (a^2*(e + f*x)^2*Sech[c + d*x])/(b*(a^
2 + b^2)*d) - (a*(e + f*x)^2*Tanh[c + d*x])/(b^2*d) + (a^3*(e + f*x)^2*Tanh
[c + d*x])/(b^2*(a^2 + b^2)*d)
```

Rubi [A] time = 1.53284, antiderivative size = 772, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 16, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5567, 5451, 4180, 2279, 2391, 5583, 4184, 3718, 2190, 5573, 3322, 2264, 2531, 2282, 6589, 6742}

$$\frac{2a^2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^{3/2}}-\frac{2a^2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2\left(a^2+b^2\right)^{3/2}}-\frac{a^3f^2\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{b^2d^3\left(a^2+b^2\right)}+\frac{2ia^2f^2\text{Po}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a*(e + f*x)^2)/(b^2*d)) + (a^3*(e + f*x)^2)/(b^2*(a^2 + b^2)*d) + (4*f*(
e + f*x)*ArcTan[E^(c + d*x)])/(b*d^2) - (4*a^2*f*(e + f*x)*ArcTan[E^(c + d*
x)])/(b*(a^2 + b^2)*d^2) + (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (a^2*(e + f*x)^2*Log[1 + (b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) + (2*a*f*(e + f*x)*Log
[1 + E^(2*(c + d*x))])/(b^2*d^2) - (2*a^3*f*(e + f*x)*Log[1 + E^(2*(c + d*x
))])/(b^2*(a^2 + b^2)*d^2) - ((2*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^
3) + ((2*I)*a^2*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^3) + ((2
*I)*f^2*PolyLog[2, I*E^(c + d*x)])/(b*d^3) - ((2*I)*a^2*f^2*PolyLog[2, I*E^
(c + d*x)])/(b*(a^2 + b^2)*d^3) + (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (2*a^2*f*(e + f*x
)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*
d^2) + (a*f^2*PolyLog[2, -E^(2*(c + d*x))])/(b^2*d^3) - (a^3*f^2*PolyLog[2,
 -E^(2*(c + d*x))])/(b^2*(a^2 + b^2)*d^3) - (2*a^2*f^2*PolyLog[3, -((b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) + (2*a^2*f^2*Poly
```

```
Log[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) -
 ((e + f*x)^2*Sech[c + d*x])/(b*d) + (a^2*(e + f*x)^2*Sech[c + d*x])/(b*(a^
2 + b^2)*d) - (a*(e + f*x)^2*Tanh[c + d*x])/(b^2*d) + (a^3*(e + f*x)^2*Tanh
[c + d*x])/(b^2*(a^2 + b^2)*d)
```

Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c +
 d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5583

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
```

```
+ d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
```

]

Rubi steps

$$\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\mathrm{sech}(c+dx)\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\mathrm{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)^2\mathrm{sech}(c+dx)}{bd} - \frac{a\int (e+fx)^2\mathrm{sech}^2(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^2\mathrm{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b^2}$$

$$= \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{(e+fx)^2\mathrm{sech}(c+dx)}{bd} - \frac{a(e+fx)^2\tanh(c+dx)}{b^2d} + \frac{a^2\int}{}$$

$$= -\frac{a(e+fx)^2}{b^2d} + \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{(e+fx)^2\mathrm{sech}(c+dx)}{bd} - \frac{a(e+fx)^2\tanh(}{b^2d}$$

$$= -\frac{a(e+fx)^2}{b^2d} + \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd^2} + \frac{2af(e+fx)\log\left(1+e^{2(c+dx)}\right)}{b^2d^2} - \frac{2if^2\mathrm{Li}_2(}{bd}$$

$$= -\frac{a(e+fx)^2}{b^2d} + \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd^2} + \frac{a^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} - \frac{a^2(e+f}{}$$

$$= -\frac{a(e+fx)^2}{b^2d} + \frac{a^3(e+fx)^2}{b^2\left(a^2+b^2\right)d} + \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{4a^2f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d^2}$$

$$= -\frac{a(e+fx)^2}{b^2d} + \frac{a^3(e+fx)^2}{b^2\left(a^2+b^2\right)d} + \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{4a^2f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d^2}$$

$$= -\frac{a(e+fx)^2}{b^2d} + \frac{a^3(e+fx)^2}{b^2\left(a^2+b^2\right)d} + \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{4a^2f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d^2}$$

$$= -\frac{a(e+fx)^2}{b^2d} + \frac{a^3(e+fx)^2}{b^2\left(a^2+b^2\right)d} + \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd^2} - \frac{4a^2f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)d^2}$$

Mathematica [A] time = 8.48947, size = 910, normalized size = 1.18

$$\left(-2e^2\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)d^2 + f^2x^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^2 + 2efx\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^2 - f^2x^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)d^2 - 2e$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*(-2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*x
*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])] + 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
```

```
- 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*
f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[3, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/((a^2 + b^2)^(3/2)*d^3) + (2*a*e
*f*Sech[c]*(Cosh[c]*Log[Cosh[c]*Cosh[d*x] + Sinh[c]*Sinh[d*x]] - d*x*Sinh[c
]))/((a^2 + b^2)*d^2*(Cosh[c]^2 - Sinh[c]^2)) + (4*b*e*f*ArcTan[(Sinh[c] +
Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]])/((a^2 + b^2)*d^2*Sqrt[
Cosh[c]^2 - Sinh[c]^2]) - (a*f^2*Csch[c]*(-((d^2*x^2)/E^ArcTanh[Coth[c]]) +
 (I*Coth[c]*(-(d*x*(-Pi + (2*I)*ArcTanh[Coth[c]])) - Pi*Log[1 + E^(2*d*x)]
- 2*(I*d*x + I*ArcTanh[Coth[c]])*Log[1 - E^((2*I)*(I*d*x + I*ArcTanh[Coth[c
]]))] + Pi*Log[Cosh[d*x]] + (2*I)*ArcTanh[Coth[c]]*Log[I*Sinh[d*x + ArcTanh
[Coth[c]]]] + I*PolyLog[2, E^((2*I)*(I*d*x + I*ArcTanh[Coth[c]]))]))/Sqrt[1
 - Coth[c]^2])*Sech[c])/((a^2 + b^2)*d^3*Sqrt[Csch[c]^2*(-Cosh[c]^2 + Sinh[
c]^2)]) + (2*b*f^2*(((-I)*Csch[c]*(I*(d*x + ArcTanh[Coth[c]])*(Log[1 - E^(-
(d*x) - ArcTanh[Coth[c]])] - Log[1 + E^(-(d*x) - ArcTanh[Coth[c]])]) + I*(P
olyLog[2, -E^(-(d*x) - ArcTanh[Coth[c]])] - PolyLog[2, E^(-(d*x) - ArcTanh[
Coth[c]])])))/Sqrt[1 - Coth[c]^2] - (2*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)
/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]]*ArcTanh[Coth[c]])/Sqrt[Cosh[c]^2 - Sinh[c
]^2]))/((a^2 + b^2)*d^3) + (Sech[c]*Sech[c + d*x]*(-(b*e^2*Cosh[c]) - 2*b*e
*f*x*Cosh[c] - b*f^2*x^2*Cosh[c] - a*e^2*Sinh[d*x] - 2*a*e*f*x*Sinh[d*x] -
a*f^2*x^2*Sinh[d*x]))/((a^2 + b^2)*d)
```

Maple [F] time = 0.576, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 (\tanh(dx+c))^2}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.41043, size = 8585, normalized size = 11.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*(a^3 + a*b^2)*d^2*e^2 - 8*(a^3 + a*b^2)*c*d*e*f + 4*(a^3 + a*b^2)*c^
2*f^2 - 4*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 +
```

```
a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*cosh(d*x + c)^2 - 4*((a^3 + a*b^2)
*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 +
 a*b^2)*c^2*f^2)*sinh(d*x + c)^2 + 4*(a^2*b*d*f^2*x + a^2*b*d*e*f + (a^2*b*
d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*f^2*x + a^2*b*d*e*f)*co
sh(d*x + c)*sinh(d*x + c) + (a^2*b*d*f^2*x + a^2*b*d*e*f)*sinh(d*x + c)^2)*
sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*
x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 4*(a^2*b*d*f^
2*x + a^2*b*d*e*f + (a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)^2 + 2*(a^2*
b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d*f^2*x + a^2
*b*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a
*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b + 1) - 2*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2 + (a^2*b*d
^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c)^2 + 2*(a^2*b*d^2*e^
2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d
^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)
/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2)
 + 2*a) + 2*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2 + (a^2*b*d^2*e
^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c)^2 + 2*(a^2*b*d^2*e^2 -
2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d^2*e
^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2
)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2
*a) + 2*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^
2*f^2 + (a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^
2*f^2)*cosh(d*x + c)^2 + 2*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b
*c*d*e*f - a^2*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d^2*f^2*x^2
+ 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*sinh(d*x + c)^2)*sqr
t((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(a^2*b*d^2*f^2*x^2
+ 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2 + (a^2*b*d^2*f^2*x^2
+ 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*cosh(d*x + c)^2 + 2*
(a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*c
osh(d*x + c)*sinh(d*x + c) + (a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2
*b*c*d*e*f - a^2*b*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*
cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2) - b)/b) - 4*(a^2*b*f^2*cosh(d*x + c)^2 + 2*a^2*b*f^2*cosh(
d*x + c)*sinh(d*x + c) + a^2*b*f^2*sinh(d*x + c)^2 + a^2*b*f^2)*sqrt((a^2 +
 b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 4*(a^2*b*f^2*cosh(d*x + c)^
2 + 2*a^2*b*f^2*cosh(d*x + c)*sinh(d*x + c) + a^2*b*f^2*sinh(d*x + c)^2 + a
^2*b*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x +
c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 4*((a^
2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*d^2*e*f*x + (a^2*b + b^3)*d^2*e^2)
*cosh(d*x + c) + 4*((a^3 + a*b^2)*f^2 + I*(a^2*b + b^3)*f^2 + ((a^3 + a*b^2
)*f^2 + I*(a^2*b + b^3)*f^2)*cosh(d*x + c)^2 + 2*((a^3 + a*b^2)*f^2 + I*(a^
2*b + b^3)*f^2)*cosh(d*x + c)*sinh(d*x + c) + ((a^3 + a*b^2)*f^2 + I*(a^2*b
 + b^3)*f^2)*sinh(d*x + c)^2)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + (4
*(a^3 + a*b^2)*f^2 - 4*I*(a^2*b + b^3)*f^2 + 4*((a^3 + a*b^2)*f^2 - I*(a^2*
b + b^3)*f^2)*cosh(d*x + c)^2 + 8*((a^3 + a*b^2)*f^2 - I*(a^2*b + b^3)*f^2)
*cosh(d*x + c)*sinh(d*x + c) + 4*((a^3 + a*b^2)*f^2 - I*(a^2*b + b^3)*f^2)*
sinh(d*x + c)^2)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + (4*(a^3 + a*b^
2)*d*e*f + 4*I*(a^2*b + b^3)*d*e*f - 4*(a^3 + a*b^2)*c*f^2 - 4*I*(a^2*b + b
^3)*c*f^2 + (4*(a^3 + a*b^2)*d*e*f + 4*I*(a^2*b + b^3)*d*e*f - 4*(a^3 + a*b
^2)*c*f^2 - 4*I*(a^2*b + b^3)*c*f^2)*cosh(d*x + c)^2 + (8*(a^3 + a*b^2)*d*e
*f + 8*I*(a^2*b + b^3)*d*e*f - 8*(a^3 + a*b^2)*c*f^2 - 8*I*(a^2*b + b^3)*c*
f^2)*cosh(d*x + c)*sinh(d*x + c) + (4*(a^3 + a*b^2)*d*e*f + 4*I*(a^2*b + b^
3)*d*e*f - 4*(a^3 + a*b^2)*c*f^2 - 4*I*(a^2*b + b^3)*c*f^2)*sinh(d*x + c)^2
)*log(cosh(d*x + c) + sinh(d*x + c) + I) + (4*(a^3 + a*b^2)*d*e*f - 4*I*(a^
2*b + b^3)*d*e*f - 4*(a^3 + a*b^2)*c*f^2 + 4*I*(a^2*b + b^3)*c*f^2 + (4*(a^
3 + a*b^2)*d*e*f - 4*I*(a^2*b + b^3)*d*e*f - 4*(a^3 + a*b^2)*c*f^2 + 4*I*(a
```

```
^2*b + b^3)*c*f^2)*cosh(d*x + c)^2 + (8*(a^3 + a*b^2)*d*e*f - 8*I*(a^2*b +
b^3)*d*e*f - 8*(a^3 + a*b^2)*c*f^2 + 8*I*(a^2*b + b^3)*c*f^2)*cosh(d*x + c)
*sinh(d*x + c) + (4*(a^3 + a*b^2)*d*e*f - 4*I*(a^2*b + b^3)*d*e*f - 4*(a^3
+ a*b^2)*c*f^2 + 4*I*(a^2*b + b^3)*c*f^2)*sinh(d*x + c)^2)*log(cosh(d*x + c
) + sinh(d*x + c) - I) + (4*(a^3 + a*b^2)*d*f^2*x - 4*I*(a^2*b + b^3)*d*f^2
*x + 4*(a^3 + a*b^2)*c*f^2 - 4*I*(a^2*b + b^3)*c*f^2 + (4*(a^3 + a*b^2)*d*f
^2*x - 4*I*(a^2*b + b^3)*d*f^2*x + 4*(a^3 + a*b^2)*c*f^2 - 4*I*(a^2*b + b^3
)*c*f^2)*cosh(d*x + c)^2 + (8*(a^3 + a*b^2)*d*f^2*x - 8*I*(a^2*b + b^3)*d*f
^2*x + 8*(a^3 + a*b^2)*c*f^2 - 8*I*(a^2*b + b^3)*c*f^2)*cosh(d*x + c)*sinh(
d*x + c) + (4*(a^3 + a*b^2)*d*f^2*x - 4*I*(a^2*b + b^3)*d*f^2*x + 4*(a^3 +
a*b^2)*c*f^2 - 4*I*(a^2*b + b^3)*c*f^2)*sinh(d*x + c)^2)*log(I*cosh(d*x + c
) + I*sinh(d*x + c) + 1) + (4*(a^3 + a*b^2)*d*f^2*x + 4*I*(a^2*b + b^3)*d*f
^2*x + 4*(a^3 + a*b^2)*c*f^2 + 4*I*(a^2*b + b^3)*c*f^2 + (4*(a^3 + a*b^2)*d
*f^2*x + 4*I*(a^2*b + b^3)*d*f^2*x + 4*(a^3 + a*b^2)*c*f^2 + 4*I*(a^2*b + b
^3)*c*f^2)*cosh(d*x + c)^2 + (8*(a^3 + a*b^2)*d*f^2*x + 8*I*(a^2*b + b^3)*d
*f^2*x + 8*(a^3 + a*b^2)*c*f^2 + 8*I*(a^2*b + b^3)*c*f^2)*cosh(d*x + c)*sin
h(d*x + c) + (4*(a^3 + a*b^2)*d*f^2*x + 4*I*(a^2*b + b^3)*d*f^2*x + 4*(a^3
+ a*b^2)*c*f^2 + 4*I*(a^2*b + b^3)*c*f^2)*sinh(d*x + c)^2)*log(-I*cosh(d*x
+ c) - I*sinh(d*x + c) + 1) - 4*((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3
)*d^2*e*f*x + (a^2*b + b^3)*d^2*e^2 + 2*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3
 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*cosh
(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c)^2 + 2*
(a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2
+ b^4)*d^3*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.384 $\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=385

$$\frac{a^2f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^{3/2}}-\frac{a^2f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2\left(a^2+b^2\right)^{3/2}}-\frac{a^2f\tan^{-1}(\sinh(c+dx))}{bd^2\left(a^2+b^2\right)}-\frac{a^3f\log(\cosh(c+dx))}{b^2d^2\left(a^2+b^2\right)}+\frac{a^2(}{}$$

```
[Out] (f*ArcTan[Sinh[c + d*x]])/(b*d^2) - (a^2*f*ArcTan[Sinh[c + d*x]])/(b*(a^2 +
 b^2)*d^2) + (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])
/((a^2 + b^2)^(3/2)*d) - (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/((a^2 + b^2)^(3/2)*d) + (a*f*Log[Cosh[c + d*x]])/(b^2*d^2) - (
a^3*f*Log[Cosh[c + d*x]])/(b^2*(a^2 + b^2)*d^2) + (a^2*f*PolyLog[2, -((b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (a^2*f*PolyLo
g[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (
(e + f*x)*Sech[c + d*x])/(b*d) + (a^2*(e + f*x)*Sech[c + d*x])/(b*(a^2 + b^
2)*d) - (a*(e + f*x)*Tanh[c + d*x])/(b^2*d) + (a^3*(e + f*x)*Tanh[c + d*x])
/(b^2*(a^2 + b^2)*d)
```

Rubi [A] time = 0.758024, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5567, 5451, 3770, 5583, 4184, 3475, 5573, 3322, 2264, 2190, 2279, 2391, 6742}

$$\frac{a^2f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2\left(a^2+b^2\right)^{3/2}}-\frac{a^2f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2\left(a^2+b^2\right)^{3/2}}-\frac{a^2f\tan^{-1}(\sinh(c+dx))}{bd^2\left(a^2+b^2\right)}-\frac{a^3f\log(\cosh(c+dx))}{b^2d^2\left(a^2+b^2\right)}+\frac{a^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (f*ArcTan[Sinh[c + d*x]])/(b*d^2) - (a^2*f*ArcTan[Sinh[c + d*x]])/(b*(a^2 +
 b^2)*d^2) + (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])
/((a^2 + b^2)^(3/2)*d) - (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/((a^2 + b^2)^(3/2)*d) + (a*f*Log[Cosh[c + d*x]])/(b^2*d^2) - (
a^3*f*Log[Cosh[c + d*x]])/(b^2*(a^2 + b^2)*d^2) + (a^2*f*PolyLog[2, -((b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (a^2*f*PolyLo
g[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (
(e + f*x)*Sech[c + d*x])/(b*d) + (a^2*(e + f*x)*Sech[c + d*x])/(b*(a^2 + b^
2)*d) - (a*(e + f*x)*Tanh[c + d*x])/(b^2*d) + (a^3*(e + f*x)*Tanh[c + d*x])
/(b^2*(a^2 + b^2)*d)
```


Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c +
 d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
```

```
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5583

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\mathbf{sech}(c+dx)\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\mathrm{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)\mathbf{sech}(c+dx)}{bd} - \frac{a\int (e+fx)\mathbf{sech}^2(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)\mathrm{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b^2} + \frac{f\ \ldots}{}$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{bd^2} - \frac{(e+fx)\mathbf{sech}(c+dx)}{bd} - \frac{a(e+fx)\tanh(c+dx)}{b^2d} + \frac{a^2\int \frac{e\ldots}{a+b\sinh\ldots}}{a^2\ldots}$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{bd^2} + \frac{af\log(\cosh(c+dx))}{b^2d^2} - \frac{(e+fx)\mathbf{sech}(c+dx)}{bd} - \frac{a(e+fx)\,\mathrm{ta}\ldots}{b^2\ldots}$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{bd^2} + \frac{af\log(\cosh(c+dx))}{b^2d^2} - \frac{(e+fx)\mathbf{sech}(c+dx)}{bd} - \frac{a(e+fx)\,\mathrm{ta}\ldots}{b^2\ldots}$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{bd^2} + \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} - \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d}\ldots$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{bd^2} - \frac{a^2f\tan^{-1}(\sinh(c+dx))}{b\left(a^2+b^2\right)d^2} + \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} - \ldots$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{bd^2} - \frac{a^2f\tan^{-1}(\sinh(c+dx))}{b\left(a^2+b^2\right)d^2} + \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d} - \ldots$$

Mathematica [A] time = 3.05218, size = 284, normalized size = 0.74

$$\frac{\frac{a^2\left(f\,\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)-f\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)-2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)+f(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)-f(c+dx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)+2cf\tanh^{-1}\left(\frac{a+be^{c\ldots}}{\sqrt{a^2+b\ldots}}\right)\ldots}{\left(a^2+b^2\right)^{3/2}}\ldots}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((2*b*f*ArcTan[Tanh[(c + d*x)/2]])/(a^2 + b^2) + (a*f*Log[Cosh[c + d*x]])/(
a^2 + b^2) + (a^2*(-2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*
```

```
c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
 - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^2 + b^2)^(3/
2) - (d*(e + f*x)*Sech[c + d*x]*(b + a*Sinh[c + d*x]))/(a^2 + b^2))/d^2
```

Maple [B] time = 0.162, size = 1928, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] -2/d^2/(a^2+b^2)^(3/2)*b^2*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)
+a)/(a+(a^2+b^2)^(1/2)))*a^2*c+2/d^2/(a^2+b^2)^(3/2)*b^2*f*c/(2*a^2+2*b^2)*
arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2+2/d/(a^2+b^2)^(3/2)*b
^2*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)
))*a^2*x-2/d/(a^2+b^2)^(3/2)*b^2*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)
^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^2*x+2/d^2/(a^2+b^2)^(3/2)*b^2*f/(2*a^2+2*b
^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^2*c-2/d*e/
(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1
/2))*a^2-2/(a^2+b^2)^(3/2)/d^2*a^4*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2
+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/(a^2+b^2)^(3/2)/d^2*a^4*f/(2*a^2+2*b^
2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2/(a^2+b^2
)^(3/2)/d*a^4*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1
/2))-2/d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)
-b)+4/d^2/(a^2+b^2)*b^3*f/(2*a^2+2*b^2)*arctan(exp(d*x+c))-2/d^2/(a^2+b^2)^
(5/2)*a^4*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/2/d^2/(a^2+
b^2)^2*a*b^2*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d^2/(a^2+b^2)*a^3*f/
(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))+2/d^2*f*c/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*
arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2-2/d^2/(a^2+b^2)^(3/2)
*b^4*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+4/d^
2/(a^2+b^2)*a^2*b*f/(2*a^2+2*b^2)*arctan(exp(d*x+c))-2/d^2/(a^2+b^2)^(5/2)*
a^2*b^2*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+4/d^2/(a^2+b^2)
^(1/2)*a^2*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2)
)+2/d^2/(a^2+b^2)*b^2*f/(2*a^2+2*b^2)*a*ln(1+exp(2*d*x+2*c))-1/d^2/(a^2+b^2
)*b^2*f/(2*a^2+2*b^2)*a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d^2/(a^2+b^
2)^(1/2)*b^2*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/
2))+2/d^2/(a^2+b^2)^(3/2)*b^2*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2
)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a^2-2/d/(a^2+b^2)^(3/2)*b^2*e/(2*a^2+2*b^2
)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2-2/(a^2+b^2)^(3/2)/d
*a^4*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)
))*x-2/d^2/(a^2+b^2)^(3/2)*b^2*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+
2*a)/(a^2+b^2)^(1/2))*a^2-2/d^2/(a^2+b^2)^(3/2)*b^2*f/(2*a^2+2*b^2)*dilog((
b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^2+2/(a^2+b^2)^(3/2)/
d^2*a^4*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
+2/(a^2+b^2)^(3/2)/d*a^4*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-
a)/(-a+(a^2+b^2)^(1/2)))*x-2/(a^2+b^2)^(3/2)/d^2*a^4*f/(2*a^2+2*b^2)*ln((b*
exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/(a^2+b^2)^(3/2)/d^2*
a^4*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2
)))*c+1/d^2/(a^2+b^2)^2*a^3*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d^2/(
a^2+b^2)*a*f*ln(exp(d*x+c))+2*(f*x+e)*(-b*exp(d*x+c)+a)/d/(a^2+b^2)/(1+exp(
2*d*x+2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.58481, size = 3274, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*(a^3 + a*b^2)*d*f*x*cosh(d*x + c)^2 + 2*(a^3 + a*b^2)*d*f*x*sinh(d*x +
c)^2 - 2*(a^3 + a*b^2)*d*e - (a^2*b*f*cosh(d*x + c)^2 + 2*a^2*b*f*cosh(d*x
+ c)*sinh(d*x + c) + a^2*b*f*sinh(d*x + c)^2 + a^2*b*f)*sqrt((a^2 + b^2)/b^
2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (a^2*b*f*cosh(d*x + c)^2 + 2*a^2
*b*f*cosh(d*x + c)*sinh(d*x + c) + a^2*b*f*sinh(d*x + c)^2 + a^2*b*f)*sqrt(
(a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (a^2*b*d*e - a^2*b
*c*f + (a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*e - a^2*b*c*f)*
cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d*e - a^2*b*c*f)*sinh(d*x + c)^2)*sqrt
((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2
 + b^2)/b^2) + 2*a) - (a^2*b*d*e - a^2*b*c*f + (a^2*b*d*e - a^2*b*c*f)*cosh
(d*x + c)^2 + 2*(a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*
b*d*e - a^2*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x
+ c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (a^2*b*d*f*x
+ a^2*b*c*f + (a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*f*x +
a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d*f*x + a^2*b*c*f)*sinh(d*x
 + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (a^2*b*d*
f*x + a^2*b*c*f + (a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*f*
x + a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d*f*x + a^2*b*c*f)*sinh
(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
- (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*((a
^2*b + b^3)*f*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*f*cosh(d*x + c)*sinh(d*x +
c) + (a^2*b + b^3)*f*sinh(d*x + c)^2 + (a^2*b + b^3)*f)*arctan(cosh(d*x + c
) + sinh(d*x + c)) + 2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e)*cosh(d*x +
 c) - ((a^3 + a*b^2)*f*cosh(d*x + c)^2 + 2*(a^3 + a*b^2)*f*cosh(d*x + c)*si
nh(d*x + c) + (a^3 + a*b^2)*f*sinh(d*x + c)^2 + (a^3 + a*b^2)*f)*log(2*cosh
(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 2*(2*(a^3 + a*b^2)*d*f*x*cosh(
d*x + c) + (a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e)*sinh(d*x + c))/((a^4 +
2*a^2*b^2 + b^4)*d^2*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(d
*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*d^2*sinh(d*x + c)^2 + (a^4
+ 2*a^2*b^2 + b^4)*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.385 $\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=90

$$-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}-\frac{a\tanh(c+dx)}{d\left(a^2+b^2\right)}-\frac{b\text{sech}(c+dx)}{d\left(a^2+b^2\right)}$$

```
[Out] (-2*a^2*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3
/2)*d) - (b*Sech[c + d*x])/((a^2 + b^2)*d) - (a*Tanh[c + d*x])/((a^2 + b^2)
*d)
```

Rubi [A] time = 0.105545, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2727, 3767, 8, 2606, 2660, 618, 204}

$$-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}-\frac{a\tanh(c+dx)}{d\left(a^2+b^2\right)}-\frac{b\text{sech}(c+dx)}{d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*a^2*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3
/2)*d) - (b*Sech[c + d*x])/((a^2 + b^2)*d) - (a*Tanh[c + d*x])/((a^2 + b^2)
*d)
```

Rule 2727

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[(b*g)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[(a^2*g^2)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{a\int \mathbf{sech}^2(c+dx)\,dx}{a^2+b^2}+\frac{a^2\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}+\frac{b\int \mathbf{sech}(c+dx)\tanh(c+dx)\,dx}{a^2+b^2}\\ &= -\frac{(ia)\,\text{Subst}(\int 1\,dx,x,-i\tanh(c+dx))}{\left(a^2+b^2\right)d}-\frac{\left(2ia^2\right)\text{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{\left(a^2+b^2\right)d}\\ &= -\frac{b\mathbf{sech}(c+dx)}{\left(a^2+b^2\right)d}-\frac{a\tanh(c+dx)}{\left(a^2+b^2\right)d}+\frac{\left(4ia^2\right)\text{Subst}\left(\int \frac{1}{-4\left(a^2+b^2\right)-x^2}\,dx,x,-2ib+2a\tan\left(\frac{1}{2}(ic+\right.\right.}{\left(a^2+b^2\right)d}\\ &= -\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d}-\frac{b\mathbf{sech}(c+dx)}{\left(a^2+b^2\right)d}-\frac{a\tanh(c+dx)}{\left(a^2+b^2\right)d}\end{aligned}$$

Mathematica [A] time = 0.195656, size = 106, normalized size = 1.18

$$-\frac{a\left(2a\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)-\sqrt{-a^2-b^2}\tanh(c+dx)\right)-b\sqrt{-a^2-b^2}\mathbf{sech}(c+dx)}{d\left(-a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((-(b*Sqrt[-a^2 - b^2]*Sech[c + d*x]) + a*(2*a*ArcTan[(b - a*Tanh[(c + d*x
)/2])/Sqrt[-a^2 - b^2]] - Sqrt[-a^2 - b^2]*Tanh[c + d*x]))/((-a^2 - b^2)^(3
/2)*d))
```

Maple [A] time = 0.001, size = 103, normalized size = 1.1

$$\frac{1}{d}\left(8\,\frac{a^2}{\left(4\,a^2+4\,b^2\right)\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\,\frac{2\,a\tanh\left(1/2\,dx+c/2\right)-2\,b}{\sqrt{a^2+b^2}}\right)+2\,\frac{-a\tanh\left(1/2\,dx+c/2\right)-b}{\left(a^2+b^2\right)\left(\left(\tanh\left(1/2\,dx+c/2\right)\right)^2+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/d*(8*a^2/(4*a^2+4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*
c)-2*b)/(a^2+b^2)^(1/2))+2/(a^2+b^2)*(-a*tanh(1/2*d*x+1/2*c)-b)/(tanh(1/2*d
*x+1/2*c)^2+1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.12362, size = 883, normalized size = 9.81

$$\frac{2\,a^{3} + 2\,ab^{2} + \left(a^{2}\cosh\left(dx + c\right)^{2} + 2\,a^{2}\cosh\left(dx + c\right)\sinh\left(dx + c\right) + a^{2}\sinh\left(dx + c\right)^{2} + a^{2}\right)\sqrt{a^{2} + b^{2}}\log\left(\frac{b^{2}\cosh(dx+c)^{2}}{}\right.}{\left(a^{4} + 2\,a^{2}b^{2} + b^{4}\right)d\cosh\left(dx + c\right)^{2} + 2\,\left(a^{4} + 2\,a^{2}b^{2} + b^{4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*a^3 + 2*a*b^2 + (a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c)
 + a^2*sinh(d*x + c)^2 + a^2)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^
2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c
) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x +
c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*
cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*(a^2*b + b^3)*cosh(d*x + c) - 2*
(a^2*b + b^3)*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + 2
*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 +
 b^4)*d*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^{2}\left(c + dx\right)}{a + b\sinh\left(c + dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [A] time = 1.37836, size = 163, normalized size = 1.81

$$-\frac{\frac{a^2\log\left(\frac{\left|-2\,be^{(dx+2\,c)}-2\,ae^c-2\,\sqrt{a^2+b^2}e^c\right|}{\left|-2\,be^{(dx+2\,c)}-2\,ae^c+2\,\sqrt{a^2+b^2}e^c\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{2\left(be^{(dx+c)}-a\right)}{\left(a^2+b^2\right)\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -(a^2*log(abs(-2*b*e^(d*x + 2*c) - 2*a*e^c - 2*sqrt(a^2 + b^2)*e^c)/abs(-2*
b*e^(d*x + 2*c) - 2*a*e^c + 2*sqrt(a^2 + b^2)*e^c))/(a^2 + b^2)^(3/2) + 2*(
b*e^(d*x + c) - a)/((a^2 + b^2)*(e^(2*d*x + 2*c) + 1)))/d
```

3.386 $\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Tanh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0729658, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]

[Out] Defer[Int][Tanh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.009, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]

[Out] $Aborted
```

Maple [A] time = 0.487, size = 0, normalized size = 0.

$$\int \frac{(\tanh(dx+c))^2}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)

[Out] int(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$2a^2\int -\frac{e^{(dx+c)}}{a^2be+b^3e+\left(a^2bf+b^3f\right)x-\left(a^2bee^{(2c)}+b^3ee^{(2c)}+\left(a^2bfe^{(2c)}+b^3fe^{(2c)}\right)x\right)e^{(2dx)}-2\left(a^3ee^c+ab^2ee^c+\left(a^3fe^c+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*a^2*integrate(-e^(d*x + c)/(a^2*b*e + b^3*e + (a^2*b*f + b^3*f)*x - (a^2*
b*e*e^(2*c) + b^3*e*e^(2*c) + (a^2*b*f*e^(2*c) + b^3*f*e^(2*c))*x)*e^(2*d*x
) - 2*(a^3*e*e^c + a*b^2*e*e^c + (a^3*f*e^c + a*b^2*f*e^c)*x)*e^(d*x)), x)
- 2*(b*e^(d*x + c) - a)/(a^2*d*e + b^2*d*e + (a^2*d*f + b^2*d*f)*x + (a^2*d
*e*e^(2*c) + b^2*d*e*e^(2*c) + (a^2*d*f*e^(2*c) + b^2*d*f*e^(2*c))*x)*e^(2*
d*x)) - integrate(2*(b*f*e^(d*x + c) - a*f)/(a^2*d*e^2 + b^2*d*e^2 + (a^2*d
*f^2 + b^2*d*f^2)*x^2 + 2*(a^2*d*e*f + b^2*d*e*f)*x + (a^2*d*e^2*e^(2*c) +
b^2*d*e^2*e^(2*c) + (a^2*d*f^2*e^(2*c) + b^2*d*f^2*e^(2*c))*x^2 + 2*(a^2*d*
e*f*e^(2*c) + b^2*d*e*f*e^(2*c))*x)*e^(2*d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\tanh(dx+c)^2}{afx+ae+\left(bfx+be\right)\sinh(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(tanh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tanh^2(c+dx)}{(a+b\sinh(c+dx))\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.387 $\int \frac{(e+fx)^2\mathbf{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1256

result too large to display

```
[Out] -((a*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^2*d)) + (2*a^3*(e + f*x)^2*ArcTan[
E^(c + d*x)])/((a^2 + b^2)^2*d) + (a^3*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^
2*(a^2 + b^2)*d) + (a*f^2*ArcTan[Sinh[c + d*x]])/(b^2*d^3) - (a^3*f^2*ArcTa
n[Sinh[c + d*x]])/(b^2*(a^2 + b^2)*d^3) + (a^2*b*(e + f*x)^2*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) + (a^2*b*(e + f*x)^2*Lo
g[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) - (a^2*b*(e
 + f*x)^2*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)^2*d) - (f^2*Log[Cosh[c + d
*x]])/(b*d^3) + (a^2*f^2*Log[Cosh[c + d*x]])/(b*(a^2 + b^2)*d^3) + (I*a*f*(
e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*d^2) - ((2*I)*a^3*f*(e + f*x)*P
olyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - (I*a^3*f*(e + f*x)*PolyL
og[2, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) - (I*a*f*(e + f*x)*PolyLog[2
, I*E^(c + d*x)])/(b^2*d^2) + ((2*I)*a^3*f*(e + f*x)*PolyLog[2, I*E^(c + d*
x)])/((a^2 + b^2)^2*d^2) + (I*a^3*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b
^2*(a^2 + b^2)*d^2) + (2*a^2*b*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) + (2*a^2*b*f*(e + f*x)*PolyLog[2,
 -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) - (a^2*b*f*
(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2)^2*d^2) - (I*a*f^2*Poly
Log[3, (-I)*E^(c + d*x)])/(b^2*d^3) + ((2*I)*a^3*f^2*PolyLog[3, (-I)*E^(c +
 d*x)])/((a^2 + b^2)^2*d^3) + (I*a^3*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b^2
*(a^2 + b^2)*d^3) + (I*a*f^2*PolyLog[3, I*E^(c + d*x)])/(b^2*d^3) - ((2*I)*
a^3*f^2*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)^2*d^3) - (I*a^3*f^2*PolyLog
[3, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) - (2*a^2*b*f^2*PolyLog[3, -((b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) - (2*a^2*b*f^2*Poly
Log[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) + (a^
2*b*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^3) - (a*f*(e + f*x
)*Sech[c + d*x])/(b^2*d^2) + (a^3*f*(e + f*x)*Sech[c + d*x])/(b^2*(a^2 + b^
2)*d^2) - ((e + f*x)^2*Sech[c + d*x]^2)/(2*b*d) + (a^2*(e + f*x)^2*Sech[c +
 d*x]^2)/(2*b*(a^2 + b^2)*d) + (f*(e + f*x)*Tanh[c + d*x])/(b*d^2) - (a^2*f
*(e + f*x)*Tanh[c + d*x])/(b*(a^2 + b^2)*d^2) - (a*(e + f*x)^2*Sech[c + d*x
]*Tanh[c + d*x])/(2*b^2*d) + (a^3*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/
(2*b^2*(a^2 + b^2)*d)
```

Rubi [A] time = 1.96754, antiderivative size = 1256, normalized size of antiderivative = 1., number of steps used = 53, number of rules used = 15, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.441, Rules used = {5583, 5451, 4184, 3475, 4186, 3770, 4180, 2531, 2282, 6589, 5573, 5561, 2190, 6742, 3718}

$$\frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)a^3}{b^2\left(a^2+b^2\right)d}+\frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)a^3}{\left(a^2+b^2\right)^2d}-\frac{f^2\tan^{-1}(\sinh(c+dx))a^3}{b^2\left(a^2+b^2\right)d^3}-\frac{if(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right)}{b^2\left(a^2+b^2\right)d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -((a*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^2*d)) + (2*a^3*(e + f*x)^2*ArcTan[
E^(c + d*x)])/((a^2 + b^2)^2*d) + (a^3*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^
2*(a^2 + b^2)*d) + (a*f^2*ArcTan[Sinh[c + d*x]])/(b^2*d^3) - (a^3*f^2*ArcTa
n[Sinh[c + d*x]])/(b^2*(a^2 + b^2)*d^3) + (a^2*b*(e + f*x)^2*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) + (a^2*b*(e + f*x)^2*Lo
```

```
g[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) - (a^2*b*(e
 + f*x)^2*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)^2*d) - (f^2*Log[Cosh[c + d
*x]])/(b*d^3) + (a^2*f^2*Log[Cosh[c + d*x]])/(b*(a^2 + b^2)*d^3) + (I*a*f*(
e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*d^2) - ((2*I)*a^3*f*(e + f*x)*P
olyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - (I*a^3*f*(e + f*x)*PolyL
og[2, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) - (I*a*f*(e + f*x)*PolyLog[2
, I*E^(c + d*x)])/(b^2*d^2) + ((2*I)*a^3*f*(e + f*x)*PolyLog[2, I*E^(c + d*
x)])/((a^2 + b^2)^2*d^2) + (I*a^3*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b
^2*(a^2 + b^2)*d^2) + (2*a^2*b*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) + (2*a^2*b*f*(e + f*x)*PolyLog[2,
 -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) - (a^2*b*f*
(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2)^2*d^2) - (I*a*f^2*Poly
Log[3, (-I)*E^(c + d*x)])/(b^2*d^3) + ((2*I)*a^3*f^2*PolyLog[3, (-I)*E^(c +
 d*x)])/((a^2 + b^2)^2*d^3) + (I*a^3*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b^2
*(a^2 + b^2)*d^3) + (I*a*f^2*PolyLog[3, I*E^(c + d*x)])/(b^2*d^3) - ((2*I)*
a^3*f^2*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)^2*d^3) - (I*a^3*f^2*PolyLog
[3, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) - (2*a^2*b*f^2*PolyLog[3, -((b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) - (2*a^2*b*f^2*Poly
Log[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) + (a^
2*b*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^3) - (a*f*(e + f*x
)*Sech[c + d*x])/(b^2*d^2) + (a^3*f*(e + f*x)*Sech[c + d*x])/(b^2*(a^2 + b^
2)*d^2) - ((e + f*x)^2*Sech[c + d*x]^2)/(2*b*d) + (a^2*(e + f*x)^2*Sech[c +
 d*x]^2)/(2*b*(a^2 + b^2)*d) + (f*(e + f*x)*Tanh[c + d*x])/(b*d^2) - (a^2*f
*(e + f*x)*Tanh[c + d*x])/(b*(a^2 + b^2)*d^2) - (a*(e + f*x)^2*Sech[c + d*x
]*Tanh[c + d*x])/(2*b^2*d) + (a^3*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/
(2*b^2*(a^2 + b^2)*d)
```

Rule 5583

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
```

```
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\mathrm{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\mathrm{sech}^2(c+dx)\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\mathrm{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,d\ldots}{b}$$

$$= -\frac{(e+fx)^2\mathrm{sech}^2(c+dx)}{2bd} - \frac{a\int (e+fx)^2\mathrm{sech}^3(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^2\mathrm{sech}\ldots}{a+b\sinh(\ldots}}{b^2}$$

$$= -\frac{af(e+fx)\mathrm{sech}(c+dx)}{b^2d^2} - \frac{(e+fx)^2\mathrm{sech}^2(c+dx)}{2bd} + \frac{f(e+fx)\tanh(c+\ldots}{bd^2}$$

$$= -\frac{a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{af^2\tan^{-1}(\sinh(c+dx))}{b^2d^3} - \frac{f^2\log(\cosh(c+d\ldots}{bd^3}$$

$$= -\frac{a^2b(e+fx)^3}{3\left(a^2+b^2\right)^2f} - \frac{a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{af^2\tan^{-1}(\sinh(c+dx))}{b^2d^3} - \ldots$$

$$= -\frac{a^2b(e+fx)^3}{3\left(a^2+b^2\right)^2f} - \frac{a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{af^2\tan^{-1}(\sinh(c+dx))}{b^2d^3} + \ldots$$

$$= -\frac{a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2d} + \frac{a^3(e+fx)^2\tan\ldots}{b^2\left(a^2+b^2\ldots}$$

$$= -\frac{a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2d} + \frac{a^3(e+fx)^2\tan\ldots}{b^2\left(a^2+b^2\ldots}$$

$$= -\frac{a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2d} + \frac{a^3(e+fx)^2\tan\ldots}{b^2\left(a^2+b^2\ldots}$$

$$= -\frac{a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2d} + \frac{a^3(e+fx)^2\tan\ldots}{b^2\left(a^2+b^2\ldots}$$

$$= -\frac{a(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2d} + \frac{a^3(e+fx)^2\tan\ldots}{b^2\left(a^2+b^2\ldots}$$

Mathematica [B] time = 31.3531, size = 3390, normalized size = 2.7

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] -(-12*a^2*b*d^3*e^2*E^(2*c)*x - 12*a^2*b*d*E^(2*c)*f^2*x - 12*b^3*d*E^(2*c)
*f^2*x - 12*a^2*b*d^3*e*E^(2*c)*f*x^2 - 4*a^2*b*d^3*E^(2*c)*f^2*x^3 - 6*a^3
*d^2*e^2*ArcTan[E^(c + d*x)] + 6*a*b^2*d^2*e^2*ArcTan[E^(c + d*x)] - 6*a^3*
d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)] + 6*a*b^2*d^2*e^2*E^(2*c)*ArcTan[E^(c +
 d*x)] - 12*a^3*f^2*ArcTan[E^(c + d*x)] - 12*a*b^2*f^2*ArcTan[E^(c + d*x)]
- 12*a^3*E^(2*c)*f^2*ArcTan[E^(c + d*x)] - 12*a*b^2*E^(2*c)*f^2*ArcTan[E^(c
 + d*x)] - (6*I)*a^3*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (6*I)*a*b^2*d^2*e*f
*x*Log[1 - I*E^(c + d*x)] - (6*I)*a^3*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*
x)] + (6*I)*a*b^2*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] - (3*I)*a^3*d^2*
f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*a*b^2*d^2*f^2*x^2*Log[1 - I*E^(c + d
*x)] - (3*I)*a^3*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*a*b^2*d
^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (6*I)*a^3*d^2*e*f*x*Log[1 + I*E
^(c + d*x)] - (6*I)*a*b^2*d^2*e*f*x*Log[1 + I*E^(c + d*x)] + (6*I)*a^3*d^2*
e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (6*I)*a*b^2*d^2*e*E^(2*c)*f*x*Log[1
+ I*E^(c + d*x)] + (3*I)*a^3*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - (3*I)*a*b
^2*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] + (3*I)*a^3*d^2*E^(2*c)*f^2*x^2*Log[1
 + I*E^(c + d*x)] - (3*I)*a*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 + I*E^(c + d*x)]
+ 6*a^2*b*d^2*e^2*Log[1 + E^(2*(c + d*x))] + 6*a^2*b*d^2*e^2*E^(2*c)*Log[1
+ E^(2*(c + d*x))] + 6*a^2*b*f^2*Log[1 + E^(2*(c + d*x))] + 6*b^3*f^2*Log[1
 + E^(2*(c + d*x))] + 6*a^2*b*E^(2*c)*f^2*Log[1 + E^(2*(c + d*x))] + 6*b^3*
E^(2*c)*f^2*Log[1 + E^(2*(c + d*x))] + 12*a^2*b*d^2*e*f*x*Log[1 + E^(2*(c +
 d*x))] + 12*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 + E^(2*(c + d*x))] + 6*a^2*b*d^2
*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 6*a^2*b*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(
2*(c + d*x))] + (6*I)*a*(a^2 - b^2)*d*(1 + E^(2*c))*f*(e + f*x)*PolyLog[2,
(-I)*E^(c + d*x)] - (6*I)*a*(a^2 - b^2)*d*(1 + E^(2*c))*f*(e + f*x)*PolyLog
[2, I*E^(c + d*x)] + 6*a^2*b*d*e*f*PolyLog[2, -E^(2*(c + d*x))] + 6*a^2*b*d
*e*E^(2*c)*f*PolyLog[2, -E^(2*(c + d*x))] + 6*a^2*b*d*f^2*x*PolyLog[2, -E^(
2*(c + d*x))] + 6*a^2*b*d*E^(2*c)*f^2*x*PolyLog[2, -E^(2*(c + d*x))] - (6*I
)*a^3*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (6*I)*a*b^2*f^2*PolyLog[3, (-I)*E^
(c + d*x)] - (6*I)*a^3*E^(2*c)*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (6*I)*a*b
^2*E^(2*c)*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (6*I)*a^3*f^2*PolyLog[3, I*E^
(c + d*x)] - (6*I)*a*b^2*f^2*PolyLog[3, I*E^(c + d*x)] + (6*I)*a^3*E^(2*c)*
f^2*PolyLog[3, I*E^(c + d*x)] - (6*I)*a*b^2*E^(2*c)*f^2*PolyLog[3, I*E^(c +
 d*x)] - 3*a^2*b*f^2*PolyLog[3, -E^(2*(c + d*x))] - 3*a^2*b*E^(2*c)*f^2*Pol
yLog[3, -E^(2*(c + d*x))])/(6*(a^2 + b^2)^2*d^3*(1 + E^(2*c))) - (a^2*b*(6*
e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2
]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d
) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-
a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[
(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(
a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-
a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))
])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d +
(6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d
- (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*
E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[
(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqr
t[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a
*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x)
```

```
)/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E
^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(-1 + E^(2*c))*f
*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)
]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a
*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*f^2*PolyLog[3, -((b*E^(2*c +
d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3
, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (6*f^2*P
olyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 +
(6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(
2*c)]))])/d^3))/(3*(a^2 + b^2)^2*(-1 + E^(2*c))) + (Csch[c]*Sech[c]*Sech[c
+ d*x]^2*(6*a^2*b*e*f + 6*b^3*e*f + 12*a^2*b*d^2*e^2*x + 6*a^2*b*f^2*x + 6*
b^3*f^2*x + 12*a^2*b*d^2*e*f*x^2 + 4*a^2*b*d^2*f^2*x^3 - 6*a^2*b*e*f*Cosh[2
*c] - 6*b^3*e*f*Cosh[2*c] - 6*a^2*b*f^2*x*Cosh[2*c] - 6*b^3*f^2*x*Cosh[2*c]
 - 6*a^2*b*e*f*Cosh[2*d*x] - 6*b^3*e*f*Cosh[2*d*x] - 6*a^2*b*f^2*x*Cosh[2*d
*x] - 6*b^3*f^2*x*Cosh[2*d*x] + 3*a^3*d*e^2*Cosh[c - d*x] + 3*a*b^2*d*e^2*C
osh[c - d*x] + 6*a^3*d*e*f*x*Cosh[c - d*x] + 6*a*b^2*d*e*f*x*Cosh[c - d*x]
+ 3*a^3*d*f^2*x^2*Cosh[c - d*x] + 3*a*b^2*d*f^2*x^2*Cosh[c - d*x] - 3*a^3*d
*e^2*Cosh[3*c + d*x] - 3*a*b^2*d*e^2*Cosh[3*c + d*x] - 6*a^3*d*e*f*x*Cosh[3
*c + d*x] - 6*a*b^2*d*e*f*x*Cosh[3*c + d*x] - 3*a^3*d*f^2*x^2*Cosh[3*c + d*
x] - 3*a*b^2*d*f^2*x^2*Cosh[3*c + d*x] + 6*a^2*b*e*f*Cosh[2*c + 2*d*x] + 6*
b^3*e*f*Cosh[2*c + 2*d*x] + 12*a^2*b*d^2*e^2*x*Cosh[2*c + 2*d*x] + 6*a^2*b*
f^2*x*Cosh[2*c + 2*d*x] + 6*b^3*f^2*x*Cosh[2*c + 2*d*x] + 12*a^2*b*d^2*e*f*
x^2*Cosh[2*c + 2*d*x] + 4*a^2*b*d^2*f^2*x^3*Cosh[2*c + 2*d*x] - 6*a^2*b*d*e
^2*Sinh[2*c] - 6*b^3*d*e^2*Sinh[2*c] - 12*a^2*b*d*e*f*x*Sinh[2*c] - 12*b^3*
d*e*f*x*Sinh[2*c] - 6*a^2*b*d*f^2*x^2*Sinh[2*c] - 6*b^3*d*f^2*x^2*Sinh[2*c]
 - 6*a^3*e*f*Sinh[c - d*x] - 6*a*b^2*e*f*Sinh[c - d*x] - 6*a^3*f^2*x*Sinh[c
 - d*x] - 6*a*b^2*f^2*x*Sinh[c - d*x] - 6*a^3*e*f*Sinh[3*c + d*x] - 6*a*b^2
*e*f*Sinh[3*c + d*x] - 6*a^3*f^2*x*Sinh[3*c + d*x] - 6*a*b^2*f^2*x*Sinh[3*c
 + d*x]))/(24*(a^2 + b^2)^2*d^2)
```

Maple [F] time = 0.448, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \operatorname{sech}\left(dx+c\right) \left(\tanh\left(dx+c\right)\right)^{2}}{a+b\sinh\left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] a^3*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*
d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b
^4*d^2), x) - a*b^2*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2
*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2
```

```
*a^2*b^2*d^2 + b^4*d^2), x) + 2*a^2*b*d^2*f^2*integrate(x^2/(a^4*d^2*e^(2*d
*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d
^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*a^3*d^2*e*f*integrate(x*e^(d*x + c)/(
a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x
+ 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a*b^2*d^2*e*f*integrate
(x*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b
^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 4*a^2*b*d
^2*e*f*integrate(x/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c)
 + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + a^2*b
*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/
((a^4 + 2*a^2*b^2 + b^4)*d^3)) + b^3*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b
^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) + (a^2*b
*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b^4)*d
) - a^2*b*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^3 - a*
b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) - (a*e^(-d*x - c) + 2
*b*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*e^(-2
*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d))*e^2 + 2*a^3*f^2*arctan(e^(d
*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3) + 2*a*b^2*f^2*arctan(e^(d*x + c))/((
a^4 + 2*a^2*b^2 + b^4)*d^3) - (2*b*f^2*x + 2*b*e*f + (a*d*f^2*x^2*e^(3*c) +
 2*a*e*f*e^(3*c) + 2*(d*e*f + f^2)*a*x*e^(3*c))*e^(3*d*x) + 2*(b*d*f^2*x^2*
e^(2*c) + b*e*f*e^(2*c) + (2*d*e*f + f^2)*b*x*e^(2*c))*e^(2*d*x) - (a*d*f^2
*x^2*e^c - 2*a*e*f*e^c + 2*(d*e*f - f^2)*a*x*e^c)*e^(d*x))/(a^2*d^2 + b^2*d
^2 + (a^2*d^2*e^(4*c) + b^2*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b
^2*d^2*e^(2*c))*e^(2*d*x)) - integrate(2*(a^2*b^2*f^2*x^2 + 2*a^2*b^2*e*f*x
 - (a^3*b*f^2*x^2*e^c + 2*a^3*b*e*f*x*e^c)*e^(d*x))/(a^4*b + 2*a^2*b^3 + b^
5 - (a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*e^
c + 2*a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x)
```

Fricas [C] time = 5.42971, size = 24496, normalized size = 19.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/2*(4*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*cosh(d*x + c)^4 + 4*((
a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*sinh(d*x + c)^4 - 4*(a^2*b + b^
3)*d*e*f + 4*(a^2*b + b^3)*c*f^2 - 2*((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 + a*
b^2)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^3 + a*b^2)*d^2*e*f + (a^3 + a*
b^2)*d*f^2)*x)*cosh(d*x + c)^3 - 2*((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 + a*b^
2)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^3 + a*b^2)*d^2*e*f + (a^3 + a*b^
2)*d*f^2)*x - 8*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*cosh(d*x + c)
)*sinh(d*x + c)^3 - 4*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 +
(a^2*b + b^3)*d*e*f - 2*(a^2*b + b^3)*c*f^2 + (2*(a^2*b + b^3)*d^2*e*f - (a
^2*b + b^3)*d*f^2)*x)*cosh(d*x + c)^2 - 2*(2*(a^2*b + b^3)*d^2*f^2*x^2 + 2*
(a^2*b + b^3)*d^2*e^2 + 2*(a^2*b + b^3)*d*e*f - 4*(a^2*b + b^3)*c*f^2 - 12*
((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*cosh(d*x + c)^2 + 2*(2*(a^2*b
 + b^3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x + 3*((a^3 + a*b^2)*d^2*f^2*x^2 + (
a^3 + a*b^2)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^3 + a*b^2)*d^2*e*f + (
a^3 + a*b^2)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*((a^3 + a*b^2)*d^
2*f^2*x^2 + (a^3 + a*b^2)*d^2*e^2 - 2*(a^3 + a*b^2)*d*e*f + 2*((a^3 + a*b^2
)*d^2*e*f - (a^3 + a*b^2)*d*f^2)*x)*cosh(d*x + c) + 4*(a^2*b*d*f^2*x + a^2*
b*d*e*f + (a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)^4 + 4*(a^2*b*d*f^2*x
+ a^2*b*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b*d*f^2*x + a^2*b*d*e*f
)*sinh(d*x + c)^4 + 2*(a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)^2 + 2*(a^
```

```
2*b*d*f^2*x + a^2*b*d*e*f + 3*(a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 4*((a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)^3 + (a^2
*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x +
 c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2) - b)/b + 1) + 4*(a^2*b*d*f^2*x + a^2*b*d*e*f + (a^2*b*d*f^2*x + a^2*
b*d*e*f)*cosh(d*x + c)^4 + 4*(a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^2*b*d*f^2*x + a^2*b*d*e*f)*sinh(d*x + c)^4 + 2*(a^2*b*d*
f^2*x + a^2*b*d*e*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*f^2*x + a^2*b*d*e*f + 3*(
a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^2*b*d
*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)^3 + (a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(
d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (4*a^2*b*d
*f^2*x + 4*a^2*b*d*e*f - 2*I*(a^3 - a*b^2)*d*f^2*x + (4*a^2*b*d*f^2*x + 4*a
^2*b*d*e*f - 2*I*(a^3 - a*b^2)*d*f^2*x - 2*I*(a^3 - a*b^2)*d*e*f)*cosh(d*x
+ c)^4 + (16*a^2*b*d*f^2*x + 16*a^2*b*d*e*f - 8*I*(a^3 - a*b^2)*d*f^2*x - 8
*I*(a^3 - a*b^2)*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (4*a^2*b*d*f^2*x +
4*a^2*b*d*e*f - 2*I*(a^3 - a*b^2)*d*f^2*x - 2*I*(a^3 - a*b^2)*d*e*f)*sinh(d
*x + c)^4 - 2*I*(a^3 - a*b^2)*d*e*f + (8*a^2*b*d*f^2*x + 8*a^2*b*d*e*f - 4*
I*(a^3 - a*b^2)*d*f^2*x - 4*I*(a^3 - a*b^2)*d*e*f)*cosh(d*x + c)^2 + (8*a^2
*b*d*f^2*x + 8*a^2*b*d*e*f - 4*I*(a^3 - a*b^2)*d*f^2*x - 4*I*(a^3 - a*b^2)*
d*e*f + (24*a^2*b*d*f^2*x + 24*a^2*b*d*e*f - 12*I*(a^3 - a*b^2)*d*f^2*x - 1
2*I*(a^3 - a*b^2)*d*e*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((16*a^2*b*d*f^
2*x + 16*a^2*b*d*e*f - 8*I*(a^3 - a*b^2)*d*f^2*x - 8*I*(a^3 - a*b^2)*d*e*f)
*cosh(d*x + c)^3 + (16*a^2*b*d*f^2*x + 16*a^2*b*d*e*f - 8*I*(a^3 - a*b^2)*d
*f^2*x - 8*I*(a^3 - a*b^2)*d*e*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(I*cos
h(d*x + c) + I*sinh(d*x + c)) - (4*a^2*b*d*f^2*x + 4*a^2*b*d*e*f + 2*I*(a^3
 - a*b^2)*d*f^2*x + (4*a^2*b*d*f^2*x + 4*a^2*b*d*e*f + 2*I*(a^3 - a*b^2)*d*
f^2*x + 2*I*(a^3 - a*b^2)*d*e*f)*cosh(d*x + c)^4 + (16*a^2*b*d*f^2*x + 16*a
^2*b*d*e*f + 8*I*(a^3 - a*b^2)*d*f^2*x + 8*I*(a^3 - a*b^2)*d*e*f)*cosh(d*x
+ c)*sinh(d*x + c)^3 + (4*a^2*b*d*f^2*x + 4*a^2*b*d*e*f + 2*I*(a^3 - a*b^2)
*d*f^2*x + 2*I*(a^3 - a*b^2)*d*e*f)*sinh(d*x + c)^4 + 2*I*(a^3 - a*b^2)*d*e
*f + (8*a^2*b*d*f^2*x + 8*a^2*b*d*e*f + 4*I*(a^3 - a*b^2)*d*f^2*x + 4*I*(a^
3 - a*b^2)*d*e*f)*cosh(d*x + c)^2 + (8*a^2*b*d*f^2*x + 8*a^2*b*d*e*f + 4*I*
(a^3 - a*b^2)*d*f^2*x + 4*I*(a^3 - a*b^2)*d*e*f + (24*a^2*b*d*f^2*x + 24*a^
2*b*d*e*f + 12*I*(a^3 - a*b^2)*d*f^2*x + 12*I*(a^3 - a*b^2)*d*e*f)*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 + ((16*a^2*b*d*f^2*x + 16*a^2*b*d*e*f + 8*I*(a^3 -
 a*b^2)*d*f^2*x + 8*I*(a^3 - a*b^2)*d*e*f)*cosh(d*x + c)^3 + (16*a^2*b*d*f^
2*x + 16*a^2*b*d*e*f + 8*I*(a^3 - a*b^2)*d*f^2*x + 8*I*(a^3 - a*b^2)*d*e*f)
*cosh(d*x + c))*sinh(d*x + c))*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) +
2*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2 + (a^2*b*d^2*e^2 - 2*a^2
*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c)^4 + 4*(a^2*b*d^2*e^2 - 2*a^2*b*c*
d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b*d^2*e^2 - 2*a
^2*b*c*d*e*f + a^2*b*c^2*f^2)*sinh(d*x + c)^4 + 2*(a^2*b*d^2*e^2 - 2*a^2*b*
c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c)^2 + 2*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e
*f + a^2*b*c^2*f^2 + 3*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*co
sh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*
b*c^2*f^2)*cosh(d*x + c)^3 + (a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f
^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c)
 + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f +
a^2*b*c^2*f^2 + (a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x
+ c)^4 + 4*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c)*
sinh(d*x + c)^3 + (a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*sinh(d*
x + c)^4 + 2*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c
)^2 + 2*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2 + 3*(a^2*b*d^2*e^2
 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((
a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c)^3 + (a^2*b*d
^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log
(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) +
 2*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2
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 + (a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2
)*cosh(d*x + c)^4 + 4*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*
e*f - a^2*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b*d^2*f^2*x^2 + 2
*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*sinh(d*x + c)^4 + 2*(a^
2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*cosh
(d*x + c)^2 + 2*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f -
a^2*b*c^2*f^2 + 3*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f
- a^2*b*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^2*b*d^2*f^2*x^2 +
 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*cosh(d*x + c)^3 + (a^
2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*cosh
(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(a^2*b*d^2*f
^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2 + (a^2*b*d^2*f
^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*cosh(d*x + c)
^4 + 4*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2
*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*
x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*sinh(d*x + c)^4 + 2*(a^2*b*d^2*f^2*x^2
 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*cosh(d*x + c)^2 + 2
*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2 +
 3*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2
)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*e*
f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*cosh(d*x + c)^3 + (a^2*b*d^2*f^2*x^2
 + 2*a^2*b*d^2*e*f*x + 2*a^2*b*c*d*e*f - a^2*b*c^2*f^2)*cosh(d*x + c))*sinh
(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*s
inh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (2*a^2*b*d^2*e^2 - 4*a^2*b*c*
d*e*f - I*(a^3 - a*b^2)*d^2*e^2 + 2*I*(a^3 - a*b^2)*c*d*e*f + (2*a^2*b*d^2*
e^2 - 4*a^2*b*c*d*e*f - I*(a^3 - a*b^2)*d^2*e^2 + 2*I*(a^3 - a*b^2)*c*d*e*f
 + 2*(a^2*b*c^2 + a^2*b + b^3)*f^2 - I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2
)*f^2)*cosh(d*x + c)^4 + (8*a^2*b*d^2*e^2 - 16*a^2*b*c*d*e*f - 4*I*(a^3 - a
*b^2)*d^2*e^2 + 8*I*(a^3 - a*b^2)*c*d*e*f + 8*(a^2*b*c^2 + a^2*b + b^3)*f^2
 - 4*I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2)*cosh(d*x + c)*sinh(d*x +
c)^3 + (2*a^2*b*d^2*e^2 - 4*a^2*b*c*d*e*f - I*(a^3 - a*b^2)*d^2*e^2 + 2*I*(
a^3 - a*b^2)*c*d*e*f + 2*(a^2*b*c^2 + a^2*b + b^3)*f^2 - I*(2*a^3 + 2*a*b^2
 + (a^3 - a*b^2)*c^2)*f^2)*sinh(d*x + c)^4 + 2*(a^2*b*c^2 + a^2*b + b^3)*f^
2 - I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2 + (4*a^2*b*d^2*e^2 - 8*a^2*
b*c*d*e*f - 2*I*(a^3 - a*b^2)*d^2*e^2 + 4*I*(a^3 - a*b^2)*c*d*e*f + 4*(a^2*
b*c^2 + a^2*b + b^3)*f^2 - 2*I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2)*c
osh(d*x + c)^2 + (4*a^2*b*d^2*e^2 - 8*a^2*b*c*d*e*f - 2*I*(a^3 - a*b^2)*d^2
*e^2 + 4*I*(a^3 - a*b^2)*c*d*e*f + 4*(a^2*b*c^2 + a^2*b + b^3)*f^2 - 2*I*(2
*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2 + (12*a^2*b*d^2*e^2 - 24*a^2*b*c*d*
e*f - 6*I*(a^3 - a*b^2)*d^2*e^2 + 12*I*(a^3 - a*b^2)*c*d*e*f + 12*(a^2*b*c^
2 + a^2*b + b^3)*f^2 - 6*I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2)*cosh(
d*x + c)^2)*sinh(d*x + c)^2 + ((8*a^2*b*d^2*e^2 - 16*a^2*b*c*d*e*f - 4*I*(a
^3 - a*b^2)*d^2*e^2 + 8*I*(a^3 - a*b^2)*c*d*e*f + 8*(a^2*b*c^2 + a^2*b + b^
3)*f^2 - 4*I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2)*cosh(d*x + c)^3 + (
8*a^2*b*d^2*e^2 - 16*a^2*b*c*d*e*f - 4*I*(a^3 - a*b^2)*d^2*e^2 + 8*I*(a^3 -
 a*b^2)*c*d*e*f + 8*(a^2*b*c^2 + a^2*b + b^3)*f^2 - 4*I*(2*a^3 + 2*a*b^2 +
(a^3 - a*b^2)*c^2)*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + s
inh(d*x + c) + I) - (2*a^2*b*d^2*e^2 - 4*a^2*b*c*d*e*f + I*(a^3 - a*b^2)*d^
2*e^2 - 2*I*(a^3 - a*b^2)*c*d*e*f + (2*a^2*b*d^2*e^2 - 4*a^2*b*c*d*e*f + I*
(a^3 - a*b^2)*d^2*e^2 - 2*I*(a^3 - a*b^2)*c*d*e*f + 2*(a^2*b*c^2 + a^2*b +
b^3)*f^2 + I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2)*cosh(d*x + c)^4 + (
8*a^2*b*d^2*e^2 - 16*a^2*b*c*d*e*f + 4*I*(a^3 - a*b^2)*d^2*e^2 - 8*I*(a^3 -
 a*b^2)*c*d*e*f + 8*(a^2*b*c^2 + a^2*b + b^3)*f^2 + 4*I*(2*a^3 + 2*a*b^2 +
(a^3 - a*b^2)*c^2)*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2*b*d^2*e^2 -
4*a^2*b*c*d*e*f + I*(a^3 - a*b^2)*d^2*e^2 - 2*I*(a^3 - a*b^2)*c*d*e*f + 2*(
a^2*b*c^2 + a^2*b + b^3)*f^2 + I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2)
*sinh(d*x + c)^4 + 2*(a^2*b*c^2 + a^2*b + b^3)*f^2 + I*(2*a^3 + 2*a*b^2 + (
a^3 - a*b^2)*c^2)*f^2 + (4*a^2*b*d^2*e^2 - 8*a^2*b*c*d*e*f + 2*I*(a^3 - a*b
```

^2)*d^2*e^2 - 4*I*(a^3 - a*b^2)*c*d*e*f + 4*(a^2*b*c^2 + a^2*b + b^3)*f^2 + 2*I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2)*cosh(d*x + c)^2 + (4*a^2*b*d^2*e^2 - 8*a^2*b*c*d*e*f + 2*I*(a^3 - a*b^2)*d^2*e^2 - 4*I*(a^3 - a*b^2)*c*d*e*f + 4*(a^2*b*c^2 + a^2*b + b^3)*f^2 + 2*I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2 + (12*a^2*b*d^2*e^2 - 24*a^2*b*c*d*e*f + 6*I*(a^3 - a*b^2)*d^2*e^2 - 12*I*(a^3 - a*b^2)*c*d*e*f + 12*(a^2*b*c^2 + a^2*b + b^3)*f^2 + 6*I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a^2*b*d^2*e^2 - 16*a^2*b*c*d*e*f + 4*I*(a^3 - a*b^2)*d^2*e^2 - 8*I*(a^3 - a*b^2)*c*d*e*f + 8*(a^2*b*c^2 + a^2*b + b^3)*f^2 + 4*I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2)*cosh(d*x + c)^3 + (8*a^2*b*d^2*e^2 - 16*a^2*b*c*d*e*f + 4*I*(a^3 - a*b^2)*d^2*e^2 - 8*I*(a^3 - a*b^2)*c*d*e*f + 8*(a^2*b*c^2 + a^2*b + b^3)*f^2 + 4*I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - I) - (2*a^2*b*d^2*f^2*x^2 + 4*a^2*b*d^2*e*f*x + 4*a^2*b*c*d*e*f - 2*a^2*b*c^2*f^2 + I*(a^3 - a*b^2)*d^2*f^2*x^2 + 2*I*(a^3 - a*b^2)*d^2*e*f*x + 2*I*(a^3 - a*b^2)*c*d*e*f - I*(a^3 - a*b^2)*c^2*f^2 + (2*a^2*b*d^2*f^2*x^2 + 4*a^2*b*d^2*e*f*x + 4*a^2*b*c*d*e*f - 2*a^2*b*c^2*f^2 + I*(a^3 - a*b^2)*d^2*f^2*x^2 + 2*I*(a^3 - a*b^2)*d^2*e*f*x + 2*I*(a^3 - a*b^2)*c*d*e*f - I*(a^3 - a*b^2)*c^2*f^2)*cosh(d*x + c)^4 + (8*a^2*b*d^2*f^2*x^2 + 16*a^2*b*d^2*e*f*x + 16*a^2*b*c*d*e*f - 8*a^2*b*c^2*f^2 + 4*I*(a^3 - a*b^2)*d^2*f^2*x^2 + 8*I*(a^3 - a*b^2)*d^2*e*f*x + 8*I*(a^3 - a*b^2)*c*d*e*f - 4*I*(a^3 - a*b^2)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2*b*d^2*f^2*x^2 + 4*a^2*b*d^2*e*f*x + 4*a^2*b*c*d*e*f - 2*a^2*b*c^2*f^2 + I*(a^3 - a*b^2)*d^2*f^2*x^2 + 2*I*(a^3 - a*b^2)*d^2*e*f*x + 2*I*(a^3 - a*b^2)*c*d*e*f - I*(a^3 - a*b^2)*c^2*f^2)*sinh(d*x + c)^4 + (4*a^2*b*d^2*f^2*x^2 + 8*a^2*b*d^2*e*f*x + 8*a^2*b*c*d*e*f - 4*a^2*b*c^2*f^2 + 2*I*(a^3 - a*b^2)*d^2*f^2*x^2 + 4*I*(a^3 - a*b^2)*d^2*e*f*x + 4*I*(a^3 - a*b^2)*c*d*e*f - 2*I*(a^3 - a*b^2)*c^2*f^2)*cosh(d*x + c)^2 + (4*a^2*b*d^2*f^2*x^2 + 8*a^2*b*d^2*e*f*x + 8*a^2*b*c*d*e*f - 4*a^2*b*c^2*f^2 + 2*I*(a^3 - a*b^2)*d^2*f^2*x^2 + 4*I*(a^3 - a*b^2)*d^2*e*f*x + 4*I*(a^3 - a*b^2)*c*d*e*f - 2*I*(a^3 - a*b^2)*c^2*f^2 + (12*a^2*b*d^2*f^2*x^2 + 24*a^2*b*d^2*e*f*x + 24*a^2*b*c*d*e*f - 12*a^2*b*c^2*f^2 + 6*I*(a^3 - a*b^2)*d^2*f^2*x^2 + 12*I*(a^3 - a*b^2)*d^2*e*f*x + 12*I*(a^3 - a*b^2)*c*d*e*f - 6*I*(a^3 - a*b^2)*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a^2*b*d^2*f^2*x^2 + 16*a^2*b*d^2*e*f*x + 16*a^2*b*c*d*e*f - 8*a^2*b*c^2*f^2 + 4*I*(a^3 - a*b^2)*d^2*f^2*x^2 + 8*I*(a^3 - a*b^2)*d^2*e*f*x + 8*I*(a^3 - a*b^2)*c*d*e*f - 4*I*(a^3 - a*b^2)*c^2*f^2)*cosh(d*x + c)^3 + (8*a^2*b*d^2*f^2*x^2 + 16*a^2*b*d^2*e*f*x + 16*a^2*b*c*d*e*f - 8*a^2*b*c^2*f^2 + 4*I*(a^3 - a*b^2)*d^2*f^2*x^2 + 8*I*(a^3 - a*b^2)*d^2*e*f*x + 8*I*(a^3 - a*b^2)*c*d*e*f - 4*I*(a^3 - a*b^2)*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) - (2*a^2*b*d^2*f^2*x^2 + 4*a^2*b*d^2*e*f*x + 4*a^2*b*c*d*e*f - 2*a^2*b*c^2*f^2 - I*(a^3 - a*b^2)*d^2*f^2*x^2 - 2*I*(a^3 - a*b^2)*d^2*e*f*x - 2*I*(a^3 - a*b^2)*c*d*e*f + I*(a^3 - a*b^2)*c^2*f^2 + (2*a^2*b*d^2*f^2*x^2 + 4*a^2*b*d^2*e*f*x + 4*a^2*b*c*d*e*f - 2*a^2*b*c^2*f^2 - I*(a^3 - a*b^2)*d^2*f^2*x^2 - 2*I*(a^3 - a*b^2)*d^2*e*f*x - 2*I*(a^3 - a*b^2)*c*d*e*f + I*(a^3 - a*b^2)*c^2*f^2)*cosh(d*x + c)^4 + (8*a^2*b*d^2*f^2*x^2 + 16*a^2*b*d^2*e*f*x + 16*a^2*b*c*d*e*f - 8*a^2*b*c^2*f^2 - 4*I*(a^3 - a*b^2)*d^2*f^2*x^2 - 8*I*(a^3 - a*b^2)*d^2*e*f*x - 8*I*(a^3 - a*b^2)*c*d*e*f + 4*I*(a^3 - a*b^2)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2*b*d^2*f^2*x^2 + 4*a^2*b*d^2*e*f*x + 4*a^2*b*c*d*e*f - 2*a^2*b*c^2*f^2 - I*(a^3 - a*b^2)*d^2*f^2*x^2 - 2*I*(a^3 - a*b^2)*d^2*e*f*x - 2*I*(a^3 - a*b^2)*c*d*e*f + I*(a^3 - a*b^2)*c^2*f^2)*sinh(d*x + c)^4 + (4*a^2*b*d^2*f^2*x^2 + 8*a^2*b*d^2*e*f*x + 8*a^2*b*c*d*e*f - 4*a^2*b*c^2*f^2 - 2*I*(a^3 - a*b^2)*d^2*f^2*x^2 - 4*I*(a^3 - a*b^2)*d^2*e*f*x - 4*I*(a^3 - a*b^2)*c*d*e*f + 2*I*(a^3 - a*b^2)*c^2*f^2)*cosh(d*x + c)^2 + (4*a^2*b*d^2*f^2*x^2 + 8*a^2*b*d^2*e*f*x + 8*a^2*b*c*d*e*f - 4*a^2*b*c^2*f^2 - 2*I*(a^3 - a*b^2)*d^2*f^2*x^2 - 4*I*(a^3 - a*b^2)*d^2*e*f*x - 4*I*(a^3 - a*b^2)*c*d*e*f + 2*I*(a^3 - a*b^2)*c^2*f^2 + (12*a^2*b*d^2*f^2*x^2 + 24*a^2*b*d^2*e*f*x + 24*a^2*b*c*d*e*f - 12*a^2*b*c^2*f^2 - 6*I*(a^3 - a*b^2)*d^2*f^2*x^2 - 12*I*(a^3 - a*b^2)*d^2*e*f*x - 12*I*(a^3 - a*b^2)*c*d*e*f + 6*I*(a^3 - a*b^2)*c^2*f^2)*cosh(d*x + c)^2)*si

```
nh(d*x + c)^2 + ((8*a^2*b*d^2*f^2*x^2 + 16*a^2*b*d^2*e*f*x + 16*a^2*b*c*d*e
*f - 8*a^2*b*c^2*f^2 - 4*I*(a^3 - a*b^2)*d^2*f^2*x^2 - 8*I*(a^3 - a*b^2)*d^
2*e*f*x - 8*I*(a^3 - a*b^2)*c*d*e*f + 4*I*(a^3 - a*b^2)*c^2*f^2)*cosh(d*x +
 c)^3 + (8*a^2*b*d^2*f^2*x^2 + 16*a^2*b*d^2*e*f*x + 16*a^2*b*c*d*e*f - 8*a^
2*b*c^2*f^2 - 4*I*(a^3 - a*b^2)*d^2*f^2*x^2 - 8*I*(a^3 - a*b^2)*d^2*e*f*x -
 8*I*(a^3 - a*b^2)*c*d*e*f + 4*I*(a^3 - a*b^2)*c^2*f^2)*cosh(d*x + c))*sinh
(d*x + c))*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1) - 4*(a^2*b*f^2*cosh(
d*x + c)^4 + 4*a^2*b*f^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*b*f^2*sinh(d*x
 + c)^4 + 2*a^2*b*f^2*cosh(d*x + c)^2 + a^2*b*f^2 + 2*(3*a^2*b*f^2*cosh(d*x
 + c)^2 + a^2*b*f^2)*sinh(d*x + c)^2 + 4*(a^2*b*f^2*cosh(d*x + c)^3 + a^2*b
*f^2*cosh(d*x + c))*sinh(d*x + c))*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x
 + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 4*(
a^2*b*f^2*cosh(d*x + c)^4 + 4*a^2*b*f^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2
*b*f^2*sinh(d*x + c)^4 + 2*a^2*b*f^2*cosh(d*x + c)^2 + a^2*b*f^2 + 2*(3*a^2
*b*f^2*cosh(d*x + c)^2 + a^2*b*f^2)*sinh(d*x + c)^2 + 4*(a^2*b*f^2*cosh(d*x
 + c)^3 + a^2*b*f^2*cosh(d*x + c))*sinh(d*x + c))*polylog(3, (a*cosh(d*x +
c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2))/b) + (4*a^2*b*f^2 + 2*(2*a^2*b*f^2 - I*(a^3 - a*b^2)*f^2)*cosh(d*x +
 c)^4 + 8*(2*a^2*b*f^2 - I*(a^3 - a*b^2)*f^2)*cosh(d*x + c)*sinh(d*x + c)^3
 + 2*(2*a^2*b*f^2 - I*(a^3 - a*b^2)*f^2)*sinh(d*x + c)^4 - 2*I*(a^3 - a*b^2
)*f^2 + 4*(2*a^2*b*f^2 - I*(a^3 - a*b^2)*f^2)*cosh(d*x + c)^2 + 4*(2*a^2*b*
f^2 - I*(a^3 - a*b^2)*f^2 + 3*(2*a^2*b*f^2 - I*(a^3 - a*b^2)*f^2)*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 + 8*((2*a^2*b*f^2 - I*(a^3 - a*b^2)*f^2)*cosh(d*x +
 c)^3 + (2*a^2*b*f^2 - I*(a^3 - a*b^2)*f^2)*cosh(d*x + c))*sinh(d*x + c))*p
olylog(3, I*cosh(d*x + c) + I*sinh(d*x + c)) + 2*(2*a^2*b*f^2 + (2*a^2*b*f^
2 + I*(a^3 - a*b^2)*f^2)*cosh(d*x + c)^4 + 4*(2*a^2*b*f^2 + I*(a^3 - a*b^2)
*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2*b*f^2 + I*(a^3 - a*b^2)*f^2)*s
inh(d*x + c)^4 + I*(a^3 - a*b^2)*f^2 + 2*(2*a^2*b*f^2 + I*(a^3 - a*b^2)*f^2
)*cosh(d*x + c)^2 + 2*(2*a^2*b*f^2 + I*(a^3 - a*b^2)*f^2 + 3*(2*a^2*b*f^2 +
 I*(a^3 - a*b^2)*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((2*a^2*b*f^2 +
I*(a^3 - a*b^2)*f^2)*cosh(d*x + c)^3 + (2*a^2*b*f^2 + I*(a^3 - a*b^2)*f^2)*
cosh(d*x + c))*sinh(d*x + c))*polylog(3, -I*cosh(d*x + c) - I*sinh(d*x + c)
) + 2*((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 + a*b^2)*d^2*e^2 - 2*(a^3 + a*b^2)*
d*e*f + 8*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*cosh(d*x + c)^3 - 3
*((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 + a*b^2)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f
 + 2*((a^3 + a*b^2)*d^2*e*f + (a^3 + a*b^2)*d*f^2)*x)*cosh(d*x + c)^2 + 2*(
(a^3 + a*b^2)*d^2*e*f - (a^3 + a*b^2)*d*f^2)*x - 4*((a^2*b + b^3)*d^2*f^2*x
^2 + (a^2*b + b^3)*d^2*e^2 + (a^2*b + b^3)*d*e*f - 2*(a^2*b + b^3)*c*f^2 +
(2*(a^2*b + b^3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x
+ c))/((a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b
^4)*d^3*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d^3*sinh(d*
x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2
 + b^4)*d^3 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c)^2 + (a^4 + 2*a
^2*b^2 + b^4)*d^3)*sinh(d*x + c)^2 + 4*((a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*
x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sech(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)

[Out] Timed out
```

<u>Giac</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.388 $\int \frac{(e+fx)\mathbf{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=760

$$-\frac{ia^3f\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2b^2d^2\left(a^2+b^2\right)}-\frac{ia^3f\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)^2}+\frac{ia^3f\text{PolyLog}\left(2,ie^{c+dx}\right)}{2b^2d^2\left(a^2+b^2\right)}+\frac{ia^3f\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)^2}+\frac{a^2bf}{}$$

```
[Out] -((a*(e + f*x)*ArcTan[E^(c + d*x)])/(b^2*d)) + (2*a^3*(e + f*x)*ArcTan[E^(c
 + d*x)])/((a^2 + b^2)^2*d) + (a^3*(e + f*x)*ArcTan[E^(c + d*x)])/(b^2*(a^2
 + b^2)*d) + (a^2*b*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
])/((a^2 + b^2)^2*d) + (a^2*b*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/((a^2 + b^2)^2*d) - (a^2*b*(e + f*x)*Log[1 + E^(2*(c + d*x))])
/((a^2 + b^2)^2*d) + ((I/2)*a*f*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*d^2) - (
I*a^3*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - ((I/2)*a^3*f*Po
lyLog[2, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) - ((I/2)*a*f*PolyLog[2, I
*E^(c + d*x)])/(b^2*d^2) + (I*a^3*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)
^2*d^2) + ((I/2)*a^3*f*PolyLog[2, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) + (
a^2*b*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^
2*d^2) + (a^2*b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a
^2 + b^2)^2*d^2) - (a^2*b*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*
d^2) - (a*f*Sech[c + d*x])/(2*b^2*d^2) + (a^3*f*Sech[c + d*x])/(2*b^2*(a^2
+ b^2)*d^2) - ((e + f*x)*Sech[c + d*x]^2)/(2*b*d) + (a^2*(e + f*x)*Sech[c +
 d*x]^2)/(2*b*(a^2 + b^2)*d) + (f*Tanh[c + d*x])/(2*b*d^2) - (a^2*f*Tanh[c
+ d*x])/(2*b*(a^2 + b^2)*d^2) - (a*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(
2*b^2*d) + (a^3*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*b^2*(a^2 + b^2)*d
)
```

Rubi [A] time = 1.14808, antiderivative size = 760, normalized size of antiderivative = 1., number of steps used = 42, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.406, Rules used = {5583, 5451, 3767, 8, 4185, 4180, 2279, 2391, 5573, 5561, 2190, 6742, 3718}

$$-\frac{ia^3f\text{PolyLog}\left(2,-ie^{c+dx}\right)}{2b^2d^2\left(a^2+b^2\right)}-\frac{ia^3f\text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)^2}+\frac{ia^3f\text{PolyLog}\left(2,ie^{c+dx}\right)}{2b^2d^2\left(a^2+b^2\right)}+\frac{ia^3f\text{PolyLog}\left(2,ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)^2}+\frac{a^2bf}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -((a*(e + f*x)*ArcTan[E^(c + d*x)])/(b^2*d)) + (2*a^3*(e + f*x)*ArcTan[E^(c
 + d*x)])/((a^2 + b^2)^2*d) + (a^3*(e + f*x)*ArcTan[E^(c + d*x)])/(b^2*(a^2
 + b^2)*d) + (a^2*b*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
])/((a^2 + b^2)^2*d) + (a^2*b*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/((a^2 + b^2)^2*d) - (a^2*b*(e + f*x)*Log[1 + E^(2*(c + d*x))])
/((a^2 + b^2)^2*d) + ((I/2)*a*f*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*d^2) - (
I*a^3*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - ((I/2)*a^3*f*Po
lyLog[2, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) - ((I/2)*a*f*PolyLog[2, I
*E^(c + d*x)])/(b^2*d^2) + (I*a^3*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)
^2*d^2) + ((I/2)*a^3*f*PolyLog[2, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) + (
a^2*b*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^
2*d^2) + (a^2*b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a
^2 + b^2)^2*d^2) - (a^2*b*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*
d^2) - (a*f*Sech[c + d*x])/(2*b^2*d^2) + (a^3*f*Sech[c + d*x])/(2*b^2*(a^2
+ b^2)*d^2) - ((e + f*x)*Sech[c + d*x]^2)/(2*b*d) + (a^2*(e + f*x)*Sech[c +
```

```
 d*x]^2)/(2*b*(a^2 + b^2)*d) + (f*Tanh[c + d*x])/(2*b*d^2) - (a^2*f*Tanh[c
+ d*x])/(2*b*(a^2 + b^2)*d^2) - (a*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(
2*b^2*d) + (a^3*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*b^2*(a^2 + b^2)*d
)
```

Rule 5583

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\text{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{sech}^2(c+dx)\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\text{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)\text{sech}^2(c+dx)}{2bd} - \frac{a\int (e+fx)\text{sech}^3(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)\text{sech}^3(c+}{a+b\sinh(c+dx}}{b^2}$$

$$= -\frac{af\text{sech}(c+dx)}{2b^2d^2} - \frac{(e+fx)\text{sech}^2(c+dx)}{2bd} - \frac{a(e+fx)\text{sech}(c+dx)\tanh(c}{2b^2d}$$

$$= -\frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} - \frac{af\text{sech}(c+dx)}{2b^2d^2} - \frac{(e+fx)\text{sech}^2(c+dx)}{2bd} + f\,\text{ta}$$

$$= -\frac{a^2b(e+fx)^2}{2\left(a^2+b^2\right)^2f} - \frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} - \frac{af\text{sech}(c+dx)}{2b^2d^2} - \frac{(e+fx)\text{sech}}{2bd}$$

$$= -\frac{a^2b(e+fx)^2}{2\left(a^2+b^2\right)^2f} - \frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{a^2b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2d}$$

$$= -\frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2d} + \frac{a^3(e+fx)\tan^{-1}\left(e^{c}\right.}{b^2\left(a^2+b^2\right)d}$$

$$= -\frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2d} + \frac{a^3(e+fx)\tan^{-1}\left(e^{c}\right.}{b^2\left(a^2+b^2\right)d}$$

$$= -\frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2d} + \frac{a^3(e+fx)\tan^{-1}\left(e^{c}\right.}{b^2\left(a^2+b^2\right)d}$$

$$= -\frac{a(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d} + \frac{2a^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2d} + \frac{a^3(e+fx)\tan^{-1}\left(e^{c}\right.}{b^2\left(a^2+b^2\right)d}$$

Mathematica [A] time = 9.50031, size = 588, normalized size = 0.77

$$a\left(-if\left(a^2-b^2\right)\text{PolyLog}\left(2,-ie^{c+dx}\right)+if\left(a^2-b^2\right)\text{PolyLog}\left(2,ie^{c+dx}\right)-abf\text{PolyLog}\left(2,-e^{2(c+dx)}\right)+2a^2de\tan^{-1}\left(e^{c+dx}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (2*a^2*b*(-(f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
 + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLog[
2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2]))]) + a*(2*a*b*d*e*(c + d*x) - 2*a*b*c*f*(c + d*x) +
a*b*f*(c + d*x)^2 + 2*a^2*d*e*ArcTan[E^(c + d*x)] - 2*b^2*d*e*ArcTan[E^(c +
 d*x)] - 2*a^2*c*f*ArcTan[E^(c + d*x)] + 2*b^2*c*f*ArcTan[E^(c + d*x)] + I*
a^2*f*(c + d*x)*Log[1 - I*E^(c + d*x)] - I*b^2*f*(c + d*x)*Log[1 - I*E^(c +
 d*x)] - I*a^2*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + I*b^2*f*(c + d*x)*Log[1
 + I*E^(c + d*x)] - 2*a*b*d*e*Log[1 + E^(2*(c + d*x))] + 2*a*b*c*f*Log[1 +
E^(2*(c + d*x))] - 2*a*b*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] - I*(a^2 - b^
```

```
2)*f*PolyLog[2, (-I)*E^(c + d*x)] + I*(a^2 - b^2)*f*PolyLog[2, I*E^(c + d*x
)] - a*b*f*PolyLog[2, -E^(2*(c + d*x))]) - (a^2 + b^2)*d*(e + f*x)*Sech[c +
 d*x]^2*(b + a*Sinh[c + d*x]) + (a^2 + b^2)*f*Sech[c + d*x]*(-a + b*Sinh[c
+ d*x]))/(2*(a^2 + b^2)^2*d^2)
```

Maple [B] time = 0.174, size = 2068, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] 2/(a^2+b^2)/d*a^3*e/(2*a^2+2*b^2)*arctan(exp(d*x+c))+I/(a^2+b^2)/d^2*a^3*f/
(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))-2/(a^2+b^2)/d^2*a^3*f*c/(2*a^2+2*b^2)*a
rctan(exp(d*x+c))-I/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))
+I/(a^2+b^2)*a/d^2*b^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c+I/(a^2+b^2)*a/d
*b^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x-I/(a^2+b^2)*a/d*b^2*f/(2*a^2+2*b^
2)*ln(1-I*exp(d*x+c))*x-1/(a^2+b^2)^(3/2)/d^2*a*b^3*f*c/(2*a^2+2*b^2)*arcta
nh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)^(3/2)/d^2*a^3*b*f*
c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^
2)^(1/2)/d^2*a*b*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^
2)^(1/2))-I/(a^2+b^2)*a/d^2*b^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c-I/(a^2
+b^2)*a/d^2*b^2*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))+2/(a^2+b^2)*a^2/d*b*f
/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x
+2/(a^2+b^2)*a^2/d^2*b*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)
/(-a+(a^2+b^2)^(1/2)))*c-2/(a^2+b^2)*a^2/d^2*b*f*c/(2*a^2+2*b^2)*ln(b*exp(2
*d*x+2*c)+2*a*exp(d*x+c)-b)+2/(a^2+b^2)*a^2/d^2*b*f*c/(2*a^2+2*b^2)*ln(1+ex
p(2*d*x+2*c))+I/(a^2+b^2)*a/d^2*b^2*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))-2
/(a^2+b^2)*a^2/d*b*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x-2/(a^2+b^2)*a^2/d^2
*b*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c-2/(a^2+b^2)*a^2/d^2*b*f/(2*a^2+2*b^
2)*ln(1+I*exp(d*x+c))*c-I/(a^2+b^2)/d*a^3*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c)
)*x-I/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c+2/(a^2+b^2)*a^
2/d*b*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2
)))*x+1/(a^2+b^2)^(3/2)/d*a*b^3*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)
+2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)^(3/2)/d*a^3*b*e/(2*a^2+2*b^2)*arctanh(1/
2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)*a^2/d^2*b*f/(2*a^2+2*b^
2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/(a^2+b^2)/d
^2*a*b^2*f*c/(2*a^2+2*b^2)*arctan(exp(d*x+c))-2/(a^2+b^2)*a^2/d*b*f/(2*a^2+
2*b^2)*ln(1+I*exp(d*x+c))*x+I/(a^2+b^2)/d*a^3*f/(2*a^2+2*b^2)*ln(1-I*exp(d*
x+c))*x+I/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c-1/(a^2+b^2
)^(1/2)/d*a*b*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1
/2))-2/(a^2+b^2)/d*a*b^2*e/(2*a^2+2*b^2)*arctan(exp(d*x+c))+2/(a^2+b^2)*a^2
/d^2*b*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2
)^(1/2)))-2/(a^2+b^2)*a^2/d^2*b*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))-2/(a^
2+b^2)*a^2/d^2*b*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))+2/(a^2+b^2)*a^2/d^2*
b*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)
))-2/(a^2+b^2)*a^2/d*b*e/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))+2/(a^2+b^2)*a^2
/d*b*e/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-(a*d*f*x*exp(3*d
*x+3*c)+a*d*e*exp(3*d*x+3*c)+2*b*d*f*x*exp(2*d*x+2*c)-a*d*f*x*exp(d*x+c)+a*
f*exp(3*d*x+3*c)+2*b*d*e*exp(2*d*x+2*c)-a*d*e*exp(d*x+c)+b*f*exp(2*d*x+2*c)
+a*f*exp(d*x+c)+b*f)/d^2/(a^2+b^2)/(1+exp(2*d*x+2*c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(\frac{a^2 b \log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{\left(a^4 + 2\,a^2b^2 + b^4\right)d} - \frac{a^2 b \log\left(e^{(-2\,dx-2\,c)} + 1\right)}{\left(a^4 + 2\,a^2b^2 + b^4\right)d} - \frac{\left(a^3 - ab^2\right)\arctan\left(e^{(-dx-c)}\right)}{\left(a^4 + 2\,a^2b^2 + b^4\right)d} - \frac{ae^{(-dx-c)} + 2}{\left(a^2 + b^2 + 2\left(a^2 + b^2\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] (a^2*b*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 +
b^4)*d) - a^2*b*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^
3 - a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) - (a*e^(-d*x -
c) + 2*b*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)
*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d))*e - f*(((a*d*x*e^(3*c
) + a*e^(3*c))*e^(3*d*x) + (2*b*d*x*e^(2*c) + b*e^(2*c))*e^(2*d*x) - (a*d*x
*e^c - a*e^c)*e^(d*x) + b)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) + b^2*d^2*
e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*x)) + 2*i
ntegrate(-(a^3*b*x*e^(d*x + c) - a^2*b^2*x)/(a^4*b + 2*a^2*b^3 + b^5 - (a^4
*b*e^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c + 2*a^
3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) - 2*integrate(1/2*(2*a^2*b*x + (a^3*e^c
 - a*b^2*e^c)*x*e^(d*x))/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(2*c) + 2*a^2*b^2*
e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x))
```

Fricas [B] time = 3.666, size = 11621, normalized size = 15.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -1/2*(2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3 + a*b^2)*f)*cosh(d*
x + c)^3 + 2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3 + a*b^2)*f)*si
nh(d*x + c)^3 + 2*(2*(a^2*b + b^3)*d*f*x + 2*(a^2*b + b^3)*d*e + (a^2*b + b
^3)*f)*cosh(d*x + c)^2 + 2*(2*(a^2*b + b^3)*d*f*x + 2*(a^2*b + b^3)*d*e + (
a^2*b + b^3)*f + 3*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3 + a*b^2)
*f)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(a^2*b + b^3)*f - 2*((a^3 + a*b^2)*d
*f*x + (a^3 + a*b^2)*d*e - (a^3 + a*b^2)*f)*cosh(d*x + c) - 2*(a^2*b*f*cosh
(d*x + c)^4 + 4*a^2*b*f*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*b*f*sinh(d*x +
c)^4 + 2*a^2*b*f*cosh(d*x + c)^2 + a^2*b*f + 2*(3*a^2*b*f*cosh(d*x + c)^2 +
 a^2*b*f)*sinh(d*x + c)^2 + 4*(a^2*b*f*cosh(d*x + c)^3 + a^2*b*f*cosh(d*x +
 c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(a^2*b*f*cosh
(d*x + c)^4 + 4*a^2*b*f*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*b*f*sinh(d*x +
c)^4 + 2*a^2*b*f*cosh(d*x + c)^2 + a^2*b*f + 2*(3*a^2*b*f*cosh(d*x + c)^2 +
 a^2*b*f)*sinh(d*x + c)^2 + 4*(a^2*b*f*cosh(d*x + c)^3 + a^2*b*f*cosh(d*x +
 c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + ((2*a^2*b*f - I
*(a^3 - a*b^2)*f)*cosh(d*x + c)^4 + 4*(2*a^2*b*f - I*(a^3 - a*b^2)*f)*cosh(
d*x + c)*sinh(d*x + c)^3 + (2*a^2*b*f - I*(a^3 - a*b^2)*f)*sinh(d*x + c)^4
+ 2*a^2*b*f + 2*(2*a^2*b*f - I*(a^3 - a*b^2)*f)*cosh(d*x + c)^2 + 2*(2*a^2*
b*f + 3*(2*a^2*b*f - I*(a^3 - a*b^2)*f)*cosh(d*x + c)^2 - I*(a^3 - a*b^2)*f
)*sinh(d*x + c)^2 - I*(a^3 - a*b^2)*f + 4*((2*a^2*b*f - I*(a^3 - a*b^2)*f)*
cosh(d*x + c)^3 + (2*a^2*b*f - I*(a^3 - a*b^2)*f)*cosh(d*x + c))*sinh(d*x +
```

c))*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + ((2*a^2*b*f + I*(a^3 - a*b^2)*f)*cosh(d*x + c)^4 + 4*(2*a^2*b*f + I*(a^3 - a*b^2)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2*b*f + I*(a^3 - a*b^2)*f)*sinh(d*x + c)^4 + 2*a^2*b*f + 2*(2*a^2*b*f + I*(a^3 - a*b^2)*f)*cosh(d*x + c)^2 + 2*(2*a^2*b*f + 3*(2*a^2*b*f + I*(a^3 - a*b^2)*f)*cosh(d*x + c)^2 + I*(a^3 - a*b^2)*f)*sinh(d*x + c)^2 + I*(a^3 - a*b^2)*f + 4*((2*a^2*b*f + I*(a^3 - a*b^2)*f)*cosh(d*x + c)^3 + (2*a^2*b*f + I*(a^3 - a*b^2)*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) - 2*(a^2*b*d*e - a^2*b*c*f + (a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c)^4 + 4*(a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b*d*e - a^2*b*c*f)*sinh(d*x + c)^4 + 2*(a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*e - a^2*b*c*f + 3*(a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c)^3 + (a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(a^2*b*d*e - a^2*b*c*f + (a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c)^4 + 4*(a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b*d*e - a^2*b*c*f)*sinh(d*x + c)^4 + 2*(a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*e - a^2*b*c*f + 3*(a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c)^3 + (a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(a^2*b*d*f*x + a^2*b*c*f + (a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^4 + 4*(a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b*d*f*x + a^2*b*c*f)*sinh(d*x + c)^4 + 2*(a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*f*x + a^2*b*c*f + 3*(a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^3 + (a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(a^2*b*d*f*x + a^2*b*c*f + (a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^4 + 4*(a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b*d*f*x + a^2*b*c*f)*sinh(d*x + c)^4 + 2*(a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*f*x + a^2*b*c*f + 3*(a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^3 + (a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (2*a^2*b*d*e - 2*a^2*b*c*f + (2*a^2*b*d*e - 2*a^2*b*c*f - I*(a^3 - a*b^2)*d*e + I*(a^3 - a*b^2)*c*f)*cosh(d*x + c)^4 + (8*a^2*b*d*e - 8*a^2*b*c*f - 4*I*(a^3 - a*b^2)*d*e + 4*I*(a^3 - a*b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2*b*d*e - 2*a^2*b*c*f - I*(a^3 - a*b^2)*d*e + I*(a^3 - a*b^2)*c*f)*sinh(d*x + c)^4 - I*(a^3 - a*b^2)*d*e + I*(a^3 - a*b^2)*c*f + (4*a^2*b*d*e - 4*a^2*b*c*f - 2*I*(a^3 - a*b^2)*d*e + 2*I*(a^3 - a*b^2)*c*f)*cosh(d*x + c)^2 + (4*a^2*b*d*e - 4*a^2*b*c*f - 2*I*(a^3 - a*b^2)*d*e + 2*I*(a^3 - a*b^2)*c*f + (12*a^2*b*d*e - 12*a^2*b*c*f - 6*I*(a^3 - a*b^2)*d*e + 6*I*(a^3 - a*b^2)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a^2*b*d*e - 8*a^2*b*c*f - 4*I*(a^3 - a*b^2)*d*e + 4*I*(a^3 - a*b^2)*c*f)*cosh(d*x + c)^3 + (8*a^2*b*d*e - 8*a^2*b*c*f - 4*I*(a^3 - a*b^2)*d*e + 4*I*(a^3 - a*b^2)*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + I) + (2*a^2*b*d*e - 2*a^2*b*c*f + (2*a^2*b*d*e - 2*a^2*b*c*f + I*(a^3 - a*b^2)*d*e - I*(a^3 - a*b^2)*c*f)*cosh(d*x + c)^4 + (8*a^2*b*d*e - 8*a^2*b*c*f + 4*I*(a^3 - a*b^2)*d*e - 4*I*(a^3 - a*b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2*b*d*e - 2*a^2*b*c*f + I*(a^3 - a*b^2)*d*e - I*(a^3 - a*b^2)*c*f)*sinh(d*x + c)^4 + I*(a^3 - a*b^2)*d*e - I*(a^3 - a*b^2)*c*f + (4*a^2*b*d*e - 4*a^2*b*c*f + 2*I*(a^3 - a*b^2)*d*e - 2*I*(a^3 - a*b^2)*c*f)*cosh(d*x + c)^2 + (4*a^2*b*d*e - 4*a^2*b*c*f + 2*I*(a^3 - a*b^2)*d*e - 2*I*(a^3 - a*b^2)*c*f + (12*a^2*b*d*e - 12*a^2*b*c*f + 6*I*(a^3 - a*b^2)*d*e - 6*I*(a^3 - a*b^2)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a^2*b*d*e - 8*a^2*b*c*f + 4*I*(a^3 - a*b^2)*d*e - 4*I*(a^3 - a*b^2)*c*f)*cosh(d*x + c)^3 + (8*a^2*b*d*e - 8*a^2*b*c*f + 4*I*(a^3 - a*b^2)*d*e - 4*I*(a^3 - a*b^2)*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - I) + (2*a^2*b*d*f*x + 2*a^2*b*c*f + (2*a^2*b*d*f*x + 2*a^2*b*c*f + I*(a^3 - a*b^

```
2)*d*f*x + I*(a^3 - a*b^2)*c*f)*cosh(d*x + c)^4 + (8*a^2*b*d*f*x + 8*a^2*b*
c*f + 4*I*(a^3 - a*b^2)*d*f*x + 4*I*(a^3 - a*b^2)*c*f)*cosh(d*x + c)*sinh(d
*x + c)^3 + (2*a^2*b*d*f*x + 2*a^2*b*c*f + I*(a^3 - a*b^2)*d*f*x + I*(a^3 -
 a*b^2)*c*f)*sinh(d*x + c)^4 + I*(a^3 - a*b^2)*d*f*x + I*(a^3 - a*b^2)*c*f
+ (4*a^2*b*d*f*x + 4*a^2*b*c*f + 2*I*(a^3 - a*b^2)*d*f*x + 2*I*(a^3 - a*b^2
)*c*f)*cosh(d*x + c)^2 + (4*a^2*b*d*f*x + 4*a^2*b*c*f + 2*I*(a^3 - a*b^2)*d
*f*x + 2*I*(a^3 - a*b^2)*c*f + (12*a^2*b*d*f*x + 12*a^2*b*c*f + 6*I*(a^3 -
a*b^2)*d*f*x + 6*I*(a^3 - a*b^2)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((
8*a^2*b*d*f*x + 8*a^2*b*c*f + 4*I*(a^3 - a*b^2)*d*f*x + 4*I*(a^3 - a*b^2)*c
*f)*cosh(d*x + c)^3 + (8*a^2*b*d*f*x + 8*a^2*b*c*f + 4*I*(a^3 - a*b^2)*d*f*
x + 4*I*(a^3 - a*b^2)*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(I*cosh(d*x + c
) + I*sinh(d*x + c) + 1) + (2*a^2*b*d*f*x + 2*a^2*b*c*f + (2*a^2*b*d*f*x +
2*a^2*b*c*f - I*(a^3 - a*b^2)*d*f*x - I*(a^3 - a*b^2)*c*f)*cosh(d*x + c)^4
+ (8*a^2*b*d*f*x + 8*a^2*b*c*f - 4*I*(a^3 - a*b^2)*d*f*x - 4*I*(a^3 - a*b^2
)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2*b*d*f*x + 2*a^2*b*c*f - I*(a^
3 - a*b^2)*d*f*x - I*(a^3 - a*b^2)*c*f)*sinh(d*x + c)^4 - I*(a^3 - a*b^2)*d
*f*x - I*(a^3 - a*b^2)*c*f + (4*a^2*b*d*f*x + 4*a^2*b*c*f - 2*I*(a^3 - a*b^
2)*d*f*x - 2*I*(a^3 - a*b^2)*c*f)*cosh(d*x + c)^2 + (4*a^2*b*d*f*x + 4*a^2*
b*c*f - 2*I*(a^3 - a*b^2)*d*f*x - 2*I*(a^3 - a*b^2)*c*f + (12*a^2*b*d*f*x +
 12*a^2*b*c*f - 6*I*(a^3 - a*b^2)*d*f*x - 6*I*(a^3 - a*b^2)*c*f)*cosh(d*x +
 c)^2)*sinh(d*x + c)^2 + ((8*a^2*b*d*f*x + 8*a^2*b*c*f - 4*I*(a^3 - a*b^2)*
d*f*x - 4*I*(a^3 - a*b^2)*c*f)*cosh(d*x + c)^3 + (8*a^2*b*d*f*x + 8*a^2*b*c
*f - 4*I*(a^3 - a*b^2)*d*f*x - 4*I*(a^3 - a*b^2)*c*f)*cosh(d*x + c))*sinh(d
*x + c))*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1) - 2*((a^3 + a*b^2)*d*f
*x + (a^3 + a*b^2)*d*e - 3*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3
+ a*b^2)*f)*cosh(d*x + c)^2 - (a^3 + a*b^2)*f - 2*(2*(a^2*b + b^3)*d*f*x +
2*(a^2*b + b^3)*d*e + (a^2*b + b^3)*f)*cosh(d*x + c))*sinh(d*x + c))/((a^4
+ 2*a^2*b^2 + b^4)*d^2*cosh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d^2*cosh
(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d^2*sinh(d*x + c)^4 + 2
*(a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2
+ 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4
)*d^2)*sinh(d*x + c)^2 + 4*((a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(d*x + c)^3 + (
a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Exception raised: RuntimeError
```

3.389 $\int \frac{\mathbf{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{a^2b\log(a+b\sinh(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{a\left(a^2-b^2\right)\tan^{-1}(\sinh(c+dx))}{2d\left(a^2+b^2\right)^2}-\frac{a^2b\log(\cosh(c+dx))}{d\left(a^2+b^2\right)^2}-\frac{\text{sech}^2(c+dx)(a\sinh(c+dx)}{2d\left(a^2+b^2\right)}$$

```
[Out] (a*(a^2 - b^2)*ArcTan[Sinh[c + d*x]])/(2*(a^2 + b^2)^2*d) - (a^2*b*Log[Cosh
[c + d*x]])/((a^2 + b^2)^2*d) + (a^2*b*Log[a + b*Sinh[c + d*x]])/((a^2 + b^
2)^2*d) - (Sech[c + d*x]^2*(b + a*Sinh[c + d*x]))/(2*(a^2 + b^2)*d)
```

Rubi [A] time = 0.229112, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2837, 12, 1647, 801, 635, 203, 260}

$$\frac{a^2b\log(a+b\sinh(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{a\left(a^2-b^2\right)\tan^{-1}(\sinh(c+dx))}{2d\left(a^2+b^2\right)^2}-\frac{a^2b\log(\cosh(c+dx))}{d\left(a^2+b^2\right)^2}-\frac{\text{sech}^2(c+dx)(a\sinh(c+dx)}{2d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(a^2 - b^2)*ArcTan[Sinh[c + d*x]])/(2*(a^2 + b^2)^2*d) - (a^2*b*Log[Cosh
[c + d*x]])/((a^2 + b^2)^2*d) + (a^2*b*Log[a + b*Sinh[c + d*x]])/((a^2 + b^
2)^2*d) - (Sech[c + d*x]^2*(b + a*Sinh[c + d*x]))/(2*(a^2 + b^2)*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{x^2}{b^2(a+x)\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= \frac{b\,\text{Subst}\left(\int \frac{x^2}{(a+x)\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2\left(a^2+b^2\right)d} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^2}{a^2+b^2}-\frac{ab^2x}{a^2+b^2}}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(c+dx)\right)}{2bd}$$

$$= -\frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2\left(a^2+b^2\right)d} - \frac{\text{Subst}\left(\int \left(-\frac{2a^2b^2}{\left(a^2+b^2\right)^2(a+x)} - \frac{ab^2\left(a^2-b^2-2ax\right)}{\left(a^2+b^2\right)^2\left(b^2+x^2\right)}\right) dx,\right.}{2bd}$$

$$= \frac{a^2b\log(a+b\sinh(c+dx))}{\left(a^2+b^2\right)^2 d} - \frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2\left(a^2+b^2\right)d} + \frac{(ab)\,\text{Subst}\left(\int \frac{a^2\cdots}{\cdots}\right.}{2\cdots}$$

$$= \frac{a^2b\log(a+b\sinh(c+dx))}{\left(a^2+b^2\right)^2 d} - \frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2\left(a^2+b^2\right)d} - \frac{\left(a^2b\right)\text{Subst}\left(\int \cdots\right.}{(\cdots}$$

$$= \frac{a\left(a^2-b^2\right)\tan^{-1}(\sinh(c+dx))}{2\left(a^2+b^2\right)^2 d} - \frac{a^2b\log(\cosh(c+dx))}{\left(a^2+b^2\right)^2 d} + \frac{a^2b\log(a+b\sinh(c+d\cdots}{\left(a^2+b^2\right)^2 d}$$

Mathematica [C] time = 0.327329, size = 130, normalized size = 1.07

$$-\frac{b\left(a^2+b^2\right)\operatorname{sech}^2(c+dx)+a\left(a^2+b^2\right)\tanh(c+dx)\operatorname{sech}(c+dx)+a\left(\left(a^2+b^2\right)\tan^{-1}(\sinh(c+dx))+a((b+ia)\log(-\cdots\right.}{2d\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(a*((a^2 + b^2)*ArcTan[Sinh[c + d*x]] + a*((I*a + b)*Log[I - Sinh[c + d*x]
] + ((-I)*a + b)*Log[I + Sinh[c + d*x]] - 2*b*Log[a + b*Sinh[c + d*x]])) +
b*(a^2 + b^2)*Sech[c + d*x]^2 + a*(a^2 + b^2)*Sech[c + d*x]*Tanh[c + d*x])/
(2*(a^2 + b^2)^2*d)
```

Maple [B] time = 0.003, size = 475, normalized size = 3.9

$$4\,\frac{a^2b\ln\left((\tanh(1/2\,dx+c/2))^2\,a-2\,\tanh(1/2\,dx+c/2)\,b-a\right)}{d\left(4\,a^4+8\,a^2b^2+4\,b^4\right)}+\frac{a^3}{d\left(a^4+2\,a^2b^2+b^4\right)}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tanh\left(\frac{dx}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] 4/d*a^2*b/(4*a^4+8*a^2*b^2+4*b^4)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x
+1/2*c)*b-a)+1/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d
*x+1/2*c)^3*a^3+1/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/
2*d*x+1/2*c)^3*a*b^2+2/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^2+1)^2*ta
nh(1/2*d*x+1/2*c)^2*a^2*b+2/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^2+1)
^2*tanh(1/2*d*x+1/2*c)^2*b^3-1/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^2
+1)^2*tanh(1/2*d*x+1/2*c)*a^3-1/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^
2+1)^2*tanh(1/2*d*x+1/2*c)*a*b^2+1/d/(a^4+2*a^2*b^2+b^4)*arctan(tanh(1/2*d*
x+1/2*c))*a^3-1/d/(a^4+2*a^2*b^2+b^4)*arctan(tanh(1/2*d*x+1/2*c))*a*b^2-1/d
/(a^4+2*a^2*b^2+b^4)*ln(tanh(1/2*d*x+1/2*c)^2+1)*a^2*b
```

Maxima [A] time = 1.77111, size = 296, normalized size = 2.45

$$\frac{a^2b\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{\left(a^4+2\,a^2b^2+b^4\right)d}-\frac{a^2b\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\left(a^4+2\,a^2b^2+b^4\right)d}-\frac{\left(a^3-ab^2\right)\arctan\left(e^{(-dx-c)}\right)}{\left(a^4+2\,a^2b^2+b^4\right)d}-\frac{ae^{(-dx-c)}+}{\left(a^2+b^2+2\left(a^2+b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] a^2*b*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b
^4)*d) - a^2*b*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^3
 - a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) - (a*e^(-d*x - c
) + 2*b*e^(-2*d*x - 2*c) - a*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*
e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d)
```

Fricas [B] time = 2.41718, size = 2241, normalized size = 18.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -((a^3 + a*b^2)*cosh(d*x + c)^3 + (a^3 + a*b^2)*sinh(d*x + c)^3 + 2*(a^2*b
+ b^3)*cosh(d*x + c)^2 + (2*a^2*b + 2*b^3 + 3*(a^3 + a*b^2)*cosh(d*x + c))*
sinh(d*x + c)^2 - ((a^3 - a*b^2)*cosh(d*x + c)^4 + 4*(a^3 - a*b^2)*cosh(d*x
 + c)*sinh(d*x + c)^3 + (a^3 - a*b^2)*sinh(d*x + c)^4 + a^3 - a*b^2 + 2*(a^
3 - a*b^2)*cosh(d*x + c)^2 + 2*(a^3 - a*b^2 + 3*(a^3 - a*b^2)*cosh(d*x + c)
^2)*sinh(d*x + c)^2 + 4*((a^3 - a*b^2)*cosh(d*x + c)^3 + (a^3 - a*b^2)*cosh
(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - (a^3 + a*
b^2)*cosh(d*x + c) - (a^2*b*cosh(d*x + c)^4 + 4*a^2*b*cosh(d*x + c)*sinh(d*
x + c)^3 + a^2*b*sinh(d*x + c)^4 + 2*a^2*b*cosh(d*x + c)^2 + a^2*b + 2*(3*a
^2*b*cosh(d*x + c)^2 + a^2*b)*sinh(d*x + c)^2 + 4*(a^2*b*cosh(d*x + c)^3 +
a^2*b*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x +
 c) - sinh(d*x + c))) + (a^2*b*cosh(d*x + c)^4 + 4*a^2*b*cosh(d*x + c)*sinh
(d*x + c)^3 + a^2*b*sinh(d*x + c)^4 + 2*a^2*b*cosh(d*x + c)^2 + a^2*b + 2*(
3*a^2*b*cosh(d*x + c)^2 + a^2*b)*sinh(d*x + c)^2 + 4*(a^2*b*cosh(d*x + c)^3
 + a^2*b*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) -
 sinh(d*x + c))) - (a^3 + a*b^2 - 3*(a^3 + a*b^2)*cosh(d*x + c)^2 - 4*(a^2*
b + b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x
+ c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 +
 2*a^2*b^2 + b^4)*d*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x
+ c)^2 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2
+ b^4)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d + 4*((a^4 + 2*a^2*b^2
 + b^4)*d*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c))*sinh(d
*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)**2*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A] time = 1.26063, size = 305, normalized size = 2.52

$$-\frac{\frac{a^2b\log\left(e^{(2dx+2c)}+1\right)}{a^4+2a^2b^2+b^4}-\frac{a^2b\log\left(\left|-be^{(2dx+2c)}-2ae^{(dx+c)}+b\right|\right)}{a^4+2a^2b^2+b^4}-\frac{\left(a^3e^c-ab^2e^c\right)\arctan\left(e^{(dx+c)}\right)e^{(-c)}}{a^4+2a^2b^2+b^4}+\frac{\left(a^3e^{(3c)}+ab^2e^{(3c)}\right)e^{(3dx)}+2\left(a^2be^{(2c)}+b^3e^{(2c)}\right)e^{(2dx)}-\left(a^3\right.}{\left(a^2+b^2\right)^2\left(e^{(2dx+2c)}+1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -(a^2*b*log(e^(2*d*x + 2*c) + 1)/(a^4 + 2*a^2*b^2 + b^4) - a^2*b*log(abs(-b
*e^(2*d*x + 2*c) - 2*a*e^(d*x + c) + b))/(a^4 + 2*a^2*b^2 + b^4) - (a^3*e^c
 - a*b^2*e^c)*arctan(e^(d*x + c))*e^(-c)/(a^4 + 2*a^2*b^2 + b^4) + ((a^3*e^
(3*c) + a*b^2*e^(3*c))*e^(3*d*x) + 2*(a^2*b*e^(2*c) + b^3*e^(2*c))*e^(2*d*x
) - (a^3*e^c + a*b^2*e^c)*e^(d*x))/((a^2 + b^2)^2*(e^(2*d*x + 2*c) + 1)^2))
/d
```

3.390 $\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{\tanh^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Sech[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [A] time = 0.0818747, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sech[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Sech[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{sech}(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Sech[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] $Aborted
```

Maple [A] time = 1.098, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(dx+c)\,(\tanh(dx+c))^2}{(fx+e)\,(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] (b*f - (a*d*f*x*e^(3*c) + (d*e - f)*a*e^(3*c))*e^(3*d*x) - (2*b*d*f*x*e^(2*
c) + (2*d*e - f)*b*e^(2*c))*e^(2*d*x) + (a*d*f*x*e^c + (d*e + f)*a*e^c)*e^(
d*x))/(a^2*d^2*e^2 + b^2*d^2*e^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*(a^2
*d^2*e*f + b^2*d^2*e*f)*x + (a^2*d^2*e^2*e^(4*c) + b^2*d^2*e^2*e^(4*c) + (a
^2*d^2*f^2*e^(4*c) + b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^2*d^2*e*f*e^(4*c) + b^
2*d^2*e*f*e^(4*c))*x)*e^(4*d*x) + 2*(a^2*d^2*e^2*e^(2*c) + b^2*d^2*e^2*e^(2
*c) + (a^2*d^2*f^2*e^(2*c) + b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^2*d^2*e*f*e^(2
*c) + b^2*d^2*e*f*e^(2*c))*x)*e^(2*d*x)) + 2*integrate(1/2*(2*a^2*b*d^2*f^2
*x^2 + 4*a^2*b*d^2*e*f*x + 2*b^3*f^2 + 2*(d^2*e^2 + f^2)*a^2*b + ((d^2*e^2
+ 2*f^2)*a^3*e^c - (d^2*e^2 - 2*f^2)*a*b^2*e^c + (a^3*d^2*f^2*e^c - a*b^2*d
^2*f^2*e^c)*x^2 + 2*(a^3*d^2*e*f*e^c - a*b^2*d^2*e*f*e^c)*x)*e^(d*x))/(a^4*
d^2*e^3 + 2*a^2*b^2*d^2*e^3 + b^4*d^2*e^3 + (a^4*d^2*f^3 + 2*a^2*b^2*d^2*f^
3 + b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*e*f^2 + 2*a^2*b^2*d^2*e*f^2 + b^4*d^2*e*f
^2)*x^2 + 3*(a^4*d^2*e^2*f + 2*a^2*b^2*d^2*e^2*f + b^4*d^2*e^2*f)*x + (a^4*
d^2*e^3*e^(2*c) + 2*a^2*b^2*d^2*e^3*e^(2*c) + b^4*d^2*e^3*e^(2*c) + (a^4*d^
2*f^3*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(2*c) + b^4*d^2*f^3*e^(2*c))*x^3 + 3*(a
^4*d^2*e*f^2*e^(2*c) + 2*a^2*b^2*d^2*e*f^2*e^(2*c) + b^4*d^2*e*f^2*e^(2*c))
*x^2 + 3*(a^4*d^2*e^2*f*e^(2*c) + 2*a^2*b^2*d^2*e^2*f*e^(2*c) + b^4*d^2*e^2
*f*e^(2*c))*x)*e^(2*d*x)), x) - 2*integrate(-(a^3*b*e^(d*x + c) - a^2*b^2)/
(a^4*b*e + 2*a^2*b^3*e + b^5*e + (a^4*b*f + 2*a^2*b^3*f + b^5*f)*x - (a^4*b
*e*e^(2*c) + 2*a^2*b^3*e*e^(2*c) + b^5*e*e^(2*c) + (a^4*b*f*e^(2*c) + 2*a^2
*b^3*f*e^(2*c) + b^5*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^5*e*e^c + 2*a^3*b^2*e*e
^c + a*b^4*e*e^c + (a^5*f*e^c + 2*a^3*b^2*f*e^c + a*b^4*f*e^c)*x)*e^(d*x)),
 x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)^2}{afx+ae+\left(bfx+be\right)\sinh(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(sech(d*x + c)*tanh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tanh^2(c+dx)\operatorname{sech}(c+dx)}{(a+b\sinh(c+dx))\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)**2*sech(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)),
x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.391 $\int \frac{(e+fx)^3 \cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)} dx$

Optimal. Leaf size=792

$$\frac{6a^3f^2(e+fx)\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^3}+\frac{6a^3f^2(e+fx)\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4d^3}-\frac{3a^3f(e+fx)^2\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}$$

```
[Out] (-3*a*f^3*x)/(8*b^2*d^3) - (a*(e + f*x)^3)/(4*b^2*d) + (a^3*(e + f*x)^4)/(4
*b^4*f) - (6*a^2*f^3*Cosh[c + d*x])/(b^3*d^4) + (14*f^3*Cosh[c + d*x])/(9*b
*d^4) - (3*a^2*f*(e + f*x)^2*Cosh[c + d*x])/(b^3*d^2) + (2*f*(e + f*x)^2*Co
sh[c + d*x])/(3*b*d^2) - (2*f^3*Cosh[c + d*x]^3)/(27*b*d^4) - (a^3*(e + f*x
)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a^3*(e + f*x
)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^4*d) - (3*a^3*f*(e +
 f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^2) - (
3*a^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
b^4*d^2) + (6*a^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2]))])/(b^4*d^3) + (6*a^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2]))])/(b^4*d^3) - (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/(b^4*d^4) - (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/(b^4*d^4) + (6*a^2*f^2*(e + f*x)*Sinh[c + d*x])/
(b^3*d^3) - (4*f^2*(e + f*x)*Sinh[c + d*x])/(3*b*d^3) + (a^2*(e + f*x)^3*Si
nh[c + d*x])/(b^3*d) + (3*a*f^3*Cosh[c + d*x]*Sinh[c + d*x])/(8*b^2*d^4) +
(3*a*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^2*d^2) - (3*a*f^2*(e +
 f*x)*Sinh[c + d*x]^2)/(4*b^2*d^3) - (a*(e + f*x)^3*Sinh[c + d*x]^2)/(2*b^2
*d) - (f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^2)/(3*b*d^2) + (2*f^2*(e +
 f*x)*Sinh[c + d*x]^3)/(9*b*d^3) + ((e + f*x)^3*Sinh[c + d*x]^3)/(3*b*d)
```

Rubi [A] time = 1.19673, antiderivative size = 792, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 15, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.441, Rules used = {5579, 5446, 3311, 3296, 2638, 2633, 32, 2635, 8, 5561, 2190, 2531, 6609, 2282, 6589}

$$\frac{6a^3f^2(e+fx)\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^3}+\frac{6a^3f^2(e+fx)\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4d^3}-\frac{3a^3f(e+fx)^2\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (-3*a*f^3*x)/(8*b^2*d^3) - (a*(e + f*x)^3)/(4*b^2*d) + (a^3*(e + f*x)^4)/(4
*b^4*f) - (6*a^2*f^3*Cosh[c + d*x])/(b^3*d^4) + (14*f^3*Cosh[c + d*x])/(9*b
*d^4) - (3*a^2*f*(e + f*x)^2*Cosh[c + d*x])/(b^3*d^2) + (2*f*(e + f*x)^2*Co
sh[c + d*x])/(3*b*d^2) - (2*f^3*Cosh[c + d*x]^3)/(27*b*d^4) - (a^3*(e + f*x
)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a^3*(e + f*x
)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^4*d) - (3*a^3*f*(e +
 f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^2) - (
3*a^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
b^4*d^2) + (6*a^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2]))])/(b^4*d^3) + (6*a^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2]))])/(b^4*d^3) - (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/(b^4*d^4) - (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/(b^4*d^4) + (6*a^2*f^2*(e + f*x)*Sinh[c + d*x])/
(b^3*d^3) - (4*f^2*(e + f*x)*Sinh[c + d*x])/(3*b*d^3) + (a^2*(e + f*x)^3*Si
nh[c + d*x])/(b^3*d) + (3*a*f^3*Cosh[c + d*x]*Sinh[c + d*x])/(8*b^2*d^4) +
(3*a*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^2*d^2) - (3*a*f^2*(e +
```

```
f*x)*Sinh[c + d*x]^2)/(4*b^2*d^3) - (a*(e + f*x)^3*Sinh[c + d*x]^2)/(2*b^2
*d) - (f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^2)/(3*b*d^2) + (2*f^2*(e +
 f*x)*Sinh[c + d*x]^3)/(9*b*d^3) + ((e + f*x)^3*Sinh[c + d*x]^3)/(3*b*d)
```

Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}dx = \frac{\int (e+fx)^3\cosh(c+dx)\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\cosh(c+dx)\sinh^2(c+d}{a+b\sinh(c+dx)}}{b}$$

$$= \frac{(e+fx)^3\sinh^3(c+dx)}{3bd} - \frac{a\int (e+fx)^3\cosh(c+dx)\sinh(c+dx)\,dx}{b^2} + $$

$$= -\frac{a(e+fx)^3\sinh^2(c+dx)}{2b^2d} - \frac{f(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{3bd^2} + \frac{2}{}$$

$$= \frac{a^3(e+fx)^4}{4b^4f} + \frac{2f(e+fx)^2\cosh(c+dx)}{3bd^2} + \frac{a^2(e+fx)^3\sinh(c+dx)}{b^3d} + 3$$

$$= -\frac{a(e+fx)^3}{4b^2d} + \frac{a^3(e+fx)^4}{4b^4f} + \frac{2f^3\cosh(c+dx)}{9bd^4} - \frac{3a^2f(e+fx)^2\cosh(c}{b^3d^2}$$

$$= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a^3(e+fx)^4}{4b^4f} + \frac{14f^3\cosh(c+dx)}{9bd^4} - \frac{3a^2f(e+f}{}$$

$$= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a^3(e+fx)^4}{4b^4f} - \frac{6a^2f^3\cosh(c+dx)}{b^3d^4} + \frac{14f^3\cosh}{9bd}$$

$$= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a^3(e+fx)^4}{4b^4f} - \frac{6a^2f^3\cosh(c+dx)}{b^3d^4} + \frac{14f^3\cosh}{9bd}$$

$$= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a^3(e+fx)^4}{4b^4f} - \frac{6a^2f^3\cosh(c+dx)}{b^3d^4} + \frac{14f^3\cosh}{9bd}$$

Mathematica [B] time = 29.333, size = 7460, normalized size = 9.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.268, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3\cosh(dx+c)(\sinh(dx+c))^3}{a+b\sinh(dx+c)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/24*e^3*(24*(d*x + c)*a^3/(b^4*d) + 24*a^3*log(-2*a*e^(-d*x - c) + b*e^(-
2*d*x - 2*c) - b)/(b^4*d) + (3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 - b^2)*e^(
-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-
3*d*x - 3*c) + 3*(4*a^2 - b^2)*e^(-d*x - c))/(b^3*d)) - 1/864*(216*a^3*d^4*
f^3*x^4*e^(3*c) + 864*a^3*d^4*e*f^2*x^3*e^(3*c) + 1296*a^3*d^4*e^2*f*x^2*e^
(3*c) - 4*(9*b^3*d^3*f^3*x^3*e^(6*c) + 9*(3*d^3*e*f^2 - d^2*f^3)*b^3*x^2*e^
(6*c) + 3*(9*d^3*e^2*f - 6*d^2*e*f^2 + 2*d*f^3)*b^3*x*e^(6*c) - (9*d^2*e^2*
f - 6*d*e*f^2 + 2*f^3)*b^3*e^(6*c))*e^(3*d*x) + 27*(4*a*b^2*d^3*f^3*x^3*e^(
5*c) + 6*(2*d^3*e*f^2 - d^2*f^3)*a*b^2*x^2*e^(5*c) + 6*(2*d^3*e^2*f - 2*d^2
*e*f^2 + d*f^3)*a*b^2*x*e^(5*c) - 3*(2*d^2*e^2*f - 2*d*e*f^2 + f^3)*a*b^2*e
^(5*c))*e^(2*d*x) + 108*(12*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a^2*b*e^(4*c) -
 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*b^3*e^(4*c) - (4*a^2*b*d^3*f^3*e^(4*c) -
 b^3*d^3*f^3*e^(4*c))*x^3 - 3*(4*(d^3*e*f^2 - d^2*f^3)*a^2*b*e^(4*c) - (d^3
*e*f^2 - d^2*f^3)*b^3*e^(4*c))*x^2 - 3*(4*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^
3)*a^2*b*e^(4*c) - (d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*b^3*e^(4*c))*x)*e^(d
*x) + 108*(12*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*a^2*b*e^(2*c) - 3*(d^2*e^2*f
+ 2*d*e*f^2 + 2*f^3)*b^3*e^(2*c) + (4*a^2*b*d^3*f^3*e^(2*c) - b^3*d^3*f^3*e
^(2*c))*x^3 + 3*(4*(d^3*e*f^2 + d^2*f^3)*a^2*b*e^(2*c) - (d^3*e*f^2 + d^2*f
^3)*b^3*e^(2*c))*x^2 + 3*(4*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*a^2*b*e^(2*
c) - (d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*b^3*e^(2*c))*x)*e^(-d*x) + 27*(4*a
*b^2*d^3*f^3*x^3*e^c + 6*(2*d^3*e*f^2 + d^2*f^3)*a*b^2*x^2*e^c + 6*(2*d^3*e
^2*f + 2*d^2*e*f^2 + d*f^3)*a*b^2*x*e^c + 3*(2*d^2*e^2*f + 2*d*e*f^2 + f^3)
*a*b^2*e^c)*e^(-2*d*x) + 4*(9*b^3*d^3*f^3*x^3 + 9*(3*d^3*e*f^2 + d^2*f^3)*b
^3*x^2 + 3*(9*d^3*e^2*f + 6*d^2*e*f^2 + 2*d*f^3)*b^3*x + (9*d^2*e^2*f + 6*d
*e*f^2 + 2*f^3)*b^3)*e^(-3*d*x))*e^(-3*c)/(b^4*d^4) + integrate(-2*(a^3*b*f
^3*x^3 + 3*a^3*b*e*f^2*x^2 + 3*a^3*b*e^2*f*x - (a^4*f^3*x^3*e^c + 3*a^4*e*f
^2*x^2*e^c + 3*a^4*e^2*f*x*e^c)*e^(d*x))/(b^5*e^(2*d*x + 2*c) + 2*a*b^4*e^(
d*x + c) - b^5), x)
```

Fricas [C] time = 3.38887, size = 14880, normalized size = 18.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/864*(36*b^3*d^3*f^3*x^3 + 36*b^3*d^3*e^3 + 36*b^3*d^2*e^2*f + 24*b^3*d*e
*f^2 - 4*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f
^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f
 - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*cosh(d*x + c)^6 - 4*(9*b^3*d^3*f^3*x^3
 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d
^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*
d*f^3)*x)*sinh(d*x + c)^6 + 8*b^3*f^3 + 27*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d
^3*e^3 - 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3
*e*f^2 - a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*
b^2*d*f^3)*x)*cosh(d*x + c)^5 + 3*(36*a*b^2*d^3*f^3*x^3 + 36*a*b^2*d^3*e^3
- 54*a*b^2*d^2*e^2*f + 54*a*b^2*d*e*f^2 - 27*a*b^2*f^3 + 54*(2*a*b^2*d^3*e*
f^2 - a*b^2*d^2*f^3)*x^2 + 54*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^
2*d*f^3)*x - 8*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3
*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3
```

*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c)^5 -
 108*((4*a^2*b - b^3)*d^3*f^3*x^3 + (4*a^2*b - b^3)*d^3*e^3 - 3*(4*a^2*b -
b^3)*d^2*e^2*f + 6*(4*a^2*b - b^3)*d*e*f^2 - 6*(4*a^2*b - b^3)*f^3 + 3*((4*
a^2*b - b^3)*d^3*e*f^2 - (4*a^2*b - b^3)*d^2*f^3)*x^2 + 3*((4*a^2*b - b^3)*
d^3*e^2*f - 2*(4*a^2*b - b^3)*d^2*e*f^2 + 2*(4*a^2*b - b^3)*d*f^3)*x)*cosh(
d*x + c)^4 - 3*(36*(4*a^2*b - b^3)*d^3*f^3*x^3 + 36*(4*a^2*b - b^3)*d^3*e^3
 - 108*(4*a^2*b - b^3)*d^2*e^2*f + 216*(4*a^2*b - b^3)*d*e*f^2 - 216*(4*a^2
*b - b^3)*f^3 + 108*((4*a^2*b - b^3)*d^3*e*f^2 - (4*a^2*b - b^3)*d^2*f^3)*x
^2 + 20*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f^
2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f
- 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x))*cosh(d*x + c)^2 + 108*((4*a^2*b - b^3)*
d^3*e^2*f - 2*(4*a^2*b - b^3)*d^2*e*f^2 + 2*(4*a^2*b - b^3)*d*f^3)*x - 45*(
4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e^3 - 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2
 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f^2 - a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3
*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c)^4
 - 216*(a^3*d^4*f^3*x^4 + 4*a^3*d^4*e*f^2*x^3 + 6*a^3*d^4*e^2*f*x^2 + 4*a^3
*d^4*e^3*x + 8*a^3*c*d^3*e^3 - 12*a^3*c^2*d^2*e^2*f + 8*a^3*c^3*d*e*f^2 - 2
*a^3*c^4*f^3)*cosh(d*x + c)^3 - 2*(108*a^3*d^4*f^3*x^4 + 432*a^3*d^4*e*f^2*
x^3 + 648*a^3*d^4*e^2*f*x^2 + 432*a^3*d^4*e^3*x + 864*a^3*c*d^3*e^3 - 1296*
a^3*c^2*d^2*e^2*f + 864*a^3*c^3*d*e*f^2 - 216*a^3*c^4*f^3 + 40*(9*b^3*d^3*f
^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3
*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f - 6*b^3*d^2*e*f^2 +
2*b^3*d*f^3)*x)*cosh(d*x + c)^3 - 135*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e^
3 - 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f^
2 - a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^2*d
*f^3)*x)*cosh(d*x + c)^2 + 216*((4*a^2*b - b^3)*d^3*f^3*x^3 + (4*a^2*b - b^
3)*d^3*e^3 - 3*(4*a^2*b - b^3)*d^2*e^2*f + 6*(4*a^2*b - b^3)*d*e*f^2 - 6*(4
*a^2*b - b^3)*f^3 + 3*((4*a^2*b - b^3)*d^3*e*f^2 - (4*a^2*b - b^3)*d^2*f^3)
x^2 + 3((4*a^2*b - b^3)*d^3*e^2*f - 2*(4*a^2*b - b^3)*d^2*e*f^2 + 2*(4*a^
2*b - b^3)*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 36*(3*b^3*d^3*e*f^2 +
 b^3*d^2*f^3)*x^2 + 108*((4*a^2*b - b^3)*d^3*f^3*x^3 + (4*a^2*b - b^3)*d^3*
e^3 + 3*(4*a^2*b - b^3)*d^2*e^2*f + 6*(4*a^2*b - b^3)*d*e*f^2 + 6*(4*a^2*b
- b^3)*f^3 + 3*((4*a^2*b - b^3)*d^3*e*f^2 + (4*a^2*b - b^3)*d^2*f^3)*x^2 +
3*((4*a^2*b - b^3)*d^3*e^2*f + 2*(4*a^2*b - b^3)*d^2*e*f^2 + 2*(4*a^2*b - b
^3)*d*f^3)*x)*cosh(d*x + c)^2 + 6*(18*(4*a^2*b - b^3)*d^3*f^3*x^3 + 18*(4*a
^2*b - b^3)*d^3*e^3 + 54*(4*a^2*b - b^3)*d^2*e^2*f + 108*(4*a^2*b - b^3)*d*
e*f^2 - 10*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e
*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2
*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*cosh(d*x + c)^4 + 108*(4*a^2*b - b^3
)*f^3 + 45*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e^3 - 6*a*b^2*d^2*e^2*f + 6*a
*b^2*d*e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f^2 - a*b^2*d^2*f^3)*x^2 + 6*
(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x)*cosh(d*x + c)^3 +
54*((4*a^2*b - b^3)*d^3*e*f^2 + (4*a^2*b - b^3)*d^2*f^3)*x^2 - 108*((4*a^2*
b - b^3)*d^3*f^3*x^3 + (4*a^2*b - b^3)*d^3*e^3 - 3*(4*a^2*b - b^3)*d^2*e^2*
f + 6*(4*a^2*b - b^3)*d*e*f^2 - 6*(4*a^2*b - b^3)*f^3 + 3*((4*a^2*b - b^3)*
d^3*e*f^2 - (4*a^2*b - b^3)*d^2*f^3)*x^2 + 3*((4*a^2*b - b^3)*d^3*e^2*f - 2
*(4*a^2*b - b^3)*d^2*e*f^2 + 2*(4*a^2*b - b^3)*d*f^3)*x)*cosh(d*x + c)^2 +
54*((4*a^2*b - b^3)*d^3*e^2*f + 2*(4*a^2*b - b^3)*d^2*e*f^2 + 2*(4*a^2*b -
b^3)*d*f^3)*x - 108*(a^3*d^4*f^3*x^4 + 4*a^3*d^4*e*f^2*x^3 + 6*a^3*d^4*e^2*
f*x^2 + 4*a^3*d^4*e^3*x + 8*a^3*c*d^3*e^3 - 12*a^3*c^2*d^2*e^2*f + 8*a^3*c^
3*d*e*f^2 - 2*a^3*c^4*f^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 12*(9*b^3*d^3*e
^2*f + 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x + 27*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2
*d^3*e^3 + 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 + 3*a*b^2*f^3 + 6*(2*a*b^2*d
^3*e*f^2 + a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*e^2*f + 2*a*b^2*d^2*e*f^2 +
a*b^2*d*f^3)*x)*cosh(d*x + c) + 2592*((a^3*d^2*f^3*x^2 + 2*a^3*d^2*e*f^2*x
+ a^3*d^2*e^2*f)*cosh(d*x + c)^3 + 3*(a^3*d^2*f^3*x^2 + 2*a^3*d^2*e*f^2*x +
 a^3*d^2*e^2*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3*d^2*f^3*x^2 + 2*a^3*
d^2*e*f^2*x + a^3*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3*d^2*f^3*x
^2 + 2*a^3*d^2*e*f^2*x + a^3*d^2*e^2*f)*sinh(d*x + c)^3)*dilog((a*cosh(d*x

+ c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2592*((a^3*d^2*f^3*x^2 + 2*a^3*d^2*e*f^2*x + a^3*d^2*e^2*f)*cosh(d*x + c)^3 + 3*(a^3*d^2*f^3*x^2 + 2*a^3*d^2*e*f^2*x + a^3*d^2*e^2*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3*d^2*f^3*x^2 + 2*a^3*d^2*e*f^2*x + a^3*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3*d^2*f^3*x^2 + 2*a^3*d^2*e*f^2*x + a^3*d^2*e^2*f)*sinh(d*x + c)^3)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 864*((a^3*d^3*e^3 - 3*a^3*c*d^2*e^2*f + 3*a^3*c^2*d*e*f^2 - a^3*c^3*f^3)*cosh(d*x + c)^3 + 3*(a^3*d^3*e^3 - 3*a^3*c*d^2*e^2*f + 3*a^3*c^2*d*e*f^2 - a^3*c^3*f^3)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3*d^3*e^3 - 3*a^3*c*d^2*e^2*f + 3*a^3*c^2*d*e*f^2 - a^3*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3*d^3*e^3 - 3*a^3*c*d^2*e^2*f + 3*a^3*c^2*d*e*f^2 - a^3*c^3*f^3)*sinh(d*x + c)^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 864*((a^3*d^3*e^3 - 3*a^3*c*d^2*e^2*f + 3*a^3*c^2*d*e*f^2 - a^3*c^3*f^3)*cosh(d*x + c)^3 + 3*(a^3*d^3*e^3 - 3*a^3*c*d^2*e^2*f + 3*a^3*c^2*d*e*f^2 - a^3*c^3*f^3)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3*d^3*e^3 - 3*a^3*c*d^2*e^2*f + 3*a^3*c^2*d*e*f^2 - a^3*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3*d^3*e^3 - 3*a^3*c*d^2*e^2*f + 3*a^3*c^2*d*e*f^2 - a^3*c^3*f^3)*sinh(d*x + c)^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 864*((a^3*d^3*f^3*x^3 + 3*a^3*d^3*e*f^2*x^2 + 3*a^3*d^3*e^2*f*x + 3*a^3*c*d^2*e^2*f - 3*a^3*c^2*d*e*f^2 + a^3*c^3*f^3)*cosh(d*x + c)^3 + 3*(a^3*d^3*f^3*x^3 + 3*a^3*d^3*e*f^2*x^2 + 3*a^3*d^3*e^2*f*x + 3*a^3*c*d^2*e^2*f - 3*a^3*c^2*d*e*f^2 + a^3*c^3*f^3)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3*d^3*f^3*x^3 + 3*a^3*d^3*e*f^2*x^2 + 3*a^3*d^3*e^2*f*x + 3*a^3*c*d^2*e^2*f - 3*a^3*c^2*d*e*f^2 + a^3*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3*d^3*f^3*x^3 + 3*a^3*d^3*e*f^2*x^2 + 3*a^3*d^3*e^2*f*x + 3*a^3*c*d^2*e^2*f - 3*a^3*c^2*d*e*f^2 + a^3*c^3*f^3)*sinh(d*x + c)^3)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 864*((a^3*d^3*f^3*x^3 + 3*a^3*d^3*e*f^2*x^2 + 3*a^3*d^3*e^2*f*x + 3*a^3*c*d^2*e^2*f - 3*a^3*c^2*d*e*f^2 + a^3*c^3*f^3)*cosh(d*x + c)^3 + 3*(a^3*d^3*f^3*x^3 + 3*a^3*d^3*e*f^2*x^2 + 3*a^3*d^3*e^2*f*x + 3*a^3*c*d^2*e^2*f - 3*a^3*c^2*d*e*f^2 + a^3*c^3*f^3)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3*d^3*f^3*x^3 + 3*a^3*d^3*e*f^2*x^2 + 3*a^3*d^3*e^2*f*x + 3*a^3*c*d^2*e^2*f - 3*a^3*c^2*d*e*f^2 + a^3*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3*d^3*f^3*x^3 + 3*a^3*d^3*e*f^2*x^2 + 3*a^3*d^3*e^2*f*x + 3*a^3*c*d^2*e^2*f - 3*a^3*c^2*d*e*f^2 + a^3*c^3*f^3)*sinh(d*x + c)^3)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 5184*(a^3*f^3*cosh(d*x + c)^3 + 3*a^3*f^3*cosh(d*x + c)^2*sinh(d*x + c) + 3*a^3*f^3*cosh(d*x + c)*sinh(d*x + c)^2 + a^3*f^3*sinh(d*x + c)^3)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 5184*(a^3*f^3*cosh(d*x + c)^3 + 3*a^3*f^3*cosh(d*x + c)^2*sinh(d*x + c) + 3*a^3*f^3*cosh(d*x + c)*sinh(d*x + c)^2 + a^3*f^3*sinh(d*x + c)^3)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 5184*((a^3*d*f^3*x + a^3*d*e*f^2)*cosh(d*x + c)^3 + 3*(a^3*d*f^3*x + a^3*d*e*f^2)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3*d*f^3*x + a^3*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3*d*f^3*x + a^3*d*e*f^2)*sinh(d*x + c)^3)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 5184*((a^3*d*f^3*x + a^3*d*e*f^2)*cosh(d*x + c)^3 + 3*(a^3*d*f^3*x + a^3*d*e*f^2)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3*d*f^3*x + a^3*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3*d*f^3*x + a^3*d*e*f^2)*sinh(d*x + c)^3)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 3*(36*a*b^2*d^3*f^3*x^3 + 36*a*b^2*d^3*e^3 + 54*a*b^2*d^2*e^2*f + 54*a*b^2*d*e*f^2 + 27*a*b^2*f^3 - 8*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*cosh(d*x + c)^5 + 45*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e^3 - 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f^2 - a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d

```
^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x)*cosh(d*x + c)^4 - 144*((4*a^
2*b - b^3)*d^3*f^3*x^3 + (4*a^2*b - b^3)*d^3*e^3 - 3*(4*a^2*b - b^3)*d^2*e^
2*f + 6*(4*a^2*b - b^3)*d*e*f^2 - 6*(4*a^2*b - b^3)*f^3 + 3*((4*a^2*b - b^3
)*d^3*e*f^2 - (4*a^2*b - b^3)*d^2*f^3)*x^2 + 3*((4*a^2*b - b^3)*d^3*e^2*f -
 2*(4*a^2*b - b^3)*d^2*e*f^2 + 2*(4*a^2*b - b^3)*d*f^3)*x)*cosh(d*x + c)^3
+ 54*(2*a*b^2*d^3*e*f^2 + a*b^2*d^2*f^3)*x^2 - 216*(a^3*d^4*f^3*x^4 + 4*a^3
*d^4*e*f^2*x^3 + 6*a^3*d^4*e^2*f*x^2 + 4*a^3*d^4*e^3*x + 8*a^3*c*d^3*e^3 -
12*a^3*c^2*d^2*e^2*f + 8*a^3*c^3*d*e*f^2 - 2*a^3*c^4*f^3)*cosh(d*x + c)^2 +
 54*(2*a*b^2*d^3*e^2*f + 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x + 72*((4*a^2*b
- b^3)*d^3*f^3*x^3 + (4*a^2*b - b^3)*d^3*e^3 + 3*(4*a^2*b - b^3)*d^2*e^2*f
+ 6*(4*a^2*b - b^3)*d*e*f^2 + 6*(4*a^2*b - b^3)*f^3 + 3*((4*a^2*b - b^3)*d^
3*e*f^2 + (4*a^2*b - b^3)*d^2*f^3)*x^2 + 3*((4*a^2*b - b^3)*d^3*e^2*f + 2*(
4*a^2*b - b^3)*d^2*e*f^2 + 2*(4*a^2*b - b^3)*d*f^3)*x)*cosh(d*x + c))*sinh(
d*x + c))/(b^4*d^4*cosh(d*x + c)^3 + 3*b^4*d^4*cosh(d*x + c)^2*sinh(d*x + c
) + 3*b^4*d^4*cosh(d*x + c)*sinh(d*x + c)^2 + b^4*d^4*sinh(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \cosh(dx+c)\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)*sinh(d*x + c)^3/(b*sinh(d*x + c) + a),
x)
```

3.392 $\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=578

$$-\frac{2a^3f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{2a^3f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4d^2}+\frac{2a^3f^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^3}+\frac{2a^3f}{}$$

```
[Out] -(a*e*f*x)/(2*b^2*d) - (a*f^2*x^2)/(4*b^2*d) + (a^3*(e + f*x)^3)/(3*b^4*f)
- (2*a^2*f*(e + f*x)*Cosh[c + d*x])/(b^3*d^2) + (4*f*(e + f*x)*Cosh[c + d*x
])/(9*b*d^2) - (a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])])/(b^4*d) - (a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
])])/(b^4*d) - (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/(b^4*d^2) - (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2]))])/(b^4*d^2) + (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/(b^4*d^3) + (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/(b^4*d^3) + (2*a^2*f^2*Sinh[c + d*x])/(b^3*d^3)
- (4*f^2*Sinh[c + d*x])/(9*b*d^3) + (a^2*(e + f*x)^2*Sinh[c + d*x])/(b^3*d)
 + (a*f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^2*d^2) - (a*f^2*Sinh[c
+ d*x]^2)/(4*b^2*d^3) - (a*(e + f*x)^2*Sinh[c + d*x]^2)/(2*b^2*d) - (2*f*(e
 + f*x)*Cosh[c + d*x]*Sinh[c + d*x]^2)/(9*b*d^2) + (2*f^2*Sinh[c + d*x]^3)/
(27*b*d^3) + ((e + f*x)^2*Sinh[c + d*x]^3)/(3*b*d)
```

Rubi [A] time = 0.928188, antiderivative size = 578, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5579, 5446, 3310, 3296, 2637, 5561, 2190, 2531, 2282, 6589}

$$-\frac{2a^3f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{2a^3f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4d^2}+\frac{2a^3f^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^3}+\frac{2a^3f}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -(a*e*f*x)/(2*b^2*d) - (a*f^2*x^2)/(4*b^2*d) + (a^3*(e + f*x)^3)/(3*b^4*f)
- (2*a^2*f*(e + f*x)*Cosh[c + d*x])/(b^3*d^2) + (4*f*(e + f*x)*Cosh[c + d*x
])/(9*b*d^2) - (a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])])/(b^4*d) - (a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
])])/(b^4*d) - (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/(b^4*d^2) - (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2]))])/(b^4*d^2) + (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/(b^4*d^3) + (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/(b^4*d^3) + (2*a^2*f^2*Sinh[c + d*x])/(b^3*d^3)
- (4*f^2*Sinh[c + d*x])/(9*b*d^3) + (a^2*(e + f*x)^2*Sinh[c + d*x])/(b^3*d)
 + (a*f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^2*d^2) - (a*f^2*Sinh[c
+ d*x]^2)/(4*b^2*d^3) - (a*(e + f*x)^2*Sinh[c + d*x]^2)/(2*b^2*d) - (2*f*(e
 + f*x)*Cosh[c + d*x]*Sinh[c + d*x]^2)/(9*b*d^2) + (2*f^2*Sinh[c + d*x]^3)/
(27*b*d^3) + ((e + f*x)^2*Sinh[c + d*x]^3)/(3*b*d)
```


Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
```

```
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh(c+dx)\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,d}{b}$$

$$= \frac{(e+fx)^2\sinh^3(c+dx)}{3bd} - \frac{a\int (e+fx)^2\cosh(c+dx)\sinh(c+dx)\,dx}{b^2} + \frac{a^2\int}{}$$

$$= -\frac{a(e+fx)^2\sinh^2(c+dx)}{2b^2d} - \frac{2f(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{9bd^2} + \frac{2f^2}{}$$

$$= \frac{a^3(e+fx)^3}{3b^4f} + \frac{4f(e+fx)\cosh(c+dx)}{9bd^2} + \frac{a^2(e+fx)^2\sinh(c+dx)}{b^3d} + \frac{af(e}{}$$

$$= -\frac{aefx}{2b^2d} - \frac{af^2x^2}{4b^2d} + \frac{a^3(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx)\cosh(c+dx)}{b^3d^2} + \frac{4f(e+fx)}{9b}$$

$$= -\frac{aefx}{2b^2d} - \frac{af^2x^2}{4b^2d} + \frac{a^3(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx)\cosh(c+dx)}{b^3d^2} + \frac{4f(e+fx)}{9b}$$

$$= -\frac{aefx}{2b^2d} - \frac{af^2x^2}{4b^2d} + \frac{a^3(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx)\cosh(c+dx)}{b^3d^2} + \frac{4f(e+fx)}{9b}$$

$$= -\frac{aefx}{2b^2d} - \frac{af^2x^2}{4b^2d} + \frac{a^3(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx)\cosh(c+dx)}{b^3d^2} + \frac{4f(e+fx)}{9b}$$

Mathematica [B] time = 14.9508, size = 3510, normalized size = 6.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] -(f^2*(2*a*x^3*(-1 + Coth[c]) - 2*a*x^3*Coth[c] - (6*a*b^2*(d^2*x^2*Log[1 +
 ((a - Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 2*d*x*PolyLog
[2, ((-a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 2*PolyLog
[3, ((-a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b]))/(Sqrt[a^2
 + b^2]*(-a + Sqrt[a^2 + b^2])*d^3) - (6*a*b^2*(d^2*x^2*Log[1 + ((a + Sqrt[
a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 2*d*x*PolyLog[2, ((a + Sq
rt[a^2 + b^2])*(-Cosh[c + d*x] + Sinh[c + d*x]))/b] - 2*PolyLog[3, ((a + Sq
rt[a^2 + b^2])*(-Cosh[c + d*x] + Sinh[c + d*x]))/b]))/(Sqrt[a^2 + b^2]*(a +
 Sqrt[a^2 + b^2])*d^3) + (6*a^2*(d^2*x^2*Log[1 + (b*(Cosh[c + d*x] + Sinh[c
 + d*x]))/(a - Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, (b*(Cosh[c + d*x] + Sin
h[c + d*x]))/(-a + Sqrt[a^2 + b^2])] - 2*PolyLog[3, (b*(Cosh[c + d*x] + Sin
h[c + d*x]))/(-a + Sqrt[a^2 + b^2])]))/(Sqrt[a^2 + b^2]*d^3) - (6*a^2*(d^2*
```

```
x^2*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])] + 2*
d*x*PolyLog[2, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))
] - 2*PolyLog[3, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]
))]))/(Sqrt[a^2 + b^2]*d^3) + (6*b*Cosh[d*x]*(-2*d*x*Cosh[c] + (2 + d^2*x^2
)*Sinh[c]))/d^3 + (6*b*((2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[c])*Sinh[d*x])/d
^3))/(12*b^2) + (e^2*((a*Log[a + b*Sinh[c + d*x]])/b^2 - Sinh[c + d*x]/b))/
(2*d) - (e*f*(-(b*Cosh[c + d*x]) - a*(-(c + d*x)^2/2 + (c + d*x)*Log[1 + (b
*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2])] - c*Log[a + b*Sinh[c + d*x]] + PolyLog[2, (b*E^(c + d*
x))/(-a + Sqrt[a^2 + b^2])] + PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))]) + b*d*x*Sinh[c + d*x]))/(b^2*d^2) + (e^2*(-3*a*(2*a^2 + b^2)*Log[a
 + b*Sinh[c + d*x]] + 3*b*(2*a^2 + b^2)*Sinh[c + d*x] - 3*a*b^2*Sinh[c + d*
x]^2 + 2*b^3*Sinh[c + d*x]^3))/(6*b^4*d) + (e*f*(-18*b*(4*a^2 + b^2)*Cosh[c
 + d*x] - 18*a*b^2*d*x*Cosh[2*(c + d*x)] - 2*b^3*Cosh[3*(c + d*x)] - 36*a*(
2*a^2 + b^2)*(-(c + d*x)^2/2 + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2])] + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - c
*Log[a + b*Sinh[c + d*x]] + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2
])] + PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]]) + 18*b*(4*a^2 +
 b^2)*d*x*Sinh[c + d*x] + 9*a*b^2*Sinh[2*(c + d*x)] + 6*b^3*d*x*Sinh[3*(c +
 d*x)]))/(36*b^4*d^2) + (f^2*((2*a*(2*a^2 + b^2)*(-1 + Coth[c])*(2*x^3 + (6
*a*(d^2*x^2*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2
])] + 2*d*x*PolyLog[2, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 +
 b^2])] - 2*PolyLog[3, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 +
 b^2])])*Sinh[c]*(Cosh[c] + Sinh[c]))/(Sqrt[a^2 + b^2]*d^3) - (3*b^2*(d^2*x
^2*Log[1 + ((a - Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 2*d
*x*PolyLog[2, ((-a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] -
 2*PolyLog[3, ((-a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b])*
(-1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 + b^2]*(-a + Sqrt[a^2 + b^2])*d^3)
- (3*b^2*(d^2*x^2*Log[1 + ((a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c +
d*x]))/b] - 2*d*x*PolyLog[2, ((a + Sqrt[a^2 + b^2])*(-Cosh[c + d*x] + Sinh[
c + d*x]))/b] - 2*PolyLog[3, ((a + Sqrt[a^2 + b^2])*(-Cosh[c + d*x] + Sinh[
c + d*x]))/b])*(-1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 + b^2]*(a + Sqrt[a^2
 + b^2])*d^3) - (3*a*(d^2*x^2*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(
a + Sqrt[a^2 + b^2])] + 2*d*x*PolyLog[2, -((b*(Cosh[c + d*x] + Sinh[c + d*x
]))/(a + Sqrt[a^2 + b^2]))] - 2*PolyLog[3, -((b*(Cosh[c + d*x] + Sinh[c + d
*x]))/(a + Sqrt[a^2 + b^2]))])*(-1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 + b^
2]*d^3)))/(3*b^4) + Csch[c]*(Cosh[3*c + 3*d*x]/(108*b^4*d^3) - Sinh[3*c + 3
*d*x]/(108*b^4*d^3))*(27*a*b^2*Cosh[d*x] + 54*a*b^2*d*x*Cosh[d*x] + 54*a*b^
2*d^2*x^2*Cosh[d*x] - 27*a*b^2*Cosh[2*c + d*x] - 54*a*b^2*d*x*Cosh[2*c + d*
x] - 54*a*b^2*d^2*x^2*Cosh[2*c + d*x] + 432*a^2*b*Cosh[c + 2*d*x] + 108*b^3
*Cosh[c + 2*d*x] + 432*a^2*b*d*x*Cosh[c + 2*d*x] + 108*b^3*d*x*Cosh[c + 2*d
*x] + 216*a^2*b*d^2*x^2*Cosh[c + 2*d*x] + 54*b^3*d^2*x^2*Cosh[c + 2*d*x] -
432*a^2*b*Cosh[3*c + 2*d*x] - 108*b^3*Cosh[3*c + 2*d*x] - 432*a^2*b*d*x*Cos
h[3*c + 2*d*x] - 108*b^3*d*x*Cosh[3*c + 2*d*x] - 216*a^2*b*d^2*x^2*Cosh[3*c
 + 2*d*x] - 54*b^3*d^2*x^2*Cosh[3*c + 2*d*x] - 144*a^3*d^3*x^3*Cosh[2*c + 3
*d*x] - 72*a*b^2*d^3*x^3*Cosh[2*c + 3*d*x] - 144*a^3*d^3*x^3*Cosh[4*c + 3*d
*x] - 72*a*b^2*d^3*x^3*Cosh[4*c + 3*d*x] - 432*a^2*b*Cosh[3*c + 4*d*x] - 10
8*b^3*Cosh[3*c + 4*d*x] + 432*a^2*b*d*x*Cosh[3*c + 4*d*x] + 108*b^3*d*x*Cos
h[3*c + 4*d*x] - 216*a^2*b*d^2*x^2*Cosh[3*c + 4*d*x] - 54*b^3*d^2*x^2*Cosh[
3*c + 4*d*x] + 432*a^2*b*Cosh[5*c + 4*d*x] + 108*b^3*Cosh[5*c + 4*d*x] - 43
2*a^2*b*d*x*Cosh[5*c + 4*d*x] - 108*b^3*d*x*Cosh[5*c + 4*d*x] + 216*a^2*b*d
^2*x^2*Cosh[5*c + 4*d*x] + 54*b^3*d^2*x^2*Cosh[5*c + 4*d*x] + 27*a*b^2*Cosh
[4*c + 5*d*x] - 54*a*b^2*d*x*Cosh[4*c + 5*d*x] + 54*a*b^2*d^2*x^2*Cosh[4*c
+ 5*d*x] - 27*a*b^2*Cosh[6*c + 5*d*x] + 54*a*b^2*d*x*Cosh[6*c + 5*d*x] - 54
*a*b^2*d^2*x^2*Cosh[6*c + 5*d*x] - 4*b^3*Cosh[5*c + 6*d*x] + 12*b^3*d*x*Cos
h[5*c + 6*d*x] - 18*b^3*d^2*x^2*Cosh[5*c + 6*d*x] + 4*b^3*Cosh[7*c + 6*d*x]
 - 12*b^3*d*x*Cosh[7*c + 6*d*x] + 18*b^3*d^2*x^2*Cosh[7*c + 6*d*x] - 8*b^3*
Sinh[c] - 24*b^3*d*x*Sinh[c] - 36*b^3*d^2*x^2*Sinh[c] + 27*a*b^2*Sinh[d*x]
+ 54*a*b^2*d*x*Sinh[d*x] + 54*a*b^2*d^2*x^2*Sinh[d*x] - 27*a*b^2*Sinh[2*c +
```

```
d*x] - 54*a*b^2*d*x*Sinh[2*c + d*x] - 54*a*b^2*d^2*x^2*Sinh[2*c + d*x] + 4
32*a^2*b*Sinh[c + 2*d*x] + 108*b^3*Sinh[c + 2*d*x] + 432*a^2*b*d*x*Sinh[c +
 2*d*x] + 108*b^3*d*x*Sinh[c + 2*d*x] + 216*a^2*b*d^2*x^2*Sinh[c + 2*d*x] +
 54*b^3*d^2*x^2*Sinh[c + 2*d*x] - 432*a^2*b*Sinh[3*c + 2*d*x] - 108*b^3*Sin
h[3*c + 2*d*x] - 432*a^2*b*d*x*Sinh[3*c + 2*d*x] - 108*b^3*d*x*Sinh[3*c + 2
*d*x] - 216*a^2*b*d^2*x^2*Sinh[3*c + 2*d*x] - 54*b^3*d^2*x^2*Sinh[3*c + 2*d
*x] - 144*a^3*d^3*x^3*Sinh[2*c + 3*d*x] - 72*a*b^2*d^3*x^3*Sinh[2*c + 3*d*x
] - 144*a^3*d^3*x^3*Sinh[4*c + 3*d*x] - 72*a*b^2*d^3*x^3*Sinh[4*c + 3*d*x]
- 432*a^2*b*Sinh[3*c + 4*d*x] - 108*b^3*Sinh[3*c + 4*d*x] + 432*a^2*b*d*x*S
inh[3*c + 4*d*x] + 108*b^3*d*x*Sinh[3*c + 4*d*x] - 216*a^2*b*d^2*x^2*Sinh[3
*c + 4*d*x] - 54*b^3*d^2*x^2*Sinh[3*c + 4*d*x] + 432*a^2*b*Sinh[5*c + 4*d*x
] + 108*b^3*Sinh[5*c + 4*d*x] - 432*a^2*b*d*x*Sinh[5*c + 4*d*x] - 108*b^3*d
*x*Sinh[5*c + 4*d*x] + 216*a^2*b*d^2*x^2*Sinh[5*c + 4*d*x] + 54*b^3*d^2*x^2
*Sinh[5*c + 4*d*x] + 27*a*b^2*Sinh[4*c + 5*d*x] - 54*a*b^2*d*x*Sinh[4*c + 5
*d*x] + 54*a*b^2*d^2*x^2*Sinh[4*c + 5*d*x] - 27*a*b^2*Sinh[6*c + 5*d*x] + 5
4*a*b^2*d*x*Sinh[6*c + 5*d*x] - 54*a*b^2*d^2*x^2*Sinh[6*c + 5*d*x] - 4*b^3*
Sinh[5*c + 6*d*x] + 12*b^3*d*x*Sinh[5*c + 6*d*x] - 18*b^3*d^2*x^2*Sinh[5*c
+ 6*d*x] + 4*b^3*Sinh[7*c + 6*d*x] - 12*b^3*d*x*Sinh[7*c + 6*d*x] + 18*b^3*
d^2*x^2*Sinh[7*c + 6*d*x])))/8
```

Maple [F] time = 0.208, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \cosh(dx+c)(\sinh(dx+c))^3}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/24*e^2*(24*(d*x + c)*a^3/(b^4*d) + 24*a^3*log(-2*a*e^(-d*x - c) + b*e^(-
2*d*x - 2*c) - b)/(b^4*d) + (3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 - b^2)*e^(
-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-
3*d*x - 3*c) + 3*(4*a^2 - b^2)*e^(-d*x - c))/(b^3*d)) - 1/432*(144*a^3*d^3*
f^2*x^3*e^(3*c) + 432*a^3*d^3*e*f*x^2*e^(3*c) - 2*(9*b^3*d^2*f^2*x^2*e^(6*c
) + 6*(3*d^2*e*f - d*f^2)*b^3*x*e^(6*c) - 2*(3*d*e*f - f^2)*b^3*e^(6*c))*e^
(3*d*x) + 27*(2*a*b^2*d^2*f^2*x^2*e^(5*c) + 2*(2*d^2*e*f - d*f^2)*a*b^2*x*e
^(5*c) - (2*d*e*f - f^2)*a*b^2*e^(5*c))*e^(2*d*x) + 54*(8*(d*e*f - f^2)*a^2
*b*e^(4*c) - 2*(d*e*f - f^2)*b^3*e^(4*c) - (4*a^2*b*d^2*f^2*e^(4*c) - b^3*d
^2*f^2*e^(4*c))*x^2 - 2*(4*(d^2*e*f - d*f^2)*a^2*b*e^(4*c) - (d^2*e*f - d*f
^2)*b^3*e^(4*c))*x)*e^(d*x) + 54*(8*(d*e*f + f^2)*a^2*b*e^(2*c) - 2*(d*e*f
+ f^2)*b^3*e^(2*c) + (4*a^2*b*d^2*f^2*e^(2*c) - b^3*d^2*f^2*e^(2*c))*x^2 +
2*(4*(d^2*e*f + d*f^2)*a^2*b*e^(2*c) - (d^2*e*f + d*f^2)*b^3*e^(2*c))*x)*e^
(-d*x) + 27*(2*a*b^2*d^2*f^2*x^2*e^c + 2*(2*d^2*e*f + d*f^2)*a*b^2*x*e^c +
```

```
(2*d*e*f + f^2)*a*b^2*e^c)*e^(-2*d*x) + 2*(9*b^3*d^2*f^2*x^2 + 6*(3*d^2*e*f
 + d*f^2)*b^3*x + 2*(3*d*e*f + f^2)*b^3)*e^(-3*d*x))*e^(-3*c)/(b^4*d^3) + i
ntegrate(-2*(a^3*b*f^2*x^2 + 2*a^3*b*e*f*x - (a^4*f^2*x^2*e^c + 2*a^4*e*f*x
*e^c)*e^(d*x))/(b^5*e^(2*d*x + 2*c) + 2*a*b^4*e^(d*x + c) - b^5), x)
```

Fricas [C] time = 2.85816, size = 9353, normalized size = 16.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/432*(18*b^3*d^2*f^2*x^2 + 18*b^3*d^2*e^2 - 2*(9*b^3*d^2*f^2*x^2 + 9*b^3*
d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d
*x + c)^6 - 2*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2
+ 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*sinh(d*x + c)^6 + 12*b^3*d*e*f + 27*(2*a
*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2
*d^2*e*f - a*b^2*d*f^2)*x)*cosh(d*x + c)^5 + 3*(18*a*b^2*d^2*f^2*x^2 + 18*a
*b^2*d^2*e^2 - 18*a*b^2*d*e*f + 9*a*b^2*f^2 + 18*(2*a*b^2*d^2*e*f - a*b^2*d
*f^2)*x - 4*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 +
6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*b^3*f^2
 - 54*((4*a^2*b - b^3)*d^2*f^2*x^2 + (4*a^2*b - b^3)*d^2*e^2 - 2*(4*a^2*b -
 b^3)*d*e*f + 2*(4*a^2*b - b^3)*f^2 + 2*((4*a^2*b - b^3)*d^2*e*f - (4*a^2*b
 - b^3)*d*f^2)*x)*cosh(d*x + c)^4 - 3*(18*(4*a^2*b - b^3)*d^2*f^2*x^2 + 18*
(4*a^2*b - b^3)*d^2*e^2 - 36*(4*a^2*b - b^3)*d*e*f + 36*(4*a^2*b - b^3)*f^2
 + 10*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b
^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*x + c)^2 + 36*((4*a^2*b - b^3)*d^2*e*f -
(4*a^2*b - b^3)*d*f^2)*x - 45*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*
b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*cosh(d*x + c))
*sinh(d*x + c)^4 - 144*(a^3*d^3*f^2*x^3 + 3*a^3*d^3*e*f*x^2 + 3*a^3*d^3*e^2
*x + 6*a^3*c*d^2*e^2 - 6*a^3*c^2*d*e*f + 2*a^3*c^3*f^2)*cosh(d*x + c)^3 - 2
*(72*a^3*d^3*f^2*x^3 + 216*a^3*d^3*e*f*x^2 + 216*a^3*d^3*e^2*x + 432*a^3*c*
d^2*e^2 - 432*a^3*c^2*d*e*f + 144*a^3*c^3*f^2 + 20*(9*b^3*d^2*f^2*x^2 + 9*b
^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cos
h(d*x + c)^3 - 135*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f +
 a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*cosh(d*x + c)^2 + 108*((4
*a^2*b - b^3)*d^2*f^2*x^2 + (4*a^2*b - b^3)*d^2*e^2 - 2*(4*a^2*b - b^3)*d*e
*f + 2*(4*a^2*b - b^3)*f^2 + 2*((4*a^2*b - b^3)*d^2*e*f - (4*a^2*b - b^3)*d
*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 54*((4*a^2*b - b^3)*d^2*f^2*x^2 +
 (4*a^2*b - b^3)*d^2*e^2 + 2*(4*a^2*b - b^3)*d*e*f + 2*(4*a^2*b - b^3)*f^2
+ 2*((4*a^2*b - b^3)*d^2*e*f + (4*a^2*b - b^3)*d*f^2)*x)*cosh(d*x + c)^2 +
6*(9*(4*a^2*b - b^3)*d^2*f^2*x^2 + 9*(4*a^2*b - b^3)*d^2*e^2 - 5*(9*b^3*d^2
*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3
*d*f^2)*x)*cosh(d*x + c)^4 + 18*(4*a^2*b - b^3)*d*e*f + 45*(2*a*b^2*d^2*f^2
*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a
*b^2*d*f^2)*x)*cosh(d*x + c)^3 + 18*(4*a^2*b - b^3)*f^2 - 54*((4*a^2*b - b^
3)*d^2*f^2*x^2 + (4*a^2*b - b^3)*d^2*e^2 - 2*(4*a^2*b - b^3)*d*e*f + 2*(4*a
^2*b - b^3)*f^2 + 2*((4*a^2*b - b^3)*d^2*e*f - (4*a^2*b - b^3)*d*f^2)*x)*co
sh(d*x + c)^2 + 18*((4*a^2*b - b^3)*d^2*e*f + (4*a^2*b - b^3)*d*f^2)*x - 72
*(a^3*d^3*f^2*x^3 + 3*a^3*d^3*e*f*x^2 + 3*a^3*d^3*e^2*x + 6*a^3*c*d^2*e^2 -
 6*a^3*c^2*d*e*f + 2*a^3*c^3*f^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 12*(3*b^
3*d^2*e*f + b^3*d*f^2)*x + 27*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 + 2*a*
b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f + a*b^2*d*f^2)*x)*cosh(d*x + c)
+ 864*((a^3*d*f^2*x + a^3*d*e*f)*cosh(d*x + c)^3 + 3*(a^3*d*f^2*x + a^3*d*e
*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3*d*f^2*x + a^3*d*e*f)*cosh(d*x +
```

```
c)*sinh(d*x + c)^2 + (a^3*d*f^2*x + a^3*d*e*f)*sinh(d*x + c)^3)*dilog((a*co
sh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b + 1) + 864*((a^3*d*f^2*x + a^3*d*e*f)*cosh(d*x + c)^3
 + 3*(a^3*d*f^2*x + a^3*d*e*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3*d*f^2
*x + a^3*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3*d*f^2*x + a^3*d*e*f)*s
inh(d*x + c)^3)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 432*((a^3*d^2*e^2 -
 2*a^3*c*d*e*f + a^3*c^2*f^2)*cosh(d*x + c)^3 + 3*(a^3*d^2*e^2 - 2*a^3*c*d*
e*f + a^3*c^2*f^2)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3*d^2*e^2 - 2*a^3*c
*d*e*f + a^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3*d^2*e^2 - 2*a^3*
c*d*e*f + a^3*c^2*f^2)*sinh(d*x + c)^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*
x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 432*((a^3*d^2*e^2 - 2*a^3*c*d*e
*f + a^3*c^2*f^2)*cosh(d*x + c)^3 + 3*(a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^
2*f^2)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3
*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^
3*c^2*f^2)*sinh(d*x + c)^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b
*sqrt((a^2 + b^2)/b^2) + 2*a) + 432*((a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2
*a^3*c*d*e*f - a^3*c^2*f^2)*cosh(d*x + c)^3 + 3*(a^3*d^2*f^2*x^2 + 2*a^3*d^
2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a
^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*cosh(d*x +
c)*sinh(d*x + c)^2 + (a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a
^3*c^2*f^2)*sinh(d*x + c)^3)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 432*((a^3*d
^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*cosh(d*x + c)^3
 + 3*(a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*cosh
(d*x + c)^2*sinh(d*x + c) + 3*(a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*
d*e*f - a^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3*d^2*f^2*x^2 + 2*a
^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*sinh(d*x + c)^3)*log(-(a*cosh(d
*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2) - b)/b) - 864*(a^3*f^2*cosh(d*x + c)^3 + 3*a^3*f^2*cosh(d*x + c)
^2*sinh(d*x + c) + 3*a^3*f^2*cosh(d*x + c)*sinh(d*x + c)^2 + a^3*f^2*sinh(d
*x + c)^3)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 864*(a^3*f^2*cosh(d*x + c)^
3 + 3*a^3*f^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*a^3*f^2*cosh(d*x + c)*sinh(
d*x + c)^2 + a^3*f^2*sinh(d*x + c)^3)*polylog(3, (a*cosh(d*x + c) + a*sinh(
d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) +
3*(18*a*b^2*d^2*f^2*x^2 + 18*a*b^2*d^2*e^2 + 18*a*b^2*d*e*f - 4*(9*b^3*d^2*
f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*
d*f^2)*x)*cosh(d*x + c)^5 + 9*a*b^2*f^2 + 45*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2
*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x)
*cosh(d*x + c)^4 - 72*((4*a^2*b - b^3)*d^2*f^2*x^2 + (4*a^2*b - b^3)*d^2*e^
2 - 2*(4*a^2*b - b^3)*d*e*f + 2*(4*a^2*b - b^3)*f^2 + 2*((4*a^2*b - b^3)*d^
2*e*f - (4*a^2*b - b^3)*d*f^2)*x)*cosh(d*x + c)^3 - 144*(a^3*d^3*f^2*x^3 +
3*a^3*d^3*e*f*x^2 + 3*a^3*d^3*e^2*x + 6*a^3*c*d^2*e^2 - 6*a^3*c^2*d*e*f + 2
*a^3*c^3*f^2)*cosh(d*x + c)^2 + 18*(2*a*b^2*d^2*e*f + a*b^2*d*f^2)*x + 36*(
(4*a^2*b - b^3)*d^2*f^2*x^2 + (4*a^2*b - b^3)*d^2*e^2 + 2*(4*a^2*b - b^3)*d
*e*f + 2*(4*a^2*b - b^3)*f^2 + 2*((4*a^2*b - b^3)*d^2*e*f + (4*a^2*b - b^3)
*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))/(b^4*d^3*cosh(d*x + c)^3 + 3*b^4*d
^3*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^4*d^3*cosh(d*x + c)*sinh(d*x + c)^2
+ b^4*d^3*sinh(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \cosh(dx+c)\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)*sinh(d*x + c)^3/(b*sinh(d*x + c) + a),
x)
```

3.393 $\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=348

$$-\frac{a^3 f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{a^3 f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4d^2}-\frac{a^2 f\cosh(c+dx)}{b^3d^2}-\frac{a^3(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^4d}-\frac{a^3(e\cdots}{\cdots}$$

```
[Out] -(a*f*x)/(4*b^2*d) + (a^3*(e + f*x)^2)/(2*b^4*f) - (a^2*f*Cosh[c + d*x])/(b
^3*d^2) + (f*Cosh[c + d*x])/(3*b*d^2) - (f*Cosh[c + d*x]^3)/(9*b*d^2) - (a^
3*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a^3*
(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^4*d) - (a^3*f*
PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^2) - (a^3*f*Po
lyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^2) + (a^2*(e + f
*x)*Sinh[c + d*x])/(b^3*d) + (a*f*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^2*d^2)
- (a*(e + f*x)*Sinh[c + d*x]^2)/(2*b^2*d) + ((e + f*x)*Sinh[c + d*x]^3)/(3*
b*d)
```

Rubi [A] time = 0.52561, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {5579, 5446, 2633, 2635, 8, 3296, 2638, 5561, 2190, 2279, 2391}

$$-\frac{a^3 f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}-\frac{a^3 f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4d^2}-\frac{a^2 f\cosh(c+dx)}{b^3d^2}-\frac{a^3(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^4d}-\frac{a^3(e\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -(a*f*x)/(4*b^2*d) + (a^3*(e + f*x)^2)/(2*b^4*f) - (a^2*f*Cosh[c + d*x])/(b
^3*d^2) + (f*Cosh[c + d*x])/(3*b*d^2) - (f*Cosh[c + d*x]^3)/(9*b*d^2) - (a^
3*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a^3*
(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^4*d) - (a^3*f*
PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^2) - (a^3*f*Po
lyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^2) + (a^2*(e + f
*x)*Sinh[c + d*x])/(b^3*d) + (a*f*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^2*d^2)
- (a*(e + f*x)*Sinh[c + d*x]^2)/(2*b^2*d) + ((e + f*x)*Sinh[c + d*x]^3)/(3*
b*d)
```


Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh(c+dx)\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\cosh(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= \frac{(e+fx)\sinh^3(c+dx)}{3bd} - \frac{a\int (e+fx)\cosh(c+dx)\sinh(c+dx)\,dx}{b^2} + \frac{a^2\int (e}{}$$

$$= -\frac{a(e+fx)\sinh^2(c+dx)}{2b^2d} + \frac{(e+fx)\sinh^3(c+dx)}{3bd} + \frac{a^2\int (e+fx)\cosh(c+}{b^3}$$

$$= \frac{a^3(e+fx)^2}{2b^4f} + \frac{f\cosh(c+dx)}{3bd^2} - \frac{f\cosh^3(c+dx)}{9bd^2} + \frac{a^2(e+fx)\sinh(c+dx)}{b^3d}$$

$$= -\frac{afx}{4b^2d} + \frac{a^3(e+fx)^2}{2b^4f} - \frac{a^2f\cosh(c+dx)}{b^3d^2} + \frac{f\cosh(c+dx)}{3bd^2} - \frac{f\cosh^3(c+d}{9bd^2}$$

$$= -\frac{afx}{4b^2d} + \frac{a^3(e+fx)^2}{2b^4f} - \frac{a^2f\cosh(c+dx)}{b^3d^2} + \frac{f\cosh(c+dx)}{3bd^2} - \frac{f\cosh^3(c+d}{9bd^2}$$

$$= -\frac{afx}{4b^2d} + \frac{a^3(e+fx)^2}{2b^4f} - \frac{a^2f\cosh(c+dx)}{b^3d^2} + \frac{f\cosh(c+dx)}{3bd^2} - \frac{f\cosh^3(c+d}{9bd^2}$$

Mathematica [A] time = 1.72985, size = 447, normalized size = 1.28

$$-\frac{72a^3f\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+72a^3f\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+72a^3cf\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)+72a^3dfx\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] -(-36*a^3*c^2*f - 72*a^3*c*d*f*x - 36*a^3*d^2*f*x^2 + 72*a^2*b*f*Cosh[c + d
*x] - 18*b^3*f*Cosh[c + d*x] + 18*a*b^2*d*f*x*Cosh[2*(c + d*x)] + 2*b^3*f*C
osh[3*(c + d*x)] + 72*a^3*c*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
] + 72*a^3*d*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 72*a^3*c*
f*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 72*a^3*d*f*x*Log[1 + (b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 72*a^3*d*e*Log[a + b*Sinh[c + d*x]] -
 72*a^3*c*f*Log[a + b*Sinh[c + d*x]] + 72*a^3*f*PolyLog[2, (b*E^(c + d*x))/
(-a + Sqrt[a^2 + b^2])] + 72*a^3*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2]))] - 72*a^2*b*d*e*Sinh[c + d*x] - 72*a^2*b*d*f*x*Sinh[c + d*x] +
18*b^3*d*f*x*Sinh[c + d*x] + 36*a*b^2*d*e*Sinh[c + d*x]^2 - 24*b^3*d*e*Sinh
[c + d*x]^3 - 9*a*b^2*f*Sinh[2*(c + d*x)] - 6*b^3*d*f*x*Sinh[3*(c + d*x)])/
(72*b^4*d^2)
```

Maple [B] time = 0.1, size = 671, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/2*a^3*f*x^2/b^4-a^3*e*x/b^4+1/72*(3*d*f*x+3*d*e-f)/d^2/b*exp(3*d*x+3*c)-1
/16*a*(2*d*f*x+2*d*e-f)/b^2/d^2*exp(2*d*x+2*c)+1/8*(4*a^2*d*f*x-b^2*d*f*x+4
*a^2*d*e-b^2*d*e-4*a^2*f+b^2*f)/b^3/d^2*exp(d*x+c)-1/8*(4*a^2-b^2)*(d*f*x+d
*e+f)/b^3/d^2*exp(-d*x-c)-1/16*a*(2*d*f*x+2*d*e+f)/b^2/d^2*exp(-2*d*x-2*c)-
1/72*(3*d*f*x+3*d*e+f)/d^2/b*exp(-3*d*x-3*c)+a^3/b^4/d^2*f*c*ln(b*exp(2*d*x
+2*c)+2*a*exp(d*x+c)-b)-2*a^3/b^4/d^2*f*c*ln(exp(d*x+c))-a^3/b^4/d*e*ln(b*e
xp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2*a^3/b^4/d*e*ln(exp(d*x+c))-a^3/b^4/d*f*ln
((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-a^3/b^4/d^2*f*ln
((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-a^3/b^4/d*f*ln((
b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-a^3/b^4/d^2*f*ln((b*
exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-a^3/b^4/d^2*f*dilog((b
*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-a^3/b^4/d^2*f*dilog((-b
*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+2*a^3/b^4/d*f*c*x+a^3/
b^4/d^2*f*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{24}e\left(\frac{24\,(dx+c)a^3}{b^4d}+\frac{24\,a^3\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{b^4d}+\frac{\left(3\,abe^{(-dx-c)}-b^2-3\left(4\,a^2-b^2\right)e^{(-2\,dx-2\,c)}\right)e^{(3\,dx+3\,c)}}{b^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/24*e*(24*(d*x + c)*a^3/(b^4*d) + 24*a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*
d*x - 2*c) - b)/(b^4*d) + (3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 - b^2)*e^(-2
*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-3*
d*x - 3*c) + 3*(4*a^2 - b^2)*e^(-d*x - c))/(b^3*d)) - 1/144*f*((72*a^3*d^2*
x^2*e^(3*c) - 2*(3*b^3*d*x*e^(6*c) - b^3*e^(6*c))*e^(3*d*x) + 9*(2*a*b^2*d*
x*e^(5*c) - a*b^2*e^(5*c))*e^(2*d*x) + 18*(4*a^2*b*e^(4*c) - b^3*e^(4*c) -
(4*a^2*b*d*e^(4*c) - b^3*d*e^(4*c))*x)*e^(d*x) + 18*(4*a^2*b*e^(2*c) - b^3*
e^(2*c) + (4*a^2*b*d*e^(2*c) - b^3*d*e^(2*c))*x)*e^(-d*x) + 9*(2*a*b^2*d*x*
e^c + a*b^2*e^c)*e^(-2*d*x) + 2*(3*b^3*d*x + b^3)*e^(-3*d*x))*e^(-3*c)/(b^4
*d^2) - 9*integrate(32*(a^4*x*e^(d*x + c) - a^3*b*x)/(b^5*e^(2*d*x + 2*c) +
 2*a*b^4*e^(d*x + c) - b^5), x))
```

Fricas [B] time = 2.53215, size = 5007, normalized size = 14.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/144*(2*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^6 + 2*(3*b^3*d*f*x
 + 3*b^3*d*e - b^3*f)*sinh(d*x + c)^6 - 6*b^3*d*f*x - 9*(2*a*b^2*d*f*x + 2*
a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^5 - 3*(6*a*b^2*d*f*x + 6*a*b^2*d*e - 3*a
*b^2*f - 4*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c))*sinh(d*x + c)^5
 - 6*b^3*d*e + 18*((4*a^2*b - b^3)*d*f*x + (4*a^2*b - b^3)*d*e - (4*a^2*b -
 b^3)*f)*cosh(d*x + c)^4 + 3*(6*(4*a^2*b - b^3)*d*f*x + 6*(4*a^2*b - b^3)*d
*e + 10*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^2 - 6*(4*a^2*b - b^
3)*f - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c))*sinh(d*x +
```

```
 c)^4 - 2*b^3*f + 72*(a^3*d^2*f*x^2 + 2*a^3*d^2*e*x + 4*a^3*c*d*e - 2*a^3*c
^2*f)*cosh(d*x + c)^3 + 2*(36*a^3*d^2*f*x^2 + 72*a^3*d^2*e*x + 144*a^3*c*d*
e - 72*a^3*c^2*f + 20*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^3 - 4
5*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^2 + 36*((4*a^2*b -
b^3)*d*f*x + (4*a^2*b - b^3)*d*e - (4*a^2*b - b^3)*f)*cosh(d*x + c))*sinh(d
*x + c)^3 - 18*((4*a^2*b - b^3)*d*f*x + (4*a^2*b - b^3)*d*e + (4*a^2*b - b^
3)*f)*cosh(d*x + c)^2 + 6*(5*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c
)^4 - 3*(4*a^2*b - b^3)*d*f*x - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*
cosh(d*x + c)^3 - 3*(4*a^2*b - b^3)*d*e + 18*((4*a^2*b - b^3)*d*f*x + (4*a^
2*b - b^3)*d*e - (4*a^2*b - b^3)*f)*cosh(d*x + c)^2 - 3*(4*a^2*b - b^3)*f +
 36*(a^3*d^2*f*x^2 + 2*a^3*d^2*e*x + 4*a^3*c*d*e - 2*a^3*c^2*f)*cosh(d*x +
c))*sinh(d*x + c)^2 - 9*(2*a*b^2*d*f*x + 2*a*b^2*d*e + a*b^2*f)*cosh(d*x +
c) - 144*(a^3*f*cosh(d*x + c)^3 + 3*a^3*f*cosh(d*x + c)^2*sinh(d*x + c) + 3
*a^3*f*cosh(d*x + c)*sinh(d*x + c)^2 + a^3*f*sinh(d*x + c)^3)*dilog((a*cosh
(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b + 1) - 144*(a^3*f*cosh(d*x + c)^3 + 3*a^3*f*cosh(d*x +
c)^2*sinh(d*x + c) + 3*a^3*f*cosh(d*x + c)*sinh(d*x + c)^2 + a^3*f*sinh(d*x
 + c)^3)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 144*((a^3*d*e - a^3*c*f)*c
osh(d*x + c)^3 + 3*(a^3*d*e - a^3*c*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a
^3*d*e - a^3*c*f)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3*d*e - a^3*c*f)*sinh(
d*x + c)^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2
)/b^2) + 2*a) - 144*((a^3*d*e - a^3*c*f)*cosh(d*x + c)^3 + 3*(a^3*d*e - a^3
*c*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3*d*e - a^3*c*f)*cosh(d*x + c)*s
inh(d*x + c)^2 + (a^3*d*e - a^3*c*f)*sinh(d*x + c)^3)*log(2*b*cosh(d*x + c)
 + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 144*((a^3*d*f*x +
 a^3*c*f)*cosh(d*x + c)^3 + 3*(a^3*d*f*x + a^3*c*f)*cosh(d*x + c)^2*sinh(d*
x + c) + 3*(a^3*d*f*x + a^3*c*f)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3*d*f*x
 + a^3*c*f)*sinh(d*x + c)^3)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 144*((a^3*d
*f*x + a^3*c*f)*cosh(d*x + c)^3 + 3*(a^3*d*f*x + a^3*c*f)*cosh(d*x + c)^2*s
inh(d*x + c) + 3*(a^3*d*f*x + a^3*c*f)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3
*d*f*x + a^3*c*f)*sinh(d*x + c)^3)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
- (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 3*(6*
a*b^2*d*f*x - 4*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^5 + 6*a*b^2
*d*e + 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^4 + 3*a*b^2
*f - 24*((4*a^2*b - b^3)*d*f*x + (4*a^2*b - b^3)*d*e - (4*a^2*b - b^3)*f)*c
osh(d*x + c)^3 - 72*(a^3*d^2*f*x^2 + 2*a^3*d^2*e*x + 4*a^3*c*d*e - 2*a^3*c^
2*f)*cosh(d*x + c)^2 + 12*((4*a^2*b - b^3)*d*f*x + (4*a^2*b - b^3)*d*e + (4
*a^2*b - b^3)*f)*cosh(d*x + c))*sinh(d*x + c))/(b^4*d^2*cosh(d*x + c)^3 + 3
*b^4*d^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^4*d^2*cosh(d*x + c)*sinh(d*x +
 c)^2 + b^4*d^2*sinh(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\cosh(dx+c)\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)*sinh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

3.394 $\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{a^2\sinh(c+dx)}{b^3d}-\frac{a^3\log(a+b\sinh(c+dx))}{b^4d}-\frac{a\sinh^2(c+dx)}{2b^2d}+\frac{\sinh^3(c+dx)}{3bd}$$

```
[Out] -((a^3*Log[a + b*Sinh[c + d*x]])/(b^4*d)) + (a^2*Sinh[c + d*x])/(b^3*d) - (
a*Sinh[c + d*x]^2)/(2*b^2*d) + Sinh[c + d*x]^3/(3*b*d)
```

Rubi [A] time = 0.0966879, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 43}

$$\frac{a^2\sinh(c+dx)}{b^3d}-\frac{a^3\log(a+b\sinh(c+dx))}{b^4d}-\frac{a\sinh^2(c+dx)}{2b^2d}+\frac{\sinh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a^3*Log[a + b*Sinh[c + d*x]])/(b^4*d)) + (a^2*Sinh[c + d*x])/(b^3*d) - (
a*Sinh[c + d*x]^2)/(2*b^2*d) + Sinh[c + d*x]^3/(3*b*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{b^3(a+x)}\,dx, x, b\sinh(c+dx)\right)}{bd}\\
&= \frac{\text{Subst}\left(\int \frac{x^3}{a+x}\,dx, x, b\sinh(c+dx)\right)}{b^4d}\\
&= \frac{\text{Subst}\left(\int \left(a^2-ax+x^2-\frac{a^3}{a+x}\right)dx, x, b\sinh(c+dx)\right)}{b^4d}\\
&= -\frac{a^3\log(a+b\sinh(c+dx))}{b^4d}+\frac{a^2\sinh(c+dx)}{b^3d}-\frac{a\sinh^2(c+dx)}{2b^2d}+\frac{\sinh^3(c+dx)}{3bd}
\end{aligned}$$

Mathematica [A] time = 0.15399, size = 66, normalized size = 0.87

$$\frac{6a^2b\sinh(c+dx)-6a^3\log(a+b\sinh(c+dx))-3ab^2\sinh^2(c+dx)+2b^3\sinh^3(c+dx)}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-6*a^3*Log[a + b*Sinh[c + d*x]] + 6*a^2*b*Sinh[c + d*x] - 3*a*b^2*Sinh[c +
 d*x]^2 + 2*b^3*Sinh[c + d*x]^3)/(6*b^4*d)
```

Maple [A] time = 0.016, size = 73, normalized size = 1.

$$-\frac{a^3\ln(a+b\sinh(dx+c))}{b^4d}+\frac{a^2\sinh(dx+c)}{b^3d}-\frac{(\sinh(dx+c))^2a}{2b^2d}+\frac{(\sinh(dx+c))^3}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] -a^3*ln(a+b*sinh(d*x+c))/b^4/d+a^2*sinh(d*x+c)/b^3/d-1/2*a*sinh(d*x+c)^2/b^
2/d+1/3*sinh(d*x+c)^3/b/d
```

Maxima [B] time = 1.12151, size = 231, normalized size = 3.04

$$-\frac{(dx+c)a^3}{b^4d}-\frac{a^3\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{b^4d}-\frac{\left(3\,abe^{(-dx-c)}-b^2-3\left(4\,a^2-b^2\right)e^{(-2\,dx-2\,c)}\right)e^{(3\,dx+3\,c)}}{24\,b^3d}-\frac{3\,abe^{(-2\,d}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -(d*x + c)*a^3/(b^4*d) - a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b
)/(b^4*d) - 1/24*(3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 - b^2)*e^(-2*d*x - 2*
c))*e^(3*d*x + 3*c)/(b^3*d) - 1/24*(3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-3*d*x
- 3*c) + 3*(4*a^2 - b^2)*e^(-d*x - c))/(b^3*d)
```

Fricas [B] time = 2.49711, size = 1484, normalized size = 19.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/24*(b^3*cosh(d*x + c)^6 + b^3*sinh(d*x + c)^6 + 24*a^3*d*x*cosh(d*x + c)^
3 - 3*a*b^2*cosh(d*x + c)^5 + 3*(2*b^3*cosh(d*x + c) - a*b^2)*sinh(d*x + c)
^5 + 3*(4*a^2*b - b^3)*cosh(d*x + c)^4 + 3*(5*b^3*cosh(d*x + c)^2 - 5*a*b^2
*cosh(d*x + c) + 4*a^2*b - b^3)*sinh(d*x + c)^4 - 3*a*b^2*cosh(d*x + c) + 2
```

```
*(10*b^3*cosh(d*x + c)^3 + 12*a^3*d*x - 15*a*b^2*cosh(d*x + c)^2 + 6*(4*a^2
*b - b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - b^3 - 3*(4*a^2*b - b^3)*cosh(d*x
 + c)^2 + 3*(5*b^3*cosh(d*x + c)^4 + 24*a^3*d*x*cosh(d*x + c) - 10*a*b^2*co
sh(d*x + c)^3 - 4*a^2*b + b^3 + 6*(4*a^2*b - b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 - 24*(a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c)^2*sinh(d*x + c) + 3
*a^3*cosh(d*x + c)*sinh(d*x + c)^2 + a^3*sinh(d*x + c)^3)*log(2*(b*sinh(d*x
 + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 3*(2*b^3*cosh(d*x + c)^5 + 24
*a^3*d*x*cosh(d*x + c)^2 - 5*a*b^2*cosh(d*x + c)^4 + 4*(4*a^2*b - b^3)*cosh
(d*x + c)^3 - a*b^2 - 2*(4*a^2*b - b^3)*cosh(d*x + c))*sinh(d*x + c))/(b^4*
d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^4*d*cosh(d*
x + c)*sinh(d*x + c)^2 + b^4*d*sinh(d*x + c)^3)
```

Sympy [A] time = 2.83729, size = 105, normalized size = 1.38

$$\begin{cases} \frac{x \sinh^{3}{\left(c \right)} \cosh{\left(c \right)}}{a} & \text{for}\: b = 0 \wedge d = 0 \\ \frac{\sinh^{4}{\left(c + d x \right)}}{4 a d} & \text{for}\: b = 0 \\ \frac{x \sinh^{3}{\left(c \right)} \cosh{\left(c \right)}}{a + b \sinh{\left(c \right)}} & \text{for}\: d = 0 \\ - \frac{a^{3} \log{\left(\frac{a}{b} + \sinh{\left(c + d x \right)} \right)}}{b^{4} d} + \frac{a^{2} \sinh{\left(c + d x \right)}}{b^{3} d} - \frac{a \sinh^{2}{\left(c + d x \right)}}{2 b^{2} d} + \frac{\sinh^{3}{\left(c + d x \right)}}{3 b d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Piecewise((x*sinh(c)**3*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)**4/
(4*a*d), Eq(b, 0)), (x*sinh(c)**3*cosh(c)/(a + b*sinh(c)), Eq(d, 0)), (-a**
3*log(a/b + sinh(c + d*x))/(b**4*d) + a**2*sinh(c + d*x)/(b**3*d) - a*sinh(
c + d*x)**2/(2*b**2*d) + sinh(c + d*x)**3/(3*b*d), True))
```

Giac [A] time = 1.25732, size = 171, normalized size = 2.25

$$-\frac{a^{3} \log\left(\left|b\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 2\,a\right|\right)}{b^{4} d} + \frac{b^{2} d^{2} \left(e^{(dx+c)} - e^{(-dx-c)}\right)^{3} - 3\,abd^{2} \left(e^{(dx+c)} - e^{(-dx-c)}\right)^{2} + 12\,a^{2} d^{2} \left(e^{(dx+c)} - e^{(-dx-c)}\right)}{24\,b^{3} d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -a^3*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(b^4*d) + 1/24*(b^2*d^2
*(e^(d*x + c) - e^(-d*x - c))^3 - 3*a*b*d^2*(e^(d*x + c) - e^(-d*x - c))^2
+ 12*a^2*d^2*(e^(d*x + c) - e^(-d*x - c)))/(b^3*d^3)
```

3.395 $\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{\sinh^3(c+dx)\cosh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Cosh[c + d*x]*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [A] time = 0.0829812, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[c + d*x]*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Cosh[c + d*x]*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.003, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[c + d*x]*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.162, size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)(\sinh(dx+c))^3}{\left(fx+e\right)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{e^{\left(-3c+\frac{3de}{f}\right)}E_1\left(\frac{3(fx+e)d}{f}\right)}{8bf}-\frac{ae^{\left(-2c+\frac{2de}{f}\right)}E_1\left(\frac{2(fx+e)d}{f}\right)}{4b^2f}+\frac{ae^{\left(2c-\frac{2de}{f}\right)}E_1\left(-\frac{2(fx+e)d}{f}\right)}{4b^2f}-\frac{e^{\left(3c-\frac{3de}{f}\right)}E_1\left(-\frac{3(fx+e)d}{f}\right)}{8bf}-\frac{(4a^2-b^2)e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/8*e^(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b*f) - 1/4*a*e^
(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b^2*f) + 1/4*a*e^(2*c
- 2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b^2*f) - 1/8*e^(3*c - 3*d*e
/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b*f) - 1/8*(4*a^2 - b^2)*e^(-c + d
*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^3*f) - 1/8*(4*a^2*e^c - b^2*e^c)*
e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^3*f) - a^3*log(f*x + e)/(b^
4*f) + 1/16*integrate(-32*(a^4*e^(d*x + c) - a^3*b)/(b^5*f*x + b^5*e - (b^5
*f*x*e^(2*c) + b^5*e*e^(2*c))*e^(2*d*x) - 2*(a*b^4*f*x*e^c + a*b^4*e*e^c)*e
^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx+c)\sinh(dx+c)^3}{afx+ae+(bfx+be)\sinh(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(cosh(d*x + c)*sinh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)\sinh(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate(cosh(d*x + c)*sinh(d*x + c)^3/((f*x + e)*(b*sinh(d*x + c) + a)),
x)
```

3.396 $\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh^3(c+dx)}{a+b\sinh(c+dx)} dx$

Optimal. Leaf size=1038

$$-\frac{(e+fx)^4}{32bf}+\frac{a^2(e+fx)^4}{8b^3f}+\frac{a^4(e+fx)^4}{4b^5f}-\frac{a\cosh^3(c+dx)(e+fx)^3}{3b^2d}-\frac{a^3\cosh(c+dx)(e+fx)^3}{b^4d}-\frac{a^3\sqrt{a^2+b^2}\log\left(\frac{e^{c+dx}}{a-\sqrt{a^2\cdots}}\right.}{b^5d}$$

```
[Out] (3*a^2*e*f^2*x)/(4*b^3*d^2) + (3*a^2*f^3*x^2)/(8*b^3*d^2) + (a^4*(e + f*x)^
4)/(4*b^5*f) + (a^2*(e + f*x)^4)/(8*b^3*f) - (e + f*x)^4/(32*b*f) - (6*a^3*
f^2*(e + f*x)*Cosh[c + d*x])/(b^4*d^3) - (4*a*f^2*(e + f*x)*Cosh[c + d*x])/
(3*b^2*d^3) - (a^3*(e + f*x)^3*Cosh[c + d*x])/(b^4*d) - (3*a^2*f^3*Cosh[c +
 d*x]^2)/(8*b^3*d^4) - (3*a^2*f*(e + f*x)^2*Cosh[c + d*x]^2)/(4*b^3*d^2) -
(2*a*f^2*(e + f*x)*Cosh[c + d*x]^3)/(9*b^2*d^3) - (a*(e + f*x)^3*Cosh[c + d
*x]^3)/(3*b^2*d) - (3*f^3*Cosh[4*c + 4*d*x])/(1024*b*d^4) - (3*f*(e + f*x)^
2*Cosh[4*c + 4*d*x])/(128*b*d^2) - (a^3*Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 +
 (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^5*d) + (a^3*Sqrt[a^2 + b^2]*(e
+ f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^5*d) - (3*a^3*S
qrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/(b^5*d^2) + (3*a^3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^2) + (6*a^3*Sqrt[a^2 + b^2]*f^2*
(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^3) -
 (6*a^3*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/(b^5*d^3) - (6*a^3*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^4) + (6*a^3*Sqrt[a^2 + b^2]*f^3*P
olyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^4) + (6*a^3*f^3
*Sinh[c + d*x])/(b^4*d^4) + (14*a*f^3*Sinh[c + d*x])/(9*b^2*d^4) + (3*a^3*f
*(e + f*x)^2*Sinh[c + d*x])/(b^4*d^2) + (2*a*f*(e + f*x)^2*Sinh[c + d*x])/(
3*b^2*d^2) + (3*a^2*f^2*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^3*d^3)
+ (a^2*(e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^3*d) + (a*f*(e + f*x)^
2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b^2*d^2) + (2*a*f^3*Sinh[c + d*x]^3)/(2
7*b^2*d^4) + (3*f^2*(e + f*x)*Sinh[4*c + 4*d*x])/(256*b*d^3) + ((e + f*x)^3
*Sinh[4*c + 4*d*x])/(32*b*d)
```

Rubi [A] time = 1.82668, antiderivative size = 1038, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 18, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5579, 5448, 3296, 2638, 5447, 3311, 2637, 2633, 32, 3310, 5565, 3322, 2264, 2190, 2531, 6609, 2282, 6589}

$$-\frac{(e+fx)^4}{32bf}+\frac{a^2(e+fx)^4}{8b^3f}+\frac{a^4(e+fx)^4}{4b^5f}-\frac{a\cosh^3(c+dx)(e+fx)^3}{3b^2d}-\frac{a^3\cosh(c+dx)(e+fx)^3}{b^4d}-\frac{a^3\sqrt{a^2+b^2}\log\left(\frac{e^{c+dx}}{a-\sqrt{a^2\cdots}}\right.}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (3*a^2*e*f^2*x)/(4*b^3*d^2) + (3*a^2*f^3*x^2)/(8*b^3*d^2) + (a^4*(e + f*x)^
4)/(4*b^5*f) + (a^2*(e + f*x)^4)/(8*b^3*f) - (e + f*x)^4/(32*b*f) - (6*a^3*
f^2*(e + f*x)*Cosh[c + d*x])/(b^4*d^3) - (4*a*f^2*(e + f*x)*Cosh[c + d*x])/
(3*b^2*d^3) - (a^3*(e + f*x)^3*Cosh[c + d*x])/(b^4*d) - (3*a^2*f^3*Cosh[c +
 d*x]^2)/(8*b^3*d^4) - (3*a^2*f*(e + f*x)^2*Cosh[c + d*x]^2)/(4*b^3*d^2) -
(2*a*f^2*(e + f*x)*Cosh[c + d*x]^3)/(9*b^2*d^3) - (a*(e + f*x)^3*Cosh[c + d
*x]^3)/(3*b^2*d) - (3*f^3*Cosh[4*c + 4*d*x])/(1024*b*d^4) - (3*f*(e + f*x)^
2*Cosh[4*c + 4*d*x])/(128*b*d^2) - (a^3*Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 +
 (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^5*d) + (a^3*Sqrt[a^2 + b^2]*(e
+ f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^5*d) - (3*a^3*S
```

```
qrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/(b^5*d^2) + (3*a^3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^2) + (6*a^3*Sqrt[a^2 + b^2]*f^2*
(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^3) -
 (6*a^3*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]))])/(b^5*d^3) - (6*a^3*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^4) + (6*a^3*Sqrt[a^2 + b^2]*f^3*P
olyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^4) + (6*a^3*f^3
*Sinh[c + d*x])/(b^4*d^4) + (14*a*f^3*Sinh[c + d*x])/(9*b^2*d^4) + (3*a^3*f
*(e + f*x)^2*Sinh[c + d*x])/(b^4*d^2) + (2*a*f*(e + f*x)^2*Sinh[c + d*x])/(
3*b^2*d^2) + (3*a^2*f^2*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^3*d^3)
+ (a^2*(e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^3*d) + (a*f*(e + f*x)^
2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b^2*d^2) + (2*a*f^3*Sinh[c + d*x]^3)/(2
7*b^2*d^4) + (3*f^2*(e + f*x)*Sinh[4*c + 4*d*x])/(256*b*d^3) + ((e + f*x)^3
*Sinh[4*c + 4*d*x])/(32*b*d)
```

Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}}{b}$$

$$= -\frac{a\int (e+fx)^3\cosh^2(c+dx)\sinh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^3\cosh^2(c+dx)\sinh(c}{a+b\sinh(c+dx)}}{b^2}$$

$$= -\frac{(e+fx)^4}{32bf} - \frac{a(e+fx)^3\cosh^3(c+dx)}{3b^2d} + \frac{a^2\int (e+fx)^3\cosh^2(c+dx)\,dx}{b^3}$$

$$= -\frac{(e+fx)^4}{32bf} - \frac{3a^2f(e+fx)^2\cosh^2(c+dx)}{4b^3d^2} - \frac{2af^2(e+fx)\cosh^3(c+dx)}{9b^2d^3}$$

$$= \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+fx)^4}{32bf} - \frac{a^3(e+fx)^3\cosh(c+dx)}{b^4d} - 3a^2f$$

$$= \frac{3a^2ef^2x}{4b^3d^2} + \frac{3a^2f^3x^2}{8b^3d^2} + \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+fx)^4}{32bf} - 4af^2(e+$$

$$= \frac{3a^2ef^2x}{4b^3d^2} + \frac{3a^2f^3x^2}{8b^3d^2} + \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+fx)^4}{32bf} - 6a^3f^2(e+$$

$$= \frac{3a^2ef^2x}{4b^3d^2} + \frac{3a^2f^3x^2}{8b^3d^2} + \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+fx)^4}{32bf} - 6a^3f^2(e+$$

$$= \frac{3a^2ef^2x}{4b^3d^2} + \frac{3a^2f^3x^2}{8b^3d^2} + \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+fx)^4}{32bf} - 6a^3f^2(e+$$

$$= \frac{3a^2ef^2x}{4b^3d^2} + \frac{3a^2f^3x^2}{8b^3d^2} + \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+fx)^4}{32bf} - 6a^3f^2(e+$$

$$= \frac{3a^2ef^2x}{4b^3d^2} + \frac{3a^2f^3x^2}{8b^3d^2} + \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+fx)^4}{32bf} - 6a^3f^2(e+$$

Mathematica [C] time = 29.1008, size = 7520, normalized size = 7.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x
]),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.238, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^3(\cosh(dx+c))^2(\sinh(dx+c))^3}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 4.20574, size = 22876, normalized size = 22.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="fricas")
```

```
[Out] -1/55296*(864*b^4*d^3*f^3*x^3 + 864*b^4*d^3*e^3 + 648*b^4*d^2*e^2*f - 27*(3
2*b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4*d^2*e^2*f + 12*b^4*d*e*f^2 - 3*
b^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3)*x^2 + 12*(8*b^4*d^3*e^2*f - 4*
b^4*d^2*e*f^2 + b^4*d*f^3)*x)*cosh(d*x + c)^8 - 27*(32*b^4*d^3*f^3*x^3 + 32
*b^4*d^3*e^3 - 24*b^4*d^2*e^2*f + 12*b^4*d*e*f^2 - 3*b^4*f^3 + 24*(4*b^4*d^
3*e*f^2 - b^4*d^2*f^3)*x^2 + 12*(8*b^4*d^3*e^2*f - 4*b^4*d^2*e*f^2 + b^4*d*
f^3)*x)*sinh(d*x + c)^8 + 324*b^4*d*e*f^2 + 256*(9*a*b^3*d^3*f^3*x^3 + 9*a*
b^3*d^3*e^3 - 9*a*b^3*d^2*e^2*f + 6*a*b^3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^
3*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2 + 3*(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2
 + 2*a*b^3*d*f^3)*x)*cosh(d*x + c)^7 + 8*(288*a*b^3*d^3*f^3*x^3 + 288*a*b^3
*d^3*e^3 - 288*a*b^3*d^2*e^2*f + 192*a*b^3*d*e*f^2 - 64*a*b^3*f^3 + 288*(3*
a*b^3*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2 + 96*(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*
e*f^2 + 2*a*b^3*d*f^3)*x - 27*(32*b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4
*d^2*e^2*f + 12*b^4*d*e*f^2 - 3*b^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3
)*x^2 + 12*(8*b^4*d^3*e^2*f - 4*b^4*d^2*e*f^2 + b^4*d*f^3)*x)*cosh(d*x + c)
)*sinh(d*x + c)^7 + 81*b^4*f^3 - 1728*(4*a^2*b^2*d^3*f^3*x^3 + 4*a^2*b^2*d^
3*e^3 - 6*a^2*b^2*d^2*e^2*f + 6*a^2*b^2*d*e*f^2 - 3*a^2*b^2*f^3 + 6*(2*a^2*
b^2*d^3*e*f^2 - a^2*b^2*d^2*f^3)*x^2 + 6*(2*a^2*b^2*d^3*e^2*f - 2*a^2*b^2*d
^2*e*f^2 + a^2*b^2*d*f^3)*x)*cosh(d*x + c)^6 - 4*(1728*a^2*b^2*d^3*f^3*x^3
+ 1728*a^2*b^2*d^3*e^3 - 2592*a^2*b^2*d^2*e^2*f + 2592*a^2*b^2*d*e*f^2 - 12
96*a^2*b^2*f^3 + 2592*(2*a^2*b^2*d^3*e*f^2 - a^2*b^2*d^2*f^3)*x^2 + 189*(32
*b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4*d^2*e^2*f + 12*b^4*d*e*f^2 - 3*b
^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3)*x^2 + 12*(8*b^4*d^3*e^2*f - 4*b
^4*d^2*e*f^2 + b^4*d*f^3)*x)*cosh(d*x + c)^2 + 2592*(2*a^2*b^2*d^3*e^2*f -
2*a^2*b^2*d^2*e*f^2 + a^2*b^2*d*f^3)*x - 448*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3
*d^3*e^3 - 9*a*b^3*d^2*e^2*f + 6*a*b^3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d
^3*e*f^2 - a*b^3*d^2*f^3)*x^2 + 3*(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 +
2*a*b^3*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c)^6 + 6912*((4*a^3*b + a*b^3)*
d^3*f^3*x^3 + (4*a^3*b + a*b^3)*d^3*e^3 - 3*(4*a^3*b + a*b^3)*d^2*e^2*f + 6
*(4*a^3*b + a*b^3)*d*e*f^2 - 6*(4*a^3*b + a*b^3)*f^3 + 3*((4*a^3*b + a*b^3)
*d^3*e*f^2 - (4*a^3*b + a*b^3)*d^2*f^3)*x^2 + 3*((4*a^3*b + a*b^3)*d^3*e^2*
f - 2*(4*a^3*b + a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + a*b^3)*d*f^3)*x)*cosh(d*x
+ c)^5 + 24*(288*(4*a^3*b + a*b^3)*d^3*f^3*x^3 + 288*(4*a^3*b + a*b^3)*d^3*
```

```
e^3 - 864*(4*a^3*b + a*b^3)*d^2*e^2*f + 1728*(4*a^3*b + a*b^3)*d*e*f^2 - 17
28*(4*a^3*b + a*b^3)*f^3 - 63*(32*b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4
*d^2*e^2*f + 12*b^4*d*e*f^2 - 3*b^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3
)*x^2 + 12*(8*b^4*d^3*e^2*f - 4*b^4*d^2*e*f^2 + b^4*d*f^3)*x)*cosh(d*x + c)
^3 + 864*((4*a^3*b + a*b^3)*d^3*e*f^2 - (4*a^3*b + a*b^3)*d^2*f^3)*x^2 + 22
4*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 - 9*a*b^3*d^2*e^2*f + 6*a*b^3*d*e*
f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2 + 3*(9*a*b^3*
d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x)*cosh(d*x + c)^2 + 864*((4
*a^3*b + a*b^3)*d^3*e^2*f - 2*(4*a^3*b + a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + a*
b^3)*d*f^3)*x - 432*(4*a^2*b^2*d^3*f^3*x^3 + 4*a^2*b^2*d^3*e^3 - 6*a^2*b^2*
d^2*e^2*f + 6*a^2*b^2*d*e*f^2 - 3*a^2*b^2*f^3 + 6*(2*a^2*b^2*d^3*e*f^2 - a^
2*b^2*d^2*f^3)*x^2 + 6*(2*a^2*b^2*d^3*e^2*f - 2*a^2*b^2*d^2*e*f^2 + a^2*b^2
*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 1728*((8*a^4 + 4*a^2*b^2 - b^4)
*d^4*f^3*x^4 + 4*(8*a^4 + 4*a^2*b^2 - b^4)*d^4*e*f^2*x^3 + 6*(8*a^4 + 4*a^2
*b^2 - b^4)*d^4*e^2*f*x^2 + 4*(8*a^4 + 4*a^2*b^2 - b^4)*d^4*e^3*x)*cosh(d*x
 + c)^4 - 2*(864*(8*a^4 + 4*a^2*b^2 - b^4)*d^4*f^3*x^4 + 3456*(8*a^4 + 4*a^
2*b^2 - b^4)*d^4*e*f^2*x^3 + 5184*(8*a^4 + 4*a^2*b^2 - b^4)*d^4*e^2*f*x^2 +
 3456*(8*a^4 + 4*a^2*b^2 - b^4)*d^4*e^3*x + 945*(32*b^4*d^3*f^3*x^3 + 32*b^
4*d^3*e^3 - 24*b^4*d^2*e^2*f + 12*b^4*d*e*f^2 - 3*b^4*f^3 + 24*(4*b^4*d^3*e
*f^2 - b^4*d^2*f^3)*x^2 + 12*(8*b^4*d^3*e^2*f - 4*b^4*d^2*e*f^2 + b^4*d*f^3
)*x)*cosh(d*x + c)^4 - 4480*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 - 9*a*b^
3*d^2*e^2*f + 6*a*b^3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 - a*b^3*
d^2*f^3)*x^2 + 3*(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x)
*cosh(d*x + c)^3 + 12960*(4*a^2*b^2*d^3*f^3*x^3 + 4*a^2*b^2*d^3*e^3 - 6*a^2
*b^2*d^2*e^2*f + 6*a^2*b^2*d*e*f^2 - 3*a^2*b^2*f^3 + 6*(2*a^2*b^2*d^3*e*f^2
 - a^2*b^2*d^2*f^3)*x^2 + 6*(2*a^2*b^2*d^3*e^2*f - 2*a^2*b^2*d^2*e*f^2 + a^
2*b^2*d*f^3)*x)*cosh(d*x + c)^2 - 17280*((4*a^3*b + a*b^3)*d^3*f^3*x^3 + (4
*a^3*b + a*b^3)*d^3*e^3 - 3*(4*a^3*b + a*b^3)*d^2*e^2*f + 6*(4*a^3*b + a*b^
3)*d*e*f^2 - 6*(4*a^3*b + a*b^3)*f^3 + 3*((4*a^3*b + a*b^3)*d^3*e*f^2 - (4*
a^3*b + a*b^3)*d^2*f^3)*x^2 + 3*((4*a^3*b + a*b^3)*d^3*e^2*f - 2*(4*a^3*b +
 a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + a*b^3)*d*f^3)*x)*cosh(d*x + c))*sinh(d*x +
 c)^4 + 6912*((4*a^3*b + a*b^3)*d^3*f^3*x^3 + (4*a^3*b + a*b^3)*d^3*e^3 + 3
*(4*a^3*b + a*b^3)*d^2*e^2*f + 6*(4*a^3*b + a*b^3)*d*e*f^2 + 6*(4*a^3*b + a
*b^3)*f^3 + 3*((4*a^3*b + a*b^3)*d^3*e*f^2 + (4*a^3*b + a*b^3)*d^2*f^3)*x^2
 + 3*((4*a^3*b + a*b^3)*d^3*e^2*f + 2*(4*a^3*b + a*b^3)*d^2*e*f^2 + 2*(4*a^
3*b + a*b^3)*d*f^3)*x)*cosh(d*x + c)^3 + 8*(864*(4*a^3*b + a*b^3)*d^3*f^3*x
^3 + 864*(4*a^3*b + a*b^3)*d^3*e^3 + 2592*(4*a^3*b + a*b^3)*d^2*e^2*f - 189
*(32*b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4*d^2*e^2*f + 12*b^4*d*e*f^2 -
 3*b^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3)*x^2 + 12*(8*b^4*d^3*e^2*f -
 4*b^4*d^2*e*f^2 + b^4*d*f^3)*x)*cosh(d*x + c)^5 + 5184*(4*a^3*b + a*b^3)*d
*e*f^2 + 1120*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 - 9*a*b^3*d^2*e^2*f +
6*a*b^3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2 +
 3*(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x)*cosh(d*x + c)
^4 + 5184*(4*a^3*b + a*b^3)*f^3 - 4320*(4*a^2*b^2*d^3*f^3*x^3 + 4*a^2*b^2*d
^3*e^3 - 6*a^2*b^2*d^2*e^2*f + 6*a^2*b^2*d*e*f^2 - 3*a^2*b^2*f^3 + 6*(2*a^2
*b^2*d^3*e*f^2 - a^2*b^2*d^2*f^3)*x^2 + 6*(2*a^2*b^2*d^3*e^2*f - 2*a^2*b^2*
d^2*e*f^2 + a^2*b^2*d*f^3)*x)*cosh(d*x + c)^3 + 2592*((4*a^3*b + a*b^3)*d^3
*e*f^2 + (4*a^3*b + a*b^3)*d^2*f^3)*x^2 + 8640*((4*a^3*b + a*b^3)*d^3*f^3*x
^3 + (4*a^3*b + a*b^3)*d^3*e^3 - 3*(4*a^3*b + a*b^3)*d^2*e^2*f + 6*(4*a^3*b
 + a*b^3)*d*e*f^2 - 6*(4*a^3*b + a*b^3)*f^3 + 3*((4*a^3*b + a*b^3)*d^3*e*f^
2 - (4*a^3*b + a*b^3)*d^2*f^3)*x^2 + 3*((4*a^3*b + a*b^3)*d^3*e^2*f - 2*(4*
a^3*b + a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + a*b^3)*d*f^3)*x)*cosh(d*x + c)^2 +
2592*((4*a^3*b + a*b^3)*d^3*e^2*f + 2*(4*a^3*b + a*b^3)*d^2*e*f^2 + 2*(4*a^
3*b + a*b^3)*d*f^3)*x - 864*((8*a^4 + 4*a^2*b^2 - b^4)*d^4*f^3*x^4 + 4*(8*a
^4 + 4*a^2*b^2 - b^4)*d^4*e*f^2*x^3 + 6*(8*a^4 + 4*a^2*b^2 - b^4)*d^4*e^2*f
*x^2 + 4*(8*a^4 + 4*a^2*b^2 - b^4)*d^4*e^3*x)*cosh(d*x + c))*sinh(d*x + c)^
3 + 648*(4*b^4*d^3*e*f^2 + b^4*d^2*f^3)*x^2 + 1728*(4*a^2*b^2*d^3*f^3*x^3 +
 4*a^2*b^2*d^3*e^3 + 6*a^2*b^2*d^2*e^2*f + 6*a^2*b^2*d*e*f^2 + 3*a^2*b^2*f^
3 + 6*(2*a^2*b^2*d^3*e*f^2 + a^2*b^2*d^2*f^3)*x^2 + 6*(2*a^2*b^2*d^3*e^2*f
```

+ 2*a^2*b^2*d^2*e*f^2 + a^2*b^2*d*f^3)*x)*cosh(d*x + c)^2 + 12*(576*a^2*b^2
*d^3*f^3*x^3 + 576*a^2*b^2*d^3*e^3 + 864*a^2*b^2*d^2*e^2*f + 864*a^2*b^2*d*
e*f^2 + 432*a^2*b^2*f^3 - 63*(32*b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4*
d^2*e^2*f + 12*b^4*d*e*f^2 - 3*b^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3)
x^2 + 12(8*b^4*d^3*e^2*f - 4*b^4*d^2*e*f^2 + b^4*d*f^3)*x)*cosh(d*x + c)^
6 + 448*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 - 9*a*b^3*d^2*e^2*f + 6*a*b^
3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2 + 3*(9*
a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x)*cosh(d*x + c)^5 - 2
160*(4*a^2*b^2*d^3*f^3*x^3 + 4*a^2*b^2*d^3*e^3 - 6*a^2*b^2*d^2*e^2*f + 6*a^
2*b^2*d*e*f^2 - 3*a^2*b^2*f^3 + 6*(2*a^2*b^2*d^3*e*f^2 - a^2*b^2*d^2*f^3)*x
^2 + 6*(2*a^2*b^2*d^3*e^2*f - 2*a^2*b^2*d^2*e*f^2 + a^2*b^2*d*f^3)*x)*cosh(
d*x + c)^4 + 5760*((4*a^3*b + a*b^3)*d^3*f^3*x^3 + (4*a^3*b + a*b^3)*d^3*e^
3 - 3*(4*a^3*b + a*b^3)*d^2*e^2*f + 6*(4*a^3*b + a*b^3)*d*e*f^2 - 6*(4*a^3*
b + a*b^3)*f^3 + 3*((4*a^3*b + a*b^3)*d^3*e*f^2 - (4*a^3*b + a*b^3)*d^2*f^3
)*x^2 + 3*((4*a^3*b + a*b^3)*d^3*e^2*f - 2*(4*a^3*b + a*b^3)*d^2*e*f^2 + 2*
(4*a^3*b + a*b^3)*d*f^3)*x)*cosh(d*x + c)^3 + 864*(2*a^2*b^2*d^3*e*f^2 + a^
2*b^2*d^2*f^3)*x^2 - 864*((8*a^4 + 4*a^2*b^2 - b^4)*d^4*f^3*x^4 + 4*(8*a^4
+ 4*a^2*b^2 - b^4)*d^4*e*f^2*x^3 + 6*(8*a^4 + 4*a^2*b^2 - b^4)*d^4*e^2*f*x^
2 + 4*(8*a^4 + 4*a^2*b^2 - b^4)*d^4*e^3*x)*cosh(d*x + c)^2 + 864*(2*a^2*b^2
*d^3*e^2*f + 2*a^2*b^2*d^2*e*f^2 + a^2*b^2*d*f^3)*x + 1728*((4*a^3*b + a*b^
3)*d^3*f^3*x^3 + (4*a^3*b + a*b^3)*d^3*e^3 + 3*(4*a^3*b + a*b^3)*d^2*e^2*f
+ 6*(4*a^3*b + a*b^3)*d*e*f^2 + 6*(4*a^3*b + a*b^3)*f^3 + 3*((4*a^3*b + a*b
^3)*d^3*e*f^2 + (4*a^3*b + a*b^3)*d^2*f^3)*x^2 + 3*((4*a^3*b + a*b^3)*d^3*e
^2*f + 2*(4*a^3*b + a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + a*b^3)*d*f^3)*x)*cosh(d
*x + c))*sinh(d*x + c)^2 + 165888*((a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x
+ a^3*b*d^2*e^2*f)*cosh(d*x + c)^4 + 4*(a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*
f^2*x + a^3*b*d^2*e^2*f)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3*b*d^2*f^3*x
^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*f)*cosh(d*x + c)^2*sinh(d*x + c)^2
+ 4*(a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*f)*cosh(d*x +
c)*sinh(d*x + c)^3 + (a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*
e^2*f)*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*si
nh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b
)/b + 1) - 165888*((a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2
*f)*cosh(d*x + c)^4 + 4*(a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^
2*e^2*f)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2
*e*f^2*x + a^3*b*d^2*e^2*f)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3*b*d^2*
f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c
)^3 + (a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*f)*sinh(d*x
+ c)^4)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 55296
*((a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^
3)*cosh(d*x + c)^4 + 4*(a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d
*e*f^2 - a^3*b*c^3*f^3)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3*b*d^3*e^3 -
3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*cosh(d*x + c)^2*
sinh(d*x + c)^2 + 4*(a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*
f^2 - a^3*b*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b*d^3*e^3 - 3*a^3
*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*sinh(d*x + c)^4)*sqrt
((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2
+ b^2)/b^2) + 2*a) + 55296*((a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b
*c^2*d*e*f^2 - a^3*b*c^3*f^3)*cosh(d*x + c)^4 + 4*(a^3*b*d^3*e^3 - 3*a^3*b*
c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*cosh(d*x + c)^3*sinh(d*x
+ c) + 6*(a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*
b*c^3*f^3)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3*b*d^3*e^3 - 3*a^3*b*c*d
^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c)
^3 + (a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3
*f^3)*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*si
nh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 55296*((a^3*b*d^3*f^3*x^3
+ 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^2*e^2*f - 3*a^3
*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*cosh(d*x + c)^4 + 4*(a^3*b*d^3*f^3*x^3 + 3*

```
a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^2*e^2*f - 3*a^3*b*c
^2*d*e*f^2 + a^3*b*c^3*f^3)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3*b*d^3*f^
3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^2*e^2*f -
 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(
a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c
*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*cosh(d*x + c)*sinh(d*x +
c)^3 + (a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3
*a^3*b*c*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*sinh(d*x + c)^4)*
sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 55296*((a^3*b*d^3*
f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^2*e^2*f
 - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*cosh(d*x + c)^4 + 4*(a^3*b*d^3*f^3*
x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^2*e^2*f - 3
*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3*
b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^2
*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*cosh(d*x + c)^2*sinh(d*x + c)
^2 + 4*(a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3
*a^3*b*c*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*cosh(d*x + c)*sin
h(d*x + c)^3 + (a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2
*f*x + 3*a^3*b*c*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*sinh(d*x
+ c)^4)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 331776*(a^
3*b*f^3*cosh(d*x + c)^4 + 4*a^3*b*f^3*cosh(d*x + c)^3*sinh(d*x + c) + 6*a^3
*b*f^3*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^3*b*f^3*cosh(d*x + c)*sinh(d*x
 + c)^3 + a^3*b*f^3*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*co
sh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2))/b) - 331776*(a^3*b*f^3*cosh(d*x + c)^4 + 4*a^3*b*f^3*cosh(d
*x + c)^3*sinh(d*x + c) + 6*a^3*b*f^3*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a
^3*b*f^3*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*b*f^3*sinh(d*x + c)^4)*sqrt((a
^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 331776*((a^3*b*d*f^3*x
+ a^3*b*d*e*f^2)*cosh(d*x + c)^4 + 4*(a^3*b*d*f^3*x + a^3*b*d*e*f^2)*cosh(d
*x + c)^3*sinh(d*x + c) + 6*(a^3*b*d*f^3*x + a^3*b*d*e*f^2)*cosh(d*x + c)^2
*sinh(d*x + c)^2 + 4*(a^3*b*d*f^3*x + a^3*b*d*e*f^2)*cosh(d*x + c)*sinh(d*x
 + c)^3 + (a^3*b*d*f^3*x + a^3*b*d*e*f^2)*sinh(d*x + c)^4)*sqrt((a^2 + b^2)
/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 331776*((a^3*b*d*f^3*x + a^3*b*d
*e*f^2)*cosh(d*x + c)^4 + 4*(a^3*b*d*f^3*x + a^3*b*d*e*f^2)*cosh(d*x + c)^3
*sinh(d*x + c) + 6*(a^3*b*d*f^3*x + a^3*b*d*e*f^2)*cosh(d*x + c)^2*sinh(d*x
 + c)^2 + 4*(a^3*b*d*f^3*x + a^3*b*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a^3*b*d*f^3*x + a^3*b*d*e*f^2)*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*pol
ylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2))/b) + 324*(8*b^4*d^3*e^2*f + 4*b^4*d^2*e*f^2 +
b^4*d*f^3)*x + 256*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 + 9*a*b^3*d^2*e^2
*f + 6*a*b^3*d*e*f^2 + 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 + a*b^3*d^2*f^3)*
x^2 + 3*(9*a*b^3*d^3*e^2*f + 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x)*cosh(d*x
 + c) + 8*(288*a*b^3*d^3*f^3*x^3 + 288*a*b^3*d^3*e^3 + 288*a*b^3*d^2*e^2*f
+ 192*a*b^3*d*e*f^2 - 27*(32*b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4*d^2*
e^2*f + 12*b^4*d*e*f^2 - 3*b^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3)*x^2
 + 12*(8*b^4*d^3*e^2*f - 4*b^4*d^2*e*f^2 + b^4*d*f^3)*x)*cosh(d*x + c)^7 +
64*a*b^3*f^3 + 224*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 - 9*a*b^3*d^2*e^2
*f + 6*a*b^3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 - a*b^3*d^2*f^3)*
x^2 + 3*(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x)*cosh(d*x
 + c)^6 - 1296*(4*a^2*b^2*d^3*f^3*x^3 + 4*a^2*b^2*d^3*e^3 - 6*a^2*b^2*d^2*e
^2*f + 6*a^2*b^2*d*e*f^2 - 3*a^2*b^2*f^3 + 6*(2*a^2*b^2*d^3*e*f^2 - a^2*b^2
*d^2*f^3)*x^2 + 6*(2*a^2*b^2*d^3*e^2*f - 2*a^2*b^2*d^2*e*f^2 + a^2*b^2*d*f^
3)*x)*cosh(d*x + c)^5 + 4320*((4*a^3*b + a*b^3)*d^3*f^3*x^3 + (4*a^3*b + a*
b^3)*d^3*e^3 - 3*(4*a^3*b + a*b^3)*d^2*e^2*f + 6*(4*a^3*b + a*b^3)*d*e*f^2
- 6*(4*a^3*b + a*b^3)*f^3 + 3*((4*a^3*b + a*b^3)*d^3*e*f^2 - (4*a^3*b + a*b
```

```
^3)*d^2*f^3)*x^2 + 3*((4*a^3*b + a*b^3)*d^3*e^2*f - 2*(4*a^3*b + a*b^3)*d^2
*e*f^2 + 2*(4*a^3*b + a*b^3)*d*f^3)*x)*cosh(d*x + c)^4 - 864*((8*a^4 + 4*a^
2*b^2 - b^4)*d^4*f^3*x^4 + 4*(8*a^4 + 4*a^2*b^2 - b^4)*d^4*e*f^2*x^3 + 6*(8
*a^4 + 4*a^2*b^2 - b^4)*d^4*e^2*f*x^2 + 4*(8*a^4 + 4*a^2*b^2 - b^4)*d^4*e^3
*x)*cosh(d*x + c)^3 + 288*(3*a*b^3*d^3*e*f^2 + a*b^3*d^2*f^3)*x^2 + 2592*((
4*a^3*b + a*b^3)*d^3*f^3*x^3 + (4*a^3*b + a*b^3)*d^3*e^3 + 3*(4*a^3*b + a*b
^3)*d^2*e^2*f + 6*(4*a^3*b + a*b^3)*d*e*f^2 + 6*(4*a^3*b + a*b^3)*f^3 + 3*(
(4*a^3*b + a*b^3)*d^3*e*f^2 + (4*a^3*b + a*b^3)*d^2*f^3)*x^2 + 3*((4*a^3*b
+ a*b^3)*d^3*e^2*f + 2*(4*a^3*b + a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + a*b^3)*d*
f^3)*x)*cosh(d*x + c)^2 + 96*(9*a*b^3*d^3*e^2*f + 6*a*b^3*d^2*e*f^2 + 2*a*b
^3*d*f^3)*x + 432*(4*a^2*b^2*d^3*f^3*x^3 + 4*a^2*b^2*d^3*e^3 + 6*a^2*b^2*d^
2*e^2*f + 6*a^2*b^2*d*e*f^2 + 3*a^2*b^2*f^3 + 6*(2*a^2*b^2*d^3*e*f^2 + a^2*
b^2*d^2*f^3)*x^2 + 6*(2*a^2*b^2*d^3*e^2*f + 2*a^2*b^2*d^2*e*f^2 + a^2*b^2*d
*f^3)*x)*cosh(d*x + c))*sinh(d*x + c))/(b^5*d^4*cosh(d*x + c)^4 + 4*b^5*d^4
*cosh(d*x + c)^3*sinh(d*x + c) + 6*b^5*d^4*cosh(d*x + c)^2*sinh(d*x + c)^2
+ 4*b^5*d^4*cosh(d*x + c)*sinh(d*x + c)^3 + b^5*d^4*sinh(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**2*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \cosh(dx+c)^2 \sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="giac")

[Out] integrate((f*x + e)^3*cosh(d*x + c)^2*sinh(d*x + c)^3/(b*sinh(d*x + c) + a)
, x)
```

3.397 $\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=755

$$-\frac{2a^3f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^2}+\frac{2a^3f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^5d^2}+\frac{2a^3f^2\sqrt{a^2+b^2}\text{Poly}}{b^5}$$

```
[Out] (a^2*f^2*x)/(4*b^3*d^2) + (a^4*(e + f*x)^3)/(3*b^5*f) + (a^2*(e + f*x)^3)/(
6*b^3*f) - (e + f*x)^3/(24*b*f) - (2*a^3*f^2*Cosh[c + d*x])/(b^4*d^3) - (4*
a*f^2*Cosh[c + d*x])/(9*b^2*d^3) - (a^3*(e + f*x)^2*Cosh[c + d*x])/(b^4*d)
- (a^2*f*(e + f*x)*Cosh[c + d*x]^2)/(2*b^3*d^2) - (2*a*f^2*Cosh[c + d*x]^3)
/(27*b^2*d^3) - (a*(e + f*x)^2*Cosh[c + d*x]^3)/(3*b^2*d) - (f*(e + f*x)*Co
sh[4*c + 4*d*x])/(64*b*d^2) - (a^3*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^5*d) + (a^3*Sqrt[a^2 + b^2]*(e + f*x
)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^5*d) - (2*a^3*Sqrt[a
^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])
/(b^5*d^2) + (2*a^3*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/(b^5*d^2) + (2*a^3*Sqrt[a^2 + b^2]*f^2*PolyLog[3
, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^3) - (2*a^3*Sqrt[a^2 +
b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^3) +
(2*a^3*f*(e + f*x)*Sinh[c + d*x])/(b^4*d^2) + (4*a*f*(e + f*x)*Sinh[c + d*x
])/(9*b^2*d^2) + (a^2*f^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^3*d^3) + (a^2*(
e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^3*d) + (2*a*f*(e + f*x)*Cosh[c
 + d*x]^2*Sinh[c + d*x])/(9*b^2*d^2) + (f^2*Sinh[4*c + 4*d*x])/(256*b*d^3)
+ ((e + f*x)^2*Sinh[4*c + 4*d*x])/(32*b*d)
```

Rubi [A] time = 1.51398, antiderivative size = 755, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 18, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5579, 5448, 3296, 2637, 5447, 3310, 2638, 3311, 32, 2635, 8, 5565, 3322, 2264, 2190, 2531, 2282, 6589}

$$-\frac{2a^3f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^2}+\frac{2a^3f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^5d^2}+\frac{2a^3f^2\sqrt{a^2+b^2}\text{Poly}}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*f^2*x)/(4*b^3*d^2) + (a^4*(e + f*x)^3)/(3*b^5*f) + (a^2*(e + f*x)^3)/(
6*b^3*f) - (e + f*x)^3/(24*b*f) - (2*a^3*f^2*Cosh[c + d*x])/(b^4*d^3) - (4*
a*f^2*Cosh[c + d*x])/(9*b^2*d^3) - (a^3*(e + f*x)^2*Cosh[c + d*x])/(b^4*d)
- (a^2*f*(e + f*x)*Cosh[c + d*x]^2)/(2*b^3*d^2) - (2*a*f^2*Cosh[c + d*x]^3)
/(27*b^2*d^3) - (a*(e + f*x)^2*Cosh[c + d*x]^3)/(3*b^2*d) - (f*(e + f*x)*Co
sh[4*c + 4*d*x])/(64*b*d^2) - (a^3*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^5*d) + (a^3*Sqrt[a^2 + b^2]*(e + f*x
)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^5*d) - (2*a^3*Sqrt[a
^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])
/(b^5*d^2) + (2*a^3*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/(b^5*d^2) + (2*a^3*Sqrt[a^2 + b^2]*f^2*PolyLog[3
, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^3) - (2*a^3*Sqrt[a^2 +
b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^3) +
(2*a^3*f*(e + f*x)*Sinh[c + d*x])/(b^4*d^2) + (4*a*f*(e + f*x)*Sinh[c + d*x
])/(9*b^2*d^2) + (a^2*f^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^3*d^3) + (a^2*(
e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^3*d) + (2*a*f*(e + f*x)*Cosh[c
 + d*x]^2*Sinh[c + d*x])/(9*b^2*d^2) + (f^2*Sinh[4*c + 4*d*x])/(256*b*d^3)
```

```
+ ((e + f*x)^2*Sinh[4*c + 4*d*x])/(32*b*d)
```

Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh^2(c+dx)\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\cosh^2(c+dx)\sinh^2(}{a+b\sinh(c+dx)}}{b}$$

$$= -\frac{a\int (e+fx)^2\cosh^2(c+dx)\sinh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^2\cosh^2(c+dx)\sin}{a+b\sinh(c+dx)}}{b^2}$$

$$= -\frac{(e+fx)^3}{24bf} - \frac{a(e+fx)^2\cosh^3(c+dx)}{3b^2d} + \frac{a^2\int (e+fx)^2\cosh^2(c+dx)\,d}{b^3}$$

$$= -\frac{(e+fx)^3}{24bf} - \frac{a^2f(e+fx)\cosh^2(c+dx)}{2b^3d^2} - \frac{2af^2\cosh^3(c+dx)}{27b^2d^3} - a(e+$$

$$= \frac{a^4(e+fx)^3}{3b^5f} + \frac{a^2(e+fx)^3}{6b^3f} - \frac{(e+fx)^3}{24bf} - \frac{a^3(e+fx)^2\cosh(c+dx)}{b^4d} - a$$

$$= \frac{a^2f^2x}{4b^3d^2} + \frac{a^4(e+fx)^3}{3b^5f} + \frac{a^2(e+fx)^3}{6b^3f} - \frac{(e+fx)^3}{24bf} - \frac{4af^2\cosh(c+dx)}{9b^2d^3} -$$

$$= \frac{a^2f^2x}{4b^3d^2} + \frac{a^4(e+fx)^3}{3b^5f} + \frac{a^2(e+fx)^3}{6b^3f} - \frac{(e+fx)^3}{24bf} - \frac{2a^3f^2\cosh(c+dx)}{b^4d^3}$$

$$= \frac{a^2f^2x}{4b^3d^2} + \frac{a^4(e+fx)^3}{3b^5f} + \frac{a^2(e+fx)^3}{6b^3f} - \frac{(e+fx)^3}{24bf} - \frac{2a^3f^2\cosh(c+dx)}{b^4d^3}$$

$$= \frac{a^2f^2x}{4b^3d^2} + \frac{a^4(e+fx)^3}{3b^5f} + \frac{a^2(e+fx)^3}{6b^3f} - \frac{(e+fx)^3}{24bf} - \frac{2a^3f^2\cosh(c+dx)}{b^4d^3}$$

$$= \frac{a^2f^2x}{4b^3d^2} + \frac{a^4(e+fx)^3}{3b^5f} + \frac{a^2(e+fx)^3}{6b^3f} - \frac{(e+fx)^3}{24bf} - \frac{2a^3f^2\cosh(c+dx)}{b^4d^3}$$

Mathematica [C] time = 19.5215, size = 5115, normalized size = 6.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x
]),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.198, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \left(\cosh \left(dx+c\right)\right)^{2} \left(\sinh \left(dx+c\right)\right)^{3}}{a+b\sinh \left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.29762, size = 14272, normalized size = 18.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="fricas")
```

```
[Out] -1/13824*(216*b^4*d^2*f^2*x^2 - 27*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b
^4*d*e*f + b^4*f^2 + 4*(4*b^4*d^2*e*f - b^4*d*f^2)*x)*cosh(d*x + c)^8 - 27*
(8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b^4*d*e*f + b^4*f^2 + 4*(4*b^4*d^2*e
*f - b^4*d*f^2)*x)*sinh(d*x + c)^8 + 216*b^4*d^2*e^2 + 64*(9*a*b^3*d^2*f^2*
x^2 + 9*a*b^3*d^2*e^2 - 6*a*b^3*d*e*f + 2*a*b^3*f^2 + 6*(3*a*b^3*d^2*e*f -
a*b^3*d*f^2)*x)*cosh(d*x + c)^7 + 8*(72*a*b^3*d^2*f^2*x^2 + 72*a*b^3*d^2*e^
2 - 48*a*b^3*d*e*f + 16*a*b^3*f^2 + 48*(3*a*b^3*d^2*e*f - a*b^3*d*f^2)*x -
27*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b^4*d*e*f + b^4*f^2 + 4*(4*b^4*d^
2*e*f - b^4*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^7 + 108*b^4*d*e*f - 864*
(2*a^2*b^2*d^2*f^2*x^2 + 2*a^2*b^2*d^2*e^2 - 2*a^2*b^2*d*e*f + a^2*b^2*f^2
+ 2*(2*a^2*b^2*d^2*e*f - a^2*b^2*d*f^2)*x)*cosh(d*x + c)^6 - 4*(432*a^2*b^2
*d^2*f^2*x^2 + 432*a^2*b^2*d^2*e^2 - 432*a^2*b^2*d*e*f + 216*a^2*b^2*f^2 +
189*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b^4*d*e*f + b^4*f^2 + 4*(4*b^4*d
^2*e*f - b^4*d*f^2)*x)*cosh(d*x + c)^2 + 432*(2*a^2*b^2*d^2*e*f - a^2*b^2*d
*f^2)*x - 112*(9*a*b^3*d^2*f^2*x^2 + 9*a*b^3*d^2*e^2 - 6*a*b^3*d*e*f + 2*a*
b^3*f^2 + 6*(3*a*b^3*d^2*e*f - a*b^3*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)
^6 + 27*b^4*f^2 + 1728*((4*a^3*b + a*b^3)*d^2*f^2*x^2 + (4*a^3*b + a*b^3)*d
^2*e^2 - 2*(4*a^3*b + a*b^3)*d*e*f + 2*(4*a^3*b + a*b^3)*f^2 + 2*((4*a^3*b
+ a*b^3)*d^2*e*f - (4*a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c)^5 + 24*(72*(4*
a^3*b + a*b^3)*d^2*f^2*x^2 + 72*(4*a^3*b + a*b^3)*d^2*e^2 - 144*(4*a^3*b +
a*b^3)*d*e*f - 63*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b^4*d*e*f + b^4*f^
2 + 4*(4*b^4*d^2*e*f - b^4*d*f^2)*x)*cosh(d*x + c)^3 + 144*(4*a^3*b + a*b^3
```

)*f^2 + 56*(9*a*b^3*d^2*f^2*x^2 + 9*a*b^3*d^2*e^2 - 6*a*b^3*d*e*f + 2*a*b^3
f^2 + 6(3*a*b^3*d^2*e*f - a*b^3*d*f^2)*x)*cosh(d*x + c)^2 + 144*((4*a^3*b
 + a*b^3)*d^2*e*f - (4*a^3*b + a*b^3)*d*f^2)*x - 216*(2*a^2*b^2*d^2*f^2*x^2
 + 2*a^2*b^2*d^2*e^2 - 2*a^2*b^2*d*e*f + a^2*b^2*f^2 + 2*(2*a^2*b^2*d^2*e*f
 - a^2*b^2*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 576*((8*a^4 + 4*a^2*b
^2 - b^4)*d^3*f^2*x^3 + 3*(8*a^4 + 4*a^2*b^2 - b^4)*d^3*e*f*x^2 + 3*(8*a^4
+ 4*a^2*b^2 - b^4)*d^3*e^2*x)*cosh(d*x + c)^4 - 2*(288*(8*a^4 + 4*a^2*b^2 -
 b^4)*d^3*f^2*x^3 + 864*(8*a^4 + 4*a^2*b^2 - b^4)*d^3*e*f*x^2 + 864*(8*a^4
+ 4*a^2*b^2 - b^4)*d^3*e^2*x + 945*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b
^4*d*e*f + b^4*f^2 + 4*(4*b^4*d^2*e*f - b^4*d*f^2)*x)*cosh(d*x + c)^4 - 112
0*(9*a*b^3*d^2*f^2*x^2 + 9*a*b^3*d^2*e^2 - 6*a*b^3*d*e*f + 2*a*b^3*f^2 + 6*
(3*a*b^3*d^2*e*f - a*b^3*d*f^2)*x)*cosh(d*x + c)^3 + 6480*(2*a^2*b^2*d^2*f^
2*x^2 + 2*a^2*b^2*d^2*e^2 - 2*a^2*b^2*d*e*f + a^2*b^2*f^2 + 2*(2*a^2*b^2*d^
2*e*f - a^2*b^2*d*f^2)*x)*cosh(d*x + c)^2 - 4320*((4*a^3*b + a*b^3)*d^2*f^2
*x^2 + (4*a^3*b + a*b^3)*d^2*e^2 - 2*(4*a^3*b + a*b^3)*d*e*f + 2*(4*a^3*b +
 a*b^3)*f^2 + 2*((4*a^3*b + a*b^3)*d^2*e*f - (4*a^3*b + a*b^3)*d*f^2)*x)*co
sh(d*x + c))*sinh(d*x + c)^4 + 1728*((4*a^3*b + a*b^3)*d^2*f^2*x^2 + (4*a^3
*b + a*b^3)*d^2*e^2 + 2*(4*a^3*b + a*b^3)*d*e*f + 2*(4*a^3*b + a*b^3)*f^2 +
 2*((4*a^3*b + a*b^3)*d^2*e*f + (4*a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c)^3
 + 8*(216*(4*a^3*b + a*b^3)*d^2*f^2*x^2 - 189*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^
2*e^2 - 4*b^4*d*e*f + b^4*f^2 + 4*(4*b^4*d^2*e*f - b^4*d*f^2)*x)*cosh(d*x +
 c)^5 + 216*(4*a^3*b + a*b^3)*d^2*e^2 + 280*(9*a*b^3*d^2*f^2*x^2 + 9*a*b^3*
d^2*e^2 - 6*a*b^3*d*e*f + 2*a*b^3*f^2 + 6*(3*a*b^3*d^2*e*f - a*b^3*d*f^2)*x
)*cosh(d*x + c)^4 + 432*(4*a^3*b + a*b^3)*d*e*f - 2160*(2*a^2*b^2*d^2*f^2*x
^2 + 2*a^2*b^2*d^2*e^2 - 2*a^2*b^2*d*e*f + a^2*b^2*f^2 + 2*(2*a^2*b^2*d^2*e
*f - a^2*b^2*d*f^2)*x)*cosh(d*x + c)^3 + 432*(4*a^3*b + a*b^3)*f^2 + 2160*(
(4*a^3*b + a*b^3)*d^2*f^2*x^2 + (4*a^3*b + a*b^3)*d^2*e^2 - 2*(4*a^3*b + a*
b^3)*d*e*f + 2*(4*a^3*b + a*b^3)*f^2 + 2*((4*a^3*b + a*b^3)*d^2*e*f - (4*a^
3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c)^2 + 432*((4*a^3*b + a*b^3)*d^2*e*f + (
4*a^3*b + a*b^3)*d*f^2)*x - 288*((8*a^4 + 4*a^2*b^2 - b^4)*d^3*f^2*x^3 + 3*
(8*a^4 + 4*a^2*b^2 - b^4)*d^3*e*f*x^2 + 3*(8*a^4 + 4*a^2*b^2 - b^4)*d^3*e^2
*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 864*(2*a^2*b^2*d^2*f^2*x^2 + 2*a^2*b^2
*d^2*e^2 + 2*a^2*b^2*d*e*f + a^2*b^2*f^2 + 2*(2*a^2*b^2*d^2*e*f + a^2*b^2*d
*f^2)*x)*cosh(d*x + c)^2 + 12*(144*a^2*b^2*d^2*f^2*x^2 + 144*a^2*b^2*d^2*e^
2 + 144*a^2*b^2*d*e*f - 63*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b^4*d*e*f
 + b^4*f^2 + 4*(4*b^4*d^2*e*f - b^4*d*f^2)*x)*cosh(d*x + c)^6 + 72*a^2*b^2*
f^2 + 112*(9*a*b^3*d^2*f^2*x^2 + 9*a*b^3*d^2*e^2 - 6*a*b^3*d*e*f + 2*a*b^3*
f^2 + 6*(3*a*b^3*d^2*e*f - a*b^3*d*f^2)*x)*cosh(d*x + c)^5 - 1080*(2*a^2*b^
2*d^2*f^2*x^2 + 2*a^2*b^2*d^2*e^2 - 2*a^2*b^2*d*e*f + a^2*b^2*f^2 + 2*(2*a^
2*b^2*d^2*e*f - a^2*b^2*d*f^2)*x)*cosh(d*x + c)^4 + 1440*((4*a^3*b + a*b^3)
*d^2*f^2*x^2 + (4*a^3*b + a*b^3)*d^2*e^2 - 2*(4*a^3*b + a*b^3)*d*e*f + 2*(4
*a^3*b + a*b^3)*f^2 + 2*((4*a^3*b + a*b^3)*d^2*e*f - (4*a^3*b + a*b^3)*d*f^
2)*x)*cosh(d*x + c)^3 - 288*((8*a^4 + 4*a^2*b^2 - b^4)*d^3*f^2*x^3 + 3*(8*a
^4 + 4*a^2*b^2 - b^4)*d^3*e*f*x^2 + 3*(8*a^4 + 4*a^2*b^2 - b^4)*d^3*e^2*x)*
cosh(d*x + c)^2 + 144*(2*a^2*b^2*d^2*e*f + a^2*b^2*d*f^2)*x + 432*((4*a^3*b
 + a*b^3)*d^2*f^2*x^2 + (4*a^3*b + a*b^3)*d^2*e^2 + 2*(4*a^3*b + a*b^3)*d*e
f + 2(4*a^3*b + a*b^3)*f^2 + 2*((4*a^3*b + a*b^3)*d^2*e*f + (4*a^3*b + a*
b^3)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 27648*((a^3*b*d*f^2*x + a^3
*b*d*e*f)*cosh(d*x + c)^4 + 4*(a^3*b*d*f^2*x + a^3*b*d*e*f)*cosh(d*x + c)^3
*sinh(d*x + c) + 6*(a^3*b*d*f^2*x + a^3*b*d*e*f)*cosh(d*x + c)^2*sinh(d*x +
 c)^2 + 4*(a^3*b*d*f^2*x + a^3*b*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^
3*b*d*f^2*x + a^3*b*d*e*f)*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*dilog((a*
cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2) - b)/b + 1) - 27648*((a^3*b*d*f^2*x + a^3*b*d*e*f)*cosh(d*
x + c)^4 + 4*(a^3*b*d*f^2*x + a^3*b*d*e*f)*cosh(d*x + c)^3*sinh(d*x + c) +
6*(a^3*b*d*f^2*x + a^3*b*d*e*f)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3*b*
d*f^2*x + a^3*b*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b*d*f^2*x + a^3
*b*d*e*f)*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a
*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)

- b)/b + 1) - 13824*((a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*cosh(d*x + c)^4 + 4*(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 13824*((a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*cosh(d*x + c)^4 + 4*(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 13824*((a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*cosh(d*x + c)^4 + 4*(a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 13824*((a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*cosh(d*x + c)^4 + 4*(a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 27648*(a^3*b*f^2*cosh(d*x + c)^4 + 4*a^3*b*f^2*cosh(d*x + c)^3*sinh(d*x + c) + 6*a^3*b*f^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^3*b*f^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*b*f^2*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 27648*(a^3*b*f^2*cosh(d*x + c)^4 + 4*a^3*b*f^2*cosh(d*x + c)^3*sinh(d*x + c) + 6*a^3*b*f^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^3*b*f^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*b*f^2*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 108*(4*b^4*d^2*e*f + b^4*d*f^2)*x + 64*(9*a*b^3*d^2*f^2*x^2 + 9*a*b^3*d^2*e^2 + 6*a*b^3*d*e*f + 2*a*b^3*f^2 + 6*(3*a*b^3*d^2*e*f + a*b^3*d*f^2)*x)*cosh(d*x + c) + 8*(72*a*b^3*d^2*f^2*x^2 + 72*a*b^3*d^2*e^2 - 27*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b^4*d*e*f + b^4*f^2 + 4*(4*b^4*d^2*e*f - b^4*d*f^2)*x)*cosh(d*x + c)^7 + 48*a*b^3*d*e*f + 56*(9*a*b^3*d^2*f^2*x^2 + 9*a*b^3*d^2*e^2 - 6*a*b^3*d*e*f + 2*a*b^3*f^2 + 6*(3*a*b^3*d^2*e*f - a*b^3*d*f^2)*x)*cosh(d*x + c)^6 + 16*a*b^3*f^2 - 648*(2*a^2*b^2*d^2*f^2*x^2 + 2*a^2*b^2*d^2*e^2 - 2*a^2*b^2*d*e*f + a^2*b^2*f^2 + 2*(2*a^2*b^2*d^2*e*f - a^2*b^2*d*f^2)*x)*cosh(d*x + c)^5 + 1080*((4*a^3*b + a*b^3)*d^2*f^2*x^2 + (4*a^3*b + a*b^3)*d^2*e^2 - 2*(4*a^3*b + a*b^3)*d*e*f + 2*(4*a^3*b + a*b^3)*f^2 + 2*((4*a^3*b + a*b^3)*d^2*e*f - (4*a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c)^4 - 288*((8*a^4 + 4*a^2*b^2 - b^4)*d^3*f^2*x^3 + 3*(8*a^4 + 4*a^2*b^2 - b^4)*d^3*e*f*x^2 + 3*(8*a^4 + 4*a^2*b^2 - b^4)*d^3*e^2*x)*cosh(d*x + c)^3 + 648*((4*a^3*b + a*b^3)*d^2*f^2*x^2 + (4*a^3*b + a*b^3)*d^2*e^2 + 2*(4*a^3*b + a*b^3)*d*e*f + 2*(4*a^3*b + a*b^3)*f^2 + 2*((4*a^3*b + a*b^3)*d^2*e*f + (4*a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c)^2 + 48*(3*a*b^3*d^2*e*f + a*b^3*d*f^2)*x + 216*(2*a^2*b^2*d^2*f^2*x^2 + 2*a^2*b^2*d^2*e^2 + 2*a^2*b^2*d*e*f + a^2*b^2*f^2 + 2*(2*a^2*b^2*d^2*e*f + a^2*b^2*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))/(b^5*d^3*cosh(d*x + c)^4 + 4*b^5*d^3*cosh(d*x + c)^3*sinh(d*x + c) + 6*b^5*d^3*cosh(d*x + c)^2*sinh(d*x + c)^

```
2 + 4*b^5*d^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^5*d^3*sinh(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**2*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2\cosh(dx+c)^2\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)^2*sinh(d*x + c)^3/(b*sinh(d*x + c) + a)
, x)
```

3.398 $\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=474

$$-\frac{a^3 f\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^2}+\frac{a^3 f\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^5d^2}+\frac{a^3 f\sinh(c+dx)}{b^4d^2}-\frac{a^2 f\cosh^2(c+dx)}{4b^3d^2}\cdot$$

```
[Out] (a^4*e*x)/b^5 + (a^2*e*x)/(2*b^3) + (a^4*f*x^2)/(2*b^5) + (a^2*f*x^2)/(4*b^
3) - (e + f*x)^2/(16*b*f) - (a^3*(e + f*x)*Cosh[c + d*x])/(b^4*d) - (a^2*f*
Cosh[c + d*x]^2)/(4*b^3*d^2) - (a*(e + f*x)*Cosh[c + d*x]^3)/(3*b^2*d) - (f
*Cosh[4*c + 4*d*x])/(128*b*d^2) - (a^3*Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b
*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^5*d) + (a^3*Sqrt[a^2 + b^2]*(e + f
*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^5*d) - (a^3*Sqrt[a^2
 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^2) +
 (a^3*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/(b^5*d^2) + (a^3*f*Sinh[c + d*x])/(b^4*d^2) + (a*f*Sinh[c + d*x])/(3*b^2
*d^2) + (a^2*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^3*d) + (a*f*Sinh[c
 + d*x]^3)/(9*b^2*d^2) + ((e + f*x)*Sinh[4*c + 4*d*x])/(32*b*d)
```

Rubi [A] time = 0.86452, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 14, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5579, 5448, 3296, 2638, 5447, 2633, 3310, 5565, 2637, 3322, 2264, 2190, 2279, 2391}

$$-\frac{a^3 f\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d^2}+\frac{a^3 f\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^5d^2}+\frac{a^3 f\sinh(c+dx)}{b^4d^2}-\frac{a^2 f\cosh^2(c+dx)}{4b^3d^2}\cdot$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (a^4*e*x)/b^5 + (a^2*e*x)/(2*b^3) + (a^4*f*x^2)/(2*b^5) + (a^2*f*x^2)/(4*b^
3) - (e + f*x)^2/(16*b*f) - (a^3*(e + f*x)*Cosh[c + d*x])/(b^4*d) - (a^2*f*
Cosh[c + d*x]^2)/(4*b^3*d^2) - (a*(e + f*x)*Cosh[c + d*x]^3)/(3*b^2*d) - (f
*Cosh[4*c + 4*d*x])/(128*b*d^2) - (a^3*Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b
*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^5*d) + (a^3*Sqrt[a^2 + b^2]*(e + f
*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^5*d) - (a^3*Sqrt[a^2
 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^2) +
 (a^3*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/(b^5*d^2) + (a^3*f*Sinh[c + d*x])/(b^4*d^2) + (a*f*Sinh[c + d*x])/(3*b^2
*d^2) + (a^2*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^3*d) + (a*f*Sinh[c
 + d*x]^3)/(9*b^2*d^2) + ((e + f*x)*Sinh[4*c + 4*d*x])/(32*b*d)
```


Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh^2(c+dx)\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\cosh^2(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}\,d}{b}$$

$$= -\frac{a\int (e+fx)\cosh^2(c+dx)\sinh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)\cosh^2(c+dx)\sinh(c+dx}{a+b\sinh(c+dx)}}{b^2}$$

$$= -\frac{(e+fx)^2}{16bf} - \frac{a(e+fx)\cosh^3(c+dx)}{3b^2d} + \frac{a^2\int (e+fx)\cosh^2(c+dx)\,dx}{b^3} - \frac{a}{}$$

$$= -\frac{(e+fx)^2}{16bf} - \frac{a^2f\cosh^2(c+dx)}{4b^3d^2} - \frac{a(e+fx)\cosh^3(c+dx)}{3b^2d} + \frac{a^2(e+fx)\mathrm{co}}{}$$

$$= \frac{a^4ex}{b^5} + \frac{a^2ex}{2b^3} + \frac{a^4fx^2}{2b^5} + \frac{a^2fx^2}{4b^3} - \frac{(e+fx)^2}{16bf} - \frac{a^3(e+fx)\cosh(c+dx)}{b^4d} - \frac{a^2}{}$$

$$= \frac{a^4ex}{b^5} + \frac{a^2ex}{2b^3} + \frac{a^4fx^2}{2b^5} + \frac{a^2fx^2}{4b^3} - \frac{(e+fx)^2}{16bf} - \frac{a^3(e+fx)\cosh(c+dx)}{b^4d} - \frac{a^2}{}$$

$$= \frac{a^4ex}{b^5} + \frac{a^2ex}{2b^3} + \frac{a^4fx^2}{2b^5} + \frac{a^2fx^2}{4b^3} - \frac{(e+fx)^2}{16bf} - \frac{a^3(e+fx)\cosh(c+dx)}{b^4d} - \frac{a^2}{}$$

$$= \frac{a^4ex}{b^5} + \frac{a^2ex}{2b^3} + \frac{a^4fx^2}{2b^5} + \frac{a^2fx^2}{4b^3} - \frac{(e+fx)^2}{16bf} - \frac{a^3(e+fx)\cosh(c+dx)}{b^4d} - \frac{a^2}{}$$

$$= \frac{a^4ex}{b^5} + \frac{a^2ex}{2b^3} + \frac{a^4fx^2}{2b^5} + \frac{a^2fx^2}{4b^3} - \frac{(e+fx)^2}{16bf} - \frac{a^3(e+fx)\cosh(c+dx)}{b^4d} - \frac{a^2}{}$$

Mathematica [C] time = 11.8889, size = 2915, normalized size = 6.15

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]

[Out] -(e*(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(Sqrt[-a^2 - b^2]*d)))/(8*b) - (f*(x^2 + (2*a*((I*Pi*ArcTanh[(-b + a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (2*((-I)*c + ArcCos[((-I)*a)/b])*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] + ((-2*I)*c + Pi - (2*I)*d*x)*ArcTanh[((a - I*b)*Tan[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] - (ArcCos[((-I)*a)/b] + (2*I)*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[((I*a + b)*(a + I*(b + Sqrt[-a^2 - b^2]))*(-I + Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(I*a + b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))] - (ArcCos[((-I)*a)/b] - (2*I)*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[((I*a + b)*(I*a - b + Sqrt[-a^2 - b^2])*(I + Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(a - I*b + Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))] + (ArcCos[((-I)*a)/b] - (2*I)*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] - (2*I)*ArcTanh[((a - I*b)*Tan[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]])*Log[-(((-1)^(3/4)*Sqrt[-a^2 - b^2]*E^(-c/2 - (d*x)/2))/(Sqrt[2]*Sqrt[(-I)*b]*Sqrt[a + b*Sinh[c + d*x]]))] + (ArcCos[((-I)*a)/b] + (2*I)*(ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] + ArcTanh[((a - I*b)*Tan[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]]))*Log[((-1)^(1/4)*Sqrt[-a^2 - b^2]*E^((c + d*x)/2))/(Sqrt[2]*Sqrt[(-I)*b]*Sqrt[a + b*Sinh[c + d*x]])] + I*(PolyLog[2, ((I*a + Sqrt[-a^2 - b^2])*(I*a + b - I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(I*a + b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))] - PolyLog[2, ((a + I*Sqrt[-a^2 - b^2])*(-a + I*b + Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(I*a + b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))]))/Sqrt[-a^2 - b^2]))/d^2))/(16*b) - (e*((4*a^2 + b^2)*(c + d*x) - (2*a*(4*a^2 + 3*b^2)*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 4*a*b*Cosh[c + d*x] + b^2*Sinh[2*(c + d*x)]))/(16*b^3*d) - (f*((4*a^2 + b^2)*(-c + d*x)*(c + d*x) - 8*a*b*d*x*Cosh[c + d*x] - b^2*Cosh[2*(c + d*x)] - (2*a*(4*a^2 + 3*b^2)*(2*c*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + (c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] - (c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])] + PolyLog[2, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])] - PolyLog[2, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))]))/Sqrt[a^2 + b^2] + 8*a*b*Sinh[c + d*x] + 2*b^2*d*x*Sinh[2*(c + d*x)]))/(32*b^3*d^2) + (e*(6*(16*a^4 + 12*a^2*b^2 + b^4)*(c + d*x) - (12*a*(16*a^4 + 20*a^2*b^2 + 5*b^4)*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 48*a*b*(2*a^2 + b^2)*Cosh[c + d*x] - 8*a*b^3*Cosh[3*(c + d*x)] + 6*b^2*(4*a^2 + b^2)*Sinh[2*(c + d*x)] + 3*b^4*Sinh[4*(c + d*x)]))/(96*b^5*d) + (f*(-576*a^4*Sqrt[a^2 + b^2]*c^2 - 432*a^2*b^2*Sqrt[a^2 + b^2]*c^2 - 36*b^4*Sqrt[a^2 + b^2]*c^2 + 576*a^4*Sqrt[a^2 + b^2]*d^2*x^2 + 432*a^2*b^2*Sqrt[a^2 + b^2]*d^2*x^2 + 36*b^4*Sqrt[a^2 + b^2]*d^2*x^2 - 2304*a^5*c*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] - 2880*a^3*b^2*c*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] - 720*a*b^4*c*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] - 1152*a^3*b*Sqrt[a^2 + b^2]*d*x*Cosh[c + d*x] - 576*a*b^3*Sqrt[a^2 + b^2]*d*x*Cosh[c + d*x] - 144*a^2*b^2*Sqrt[a^2 + b^2]*Cosh[2*(c + d*x)] - 36*b^4*Sqrt[a^2 + b^2]*Cosh[2*(c + d*x)] - 96*a*b^3*Sqrt[a^2 + b^2]*d*x*Cosh[3*(c + d*x)] - 9*b^4*Sqrt[a^2 + b^2]*Cosh[4*(c + d*x)] - 1152*a^5*c*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] - 1440*a^3*b^2*c*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] - 360*a*b^4*c*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] - 1152*a^5*d*x*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] - 1440*a^3*b^2*d*x*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 +

```
b^2])] - 360*a*b^4*d*x*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqr
t[a^2 + b^2])] + 1152*a^5*c*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a
+ Sqrt[a^2 + b^2])] + 1440*a^3*b^2*c*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d
*x]))/(a + Sqrt[a^2 + b^2])] + 360*a*b^4*c*Log[1 + (b*(Cosh[c + d*x] + Sinh
[c + d*x]))/(a + Sqrt[a^2 + b^2])] + 1152*a^5*d*x*Log[1 + (b*(Cosh[c + d*x]
 + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])] + 1440*a^3*b^2*d*x*Log[1 + (b*(Co
sh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])] + 360*a*b^4*d*x*Log[1
+ (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])] - 72*a*(16*a^4
 + 20*a^2*b^2 + 5*b^4)*PolyLog[2, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a +
 Sqrt[a^2 + b^2])] + 72*a*(16*a^4 + 20*a^2*b^2 + 5*b^4)*PolyLog[2, -((b*(Co
sh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))] + 1152*a^3*b*Sqrt[a^2
 + b^2]*Sinh[c + d*x] + 576*a*b^3*Sqrt[a^2 + b^2]*Sinh[c + d*x] + 288*a^2*b
^2*Sqrt[a^2 + b^2]*d*x*Sinh[2*(c + d*x)] + 72*b^4*Sqrt[a^2 + b^2]*d*x*Sinh[
2*(c + d*x)] + 32*a*b^3*Sqrt[a^2 + b^2]*Sinh[3*(c + d*x)] + 36*b^4*Sqrt[a^2
 + b^2]*d*x*Sinh[4*(c + d*x)]))/(1152*b^5*Sqrt[a^2 + b^2]*d^2)
```

Maple [B] time = 0.106, size = 1213, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/16*f*x^2/b-a^5/b^5/d^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^
(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+a^5/b^5/d^2*f/(a^2+b^2)^(1/2)*dilog((b*exp(d
*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2*a^5/b^5/d*e/(a^2+b^2)^(1/2)
*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/2*a^4*f*x^2/b^5+1/4*a^
2*f*x^2/b^3-1/8*a*(4*a^2+b^2)*(d*f*x+d*e+f)/b^4/d^2*exp(-d*x-c)+a^4*e*x/b^5
+1/2*a^2*e*x/b^3-1/8*e*x/b+2*a^3/b^3/d*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*e
xp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-a^3/b^3/d^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp
(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+a^3/b^3/d^2*f/(a^2+b^2)^(1
/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2*a^5/b^5/d
^2*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-a^
5/b^5/d*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2
)^(1/2)))*x-a^5/b^5/d^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))*c+a^5/b^5/d*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a
^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+a^5/b^5/d^2*f/(a^2+b^2)^(1/2)*ln((b
*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-a^3/b^3/d*f/(a^2+b^2)
^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-a^3/b^3
/d^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(
1/2)))*c+a^3/b^3/d*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a
+(a^2+b^2)^(1/2)))*x+a^3/b^3/d^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^
2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-2*a^3/b^3/d^2*f*c/(a^2+b^2)^(1/2)*arctan
h(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/256*(4*d*f*x+4*d*e-f)/d^2/b*e
xp(4*d*x+4*c)-1/72*a*(3*d*f*x+3*d*e-f)/b^2/d^2*exp(3*d*x+3*c)-1/8*a*(4*a^2*
d*f*x+b^2*d*f*x+4*a^2*d*e+b^2*d*e-4*a^2*f-b^2*f)/b^4/d^2*exp(d*x+c)-1/256*(
4*d*f*x+4*d*e+f)/d^2/b*exp(-4*d*x-4*c)-1/72*a*(3*d*f*x+3*d*e+f)/b^2/d^2*exp
(-3*d*x-3*c)+1/16*a^2*(2*d*f*x+2*d*e-f)/b^3/d^2*exp(2*d*x+2*c)-1/16*a^2*(2*
d*f*x+2*d*e+f)/b^3/d^2*exp(-2*d*x-2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.79816, size = 7611, normalized size = 16.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/2304*(9*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*cosh(d*x + c)^8 + 9*(4*b^4*d*f*
x + 4*b^4*d*e - b^4*f)*sinh(d*x + c)^8 - 32*(3*a*b^3*d*f*x + 3*a*b^3*d*e -
a*b^3*f)*cosh(d*x + c)^7 - 8*(12*a*b^3*d*f*x + 12*a*b^3*d*e - 4*a*b^3*f - 9
*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*cosh(d*x + c))*sinh(d*x + c)^7 - 36*b^4*
d*f*x + 144*(2*a^2*b^2*d*f*x + 2*a^2*b^2*d*e - a^2*b^2*f)*cosh(d*x + c)^6 +
 4*(72*a^2*b^2*d*f*x + 72*a^2*b^2*d*e - 36*a^2*b^2*f + 63*(4*b^4*d*f*x + 4*
b^4*d*e - b^4*f)*cosh(d*x + c)^2 - 56*(3*a*b^3*d*f*x + 3*a*b^3*d*e - a*b^3*
f)*cosh(d*x + c))*sinh(d*x + c)^6 - 36*b^4*d*e - 288*((4*a^3*b + a*b^3)*d*f
*x + (4*a^3*b + a*b^3)*d*e - (4*a^3*b + a*b^3)*f)*cosh(d*x + c)^5 - 24*(12*
(4*a^3*b + a*b^3)*d*f*x - 21*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*cosh(d*x + c
)^3 + 12*(4*a^3*b + a*b^3)*d*e + 28*(3*a*b^3*d*f*x + 3*a*b^3*d*e - a*b^3*f)
*cosh(d*x + c)^2 - 12*(4*a^3*b + a*b^3)*f - 36*(2*a^2*b^2*d*f*x + 2*a^2*b^2
*d*e - a^2*b^2*f)*cosh(d*x + c))*sinh(d*x + c)^5 - 9*b^4*f + 144*((8*a^4 +
4*a^2*b^2 - b^4)*d^2*f*x^2 + 2*(8*a^4 + 4*a^2*b^2 - b^4)*d^2*e*x)*cosh(d*x
+ c)^4 + 2*(72*(8*a^4 + 4*a^2*b^2 - b^4)*d^2*f*x^2 + 144*(8*a^4 + 4*a^2*b^2
 - b^4)*d^2*e*x + 315*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*cosh(d*x + c)^4 - 5
60*(3*a*b^3*d*f*x + 3*a*b^3*d*e - a*b^3*f)*cosh(d*x + c)^3 + 1080*(2*a^2*b^
2*d*f*x + 2*a^2*b^2*d*e - a^2*b^2*f)*cosh(d*x + c)^2 - 720*((4*a^3*b + a*b^
3)*d*f*x + (4*a^3*b + a*b^3)*d*e - (4*a^3*b + a*b^3)*f)*cosh(d*x + c))*sinh
(d*x + c)^4 - 288*((4*a^3*b + a*b^3)*d*f*x + (4*a^3*b + a*b^3)*d*e + (4*a^3
*b + a*b^3)*f)*cosh(d*x + c)^3 + 8*(63*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*co
sh(d*x + c)^5 - 140*(3*a*b^3*d*f*x + 3*a*b^3*d*e - a*b^3*f)*cosh(d*x + c)^4
 - 36*(4*a^3*b + a*b^3)*d*f*x + 360*(2*a^2*b^2*d*f*x + 2*a^2*b^2*d*e - a^2*
b^2*f)*cosh(d*x + c)^3 - 36*(4*a^3*b + a*b^3)*d*e - 360*((4*a^3*b + a*b^3)*
d*f*x + (4*a^3*b + a*b^3)*d*e - (4*a^3*b + a*b^3)*f)*cosh(d*x + c)^2 - 36*(
4*a^3*b + a*b^3)*f + 72*((8*a^4 + 4*a^2*b^2 - b^4)*d^2*f*x^2 + 2*(8*a^4 + 4
*a^2*b^2 - b^4)*d^2*e*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 144*(2*a^2*b^2*d*
f*x + 2*a^2*b^2*d*e + a^2*b^2*f)*cosh(d*x + c)^2 - 12*(24*a^2*b^2*d*f*x - 2
1*(4*b^4*d*f*x + 4*b^4*d*e - b^4*f)*cosh(d*x + c)^6 + 24*a^2*b^2*d*e + 56*(
3*a*b^3*d*f*x + 3*a*b^3*d*e - a*b^3*f)*cosh(d*x + c)^5 + 12*a^2*b^2*f - 180
*(2*a^2*b^2*d*f*x + 2*a^2*b^2*d*e - a^2*b^2*f)*cosh(d*x + c)^4 + 240*((4*a^
3*b + a*b^3)*d*f*x + (4*a^3*b + a*b^3)*d*e - (4*a^3*b + a*b^3)*f)*cosh(d*x
+ c)^3 - 72*((8*a^4 + 4*a^2*b^2 - b^4)*d^2*f*x^2 + 2*(8*a^4 + 4*a^2*b^2 - b
^4)*d^2*e*x)*cosh(d*x + c)^2 + 72*((4*a^3*b + a*b^3)*d*f*x + (4*a^3*b + a*b
^3)*d*e + (4*a^3*b + a*b^3)*f)*cosh(d*x + c))*sinh(d*x + c)^2 - 2304*(a^3*b
*f*cosh(d*x + c)^4 + 4*a^3*b*f*cosh(d*x + c)^3*sinh(d*x + c) + 6*a^3*b*f*co
sh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^3*b*f*cosh(d*x + c)*sinh(d*x + c)^3 + a
^3*b*f*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*si
nh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b
)/b + 1) + 2304*(a^3*b*f*cosh(d*x + c)^4 + 4*a^3*b*f*cosh(d*x + c)^3*sinh(d
*x + c) + 6*a^3*b*f*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^3*b*f*cosh(d*x +
c)*sinh(d*x + c)^3 + a^3*b*f*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*dilog((
a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqr
```

```
t((a^2 + b^2)/b^2) - b)/b + 1) + 2304*((a^3*b*d*e - a^3*b*c*f)*cosh(d*x + c
)^4 + 4*(a^3*b*d*e - a^3*b*c*f)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3*b*d*
e - a^3*b*c*f)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3*b*d*e - a^3*b*c*f)*
cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b*d*e - a^3*b*c*f)*sinh(d*x + c)^4)*sq
rt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a
^2 + b^2)/b^2) + 2*a) - 2304*((a^3*b*d*e - a^3*b*c*f)*cosh(d*x + c)^4 + 4*(
a^3*b*d*e - a^3*b*c*f)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3*b*d*e - a^3*b
*c*f)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3*b*d*e - a^3*b*c*f)*cosh(d*x
+ c)*sinh(d*x + c)^3 + (a^3*b*d*e - a^3*b*c*f)*sinh(d*x + c)^4)*sqrt((a^2 +
 b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)
/b^2) + 2*a) - 2304*((a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x + c)^4 + 4*(a^3*b*d
*f*x + a^3*b*c*f)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3*b*d*f*x + a^3*b*c*
f)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a^3*b*d*f*x + a^3*b*c*f)*sinh(d*x + c)^4)*sqrt((a^2
+ b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2304*((a^3*b*d*f*x + a^3*b*c
*f)*cosh(d*x + c)^4 + 4*(a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x + c)^3*sinh(d*x
+ c) + 6*(a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3
*b*d*f*x + a^3*b*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b*d*f*x + a^3*b*
c*f)*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(
d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b
) - 32*(3*a*b^3*d*f*x + 3*a*b^3*d*e + a*b^3*f)*cosh(d*x + c) + 8*(9*(4*b^4*
d*f*x + 4*b^4*d*e - b^4*f)*cosh(d*x + c)^7 - 12*a*b^3*d*f*x - 28*(3*a*b^3*d
*f*x + 3*a*b^3*d*e - a*b^3*f)*cosh(d*x + c)^6 - 12*a*b^3*d*e + 108*(2*a^2*b
^2*d*f*x + 2*a^2*b^2*d*e - a^2*b^2*f)*cosh(d*x + c)^5 - 4*a*b^3*f - 180*((4
*a^3*b + a*b^3)*d*f*x + (4*a^3*b + a*b^3)*d*e - (4*a^3*b + a*b^3)*f)*cosh(d
*x + c)^4 + 72*((8*a^4 + 4*a^2*b^2 - b^4)*d^2*f*x^2 + 2*(8*a^4 + 4*a^2*b^2
- b^4)*d^2*e*x)*cosh(d*x + c)^3 - 108*((4*a^3*b + a*b^3)*d*f*x + (4*a^3*b +
 a*b^3)*d*e + (4*a^3*b + a*b^3)*f)*cosh(d*x + c)^2 - 36*(2*a^2*b^2*d*f*x +
2*a^2*b^2*d*e + a^2*b^2*f)*cosh(d*x + c))*sinh(d*x + c))/(b^5*d^2*cosh(d*x
+ c)^4 + 4*b^5*d^2*cosh(d*x + c)^3*sinh(d*x + c) + 6*b^5*d^2*cosh(d*x + c)^
2*sinh(d*x + c)^2 + 4*b^5*d^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^5*d^2*sinh(
d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**2*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\cosh(dx+c)^2\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)^2*sinh(d*x + c)^3/(b*sinh(d*x + c) + a),
x)
```

3.399 $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=184

$$-\frac{a(3a^2+b^2)\cosh(c+dx)}{3b^4d}+\frac{2a^3\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^5d}+\frac{(4a^2+b^2)\sinh(c+dx)\cosh(c+dx)}{8b^3d}+\frac{x(4a^2b^2}{8}$$

```
[Out] ((8*a^4 + 4*a^2*b^2 - b^4)*x)/(8*b^5) + (2*a^3*Sqrt[a^2 + b^2]*ArcTanh[(b -
 a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b^5*d) - (a*(3*a^2 + b^2)*Cosh[c +
 d*x])/(3*b^4*d) + ((4*a^2 + b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(8*b^3*d) -
(a*Cosh[c + d*x]*Sinh[c + d*x]^2)/(3*b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x]^
3)/(4*b*d)
```

Rubi [A] time = 0.79298, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2889, 3050, 3049, 3023, 2735, 2660, 618, 204}

$$-\frac{a(3a^2+b^2)\cosh(c+dx)}{3b^4d}+\frac{2a^3\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^5d}+\frac{(4a^2+b^2)\sinh(c+dx)\cosh(c+dx)}{8b^3d}+\frac{x(4a^2b^2}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((8*a^4 + 4*a^2*b^2 - b^4)*x)/(8*b^5) + (2*a^3*Sqrt[a^2 + b^2]*ArcTanh[(b -
 a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b^5*d) - (a*(3*a^2 + b^2)*Cosh[c +
 d*x])/(3*b^4*d) + ((4*a^2 + b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(8*b^3*d) -
(a*Cosh[c + d*x]*Sinh[c + d*x]^2)/(3*b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x]^
3)/(4*b*d)
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\sinh^3(c+dx)\left(1+\sinh^2(c+dx)\right)}{a+b\sinh(c+dx)}\,dx$$

$$= \frac{\cosh(c+dx)\sinh^3(c+dx)}{4bd} + \frac{\int \frac{\sinh^2(c+dx)\left(-3a+b\sinh(c+dx)-4a\sinh^2(c+dx)\right)}{a+b\sinh(c+dx)}\,dx}{4b}$$

$$= -\frac{a\cosh(c+dx)\sinh^2(c+dx)}{3b^2d} + \frac{\cosh(c+dx)\sinh^3(c+dx)}{4bd} + \frac{\int \frac{\sinh(c+dx)\left(8a^2-ab\sin\right.}{a+}}{}$$

$$= \frac{\left(4a^2+b^2\right)\cosh(c+dx)\sinh(c+dx)}{8b^3d} - \frac{a\cosh(c+dx)\sinh^2(c+dx)}{3b^2d} + \frac{\cosh(c+dx}{}$$

$$= -\frac{a\left(3a^2+b^2\right)\cosh(c+dx)}{3b^4d} + \frac{\left(4a^2+b^2\right)\cosh(c+dx)\sinh(c+dx)}{8b^3d} - \frac{a\cosh(c+dx}{3}$$

$$= \frac{\left(8a^4+4a^2b^2-b^4\right)x}{8b^5} - \frac{a\left(3a^2+b^2\right)\cosh(c+dx)}{3b^4d} + \frac{\left(4a^2+b^2\right)\cosh(c+dx)\sinh(c}{8b^3d}$$

$$= \frac{\left(8a^4+4a^2b^2-b^4\right)x}{8b^5} - \frac{a\left(3a^2+b^2\right)\cosh(c+dx)}{3b^4d} + \frac{\left(4a^2+b^2\right)\cosh(c+dx)\sinh(c}{8b^3d}$$

$$= \frac{\left(8a^4+4a^2b^2-b^4\right)x}{8b^5} - \frac{a\left(3a^2+b^2\right)\cosh(c+dx)}{3b^4d} + \frac{\left(4a^2+b^2\right)\cosh(c+dx)\sinh(c}{8b^3d}$$

$$= \frac{\left(8a^4+4a^2b^2-b^4\right)x}{8b^5} + \frac{2a^3\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^5d} - \frac{a\left(3a^2+b^2\right)\cosh(}{3b^4d}$$

Mathematica [A] time = 1.98925, size = 153, normalized size = 0.83

$$\frac{-24ab\left(4a^2+b^2\right)\cosh(c+dx)+3\left(4\left(4a^2b^2+8a^4-b^4\right)(c+dx)+8a^2b^2\sinh(2(c+dx))+64a^3\sqrt{-a^2-b^2}\tan^{-1}\left(\frac{b-a\tan}{\sqrt{}}\right.\right.}{96b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-24*a*b*(4*a^2 + b^2)*Cosh[c + d*x] - 8*a*b^3*Cosh[3*(c + d*x)] + 3*(4*(8*
a^4 + 4*a^2*b^2 - b^4)*(c + d*x) + 64*a^3*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Ta
nh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] + 8*a^2*b^2*Sinh[2*(c + d*x)] + b^4*Sinh
[4*(c + d*x)]))/(96*b^5*d)
```

Maple [B] time = 0.049, size = 624, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/8/d/b*ln(tanh(1/2*d*x+1/2*c)+1)+1/8/d/b*ln(tanh(1/2*d*x+1/2*c)-1)-3/8/d/
b/(tanh(1/2*d*x+1/2*c)+1)^2+3/8/d/b/(tanh(1/2*d*x+1/2*c)-1)^2-1/d*a^4/b^5*l
n(tanh(1/2*d*x+1/2*c)-1)-1/2/d/b^3/(tanh(1/2*d*x+1/2*c)+1)^2*a^2-1/d/b^4/(t
```

```
anh(1/2*d*x+1/2*c)+1)*a^3+1/d*a^4/b^5*ln(tanh(1/2*d*x+1/2*c)+1)-1/3/d/b^2/(
tanh(1/2*d*x+1/2*c)+1)^3*a+1/3/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^3*a+1/2/d/b^3/
(tanh(1/2*d*x+1/2*c)-1)^2*a^2+1/d/b^4/(tanh(1/2*d*x+1/2*c)-1)*a^3+1/2/d/b^2
/(tanh(1/2*d*x+1/2*c)+1)^2*a+1/2/d/b^3/(tanh(1/2*d*x+1/2*c)+1)*a^2+1/2/d/b^
2/(tanh(1/2*d*x+1/2*c)-1)^2*a+1/2/d/b^3/(tanh(1/2*d*x+1/2*c)-1)*a^2-1/2/d/b
^2/(tanh(1/2*d*x+1/2*c)+1)*a+1/2/d/b^3*ln(tanh(1/2*d*x+1/2*c)+1)*a^2+1/2/d/
b^2/(tanh(1/2*d*x+1/2*c)-1)*a-1/2/d/b^3*ln(tanh(1/2*d*x+1/2*c)-1)*a^2-2/d*a
^3*(a^2+b^2)^(1/2)/b^5*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^
(1/2))+1/4/d/b/(tanh(1/2*d*x+1/2*c)-1)^4-1/4/d/b/(tanh(1/2*d*x+1/2*c)+1)^4+
1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^3+1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^3+1/8/d/b
/(tanh(1/2*d*x+1/2*c)+1)+1/8/d/b/(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxim
a")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.29543, size = 2803, normalized size = 15.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="frica
s")

[Out] 1/192*(3*b^4*cosh(d*x + c)^8 + 3*b^4*sinh(d*x + c)^8 - 8*a*b^3*cosh(d*x + c
)^7 + 24*a^2*b^2*cosh(d*x + c)^6 + 8*(3*b^4*cosh(d*x + c) - a*b^3)*sinh(d*x
 + c)^7 + 24*(8*a^4 + 4*a^2*b^2 - b^4)*d*x*cosh(d*x + c)^4 + 4*(21*b^4*cosh
(d*x + c)^2 - 14*a*b^3*cosh(d*x + c) + 6*a^2*b^2)*sinh(d*x + c)^6 - 24*a^2*
b^2*cosh(d*x + c)^2 - 24*(4*a^3*b + a*b^3)*cosh(d*x + c)^5 + 24*(7*b^4*cosh
(d*x + c)^3 - 7*a*b^3*cosh(d*x + c)^2 + 6*a^2*b^2*cosh(d*x + c) - 4*a^3*b -
 a*b^3)*sinh(d*x + c)^5 - 8*a*b^3*cosh(d*x + c) + 2*(105*b^4*cosh(d*x + c)^
4 - 140*a*b^3*cosh(d*x + c)^3 + 180*a^2*b^2*cosh(d*x + c)^2 + 12*(8*a^4 + 4
*a^2*b^2 - b^4)*d*x - 60*(4*a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 -
 3*b^4 - 24*(4*a^3*b + a*b^3)*cosh(d*x + c)^3 + 8*(21*b^4*cosh(d*x + c)^5 -
 35*a*b^3*cosh(d*x + c)^4 + 60*a^2*b^2*cosh(d*x + c)^3 - 12*a^3*b - 3*a*b^3
 + 12*(8*a^4 + 4*a^2*b^2 - b^4)*d*x*cosh(d*x + c) - 30*(4*a^3*b + a*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c)^3 + 12*(7*b^4*cosh(d*x + c)^6 - 14*a*b^3*cosh(
d*x + c)^5 + 30*a^2*b^2*cosh(d*x + c)^4 + 12*(8*a^4 + 4*a^2*b^2 - b^4)*d*x*
cosh(d*x + c)^2 - 2*a^2*b^2 - 20*(4*a^3*b + a*b^3)*cosh(d*x + c)^3 - 6*(4*a
^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 192*(a^3*cosh(d*x + c)^4 + 4
*a^3*cosh(d*x + c)^3*sinh(d*x + c) + 6*a^3*cosh(d*x + c)^2*sinh(d*x + c)^2
+ 4*a^3*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*sinh(d*x + c)^4)*sqrt(a^2 + b^2
)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*
a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(
b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c
)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 8*(
3*b^4*cosh(d*x + c)^7 - 7*a*b^3*cosh(d*x + c)^6 + 18*a^2*b^2*cosh(d*x + c)^
```

```
5 + 12*(8*a^4 + 4*a^2*b^2 - b^4)*d*x*cosh(d*x + c)^3 - 6*a^2*b^2*cosh(d*x +
 c) - 15*(4*a^3*b + a*b^3)*cosh(d*x + c)^4 - a*b^3 - 9*(4*a^3*b + a*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c))/(b^5*d*cosh(d*x + c)^4 + 4*b^5*d*cosh(d*x + c
)^3*sinh(d*x + c) + 6*b^5*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^5*d*cosh(
d*x + c)*sinh(d*x + c)^3 + b^5*d*sinh(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20274, size = 379, normalized size = 2.06

$$\frac{\left(8\,a^4+4\,a^2b^2-b^4\right)(dx+c)}{8\,b^5d}-\frac{\left(24\,a^2b^2e^{(2\,dx+2\,c)}+8\,ab^3e^{(dx+c)}+3\,b^4+24\left(4\,a^3b+ab^3\right)e^{(3\,dx+3\,c)}\right)e^{(-4\,dx-4\,c)}}{192\,b^5d}-\frac{\left(a^5+a^3b^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac"
)
```

```
[Out] 1/8*(8*a^4 + 4*a^2*b^2 - b^4)*(d*x + c)/(b^5*d) - 1/192*(24*a^2*b^2*e^(2*d*
x + 2*c) + 8*a*b^3*e^(d*x + c) + 3*b^4 + 24*(4*a^3*b + a*b^3)*e^(3*d*x + 3*
c))*e^(-4*d*x - 4*c)/(b^5*d) - (a^5 + a^3*b^2)*log(abs(2*b*e^(d*x + c) + 2*
a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(sqr
t(a^2 + b^2)*b^5*d) + 1/192*(3*b^3*d^3*e^(4*d*x + 4*c) - 8*a*b^2*d^3*e^(3*d
*x + 3*c) + 24*a^2*b*d^3*e^(2*d*x + 2*c) - 96*a^3*d^3*e^(d*x + c) - 24*a*b^
2*d^3*e^(d*x + c))/(b^4*d^4)
```

3.400 $\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=38

$$\text{Unintegrable}\left(\frac{\sinh^3(c+dx)\cosh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Cosh[c + d*x]^2*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d
*x])), x]
```

Rubi [A] time = 0.127051, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[c + d*x]^2*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Cosh[c + d*x]^2*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh^2(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[c + d*x]^2*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x]
)),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.123, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx+c))^2(\sinh(dx+c))^3}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

[Out] int(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-2\left(a^5e^c+a^3b^2e^c\right)\int -\frac{e^{(dx)}}{b^6fx+b^6e-\left(b^6fxe^{(2c)}+b^6ee^{(2c)}\right)e^{(2dx)}-2\left(ab^5fxe^c+ab^5ee^c\right)e^{(dx)}}\,dx-\frac{e^{\left(-4c+\frac{4de}{f}\right)}E_1\left(\frac{4(fx+e)d}{f}\right)}{16bf}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -2*(a^5*e^c + a^3*b^2*e^c)*integrate(-e^(d*x)/(b^6*f*x + b^6*e - (b^6*f*x*e
^(2*c) + b^6*e*e^(2*c))*e^(2*d*x) - 2*(a*b^5*f*x*e^c + a*b^5*e*e^c)*e^(d*x)
), x) - 1/16*e^(-4*c + 4*d*e/f)*exp_integral_e(1, 4*(f*x + e)*d/f)/(b*f) -
1/8*a*e^(-3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b^2*f) - 1/4*a
^2*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b^3*f) - 1/4*a^2*
e^(2*c - 2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b^3*f) + 1/8*a*e^(3*
c - 3*d*e/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b^2*f) - 1/16*e^(4*c - 4*
d*e/f)*exp_integral_e(1, -4*(f*x + e)*d/f)/(b*f) - 1/8*(4*a^3 + a*b^2)*e^(-
c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^4*f) + 1/8*(4*a^3*e^c + a*b^
2*e^c)*e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^4*f) + 1/8*(8*a^4 +
4*a^2*b^2 - b^4)*log(f*x + e)/(b^5*f)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx+c)^2\sinh(dx+c)^3}{afx+ae+\left(bfx+be\right)\sinh(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] integral(cosh(d*x + c)^2*sinh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(
d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*sinh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^2\sinh(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate(cosh(d*x + c)^2*sinh(d*x + c)^3/((f*x + e)*(b*sinh(d*x + c) + a))
, x)
```

3.401 $\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1443

result too large to display

```
[Out] (-3*a^3*f^3*x)/(8*b^4*d^3) + (45*a*f^3*x)/(256*b^2*d^3) - (a^3*(e + f*x)^3)
/(4*b^4*d) + (3*a*(e + f*x)^3)/(32*b^2*d) + (a^3*(a^2 + b^2)*(e + f*x)^4)/(
4*b^6*f) - (6*a^4*f^3*Cosh[c + d*x])/(b^5*d^4) - (40*a^2*f^3*Cosh[c + d*x])
/(9*b^3*d^4) + (3*f^3*Cosh[c + d*x])/(4*b*d^4) - (3*a^4*f*(e + f*x)^2*Cosh[
c + d*x])/(b^5*d^2) - (2*a^2*f*(e + f*x)^2*Cosh[c + d*x])/(b^3*d^2) + (3*f*
(e + f*x)^2*Cosh[c + d*x])/(8*b*d^2) - (9*a*f^2*(e + f*x)*Cosh[c + d*x]^2)/
(32*b^2*d^3) - (2*a^2*f^3*Cosh[c + d*x]^3)/(27*b^3*d^4) - (a^2*f*(e + f*x)^
2*Cosh[c + d*x]^3)/(3*b^3*d^2) - (3*a*f^2*(e + f*x)*Cosh[c + d*x]^4)/(32*b^
2*d^3) - (a*(e + f*x)^3*Cosh[c + d*x]^4)/(4*b^2*d) - (f^3*Cosh[3*c + 3*d*x]
)/(216*b*d^4) - (f*(e + f*x)^2*Cosh[3*c + 3*d*x])/(48*b*d^2) - (3*f^3*Cosh[
5*c + 5*d*x])/(5000*b*d^4) - (3*f*(e + f*x)^2*Cosh[5*c + 5*d*x])/(400*b*d^2
) - (a^3*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])])/(b^6*d) - (a^3*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])])/(b^6*d) - (3*a^3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^6*d^2) - (3*a^3*(a^2 + b^2)*f*(e
 + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^6*d^2) +
 (6*a^3*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/(b^6*d^3) + (6*a^3*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^6*d^3) - (6*a^3*(a^2 + b^2)*f^3*Pol
yLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^6*d^4) - (6*a^3*(a^2
+ b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^6*d^4)
+ (6*a^4*f^2*(e + f*x)*Sinh[c + d*x])/(b^5*d^3) + (40*a^2*f^2*(e + f*x)*Sin
h[c + d*x])/(9*b^3*d^3) - (3*f^2*(e + f*x)*Sinh[c + d*x])/(4*b*d^3) + (a^4*
(e + f*x)^3*Sinh[c + d*x])/(b^5*d) + (2*a^2*(e + f*x)^3*Sinh[c + d*x])/(3*b
^3*d) - ((e + f*x)^3*Sinh[c + d*x])/(8*b*d) + (3*a^3*f^3*Cosh[c + d*x]*Sinh
[c + d*x])/(8*b^4*d^4) + (45*a*f^3*Cosh[c + d*x]*Sinh[c + d*x])/(256*b^2*d^
4) + (3*a^3*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^4*d^2) + (9*a*f
*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(32*b^2*d^2) + (2*a^2*f^2*(e + f*
x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(9*b^3*d^3) + (a^2*(e + f*x)^3*Cosh[c + d
*x]^2*Sinh[c + d*x])/(3*b^3*d) + (3*a*f^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(1
28*b^2*d^4) + (3*a*f*(e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(16*b^2*d^2
) - (3*a^3*f^2*(e + f*x)*Sinh[c + d*x]^2)/(4*b^4*d^3) - (a^3*(e + f*x)^3*Si
nh[c + d*x]^2)/(2*b^4*d) + (f^2*(e + f*x)*Sinh[3*c + 3*d*x])/(72*b*d^3) + (
(e + f*x)^3*Sinh[3*c + 3*d*x])/(48*b*d) + (3*f^2*(e + f*x)*Sinh[5*c + 5*d*x
])/(1000*b*d^3) + ((e + f*x)^3*Sinh[5*c + 5*d*x])/(80*b*d)
```

Rubi [A] time = 2.18695, antiderivative size = 1443, normalized size of antiderivative = 1., number of steps used = 55, number of rules used = 18, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5579, 5448, 3296, 2638, 5447, 3311, 32, 2635, 8, 3310, 5565, 5446, 5561, 2190, 2531, 6609, 2282, 6589}

$$-\frac{6f^3\cosh(c+dx)a^4}{b^5d^4}-\frac{3f(e+fx)^2\cosh(c+dx)a^4}{b^5d^2}+\frac{(e+fx)^3\sinh(c+dx)a^4}{b^5d}+\frac{6f^2(e+fx)\sinh(c+dx)a^4}{b^5d^3}+\frac{(a^2+b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-3*a^3*f^3*x)/(8*b^4*d^3) + (45*a*f^3*x)/(256*b^2*d^3) - (a^3*(e + f*x)^3)
/(4*b^4*d) + (3*a*(e + f*x)^3)/(32*b^2*d) + (a^3*(a^2 + b^2)*(e + f*x)^4)/(
```

```
4*b^6*f) - (6*a^4*f^3*Cosh[c + d*x])/(b^5*d^4) - (40*a^2*f^3*Cosh[c + d*x])
/(9*b^3*d^4) + (3*f^3*Cosh[c + d*x])/(4*b*d^4) - (3*a^4*f*(e + f*x)^2*Cosh[
c + d*x])/(b^5*d^2) - (2*a^2*f*(e + f*x)^2*Cosh[c + d*x])/(b^3*d^2) + (3*f*
(e + f*x)^2*Cosh[c + d*x])/(8*b*d^2) - (9*a*f^2*(e + f*x)*Cosh[c + d*x]^2)/
(32*b^2*d^3) - (2*a^2*f^3*Cosh[c + d*x]^3)/(27*b^3*d^4) - (a^2*f*(e + f*x)^
2*Cosh[c + d*x]^3)/(3*b^3*d^2) - (3*a*f^2*(e + f*x)*Cosh[c + d*x]^4)/(32*b^
2*d^3) - (a*(e + f*x)^3*Cosh[c + d*x]^4)/(4*b^2*d) - (f^3*Cosh[3*c + 3*d*x]
)/(216*b*d^4) - (f*(e + f*x)^2*Cosh[3*c + 3*d*x])/(48*b*d^2) - (3*f^3*Cosh[
5*c + 5*d*x])/(5000*b*d^4) - (3*f*(e + f*x)^2*Cosh[5*c + 5*d*x])/(400*b*d^2
) - (a^3*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])])/(b^6*d) - (a^3*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])])/(b^6*d) - (3*a^3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^6*d^2) - (3*a^3*(a^2 + b^2)*f*(e
 + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^6*d^2) +
 (6*a^3*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/(b^6*d^3) + (6*a^3*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^6*d^3) - (6*a^3*(a^2 + b^2)*f^3*Pol
yLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^6*d^4) - (6*a^3*(a^2
+ b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^6*d^4)
+ (6*a^4*f^2*(e + f*x)*Sinh[c + d*x])/(b^5*d^3) + (40*a^2*f^2*(e + f*x)*Sin
h[c + d*x])/(9*b^3*d^3) - (3*f^2*(e + f*x)*Sinh[c + d*x])/(4*b*d^3) + (a^4*
(e + f*x)^3*Sinh[c + d*x])/(b^5*d) + (2*a^2*(e + f*x)^3*Sinh[c + d*x])/(3*b
^3*d) - ((e + f*x)^3*Sinh[c + d*x])/(8*b*d) + (3*a^3*f^3*Cosh[c + d*x]*Sinh
[c + d*x])/(8*b^4*d^4) + (45*a*f^3*Cosh[c + d*x]*Sinh[c + d*x])/(256*b^2*d^
4) + (3*a^3*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^4*d^2) + (9*a*f
*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(32*b^2*d^2) + (2*a^2*f^2*(e + f*
x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(9*b^3*d^3) + (a^2*(e + f*x)^3*Cosh[c + d
*x]^2*Sinh[c + d*x])/(3*b^3*d) + (3*a*f^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(1
28*b^2*d^4) + (3*a*f*(e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(16*b^2*d^2
) - (3*a^3*f^2*(e + f*x)*Sinh[c + d*x]^2)/(4*b^4*d^3) - (a^3*(e + f*x)^3*Si
nh[c + d*x]^2)/(2*b^4*d) + (f^2*(e + f*x)*Sinh[3*c + 3*d*x])/(72*b*d^3) + (
(e + f*x)^3*Sinh[3*c + 3*d*x])/(48*b*d) + (3*f^2*(e + f*x)*Sinh[5*c + 5*d*x
])/(1000*b*d^3) + ((e + f*x)^3*Sinh[5*c + 5*d*x])/(80*b*d)
```

Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
```

```
[{c, d}, x]
```

Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh^2(c+dx)}{a+b\sinh(c+dx)}}{b}$$

$$= -\frac{a\int (e+fx)^3\cosh^3(c+dx)\sinh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^3\cosh^3(c+dx)\sinh(c}{a+b\sinh(c+dx)}}{b^2}$$

$$= -\frac{a(e+fx)^3\cosh^4(c+dx)}{4b^2d} + \frac{a^2\int (e+fx)^3\cosh^3(c+dx)\,dx}{b^3} - \frac{a^3\int \frac{(e+fx)^3}{a+b}}{}$$

$$= -\frac{a^2f(e+fx)^2\cosh^3(c+dx)}{3b^3d^2} - \frac{3af^2(e+fx)\cosh^4(c+dx)}{32b^2d^3} - \frac{a(e+fx)^3 c}{4}$$

$$= \frac{a^3\left(a^2+b^2\right)(e+fx)^4}{4b^6f} + \frac{3f(e+fx)^2\cosh(c+dx)}{8bd^2} - \frac{9af^2(e+fx)\cosh^2(c}{32b^2d^3}$$

$$= \frac{3a(e+fx)^3}{32b^2d} + \frac{a^3\left(a^2+b^2\right)(e+fx)^4}{4b^6f} - \frac{3a^4f(e+fx)^2\cosh(c+dx)}{b^5d^2} - \frac{2a^2}{}$$

$$= \frac{45af^3x}{256b^2d^3} - \frac{a^3(e+fx)^3}{4b^4d} + \frac{3a(e+fx)^3}{32b^2d} + \frac{a^3\left(a^2+b^2\right)(e+fx)^4}{4b^6f} - \frac{4a^2f^3 co}{9b}$$

$$= -\frac{3a^3f^3x}{8b^4d^3} + \frac{45af^3x}{256b^2d^3} - \frac{a^3(e+fx)^3}{4b^4d} + \frac{3a(e+fx)^3}{32b^2d} + \frac{a^3\left(a^2+b^2\right)(e+fx)^4}{4b^6f}$$

$$= -\frac{3a^3f^3x}{8b^4d^3} + \frac{45af^3x}{256b^2d^3} - \frac{a^3(e+fx)^3}{4b^4d} + \frac{3a(e+fx)^3}{32b^2d} + \frac{a^3\left(a^2+b^2\right)(e+fx)^4}{4b^6f}$$

$$= -\frac{3a^3f^3x}{8b^4d^3} + \frac{45af^3x}{256b^2d^3} - \frac{a^3(e+fx)^3}{4b^4d} + \frac{3a(e+fx)^3}{32b^2d} + \frac{a^3\left(a^2+b^2\right)(e+fx)^4}{4b^6f}$$

Mathematica [B] time = 21.8732, size = 5157, normalized size = 3.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x
]),x]

[Out] Result too large to show
```

Maple [F] time = 0.23, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3(\cosh(dx+c))^3(\sinh(dx+c))^3}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)

[Out] int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="maxima")
```

```
[Out] -1/960*e^3*((15*a*b^3*e^(-d*x - c) - 6*b^4 - 10*(4*a^2*b^2 + b^4)*e^(-2*d*x
 - 2*c) + 60*(2*a^3*b + a*b^3)*e^(-3*d*x - 3*c) - 60*(8*a^4 + 6*a^2*b^2 - b
^4)*e^(-4*d*x - 4*c))*e^(5*d*x + 5*c)/(b^5*d) + 960*(a^5 + a^3*b^2)*(d*x +
c)/(b^6*d) + (15*a*b^3*e^(-4*d*x - 4*c) + 6*b^4*e^(-5*d*x - 5*c) + 60*(8*a^
4 + 6*a^2*b^2 - b^4)*e^(-d*x - c) + 60*(2*a^3*b + a*b^3)*e^(-2*d*x - 2*c) +
 10*(4*a^2*b^2 + b^4)*e^(-3*d*x - 3*c))/(b^5*d) + 960*(a^5 + a^3*b^2)*log(-
2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^6*d)) - 1/34560000*(8640000*(
a^5*d^4*f^3*e^(5*c) + a^3*b^2*d^4*f^3*e^(5*c))*x^4 + 34560000*(a^5*d^4*e*f^
2*e^(5*c) + a^3*b^2*d^4*e*f^2*e^(5*c))*x^3 + 51840000*(a^5*d^4*e^2*f*e^(5*c
) + a^3*b^2*d^4*e^2*f*e^(5*c))*x^2 - 1728*(125*b^5*d^3*f^3*x^3*e^(10*c) + 7
5*(5*d^3*e*f^2 - d^2*f^3)*b^5*x^2*e^(10*c) + 15*(25*d^3*e^2*f - 10*d^2*e*f^
2 + 2*d*f^3)*b^5*x*e^(10*c) - 3*(25*d^2*e^2*f - 10*d*e*f^2 + 2*f^3)*b^5*e^(
10*c))*e^(5*d*x) + 16875*(32*a*b^4*d^3*f^3*x^3*e^(9*c) + 24*(4*d^3*e*f^2 -
d^2*f^3)*a*b^4*x^2*e^(9*c) + 12*(8*d^3*e^2*f - 4*d^2*e*f^2 + d*f^3)*a*b^4*x
*e^(9*c) - 3*(8*d^2*e^2*f - 4*d*e*f^2 + f^3)*a*b^4*e^(9*c))*e^(4*d*x) + 400
00*(4*(9*d^2*e^2*f - 6*d*e*f^2 + 2*f^3)*a^2*b^3*e^(8*c) + (9*d^2*e^2*f - 6*
d*e*f^2 + 2*f^3)*b^5*e^(8*c) - 9*(4*a^2*b^3*d^3*f^3*e^(8*c) + b^5*d^3*f^3*e
^(8*c))*x^3 - 9*(4*(3*d^3*e*f^2 - d^2*f^3)*a^2*b^3*e^(8*c) + (3*d^3*e*f^2 -
 d^2*f^3)*b^5*e^(8*c))*x^2 - 3*(4*(9*d^3*e^2*f - 6*d^2*e*f^2 + 2*d*f^3)*a^2
*b^3*e^(8*c) + (9*d^3*e^2*f - 6*d^2*e*f^2 + 2*d*f^3)*b^5*e^(8*c))*x)*e^(3*d
*x) - 540000*(6*(2*d^2*e^2*f - 2*d*e*f^2 + f^3)*a^3*b^2*e^(7*c) + 3*(2*d^2*
e^2*f - 2*d*e*f^2 + f^3)*a*b^4*e^(7*c) - 4*(2*a^3*b^2*d^3*f^3*e^(7*c) + a*b
^4*d^3*f^3*e^(7*c))*x^3 - 6*(2*(2*d^3*e*f^2 - d^2*f^3)*a^3*b^2*e^(7*c) + (2
*d^3*e*f^2 - d^2*f^3)*a*b^4*e^(7*c))*x^2 - 6*(2*(2*d^3*e^2*f - 2*d^2*e*f^2
+ d*f^3)*a^3*b^2*e^(7*c) + (2*d^3*e^2*f - 2*d^2*e*f^2 + d*f^3)*a*b^4*e^(7*c
))*x)*e^(2*d*x) + 2160000*(24*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a^4*b*e^(6*c)
 + 18*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a^2*b^3*e^(6*c) - 3*(d^2*e^2*f - 2*d*
e*f^2 + 2*f^3)*b^5*e^(6*c) - (8*a^4*b*d^3*f^3*e^(6*c) + 6*a^2*b^3*d^3*f^3*e
^(6*c) - b^5*d^3*f^3*e^(6*c))*x^3 - 3*(8*(d^3*e*f^2 - d^2*f^3)*a^4*b*e^(6*c
) + 6*(d^3*e*f^2 - d^2*f^3)*a^2*b^3*e^(6*c) - (d^3*e*f^2 - d^2*f^3)*b^5*e^(
6*c))*x^2 - 3*(8*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*a^4*b*e^(6*c) + 6*(d^3
*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*a^2*b^3*e^(6*c) - (d^3*e^2*f - 2*d^2*e*f^2
+ 2*d*f^3)*b^5*e^(6*c))*x)*e^(d*x) + 2160000*(24*(d^2*e^2*f + 2*d*e*f^2 + 2
*f^3)*a^4*b*e^(4*c) + 18*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*a^2*b^3*e^(4*c) -
3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*b^5*e^(4*c) + (8*a^4*b*d^3*f^3*e^(4*c) +
6*a^2*b^3*d^3*f^3*e^(4*c) - b^5*d^3*f^3*e^(4*c))*x^3 + 3*(8*(d^3*e*f^2 + d^
2*f^3)*a^4*b*e^(4*c) + 6*(d^3*e*f^2 + d^2*f^3)*a^2*b^3*e^(4*c) - (d^3*e*f^2
 + d^2*f^3)*b^5*e^(4*c))*x^2 + 3*(8*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*a^4
*b*e^(4*c) + 6*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*a^2*b^3*e^(4*c) - (d^3*e
^2*f + 2*d^2*e*f^2 + 2*d*f^3)*b^5*e^(4*c))*x)*e^(-d*x) + 540000*(6*(2*d^2*e
^2*f + 2*d*e*f^2 + f^3)*a^3*b^2*e^(3*c) + 3*(2*d^2*e^2*f + 2*d*e*f^2 + f^3)
*a*b^4*e^(3*c) + 4*(2*a^3*b^2*d^3*f^3*e^(3*c) + a*b^4*d^3*f^3*e^(3*c))*x^3
+ 6*(2*(2*d^3*e*f^2 + d^2*f^3)*a^3*b^2*e^(3*c) + (2*d^3*e*f^2 + d^2*f^3)*a*
b^4*e^(3*c))*x^2 + 6*(2*(2*d^3*e^2*f + 2*d^2*e*f^2 + d*f^3)*a^3*b^2*e^(3*c)
 + (2*d^3*e^2*f + 2*d^2*e*f^2 + d*f^3)*a*b^4*e^(3*c))*x)*e^(-2*d*x) + 40000
*(4*(9*d^2*e^2*f + 6*d*e*f^2 + 2*f^3)*a^2*b^3*e^(2*c) + (9*d^2*e^2*f + 6*d*
e*f^2 + 2*f^3)*b^5*e^(2*c) + 9*(4*a^2*b^3*d^3*f^3*e^(2*c) + b^5*d^3*f^3*e^(
2*c))*x^3 + 9*(4*(3*d^3*e*f^2 + d^2*f^3)*a^2*b^3*e^(2*c) + (3*d^3*e*f^2 + d
^2*f^3)*b^5*e^(2*c))*x^2 + 3*(4*(9*d^3*e^2*f + 6*d^2*e*f^2 + 2*d*f^3)*a^2*b
```

```
^3*e^(2*c) + (9*d^3*e^2*f + 6*d^2*e*f^2 + 2*d*f^3)*b^5*e^(2*c))*x)*e^(-3*d*
x) + 16875*(32*a*b^4*d^3*f^3*x^3*e^c + 24*(4*d^3*e*f^2 + d^2*f^3)*a*b^4*x^2
*e^c + 12*(8*d^3*e^2*f + 4*d^2*e*f^2 + d*f^3)*a*b^4*x*e^c + 3*(8*d^2*e^2*f
+ 4*d*e*f^2 + f^3)*a*b^4*e^c)*e^(-4*d*x) + 1728*(125*b^5*d^3*f^3*x^3 + 75*(
5*d^3*e*f^2 + d^2*f^3)*b^5*x^2 + 15*(25*d^3*e^2*f + 10*d^2*e*f^2 + 2*d*f^3)
*b^5*x + 3*(25*d^2*e^2*f + 10*d*e*f^2 + 2*f^3)*b^5)*e^(-5*d*x))*e^(-5*c)/(b
^6*d^4) + integrate(-2*((a^5*b*f^3 + a^3*b^3*f^3)*x^3 + 3*(a^5*b*e*f^2 + a^
3*b^3*e*f^2)*x^2 + 3*(a^5*b*e^2*f + a^3*b^3*e^2*f)*x - ((a^6*f^3*e^c + a^4*
b^2*f^3*e^c)*x^3 + 3*(a^6*e*f^2*e^c + a^4*b^2*e*f^2*e^c)*x^2 + 3*(a^6*e^2*f
*e^c + a^4*b^2*e^2*f*e^c)*x)*e^(d*x))/(b^7*e^(2*d*x + 2*c) + 2*a*b^6*e^(d*x
 + c) - b^7), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**3*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^3 \cosh\left(dx+c\right)^3 \sinh\left(dx+c\right)^3}{b\sinh\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)^3*sinh(d*x + c)^3/(b*sinh(d*x + c) + a)
, x)
```

3.402 $\int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$

Optimal. Leaf size=1049

result too large to display

```
[Out] -(a^3*e*f*x)/(2*b^4*d) + (3*a*e*f*x)/(16*b^2*d) - (a^3*f^2*x^2)/(4*b^4*d) +
 (3*a*f^2*x^2)/(32*b^2*d) + (a^3*(a^2 + b^2)*(e + f*x)^3)/(3*b^6*f) - (2*a^
4*f*(e + f*x)*Cosh[c + d*x])/(b^5*d^2) - (4*a^2*f*(e + f*x)*Cosh[c + d*x])/
(3*b^3*d^2) + (f*(e + f*x)*Cosh[c + d*x])/(4*b*d^2) - (3*a*f^2*Cosh[c + d*x
]^2)/(32*b^2*d^3) - (2*a^2*f*(e + f*x)*Cosh[c + d*x]^3)/(9*b^3*d^2) - (a*f^
2*Cosh[c + d*x]^4)/(32*b^2*d^3) - (a*(e + f*x)^2*Cosh[c + d*x]^4)/(4*b^2*d)
 - (f*(e + f*x)*Cosh[3*c + 3*d*x])/(72*b*d^2) - (f*(e + f*x)*Cosh[5*c + 5*d
*x])/(200*b*d^2) - (a^3*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2])])/(b^6*d) - (a^3*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2])])/(b^6*d) - (2*a^3*(a^2 + b^2)*f*(e + f*x)*P
olyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^6*d^2) - (2*a^3*(a^
2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/
(b^6*d^2) + (2*a^3*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(b^6*d^3) + (2*a^3*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))])/(b^6*d^3) + (2*a^4*f^2*Sinh[c + d*x])/(b^5*d^3)
 + (14*a^2*f^2*Sinh[c + d*x])/(9*b^3*d^3) - (f^2*Sinh[c + d*x])/(4*b*d^3) +
 (a^4*(e + f*x)^2*Sinh[c + d*x])/(b^5*d) + (2*a^2*(e + f*x)^2*Sinh[c + d*x]
)/(3*b^3*d) - ((e + f*x)^2*Sinh[c + d*x])/(8*b*d) + (a^3*f*(e + f*x)*Cosh[c
 + d*x]*Sinh[c + d*x])/(2*b^4*d^2) + (3*a*f*(e + f*x)*Cosh[c + d*x]*Sinh[c
+ d*x])/(16*b^2*d^2) + (a^2*(e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b
^3*d) + (a*f*(e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x])/(8*b^2*d^2) - (a^3*f^
2*Sinh[c + d*x]^2)/(4*b^4*d^3) - (a^3*(e + f*x)^2*Sinh[c + d*x]^2)/(2*b^4*d
) + (2*a^2*f^2*Sinh[c + d*x]^3)/(27*b^3*d^3) + (f^2*Sinh[3*c + 3*d*x])/(216
*b*d^3) + ((e + f*x)^2*Sinh[3*c + 3*d*x])/(48*b*d) + (f^2*Sinh[5*c + 5*d*x]
)/(1000*b*d^3) + ((e + f*x)^2*Sinh[5*c + 5*d*x])/(80*b*d)
```

Rubi [A] time = 1.62347, antiderivative size = 1049, normalized size of antiderivative = 1., number of steps used = 40, number of rules used = 15, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5579, 5448, 3296, 2637, 5447, 3310, 3311, 2633, 5565, 5446, 5561, 2190, 2531, 2282, 6589}

$$-\frac{2f(e+fx)\cosh(c+dx)a^4}{b^5d^2}+\frac{2f^2\sinh(c+dx)a^4}{b^5d^3}+\frac{(e+fx)^2\sinh(c+dx)a^4}{b^5d}+\frac{\left(a^2+b^2\right)(e+fx)^3a^3}{3b^6f}-\frac{f^2x^2a^3}{4b^4d}-\frac{f^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -(a^3*e*f*x)/(2*b^4*d) + (3*a*e*f*x)/(16*b^2*d) - (a^3*f^2*x^2)/(4*b^4*d) +
 (3*a*f^2*x^2)/(32*b^2*d) + (a^3*(a^2 + b^2)*(e + f*x)^3)/(3*b^6*f) - (2*a^
4*f*(e + f*x)*Cosh[c + d*x])/(b^5*d^2) - (4*a^2*f*(e + f*x)*Cosh[c + d*x])/
(3*b^3*d^2) + (f*(e + f*x)*Cosh[c + d*x])/(4*b*d^2) - (3*a*f^2*Cosh[c + d*x
]^2)/(32*b^2*d^3) - (2*a^2*f*(e + f*x)*Cosh[c + d*x]^3)/(9*b^3*d^2) - (a*f^
2*Cosh[c + d*x]^4)/(32*b^2*d^3) - (a*(e + f*x)^2*Cosh[c + d*x]^4)/(4*b^2*d)
 - (f*(e + f*x)*Cosh[3*c + 3*d*x])/(72*b*d^2) - (f*(e + f*x)*Cosh[5*c + 5*d
*x])/(200*b*d^2) - (a^3*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2])])/(b^6*d) - (a^3*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2])])/(b^6*d) - (2*a^3*(a^2 + b^2)*f*(e + f*x)*P
olyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^6*d^2) - (2*a^3*(a^
```

```
2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/
(b^6*d^2) + (2*a^3*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(b^6*d^3) + (2*a^3*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))])/(b^6*d^3) + (2*a^4*f^2*Sinh[c + d*x])/(b^5*d^3)
 + (14*a^2*f^2*Sinh[c + d*x])/(9*b^3*d^3) - (f^2*Sinh[c + d*x])/(4*b*d^3) +
 (a^4*(e + f*x)^2*Sinh[c + d*x])/(b^5*d) + (2*a^2*(e + f*x)^2*Sinh[c + d*x]
)/(3*b^3*d) - ((e + f*x)^2*Sinh[c + d*x])/(8*b*d) + (a^3*f*(e + f*x)*Cosh[c
 + d*x]*Sinh[c + d*x])/(2*b^4*d^2) + (3*a*f*(e + f*x)*Cosh[c + d*x]*Sinh[c
+ d*x])/(16*b^2*d^2) + (a^2*(e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b
^3*d) + (a*f*(e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x])/(8*b^2*d^2) - (a^3*f^
2*Sinh[c + d*x]^2)/(4*b^4*d^3) - (a^3*(e + f*x)^2*Sinh[c + d*x]^2)/(2*b^4*d
) + (2*a^2*f^2*Sinh[c + d*x]^3)/(27*b^3*d^3) + (f^2*Sinh[3*c + 3*d*x])/(216
*b*d^3) + ((e + f*x)^2*Sinh[3*c + 3*d*x])/(48*b*d) + (f^2*Sinh[5*c + 5*d*x]
)/(1000*b*d^3) + ((e + f*x)^2*Sinh[5*c + 5*d*x])/(80*b*d)
```

Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh^2(c+dx}{a+b\sinh(c+dx)}}{b}$$

$$= -\frac{a\int (e+fx)^2\cosh^3(c+dx)\sinh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^2\cosh^3(c+dx)\sinh(c}{a+b\sinh(c+dx)}}{b^2}$$

$$= -\frac{a(e+fx)^2\cosh^4(c+dx)}{4b^2d} + \frac{a^2\int (e+fx)^2\cosh^3(c+dx)\,dx}{b^3} - \frac{a^3\int \frac{(e+fx)^2}{a+b}}{}$$

$$= -\frac{2a^2f(e+fx)\cosh^3(c+dx)}{9b^3d^2} - \frac{af^2\cosh^4(c+dx)}{32b^2d^3} - \frac{a(e+fx)^2\cosh^4(c+}{4b^2d}$$

$$= \frac{a^3(a^2+b^2)(e+fx)^3}{3b^6f} + \frac{f(e+fx)\cosh(c+dx)}{4bd^2} - \frac{3af^2\cosh^2(c+dx)}{32b^2d^3} - 2$$

$$= \frac{3aefx}{16b^2d} + \frac{3af^2x^2}{32b^2d} + \frac{a^3(a^2+b^2)(e+fx)^3}{3b^6f} - \frac{2a^4f(e+fx)\cosh(c+dx)}{b^5d^2} -$$

$$= -\frac{a^3efx}{2b^4d} + \frac{3aefx}{16b^2d} - \frac{a^3f^2x^2}{4b^4d} + \frac{3af^2x^2}{32b^2d} + \frac{a^3(a^2+b^2)(e+fx)^3}{3b^6f} - \frac{2a^4f(e+}{}$$

$$= -\frac{a^3efx}{2b^4d} + \frac{3aefx}{16b^2d} - \frac{a^3f^2x^2}{4b^4d} + \frac{3af^2x^2}{32b^2d} + \frac{a^3(a^2+b^2)(e+fx)^3}{3b^6f} - \frac{2a^4f(e+}{}$$

$$= -\frac{a^3efx}{2b^4d} + \frac{3aefx}{16b^2d} - \frac{a^3f^2x^2}{4b^4d} + \frac{3af^2x^2}{32b^2d} + \frac{a^3(a^2+b^2)(e+fx)^3}{3b^6f} - \frac{2a^4f(e+}{}$$

Mathematica [B] time = 14.9595, size = 3179, normalized size = 3.03

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x
]),x]
```

```
[Out] ((8*a^3*(a^2 + b^2)*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^
3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/
(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a
+ b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2
 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^
(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x)
)/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b
*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 +
E^(2*(c + d*x)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^
2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c
```

```
- Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*
E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c
 + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2
*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1
 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*L
og[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*
c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/
d - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - S
qrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2,
 -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*f^2*Po
lyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (
6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)]))])/d^3 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b
^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c
 + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3))/(3*b^6*(-1 + E^(2*c))) - (8*a^3*(a^2
 + b^2)*e^2*x*(1 + Cosh[2*c] + Sinh[2*c]))/(b^6*(-1 + Cosh[2*c] + Sinh[2*c]
)) - (8*a^3*(a^2 + b^2)*e*f*x^2*(1 + Cosh[2*c] + Sinh[2*c]))/(b^6*(-1 + Cos
h[2*c] + Sinh[2*c])) - (8*a^3*(a^2 + b^2)*f^2*x^3*(1 + Cosh[2*c] + Sinh[2*c
]))/(3*b^6*(-1 + Cosh[2*c] + Sinh[2*c])) + ((-8*a^4 - 6*a^2*b^2 + b^4)*(d^2
*e^2 + 2*d*e*f + 2*f^2)*(Cosh[c]/(2*b^5*d^3) - Sinh[c]/(2*b^5*d^3)) + (8*a^
4*d*e*f + 6*a^2*b^2*d*e*f - b^4*d*e*f + 8*a^4*f^2 + 6*a^2*b^2*f^2 - b^4*f^2
)*(-((x*Cosh[c])/(b^5*d^2)) + (x*Sinh[c])/(b^5*d^2)) + (-8*a^4 - 6*a^2*b^2
+ b^4)*((f^2*x^2*Cosh[c])/(2*b^5*d) - (f^2*x^2*Sinh[c])/(2*b^5*d)))*(Cosh[d
*x] - Sinh[d*x]) + ((-8*a^4 - 6*a^2*b^2 + b^4)*(d^2*e^2 - 2*d*e*f + 2*f^2)*
(-Cosh[c]/(2*b^5*d^3) - Sinh[c]/(2*b^5*d^3)) + (x*(8*a^4*d*e*f*Cosh[c] + 6*
a^2*b^2*d*e*f*Cosh[c] - b^4*d*e*f*Cosh[c] - 8*a^4*f^2*Cosh[c] - 6*a^2*b^2*f
^2*Cosh[c] + b^4*f^2*Cosh[c] + 8*a^4*d*e*f*Sinh[c] + 6*a^2*b^2*d*e*f*Sinh[c
] - b^4*d*e*f*Sinh[c] - 8*a^4*f^2*Sinh[c] - 6*a^2*b^2*f^2*Sinh[c] + b^4*f^2
*Sinh[c]))/(b^5*d^2) + (-8*a^4 - 6*a^2*b^2 + b^4)*(-(f^2*x^2*Cosh[c])/(2*b^
5*d) - (f^2*x^2*Sinh[c])/(2*b^5*d)))*(Cosh[d*x] + Sinh[d*x]) + ((2*a^2 + b^
2)*(2*d^2*e^2 + 2*d*e*f + f^2)*(-(a*Cosh[2*c])/(4*b^4*d^3) + (a*Sinh[2*c])/
(4*b^4*d^3)) + (4*a^3*d*e*f + 2*a*b^2*d*e*f + 2*a^3*f^2 + a*b^2*f^2)*(-(x*C
osh[2*c])/(2*b^4*d^2) + (x*Sinh[2*c])/(2*b^4*d^2)) + (2*a^2 + b^2)*(-(a*f^2
*x^2*Cosh[2*c])/(2*b^4*d) + (a*f^2*x^2*Sinh[2*c])/(2*b^4*d)))*(Cosh[2*d*x]
 - Sinh[2*d*x]) + ((2*a^2 + b^2)*(2*d^2*e^2 - 2*d*e*f + f^2)*(-(a*Cosh[2*c])
/(4*b^4*d^3) - (a*Sinh[2*c])/(4*b^4*d^3)) + (x*(-4*a^3*d*e*f*Cosh[2*c] - 2*
a*b^2*d*e*f*Cosh[2*c] + 2*a^3*f^2*Cosh[2*c] + a*b^2*f^2*Cosh[2*c] - 4*a^3*d
*e*f*Sinh[2*c] - 2*a*b^2*d*e*f*Sinh[2*c] + 2*a^3*f^2*Sinh[2*c] + a*b^2*f^2*
Sinh[2*c]))/(2*b^4*d^2) + (2*a^2 + b^2)*(-(a*f^2*x^2*Cosh[2*c])/(2*b^4*d) -
 (a*f^2*x^2*Sinh[2*c])/(2*b^4*d)))*(Cosh[2*d*x] + Sinh[2*d*x]) + ((4*a^2 +
b^2)*(9*d^2*e^2 + 6*d*e*f + 2*f^2)*(-Cosh[3*c]/(108*b^3*d^3) + Sinh[3*c]/(1
08*b^3*d^3)) + (12*a^2*d*e*f + 3*b^2*d*e*f + 4*a^2*f^2 + b^2*f^2)*(-(x*Cosh
[3*c])/(18*b^3*d^2) + (x*Sinh[3*c])/(18*b^3*d^2)) + (4*a^2 + b^2)*(-(f^2*x^
2*Cosh[3*c])/(12*b^3*d) + (f^2*x^2*Sinh[3*c])/(12*b^3*d)))*(Cosh[3*d*x] - S
inh[3*d*x]) + ((4*a^2 + b^2)*(9*d^2*e^2 - 6*d*e*f + 2*f^2)*(Cosh[3*c]/(108*
b^3*d^3) + Sinh[3*c]/(108*b^3*d^3)) + (x*(12*a^2*d*e*f*Cosh[3*c] + 3*b^2*d*
e*f*Cosh[3*c] - 4*a^2*f^2*Cosh[3*c] - b^2*f^2*Cosh[3*c] + 12*a^2*d*e*f*Sinh
[3*c] + 3*b^2*d*e*f*Sinh[3*c] - 4*a^2*f^2*Sinh[3*c] - b^2*f^2*Sinh[3*c]))/(
18*b^3*d^2) + (4*a^2 + b^2)*((f^2*x^2*Cosh[3*c])/(12*b^3*d) + (f^2*x^2*Sinh
[3*c])/(12*b^3*d)))*(Cosh[3*d*x] + Sinh[3*d*x]) + (-(a*f^2*x^2*Cosh[4*c])/(
8*b^2*d) + (a*f^2*x^2*Sinh[4*c])/(8*b^2*d) + (8*d^2*e^2 + 4*d*e*f + f^2)*(-
(a*Cosh[4*c])/(64*b^2*d^3) + (a*Sinh[4*c])/(64*b^2*d^3)) + (4*a*d*e*f + a*f
^2)*(-(x*Cosh[4*c])/(16*b^2*d^2) + (x*Sinh[4*c])/(16*b^2*d^2)))*(Cosh[4*d*x
] - Sinh[4*d*x]) + (-(a*f^2*x^2*Cosh[4*c])/(8*b^2*d) - (a*f^2*x^2*Sinh[4*c]
)/(8*b^2*d) + (8*d^2*e^2 - 4*d*e*f + f^2)*(-(a*Cosh[4*c])/(64*b^2*d^3) - (a
*Sinh[4*c])/(64*b^2*d^3)) + (x*(-4*a*d*e*f*Cosh[4*c] + a*f^2*Cosh[4*c] - 4*
a*d*e*f*Sinh[4*c] + a*f^2*Sinh[4*c]))/(16*b^2*d^2))*(Cosh[4*d*x] + Sinh[4*d
*x]) + (-(f^2*x^2*Cosh[5*c])/(20*b*d) + (f^2*x^2*Sinh[5*c])/(20*b*d) + (25*
d^2*e^2 + 10*d*e*f + 2*f^2)*(-Cosh[5*c]/(500*b*d^3) + Sinh[5*c]/(500*b*d^3)
```

```
) + (5*d*e*f + f^2)*(-(x*Cosh[5*c])/(50*b*d^2) + (x*Sinh[5*c])/(50*b*d^2)))
*(Cosh[5*d*x] - Sinh[5*d*x]) + ((f^2*x^2*Cosh[5*c])/(20*b*d) + (f^2*x^2*Sin
h[5*c])/(20*b*d) + (25*d^2*e^2 - 10*d*e*f + 2*f^2)*(Cosh[5*c]/(500*b*d^3) +
 Sinh[5*c]/(500*b*d^3)) + (x*(5*d*e*f*Cosh[5*c] - f^2*Cosh[5*c] + 5*d*e*f*S
inh[5*c] - f^2*Sinh[5*c]))/(50*b*d^2))*(Cosh[5*d*x] + Sinh[5*d*x]))/8
```

Maple [F] time = 0.211, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 (\cosh(dx+c))^3 (\sinh(dx+c))^3}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="maxima")
```

```
[Out] -1/960*e^2*((15*a*b^3*e^(-d*x - c) - 6*b^4 - 10*(4*a^2*b^2 + b^4)*e^(-2*d*x
 - 2*c) + 60*(2*a^3*b + a*b^3)*e^(-3*d*x - 3*c) - 60*(8*a^4 + 6*a^2*b^2 - b
^4)*e^(-4*d*x - 4*c))*e^(5*d*x + 5*c)/(b^5*d) + 960*(a^5 + a^3*b^2)*(d*x +
c)/(b^6*d) + (15*a*b^3*e^(-4*d*x - 4*c) + 6*b^4*e^(-5*d*x - 5*c) + 60*(8*a^
4 + 6*a^2*b^2 - b^4)*e^(-d*x - c) + 60*(2*a^3*b + a*b^3)*e^(-2*d*x - 2*c) +
 10*(4*a^2*b^2 + b^4)*e^(-3*d*x - 3*c))/(b^5*d) + 960*(a^5 + a^3*b^2)*log(-
2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^6*d)) - 1/1728000*(576000*(a^
5*d^3*f^2*e^(5*c) + a^3*b^2*d^3*f^2*e^(5*c))*x^3 + 1728000*(a^5*d^3*e*f*e^(
5*c) + a^3*b^2*d^3*e*f*e^(5*c))*x^2 - 432*(25*b^5*d^2*f^2*x^2*e^(10*c) + 10
*(5*d^2*e*f - d*f^2)*b^5*x*e^(10*c) - 2*(5*d*e*f - f^2)*b^5*e^(10*c))*e^(5*
d*x) + 3375*(8*a*b^4*d^2*f^2*x^2*e^(9*c) + 4*(4*d^2*e*f - d*f^2)*a*b^4*x*e^
(9*c) - (4*d*e*f - f^2)*a*b^4*e^(9*c))*e^(4*d*x) + 2000*(8*(3*d*e*f - f^2)*
a^2*b^3*e^(8*c) + 2*(3*d*e*f - f^2)*b^5*e^(8*c) - 9*(4*a^2*b^3*d^2*f^2*e^(8
*c) + b^5*d^2*f^2*e^(8*c))*x^2 - 6*(4*(3*d^2*e*f - d*f^2)*a^2*b^3*e^(8*c) +
 (3*d^2*e*f - d*f^2)*b^5*e^(8*c))*x)*e^(3*d*x) - 54000*(2*(2*d*e*f - f^2)*a
^3*b^2*e^(7*c) + (2*d*e*f - f^2)*a*b^4*e^(7*c) - 2*(2*a^3*b^2*d^2*f^2*e^(7*
c) + a*b^4*d^2*f^2*e^(7*c))*x^2 - 2*(2*(2*d^2*e*f - d*f^2)*a^3*b^2*e^(7*c)
+ (2*d^2*e*f - d*f^2)*a*b^4*e^(7*c))*x)*e^(2*d*x) + 108000*(16*(d*e*f - f^2
)*a^4*b*e^(6*c) + 12*(d*e*f - f^2)*a^2*b^3*e^(6*c) - 2*(d*e*f - f^2)*b^5*e^
(6*c) - (8*a^4*b*d^2*f^2*e^(6*c) + 6*a^2*b^3*d^2*f^2*e^(6*c) - b^5*d^2*f^2*
e^(6*c))*x^2 - 2*(8*(d^2*e*f - d*f^2)*a^4*b*e^(6*c) + 6*(d^2*e*f - d*f^2)*a
^2*b^3*e^(6*c) - (d^2*e*f - d*f^2)*b^5*e^(6*c))*x)*e^(d*x) + 108000*(16*(d*
e*f + f^2)*a^4*b*e^(4*c) + 12*(d*e*f + f^2)*a^2*b^3*e^(4*c) - 2*(d*e*f + f^
2)*b^5*e^(4*c) + (8*a^4*b*d^2*f^2*e^(4*c) + 6*a^2*b^3*d^2*f^2*e^(4*c) - b^5
*d^2*f^2*e^(4*c))*x^2 + 2*(8*(d^2*e*f + d*f^2)*a^4*b*e^(4*c) + 6*(d^2*e*f +
 d*f^2)*a^2*b^3*e^(4*c) - (d^2*e*f + d*f^2)*b^5*e^(4*c))*x)*e^(-d*x) + 5400
0*(2*(2*d*e*f + f^2)*a^3*b^2*e^(3*c) + (2*d*e*f + f^2)*a*b^4*e^(3*c) + 2*(2
*a^3*b^2*d^2*f^2*e^(3*c) + a*b^4*d^2*f^2*e^(3*c))*x^2 + 2*(2*(2*d^2*e*f + d
```

```
*f^2)*a^3*b^2*e^(3*c) + (2*d^2*e*f + d*f^2)*a*b^4*e^(3*c))*x)*e^(-2*d*x) +
2000*(8*(3*d*e*f + f^2)*a^2*b^3*e^(2*c) + 2*(3*d*e*f + f^2)*b^5*e^(2*c) + 9
*(4*a^2*b^3*d^2*f^2*e^(2*c) + b^5*d^2*f^2*e^(2*c))*x^2 + 6*(4*(3*d^2*e*f +
d*f^2)*a^2*b^3*e^(2*c) + (3*d^2*e*f + d*f^2)*b^5*e^(2*c))*x)*e^(-3*d*x) + 3
375*(8*a*b^4*d^2*f^2*x^2*e^c + 4*(4*d^2*e*f + d*f^2)*a*b^4*x*e^c + (4*d*e*f
 + f^2)*a*b^4*e^c)*e^(-4*d*x) + 432*(25*b^5*d^2*f^2*x^2 + 10*(5*d^2*e*f + d
*f^2)*b^5*x + 2*(5*d*e*f + f^2)*b^5)*e^(-5*d*x))*e^(-5*c)/(b^6*d^3) + integ
rate(-2*((a^5*b*f^2 + a^3*b^3*f^2)*x^2 + 2*(a^5*b*e*f + a^3*b^3*e*f)*x - ((
a^6*f^2*e^c + a^4*b^2*f^2*e^c)*x^2 + 2*(a^6*e*f*e^c + a^4*b^2*e*f*e^c)*x)*e
^(d*x))/(b^7*e^(2*d*x + 2*c) + 2*a*b^6*e^(d*x + c) - b^7), x)
```

Fricas [C] time = 4.60862, size = 24563, normalized size = 23.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="fricas")
```

```
[Out] -1/1728000*(10800*b^5*d^2*f^2*x^2 - 432*(25*b^5*d^2*f^2*x^2 + 25*b^5*d^2*e^
2 - 10*b^5*d*e*f + 2*b^5*f^2 + 10*(5*b^5*d^2*e*f - b^5*d*f^2)*x)*cosh(d*x +
 c)^10 - 432*(25*b^5*d^2*f^2*x^2 + 25*b^5*d^2*e^2 - 10*b^5*d*e*f + 2*b^5*f^
2 + 10*(5*b^5*d^2*e*f - b^5*d*f^2)*x)*sinh(d*x + c)^10 + 3375*(8*a*b^4*d^2*
f^2*x^2 + 8*a*b^4*d^2*e^2 - 4*a*b^4*d*e*f + a*b^4*f^2 + 4*(4*a*b^4*d^2*e*f
- a*b^4*d*f^2)*x)*cosh(d*x + c)^9 + 135*(200*a*b^4*d^2*f^2*x^2 + 200*a*b^4*
d^2*e^2 - 100*a*b^4*d*e*f + 25*a*b^4*f^2 + 100*(4*a*b^4*d^2*e*f - a*b^4*d*f
^2)*x - 32*(25*b^5*d^2*f^2*x^2 + 25*b^5*d^2*e^2 - 10*b^5*d*e*f + 2*b^5*f^2
+ 10*(5*b^5*d^2*e*f - b^5*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^9 + 10800*
b^5*d^2*e^2 - 2000*(9*(4*a^2*b^3 + b^5)*d^2*f^2*x^2 + 9*(4*a^2*b^3 + b^5)*d
^2*e^2 - 6*(4*a^2*b^3 + b^5)*d*e*f + 2*(4*a^2*b^3 + b^5)*f^2 + 6*(3*(4*a^2*
b^3 + b^5)*d^2*e*f - (4*a^2*b^3 + b^5)*d*f^2)*x)*cosh(d*x + c)^8 - 5*(3600*
(4*a^2*b^3 + b^5)*d^2*f^2*x^2 + 3600*(4*a^2*b^3 + b^5)*d^2*e^2 - 2400*(4*a^
2*b^3 + b^5)*d*e*f + 800*(4*a^2*b^3 + b^5)*f^2 + 3888*(25*b^5*d^2*f^2*x^2 +
 25*b^5*d^2*e^2 - 10*b^5*d*e*f + 2*b^5*f^2 + 10*(5*b^5*d^2*e*f - b^5*d*f^2)
*x)*cosh(d*x + c)^2 + 2400*(3*(4*a^2*b^3 + b^5)*d^2*e*f - (4*a^2*b^3 + b^5)
*d*f^2)*x - 6075*(8*a*b^4*d^2*f^2*x^2 + 8*a*b^4*d^2*e^2 - 4*a*b^4*d*e*f + a
*b^4*f^2 + 4*(4*a*b^4*d^2*e*f - a*b^4*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c
)^8 + 4320*b^5*d*e*f + 54000*(2*(2*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^3*
b^2 + a*b^4)*d^2*e^2 - 2*(2*a^3*b^2 + a*b^4)*d*e*f + (2*a^3*b^2 + a*b^4)*f^
2 + 2*(2*(2*a^3*b^2 + a*b^4)*d^2*e*f - (2*a^3*b^2 + a*b^4)*d*f^2)*x)*cosh(d
*x + c)^7 + 20*(5400*(2*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 5400*(2*a^3*b^2 + a*
b^4)*d^2*e^2 - 5400*(2*a^3*b^2 + a*b^4)*d*e*f - 2592*(25*b^5*d^2*f^2*x^2 +
25*b^5*d^2*e^2 - 10*b^5*d*e*f + 2*b^5*f^2 + 10*(5*b^5*d^2*e*f - b^5*d*f^2)*
x)*cosh(d*x + c)^3 + 2700*(2*a^3*b^2 + a*b^4)*f^2 + 6075*(8*a*b^4*d^2*f^2*x
^2 + 8*a*b^4*d^2*e^2 - 4*a*b^4*d*e*f + a*b^4*f^2 + 4*(4*a*b^4*d^2*e*f - a*b
^4*d*f^2)*x)*cosh(d*x + c)^2 + 5400*(2*(2*a^3*b^2 + a*b^4)*d^2*e*f - (2*a^3
*b^2 + a*b^4)*d*f^2)*x - 800*(9*(4*a^2*b^3 + b^5)*d^2*f^2*x^2 + 9*(4*a^2*b^
3 + b^5)*d^2*e^2 - 6*(4*a^2*b^3 + b^5)*d*e*f + 2*(4*a^2*b^3 + b^5)*f^2 + 6*
(3*(4*a^2*b^3 + b^5)*d^2*e*f - (4*a^2*b^3 + b^5)*d*f^2)*x)*cosh(d*x + c))*s
inh(d*x + c)^7 + 864*b^5*f^2 - 108000*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*f^2*
x^2 + (8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e^2 - 2*(8*a^4*b + 6*a^2*b^3 - b^5)*d
*e*f + 2*(8*a^4*b + 6*a^2*b^3 - b^5)*f^2 + 2*((8*a^4*b + 6*a^2*b^3 - b^5)*d
^2*e*f - (8*a^4*b + 6*a^2*b^3 - b^5)*d*f^2)*x)*cosh(d*x + c)^6 - 20*(5400*(
8*a^4*b + 6*a^2*b^3 - b^5)*d^2*f^2*x^2 + 5400*(8*a^4*b + 6*a^2*b^3 - b^5)*d
^2*e^2 + 4536*(25*b^5*d^2*f^2*x^2 + 25*b^5*d^2*e^2 - 10*b^5*d*e*f + 2*b^5*f
^2 + 10*(5*b^5*d^2*e*f - b^5*d*f^2)*x)*cosh(d*x + c)^4 - 10800*(8*a^4*b + 6
```

*a^2*b^3 - b^5)*d*e*f - 14175*(8*a*b^4*d^2*f^2*x^2 + 8*a*b^4*d^2*e^2 - 4*a*
b^4*d*e*f + a*b^4*f^2 + 4*(4*a*b^4*d^2*e*f - a*b^4*d*f^2)*x)*cosh(d*x + c)^
3 + 10800*(8*a^4*b + 6*a^2*b^3 - b^5)*f^2 + 2800*(9*(4*a^2*b^3 + b^5)*d^2*f
^2*x^2 + 9*(4*a^2*b^3 + b^5)*d^2*e^2 - 6*(4*a^2*b^3 + b^5)*d*e*f + 2*(4*a^2
*b^3 + b^5)*f^2 + 6*(3*(4*a^2*b^3 + b^5)*d^2*e*f - (4*a^2*b^3 + b^5)*d*f^2)
*x)*cosh(d*x + c)^2 + 10800*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e*f - (8*a^4*b
 + 6*a^2*b^3 - b^5)*d*f^2)*x - 18900*(2*(2*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2
*(2*a^3*b^2 + a*b^4)*d^2*e^2 - 2*(2*a^3*b^2 + a*b^4)*d*e*f + (2*a^3*b^2 + a
*b^4)*f^2 + 2*(2*(2*a^3*b^2 + a*b^4)*d^2*e*f - (2*a^3*b^2 + a*b^4)*d*f^2)*x
)*cosh(d*x + c))*sinh(d*x + c)^6 - 576000*((a^5 + a^3*b^2)*d^3*f^2*x^3 + 3*
(a^5 + a^3*b^2)*d^3*e*f*x^2 + 3*(a^5 + a^3*b^2)*d^3*e^2*x + 6*(a^5 + a^3*b^
2)*c*d^2*e^2 - 6*(a^5 + a^3*b^2)*c^2*d*e*f + 2*(a^5 + a^3*b^2)*c^3*f^2)*cos
h(d*x + c)^5 - 2*(288000*(a^5 + a^3*b^2)*d^3*f^2*x^3 + 864000*(a^5 + a^3*b^
2)*d^3*e*f*x^2 + 864000*(a^5 + a^3*b^2)*d^3*e^2*x + 1728000*(a^5 + a^3*b^2)
*c*d^2*e^2 - 1728000*(a^5 + a^3*b^2)*c^2*d*e*f + 576000*(a^5 + a^3*b^2)*c^3
f^2 + 54432(25*b^5*d^2*f^2*x^2 + 25*b^5*d^2*e^2 - 10*b^5*d*e*f + 2*b^5*f^
2 + 10*(5*b^5*d^2*e*f - b^5*d*f^2)*x)*cosh(d*x + c)^5 - 212625*(8*a*b^4*d^2
*f^2*x^2 + 8*a*b^4*d^2*e^2 - 4*a*b^4*d*e*f + a*b^4*f^2 + 4*(4*a*b^4*d^2*e*f
 - a*b^4*d*f^2)*x)*cosh(d*x + c)^4 + 56000*(9*(4*a^2*b^3 + b^5)*d^2*f^2*x^2
 + 9*(4*a^2*b^3 + b^5)*d^2*e^2 - 6*(4*a^2*b^3 + b^5)*d*e*f + 2*(4*a^2*b^3 +
 b^5)*f^2 + 6*(3*(4*a^2*b^3 + b^5)*d^2*e*f - (4*a^2*b^3 + b^5)*d*f^2)*x)*co
sh(d*x + c)^3 - 567000*(2*(2*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^3*b^2 +
a*b^4)*d^2*e^2 - 2*(2*a^3*b^2 + a*b^4)*d*e*f + (2*a^3*b^2 + a*b^4)*f^2 + 2*
(2*(2*a^3*b^2 + a*b^4)*d^2*e*f - (2*a^3*b^2 + a*b^4)*d*f^2)*x)*cosh(d*x + c
)^2 + 324000*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*f^2*x^2 + (8*a^4*b + 6*a^2*b^
3 - b^5)*d^2*e^2 - 2*(8*a^4*b + 6*a^2*b^3 - b^5)*d*e*f + 2*(8*a^4*b + 6*a^2
*b^3 - b^5)*f^2 + 2*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e*f - (8*a^4*b + 6*a^2
*b^3 - b^5)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 108000*((8*a^4*b + 6
*a^2*b^3 - b^5)*d^2*f^2*x^2 + (8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e^2 + 2*(8*a^
4*b + 6*a^2*b^3 - b^5)*d*e*f + 2*(8*a^4*b + 6*a^2*b^3 - b^5)*f^2 + 2*((8*a^
4*b + 6*a^2*b^3 - b^5)*d^2*e*f + (8*a^4*b + 6*a^2*b^3 - b^5)*d*f^2)*x)*cosh
(d*x + c)^4 + 10*(10800*(8*a^4*b + 6*a^2*b^3 - b^5)*d^2*f^2*x^2 - 9072*(25*
b^5*d^2*f^2*x^2 + 25*b^5*d^2*e^2 - 10*b^5*d*e*f + 2*b^5*f^2 + 10*(5*b^5*d^2
*e*f - b^5*d*f^2)*x)*cosh(d*x + c)^6 + 42525*(8*a*b^4*d^2*f^2*x^2 + 8*a*b^4
*d^2*e^2 - 4*a*b^4*d*e*f + a*b^4*f^2 + 4*(4*a*b^4*d^2*e*f - a*b^4*d*f^2)*x)
*cosh(d*x + c)^5 + 10800*(8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e^2 - 14000*(9*(4*
a^2*b^3 + b^5)*d^2*f^2*x^2 + 9*(4*a^2*b^3 + b^5)*d^2*e^2 - 6*(4*a^2*b^3 + b
^5)*d*e*f + 2*(4*a^2*b^3 + b^5)*f^2 + 6*(3*(4*a^2*b^3 + b^5)*d^2*e*f - (4*a
^2*b^3 + b^5)*d*f^2)*x)*cosh(d*x + c)^4 + 21600*(8*a^4*b + 6*a^2*b^3 - b^5)
*d*e*f + 189000*(2*(2*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^3*b^2 + a*b^4)*
d^2*e^2 - 2*(2*a^3*b^2 + a*b^4)*d*e*f + (2*a^3*b^2 + a*b^4)*f^2 + 2*(2*(2*a
^3*b^2 + a*b^4)*d^2*e*f - (2*a^3*b^2 + a*b^4)*d*f^2)*x)*cosh(d*x + c)^3 + 2
1600*(8*a^4*b + 6*a^2*b^3 - b^5)*f^2 - 162000*((8*a^4*b + 6*a^2*b^3 - b^5)*
d^2*f^2*x^2 + (8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e^2 - 2*(8*a^4*b + 6*a^2*b^3
- b^5)*d*e*f + 2*(8*a^4*b + 6*a^2*b^3 - b^5)*f^2 + 2*((8*a^4*b + 6*a^2*b^3
- b^5)*d^2*e*f - (8*a^4*b + 6*a^2*b^3 - b^5)*d*f^2)*x)*cosh(d*x + c)^2 + 21
600*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e*f + (8*a^4*b + 6*a^2*b^3 - b^5)*d*f^
2)*x - 288000*((a^5 + a^3*b^2)*d^3*f^2*x^3 + 3*(a^5 + a^3*b^2)*d^3*e*f*x^2
+ 3*(a^5 + a^3*b^2)*d^3*e^2*x + 6*(a^5 + a^3*b^2)*c*d^2*e^2 - 6*(a^5 + a^3*
b^2)*c^2*d*e*f + 2*(a^5 + a^3*b^2)*c^3*f^2)*cosh(d*x + c))*sinh(d*x + c)^4
+ 54000*(2*(2*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^3*b^2 + a*b^4)*d^2*e^2
+ 2*(2*a^3*b^2 + a*b^4)*d*e*f + (2*a^3*b^2 + a*b^4)*f^2 + 2*(2*(2*a^3*b^2 +
 a*b^4)*d^2*e*f + (2*a^3*b^2 + a*b^4)*d*f^2)*x)*cosh(d*x + c)^3 - 20*(2592*
(25*b^5*d^2*f^2*x^2 + 25*b^5*d^2*e^2 - 10*b^5*d*e*f + 2*b^5*f^2 + 10*(5*b^5
*d^2*e*f - b^5*d*f^2)*x)*cosh(d*x + c)^7 - 5400*(2*a^3*b^2 + a*b^4)*d^2*f^2
x^2 - 14175(8*a*b^4*d^2*f^2*x^2 + 8*a*b^4*d^2*e^2 - 4*a*b^4*d*e*f + a*b^4
f^2 + 4(4*a*b^4*d^2*e*f - a*b^4*d*f^2)*x)*cosh(d*x + c)^6 + 5600*(9*(4*a^
2*b^3 + b^5)*d^2*f^2*x^2 + 9*(4*a^2*b^3 + b^5)*d^2*e^2 - 6*(4*a^2*b^3 + b^5
)*d*e*f + 2*(4*a^2*b^3 + b^5)*f^2 + 6*(3*(4*a^2*b^3 + b^5)*d^2*e*f - (4*a^2

*b^3 + b^5)*d*f^2)*x)*cosh(d*x + c)^5 - 5400*(2*a^3*b^2 + a*b^4)*d^2*e^2 -
94500*(2*(2*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^3*b^2 + a*b^4)*d^2*e^2 -
2*(2*a^3*b^2 + a*b^4)*d*e*f + (2*a^3*b^2 + a*b^4)*f^2 + 2*(2*(2*a^3*b^2 + a
*b^4)*d^2*e*f - (2*a^3*b^2 + a*b^4)*d*f^2)*x)*cosh(d*x + c)^4 - 5400*(2*a^3
*b^2 + a*b^4)*d*e*f + 108000*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*f^2*x^2 + (8*
a^4*b + 6*a^2*b^3 - b^5)*d^2*e^2 - 2*(8*a^4*b + 6*a^2*b^3 - b^5)*d*e*f + 2*
(8*a^4*b + 6*a^2*b^3 - b^5)*f^2 + 2*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e*f -
(8*a^4*b + 6*a^2*b^3 - b^5)*d*f^2)*x)*cosh(d*x + c)^3 - 2700*(2*a^3*b^2 + a
*b^4)*f^2 + 288000*((a^5 + a^3*b^2)*d^3*f^2*x^3 + 3*(a^5 + a^3*b^2)*d^3*e*f
x^2 + 3(a^5 + a^3*b^2)*d^3*e^2*x + 6*(a^5 + a^3*b^2)*c*d^2*e^2 - 6*(a^5 +
a^3*b^2)*c^2*d*e*f + 2*(a^5 + a^3*b^2)*c^3*f^2)*cosh(d*x + c)^2 - 5400*(2*
(2*a^3*b^2 + a*b^4)*d^2*e*f + (2*a^3*b^2 + a*b^4)*d*f^2)*x - 21600*((8*a^4*
b + 6*a^2*b^3 - b^5)*d^2*f^2*x^2 + (8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e^2 + 2*
(8*a^4*b + 6*a^2*b^3 - b^5)*d*e*f + 2*(8*a^4*b + 6*a^2*b^3 - b^5)*f^2 + 2*(
(8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e*f + (8*a^4*b + 6*a^2*b^3 - b^5)*d*f^2)*x)
*cosh(d*x + c))*sinh(d*x + c)^3 + 2000*(9*(4*a^2*b^3 + b^5)*d^2*f^2*x^2 + 9
*(4*a^2*b^3 + b^5)*d^2*e^2 + 6*(4*a^2*b^3 + b^5)*d*e*f + 2*(4*a^2*b^3 + b^5
)*f^2 + 6*(3*(4*a^2*b^3 + b^5)*d^2*e*f + (4*a^2*b^3 + b^5)*d*f^2)*x)*cosh(d
x + c)^2 - 20(972*(25*b^5*d^2*f^2*x^2 + 25*b^5*d^2*e^2 - 10*b^5*d*e*f + 2
*b^5*f^2 + 10*(5*b^5*d^2*e*f - b^5*d*f^2)*x)*cosh(d*x + c)^8 - 6075*(8*a*b^
4*d^2*f^2*x^2 + 8*a*b^4*d^2*e^2 - 4*a*b^4*d*e*f + a*b^4*f^2 + 4*(4*a*b^4*d^
2*e*f - a*b^4*d*f^2)*x)*cosh(d*x + c)^7 - 900*(4*a^2*b^3 + b^5)*d^2*f^2*x^2
+ 2800*(9*(4*a^2*b^3 + b^5)*d^2*f^2*x^2 + 9*(4*a^2*b^3 + b^5)*d^2*e^2 - 6*
(4*a^2*b^3 + b^5)*d*e*f + 2*(4*a^2*b^3 + b^5)*f^2 + 6*(3*(4*a^2*b^3 + b^5)*
d^2*e*f - (4*a^2*b^3 + b^5)*d*f^2)*x)*cosh(d*x + c)^6 - 56700*(2*(2*a^3*b^2
+ a*b^4)*d^2*f^2*x^2 + 2*(2*a^3*b^2 + a*b^4)*d^2*e^2 - 2*(2*a^3*b^2 + a*b^
4)*d*e*f + (2*a^3*b^2 + a*b^4)*f^2 + 2*(2*(2*a^3*b^2 + a*b^4)*d^2*e*f - (2*
a^3*b^2 + a*b^4)*d*f^2)*x)*cosh(d*x + c)^5 - 900*(4*a^2*b^3 + b^5)*d^2*e^2
+ 81000*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*f^2*x^2 + (8*a^4*b + 6*a^2*b^3 - b
^5)*d^2*e^2 - 2*(8*a^4*b + 6*a^2*b^3 - b^5)*d*e*f + 2*(8*a^4*b + 6*a^2*b^3
- b^5)*f^2 + 2*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e*f - (8*a^4*b + 6*a^2*b^3
- b^5)*d*f^2)*x)*cosh(d*x + c)^4 - 600*(4*a^2*b^3 + b^5)*d*e*f + 288000*((a
^5 + a^3*b^2)*d^3*f^2*x^3 + 3*(a^5 + a^3*b^2)*d^3*e*f*x^2 + 3*(a^5 + a^3*b^
2)*d^3*e^2*x + 6*(a^5 + a^3*b^2)*c*d^2*e^2 - 6*(a^5 + a^3*b^2)*c^2*d*e*f +
2*(a^5 + a^3*b^2)*c^3*f^2)*cosh(d*x + c)^3 - 200*(4*a^2*b^3 + b^5)*f^2 - 32
400*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*f^2*x^2 + (8*a^4*b + 6*a^2*b^3 - b^5)*
d^2*e^2 + 2*(8*a^4*b + 6*a^2*b^3 - b^5)*d*e*f + 2*(8*a^4*b + 6*a^2*b^3 - b^
5)*f^2 + 2*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e*f + (8*a^4*b + 6*a^2*b^3 - b^
5)*d*f^2)*x)*cosh(d*x + c)^2 - 600*(3*(4*a^2*b^3 + b^5)*d^2*e*f + (4*a^2*b^
3 + b^5)*d*f^2)*x - 8100*(2*(2*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^3*b^2
+ a*b^4)*d^2*e^2 + 2*(2*a^3*b^2 + a*b^4)*d*e*f + (2*a^3*b^2 + a*b^4)*f^2 +
2*(2*(2*a^3*b^2 + a*b^4)*d^2*e*f + (2*a^3*b^2 + a*b^4)*d*f^2)*x)*cosh(d*x +
c))*sinh(d*x + c)^2 + 4320*(5*b^5*d^2*e*f + b^5*d*f^2)*x + 3375*(8*a*b^4*d
^2*f^2*x^2 + 8*a*b^4*d^2*e^2 + 4*a*b^4*d*e*f + a*b^4*f^2 + 4*(4*a*b^4*d^2*e
*f + a*b^4*d*f^2)*x)*cosh(d*x + c) + 3456000*(((a^5 + a^3*b^2)*d*f^2*x + (a
^5 + a^3*b^2)*d*e*f)*cosh(d*x + c)^5 + 5*((a^5 + a^3*b^2)*d*f^2*x + (a^5 +
a^3*b^2)*d*e*f)*cosh(d*x + c)^4*sinh(d*x + c) + 10*((a^5 + a^3*b^2)*d*f^2*x
+ (a^5 + a^3*b^2)*d*e*f)*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*((a^5 + a^3*
b^2)*d*f^2*x + (a^5 + a^3*b^2)*d*e*f)*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*(
(a^5 + a^3*b^2)*d*f^2*x + (a^5 + a^3*b^2)*d*e*f)*cosh(d*x + c)*sinh(d*x + c
)^4 + ((a^5 + a^3*b^2)*d*f^2*x + (a^5 + a^3*b^2)*d*e*f)*sinh(d*x + c)^5)*di
log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 3456000*(((a^5 + a^3*b^2)*d*f^2*x + (
a^5 + a^3*b^2)*d*e*f)*cosh(d*x + c)^5 + 5*((a^5 + a^3*b^2)*d*f^2*x + (a^5 +
a^3*b^2)*d*e*f)*cosh(d*x + c)^4*sinh(d*x + c) + 10*((a^5 + a^3*b^2)*d*f^2*
x + (a^5 + a^3*b^2)*d*e*f)*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*((a^5 + a^3
*b^2)*d*f^2*x + (a^5 + a^3*b^2)*d*e*f)*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*
((a^5 + a^3*b^2)*d*f^2*x + (a^5 + a^3*b^2)*d*e*f)*cosh(d*x + c)*sinh(d*x +
c)^4 + ((a^5 + a^3*b^2)*d*f^2*x + (a^5 + a^3*b^2)*d*e*f)*sinh(d*x + c)^5)*d

ilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 1728000*(((a^5 + a^3*b^2)*d^2*e^2 -
2*(a^5 + a^3*b^2)*c*d*e*f + (a^5 + a^3*b^2)*c^2*f^2)*cosh(d*x + c)^5 + 5*((
a^5 + a^3*b^2)*d^2*e^2 - 2*(a^5 + a^3*b^2)*c*d*e*f + (a^5 + a^3*b^2)*c^2*f^
2)*cosh(d*x + c)^4*sinh(d*x + c) + 10*((a^5 + a^3*b^2)*d^2*e^2 - 2*(a^5 + a
^3*b^2)*c*d*e*f + (a^5 + a^3*b^2)*c^2*f^2)*cosh(d*x + c)^3*sinh(d*x + c)^2
+ 10*((a^5 + a^3*b^2)*d^2*e^2 - 2*(a^5 + a^3*b^2)*c*d*e*f + (a^5 + a^3*b^2)
*c^2*f^2)*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*((a^5 + a^3*b^2)*d^2*e^2 - 2*
(a^5 + a^3*b^2)*c*d*e*f + (a^5 + a^3*b^2)*c^2*f^2)*cosh(d*x + c)*sinh(d*x +
c)^4 + ((a^5 + a^3*b^2)*d^2*e^2 - 2*(a^5 + a^3*b^2)*c*d*e*f + (a^5 + a^3*b
^2)*c^2*f^2)*sinh(d*x + c)^5)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2
*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 1728000*(((a^5 + a^3*b^2)*d^2*e^2 - 2*(a^
5 + a^3*b^2)*c*d*e*f + (a^5 + a^3*b^2)*c^2*f^2)*cosh(d*x + c)^5 + 5*((a^5 +
a^3*b^2)*d^2*e^2 - 2*(a^5 + a^3*b^2)*c*d*e*f + (a^5 + a^3*b^2)*c^2*f^2)*co
sh(d*x + c)^4*sinh(d*x + c) + 10*((a^5 + a^3*b^2)*d^2*e^2 - 2*(a^5 + a^3*b^
2)*c*d*e*f + (a^5 + a^3*b^2)*c^2*f^2)*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*
((a^5 + a^3*b^2)*d^2*e^2 - 2*(a^5 + a^3*b^2)*c*d*e*f + (a^5 + a^3*b^2)*c^2*
f^2)*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*((a^5 + a^3*b^2)*d^2*e^2 - 2*(a^5
+ a^3*b^2)*c*d*e*f + (a^5 + a^3*b^2)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^4
+ ((a^5 + a^3*b^2)*d^2*e^2 - 2*(a^5 + a^3*b^2)*c*d*e*f + (a^5 + a^3*b^2)*c
^2*f^2)*sinh(d*x + c)^5)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sq
rt((a^2 + b^2)/b^2) + 2*a) + 1728000*(((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5
+ a^3*b^2)*d^2*e*f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2
)*cosh(d*x + c)^5 + 5*((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5 + a^3*b^2)*d^2*
e*f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2)*cosh(d*x + c)^
4*sinh(d*x + c) + 10*((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5 + a^3*b^2)*d^2*e
*f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2)*cosh(d*x + c)^3
*sinh(d*x + c)^2 + 10*((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5 + a^3*b^2)*d^2*
e*f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2)*cosh(d*x + c)^
2*sinh(d*x + c)^3 + 5*((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5 + a^3*b^2)*d^2*
e*f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2)*cosh(d*x + c)*
sinh(d*x + c)^4 + ((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5 + a^3*b^2)*d^2*e*f*
x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2)*sinh(d*x + c)^5)*l
og(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2) - b)/b) + 1728000*(((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2
*(a^5 + a^3*b^2)*d^2*e*f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^
2*f^2)*cosh(d*x + c)^5 + 5*((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5 + a^3*b^2)
*d^2*e*f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2)*cosh(d*x
+ c)^4*sinh(d*x + c) + 10*((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5 + a^3*b^2)*
d^2*e*f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2)*cosh(d*x +
c)^3*sinh(d*x + c)^2 + 10*((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5 + a^3*b^2)
*d^2*e*f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2)*cosh(d*x
+ c)^2*sinh(d*x + c)^3 + 5*((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5 + a^3*b^2)
*d^2*e*f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2)*cosh(d*x
+ c)*sinh(d*x + c)^4 + ((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5 + a^3*b^2)*d^2
*e*f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2)*sinh(d*x + c)
^5)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 3456000*((a^5 + a^3*b^2)*f^2*cosh(d*
x + c)^5 + 5*(a^5 + a^3*b^2)*f^2*cosh(d*x + c)^4*sinh(d*x + c) + 10*(a^5 +
a^3*b^2)*f^2*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*(a^5 + a^3*b^2)*f^2*cosh(
d*x + c)^2*sinh(d*x + c)^3 + 5*(a^5 + a^3*b^2)*f^2*cosh(d*x + c)*sinh(d*x +
c)^4 + (a^5 + a^3*b^2)*f^2*sinh(d*x + c)^5)*polylog(3, (a*cosh(d*x + c) +
a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
)/b) - 3456000*((a^5 + a^3*b^2)*f^2*cosh(d*x + c)^5 + 5*(a^5 + a^3*b^2)*f^2
*cosh(d*x + c)^4*sinh(d*x + c) + 10*(a^5 + a^3*b^2)*f^2*cosh(d*x + c)^3*sin
h(d*x + c)^2 + 10*(a^5 + a^3*b^2)*f^2*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*(
a^5 + a^3*b^2)*f^2*cosh(d*x + c)*sinh(d*x + c)^4 + (a^5 + a^3*b^2)*f^2*sinh
(d*x + c)^5)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 5*(5400*a*b^4*d^2*f^2*x^2

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 - 864*(25*b^5*d^2*f^2*x^2 + 25*b^5*d^2*e^2 - 10*b^5*d*e*f + 2*b^5*f^2 + 10
*(5*b^5*d^2*e*f - b^5*d*f^2)*x)*cosh(d*x + c)^9 + 5400*a*b^4*d^2*e^2 + 6075
*(8*a*b^4*d^2*f^2*x^2 + 8*a*b^4*d^2*e^2 - 4*a*b^4*d*e*f + a*b^4*f^2 + 4*(4*
a*b^4*d^2*e*f - a*b^4*d*f^2)*x)*cosh(d*x + c)^8 + 2700*a*b^4*d*e*f - 3200*(
9*(4*a^2*b^3 + b^5)*d^2*f^2*x^2 + 9*(4*a^2*b^3 + b^5)*d^2*e^2 - 6*(4*a^2*b^
3 + b^5)*d*e*f + 2*(4*a^2*b^3 + b^5)*f^2 + 6*(3*(4*a^2*b^3 + b^5)*d^2*e*f -
 (4*a^2*b^3 + b^5)*d*f^2)*x)*cosh(d*x + c)^7 + 675*a*b^4*f^2 + 75600*(2*(2*
a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^3*b^2 + a*b^4)*d^2*e^2 - 2*(2*a^3*b^2
 + a*b^4)*d*e*f + (2*a^3*b^2 + a*b^4)*f^2 + 2*(2*(2*a^3*b^2 + a*b^4)*d^2*e*
f - (2*a^3*b^2 + a*b^4)*d*f^2)*x)*cosh(d*x + c)^6 - 129600*((8*a^4*b + 6*a^
2*b^3 - b^5)*d^2*f^2*x^2 + (8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e^2 - 2*(8*a^4*b
 + 6*a^2*b^3 - b^5)*d*e*f + 2*(8*a^4*b + 6*a^2*b^3 - b^5)*f^2 + 2*((8*a^4*b
 + 6*a^2*b^3 - b^5)*d^2*e*f - (8*a^4*b + 6*a^2*b^3 - b^5)*d*f^2)*x)*cosh(d*
x + c)^5 - 576000*((a^5 + a^3*b^2)*d^3*f^2*x^3 + 3*(a^5 + a^3*b^2)*d^3*e*f*
x^2 + 3*(a^5 + a^3*b^2)*d^3*e^2*x + 6*(a^5 + a^3*b^2)*c*d^2*e^2 - 6*(a^5 +
a^3*b^2)*c^2*d*e*f + 2*(a^5 + a^3*b^2)*c^3*f^2)*cosh(d*x + c)^4 + 86400*((8
*a^4*b + 6*a^2*b^3 - b^5)*d^2*f^2*x^2 + (8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e^2
 + 2*(8*a^4*b + 6*a^2*b^3 - b^5)*d*e*f + 2*(8*a^4*b + 6*a^2*b^3 - b^5)*f^2
+ 2*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e*f + (8*a^4*b + 6*a^2*b^3 - b^5)*d*f^
2)*x)*cosh(d*x + c)^3 + 32400*(2*(2*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^3
*b^2 + a*b^4)*d^2*e^2 + 2*(2*a^3*b^2 + a*b^4)*d*e*f + (2*a^3*b^2 + a*b^4)*f
^2 + 2*(2*(2*a^3*b^2 + a*b^4)*d^2*e*f + (2*a^3*b^2 + a*b^4)*d*f^2)*x)*cosh(
d*x + c)^2 + 2700*(4*a*b^4*d^2*e*f + a*b^4*d*f^2)*x + 800*(9*(4*a^2*b^3 + b
^5)*d^2*f^2*x^2 + 9*(4*a^2*b^3 + b^5)*d^2*e^2 + 6*(4*a^2*b^3 + b^5)*d*e*f +
 2*(4*a^2*b^3 + b^5)*f^2 + 6*(3*(4*a^2*b^3 + b^5)*d^2*e*f + (4*a^2*b^3 + b^
5)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))/(b^6*d^3*cosh(d*x + c)^5 + 5*b^6
*d^3*cosh(d*x + c)^4*sinh(d*x + c) + 10*b^6*d^3*cosh(d*x + c)^3*sinh(d*x +
c)^2 + 10*b^6*d^3*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*b^6*d^3*cosh(d*x + c)
*sinh(d*x + c)^4 + b^6*d^3*sinh(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**3*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2\cosh(dx+c)^3\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)^3*sinh(d*x + c)^3/(b*sinh(d*x + c) + a)
, x)
```

3.403 $\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=641

$$-\frac{a^3 f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^6d^2}-\frac{a^3 f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^6d^2}-\frac{a^2 f\cosh^3(c+dx)}{9b^3d^2}-\frac{a^4 f\cosh(c+dx)}{b^5d^2}$$

```
[Out] -(a^3*f*x)/(4*b^4*d) + (3*a*f*x)/(32*b^2*d) + (a^3*(a^2 + b^2)*(e + f*x)^2)
/(2*b^6*f) - (a^4*f*Cosh[c + d*x])/(b^5*d^2) - (2*a^2*f*Cosh[c + d*x])/(3*b
^3*d^2) + (f*Cosh[c + d*x])/(8*b*d^2) - (a^2*f*Cosh[c + d*x]^3)/(9*b^3*d^2)
 - (a*(e + f*x)*Cosh[c + d*x]^4)/(4*b^2*d) - (f*Cosh[3*c + 3*d*x])/(144*b*d
^2) - (f*Cosh[5*c + 5*d*x])/(400*b*d^2) - (a^3*(a^2 + b^2)*(e + f*x)*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^6*d) - (a^3*(a^2 + b^2)*(e + f
*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^6*d) - (a^3*(a^2 + b
^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^6*d^2) - (a^
3*(a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^6*
d^2) + (a^4*(e + f*x)*Sinh[c + d*x])/(b^5*d) + (2*a^2*(e + f*x)*Sinh[c + d*
x])/(3*b^3*d) - ((e + f*x)*Sinh[c + d*x])/(8*b*d) + (a^3*f*Cosh[c + d*x]*Si
nh[c + d*x])/(4*b^4*d^2) + (3*a*f*Cosh[c + d*x]*Sinh[c + d*x])/(32*b^2*d^2)
 + (a^2*(e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b^3*d) + (a*f*Cosh[c +
d*x]^3*Sinh[c + d*x])/(16*b^2*d^2) - (a^3*(e + f*x)*Sinh[c + d*x]^2)/(2*b^4
*d) + ((e + f*x)*Sinh[3*c + 3*d*x])/(48*b*d) + ((e + f*x)*Sinh[5*c + 5*d*x]
)/(80*b*d)
```

Rubi [A] time = 0.944258, antiderivative size = 641, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 14, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5579, 5448, 3296, 2638, 5447, 2635, 8, 3310, 5565, 5446, 5561, 2190, 2279, 2391}

$$-\frac{a^3 f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^6d^2}-\frac{a^3 f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^6d^2}-\frac{a^2 f\cosh^3(c+dx)}{9b^3d^2}-\frac{a^4 f\cosh(c+dx)}{b^5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -(a^3*f*x)/(4*b^4*d) + (3*a*f*x)/(32*b^2*d) + (a^3*(a^2 + b^2)*(e + f*x)^2)
/(2*b^6*f) - (a^4*f*Cosh[c + d*x])/(b^5*d^2) - (2*a^2*f*Cosh[c + d*x])/(3*b
^3*d^2) + (f*Cosh[c + d*x])/(8*b*d^2) - (a^2*f*Cosh[c + d*x]^3)/(9*b^3*d^2)
 - (a*(e + f*x)*Cosh[c + d*x]^4)/(4*b^2*d) - (f*Cosh[3*c + 3*d*x])/(144*b*d
^2) - (f*Cosh[5*c + 5*d*x])/(400*b*d^2) - (a^3*(a^2 + b^2)*(e + f*x)*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^6*d) - (a^3*(a^2 + b^2)*(e + f
*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^6*d) - (a^3*(a^2 + b
^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^6*d^2) - (a^
3*(a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^6*
d^2) + (a^4*(e + f*x)*Sinh[c + d*x])/(b^5*d) + (2*a^2*(e + f*x)*Sinh[c + d*
x])/(3*b^3*d) - ((e + f*x)*Sinh[c + d*x])/(8*b*d) + (a^3*f*Cosh[c + d*x]*Si
nh[c + d*x])/(4*b^4*d^2) + (3*a*f*Cosh[c + d*x]*Sinh[c + d*x])/(32*b^2*d^2)
 + (a^2*(e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b^3*d) + (a*f*Cosh[c +
d*x]^3*Sinh[c + d*x])/(16*b^2*d^2) - (a^3*(e + f*x)*Sinh[c + d*x]^2)/(2*b^4
*d) + ((e + f*x)*Sinh[3*c + 3*d*x])/(48*b*d) + ((e + f*x)*Sinh[5*c + 5*d*x]
)/(80*b*d)
```


Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
```

```
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(e+fx)\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh^3(c+dx)\sinh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\cosh^3(c+dx)\sinh^2(c+d}{a+b\sinh(c+dx)}}{b}$$

$$= -\frac{a\int (e+fx)\cosh^3(c+dx)\sinh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)\cosh^3(c+dx)\sinh(c}{a+b\sinh(c+dx)}}{b^2}$$

$$= -\frac{a(e+fx)\cosh^4(c+dx)}{4b^2d} + \frac{a^2\int (e+fx)\cosh^3(c+dx)\,dx}{b^3} - \frac{a^3\int \frac{(e+fx)}{a+b s}}{}$$

$$= -\frac{a^2f\cosh^3(c+dx)}{9b^3d^2} - \frac{a(e+fx)\cosh^4(c+dx)}{4b^2d} - \frac{(e+fx)\sinh(c+dx)}{8bd}$$

$$= \frac{a^3\left(a^2+b^2\right)(e+fx)^2}{2b^6f} + \frac{f\cosh(c+dx)}{8bd^2} - \frac{a^2f\cosh^3(c+dx)}{9b^3d^2} - \frac{a(e+fx}{}$$

$$= \frac{3afx}{32b^2d} + \frac{a^3\left(a^2+b^2\right)(e+fx)^2}{2b^6f} - \frac{a^4f\cosh(c+dx)}{b^5d^2} - \frac{2a^2f\cosh(c+dx)}{3b^3d^2}$$

$$= -\frac{a^3fx}{4b^4d} + \frac{3afx}{32b^2d} + \frac{a^3\left(a^2+b^2\right)(e+fx)^2}{2b^6f} - \frac{a^4f\cosh(c+dx)}{b^5d^2} - \frac{2a^2f\cos}{3b}$$

$$= -\frac{a^3fx}{4b^4d} + \frac{3afx}{32b^2d} + \frac{a^3\left(a^2+b^2\right)(e+fx)^2}{2b^6f} - \frac{a^4f\cosh(c+dx)}{b^5d^2} - \frac{2a^2f\cos}{3b}$$

Mathematica [A] time = 4.34045, size = 958, normalized size = 1.49

$$-\frac{-14400d^2fx^2a^5 - 14400c^2fa^5 - 28800cdfxa^5 + 28800cf\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)a^5 + 28800dfx\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)a^5 + }{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] -(-14400*a^5*c^2*f - 14400*a^3*b^2*c^2*f - 28800*a^5*c*d*f*x - 28800*a^3*b^
2*c*d*f*x - 14400*a^5*d^2*f*x^2 - 14400*a^3*b^2*d^2*f*x^2 + 28800*a^4*b*f*C
osh[c + d*x] + 21600*a^2*b^3*f*Cosh[c + d*x] - 3600*b^5*f*Cosh[c + d*x] + 7
200*a^3*b^2*d*e*Cosh[2*(c + d*x)] + 3600*a*b^4*d*e*Cosh[2*(c + d*x)] + 7200
*a^3*b^2*d*f*x*Cosh[2*(c + d*x)] + 3600*a*b^4*d*f*x*Cosh[2*(c + d*x)] + 800
*a^2*b^3*f*Cosh[3*(c + d*x)] + 200*b^5*f*Cosh[3*(c + d*x)] + 900*a*b^4*d*e*
Cosh[4*(c + d*x)] + 900*a*b^4*d*f*x*Cosh[4*(c + d*x)] + 72*b^5*f*Cosh[5*(c
+ d*x)] + 28800*a^5*c*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 28
800*a^3*b^2*c*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 28800*a^5*
d*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 28800*a^3*b^2*d*f*x*
Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 28800*a^5*c*f*Log[1 + (b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 28800*a^3*b^2*c*f*Log[1 + (b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2])] + 28800*a^5*d*f*x*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])] + 28800*a^3*b^2*d*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2])] + 28800*a^5*d*e*Log[a + b*Sinh[c + d*x]] + 28800*a^3*b^2*d*e*Log
[a + b*Sinh[c + d*x]] - 28800*a^5*c*f*Log[a + b*Sinh[c + d*x]] - 28800*a^3*
b^2*c*f*Log[a + b*Sinh[c + d*x]] + 28800*a^3*(a^2 + b^2)*f*PolyLog[2, (b*E^
(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 28800*a^3*(a^2 + b^2)*f*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 28800*a^4*b*d*e*Sinh[c + d*x] - 21
600*a^2*b^3*d*e*Sinh[c + d*x] + 3600*b^5*d*e*Sinh[c + d*x] - 28800*a^4*b*d*
f*x*Sinh[c + d*x] - 21600*a^2*b^3*d*f*x*Sinh[c + d*x] + 3600*b^5*d*f*x*Sinh
```

```
[c + d*x] - 3600*a^3*b^2*f*Sinh[2*(c + d*x)] - 1800*a*b^4*f*Sinh[2*(c + d*x
)] - 2400*a^2*b^3*d*e*Sinh[3*(c + d*x)] - 600*b^5*d*e*Sinh[3*(c + d*x)] - 2
400*a^2*b^3*d*f*x*Sinh[3*(c + d*x)] - 600*b^5*d*f*x*Sinh[3*(c + d*x)] - 225
*a*b^4*f*Sinh[4*(c + d*x)] - 360*b^5*d*e*Sinh[5*(c + d*x)] - 360*b^5*d*f*x*
Sinh[5*(c + d*x)])/(28800*b^6*d^2)
```

Maple [B] time = 0.115, size = 1363, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] a^5/b^6/d^2*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2*a^5/b^6/d^2*f*c*ln(
exp(d*x+c))-a^5/b^6/d*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^
(1/2)))*x-a^5/b^6/d^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^
(1/2)))*c-a^5/b^6/d*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2
)))*x-a^5/b^6/d^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))
)*c+2*a^5/b^6/d*f*c*x+1/2*a^3*f*x^2/b^4-1/32*a*(2*a^2+b^2)*(2*d*f*x+2*d*e+f
)/b^4/d^2*exp(-2*d*x-2*c)-a^3/b^4/d*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
(-a+(a^2+b^2)^(1/2)))*x-a^3/b^4/d^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
(-a+(a^2+b^2)^(1/2)))*c-a^3/b^4/d*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+
(a^2+b^2)^(1/2)))*x-a^3/b^4/d^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a
^2+b^2)^(1/2)))*c+a^3/b^4/d^2*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2*a
^3/b^4/d^2*f*c*ln(exp(d*x+c))+2*a^3/b^4/d*f*c*x-a^3*e*x/b^4+1/800*(5*d*f*x+
5*d*e-f)/d^2/b*exp(5*d*x+5*c)+1/288*(12*a^2*d*f*x+3*b^2*d*f*x+12*a^2*d*e+3*
b^2*d*e-4*a^2*f-b^2*f)/b^3/d^2*exp(3*d*x+3*c)+a^5/b^6/d^2*f*c^2+1/16*(8*a^4
*d*f*x+6*a^2*b^2*d*f*x-b^4*d*f*x+8*a^4*d*e+6*a^2*b^2*d*e-b^4*d*e-8*a^4*f-6*
a^2*b^2*f+b^4*f)/b^5/d^2*exp(d*x+c)-1/800*(5*d*f*x+5*d*e+f)/d^2/b*exp(-5*d*
x-5*c)-1/16*(8*a^4+6*a^2*b^2-b^4)*(d*f*x+d*e+f)/b^5/d^2*exp(-d*x-c)-1/288*(
4*a^2+b^2)*(3*d*f*x+3*d*e+f)/b^3/d^2*exp(-3*d*x-3*c)-1/256*a*(4*d*f*x+4*d*e
+f)/b^2/d^2*exp(-4*d*x-4*c)-1/256*a*(4*d*f*x+4*d*e-f)/b^2/d^2*exp(4*d*x+4*c
)-1/32*a*(4*a^2*d*f*x+2*b^2*d*f*x+4*a^2*d*e+2*b^2*d*e-2*a^2*f-b^2*f)/b^4/d^
2*exp(2*d*x+2*c)+a^3/b^4/d^2*f*c^2-a^3/b^4/d^2*f*dilog((b*exp(d*x+c)+(a^2+b
^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-a^3/b^4/d^2*f*dilog((-b*exp(d*x+c)+(a^2+b
^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-a^3/b^4/d*e*ln(b*exp(2*d*x+2*c)+2*a*exp(
d*x+c)-b)+2*a^3/b^4/d*e*ln(exp(d*x+c))+1/2*a^5/b^6*f*x^2-a^5/b^6*e*x-a^5/b^
6/d*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2*a^5/b^6/d*e*ln(exp(d*x+c))-a^
5/b^6/d^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-a^5
/b^6/d^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/960*e*((15*a*b^3*e^(-d*x - c) - 6*b^4 - 10*(4*a^2*b^2 + b^4)*e^(-2*d*x -
 2*c) + 60*(2*a^3*b + a*b^3)*e^(-3*d*x - 3*c) - 60*(8*a^4 + 6*a^2*b^2 - b^4
)*e^(-4*d*x - 4*c))*e^(5*d*x + 5*c)/(b^5*d) + 960*(a^5 + a^3*b^2)*(d*x + c)
```

```
/(b^6*d) + (15*a*b^3*e^(-4*d*x - 4*c) + 6*b^4*e^(-5*d*x - 5*c) + 60*(8*a^4
+ 6*a^2*b^2 - b^4)*e^(-d*x - c) + 60*(2*a^3*b + a*b^3)*e^(-2*d*x - 2*c) + 1
0*(4*a^2*b^2 + b^4)*e^(-3*d*x - 3*c))/(b^5*d) + 960*(a^5 + a^3*b^2)*log(-2*
a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^6*d)) - 1/57600*f*((28800*(a^5*
d^2*e^(5*c) + a^3*b^2*d^2*e^(5*c))*x^2 - 72*(5*b^5*d*x*e^(10*c) - b^5*e^(10
*c))*e^(5*d*x) + 225*(4*a*b^4*d*x*e^(9*c) - a*b^4*e^(9*c))*e^(4*d*x) + 200*
(4*a^2*b^3*e^(8*c) + b^5*e^(8*c) - 3*(4*a^2*b^3*d*e^(8*c) + b^5*d*e^(8*c))*
x)*e^(3*d*x) - 1800*(2*a^3*b^2*e^(7*c) + a*b^4*e^(7*c) - 2*(2*a^3*b^2*d*e^(
7*c) + a*b^4*d*e^(7*c))*x)*e^(2*d*x) + 3600*(8*a^4*b*e^(6*c) + 6*a^2*b^3*e^
(6*c) - b^5*e^(6*c) - (8*a^4*b*d*e^(6*c) + 6*a^2*b^3*d*e^(6*c) - b^5*d*e^(6
*c))*x)*e^(d*x) + 3600*(8*a^4*b*e^(4*c) + 6*a^2*b^3*e^(4*c) - b^5*e^(4*c) +
 (8*a^4*b*d*e^(4*c) + 6*a^2*b^3*d*e^(4*c) - b^5*d*e^(4*c))*x)*e^(-d*x) + 18
00*(2*a^3*b^2*e^(3*c) + a*b^4*e^(3*c) + 2*(2*a^3*b^2*d*e^(3*c) + a*b^4*d*e^
(3*c))*x)*e^(-2*d*x) + 200*(4*a^2*b^3*e^(2*c) + b^5*e^(2*c) + 3*(4*a^2*b^3*
d*e^(2*c) + b^5*d*e^(2*c))*x)*e^(-3*d*x) + 225*(4*a*b^4*d*x*e^c + a*b^4*e^c
)*e^(-4*d*x) + 72*(5*b^5*d*x + b^5)*e^(-5*d*x))*e^(-5*c)/(b^6*d^2) - 900*in
tegrate(128*((a^6*e^c + a^4*b^2*e^c)*x*e^(d*x) - (a^5*b + a^3*b^3)*x)/(b^7*
e^(2*d*x + 2*c) + 2*a*b^6*e^(d*x + c) - b^7), x))
```

Fricas [B] time = 4.19557, size = 12709, normalized size = 19.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/57600*(72*(5*b^5*d*f*x + 5*b^5*d*e - b^5*f)*cosh(d*x + c)^10 + 72*(5*b^5*
d*f*x + 5*b^5*d*e - b^5*f)*sinh(d*x + c)^10 - 225*(4*a*b^4*d*f*x + 4*a*b^4*
d*e - a*b^4*f)*cosh(d*x + c)^9 - 45*(20*a*b^4*d*f*x + 20*a*b^4*d*e - 5*a*b^
4*f - 16*(5*b^5*d*f*x + 5*b^5*d*e - b^5*f)*cosh(d*x + c))*sinh(d*x + c)^9 +
 200*(3*(4*a^2*b^3 + b^5)*d*f*x + 3*(4*a^2*b^3 + b^5)*d*e - (4*a^2*b^3 + b^
5)*f)*cosh(d*x + c)^8 + 5*(120*(4*a^2*b^3 + b^5)*d*f*x + 120*(4*a^2*b^3 + b
^5)*d*e + 648*(5*b^5*d*f*x + 5*b^5*d*e - b^5*f)*cosh(d*x + c)^2 - 40*(4*a^2
*b^3 + b^5)*f - 405*(4*a*b^4*d*f*x + 4*a*b^4*d*e - a*b^4*f)*cosh(d*x + c))*
sinh(d*x + c)^8 - 360*b^5*d*f*x - 1800*(2*(2*a^3*b^2 + a*b^4)*d*f*x + 2*(2*
a^3*b^2 + a*b^4)*d*e - (2*a^3*b^2 + a*b^4)*f)*cosh(d*x + c)^7 - 20*(180*(2*
a^3*b^2 + a*b^4)*d*f*x - 432*(5*b^5*d*f*x + 5*b^5*d*e - b^5*f)*cosh(d*x + c
)^3 + 180*(2*a^3*b^2 + a*b^4)*d*e + 405*(4*a*b^4*d*f*x + 4*a*b^4*d*e - a*b^
4*f)*cosh(d*x + c)^2 - 90*(2*a^3*b^2 + a*b^4)*f - 80*(3*(4*a^2*b^3 + b^5)*d
*f*x + 3*(4*a^2*b^3 + b^5)*d*e - (4*a^2*b^3 + b^5)*f)*cosh(d*x + c))*sinh(d
*x + c)^7 - 360*b^5*d*e + 3600*((8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x + (8*a^4*
b + 6*a^2*b^3 - b^5)*d*e - (8*a^4*b + 6*a^2*b^3 - b^5)*f)*cosh(d*x + c)^6 +
 20*(756*(5*b^5*d*f*x + 5*b^5*d*e - b^5*f)*cosh(d*x + c)^4 + 180*(8*a^4*b +
 6*a^2*b^3 - b^5)*d*f*x - 945*(4*a*b^4*d*f*x + 4*a*b^4*d*e - a*b^4*f)*cosh(
d*x + c)^3 + 180*(8*a^4*b + 6*a^2*b^3 - b^5)*d*e + 280*(3*(4*a^2*b^3 + b^5)
*d*f*x + 3*(4*a^2*b^3 + b^5)*d*e - (4*a^2*b^3 + b^5)*f)*cosh(d*x + c)^2 - 1
80*(8*a^4*b + 6*a^2*b^3 - b^5)*f - 630*(2*(2*a^3*b^2 + a*b^4)*d*f*x + 2*(2*
a^3*b^2 + a*b^4)*d*e - (2*a^3*b^2 + a*b^4)*f)*cosh(d*x + c))*sinh(d*x + c)^
6 - 72*b^5*f + 28800*((a^5 + a^3*b^2)*d^2*f*x^2 + 2*(a^5 + a^3*b^2)*d^2*e*x
 + 4*(a^5 + a^3*b^2)*c*d*e - 2*(a^5 + a^3*b^2)*c^2*f)*cosh(d*x + c)^5 + 2*(
14400*(a^5 + a^3*b^2)*d^2*f*x^2 + 9072*(5*b^5*d*f*x + 5*b^5*d*e - b^5*f)*co
sh(d*x + c)^5 + 28800*(a^5 + a^3*b^2)*d^2*e*x - 14175*(4*a*b^4*d*f*x + 4*a*
b^4*d*e - a*b^4*f)*cosh(d*x + c)^4 + 57600*(a^5 + a^3*b^2)*c*d*e - 28800*(a
^5 + a^3*b^2)*c^2*f + 5600*(3*(4*a^2*b^3 + b^5)*d*f*x + 3*(4*a^2*b^3 + b^5)
*d*e - (4*a^2*b^3 + b^5)*f)*cosh(d*x + c)^3 - 18900*(2*(2*a^3*b^2 + a*b^4)*
```

d*f*x + 2*(2*a^3*b^2 + a*b^4)*d*e - (2*a^3*b^2 + a*b^4)*f)*cosh(d*x + c)^2 + 10800*((8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x + (8*a^4*b + 6*a^2*b^3 - b^5)*d*e - (8*a^4*b + 6*a^2*b^3 - b^5)*f)*cosh(d*x + c))*sinh(d*x + c)^5 - 3600*((8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x + (8*a^4*b + 6*a^2*b^3 - b^5)*d*e + (8*a^4*b + 6*a^2*b^3 - b^5)*f)*cosh(d*x + c)^4 + 10*(1512*(5*b^5*d*f*x + 5*b^5*d*e - b^5*f)*cosh(d*x + c)^6 - 2835*(4*a*b^4*d*f*x + 4*a*b^4*d*e - a*b^4*f)*cosh(d*x + c)^5 + 1400*(3*(4*a^2*b^3 + b^5)*d*f*x + 3*(4*a^2*b^3 + b^5)*d*e - (4*a^2*b^3 + b^5)*f)*cosh(d*x + c)^4 - 360*(8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x - 6300*(2*(2*a^3*b^2 + a*b^4)*d*f*x + 2*(2*a^3*b^2 + a*b^4)*d*e - (2*a^3*b^2 + a*b^4)*f)*cosh(d*x + c)^3 - 360*(8*a^4*b + 6*a^2*b^3 - b^5)*d*e + 5400*((8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x + (8*a^4*b + 6*a^2*b^3 - b^5)*d*e - (8*a^4*b + 6*a^2*b^3 - b^5)*f)*cosh(d*x + c)^2 - 360*(8*a^4*b + 6*a^2*b^3 - b^5)*f + 14400*((a^5 + a^3*b^2)*d^2*f*x^2 + 2*(a^5 + a^3*b^2)*d^2*e*x + 4*(a^5 + a^3*b^2)*c*d*e - 2*(a^5 + a^3*b^2)*c^2*f)*cosh(d*x + c))*sinh(d*x + c)^4 - 1800*(2*(2*a^3*b^2 + a*b^4)*d*f*x + 2*(2*a^3*b^2 + a*b^4)*d*e + (2*a^3*b^2 + a*b^4)*f)*cosh(d*x + c)^3 + 20*(432*(5*b^5*d*f*x + 5*b^5*d*e - b^5*f)*cosh(d*x + c)^7 - 945*(4*a*b^4*d*f*x + 4*a*b^4*d*e - a*b^4*f)*cosh(d*x + c)^6 + 560*(3*(4*a^2*b^3 + b^5)*d*f*x + 3*(4*a^2*b^3 + b^5)*d*e - (4*a^2*b^3 + b^5)*f)*cosh(d*x + c)^5 - 3150*(2*(2*a^3*b^2 + a*b^4)*d*f*x + 2*(2*a^3*b^2 + a*b^4)*d*e - (2*a^3*b^2 + a*b^4)*f)*cosh(d*x + c)^4 - 180*(2*a^3*b^2 + a*b^4)*d*f*x + 3600*((8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x + (8*a^4*b + 6*a^2*b^3 - b^5)*d*e - (8*a^4*b + 6*a^2*b^3 - b^5)*f)*cosh(d*x + c)^3 - 180*(2*a^3*b^2 + a*b^4)*d*e + 14400*((a^5 + a^3*b^2)*d^2*f*x^2 + 2*(a^5 + a^3*b^2)*d^2*e*x + 4*(a^5 + a^3*b^2)*c*d*e - 2*(a^5 + a^3*b^2)*c^2*f)*cosh(d*x + c)^2 - 90*(2*a^3*b^2 + a*b^4)*f - 720*((8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x + (8*a^4*b + 6*a^2*b^3 - b^5)*d*e + (8*a^4*b + 6*a^2*b^3 - b^5)*f)*cosh(d*x + c))*sinh(d*x + c)^3 - 200*(3*(4*a^2*b^3 + b^5)*d*f*x + 3*(4*a^2*b^3 + b^5)*d*e + (4*a^2*b^3 + b^5)*f)*cosh(d*x + c)^2 + 20*(162*(5*b^5*d*f*x + 5*b^5*d*e - b^5*f)*cosh(d*x + c)^8 - 405*(4*a*b^4*d*f*x + 4*a*b^4*d*e - a*b^4*f)*cosh(d*x + c)^7 + 280*(3*(4*a^2*b^3 + b^5)*d*f*x + 3*(4*a^2*b^3 + b^5)*d*e - (4*a^2*b^3 + b^5)*f)*cosh(d*x + c)^6 - 1890*(2*(2*a^3*b^2 + a*b^4)*d*f*x + 2*(2*a^3*b^2 + a*b^4)*d*e - (2*a^3*b^2 + a*b^4)*f)*cosh(d*x + c)^5 + 2700*((8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x + (8*a^4*b + 6*a^2*b^3 - b^5)*d*e - (8*a^4*b + 6*a^2*b^3 - b^5)*f)*cosh(d*x + c)^4 - 30*(4*a^2*b^3 + b^5)*d*f*x + 14400*((a^5 + a^3*b^2)*d^2*f*x^2 + 2*(a^5 + a^3*b^2)*d^2*e*x + 4*(a^5 + a^3*b^2)*c*d*e - 2*(a^5 + a^3*b^2)*c^2*f)*cosh(d*x + c)^3 - 30*(4*a^2*b^3 + b^5)*d*e - 1080*((8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x + (8*a^4*b + 6*a^2*b^3 - b^5)*d*e + (8*a^4*b + 6*a^2*b^3 - b^5)*f)*cosh(d*x + c)^2 - 10*(4*a^2*b^3 + b^5)*f - 270*(2*(2*a^3*b^2 + a*b^4)*d*f*x + 2*(2*a^3*b^2 + a*b^4)*d*e + (2*a^3*b^2 + a*b^4)*f)*cosh(d*x + c))*sinh(d*x + c)^2 - 225*(4*a*b^4*d*f*x + 4*a*b^4*d*e + a*b^4*f)*cosh(d*x + c) - 57600*((a^5 + a^3*b^2)*f*cosh(d*x + c)^5 + 5*(a^5 + a^3*b^2)*f*cosh(d*x + c)^4*sinh(d*x + c) + 10*(a^5 + a^3*b^2)*f*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*(a^5 + a^3*b^2)*f*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*(a^5 + a^3*b^2)*f*cosh(d*x + c)*sinh(d*x + c)^4 + (a^5 + a^3*b^2)*f*sinh(d*x + c)^5)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 57600*((a^5 + a^3*b^2)*f*cosh(d*x + c)^5 + 5*(a^5 + a^3*b^2)*f*cosh(d*x + c)^4*sinh(d*x + c) + 10*(a^5 + a^3*b^2)*f*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*(a^5 + a^3*b^2)*f*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*(a^5 + a^3*b^2)*f*cosh(d*x + c)*sinh(d*x + c)^4 + (a^5 + a^3*b^2)*f*sinh(d*x + c)^5)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 57600*(((a^5 + a^3*b^2)*d*e - (a^5 + a^3*b^2)*c*f)*cosh(d*x + c)^5 + 5*((a^5 + a^3*b^2)*d*e - (a^5 + a^3*b^2)*c*f)*cosh(d*x + c)^4*sinh(d*x + c) + 10*((a^5 + a^3*b^2)*d*e - (a^5 + a^3*b^2)*c*f)*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*((a^5 + a^3*b^2)*d*e - (a^5 + a^3*b^2)*c*f)*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*((a^5 + a^3*b^2)*d*e - (a^5 + a^3*b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)^4 + ((a^5 + a^3*b^2)*d*e - (a^5 + a^3*b^2)*c*f)*sinh(d*x + c)^5)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 57600*(((a^5 + a^3*b^2)*d*e - (a^5 + a^3*b^2)*c

```
*f)*cosh(d*x + c)^5 + 5*((a^5 + a^3*b^2)*d*e - (a^5 + a^3*b^2)*c*f)*cosh(d*
x + c)^4*sinh(d*x + c) + 10*((a^5 + a^3*b^2)*d*e - (a^5 + a^3*b^2)*c*f)*cos
h(d*x + c)^3*sinh(d*x + c)^2 + 10*((a^5 + a^3*b^2)*d*e - (a^5 + a^3*b^2)*c*
f)*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*((a^5 + a^3*b^2)*d*e - (a^5 + a^3*b^
2)*c*f)*cosh(d*x + c)*sinh(d*x + c)^4 + ((a^5 + a^3*b^2)*d*e - (a^5 + a^3*b
^2)*c*f)*sinh(d*x + c)^5)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*s
qrt((a^2 + b^2)/b^2) + 2*a) - 57600*(((a^5 + a^3*b^2)*d*f*x + (a^5 + a^3*b^
2)*c*f)*cosh(d*x + c)^5 + 5*((a^5 + a^3*b^2)*d*f*x + (a^5 + a^3*b^2)*c*f)*c
osh(d*x + c)^4*sinh(d*x + c) + 10*((a^5 + a^3*b^2)*d*f*x + (a^5 + a^3*b^2)*
c*f)*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*((a^5 + a^3*b^2)*d*f*x + (a^5 + a
^3*b^2)*c*f)*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*((a^5 + a^3*b^2)*d*f*x + (
a^5 + a^3*b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)^4 + ((a^5 + a^3*b^2)*d*f*x
+ (a^5 + a^3*b^2)*c*f)*sinh(d*x + c)^5)*log(-(a*cosh(d*x + c) + a*sinh(d*x
+ c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) -
57600*(((a^5 + a^3*b^2)*d*f*x + (a^5 + a^3*b^2)*c*f)*cosh(d*x + c)^5 + 5*((
a^5 + a^3*b^2)*d*f*x + (a^5 + a^3*b^2)*c*f)*cosh(d*x + c)^4*sinh(d*x + c) +
 10*((a^5 + a^3*b^2)*d*f*x + (a^5 + a^3*b^2)*c*f)*cosh(d*x + c)^3*sinh(d*x
+ c)^2 + 10*((a^5 + a^3*b^2)*d*f*x + (a^5 + a^3*b^2)*c*f)*cosh(d*x + c)^2*s
inh(d*x + c)^3 + 5*((a^5 + a^3*b^2)*d*f*x + (a^5 + a^3*b^2)*c*f)*cosh(d*x +
 c)*sinh(d*x + c)^4 + ((a^5 + a^3*b^2)*d*f*x + (a^5 + a^3*b^2)*c*f)*sinh(d*
x + c)^5)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 5*(144*(5*b^5*d*f*x + 5*b^5*d*
e - b^5*f)*cosh(d*x + c)^9 - 405*(4*a*b^4*d*f*x + 4*a*b^4*d*e - a*b^4*f)*co
sh(d*x + c)^8 - 180*a*b^4*d*f*x + 320*(3*(4*a^2*b^3 + b^5)*d*f*x + 3*(4*a^2
*b^3 + b^5)*d*e - (4*a^2*b^3 + b^5)*f)*cosh(d*x + c)^7 - 180*a*b^4*d*e - 25
20*(2*(2*a^3*b^2 + a*b^4)*d*f*x + 2*(2*a^3*b^2 + a*b^4)*d*e - (2*a^3*b^2 +
a*b^4)*f)*cosh(d*x + c)^6 - 45*a*b^4*f + 4320*((8*a^4*b + 6*a^2*b^3 - b^5)*
d*f*x + (8*a^4*b + 6*a^2*b^3 - b^5)*d*e - (8*a^4*b + 6*a^2*b^3 - b^5)*f)*co
sh(d*x + c)^5 + 28800*((a^5 + a^3*b^2)*d^2*f*x^2 + 2*(a^5 + a^3*b^2)*d^2*e*
x + 4*(a^5 + a^3*b^2)*c*d*e - 2*(a^5 + a^3*b^2)*c^2*f)*cosh(d*x + c)^4 - 28
80*((8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x + (8*a^4*b + 6*a^2*b^3 - b^5)*d*e + (
8*a^4*b + 6*a^2*b^3 - b^5)*f)*cosh(d*x + c)^3 - 1080*(2*(2*a^3*b^2 + a*b^4)
*d*f*x + 2*(2*a^3*b^2 + a*b^4)*d*e + (2*a^3*b^2 + a*b^4)*f)*cosh(d*x + c)^2
 - 80*(3*(4*a^2*b^3 + b^5)*d*f*x + 3*(4*a^2*b^3 + b^5)*d*e + (4*a^2*b^3 + b
^5)*f)*cosh(d*x + c))*sinh(d*x + c))/(b^6*d^2*cosh(d*x + c)^5 + 5*b^6*d^2*c
osh(d*x + c)^4*sinh(d*x + c) + 10*b^6*d^2*cosh(d*x + c)^3*sinh(d*x + c)^2 +
 10*b^6*d^2*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*b^6*d^2*cosh(d*x + c)*sinh(
d*x + c)^4 + b^6*d^2*sinh(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**3*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\cosh(dx+c)^3\sinh(dx+c)^3}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)^3*sinh(d*x + c)^3/(b*sinh(d*x + c) + a),
x)
```

3.404 $\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=141

$$\frac{(a^2+b^2)\sinh^3(c+dx)}{3b^3d}-\frac{a(a^2+b^2)\sinh^2(c+dx)}{2b^4d}+\frac{a^2(a^2+b^2)\sinh(c+dx)}{b^5d}-\frac{a^3(a^2+b^2)\log(a+b\sinh(c+dx))}{b^6d}$$

```
[Out] -((a^3*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/(b^6*d)) + (a^2*(a^2 + b^2)*Si
nh[c + d*x])/(b^5*d) - (a*(a^2 + b^2)*Sinh[c + d*x]^2)/(2*b^4*d) + ((a^2 +
b^2)*Sinh[c + d*x]^3)/(3*b^3*d) - (a*Sinh[c + d*x]^4)/(4*b^2*d) + Sinh[c +
d*x]^5/(5*b*d)
```

Rubi [A] time = 0.223435, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2837, 12, 894}

$$\frac{(a^2+b^2)\sinh^3(c+dx)}{3b^3d}-\frac{a(a^2+b^2)\sinh^2(c+dx)}{2b^4d}+\frac{a^2(a^2+b^2)\sinh(c+dx)}{b^5d}-\frac{a^3(a^2+b^2)\log(a+b\sinh(c+dx))}{b^6d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a^3*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/(b^6*d)) + (a^2*(a^2 + b^2)*Si
nh[c + d*x])/(b^5*d) - (a*(a^2 + b^2)*Sinh[c + d*x]^2)/(2*b^4*d) + ((a^2 +
b^2)*Sinh[c + d*x]^3)/(3*b^3*d) - (a*Sinh[c + d*x]^4)/(4*b^2*d) + Sinh[c +
d*x]^5/(5*b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^3(-b^2-x^2)}{b^3(a+x)}\,dx, x, b\sinh(c+dx)\right)}{b^3d}$$
$$= -\frac{\text{Subst}\left(\int \frac{x^3(-b^2-x^2)}{a+x}\,dx, x, b\sinh(c+dx)\right)}{b^6d}$$
$$= -\frac{\text{Subst}\left(\int \left(-a^2\left(a^2+b^2\right)+a\left(a^2+b^2\right)x-\left(a^2+b^2\right)x^2+ax^3-x^4+\frac{a^3(a^2+b^2)}{a+x}\right)dx,\right.}{b^6d}$$
$$= -\frac{a^3\left(a^2+b^2\right)\log(a+b\sinh(c+dx))}{b^6d}+\frac{a^2\left(a^2+b^2\right)\sinh(c+dx)}{b^5d}-\frac{a\left(a^2+b^2\right)\sinh}{2b^4d}$$

Mathematica [A] time = 0.387743, size = 123, normalized size = 0.87

$$-\frac{-\frac{20(a^2+b^2)\sinh^3(c+dx)}{b^3}+\frac{30a(a^2+b^2)\sinh^2(c+dx)}{b^4}-\frac{60a^2(a^2+b^2)\sinh(c+dx)}{b^5}+\frac{60a^3(a^2+b^2)\log(a+b\sinh(c+dx))}{b^6}+\frac{15a\sinh^4(c+dx)}{b^2}-\frac{12\sinh^5(c+}{b}}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((60*a^3*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/b^6 - (60*a^2*(a^2 + b^2)*S
inh[c + d*x]])/b^5 + (30*a*(a^2 + b^2)*Sinh[c + d*x]^2)/b^4 - (20*(a^2 + b^2
)*Sinh[c + d*x]^3)/b^3 + (15*a*Sinh[c + d*x]^4)/b^2 - (12*Sinh[c + d*x]^5)/
b)/(60*d)
```

Maple [B] time = 0.052, size = 804, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] 3/8/d/b/(tanh(1/2*d*x+1/2*c)+1)^2-3/8/d/b/(tanh(1/2*d*x+1/2*c)-1)^2-1/d*a^5
/b^6*ln(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)*b-a)+1/d*a^5/b^6*ln(t
anh(1/2*d*x+1/2*c)+1)-1/4/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^4*a-1/3/d/b^3/(tanh
(1/2*d*x+1/2*c)-1)^3*a^2-1/2/d/b^4/(tanh(1/2*d*x+1/2*c)-1)^2*a^3-1/4/d/b^2/
(tanh(1/2*d*x+1/2*c)+1)^4*a-1/3/d/b^3/(tanh(1/2*d*x+1/2*c)+1)^3*a^2-1/2/d/b
^4/(tanh(1/2*d*x+1/2*c)+1)^2*a^3-1/d*a^4/b^5/(tanh(1/2*d*x+1/2*c)+1)-1/d*a^
4/b^5/(tanh(1/2*d*x+1/2*c)-1)+1/d*a^5/b^6*ln(tanh(1/2*d*x+1/2*c)-1)+1/2/d/b
^3/(tanh(1/2*d*x+1/2*c)+1)^2*a^2+1/2/d/b^4/(tanh(1/2*d*x+1/2*c)+1)*a^3+1/2/
d/b^2/(tanh(1/2*d*x+1/2*c)+1)^3*a-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^3*a-1/2
/d/b^3/(tanh(1/2*d*x+1/2*c)-1)^2*a^2-1/2/d/b^4/(tanh(1/2*d*x+1/2*c)-1)*a^3-
1/5/d/b/(tanh(1/2*d*x+1/2*c)+1)^5-1/5/d/b/(tanh(1/2*d*x+1/2*c)-1)^5-1/d*a^3
/b^4*ln(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)*b-a)-5/8/d/b^2/(tanh(
1/2*d*x+1/2*c)+1)^2*a-1/d/b^3/(tanh(1/2*d*x+1/2*c)+1)*a^2+1/d*a^3/b^4*ln(ta
nh(1/2*d*x+1/2*c)+1)-5/8/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^2*a-1/d/b^3/(tanh(1/
2*d*x+1/2*c)-1)*a^2+1/d*a^3/b^4*ln(tanh(1/2*d*x+1/2*c)-1)+3/8/d/b^2/(tanh(1
/2*d*x+1/2*c)+1)*a-3/8/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*a-1/2/d/b/(tanh(1/2*d*
x+1/2*c)-1)^4+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^4-7/12/d/b/(tanh(1/2*d*x+1/2*
c)+1)^3-7/12/d/b/(tanh(1/2*d*x+1/2*c)-1)^3
```

Maxima [B] time = 1.31775, size = 405, normalized size = 2.87

$$-\frac{\left(15\,ab^3e^{(-dx-c)} - 6\,b^4 - 10\left(4\,a^2b^2 + b^4\right)e^{(-2\,dx-2\,c)} + 60\left(2\,a^3b + ab^3\right)e^{(-3\,dx-3\,c)} - 60\left(8\,a^4 + 6\,a^2b^2 - b^4\right)e^{(-4\,dx-4\,c)}\right)e^{(5\,d}}{960\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxim
a")
```

```
[Out] -1/960*(15*a*b^3*e^(-d*x - c) - 6*b^4 - 10*(4*a^2*b^2 + b^4)*e^(-2*d*x - 2*
c) + 60*(2*a^3*b + a*b^3)*e^(-3*d*x - 3*c) - 60*(8*a^4 + 6*a^2*b^2 - b^4)*e
^(-4*d*x - 4*c))*e^(5*d*x + 5*c)/(b^5*d) - (a^5 + a^3*b^2)*(d*x + c)/(b^6*d
) - 1/960*(15*a*b^3*e^(-4*d*x - 4*c) + 6*b^4*e^(-5*d*x - 5*c) + 60*(8*a^4 +
 6*a^2*b^2 - b^4)*e^(-d*x - c) + 60*(2*a^3*b + a*b^3)*e^(-2*d*x - 2*c) + 10
*(4*a^2*b^2 + b^4)*e^(-3*d*x - 3*c))/(b^5*d) - (a^5 + a^3*b^2)*log(-2*a*e^(
-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^6*d)
```

Fricas [B] time = 2.58708, size = 4018, normalized size = 28.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="frica
s")
```

```
[Out] 1/960*(6*b^5*cosh(d*x + c)^10 + 6*b^5*sinh(d*x + c)^10 - 15*a*b^4*cosh(d*x
+ c)^9 + 15*(4*b^5*cosh(d*x + c) - a*b^4)*sinh(d*x + c)^9 + 10*(4*a^2*b^3 +
 b^5)*cosh(d*x + c)^8 + 5*(54*b^5*cosh(d*x + c)^2 - 27*a*b^4*cosh(d*x + c)
+ 8*a^2*b^3 + 2*b^5)*sinh(d*x + c)^8 + 960*(a^5 + a^3*b^2)*d*x*cosh(d*x + c
)^5 - 60*(2*a^3*b^2 + a*b^4)*cosh(d*x + c)^7 + 20*(36*b^5*cosh(d*x + c)^3 -
 27*a*b^4*cosh(d*x + c)^2 - 6*a^3*b^2 - 3*a*b^4 + 4*(4*a^2*b^3 + b^5)*cosh(
d*x + c))*sinh(d*x + c)^7 + 60*(8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x + c)^6
+ 20*(63*b^5*cosh(d*x + c)^4 - 63*a*b^4*cosh(d*x + c)^3 + 24*a^4*b + 18*a^2
*b^3 - 3*b^5 + 14*(4*a^2*b^3 + b^5)*cosh(d*x + c)^2 - 21*(2*a^3*b^2 + a*b^4
)*cosh(d*x + c))*sinh(d*x + c)^6 - 15*a*b^4*cosh(d*x + c) + 2*(756*b^5*cosh
(d*x + c)^5 - 945*a*b^4*cosh(d*x + c)^4 + 280*(4*a^2*b^3 + b^5)*cosh(d*x +
c)^3 + 480*(a^5 + a^3*b^2)*d*x - 630*(2*a^3*b^2 + a*b^4)*cosh(d*x + c)^2 +
180*(8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x + c))*sinh(d*x + c)^5 - 6*b^5 - 60
*(8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x + c)^4 + 10*(126*b^5*cosh(d*x + c)^6
- 189*a*b^4*cosh(d*x + c)^5 - 48*a^4*b - 36*a^2*b^3 + 6*b^5 + 70*(4*a^2*b^3
 + b^5)*cosh(d*x + c)^4 + 480*(a^5 + a^3*b^2)*d*x*cosh(d*x + c) - 210*(2*a^
3*b^2 + a*b^4)*cosh(d*x + c)^3 + 90*(8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x +
c)^2)*sinh(d*x + c)^4 - 60*(2*a^3*b^2 + a*b^4)*cosh(d*x + c)^3 + 20*(36*b^5
*cosh(d*x + c)^7 - 63*a*b^4*cosh(d*x + c)^6 + 28*(4*a^2*b^3 + b^5)*cosh(d*x
 + c)^5 - 6*a^3*b^2 - 3*a*b^4 + 480*(a^5 + a^3*b^2)*d*x*cosh(d*x + c)^2 - 1
05*(2*a^3*b^2 + a*b^4)*cosh(d*x + c)^4 + 60*(8*a^4*b + 6*a^2*b^3 - b^5)*cos
h(d*x + c)^3 - 12*(8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x + c))*sinh(d*x + c)^
3 - 10*(4*a^2*b^3 + b^5)*cosh(d*x + c)^2 + 10*(27*b^5*cosh(d*x + c)^8 - 54*
a*b^4*cosh(d*x + c)^7 + 28*(4*a^2*b^3 + b^5)*cosh(d*x + c)^6 + 960*(a^5 + a
^3*b^2)*d*x*cosh(d*x + c)^3 - 126*(2*a^3*b^2 + a*b^4)*cosh(d*x + c)^5 - 4*a
^2*b^3 - b^5 + 90*(8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x + c)^4 - 36*(8*a^4*b
 + 6*a^2*b^3 - b^5)*cosh(d*x + c)^2 - 18*(2*a^3*b^2 + a*b^4)*cosh(d*x + c))
*sinh(d*x + c)^2 - 960*((a^5 + a^3*b^2)*cosh(d*x + c)^5 + 5*(a^5 + a^3*b^2)
```

```
*cosh(d*x + c)^4*sinh(d*x + c) + 10*(a^5 + a^3*b^2)*cosh(d*x + c)^3*sinh(d*
x + c)^2 + 10*(a^5 + a^3*b^2)*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*(a^5 + a^
3*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 + (a^5 + a^3*b^2)*sinh(d*x + c)^5)*log
(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 5*(12*b^5*cosh(
d*x + c)^9 - 27*a*b^4*cosh(d*x + c)^8 + 16*(4*a^2*b^3 + b^5)*cosh(d*x + c)^
7 + 960*(a^5 + a^3*b^2)*d*x*cosh(d*x + c)^4 - 84*(2*a^3*b^2 + a*b^4)*cosh(d
*x + c)^6 + 72*(8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x + c)^5 - 3*a*b^4 - 48*(
8*a^4*b + 6*a^2*b^3 - b^5)*cosh(d*x + c)^3 - 36*(2*a^3*b^2 + a*b^4)*cosh(d*
x + c)^2 - 4*(4*a^2*b^3 + b^5)*cosh(d*x + c))*sinh(d*x + c))/(b^6*d*cosh(d*
x + c)^5 + 5*b^6*d*cosh(d*x + c)^4*sinh(d*x + c) + 10*b^6*d*cosh(d*x + c)^3
*sinh(d*x + c)^2 + 10*b^6*d*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*b^6*d*cosh(
d*x + c)*sinh(d*x + c)^4 + b^6*d*sinh(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19815, size = 383, normalized size = 2.72

$$-\frac{(a^5+a^3b^2)\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{b^6d}+\frac{6\,b^4d^4\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5-15\,ab^3d^4\left(e^{(dx+c)}-e^{(-dx-c)}\right)^4+40\,a^2b^2d^4\left(e^{(dx+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac"
)
```

```
[Out] -(a^5 + a^3*b^2)*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(b^6*d) + 1
/960*(6*b^4*d^4*(e^(d*x + c) - e^(-d*x - c))^5 - 15*a*b^3*d^4*(e^(d*x + c)
- e^(-d*x - c))^4 + 40*a^2*b^2*d^4*(e^(d*x + c) - e^(-d*x - c))^3 + 40*b^4*
d^4*(e^(d*x + c) - e^(-d*x - c))^3 - 120*a^3*b*d^4*(e^(d*x + c) - e^(-d*x -
 c))^2 - 120*a*b^3*d^4*(e^(d*x + c) - e^(-d*x - c))^2 + 480*a^4*d^4*(e^(d*x
 + c) - e^(-d*x - c)) + 480*a^2*b^2*d^4*(e^(d*x + c) - e^(-d*x - c)))/(b^5*
d^5)
```

3.405 $\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=38

$$\text{Unintegrable}\left(\frac{\sinh^3(c+dx)\cosh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Cosh[c + d*x]^3*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d
*x])), x]
```

Rubi [A] time = 0.126685, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[c + d*x]^3*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Cosh[c + d*x]^3*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh^3(c+dx)\sinh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.002, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x]
)),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.223, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx+c))^3(\sinh(dx+c))^3}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/32*e^(-5*c + 5*d*e/f)*exp_integral_e(1, 5*(f*x + e)*d/f)/(b*f) - 1/16*a*
e^(-4*c + 4*d*e/f)*exp_integral_e(1, 4*(f*x + e)*d/f)/(b^2*f) + 1/16*a*e^(4
*c - 4*d*e/f)*exp_integral_e(1, -4*(f*x + e)*d/f)/(b^2*f) - 1/32*e^(5*c - 5
*d*e/f)*exp_integral_e(1, -5*(f*x + e)*d/f)/(b*f) - 1/32*(4*a^2 + b^2)*e^(-
3*c + 3*d*e/f)*exp_integral_e(1, 3*(f*x + e)*d/f)/(b^3*f) - 1/32*(4*a^2*e^(
3*c) + b^2*e^(3*c))*e^(-3*d*e/f)*exp_integral_e(1, -3*(f*x + e)*d/f)/(b^3*f
) - 1/8*(2*a^3 + a*b^2)*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/
f)/(b^4*f) + 1/8*(2*a^3*e^(2*c) + a*b^2*e^(2*c))*e^(-2*d*e/f)*exp_integral_
e(1, -2*(f*x + e)*d/f)/(b^4*f) - 1/16*(8*a^4 + 6*a^2*b^2 - b^4)*e^(-c + d*e
/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^5*f) - 1/16*(8*a^4*e^c + 6*a^2*b^2*
e^c - b^4*e^c)*e^(-d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^5*f) - (a^5
+ a^3*b^2)*log(f*x + e)/(b^6*f) + 1/64*integrate(128*(a^5*b + a^3*b^3 - (a^
6*e^c + a^4*b^2*e^c)*e^(d*x))/(b^7*f*x + b^7*e - (b^7*f*x*e^(2*c) + b^7*e*e
^(2*c))*e^(2*d*x) - 2*(a*b^6*f*x*e^c + a*b^6*e*e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh\left(dx+c\right)^{3}\sinh\left(dx+c\right)^{3}}{afx+ae+\left(bfx+be\right)\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] integral(cosh(d*x + c)^3*sinh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(
d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*sinh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^3 \sinh(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate(cosh(d*x + c)^3*sinh(d*x + c)^3/((f*x + e)*(b*sinh(d*x + c) + a))
, x)
```

3.406 $\int \frac{(e+fx)^3 \sinh^2(c+dx) \tanh(c+dx)}{a+b\sinh(c+dx)} dx$

Optimal. Leaf size=1519

result too large to display

```
[Out] (a*(e + f*x)^4)/(4*b^2*f) + (2*a^2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b^3*d)
 - (2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b*d) - (2*a^4*(e + f*x)^3*ArcTan[E^
(c + d*x)])/(b^3*(a^2 + b^2)*d) - (6*f^3*Cosh[c + d*x])/(b*d^4) - (3*f*(e +
 f*x)^2*Cosh[c + d*x])/(b*d^2) - (a^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d) - (a^3*(e + f*x)^3*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d) - (a*(e + f*x)^3*Log
[1 + E^(2*(c + d*x))])/(b^2*d) + (a^3*(e + f*x)^3*Log[1 + E^(2*(c + d*x))])
/(b^2*(a^2 + b^2)*d) - ((3*I)*a^2*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)
])/(b^3*d^2) + ((3*I)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) +
 ((3*I)*a^4*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^
2) + ((3*I)*a^2*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b^3*d^2) - ((3*I)
*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - ((3*I)*a^4*f*(e + f*x)^
2*PolyLog[2, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) - (3*a^3*f*(e + f*x)^2*P
olyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^2)
 - (3*a^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/(b^2*(a^2 + b^2)*d^2) - (3*a*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/
(2*b^2*d^2) + (3*a^3*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*b^2*(a^
2 + b^2)*d^2) + ((6*I)*a^2*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(b^3
*d^3) - ((6*I)*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(b*d^3) - ((6*I)
*a^4*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) - ((
6*I)*a^2*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(b^3*d^3) + ((6*I)*f^2*(e
 + f*x)*PolyLog[3, I*E^(c + d*x)])/(b*d^3) + ((6*I)*a^4*f^2*(e + f*x)*PolyL
og[3, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) + (6*a^3*f^2*(e + f*x)*PolyLog[
3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^3) + (6*a^
3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*
(a^2 + b^2)*d^3) + (3*a*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*
d^3) - (3*a^3*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*(a^2 + b^2
)*d^3) - ((6*I)*a^2*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(b^3*d^4) + ((6*I)*f^
3*PolyLog[4, (-I)*E^(c + d*x)])/(b*d^4) + ((6*I)*a^4*f^3*PolyLog[4, (-I)*E^
(c + d*x)])/(b^3*(a^2 + b^2)*d^4) + ((6*I)*a^2*f^3*PolyLog[4, I*E^(c + d*x)
])/(b^3*d^4) - ((6*I)*f^3*PolyLog[4, I*E^(c + d*x)])/(b*d^4) - ((6*I)*a^4*f
^3*PolyLog[4, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^4) - (6*a^3*f^3*PolyLog[4,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^4) - (6*a^3*
f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*
d^4) - (3*a*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*b^2*d^4) + (3*a^3*f^3*Poly
Log[4, -E^(2*(c + d*x))])/(4*b^2*(a^2 + b^2)*d^4) + (6*f^2*(e + f*x)*Sinh[c
 + d*x])/(b*d^3) + ((e + f*x)^3*Sinh[c + d*x])/(b*d)
```

Rubi [A] time = 2.15352, antiderivative size = 1519, normalized size of antiderivative = 1., number of steps used = 61, number of rules used = 15, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.441, Rules used = {5581, 5449, 3296, 2638, 4180, 2531, 6609, 2282, 6589, 3718, 2190, 5567, 5573, 5561, 6742}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (a*(e + f*x)^4)/(4*b^2*f) + (2*a^2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b^3*d)
 - (2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b*d) - (2*a^4*(e + f*x)^3*ArcTan[E^
```

```
(c + d*x)])/(b^3*(a^2 + b^2)*d) - (6*f^3*Cosh[c + d*x])/(b*d^4) - (3*f*(e +
 f*x)^2*Cosh[c + d*x])/(b*d^2) - (a^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d) - (a^3*(e + f*x)^3*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d) - (a*(e + f*x)^3*Log
[1 + E^(2*(c + d*x))])/(b^2*d) + (a^3*(e + f*x)^3*Log[1 + E^(2*(c + d*x))])
/(b^2*(a^2 + b^2)*d) - ((3*I)*a^2*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)
])/(b^3*d^2) + ((3*I)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) +
 ((3*I)*a^4*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^
2) + ((3*I)*a^2*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b^3*d^2) - ((3*I)
*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - ((3*I)*a^4*f*(e + f*x)^
2*PolyLog[2, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) - (3*a^3*f*(e + f*x)^2*P
olyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^2)
- (3*a^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/(b^2*(a^2 + b^2)*d^2) - (3*a*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/
(2*b^2*d^2) + (3*a^3*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*b^2*(a^
2 + b^2)*d^2) + ((6*I)*a^2*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(b^3
*d^3) - ((6*I)*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(b*d^3) - ((6*I)
*a^4*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) - ((
6*I)*a^2*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(b^3*d^3) + ((6*I)*f^2*(e
 + f*x)*PolyLog[3, I*E^(c + d*x)])/(b*d^3) + ((6*I)*a^4*f^2*(e + f*x)*PolyL
og[3, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) + (6*a^3*f^2*(e + f*x)*PolyLog[
3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^3) + (6*a^
3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*
(a^2 + b^2)*d^3) + (3*a*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*
d^3) - (3*a^3*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*(a^2 + b^2
)*d^3) - ((6*I)*a^2*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(b^3*d^4) + ((6*I)*f^
3*PolyLog[4, (-I)*E^(c + d*x)])/(b*d^4) + ((6*I)*a^4*f^3*PolyLog[4, (-I)*E^
(c + d*x)])/(b^3*(a^2 + b^2)*d^4) + ((6*I)*a^2*f^3*PolyLog[4, I*E^(c + d*x)
])/(b^3*d^4) - ((6*I)*f^3*PolyLog[4, I*E^(c + d*x)])/(b*d^4) - ((6*I)*a^4*f
^3*PolyLog[4, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^4) - (6*a^3*f^3*PolyLog[4,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^4) - (6*a^3*
f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*
d^4) - (3*a*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*b^2*d^4) + (3*a^3*f^3*Poly
Log[4, -E^(2*(c + d*x))])/(4*b^2*(a^2 + b^2)*d^4) + (6*f^2*(e + f*x)*Sinh[c
 + d*x])/(b*d^3) + ((e + f*x)^3*Sinh[c + d*x])/(b*d)
```

Rule 5581

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - Di
st[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n)/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c +
 d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^3\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\sinh(c+dx)\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{a\int (e+fx)^3\tanh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^3\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b^2} + \frac{\int (e+fx)^3\cos}{b}$$

$$= \frac{a(e+fx)^4}{4b^2f} - \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} + \frac{(e+fx)^3\sinh(c+dx)}{bd} + \frac{a^2\int (e+}{}$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{3f(e}{}$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{a^3(e+fx)^4}{4b^2\left(a^2+b^2\right)f} + \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^3\tan}{bd}$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{a^3(e+fx)^4}{4b^2\left(a^2+b^2\right)f} + \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^3\tan}{bd}$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(}{}$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(}{}$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(}{}$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(}{}$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(}{}$$

$$= \frac{a(e+fx)^4}{4b^2f} + \frac{2a^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(}{}$$

Mathematica [A] time = 26.2135, size = 2861, normalized size = 1.88

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] -(8*b*d^3*e^3*(1 + E^(2*c))*ArcTan[E^(c + d*x)] - 4*a*d^3*e^3*E^(2*c)*(2*d*
x - Log[1 + E^(2*(c + d*x))]) + 4*a*d^3*e^3*Log[1 + E^(2*(c + d*x))] + (12*
I)*b*d^2*e^2*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c
+ d*x)]) - PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*
d^2*e^2*E^(2*c)*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(
2*(c + d*x))]) + 6*a*d^2*e^2*f*(2*d*x*Log[1 + E^(2*(c + d*x))] + PolyLog[2,
 -E^(2*(c + d*x))]) + (12*I)*b*d*e*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(
c + d*x)] - d^2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d
```

*x)] + 2*d*x*PolyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2
*PolyLog[3, I*E^(c + d*x)]) + 6*a*d*e*f^2*(2*d^2*x^2*Log[1 + E^(2*(c + d*x)
)] + 2*d*x*PolyLog[2, -E^(2*(c + d*x))] - PolyLog[3, -E^(2*(c + d*x))]) - 2
*a*d*e*E^(2*c)*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))]) - 6*d*x*
PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))]) + (4*I)*b*(1
 + E^(2*c))*f^3*(d^3*x^3*Log[1 - I*E^(c + d*x)] - d^3*x^3*Log[1 + I*E^(c +
d*x)] - 3*d^2*x^2*PolyLog[2, (-I)*E^(c + d*x)] + 3*d^2*x^2*PolyLog[2, I*E^(
c + d*x)] + 6*d*x*PolyLog[3, (-I)*E^(c + d*x)] - 6*d*x*PolyLog[3, I*E^(c +
d*x)] - 6*PolyLog[4, (-I)*E^(c + d*x)] + 6*PolyLog[4, I*E^(c + d*x)]) - a*E
^(2*c)*f^3*(2*d^4*x^4 - 4*d^3*x^3*Log[1 + E^(2*(c + d*x))] - 6*d^2*x^2*Poly
Log[2, -E^(2*(c + d*x))] + 6*d*x*PolyLog[3, -E^(2*(c + d*x))] - 3*PolyLog[4
, -E^(2*(c + d*x))]) + a*f^3*(4*d^3*x^3*Log[1 + E^(2*(c + d*x))] + 6*d^2*x^
2*PolyLog[2, -E^(2*(c + d*x))] - 6*d*x*PolyLog[3, -E^(2*(c + d*x))] + 3*Pol
yLog[4, -E^(2*(c + d*x))])))/(4*(a^2 + b^2)*d^4*(1 + E^(2*c))) + (a^3*(4*e^3
*E^(2*c)*x + 6*e^2*E^(2*c)*f*x^2 + 4*e*E^(2*c)*f^2*x^3 + E^(2*c)*f^3*x^4 +
(4*a*Sqrt[-(a^2 + b^2)^2]*e^3*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2
- b^2]])/((a^2 + b^2)^(3/2)*d) + (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*E^(2*c)*ArcT
anh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (2*e^3*L
og[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))])/d - (2*e^3*E^(2*c)*Log[2*a*E^(
c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*
x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b
*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[
1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c
)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d
 + (2*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])
])/d - (2*E^(2*c)*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^
2)*E^(2*c)])])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2
+ b^2)*E^(2*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c
+ Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(
a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f^2*x^2*Log[1 + (b*E^
(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (2*f^3*x^3*Log[1 + (
b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (2*E^(2*c)*f^3*x
^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(
-1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a
^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, -((
b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (12*e*f^2*Pol
yLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (1
2*e*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^
(2*c)]))])/d^3 - (12*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^
2 + b^2)*E^(2*c)]))])/d^3 + (12*E^(2*c)*f^3*x*PolyLog[3, -((b*E^(2*c + d*x)
)/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (12*e*f^2*PolyLog[3, -((b*E^
(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*e*E^(2*c)*f^2
*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3
- (12*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c
)]))])/d^3 + (12*E^(2*c)*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt
[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E
^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^4 - (12*E^(2*c)*f^3*PolyLog[4, -((b*E^
(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^4 + (12*f^3*PolyLog[4
, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^4 - (12*E^(2
*c)*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))
])/d^4))/(2*b^2*(a^2 + b^2)*(-1 + E^(2*c))) - (a*e^3*x*(a^2 - b^2 + (a^2 +
b^2)*Cosh[2*c])*Csch[c]*Sech[c])/(2*b^2*(a^2 + b^2)) - (3*a*e^2*f*x^2*(a^2
- b^2 + (a^2 + b^2)*Cosh[2*c])*Csch[c]*Sech[c])/(4*b^2*(a^2 + b^2)) - (a*e*
f^2*x^3*(a^2 - b^2 + (a^2 + b^2)*Cosh[2*c])*Csch[c]*Sech[c])/(2*b^2*(a^2 +
b^2)) - (a*f^3*x^4*(a^2 - b^2 + (a^2 + b^2)*Cosh[2*c])*Csch[c]*Sech[c])/(8*
b^2*(a^2 + b^2)) + ((6*f^3 + 6*d*f^2*(e + f*x) + 3*d^2*f*(e + f*x)^2 + d^3*
(e + f*x)^3)*(-Cosh[c + d*x] + Sinh[c + d*x]))/(2*b*d^4) + ((-6*f^3 + 6*d*f
^2*(e + f*x) - 3*d^2*f*(e + f*x)^2 + d^3*(e + f*x)^3)*(Cosh[c + d*x] + Sinh
[c + d*x]))/(2*b*d^4)

Maple [F] time = 1.115, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 (\sinh(dx+c))^2 \tanh(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/2*(2*a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b^2 + b^4
)*d) - 4*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + 2*a*log(e^(-2*d*x - 2*c)
+ 1)/((a^2 + b^2)*d) + 2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x
- c)/(b*d))*e^3 - 1/4*(a*d^4*f^3*x^4*e^c + 4*a*d^4*e*f^2*x^3*e^c + 6*a*d^4*
e^2*f*x^2*e^c - 2*(b*d^3*f^3*x^3*e^(2*c) + 3*(d^3*e*f^2 - d^2*f^3)*b*x^2*e^
(2*c) + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*b*x*e^(2*c) - 3*(d^2*e^2*f -
2*d*e*f^2 + 2*f^3)*b*e^(2*c))*e^(d*x) + 2*(b*d^3*f^3*x^3 + 3*(d^3*e*f^2 + d
^2*f^3)*b*x^2 + 3*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*b*x + 3*(d^2*e^2*f +
2*d*e*f^2 + 2*f^3)*b)*e^(-d*x))*e^(-c)/(b^2*d^4) + integrate(2*(a^3*b*f^3*x
^3 + 3*a^3*b*e*f^2*x^2 + 3*a^3*b*e^2*f*x - (a^4*f^3*x^3*e^c + 3*a^4*e*f^2*x
^2*e^c + 3*a^4*e^2*f*x*e^c)*e^(d*x))/(a^2*b^3 + b^5 - (a^2*b^3*e^(2*c) + b^
5*e^(2*c))*e^(2*d*x) - 2*(a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) - integrate
(-2*(a*f^3*x^3 + 3*a*e*f^2*x^2 + 3*a*e^2*f*x - (b*f^3*x^3*e^c + 3*b*e*f^2*x
^2*e^c + 3*b*e^2*f*x*e^c)*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))
*e^(2*d*x)), x)
```

Fricas [C] time = 4.30697, size = 10368, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/4*(2*(a^2*b + b^3)*d^3*f^3*x^3 + 2*(a^2*b + b^3)*d^3*e^3 + 6*(a^2*b + b^
3)*d^2*e^2*f + 12*(a^2*b + b^3)*d*e*f^2 + 12*(a^2*b + b^3)*f^3 + 6*((a^2*b
+ b^3)*d^3*e*f^2 + (a^2*b + b^3)*d^2*f^3)*x^2 - 2*((a^2*b + b^3)*d^3*f^3*x^
3 + (a^2*b + b^3)*d^3*e^3 - 3*(a^2*b + b^3)*d^2*e^2*f + 6*(a^2*b + b^3)*d*e
*f^2 - 6*(a^2*b + b^3)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^2*b + b^3)*d^2
```

```
*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^2*b + b^3)*d^2*e*f^2 + 2*(a^2
*b + b^3)*d*f^3)*x)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d^3*f^3*x^3 + (a^2*b
 + b^3)*d^3*e^3 - 3*(a^2*b + b^3)*d^2*e^2*f + 6*(a^2*b + b^3)*d*e*f^2 - 6*(
a^2*b + b^3)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^2*b + b^3)*d^2*f^3)*x^2
+ 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^2*b + b^3)*d^2*e*f^2 + 2*(a^2*b + b^3)*
d*f^3)*x)*sinh(d*x + c)^2 + 6*((a^2*b + b^3)*d^3*e^2*f + 2*(a^2*b + b^3)*d^
2*e*f^2 + 2*(a^2*b + b^3)*d*f^3)*x - ((a^3 + a*b^2)*d^4*f^3*x^4 + 4*(a^3 +
a*b^2)*d^4*e*f^2*x^3 + 6*(a^3 + a*b^2)*d^4*e^2*f*x^2 + 4*(a^3 + a*b^2)*d^4*
e^3*x + 8*(a^3 + a*b^2)*c*d^3*e^3 - 12*(a^3 + a*b^2)*c^2*d^2*e^2*f + 8*(a^3
 + a*b^2)*c^3*d*e*f^2 - 2*(a^3 + a*b^2)*c^4*f^3)*cosh(d*x + c) + 12*((a^3*d
^2*f^3*x^2 + 2*a^3*d^2*e*f^2*x + a^3*d^2*e^2*f)*cosh(d*x + c) + (a^3*d^2*f^
3*x^2 + 2*a^3*d^2*e*f^2*x + a^3*d^2*e^2*f)*sinh(d*x + c))*dilog((a*cosh(d*x
 + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2) - b)/b + 1) + 12*((a^3*d^2*f^3*x^2 + 2*a^3*d^2*e*f^2*x + a^3*d^2*e
^2*f)*cosh(d*x + c) + (a^3*d^2*f^3*x^2 + 2*a^3*d^2*e*f^2*x + a^3*d^2*e^2*f)
*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + ((12*a*b^2*d^2*f^3*
x^2 + 12*I*b^3*d^2*f^3*x^2 + 24*a*b^2*d^2*e*f^2*x + 24*I*b^3*d^2*e*f^2*x +
12*a*b^2*d^2*e^2*f + 12*I*b^3*d^2*e^2*f)*cosh(d*x + c) + (12*a*b^2*d^2*f^3*
x^2 + 12*I*b^3*d^2*f^3*x^2 + 24*a*b^2*d^2*e*f^2*x + 24*I*b^3*d^2*e*f^2*x +
12*a*b^2*d^2*e^2*f + 12*I*b^3*d^2*e^2*f)*sinh(d*x + c))*dilog(I*cosh(d*x +
c) + I*sinh(d*x + c)) + ((12*a*b^2*d^2*f^3*x^2 - 12*I*b^3*d^2*f^3*x^2 + 24*
a*b^2*d^2*e*f^2*x - 24*I*b^3*d^2*e*f^2*x + 12*a*b^2*d^2*e^2*f - 12*I*b^3*d^
2*e^2*f)*cosh(d*x + c) + (12*a*b^2*d^2*f^3*x^2 - 12*I*b^3*d^2*f^3*x^2 + 24*
a*b^2*d^2*e*f^2*x - 24*I*b^3*d^2*e*f^2*x + 12*a*b^2*d^2*e^2*f - 12*I*b^3*d^
2*e^2*f)*sinh(d*x + c))*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + 4*((a^3
*d^3*e^3 - 3*a^3*c*d^2*e^2*f + 3*a^3*c^2*d*e*f^2 - a^3*c^3*f^3)*cosh(d*x +
c) + (a^3*d^3*e^3 - 3*a^3*c*d^2*e^2*f + 3*a^3*c^2*d*e*f^2 - a^3*c^3*f^3)*si
nh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^
2)/b^2) + 2*a) + 4*((a^3*d^3*e^3 - 3*a^3*c*d^2*e^2*f + 3*a^3*c^2*d*e*f^2 -
a^3*c^3*f^3)*cosh(d*x + c) + (a^3*d^3*e^3 - 3*a^3*c*d^2*e^2*f + 3*a^3*c^2*d
*e*f^2 - a^3*c^3*f^3)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
 c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 4*((a^3*d^3*f^3*x^3 + 3*a^3*d^3*e*
f^2*x^2 + 3*a^3*d^3*e^2*f*x + 3*a^3*c*d^2*e^2*f - 3*a^3*c^2*d*e*f^2 + a^3*c
^3*f^3)*cosh(d*x + c) + (a^3*d^3*f^3*x^3 + 3*a^3*d^3*e*f^2*x^2 + 3*a^3*d^3*
e^2*f*x + 3*a^3*c*d^2*e^2*f - 3*a^3*c^2*d*e*f^2 + a^3*c^3*f^3)*sinh(d*x + c
))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*((a^3*d^3*f^3*x^3 + 3*a^3*d^3*e*f^2
*x^2 + 3*a^3*d^3*e^2*f*x + 3*a^3*c*d^2*e^2*f - 3*a^3*c^2*d*e*f^2 + a^3*c^3*
f^3)*cosh(d*x + c) + (a^3*d^3*f^3*x^3 + 3*a^3*d^3*e*f^2*x^2 + 3*a^3*d^3*e^2
*f*x + 3*a^3*c*d^2*e^2*f - 3*a^3*c^2*d*e*f^2 + a^3*c^3*f^3)*sinh(d*x + c))*
log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b) + ((4*a*b^2*d^3*e^3 + 4*I*b^3*d^3*e^3 - 12
*a*b^2*c*d^2*e^2*f - 12*I*b^3*c*d^2*e^2*f + 12*a*b^2*c^2*d*e*f^2 + 12*I*b^3
*c^2*d*e*f^2 - 4*a*b^2*c^3*f^3 - 4*I*b^3*c^3*f^3)*cosh(d*x + c) + (4*a*b^2*
d^3*e^3 + 4*I*b^3*d^3*e^3 - 12*a*b^2*c*d^2*e^2*f - 12*I*b^3*c*d^2*e^2*f + 1
2*a*b^2*c^2*d*e*f^2 + 12*I*b^3*c^2*d*e*f^2 - 4*a*b^2*c^3*f^3 - 4*I*b^3*c^3*
f^3)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + I) + ((4*a*b^2*d^3*
e^3 - 4*I*b^3*d^3*e^3 - 12*a*b^2*c*d^2*e^2*f + 12*I*b^3*c*d^2*e^2*f + 12*a*
b^2*c^2*d*e*f^2 - 12*I*b^3*c^2*d*e*f^2 - 4*a*b^2*c^3*f^3 + 4*I*b^3*c^3*f^3)
*cosh(d*x + c) + (4*a*b^2*d^3*e^3 - 4*I*b^3*d^3*e^3 - 12*a*b^2*c*d^2*e^2*f
+ 12*I*b^3*c*d^2*e^2*f + 12*a*b^2*c^2*d*e*f^2 - 12*I*b^3*c^2*d*e*f^2 - 4*a*
b^2*c^3*f^3 + 4*I*b^3*c^3*f^3)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x
+ c) - I) + ((4*a*b^2*d^3*f^3*x^3 - 4*I*b^3*d^3*f^3*x^3 + 12*a*b^2*d^3*e*f^
2*x^2 - 12*I*b^3*d^3*e*f^2*x^2 + 12*a*b^2*d^3*e^2*f*x - 12*I*b^3*d^3*e^2*f*
x + 12*a*b^2*c*d^2*e^2*f - 12*I*b^3*c*d^2*e^2*f - 12*a*b^2*c^2*d*e*f^2 + 12
*I*b^3*c^2*d*e*f^2 + 4*a*b^2*c^3*f^3 - 4*I*b^3*c^3*f^3)*cosh(d*x + c) + (4*
a*b^2*d^3*f^3*x^3 - 4*I*b^3*d^3*f^3*x^3 + 12*a*b^2*d^3*e*f^2*x^2 - 12*I*b^3
*d^3*e*f^2*x^2 + 12*a*b^2*d^3*e^2*f*x - 12*I*b^3*d^3*e^2*f*x + 12*a*b^2*c*d
```

```
^2*e^2*f - 12*I*b^3*c*d^2*e^2*f - 12*a*b^2*c^2*d*e*f^2 + 12*I*b^3*c^2*d*e*f
^2 + 4*a*b^2*c^3*f^3 - 4*I*b^3*c^3*f^3)*sinh(d*x + c))*log(I*cosh(d*x + c)
+ I*sinh(d*x + c) + 1) + ((4*a*b^2*d^3*f^3*x^3 + 4*I*b^3*d^3*f^3*x^3 + 12*a
*b^2*d^3*e*f^2*x^2 + 12*I*b^3*d^3*e*f^2*x^2 + 12*a*b^2*d^3*e^2*f*x + 12*I*b
^3*d^3*e^2*f*x + 12*a*b^2*c*d^2*e^2*f + 12*I*b^3*c*d^2*e^2*f - 12*a*b^2*c^2
*d*e*f^2 - 12*I*b^3*c^2*d*e*f^2 + 4*a*b^2*c^3*f^3 + 4*I*b^3*c^3*f^3)*cosh(d
*x + c) + (4*a*b^2*d^3*f^3*x^3 + 4*I*b^3*d^3*f^3*x^3 + 12*a*b^2*d^3*e*f^2*x
^2 + 12*I*b^3*d^3*e*f^2*x^2 + 12*a*b^2*d^3*e^2*f*x + 12*I*b^3*d^3*e^2*f*x +
 12*a*b^2*c*d^2*e^2*f + 12*I*b^3*c*d^2*e^2*f - 12*a*b^2*c^2*d*e*f^2 - 12*I*
b^3*c^2*d*e*f^2 + 4*a*b^2*c^3*f^3 + 4*I*b^3*c^3*f^3)*sinh(d*x + c))*log(-I*
cosh(d*x + c) - I*sinh(d*x + c) + 1) + 24*(a^3*f^3*cosh(d*x + c) + a^3*f^3*
sinh(d*x + c))*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*(a^3*f^3*cosh(d*x +
c) + a^3*f^3*sinh(d*x + c))*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) -
 (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + ((24*a*b^2
*f^3 + 24*I*b^3*f^3)*cosh(d*x + c) + (24*a*b^2*f^3 + 24*I*b^3*f^3)*sinh(d*x
 + c))*polylog(4, I*cosh(d*x + c) + I*sinh(d*x + c)) + ((24*a*b^2*f^3 - 24*
I*b^3*f^3)*cosh(d*x + c) + (24*a*b^2*f^3 - 24*I*b^3*f^3)*sinh(d*x + c))*pol
ylog(4, -I*cosh(d*x + c) - I*sinh(d*x + c)) - 24*((a^3*d*f^3*x + a^3*d*e*f^
2)*cosh(d*x + c) + (a^3*d*f^3*x + a^3*d*e*f^2)*sinh(d*x + c))*polylog(3, (a
*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2))/b) - 24*((a^3*d*f^3*x + a^3*d*e*f^2)*cosh(d*x + c) + (a^
3*d*f^3*x + a^3*d*e*f^2)*sinh(d*x + c))*polylog(3, (a*cosh(d*x + c) + a*sin
h(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b)
- ((24*a*b^2*d*f^3*x + 24*I*b^3*d*f^3*x + 24*a*b^2*d*e*f^2 + 24*I*b^3*d*e*f
^2)*cosh(d*x + c) + (24*a*b^2*d*f^3*x + 24*I*b^3*d*f^3*x + 24*a*b^2*d*e*f^2
 + 24*I*b^3*d*e*f^2)*sinh(d*x + c))*polylog(3, I*cosh(d*x + c) + I*sinh(d*x
 + c)) - ((24*a*b^2*d*f^3*x - 24*I*b^3*d*f^3*x + 24*a*b^2*d*e*f^2 - 24*I*b^
3*d*e*f^2)*cosh(d*x + c) + (24*a*b^2*d*f^3*x - 24*I*b^3*d*f^3*x + 24*a*b^2*
d*e*f^2 - 24*I*b^3*d*e*f^2)*sinh(d*x + c))*polylog(3, -I*cosh(d*x + c) - I*
sinh(d*x + c)) - ((a^3 + a*b^2)*d^4*f^3*x^4 + 4*(a^3 + a*b^2)*d^4*e*f^2*x^3
 + 6*(a^3 + a*b^2)*d^4*e^2*f*x^2 + 4*(a^3 + a*b^2)*d^4*e^3*x + 8*(a^3 + a*b
^2)*c*d^3*e^3 - 12*(a^3 + a*b^2)*c^2*d^2*e^2*f + 8*(a^3 + a*b^2)*c^3*d*e*f^
2 - 2*(a^3 + a*b^2)*c^4*f^3 + 4*((a^2*b + b^3)*d^3*f^3*x^3 + (a^2*b + b^3)*
d^3*e^3 - 3*(a^2*b + b^3)*d^2*e^2*f + 6*(a^2*b + b^3)*d*e*f^2 - 6*(a^2*b +
b^3)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^2*b + b^3)*d^2*f^3)*x^2 + 3*((a^
2*b + b^3)*d^3*e^2*f - 2*(a^2*b + b^3)*d^2*e*f^2 + 2*(a^2*b + b^3)*d*f^3)*x
)*cosh(d*x + c))*sinh(d*x + c))/((a^2*b^2 + b^4)*d^4*cosh(d*x + c) + (a^2*b
^2 + b^4)*d^4*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^3 \sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*sinh(c + d*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x)),
 x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.407 $\int \frac{(e+fx)^2 \sinh^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal. Leaf size=1067

result too large to display

```
[Out] (a*(e + f*x)^3)/(3*b^2*f) + (2*a^2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^3*d)
 - (2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b*d) - (2*a^4*(e + f*x)^2*ArcTan[E^
(c + d*x)])/(b^3*(a^2 + b^2)*d) - (2*f*(e + f*x)*Cosh[c + d*x])/(b*d^2) - (
a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*(a^2 +
 b^2)*d) - (a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])
/(b^2*(a^2 + b^2)*d) - (a*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b^2*d) + (
a^3*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b^2*(a^2 + b^2)*d) - ((2*I)*a^2*
f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^3*d^2) + ((2*I)*f*(e + f*x)*Po
lyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + ((2*I)*a^4*f*(e + f*x)*PolyLog[2, (-I
)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) + ((2*I)*a^2*f*(e + f*x)*PolyLog[2, I
*E^(c + d*x)])/(b^3*d^2) - ((2*I)*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b
*d^2) - ((2*I)*a^4*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*
d^2) - (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
))])/(b^2*(a^2 + b^2)*d^2) - (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^2) - (a*f*(e + f*x)*PolyLog[2
, -E^(2*(c + d*x))])/(b^2*d^2) + (a^3*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x
))])/(b^2*(a^2 + b^2)*d^2) + ((2*I)*a^2*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(
b^3*d^3) - ((2*I)*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b*d^3) - ((2*I)*a^4*f^
2*PolyLog[3, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) - ((2*I)*a^2*f^2*Poly
Log[3, I*E^(c + d*x)])/(b^3*d^3) + ((2*I)*f^2*PolyLog[3, I*E^(c + d*x)])/(b
*d^3) + ((2*I)*a^4*f^2*PolyLog[3, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) + (
2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*(a^2 +
 b^2)*d^3) + (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/(b^2*(a^2 + b^2)*d^3) + (a*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*d^3)
 - (a^3*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*(a^2 + b^2)*d^3) + (2*f^2*
Sinh[c + d*x])/(b*d^3) + ((e + f*x)^2*Sinh[c + d*x])/(b*d)
```

Rubi [A] time = 1.69094, antiderivative size = 1067, normalized size of antiderivative = 1., number of steps used = 50, number of rules used = 14, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5581, 5449, 3296, 2637, 4180, 2531, 2282, 6589, 3718, 2190, 5567, 5573, 5561, 6742}

$$-\frac{2(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right) a^4}{b^3\left(a^2+b^2\right) d}+\frac{2if(e+fx)\text{PolyLog}\left(2,-ie^{c+dx}\right) a^4}{b^3\left(a^2+b^2\right) d^2}-\frac{2if(e+fx)\text{PolyLog}\left(2,ie^{c+dx}\right) a^4}{b^3\left(a^2+b^2\right) d^2}-\frac{2if^2\text{PolyLog}}{b^3\left(a^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (a*(e + f*x)^3)/(3*b^2*f) + (2*a^2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^3*d)
 - (2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b*d) - (2*a^4*(e + f*x)^2*ArcTan[E^
(c + d*x)])/(b^3*(a^2 + b^2)*d) - (2*f*(e + f*x)*Cosh[c + d*x])/(b*d^2) - (
a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*(a^2 +
 b^2)*d) - (a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])
/(b^2*(a^2 + b^2)*d) - (a*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b^2*d) + (
a^3*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b^2*(a^2 + b^2)*d) - ((2*I)*a^2*
f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^3*d^2) + ((2*I)*f*(e + f*x)*Po
lyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + ((2*I)*a^4*f*(e + f*x)*PolyLog[2, (-I
)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) + ((2*I)*a^2*f*(e + f*x)*PolyLog[2, I
```

```
*E^(c + d*x)])/(b^3*d^2) - ((2*I)*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b
*d^2) - ((2*I)*a^4*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*
d^2) - (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
))])/(b^2*(a^2 + b^2)*d^2) - (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^2) - (a*f*(e + f*x)*PolyLog[2
, -E^(2*(c + d*x))])/(b^2*d^2) + (a^3*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x
))])/(b^2*(a^2 + b^2)*d^2) + ((2*I)*a^2*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(
b^3*d^3) - ((2*I)*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b*d^3) - ((2*I)*a^4*f^
2*PolyLog[3, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) - ((2*I)*a^2*f^2*Poly
Log[3, I*E^(c + d*x)])/(b^3*d^3) + ((2*I)*f^2*PolyLog[3, I*E^(c + d*x)])/(b
*d^3) + ((2*I)*a^4*f^2*PolyLog[3, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) + (
2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*(a^2 +
 b^2)*d^3) + (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/(b^2*(a^2 + b^2)*d^3) + (a*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*d^3)
 - (a^3*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*(a^2 + b^2)*d^3) + (2*f^2*
Sinh[c + d*x])/(b*d^3) + ((e + f*x)^2*Sinh[c + d*x])/(b*d)
```

Rule 5581

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - Di
st[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n)/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c +
 d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2 \sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2 \sinh(c+dx)\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2 \sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}}{b}$$

$$= -\frac{a\int (e+fx)^2 \tanh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^2 \tanh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b^2} + \frac{\int (e+fx)^2}{}$$

$$= \frac{a(e+fx)^3}{3b^2 f} - \frac{2(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{bd} + \frac{(e+fx)^2 \sinh(c+dx)}{bd} + \frac{a^2\int (}{}$$

$$= \frac{a(e+fx)^3}{3b^2 f} + \frac{2a^2(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{b^3 d} - \frac{2(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2}{}$$

$$= \frac{a(e+fx)^3}{3b^2 f} + \frac{a^3(e+fx)^3}{3b^2\left(a^2+b^2\right)f} + \frac{2a^2(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{b^3 d} - \frac{2(e+fx)^2}{}$$

$$= \frac{a(e+fx)^3}{3b^2 f} + \frac{a^3(e+fx)^3}{3b^2\left(a^2+b^2\right)f} + \frac{2a^2(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{b^3 d} - \frac{2(e+fx)^2}{}$$

$$= \frac{a(e+fx)^3}{3b^2 f} + \frac{2a^2(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{b^3 d} - \frac{2(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2}{}$$

$$= \frac{a(e+fx)^3}{3b^2 f} + \frac{2a^2(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{b^3 d} - \frac{2(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2}{}$$

$$= \frac{a(e+fx)^3}{3b^2 f} + \frac{2a^2(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{b^3 d} - \frac{2(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2}{}$$

$$= \frac{a(e+fx)^3}{3b^2 f} + \frac{2a^2(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{b^3 d} - \frac{2(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2}{}$$

$$= \frac{a(e+fx)^3}{3b^2 f} + \frac{2a^2(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{b^3 d} - \frac{2(e+fx)^2 \tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2}{}$$

Mathematica [B] time = 19.6768, size = 3418, normalized size = 3.2

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] -(-12*a*d^3*e^2*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*d^3*e*f*x^2
 + 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] - 6*a*d
^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*b*d*e*(1 +
 E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - PolyLo
g[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*e*(1 + E^(2*c))
*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))])
+ (6*I)*b*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log[1
 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2, I
```

```
*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*x)
]) - a*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))]) -
6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))]))/(6*(a
^2 + b^2)*d^3*(1 + E^(2*c))) + (a^3*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 +
2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqr
t[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^
(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) -
 (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]]
)/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a
 + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a
*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c +
 d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(
a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c
 + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^
(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*L
og[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x
*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E
^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/
d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]
)])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b
^2)*E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c +
d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e +
 f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])
/d^2 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[
(a^2 + b^2)*E^(2*c)]))])/d^3 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c
 + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*
c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3))/(3*b^2*(a^2 + b^2)*(
-1 + E^(2*c))) + (Csch[c]*Sech[c]*(Cosh[c + d*x]/(24*b^2*d^3) - Sinh[c + d*
x]/(24*b^2*d^3))*(-3*a^2*b*d^2*e^2*Cosh[2*d*x] - 3*b^3*d^2*e^2*Cosh[2*d*x]
+ 6*a^2*b*d*e*f*Cosh[2*d*x] + 6*b^3*d*e*f*Cosh[2*d*x] - 6*a^2*b*f^2*Cosh[2*
d*x] - 6*b^3*f^2*Cosh[2*d*x] - 6*a^2*b*d^2*e*f*x*Cosh[2*d*x] - 6*b^3*d^2*e*
f*x*Cosh[2*d*x] + 6*a^2*b*d*f^2*x*Cosh[2*d*x] + 6*b^3*d*f^2*x*Cosh[2*d*x] -
 3*a^2*b*d^2*f^2*x^2*Cosh[2*d*x] - 3*b^3*d^2*f^2*x^2*Cosh[2*d*x] - 6*a^3*d^
3*e^2*x*Cosh[c - d*x] - 6*a*b^2*d^3*e^2*x*Cosh[c - d*x] - 6*a^3*d^3*e*f*x^2
*Cosh[c - d*x] - 6*a*b^2*d^3*e*f*x^2*Cosh[c - d*x] - 2*a^3*d^3*f^2*x^3*Cosh
[c - d*x] - 2*a*b^2*d^3*f^2*x^3*Cosh[c - d*x] - 12*a^3*d^3*e^2*x*Cosh[c + d
*x] + 12*a*b^2*d^3*e^2*x*Cosh[c + d*x] - 12*a^3*d^3*e*f*x^2*Cosh[c + d*x] +
 12*a*b^2*d^3*e*f*x^2*Cosh[c + d*x] - 4*a^3*d^3*f^2*x^3*Cosh[c + d*x] + 4*a
*b^2*d^3*f^2*x^3*Cosh[c + d*x] - 6*a^3*d^3*e^2*x*Cosh[3*c + d*x] - 6*a*b^2*
d^3*e^2*x*Cosh[3*c + d*x] - 6*a^3*d^3*e*f*x^2*Cosh[3*c + d*x] - 6*a*b^2*d^3
*e*f*x^2*Cosh[3*c + d*x] - 2*a^3*d^3*f^2*x^3*Cosh[3*c + d*x] - 2*a*b^2*d^3*
f^2*x^3*Cosh[3*c + d*x] + 3*a^2*b*d^2*e^2*Cosh[4*c + 2*d*x] + 3*b^3*d^2*e^2
*Cosh[4*c + 2*d*x] - 6*a^2*b*d*e*f*Cosh[4*c + 2*d*x] - 6*b^3*d*e*f*Cosh[4*c
 + 2*d*x] + 6*a^2*b*f^2*Cosh[4*c + 2*d*x] + 6*b^3*f^2*Cosh[4*c + 2*d*x] + 6
*a^2*b*d^2*e*f*x*Cosh[4*c + 2*d*x] + 6*b^3*d^2*e*f*x*Cosh[4*c + 2*d*x] - 6*
a^2*b*d*f^2*x*Cosh[4*c + 2*d*x] - 6*b^3*d*f^2*x*Cosh[4*c + 2*d*x] + 3*a^2*b
*d^2*f^2*x^2*Cosh[4*c + 2*d*x] + 3*b^3*d^2*f^2*x^2*Cosh[4*c + 2*d*x] - 6*a^
2*b*d^2*e^2*Sinh[2*c] - 6*b^3*d^2*e^2*Sinh[2*c] - 12*a^2*b*d*e*f*Sinh[2*c]
- 12*b^3*d*e*f*Sinh[2*c] - 12*a^2*b*f^2*Sinh[2*c] - 12*b^3*f^2*Sinh[2*c] -
12*a^2*b*d^2*e*f*x*Sinh[2*c] - 12*b^3*d^2*e*f*x*Sinh[2*c] - 12*a^2*b*d*f^2*
x*Sinh[2*c] - 12*b^3*d*f^2*x*Sinh[2*c] - 6*a^2*b*d^2*f^2*x^2*Sinh[2*c] - 6*
b^3*d^2*f^2*x^2*Sinh[2*c] - 3*a^2*b*d^2*e^2*Sinh[2*d*x] - 3*b^3*d^2*e^2*Sin
h[2*d*x] + 6*a^2*b*d*e*f*Sinh[2*d*x] + 6*b^3*d*e*f*Sinh[2*d*x] - 6*a^2*b*f^
2*Sinh[2*d*x] - 6*b^3*f^2*Sinh[2*d*x] - 6*a^2*b*d^2*e*f*x*Sinh[2*d*x] - 6*b
^3*d^2*e*f*x*Sinh[2*d*x] + 6*a^2*b*d*f^2*x*Sinh[2*d*x] + 6*b^3*d*f^2*x*Sinh
[2*d*x] - 3*a^2*b*d^2*f^2*x^2*Sinh[2*d*x] - 3*b^3*d^2*f^2*x^2*Sinh[2*d*x] +
 6*a^3*d^3*e^2*x*Sinh[c - d*x] + 6*a*b^2*d^3*e^2*x*Sinh[c - d*x] + 6*a^3*d^
3*e*f*x^2*Sinh[c - d*x] + 6*a*b^2*d^3*e*f*x^2*Sinh[c - d*x] + 2*a^3*d^3*f^2
```

```
*x^3*Sinh[c - d*x] + 2*a*b^2*d^3*f^2*x^3*Sinh[c - d*x] - 12*a^3*d^3*e^2*x*S
inh[c + d*x] + 12*a*b^2*d^3*e^2*x*Sinh[c + d*x] - 12*a^3*d^3*e*f*x^2*Sinh[c
 + d*x] + 12*a*b^2*d^3*e*f*x^2*Sinh[c + d*x] - 4*a^3*d^3*f^2*x^3*Sinh[c + d
*x] + 4*a*b^2*d^3*f^2*x^3*Sinh[c + d*x] - 6*a^3*d^3*e^2*x*Sinh[3*c + d*x] -
 6*a*b^2*d^3*e^2*x*Sinh[3*c + d*x] - 6*a^3*d^3*e*f*x^2*Sinh[3*c + d*x] - 6*
a*b^2*d^3*e*f*x^2*Sinh[3*c + d*x] - 2*a^3*d^3*f^2*x^3*Sinh[3*c + d*x] - 2*a
*b^2*d^3*f^2*x^3*Sinh[3*c + d*x] + 3*a^2*b*d^2*e^2*Sinh[4*c + 2*d*x] + 3*b^
3*d^2*e^2*Sinh[4*c + 2*d*x] - 6*a^2*b*d*e*f*Sinh[4*c + 2*d*x] - 6*b^3*d*e*f
*Sinh[4*c + 2*d*x] + 6*a^2*b*f^2*Sinh[4*c + 2*d*x] + 6*b^3*f^2*Sinh[4*c + 2
*d*x] + 6*a^2*b*d^2*e*f*x*Sinh[4*c + 2*d*x] + 6*b^3*d^2*e*f*x*Sinh[4*c + 2*
d*x] - 6*a^2*b*d*f^2*x*Sinh[4*c + 2*d*x] - 6*b^3*d*f^2*x*Sinh[4*c + 2*d*x]
+ 3*a^2*b*d^2*f^2*x^2*Sinh[4*c + 2*d*x] + 3*b^3*d^2*f^2*x^2*Sinh[4*c + 2*d*
x]))/(a^2 + b^2)
```

Maple [F] time = 0.853, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 (\sinh(dx+c))^2 \tanh(dx+c)}{a+b\sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(\frac{2\,a^3\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{\left(a^2b^2+b^4\right)d}-\frac{4\,b\arctan\left(e^{(-dx-c)}\right)}{\left(a^2+b^2\right)d}+\frac{2\,a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\left(a^2+b^2\right)d}+\frac{2\,(dx+c)a}{b^2d}-\frac{e^{(dx+c)}}{bd}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/2*(2*a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b^2 + b^4
)*d) - 4*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + 2*a*log(e^(-2*d*x - 2*c)
+ 1)/((a^2 + b^2)*d) + 2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x
- c)/(b*d))*e^2 - 1/6*(2*a*d^3*f^2*x^3*e^c + 6*a*d^3*e*f*x^2*e^c - 3*(b*d^2
*f^2*x^2*e^(2*c) + 2*(d^2*e*f - d*f^2)*b*x*e^(2*c) - 2*(d*e*f - f^2)*b*e^(2
*c))*e^(d*x) + 3*(b*d^2*f^2*x^2 + 2*(d^2*e*f + d*f^2)*b*x + 2*(d*e*f + f^2)
*b)*e^(-d*x))*e^(-c)/(b^2*d^3) + integrate(2*(a^3*b*f^2*x^2 + 2*a^3*b*e*f*x
 - (a^4*f^2*x^2*e^c + 2*a^4*e*f*x*e^c)*e^(d*x))/(a^2*b^3 + b^5 - (a^2*b^3*e
^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x)
- integrate(-2*(a*f^2*x^2 + 2*a*e*f*x - (b*f^2*x^2*e^c + 2*b*e*f*x*e^c)*e^(
d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [C] time = 3.65942, size = 6611, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/6*(3*(a^2*b + b^3)*d^2*f^2*x^2 + 3*(a^2*b + b^3)*d^2*e^2 + 6*(a^2*b + b^
3)*d*e*f + 6*(a^2*b + b^3)*f^2 - 3*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^
3)*d^2*e^2 - 2*(a^2*b + b^3)*d*e*f + 2*(a^2*b + b^3)*f^2 + 2*((a^2*b + b^3)
*d^2*e*f - (a^2*b + b^3)*d*f^2)*x)*cosh(d*x + c)^2 - 3*((a^2*b + b^3)*d^2*f
^2*x^2 + (a^2*b + b^3)*d^2*e^2 - 2*(a^2*b + b^3)*d*e*f + 2*(a^2*b + b^3)*f^
2 + 2*((a^2*b + b^3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x)*sinh(d*x + c)^2 + 6*
((a^2*b + b^3)*d^2*e*f + (a^2*b + b^3)*d*f^2)*x - 2*((a^3 + a*b^2)*d^3*f^2*
x^3 + 3*(a^3 + a*b^2)*d^3*e*f*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*x + 6*(a^3 + a*
b^2)*c*d^2*e^2 - 6*(a^3 + a*b^2)*c^2*d*e*f + 2*(a^3 + a*b^2)*c^3*f^2)*cosh(
d*x + c) + 12*((a^3*d*f^2*x + a^3*d*e*f)*cosh(d*x + c) + (a^3*d*f^2*x + a^3
*d*e*f)*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*((a^3*d*f
^2*x + a^3*d*e*f)*cosh(d*x + c) + (a^3*d*f^2*x + a^3*d*e*f)*sinh(d*x + c))*
dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + ((12*a*b^2*d*f^2*x + 12*I*b^3*d*f^2
*x + 12*a*b^2*d*e*f + 12*I*b^3*d*e*f)*cosh(d*x + c) + (12*a*b^2*d*f^2*x + 1
2*I*b^3*d*f^2*x + 12*a*b^2*d*e*f + 12*I*b^3*d*e*f)*sinh(d*x + c))*dilog(I*c
osh(d*x + c) + I*sinh(d*x + c)) + ((12*a*b^2*d*f^2*x - 12*I*b^3*d*f^2*x + 1
2*a*b^2*d*e*f - 12*I*b^3*d*e*f)*cosh(d*x + c) + (12*a*b^2*d*f^2*x - 12*I*b^
3*d*f^2*x + 12*a*b^2*d*e*f - 12*I*b^3*d*e*f)*sinh(d*x + c))*dilog(-I*cosh(d
*x + c) - I*sinh(d*x + c)) + 6*((a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)
*cosh(d*x + c) + (a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)*sinh(d*x + c))
*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*
a) + 6*((a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)*cosh(d*x + c) + (a^3*d^
2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)*sinh(d*x + c))*log(2*b*cosh(d*x + c) +
 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 6*((a^3*d^2*f^2*x^2
 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*cosh(d*x + c) + (a^3*d^2*
f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*sinh(d*x + c))*log
(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2) - b)/b) + 6*((a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a
^3*c*d*e*f - a^3*c^2*f^2)*cosh(d*x + c) + (a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*
x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*s
inh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
b)/b) + ((6*a*b^2*d^2*e^2 + 6*I*b^3*d^2*e^2 - 12*a*b^2*c*d*e*f - 12*I*b^3*c
*d*e*f + 6*a*b^2*c^2*f^2 + 6*I*b^3*c^2*f^2)*cosh(d*x + c) + (6*a*b^2*d^2*e^
2 + 6*I*b^3*d^2*e^2 - 12*a*b^2*c*d*e*f - 12*I*b^3*c*d*e*f + 6*a*b^2*c^2*f^2
 + 6*I*b^3*c^2*f^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + I) +
 ((6*a*b^2*d^2*e^2 - 6*I*b^3*d^2*e^2 - 12*a*b^2*c*d*e*f + 12*I*b^3*c*d*e*f
+ 6*a*b^2*c^2*f^2 - 6*I*b^3*c^2*f^2)*cosh(d*x + c) + (6*a*b^2*d^2*e^2 - 6*I
*b^3*d^2*e^2 - 12*a*b^2*c*d*e*f + 12*I*b^3*c*d*e*f + 6*a*b^2*c^2*f^2 - 6*I*
b^3*c^2*f^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - I) + ((6*a*
b^2*d^2*f^2*x^2 - 6*I*b^3*d^2*f^2*x^2 + 12*a*b^2*d^2*e*f*x - 12*I*b^3*d^2*e
*f*x + 12*a*b^2*c*d*e*f - 12*I*b^3*c*d*e*f - 6*a*b^2*c^2*f^2 + 6*I*b^3*c^2*
f^2)*cosh(d*x + c) + (6*a*b^2*d^2*f^2*x^2 - 6*I*b^3*d^2*f^2*x^2 + 12*a*b^2*
d^2*e*f*x - 12*I*b^3*d^2*e*f*x + 12*a*b^2*c*d*e*f - 12*I*b^3*c*d*e*f - 6*a*
b^2*c^2*f^2 + 6*I*b^3*c^2*f^2)*sinh(d*x + c))*log(I*cosh(d*x + c) + I*sinh(
d*x + c) + 1) + ((6*a*b^2*d^2*f^2*x^2 + 6*I*b^3*d^2*f^2*x^2 + 12*a*b^2*d^2*
e*f*x + 12*I*b^3*d^2*e*f*x + 12*a*b^2*c*d*e*f + 12*I*b^3*c*d*e*f - 6*a*b^2*
c^2*f^2 - 6*I*b^3*c^2*f^2)*cosh(d*x + c) + (6*a*b^2*d^2*f^2*x^2 + 6*I*b^3*d
^2*f^2*x^2 + 12*a*b^2*d^2*e*f*x + 12*I*b^3*d^2*e*f*x + 12*a*b^2*c*d*e*f + 1
2*I*b^3*c*d*e*f - 6*a*b^2*c^2*f^2 - 6*I*b^3*c^2*f^2)*sinh(d*x + c))*log(-I*
cosh(d*x + c) - I*sinh(d*x + c) + 1) - 12*(a^3*f^2*cosh(d*x + c) + a^3*f^2*
sinh(d*x + c))*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 12*(a^3*f^2*cosh(d*x +
c) + a^3*f^2*sinh(d*x + c))*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) -
 (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - ((12*a*b^2
```

```
*f^2 + 12*I*b^3*f^2)*cosh(d*x + c) + (12*a*b^2*f^2 + 12*I*b^3*f^2)*sinh(d*x
 + c))*polylog(3, I*cosh(d*x + c) + I*sinh(d*x + c)) - ((12*a*b^2*f^2 - 12*
I*b^3*f^2)*cosh(d*x + c) + (12*a*b^2*f^2 - 12*I*b^3*f^2)*sinh(d*x + c))*pol
ylog(3, -I*cosh(d*x + c) - I*sinh(d*x + c)) - 2*((a^3 + a*b^2)*d^3*f^2*x^3
+ 3*(a^3 + a*b^2)*d^3*e*f*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*x + 6*(a^3 + a*b^2)
*c*d^2*e^2 - 6*(a^3 + a*b^2)*c^2*d*e*f + 2*(a^3 + a*b^2)*c^3*f^2 + 3*((a^2*
b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 - 2*(a^2*b + b^3)*d*e*f + 2*(a
^2*b + b^3)*f^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x)*cosh(d
*x + c))*sinh(d*x + c))/((a^2*b^2 + b^4)*d^3*cosh(d*x + c) + (a^2*b^2 + b^4
)*d^3*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e+fx\right)^2\sinh^2\left(c+dx\right)\tanh\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*sinh(c + d*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x)),
 x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.408 $\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=631

$$\frac{ia^4 f\text{PolyLog}\left(2,-ie^{c+dx}\right)}{b^3d^2\left(a^2+b^2\right)}-\frac{ia^4 f\text{PolyLog}\left(2,ie^{c+dx}\right)}{b^3d^2\left(a^2+b^2\right)}-\frac{a^3 f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2\left(a^2+b^2\right)}-\frac{a^3 f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^2\left(a^2+b^2\right)}+a^3 f$$

```
[Out] (a*(e + f*x)^2)/(2*b^2*f) + (2*a^2*(e + f*x)*ArcTan[E^(c + d*x)])/(b^3*d) -
 (2*(e + f*x)*ArcTan[E^(c + d*x)])/(b*d) - (2*a^4*(e + f*x)*ArcTan[E^(c + d
*x)])/(b^3*(a^2 + b^2)*d) - (f*Cosh[c + d*x])/(b*d^2) - (a^3*(e + f*x)*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d) - (a^3*(e +
 f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d) -
 (a*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(b^2*d) + (a^3*(e + f*x)*Log[1 + E^
(2*(c + d*x))])/(b^2*(a^2 + b^2)*d) - (I*a^2*f*PolyLog[2, (-I)*E^(c + d*x)]
)/(b^3*d^2) + (I*f*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + (I*a^4*f*PolyLog
[2, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) + (I*a^2*f*PolyLog[2, I*E^(c +
 d*x)])/(b^3*d^2) - (I*f*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - (I*a^4*f*Poly
Log[2, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) - (a^3*f*PolyLog[2, -((b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^2) - (a^3*f*PolyLog[2,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^2) - (a*f*Pol
yLog[2, -E^(2*(c + d*x))])/(2*b^2*d^2) + (a^3*f*PolyLog[2, -E^(2*(c + d*x))
])/(2*b^2*(a^2 + b^2)*d^2) + ((e + f*x)*Sinh[c + d*x])/(b*d)
```

Rubi [A] time = 0.957045, antiderivative size = 631, normalized size of antiderivative = 1., number of steps used = 39, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.406, Rules used = {5581, 5449, 3296, 2638, 4180, 2279, 2391, 3718, 2190, 5567, 5573, 5561, 6742}

$$\frac{ia^4 f\text{PolyLog}\left(2,-ie^{c+dx}\right)}{b^3d^2\left(a^2+b^2\right)}-\frac{ia^4 f\text{PolyLog}\left(2,ie^{c+dx}\right)}{b^3d^2\left(a^2+b^2\right)}-\frac{a^3 f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2\left(a^2+b^2\right)}-\frac{a^3 f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^2\left(a^2+b^2\right)}+a^3 f$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (a*(e + f*x)^2)/(2*b^2*f) + (2*a^2*(e + f*x)*ArcTan[E^(c + d*x)])/(b^3*d) -
 (2*(e + f*x)*ArcTan[E^(c + d*x)])/(b*d) - (2*a^4*(e + f*x)*ArcTan[E^(c + d
*x)])/(b^3*(a^2 + b^2)*d) - (f*Cosh[c + d*x])/(b*d^2) - (a^3*(e + f*x)*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d) - (a^3*(e +
 f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d) -
 (a*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(b^2*d) + (a^3*(e + f*x)*Log[1 + E^
(2*(c + d*x))])/(b^2*(a^2 + b^2)*d) - (I*a^2*f*PolyLog[2, (-I)*E^(c + d*x)]
)/(b^3*d^2) + (I*f*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + (I*a^4*f*PolyLog
[2, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) + (I*a^2*f*PolyLog[2, I*E^(c +
 d*x)])/(b^3*d^2) - (I*f*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - (I*a^4*f*Poly
Log[2, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) - (a^3*f*PolyLog[2, -((b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^2) - (a^3*f*PolyLog[2,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^2) - (a*f*Pol
yLog[2, -E^(2*(c + d*x))])/(2*b^2*d^2) + (a^3*f*PolyLog[2, -E^(2*(c + d*x))
])/(2*b^2*(a^2 + b^2)*d^2) + ((e + f*x)*Sinh[c + d*x])/(b*d)
```


Rule 5581

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
```

```
ist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - Di
st[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n)/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5449

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*
x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c +
 d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\sinh(c+dx)\tanh(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{a\int (e+fx)\tanh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b^2} + \frac{\int (e+fx)\cos}{b}$$

$$= \frac{a(e+fx)^2}{2b^2f} - \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} + \frac{(e+fx)\sinh(c+dx)}{bd} + \frac{a^2\int (e+}{}$$

$$= \frac{a(e+fx)^2}{2b^2f} + \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{f\cos}{}$$

$$= \frac{a(e+fx)^2}{2b^2f} + \frac{a^3(e+fx)^2}{2b^2\left(a^2+b^2\right)f} + \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)\tan}{bd}$$

$$= \frac{a(e+fx)^2}{2b^2f} + \frac{a^3(e+fx)^2}{2b^2\left(a^2+b^2\right)f} + \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)\tan}{bd}$$

$$= \frac{a(e+fx)^2}{2b^2f} + \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(}{}$$

$$= \frac{a(e+fx)^2}{2b^2f} + \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(}{}$$

$$= \frac{a(e+fx)^2}{2b^2f} + \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(}{}$$

$$= \frac{a(e+fx)^2}{2b^2f} + \frac{2a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} - \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(}{}$$

Mathematica [A] time = 4.78708, size = 481, normalized size = 0.76

$$-\frac{a^3\left(f\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+f(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)+f(c+dx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)+de\log(a+b\sinh(c+dx))-cf\log(a+b\sinh}{b^2\left(a^2+b^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] -(((f*Cosh[c + d*x])/b + (a^3*(-(f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c
 + d*x]] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog
[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(b^2*(a^2 + b^2)) + (-(a*d*
e*(c + d*x)) + a*c*f*(c + d*x) + (a*f*(c + d*x)^2)/2 + 2*b*d*e*ArcTan[Cosh[
c + d*x] + Sinh[c + d*x]] - 2*b*c*f*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] +
 2*b*f*(c + d*x)*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] + a*f*(c + d*x)*Log[
2*Cosh[c + d*x]*(Cosh[c + d*x] - Sinh[c + d*x])] + a*d*e*Log[1 + Cosh[2*(c
+ d*x)] + Sinh[2*(c + d*x)]] - a*c*f*Log[1 + Cosh[2*(c + d*x)] + Sinh[2*(c
+ d*x)]] - I*b*f*PolyLog[2, (-I)*(Cosh[c + d*x] + Sinh[c + d*x])] + I*b*f*P
olyLog[2, I*(Cosh[c + d*x] + Sinh[c + d*x])] - (a*f*PolyLog[2, -Cosh[2*(c +
```

```
d*x)] + Sinh[2*(c + d*x)]])/2)/(a^2 + b^2) - (d*(e + f*x)*Sinh[c + d*x])/b
)/d^2)
```

Maple [B] time = 0.309, size = 4066, normalized size = 6.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/b^2/d^2*a^4*f/(a^2+b^2)^(3/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(
a^2+b^2)^(1/2)))-1/b^2/d^2*a^4*f/(a^2+b^2)^(3/2)*dilog((-b*exp(d*x+c)+(a^2+
b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/b^2/d^2*a^3*f/(a^2+b^2)*dilog((b*exp(
d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/b^2/d^2*a^3*f/(a^2+b^2)*di
log((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2/d^2*f/(2*a^2+
2*b^2)*ln(1-I*exp(d*x+c))*a*c+1/d*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^
2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a*x+1/d^2*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+
c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a*c+1/d*f/(2*a^2+2*b^2)*ln((b*e
xp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a*x+1/d^2*f/(2*a^2+2*b^2)
*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a*c-2/d*f/(2*a^2+
2*b^2)*ln(1+I*exp(d*x+c))*a*x+4/d^2*f*c/(2*a^2+2*b^2)*b*arctan(exp(d*x+c))-
1/d^2*f*c/(2*a^2+2*b^2)*a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d^2*f*c/(
2*a^2+2*b^2)*(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/
2))+2/d^2*f*c/(2*a^2+2*b^2)*a*ln(1+exp(2*d*x+2*c))-2/d^2*f/(2*a^2+2*b^2)*ln
(1+I*exp(d*x+c))*a*c-2/d*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*x+1/2*(d*f*x+
d*e-f)/d^2/b*exp(d*x+c)-1/2*(d*f*x+d*e+f)/d^2/b*exp(-d*x-c)+1/2*a*f*x^2/b^2
+2/d/(a^2+b^2)^(1/2)*b^2*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(
a^2+b^2)^(1/2))-a*e*x/b^2-2*a/b^2/d^2*f*c*ln(exp(d*x+c))+2*a/b^2/d*f*c*x+2/
d*a^2*e/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1
/2/d^2*a*f/(a^2+b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^
(1/2)))+1/d^2*a^2*f/(a^2+b^2)^(3/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/
(a+(a^2+b^2)^(1/2)))-1/d^2*a^2*f/(a^2+b^2)^(3/2)*dilog((-b*exp(d*x+c)+(a^2+
b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/2/d^2*a*f/(a^2+b^2)*dilog((b*exp(d*x+
c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/2/d*a*e/(a^2+b^2)*ln(b*exp(2*d
*x+2*c)+2*a*exp(d*x+c)-b)-2/d^2/(a^2+b^2)^(1/2)*b^2*f*c/(2*a^2+2*b^2)*arcta
nh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d^2*f/(2*a^2+2*b^2)*dilog(1-
I*exp(d*x+c))*a+1/d^2*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a
)/(a+(a^2+b^2)^(1/2)))*a+1/d^2*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^
2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a+a/b^2/d^2*f*c^2+2*a/b^2/d*e*ln(exp(d*x+
c))-2/b^2/d*a^4*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^
(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-2/b^2/d^2*a^4*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/
2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/b^2/d^2*a^4
*f*c/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^
2)^(1/2))+2/b^2/d^2*a^2*f*c/(2*a^2+2*b^2)*(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*
exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/b^2/d*a^4*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2
)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+2/b^2/d^2*a^
4*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+
(a^2+b^2)^(1/2)))*c-2/b^2/d^2*a^4*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*dilog((b*
exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/b^2/d^2*a^4*f/(2*a^2+2
*b^2)/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)
^(1/2)))-2/b^2/d^2*a^4*f*c/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)
/(a^2+b^2)^(1/2))+1/b^2/d^2*a^4*f/(a^2+b^2)^(3/2)*ln((b*exp(d*x+c)+(a^2+b^2
)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/b^2/d*a^3*f/(a^2+b^2)*ln((b*exp(d*x+c)+
(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/b^2/d^2*a^3*f/(a^2+b^2)*ln((b*e
xp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/b^2/d*a^4*f/(a^2+b^2)
^(3/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/b^2/d
```

```
^2*a^4*f/(a^2+b^2)^(3/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)
^(1/2)))*c+2/d*a^2*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b
^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+2/d^2*a^2*f/(2*a^2+2*b^2)/(a^2+b^2)^(1
/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/d*a^2*f/
(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b
^2)^(1/2)))*x-2/d^2*a^2*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a
^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/b^2/d*a^3*f/(a^2+b^2)*ln((-b*exp(
d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/b^2/d^2*a^3*f/(a^2+b^2)
*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/b^2/d*a^4*f
/(a^2+b^2)^(3/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x
-2*I*b/d*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x-2*I*b/d^2*f/(2*a^2+2*b^2)*ln(
1-I*exp(d*x+c))*c+2*I*b/d*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x+2*I*b/d^2*f/
(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c-2/d*e/(2*a^2+2*b^2)*a*ln(1+exp(2*d*x+2*c
))-4/d*e/(2*a^2+2*b^2)*b*arctan(exp(d*x+c))+1/d*e/(2*a^2+2*b^2)*a*ln(b*exp(
2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d*e/(2*a^2+2*b^2)*(a^2+b^2)^(1/2)*arctanh(1/
2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d^2*f/(2*a^2+2*b^2)*dilog(1+I*exp
(d*x+c))*a-2/b^2/d*a^2*e/(2*a^2+2*b^2)*(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp
(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/b^2/d*a^4*e/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*a
rctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/b^2/d^2*a^3*f*c/(a^2+b^2
)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/b^2/d*a^4*e/(a^2+b^2)^(3/2)*arcta
nh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/b^2/d*a^3*e/(a^2+b^2)*ln(b*e
xp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d^2*a^2*f/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*d
ilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/d^2*a^2*f/(2*a
^2+2*b^2)/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+
b^2)^(1/2)))+1/2/d^2*a*f*c/(a^2+b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-
2/d^2*a^2*f*c/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1
/2))-1/2/d*a*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)
^(1/2)))*x-1/2/d^2*a*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(
a^2+b^2)^(1/2)))*c-1/2/d*a*f/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/
(a+(a^2+b^2)^(1/2)))*x-1/2/d^2*a*f/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/
2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d*a^2*f/(a^2+b^2)^(3/2)*ln((b*exp(d*x+c)+(a^
2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2*a^2*f/(a^2+b^2)^(3/2)*ln((b*ex
p(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/d*a^2*f/(a^2+b^2)^(3/2
)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d^2*a^2*f/
(a^2+b^2)^(3/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*
c-2*I*b/d^2*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))+2*I*b/d^2*f/(2*a^2+2*b^2)
*dilog(1+I*exp(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(\frac{2\,a^3\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{\left(a^2b^2+b^4\right)d}-\frac{4\,b\arctan\left(e^{(-dx-c)}\right)}{\left(a^2+b^2\right)d}+\frac{2\,a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\left(a^2+b^2\right)d}+\frac{2\,(dx+c)a}{b^2d}-\frac{e^{(dx+c)}}{bd}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/2*(2*a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b^2 + b^4
)*d) - 4*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + 2*a*log(e^(-2*d*x - 2*c)
+ 1)/((a^2 + b^2)*d) + 2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x
- c)/(b*d))*e - 1/4*f*(2*(a*d^2*x^2*e^c - (b*d*x*e^(2*c) - b*e^(2*c))*e^(d*
x) + (b*d*x + b)*e^(-d*x))*e^(-c)/(b^2*d^2) - integrate(-8*(a^4*x*e^(d*x +
c) - a^3*b*x)/(a^2*b^3 + b^5 - (a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) -
2*(a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) + integrate(8*(b*x*e^(d*x + c) - a
*x)/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x))
```

Fricas [B] time = 3.11505, size = 3537, normalized size = 5.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -1/2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e - ((a^2*b + b^3)*d*f*x + (a^2
*b + b^3)*d*e - (a^2*b + b^3)*f)*cosh(d*x + c)^2 - ((a^2*b + b^3)*d*f*x + (
a^2*b + b^3)*d*e - (a^2*b + b^3)*f)*sinh(d*x + c)^2 + (a^2*b + b^3)*f - ((a
^3 + a*b^2)*d^2*f*x^2 + 2*(a^3 + a*b^2)*d^2*e*x + 4*(a^3 + a*b^2)*c*d*e - 2
*(a^3 + a*b^2)*c^2*f)*cosh(d*x + c) + 2*(a^3*f*cosh(d*x + c) + a^3*f*sinh(d
*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a^3*f*cosh(d*x + c) + a
^3*f*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + ((2*a*b^2*f + 2
*I*b^3*f)*cosh(d*x + c) + (2*a*b^2*f + 2*I*b^3*f)*sinh(d*x + c))*dilog(I*co
sh(d*x + c) + I*sinh(d*x + c)) + ((2*a*b^2*f - 2*I*b^3*f)*cosh(d*x + c) + (
2*a*b^2*f - 2*I*b^3*f)*sinh(d*x + c))*dilog(-I*cosh(d*x + c) - I*sinh(d*x +
 c)) + 2*((a^3*d*e - a^3*c*f)*cosh(d*x + c) + (a^3*d*e - a^3*c*f)*sinh(d*x
+ c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2)
 + 2*a) + 2*((a^3*d*e - a^3*c*f)*cosh(d*x + c) + (a^3*d*e - a^3*c*f)*sinh(d
*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b
^2) + 2*a) + 2*((a^3*d*f*x + a^3*c*f)*cosh(d*x + c) + (a^3*d*f*x + a^3*c*f)
*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*((a^3*d*f*x + a^3*c*f)
*cosh(d*x + c) + (a^3*d*f*x + a^3*c*f)*sinh(d*x + c))*log(-(a*cosh(d*x + c)
 + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2) - b)/b) + ((2*a*b^2*d*e + 2*I*b^3*d*e - 2*a*b^2*c*f - 2*I*b^3*c*f)*cosh
(d*x + c) + (2*a*b^2*d*e + 2*I*b^3*d*e - 2*a*b^2*c*f - 2*I*b^3*c*f)*sinh(d*
x + c))*log(cosh(d*x + c) + sinh(d*x + c) + I) + ((2*a*b^2*d*e - 2*I*b^3*d*
e - 2*a*b^2*c*f + 2*I*b^3*c*f)*cosh(d*x + c) + (2*a*b^2*d*e - 2*I*b^3*d*e -
 2*a*b^2*c*f + 2*I*b^3*c*f)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c
) - I) + ((2*a*b^2*d*f*x - 2*I*b^3*d*f*x + 2*a*b^2*c*f - 2*I*b^3*c*f)*cosh(
d*x + c) + (2*a*b^2*d*f*x - 2*I*b^3*d*f*x + 2*a*b^2*c*f - 2*I*b^3*c*f)*sinh
(d*x + c))*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) + ((2*a*b^2*d*f*x + 2
*I*b^3*d*f*x + 2*a*b^2*c*f + 2*I*b^3*c*f)*cosh(d*x + c) + (2*a*b^2*d*f*x +
2*I*b^3*d*f*x + 2*a*b^2*c*f + 2*I*b^3*c*f)*sinh(d*x + c))*log(-I*cosh(d*x +
 c) - I*sinh(d*x + c) + 1) - ((a^3 + a*b^2)*d^2*f*x^2 + 2*(a^3 + a*b^2)*d^2
*e*x + 4*(a^3 + a*b^2)*c*d*e - 2*(a^3 + a*b^2)*c^2*f + 2*((a^2*b + b^3)*d*f
*x + (a^2*b + b^3)*d*e - (a^2*b + b^3)*f)*cosh(d*x + c))*sinh(d*x + c))/((a
^2*b^2 + b^4)*d^2*cosh(d*x + c) + (a^2*b^2 + b^4)*d^2*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e + fx\right)\sinh^{2}\left(c + dx\right)\tanh\left(c + dx\right)}{a + b\sinh\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*sinh(c + d*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")

[Out] Timed out
```

3.409 $\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=89

$$-\frac{a^3\log(a+b\sinh(c+dx))}{b^2d\left(a^2+b^2\right)}-\frac{b\tan^{-1}(\sinh(c+dx))}{d\left(a^2+b^2\right)}-\frac{a\log(\cosh(c+dx))}{d\left(a^2+b^2\right)}+\frac{\sinh(c+dx)}{bd}$$

```
[Out] -((b*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d)) - (a*Log[Cosh[c + d*x]])/((a^2
 + b^2)*d) - (a^3*Log[a + b*Sinh[c + d*x]])/(b^2*(a^2 + b^2)*d) + Sinh[c +
d*x]/(b*d)
```

Rubi [A] time = 0.195858, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2837, 12, 1629, 635, 203, 260}

$$-\frac{a^3\log(a+b\sinh(c+dx))}{b^2d\left(a^2+b^2\right)}-\frac{b\tan^{-1}(\sinh(c+dx))}{d\left(a^2+b^2\right)}-\frac{a\log(\cosh(c+dx))}{d\left(a^2+b^2\right)}+\frac{\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((b*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d)) - (a*Log[Cosh[c + d*x]])/((a^2
 + b^2)*d) - (a^3*Log[a + b*Sinh[c + d*x]])/(b^2*(a^2 + b^2)*d) + Sinh[c +
d*x]/(b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b\,\text{Subst}\left(\int \frac{x^3}{b^3(a+x)(-b^2-x^2)}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^3}{(a+x)(-b^2-x^2)}\,dx, x, b\sinh(c+dx)\right)}{b^2 d}$$

$$= -\frac{\text{Subst}\left(\int \left(-1+\frac{a^3}{(a^2+b^2)(a+x)}+\frac{b^4+ab^2x}{(a^2+b^2)(b^2+x^2)}\right) dx, x, b\sinh(c+dx)\right)}{b^2 d}$$

$$= -\frac{a^3\log(a+b\sinh(c+dx))}{b^2(a^2+b^2)d} + \frac{\sinh(c+dx)}{bd} - \frac{\text{Subst}\left(\int \frac{b^4+ab^2x}{b^2+x^2}\,dx, x, b\sinh(c+ \ldots$$

$$= -\frac{a^3\log(a+b\sinh(c+dx))}{b^2(a^2+b^2)d} + \frac{\sinh(c+dx)}{bd} - \frac{a\,\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(c+ \ldots}{(a^2+b^2)d}$$

$$= -\frac{b\tan^{-1}(\sinh(c+dx))}{(a^2+b^2)d} - \frac{a\log(\cosh(c+dx))}{(a^2+b^2)d} - \frac{a^3\log(a+b\sinh(c+dx))}{b^2(a^2+b^2)d} + \ldots$$

Mathematica [C] time = 0.175769, size = 91, normalized size = 1.02

$$-\frac{\frac{2a^3\log(a+b\sinh(c+dx))}{b^2(a^2+b^2)}+\frac{\log(-\sinh(c+dx)+i)}{a+ib}+\frac{\log(\sinh(c+dx)+i)}{a-ib}-\frac{2\sinh(c+dx)}{b}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(Log[I - Sinh[c + d*x]]/(a + I*b) + Log[I + Sinh[c + d*x]]/(a - I*b) + (2*
a^3*Log[a + b*Sinh[c + d*x]])/(b^2*(a^2 + b^2)) - (2*Sinh[c + d*x])/b)/(2*d
)
```

Maple [B] time = 0.06, size = 196, normalized size = 2.2

$$-\frac{1}{bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+\frac{a}{b^2d}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{1}{bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+\frac{a}{b^2d}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)- \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/d/b/(tanh(1/2*d*x+1/2*c)+1)+1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/b/(t
anh(1/2*d*x+1/2*c)-1)+1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)-1/d*a^3/b^2/(a^2+
```

```
b^2)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)*b-a)-8/d/(8*a^2+8*b^2
)*a*ln(tanh(1/2*d*x+1/2*c)^2+1)-16/d/(8*a^2+8*b^2)*b*arctan(tanh(1/2*d*x+1/
2*c))
```

Maxima [A] time = 1.56604, size = 198, normalized size = 2.22

$$-\frac{a^3\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{\left(a^2b^2+b^4\right)d}+\frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{\left(a^2+b^2\right)d}-\frac{a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\left(a^2+b^2\right)d}-\frac{(dx+c)a}{b^2d}+\frac{e^{(dx+c)}}{2\,bd}-\frac{e^{(-dx-c)}}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b^2 + b^4)*d) +
2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a^
2 + b^2)*d) - (d*x + c)*a/(b^2*d) + 1/2*e^(d*x + c)/(b*d) - 1/2*e^(-d*x - c
)/(b*d)
```

Fricas [B] time = 2.72294, size = 733, normalized size = 8.24

$$2\left(a^3+ab^2\right)dx\cosh\left(dx+c\right)-a^2b-b^3+\left(a^2b+b^3\right)\cosh\left(dx+c\right)^2+\left(a^2b+b^3\right)\sinh\left(dx+c\right)^2-4\left(b^3\cosh\left(dx+c\right)+b^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*(a^3 + a*b^2)*d*x*cosh(d*x + c) - a^2*b - b^3 + (a^2*b + b^3)*cosh(d
*x + c)^2 + (a^2*b + b^3)*sinh(d*x + c)^2 - 4*(b^3*cosh(d*x + c) + b^3*sinh
(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*(a^3*cosh(d*x + c) + a
^3*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c
))) - 2*(a*b^2*cosh(d*x + c) + a*b^2*sinh(d*x + c))*log(2*cosh(d*x + c)/(co
sh(d*x + c) - sinh(d*x + c))) + 2*((a^3 + a*b^2)*d*x + (a^2*b + b^3)*cosh(d
*x + c))*sinh(d*x + c))/((a^2*b^2 + b^4)*d*cosh(d*x + c) + (a^2*b^2 + b^4)*
d*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh^2\left(c+dx\right)\tanh\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sinh(c + d*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A] time = 1.41449, size = 169, normalized size = 1.9

$$-\frac{\frac{2a^3\log\left(\left|be^{(2dx+2c)}+2ae^{(dx+c)}-b\right|\right)}{a^2b^2+b^4}-\frac{2adx}{b^2}+\frac{4b\arctan\left(e^{(dx+c)}\right)}{a^2+b^2}+\frac{2a\log\left(e^{(2dx+2c)}+1\right)}{a^2+b^2}-\frac{e^{(dx+c)}}{b}+\frac{e^{(-dx-c)}}{b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^3*log(abs(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - b))/(a^2*b^2 + b^
4) - 2*a*d*x/b^2 + 4*b*arctan(e^(d*x + c))/(a^2 + b^2) + 2*a*log(e^(2*d*x +
 2*c) + 1)/(a^2 + b^2) - e^(d*x + c)/b + e^(-d*x - c)/b)/d
```

3.410 $\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Sinh[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [A] time = 0.0891297, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sinh[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Sinh[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\sinh^2(c+dx)\tanh(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.002, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Sinh[c + d*x]^2*Tanh[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.934, size = 0, normalized size = 0.

$$\int \frac{(\sinh(dx+c))^2\tanh(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{e^{\left(-c+\frac{de}{f}\right)}E_1\left(\frac{(fx+e)d}{f}\right)}{2\,bf}-\frac{e^{\left(c-\frac{de}{f}\right)}E_1\left(-\frac{(fx+e)d}{f}\right)}{2\,bf}-\frac{a\log\left(fx+e\right)}{b^2f}+\frac{1}{4}\int-\frac{}{a^2b^3e+b^5e+\left(a^2b^3f+b^5f\right)x-\left(a^2b^3ee^{(2\,c)}+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/2*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b*f) - 1/2*e^(c - d*e
/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b*f) - a*log(f*x + e)/(b^2*f) + 1/4*
integrate(-8*(a^4*e^(d*x + c) - a^3*b)/(a^2*b^3*e + b^5*e + (a^2*b^3*f + b^
5*f)*x - (a^2*b^3*e*e^(2*c) + b^5*e*e^(2*c) + (a^2*b^3*f*e^(2*c) + b^5*f*e^
(2*c))*x)*e^(2*d*x) - 2*(a^3*b^2*e*e^c + a*b^4*e*e^c + (a^3*b^2*f*e^c + a*b
^4*f*e^c)*x)*e^(d*x)), x) - 1/4*integrate(8*(b*e^(d*x + c) - a)/(a^2*e + b^
2*e + (a^2*f + b^2*f)*x + (a^2*e*e^(2*c) + b^2*e*e^(2*c) + (a^2*f*e^(2*c) +
 b^2*f*e^(2*c))*x)*e^(2*d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh\left(dx+c\right)^2\tanh\left(dx+c\right)}{afx+ae+\left(bfx+be\right)\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(sinh(d*x + c)^2*tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh^2\left(c+dx\right)\tanh\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sinh(c + d*x)**2*tanh(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)),
x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.411 $\int \frac{(e+fx)^3 \sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1294

result too large to display

```
[Out] (a^2*(e + f*x)^3)/(b^3*d) - (e + f*x)^3/(b*d) - (a^4*(e + f*x)^3)/(b^3*(a^2
 + b^2)*d) + (e + f*x)^4/(4*b*f) - (6*a*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/
(b^2*d^2) + (6*a^3*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2)
 - (a^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2
 + b^2)^(3/2)*d) + (a^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(b*(a^2 + b^2)^(3/2)*d) - (3*a^2*f*(e + f*x)^2*Log[1 + E^(2*(c + d
*x))])/(b^3*d^2) + (3*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b*d^2) + (3*
a^4*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b^3*(a^2 + b^2)*d^2) + ((6*I)*
a*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*d^3) - ((6*I)*a^3*f^2*(e
 + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) - ((6*I)*a*f^2*
(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b^2*d^3) + ((6*I)*a^3*f^2*(e + f*x)*P
olyLog[2, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) - (3*a^3*f*(e + f*x)^2*Poly
Log[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^2)
 + (3*a^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/(b*(a^2 + b^2)^(3/2)*d^2) - (3*a^2*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d
*x))])/(b^3*d^3) + (3*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(b*d^3) +
 (3*a^4*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(b^3*(a^2 + b^2)*d^3) -
 ((6*I)*a*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(b^2*d^4) + ((6*I)*a^3*f^3*Poly
Log[3, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^4) + ((6*I)*a*f^3*PolyLog[3, I
*E^(c + d*x)])/(b^2*d^4) - ((6*I)*a^3*f^3*PolyLog[3, I*E^(c + d*x)])/(b^2*(
a^2 + b^2)*d^4) + (6*a^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) - (6*a^3*f^2*(e + f*x)*PolyLog[
3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) + (
3*a^2*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*b^3*d^4) - (3*f^3*PolyLog[3, -E^
(2*(c + d*x))])/(2*b*d^4) - (3*a^4*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*b^3
*(a^2 + b^2)*d^4) - (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/(b*(a^2 + b^2)^(3/2)*d^4) + (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^4) + (a*(e + f*x)^3*Sech
[c + d*x])/(b^2*d) - (a^3*(e + f*x)^3*Sech[c + d*x])/(b^2*(a^2 + b^2)*d) +
(a^2*(e + f*x)^3*Tanh[c + d*x])/(b^3*d) - ((e + f*x)^3*Tanh[c + d*x])/(b*d)
 - (a^4*(e + f*x)^3*Tanh[c + d*x])/(b^3*(a^2 + b^2)*d)
```

Rubi [A] time = 2.45464, antiderivative size = 1294, normalized size of antiderivative = 1., number of steps used = 53, number of rules used = 18, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {5581, 3720, 3718, 2190, 2531, 2282, 6589, 32, 5567, 5451, 4180, 5583, 4184, 5573, 3322, 2264, 6609, 6742}

$$-\frac{(e+fx)^3a^4}{b^3\left(a^2+b^2\right)d}+\frac{3f(e+fx)^2\log\left(1+e^{2(c+dx)}\right)a^4}{b^3\left(a^2+b^2\right)d^2}+\frac{3f^2(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)a^4}{b^3\left(a^2+b^2\right)d^3}-\frac{3f^3\text{PolyLog}\left(3,-e^{2(c+dx}\right.}{2b^3\left(a^2+b^2\right)d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*(e + f*x)^3)/(b^3*d) - (e + f*x)^3/(b*d) - (a^4*(e + f*x)^3)/(b^3*(a^2
 + b^2)*d) + (e + f*x)^4/(4*b*f) - (6*a*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/
(b^2*d^2) + (6*a^3*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2)
 - (a^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2
 + b^2)^(3/2)*d) + (a^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
```

```
b^2])])/(b*(a^2 + b^2)^(3/2)*d) - (3*a^2*f*(e + f*x)^2*Log[1 + E^(2*(c + d
*x))])/(b^3*d^2) + (3*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b*d^2) + (3*
a^4*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b^3*(a^2 + b^2)*d^2) + ((6*I)*
a*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*d^3) - ((6*I)*a^3*f^2*(e
 + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) - ((6*I)*a*f^2*
(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b^2*d^3) + ((6*I)*a^3*f^2*(e + f*x)*P
olyLog[2, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) - (3*a^3*f*(e + f*x)^2*Poly
Log[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^2)
 + (3*a^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/(b*(a^2 + b^2)^(3/2)*d^2) - (3*a^2*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d
*x))])/(b^3*d^3) + (3*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(b*d^3) +
 (3*a^4*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(b^3*(a^2 + b^2)*d^3) -
 ((6*I)*a*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(b^2*d^4) + ((6*I)*a^3*f^3*Poly
Log[3, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^4) + ((6*I)*a*f^3*PolyLog[3, I
*E^(c + d*x)])/(b^2*d^4) - ((6*I)*a^3*f^3*PolyLog[3, I*E^(c + d*x)])/(b^2*(
a^2 + b^2)*d^4) + (6*a^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) - (6*a^3*f^2*(e + f*x)*PolyLog[
3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) + (
3*a^2*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*b^3*d^4) - (3*f^3*PolyLog[3, -E^
(2*(c + d*x))])/(2*b*d^4) - (3*a^4*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*b^3
*(a^2 + b^2)*d^4) - (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/(b*(a^2 + b^2)^(3/2)*d^4) + (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^4) + (a*(e + f*x)^3*Sech
[c + d*x])/(b^2*d) - (a^3*(e + f*x)^3*Sech[c + d*x])/(b^2*(a^2 + b^2)*d) +
(a^2*(e + f*x)^3*Tanh[c + d*x])/(b^3*d) - ((e + f*x)^3*Tanh[c + d*x])/(b*d)
 - (a^4*(e + f*x)^3*Tanh[c + d*x])/(b^3*(a^2 + b^2)*d)
```

Rule 5581

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - Di
st[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n)/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c +
 d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5583

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^3\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\tanh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^3\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)^3\tanh(c+dx)}{bd} - \frac{a\int (e+fx)^3\text{sech}(c+dx)\tanh(c+dx)\,dx}{b^2} +$$

$$= -\frac{(e+fx)^3}{bd} + \frac{(e+fx)^4}{4bf} + \frac{a(e+fx)^3\text{sech}(c+dx)}{b^2d} - \frac{(e+fx)^3\tanh(c+}{bd}$$

$$= -\frac{(e+fx)^3}{bd} + \frac{(e+fx)^4}{4bf} - \frac{6af(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d^2} + \frac{3f(e+fx)^2\log}{bd}$$

$$= \frac{a^2(e+fx)^3}{b^3d} - \frac{(e+fx)^3}{bd} + \frac{(e+fx)^4}{4bf} - \frac{6af(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d^2} + \frac{3f}{}$$

$$= \frac{a^2(e+fx)^3}{b^3d} - \frac{(e+fx)^3}{bd} + \frac{(e+fx)^4}{4bf} - \frac{6af(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d^2} - \frac{3a}{}$$

$$= \frac{a^2(e+fx)^3}{b^3d} - \frac{(e+fx)^3}{bd} + \frac{(e+fx)^4}{4bf} - \frac{6af(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^2d^2} - \frac{a^3}{}$$

$$= \frac{a^2(e+fx)^3}{b^3d} - \frac{(e+fx)^3}{bd} - \frac{a^4(e+fx)^3}{b^3\left(a^2+b^2\right)d} + \frac{(e+fx)^4}{4bf} - \frac{6af(e+fx)^2\,\text{ta}}{b^2d^2}$$

$$= \frac{a^2(e+fx)^3}{b^3d} - \frac{(e+fx)^3}{bd} - \frac{a^4(e+fx)^3}{b^3\left(a^2+b^2\right)d} + \frac{(e+fx)^4}{4bf} - \frac{6af(e+fx)^2\,\text{ta}}{b^2d^2}$$

$$= \frac{a^2(e+fx)^3}{b^3d} - \frac{(e+fx)^3}{bd} - \frac{a^4(e+fx)^3}{b^3\left(a^2+b^2\right)d} + \frac{(e+fx)^4}{4bf} - \frac{6af(e+fx)^2\,\text{ta}}{b^2d^2}$$

$$= \frac{a^2(e+fx)^3}{b^3d} - \frac{(e+fx)^3}{bd} - \frac{a^4(e+fx)^3}{b^3\left(a^2+b^2\right)d} + \frac{(e+fx)^4}{4bf} - \frac{6af(e+fx)^2\,\text{ta}}{b^2d^2}$$

$$= \frac{a^2(e+fx)^3}{b^3d} - \frac{(e+fx)^3}{bd} - \frac{a^4(e+fx)^3}{b^3\left(a^2+b^2\right)d} + \frac{(e+fx)^4}{4bf} - \frac{6af(e+fx)^2\,\text{ta}}{b^2d^2}$$

Mathematica [A] time = 13.0382, size = 1111, normalized size = 0.86

$$\left(2e^3\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)d^3 - f^3x^3\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^3 - 3ef^2x^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^3 - 3e^2fx\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^3 + \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(4*b) - (f*(12*b*d^3*e^2*E^
(2*c)*x - 12*b*d^3*e^2*(1 + E^(2*c))*x - 12*b*d^3*e*f*x^2 - 4*b*d^3*f^2*x^3
 + 12*a*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] + 6*b*d^2*e^2*(1 + E^(2*c
))*(2*d*x - Log[1 + E^(2*(c + d*x))]]) + (12*I)*a*d*e*(1 + E^(2*c))*f*(d*x*(
```

```
Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - PolyLog[2, (-I)*E^(c + d
*x)] + PolyLog[2, I*E^(c + d*x)]) + 6*b*d*e*(1 + E^(2*c))*f*(2*d*x*(d*x - L
og[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]) + (6*I)*a*(1 + E^(
2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log[1 + I*E^(c + d*x)]
- 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2, I*E^(c + d*x)] + 2*
PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*x)]) + b*(1 + E^(2*c
))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))]) - 6*d*x*PolyLog[2, -
E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))]))))/(2*(a^2 + b^2)*d^4*(1
+ E^(2*c))) + (a^3*(2*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]]
- 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 3*d^3*e*f^
2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^3*f^3*x^3*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])] + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)] - 3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
 + 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
 + 6*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*d*f^3*x
*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*d*e*f^2*PolyLog[3,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*f^3*x*PolyLog[3, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))] - 6*f^3*PolyLog[4, (b*E^(c + d*x))/(-a + Sq
rt[a^2 + b^2])] + 6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
]))/(b*(a^2 + b^2)^(3/2)*d^4) + ((e + f*x)^3*Sech[c + d*x]*(a - b*Sech[c]*S
inh[d*x]))/((a^2 + b^2)*d)
```

Maple [F] time = 0.785, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3} \sinh \left(dx+c\right) \left(\tanh \left(dx+c\right)\right)^{2}}{a+b \sinh \left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)

[Out] int((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")

[Out] Exception raised: ValueError
```

Fricas [C] time = 5.39078, size = 16386, normalized size = 12.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/4*((a^4 + 2*a^2*b^2 + b^4)*d^4*f^3*x^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d^4*e*
f^2*x^3 + 6*(a^4 + 2*a^2*b^2 + b^4)*d^4*e^2*f*x^2 + 4*(a^4 + 2*a^2*b^2 + b^
4)*d^4*e^3*x + 8*(a^2*b^2 + b^4)*d^3*e^3 - 24*(a^2*b^2 + b^4)*c*d^2*e^2*f +
 24*(a^2*b^2 + b^4)*c^2*d*e*f^2 - 8*(a^2*b^2 + b^4)*c^3*f^3 + ((a^4 + 2*a^2
*b^2 + b^4)*d^4*f^3*x^4 - 24*(a^2*b^2 + b^4)*c*d^2*e^2*f + 24*(a^2*b^2 + b^
4)*c^2*d*e*f^2 - 8*(a^2*b^2 + b^4)*c^3*f^3 + 4*((a^4 + 2*a^2*b^2 + b^4)*d^4
*e*f^2 - 2*(a^2*b^2 + b^4)*d^3*f^3)*x^3 + 6*((a^4 + 2*a^2*b^2 + b^4)*d^4*e^
2*f - 4*(a^2*b^2 + b^4)*d^3*e*f^2)*x^2 + 4*((a^4 + 2*a^2*b^2 + b^4)*d^4*e^3
 - 6*(a^2*b^2 + b^4)*d^3*e^2*f)*x)*cosh(d*x + c)^2 + ((a^4 + 2*a^2*b^2 + b^
4)*d^4*f^3*x^4 - 24*(a^2*b^2 + b^4)*c*d^2*e^2*f + 24*(a^2*b^2 + b^4)*c^2*d*
e*f^2 - 8*(a^2*b^2 + b^4)*c^3*f^3 + 4*((a^4 + 2*a^2*b^2 + b^4)*d^4*e*f^2 -
2*(a^2*b^2 + b^4)*d^3*f^3)*x^3 + 6*((a^4 + 2*a^2*b^2 + b^4)*d^4*e^2*f - 4*(
a^2*b^2 + b^4)*d^3*e*f^2)*x^2 + 4*((a^4 + 2*a^2*b^2 + b^4)*d^4*e^3 - 6*(a^2
*b^2 + b^4)*d^3*e^2*f)*x)*sinh(d*x + c)^2 - 12*(a^3*b*d^2*f^3*x^2 + 2*a^3*b
*d^2*e*f^2*x + a^3*b*d^2*e^2*f + (a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x +
 a^3*b*d^2*e^2*f)*cosh(d*x + c)^2 + 2*(a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^
2*x + a^3*b*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d^2*f^3*x^2 + 2
*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2
)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*(a^3*b*d^2*f^3*x^2 + 2*a^3*b*d
^2*e*f^2*x + a^3*b*d^2*e^2*f + (a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a
^3*b*d^2*e^2*f)*cosh(d*x + c)^2 + 2*(a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*
x + a^3*b*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d^2*f^3*x^2 + 2*a
^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*
dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 4*(a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^
2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3 + (a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*
e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*cosh(d*x + c)^2 + 2*(a^3*b*d^3
*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*cosh(d*x
+ c)*sinh(d*x + c) + (a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e
*f^2 - a^3*b*c^3*f^3)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d
*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 4*(a^3*b*d
^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3 + (a^3*b
*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*cosh(
d*x + c)^2 + 2*(a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 -
 a^3*b*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d^3*e^3 - 3*a^3*b*c*d^
2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*sinh(d*x + c)^2)*sqrt((a^2 +
 b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)
/b^2) + 2*a) - 4*(a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e
^2*f*x + 3*a^3*b*c*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3 + (a^3*b
*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^2*
e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*cosh(d*x + c)^2 + 2*(a^3*b*d^3
*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^2*e^2*
f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) + (a^3
*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^
2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*sinh(d*x + c)^2)*sqrt((a^2 +
 b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*s
inh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*(a^3*b*d^3*f^3*x^3 + 3*a^3*
b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^2*e^2*f - 3*a^3*b*c^2*d
*e*f^2 + a^3*b*c^3*f^3 + (a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3
*b*d^3*e^2*f*x + 3*a^3*b*c*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)
*cosh(d*x + c)^2 + 2*(a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d
^3*e^2*f*x + 3*a^3*b*c*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*cos
h(d*x + c)*sinh(d*x + c) + (a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a
^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^
3)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*
```

x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b)
- 24*(a^3*b*f^3*cosh(d*x + c)^2 + 2*a^3*b*f^3*cosh(d*x + c)*sinh(d*x + c) +
 a^3*b*f^3*sinh(d*x + c)^2 + a^3*b*f^3)*sqrt((a^2 + b^2)/b^2)*polylog(4, (a
*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2))/b) + 24*(a^3*b*f^3*cosh(d*x + c)^2 + 2*a^3*b*f^3*cosh(d*
x + c)*sinh(d*x + c) + a^3*b*f^3*sinh(d*x + c)^2 + a^3*b*f^3)*sqrt((a^2 + b
^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*(a^3*b*d*f^3*x + a^3*b*d*e
*f^2 + (a^3*b*d*f^3*x + a^3*b*d*e*f^2)*cosh(d*x + c)^2 + 2*(a^3*b*d*f^3*x +
 a^3*b*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d*f^3*x + a^3*b*d*e*f^
2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*s
inh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b
) - 24*(a^3*b*d*f^3*x + a^3*b*d*e*f^2 + (a^3*b*d*f^3*x + a^3*b*d*e*f^2)*cos
h(d*x + c)^2 + 2*(a^3*b*d*f^3*x + a^3*b*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c
) + (a^3*b*d*f^3*x + a^3*b*d*e*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*
polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 8*((a^3*b + a*b^3)*d^3*f^3*x^3 + 3*(a^
3*b + a*b^3)*d^3*e*f^2*x^2 + 3*(a^3*b + a*b^3)*d^3*e^2*f*x + (a^3*b + a*b^3
)*d^3*e^3)*cosh(d*x + c) + (-24*I*(a^3*b + a*b^3)*d*f^3*x + 24*(a^2*b^2 + b
^4)*d*f^3*x - 24*I*(a^3*b + a*b^3)*d*e*f^2 + 24*(a^2*b^2 + b^4)*d*e*f^2 + (
-24*I*(a^3*b + a*b^3)*d*f^3*x + 24*(a^2*b^2 + b^4)*d*f^3*x - 24*I*(a^3*b +
a*b^3)*d*e*f^2 + 24*(a^2*b^2 + b^4)*d*e*f^2)*cosh(d*x + c)^2 + (-48*I*(a^3*
b + a*b^3)*d*f^3*x + 48*(a^2*b^2 + b^4)*d*f^3*x - 48*I*(a^3*b + a*b^3)*d*e*
f^2 + 48*(a^2*b^2 + b^4)*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) + (-24*I*(a^3
*b + a*b^3)*d*f^3*x + 24*(a^2*b^2 + b^4)*d*f^3*x - 24*I*(a^3*b + a*b^3)*d*e
f^2 + 24(a^2*b^2 + b^4)*d*e*f^2)*sinh(d*x + c)^2)*dilog(I*cosh(d*x + c) +
 I*sinh(d*x + c)) + (24*I*(a^3*b + a*b^3)*d*f^3*x + 24*(a^2*b^2 + b^4)*d*f^
3*x + 24*I*(a^3*b + a*b^3)*d*e*f^2 + 24*(a^2*b^2 + b^4)*d*e*f^2 + (24*I*(a^
3*b + a*b^3)*d*f^3*x + 24*(a^2*b^2 + b^4)*d*f^3*x + 24*I*(a^3*b + a*b^3)*d*
e*f^2 + 24*(a^2*b^2 + b^4)*d*e*f^2)*cosh(d*x + c)^2 + (48*I*(a^3*b + a*b^3)
*d*f^3*x + 48*(a^2*b^2 + b^4)*d*f^3*x + 48*I*(a^3*b + a*b^3)*d*e*f^2 + 48*(
a^2*b^2 + b^4)*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) + (24*I*(a^3*b + a*b^3)
*d*f^3*x + 24*(a^2*b^2 + b^4)*d*f^3*x + 24*I*(a^3*b + a*b^3)*d*e*f^2 + 24*(
a^2*b^2 + b^4)*d*e*f^2)*sinh(d*x + c)^2)*dilog(-I*cosh(d*x + c) - I*sinh(d*
x + c)) + (-12*I*(a^3*b + a*b^3)*d^2*e^2*f + 12*(a^2*b^2 + b^4)*d^2*e^2*f +
 24*I*(a^3*b + a*b^3)*c*d*e*f^2 - 24*(a^2*b^2 + b^4)*c*d*e*f^2 - 12*I*(a^3*
b + a*b^3)*c^2*f^3 + 12*(a^2*b^2 + b^4)*c^2*f^3 + (-12*I*(a^3*b + a*b^3)*d^
2*e^2*f + 12*(a^2*b^2 + b^4)*d^2*e^2*f + 24*I*(a^3*b + a*b^3)*c*d*e*f^2 - 2
4*(a^2*b^2 + b^4)*c*d*e*f^2 - 12*I*(a^3*b + a*b^3)*c^2*f^3 + 12*(a^2*b^2 +
b^4)*c^2*f^3)*cosh(d*x + c)^2 + (-24*I*(a^3*b + a*b^3)*d^2*e^2*f + 24*(a^2*
b^2 + b^4)*d^2*e^2*f + 48*I*(a^3*b + a*b^3)*c*d*e*f^2 - 48*(a^2*b^2 + b^4)*
c*d*e*f^2 - 24*I*(a^3*b + a*b^3)*c^2*f^3 + 24*(a^2*b^2 + b^4)*c^2*f^3)*cosh
(d*x + c)*sinh(d*x + c) + (-12*I*(a^3*b + a*b^3)*d^2*e^2*f + 12*(a^2*b^2 +
b^4)*d^2*e^2*f + 24*I*(a^3*b + a*b^3)*c*d*e*f^2 - 24*(a^2*b^2 + b^4)*c*d*e*
f^2 - 12*I*(a^3*b + a*b^3)*c^2*f^3 + 12*(a^2*b^2 + b^4)*c^2*f^3)*sinh(d*x +
 c)^2)*log(cosh(d*x + c) + sinh(d*x + c) + I) + (12*I*(a^3*b + a*b^3)*d^2*e
^2*f + 12*(a^2*b^2 + b^4)*d^2*e^2*f - 24*I*(a^3*b + a*b^3)*c*d*e*f^2 - 24*(
a^2*b^2 + b^4)*c*d*e*f^2 + 12*I*(a^3*b + a*b^3)*c^2*f^3 + 12*(a^2*b^2 + b^4
)*c^2*f^3 + (12*I*(a^3*b + a*b^3)*d^2*e^2*f + 12*(a^2*b^2 + b^4)*d^2*e^2*f
- 24*I*(a^3*b + a*b^3)*c*d*e*f^2 - 24*(a^2*b^2 + b^4)*c*d*e*f^2 + 12*I*(a^3
*b + a*b^3)*c^2*f^3 + 12*(a^2*b^2 + b^4)*c^2*f^3)*cosh(d*x + c)^2 + (24*I*(
a^3*b + a*b^3)*d^2*e^2*f + 24*(a^2*b^2 + b^4)*d^2*e^2*f - 48*I*(a^3*b + a*b
^3)*c*d*e*f^2 - 48*(a^2*b^2 + b^4)*c*d*e*f^2 + 24*I*(a^3*b + a*b^3)*c^2*f^3
 + 24*(a^2*b^2 + b^4)*c^2*f^3)*cosh(d*x + c)*sinh(d*x + c) + (12*I*(a^3*b +
 a*b^3)*d^2*e^2*f + 12*(a^2*b^2 + b^4)*d^2*e^2*f - 24*I*(a^3*b + a*b^3)*c*d
*e*f^2 - 24*(a^2*b^2 + b^4)*c*d*e*f^2 + 12*I*(a^3*b + a*b^3)*c^2*f^3 + 12*(
a^2*b^2 + b^4)*c^2*f^3)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c)
- I) + (12*I*(a^3*b + a*b^3)*d^2*f^3*x^2 + 12*(a^2*b^2 + b^4)*d^2*f^3*x^2 +
 24*I*(a^3*b + a*b^3)*d^2*e*f^2*x + 24*(a^2*b^2 + b^4)*d^2*e*f^2*x + 24*I*(

```
a^3*b + a*b^3)*c*d*e*f^2 + 24*(a^2*b^2 + b^4)*c*d*e*f^2 - 12*I*(a^3*b + a*b
^3)*c^2*f^3 - 12*(a^2*b^2 + b^4)*c^2*f^3 + (12*I*(a^3*b + a*b^3)*d^2*f^3*x^
2 + 12*(a^2*b^2 + b^4)*d^2*f^3*x^2 + 24*I*(a^3*b + a*b^3)*d^2*e*f^2*x + 24*
(a^2*b^2 + b^4)*d^2*e*f^2*x + 24*I*(a^3*b + a*b^3)*c*d*e*f^2 + 24*(a^2*b^2
+ b^4)*c*d*e*f^2 - 12*I*(a^3*b + a*b^3)*c^2*f^3 - 12*(a^2*b^2 + b^4)*c^2*f^
3)*cosh(d*x + c)^2 + (24*I*(a^3*b + a*b^3)*d^2*f^3*x^2 + 24*(a^2*b^2 + b^4)
*d^2*f^3*x^2 + 48*I*(a^3*b + a*b^3)*d^2*e*f^2*x + 48*(a^2*b^2 + b^4)*d^2*e*
f^2*x + 48*I*(a^3*b + a*b^3)*c*d*e*f^2 + 48*(a^2*b^2 + b^4)*c*d*e*f^2 - 24*
I*(a^3*b + a*b^3)*c^2*f^3 - 24*(a^2*b^2 + b^4)*c^2*f^3)*cosh(d*x + c)*sinh(
d*x + c) + (12*I*(a^3*b + a*b^3)*d^2*f^3*x^2 + 12*(a^2*b^2 + b^4)*d^2*f^3*x
^2 + 24*I*(a^3*b + a*b^3)*d^2*e*f^2*x + 24*(a^2*b^2 + b^4)*d^2*e*f^2*x + 24
*I*(a^3*b + a*b^3)*c*d*e*f^2 + 24*(a^2*b^2 + b^4)*c*d*e*f^2 - 12*I*(a^3*b +
 a*b^3)*c^2*f^3 - 12*(a^2*b^2 + b^4)*c^2*f^3)*sinh(d*x + c)^2)*log(I*cosh(d
*x + c) + I*sinh(d*x + c) + 1) + (-12*I*(a^3*b + a*b^3)*d^2*f^3*x^2 + 12*(a
^2*b^2 + b^4)*d^2*f^3*x^2 - 24*I*(a^3*b + a*b^3)*d^2*e*f^2*x + 24*(a^2*b^2
+ b^4)*d^2*e*f^2*x - 24*I*(a^3*b + a*b^3)*c*d*e*f^2 + 24*(a^2*b^2 + b^4)*c*
d*e*f^2 + 12*I*(a^3*b + a*b^3)*c^2*f^3 - 12*(a^2*b^2 + b^4)*c^2*f^3 + (-12*
I*(a^3*b + a*b^3)*d^2*f^3*x^2 + 12*(a^2*b^2 + b^4)*d^2*f^3*x^2 - 24*I*(a^3*
b + a*b^3)*d^2*e*f^2*x + 24*(a^2*b^2 + b^4)*d^2*e*f^2*x - 24*I*(a^3*b + a*b
^3)*c*d*e*f^2 + 24*(a^2*b^2 + b^4)*c*d*e*f^2 + 12*I*(a^3*b + a*b^3)*c^2*f^3
 - 12*(a^2*b^2 + b^4)*c^2*f^3)*cosh(d*x + c)^2 + (-24*I*(a^3*b + a*b^3)*d^2
*f^3*x^2 + 24*(a^2*b^2 + b^4)*d^2*f^3*x^2 - 48*I*(a^3*b + a*b^3)*d^2*e*f^2*
x + 48*(a^2*b^2 + b^4)*d^2*e*f^2*x - 48*I*(a^3*b + a*b^3)*c*d*e*f^2 + 48*(a
^2*b^2 + b^4)*c*d*e*f^2 + 24*I*(a^3*b + a*b^3)*c^2*f^3 - 24*(a^2*b^2 + b^4)
*c^2*f^3)*cosh(d*x + c)*sinh(d*x + c) + (-12*I*(a^3*b + a*b^3)*d^2*f^3*x^2
+ 12*(a^2*b^2 + b^4)*d^2*f^3*x^2 - 24*I*(a^3*b + a*b^3)*d^2*e*f^2*x + 24*(a
^2*b^2 + b^4)*d^2*e*f^2*x - 24*I*(a^3*b + a*b^3)*c*d*e*f^2 + 24*(a^2*b^2 +
b^4)*c*d*e*f^2 + 12*I*(a^3*b + a*b^3)*c^2*f^3 - 12*(a^2*b^2 + b^4)*c^2*f^3)
*sinh(d*x + c)^2)*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1) + (24*I*(a^3*
b + a*b^3)*f^3 - 24*(a^2*b^2 + b^4)*f^3 + (24*I*(a^3*b + a*b^3)*f^3 - 24*(a
^2*b^2 + b^4)*f^3)*cosh(d*x + c)^2 + (48*I*(a^3*b + a*b^3)*f^3 - 48*(a^2*b^
2 + b^4)*f^3)*cosh(d*x + c)*sinh(d*x + c) + (24*I*(a^3*b + a*b^3)*f^3 - 24*
(a^2*b^2 + b^4)*f^3)*sinh(d*x + c)^2)*polylog(3, I*cosh(d*x + c) + I*sinh(d
*x + c)) + (-24*I*(a^3*b + a*b^3)*f^3 - 24*(a^2*b^2 + b^4)*f^3 + (-24*I*(a^
3*b + a*b^3)*f^3 - 24*(a^2*b^2 + b^4)*f^3)*cosh(d*x + c)^2 + (-48*I*(a^3*b
+ a*b^3)*f^3 - 48*(a^2*b^2 + b^4)*f^3)*cosh(d*x + c)*sinh(d*x + c) + (-24*I
*(a^3*b + a*b^3)*f^3 - 24*(a^2*b^2 + b^4)*f^3)*sinh(d*x + c)^2)*polylog(3,
-I*cosh(d*x + c) - I*sinh(d*x + c)) + 2*(4*(a^3*b + a*b^3)*d^3*f^3*x^3 + 12
*(a^3*b + a*b^3)*d^3*e*f^2*x^2 + 12*(a^3*b + a*b^3)*d^3*e^2*f*x + 4*(a^3*b
+ a*b^3)*d^3*e^3 + ((a^4 + 2*a^2*b^2 + b^4)*d^4*f^3*x^4 - 24*(a^2*b^2 + b^4
)*c*d^2*e^2*f + 24*(a^2*b^2 + b^4)*c^2*d*e*f^2 - 8*(a^2*b^2 + b^4)*c^3*f^3
+ 4*((a^4 + 2*a^2*b^2 + b^4)*d^4*e*f^2 - 2*(a^2*b^2 + b^4)*d^3*f^3)*x^3 + 6
*((a^4 + 2*a^2*b^2 + b^4)*d^4*e^2*f - 4*(a^2*b^2 + b^4)*d^3*e*f^2)*x^2 + 4*
((a^4 + 2*a^2*b^2 + b^4)*d^4*e^3 - 6*(a^2*b^2 + b^4)*d^3*e^2*f)*x)*cosh(d*x
 + c))*sinh(d*x + c))/((a^4*b + 2*a^2*b^3 + b^5)*d^4*cosh(d*x + c)^2 + 2*(a
^4*b + 2*a^2*b^3 + b^5)*d^4*cosh(d*x + c)*sinh(d*x + c) + (a^4*b + 2*a^2*b^
3 + b^5)*d^4*sinh(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^3 \sinh(c + dx) \tanh^2(c + dx)}{a + b \sinh(c + dx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*sinh(c + d*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x)),
 x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.412 $\int \frac{(e+fx)^2 \sinh(c+dx) \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=904

$$-\frac{(e+fx)^2a^4}{b^3\left(a^2+b^2\right)d}+\frac{2f(e+fx)\log\left(1+e^{2(c+dx)}\right)a^4}{b^3\left(a^2+b^2\right)d^2}+\frac{f^2\text{PolyLog}\left(2,-e^{2(c+dx)}\right)a^4}{b^3\left(a^2+b^2\right)d^3}-\frac{(e+fx)^2\tanh(c+dx)a^4}{b^3\left(a^2+b^2\right)d}+4f(e+$$

```
[Out] (a^2*(e + f*x)^2)/(b^3*d) - (e + f*x)^2/(b*d) - (a^4*(e + f*x)^2)/(b^3*(a^2
 + b^2)*d) + (e + f*x)^3/(3*b*f) - (4*a*f*(e + f*x)*ArcTan[E^(c + d*x)])/(b
^2*d^2) + (4*a^3*f*(e + f*x)*ArcTan[E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) - (
a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b
^2)^(3/2)*d) + (a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
])])/(b*(a^2 + b^2)^(3/2)*d) - (2*a^2*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])
/(b^3*d^2) + (2*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(b*d^2) + (2*a^4*f*(e
 + f*x)*Log[1 + E^(2*(c + d*x))])/(b^3*(a^2 + b^2)*d^2) + ((2*I)*a*f^2*Poly
Log[2, (-I)*E^(c + d*x)])/(b^2*d^3) - ((2*I)*a^3*f^2*PolyLog[2, (-I)*E^(c +
 d*x)])/(b^2*(a^2 + b^2)*d^3) - ((2*I)*a*f^2*PolyLog[2, I*E^(c + d*x)])/(b^
2*d^3) + ((2*I)*a^3*f^2*PolyLog[2, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) -
(2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b
*(a^2 + b^2)^(3/2)*d^2) + (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^2) - (a^2*f^2*PolyLog[2, -E^
(2*(c + d*x))])/(b^3*d^3) + (f^2*PolyLog[2, -E^(2*(c + d*x))])/(b*d^3) + (a
^4*f^2*PolyLog[2, -E^(2*(c + d*x))])/(b^3*(a^2 + b^2)*d^3) + (2*a^3*f^2*Pol
yLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3
) - (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^
2 + b^2)^(3/2)*d^3) + (a*(e + f*x)^2*Sech[c + d*x])/(b^2*d) - (a^3*(e + f*x
)^2*Sech[c + d*x])/(b^2*(a^2 + b^2)*d) + (a^2*(e + f*x)^2*Tanh[c + d*x])/(b
^3*d) - ((e + f*x)^2*Tanh[c + d*x])/(b*d) - (a^4*(e + f*x)^2*Tanh[c + d*x])
/(b^3*(a^2 + b^2)*d)
```

Rubi [A] time = 1.82137, antiderivative size = 904, normalized size of antiderivative = 1., number of steps used = 44, number of rules used = 19, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.559, Rules used = {5581, 3720, 3718, 2190, 2279, 2391, 32, 5567, 5451, 4180, 5583, 4184, 5573, 3322, 2264, 2531, 2282, 6589, 6742}

$$-\frac{(e+fx)^2a^4}{b^3\left(a^2+b^2\right)d}+\frac{2f(e+fx)\log\left(1+e^{2(c+dx)}\right)a^4}{b^3\left(a^2+b^2\right)d^2}+\frac{f^2\text{PolyLog}\left(2,-e^{2(c+dx)}\right)a^4}{b^3\left(a^2+b^2\right)d^3}-\frac{(e+fx)^2\tanh(c+dx)a^4}{b^3\left(a^2+b^2\right)d}+4f(e+$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*(e + f*x)^2)/(b^3*d) - (e + f*x)^2/(b*d) - (a^4*(e + f*x)^2)/(b^3*(a^2
 + b^2)*d) + (e + f*x)^3/(3*b*f) - (4*a*f*(e + f*x)*ArcTan[E^(c + d*x)])/(b
^2*d^2) + (4*a^3*f*(e + f*x)*ArcTan[E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) - (
a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b
^2)^(3/2)*d) + (a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
])])/(b*(a^2 + b^2)^(3/2)*d) - (2*a^2*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])
/(b^3*d^2) + (2*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(b*d^2) + (2*a^4*f*(e
 + f*x)*Log[1 + E^(2*(c + d*x))])/(b^3*(a^2 + b^2)*d^2) + ((2*I)*a*f^2*Poly
Log[2, (-I)*E^(c + d*x)])/(b^2*d^3) - ((2*I)*a^3*f^2*PolyLog[2, (-I)*E^(c +
 d*x)])/(b^2*(a^2 + b^2)*d^3) - ((2*I)*a*f^2*PolyLog[2, I*E^(c + d*x)])/(b^
2*d^3) + ((2*I)*a^3*f^2*PolyLog[2, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) -
(2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b
```

```
*(a^2 + b^2)^(3/2)*d^2) + (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^2) - (a^2*f^2*PolyLog[2, -E^
(2*(c + d*x))])/(b^3*d^3) + (f^2*PolyLog[2, -E^(2*(c + d*x))])/(b*d^3) + (a
^4*f^2*PolyLog[2, -E^(2*(c + d*x))])/(b^3*(a^2 + b^2)*d^3) + (2*a^3*f^2*Pol
yLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3
) - (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^
2 + b^2)^(3/2)*d^3) + (a*(e + f*x)^2*Sech[c + d*x])/(b^2*d) - (a^3*(e + f*x
)^2*Sech[c + d*x])/(b^2*(a^2 + b^2)*d) + (a^2*(e + f*x)^2*Tanh[c + d*x])/(b
^3*d) - ((e + f*x)^2*Tanh[c + d*x])/(b*d) - (a^4*(e + f*x)^2*Tanh[c + d*x])
/(b^3*(a^2 + b^2)*d)
```

Rule 5581

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - Di
st[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n)/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c +
 d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5583

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)^2\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\tanh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)^2\tanh(c+dx)}{bd} - \frac{a\int (e+fx)^2\text{sech}(c+dx)\tanh(c+dx)\,dx}{b^2} +$$

$$= -\frac{(e+fx)^2}{bd} + \frac{(e+fx)^3}{3bf} + \frac{a(e+fx)^2\text{sech}(c+dx)}{b^2d} - \frac{(e+fx)^2\tanh(c+}{bd}$$

$$= -\frac{(e+fx)^2}{bd} + \frac{(e+fx)^3}{3bf} - \frac{4af(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d^2} + \frac{2f(e+fx)\log(}{bd^2}$$

$$= \frac{a^2(e+fx)^2}{b^3d} - \frac{(e+fx)^2}{bd} + \frac{(e+fx)^3}{3bf} - \frac{4af(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d^2} + 2f($$

$$= \frac{a^2(e+fx)^2}{b^3d} - \frac{(e+fx)^2}{bd} + \frac{(e+fx)^3}{3bf} - \frac{4af(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d^2} - 2a^2$$

$$= \frac{a^2(e+fx)^2}{b^3d} - \frac{(e+fx)^2}{bd} + \frac{(e+fx)^3}{3bf} - \frac{4af(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^2d^2} - a^3($$

$$= \frac{a^2(e+fx)^2}{b^3d} - \frac{(e+fx)^2}{bd} - \frac{a^4(e+fx)^2}{b^3\left(a^2+b^2\right)d} + \frac{(e+fx)^3}{3bf} - \frac{4af(e+fx)\tan}{b^2d^2}$$

$$= \frac{a^2(e+fx)^2}{b^3d} - \frac{(e+fx)^2}{bd} - \frac{a^4(e+fx)^2}{b^3\left(a^2+b^2\right)d} + \frac{(e+fx)^3}{3bf} - \frac{4af(e+fx)\tan}{b^2d^2}$$

$$= \frac{a^2(e+fx)^2}{b^3d} - \frac{(e+fx)^2}{bd} - \frac{a^4(e+fx)^2}{b^3\left(a^2+b^2\right)d} + \frac{(e+fx)^3}{3bf} - \frac{4af(e+fx)\tan}{b^2d^2}$$

$$= \frac{a^2(e+fx)^2}{b^3d} - \frac{(e+fx)^2}{bd} - \frac{a^4(e+fx)^2}{b^3\left(a^2+b^2\right)d} + \frac{(e+fx)^3}{3bf} - \frac{4af(e+fx)\tan}{b^2d^2}$$

Mathematica [A] time = 8.54056, size = 937, normalized size = 1.04

$$\left(2e^2\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)d^2 - f^2x^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^2 - 2efx\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^2 + f^2x^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)d^2 + 2ef$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2))/(3*b) + (a^3*(2*d^2*e^2*ArcTanh[(a + b*E^(c
 + d*x))/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] +
2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Lo
g[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, (
```

```
b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a +
 Sqrt[a^2 + b^2])] - 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
]))]))/(b*(a^2 + b^2)^(3/2)*d^3) + (2*b*e*f*Sech[c]*(Cosh[c]*Log[Cosh[c]*Co
sh[d*x] + Sinh[c]*Sinh[d*x]] - d*x*Sinh[c]))/((a^2 + b^2)*d^2*(Cosh[c]^2 -
Sinh[c]^2)) - (4*a*e*f*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c
]^2 - Sinh[c]^2]])/((a^2 + b^2)*d^2*Sqrt[Cosh[c]^2 - Sinh[c]^2]) - (b*f^2*C
sch[c]*(-((d^2*x^2)/E^ArcTanh[Coth[c]]) + (I*Coth[c]*(-(d*x*(-Pi + (2*I)*Ar
cTanh[Coth[c]])) - Pi*Log[1 + E^(2*d*x)] - 2*(I*d*x + I*ArcTanh[Coth[c]])*L
og[1 - E^((2*I)*(I*d*x + I*ArcTanh[Coth[c]]))] + Pi*Log[Cosh[d*x]] + (2*I)*
ArcTanh[Coth[c]]*Log[I*Sinh[d*x + ArcTanh[Coth[c]]]] + I*PolyLog[2, E^((2*I
)*(I*d*x + I*ArcTanh[Coth[c]]))]))/Sqrt[1 - Coth[c]^2])*Sech[c])/((a^2 + b^
2)*d^3*Sqrt[Csch[c]^2*(-Cosh[c]^2 + Sinh[c]^2)]) - (2*a*f^2*(((-I)*Csch[c]*
(I*(d*x + ArcTanh[Coth[c]])*(Log[1 - E^(-(d*x) - ArcTanh[Coth[c]])] - Log[1
 + E^(-(d*x) - ArcTanh[Coth[c]])]) + I*(PolyLog[2, -E^(-(d*x) - ArcTanh[Cot
h[c]])] - PolyLog[2, E^(-(d*x) - ArcTanh[Coth[c]])])))/Sqrt[1 - Coth[c]^2]
- (2*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]]*
ArcTanh[Coth[c]])/Sqrt[Cosh[c]^2 - Sinh[c]^2]))/((a^2 + b^2)*d^3) + (Sech[c
]*Sech[c + d*x]*(a*e^2*Cosh[c] + 2*a*e*f*x*Cosh[c] + a*f^2*x^2*Cosh[c] - b*
e^2*Sinh[d*x] - 2*b*e*f*x*Sinh[d*x] - b*f^2*x^2*Sinh[d*x]))/((a^2 + b^2)*d)
```

Maple [F] time = 0.651, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \sinh(dx+c) (\tanh(dx+c))^2}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 4.13807, size = 9671, normalized size = 10.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/6*(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*f^2*x^3 + 6*(a^4 + 2*a^2*b^2 + b^4)*d^3*
e*f*x^2 + 6*(a^4 + 2*a^2*b^2 + b^4)*d^3*e^2*x + 12*(a^2*b^2 + b^4)*d^2*e^2
- 24*(a^2*b^2 + b^4)*c*d*e*f + 12*(a^2*b^2 + b^4)*c^2*f^2 + 2*((a^4 + 2*a^2
*b^2 + b^4)*d^3*f^2*x^3 - 12*(a^2*b^2 + b^4)*c*d*e*f + 6*(a^2*b^2 + b^4)*c^
2*f^2 + 3*((a^4 + 2*a^2*b^2 + b^4)*d^3*e*f - 2*(a^2*b^2 + b^4)*d^2*f^2)*x^2
 + 3*((a^4 + 2*a^2*b^2 + b^4)*d^3*e^2 - 4*(a^2*b^2 + b^4)*d^2*e*f)*x)*cosh(
d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d^3*f^2*x^3 - 12*(a^2*b^2 + b^4)*c*
d*e*f + 6*(a^2*b^2 + b^4)*c^2*f^2 + 3*((a^4 + 2*a^2*b^2 + b^4)*d^3*e*f - 2*
(a^2*b^2 + b^4)*d^2*f^2)*x^2 + 3*((a^4 + 2*a^2*b^2 + b^4)*d^3*e^2 - 4*(a^2*
b^2 + b^4)*d^2*e*f)*x)*sinh(d*x + c)^2 - 12*(a^3*b*d*f^2*x + a^3*b*d*e*f +
(a^3*b*d*f^2*x + a^3*b*d*e*f)*cosh(d*x + c)^2 + 2*(a^3*b*d*f^2*x + a^3*b*d*
e*f)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d*f^2*x + a^3*b*d*e*f)*sinh(d*x +
 c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*(a^
3*b*d*f^2*x + a^3*b*d*e*f + (a^3*b*d*f^2*x + a^3*b*d*e*f)*cosh(d*x + c)^2 +
 2*(a^3*b*d*f^2*x + a^3*b*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d*f^2
*x + a^3*b*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2) - b)/b + 1) + 6*(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2 +
(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*cosh(d*x + c)^2 + 2*(a^3*
b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) +
(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^
2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b
^2)/b^2) + 2*a) - 6*(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2 + (a^3
*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*cosh(d*x + c)^2 + 2*(a^3*b*d^
2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a^3
*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 +
b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/
b^2) + 2*a) - 6*(a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f -
a^3*b*c^2*f^2 + (a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f -
a^3*b*c^2*f^2)*cosh(d*x + c)^2 + 2*(a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x +
 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d^2*
f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*sinh(d*x + c
)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 6*(a^3*b*d^2*
f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2 + (a^3*b*d^2*
f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*cosh(d*x + c
)^2 + 2*(a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^
2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x
 + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*
log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b) + 12*(a^3*b*f^2*cosh(d*x + c)^2 + 2*a^3*b*
f^2*cosh(d*x + c)*sinh(d*x + c) + a^3*b*f^2*sinh(d*x + c)^2 + a^3*b*f^2)*sq
rt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 12*(a^3*b*f^2*cosh
(d*x + c)^2 + 2*a^3*b*f^2*cosh(d*x + c)*sinh(d*x + c) + a^3*b*f^2*sinh(d*x
+ c)^2 + a^3*b*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*s
inh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b
) + 12*((a^3*b + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x + (a^3*b
+ a*b^3)*d^2*e^2)*cosh(d*x + c) + (-12*I*(a^3*b + a*b^3)*f^2 + 12*(a^2*b^2
+ b^4)*f^2 + (-12*I*(a^3*b + a*b^3)*f^2 + 12*(a^2*b^2 + b^4)*f^2)*cosh(d*x
+ c)^2 + (-24*I*(a^3*b + a*b^3)*f^2 + 24*(a^2*b^2 + b^4)*f^2)*cosh(d*x + c)
*sinh(d*x + c) + (-12*I*(a^3*b + a*b^3)*f^2 + 12*(a^2*b^2 + b^4)*f^2)*sinh(
d*x + c)^2)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + (12*I*(a^3*b + a*b^3
)*f^2 + 12*(a^2*b^2 + b^4)*f^2 + (12*I*(a^3*b + a*b^3)*f^2 + 12*(a^2*b^2 +
b^4)*f^2)*cosh(d*x + c)^2 + (24*I*(a^3*b + a*b^3)*f^2 + 24*(a^2*b^2 + b^4)*
f^2)*cosh(d*x + c)*sinh(d*x + c) + (12*I*(a^3*b + a*b^3)*f^2 + 12*(a^2*b^2
+ b^4)*f^2)*sinh(d*x + c)^2)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + (-
12*I*(a^3*b + a*b^3)*d*e*f + 12*(a^2*b^2 + b^4)*d*e*f + 12*I*(a^3*b + a*b^3
```

```
)*c*f^2 - 12*(a^2*b^2 + b^4)*c*f^2 + (-12*I*(a^3*b + a*b^3)*d*e*f + 12*(a^2
*b^2 + b^4)*d*e*f + 12*I*(a^3*b + a*b^3)*c*f^2 - 12*(a^2*b^2 + b^4)*c*f^2)*
cosh(d*x + c)^2 + (-24*I*(a^3*b + a*b^3)*d*e*f + 24*(a^2*b^2 + b^4)*d*e*f +
 24*I*(a^3*b + a*b^3)*c*f^2 - 24*(a^2*b^2 + b^4)*c*f^2)*cosh(d*x + c)*sinh(
d*x + c) + (-12*I*(a^3*b + a*b^3)*d*e*f + 12*(a^2*b^2 + b^4)*d*e*f + 12*I*(
a^3*b + a*b^3)*c*f^2 - 12*(a^2*b^2 + b^4)*c*f^2)*sinh(d*x + c)^2)*log(cosh(
d*x + c) + sinh(d*x + c) + I) + (12*I*(a^3*b + a*b^3)*d*e*f + 12*(a^2*b^2 +
 b^4)*d*e*f - 12*I*(a^3*b + a*b^3)*c*f^2 - 12*(a^2*b^2 + b^4)*c*f^2 + (12*I
*(a^3*b + a*b^3)*d*e*f + 12*(a^2*b^2 + b^4)*d*e*f - 12*I*(a^3*b + a*b^3)*c*
f^2 - 12*(a^2*b^2 + b^4)*c*f^2)*cosh(d*x + c)^2 + (24*I*(a^3*b + a*b^3)*d*e
*f + 24*(a^2*b^2 + b^4)*d*e*f - 24*I*(a^3*b + a*b^3)*c*f^2 - 24*(a^2*b^2 +
b^4)*c*f^2)*cosh(d*x + c)*sinh(d*x + c) + (12*I*(a^3*b + a*b^3)*d*e*f + 12*
(a^2*b^2 + b^4)*d*e*f - 12*I*(a^3*b + a*b^3)*c*f^2 - 12*(a^2*b^2 + b^4)*c*f
^2)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) - I) + (12*I*(a^3*b
+ a*b^3)*d*f^2*x + 12*(a^2*b^2 + b^4)*d*f^2*x + 12*I*(a^3*b + a*b^3)*c*f^2
+ 12*(a^2*b^2 + b^4)*c*f^2 + (12*I*(a^3*b + a*b^3)*d*f^2*x + 12*(a^2*b^2 +
b^4)*d*f^2*x + 12*I*(a^3*b + a*b^3)*c*f^2 + 12*(a^2*b^2 + b^4)*c*f^2)*cosh(
d*x + c)^2 + (24*I*(a^3*b + a*b^3)*d*f^2*x + 24*(a^2*b^2 + b^4)*d*f^2*x + 2
4*I*(a^3*b + a*b^3)*c*f^2 + 24*(a^2*b^2 + b^4)*c*f^2)*cosh(d*x + c)*sinh(d*
x + c) + (12*I*(a^3*b + a*b^3)*d*f^2*x + 12*(a^2*b^2 + b^4)*d*f^2*x + 12*I*
(a^3*b + a*b^3)*c*f^2 + 12*(a^2*b^2 + b^4)*c*f^2)*sinh(d*x + c)^2)*log(I*co
sh(d*x + c) + I*sinh(d*x + c) + 1) + (-12*I*(a^3*b + a*b^3)*d*f^2*x + 12*(a
^2*b^2 + b^4)*d*f^2*x - 12*I*(a^3*b + a*b^3)*c*f^2 + 12*(a^2*b^2 + b^4)*c*f
^2 + (-12*I*(a^3*b + a*b^3)*d*f^2*x + 12*(a^2*b^2 + b^4)*d*f^2*x - 12*I*(a^
3*b + a*b^3)*c*f^2 + 12*(a^2*b^2 + b^4)*c*f^2)*cosh(d*x + c)^2 + (-24*I*(a^
3*b + a*b^3)*d*f^2*x + 24*(a^2*b^2 + b^4)*d*f^2*x - 24*I*(a^3*b + a*b^3)*c*
f^2 + 24*(a^2*b^2 + b^4)*c*f^2)*cosh(d*x + c)*sinh(d*x + c) + (-12*I*(a^3*b
 + a*b^3)*d*f^2*x + 12*(a^2*b^2 + b^4)*d*f^2*x - 12*I*(a^3*b + a*b^3)*c*f^2
 + 12*(a^2*b^2 + b^4)*c*f^2)*sinh(d*x + c)^2)*log(-I*cosh(d*x + c) - I*sinh
(d*x + c) + 1) + 4*(3*(a^3*b + a*b^3)*d^2*f^2*x^2 + 6*(a^3*b + a*b^3)*d^2*e
*f*x + 3*(a^3*b + a*b^3)*d^2*e^2 + ((a^4 + 2*a^2*b^2 + b^4)*d^3*f^2*x^3 - 1
2*(a^2*b^2 + b^4)*c*d*e*f + 6*(a^2*b^2 + b^4)*c^2*f^2 + 3*((a^4 + 2*a^2*b^2
 + b^4)*d^3*e*f - 2*(a^2*b^2 + b^4)*d^2*f^2)*x^2 + 3*((a^4 + 2*a^2*b^2 + b^
4)*d^3*e^2 - 4*(a^2*b^2 + b^4)*d^2*e*f)*x)*cosh(d*x + c))*sinh(d*x + c))/((
a^4*b + 2*a^2*b^3 + b^5)*d^3*cosh(d*x + c)^2 + 2*(a^4*b + 2*a^2*b^3 + b^5)*
d^3*cosh(d*x + c)*sinh(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d^3*sinh(d*x +
c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^2 \sinh(c+dx) \tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*sinh(c + d*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x)),
 x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.413 $\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=454

$$-\frac{a^3f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2\left(a^2+b^2\right)^{3/2}}+\frac{a^3f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2\left(a^2+b^2\right)^{3/2}}+\frac{a^3f\tan^{-1}(\sinh(c+dx))}{b^2d^2\left(a^2+b^2\right)}+\frac{a^4f\log(\cosh(c+dx))}{b^3d^2\left(a^2+b^2\right)}-\frac{a^2f\,\text{l}}{}$$

```
[Out] (e*x)/b + (f*x^2)/(2*b) - (a*f*ArcTan[Sinh[c + d*x]])/(b^2*d^2) + (a^3*f*Ar
cTan[Sinh[c + d*x]])/(b^2*(a^2 + b^2)*d^2) - (a^3*(e + f*x)*Log[1 + (b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)^(3/2)*d) + (a^3*(e + f*x)*L
og[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)^(3/2)*d) - (a
^2*f*Log[Cosh[c + d*x]])/(b^3*d^2) + (f*Log[Cosh[c + d*x]])/(b*d^2) + (a^4*
f*Log[Cosh[c + d*x]])/(b^3*(a^2 + b^2)*d^2) - (a^3*f*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^2) + (a^3*f*PolyLog[
2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^2) + (
a*(e + f*x)*Sech[c + d*x])/(b^2*d) - (a^3*(e + f*x)*Sech[c + d*x])/(b^2*(a^
2 + b^2)*d) + (a^2*(e + f*x)*Tanh[c + d*x])/(b^3*d) - ((e + f*x)*Tanh[c + d
*x])/(b*d) - (a^4*(e + f*x)*Tanh[c + d*x])/(b^3*(a^2 + b^2)*d)
```

Rubi [A] time = 0.891763, antiderivative size = 454, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 15, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.469, Rules used = {5581, 3720, 3475, 5567, 5451, 3770, 5583, 4184, 5573, 3322, 2264, 2190, 2279, 2391, 6742}

$$-\frac{a^3f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2\left(a^2+b^2\right)^{3/2}}+\frac{a^3f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2\left(a^2+b^2\right)^{3/2}}+\frac{a^3f\tan^{-1}(\sinh(c+dx))}{b^2d^2\left(a^2+b^2\right)}+\frac{a^4f\log(\cosh(c+dx))}{b^3d^2\left(a^2+b^2\right)}-\frac{a^2f\,\text{l}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e*x)/b + (f*x^2)/(2*b) - (a*f*ArcTan[Sinh[c + d*x]])/(b^2*d^2) + (a^3*f*Ar
cTan[Sinh[c + d*x]])/(b^2*(a^2 + b^2)*d^2) - (a^3*(e + f*x)*Log[1 + (b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)^(3/2)*d) + (a^3*(e + f*x)*L
og[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)^(3/2)*d) - (a
^2*f*Log[Cosh[c + d*x]])/(b^3*d^2) + (f*Log[Cosh[c + d*x]])/(b*d^2) + (a^4*
f*Log[Cosh[c + d*x]])/(b^3*(a^2 + b^2)*d^2) - (a^3*f*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^2) + (a^3*f*PolyLog[
2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^2) + (
a*(e + f*x)*Sech[c + d*x])/(b^2*d) - (a^3*(e + f*x)*Sech[c + d*x])/(b^2*(a^
2 + b^2)*d) + (a^2*(e + f*x)*Tanh[c + d*x])/(b^3*d) - ((e + f*x)*Tanh[c + d
*x])/(b*d) - (a^4*(e + f*x)*Tanh[c + d*x])/(b^3*(a^2 + b^2)*d)
```

Rule 5581

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - Di
st[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n)/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c +
 d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5583

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\tanh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{(e+fx)\tanh(c+dx)}{bd} - \frac{a\int (e+fx)\mathbf{sech}(c+dx)\tanh(c+dx)\,dx}{b^2} + \frac{a^2}{}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} + \frac{f\log(\cosh(c+dx))}{bd^2} + \frac{a(e+fx)\mathbf{sech}(c+dx)}{b^2d} - \frac{(e+fx)\tan}{bd}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{af\tan^{-1}(\sinh(c+dx))}{b^2d^2} + \frac{f\log(\cosh(c+dx))}{bd^2} + \frac{a(e+fx)s}{b}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{af\tan^{-1}(\sinh(c+dx))}{b^2d^2} - \frac{a^2f\log(\cosh(c+dx))}{b^3d^2} + \frac{f\log(co}{}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{af\tan^{-1}(\sinh(c+dx))}{b^2d^2} - \frac{a^2f\log(\cosh(c+dx))}{b^3d^2} + \frac{f\log(co}{}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{af\tan^{-1}(\sinh(c+dx))}{b^2d^2} - \frac{a^3(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d} + \frac{a}{}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{af\tan^{-1}(\sinh(c+dx))}{b^2d^2} + \frac{a^3f\tan^{-1}(\sinh(c+dx))}{b^2\left(a^2+b^2\right)d^2} - \frac{a^3(e+}{}$$

$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{af\tan^{-1}(\sinh(c+dx))}{b^2d^2} + \frac{a^3f\tan^{-1}(\sinh(c+dx))}{b^2\left(a^2+b^2\right)d^2} - \frac{a^3(e+}{}$$

Mathematica [A] time = 4.54347, size = 317, normalized size = 0.7

$$\frac{\frac{2a^3\left(-f\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-f(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)+f(c+dx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)-2cf\tanh^{-1}\left(\frac{a+be}{\sqrt{a^2}}\right.\right.}{b\left(a^2+b^2\right)^{3/2}}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (-(((c + d*x)*(c*f - d*(2*e + f*x)))/b) - (4*a*f*ArcTan[Tanh[(c + d*x)/2]])
/(a^2 + b^2) + (2*b*f*Log[Cosh[c + d*x]])/(a^2 + b^2) + (2*a^3*(2*d*e*ArcTa
nh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*c*f*ArcTanh[(a + b*E^(c + d*x))
/Sqrt[a^2 + b^2]] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - f*PolyLo
g[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))]))/(b*(a^2 + b^2)^(3/2)) + (2*d*(e + f*x)*Sech[c
+ d*x]*(a - b*Sinh[c + d*x]))/(a^2 + b^2))/(2*d^2)
```

Maple [B] time = 0.214, size = 1897, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/2*f*x^2/b+1/(a^2+b^2)^2/d^2*f*b*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)*a^2
+2/(a^2+b^2)/d^2*f*b^3/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))-1/(a^2+b^2)/d^2*f
*b^3/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-4/(a^2+b^2)/d^2*a^
3*f/(2*a^2+2*b^2)*arctan(exp(d*x+c))-2/(a^2+b^2)^(5/2)/d^2*f*b*arctanh(1/2*
(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^3-2/(a^2+b^2)^(5/2)/d^2*f*b^3*arcta
nh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a-4/(a^2+b^2)/d^2*f*b^2/(2*a^2
+2*b^2)*a*arctan(exp(d*x+c))+2/(a^2+b^2)/d^2*a^2*b*f/(2*a^2+2*b^2)*ln(1+exp
(2*d*x+2*c))-2/(a^2+b^2)/d^2*a^2*b*f/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*
exp(d*x+c)-b)+e*x/b+2/(a^2+b^2)^(3/2)/b/d^2*a^5*f/(2*a^2+2*b^2)*ln((b*exp(d
*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-2/(a^2+b^2)^(3/2)/b/d^2*a^5
*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/(a^2
+b^2)^(3/2)/d^2*a^3*b*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a
^2+b^2)^(1/2))-2/(a^2+b^2)^(3/2)/b/d*a^5*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+
(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+2/(a^2+b^2)^(3/2)/b/d*a^5*f/(2*a
^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-2/(a^2
+b^2)^(3/2)/d^2*a^3*b*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
(-a+(a^2+b^2)^(1/2)))*c+2/(a^2+b^2)^(3/2)/d^2*a^3*b*f/(2*a^2+2*b^2)*ln((b*e
xp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/(a^2+b^2)^(3/2)/d*a^3
*b*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))
*x-2/(a^2+b^2)^(3/2)/d*a^3*b*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1
/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-2/(a^2+b^2)^(3/2)/b/d^2*a^5*f/(2*a^2+2*b^2)*
ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/b/d^2*a^3*f*
c/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^
(1/2))+2/(a^2+b^2)^(3/2)/d*a^3*b*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c
)+2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)^(3/2)/d^2*a^3*b*f/(2*a^2+2*b^2)*dilog((
b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2/(a^2+b^2)^(3/2)/d^2*
a^3*b*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)
^(1/2)))+2/(a^2+b^2)^(1/2)/d^2*a*b*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x
+c)+2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)^(3/2)/d^2*a*b^3*f/(2*a^2+2*b^2)*arcta
nh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)^(3/2)/d^2*a^3*b*f/
(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)
^(3/2)/b/d*a^5*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(
1/2))+2/(a^2+b^2)^(3/2)/b/d^2*a^5*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+
b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2/(a^2+b^2)^(3/2)/b/d^2*a^5*f/(2*a^2+2*b
^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2/(a^2+b^
2)/d^2*b*f*ln(exp(d*x+c))+1/2/(a^2+b^2)^2/d^2*f*b^3*ln(b*exp(2*d*x+2*c)+2*a
*exp(d*x+c)-b)+2*(f*x+e)*(a*exp(d*x+c)+b)/d/(a^2+b^2)/(1+exp(2*d*x+2*c))+2/
b/d*a^3*e/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a
^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.01406, size = 3753, normalized size = 8.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/2*((a^4 + 2*a^2*b^2 + b^4)*d^2*f*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*x
+ 4*(a^2*b^2 + b^4)*d*e + ((a^4 + 2*a^2*b^2 + b^4)*d^2*f*x^2 + 2*((a^4 + 2*
a^2*b^2 + b^4)*d^2*e - 2*(a^2*b^2 + b^4)*d*f)*x)*cosh(d*x + c)^2 + ((a^4 +
2*a^2*b^2 + b^4)*d^2*f*x^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*e - 2*(a^2*b^2
+ b^4)*d*f)*x)*sinh(d*x + c)^2 - 2*(a^3*b*f*cosh(d*x + c)^2 + 2*a^3*b*f*cos
h(d*x + c)*sinh(d*x + c) + a^3*b*f*sinh(d*x + c)^2 + a^3*b*f)*sqrt((a^2 + b
^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a^3*b*f*cosh(d*x + c)^2
 + 2*a^3*b*f*cosh(d*x + c)*sinh(d*x + c) + a^3*b*f*sinh(d*x + c)^2 + a^3*b*
f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a^3*b*d
*e - a^3*b*c*f + (a^3*b*d*e - a^3*b*c*f)*cosh(d*x + c)^2 + 2*(a^3*b*d*e - a
^3*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d*e - a^3*b*c*f)*sinh(d*x +
c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b
*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(a^3*b*d*e - a^3*b*c*f + (a^3*b*d*e - a^3
*b*c*f)*cosh(d*x + c)^2 + 2*(a^3*b*d*e - a^3*b*c*f)*cosh(d*x + c)*sinh(d*x
+ c) + (a^3*b*d*e - a^3*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2
*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2
*(a^3*b*d*f*x + a^3*b*c*f + (a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x + c)^2 + 2*(
a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d*f*x + a^3*b
*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh
(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/
b) + 2*(a^3*b*d*f*x + a^3*b*c*f + (a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x + c)^2
 + 2*(a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d*f*x +
 a^3*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) +
a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
 - b)/b) - 4*((a^3*b + a*b^3)*f*cosh(d*x + c)^2 + 2*(a^3*b + a*b^3)*f*cosh(
d*x + c)*sinh(d*x + c) + (a^3*b + a*b^3)*f*sinh(d*x + c)^2 + (a^3*b + a*b^3
)*f)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 4*((a^3*b + a*b^3)*d*f*x + (a^
3*b + a*b^3)*d*e)*cosh(d*x + c) + 2*((a^2*b^2 + b^4)*f*cosh(d*x + c)^2 + 2*
(a^2*b^2 + b^4)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2*b^2 + b^4)*f*sinh(d*x
+ c)^2 + (a^2*b^2 + b^4)*f)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x +
 c))) + 2*(2*(a^3*b + a*b^3)*d*f*x + 2*(a^3*b + a*b^3)*d*e + ((a^4 + 2*a^2*
b^2 + b^4)*d^2*f*x^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*e - 2*(a^2*b^2 + b^4)
*d*f)*x)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b + 2*a^2*b^3 + b^5)*d^2*cosh(
d*x + c)^2 + 2*(a^4*b + 2*a^2*b^3 + b^5)*d^2*cosh(d*x + c)*sinh(d*x + c) +
(a^4*b + 2*a^2*b^3 + b^5)*d^2*sinh(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*d
^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)\sinh(c + dx)\tanh^2(c + dx)}{a + b\sinh(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*sinh(c + d*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.414 $\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd\left(a^2+b^2\right)^{3/2}}-\frac{b\tanh(c+dx)}{d\left(a^2+b^2\right)}+\frac{a\operatorname{sech}(c+dx)}{d\left(a^2+b^2\right)}+\frac{a^2x}{b\left(a^2+b^2\right)}+\frac{bx}{a^2+b^2}$$

```
[Out] (a^2*x)/(b*(a^2 + b^2)) + (b*x)/(a^2 + b^2) + (2*a^3*ArcTanh[(b - a*Tanh[(c
 + d*x)/2])/Sqrt[a^2 + b^2]])/(b*(a^2 + b^2)^(3/2)*d) + (a*Sech[c + d*x])/(
(a^2 + b^2)*d) - (b*Tanh[c + d*x])/((a^2 + b^2)*d)
```

Rubi [A] time = 0.20893, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2902, 2606, 8, 3473, 2735, 2660, 618, 204}

$$\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd\left(a^2+b^2\right)^{3/2}}-\frac{b\tanh(c+dx)}{d\left(a^2+b^2\right)}+\frac{a\operatorname{sech}(c+dx)}{d\left(a^2+b^2\right)}+\frac{a^2x}{b\left(a^2+b^2\right)}+\frac{bx}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*x)/(b*(a^2 + b^2)) + (b*x)/(a^2 + b^2) + (2*a^3*ArcTanh[(b - a*Tanh[(c
 + d*x)/2])/Sqrt[a^2 + b^2]])/(b*(a^2 + b^2)^(3/2)*d) + (a*Sech[c + d*x])/(
(a^2 + b^2)*d) - (b*Tanh[c + d*x])/((a^2 + b^2)*d)
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{a\int \operatorname{sech}(c+dx)\tanh(c+dx)\,dx}{a^2+b^2}+\frac{a^2\int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}+\frac{b\int \tanh^2(c+dx)\,dx}{a^2+b^2} \\
&= \frac{a^2x}{b\left(a^2+b^2\right)}-\frac{b\tanh(c+dx)}{\left(a^2+b^2\right)d}-\frac{a^3\int \frac{1}{a+b\sinh(c+dx)}\,dx}{b\left(a^2+b^2\right)}+\frac{b\int 1\,dx}{a^2+b^2}+\frac{a\,\mathrm{Subst}(\int 1\,dx,x,
}{\left(a^2+b
} \\
&= \frac{a^2x}{b\left(a^2+b^2\right)}+\frac{bx}{a^2+b^2}+\frac{a\operatorname{sech}(c+dx)}{\left(a^2+b^2\right)d}-\frac{b\tanh(c+dx)}{\left(a^2+b^2\right)d}+\frac{\left(2ia^3\right)\mathrm{Subst}\left(\int \frac{1}{a-2ibx+a
}}{b\left(a^2
} \\
&= \frac{a^2x}{b\left(a^2+b^2\right)}+\frac{bx}{a^2+b^2}+\frac{a\operatorname{sech}(c+dx)}{\left(a^2+b^2\right)d}-\frac{b\tanh(c+dx)}{\left(a^2+b^2\right)d}-\frac{\left(4ia^3\right)\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2+b^2
}}{} \\
&= \frac{a^2x}{b\left(a^2+b^2\right)}+\frac{bx}{a^2+b^2}+\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\left(a^2+b^2\right)^{3/2}d}+\frac{a\operatorname{sech}(c+dx)}{\left(a^2+b^2\right)d}-\frac{b\tanh(c-
}{\left(a^2+b^2
}
\end{aligned}
$$

Mathematica [A] time = 0.502191, size = 96, normalized size = 0.79

$$
\frac{\frac{2a^3\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{b\left(-a^2-b^2\right)^{3/2}}+\frac{\operatorname{sech}(c+dx)(a-b\sinh(c+dx))}{a^2+b^2}+\frac{c+dx}{b}}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((c + d*x)/b + (2*a^3*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(
b*(-a^2 - b^2)^(3/2)) + (Sech[c + d*x]*(a - b*Sinh[c + d*x]))/(a^2 + b^2))/
d
```

Maple [A] time = 0.003, size = 158, normalized size = 1.3

$$\frac{1}{bd}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{1}{bd}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-2\frac{a^3}{bd\left(a^2+b^2\right)^{3/2}}\mathrm{Artanh}\left(1/2\,\frac{2\,a\tanh\left(1/2\,dx+c/2\right)-2\,b}{\sqrt{a^2+b^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/d/b*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/b*ln(tanh(1/2*d*x+1/2*c)-1)-2/d/b*a^3/(
a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-2
/d/(a^2+b^2)/(tanh(1/2*d*x+1/2*c)^2+1)*tanh(1/2*d*x+1/2*c)*b+2/d/(a^2+b^2)/
(tanh(1/2*d*x+1/2*c)^2+1)*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.53876, size = 1126, normalized size = 9.31

$$\frac{\left(a^4+2\,a^2b^2+b^4\right)dx\cosh\left(dx+c\right)^2+\left(a^4+2\,a^2b^2+b^4\right)dx\sinh\left(dx+c\right)^2+2\,a^2b^2+2\,b^4+\left(a^4+2\,a^2b^2+b^4\right)dx+\left(a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] ((a^4 + 2*a^2*b^2 + b^4)*d*x*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d*x*
sinh(d*x + c)^2 + 2*a^2*b^2 + 2*b^4 + (a^4 + 2*a^2*b^2 + b^4)*d*x + (a^3*co
sh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*x + c)^2 + a
^3)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*
cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2
*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^
2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*
x + c) - b)) + 2*(a^3*b + a*b^3)*cosh(d*x + c) + 2*(a^3*b + a*b^3 + (a^4 +
2*a^2*b^2 + b^4)*d*x*cosh(d*x + c))*sinh(d*x + c))/((a^4*b + 2*a^2*b^3 + b^
5)*d*cosh(d*x + c)^2 + 2*(a^4*b + 2*a^2*b^3 + b^5)*d*cosh(d*x + c)*sinh(d*x
 + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*sinh(d*x + c)^2 + (a^4*b + 2*a^2*b^3 +
```

```
b^5)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sinh(c + d*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [A] time = 1.43235, size = 182, normalized size = 1.5

$$\frac{\frac{a^3\log\left(\frac{\left|-2\,be^{(dx+2\,c)}-2\,ae^{c}-2\,\sqrt{a^2+b^2}e^{c}\right|}{\left|-2\,be^{(dx+2\,c)}-2\,ae^{c}+2\,\sqrt{a^2+b^2}e^{c}\right|}\right)}{\left(a^2b+b^3\right)\sqrt{a^2+b^2}}+\frac{dx}{b}+\frac{2\left(ae^{(dx+c)}+b\right)}{\left(a^2+b^2\right)\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (a^3*log(abs(-2*b*e^(d*x + 2*c) - 2*a*e^c - 2*sqrt(a^2 + b^2)*e^c)/abs(-2*b
*e^(d*x + 2*c) - 2*a*e^c + 2*sqrt(a^2 + b^2)*e^c))/((a^2*b + b^3)*sqrt(a^2
+ b^2)) + d*x/b + 2*(a*e^(d*x + c) + b)/((a^2 + b^2)*(e^(2*d*x + 2*c) + 1))
)/d
```

3.415 $\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Sinh[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [A] time = 0.0811154, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sinh[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]

[Out] Defer[Int][(Sinh[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\sinh(c+dx)\tanh^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.003, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Sinh[c + d*x]*Tanh[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]

[Out] $Aborted
```

Maple [A] time = 0.753, size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)\left(\tanh(dx+c)\right)^2}{\left(fx+e\right)\left(a+b\sinh(dx+c)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-2\,a^3\int -\frac{e^{(dx+c)}}{a^2b^2e + b^4e + \left(a^2b^2f + b^4f\right)x - \left(a^2b^2ee^{(2\,c)} + b^4ee^{(2\,c)} + \left(a^2b^2fe^{(2\,c)} + b^4fe^{(2\,c)}\right)x\right)e^{(2\,dx)} - 2\left(a^3bee^c + ab^3ee^c + (\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -2*a^3*integrate(-e^(d*x + c)/(a^2*b^2*e + b^4*e + (a^2*b^2*f + b^4*f)*x -
(a^2*b^2*e*e^(2*c) + b^4*e*e^(2*c) + (a^2*b^2*f*e^(2*c) + b^4*f*e^(2*c))*x)
*e^(2*d*x) - 2*(a^3*b*e*e^c + a*b^3*e*e^c + (a^3*b*f*e^c + a*b^3*f*e^c)*x)*
e^(d*x)), x) + 2*(a*e^(d*x + c) + b)/(a^2*d*e + b^2*d*e + (a^2*d*f + b^2*d*
f)*x + (a^2*d*e*e^(2*c) + b^2*d*e*e^(2*c) + (a^2*d*f*e^(2*c) + b^2*d*f*e^(2
*c))*x)*e^(2*d*x)) + log(f*x + e)/(b*f) + 1/2*integrate(4*(a*f*e^(d*x + c)
+ b*f)/(a^2*d*e^2 + b^2*d*e^2 + (a^2*d*f^2 + b^2*d*f^2)*x^2 + 2*(a^2*d*e*f
+ b^2*d*e*f)*x + (a^2*d*e^2*e^(2*c) + b^2*d*e^2*e^(2*c) + (a^2*d*f^2*e^(2*c
) + b^2*d*f^2*e^(2*c))*x^2 + 2*(a^2*d*e*f*e^(2*c) + b^2*d*e*f*e^(2*c))*x)*e
^(2*d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh\left(dx+c\right)\tanh\left(dx+c\right)^2}{afx + ae + \left(bfx + be\right)\sinh\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(sinh(d*x + c)*tanh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(c+dx\right)\tanh^2\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sinh(c + d*x)*tanh(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)),
x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.416 $\int \frac{(e+fx)^2\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1479

result too large to display

```
[Out] (a^2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^3*d) + ((e + f*x)^2*ArcTan[E^(c +
d*x)])/(b*d) - (2*a^4*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b*(a^2 + b^2)^2*d)
- (a^4*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^3*(a^2 + b^2)*d) - (a^2*f^2*ArcT
an[Sinh[c + d*x]])/(b^3*d^3) + (f^2*ArcTan[Sinh[c + d*x]])/(b*d^3) + (a^4*f
^2*ArcTan[Sinh[c + d*x]])/(b^3*(a^2 + b^2)*d^3) - (a^3*(e + f*x)^2*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) - (a^3*(e + f*x)^
2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) + (a^3*
(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)^2*d) + (a*f^2*Log[Cosh[c
 + d*x]])/(b^2*d^3) - (a^3*f^2*Log[Cosh[c + d*x]])/(b^2*(a^2 + b^2)*d^3) -
(I*a^2*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^3*d^2) - (I*f*(e + f*x)
*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + ((2*I)*a^4*f*(e + f*x)*PolyLog[2,
(-I)*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^2) + (I*a^4*f*(e + f*x)*PolyLog[2, (-
I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) + (I*a^2*f*(e + f*x)*PolyLog[2, I*E^
(c + d*x)])/(b^3*d^2) + (I*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b*d^2) -
 ((2*I)*a^4*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^2) -
(I*a^4*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) - (2*a^
3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 +
 b^2)^2*d^2) - (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/((a^2 + b^2)^2*d^2) + (a^3*f*(e + f*x)*PolyLog[2, -E^(2*(c + d
*x))])/((a^2 + b^2)^2*d^2) + (I*a^2*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b^3*
d^3) + (I*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b*d^3) - ((2*I)*a^4*f^2*PolyLo
g[3, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^3) - (I*a^4*f^2*PolyLog[3, (-I)*
E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) - (I*a^2*f^2*PolyLog[3, I*E^(c + d*x)])
/(b^3*d^3) - (I*f^2*PolyLog[3, I*E^(c + d*x)])/(b*d^3) + ((2*I)*a^4*f^2*Pol
yLog[3, I*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^3) + (I*a^4*f^2*PolyLog[3, I*E^(
c + d*x)])/(b^3*(a^2 + b^2)*d^3) + (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) + (2*a^3*f^2*PolyLog[3, -((b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) - (a^3*f^2*PolyLog
[3, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^3) + (a^2*f*(e + f*x)*Sech[c + d*
x])/(b^3*d^2) - (f*(e + f*x)*Sech[c + d*x])/(b*d^2) - (a^4*f*(e + f*x)*Sech
[c + d*x])/(b^3*(a^2 + b^2)*d^2) + (a*(e + f*x)^2*Sech[c + d*x]^2)/(2*b^2*d
) - (a^3*(e + f*x)^2*Sech[c + d*x]^2)/(2*b^2*(a^2 + b^2)*d) - (a*f*(e + f*x
)*Tanh[c + d*x])/(b^2*d^2) + (a^3*f*(e + f*x)*Tanh[c + d*x])/(b^2*(a^2 + b^
2)*d^2) + (a^2*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*b^3*d) - ((e + f
*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*b*d) - (a^4*(e + f*x)^2*Sech[c + d*x]
*Tanh[c + d*x])/(2*b^3*(a^2 + b^2)*d)
```

Rubi [A] time = 2.45324, antiderivative size = 1479, normalized size of antiderivative = 1., number of steps used = 71, number of rules used = 17, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.607, Rules used = {5567, 5455, 4180, 2531, 2282, 6589, 4186, 3770, 5583, 5451, 4184, 3475, 5573, 5561, 2190, 6742, 3718}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Tanh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^3*d) + ((e + f*x)^2*ArcTan[E^(c +
d*x)])/(b*d) - (2*a^4*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b*(a^2 + b^2)^2*d)
```

```
- (a^4*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^3*(a^2 + b^2)*d) - (a^2*f^2*ArcT
an[Sinh[c + d*x]])/(b^3*d^3) + (f^2*ArcTan[Sinh[c + d*x]])/(b*d^3) + (a^4*f
^2*ArcTan[Sinh[c + d*x]])/(b^3*(a^2 + b^2)*d^3) - (a^3*(e + f*x)^2*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) - (a^3*(e + f*x)^
2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) + (a^3*
(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)^2*d) + (a*f^2*Log[Cosh[c
 + d*x]])/(b^2*d^3) - (a^3*f^2*Log[Cosh[c + d*x]])/(b^2*(a^2 + b^2)*d^3) -
(I*a^2*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^3*d^2) - (I*f*(e + f*x)
*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + ((2*I)*a^4*f*(e + f*x)*PolyLog[2,
(-I)*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^2) + (I*a^4*f*(e + f*x)*PolyLog[2, (-
I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) + (I*a^2*f*(e + f*x)*PolyLog[2, I*E^
(c + d*x)])/(b^3*d^2) + (I*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b*d^2) -
 ((2*I)*a^4*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^2) -
(I*a^4*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) - (2*a^
3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 +
 b^2)^2*d^2) - (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/((a^2 + b^2)^2*d^2) + (a^3*f*(e + f*x)*PolyLog[2, -E^(2*(c + d
*x))])/((a^2 + b^2)^2*d^2) + (I*a^2*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b^3*
d^3) + (I*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b*d^3) - ((2*I)*a^4*f^2*PolyLo
g[3, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^3) - (I*a^4*f^2*PolyLog[3, (-I)*
E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) - (I*a^2*f^2*PolyLog[3, I*E^(c + d*x)])
/(b^3*d^3) - (I*f^2*PolyLog[3, I*E^(c + d*x)])/(b*d^3) + ((2*I)*a^4*f^2*Pol
yLog[3, I*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^3) + (I*a^4*f^2*PolyLog[3, I*E^(
c + d*x)])/(b^3*(a^2 + b^2)*d^3) + (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) + (2*a^3*f^2*PolyLog[3, -((b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) - (a^3*f^2*PolyLog
[3, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^3) + (a^2*f*(e + f*x)*Sech[c + d*
x])/(b^3*d^2) - (f*(e + f*x)*Sech[c + d*x])/(b*d^2) - (a^4*f*(e + f*x)*Sech
[c + d*x])/(b^3*(a^2 + b^2)*d^2) + (a*(e + f*x)^2*Sech[c + d*x]^2)/(2*b^2*d
) - (a^3*(e + f*x)^2*Sech[c + d*x]^2)/(2*b^2*(a^2 + b^2)*d) - (a*f*(e + f*x
)*Tanh[c + d*x])/(b^2*d^2) + (a^3*f*(e + f*x)*Tanh[c + d*x])/(b^2*(a^2 + b^
2)*d^2) + (a^2*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*b^3*d) - ((e + f
*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*b*d) - (a^4*(e + f*x)^2*Sech[c + d*x]
*Tanh[c + d*x])/(2*b^3*(a^2 + b^2)*d)
```

Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c +
 d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5455

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5583

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2 \tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\mathrm{sech}(c+dx)\tanh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)^2\mathrm{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{a\int (e+fx)^2\mathrm{sech}^2(c+dx)\tanh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)^2\mathrm{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b^2} + \int(e$$

$$= \frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{f(e+fx)\mathrm{sech}(c+dx)}{bd^2} + \frac{a(e+fx)^2\mathrm{sech}^2(c+dx)}{2b^2d} - (e+fx$$

$$= \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} + \frac{f^2\tan^{-1}(\sinh(c+dx))}{bd^3} - \frac{2if(e+fx)\mathrm{Li}_2\left(-ie^{c+dx}\right)}{bd^2} + \frac{2if(e+}{}$$

$$= \frac{a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a^2f^2\tan^{-1}(\sinh(c+dx))}{b^3d^3} + \frac{f^2\,\mathrm{ta}}{}$$

$$= \frac{a^3(e+fx)^3}{3\left(a^2+b^2\right)^2f} + \frac{a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a^2f^2\tan^{-1}(\sinh}{b^3d^3}$$

$$= \frac{a^3(e+fx)^3}{3\left(a^2+b^2\right)^2f} + \frac{a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a^2f^2\tan^{-1}(\sinh}{b^3d^3}$$

$$= \frac{a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)^2d} - \frac{a^4(e}{}$$

$$= \frac{a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)^2d} - \frac{a^4(e}{}$$

$$= \frac{a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)^2d} - \frac{a^4(e}{}$$

$$= \frac{a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)^2d} - \frac{a^4(e}{}$$

$$= \frac{a^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)^2d} - \frac{a^4(e}{}$$

Mathematica [B] time = 30.8825, size = 3368, normalized size = 2.28

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Tanh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-12*a^3*d^3*e^2*E^(2*c)*x - 12*a^3*d*E^(2*c)*f^2*x - 12*a*b^2*d*E^(2*c)*f^
2*x - 12*a^3*d^3*e*E^(2*c)*f*x^2 - 4*a^3*d^3*E^(2*c)*f^2*x^3 + 18*a^2*b*d^2
*e^2*ArcTan[E^(c + d*x)] + 6*b^3*d^2*e^2*ArcTan[E^(c + d*x)] + 18*a^2*b*d^2
*e^2*E^(2*c)*ArcTan[E^(c + d*x)] + 6*b^3*d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)
] + 12*a^2*b*f^2*ArcTan[E^(c + d*x)] + 12*b^3*f^2*ArcTan[E^(c + d*x)] + 12*
a^2*b*E^(2*c)*f^2*ArcTan[E^(c + d*x)] + 12*b^3*E^(2*c)*f^2*ArcTan[E^(c + d*
x)] + (18*I)*a^2*b*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (6*I)*b^3*d^2*e*f*x*L
```

og[1 - I*E^(c + d*x)] + (18*I)*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x
)] + (6*I)*b^3*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (9*I)*a^2*b*d^2*f
^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*b^3*d^2*f^2*x^2*Log[1 - I*E^(c + d*x)
] + (9*I)*a^2*b*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*b^3*d^2*
E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] - (18*I)*a^2*b*d^2*e*f*x*Log[1 + I*E
^(c + d*x)] - (6*I)*b^3*d^2*e*f*x*Log[1 + I*E^(c + d*x)] - (18*I)*a^2*b*d^2
*e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (6*I)*b^3*d^2*e*E^(2*c)*f*x*Log[1 +
 I*E^(c + d*x)] - (9*I)*a^2*b*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - (3*I)*b^
3*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - (9*I)*a^2*b*d^2*E^(2*c)*f^2*x^2*Log[
1 + I*E^(c + d*x)] - (3*I)*b^3*d^2*E^(2*c)*f^2*x^2*Log[1 + I*E^(c + d*x)] +
 6*a^3*d^2*e^2*Log[1 + E^(2*(c + d*x))] + 6*a^3*d^2*e^2*E^(2*c)*Log[1 + E^(
2*(c + d*x))] + 6*a^3*f^2*Log[1 + E^(2*(c + d*x))] + 6*a*b^2*f^2*Log[1 + E^
(2*(c + d*x))] + 6*a^3*E^(2*c)*f^2*Log[1 + E^(2*(c + d*x))] + 6*a*b^2*E^(2*
c)*f^2*Log[1 + E^(2*(c + d*x))] + 12*a^3*d^2*e*f*x*Log[1 + E^(2*(c + d*x))]
 + 12*a^3*d^2*e*E^(2*c)*f*x*Log[1 + E^(2*(c + d*x))] + 6*a^3*d^2*f^2*x^2*Lo
g[1 + E^(2*(c + d*x))] + 6*a^3*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(2*(c + d*x))]
 - (6*I)*b*(3*a^2 + b^2)*d*(1 + E^(2*c))*f*(e + f*x)*PolyLog[2, (-I)*E^(c +
 d*x)] + (6*I)*b*(3*a^2 + b^2)*d*(1 + E^(2*c))*f*(e + f*x)*PolyLog[2, I*E^(
c + d*x)] + 6*a^3*d*e*f*PolyLog[2, -E^(2*(c + d*x))] + 6*a^3*d*e*E^(2*c)*f*
PolyLog[2, -E^(2*(c + d*x))] + 6*a^3*d*f^2*x*PolyLog[2, -E^(2*(c + d*x))] +
 6*a^3*d*E^(2*c)*f^2*x*PolyLog[2, -E^(2*(c + d*x))] + (18*I)*a^2*b*f^2*Poly
Log[3, (-I)*E^(c + d*x)] + (6*I)*b^3*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (18
*I)*a^2*b*E^(2*c)*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (6*I)*b^3*E^(2*c)*f^2*
PolyLog[3, (-I)*E^(c + d*x)] - (18*I)*a^2*b*f^2*PolyLog[3, I*E^(c + d*x)] -
 (6*I)*b^3*f^2*PolyLog[3, I*E^(c + d*x)] - (18*I)*a^2*b*E^(2*c)*f^2*PolyLog
[3, I*E^(c + d*x)] - (6*I)*b^3*E^(2*c)*f^2*PolyLog[3, I*E^(c + d*x)] - 3*a^
3*f^2*PolyLog[3, -E^(2*(c + d*x))] - 3*a^3*E^(2*c)*f^2*PolyLog[3, -E^(2*(c
+ d*x))]])/(6*(a^2 + b^2)^2*d^3*(1 + E^(2*c))) + (a^3*(6*e^2*E^(2*c)*x + 6*e
*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b
*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2
 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2
+ b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))
/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E
^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d)
 + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E^(2*
c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Log[1 + (b
*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*
Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*
x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*
E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)
])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*
c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 +
b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^
2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^
c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[
2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1
+ E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 +
b^2)*E^(2*c)]))])/d^2 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt
[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x
))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (6*f^2*PolyLog[3, -((b*E^(2
*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*Poly
Log[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3))/(3*
(a^2 + b^2)^2*(-1 + E^(2*c))) + (Csch[c]*Sech[c]*Sech[c + d*x]^2*(-6*a^3*e*
f - 6*a*b^2*e*f - 12*a^3*d^2*e^2*x - 6*a^3*f^2*x - 6*a*b^2*f^2*x - 12*a^3*d
^2*e*f*x^2 - 4*a^3*d^2*f^2*x^3 + 6*a^3*e*f*Cosh[2*c] + 6*a*b^2*e*f*Cosh[2*c
] + 6*a^3*f^2*x*Cosh[2*c] + 6*a*b^2*f^2*x*Cosh[2*c] + 6*a^3*e*f*Cosh[2*d*x]
 + 6*a*b^2*e*f*Cosh[2*d*x] + 6*a^3*f^2*x*Cosh[2*d*x] + 6*a*b^2*f^2*x*Cosh[2
*d*x] + 3*a^2*b*d*e^2*Cosh[c - d*x] + 3*b^3*d*e^2*Cosh[c - d*x] + 6*a^2*b*d
*e*f*x*Cosh[c - d*x] + 6*b^3*d*e*f*x*Cosh[c - d*x] + 3*a^2*b*d*f^2*x^2*Cosh

```
[c - d*x] + 3*b^3*d*f^2*x^2*Cosh[c - d*x] - 3*a^2*b*d*e^2*Cosh[3*c + d*x] -
 3*b^3*d*e^2*Cosh[3*c + d*x] - 6*a^2*b*d*e*f*x*Cosh[3*c + d*x] - 6*b^3*d*e*
f*x*Cosh[3*c + d*x] - 3*a^2*b*d*f^2*x^2*Cosh[3*c + d*x] - 3*b^3*d*f^2*x^2*C
osh[3*c + d*x] - 6*a^3*e*f*Cosh[2*c + 2*d*x] - 6*a*b^2*e*f*Cosh[2*c + 2*d*x
] - 12*a^3*d^2*e^2*x*Cosh[2*c + 2*d*x] - 6*a^3*f^2*x*Cosh[2*c + 2*d*x] - 6*
a*b^2*f^2*x*Cosh[2*c + 2*d*x] - 12*a^3*d^2*e*f*x^2*Cosh[2*c + 2*d*x] - 4*a^
3*d^2*f^2*x^3*Cosh[2*c + 2*d*x] + 6*a^3*d*e^2*Sinh[2*c] + 6*a*b^2*d*e^2*Sin
h[2*c] + 12*a^3*d*e*f*x*Sinh[2*c] + 12*a*b^2*d*e*f*x*Sinh[2*c] + 6*a^3*d*f^
2*x^2*Sinh[2*c] + 6*a*b^2*d*f^2*x^2*Sinh[2*c] - 6*a^2*b*e*f*Sinh[c - d*x] -
 6*b^3*e*f*Sinh[c - d*x] - 6*a^2*b*f^2*x*Sinh[c - d*x] - 6*b^3*f^2*x*Sinh[c
 - d*x] - 6*a^2*b*e*f*Sinh[3*c + d*x] - 6*b^3*e*f*Sinh[3*c + d*x] - 6*a^2*b
*f^2*x*Sinh[3*c + d*x] - 6*b^3*f^2*x*Sinh[3*c + d*x]))/(24*(a^2 + b^2)^2*d^
2)
```

Maple [F] time = 0.447, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 (\tanh (dx+c))^3}{a+b\sinh (dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 3*a^2*b*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*
b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2
 + b^4*d^2), x) + b^3*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x +
 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 +
 2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a^3*d^2*f^2*integrate(x^2/(a^4*d^2*e^(2*d
*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d
^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 6*a^2*b*d^2*e*f*integrate(x*e^(d*x + c)
/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*
x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*b^3*d^2*e*f*integrate
(x*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b
^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 4*a^3*d^2
*e*f*integrate(x/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) +
 b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - a^3*f^2
*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/((a^
4 + 2*a^2*b^2 + b^4)*d^3)) - a*b^2*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4
)*d^3) - log(e^(2*d*x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) - (a^3*log
(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b^4)*d) -
a^3*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) + (3*a^2*b + b^3)
*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (b*e^(-d*x - c) - 2*a*e
^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*e^(-2*d*x
 - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d))*e^2 + 2*a^2*b*f^2*arctan(e^(d*x
```

```
+ c))/((a^4 + 2*a^2*b^2 + b^4)*d^3) + 2*b^3*f^2*arctan(e^(d*x + c))/((a^4
+ 2*a^2*b^2 + b^4)*d^3) + (2*a*f^2*x + 2*a*e*f - (b*d*f^2*x^2*e^(3*c) + 2*b
*e*f*e^(3*c) + 2*(d*e*f + f^2)*b*x*e^(3*c))*e^(3*d*x) + 2*(a*d*f^2*x^2*e^(2
*c) + a*e*f*e^(2*c) + (2*d*e*f + f^2)*a*x*e^(2*c))*e^(2*d*x) + (b*d*f^2*x^2
*e^c - 2*b*e*f*e^c + 2*(d*e*f - f^2)*b*x*e^c)*e^(d*x))/(a^2*d^2 + b^2*d^2 +
 (a^2*d^2*e^(4*c) + b^2*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d
^2*e^(2*c))*e^(2*d*x)) + integrate(2*(a^3*b*f^2*x^2 + 2*a^3*b*e*f*x - (a^4*
f^2*x^2*e^c + 2*a^4*e*f*x*e^c)*e^(d*x))/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b*e
^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c + 2*a^3*b^
2*e^c + a*b^4*e^c)*e^(d*x)), x)
```

Fricas [C] time = 5.78727, size = 24035, normalized size = 16.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*cosh(d*x + c)^4 + 4*(
(a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*sinh(d*x + c)^4 - 4*(a^3 + a*b
^2)*d*e*f + 4*(a^3 + a*b^2)*c*f^2 + 2*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b +
 b^3)*d^2*e^2 + 2*(a^2*b + b^3)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + (a^2*b +
 b^3)*d*f^2)*x)*cosh(d*x + c)^3 + 2*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b
^3)*d^2*e^2 + 2*(a^2*b + b^3)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + (a^2*b + b
^3)*d*f^2)*x + 8*((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*cosh(d*x + c
))*sinh(d*x + c)^3 - 4*((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 + a*b^2)*d^2*e^2 +
 (a^3 + a*b^2)*d*e*f - 2*(a^3 + a*b^2)*c*f^2 + (2*(a^3 + a*b^2)*d^2*e*f - (
a^3 + a*b^2)*d*f^2)*x)*cosh(d*x + c)^2 - 2*(2*(a^3 + a*b^2)*d^2*f^2*x^2 + 2
*(a^3 + a*b^2)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f - 4*(a^3 + a*b^2)*c*f^2 - 12
*((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*cosh(d*x + c)^2 + 2*(2*(a^3
+ a*b^2)*d^2*e*f - (a^3 + a*b^2)*d*f^2)*x - 3*((a^2*b + b^3)*d^2*f^2*x^2 +
(a^2*b + b^3)*d^2*e^2 + 2*(a^2*b + b^3)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f +
(a^2*b + b^3)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*((a^2*b + b^3)*d
^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 - 2*(a^2*b + b^3)*d*e*f + 2*((a^2*b + b^
3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x)*cosh(d*x + c) + 4*(a^3*d*f^2*x + a^3*d
*e*f + (a^3*d*f^2*x + a^3*d*e*f)*cosh(d*x + c)^4 + 4*(a^3*d*f^2*x + a^3*d*e
*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*d*f^2*x + a^3*d*e*f)*sinh(d*x + c)
^4 + 2*(a^3*d*f^2*x + a^3*d*e*f)*cosh(d*x + c)^2 + 2*(a^3*d*f^2*x + a^3*d*e
*f + 3*(a^3*d*f^2*x + a^3*d*e*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3
*d*f^2*x + a^3*d*e*f)*cosh(d*x + c)^3 + (a^3*d*f^2*x + a^3*d*e*f)*cosh(d*x
+ c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 4*(a^3*d*f^2*x
 + a^3*d*e*f + (a^3*d*f^2*x + a^3*d*e*f)*cosh(d*x + c)^4 + 4*(a^3*d*f^2*x +
 a^3*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*d*f^2*x + a^3*d*e*f)*sinh(
d*x + c)^4 + 2*(a^3*d*f^2*x + a^3*d*e*f)*cosh(d*x + c)^2 + 2*(a^3*d*f^2*x +
 a^3*d*e*f + 3*(a^3*d*f^2*x + a^3*d*e*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
 4*((a^3*d*f^2*x + a^3*d*e*f)*cosh(d*x + c)^3 + (a^3*d*f^2*x + a^3*d*e*f)*c
osh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (4*a^3
*d*f^2*x + 4*a^3*d*e*f + 2*I*(3*a^2*b + b^3)*d*f^2*x + (4*a^3*d*f^2*x + 4*a
^3*d*e*f + 2*I*(3*a^2*b + b^3)*d*f^2*x + 2*I*(3*a^2*b + b^3)*d*e*f)*cosh(d*
x + c)^4 + (16*a^3*d*f^2*x + 16*a^3*d*e*f + 8*I*(3*a^2*b + b^3)*d*f^2*x + 8
*I*(3*a^2*b + b^3)*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (4*a^3*d*f^2*x +
4*a^3*d*e*f + 2*I*(3*a^2*b + b^3)*d*f^2*x + 2*I*(3*a^2*b + b^3)*d*e*f)*sinh
(d*x + c)^4 + 2*I*(3*a^2*b + b^3)*d*e*f + (8*a^3*d*f^2*x + 8*a^3*d*e*f + 4*
I*(3*a^2*b + b^3)*d*f^2*x + 4*I*(3*a^2*b + b^3)*d*e*f)*cosh(d*x + c)^2 + (8
```

*a^3*d*f^2*x + 8*a^3*d*e*f + 4*I*(3*a^2*b + b^3)*d*f^2*x + 4*I*(3*a^2*b + b
^3)*d*e*f + (24*a^3*d*f^2*x + 24*a^3*d*e*f + 12*I*(3*a^2*b + b^3)*d*f^2*x +
 12*I*(3*a^2*b + b^3)*d*e*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((16*a^3*d*
f^2*x + 16*a^3*d*e*f + 8*I*(3*a^2*b + b^3)*d*f^2*x + 8*I*(3*a^2*b + b^3)*d*
e*f)*cosh(d*x + c)^3 + (16*a^3*d*f^2*x + 16*a^3*d*e*f + 8*I*(3*a^2*b + b^3)
*d*f^2*x + 8*I*(3*a^2*b + b^3)*d*e*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(I
*cosh(d*x + c) + I*sinh(d*x + c)) - (4*a^3*d*f^2*x + 4*a^3*d*e*f - 2*I*(3*a
^2*b + b^3)*d*f^2*x + (4*a^3*d*f^2*x + 4*a^3*d*e*f - 2*I*(3*a^2*b + b^3)*d*
f^2*x - 2*I*(3*a^2*b + b^3)*d*e*f)*cosh(d*x + c)^4 + (16*a^3*d*f^2*x + 16*a
^3*d*e*f - 8*I*(3*a^2*b + b^3)*d*f^2*x - 8*I*(3*a^2*b + b^3)*d*e*f)*cosh(d*
x + c)*sinh(d*x + c)^3 + (4*a^3*d*f^2*x + 4*a^3*d*e*f - 2*I*(3*a^2*b + b^3)
*d*f^2*x - 2*I*(3*a^2*b + b^3)*d*e*f)*sinh(d*x + c)^4 - 2*I*(3*a^2*b + b^3)
*d*e*f + (8*a^3*d*f^2*x + 8*a^3*d*e*f - 4*I*(3*a^2*b + b^3)*d*f^2*x - 4*I*(
3*a^2*b + b^3)*d*e*f)*cosh(d*x + c)^2 + (8*a^3*d*f^2*x + 8*a^3*d*e*f - 4*I*
(3*a^2*b + b^3)*d*f^2*x - 4*I*(3*a^2*b + b^3)*d*e*f + (24*a^3*d*f^2*x + 24*
a^3*d*e*f - 12*I*(3*a^2*b + b^3)*d*f^2*x - 12*I*(3*a^2*b + b^3)*d*e*f)*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + ((16*a^3*d*f^2*x + 16*a^3*d*e*f - 8*I*(3*a^2
*b + b^3)*d*f^2*x - 8*I*(3*a^2*b + b^3)*d*e*f)*cosh(d*x + c)^3 + (16*a^3*d*
f^2*x + 16*a^3*d*e*f - 8*I*(3*a^2*b + b^3)*d*f^2*x - 8*I*(3*a^2*b + b^3)*d*
e*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)
) + 2*(a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2 + (a^3*d^2*e^2 - 2*a^3*c*d
*e*f + a^3*c^2*f^2)*cosh(d*x + c)^4 + 4*(a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*
c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3
*c^2*f^2)*sinh(d*x + c)^4 + 2*(a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)*c
osh(d*x + c)^2 + 2*(a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2 + 3*(a^3*d^2*
e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a
^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)*cosh(d*x + c)^3 + (a^3*d^2*e^2 -
2*a^3*c*d*e*f + a^3*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x
 + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(a^3*d^2*e
^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2 + (a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f
^2)*cosh(d*x + c)^4 + 4*(a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)*cosh(d*
x + c)*sinh(d*x + c)^3 + (a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)*sinh(d
x + c)^4 + 2(a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)*cosh(d*x + c)^2 +
 2*(a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2 + 3*(a^3*d^2*e^2 - 2*a^3*c*d*
e*f + a^3*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3*d^2*e^2 - 2*a
^3*c*d*e*f + a^3*c^2*f^2)*cosh(d*x + c)^3 + (a^3*d^2*e^2 - 2*a^3*c*d*e*f +
a^3*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh
(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(a^3*d^2*f^2*x^2 + 2*a^3*d
^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2 + (a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x
 + 2*a^3*c*d*e*f - a^3*c^2*f^2)*cosh(d*x + c)^4 + 4*(a^3*d^2*f^2*x^2 + 2*a^
3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 +
(a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*sinh(d*x
+ c)^4 + 2*(a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2
)*cosh(d*x + c)^2 + 2*(a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f -
a^3*c^2*f^2 + 3*(a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^
2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3*d^2*f^2*x^2 + 2*a^3*d^2*e
*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*cosh(d*x + c)^3 + (a^3*d^2*f^2*x^2 + 2*
a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*
log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2
*a^3*c*d*e*f - a^3*c^2*f^2 + (a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d
*e*f - a^3*c^2*f^2)*cosh(d*x + c)^4 + 4*(a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x
+ 2*a^3*c*d*e*f - a^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*d^2*f^2
*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*sinh(d*x + c)^4 + 2*(
a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*cosh(d*x +
 c)^2 + 2*(a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2
+ 3*(a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*cosh(
d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3
*c*d*e*f - a^3*c^2*f^2)*cosh(d*x + c)^3 + (a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*

```
x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(-(a*cosh
(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b) - (2*a^3*d^2*e^2 - 4*a^3*c*d*e*f + I*(3*a^2*b + b^3)*d
^2*e^2 - 2*I*(3*a^2*b + b^3)*c*d*e*f + (2*a^3*d^2*e^2 - 4*a^3*c*d*e*f + I*(
3*a^2*b + b^3)*d^2*e^2 - 2*I*(3*a^2*b + b^3)*c*d*e*f + 2*(a^3*c^2 + a^3 + a
*b^2)*f^2 + I*(2*a^2*b + 2*b^3 + (3*a^2*b + b^3)*c^2)*f^2)*cosh(d*x + c)^4
+ (8*a^3*d^2*e^2 - 16*a^3*c*d*e*f + 4*I*(3*a^2*b + b^3)*d^2*e^2 - 8*I*(3*a^
2*b + b^3)*c*d*e*f + 8*(a^3*c^2 + a^3 + a*b^2)*f^2 + 4*I*(2*a^2*b + 2*b^3 +
 (3*a^2*b + b^3)*c^2)*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^3*d^2*e^2 -
 4*a^3*c*d*e*f + I*(3*a^2*b + b^3)*d^2*e^2 - 2*I*(3*a^2*b + b^3)*c*d*e*f +
2*(a^3*c^2 + a^3 + a*b^2)*f^2 + I*(2*a^2*b + 2*b^3 + (3*a^2*b + b^3)*c^2)*f
^2)*sinh(d*x + c)^4 + 2*(a^3*c^2 + a^3 + a*b^2)*f^2 + I*(2*a^2*b + 2*b^3 +
(3*a^2*b + b^3)*c^2)*f^2 + (4*a^3*d^2*e^2 - 8*a^3*c*d*e*f + 2*I*(3*a^2*b +
b^3)*d^2*e^2 - 4*I*(3*a^2*b + b^3)*c*d*e*f + 4*(a^3*c^2 + a^3 + a*b^2)*f^2
+ 2*I*(2*a^2*b + 2*b^3 + (3*a^2*b + b^3)*c^2)*f^2)*cosh(d*x + c)^2 + (4*a^3
*d^2*e^2 - 8*a^3*c*d*e*f + 2*I*(3*a^2*b + b^3)*d^2*e^2 - 4*I*(3*a^2*b + b^3
)*c*d*e*f + 4*(a^3*c^2 + a^3 + a*b^2)*f^2 + 2*I*(2*a^2*b + 2*b^3 + (3*a^2*b
 + b^3)*c^2)*f^2 + (12*a^3*d^2*e^2 - 24*a^3*c*d*e*f + 6*I*(3*a^2*b + b^3)*d
^2*e^2 - 12*I*(3*a^2*b + b^3)*c*d*e*f + 12*(a^3*c^2 + a^3 + a*b^2)*f^2 + 6*
I*(2*a^2*b + 2*b^3 + (3*a^2*b + b^3)*c^2)*f^2)*cosh(d*x + c)^2)*sinh(d*x +
c)^2 + ((8*a^3*d^2*e^2 - 16*a^3*c*d*e*f + 4*I*(3*a^2*b + b^3)*d^2*e^2 - 8*I
*(3*a^2*b + b^3)*c*d*e*f + 8*(a^3*c^2 + a^3 + a*b^2)*f^2 + 4*I*(2*a^2*b + 2
*b^3 + (3*a^2*b + b^3)*c^2)*f^2)*cosh(d*x + c)^3 + (8*a^3*d^2*e^2 - 16*a^3*
c*d*e*f + 4*I*(3*a^2*b + b^3)*d^2*e^2 - 8*I*(3*a^2*b + b^3)*c*d*e*f + 8*(a^
3*c^2 + a^3 + a*b^2)*f^2 + 4*I*(2*a^2*b + 2*b^3 + (3*a^2*b + b^3)*c^2)*f^2)
*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + I) - (2*
a^3*d^2*e^2 - 4*a^3*c*d*e*f - I*(3*a^2*b + b^3)*d^2*e^2 + 2*I*(3*a^2*b + b^
3)*c*d*e*f + (2*a^3*d^2*e^2 - 4*a^3*c*d*e*f - I*(3*a^2*b + b^3)*d^2*e^2 + 2
*I*(3*a^2*b + b^3)*c*d*e*f + 2*(a^3*c^2 + a^3 + a*b^2)*f^2 - I*(2*a^2*b + 2
*b^3 + (3*a^2*b + b^3)*c^2)*f^2)*cosh(d*x + c)^4 + (8*a^3*d^2*e^2 - 16*a^3*
c*d*e*f - 4*I*(3*a^2*b + b^3)*d^2*e^2 + 8*I*(3*a^2*b + b^3)*c*d*e*f + 8*(a^
3*c^2 + a^3 + a*b^2)*f^2 - 4*I*(2*a^2*b + 2*b^3 + (3*a^2*b + b^3)*c^2)*f^2)
*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^3*d^2*e^2 - 4*a^3*c*d*e*f - I*(3*a^2*
b + b^3)*d^2*e^2 + 2*I*(3*a^2*b + b^3)*c*d*e*f + 2*(a^3*c^2 + a^3 + a*b^2)*
f^2 - I*(2*a^2*b + 2*b^3 + (3*a^2*b + b^3)*c^2)*f^2)*sinh(d*x + c)^4 + 2*(a
^3*c^2 + a^3 + a*b^2)*f^2 - I*(2*a^2*b + 2*b^3 + (3*a^2*b + b^3)*c^2)*f^2 +
 (4*a^3*d^2*e^2 - 8*a^3*c*d*e*f - 2*I*(3*a^2*b + b^3)*d^2*e^2 + 4*I*(3*a^2*
b + b^3)*c*d*e*f + 4*(a^3*c^2 + a^3 + a*b^2)*f^2 - 2*I*(2*a^2*b + 2*b^3 + (
3*a^2*b + b^3)*c^2)*f^2)*cosh(d*x + c)^2 + (4*a^3*d^2*e^2 - 8*a^3*c*d*e*f -
 2*I*(3*a^2*b + b^3)*d^2*e^2 + 4*I*(3*a^2*b + b^3)*c*d*e*f + 4*(a^3*c^2 + a
^3 + a*b^2)*f^2 - 2*I*(2*a^2*b + 2*b^3 + (3*a^2*b + b^3)*c^2)*f^2 + (12*a^3
*d^2*e^2 - 24*a^3*c*d*e*f - 6*I*(3*a^2*b + b^3)*d^2*e^2 + 12*I*(3*a^2*b + b
^3)*c*d*e*f + 12*(a^3*c^2 + a^3 + a*b^2)*f^2 - 6*I*(2*a^2*b + 2*b^3 + (3*a^
2*b + b^3)*c^2)*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a^3*d^2*e^2 - 1
6*a^3*c*d*e*f - 4*I*(3*a^2*b + b^3)*d^2*e^2 + 8*I*(3*a^2*b + b^3)*c*d*e*f +
 8*(a^3*c^2 + a^3 + a*b^2)*f^2 - 4*I*(2*a^2*b + 2*b^3 + (3*a^2*b + b^3)*c^2
)*f^2)*cosh(d*x + c)^3 + (8*a^3*d^2*e^2 - 16*a^3*c*d*e*f - 4*I*(3*a^2*b + b
^3)*d^2*e^2 + 8*I*(3*a^2*b + b^3)*c*d*e*f + 8*(a^3*c^2 + a^3 + a*b^2)*f^2 -
 4*I*(2*a^2*b + 2*b^3 + (3*a^2*b + b^3)*c^2)*f^2)*cosh(d*x + c))*sinh(d*x +
 c))*log(cosh(d*x + c) + sinh(d*x + c) - I) - (2*a^3*d^2*f^2*x^2 + 4*a^3*d^
2*e*f*x + 4*a^3*c*d*e*f - 2*a^3*c^2*f^2 - I*(3*a^2*b + b^3)*d^2*f^2*x^2 - 2
*I*(3*a^2*b + b^3)*d^2*e*f*x - 2*I*(3*a^2*b + b^3)*c*d*e*f + I*(3*a^2*b + b
^3)*c^2*f^2 + (2*a^3*d^2*f^2*x^2 + 4*a^3*d^2*e*f*x + 4*a^3*c*d*e*f - 2*a^3*
c^2*f^2 - I*(3*a^2*b + b^3)*d^2*f^2*x^2 - 2*I*(3*a^2*b + b^3)*d^2*e*f*x - 2
*I*(3*a^2*b + b^3)*c*d*e*f + I*(3*a^2*b + b^3)*c^2*f^2)*cosh(d*x + c)^4 + (
8*a^3*d^2*f^2*x^2 + 16*a^3*d^2*e*f*x + 16*a^3*c*d*e*f - 8*a^3*c^2*f^2 - 4*I
*(3*a^2*b + b^3)*d^2*f^2*x^2 - 8*I*(3*a^2*b + b^3)*d^2*e*f*x - 8*I*(3*a^2*b
 + b^3)*c*d*e*f + 4*I*(3*a^2*b + b^3)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^
3 + (2*a^3*d^2*f^2*x^2 + 4*a^3*d^2*e*f*x + 4*a^3*c*d*e*f - 2*a^3*c^2*f^2 -
```

```
I*(3*a^2*b + b^3)*d^2*f^2*x^2 - 2*I*(3*a^2*b + b^3)*d^2*e*f*x - 2*I*(3*a^2*
b + b^3)*c*d*e*f + I*(3*a^2*b + b^3)*c^2*f^2)*sinh(d*x + c)^4 + (4*a^3*d^2*
f^2*x^2 + 8*a^3*d^2*e*f*x + 8*a^3*c*d*e*f - 4*a^3*c^2*f^2 - 2*I*(3*a^2*b +
b^3)*d^2*f^2*x^2 - 4*I*(3*a^2*b + b^3)*d^2*e*f*x - 4*I*(3*a^2*b + b^3)*c*d*
e*f + 2*I*(3*a^2*b + b^3)*c^2*f^2)*cosh(d*x + c)^2 + (4*a^3*d^2*f^2*x^2 + 8
*a^3*d^2*e*f*x + 8*a^3*c*d*e*f - 4*a^3*c^2*f^2 - 2*I*(3*a^2*b + b^3)*d^2*f^
2*x^2 - 4*I*(3*a^2*b + b^3)*d^2*e*f*x - 4*I*(3*a^2*b + b^3)*c*d*e*f + 2*I*(
3*a^2*b + b^3)*c^2*f^2 + (12*a^3*d^2*f^2*x^2 + 24*a^3*d^2*e*f*x + 24*a^3*c*
d*e*f - 12*a^3*c^2*f^2 - 6*I*(3*a^2*b + b^3)*d^2*f^2*x^2 - 12*I*(3*a^2*b +
b^3)*d^2*e*f*x - 12*I*(3*a^2*b + b^3)*c*d*e*f + 6*I*(3*a^2*b + b^3)*c^2*f^2
)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a^3*d^2*f^2*x^2 + 16*a^3*d^2*e*f*x
 + 16*a^3*c*d*e*f - 8*a^3*c^2*f^2 - 4*I*(3*a^2*b + b^3)*d^2*f^2*x^2 - 8*I*(
3*a^2*b + b^3)*d^2*e*f*x - 8*I*(3*a^2*b + b^3)*c*d*e*f + 4*I*(3*a^2*b + b^3
)*c^2*f^2)*cosh(d*x + c)^3 + (8*a^3*d^2*f^2*x^2 + 16*a^3*d^2*e*f*x + 16*a^3
*c*d*e*f - 8*a^3*c^2*f^2 - 4*I*(3*a^2*b + b^3)*d^2*f^2*x^2 - 8*I*(3*a^2*b +
 b^3)*d^2*e*f*x - 8*I*(3*a^2*b + b^3)*c*d*e*f + 4*I*(3*a^2*b + b^3)*c^2*f^2
)*cosh(d*x + c))*sinh(d*x + c))*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1)
- (2*a^3*d^2*f^2*x^2 + 4*a^3*d^2*e*f*x + 4*a^3*c*d*e*f - 2*a^3*c^2*f^2 + I*
(3*a^2*b + b^3)*d^2*f^2*x^2 + 2*I*(3*a^2*b + b^3)*d^2*e*f*x + 2*I*(3*a^2*b
+ b^3)*c*d*e*f - I*(3*a^2*b + b^3)*c^2*f^2 + (2*a^3*d^2*f^2*x^2 + 4*a^3*d^2
*e*f*x + 4*a^3*c*d*e*f - 2*a^3*c^2*f^2 + I*(3*a^2*b + b^3)*d^2*f^2*x^2 + 2*
I*(3*a^2*b + b^3)*d^2*e*f*x + 2*I*(3*a^2*b + b^3)*c*d*e*f - I*(3*a^2*b + b^
3)*c^2*f^2)*cosh(d*x + c)^4 + (8*a^3*d^2*f^2*x^2 + 16*a^3*d^2*e*f*x + 16*a^
3*c*d*e*f - 8*a^3*c^2*f^2 + 4*I*(3*a^2*b + b^3)*d^2*f^2*x^2 + 8*I*(3*a^2*b
+ b^3)*d^2*e*f*x + 8*I*(3*a^2*b + b^3)*c*d*e*f - 4*I*(3*a^2*b + b^3)*c^2*f^
2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^3*d^2*f^2*x^2 + 4*a^3*d^2*e*f*x + 4
*a^3*c*d*e*f - 2*a^3*c^2*f^2 + I*(3*a^2*b + b^3)*d^2*f^2*x^2 + 2*I*(3*a^2*b
 + b^3)*d^2*e*f*x + 2*I*(3*a^2*b + b^3)*c*d*e*f - I*(3*a^2*b + b^3)*c^2*f^2
)*sinh(d*x + c)^4 + (4*a^3*d^2*f^2*x^2 + 8*a^3*d^2*e*f*x + 8*a^3*c*d*e*f -
4*a^3*c^2*f^2 + 2*I*(3*a^2*b + b^3)*d^2*f^2*x^2 + 4*I*(3*a^2*b + b^3)*d^2*e
*f*x + 4*I*(3*a^2*b + b^3)*c*d*e*f - 2*I*(3*a^2*b + b^3)*c^2*f^2)*cosh(d*x
+ c)^2 + (4*a^3*d^2*f^2*x^2 + 8*a^3*d^2*e*f*x + 8*a^3*c*d*e*f - 4*a^3*c^2*f
^2 + 2*I*(3*a^2*b + b^3)*d^2*f^2*x^2 + 4*I*(3*a^2*b + b^3)*d^2*e*f*x + 4*I*
(3*a^2*b + b^3)*c*d*e*f - 2*I*(3*a^2*b + b^3)*c^2*f^2 + (12*a^3*d^2*f^2*x^2
 + 24*a^3*d^2*e*f*x + 24*a^3*c*d*e*f - 12*a^3*c^2*f^2 + 6*I*(3*a^2*b + b^3)
*d^2*f^2*x^2 + 12*I*(3*a^2*b + b^3)*d^2*e*f*x + 12*I*(3*a^2*b + b^3)*c*d*e*
f - 6*I*(3*a^2*b + b^3)*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a^3
*d^2*f^2*x^2 + 16*a^3*d^2*e*f*x + 16*a^3*c*d*e*f - 8*a^3*c^2*f^2 + 4*I*(3*a
^2*b + b^3)*d^2*f^2*x^2 + 8*I*(3*a^2*b + b^3)*d^2*e*f*x + 8*I*(3*a^2*b + b^
3)*c*d*e*f - 4*I*(3*a^2*b + b^3)*c^2*f^2)*cosh(d*x + c)^3 + (8*a^3*d^2*f^2*
x^2 + 16*a^3*d^2*e*f*x + 16*a^3*c*d*e*f - 8*a^3*c^2*f^2 + 4*I*(3*a^2*b + b^
3)*d^2*f^2*x^2 + 8*I*(3*a^2*b + b^3)*d^2*e*f*x + 8*I*(3*a^2*b + b^3)*c*d*e*
f - 4*I*(3*a^2*b + b^3)*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(-I*cosh(
d*x + c) - I*sinh(d*x + c) + 1) - 4*(a^3*f^2*cosh(d*x + c)^4 + 4*a^3*f^2*co
sh(d*x + c)*sinh(d*x + c)^3 + a^3*f^2*sinh(d*x + c)^4 + 2*a^3*f^2*cosh(d*x
+ c)^2 + a^3*f^2 + 2*(3*a^3*f^2*cosh(d*x + c)^2 + a^3*f^2)*sinh(d*x + c)^2
+ 4*(a^3*f^2*cosh(d*x + c)^3 + a^3*f^2*cosh(d*x + c))*sinh(d*x + c))*polylo
g(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2))/b) - 4*(a^3*f^2*cosh(d*x + c)^4 + 4*a^3*f^2*cosh(
d*x + c)*sinh(d*x + c)^3 + a^3*f^2*sinh(d*x + c)^4 + 2*a^3*f^2*cosh(d*x + c
)^2 + a^3*f^2 + 2*(3*a^3*f^2*cosh(d*x + c)^2 + a^3*f^2)*sinh(d*x + c)^2 + 4
*(a^3*f^2*cosh(d*x + c)^3 + a^3*f^2*cosh(d*x + c))*sinh(d*x + c))*polylog(3
, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2))/b) + (4*a^3*f^2 + (4*a^3*f^2 + 2*I*(3*a^2*b + b^3)*f
^2)*cosh(d*x + c)^4 + (16*a^3*f^2 + 8*I*(3*a^2*b + b^3)*f^2)*cosh(d*x + c)*
sinh(d*x + c)^3 + (4*a^3*f^2 + 2*I*(3*a^2*b + b^3)*f^2)*sinh(d*x + c)^4 + 2
*I*(3*a^2*b + b^3)*f^2 + (8*a^3*f^2 + 4*I*(3*a^2*b + b^3)*f^2)*cosh(d*x + c
)^2 + (8*a^3*f^2 + 4*I*(3*a^2*b + b^3)*f^2 + (24*a^3*f^2 + 12*I*(3*a^2*b +
b^3)*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((16*a^3*f^2 + 8*I*(3*a^2*b +
```

```
b^3)*f^2)*cosh(d*x + c)^3 + (16*a^3*f^2 + 8*I*(3*a^2*b + b^3)*f^2)*cosh(d*x
 + c))*sinh(d*x + c))*polylog(3, I*cosh(d*x + c) + I*sinh(d*x + c)) + (4*a^
3*f^2 + (4*a^3*f^2 - 2*I*(3*a^2*b + b^3)*f^2)*cosh(d*x + c)^4 + (16*a^3*f^2
 - 8*I*(3*a^2*b + b^3)*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (4*a^3*f^2 - 2*
I*(3*a^2*b + b^3)*f^2)*sinh(d*x + c)^4 - 2*I*(3*a^2*b + b^3)*f^2 + (8*a^3*f
^2 - 4*I*(3*a^2*b + b^3)*f^2)*cosh(d*x + c)^2 + (8*a^3*f^2 - 4*I*(3*a^2*b +
 b^3)*f^2 + (24*a^3*f^2 - 12*I*(3*a^2*b + b^3)*f^2)*cosh(d*x + c)^2)*sinh(d
*x + c)^2 + ((16*a^3*f^2 - 8*I*(3*a^2*b + b^3)*f^2)*cosh(d*x + c)^3 + (16*a
^3*f^2 - 8*I*(3*a^2*b + b^3)*f^2)*cosh(d*x + c))*sinh(d*x + c))*polylog(3,
-I*cosh(d*x + c) - I*sinh(d*x + c)) - 2*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b
 + b^3)*d^2*e^2 - 2*(a^2*b + b^3)*d*e*f - 8*((a^3 + a*b^2)*d*f^2*x + (a^3 +
 a*b^2)*c*f^2)*cosh(d*x + c)^3 - 3*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^
3)*d^2*e^2 + 2*(a^2*b + b^3)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + (a^2*b + b^
3)*d*f^2)*x)*cosh(d*x + c)^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^2*b + b^3)*d*f
^2)*x + 4*((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 + a*b^2)*d^2*e^2 + (a^3 + a*b^2
)*d*e*f - 2*(a^3 + a*b^2)*c*f^2 + (2*(a^3 + a*b^2)*d^2*e*f - (a^3 + a*b^2)*
d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d
*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c)*sinh(d*x + c)^3 + (
a^4 + 2*a^2*b^2 + b^4)*d^3*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*
cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^3 + 2*(3*(a^4 + 2*a^2*b^2 + b^4
)*d^3*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^3)*sinh(d*x + c)^2 + 4*((
a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d^3*co
sh(d*x + c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^2 \tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*tanh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*tanh(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.417 $\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=894

result too large to display

```
[Out] (a^2*(e + f*x)*ArcTan[E^(c + d*x)])/(b^3*d) + ((e + f*x)*ArcTan[E^(c + d*x)
])/(b*d) - (2*a^4*(e + f*x)*ArcTan[E^(c + d*x)])/(b*(a^2 + b^2)^2*d) - (a^4
*(e + f*x)*ArcTan[E^(c + d*x)])/(b^3*(a^2 + b^2)*d) - (a^3*(e + f*x)*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) - (a^3*(e + f*x
)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) + (a^3*
(e + f*x)*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)^2*d) - ((I/2)*a^2*f*PolyLo
g[2, (-I)*E^(c + d*x)])/(b^3*d^2) - ((I/2)*f*PolyLog[2, (-I)*E^(c + d*x)])/
(b*d^2) + (I*a^4*f*PolyLog[2, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^2) + ((
I/2)*a^4*f*PolyLog[2, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) + ((I/2)*a^2
*f*PolyLog[2, I*E^(c + d*x)])/(b^3*d^2) + ((I/2)*f*PolyLog[2, I*E^(c + d*x)
])/(b*d^2) - (I*a^4*f*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^2) - ((
I/2)*a^4*f*PolyLog[2, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) - (a^3*f*PolyLo
g[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) - (a^3*
f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2)
 + (a^3*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^2) + (a^2*f*Sech
[c + d*x])/(2*b^3*d^2) - (f*Sech[c + d*x])/(2*b*d^2) - (a^4*f*Sech[c + d*x]
)/(2*b^3*(a^2 + b^2)*d^2) + (a*(e + f*x)*Sech[c + d*x]^2)/(2*b^2*d) - (a^3*
(e + f*x)*Sech[c + d*x]^2)/(2*b^2*(a^2 + b^2)*d) - (a*f*Tanh[c + d*x])/(2*b
^2*d^2) + (a^3*f*Tanh[c + d*x])/(2*b^2*(a^2 + b^2)*d^2) + (a^2*(e + f*x)*Se
ch[c + d*x]*Tanh[c + d*x])/(2*b^3*d) - ((e + f*x)*Sech[c + d*x]*Tanh[c + d*
x])/(2*b*d) - (a^4*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*b^3*(a^2 + b^2
)*d)
```

Rubi [A] time = 1.41524, antiderivative size = 894, normalized size of antiderivative = 1., number of steps used = 55, number of rules used = 15, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {5567, 5455, 4180, 2279, 2391, 4185, 5583, 5451, 3767, 8, 5573, 5561, 2190, 6742, 3718}

$$-\frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)a^4}{b^3\left(a^2+b^2\right)d}-\frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)a^4}{b\left(a^2+b^2\right)^2d}+\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)a^4}{2b^3\left(a^2+b^2\right)d^2}+\frac{if\text{PolyLog}\left(2,-ie^{c+dx}\right)a^4}{b\left(a^2+b^2\right)^2d^2}-\frac{if\text{Po}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Tanh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*(e + f*x)*ArcTan[E^(c + d*x)])/(b^3*d) + ((e + f*x)*ArcTan[E^(c + d*x)
])/(b*d) - (2*a^4*(e + f*x)*ArcTan[E^(c + d*x)])/(b*(a^2 + b^2)^2*d) - (a^4
*(e + f*x)*ArcTan[E^(c + d*x)])/(b^3*(a^2 + b^2)*d) - (a^3*(e + f*x)*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) - (a^3*(e + f*x
)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) + (a^3*
(e + f*x)*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)^2*d) - ((I/2)*a^2*f*PolyLo
g[2, (-I)*E^(c + d*x)])/(b^3*d^2) - ((I/2)*f*PolyLog[2, (-I)*E^(c + d*x)])/
(b*d^2) + (I*a^4*f*PolyLog[2, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^2) + ((
I/2)*a^4*f*PolyLog[2, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) + ((I/2)*a^2
*f*PolyLog[2, I*E^(c + d*x)])/(b^3*d^2) + ((I/2)*f*PolyLog[2, I*E^(c + d*x)
])/(b*d^2) - (I*a^4*f*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^2) - ((
I/2)*a^4*f*PolyLog[2, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) - (a^3*f*PolyLo
g[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) - (a^3*
f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2)
```

```
 + (a^3*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^2) + (a^2*f*Sech
[c + d*x])/(2*b^3*d^2) - (f*Sech[c + d*x])/(2*b*d^2) - (a^4*f*Sech[c + d*x]
)/(2*b^3*(a^2 + b^2)*d^2) + (a*(e + f*x)*Sech[c + d*x]^2)/(2*b^2*d) - (a^3*
(e + f*x)*Sech[c + d*x]^2)/(2*b^2*(a^2 + b^2)*d) - (a*f*Tanh[c + d*x])/(2*b
^2*d^2) + (a^3*f*Tanh[c + d*x])/(2*b^2*(a^2 + b^2)*d^2) + (a^2*(e + f*x)*Se
ch[c + d*x]*Tanh[c + d*x])/(2*b^3*d) - ((e + f*x)*Sech[c + d*x]*Tanh[c + d*
x])/(2*b*d) - (a^4*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*b^3*(a^2 + b^2
)*d)
```

Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Sech[c
 + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c +
 d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5455

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]*Tanh[(a_.) + (b_.)*
(x_)]^(p_), x_Symbol] :> Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5583

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
 (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
```

```
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\mathrm{sech}(c+dx)\tanh^2(c+dx)\,dx}{b} - \frac{a\int \frac{(e+fx)\mathrm{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{b}$$

$$= -\frac{a\int (e+fx)\mathrm{sech}^2(c+dx)\tanh(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{(e+fx)\mathrm{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)}\,dx}{b^2} + \int(e$$

$$= \frac{2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{f\mathrm{sech}(c+dx)}{2bd^2} + \frac{a(e+fx)\mathrm{sech}^2(c+dx)}{2b^2d} - (e+fx)\mathrm{sech}(c$$

$$= \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} + \frac{a^2f\mathrm{sech}(c+dx)}{2b^3d^2} - \frac{f\mathrm{sech}(c+dx)}{2bd^2} + \frac{a(e+fx)\mathrm{sech}^2(c+dx)}{2b^2d}$$

$$= \frac{a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{if\mathrm{Li}_2\left(-ie^{c+dx}\right)}{bd^2} + \frac{if\mathrm{Li}_2\left(ie^{c+dx}\right)}{bd^2} +$$

$$= \frac{a^3(e+fx)^2}{2\left(a^2+b^2\right)^2 f} + \frac{a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{if\mathrm{Li}_2\left(-ie^{c+dx}\right)}{2bd^2} -$$

$$= \frac{a^3(e+fx)^2}{2\left(a^2+b^2\right)^2 f} + \frac{a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{a^3(e+fx)\log\left(1\right.}{\left(a^2+b^2\right.}$$

$$= \frac{a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)^2 d} - \frac{a^4(e}{}$$

$$= \frac{a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)^2 d} - \frac{a^4(e}{}$$

$$= \frac{a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)^2 d} - \frac{a^4(e}{}$$

$$= \frac{a^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b^3d} + \frac{(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{bd} - \frac{2a^4(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{b\left(a^2+b^2\right)^2 d} - \frac{a^4(e}{}$$

Mathematica [A] time = 7.86361, size = 588, normalized size = 0.66

$$\frac{-2a^3f\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) - 2a^3f\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) - ibf\left(3a^2+b^2\right)\mathrm{PolyLog}\left(2,-ie^{c+dx}\right) + ibf\left(3a^2+b^2\right)\mathrm{P}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Tanh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*a^3*d*e*(c + d*x) + 2*a^3*c*f*(c + d*x) + 6*a^2*b*d*e*ArcTan[E^(c + d*x
)] + 2*b^3*d*e*ArcTan[E^(c + d*x)] - 6*a^2*b*c*f*ArcTan[E^(c + d*x)] - 2*b^
3*c*f*ArcTan[E^(c + d*x)] + (3*I)*a^2*b*f*(c + d*x)*Log[1 - I*E^(c + d*x)]
+ I*b^3*f*(c + d*x)*Log[1 - I*E^(c + d*x)] - (3*I)*a^2*b*f*(c + d*x)*Log[1
+ I*E^(c + d*x)] - I*b^3*f*(c + d*x)*Log[1 + I*E^(c + d*x)] - 2*a^3*f*(c +
d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*a^3*f*(c + d*x)*Log
[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*a^3*d*e*Log[1 + E^(2*(c + d
*x))] - 2*a^3*c*f*Log[1 + E^(2*(c + d*x))] + 2*a^3*f*(c + d*x)*Log[1 + E^(2
*(c + d*x))] - 2*a^3*d*e*Log[a + b*Sinh[c + d*x]] + 2*a^3*c*f*Log[a + b*Sin
```

```
h[c + d*x]] - I*b*(3*a^2 + b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] + I*b*(3*a^2
 + b^2)*f*PolyLog[2, I*E^(c + d*x)] - 2*a^3*f*PolyLog[2, (b*E^(c + d*x))/(-
a + Sqrt[a^2 + b^2])] - 2*a^3*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))] + a^3*f*PolyLog[2, -E^(2*(c + d*x))] - (a^2 + b^2)*f*Sech[c + d*x
]*(b + a*Sinh[c + d*x]) + (a^2 + b^2)*d*(e + f*x)*Sech[c + d*x]^2*(a - b*Si
nh[c + d*x]))/(2*(a^2 + b^2)^2*d^2)
```

Maple [B] time = 0.202, size = 2284, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] 3/d^2/(a^2+b^2)^(3/2)*b^2*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a
)/(a^2+b^2)^(1/2))*a^2+2/d*e/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b
*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2-2/(a^2+b^2)^(3/2)/d*a^4*e/(2*a^2+2*b^
2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/d^2*f*c/(2*a^2+2*b^2
)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2-1/d
/(a^2+b^2)^(3/2)*b^4*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+
b^2)^(1/2))+1/d/(a^2+b^2)^(1/2)*b^2*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*
x+c)+2*a)/(a^2+b^2)^(1/2))-2/(a^2+b^2)/d*a^3*f/(2*a^2+2*b^2)*ln((-b*exp(d*x
+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-2/(a^2+b^2)/d^2*a^3*f/(2*a^2
+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/(a^2
+b^2)/d*a^3*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2
)^(1/2)))*x-2/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^
(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/(a^2+b^2)/d*a^3*f/(2*a^2+2*b^2)*ln(1+I*ex
p(d*x+c))*x+2/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c-2/(a^2
+b^2)/d^2*a^3*f*c/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))+2/(a^2+b^2)/d^2*a^3*f*
c/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/(a^2+b^2)/d*a^3*f/(
2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x+2/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*ln(1-
I*exp(d*x+c))*c-3*I*b/(a^2+b^2)/d*a^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x-
3*I*b/(a^2+b^2)/d^2*a^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c+3*I/(a^2+b^2)/
d*a^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*b*x+3*I/(a^2+b^2)/d^2*a^2*f/(2*a^2
+2*b^2)*ln(1-I*exp(d*x+c))*b*c-I*b^3/(a^2+b^2)/d*f/(2*a^2+2*b^2)*ln(1+I*exp
(d*x+c))*x-I*b^3/(a^2+b^2)/d^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c+I*b^3/(
a^2+b^2)/d*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x+I*b^3/(a^2+b^2)/d^2*f/(2*a^
2+2*b^2)*ln(1-I*exp(d*x+c))*c-6*b/(a^2+b^2)/d^2*a^2*f*c/(2*a^2+2*b^2)*arcta
n(exp(d*x+c))-2*b^3/(a^2+b^2)/d^2*f*c/(2*a^2+2*b^2)*arctan(exp(d*x+c))+I*b^
3/(a^2+b^2)/d^2*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))+6*b/(a^2+b^2)/d*a^2*e
/(2*a^2+2*b^2)*arctan(exp(d*x+c))-I*b^3/(a^2+b^2)/d^2*f/(2*a^2+2*b^2)*dilog
(1+I*exp(d*x+c))+3*I/(a^2+b^2)/d^2*a^2*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c)
)*b-3*I*b/(a^2+b^2)/d^2*a^2*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))+1/d^2/(a^
2+b^2)^(3/2)*b^4*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^
2)^(1/2))-3/d/(a^2+b^2)^(3/2)*b^2*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+
c)+2*a)/(a^2+b^2)^(1/2))*a^2-1/d^2/(a^2+b^2)^(1/2)*b^2*f*c/(2*a^2+2*b^2)*ar
ctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)^(3/2)/d^2*a^4*f
*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+(-b*d*f*
x*exp(3*d*x+3*c)+2*a*d*f*x*exp(2*d*x+2*c)-b*d*e*exp(3*d*x+3*c)+2*a*d*e*exp(
2*d*x+2*c)+b*d*f*x*exp(d*x+c)-b*f*exp(3*d*x+3*c)+a*f*exp(2*d*x+2*c)+b*d*e*e
xp(d*x+c)-f*b*exp(d*x+c)+a*f)/d^2/(a^2+b^2)/(1+exp(2*d*x+2*c))^2+2/(a^2+b^2
)/d^2*a^3*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))+2*b^3/(a^2+b^2)/d*e/(2*a^2+
2*b^2)*arctan(exp(d*x+c))+2/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*dilog(1-I*exp
(d*x+c))-2/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^
(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*dilog((-b
*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+2/(a^2+b^2)/d*a^3*e/(2
```

```
*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))-2/(a^2+b^2)/d*a^3*e/(2*a^2+2*b^2)*ln(b*exp
(2*d*x+2*c)+2*a*exp(d*x+c)-b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\left(\frac{a^3 \log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{\left(a^4 + 2\,a^2b^2 + b^4\right)d} - \frac{a^3 \log\left(e^{(-2\,dx-2\,c)} + 1\right)}{\left(a^4 + 2\,a^2b^2 + b^4\right)d} + \frac{\left(3\,a^2b + b^3\right)\arctan\left(e^{(-dx-c)}\right)}{\left(a^4 + 2\,a^2b^2 + b^4\right)d} + \frac{be^{(-dx-c)}}{\left(a^2 + b^2 + 2\left(a^2 + \right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b
^4)*d) - a^3*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) + (3*a^2
*b + b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (b*e^(-d*x - c
) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*
e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d))*e - f*(((b*d*x*e^(3*c)
 + b*e^(3*c))*e^(3*d*x) - (2*a*d*x*e^(2*c) + a*e^(2*c))*e^(2*d*x) - (b*d*x*
e^c - b*e^c)*e^(d*x) - a)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) + b^2*d^2*e
^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*x)) - inte
grate(-2*(a^4*x*e^(d*x + c) - a^3*b*x)/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b*e^
(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c + 2*a^3*b^2
*e^c + a*b^4*e^c)*e^(d*x)), x) - integrate(-(2*a^3*x - (3*a^2*b*e^c + b^3*e
^c)*x*e^(d*x))/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(2*c) + 2*a^2*b^2*e^(2*c) +
b^4*e^(2*c))*e^(2*d*x)), x))
```

Fricas [B] time = 4.00183, size = 11386, normalized size = 12.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^2*b + b^3)*f)*cosh(d*
x + c)^3 + 2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^2*b + b^3)*f)*si
nh(d*x + c)^3 - 2*(2*(a^3 + a*b^2)*d*f*x + 2*(a^3 + a*b^2)*d*e + (a^3 + a*b
^2)*f)*cosh(d*x + c)^2 - 2*(2*(a^3 + a*b^2)*d*f*x + 2*(a^3 + a*b^2)*d*e + (
a^3 + a*b^2)*f - 3*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^2*b + b^3)
*f)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(a^3 + a*b^2)*f - 2*((a^2*b + b^3)*d
*f*x + (a^2*b + b^3)*d*e - (a^2*b + b^3)*f)*cosh(d*x + c) + 2*(a^3*f*cosh(d
*x + c)^4 + 4*a^3*f*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*f*sinh(d*x + c)^4 +
 2*a^3*f*cosh(d*x + c)^2 + a^3*f + 2*(3*a^3*f*cosh(d*x + c)^2 + a^3*f)*sinh
(d*x + c)^2 + 4*(a^3*f*cosh(d*x + c)^3 + a^3*f*cosh(d*x + c))*sinh(d*x + c)
)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a^3*f*cosh(d*x + c)^4 + 4*a^3*
f*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*f*sinh(d*x + c)^4 + 2*a^3*f*cosh(d*x
+ c)^2 + a^3*f + 2*(3*a^3*f*cosh(d*x + c)^2 + a^3*f)*sinh(d*x + c)^2 + 4*(a
^3*f*cosh(d*x + c)^3 + a^3*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*
x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b + 1) - ((2*a^3*f + I*(3*a^2*b + b^3)*f)*cosh(d*x + c)^4 +
(8*a^3*f + 4*I*(3*a^2*b + b^3)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^3*f
+ I*(3*a^2*b + b^3)*f)*sinh(d*x + c)^4 + 2*a^3*f + (4*a^3*f + 2*I*(3*a^2*b
+ b^3)*f)*cosh(d*x + c)^2 + (4*a^3*f + (12*a^3*f + 6*I*(3*a^2*b + b^3)*f)*c
```

osh(d*x + c)^2 + 2*I*(3*a^2*b + b^3)*f)*sinh(d*x + c)^2 + I*(3*a^2*b + b^3)
*f + ((8*a^3*f + 4*I*(3*a^2*b + b^3)*f)*cosh(d*x + c)^3 + (8*a^3*f + 4*I*(3
*a^2*b + b^3)*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(I*cosh(d*x + c) + I*si
nh(d*x + c)) - ((2*a^3*f - I*(3*a^2*b + b^3)*f)*cosh(d*x + c)^4 + (8*a^3*f
- 4*I*(3*a^2*b + b^3)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^3*f - I*(3*a^
2*b + b^3)*f)*sinh(d*x + c)^4 + 2*a^3*f + (4*a^3*f - 2*I*(3*a^2*b + b^3)*f)
*cosh(d*x + c)^2 + (4*a^3*f + (12*a^3*f - 6*I*(3*a^2*b + b^3)*f)*cosh(d*x +
c)^2 - 2*I*(3*a^2*b + b^3)*f)*sinh(d*x + c)^2 - I*(3*a^2*b + b^3)*f + ((8*
a^3*f - 4*I*(3*a^2*b + b^3)*f)*cosh(d*x + c)^3 + (8*a^3*f - 4*I*(3*a^2*b +
b^3)*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(-I*cosh(d*x + c) - I*sinh(d*x +
c)) + 2*(a^3*d*e - a^3*c*f + (a^3*d*e - a^3*c*f)*cosh(d*x + c)^4 + 4*(a^3*
d*e - a^3*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*d*e - a^3*c*f)*sinh(d*x
+ c)^4 + 2*(a^3*d*e - a^3*c*f)*cosh(d*x + c)^2 + 2*(a^3*d*e - a^3*c*f + 3*
(a^3*d*e - a^3*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3*d*e - a^3*c*
f)*cosh(d*x + c)^3 + (a^3*d*e - a^3*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(
2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) +
2*(a^3*d*e - a^3*c*f + (a^3*d*e - a^3*c*f)*cosh(d*x + c)^4 + 4*(a^3*d*e - a
^3*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*d*e - a^3*c*f)*sinh(d*x + c)^4
+ 2*(a^3*d*e - a^3*c*f)*cosh(d*x + c)^2 + 2*(a^3*d*e - a^3*c*f + 3*(a^3*d*
e - a^3*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3*d*e - a^3*c*f)*cosh
(d*x + c)^3 + (a^3*d*e - a^3*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cos
h(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(a^3*
d*f*x + a^3*c*f + (a^3*d*f*x + a^3*c*f)*cosh(d*x + c)^4 + 4*(a^3*d*f*x + a^
3*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*d*f*x + a^3*c*f)*sinh(d*x + c)^
4 + 2*(a^3*d*f*x + a^3*c*f)*cosh(d*x + c)^2 + 2*(a^3*d*f*x + a^3*c*f + 3*(a
^3*d*f*x + a^3*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3*d*f*x + a^3*
c*f)*cosh(d*x + c)^3 + (a^3*d*f*x + a^3*c*f)*cosh(d*x + c))*sinh(d*x + c))*
log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(a^3*d*f*x + a^3*c*f + (a^3*d*f*x + a^
3*c*f)*cosh(d*x + c)^4 + 4*(a^3*d*f*x + a^3*c*f)*cosh(d*x + c)*sinh(d*x + c
)^3 + (a^3*d*f*x + a^3*c*f)*sinh(d*x + c)^4 + 2*(a^3*d*f*x + a^3*c*f)*cosh(
d*x + c)^2 + 2*(a^3*d*f*x + a^3*c*f + 3*(a^3*d*f*x + a^3*c*f)*cosh(d*x + c)
^2)*sinh(d*x + c)^2 + 4*((a^3*d*f*x + a^3*c*f)*cosh(d*x + c)^3 + (a^3*d*f*x
+ a^3*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*
x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b)
- (2*a^3*d*e - 2*a^3*c*f + (2*a^3*d*e - 2*a^3*c*f + I*(3*a^2*b + b^3)*d*e -
I*(3*a^2*b + b^3)*c*f)*cosh(d*x + c)^4 + (8*a^3*d*e - 8*a^3*c*f + 4*I*(3*a
^2*b + b^3)*d*e - 4*I*(3*a^2*b + b^3)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 +
(2*a^3*d*e - 2*a^3*c*f + I*(3*a^2*b + b^3)*d*e - I*(3*a^2*b + b^3)*c*f)*sin
h(d*x + c)^4 + I*(3*a^2*b + b^3)*d*e - I*(3*a^2*b + b^3)*c*f + (4*a^3*d*e -
4*a^3*c*f + 2*I*(3*a^2*b + b^3)*d*e - 2*I*(3*a^2*b + b^3)*c*f)*cosh(d*x +
c)^2 + (4*a^3*d*e - 4*a^3*c*f + 2*I*(3*a^2*b + b^3)*d*e - 2*I*(3*a^2*b + b^
3)*c*f + (12*a^3*d*e - 12*a^3*c*f + 6*I*(3*a^2*b + b^3)*d*e - 6*I*(3*a^2*b
+ b^3)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a^3*d*e - 8*a^3*c*f + 4*
I*(3*a^2*b + b^3)*d*e - 4*I*(3*a^2*b + b^3)*c*f)*cosh(d*x + c)^3 + (8*a^3*d
*e - 8*a^3*c*f + 4*I*(3*a^2*b + b^3)*d*e - 4*I*(3*a^2*b + b^3)*c*f)*cosh(d*
x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + I) - (2*a^3*d*e
- 2*a^3*c*f + (2*a^3*d*e - 2*a^3*c*f - I*(3*a^2*b + b^3)*d*e + I*(3*a^2*b +
b^3)*c*f)*cosh(d*x + c)^4 + (8*a^3*d*e - 8*a^3*c*f - 4*I*(3*a^2*b + b^3)*d
*e + 4*I*(3*a^2*b + b^3)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^3*d*e -
2*a^3*c*f - I*(3*a^2*b + b^3)*d*e + I*(3*a^2*b + b^3)*c*f)*sinh(d*x + c)^4
- I*(3*a^2*b + b^3)*d*e + I*(3*a^2*b + b^3)*c*f + (4*a^3*d*e - 4*a^3*c*f -
2*I*(3*a^2*b + b^3)*d*e + 2*I*(3*a^2*b + b^3)*c*f)*cosh(d*x + c)^2 + (4*a^3
*d*e - 4*a^3*c*f - 2*I*(3*a^2*b + b^3)*d*e + 2*I*(3*a^2*b + b^3)*c*f + (12*
a^3*d*e - 12*a^3*c*f - 6*I*(3*a^2*b + b^3)*d*e + 6*I*(3*a^2*b + b^3)*c*f)*c
osh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a^3*d*e - 8*a^3*c*f - 4*I*(3*a^2*b +
b^3)*d*e + 4*I*(3*a^2*b + b^3)*c*f)*cosh(d*x + c)^3 + (8*a^3*d*e - 8*a^3*c*
f - 4*I*(3*a^2*b + b^3)*d*e + 4*I*(3*a^2*b + b^3)*c*f)*cosh(d*x + c))*sinh(
d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - I) - (2*a^3*d*f*x + 2*a^3*c*f

```
 + (2*a^3*d*f*x + 2*a^3*c*f - I*(3*a^2*b + b^3)*d*f*x - I*(3*a^2*b + b^3)*c
*f)*cosh(d*x + c)^4 + (8*a^3*d*f*x + 8*a^3*c*f - 4*I*(3*a^2*b + b^3)*d*f*x
- 4*I*(3*a^2*b + b^3)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^3*d*f*x + 2
*a^3*c*f - I*(3*a^2*b + b^3)*d*f*x - I*(3*a^2*b + b^3)*c*f)*sinh(d*x + c)^4
 - I*(3*a^2*b + b^3)*d*f*x - I*(3*a^2*b + b^3)*c*f + (4*a^3*d*f*x + 4*a^3*c
*f - 2*I*(3*a^2*b + b^3)*d*f*x - 2*I*(3*a^2*b + b^3)*c*f)*cosh(d*x + c)^2 +
 (4*a^3*d*f*x + 4*a^3*c*f - 2*I*(3*a^2*b + b^3)*d*f*x - 2*I*(3*a^2*b + b^3)
*c*f + (12*a^3*d*f*x + 12*a^3*c*f - 6*I*(3*a^2*b + b^3)*d*f*x - 6*I*(3*a^2*
b + b^3)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a^3*d*f*x + 8*a^3*c*f
- 4*I*(3*a^2*b + b^3)*d*f*x - 4*I*(3*a^2*b + b^3)*c*f)*cosh(d*x + c)^3 + (8
*a^3*d*f*x + 8*a^3*c*f - 4*I*(3*a^2*b + b^3)*d*f*x - 4*I*(3*a^2*b + b^3)*c*
f)*cosh(d*x + c))*sinh(d*x + c))*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1)
 - (2*a^3*d*f*x + 2*a^3*c*f + (2*a^3*d*f*x + 2*a^3*c*f + I*(3*a^2*b + b^3)*
d*f*x + I*(3*a^2*b + b^3)*c*f)*cosh(d*x + c)^4 + (8*a^3*d*f*x + 8*a^3*c*f +
 4*I*(3*a^2*b + b^3)*d*f*x + 4*I*(3*a^2*b + b^3)*c*f)*cosh(d*x + c)*sinh(d*
x + c)^3 + (2*a^3*d*f*x + 2*a^3*c*f + I*(3*a^2*b + b^3)*d*f*x + I*(3*a^2*b
+ b^3)*c*f)*sinh(d*x + c)^4 + I*(3*a^2*b + b^3)*d*f*x + I*(3*a^2*b + b^3)*c
*f + (4*a^3*d*f*x + 4*a^3*c*f + 2*I*(3*a^2*b + b^3)*d*f*x + 2*I*(3*a^2*b +
b^3)*c*f)*cosh(d*x + c)^2 + (4*a^3*d*f*x + 4*a^3*c*f + 2*I*(3*a^2*b + b^3)*
d*f*x + 2*I*(3*a^2*b + b^3)*c*f + (12*a^3*d*f*x + 12*a^3*c*f + 6*I*(3*a^2*b
 + b^3)*d*f*x + 6*I*(3*a^2*b + b^3)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
 ((8*a^3*d*f*x + 8*a^3*c*f + 4*I*(3*a^2*b + b^3)*d*f*x + 4*I*(3*a^2*b + b^3
)*c*f)*cosh(d*x + c)^3 + (8*a^3*d*f*x + 8*a^3*c*f + 4*I*(3*a^2*b + b^3)*d*f
*x + 4*I*(3*a^2*b + b^3)*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-I*cosh(d*x
 + c) - I*sinh(d*x + c) + 1) - 2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e -
 3*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^2*b + b^3)*f)*cosh(d*x + c
)^2 - (a^2*b + b^3)*f + 2*(2*(a^3 + a*b^2)*d*f*x + 2*(a^3 + a*b^2)*d*e + (a
^3 + a*b^2)*f)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^2*c
osh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(d*x + c)*sinh(d*x + c)^
3 + (a^4 + 2*a^2*b^2 + b^4)*d^2*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)
*d^2*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2 + 2*(3*(a^4 + 2*a^2*b^2
+ b^4)*d^2*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)*sinh(d*x + c)^2 +
 4*((a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d
^2*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*tanh(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.418 $\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=120

$$-\frac{a^3\log(a+b\sinh(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{b\left(3a^2+b^2\right)\tan^{-1}(\sinh(c+dx))}{2d\left(a^2+b^2\right)^2}+\frac{a^3\log(\cosh(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+}{2d\left(a^2+b^2\right)}$$

```
[Out] (b*(3*a^2 + b^2)*ArcTan[Sinh[c + d*x]])/(2*(a^2 + b^2)^2*d) + (a^3*Log[Cosh
[c + d*x]])/((a^2 + b^2)^2*d) - (a^3*Log[a + b*Sinh[c + d*x]])/((a^2 + b^2)
^2*d) + (Sech[c + d*x]^2*(a - b*Sinh[c + d*x]))/(2*(a^2 + b^2)*d)
```

Rubi [A] time = 0.201183, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2721, 1647, 801, 635, 203, 260}

$$-\frac{a^3\log(a+b\sinh(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{b\left(3a^2+b^2\right)\tan^{-1}(\sinh(c+dx))}{2d\left(a^2+b^2\right)^2}+\frac{a^3\log(\cosh(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+}{2d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*(3*a^2 + b^2)*ArcTan[Sinh[c + d*x]])/(2*(a^2 + b^2)^2*d) + (a^3*Log[Cosh
[c + d*x]])/((a^2 + b^2)^2*d) - (a^3*Log[a + b*Sinh[c + d*x]])/((a^2 + b^2)
^2*d) + (Sech[c + d*x]^2*(a - b*Sinh[c + d*x]))/(2*(a^2 + b^2)*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= \frac{\text{sech}^2(c+dx)(a-b\sinh(c+dx))}{2\left(a^2+b^2\right)d} - \frac{\text{Subst}\left(\int \frac{\frac{ab^4}{a^2+b^2}+\frac{b^2\left(2a^2+b^2\right)x}{a^2+b^2}}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(c+dx)\right)}{2b^2d}$$

$$= \frac{\text{sech}^2(c+dx)(a-b\sinh(c+dx))}{2\left(a^2+b^2\right)d} - \frac{\text{Subst}\left(\int \left(\frac{2a^3b^2}{\left(a^2+b^2\right)^2(a+x)} - \frac{b^2\left(3a^2b^2+b^4+2a^3x\right)}{\left(a^2+b^2\right)^2\left(b^2+x^2\right)}\right)dx, x, b\sin \cdots}{2b^2d}$$

$$= -\frac{a^3\log(a+b\sinh(c+dx))}{\left(a^2+b^2\right)^2d} + \frac{\text{sech}^2(c+dx)(a-b\sinh(c+dx))}{2\left(a^2+b^2\right)d} + \frac{\text{Subst}\left(\int \frac{3a^2b^2+b^4+2a^3x}{b^2+x^2} \cdots\right.}{2\left(a^2+ \cdots\right.}$$

$$= -\frac{a^3\log(a+b\sinh(c+dx))}{\left(a^2+b^2\right)^2d} + \frac{\text{sech}^2(c+dx)(a-b\sinh(c+dx))}{2\left(a^2+b^2\right)d} + \frac{a^3\,\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x \cdots\right.}{\left(a^2+b^2\right. \cdots}$$

$$= \frac{b\left(3a^2+b^2\right)\tan^{-1}(\sinh(c+dx))}{2\left(a^2+b^2\right)^2d} + \frac{a^3\log(\cosh(c+dx))}{\left(a^2+b^2\right)^2d} - \frac{a^3\log(a+b\sinh(c+dx))}{\left(a^2+b^2\right)^2d} + \cdots$$

Mathematica [C] time = 0.402208, size = 152, normalized size = 1.27

$$-\frac{-a\left(a^2+b^2\right)\text{sech}^2(c+dx) - \left(a^3-i\left(2a^2b+b^3\right)\right)\log(-\sinh(c+dx)+i) - \left(a^3+i\left(2a^2b+b^3\right)\right)\log(\sinh(c+dx)+i) \cdots}{2d\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(b*(a^2 + b^2)*ArcTan[Sinh[c + d*x]] - (a^3 - I*(2*a^2*b + b^3))*Log[I - S
inh[c + d*x]] - (a^3 + I*(2*a^2*b + b^3))*Log[I + Sinh[c + d*x]] + 2*a^3*Lo
g[a + b*Sinh[c + d*x]] - a*(a^2 + b^2)*Sech[c + d*x]^2 + b*(a^2 + b^2)*Sech
[c + d*x]*Tanh[c + d*x])/(2*(a^2 + b^2)^2*d)
```

Maple [B] time = 0.002, size = 472, normalized size = 3.9

$$-8\frac{a^{3}\ln\left(\left(\tanh\left(1/2\,dx+c/2\right)\right)^{2}a-2\,\tanh\left(1/2\,dx+c/2\right)b-a\right)}{d\left(8\,a^{4}+16\,a^{2}b^{2}+8\,b^{4}\right)}+\frac{a^{2}b}{d\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}\left(\left(\tanh\left(\frac{dx}{2}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] -8/d*a^3/(8*a^4+16*a^2*b^2+8*b^4)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x
+1/2*c)*b-a)+1/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d
*x+1/2*c)^3*a^2*b+1/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(
1/2*d*x+1/2*c)^3*b^3-2/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^2+1)^2*ta
nh(1/2*d*x+1/2*c)^2*a^3-2/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^2+1)^2
*tanh(1/2*d*x+1/2*c)^2*a*b^2-1/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c)^2
+1)^2*tanh(1/2*d*x+1/2*c)*a^2*b-1/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*x+1/2*c
)^2+1)^2*tanh(1/2*d*x+1/2*c)*b^3+1/d/(a^4+2*a^2*b^2+b^4)*a^3*ln(tanh(1/2*d*
x+1/2*c)^2+1)+3/d/(a^4+2*a^2*b^2+b^4)*arctan(tanh(1/2*d*x+1/2*c))*a^2*b+1/d
/(a^4+2*a^2*b^2+b^4)*arctan(tanh(1/2*d*x+1/2*c))*b^3
```

Maxima [A] time = 1.67531, size = 293, normalized size = 2.44

$$-\frac{a^{3}\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)d}+\frac{a^{3}\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)d}-\frac{\left(3\,a^{2}b+b^{3}\right)\arctan\left(e^{(-dx-c)}\right)}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)d}-\frac{be^{(-dx-c)}-2}{\left(a^{2}+b^{2}+2\left(a^{2}+b^{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + 2*a^2*b^2 + b^
4)*d) + a^3*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (3*a^2*
b + b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) - (b*e^(-d*x - c)
 - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*e
^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d)
```

Fricas [B] time = 2.90572, size = 2217, normalized size = 18.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((a^2*b + b^3)*cosh(d*x + c)^3 + (a^2*b + b^3)*sinh(d*x + c)^3 - 2*(a^3 +
a*b^2)*cosh(d*x + c)^2 - (2*a^3 + 2*a*b^2 - 3*(a^2*b + b^3)*cosh(d*x + c))*
sinh(d*x + c)^2 - ((3*a^2*b + b^3)*cosh(d*x + c)^4 + 4*(3*a^2*b + b^3)*cosh
(d*x + c)*sinh(d*x + c)^3 + (3*a^2*b + b^3)*sinh(d*x + c)^4 + 3*a^2*b + b^3
 + 2*(3*a^2*b + b^3)*cosh(d*x + c)^2 + 2*(3*a^2*b + b^3 + 3*(3*a^2*b + b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((3*a^2*b + b^3)*cosh(d*x + c)^3 + (3
*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x
 + c)) - (a^2*b + b^3)*cosh(d*x + c) + (a^3*cosh(d*x + c)^4 + 4*a^3*cosh(d*
x + c)*sinh(d*x + c)^3 + a^3*sinh(d*x + c)^4 + 2*a^3*cosh(d*x + c)^2 + a^3
+ 2*(3*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^2 + 4*(a^3*cosh(d*x + c)^3
```

```
+ a^3*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x +
 c) - sinh(d*x + c))) - (a^3*cosh(d*x + c)^4 + 4*a^3*cosh(d*x + c)*sinh(d*x
 + c)^3 + a^3*sinh(d*x + c)^4 + 2*a^3*cosh(d*x + c)^2 + a^3 + 2*(3*a^3*cosh
(d*x + c)^2 + a^3)*sinh(d*x + c)^2 + 4*(a^3*cosh(d*x + c)^3 + a^3*cosh(d*x
+ c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) -
 (a^2*b + b^3 - 3*(a^2*b + b^3)*cosh(d*x + c)^2 + 4*(a^3 + a*b^2)*cosh(d*x
+ c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^4 + 4*(a^4 +
2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*
d*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4
 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d)*sinh(d*x
 + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d + 4*((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x
 + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Giac [A] time = 1.4486, size = 301, normalized size = 2.51

$$\frac{\frac{a^3\log\left(e^{(2dx+2c)}+1\right)}{a^4+2a^2b^2+b^4}-\frac{a^3\log\left(\left|-be^{(2dx+2c)}-2ae^{(dx+c)}+b\right|\right)}{a^4+2a^2b^2+b^4}+\frac{\left(3a^2be^c+b^3e^c\right)\arctan\left(e^{(dx+c)}\right)e^{(-c)}}{a^4+2a^2b^2+b^4}-\frac{\left(a^2be^{(3c)}+b^3e^{(3c)}\right)e^{(3dx)}-2\left(a^3e^{(2c)}+ab^2e^{(2c)}\right)e^{(2dx)}-(}{\left(a^2+b^2\right)^2\left(e^{(2dx+2c)}+1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (a^3*log(e^(2*d*x + 2*c) + 1)/(a^4 + 2*a^2*b^2 + b^4) - a^3*log(abs(-b*e^(2
*d*x + 2*c) - 2*a*e^(d*x + c) + b))/(a^4 + 2*a^2*b^2 + b^4) + (3*a^2*b*e^c
+ b^3*e^c)*arctan(e^(d*x + c))*e^(-c)/(a^4 + 2*a^2*b^2 + b^4) - ((a^2*b*e^(
3*c) + b^3*e^(3*c))*e^(3*d*x) - 2*(a^3*e^(2*c) + a*b^2*e^(2*c))*e^(2*d*x) -
 (a^2*b*e^c + b^3*e^c)*e^(d*x))/((a^2 + b^2)^2*(e^(2*d*x + 2*c) + 1)^2))/d
```

3.419 $\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Tanh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0757226, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Tanh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.905, size = 0, normalized size = 0.

$$\int \frac{(\tanh(dx+c))^3}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a*f + (b*d*f*x*e^(3*c) + (d*e - f)*b*e^(3*c))*e^(3*d*x) - (2*a*d*f*x*e^(2
*c) + (2*d*e - f)*a*e^(2*c))*e^(2*d*x) - (b*d*f*x*e^c + (d*e + f)*b*e^c)*e^
(d*x))/(a^2*d^2*e^2 + b^2*d^2*e^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*(a^
2*d^2*e*f + b^2*d^2*e*f)*x + (a^2*d^2*e^2*e^(4*c) + b^2*d^2*e^2*e^(4*c) + (
a^2*d^2*f^2*e^(4*c) + b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^2*d^2*e*f*e^(4*c) + b
^2*d^2*e*f*e^(4*c))*x)*e^(4*d*x) + 2*(a^2*d^2*e^2*e^(2*c) + b^2*d^2*e^2*e^(
2*c) + (a^2*d^2*f^2*e^(2*c) + b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^2*d^2*e*f*e^(
2*c) + b^2*d^2*e*f*e^(2*c))*x)*e^(2*d*x)) + integrate(-(2*a^3*d^2*f^2*x^2 +
 4*a^3*d^2*e*f*x + 2*a*b^2*f^2 + 2*(d^2*e^2 + f^2)*a^3 - ((3*d^2*e^2 + 2*f^
2)*a^2*b*e^c + (d^2*e^2 + 2*f^2)*b^3*e^c + (3*a^2*b*d^2*f^2*e^c + b^3*d^2*f
^2*e^c)*x^2 + 2*(3*a^2*b*d^2*e*f*e^c + b^3*d^2*e*f*e^c)*x)*e^(d*x))/(a^4*d^
2*e^3 + 2*a^2*b^2*d^2*e^3 + b^4*d^2*e^3 + (a^4*d^2*f^3 + 2*a^2*b^2*d^2*f^3
+ b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*e*f^2 + 2*a^2*b^2*d^2*e*f^2 + b^4*d^2*e*f^2
)*x^2 + 3*(a^4*d^2*e^2*f + 2*a^2*b^2*d^2*e^2*f + b^4*d^2*e^2*f)*x + (a^4*d^
2*e^3*e^(2*c) + 2*a^2*b^2*d^2*e^3*e^(2*c) + b^4*d^2*e^3*e^(2*c) + (a^4*d^2*
f^3*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(2*c) + b^4*d^2*f^3*e^(2*c))*x^3 + 3*(a^4
*d^2*e*f^2*e^(2*c) + 2*a^2*b^2*d^2*e*f^2*e^(2*c) + b^4*d^2*e*f^2*e^(2*c))*x
^2 + 3*(a^4*d^2*e^2*f*e^(2*c) + 2*a^2*b^2*d^2*e^2*f*e^(2*c) + b^4*d^2*e^2*f
*e^(2*c))*x)*e^(2*d*x)), x) + integrate(-2*(a^4*e^(d*x + c) - a^3*b)/(a^4*b
*e + 2*a^2*b^3*e + b^5*e + (a^4*b*f + 2*a^2*b^3*f + b^5*f)*x - (a^4*b*e*e^(
2*c) + 2*a^2*b^3*e*e^(2*c) + b^5*e*e^(2*c) + (a^4*b*f*e^(2*c) + 2*a^2*b^3*f
*e^(2*c) + b^5*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^5*e*e^c + 2*a^3*b^2*e*e^c + a
*b^4*e*e^c + (a^5*f*e^c + 2*a^3*b^2*f*e^c + a*b^4*f*e^c)*x)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tanh\left(dx+c\right)^{3}}{afx+ae+\left(bfx+be\right)\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(tanh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^{3}\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)**3/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.420 $\int \frac{(e+fx)^3 \coth(c+dx)}{a+b\sinh(c+dx)} \, dx$

Optimal. Leaf size=451

$$\frac{6f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3}+\frac{6f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^3}-\frac{3f(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2}-$$

```
[Out] -(((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*d)) - ((e
 + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*d) + ((e + f*x
)^3*Log[1 - E^(2*(c + d*x))])/(a*d) - (3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*d^2) - (3*f*(e + f*x)^2*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*d^2) + (3*f*(e + f*x)^2*PolyLog[
2, E^(2*(c + d*x))])/(2*a*d^2) + (6*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2]))])/(a*d^3) + (6*f^2*(e + f*x)*PolyLog[3, -((b*E^(
c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*d^3) - (3*f^2*(e + f*x)*PolyLog[3, E^
(2*(c + d*x))])/(2*a*d^3) - (6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(a*d^4) - (6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/(a*d^4) + (3*f^3*PolyLog[4, E^(2*(c + d*x))])/(4*a*d^4)
```

Rubi [A] time = 0.769999, antiderivative size = 451, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5569, 3716, 2190, 2531, 6609, 2282, 6589, 5561}

$$\frac{6f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3}+\frac{6f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^3}-\frac{3f(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2}-$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -(((e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*d)) - ((e
 + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*d) + ((e + f*x
)^3*Log[1 - E^(2*(c + d*x))])/(a*d) - (3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*d^2) - (3*f*(e + f*x)^2*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*d^2) + (3*f*(e + f*x)^2*PolyLog[
2, E^(2*(c + d*x))])/(2*a*d^2) + (6*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2]))])/(a*d^3) + (6*f^2*(e + f*x)*PolyLog[3, -((b*E^(
c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*d^3) - (3*f^2*(e + f*x)*PolyLog[3, E^
(2*(c + d*x))])/(2*a*d^3) - (6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(a*d^4) - (6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/(a*d^4) + (3*f^3*PolyLog[4, E^(2*(c + d*x))])/(4*a*d^4)
```


Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
```

```
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\coth(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{2\int \frac{e^{2(c+dx)}(e+fx)^3}{1-e^{2(c+dx)}}\,dx}{a} - \frac{b\int \frac{e^{c+dx}(e+fx)^3}{a-\sqrt{a^2+b^2}+be^{c+dx}}\,dx}{a} - \frac{b\int \frac{e^{c+dx}(e+fx)^3}{a+\sqrt{a^2+b^2}+be^{c+dx}}\,dx}{a}$$

$$= -\frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e+fx)^3\log\left(1-e^{2(c+\right.}}{ad}$$

$$= -\frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e+fx)^3\log\left(1-e^{2(c+\right.}}{ad}$$

$$= -\frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e+fx)^3\log\left(1-e^{2(c+\right.}}{ad}$$

$$= -\frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e+fx)^3\log\left(1-e^{2(c+\right.}}{ad}$$

$$= -\frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e+fx)^3\log\left(1-e^{2(c+\right.}}{ad}$$

Mathematica [B] time = 19.0461, size = 1924, normalized size = 4.27

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(E^(2*c)*((e + f*x)^4/(E^(2*c)*f) - (2*(1 - E^(-2*c))*(e + f*x)^3*Log[1 -
E^(-c - d*x)])/d - (2*(1 - E^(-2*c))*(e + f*x)^3*Log[1 + E^(-c - d*x)])/d +
 (6*(-1 + E^(2*c))*f*(d^2*(e + f*x)^2*PolyLog[2, -E^(-c - d*x)] + 2*f*(d*(e
 + f*x)*PolyLog[3, -E^(-c - d*x)] + f*PolyLog[4, -E^(-c - d*x)])))/(d^4*E^(
2*c)) + (6*(-1 + E^(2*c))*f*(d^2*(e + f*x)^2*PolyLog[2, E^(-c - d*x)] + 2*f
*(d*(e + f*x)*PolyLog[3, E^(-c - d*x)] + f*PolyLog[4, E^(-c - d*x)])))/(d^4
*E^(2*c))))/(2*a*(-1 + E^(2*c))) + (4*e^3*E^(2*c)*x + 6*e^2*E^(2*c)*f*x^2 +
 4*e*E^(2*c)*f^2*x^3 + E^(2*c)*f^3*x^4 + (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*E^(2
*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) + (
4*a*Sqrt[-(a^2 + b^2)^2]*e^3*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 +
 b^2]])/((-a^2 - b^2)^(3/2)*d) + (2*e^3*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c
 + d*x))])/d - (2*e^3*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x))
)])/d + (6*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)])])/d - (6*e^2*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2
 + b^2)*E^(2*c)])])/d + (6*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqr
t[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x)
)/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (2*f^3*x^3*Log[1 + (b*E^(2*c +
d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (2*E^(2*c)*f^3*x^3*Log[1 +
(b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e^2*f*x*Log[
1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e^2*E^(2
*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d +
 (6*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])
])/d - (6*e*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 +
b^2)*E^(2*c)])])/d + (2*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^
2 + b^2)*E^(2*c)])])/d - (2*E^(2*c)*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^
c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(e + f*x)^2*PolyLo
g[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-
```

```
1 + E^(2*c))*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^
2 + b^2)*E^(2*c)]))])/d^2 - (12*e*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c
 - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*e*E^(2*c)*f^2*PolyLog[3, -((b*E^
(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (12*f^3*x*PolyLog
[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*E^
(2*c)*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c
)]))])/d^3 - (12*e*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 +
b^2)*E^(2*c)]))])/d^3 + (12*e*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a
*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (12*f^3*x*PolyLog[3, -((b*E^(2*c
 + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (12*E^(2*c)*f^3*x*Pol
yLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (1
2*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])
/d^4 - (12*E^(2*c)*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 +
b^2)*E^(2*c)]))])/d^4 + (12*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c + Sqr
t[(a^2 + b^2)*E^(2*c)]))])/d^4 - (12*E^(2*c)*f^3*PolyLog[4, -((b*E^(2*c + d
*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^4)/(2*a*(-1 + E^(2*c)))
```

Maple [F] time = 0.495, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3} \coth \left(dx+c\right)}{a+b \sinh \left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-e^{3}\left(\frac{\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{ad}-\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}-\frac{\log\left(e^{(-dx-c)}-1\right)}{ad}\right)+\frac{3\left(dx\log\left(e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(-e^{(dx+c)}\right)\right)}{ad^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -e^3*(log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*d) - log(e^(-d*x -
 c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d)) + 3*(d*x*log(e^(d*x + c) + 1)
 + dilog(-e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d*x*log(-e^(d*x + c) + 1) + dilo
g(e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dil
og(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*e*f^2/(a*d^3) + 3*(d^2*x^2*l
og(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c))
)*e*f^2/(a*d^3) + (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x +
 c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*f^3/(a*
d^4) + (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*
x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*f^3/(a*d^4) - 1/2*(d
^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*d^4*e^2*f*x^2)/(a*d^4) + integrate(-2*(b*f
^3*x^3 + 3*b*e*f^2*x^2 + 3*b*e^2*f*x - (a*f^3*x^3*e^c + 3*a*e*f^2*x^2*e^c +
 3*a*e^2*f*x*e^c)*e^(d*x))/(a*b*e^(2*d*x + 2*c) + 2*a^2*e^(d*x + c) - a*b),
 x)
```

Fricas [C] time = 2.85818, size = 2996, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(6*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*f^3*polylog(4, (a*cosh(d*x +
 c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2))/b) - 6*f^3*polylog(4, cosh(d*x + c) + sinh(d*x + c)) - 6*f^3*polylo
g(4, -cosh(d*x + c) - sinh(d*x + c)) + 3*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2
*e^2*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 3*(d^2*f^3*x^2 + 2*d^2*e*f^
2*x + d^2*e^2*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(d^2*f^3*x^2 + 2
*d^2*e*f^2*x + d^2*e^2*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) - 3*(d^2*f^3
*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) + (
d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*log(2*b*cosh(d*x + c) +
2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (d^3*e^3 - 3*c*d^2*e
^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) -
 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*
e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*log(-(a*cosh(d*x + c) +
a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
 - b)/b) + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f -
 3*c^2*d*e*f^2 + c^3*f^3)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (d^3*f^3*x^3 +
 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + d^3*e^3)*log(cosh(d*x + c) + sinh(d*x +
c) + 1) - (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*log(cosh(d*x
+ c) + sinh(d*x + c) - 1) - (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x
+ 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*log(-cosh(d*x + c) - sinh(d*x +
c) + 1) - 6*(d*f^3*x + d*e*f^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x +
c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*(d*f
^3*x + d*e*f^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*(d*f^3*x + d*e*f^2)*
polylog(3, cosh(d*x + c) + sinh(d*x + c)) + 6*(d*f^3*x + d*e*f^2)*polylog(3
, -cosh(d*x + c) - sinh(d*x + c)))/(a*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^3 \coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \coth(dx+c)}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*coth(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.421 $\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=325

$$-\frac{2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2}-\frac{2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2}+\frac{2f^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3}+\frac{2f^2\text{Poly}}{}$$

```
[Out] -(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*d)) - ((e
 + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*d) + ((e + f*x
)^2*Log[1 - E^(2*(c + d*x))])/(a*d) - (2*f*(e + f*x)*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(a*d^2) - (2*f*(e + f*x)*PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*d^2) + (f*(e + f*x)*PolyLog[2, E^(2*
(c + d*x))])/(a*d^2) + (2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2]))])/(a*d^3) + (2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
))])/(a*d^3) - (f^2*PolyLog[3, E^(2*(c + d*x))])/(2*a*d^3)
```

Rubi [A] time = 0.653868, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5569, 3716, 2190, 2531, 2282, 6589, 5561}

$$-\frac{2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2}-\frac{2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2}+\frac{2f^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3}+\frac{2f^2\text{Poly}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*d)) - ((e
 + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*d) + ((e + f*x
)^2*Log[1 - E^(2*(c + d*x))])/(a*d) - (2*f*(e + f*x)*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(a*d^2) - (2*f*(e + f*x)*PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*d^2) + (f*(e + f*x)*PolyLog[2, E^(2*
(c + d*x))])/(a*d^2) + (2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2]))])/(a*d^3) + (2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
))])/(a*d^3) - (f^2*PolyLog[3, E^(2*(c + d*x))])/(2*a*d^3)
```

Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\coth(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{2\int \frac{e^{2(c+dx)}(e+fx)^2}{1-e^{2(c+dx)}}\,dx}{a} - \frac{b\int \frac{e^{c+dx}(e+fx)^2}{a-\sqrt{a^2+b^2}+be^{c+dx}}\,dx}{a} - \frac{b\int \frac{e^{c+dx}(e+fx)^2}{a+\sqrt{a^2+b^2}+be^{c+dx}}\,dx}{a}$$

$$= -\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{ad}$$

$$= -\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{ad}$$

$$= -\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{ad}$$

$$= -\frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e+fx)^2\log\left(1-e^{2(c+dx)}\right)}{ad}$$

Mathematica [B] time = 14.1565, size = 1296, normalized size = 3.99

$$2e^{2c}f^2x^3+6ee^{2c}fx^2-\frac{3e^{2c}f^2\log\left(\frac{e^{2c+dx}b}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}+1\right)x^2}{d}+\frac{3f^2\log\left(\frac{e^{2c+dx}b}{ae^c-\sqrt{(a^2+b^2)e^{2c}}}+1\right)x^2}{d}-\frac{3e^{2c}f^2\log\left(\frac{e^{2c+dx}b}{e^ca+\sqrt{(a^2+b^2)e^{2c}}}+1\right)x^2}{d}+\frac{3f^2\log\left(\frac{e^{2c}\cdots}{e^ca+\sqrt{(\cdots}}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 - (2*(e + f*x)^3)/
f + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/
(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a
+ b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2
 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^
(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x)
)/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*(-1 + E^(2*c))*(e + f*x)^2*
Log[1 - E^(-c - d*x)])/d + (3*(-1 + E^(2*c))*(e + f*x)^2*Log[1 + E^(-c - d*
x)])/d + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2
*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Log[
1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c
)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (
3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d
 - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E
^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)
*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(
a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sq
rt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))
/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(e + f*x)*Po
lyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (
6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(
a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(d*(e + f*x)*PolyLog[2, -
E^(-c - d*x)] + f*PolyLog[3, -E^(-c - d*x)]))/d^3 - (6*(-1 + E^(2*c))*f*(d*
(e + f*x)*PolyLog[2, E^(-c - d*x)] + f*PolyLog[3, E^(-c - d*x)]))/d^3 - (6*
f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d
^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2
)*E^(2*c)]))])/d^3 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a
^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/
(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3)/(3*a*(-1 + E^(2*c)))
```

Maple [F] time = 0.39, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2\coth(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-e^2\left(\frac{\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{ad}-\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}-\frac{\log\left(e^{(-dx-c)}-1\right)}{ad}\right)+\frac{2\left(dx\log\left(e^{(dx+c)}+1\right)+\mathrm{Li}_2\left(-e^{(dx+c)}\right)\right)}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -e^2*(log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*d) - log(e^(-d*x -
 c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d)) + 2*(d*x*log(e^(d*x + c) + 1)
 + dilog(-e^(d*x + c)))*e*f/(a*d^2) + 2*(d*x*log(-e^(d*x + c) + 1) + dilog(
e^(d*x + c)))*e*f/(a*d^2) + (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^
(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2/(a*d^3) + (d^2*x^2*log(-e^(d*x
 + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*f^2/(a*d
^3) - 2/3*(d^3*f^2*x^3 + 3*d^3*e*f*x^2)/(a*d^3) + integrate(-2*(b*f^2*x^2 +
 2*b*e*f*x - (a*f^2*x^2*e^c + 2*a*e*f*x*e^c)*e^(d*x))/(a*b*e^(2*d*x + 2*c)
+ 2*a^2*e^(d*x + c) - a*b), x)
```

Fricas [C] time = 2.61739, size = 2053, normalized size = 6.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*f^2*polylog(3, (a*cosh(d*x +
c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2))/b) - 2*f^2*polylog(3, cosh(d*x + c) + sinh(d*x + c)) - 2*f^2*polylog
(3, -cosh(d*x + c) - sinh(d*x + c)) - 2*(d*f^2*x + d*e*f)*dilog((a*cosh(d*x
 + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2) - b)/b + 1) - 2*(d*f^2*x + d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(
d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b
 + 1) + 2*(d*f^2*x + d*e*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) + 2*(d*f^2
*x + d*e*f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) - (d^2*e^2 - 2*c*d*e*f +
c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b
^2) + 2*a) - (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b*si
nh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (d^2*f^2*x^2 + 2*d^2*e*f*x
 + 2*c*d*e*f - c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (d^2*f^2*x^2 + 2
*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) -
 (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (d^2*f
^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (d
^2*e^2 - 2*c*d*e*f + c^2*f^2)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + (d^2
*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*log(-cosh(d*x + c) - sinh(d*x
 + c) + 1))/(a*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \coth(dx+c)}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*coth(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.422 $\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=205

$$-\frac{f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2}-\frac{f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2}+\frac{f\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{2ad^2}-\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{ad}-\frac{(e+f}{}$$

```
[Out] -(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*d)) - ((e +
 f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*d) + ((e + f*x)*Lo
g[1 - E^(2*(c + d*x))])/(a*d) - (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(a*d^2) - (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
]))])/(a*d^2) + (f*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^2)
```

Rubi [A] time = 0.380062, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5569, 3716, 2190, 2279, 2391, 5561}

$$-\frac{f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2}-\frac{f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2}+\frac{f\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{2ad^2}-\frac{(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{ad}-\frac{(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*d)) - ((e +
 f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*d) + ((e + f*x)*Lo
g[1 - E^(2*(c + d*x))])/(a*d) - (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(a*d^2) - (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
]))])/(a*d^2) + (f*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^2)
```


Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\coth(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$
$$= -\frac{2\int \frac{e^{2(c+dx)}(e+fx)}{1-e^{2(c+dx)}}\,dx}{a} - \frac{b\int \frac{e^{c+dx}(e+fx)}{a-\sqrt{a^2+b^2}+be^{c+dx}}\,dx}{a} - \frac{b\int \frac{e^{c+dx}(e+fx)}{a+\sqrt{a^2+b^2}+be^{c+dx}}\,dx}{a}$$
$$= -\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad}$$
$$= -\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad}$$
$$= -\frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e+fx)\log\left(1-e^{2(c+dx)}\right)}{ad}$$

Mathematica [A] time = 0.945634, size = 236, normalized size = 1.15

$$-\frac{f\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+\frac{1}{2}f\text{PolyLog}\left(2,e^{-2(c+dx)}\right)+f(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)+f(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((-(f*(c + d*x)^2) - f*(c + d*x)*Log[1 - E^(-2*(c + d*x))] + f*(c + d*x)*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2])] - d*e*Log[Sinh[c + d*x]] + c*f*Log[Sinh[c +
 d*x]] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + (f*P
olyLog[2, E^(-2*(c + d*x))])/2 + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^
2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*d^2
))
```

Maple [B] time = 0.134, size = 451, normalized size = 2.2

$$\frac{e\ln\left(e^{dx+c}-1\right)}{da} - \frac{e\ln\left(be^{2dx+2c}+2ae^{dx+c}-b\right)}{da} + \frac{e\ln\left(e^{dx+c}+1\right)}{da} - \frac{f\,\text{dilog}\left(e^{dx+c}\right)}{ad^2} - \frac{f}{ad^2}\text{dilog}\left(\left(-be^{dx+c}+\sqrt{a^2+b^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/d/a*e*ln(exp(d*x+c)-1)-1/d*e/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/d/
a*e*ln(exp(d*x+c)+1)-1/d^2*f/a*dilog(exp(d*x+c))-1/d^2*f/a*dilog((-b*exp(d*
x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*f/a*dilog((b*exp(d*x+c)
+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/d^2*f/a*dilog(exp(d*x+c)+1)-1/d*
f/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d^2*f/a*
ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d*f/a*ln((b*
exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*f/a*ln((b*exp(d*
x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d/a*ln(exp(d*x+c)+1)*f*x-1
/d^2/a*f*c*ln(exp(d*x+c)-1)+1/d^2*f*c/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-
b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-e\left(\frac{\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{ad}-\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}-\frac{\log\left(e^{(-dx-c)}-1\right)}{ad}\right)+f\int\frac{2\,x\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right)\left(e^{(dx+c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -e*(log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*d) - log(e^(-d*x - c
) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d)) + f*integrate(2*x*(e^(d*x + c)
+ e^(-d*x - c))/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c) - e^(-
d*x - c))), x)
```

Fricas [B] time = 2.48589, size = 1243, normalized size = 6.06

$$-\frac{f\mathrm{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}}-b}{b}+1\right)+f\mathrm{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)-(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{}}{b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + f*dilog((a*cosh(d*x + c) + a*si
nh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b
)/b + 1) - f*dilog(cosh(d*x + c) + sinh(d*x + c)) - f*dilog(-cosh(d*x + c)
- sinh(d*x + c)) + (d*e - c*f)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) +
2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (d*e - c*f)*log(2*b*cosh(d*x + c) + 2*b*
sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (d*f*x + c*f)*log(-(a*co
sh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b) + (d*f*x + c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x +
 c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (
d*f*x + d*e)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (d*e - c*f)*log(cosh(
d*x + c) + sinh(d*x + c) - 1) - (d*f*x + c*f)*log(-cosh(d*x + c) - sinh(d*x
 + c) + 1))/(a*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\coth(dx+c)}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*coth(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.423 $\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{\log(\sinh(c+dx))}{ad}-\frac{\log(a+b\sinh(c+dx))}{ad}$$

```
[Out] Log[Sinh[c + d*x]]/(a*d) - Log[a + b*Sinh[c + d*x]]/(a*d)
```

Rubi [A] time = 0.0473075, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2721, 36, 29, 31}

$$\frac{\log(\sinh(c+dx))}{ad}-\frac{\log(a+b\sinh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Log[Sinh[c + d*x]]/(a*d) - Log[a + b*Sinh[c + d*x]]/(a*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\sinh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\sinh(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(c+dx)\right)}{ad}\\
&= \frac{\log(\sinh(c+dx))}{ad}-\frac{\log(a+b\sinh(c+dx))}{ad}
\end{aligned}$$

Mathematica [A] time = 0.0213791, size = 28, normalized size = 0.82

$$\frac{\log(\sinh(c+dx))-\log(a+b\sinh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (Log[Sinh[c + d*x]] - Log[a + b*Sinh[c + d*x]])/(a*d)
```

Maple [A] time = 0.001, size = 35, normalized size = 1.

$$\frac{\ln(\sinh(dx+c))}{da}-\frac{\ln(a+b\sinh(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] ln(sinh(d*x+c))/a/d-ln(a+b*sinh(d*x+c))/a/d
```

Maxima [B] time = 1.08305, size = 101, normalized size = 2.97

$$-\frac{\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{ad}+\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}+\frac{\log\left(e^{(-dx-c)}-1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*d) + log(e^(-d*x - c) +
 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d)
```

Fricas [A] time = 2.39192, size = 170, normalized size = 5.

$$-\frac{\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)-\log\left(\frac{2\,\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) - log(2*sinh
(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A] time = 1.32442, size = 85, normalized size = 2.5

$$\frac{\frac{\log\left(e^{(dx+c)}+1\right)}{a} - \frac{\log\left(\left|be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}-b\right|\right)}{a} + \frac{\log\left(\left|e^{(dx+c)}-1\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (log(e^(d*x + c) + 1)/a - log(abs(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - b))
/a + log(abs(e^(d*x + c) - 1))/a)/d
```

3.424 $\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left(\frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Coth[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.049565, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Coth[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A] time = 29.0795, size = 0, normalized size = 0.

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Coth[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Integrate[Coth[c + d*x]/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Maple [A] time = 0.379, size = 0, normalized size = 0.

$$\int \frac{\coth(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(coth(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\coth(dx+c)}{afx+ae+(bfx+be)\sinh(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(coth(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(c+dx)}{(a+b\sinh(c+dx))(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(coth(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

3.425 $\int \frac{(e+fx)^3 \cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=638

$$\frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^3}-\frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{abd^3}-\frac{3f\sqrt{a^2+b^2}(e+fx)^2}{a}$$

```
[Out] (e + f*x)^4/(4*b*f) - (2*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (Sqrt[a^
2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*b*d
) + (Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(a*b*d) - (3*f*(e + f*x)^2*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (3*f*(
e + f*x)^2*PolyLog[2, E^(c + d*x)])/(a*d^2) - (3*Sqrt[a^2 + b^2]*f*(e + f*x
)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b*d^2) + (3*Sq
rt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2]))])/(a*b*d^2) + (6*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) - (6*
f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a*d^3) + (6*Sqrt[a^2 + b^2]*f^2*(e
+ f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b*d^3) - (6
*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/(a*b*d^3) - (6*f^3*PolyLog[4, -E^(c + d*x)])/(a*d^4) + (6*f^3*Pol
yLog[4, E^(c + d*x)])/(a*d^4) - (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b*d^4) + (6*Sqrt[a^2 + b^2]*f^3*PolyLo
g[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*b*d^4)
```

Rubi [A] time = 1.27521, antiderivative size = 638, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 14, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5585, 5450, 3296, 2637, 4182, 2531, 6609, 2282, 6589, 5565, 32, 3322, 2264, 2190}

$$\frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^3}-\frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{abd^3}-\frac{3f\sqrt{a^2+b^2}(e+fx)^2}{a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (e + f*x)^4/(4*b*f) - (2*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (Sqrt[a^
2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*b*d
) + (Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(a*b*d) - (3*f*(e + f*x)^2*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (3*f*(
e + f*x)^2*PolyLog[2, E^(c + d*x)])/(a*d^2) - (3*Sqrt[a^2 + b^2]*f*(e + f*x
)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b*d^2) + (3*Sq
rt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2]))])/(a*b*d^2) + (6*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) - (6*
f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a*d^3) + (6*Sqrt[a^2 + b^2]*f^2*(e
+ f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b*d^3) - (6
*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))])/(a*b*d^3) - (6*f^3*PolyLog[4, -E^(c + d*x)])/(a*d^4) + (6*f^3*Pol
yLog[4, E^(c + d*x)])/(a*d^4) - (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b*d^4) + (6*Sqrt[a^2 + b^2]*f^3*PolyLo
g[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*b*d^4)
```


Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
```

```
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\cosh(c+dx)\coth(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)^3\mathbf{csch}(c+dx)\,dx}{a} + \frac{\int (e+fx)^3\,dx}{b} - \frac{\left(a^2+b^2\right)\int \frac{(e+fx)^3}{a+b\sinh(c+dx)}\,dx}{ab}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\left(2\left(a^2+b^2\right)\right)\int \frac{e^{c+dx}(e+fx)^3}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{ab}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{3f(e+fx)^2\mathbf{Li}_2\left(-e^{c+dx}\right)}{ad^2} + 3f(e+$$

$$= \frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b}}\right.}{abd}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b}}\right.}{abd}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b}}\right.}{abd}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b}}\right.}{abd}$$

$$= \frac{(e+fx)^4}{4bf} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b}}\right.}{abd}$$

Mathematica [A] time = 2.47645, size = 802, normalized size = 1.26

$$ax\left(4e^3+6fxe^2+4f^2x^2e+f^3x^3\right)d^4+4\sqrt{a^2+b^2}\left(2e^3\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)d^3-f^3x^3\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^3-3ef^2x^2\log\left(\frac{e}{a-}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (a*d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) + 4*Sqrt[a^2 + b^2]*(2
*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 3*d^3*e^2*f*x*Log[1
 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2])] - d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2])] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^
3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x
)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^
2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*d*e*f^2*PolyLog[
3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*d*f^3*x*PolyLog[3, (b*E^(c +
 d*x))/(-a + Sqrt[a^2 + b^2])] - 6*d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))] - 6*d*f^3*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2]))] - 6*f^3*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*f
^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) - 4*b*(2*d^3*(e +
f*x)^3*ArcTanh[Cosh[c + d*x] + Sinh[c + d*x]] + 3*f*(d^2*(e + f*x)^2*PolyLo
g[2, -Cosh[c + d*x] - Sinh[c + d*x]] - 2*d*f*(e + f*x)*PolyLog[3, -Cosh[c +
 d*x] - Sinh[c + d*x]] + 2*f^2*PolyLog[4, -Cosh[c + d*x] - Sinh[c + d*x]])
- 3*f*(d^2*(e + f*x)^2*PolyLog[2, Cosh[c + d*x] + Sinh[c + d*x]] - 2*d*f*(e
```

```
 + f*x)*PolyLog[3, Cosh[c + d*x] + Sinh[c + d*x]] + 2*f^2*PolyLog[4, Cosh[c
 + d*x] + Sinh[c + d*x]])))/(4*a*b*d^4)
```

Maple [F] time = 0.734, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \cosh(dx+c)\coth(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.41837, size = 3588, normalized size = 5.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/4*(a*d^4*f^3*x^4 + 4*a*d^4*e*f^2*x^3 + 6*a*d^4*e^2*f*x^2 + 4*a*d^4*e^3*x
- 24*b*f^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x +
 c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*b*
f^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*b*f^3*pol
ylog(4, cosh(d*x + c) + sinh(d*x + c)) - 24*b*f^3*polylog(4, -cosh(d*x + c)
 - sinh(d*x + c)) - 12*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*sqrt
((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*(b*d^2*f^3*x^2
 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x +
 c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2) - b)/b + 1) + 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c
^3*f^3)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2
*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*
d*e*f^2 - b*c^3*f^3)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh
(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 4*(b*d^3*f^3*x^3 + 3*b*d^3*e
*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)
```

```
*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*
x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*(b*d^3*f^3*x^3
+ 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 +
 b*c^3*f^3)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) -
 (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 24*(b*
d*f^3*x + b*d*e*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*
sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/
b) - 24*(b*d*f^3*x + b*d*e*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*
x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2))/b) + 12*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*dilog(co
sh(d*x + c) + sinh(d*x + c)) - 12*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*
e^2*f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) - 4*(b*d^3*f^3*x^3 + 3*b*d^3*e
*f^2*x^2 + 3*b*d^3*e^2*f*x + b*d^3*e^3)*log(cosh(d*x + c) + sinh(d*x + c) +
 1) + 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(cos
h(d*x + c) + sinh(d*x + c) - 1) + 4*(b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*
b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*log(-cosh(d*
x + c) - sinh(d*x + c) + 1) - 24*(b*d*f^3*x + b*d*e*f^2)*polylog(3, cosh(d*
x + c) + sinh(d*x + c)) + 24*(b*d*f^3*x + b*d*e*f^2)*polylog(3, -cosh(d*x +
 c) - sinh(d*x + c)))/(a*b*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.426 $\int \frac{(e+fx)^2 \cosh(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} \, dx$

Optimal. Leaf size=462

$$-\frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^2}+\frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{abd^2}+\frac{2f^2\sqrt{a^2+b^2}\text{PolyLog}}{abd^3}$$

```
[Out] (e + f*x)^3/(3*b*f) - (2*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (Sqrt[a^
2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*b*d
) + (Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(a*b*d) - (2*f*(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (2*f*(e
+ f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) - (2*Sqrt[a^2 + b^2]*f*(e + f*x)*Po
lyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b*d^2) + (2*Sqrt[a^2
 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
a*b*d^2) + (2*f^2*PolyLog[3, -E^(c + d*x)])/(a*d^3) - (2*f^2*PolyLog[3, E^(
c + d*x)])/(a*d^3) + (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2]))])/(a*b*d^3) - (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*b*d^3)
```

Rubi [A] time = 1.04022, antiderivative size = 462, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.406, Rules used = {5585, 5450, 3296, 2638, 4182, 2531, 2282, 6589, 5565, 32, 3322, 2264, 2190}

$$-\frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^2}+\frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{abd^2}+\frac{2f^2\sqrt{a^2+b^2}\text{PolyLog}}{abd^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (e + f*x)^3/(3*b*f) - (2*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (Sqrt[a^
2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*b*d
) + (Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2])])/(a*b*d) - (2*f*(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (2*f*(e
+ f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) - (2*Sqrt[a^2 + b^2]*f*(e + f*x)*Po
lyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b*d^2) + (2*Sqrt[a^2
 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
a*b*d^2) + (2*f^2*PolyLog[3, -E^(c + d*x)])/(a*d^3) - (2*f^2*PolyLog[3, E^(
c + d*x)])/(a*d^3) + (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2]))])/(a*b*d^3) - (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*b*d^3)
```


Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh(c+dx)\coth(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)^2\mathbf{csch}(c+dx)\,dx}{a} + \frac{\int (e+fx)^2\,dx}{b} - \frac{\left(a^2+b^2\right)\int \frac{(e+fx)^2}{a+b\sinh(c+dx)}\,d}{ab}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\left(2\left(a^2+b^2\right)\right)\int \frac{e^{c+dx}(e+fx)^2}{-b+2ae^{c+dx}+be^{2(c+dx}}}{ab}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2f(e+fx)\mathrm{Li}_2\left(-e^{c+dx}\right)}{ad^2} + \frac{2f(e}{}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c}}{a-\sqrt{a}}\right.}{abd}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c}}{a-\sqrt{a}}\right.}{abd}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c}}{a-\sqrt{a}}\right.}{abd}$$

$$= \frac{(e+fx)^3}{3bf} - \frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}(e+fx)^2\log\left(1+\frac{be^{c}}{a-\sqrt{a}}\right.}{abd}$$

Mathematica [A] time = 1.87117, size = 489, normalized size = 1.06

$$\frac{\sqrt{a^2+b^2}\left(-2df(e+fx)\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+2df(e+fx)\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+2f^2\mathrm{PolyLog}\left(3,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2))/(3*b) + ((e + f*x)^2*Log[1 - E^(c + d*x)] -
 (e + f*x)^2*Log[1 + E^(c + d*x)] - (2*f*(d*(e + f*x)*PolyLog[2, -E^(c + d*
x)] - f*PolyLog[3, -E^(c + d*x)]))/d^2 + (2*f*(d*(e + f*x)*PolyLog[2, E^(c
+ d*x)] - f*PolyLog[3, E^(c + d*x)]))/d^2)/(a*d) + (Sqrt[a^2 + b^2]*(2*d^2*
e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a
 - Sqrt[a^2 + b^2])] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*d*f
*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*d*f*(e +
f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 2*f^2*PolyLog[3
, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*f^2*PolyLog[3, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))]))/(a*b*d^3)
```

Maple [F] time = 0.594, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \cosh \left(dx+c\right) \coth \left(dx+c\right)}{a+b \sinh \left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.73447, size = 2483, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/3*(a*d^3*f^2*x^3 + 3*a*d^3*e*f*x^2 + 3*a*d^3*e^2*x + 6*b*f^2*sqrt((a^2 +
b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*b*f^2*sqrt((a^2 + b^2)/b^2
)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh
```

```
(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*b*f^2*polylog(3, cosh(d*x + c) + s
inh(d*x + c)) + 6*b*f^2*polylog(3, -cosh(d*x + c) - sinh(d*x + c)) - 6*(b*d
*f^2*x + b*d*e*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x
 + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b +
1) + 6*(b*d*f^2*x + b*d*e*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) +
 a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
) - b)/b + 1) + 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt((a^2 + b^2)/b^
2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) +
2*a) - 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(2*
b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*
(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*sqrt((a^2 + b^2)/
b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 3*(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x +
2*b*c*d*e*f - b*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*si
nh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b
)/b) + 6*(b*d*f^2*x + b*d*e*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) - 6*(b*
d*f^2*x + b*d*e*f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) - 3*(b*d^2*f^2*x^2
 + 2*b*d^2*e*f*x + b*d^2*e^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 3*(b
*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(cosh(d*x + c) + sinh(d*x + c) - 1)
+ 3*(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*log(-cosh(d*x
 + c) - sinh(d*x + c) + 1))/(a*b*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)^2 \cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*cosh(c + d*x)*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.427 $\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=286

$$-\frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^2}+\frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{abd^2}-\frac{f\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}+\frac{f\text{PolyLog}\left(2,e^{c+d\cdot}\right.}{ad^2}$$

```
[Out] (e*x)/b + (f*x^2)/(2*b) - (2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d) - (Sqrt[
a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*b*d
) + (Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)])/(a*b*d) - (f*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (f*PolyLog[2, E^(c + d
*x)])/(a*d^2) - (Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(a*b*d^2) + (Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(a*b*d^2)
```

Rubi [A] time = 0.579288, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {5585, 5450, 3296, 2637, 4182, 2279, 2391, 5565, 3322, 2264, 2190}

$$-\frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^2}+\frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{abd^2}-\frac{f\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}+\frac{f\text{PolyLog}\left(2,e^{c+d\cdot}\right.}{ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (e*x)/b + (f*x^2)/(2*b) - (2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d) - (Sqrt[
a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*b*d
) + (Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)])/(a*b*d) - (f*PolyLog[2, -E^(c + d*x)])/(a*d^2) + (f*PolyLog[2, E^(c + d
*x)])/(a*d^2) - (Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(a*b*d^2) + (Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(a*b*d^2)
```


Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh(c+dx)\coth(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cosh^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$
$$= \frac{\int (e+fx)\text{csch}(c+dx)\,dx}{a} + \frac{\int (e+fx)\,dx}{b} - \frac{(a^2+b^2)\int \frac{e+fx}{a+b\sinh(c+dx)}\,dx}{ab}$$
$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\left(2\left(a^2+b^2\right)\right)\int \frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{ab}$$
$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\left(2\sqrt{a^2+b^2}\right)\int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}}\,dx}{a}$$
$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd}$$
$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd}$$
$$= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd}$$

Mathematica [A] time = 1.83532, size = 339, normalized size = 1.19

$$2\sqrt{a^2+b^2}\left(-f\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-f(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-(a*(c + d*x)*(c*f - d*(2*e + f*x))) + 2*b*d*e*Log[Tanh[(c + d*x)/2]] - 2*
b*c*f*Log[Tanh[(c + d*x)/2]] + 2*b*f*((c + d*x)*(Log[1 - E^(-c - d*x)] - Lo
g[1 + E^(-c - d*x)]) + PolyLog[2, -E^(-c - d*x)] - PolyLog[2, E^(-c - d*x)]
) + 2*Sqrt[a^2 + b^2]*(2*d*e*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] -
 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - f*(c + d*x)*Log[1 + (
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2])] - f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2
])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(2*a*b*d^2)
```

Maple [B] time = 0.187, size = 970, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/2*f*x^2/b+e*x/b-1/d*f*b/a/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/
2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d^2*f*b/a/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)
+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d*f*b/a/(a^2+b^2)^(1/2)*ln((b
*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2*f*b/a/(a^2+b^2)
^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d^2*f*b
/a/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2
```

```
)))-1/d/a*ln(exp(d*x+c)+1)*f*x-1/d^2*f*b/a/(a^2+b^2)^(1/2)*dilog((-b*exp(d*
x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2/a*f*c*ln(exp(d*x+c)-1)-
2*a/b/d^2*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1
/2))-2/d^2*f*c*b/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^
2)^(1/2))-a/b/d*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+
(a^2+b^2)^(1/2)))*x-a/b/d^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(
1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+a/b/d*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a
^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+a/b/d^2*f/(a^2+b^2)^(1/2)*ln((b*exp
(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d/a*e*ln(exp(d*x+c)-1)-
1/d/a*e*ln(exp(d*x+c)+1)-a/b/d^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^
2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+a/b/d^2*f/(a^2+b^2)^(1/2)*dilog((b*ex
p(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2*a/b/d*e/(a^2+b^2)^(1/2)*
arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d*e*b/a/(a^2+b^2)^(1/2)
*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d^2*f/a*dilog(exp(d*x+
c))-1/d^2*f/a*dilog(exp(d*x+c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.72461, size = 1544, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] 1/2*(a*d^2*f*x^2 + 2*a*d^2*e*x - 2*b*f*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(
d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2) - b)/b + 1) + 2*b*f*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c
) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2) - b)/b + 1) + 2*b*f*dilog(cosh(d*x + c) + sinh(d*x + c)) - 2*b*f*dilog
(-cosh(d*x + c) - sinh(d*x + c)) + 2*(b*d*e - b*c*f)*sqrt((a^2 + b^2)/b^2)*
log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a
) - 2*(b*d*e - b*c*f)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sin
h(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(b*d*f*x + b*c*f)*sqrt((a
^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(b*d*f*x + b*c*f)*sqrt(
(a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(b*d*f*x + b*d*e)*log
(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*(b*d*e - b*c*f)*log(cosh(d*x + c) +
 sinh(d*x + c) - 1) + 2*(b*d*f*x + b*c*f)*log(-cosh(d*x + c) - sinh(d*x + c
) + 1))/(a*b*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*cosh(c + d*x)*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.428 $\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=71

$$\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{abd}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{x}{b}$$

```
[Out] x/b - ArcTanh[Cosh[c + d*x]]/(a*d) + (2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh
[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a*b*d)
```

Rubi [A] time = 0.212942, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2889, 3058, 2660, 618, 204, 3770}

$$\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{abd}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] x/b - ArcTanh[Cosh[c + d*x]]/(a*d) + (2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh
[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a*b*d)
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3058

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(C*x
)/(b*d), x] + (Dist[(A*b^2 + a^2*C)/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f
*x]), x], x] - Dist[(c^2*C + A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f
*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx &= \int \frac{\operatorname{csch}(c+dx)\left(1+\sinh^2(c+dx)\right)}{a+b\sinh(c+dx)}\,dx \\
&= \frac{x}{b}+\frac{\int \operatorname{csch}(c+dx)\,dx}{a}-\frac{\left(a^2+b^2\right)\int \frac{1}{a+b\sinh(c+dx)}\,dx}{ab} \\
&= \frac{x}{b}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{\left(2i\left(a^2+b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx\right.\right.}{abd} \\
&= \frac{x}{b}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}-\frac{\left(4i\left(a^2+b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^2+b^2\right)-x^2}\,dx,x,-2ib+2a\tan\right.}{abd} \\
&= \frac{x}{b}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{abd}
\end{aligned}$$

Mathematica [A]   time = 0.146125, size = 80, normalized size = 1.13

$$\frac{2\sqrt{-a^2-b^2}\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)+a(c+dx)+b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(c + d*x) + 2*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^
2 - b^2]] + b*Log[Tanh[(c + d*x)/2]])/(a*b*d)
```

Maple [B]   time = 0.002, size = 150, normalized size = 2.1

$$\frac{1}{bd}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{1}{bd}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\frac{a}{bd\sqrt{a^2+b^2}}\operatorname{Artanh}\left(1/2\,\frac{2\,a\tanh\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/d/b*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/b*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a*ln(ta
nh(1/2*d*x+1/2*c))-2/d*a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/
2*c)-2*b)/(a^2+b^2)^(1/2))-2/d*b/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/
```

```
2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.58144, size = 559, normalized size = 7.87

$$\frac{adx - b\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right) + b\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right) + \sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(dx+c)^2 \cdots}{\cdots}\right)}{abd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a*d*x - b*log(cosh(d*x + c) + sinh(d*x + c) + 1) + b*log(cosh(d*x + c) + s
inh(d*x + c) - 1) + sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x
 + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*s
inh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(
b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x +
 c) + a)*sinh(d*x + c) - b)))/(a*b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(c+dx\right)\coth\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(cosh(c + d*x)*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [B] time = 2.39724, size = 188, normalized size = 2.65

$$\frac{\frac{dx}{b} - \frac{\left(a^2e^c+b^2e^c\right)e^{(-c)}\log\left(\frac{\left|2\,be^{(dx+2\,c)}+2\,ae^c-2\,\sqrt{a^2+b^2}e^c\right|}{\left|2\,be^{(dx+2\,c)}+2\,ae^c+2\,\sqrt{a^2+b^2}e^c\right|}\right)}{\sqrt{a^2+b^2}ab} - \frac{\log\left(e^{(dx+c)}+1\right)}{a} + \frac{\log\left(\left|e^{(dx+c)}-1\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (d*x/b - (a^2*e^c + b^2*e^c)*e^(-c)*log(abs(2*b*e^(d*x + 2*c) + 2*a*e^c - 2
*sqrt(a^2 + b^2)*e^c)/abs(2*b*e^(d*x + 2*c) + 2*a*e^c + 2*sqrt(a^2 + b^2)*e
^c))/(sqrt(a^2 + b^2)*a*b) - log(e^(d*x + c) + 1)/a + log(abs(e^(d*x + c) -
 1))/a)/d
```

3.429 $\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$

Optimal. Leaf size=34

$$\text{Unintegrable}\left(\frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Cosh[c + d*x]*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [A] time = 0.059309, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[c + d*x]*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Cosh[c + d*x]*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

Rubi steps

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Mathematica [A] time = 57.9944, size = 0, normalized size = 0.

$$\int \frac{\cosh(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[c + d*x]*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x
]
```

```
[Out] Integrate[(Cosh[c + d*x]*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

Maple [A] time = 0.826, size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)\coth(dx+c)}{(fx+e)(a+b\sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-2\left(a^2e^c+b^2e^c\right)\int-\frac{e^{(dx)}}{ab^2fx+ab^2e-\left(ab^2fxe^{(2c)}+ab^2ee^{(2c)}\right)e^{(2dx)}-2\left(a^2bfxe^c+a^2bee^c\right)e^{(dx)}}\,dx+\frac{\log\left(fx+e\right)}{bf}+\int\frac{}{afx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] -2*(a^2*e^c + b^2*e^c)*integrate(-e^(d*x)/(a*b^2*f*x + a*b^2*e - (a*b^2*f*x
*e^(2*c) + a*b^2*e*e^(2*c))*e^(2*d*x) - 2*(a^2*b*f*x*e^c + a^2*b*e*e^c)*e^(
d*x)), x) + log(f*x + e)/(b*f) + integrate(1/(a*f*x + a*e + (a*f*x*e^c + a*
e*e^c)*e^(d*x)), x) + integrate(-1/(a*f*x + a*e - (a*f*x*e^c + a*e*e^c)*e^(
d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cosh\left(dx+c\right)\coth\left(dx+c\right)}{afx+ae+\left(bfx+be\right)\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] integral(cosh(d*x + c)*coth(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
+ c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh\left(c+dx\right)\coth\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(cosh(c + d*x)*coth(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.430 $\int \frac{(e+fx)^3 \cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=656

$$\frac{6f^2\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^3}+\frac{6f^2\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ab^2d^3}-\frac{3f\left(a^2+b^2\right)(e+fx)^2\text{I}}{ab}$$

```
[Out] -(e + f*x)^4/(4*a*f) + ((a^2 + b^2)*(e + f*x)^4)/(4*a*b^2*f) - (6*f^3*Cosh[
c + d*x])/(b*d^4) - (3*f*(e + f*x)^2*Cosh[c + d*x])/(b*d^2) - ((a^2 + b^2)*
(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*b^2*d) - ((a
^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*b^
2*d) + ((e + f*x)^3*Log[1 - E^(2*(c + d*x))])/(a*d) - (3*(a^2 + b^2)*f*(e +
 f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b^2*d^2) -
 (3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))])/(a*b^2*d^2) + (3*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*x))])/(2*a*d
^2) + (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(a*b^2*d^3) + (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*b^2*d^3) - (3*f^2*(e + f*x)*PolyLog[
3, E^(2*(c + d*x))])/(2*a*d^3) - (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b^2*d^4) - (6*(a^2 + b^2)*f^3*PolyLog[4,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*b^2*d^4) + (3*f^3*PolyLog[4,
E^(2*(c + d*x))])/(4*a*d^4) + (6*f^2*(e + f*x)*Sinh[c + d*x])/(b*d^3) + ((e
 + f*x)^3*Sinh[c + d*x])/(b*d)
```

Rubi [A] time = 1.22967, antiderivative size = 656, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 17, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5585, 5450, 5446, 3311, 32, 2635, 8, 3716, 2190, 2531, 6609, 2282, 6589, 5565, 3296, 2638, 5561}

$$\frac{6f^2\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^3}+\frac{6f^2\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ab^2d^3}-\frac{3f\left(a^2+b^2\right)(e+fx)^2\text{I}}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -(e + f*x)^4/(4*a*f) + ((a^2 + b^2)*(e + f*x)^4)/(4*a*b^2*f) - (6*f^3*Cosh[
c + d*x])/(b*d^4) - (3*f*(e + f*x)^2*Cosh[c + d*x])/(b*d^2) - ((a^2 + b^2)*
(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*b^2*d) - ((a
^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*b^
2*d) + ((e + f*x)^3*Log[1 - E^(2*(c + d*x))])/(a*d) - (3*(a^2 + b^2)*f*(e +
 f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b^2*d^2) -
 (3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))])/(a*b^2*d^2) + (3*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*x))])/(2*a*d
^2) + (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(a*b^2*d^3) + (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*b^2*d^3) - (3*f^2*(e + f*x)*PolyLog[
3, E^(2*(c + d*x))])/(2*a*d^3) - (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b^2*d^4) - (6*(a^2 + b^2)*f^3*PolyLog[4,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*b^2*d^4) + (3*f^3*PolyLog[4,
E^(2*(c + d*x))])/(4*a*d^4) + (6*f^2*(e + f*x)*Sinh[c + d*x])/(b*d^3) + ((e
 + f*x)^3*Sinh[c + d*x])/(b*d)
```


Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
```

```
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3 \cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\cosh^2(c+dx)\coth(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)^3\coth(c+dx)\,dx}{a} + \frac{\int (e+fx)^3\cosh(c+dx)\,dx}{b} - \frac{\left(a^2+b^2\right)\int}{}$$

$$= -\frac{(e+fx)^4}{4af} + \frac{\left(a^2+b^2\right)(e+fx)^4}{4ab^2f} + \frac{(e+fx)^3\sinh(c+dx)}{bd} - \frac{2\int \frac{e^{2(c+dx)}(}{1-e^{2(}}}{a}$$

$$= -\frac{(e+fx)^4}{4af} + \frac{\left(a^2+b^2\right)(e+fx)^4}{4ab^2f} - \frac{3f(e+fx)^2\cosh(c+dx)}{bd^2} - \frac{\left(a^2+b\right.}{}$$

$$= -\frac{(e+fx)^4}{4af} + \frac{\left(a^2+b^2\right)(e+fx)^4}{4ab^2f} - \frac{3f(e+fx)^2\cosh(c+dx)}{bd^2} - \frac{\left(a^2+b\right.}{}$$

$$= -\frac{(e+fx)^4}{4af} + \frac{\left(a^2+b^2\right)(e+fx)^4}{4ab^2f} - \frac{6f^3\cosh(c+dx)}{bd^4} - \frac{3f(e+fx)^2\cos}{bd^2}$$

$$= -\frac{(e+fx)^4}{4af} + \frac{\left(a^2+b^2\right)(e+fx)^4}{4ab^2f} - \frac{6f^3\cosh(c+dx)}{bd^4} - \frac{3f(e+fx)^2\cos}{bd^2}$$

$$= -\frac{(e+fx)^4}{4af} + \frac{\left(a^2+b^2\right)(e+fx)^4}{4ab^2f} - \frac{6f^3\cosh(c+dx)}{bd^4} - \frac{3f(e+fx)^2\cos}{bd^2}$$

Mathematica [B] time = 18.4519, size = 2574, normalized size = 3.92

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] ((a^2 + b^2)*E^(2*c)*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(2*a*b^
2*(-1 + E^(2*c))) - (E^(2*c)*((e + f*x)^4/(E^(2*c)*f) - (2*(1 - E^(-2*c))*(
e + f*x)^3*Log[1 - E^(-c - d*x)])/d - (2*(1 - E^(-2*c))*(e + f*x)^3*Log[1 +
 E^(-c - d*x)])/d + (6*(-1 + E^(2*c))*f*(d^2*(e + f*x)^2*PolyLog[2, -E^(-c
- d*x)] + 2*f*(d*(e + f*x)*PolyLog[3, -E^(-c - d*x)] + f*PolyLog[4, -E^(-c
- d*x)])))/(d^4*E^(2*c)) + (6*(-1 + E^(2*c))*f*(d^2*(e + f*x)^2*PolyLog[2,
E^(-c - d*x)] + 2*f*(d*(e + f*x)*PolyLog[3, E^(-c - d*x)] + f*PolyLog[4, E^
(-c - d*x)])))/(d^4*E^(2*c))))/(2*a*(-1 + E^(2*c))) - ((a^2 + b^2)*(2*a*Sqr
t[a^2 + b^2]*d^3*e^3*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]] + 2*a*Sqr
t[-a^2 - b^2]*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + Sqrt[-
(a^2 + b^2)^2]*d^3*e^3*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] + 3*
Sqrt[-(a^2 + b^2)^2]*d^3*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a
^2 + b^2)*E^(2*c)])] + 3*Sqrt[-(a^2 + b^2)^2]*d^3*e*f^2*x^2*Log[1 + (b*E^(2
*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + Sqrt[-(a^2 + b^2)^2]*d^3*
f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 3*
Sqrt[-(a^2 + b^2)^2]*d^3*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a
^2 + b^2)*E^(2*c)])] + 3*Sqrt[-(a^2 + b^2)^2]*d^3*e*f^2*x^2*Log[1 + (b*E^(2
*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + Sqrt[-(a^2 + b^2)^2]*d^3*
```

```
f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] + 3*
Sqrt[-(a^2 + b^2)^2]*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^
c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 3*Sqrt[-(a^2 + b^2)^2]*d^2*f*(e + f*x)^2
*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*S
qrt[-(a^2 + b^2)^2]*d*e*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a
^2 + b^2)*E^(2*c)]))] - 6*Sqrt[-(a^2 + b^2)^2]*d*f^3*x*PolyLog[3, -((b*E^(2
*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*Sqrt[-(a^2 + b^2)^2]*d
*e*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]
 - 6*Sqrt[-(a^2 + b^2)^2]*d*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + S
qrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[-(a^2 + b^2)^2]*f^3*PolyLog[4, -((b*E^
(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*Sqrt[-(a^2 + b^2)^2]
*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))
/(a*b^2*Sqrt[-(a^2 + b^2)^2]*d^4) + Csch[c]*(Cosh[c + d*x]/(8*b^2*d^4) - Si
nh[c + d*x]/(8*b^2*d^4))*(-4*a*d^4*e^3*x*Cosh[d*x] - 6*a*d^4*e^2*f*x^2*Cosh
[d*x] - 4*a*d^4*e*f^2*x^3*Cosh[d*x] - a*d^4*f^3*x^4*Cosh[d*x] - 4*a*d^4*e^3
*x*Cosh[2*c + d*x] - 6*a*d^4*e^2*f*x^2*Cosh[2*c + d*x] - 4*a*d^4*e*f^2*x^3*
Cosh[2*c + d*x] - a*d^4*f^3*x^4*Cosh[2*c + d*x] - 2*b*d^3*e^3*Cosh[c + 2*d*
x] + 6*b*d^2*e^2*f*Cosh[c + 2*d*x] - 12*b*d*e*f^2*Cosh[c + 2*d*x] + 12*b*f^
3*Cosh[c + 2*d*x] - 6*b*d^3*e^2*f*x*Cosh[c + 2*d*x] + 12*b*d^2*e*f^2*x*Cosh
[c + 2*d*x] - 12*b*d*f^3*x*Cosh[c + 2*d*x] - 6*b*d^3*e*f^2*x^2*Cosh[c + 2*d
*x] + 6*b*d^2*f^3*x^2*Cosh[c + 2*d*x] - 2*b*d^3*f^3*x^3*Cosh[c + 2*d*x] + 2
*b*d^3*e^3*Cosh[3*c + 2*d*x] - 6*b*d^2*e^2*f*Cosh[3*c + 2*d*x] + 12*b*d*e*f
^2*Cosh[3*c + 2*d*x] - 12*b*f^3*Cosh[3*c + 2*d*x] + 6*b*d^3*e^2*f*x*Cosh[3*
c + 2*d*x] - 12*b*d^2*e*f^2*x*Cosh[3*c + 2*d*x] + 12*b*d*f^3*x*Cosh[3*c + 2
*d*x] + 6*b*d^3*e*f^2*x^2*Cosh[3*c + 2*d*x] - 6*b*d^2*f^3*x^2*Cosh[3*c + 2*
d*x] + 2*b*d^3*f^3*x^3*Cosh[3*c + 2*d*x] - 4*b*d^3*e^3*Sinh[c] - 12*b*d^2*e
^2*f*Sinh[c] - 24*b*d*e*f^2*Sinh[c] - 24*b*f^3*Sinh[c] - 12*b*d^3*e^2*f*x*S
inh[c] - 24*b*d^2*e*f^2*x*Sinh[c] - 24*b*d*f^3*x*Sinh[c] - 12*b*d^3*e*f^2*x
^2*Sinh[c] - 12*b*d^2*f^3*x^2*Sinh[c] - 4*b*d^3*f^3*x^3*Sinh[c] - 4*a*d^4*e
^3*x*Sinh[d*x] - 6*a*d^4*e^2*f*x^2*Sinh[d*x] - 4*a*d^4*e*f^2*x^3*Sinh[d*x]
- a*d^4*f^3*x^4*Sinh[d*x] - 4*a*d^4*e^3*x*Sinh[2*c + d*x] - 6*a*d^4*e^2*f*x
^2*Sinh[2*c + d*x] - 4*a*d^4*e*f^2*x^3*Sinh[2*c + d*x] - a*d^4*f^3*x^4*Sinh
[2*c + d*x] - 2*b*d^3*e^3*Sinh[c + 2*d*x] + 6*b*d^2*e^2*f*Sinh[c + 2*d*x] -
 12*b*d*e*f^2*Sinh[c + 2*d*x] + 12*b*f^3*Sinh[c + 2*d*x] - 6*b*d^3*e^2*f*x*
Sinh[c + 2*d*x] + 12*b*d^2*e*f^2*x*Sinh[c + 2*d*x] - 12*b*d*f^3*x*Sinh[c +
2*d*x] - 6*b*d^3*e*f^2*x^2*Sinh[c + 2*d*x] + 6*b*d^2*f^3*x^2*Sinh[c + 2*d*x
] - 2*b*d^3*f^3*x^3*Sinh[c + 2*d*x] + 2*b*d^3*e^3*Sinh[3*c + 2*d*x] - 6*b*d
^2*e^2*f*Sinh[3*c + 2*d*x] + 12*b*d*e*f^2*Sinh[3*c + 2*d*x] - 12*b*f^3*Sinh
[3*c + 2*d*x] + 6*b*d^3*e^2*f*x*Sinh[3*c + 2*d*x] - 12*b*d^2*e*f^2*x*Sinh[3
*c + 2*d*x] + 12*b*d*f^3*x*Sinh[3*c + 2*d*x] + 6*b*d^3*e*f^2*x^2*Sinh[3*c +
 2*d*x] - 6*b*d^2*f^3*x^2*Sinh[3*c + 2*d*x] + 2*b*d^3*f^3*x^3*Sinh[3*c + 2*
d*x])
```

Maple [F] time = 1.22, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3} \left(\cosh \left(dx+c\right)\right)^{2} \coth \left(dx+c\right)}{a+b\sinh \left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/2*e^3*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) -
2*log(e^(-d*x - c) + 1)/(a*d) - 2*log(e^(-d*x - c) - 1)/(a*d) + 2*(a^2 + b^
2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*b^2*d)) + 3*(d*x*log(
e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d*x*log(-e^(d*x
+ c) + 1) + dilog(e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d^2*x^2*log(e^(d*x + c)
+ 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*e*f^2/(a*d^3
) + 3*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog
(3, e^(d*x + c)))*e*f^2/(a*d^3) + (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2
*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*
x + c)))*f^3/(a*d^4) + (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(
d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*f^3/
(a*d^4) - 1/2*(d^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*d^4*e^2*f*x^2)/(a*d^4) - 1
/4*(a*d^4*f^3*x^4*e^c + 4*a*d^4*e*f^2*x^3*e^c + 6*a*d^4*e^2*f*x^2*e^c - 2*(
b*d^3*f^3*x^3*e^(2*c) + 3*(d^3*e*f^2 - d^2*f^3)*b*x^2*e^(2*c) + 3*(d^3*e^2*
f - 2*d^2*e*f^2 + 2*d*f^3)*b*x*e^(2*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*
b*e^(2*c))*e^(d*x) + 2*(b*d^3*f^3*x^3 + 3*(d^3*e*f^2 + d^2*f^3)*b*x^2 + 3*(
d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*b*x + 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*
b)*e^(-d*x))*e^(-c)/(b^2*d^4) + integrate(-2*((a^2*b*f^3 + b^3*f^3)*x^3 + 3
*(a^2*b*e*f^2 + b^3*e*f^2)*x^2 + 3*(a^2*b*e^2*f + b^3*e^2*f)*x - ((a^3*f^3*
e^c + a*b^2*f^3*e^c)*x^3 + 3*(a^3*e*f^2*e^c + a*b^2*e*f^2*e^c)*x^2 + 3*(a^3
*e^2*f*e^c + a*b^2*e^2*f*e^c)*x)*e^(d*x))/(a*b^3*e^(2*d*x + 2*c) + 2*a^2*b^
2*e^(d*x + c) - a*b^3), x)
```

Fricas [C] time = 3.65833, size = 7715, normalized size = 11.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/4*(2*a*b*d^3*f^3*x^3 + 2*a*b*d^3*e^3 + 6*a*b*d^2*e^2*f + 12*a*b*d*e*f^2
+ 12*a*b*f^3 + 6*(a*b*d^3*e*f^2 + a*b*d^2*f^3)*x^2 - 2*(a*b*d^3*f^3*x^3 + a
*b*d^3*e^3 - 3*a*b*d^2*e^2*f + 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b*d^3*e*f^2
 - a*b*d^2*f^3)*x^2 + 3*(a*b*d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x)*
cosh(d*x + c)^2 - 2*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3 - 3*a*b*d^2*e^2*f + 6*a*
b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(a*b*d^3*e^
2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x)*sinh(d*x + c)^2 + 6*(a*b*d^3*e^2*f
+ 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x - (a^2*d^4*f^3*x^4 + 4*a^2*d^4*e*f^2*x^3
 + 6*a^2*d^4*e^2*f*x^2 + 4*a^2*d^4*e^3*x + 8*a^2*c*d^3*e^3 - 12*a^2*c^2*d^2
*e^2*f + 8*a^2*c^3*d*e*f^2 - 2*a^2*c^4*f^3)*cosh(d*x + c) + 12*(((a^2 + b^2
)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*cosh(d*x
 + c) + ((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*
d^2*e^2*f)*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*(((a^2
 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*co
sh(d*x + c) + ((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 +
```

b^2)*d^2*e^2*f)*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 12*((b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*cosh(d*x + c) + (b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)) - 12*((b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*cosh(d*x + c) + (b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*sinh(d*x + c))*dilog(-cosh(d*x + c) - sinh(d*x + c)) + 4*(((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*cosh(d*x + c) + ((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 4*(((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*cosh(d*x + c) + ((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 4*(((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*f^3)*cosh(d*x + c) + ((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*f^3)*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*(((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*f^3)*cosh(d*x + c) + ((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*f^3)*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 4*((b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + b^2*d^3*e^3)*cosh(d*x + c) + (b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + b^2*d^3*e^3)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 4*((b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*cosh(d*x + c) + (b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) - 4*((b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*cosh(d*x + c) + (b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*sinh(d*x + c))*log(-cosh(d*x + c) - sinh(d*x + c) + 1) + 24*((a^2 + b^2)*f^3*cosh(d*x + c) + (a^2 + b^2)*f^3*sinh(d*x + c))*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*((a^2 + b^2)*f^3*cosh(d*x + c) + (a^2 + b^2)*f^3*sinh(d*x + c))*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 24*(b^2*f^3*cosh(d*x + c) + b^2*f^3*sinh(d*x + c))*polylog(4, cosh(d*x + c) + sinh(d*x + c)) - 24*(b^2*f^3*cosh(d*x + c) + b^2*f^3*sinh(d*x + c))*polylog(4, -cosh(d*x + c) - sinh(d*x + c)) - 24*(((a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*cosh(d*x + c) + ((a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*sinh(d*x + c))*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 24*(((a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*cosh(d*x + c) + ((a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*sinh(d*x + c))*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*((b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c) + (b^2*d*f^3*x + b^2*d*e*f^2)*sinh(d*x + c))*polylog(3, cosh(d*x + c) + sinh(d*x + c)) + 24*((b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c) + (b^2*d*f^3*x + b^2*d*e*f^2)*sinh(d*x + c))*polylog(3, -cosh(d*x + c) - sinh(d*x + c)) - (a^2*d^4*f^3*x^4 + 4*a^2*d^4*e*f^2*x^3 + 6*a^2*d^4*e^2*f*x^2 + 4*a^2*d^4*e^3*x + 8*a^2*c*d^3*e^3 - 12*a^2*c^2*d^2*e^2*f + 8*a^2*c^3*d*e*f^2 - 2*a^2*c^4*f^3 + 4*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3 - 3*a*b*d^2*e^2*f + 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(a*b*d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d*f

```
^3)*x)*cosh(d*x + c))*sinh(d*x + c))/(a*b^2*d^4*cosh(d*x + c) + a*b^2*d^4*s
inh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.431 $\int \frac{(e+fx)^2 \cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=486

$$-\frac{2f\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^2}-\frac{2f\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ab^2d^2}+\frac{2f^2\left(a^2+b^2\right)\text{PolyLog}}{ab^2d^3}$$

```
[Out] -(e + f*x)^3/(3*a*f) + ((a^2 + b^2)*(e + f*x)^3)/(3*a*b^2*f) - (2*f*(e + f*
x)*Cosh[c + d*x])/(b*d^2) - ((a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2])])/(a*b^2*d) - ((a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*b^2*d) + ((e + f*x)^2*Log[1 - E^(2*(
c + d*x))])/(a*d) - (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/(a*b^2*d^2) - (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[
2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*b^2*d^2) + (f*(e + f*x)*Po
lyLog[2, E^(2*(c + d*x))])/(a*d^2) + (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b^2*d^3) + (2*(a^2 + b^2)*f^2*PolyLog
[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*b^2*d^3) - (f^2*PolyLog[3
, E^(2*(c + d*x))])/(2*a*d^3) + (2*f^2*Sinh[c + d*x])/(b*d^3) + ((e + f*x)^
2*Sinh[c + d*x])/(b*d)
```

Rubi [A] time = 1.02531, antiderivative size = 486, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 13, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.382, Rules used = {5585, 5450, 5446, 3310, 3716, 2190, 2531, 2282, 6589, 5565, 3296, 2637, 5561}

$$-\frac{2f\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^2}-\frac{2f\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ab^2d^2}+\frac{2f^2\left(a^2+b^2\right)\text{PolyLog}}{ab^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -(e + f*x)^3/(3*a*f) + ((a^2 + b^2)*(e + f*x)^3)/(3*a*b^2*f) - (2*f*(e + f*
x)*Cosh[c + d*x])/(b*d^2) - ((a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2])])/(a*b^2*d) - ((a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*b^2*d) + ((e + f*x)^2*Log[1 - E^(2*(
c + d*x))])/(a*d) - (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/(a*b^2*d^2) - (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[
2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*b^2*d^2) + (f*(e + f*x)*Po
lyLog[2, E^(2*(c + d*x))])/(a*d^2) + (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b^2*d^3) + (2*(a^2 + b^2)*f^2*PolyLog
[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*b^2*d^3) - (f^2*PolyLog[3
, E^(2*(c + d*x))])/(2*a*d^3) + (2*f^2*Sinh[c + d*x])/(b*d^3) + ((e + f*x)^
2*Sinh[c + d*x])/(b*d)
```


Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh^2(c+dx)\coth(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)^2\coth(c+dx)\,dx}{a} + \frac{\int (e+fx)^2\cosh(c+dx)\,dx}{b} - \frac{\left(a^2+b^2\right)\int \frac{(e+\cdots}{\cdots}}{\cdots}$$

$$= -\frac{(e+fx)^3}{3af} + \frac{\left(a^2+b^2\right)(e+fx)^3}{3ab^2f} + \frac{(e+fx)^2\sinh(c+dx)}{bd} - \frac{2\int \frac{e^{2(c+dx)}(e+f\cdots}{1-e^{2(c+dx)}}}{a}$$

$$= -\frac{(e+fx)^3}{3af} + \frac{\left(a^2+b^2\right)(e+fx)^3}{3ab^2f} - \frac{2f(e+fx)\cosh(c+dx)}{bd^2} - \frac{\left(a^2+b^2\right)(e\cdots}{\cdots}$$

$$= -\frac{(e+fx)^3}{3af} + \frac{\left(a^2+b^2\right)(e+fx)^3}{3ab^2f} - \frac{2f(e+fx)\cosh(c+dx)}{bd^2} - \frac{\left(a^2+b^2\right)(e\cdots}{\cdots}$$

$$= -\frac{(e+fx)^3}{3af} + \frac{\left(a^2+b^2\right)(e+fx)^3}{3ab^2f} - \frac{2f(e+fx)\cosh(c+dx)}{bd^2} - \frac{\left(a^2+b^2\right)(e\cdots}{\cdots}$$

$$= -\frac{(e+fx)^3}{3af} + \frac{\left(a^2+b^2\right)(e+fx)^3}{3ab^2f} - \frac{2f(e+fx)\cosh(c+dx)}{bd^2} - \frac{\left(a^2+b^2\right)(e\cdots}{\cdots}$$

Mathematica [B] time = 14.3014, size = 1521, normalized size = 3.13

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] -(E^(2*c)*((2*(e + f*x)^3)/(E^(2*c)*f) - (3*(1 - E^(-2*c))*(e + f*x)^2*Log[
1 - E^(-c - d*x)])/d - (3*(1 - E^(-2*c))*(e + f*x)^2*Log[1 + E^(-c - d*x)])
/d + (6*(-1 + E^(2*c))*f*(d*(e + f*x)*PolyLog[2, -E^(-c - d*x)] + f*PolyLog
[3, -E^(-c - d*x)]))/(d^3*E^(2*c)) + (6*(-1 + E^(2*c))*f*(d*(e + f*x)*PolyL
og[2, E^(-c - d*x)] + f*PolyLog[3, E^(-c - d*x)]))/(d^3*E^(2*c))))/(3*a*(-1
 + E^(2*c))) + ((a^2 + b^2)*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c
)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 -
 b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*Ar
cTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sq
rt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2
 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(
c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c +
d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) +
b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c -
Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))
/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d
*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (
b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 +
 (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f
*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f
^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d -
(3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2
*c)])])/d - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a
*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*Po
lyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (
6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])
/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b
^2)*E^(2*c)]))])/d^3 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[
(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x)
)/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3))/(3*a*b^2*(-1 + E^(2*c))) - (
a*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Cosh[c]*Csch[c/2]*Sech[c/2])/(6*b^2) + (Cos
h[d*x]*(-2*d*e*f*Cosh[c] - 2*d*f^2*x*Cosh[c] + d^2*e^2*Sinh[c] + 2*f^2*Sinh
[c] + 2*d^2*e*f*x*Sinh[c] + d^2*f^2*x^2*Sinh[c]))/(b*d^3) + ((d^2*e^2*Cosh[
c] + 2*f^2*Cosh[c] + 2*d^2*e*f*x*Cosh[c] + d^2*f^2*x^2*Cosh[c] - 2*d*e*f*Si
nh[c] - 2*d*f^2*x*Sinh[c])*Sinh[d*x])/(b*d^3)
```

Maple [F] time = 1.063, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \left(\cosh \left(dx+c\right)\right)^{2} \coth \left(dx+c\right)}{a+b\sinh \left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -1/2*e^2*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) -
2*log(e^(-d*x - c) + 1)/(a*d) - 2*log(e^(-d*x - c) - 1)/(a*d) + 2*(a^2 + b^
2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*b^2*d)) + 2*(d*x*log(
e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*e*f/(a*d^2) + 2*(d*x*log(-e^(d*x +
c) + 1) + dilog(e^(d*x + c)))*e*f/(a*d^2) + (d^2*x^2*log(e^(d*x + c) + 1) +
 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2/(a*d^3) + (d^2
*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x
 + c)))*f^2/(a*d^3) - 2/3*(d^3*f^2*x^3 + 3*d^3*e*f*x^2)/(a*d^3) - 1/6*(2*a*
d^3*f^2*x^3*e^c + 6*a*d^3*e*f*x^2*e^c - 3*(b*d^2*f^2*x^2*e^(2*c) + 2*(d^2*e
*f - d*f^2)*b*x*e^(2*c) - 2*(d*e*f - f^2)*b*e^(2*c))*e^(d*x) + 3*(b*d^2*f^2
*x^2 + 2*(d^2*e*f + d*f^2)*b*x + 2*(d*e*f + f^2)*b)*e^(-d*x))*e^(-c)/(b^2*d
^3) + integrate(-2*((a^2*b*f^2 + b^3*f^2)*x^2 + 2*(a^2*b*e*f + b^3*e*f)*x -
 ((a^3*f^2*e^c + a*b^2*f^2*e^c)*x^2 + 2*(a^3*e*f*e^c + a*b^2*e*f*e^c)*x)*e^
(d*x))/(a*b^3*e^(2*d*x + 2*c) + 2*a^2*b^2*e^(d*x + c) - a*b^3), x)
```

Fricas [C] time = 2.86187, size = 5030, normalized size = 10.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/6*(3*a*b*d^2*f^2*x^2 + 3*a*b*d^2*e^2 + 6*a*b*d*e*f + 6*a*b*f^2 - 3*(a*b*
d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*
d*f^2)*x)*cosh(d*x + c)^2 - 3*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f
+ 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*x)*sinh(d*x + c)^2 + 6*(a*b*d^2*e
*f + a*b*d*f^2)*x - 2*(a^2*d^3*f^2*x^3 + 3*a^2*d^3*e*f*x^2 + 3*a^2*d^3*e^2*
x + 6*a^2*c*d^2*e^2 - 6*a^2*c^2*d*e*f + 2*a^2*c^3*f^2)*cosh(d*x + c) + 12*(
((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c) + ((a^2 + b^2)*d*f^
2*x + (a^2 + b^2)*d*e*f)*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x
 + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b +
1) + 12*(((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c) + ((a^2 +
b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a
*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b + 1) - 12*((b^2*d*f^2*x + b^2*d*e*f)*cosh(d*x + c) + (b^2*d*f^2*x +
b^2*d*e*f)*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)) - 12*((b^2*d
*f^2*x + b^2*d*e*f)*cosh(d*x + c) + (b^2*d*f^2*x + b^2*d*e*f)*sinh(d*x + c)
)*dilog(-cosh(d*x + c) - sinh(d*x + c)) + 6*(((a^2 + b^2)*d^2*e^2 - 2*(a^2
+ b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*cosh(d*x + c) + ((a^2 + b^2)*d^2*e^2
- 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*sinh(d*x + c))*log(2*b*cosh(
d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 6*(((a^2
+ b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)
 + ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*sinh
(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)
/b^2) + 2*a) + 6*(((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a
^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*cosh(d*x + c) + ((a^2 + b^2)*d^2*f
^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*
f^2)*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
 c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 6*(((a^2 + b^2)*d^2*
f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2
*f^2)*cosh(d*x + c) + ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x +
2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*sinh(d*x + c))*log(-(a*cosh(d*
```

```
x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b) - 6*((b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + b^2*d^2*e^2)*co
sh(d*x + c) + (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + b^2*d^2*e^2)*sinh(d*x +
c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 6*((b^2*d^2*e^2 - 2*b^2*c*d*e*
f + b^2*c^2*f^2)*cosh(d*x + c) + (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2
)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) - 6*((b^2*d^2*f^2*x
^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*cosh(d*x + c) + (b^2*d^
2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*sinh(d*x + c))*l
og(-cosh(d*x + c) - sinh(d*x + c) + 1) - 12*((a^2 + b^2)*f^2*cosh(d*x + c)
+ (a^2 + b^2)*f^2*sinh(d*x + c))*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x +
 c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 12*((
a^2 + b^2)*f^2*cosh(d*x + c) + (a^2 + b^2)*f^2*sinh(d*x + c))*polylog(3, (a
*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2))/b) + 12*(b^2*f^2*cosh(d*x + c) + b^2*f^2*sinh(d*x + c))*
polylog(3, cosh(d*x + c) + sinh(d*x + c)) + 12*(b^2*f^2*cosh(d*x + c) + b^2
*f^2*sinh(d*x + c))*polylog(3, -cosh(d*x + c) - sinh(d*x + c)) - 2*(a^2*d^3
*f^2*x^3 + 3*a^2*d^3*e*f*x^2 + 3*a^2*d^3*e^2*x + 6*a^2*c*d^2*e^2 - 6*a^2*c^
2*d*e*f + 2*a^2*c^3*f^2 + 3*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f +
2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))/(a
*b^2*d^3*cosh(d*x + c) + a*b^2*d^3*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.432 $\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=322

$$-\frac{f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^2}-\frac{f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ab^2d^2}+\frac{f\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{2ad^2}-\frac{\left(a^2+b^2\right)(e+f\cdots}{\cdots}$$

```
[Out] -(e + f*x)^2/(2*a*f) + ((a^2 + b^2)*(e + f*x)^2)/(2*a*b^2*f) - (f*Cosh[c +
d*x])/(b*d^2) - ((a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2])])/(a*b^2*d) - ((a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2])])/(a*b^2*d) + ((e + f*x)*Log[1 - E^(2*(c + d*x))])/(a*d)
- ((a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b
^2*d^2) - ((a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)])/(a*b^2*d^2) + (f*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^2) + ((e + f*x)*Si
nh[c + d*x])/(b*d)
```

Rubi [A] time = 0.591985, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.406, Rules used = {5585, 5450, 5446, 2635, 8, 3716, 2190, 2279, 2391, 5565, 3296, 2638, 5561}

$$-\frac{f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^2}-\frac{f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ab^2d^2}+\frac{f\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{2ad^2}-\frac{\left(a^2+b^2\right)(e+f\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -(e + f*x)^2/(2*a*f) + ((a^2 + b^2)*(e + f*x)^2)/(2*a*b^2*f) - (f*Cosh[c +
d*x])/(b*d^2) - ((a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2])])/(a*b^2*d) - ((a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2])])/(a*b^2*d) + ((e + f*x)*Log[1 - E^(2*(c + d*x))])/(a*d)
- ((a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b
^2*d^2) - ((a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)])/(a*b^2*d^2) + (f*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^2) + ((e + f*x)*Si
nh[c + d*x])/(b*d)
```


Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh^2(c+dx)\coth(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cosh^3(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)\coth(c+dx)\,dx}{a} + \frac{\int (e+fx)\cosh(c+dx)\,dx}{b} - \frac{\left(a^2+b^2\right)\int \frac{(e+fx)}{a+b\,\mathrm{s}}}{ab}$$

$$= -\frac{(e+fx)^2}{2af} + \frac{\left(a^2+b^2\right)(e+fx)^2}{2ab^2f} + \frac{(e+fx)\sinh(c+dx)}{bd} - \frac{2\int \frac{e^{2(c+dx)}(e+fx)}{1-e^{2(c+dx)}}}{a}$$

$$= -\frac{(e+fx)^2}{2af} + \frac{\left(a^2+b^2\right)(e+fx)^2}{2ab^2f} - \frac{f\cosh(c+dx)}{bd^2} - \frac{\left(a^2+b^2\right)(e+fx)\log}{ab^2d}$$

$$= -\frac{(e+fx)^2}{2af} + \frac{\left(a^2+b^2\right)(e+fx)^2}{2ab^2f} - \frac{f\cosh(c+dx)}{bd^2} - \frac{\left(a^2+b^2\right)(e+fx)\log}{ab^2d}$$

$$= -\frac{(e+fx)^2}{2af} + \frac{\left(a^2+b^2\right)(e+fx)^2}{2ab^2f} - \frac{f\cosh(c+dx)}{bd^2} - \frac{\left(a^2+b^2\right)(e+fx)\log}{ab^2d}$$

Mathematica [A] time = 1.92519, size = 296, normalized size = 0.92

$$\left(a^2+b^2\right)\left(-f\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)-f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)-f(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)-f(c+dx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (-(a*b*f*Cosh[c + d*x]) + b^2*d*e*Log[Sinh[c + d*x]] - b^2*c*f*Log[Sinh[c +
 d*x]] + (b^2*f*((c + d*x)*(c + d*x + 2*Log[1 - E^(-2*(c + d*x))]) - PolyLo
g[2, E^(-2*(c + d*x))]))/2 + (a^2 + b^2)*((f*(c + d*x)^2)/2 - f*(c + d*x)*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2])] - d*e*Log[a + b*Sinh[c + d*x]] + c*f*Log[a
+ b*Sinh[c + d*x]] - f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] -
 f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) + a*b*d*(e + f*x)*
Sinh[c + d*x])/(a*b^2*d^2)
```

Maple [B] time = 0.255, size = 932, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/2*(d*f*x+d*e-f)/d^2/b*exp(d*x+c)-1/2*(d*f*x+d*e+f)/d^2/b*exp(-d*x-c)+1/2*
a*f*x^2/b^2-1/d*f/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2
)))*x-1/d^2*f/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*
c-1/d*f/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*
f/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d^2*f*c/a*
ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-a*e*x/b^2+a/b^2/d^2*f*c*ln(b*exp(2*d*
x+2*c)+2*a*exp(d*x+c)-b)-2*a/b^2/d^2*f*c*ln(exp(d*x+c))-a/b^2/d*f*ln((-b*ex
p(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-a/b^2/d^2*f*ln((-b*exp(
d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-a/b^2/d*f*ln((b*exp(d*x+c
)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-a/b^2/d^2*f*ln((b*exp(d*x+c)+(a
^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2*a/b^2/d*f*c*x-1/d^2*f/a*dilog((-b
*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*f/a*dilog((b*exp
(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/d^2*f/a*dilog(exp(d*x+c))
+1/d^2*f/a*dilog(exp(d*x+c)+1)+1/d/a*e*ln(exp(d*x+c)-1)+1/d/a*e*ln(exp(d*x+
c)+1)+a/b^2/d^2*f*c^2-a/b^2/d*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2*a/b
^2/d*e*ln(exp(d*x+c))-a/b^2/d^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a
+(a^2+b^2)^(1/2)))-a/b^2/d^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+
(a^2+b^2)^(1/2)))+1/d/a*ln(exp(d*x+c)+1)*f*x-1/d^2/a*f*c*ln(exp(d*x+c)-1)-1
/d*e/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}e\left(\frac{2(dx+c)a}{b^2d}-\frac{e^{(dx+c)}}{bd}+\frac{e^{(-dx-c)}}{bd}-\frac{2\log\left(e^{(-dx-c)}+1\right)}{ad}-\frac{2\log\left(e^{(-dx-c)}-1\right)}{ad}+\frac{2\left(a^2+b^2\right)\log\left(-2ae^{(-dx-c)}+be^{(-\cdots}\right.}{ab^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/2*e*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) - 2*
log(e^(-d*x - c) + 1)/(a*d) - 2*log(e^(-d*x - c) - 1)/(a*d) + 2*(a^2 + b^2)
*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*b^2*d)) - 1/4*f*(2*(a*d
^2*x^2*e^c - (b*d*x*e^(2*c) - b*e^(2*c))*e^(d*x) + (b*d*x + b)*e^(-d*x))*e^
(-c)/(b^2*d^2) - integrate(8*((a^3*e^c + a*b^2*e^c)*x*e^(d*x) - (a^2*b + b^
3)*x)/(a*b^3*e^(2*d*x + 2*c) + 2*a^2*b^2*e^(d*x + c) - a*b^3), x) + 4*integ
rate(x/(a*e^(d*x + c) + a), x) - 4*integrate(x/(a*e^(d*x + c) - a), x))
```

Fricas [B] time = 2.99753, size = 2799, normalized size = 8.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -1/2*(a*b*d*f*x + a*b*d*e + a*b*f - (a*b*d*f*x + a*b*d*e - a*b*f)*cosh(d*x
+ c)^2 - (a*b*d*f*x + a*b*d*e - a*b*f)*sinh(d*x + c)^2 - (a^2*d^2*f*x^2 + 2
*a^2*d^2*e*x + 4*a^2*c*d*e - 2*a^2*c^2*f)*cosh(d*x + c) + 2*((a^2 + b^2)*f*
```

```
cosh(d*x + c) + (a^2 + b^2)*f*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sin
h(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)
/b + 1) + 2*((a^2 + b^2)*f*cosh(d*x + c) + (a^2 + b^2)*f*sinh(d*x + c))*dil
og((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b^2*f*cosh(d*x + c) + b^2*f*sinh(d*
x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)) - 2*(b^2*f*cosh(d*x + c) + b^2
*f*sinh(d*x + c))*dilog(-cosh(d*x + c) - sinh(d*x + c)) + 2*(((a^2 + b^2)*d
*e - (a^2 + b^2)*c*f)*cosh(d*x + c) + ((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*s
inh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b
^2)/b^2) + 2*a) + 2*(((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c) + ((
a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*
b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(((a^2 + b^2)*d*f*x
+ (a^2 + b^2)*c*f)*cosh(d*x + c) + ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*si
nh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(((a^2 + b^2)*d*f*x + (a^
2 + b^2)*c*f)*cosh(d*x + c) + ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*sinh(d*
x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*((b^2*d*f*x + b^2*d*e)*cosh(d*
x + c) + (b^2*d*f*x + b^2*d*e)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x
+ c) + 1) - 2*((b^2*d*e - b^2*c*f)*cosh(d*x + c) + (b^2*d*e - b^2*c*f)*sinh
(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) - 2*((b^2*d*f*x + b^2*c*f
)*cosh(d*x + c) + (b^2*d*f*x + b^2*c*f)*sinh(d*x + c))*log(-cosh(d*x + c) -
 sinh(d*x + c) + 1) - (a^2*d^2*f*x^2 + 2*a^2*d^2*e*x + 4*a^2*c*d*e - 2*a^2*
c^2*f + 2*(a*b*d*f*x + a*b*d*e - a*b*f)*cosh(d*x + c))*sinh(d*x + c))/(a*b^
2*d^2*cosh(d*x + c) + a*b^2*d^2*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*cosh(c + d*x)**2*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.433 $\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=57

$$-\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{ab^2d}+\frac{\log(\sinh(c+dx))}{ad}+\frac{\sinh(c+dx)}{bd}$$

```
[Out] Log[Sinh[c + d*x]]/(a*d) - ((a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/(a*b^2*d)
 + Sinh[c + d*x]/(b*d)
```

Rubi [A] time = 0.128075, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$-\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{ab^2d}+\frac{\log(\sinh(c+dx))}{ad}+\frac{\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Log[Sinh[c + d*x]]/(a*d) - ((a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/(a*b^2*d)
 + Sinh[c + d*x]/(b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{b(-b^2-x^2)}{x(a+x)}\,dx,x,b\sinh(c+dx)\right)}{b^3d}$$
$$= -\frac{\text{Subst}\left(\int \frac{-b^2-x^2}{x(a+x)}\,dx,x,b\sinh(c+dx)\right)}{b^2d}$$
$$= -\frac{\text{Subst}\left(\int \left(-1-\frac{b^2}{ax}+\frac{a^2+b^2}{a(a+x)}\right)dx,x,b\sinh(c+dx)\right)}{b^2d}$$
$$= \frac{\log(\sinh(c+dx))}{ad} - \frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{ab^2d} + \frac{\sinh(c+dx)}{bd}$$

Mathematica [A] time = 0.0817595, size = 48, normalized size = 0.84

$$\frac{-\left(\frac{a}{b^2}+\frac{1}{a}\right)\log(a+b\sinh(c+dx)) + \frac{\log(\sinh(c+dx))}{a} + \frac{\sinh(c+dx)}{b}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (Log[Sinh[c + d*x]]/a - (a^(-1) + a/b^2)*Log[a + b*Sinh[c + d*x]] + Sinh[c
+ d*x]/b)/d
```

Maple [B] time = 0.078, size = 178, normalized size = 3.1

$$-\frac{1}{bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} + \frac{a}{b^2d}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) - \frac{1}{bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} + \frac{a}{b^2d}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/d/b/(tanh(1/2*d*x+1/2*c)+1)+1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/b/(t
anh(1/2*d*x+1/2*c)-1)+1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a*ln(tanh(1/2
*d*x+1/2*c))-1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)*b-a
)-1/d/a*ln(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)*b-a)
```

Maxima [B] time = 1.08799, size = 176, normalized size = 3.09

$$-\frac{(dx+c)a}{b^2d} + \frac{e^{(dx+c)}}{2\,bd} - \frac{e^{(-dx-c)}}{2\,bd} + \frac{\log\left(e^{(-dx-c)}+1\right)}{ad} + \frac{\log\left(e^{(-dx-c)}-1\right)}{ad} - \frac{(a^2+b^2)\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{ab^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -(d*x + c)*a/(b^2*d) + 1/2*e^(d*x + c)/(b*d) - 1/2*e^(-d*x - c)/(b*d) + log
(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d) - (a^2 + b^2)*log(-2
```

```
*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a*b^2*d)
```

Fricas [B] time = 3.20018, size = 533, normalized size = 9.35

$$\frac{2\,a^{2}dx\cosh\left(dx+c\right)+ab\cosh\left(dx+c\right)^{2}+ab\sinh\left(dx+c\right)^{2}-ab-2\left(\left(a^{2}+b^{2}\right)\cosh\left(dx+c\right)+\left(a^{2}+b^{2}\right)\sinh\left(dx+c\right.\right.}{2\left(ab^{2}d\cosh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*a^2*d*x*cosh(d*x + c) + a*b*cosh(d*x + c)^2 + a*b*sinh(d*x + c)^2 -
a*b - 2*((a^2 + b^2)*cosh(d*x + c) + (a^2 + b^2)*sinh(d*x + c))*log(2*(b*si
nh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 2*(b^2*cosh(d*x + c) +
b^2*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 2
*(a^2*d*x + a*b*cosh(d*x + c))*sinh(d*x + c))/(a*b^2*d*cosh(d*x + c) + a*b^
2*d*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^{2}\left(c+dx\right)\coth\left(c+dx\right)}{a+b\sinh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(cosh(c + d*x)**2*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A] time = 1.5483, size = 146, normalized size = 2.56

$$\frac{\frac{2\,adx}{b^{2}}+\frac{e^{\left(dx+c\right)}}{b}-\frac{e^{\left(-dx-c\right)}}{b}+\frac{2\,\log\left(e^{\left(dx+c\right)}+1\right)}{a}+\frac{2\,\log\left(\left|e^{\left(dx+c\right)}-1\right|\right)}{a}-\frac{2\left(a^{2}+b^{2}\right)\log\left(\left|be^{\left(2\,dx+2\,c\right)}+2\,ae^{\left(dx+c\right)}-b\right|\right)}{ab^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*d*x/b^2 + e^(d*x + c)/b - e^(-d*x - c)/b + 2*log(e^(d*x + c) + 1)/
a + 2*log(abs(e^(d*x + c) - 1))/a - 2*(a^2 + b^2)*log(abs(b*e^(2*d*x + 2*c)
 + 2*a*e^(d*x + c) - b))/(a*b^2))/d
```

3.434 $\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Cosh[c + d*x]^2*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [A] time = 0.0834457, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[c + d*x]^2*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Cosh[c + d*x]^2*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\cosh^2(c+dx)\coth(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.002, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[c + d*x]^2*Coth[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] $Aborted
```

Maple [A] time = 1.079, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx+c))^2\coth(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{e^{\left(-c+\frac{de}{f}\right)}E_1\left(\frac{(fx+e)d}{f}\right)}{2\,bf}-\frac{e^{\left(c-\frac{de}{f}\right)}E_1\left(-\frac{(fx+e)d}{f}\right)}{2\,bf}-\frac{a\log\left(fx+e\right)}{b^2f}+\frac{1}{4}\int\frac{8\left(a^2b+b^3-\left(a^3e^c+ab^2e^c\right)\right.}{ab^3fx+ab^3e-\left(ab^3fxe^{(2\,c)}+ab^3ee^{(2\,c)}\right)e^{(2\,dx)}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/2*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b*f) - 1/2*e^(c - d*e
/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b*f) - a*log(f*x + e)/(b^2*f) + 1/4*
integrate(8*(a^2*b + b^3 - (a^3*e^c + a*b^2*e^c)*e^(d*x))/(a*b^3*f*x + a*b^
3*e - (a*b^3*f*x*e^(2*c) + a*b^3*e*e^(2*c))*e^(2*d*x) - 2*(a^2*b^2*f*x*e^c
+ a^2*b^2*e*e^c)*e^(d*x)), x) - integrate(1/(a*f*x + a*e + (a*f*x*e^c + a*e
*e^c)*e^(d*x)), x) + integrate(-1/(a*f*x + a*e - (a*f*x*e^c + a*e*e^c)*e^(d
*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh\left(dx+c\right)^2\coth\left(dx+c\right)}{afx+ae+\left(bfx+be\right)\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(cosh(d*x + c)^2*coth(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh^2\left(c+dx\right)\coth\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(cosh(c + d*x)**2*coth(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)),
x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] sage0*x
```

3.435 $\int \frac{(e+fx)^3\mathbf{csch}(c+dx)\mathbf{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1049

result too large to display

```
[Out] (-2*b*(e + f*x)^3*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (2*(e + f*x)^3*Arc
Tanh[E^(2*c + 2*d*x)])/(a*d) - (b^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) - (b^2*(e + f*x)^3*Log[1 + (b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) + (b^2*(e + f*x)^3*Log[1 +
 E^(2*(c + d*x))])/(a*(a^2 + b^2)*d) + ((3*I)*b*f*(e + f*x)^2*PolyLog[2, (-
I)*E^(c + d*x)])/((a^2 + b^2)*d^2) - ((3*I)*b*f*(e + f*x)^2*PolyLog[2, I*E^
(c + d*x)])/((a^2 + b^2)*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^2) - (3*b^2*f*(e + f*x)^2*
PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^2) +
 (3*b^2*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*a*(a^2 + b^2)*d^2) -
 (3*f*(e + f*x)^2*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a*d^2) + (3*f*(e + f*x)^
2*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d^2) - ((6*I)*b*f^2*(e + f*x)*PolyLog[3
, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) + ((6*I)*b*f^2*(e + f*x)*PolyLog[3,
I*E^(c + d*x)])/((a^2 + b^2)*d^3) + (6*b^2*f^2*(e + f*x)*PolyLog[3, -((b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^3) + (6*b^2*f^2*(e + f
*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^
3) - (3*b^2*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*a*(a^2 + b^2)*d^
3) + (3*f^2*(e + f*x)*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a*d^3) - (3*f^2*(e +
 f*x)*PolyLog[3, E^(2*c + 2*d*x)])/(2*a*d^3) + ((6*I)*b*f^3*PolyLog[4, (-I)
*E^(c + d*x)])/((a^2 + b^2)*d^4) - ((6*I)*b*f^3*PolyLog[4, I*E^(c + d*x)])/
((a^2 + b^2)*d^4) - (6*b^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/(a*(a^2 + b^2)*d^4) - (6*b^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^4) + (3*b^2*f^3*PolyLog[4, -E^(2*(c
+ d*x))])/(4*a*(a^2 + b^2)*d^4) - (3*f^3*PolyLog[4, -E^(2*c + 2*d*x)])/(4*a
*d^4) + (3*f^3*PolyLog[4, E^(2*c + 2*d*x)])/(4*a*d^4)
```

Rubi [A] time = 1.34083, antiderivative size = 1049, normalized size of antiderivative = 1., number of steps used = 40, number of rules used = 13, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.406, Rules used = {5589, 5461, 4182, 2531, 6609, 2282, 6589, 5573, 5561, 2190, 6742, 4180, 3718}

$$\frac{6ib\text{PolyLog}\left(4,-ie^{c+dx}\right)f^3}{\left(a^2+b^2\right)d^4}-\frac{6ib\text{PolyLog}\left(4,ie^{c+dx}\right)f^3}{\left(a^2+b^2\right)d^4}-\frac{6b^2\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)f^3}{a\left(a^2+b^2\right)d^4}-\frac{6b^2\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2+b}}\right.}{a\left(a^2+b^2\right)d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (-2*b*(e + f*x)^3*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (2*(e + f*x)^3*Arc
Tanh[E^(2*c + 2*d*x)])/(a*d) - (b^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) - (b^2*(e + f*x)^3*Log[1 + (b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) + (b^2*(e + f*x)^3*Log[1 +
 E^(2*(c + d*x))])/(a*(a^2 + b^2)*d) + ((3*I)*b*f*(e + f*x)^2*PolyLog[2, (-
I)*E^(c + d*x)])/((a^2 + b^2)*d^2) - ((3*I)*b*f*(e + f*x)^2*PolyLog[2, I*E^
(c + d*x)])/((a^2 + b^2)*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^2) - (3*b^2*f*(e + f*x)^2*
PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^2) +
 (3*b^2*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*a*(a^2 + b^2)*d^2) -
 (3*f*(e + f*x)^2*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a*d^2) + (3*f*(e + f*x)^
```

```
2*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d^2) - ((6*I)*b*f^2*(e + f*x)*PolyLog[3
, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) + ((6*I)*b*f^2*(e + f*x)*PolyLog[3,
I*E^(c + d*x)])/((a^2 + b^2)*d^3) + (6*b^2*f^2*(e + f*x)*PolyLog[3, -((b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^3) + (6*b^2*f^2*(e + f
*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^
3) - (3*b^2*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*a*(a^2 + b^2)*d^
3) + (3*f^2*(e + f*x)*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a*d^3) - (3*f^2*(e +
 f*x)*PolyLog[3, E^(2*c + 2*d*x)])/(2*a*d^3) + ((6*I)*b*f^3*PolyLog[4, (-I)
*E^(c + d*x)])/((a^2 + b^2)*d^4) - ((6*I)*b*f^3*PolyLog[4, I*E^(c + d*x)])/
((a^2 + b^2)*d^4) - (6*b^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/(a*(a^2 + b^2)*d^4) - (6*b^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^4) + (3*b^2*f^3*PolyLog[4, -E^(2*(c
+ d*x))])/(4*a*(a^2 + b^2)*d^4) - (3*f^3*PolyLog[4, -E^(2*c + 2*d*x)])/(4*a
*d^4) + (3*f^3*PolyLog[4, E^(2*c + 2*d*x)])/(4*a*d^4)
```

Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\text{csch}(c+dx)\text{sech}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= \frac{2\int (e+fx)^3\text{csch}(2c+2dx)\,dx}{a} - \frac{b\int (e+fx)^3\text{sech}(c+dx)(a-b\sinh(c+d}{a\left(a^2+b^2\right)}$$

$$= \frac{b^2(e+fx)^4}{4a\left(a^2+b^2\right)f} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b\int \left(a(e+fx)^3\text{sech}(c+dx)\right.}{a\left(a^2\right.}$$

$$= \frac{b^2(e+fx)^4}{4a\left(a^2+b^2\right)f} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{a\left(a^2+b^2\right)d}$$

$$= -\frac{2b(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(e+fx)^3\log\left(\right.}{a\left(a^2+\right.}$$

$$= -\frac{2b(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(e+fx)^3\log\left(\right.}{a\left(a^2+\right.}$$

$$= -\frac{2b(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(e+fx)^3\log\left(\right.}{a\left(a^2+\right.}$$

$$= -\frac{2b(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(e+fx)^3\log\left(\right.}{a\left(a^2+\right.}$$

$$= -\frac{2b(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(e+fx)^3\log\left(\right.}{a\left(a^2+\right.}$$

$$= -\frac{2b(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} - \frac{2(e+fx)^3\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(e+fx)^3\log\left(\right.}{a\left(a^2+\right.}$$

Mathematica [B] time = 32.8475, size = 3000, normalized size = 2.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] 2*((a*E^c*((e + f*x)^4/(4*E^c*f) + ((1 + E^(-c))*(e + f*x)^3*Log[1 + E^(-c
- d*x)])/d - (3*(1 + E^c)*f*(d^2*(e + f*x)^2*PolyLog[2, -E^(-c - d*x)] + 2*
f*(d*(e + f*x)*PolyLog[3, -E^(-c - d*x)] + f*PolyLog[4, -E^(-c - d*x)])))/(
d^4*E^c)))/(2*(a^2 + b^2)*(1 + E^c)) + ((I/2)*a*E^c*((e + f*x)^4/(4*E^c*f)
+ ((I + E^(-c))*(e + f*x)^3*Log[1 - I*E^(-c - d*x)])/d - (3*(1 + I*E^c)*f*(
d^2*(e + f*x)^2*PolyLog[2, I*E^(-c - d*x)] + 2*f*(d*(e + f*x)*PolyLog[3, I*
E^(-c - d*x)] + f*PolyLog[4, I*E^(-c - d*x)])))/(d^4*E^c)))/((a^2 + b^2)*(-
I + E^c)) - (b^2*E^(2*c)*((e + f*x)^4/(E^(2*c)*f) - (2*(1 - E^(-2*c))*(e +
f*x)^3*Log[1 - E^(-c - d*x)])/d - (2*(1 - E^(-2*c))*(e + f*x)^3*Log[1 + E^(
-c - d*x)])/d + (6*(-1 + E^(2*c))*f*(d^2*(e + f*x)^2*PolyLog[2, -E^(-c - d*
x)] + 2*f*(d*(e + f*x)*PolyLog[3, -E^(-c - d*x)] + f*PolyLog[4, -E^(-c - d*
x)])))/(d^4*E^(2*c)) + (6*(-1 + E^(2*c))*f*(d^2*(e + f*x)^2*PolyLog[2, E^(-
c - d*x)] + 2*f*(d*(e + f*x)*PolyLog[3, E^(-c - d*x)] + f*PolyLog[4, E^(-c
- d*x)])))/(d^4*E^(2*c))))/(4*a*(a^2 + b^2)*(-1 + E^(2*c))) - ((I/2)*b*((-2
```

```
*I)*d^3*e^3*ArcTan[E^(c + d*x)] + 3*d^3*e^2*f*x*Log[1 - I*E^(c + d*x)] + 3*
d^3*e*f^2*x^2*Log[1 - I*E^(c + d*x)] + d^3*f^3*x^3*Log[1 - I*E^(c + d*x)] -
 3*d^3*e^2*f*x*Log[1 + I*E^(c + d*x)] - 3*d^3*e*f^2*x^2*Log[1 + I*E^(c + d*
x)] - d^3*f^3*x^3*Log[1 + I*E^(c + d*x)] - 3*d^2*f*(e + f*x)^2*PolyLog[2, (
-I)*E^(c + d*x)] + 3*d^2*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)] + 6*d*e*f^
2*PolyLog[3, (-I)*E^(c + d*x)] + 6*d*f^3*x*PolyLog[3, (-I)*E^(c + d*x)] - 6
*d*e*f^2*PolyLog[3, I*E^(c + d*x)] - 6*d*f^3*x*PolyLog[3, I*E^(c + d*x)] -
6*f^3*PolyLog[4, (-I)*E^(c + d*x)] + 6*f^3*PolyLog[4, I*E^(c + d*x)]))/((a^
2 + b^2)*d^4) - (a*(-((e + f*x)^3*Log[1 - E^(c + d*x)]) + (e + f*x)^3*Log[1
 - I*E^(c + d*x)] + (3*f*(d^2*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)] - 2*d*f
*(e + f*x)*PolyLog[3, I*E^(c + d*x)] + 2*f^2*PolyLog[4, I*E^(c + d*x)]))/d^
3 - (3*f*(d^2*(e + f*x)^2*PolyLog[2, E^(c + d*x)] - 2*d*f*(e + f*x)*PolyLog
[3, E^(c + d*x)] + 2*f^2*PolyLog[4, E^(c + d*x)]))/d^3))/(2*(a^2 + b^2)*d)
+ (b^2*((E^(2*c)*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(-1 + E^(2*
c)) - (2*(d^3*e^3*Log[b - 2*a*E^(c + d*x) - b*E^(2*(c + d*x))] + 3*d^3*e^2*
f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] + 3*d^3*
e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] +
d^3*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])]
+ 3*d^3*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)
])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E
^(2*c)])] + d^3*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)
*E^(2*c)])] + 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - S
qrt[(a^2 + b^2)*E^(2*c)]))] + 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c +
d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*d*e*f^2*PolyLog[3, -((b*E^(
2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*d*f^3*x*PolyLog[3, -(
(b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*d*e*f^2*PolyLog
[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 6*d*f^3*x*P
olyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*f^3
*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 6*f
^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]))/d
^4))/(4*a*(a^2 + b^2)) - (b^2*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)
*Csch[c/2]*Sech[c/2]*Sech[c])/(32*a*(a^2 + b^2)) + (3*a*e^2*f*x^2*Csch[c/2]
*Sech[c/2])/(16*(a^2 + b^2)*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2] + I*Sinh[c
/2])) + (3*b^2*e^2*f*x^2*Csch[c/2]*Sech[c/2])/(16*a*(a^2 + b^2)*(Cosh[c/2]
- I*Sinh[c/2])*(Cosh[c/2] + I*Sinh[c/2])) + (a*e*f^2*x^3*Csch[c/2]*Sech[c/2
])/(8*(a^2 + b^2)*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2] + I*Sinh[c/2])) + (b
^2*e*f^2*x^3*Csch[c/2]*Sech[c/2])/(8*a*(a^2 + b^2)*(Cosh[c/2] - I*Sinh[c/2]
)*(Cosh[c/2] + I*Sinh[c/2])) + (a*f^3*x^4*Csch[c/2]*Sech[c/2])/(32*(a^2 + b
^2)*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2] + I*Sinh[c/2])) + (b^2*f^3*x^4*Csc
h[c/2]*Sech[c/2])/(32*a*(a^2 + b^2)*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2] +
I*Sinh[c/2])) - (3*a*e^2*f*x^2*Cosh[c]*Csch[c/2]*Sech[c/2])/(16*(a^2 + b^2)
*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2] + I*Sinh[c/2])) - (a*e*f^2*x^3*Cosh[c
]*Csch[c/2]*Sech[c/2])/(8*(a^2 + b^2)*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2]
+ I*Sinh[c/2])) - (a*f^3*x^4*Cosh[c]*Csch[c/2]*Sech[c/2])/(32*(a^2 + b^2)*(
Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2] + I*Sinh[c/2])) - (((3*I)/16)*a*e^2*f*x
^2*Csch[c/2]*Sech[c/2]*Sinh[c])/((a^2 + b^2)*(Cosh[c/2] - I*Sinh[c/2])*(Cos
h[c/2] + I*Sinh[c/2])) - ((I/8)*a*e*f^2*x^3*Csch[c/2]*Sech[c/2]*Sinh[c])/((
a^2 + b^2)*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2] + I*Sinh[c/2])) - ((I/32)*a
*f^3*x^4*Csch[c/2]*Sech[c/2]*Sinh[c])/((a^2 + b^2)*(Cosh[c/2] - I*Sinh[c/2]
)*(Cosh[c/2] + I*Sinh[c/2])) - (e^3*x*Csch[c/2]*Sech[c/2]*(-a^2 - b^2 + a^2
*Cosh[c] + I*a^2*Sinh[c]))/(8*a*(a^2 + b^2)*(Cosh[c/2] - I*Sinh[c/2])*(Cosh
[c/2] + I*Sinh[c/2])))
```

Maple [F] time = 0.756, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3} \operatorname{csch}\left(dx+c\right) \operatorname{sech}\left(dx+c\right)}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -e^3*(b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^3 + a*b^2)*d)
 - 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(e^(-2*d*x - 2*c) + 1)/(
(a^2 + b^2)*d) - log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d))
 + 3*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d*
x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))*e^2*f/(a*d^2) + 3*(d^2*x^2*lo
g(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c))
)*e*f^2/(a*d^3) + 3*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c
)) - 2*polylog(3, e^(d*x + c)))*e*f^2/(a*d^3) + (d^3*x^3*log(e^(d*x + c) +
1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*pol
ylog(4, -e^(d*x + c)))*f^3/(a*d^4) + (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2
*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d
*x + c)))*f^3/(a*d^4) - 1/2*(d^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*d^4*e^2*f*x^
2)/(a*d^4) + integrate(2*(b^3*f^3*x^3 + 3*b^3*e*f^2*x^2 + 3*b^3*e^2*f*x - (
a*b^2*f^3*x^3*e^c + 3*a*b^2*e*f^2*x^2*e^c + 3*a*b^2*e^2*f*x*e^c)*e^(d*x))/(
a^3*b + a*b^3 - (a^3*b*e^(2*c) + a*b^3*e^(2*c))*e^(2*d*x) - 2*(a^4*e^c + a^
2*b^2*e^c)*e^(d*x)), x) - integrate(-2*(a*f^3*x^3 + 3*a*e*f^2*x^2 + 3*a*e^2
*f*x - (b*f^3*x^3*e^c + 3*b*e*f^2*x^2*e^c + 3*b*e^2*f*x*e^c)*e^(d*x))/(a^2
+ b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [C] time = 3.69805, size = 5839, normalized size = 5.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -(6*b^2*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*b^2*f^3*polylog(4, (a*co
sh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2))/b) - 6*(a^2 + b^2)*f^3*polylog(4, cosh(d*x + c) + sinh(d*x
+ c)) - 6*(a^2 + b^2)*f^3*polylog(4, -cosh(d*x + c) - sinh(d*x + c)) + 3*(b
^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*dilog((a*cosh(d*x + c)
+ a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2) - b)/b + 1) + 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*di
log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 +
 b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*dilog(cosh(d*x + c) + sinh(d*x +
```

```
 c)) + (3*a^2*d^2*f^3*x^2 + 3*I*a*b*d^2*f^3*x^2 + 6*a^2*d^2*e*f^2*x + 6*I*a
*b*d^2*e*f^2*x + 3*a^2*d^2*e^2*f + 3*I*a*b*d^2*e^2*f)*dilog(I*cosh(d*x + c)
 + I*sinh(d*x + c)) + (3*a^2*d^2*f^3*x^2 - 3*I*a*b*d^2*f^3*x^2 + 6*a^2*d^2*
e*f^2*x - 6*I*a*b*d^2*e*f^2*x + 3*a^2*d^2*e^2*f - 3*I*a*b*d^2*e^2*f)*dilog(
-I*cosh(d*x + c) - I*sinh(d*x + c)) - 3*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 +
 b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*dilog(-cosh(d*x + c) - sinh(d*x
+ c)) + (b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)
*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*
a) + (b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*lo
g(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
+ (b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*
e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*log(-(a*cosh(d*x + c) + a*sinh(d*x
 + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) +
 (b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e
^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*log(-(a*cosh(d*x + c) + a*sinh(d*x
+ c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) -
((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*
e^2*f*x + (a^2 + b^2)*d^3*e^3)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (a^
2*d^3*e^3 + I*a*b*d^3*e^3 - 3*a^2*c*d^2*e^2*f - 3*I*a*b*c*d^2*e^2*f + 3*a^2
*c^2*d*e*f^2 + 3*I*a*b*c^2*d*e*f^2 - a^2*c^3*f^3 - I*a*b*c^3*f^3)*log(cosh(
d*x + c) + sinh(d*x + c) + I) + (a^2*d^3*e^3 - I*a*b*d^3*e^3 - 3*a^2*c*d^2*
e^2*f + 3*I*a*b*c*d^2*e^2*f + 3*a^2*c^2*d*e*f^2 - 3*I*a*b*c^2*d*e*f^2 - a^2
*c^3*f^3 + I*a*b*c^3*f^3)*log(cosh(d*x + c) + sinh(d*x + c) - I) - ((a^2 +
b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2
 + b^2)*c^3*f^3)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + (a^2*d^3*f^3*x^3
- I*a*b*d^3*f^3*x^3 + 3*a^2*d^3*e*f^2*x^2 - 3*I*a*b*d^3*e*f^2*x^2 + 3*a^2*d
^3*e^2*f*x - 3*I*a*b*d^3*e^2*f*x + 3*a^2*c*d^2*e^2*f - 3*I*a*b*c*d^2*e^2*f
- 3*a^2*c^2*d*e*f^2 + 3*I*a*b*c^2*d*e*f^2 + a^2*c^3*f^3 - I*a*b*c^3*f^3)*lo
g(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) + (a^2*d^3*f^3*x^3 + I*a*b*d^3*f^3
*x^3 + 3*a^2*d^3*e*f^2*x^2 + 3*I*a*b*d^3*e*f^2*x^2 + 3*a^2*d^3*e^2*f*x + 3*
I*a*b*d^3*e^2*f*x + 3*a^2*c*d^2*e^2*f + 3*I*a*b*c*d^2*e^2*f - 3*a^2*c^2*d*e
*f^2 - 3*I*a*b*c^2*d*e*f^2 + a^2*c^3*f^3 + I*a*b*c^3*f^3)*log(-I*cosh(d*x +
 c) - I*sinh(d*x + c) + 1) - ((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e
*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 +
 b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*f^3)*log(-cosh(d*x + c) - sinh(d*x + c)
 + 1) + (6*a^2*f^3 + 6*I*a*b*f^3)*polylog(4, I*cosh(d*x + c) + I*sinh(d*x +
 c)) + (6*a^2*f^3 - 6*I*a*b*f^3)*polylog(4, -I*cosh(d*x + c) - I*sinh(d*x +
 c)) - 6*(b^2*d*f^3*x + b^2*d*e*f^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d
*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6
*(b^2*d*f^3*x + b^2*d*e*f^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c)
- (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*((a^2 +
 b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*polylog(3, cosh(d*x + c) + sinh(d*x +
c)) - (6*a^2*d*f^3*x + 6*I*a*b*d*f^3*x + 6*a^2*d*e*f^2 + 6*I*a*b*d*e*f^2)*p
olylog(3, I*cosh(d*x + c) + I*sinh(d*x + c)) - (6*a^2*d*f^3*x - 6*I*a*b*d*f
^3*x + 6*a^2*d*e*f^2 - 6*I*a*b*d*e*f^2)*polylog(3, -I*cosh(d*x + c) - I*sin
h(d*x + c)) + 6*((a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*polylog(3, -cos
h(d*x + c) - sinh(d*x + c)))/((a^3 + a*b^2)*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \operatorname{csch}(dx+c)\operatorname{sech}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate((f*x + e)^3*csch(d*x + c)*sech(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.436 $\int \frac{(e+fx)^2\mathbf{csch}(c+dx)\mathbf{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=734

$$-\frac{2b^2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\left(a^2+b^2\right)}-\frac{2b^2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2\left(a^2+b^2\right)}+\frac{b^2f(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{ad^2\left(a^2+b^2\right)}$$

[Out] (-2*b*(e + f*x)^2*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (2*(e + f*x)^2*ArcTanh[E^(2*c + 2*d*x)])/(a*d) - (b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) - (b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) + (b^2*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(a*(a^2 + b^2)*d) + ((2*I)*b*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) - ((2*I)*b*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^2) + (b^2*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(a*(a^2 + b^2)*d^2) - (f*(e + f*x)*PolyLog[2, -E^(2*c + 2*d*x)])/(a*d^2) + (f*(e + f*x)*PolyLog[2, E^(2*c + 2*d*x)])/(a*d^2) - ((2*I)*b*f^2*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) + ((2*I)*b*f^2*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)*d^3) + (2*b^2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^3) + (2*b^2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^3) - (b^2*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*a*(a^2 + b^2)*d^3) + (f^2*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a*d^3) - (f^2*PolyLog[3, E^(2*c + 2*d*x)])/(2*a*d^3)

Rubi [A] time = 1.09728, antiderivative size = 734, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5589, 5461, 4182, 2531, 2282, 6589, 5573, 5561, 2190, 6742, 4180, 3718}

$$-\frac{2b^2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\left(a^2+b^2\right)}-\frac{2b^2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2\left(a^2+b^2\right)}+\frac{b^2f(e+fx)\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{ad^2\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

[In] Int[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]

[Out] (-2*b*(e + f*x)^2*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (2*(e + f*x)^2*ArcTanh[E^(2*c + 2*d*x)])/(a*d) - (b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) - (b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) + (b^2*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(a*(a^2 + b^2)*d) + ((2*I)*b*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) - ((2*I)*b*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^2) + (b^2*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(a*(a^2 + b^2)*d^2) - (f*(e + f*x)*PolyLog[2, -E^(2*c + 2*d*x)])/(a*d^2) + (f*(e + f*x)*PolyLog[2, E^(2*c + 2*d*x)])/(a*d^2) - ((2*I)*b*f^2*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) + ((2*I)*b*f^2*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)*d^3) + (2*b^2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^3) + (2*b^2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^3) - (b^2*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*a*(a^2 + b^2)*d^3)

```
+ (f^2*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a*d^3) - (f^2*PolyLog[3, E^(2*c + 2
*d*x)])/(2*a*d^3)
```

Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
```

```
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= \frac{2\int (e+fx)^2\text{csch}(2c+2dx)\,dx}{a} - \frac{b\int (e+fx)^2\text{sech}(c+dx)(a-b\sinh(c+d$$

$$= \frac{b^2(e+fx)^3}{3a(a^2+b^2)f} - \frac{2(e+fx)^2\tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b\int (a(e+fx)^2\text{sech}(c+dx)}{a(a^2}$$

$$= \frac{b^2(e+fx)^3}{3a(a^2+b^2)f} - \frac{2(e+fx)^2\tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{a(a^2+b^2)d}$$

$$= -\frac{2b(e+fx)^2\tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^2\tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)^2\log(}{a(a^2+}$$

$$= -\frac{2b(e+fx)^2\tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^2\tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)^2\log(}{a(a^2+}$$

$$= -\frac{2b(e+fx)^2\tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^2\tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)^2\log(}{a(a^2+}$$

$$= -\frac{2b(e+fx)^2\tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^2\tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)^2\log(}{a(a^2+}$$

$$= -\frac{2b(e+fx)^2\tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^2\tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)^2\log(}{a(a^2+}$$

Mathematica [B] time = 33.66, size = 3268, normalized size = 4.45

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] 2*(-(b^2*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Csch[2*c])/(6*a*(a^2 + b^2)) + (a*E^
c*((e + f*x)^3/(3*E^c*f) + ((1 + E^(-c))*(e + f*x)^2*Log[1 + E^(-c - d*x)])
/d - (2*(1 + E^c)*f*(d*(e + f*x)*PolyLog[2, -E^(-c - d*x)] + f*PolyLog[3, -
E^(-c - d*x)]))/(d^3*E^c)))/(2*(a^2 + b^2)*(1 + E^c)) + (d^2*(d*x*((-3*I)*b
*e*f*x + a*((-3*I)*e^2*E^c + 3*e*f*x + f^2*x^2)) + 3*(1 + I*E^c)*f*x*(2*a*e
 - (2*I)*b*e + a*f*x)*Log[1 - I*E^(-c - d*x)] + 3*a*e^2*(1 + I*E^c)*Log[I -
 E^(c + d*x)]) - (6*I)*d*(-I + E^c)*f*((-I)*b*e + a*(e + f*x))*PolyLog[2, I
*E^(-c - d*x)] - (6*I)*a*(-I + E^c)*f^2*PolyLog[3, I*E^(-c - d*x)])/(6*(a -
 I*b)*((-I)*a + b)*d^3*(-I + E^c)) - (b^2*E^(2*c)*((2*(e + f*x)^3)/(E^(2*c)
*f) - (3*(1 - E^(-2*c))*(e + f*x)^2*Log[1 - E^(-c - d*x)])/d - (3*(1 - E^(-
2*c))*(e + f*x)^2*Log[1 + E^(-c - d*x)])/d + (6*(-1 + E^(2*c))*f*(d*(e + f*
x)*PolyLog[2, -E^(-c - d*x)] + f*PolyLog[3, -E^(-c - d*x)]))/(d^3*E^(2*c))
+ (6*(-1 + E^(2*c))*f*(d*(e + f*x)*PolyLog[2, E^(-c - d*x)] + f*PolyLog[3,
E^(-c - d*x)]))/(d^3*E^(2*c))))/(6*a*(a^2 + b^2)*(-1 + E^(2*c))) - ((I/2)*b
*((-2*I)*d^2*e^2*ArcTan[E^(c + d*x)] + d^2*f^2*x^2*Log[1 - I*E^(c + d*x)] -
 d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*f^2*x*PolyLog[2, (-I)*E^(c + d*x)
] + 2*d*f^2*x*PolyLog[2, I*E^(c + d*x)] + 2*f^2*PolyLog[3, (-I)*E^(c + d*x)
```

] - 2*f^2*PolyLog[3, I*E^(c + d*x)]))/((a^2 + b^2)*d^3) - ((-I)*b*d^3*e*E^(2*c)*f*x^2 + 2*a*d^2*e^2*ArcTan[1 - (1 + I)*E^(c + d*x)] + (2*I)*a*d^2*e^2*E^(2*c)*ArcTan[1 - (1 + I)*E^(c + d*x)] + (2*I)*a*d^2*e*f*x*Log[1 - E^(c + d*x)] - 2*a*d^2*e*E^(2*c)*f*x*Log[1 - E^(c + d*x)] + I*a*d^2*f^2*x^2*Log[1 - E^(c + d*x)] - a*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] - (2*I)*a*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + 2*b*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + 2*a*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (2*I)*b*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] - I*a*d^2*f^2*x^2*Log[1 - I*E^(c + d*x)] + a*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + 2*d*(-I + E^(2*c))*f*(I*b*e + a*(e + f*x))*PolyLog[2, I*E^(c + d*x)] - 2*a*d*(-I + E^(2*c))*f*(e + f*x)*PolyLog[2, E^(c + d*x)] + (2*I)*a*f^2*PolyLog[3, I*E^(c + d*x)] - 2*a*E^(2*c)*f^2*PolyLog[3, I*E^(c + d*x)] - (2*I)*a*f^2*PolyLog[3, E^(c + d*x)] + 2*a*E^(2*c)*f^2*PolyLog[3, E^(c + d*x)])/(2*(a^2 + b^2)*d^3*(-I + E^(2*c))) + (b^2*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3))/(6*a*(a^2 + b^2)*(-1 + E^(2*c))) + (b^2*e*f*x^2)/(2*a*(a^2 + b^2)*(Cosh[c] + Sinh[c])^2) + (b^2*f^2*x^3)/(6*a*(a^2 + b^2)*(Cosh[c] + Sinh[c])^2) + (b^2*f^2*x^3*Coth[2*c])/(6*a*(a^2 + b^2)*(Cosh[c] + Sinh[c])^2) + (b^2*e*f*x^2*Cosh[2*c]*Csch[c]*Sech[c])/(4*a*(a^2 + b^2)*(Cosh[c] + Sinh[c])^2) + (e^2*x*Csch[c]^2*(-(a^2*Coth[c]) + Csch[c]*(a^2 + b^2 - I*a^2*Sinh[c])))/(a*(a^2 + b^2)*(Csch[c/2] - I*Sech[c/2])*(Csch[c/2] + I*Sech[c/2])) - ((1/4 - I/4)*a*e*f*x^2*Cosh[c])/((a^2 + b^2)*(Cosh[c] + Sinh[c])^2*(-I - I*Cosh[c] + Sinh[c] + Sinh[2*c])) + (b*e*f*x^2*Cosh[c])/(4*(a^2 + b^2)*(Cosh[c] + Sinh[c])^2*(-I - I*Cosh[c] + Sinh[c] + Sinh[2*c])) - ((1/12 - I/12)*a*f^2*x^3*Cosh[c])/((a^2 + b^2)*(Cosh[c] + Sinh[c])^2*(-I - I*Cosh[c] + Sinh[c] + Sinh[2*c])) - (b*e*f*x^2*Cosh[3*c])/(4*(a^2 + b^2)*(Cosh[c] + Sinh[c])^2*(-I - I*Cosh[c] + Sinh[c] + Sinh[2*c])) - ((1/4 - I/4)*a*e*f*x^2*Sinh[c])/((a^2 + b^2)*(Cosh[c] + Sinh[c])^2*(-I - I*Cosh[c] + Sinh[c] + Sinh[2*c])) + (b*e*f*x^2*Sinh[c])/(4*(a^2 + b^2)*(Cosh[c] + Sinh[c])^2*(-I - I*Cosh[c] + Sinh[c] + Sinh[2*c])) - ((1/12 - I/12)*a*f^2*x^3*Sinh[c])/((a^2 + b^2)*(Cosh[c] + Sinh[c])^2*(-I - I*Cosh[c] + Sinh[c] + Sinh[2*c])) + ((1/4 - I/4)*a*e*f*x^2*Cosh[3*c])/((a^2 + b^2)*(Cosh[c] + Sinh[c])^2*(Cosh[c] + I*(-I + Sinh[c] + Sinh[2*c]))) + ((1/12 - I/12)*a*f^2*x^3*Cosh[3*c])/((a^2 + b^2)*(Cosh[c] + Sinh[c])^2*(Cosh[c] + I*(-I + Sinh[c] + Sinh[2*c]))) - (b*e*f*x^2*Sinh[3*c])/(4*(a^2 + b^2)*(Cosh[c] + Sinh[c])^2*(-I - I*Cosh[c] + Sinh[c] + Sinh[2*c])) + ((1/4 - I/4)*a*e*f*x^2*Sinh[3*c])/((a^2 + b^2)*(Cosh[c] + Sinh[c])^2*(Cosh[c] + I*(-I + Sinh[c] + Sinh[2*c]))) + ((1/12 - I/12)*a*f^2*x

```
^3*Sinh[3*c])/((a^2 + b^2)*(Cosh[c] + Sinh[c])^2*(Cosh[c] + I*(-I + Sinh[c]
 + Sinh[2*c]))))
```

Maple [F] time = 0.58, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)\operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-e^2\left(\frac{b^2\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{\left(a^3+ab^2\right)d}-\frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{\left(a^2+b^2\right)d}+\frac{a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\left(a^2+b^2\right)d}-\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}-\frac{\log\left(e^{(-dx}\right.}{ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -e^2*(b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^3 + a*b^2)*d)
 - 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(e^(-2*d*x - 2*c) + 1)/(
(a^2 + b^2)*d) - log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d))
 + 2*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))*e*f/(a*d^2) + 2*(d*x*
log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))*e*f/(a*d^2) + (d^2*x^2*log(e^(d
*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2/
(a*d^3) + (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*pol
ylog(3, e^(d*x + c)))*f^2/(a*d^3) - 2/3*(d^3*f^2*x^3 + 3*d^3*e*f*x^2)/(a*d^
3) + integrate(2*(b^3*f^2*x^2 + 2*b^3*e*f*x - (a*b^2*f^2*x^2*e^c + 2*a*b^2*
e*f*x*e^c)*e^(d*x))/(a^3*b + a*b^3 - (a^3*b*e^(2*c) + a*b^3*e^(2*c))*e^(2*d
*x) - 2*(a^4*e^c + a^2*b^2*e^c)*e^(d*x)), x) - integrate(-2*(a*f^2*x^2 + 2*
a*e*f*x - (b*f^2*x^2*e^c + 2*b*e*f*x*e^c)*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c
) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [C] time = 3.05557, size = 3794, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] (2*b^2*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*b^2*f^2*polylog(3, (a*cos
h(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2))/b) - 2*(a^2 + b^2)*f^2*polylog(3, cosh(d*x + c) + sinh(d*x +
```

```
c)) - 2*(a^2 + b^2)*f^2*polylog(3, -cosh(d*x + c) - sinh(d*x + c)) - 2*(b^
2*d*f^2*x + b^2*d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b^2*d*f^2
*x + b^2*d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*((a^2 + b^2)*d*f^
2*x + (a^2 + b^2)*d*e*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) - (2*a^2*d*f^
2*x + 2*I*a*b*d*f^2*x + 2*a^2*d*e*f + 2*I*a*b*d*e*f)*dilog(I*cosh(d*x + c)
+ I*sinh(d*x + c)) - (2*a^2*d*f^2*x - 2*I*a*b*d*f^2*x + 2*a^2*d*e*f - 2*I*a
*b*d*e*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + 2*((a^2 + b^2)*d*f^2*
x + (a^2 + b^2)*d*e*f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) - (b^2*d^2*e^2
 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) +
 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*
f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2)
+ 2*a) - (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*
log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b
^2*c*d*e*f - b^2*c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + ((a^2 + b^2)*d
^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + (a^2 + b^2)*d^2*e^2)*log(cosh(d*x +
c) + sinh(d*x + c) + 1) - (a^2*d^2*e^2 + I*a*b*d^2*e^2 - 2*a^2*c*d*e*f - 2*
I*a*b*c*d*e*f + a^2*c^2*f^2 + I*a*b*c^2*f^2)*log(cosh(d*x + c) + sinh(d*x +
 c) + I) - (a^2*d^2*e^2 - I*a*b*d^2*e^2 - 2*a^2*c*d*e*f + 2*I*a*b*c*d*e*f +
 a^2*c^2*f^2 - I*a*b*c^2*f^2)*log(cosh(d*x + c) + sinh(d*x + c) - I) + ((a^
2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*log(cosh(d*
x + c) + sinh(d*x + c) - 1) - (a^2*d^2*f^2*x^2 - I*a*b*d^2*f^2*x^2 + 2*a^2*
d^2*e*f*x - 2*I*a*b*d^2*e*f*x + 2*a^2*c*d*e*f - 2*I*a*b*c*d*e*f - a^2*c^2*f
^2 + I*a*b*c^2*f^2)*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) - (a^2*d^2*f
^2*x^2 + I*a*b*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + 2*I*a*b*d^2*e*f*x + 2*a^2*c*
d*e*f + 2*I*a*b*c*d*e*f - a^2*c^2*f^2 - I*a*b*c^2*f^2)*log(-I*cosh(d*x + c)
 - I*sinh(d*x + c) + 1) + ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*
x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*log(-cosh(d*x + c) - sinh(
d*x + c) + 1) + (2*a^2*f^2 + 2*I*a*b*f^2)*polylog(3, I*cosh(d*x + c) + I*si
nh(d*x + c)) + (2*a^2*f^2 - 2*I*a*b*f^2)*polylog(3, -I*cosh(d*x + c) - I*si
nh(d*x + c)))/((a^3 + a*b^2)*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)\operatorname{sech}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate((f*x + e)^2*csch(d*x + c)*sech(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.437 $\int \frac{(e+fx)\mathbf{csch}(c+dx)\mathbf{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=439

$$-\frac{b^2 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\left(a^2+b^2\right)}-\frac{b^2 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2\left(a^2+b^2\right)}+\frac{b^2 f \text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2ad^2\left(a^2+b^2\right)}+\frac{ibf \text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)}$$

```
[Out] (-2*b*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (2*(e + f*x)*ArcTanh
[E^(2*c + 2*d*x)])/(a*d) - (b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2])])/(a*(a^2 + b^2)*d) - (b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) + (b^2*(e + f*x)*Log[1 + E^(2*(c +
 d*x))])/(a*(a^2 + b^2)*d) + (I*b*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b
^2)*d^2) - (I*b*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) - (b^2*f*Pol
yLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^2) - (b
^2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d
^2) + (b^2*f*PolyLog[2, -E^(2*(c + d*x))])/(2*a*(a^2 + b^2)*d^2) - (f*PolyL
og[2, -E^(2*c + 2*d*x)])/(2*a*d^2) + (f*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d
^2)
```

Rubi [A] time = 0.636683, antiderivative size = 439, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {5589, 5461, 4182, 2279, 2391, 5573, 5561, 2190, 6742, 4180, 3718}

$$-\frac{b^2 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\left(a^2+b^2\right)}-\frac{b^2 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2\left(a^2+b^2\right)}+\frac{b^2 f \text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2ad^2\left(a^2+b^2\right)}+\frac{ibf \text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (-2*b*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (2*(e + f*x)*ArcTanh
[E^(2*c + 2*d*x)])/(a*d) - (b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt
[a^2 + b^2])])/(a*(a^2 + b^2)*d) - (b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) + (b^2*(e + f*x)*Log[1 + E^(2*(c +
 d*x))])/(a*(a^2 + b^2)*d) + (I*b*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b
^2)*d^2) - (I*b*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) - (b^2*f*Pol
yLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^2) - (b
^2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d
^2) + (b^2*f*PolyLog[2, -E^(2*(c + d*x))])/(2*a*(a^2 + b^2)*d^2) - (f*PolyL
og[2, -E^(2*c + 2*d*x)])/(2*a*d^2) + (f*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d
^2)
```


Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
```

```
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= \frac{2\int (e+fx)\operatorname{csch}(2c+2dx)\,dx}{a} - \frac{b\int (e+fx)\operatorname{sech}(c+dx)(a-b\sinh(c+d}{a\left(a^2+b^2\right)}$$

$$= \frac{b^2(e+fx)^2}{2a\left(a^2+b^2\right)f} - \frac{2(e+fx)\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b\int (a(e+fx)\operatorname{sech}(c+dx) -}{a\left(a^2\right.}$$

$$= \frac{b^2(e+fx)^2}{2a\left(a^2+b^2\right)f} - \frac{2(e+fx)\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{a\left(a^2+b^2\right)d}$$

$$= -\frac{2b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} - \frac{2(e+fx)\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(e+fx)\log\left(1\right.}{a\left(a^2+\right.}$$

$$= -\frac{2b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} - \frac{2(e+fx)\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(e+fx)\log\left(1\right.}{a\left(a^2+\right.}$$

$$= -\frac{2b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} - \frac{2(e+fx)\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(e+fx)\log\left(1\right.}{a\left(a^2+\right.}$$

$$= -\frac{2b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} - \frac{2(e+fx)\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} - \frac{b^2(e+fx)\log\left(1\right.}{a\left(a^2+\right.}$$

Mathematica [B] time = 2.68984, size = 1540, normalized size = 3.51

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] 2*(((-I/4)*(a^2 - b^2)*(d*e - c*f)*(c + d*x))/(a*(a^2 + b^2)*d^2) - ((I/8)*
(a^2 - b^2)*f*(c + d*x)^2)/(a*(a^2 + b^2)*d^2) - (e*ArcTanh[1 - (2*I)*Tanh[
(c + d*x)/2]])/((a - I*b)*d) + (I*b*e*ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]])
/(a*(a - I*b)*d) + (c*f*ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]])/((a - I*b)*d^
2) - (I*b*c*f*ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]])/(a*(a - I*b)*d^2) + (e*
Log[Cosh[(c + d*x)/2]])/(2*a*d) - (c*f*Log[Cosh[(c + d*x)/2]])/(2*a*d^2) -
(e*((-I/2)*(c + d*x) + Log[Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]]))/(2*(a
 + I*b)*d) + (c*f*((-I/2)*(c + d*x) + Log[Cosh[(c + d*x)/2] + I*Sinh[(c + d
*x)/2]]))/(2*(a + I*b)*d^2) - ((I/4)*b*e*((-I)*(c + d*x) + 2*ArcTanh[1 - (2
*I)*Tanh[(c + d*x)/2]] + Log[-1 + Cosh[c + d*x] + I*Sinh[c + d*x]]))/(a*(a
- I*b)*d) + ((I/4)*b*c*f*((-I)*(c + d*x) + 2*ArcTanh[1 - (2*I)*Tanh[(c + d*
x)/2]] + Log[-1 + Cosh[c + d*x] + I*Sinh[c + d*x]]))/(a*(a - I*b)*d^2) + (I
*f*((-I/8)*(c + d*x)^2 - (I/2)*(c + d*x)*Log[1 + E^(-c - d*x)] + (I/2)*Poly
```

```
Log[2, -E^(-c - d*x)]))/(a*d^2) - ((I/2)*b*f*((-I/2)*(c + d*x)^2 + (I/4)*(3
*Pi*(c + d*x) + (1 - I)*(c + d*x)^2 + 2*(Pi - (2*I)*(c + d*x))*Log[1 + I*E^
(-c - d*x)] - 4*Pi*Log[1 + E^(c + d*x)] - 2*Pi*Log[-Cos[(Pi + (2*I)*(c + d*
x))/4]] + 4*Pi*Log[Cosh[(c + d*x)/2]] + (4*I)*PolyLog[2, (-I)*E^(-c - d*x)]
)))/(a*(a - I*b)*d^2) + ((I/2)*f*((c + d*x)^2/4 + (-3*Pi*(c + d*x) - (1 - I
)*(c + d*x)^2 - 2*(Pi - (2*I)*(c + d*x))*Log[1 + I*E^(-c - d*x)] + 4*Pi*Log
[1 + E^(c + d*x)] + 2*Pi*Log[-Cos[(Pi + (2*I)*(c + d*x))/4]] - 4*Pi*Log[Cos
h[(c + d*x)/2]] - (4*I)*PolyLog[2, (-I)*E^(-c - d*x)])/4 - (I/2)*(-(c + d*x
)^2/2 + 2*(c + d*x)*Log[1 - E^(c + d*x)] + 2*PolyLog[2, E^(c + d*x)])))/((a
 - I*b)*d^2) + (b*f*((c + d*x)^2/4 + (-3*Pi*(c + d*x) - (1 - I)*(c + d*x)^2
 - 2*(Pi - (2*I)*(c + d*x))*Log[1 + I*E^(-c - d*x)] + 4*Pi*Log[1 + E^(c + d
*x)] + 2*Pi*Log[-Cos[(Pi + (2*I)*(c + d*x))/4]] - 4*Pi*Log[Cosh[(c + d*x)/2
]] - (4*I)*PolyLog[2, (-I)*E^(-c - d*x)])/4 - (I/2)*(-(c + d*x)^2/2 + 2*(c
+ d*x)*Log[1 - E^(c + d*x)] + 2*PolyLog[2, E^(c + d*x)])))/(2*a*(a - I*b)*d
^2) - (b^2*(-(f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLo
g[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))]))/(2*a*(a^2 + b^2)*d^2) - (I*f*(-(E^((I/4)*Pi)*(
c + d*x)^2)/4 + ((Pi*(c + d*x))/4 - Pi*Log[1 + E^(c + d*x)] - 2*(Pi/4 + (I/
2)*(c + d*x))*Log[1 - E^((2*I)*(Pi/4 + (I/2)*(c + d*x)))] + Pi*Log[Cosh[(c
+ d*x)/2]] + (Pi*Log[Sin[Pi/4 + (I/2)*(c + d*x)]])/2 + I*PolyLog[2, E^((2*I
)*(Pi/4 + (I/2)*(c + d*x)))])/Sqrt[2]))/(Sqrt[2]*(a + I*b)*d^2))
```

Maple [B] time = 0.171, size = 1065, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/d^2*f*c*b^2/(a^2+b^2)/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d^2*f*b^2
/(a^2+b^2)/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1
/d*f*b^2/(a^2+b^2)/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/
2)))*x-1/d*f*b^2/(a^2+b^2)/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^
2)^(1/2)))*x-1/d^2*f*b^2/(a^2+b^2)/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a
+(a^2+b^2)^(1/2)))*c+4*I/d^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*b*c-4*I/d*f
/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*b*x-4*I/d^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*
x+c))*b*c+4*I/d*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*b*x-4/d^2*f/(4*a^2+4*b^2
)*dilog(1+I*exp(d*x+c))*a-4/d^2*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))*a-8/d
*e/(4*a^2+4*b^2)*b*arctan(exp(d*x+c))-4/d*e/(4*a^2+4*b^2)*a*ln(1+exp(2*d*x+
2*c))-1/d^2*f/a*dilog(exp(d*x+c))+1/d^2*f/a*dilog(exp(d*x+c)+1)-1/d^2*f*b^2
/(a^2+b^2)/a*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-
1/d^2*f*b^2/(a^2+b^2)/a*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)
^(1/2)))+8/d^2*f*c/(4*a^2+4*b^2)*b*arctan(exp(d*x+c))-4/d*f/(4*a^2+4*b^2)*l
n(1+I*exp(d*x+c))*a*x-4/d^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*a*c-4/d*f/(4
*a^2+4*b^2)*ln(1-I*exp(d*x+c))*a*x-4/d^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))
*a*c-1/d*e*b^2/(a^2+b^2)/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+4/d^2*f*c/
(4*a^2+4*b^2)*a*ln(1+exp(2*d*x+2*c))+4*I/d^2*f/(4*a^2+4*b^2)*dilog(1+I*exp(
d*x+c))*b-4*I/d^2*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))*b+1/d/a*e*ln(exp(d*
x+c)-1)+1/d/a*e*ln(exp(d*x+c)+1)+1/d/a*ln(exp(d*x+c)+1)*f*x-1/d^2/a*f*c*ln(
exp(d*x+c)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-e\left(\frac{b^2\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{\left(a^3+ab^2\right)d}-\frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{\left(a^2+b^2\right)d}+\frac{a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\left(a^2+b^2\right)d}-\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}-\frac{\log\left(e^{(\cdot}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] -e*(b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^3 + a*b^2)*d) -
 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(e^(-2*d*x - 2*c) + 1)/((a
^2 + b^2)*d) - log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d)) +
 4*f*integrate(2*x/((b*(e^(d*x + c) - e^(-d*x - c)) + 2*a)*(e^(d*x + c) + e
^(-d*x - c))*(e^(d*x + c) - e^(-d*x - c))), x)
```

Fricas [B] time = 2.52182, size = 2091, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] -(b^2*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + b^2*f*dilog((a*cosh(d*x + c
) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2) - b)/b + 1) - (a^2 + b^2)*f*dilog(cosh(d*x + c) + sinh(d*x + c)) - (a^
2 + b^2)*f*dilog(-cosh(d*x + c) - sinh(d*x + c)) + (a^2*f + I*a*b*f)*dilog(
I*cosh(d*x + c) + I*sinh(d*x + c)) + (a^2*f - I*a*b*f)*dilog(-I*cosh(d*x +
c) - I*sinh(d*x + c)) + (b^2*d*e - b^2*c*f)*log(2*b*cosh(d*x + c) + 2*b*sin
h(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^2*d*e - b^2*c*f)*log(2*b
*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^
2*d*f*x + b^2*c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b^2*d*f*x + b^2*c*f)
*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b) - ((a^2 + b^2)*d*f*x + (a^2 + b^2)*d*e)*l
og(cosh(d*x + c) + sinh(d*x + c) + 1) + (a^2*d*e + I*a*b*d*e - a^2*c*f - I*
a*b*c*f)*log(cosh(d*x + c) + sinh(d*x + c) + I) + (a^2*d*e - I*a*b*d*e - a^
2*c*f + I*a*b*c*f)*log(cosh(d*x + c) + sinh(d*x + c) - I) - ((a^2 + b^2)*d*
e - (a^2 + b^2)*c*f)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + (a^2*d*f*x -
I*a*b*d*f*x + a^2*c*f - I*a*b*c*f)*log(I*cosh(d*x + c) + I*sinh(d*x + c) +
1) + (a^2*d*f*x + I*a*b*d*f*x + a^2*c*f + I*a*b*c*f)*log(-I*cosh(d*x + c) -
 I*sinh(d*x + c) + 1) - ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*log(-cosh(d*x
 + c) - sinh(d*x + c) + 1))/((a^3 + a*b^2)*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] integrate((f*x + e)*csch(d*x + c)*sech(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.438 $\int \frac{\mathbf{csch}(c+dx)\mathbf{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=90

$$-\frac{b^2\log(a+b\sinh(c+dx))}{ad\left(a^2+b^2\right)}-\frac{b\tan^{-1}(\sinh(c+dx))}{d\left(a^2+b^2\right)}-\frac{a\log(\cosh(c+dx))}{d\left(a^2+b^2\right)}+\frac{\log(\sinh(c+dx))}{ad}$$

```
[Out] -((b*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d)) - (a*Log[Cosh[c + d*x]])/((a^2
 + b^2)*d) + Log[Sinh[c + d*x]]/(a*d) - (b^2*Log[a + b*Sinh[c + d*x]])/(a*(
a^2 + b^2)*d)
```

Rubi [A] time = 0.169638, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2837, 12, 894, 635, 203, 260}

$$-\frac{b^2\log(a+b\sinh(c+dx))}{ad\left(a^2+b^2\right)}-\frac{b\tan^{-1}(\sinh(c+dx))}{d\left(a^2+b^2\right)}-\frac{a\log(\cosh(c+dx))}{d\left(a^2+b^2\right)}+\frac{\log(\sinh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((b*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d)) - (a*Log[Cosh[c + d*x]])/((a^2
 + b^2)*d) + Log[Sinh[c + d*x]]/(a*d) - (b^2*Log[a + b*Sinh[c + d*x]])/(a*(
a^2 + b^2)*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b\,\text{Subst}\left(\int \frac{b}{x(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= -\frac{b^2\,\text{Subst}\left(\int \frac{1}{x(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= -\frac{b^2\,\text{Subst}\left(\int \left(-\frac{1}{ab^2x}+\frac{1}{a\left(a^2+b^2\right)(a+x)}+\frac{b^2+ax}{b^2\left(a^2+b^2\right)\left(b^2+x^2\right)}\right) dx, x, b\sinh(c+dx)\right)}{d}$$

$$= \frac{\log(\sinh(c+dx))}{ad} - \frac{b^2\log(a+b\sinh(c+dx))}{a\left(a^2+b^2\right)d} - \frac{\text{Subst}\left(\int \frac{b^2+ax}{b^2+x^2}\,dx, x, b\sinh(c+dx)\right)}{\left(a^2+b^2\right)d}$$

$$= \frac{\log(\sinh(c+dx))}{ad} - \frac{b^2\log(a+b\sinh(c+dx))}{a\left(a^2+b^2\right)d} - \frac{a\,\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(c+dx)\right)}{\left(a^2+b^2\right)d}$$

$$= -\frac{b\tan^{-1}(\sinh(c+dx))}{\left(a^2+b^2\right)d} - \frac{a\log(\cosh(c+dx))}{\left(a^2+b^2\right)d} + \frac{\log(\sinh(c+dx))}{ad} - \frac{b^2\log(a+b\sin}{a\left(a^2+b\right.}$$

Mathematica [C] time = 0.133677, size = 92, normalized size = 1.02

$$-\frac{\frac{2b^2\log(a+b\sinh(c+dx))}{a\left(a^2+b^2\right)}+\frac{\log(-\sinh(c+dx)+i)}{a+ib}+\frac{\log(\sinh(c+dx)+i)}{a-ib}-\frac{2\log(\sinh(c+dx))}{a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(Log[I - Sinh[c + d*x]]/(a + I*b) - (2*Log[Sinh[c + d*x]])/a + Log[I + Sin
h[c + d*x]]/(a - I*b) + (2*b^2*Log[a + b*Sinh[c + d*x]])/(a*(a^2 + b^2)))/(
2*d)
```

Maple [A] time = 0.003, size = 123, normalized size = 1.4

$$\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right) - \frac{b^2}{d\left(a^2+b^2\right)a}\ln\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 a - 2\tanh(1/2\,dx+c/2)\,b - a\right) - \frac{a}{d\left(a^2+b^2\right)}\ln\left(\left(\tanh\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/d/a*ln(tanh(1/2*d*x+1/2*c))-1/d*b^2/(a^2+b^2)/a*ln(tanh(1/2*d*x+1/2*c)^2*
a-2*tanh(1/2*d*x+1/2*c)*b-a)-1/d/(a^2+b^2)*a*ln(tanh(1/2*d*x+1/2*c)^2+1)-2/
d/(a^2+b^2)*b*arctan(tanh(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.72629, size = 186, normalized size = 2.07

$$-\frac{b^2\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{\left(a^3+ab^2\right)d}+\frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{\left(a^2+b^2\right)d}-\frac{a\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\left(a^2+b^2\right)d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{ad}+\frac{\log\left(e^{(-d}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^3 + a*b^2)*d) + 2*
b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a^2
+ b^2)*d) + log(e^(-d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d)
```

Fricas [A] time = 2.6949, size = 350, normalized size = 3.89

$$-\frac{2\,ab\arctan\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)\right)+b^2\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)+a^2\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)-\left(a^2+b^2\right)}{\left(a^3+ab^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*a*b*arctan(cosh(d*x + c) + sinh(d*x + c)) + b^2*log(2*(b*sinh(d*x + c)
+ a)/(cosh(d*x + c) - sinh(d*x + c))) + a^2*log(2*cosh(d*x + c)/(cosh(d*x +
 c) - sinh(d*x + c))) - (a^2 + b^2)*log(2*sinh(d*x + c)/(cosh(d*x + c) - si
nh(d*x + c))))/((a^3 + a*b^2)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27236, size = 211, normalized size = 2.34

$$-\frac{b^3\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^3bd+ab^3d}-\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)b}{2\left(a^2d+b^2d\right)}-\frac{a\log\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)}{2\left(a^2d+b^2d\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^3*b*d + a*b^3*d) - 1
/2*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*b/(a^2*d + b^2*d
) - 1/2*a*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2*d + b^2*d) + log(abs
(e^(d*x + c) - e^(-d*x - c)))/(a*d)
```

3.439 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=34

$$\text{Unintegrable}\left(\frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Csch[c + d*x]*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [A] time = 0.0632793, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[c + d*x]*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Csch[c + d*x]*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A] time = 27.5799, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[c + d*x]*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x
]
```

```
[Out] Integrate[(Csch[c + d*x]*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
x]
```

Maple [A] time = 0.098, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] integrate(csch(d*x + c)*sech(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)}{afx+ae+(bfx+be)\sinh(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] integral(csch(d*x + c)*sech(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
+ c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx+c)\operatorname{sech}(dx+c)}{(fx+e)(b\sinh(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] integrate(csch(d*x + c)*sech(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

3.440 $\int \frac{(e+fx)^3\mathbf{csch}(c+dx)\mathbf{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1164

result too large to display

```
[Out] -((b*(e + f*x)^3)/((a^2 + b^2)*d)) - (6*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/
(a*d^2) + (6*b^2*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/(a*(a^2 + b^2)*d^2) - (
2*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (b^3*(e + f*x)^3*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^(3/2)*d) + (b^3*(e + f*x)^
3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^(3/2)*d) +
 (3*b*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)*d^2) - (3*f*(e +
 f*x)^2*PolyLog[2, -E^(c + d*x)])/(a*d^2) + ((6*I)*f^2*(e + f*x)*PolyLog[2,
 (-I)*E^(c + d*x)])/(a*d^3) - ((6*I)*b^2*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c
 + d*x)])/(a*(a^2 + b^2)*d^3) - ((6*I)*f^2*(e + f*x)*PolyLog[2, I*E^(c + d*
x)])/(a*d^3) + ((6*I)*b^2*f^2*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(a*(a^2
+ b^2)*d^3) + (3*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)])/(a*d^2) - (3*b^3*f*
(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 +
 b^2)^(3/2)*d^2) + (3*b^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^2) + (3*b*f^2*(e + f*x)*PolyLog[2
, -E^(2*(c + d*x))])/((a^2 + b^2)*d^3) + (6*f^2*(e + f*x)*PolyLog[3, -E^(c
+ d*x)])/(a*d^3) - ((6*I)*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^4) + ((6*I
)*b^2*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^4) + ((6*I)*f^3*Po
lyLog[3, I*E^(c + d*x)])/(a*d^4) - ((6*I)*b^2*f^3*PolyLog[3, I*E^(c + d*x)]
)/(a*(a^2 + b^2)*d^4) - (6*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a*d^3) +
 (6*b^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])
/(a*(a^2 + b^2)^(3/2)*d^3) - (6*b^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^3) - (3*b*f^3*PolyLog[3
, -E^(2*(c + d*x))])/(2*(a^2 + b^2)*d^4) - (6*f^3*PolyLog[4, -E^(c + d*x)])
/(a*d^4) + (6*f^3*PolyLog[4, E^(c + d*x)])/(a*d^4) - (6*b^3*f^3*PolyLog[4,
-((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^4) + (6*b
^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)
^(3/2)*d^4) + ((e + f*x)^3*Sech[c + d*x])/(a*d) - (b^2*(e + f*x)^3*Sech[c +
 d*x])/(a*(a^2 + b^2)*d) - (b*(e + f*x)^3*Tanh[c + d*x])/((a^2 + b^2)*d)
```

Rubi [A] time = 2.19887, antiderivative size = 1164, normalized size of antiderivative = 1., number of steps used = 53, number of rules used = 22, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {5589, 2622, 321, 207, 5462, 6741, 12, 6742, 6273, 4182, 2531, 6609, 2282, 6589, 4180, 5573, 3322, 2264, 2190, 4184, 3718, 5451}

$$-\frac{(e+fx)^3\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)b^3}{a\left(a^2+b^2\right)^{3/2}d}+\frac{(e+fx)^3\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)b^3}{a\left(a^2+b^2\right)^{3/2}d}-\frac{3f(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)b^3}{a\left(a^2+b^2\right)^{3/2}d^2}+ 3f(e+$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -((b*(e + f*x)^3)/((a^2 + b^2)*d)) - (6*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/
(a*d^2) + (6*b^2*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/(a*(a^2 + b^2)*d^2) - (
2*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (b^3*(e + f*x)^3*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^(3/2)*d) + (b^3*(e + f*x)^
3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^(3/2)*d) +
 (3*b*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)*d^2) - (3*f*(e +
 f*x)^2*PolyLog[2, -E^(c + d*x)])/(a*d^2) + ((6*I)*f^2*(e + f*x)*PolyLog[2,
 (-I)*E^(c + d*x)])/(a*d^3) - ((6*I)*b^2*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c
```

```
 + d*x)])/(a*(a^2 + b^2)*d^3) - ((6*I)*f^2*(e + f*x)*PolyLog[2, I*E^(c + d*
x)])/(a*d^3) + ((6*I)*b^2*f^2*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(a*(a^2
+ b^2)*d^3) + (3*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)])/(a*d^2) - (3*b^3*f*
(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 +
 b^2)^(3/2)*d^2) + (3*b^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^2) + (3*b*f^2*(e + f*x)*PolyLog[2
, -E^(2*(c + d*x))])/((a^2 + b^2)*d^3) + (6*f^2*(e + f*x)*PolyLog[3, -E^(c
+ d*x)])/(a*d^3) - ((6*I)*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^4) + ((6*I
)*b^2*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^4) + ((6*I)*f^3*Po
lyLog[3, I*E^(c + d*x)])/(a*d^4) - ((6*I)*b^2*f^3*PolyLog[3, I*E^(c + d*x)]
)/(a*(a^2 + b^2)*d^4) - (6*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a*d^3) +
 (6*b^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])
/(a*(a^2 + b^2)^(3/2)*d^3) - (6*b^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^3) - (3*b*f^3*PolyLog[3
, -E^(2*(c + d*x))])/(2*(a^2 + b^2)*d^4) - (6*f^3*PolyLog[4, -E^(c + d*x)])
/(a*d^4) + (6*f^3*PolyLog[4, E^(c + d*x)])/(a*d^4) - (6*b^3*f^3*PolyLog[4,
-((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^4) + (6*b
^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)
^(3/2)*d^4) + ((e + f*x)^3*Sech[c + d*x])/(a*d) - (b^2*(e + f*x)^3*Sech[c +
 d*x])/(a*(a^2 + b^2)*d) - (b*(e + f*x)^3*Tanh[c + d*x])/((a^2 + b^2)*d)
```

Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
```

```
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^3\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(e+fx)^3\operatorname{sech}(c+dx)}{ad} - \frac{b\int (e+fx$$

$$= -\frac{(e+fx)^3\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(e+fx)^3\operatorname{sech}(c+dx)}{ad} - \frac{b\int \left(a(e+\right.$$

$$= -\frac{(e+fx)^3\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(e+fx)^3\operatorname{sech}(c+dx)}{ad} - \frac{\left(2b^4\right)\int \frac{}{2a-}}{a\left(a\right.}$$

$$= -\frac{(e+fx)^3\tanh^{-1}(\cosh(c+dx))}{ad} - \frac{b^3(e+fx)^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{3/2}d} + \frac{b^3(}{}$$

$$= -\frac{b(e+fx)^3}{\left(a^2+b^2\right)d} + \frac{6b^2f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d^2} - \frac{(e+fx)^3\tanh^{-1}(\cosh(c+}{ad}$$

$$= -\frac{b(e+fx)^3}{\left(a^2+b^2\right)d} - \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{6b^2f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d^2}$$

$$= -\frac{b(e+fx)^3}{\left(a^2+b^2\right)d} - \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{6b^2f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d^2}$$

$$= -\frac{b(e+fx)^3}{\left(a^2+b^2\right)d} - \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{6b^2f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d^2}$$

$$= -\frac{b(e+fx)^3}{\left(a^2+b^2\right)d} - \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{6b^2f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d^2}$$

$$= -\frac{b(e+fx)^3}{\left(a^2+b^2\right)d} - \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{6b^2f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d^2}$$

$$= -\frac{b(e+fx)^3}{\left(a^2+b^2\right)d} - \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{6b^2f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d^2}$$

$$= -\frac{b(e+fx)^3}{\left(a^2+b^2\right)d} - \frac{6f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{6b^2f(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d^2}$$

Mathematica [A] time = 16.9409, size = 1467, normalized size = 1.26

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] 4*(-(f*Csch[c + d*x]*(12*b*d^3*e^2*E^(2*c)*x - 12*b*d^3*e^2*(1 + E^(2*c))*x
 - 12*b*d^3*e*f*x^2 - 4*b*d^3*f^2*x^3 + 12*a*d^2*e^2*(1 + E^(2*c))*ArcTan[E
^(c + d*x)] + 6*b*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))])
+ (12*I)*a*d*e*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(
c + d*x)]) - PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) + 6*
b*d*e*(1 + E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2,
-E^(2*(c + d*x))]) + (6*I)*a*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*
x)] - d^2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] +
 2*d*x*PolyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyL
og[3, I*E^(c + d*x)]) + b*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E
^(2*(c + d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*
(c + d*x))]))*(a + b*Sinh[c + d*x]))/(8*(a^2 + b^2)*d^4*(1 + E^(2*c))*(b +
a*Csch[c + d*x])) + (b^3*Csch[c + d*x]*(2*d^3*e^3*ArcTanh[(a + b*E^(c + d*x
))/Sqrt[a^2 + b^2]] - 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])] - 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] -
d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 3*d^3*e^2*f*x*
Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a
 + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))] + 6*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2
+ b^2])] + 6*d*f^3*x*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6
*d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*f^3*x*P
olyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*f^3*PolyLog[4, (b*E
^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2]))])*(a + b*Sinh[c + d*x]))/(4*a*(a^2 + b^2)^(3/2)*d^4*(b
 + a*Csch[c + d*x])) + (Csch[c + d*x]*(-2*(e + f*x)^3*ArcTanh[Cosh[c + d*x]
 + Sinh[c + d*x]] - (3*f*(d^2*(e + f*x)^2*PolyLog[2, -Cosh[c + d*x] - Sinh[
c + d*x]] - 2*d*f*(e + f*x)*PolyLog[3, -Cosh[c + d*x] - Sinh[c + d*x]] + 2*
f^2*PolyLog[4, -Cosh[c + d*x] - Sinh[c + d*x]]))/d^3 + (3*f*(d^2*(e + f*x)^
2*PolyLog[2, Cosh[c + d*x] + Sinh[c + d*x]] - 2*d*f*(e + f*x)*PolyLog[3, Co
sh[c + d*x] + Sinh[c + d*x]] + 2*f^2*PolyLog[4, Cosh[c + d*x] + Sinh[c + d*
x]]))/d^3)*(a + b*Sinh[c + d*x]))/(4*a*d*(b + a*Csch[c + d*x])) + (Csch[c +
 d*x]*Sech[c]*Sech[c + d*x]*(a*e^3*Cosh[c] + 3*a*e^2*f*x*Cosh[c] + 3*a*e*f^
2*x^2*Cosh[c] + a*f^3*x^3*Cosh[c] - b*e^3*Sinh[d*x] - 3*b*e^2*f*x*Sinh[d*x]
 - 3*b*e*f^2*x^2*Sinh[d*x] - b*f^3*x^3*Sinh[d*x])*(a + b*Sinh[c + d*x]))/(4
*(a^2 + b^2)*d*(b + a*Csch[c + d*x])))
```

Maple [F] time = 1.674, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3} \operatorname{csch}\left(dx+c\right) \left(\operatorname{sech}\left(dx+c\right)\right)^{2}}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 5.446, size = 21639, normalized size = 18.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/2*(4*(a^3*b + a*b^3)*d^3*e^3 - 12*(a^3*b + a*b^3)*c*d^2*e^2*f + 12*(a^3*b
 + a*b^3)*c^2*d*e*f^2 - 4*(a^3*b + a*b^3)*c^3*f^3 - 4*((a^3*b + a*b^3)*d^3*
f^3*x^3 + 3*(a^3*b + a*b^3)*d^3*e*f^2*x^2 + 3*(a^3*b + a*b^3)*d^3*e^2*f*x +
 3*(a^3*b + a*b^3)*c*d^2*e^2*f - 3*(a^3*b + a*b^3)*c^2*d*e*f^2 + (a^3*b + a
*b^3)*c^3*f^3)*cosh(d*x + c)^2 - 4*((a^3*b + a*b^3)*d^3*f^3*x^3 + 3*(a^3*b
+ a*b^3)*d^3*e*f^2*x^2 + 3*(a^3*b + a*b^3)*d^3*e^2*f*x + 3*(a^3*b + a*b^3)*
c*d^2*e^2*f - 3*(a^3*b + a*b^3)*c^2*d*e*f^2 + (a^3*b + a*b^3)*c^3*f^3)*sinh
(d*x + c)^2 - 6*(b^4*d^2*f^3*x^2 + 2*b^4*d^2*e*f^2*x + b^4*d^2*e^2*f + (b^4
*d^2*f^3*x^2 + 2*b^4*d^2*e*f^2*x + b^4*d^2*e^2*f)*cosh(d*x + c)^2 + 2*(b^4*
d^2*f^3*x^2 + 2*b^4*d^2*e*f^2*x + b^4*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c
) + (b^4*d^2*f^3*x^2 + 2*b^4*d^2*e*f^2*x + b^4*d^2*e^2*f)*sinh(d*x + c)^2)*
sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*
x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 6*(b^4*d^2*f^
3*x^2 + 2*b^4*d^2*e*f^2*x + b^4*d^2*e^2*f + (b^4*d^2*f^3*x^2 + 2*b^4*d^2*e*
f^2*x + b^4*d^2*e^2*f)*cosh(d*x + c)^2 + 2*(b^4*d^2*f^3*x^2 + 2*b^4*d^2*e*f
^2*x + b^4*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) + (b^4*d^2*f^3*x^2 + 2*b^
4*d^2*e*f^2*x + b^4*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog
((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^4*d^3*e^3 - 3*b^4*c*d^2*e^2*f + 3*b
^4*c^2*d*e*f^2 - b^4*c^3*f^3 + (b^4*d^3*e^3 - 3*b^4*c*d^2*e^2*f + 3*b^4*c^2
*d*e*f^2 - b^4*c^3*f^3)*cosh(d*x + c)^2 + 2*(b^4*d^3*e^3 - 3*b^4*c*d^2*e^2*
f + 3*b^4*c^2*d*e*f^2 - b^4*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) + (b^4*d^3
*e^3 - 3*b^4*c*d^2*e^2*f + 3*b^4*c^2*d*e*f^2 - b^4*c^3*f^3)*sinh(d*x + c)^2
)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqr
t((a^2 + b^2)/b^2) + 2*a) - 2*(b^4*d^3*e^3 - 3*b^4*c*d^2*e^2*f + 3*b^4*c^2*
d*e*f^2 - b^4*c^3*f^3 + (b^4*d^3*e^3 - 3*b^4*c*d^2*e^2*f + 3*b^4*c^2*d*e*f^
2 - b^4*c^3*f^3)*cosh(d*x + c)^2 + 2*(b^4*d^3*e^3 - 3*b^4*c*d^2*e^2*f + 3*b
^4*c^2*d*e*f^2 - b^4*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) + (b^4*d^3*e^3 -
3*b^4*c*d^2*e^2*f + 3*b^4*c^2*d*e*f^2 - b^4*c^3*f^3)*sinh(d*x + c)^2)*sqrt(
(a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2
+ b^2)/b^2) + 2*a) - 2*(b^4*d^3*f^3*x^3 + 3*b^4*d^3*e*f^2*x^2 + 3*b^4*d^3*e
^2*f*x + 3*b^4*c*d^2*e^2*f - 3*b^4*c^2*d*e*f^2 + b^4*c^3*f^3 + (b^4*d^3*f^3
*x^3 + 3*b^4*d^3*e*f^2*x^2 + 3*b^4*d^3*e^2*f*x + 3*b^4*c*d^2*e^2*f - 3*b^4*
c^2*d*e*f^2 + b^4*c^3*f^3)*cosh(d*x + c)^2 + 2*(b^4*d^3*f^3*x^3 + 3*b^4*d^3
*e*f^2*x^2 + 3*b^4*d^3*e^2*f*x + 3*b^4*c*d^2*e^2*f - 3*b^4*c^2*d*e*f^2 + b^
4*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) + (b^4*d^3*f^3*x^3 + 3*b^4*d^3*e*f^2
*x^2 + 3*b^4*d^3*e^2*f*x + 3*b^4*c*d^2*e^2*f - 3*b^4*c^2*d*e*f^2 + b^4*c^3*
f^3)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(
d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b
) + 2*(b^4*d^3*f^3*x^3 + 3*b^4*d^3*e*f^2*x^2 + 3*b^4*d^3*e^2*f*x + 3*b^4*c*
d^2*e^2*f - 3*b^4*c^2*d*e*f^2 + b^4*c^3*f^3 + (b^4*d^3*f^3*x^3 + 3*b^4*d^3*
```

e*f^2*x^2 + 3*b^4*d^3*e^2*f*x + 3*b^4*c*d^2*e^2*f - 3*b^4*c^2*d*e*f^2 + b^4
*c^3*f^3)*cosh(d*x + c)^2 + 2*(b^4*d^3*f^3*x^3 + 3*b^4*d^3*e*f^2*x^2 + 3*b^
4*d^3*e^2*f*x + 3*b^4*c*d^2*e^2*f - 3*b^4*c^2*d*e*f^2 + b^4*c^3*f^3)*cosh(d
*x + c)*sinh(d*x + c) + (b^4*d^3*f^3*x^3 + 3*b^4*d^3*e*f^2*x^2 + 3*b^4*d^3*
e^2*f*x + 3*b^4*c*d^2*e^2*f - 3*b^4*c^2*d*e*f^2 + b^4*c^3*f^3)*sinh(d*x + c
)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 12*(b^4*f^3*c
osh(d*x + c)^2 + 2*b^4*f^3*cosh(d*x + c)*sinh(d*x + c) + b^4*f^3*sinh(d*x +
c)^2 + b^4*f^3)*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh
(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) +
12*(b^4*f^3*cosh(d*x + c)^2 + 2*b^4*f^3*cosh(d*x + c)*sinh(d*x + c) + b^4*
f^3*sinh(d*x + c)^2 + b^4*f^3)*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2))/b) + 12*(b^4*d*f^3*x + b^4*d*e*f^2 + (b^4*d*f^3*x + b^4*d*e*f^2)*
cosh(d*x + c)^2 + 2*(b^4*d*f^3*x + b^4*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c)
+ (b^4*d*f^3*x + b^4*d*e*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*polyl
og(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2))/b) - 12*(b^4*d*f^3*x + b^4*d*e*f^2 + (b^4*d*f^3*
x + b^4*d*e*f^2)*cosh(d*x + c)^2 + 2*(b^4*d*f^3*x + b^4*d*e*f^2)*cosh(d*x +
c)*sinh(d*x + c) + (b^4*d*f^3*x + b^4*d*e*f^2)*sinh(d*x + c)^2)*sqrt((a^2
+ b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 4*((a^4 + a^2*b^2)*d^3*f^3
x^3 + 3(a^4 + a^2*b^2)*d^3*e*f^2*x^2 + 3*(a^4 + a^2*b^2)*d^3*e^2*f*x + (a
^4 + a^2*b^2)*d^3*e^3)*cosh(d*x + c) + 6*((a^4 + 2*a^2*b^2 + b^4)*d^2*f^3*x
^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d^2*e^
2*f + ((a^4 + 2*a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*
e*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d^2*e^2*f)*cosh(d*x + c)^2 + 2*((a^4 + 2*
a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f^2*x + (a^4 +
2*a^2*b^2 + b^4)*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) + ((a^4 + 2*a^2*b^
2 + b^4)*d^2*f^3*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f^2*x + (a^4 + 2*a^2
*b^2 + b^4)*d^2*e^2*f)*sinh(d*x + c)^2)*dilog(cosh(d*x + c) + sinh(d*x + c)
) + (-12*I*(a^4 + a^2*b^2)*d*f^3*x + 12*(a^3*b + a*b^3)*d*f^3*x - 12*I*(a^4
+ a^2*b^2)*d*e*f^2 + 12*(a^3*b + a*b^3)*d*e*f^2 + (-12*I*(a^4 + a^2*b^2)*d
*f^3*x + 12*(a^3*b + a*b^3)*d*f^3*x - 12*I*(a^4 + a^2*b^2)*d*e*f^2 + 12*(a^
3*b + a*b^3)*d*e*f^2)*cosh(d*x + c)^2 + (-24*I*(a^4 + a^2*b^2)*d*f^3*x + 24
*(a^3*b + a*b^3)*d*f^3*x - 24*I*(a^4 + a^2*b^2)*d*e*f^2 + 24*(a^3*b + a*b^3
)*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) + (-12*I*(a^4 + a^2*b^2)*d*f^3*x + 1
2*(a^3*b + a*b^3)*d*f^3*x - 12*I*(a^4 + a^2*b^2)*d*e*f^2 + 12*(a^3*b + a*b^
3)*d*e*f^2)*sinh(d*x + c)^2)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + (12
I(a^4 + a^2*b^2)*d*f^3*x + 12*(a^3*b + a*b^3)*d*f^3*x + 12*I*(a^4 + a^2*b
^2)*d*e*f^2 + 12*(a^3*b + a*b^3)*d*e*f^2 + (12*I*(a^4 + a^2*b^2)*d*f^3*x +
12*(a^3*b + a*b^3)*d*f^3*x + 12*I*(a^4 + a^2*b^2)*d*e*f^2 + 12*(a^3*b + a*b
^3)*d*e*f^2)*cosh(d*x + c)^2 + (24*I*(a^4 + a^2*b^2)*d*f^3*x + 24*(a^3*b +
a*b^3)*d*f^3*x + 24*I*(a^4 + a^2*b^2)*d*e*f^2 + 24*(a^3*b + a*b^3)*d*e*f^2)
*cosh(d*x + c)*sinh(d*x + c) + (12*I*(a^4 + a^2*b^2)*d*f^3*x + 12*(a^3*b +
a*b^3)*d*f^3*x + 12*I*(a^4 + a^2*b^2)*d*e*f^2 + 12*(a^3*b + a*b^3)*d*e*f^2)
*sinh(d*x + c)^2)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) - 6*((a^4 + 2*a
^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f^2*x + (a^4 +
2*a^2*b^2 + b^4)*d^2*e^2*f + ((a^4 + 2*a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^4
+ 2*a^2*b^2 + b^4)*d^2*e*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d^2*e^2*f)*cosh(d*
x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^4 + 2*a^2*b^2 + b^
4)*d^2*e*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x
+ c) + ((a^4 + 2*a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2
*e*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d^2*e^2*f)*sinh(d*x + c)^2)*dilog(-cosh(
d*x + c) - sinh(d*x + c)) - 2*((a^4 + 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 3*(a^4
+ 2*a^2*b^2 + b^4)*d^3*e*f^2*x^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^3*e^2*f*x +
(a^4 + 2*a^2*b^2 + b^4)*d^3*e^3 + ((a^4 + 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 3
*(a^4 + 2*a^2*b^2 + b^4)*d^3*e*f^2*x^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^3*e^2*
f*x + (a^4 + 2*a^2*b^2 + b^4)*d^3*e^3)*cosh(d*x + c)^2 + 2*((a^4 + 2*a^2*b^

```
2 + b^4)*d^3*f^3*x^3 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^3*e*f^2*x^2 + 3*(a^4 + 2
*a^2*b^2 + b^4)*d^3*e^2*f*x + (a^4 + 2*a^2*b^2 + b^4)*d^3*e^3)*cosh(d*x + c
)*sinh(d*x + c) + ((a^4 + 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 3*(a^4 + 2*a^2*b^2
 + b^4)*d^3*e*f^2*x^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^3*e^2*f*x + (a^4 + 2*a^
2*b^2 + b^4)*d^3*e^3)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) +
1) + (-6*I*(a^4 + a^2*b^2)*d^2*e^2*f + 6*(a^3*b + a*b^3)*d^2*e^2*f + 12*I*(
a^4 + a^2*b^2)*c*d*e*f^2 - 12*(a^3*b + a*b^3)*c*d*e*f^2 - 6*I*(a^4 + a^2*b^
2)*c^2*f^3 + 6*(a^3*b + a*b^3)*c^2*f^3 + (-6*I*(a^4 + a^2*b^2)*d^2*e^2*f +
6*(a^3*b + a*b^3)*d^2*e^2*f + 12*I*(a^4 + a^2*b^2)*c*d*e*f^2 - 12*(a^3*b +
a*b^3)*c*d*e*f^2 - 6*I*(a^4 + a^2*b^2)*c^2*f^3 + 6*(a^3*b + a*b^3)*c^2*f^3)
*cosh(d*x + c)^2 + (-12*I*(a^4 + a^2*b^2)*d^2*e^2*f + 12*(a^3*b + a*b^3)*d^
2*e^2*f + 24*I*(a^4 + a^2*b^2)*c*d*e*f^2 - 24*(a^3*b + a*b^3)*c*d*e*f^2 - 1
2*I*(a^4 + a^2*b^2)*c^2*f^3 + 12*(a^3*b + a*b^3)*c^2*f^3)*cosh(d*x + c)*sin
h(d*x + c) + (-6*I*(a^4 + a^2*b^2)*d^2*e^2*f + 6*(a^3*b + a*b^3)*d^2*e^2*f
+ 12*I*(a^4 + a^2*b^2)*c*d*e*f^2 - 12*(a^3*b + a*b^3)*c*d*e*f^2 - 6*I*(a^4
+ a^2*b^2)*c^2*f^3 + 6*(a^3*b + a*b^3)*c^2*f^3)*sinh(d*x + c)^2)*log(cosh(d
*x + c) + sinh(d*x + c) + I) + (6*I*(a^4 + a^2*b^2)*d^2*e^2*f + 6*(a^3*b +
a*b^3)*d^2*e^2*f - 12*I*(a^4 + a^2*b^2)*c*d*e*f^2 - 12*(a^3*b + a*b^3)*c*d*
e*f^2 + 6*I*(a^4 + a^2*b^2)*c^2*f^3 + 6*(a^3*b + a*b^3)*c^2*f^3 + (6*I*(a^4
 + a^2*b^2)*d^2*e^2*f + 6*(a^3*b + a*b^3)*d^2*e^2*f - 12*I*(a^4 + a^2*b^2)*
c*d*e*f^2 - 12*(a^3*b + a*b^3)*c*d*e*f^2 + 6*I*(a^4 + a^2*b^2)*c^2*f^3 + 6*
(a^3*b + a*b^3)*c^2*f^3)*cosh(d*x + c)^2 + (12*I*(a^4 + a^2*b^2)*d^2*e^2*f
+ 12*(a^3*b + a*b^3)*d^2*e^2*f - 24*I*(a^4 + a^2*b^2)*c*d*e*f^2 - 24*(a^3*b
 + a*b^3)*c*d*e*f^2 + 12*I*(a^4 + a^2*b^2)*c^2*f^3 + 12*(a^3*b + a*b^3)*c^2
*f^3)*cosh(d*x + c)*sinh(d*x + c) + (6*I*(a^4 + a^2*b^2)*d^2*e^2*f + 6*(a^3
*b + a*b^3)*d^2*e^2*f - 12*I*(a^4 + a^2*b^2)*c*d*e*f^2 - 12*(a^3*b + a*b^3)
*c*d*e*f^2 + 6*I*(a^4 + a^2*b^2)*c^2*f^3 + 6*(a^3*b + a*b^3)*c^2*f^3)*sinh(
d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) - I) + 2*((a^4 + 2*a^2*b^2 +
b^4)*d^3*e^3 - 3*(a^4 + 2*a^2*b^2 + b^4)*c*d^2*e^2*f + 3*(a^4 + 2*a^2*b^2 +
 b^4)*c^2*d*e*f^2 - (a^4 + 2*a^2*b^2 + b^4)*c^3*f^3 + ((a^4 + 2*a^2*b^2 + b
^4)*d^3*e^3 - 3*(a^4 + 2*a^2*b^2 + b^4)*c*d^2*e^2*f + 3*(a^4 + 2*a^2*b^2 +
b^4)*c^2*d*e*f^2 - (a^4 + 2*a^2*b^2 + b^4)*c^3*f^3)*cosh(d*x + c)^2 + 2*((a
^4 + 2*a^2*b^2 + b^4)*d^3*e^3 - 3*(a^4 + 2*a^2*b^2 + b^4)*c*d^2*e^2*f + 3*(
a^4 + 2*a^2*b^2 + b^4)*c^2*d*e*f^2 - (a^4 + 2*a^2*b^2 + b^4)*c^3*f^3)*cosh(
d*x + c)*sinh(d*x + c) + ((a^4 + 2*a^2*b^2 + b^4)*d^3*e^3 - 3*(a^4 + 2*a^2*
b^2 + b^4)*c*d^2*e^2*f + 3*(a^4 + 2*a^2*b^2 + b^4)*c^2*d*e*f^2 - (a^4 + 2*a
^2*b^2 + b^4)*c^3*f^3)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) -
 1) + (6*I*(a^4 + a^2*b^2)*d^2*f^3*x^2 + 6*(a^3*b + a*b^3)*d^2*f^3*x^2 + 12
*I*(a^4 + a^2*b^2)*d^2*e*f^2*x + 12*(a^3*b + a*b^3)*d^2*e*f^2*x + 12*I*(a^4
 + a^2*b^2)*c*d*e*f^2 + 12*(a^3*b + a*b^3)*c*d*e*f^2 - 6*I*(a^4 + a^2*b^2)*
c^2*f^3 - 6*(a^3*b + a*b^3)*c^2*f^3 + (6*I*(a^4 + a^2*b^2)*d^2*f^3*x^2 + 6*
(a^3*b + a*b^3)*d^2*f^3*x^2 + 12*I*(a^4 + a^2*b^2)*d^2*e*f^2*x + 12*(a^3*b
+ a*b^3)*d^2*e*f^2*x + 12*I*(a^4 + a^2*b^2)*c*d*e*f^2 + 12*(a^3*b + a*b^3)*
c*d*e*f^2 - 6*I*(a^4 + a^2*b^2)*c^2*f^3 - 6*(a^3*b + a*b^3)*c^2*f^3)*cosh(d
*x + c)^2 + (12*I*(a^4 + a^2*b^2)*d^2*f^3*x^2 + 12*(a^3*b + a*b^3)*d^2*f^3*
x^2 + 24*I*(a^4 + a^2*b^2)*d^2*e*f^2*x + 24*(a^3*b + a*b^3)*d^2*e*f^2*x + 2
4*I*(a^4 + a^2*b^2)*c*d*e*f^2 + 24*(a^3*b + a*b^3)*c*d*e*f^2 - 12*I*(a^4 +
a^2*b^2)*c^2*f^3 - 12*(a^3*b + a*b^3)*c^2*f^3)*cosh(d*x + c)*sinh(d*x + c)
+ (6*I*(a^4 + a^2*b^2)*d^2*f^3*x^2 + 6*(a^3*b + a*b^3)*d^2*f^3*x^2 + 12*I*(
a^4 + a^2*b^2)*d^2*e*f^2*x + 12*(a^3*b + a*b^3)*d^2*e*f^2*x + 12*I*(a^4 + a
^2*b^2)*c*d*e*f^2 + 12*(a^3*b + a*b^3)*c*d*e*f^2 - 6*I*(a^4 + a^2*b^2)*c^2*
f^3 - 6*(a^3*b + a*b^3)*c^2*f^3)*sinh(d*x + c)^2)*log(I*cosh(d*x + c) + I*s
inh(d*x + c) + 1) + (-6*I*(a^4 + a^2*b^2)*d^2*f^3*x^2 + 6*(a^3*b + a*b^3)*d
^2*f^3*x^2 - 12*I*(a^4 + a^2*b^2)*d^2*e*f^2*x + 12*(a^3*b + a*b^3)*d^2*e*f^
2*x - 12*I*(a^4 + a^2*b^2)*c*d*e*f^2 + 12*(a^3*b + a*b^3)*c*d*e*f^2 + 6*I*(
a^4 + a^2*b^2)*c^2*f^3 - 6*(a^3*b + a*b^3)*c^2*f^3 + (-6*I*(a^4 + a^2*b^2)*
d^2*f^3*x^2 + 6*(a^3*b + a*b^3)*d^2*f^3*x^2 - 12*I*(a^4 + a^2*b^2)*d^2*e*f^
2*x + 12*(a^3*b + a*b^3)*d^2*e*f^2*x - 12*I*(a^4 + a^2*b^2)*c*d*e*f^2 + 12*
(a^3*b + a*b^3)*c*d*e*f^2 + 6*I*(a^4 + a^2*b^2)*c^2*f^3 - 6*(a^3*b + a*b^3)
```

```
*c^2*f^3)*cosh(d*x + c)^2 + (-12*I*(a^4 + a^2*b^2)*d^2*f^3*x^2 + 12*(a^3*b
+ a*b^3)*d^2*f^3*x^2 - 24*I*(a^4 + a^2*b^2)*d^2*e*f^2*x + 24*(a^3*b + a*b^3
)*d^2*e*f^2*x - 24*I*(a^4 + a^2*b^2)*c*d*e*f^2 + 24*(a^3*b + a*b^3)*c*d*e*f
^2 + 12*I*(a^4 + a^2*b^2)*c^2*f^3 - 12*(a^3*b + a*b^3)*c^2*f^3)*cosh(d*x +
c)*sinh(d*x + c) + (-6*I*(a^4 + a^2*b^2)*d^2*f^3*x^2 + 6*(a^3*b + a*b^3)*d^
2*f^3*x^2 - 12*I*(a^4 + a^2*b^2)*d^2*e*f^2*x + 12*(a^3*b + a*b^3)*d^2*e*f^2
*x - 12*I*(a^4 + a^2*b^2)*c*d*e*f^2 + 12*(a^3*b + a*b^3)*c*d*e*f^2 + 6*I*(a
^4 + a^2*b^2)*c^2*f^3 - 6*(a^3*b + a*b^3)*c^2*f^3)*sinh(d*x + c)^2)*log(-I*
cosh(d*x + c) - I*sinh(d*x + c) + 1) + 2*((a^4 + 2*a^2*b^2 + b^4)*d^3*f^3*x
^3 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^3*e*f^2*x^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^
3*e^2*f*x + 3*(a^4 + 2*a^2*b^2 + b^4)*c*d^2*e^2*f - 3*(a^4 + 2*a^2*b^2 + b^
4)*c^2*d*e*f^2 + (a^4 + 2*a^2*b^2 + b^4)*c^3*f^3 + ((a^4 + 2*a^2*b^2 + b^4)
*d^3*f^3*x^3 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^3*e*f^2*x^2 + 3*(a^4 + 2*a^2*b^2
 + b^4)*d^3*e^2*f*x + 3*(a^4 + 2*a^2*b^2 + b^4)*c*d^2*e^2*f - 3*(a^4 + 2*a^
2*b^2 + b^4)*c^2*d*e*f^2 + (a^4 + 2*a^2*b^2 + b^4)*c^3*f^3)*cosh(d*x + c)^2
 + 2*((a^4 + 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^3*e
*f^2*x^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^3*e^2*f*x + 3*(a^4 + 2*a^2*b^2 + b^4
)*c*d^2*e^2*f - 3*(a^4 + 2*a^2*b^2 + b^4)*c^2*d*e*f^2 + (a^4 + 2*a^2*b^2 +
b^4)*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) + ((a^4 + 2*a^2*b^2 + b^4)*d^3*f^
3*x^3 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^3*e*f^2*x^2 + 3*(a^4 + 2*a^2*b^2 + b^4)
*d^3*e^2*f*x + 3*(a^4 + 2*a^2*b^2 + b^4)*c*d^2*e^2*f - 3*(a^4 + 2*a^2*b^2 +
 b^4)*c^2*d*e*f^2 + (a^4 + 2*a^2*b^2 + b^4)*c^3*f^3)*sinh(d*x + c)^2)*log(-
cosh(d*x + c) - sinh(d*x + c) + 1) + 12*((a^4 + 2*a^2*b^2 + b^4)*f^3*cosh(d
*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*f^3*cosh(d*x + c)*sinh(d*x + c) + (a^
4 + 2*a^2*b^2 + b^4)*f^3*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*f^3)*pol
ylog(4, cosh(d*x + c) + sinh(d*x + c)) - 12*((a^4 + 2*a^2*b^2 + b^4)*f^3*co
sh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*f^3*cosh(d*x + c)*sinh(d*x + c) +
 (a^4 + 2*a^2*b^2 + b^4)*f^3*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*f^3)
*polylog(4, -cosh(d*x + c) - sinh(d*x + c)) - 12*((a^4 + 2*a^2*b^2 + b^4)*d
*f^3*x + (a^4 + 2*a^2*b^2 + b^4)*d*e*f^2 + ((a^4 + 2*a^2*b^2 + b^4)*d*f^3*x
 + (a^4 + 2*a^2*b^2 + b^4)*d*e*f^2)*cosh(d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 +
 b^4)*d*f^3*x + (a^4 + 2*a^2*b^2 + b^4)*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c
) + ((a^4 + 2*a^2*b^2 + b^4)*d*f^3*x + (a^4 + 2*a^2*b^2 + b^4)*d*e*f^2)*sin
h(d*x + c)^2)*polylog(3, cosh(d*x + c) + sinh(d*x + c)) + (12*I*(a^4 + a^2*
b^2)*f^3 - 12*(a^3*b + a*b^3)*f^3 + (12*I*(a^4 + a^2*b^2)*f^3 - 12*(a^3*b +
 a*b^3)*f^3)*cosh(d*x + c)^2 + (24*I*(a^4 + a^2*b^2)*f^3 - 24*(a^3*b + a*b^
3)*f^3)*cosh(d*x + c)*sinh(d*x + c) + (12*I*(a^4 + a^2*b^2)*f^3 - 12*(a^3*b
 + a*b^3)*f^3)*sinh(d*x + c)^2)*polylog(3, I*cosh(d*x + c) + I*sinh(d*x + c
)) + (-12*I*(a^4 + a^2*b^2)*f^3 - 12*(a^3*b + a*b^3)*f^3 + (-12*I*(a^4 + a^
2*b^2)*f^3 - 12*(a^3*b + a*b^3)*f^3)*cosh(d*x + c)^2 + (-24*I*(a^4 + a^2*b^
2)*f^3 - 24*(a^3*b + a*b^3)*f^3)*cosh(d*x + c)*sinh(d*x + c) + (-12*I*(a^4
+ a^2*b^2)*f^3 - 12*(a^3*b + a*b^3)*f^3)*sinh(d*x + c)^2)*polylog(3, -I*cos
h(d*x + c) - I*sinh(d*x + c)) + 12*((a^4 + 2*a^2*b^2 + b^4)*d*f^3*x + (a^4
+ 2*a^2*b^2 + b^4)*d*e*f^2 + ((a^4 + 2*a^2*b^2 + b^4)*d*f^3*x + (a^4 + 2*a^
2*b^2 + b^4)*d*e*f^2)*cosh(d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d*f^3*x
+ (a^4 + 2*a^2*b^2 + b^4)*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) + ((a^4 + 2*
a^2*b^2 + b^4)*d*f^3*x + (a^4 + 2*a^2*b^2 + b^4)*d*e*f^2)*sinh(d*x + c)^2)*
polylog(3, -cosh(d*x + c) - sinh(d*x + c)) + 4*((a^4 + a^2*b^2)*d^3*f^3*x^3
 + 3*(a^4 + a^2*b^2)*d^3*e*f^2*x^2 + 3*(a^4 + a^2*b^2)*d^3*e^2*f*x + (a^4 +
 a^2*b^2)*d^3*e^3 - 2*((a^3*b + a*b^3)*d^3*f^3*x^3 + 3*(a^3*b + a*b^3)*d^3*
e*f^2*x^2 + 3*(a^3*b + a*b^3)*d^3*e^2*f*x + 3*(a^3*b + a*b^3)*c*d^2*e^2*f -
 3*(a^3*b + a*b^3)*c^2*d*e*f^2 + (a^3*b + a*b^3)*c^3*f^3)*cosh(d*x + c))*si
nh(d*x + c))/((a^5 + 2*a^3*b^2 + a*b^4)*d^4*cosh(d*x + c)^2 + 2*(a^5 + 2*a^
3*b^2 + a*b^4)*d^4*cosh(d*x + c)*sinh(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4)*
d^4*sinh(d*x + c)^2 + (a^5 + 2*a^3*b^2 + a*b^4)*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.441 $\int \frac{(e+fx)^2\mathbf{csch}(c+dx)\mathbf{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=795

$$-\frac{(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)b^3}{a\left(a^2+b^2\right)^{3/2}d}+\frac{(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)b^3}{a\left(a^2+b^2\right)^{3/2}d}-\frac{2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)b^3}{a\left(a^2+b^2\right)^{3/2}d^2}+\frac{2f(e+fx)}{}$$

```
[Out] -((b*(e + f*x)^2)/((a^2 + b^2)*d)) - (4*f*(e + f*x)*ArcTan[E^(c + d*x)])/(a
*d^2) + (4*b^2*f*(e + f*x)*ArcTan[E^(c + d*x)])/(a*(a^2 + b^2)*d^2) - (2*(e
 + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (b^3*(e + f*x)^2*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^(3/2)*d) + (b^3*(e + f*x)^2*Lo
g[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^(3/2)*d) + (2*
b*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)*d^2) - (2*f*(e + f*x)*
PolyLog[2, -E^(c + d*x)])/(a*d^2) + ((2*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)]
)/(a*d^3) - ((2*I)*b^2*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^3
) - ((2*I)*f^2*PolyLog[2, I*E^(c + d*x)])/(a*d^3) + ((2*I)*b^2*f^2*PolyLog[
2, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^3) + (2*f*(e + f*x)*PolyLog[2, E^(c + d
*x)])/(a*d^2) - (2*b^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^2) + (2*b^3*f*(e + f*x)*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^2) + (b*f^2*
PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2)*d^3) + (2*f^2*PolyLog[3, -E^(c +
 d*x)])/(a*d^3) - (2*f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) + (2*b^3*f^2*Poly
Log[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^3)
 - (2*b^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2
 + b^2)^(3/2)*d^3) + ((e + f*x)^2*Sech[c + d*x])/(a*d) - (b^2*(e + f*x)^2*S
ech[c + d*x])/(a*(a^2 + b^2)*d) - (b*(e + f*x)^2*Tanh[c + d*x])/((a^2 + b^2
)*d)
```

Rubi [A] time = 1.61409, antiderivative size = 795, normalized size of antiderivative = 1., number of steps used = 44, number of rules used = 23, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.676, Rules used = {5589, 2622, 321, 207, 5462, 6741, 12, 6742, 6273, 4182, 2531, 2282, 6589, 4180, 2279, 2391, 5573, 3322, 2264, 2190, 4184, 3718, 5451}

$$-\frac{(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)b^3}{a\left(a^2+b^2\right)^{3/2}d}+\frac{(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)b^3}{a\left(a^2+b^2\right)^{3/2}d}-\frac{2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)b^3}{a\left(a^2+b^2\right)^{3/2}d^2}+\frac{2f(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -((b*(e + f*x)^2)/((a^2 + b^2)*d)) - (4*f*(e + f*x)*ArcTan[E^(c + d*x)])/(a
*d^2) + (4*b^2*f*(e + f*x)*ArcTan[E^(c + d*x)])/(a*(a^2 + b^2)*d^2) - (2*(e
 + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (b^3*(e + f*x)^2*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^(3/2)*d) + (b^3*(e + f*x)^2*Lo
g[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^(3/2)*d) + (2*
b*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)*d^2) - (2*f*(e + f*x)*
PolyLog[2, -E^(c + d*x)])/(a*d^2) + ((2*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)]
)/(a*d^3) - ((2*I)*b^2*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^3
) - ((2*I)*f^2*PolyLog[2, I*E^(c + d*x)])/(a*d^3) + ((2*I)*b^2*f^2*PolyLog[
2, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^3) + (2*f*(e + f*x)*PolyLog[2, E^(c + d
*x)])/(a*d^2) - (2*b^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^2) + (2*b^3*f*(e + f*x)*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^2) + (b*f^2*
PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2)*d^3) + (2*f^2*PolyLog[3, -E^(c +
```

```
d*x)])/(a*d^3) - (2*f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) + (2*b^3*f^2*Poly
Log[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^3)
 - (2*b^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2
 + b^2)^(3/2)*d^3) + ((e + f*x)^2*Sech[c + d*x])/(a*d) - (b^2*(e + f*x)^2*S
ech[c + d*x])/(a*(a^2 + b^2)*d) - (b*(e + f*x)^2*Tanh[c + d*x])/((a^2 + b^2
)*d)
```

Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx = \frac{\int (e+fx)^2\text{csch}(c+dx)\text{sech}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$= -\frac{(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(e+fx)^2\text{sech}(c+dx)}{ad} - \frac{b\int (e+fx)^2\text{se}}{}$$

$$= -\frac{(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(e+fx)^2\text{sech}(c+dx)}{ad} - \frac{b\int \left(a(e+fx)\right.}{}$$

$$= -\frac{(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(e+fx)^2\text{sech}(c+dx)}{ad} - \frac{\left(2b^4\right)\int \frac{e^{c+}}{2a-2\sqrt{a}}}{a\left(a^2+\right.}$$

$$= -\frac{(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{ad} - \frac{b^3(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{3/2}d} + \frac{b^3(e+}{}$$

$$= -\frac{b(e+fx)^2}{\left(a^2+b^2\right)d} + \frac{4b^2f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d^2} - \frac{(e+fx)^2\tanh^{-1}(\cosh(c+dx)}{ad}$$

$$= -\frac{b(e+fx)^2}{\left(a^2+b^2\right)d} - \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{4b^2f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d^2} - \frac{b^3(}{}$$

$$= -\frac{b(e+fx)^2}{\left(a^2+b^2\right)d} - \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{4b^2f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d^2} - \frac{b^3(}{}$$

$$= -\frac{b(e+fx)^2}{\left(a^2+b^2\right)d} - \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{4b^2f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d^2} - \frac{2(e}{}$$

$$= -\frac{b(e+fx)^2}{\left(a^2+b^2\right)d} - \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{4b^2f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d^2} - \frac{2(e}{}$$

$$= -\frac{b(e+fx)^2}{\left(a^2+b^2\right)d} - \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{4b^2f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d^2} - \frac{2(e}{}$$

$$= -\frac{b(e+fx)^2}{\left(a^2+b^2\right)d} - \frac{4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{4b^2f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d^2} - \frac{2(e}{}$$

Mathematica [A] time = 11.5366, size = 1245, normalized size = 1.57

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] 4*((Csch[c + d*x]*((e + f*x)^2*Log[1 - E^(c + d*x)] - (e + f*x)^2*Log[1 + E
^(c + d*x)] - (2*f*(d*(e + f*x)*PolyLog[2, -E^(c + d*x)] - f*PolyLog[3, -E^
(c + d*x)]))/d^2 + (2*f*(d*(e + f*x)*PolyLog[2, E^(c + d*x)] - f*PolyLog[3,
 E^(c + d*x)]))/d^2)*(a + b*Sinh[c + d*x]))/(4*a*d*(b + a*Csch[c + d*x])) +
 (b^3*Csch[c + d*x]*(2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]]
```

```
 - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2*f^2*x^2
*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*d^2*e*f*x*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt
[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))] + 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*f^
2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])*(a + b*Sinh[c + d*x
]))/(4*a*(a^2 + b^2)^(3/2)*d^3*(b + a*Csch[c + d*x])) + (b*e*f*Csch[c + d*x
]*Sech[c]*(Cosh[c]*Log[Cosh[c]*Cosh[d*x] + Sinh[c]*Sinh[d*x]] - d*x*Sinh[c]
)*(a + b*Sinh[c + d*x]))/(2*(a^2 + b^2)*d^2*(b + a*Csch[c + d*x])*(Cosh[c]^
2 - Sinh[c]^2)) - (a*e*f*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh
[c]^2 - Sinh[c]^2]]*Csch[c + d*x]*(a + b*Sinh[c + d*x]))/((a^2 + b^2)*d^2*(
b + a*Csch[c + d*x])*Sqrt[Cosh[c]^2 - Sinh[c]^2]) - (b*f^2*Csch[c]*Csch[c +
 d*x]*(-((d^2*x^2)/E^ArcTanh[Coth[c]]) + (I*Coth[c]*(-(d*x*(-Pi + (2*I)*Arc
Tanh[Coth[c]])) - Pi*Log[1 + E^(2*d*x)] - 2*(I*d*x + I*ArcTanh[Coth[c]])*Lo
g[1 - E^((2*I)*(I*d*x + I*ArcTanh[Coth[c]]))] + Pi*Log[Cosh[d*x]] + (2*I)*A
rcTanh[Coth[c]]*Log[I*Sinh[d*x + ArcTanh[Coth[c]]]] + I*PolyLog[2, E^((2*I)
*(I*d*x + I*ArcTanh[Coth[c]]))]))/Sqrt[1 - Coth[c]^2])*Sech[c]*(a + b*Sinh[
c + d*x]))/(4*(a^2 + b^2)*d^3*(b + a*Csch[c + d*x])*Sqrt[Csch[c]^2*(-Cosh[c
]^2 + Sinh[c]^2)]) - (a*f^2*Csch[c + d*x]*(((-I)*Csch[c]*(I*(d*x + ArcTanh[
Coth[c]])*(Log[1 - E^(-(d*x) - ArcTanh[Coth[c]])] - Log[1 + E^(-(d*x) - Arc
Tanh[Coth[c]])]) + I*(PolyLog[2, -E^(-(d*x) - ArcTanh[Coth[c]])] - PolyLog[
2, E^(-(d*x) - ArcTanh[Coth[c]])])))/Sqrt[1 - Coth[c]^2] - (2*ArcTan[(Sinh[
c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]]*ArcTanh[Coth[c]])/
Sqrt[Cosh[c]^2 - Sinh[c]^2])*(a + b*Sinh[c + d*x]))/(2*(a^2 + b^2)*d^3*(b +
 a*Csch[c + d*x])) + (Csch[c + d*x]*Sech[c]*Sech[c + d*x]*(a*e^2*Cosh[c] +
2*a*e*f*x*Cosh[c] + a*f^2*x^2*Cosh[c] - b*e^2*Sinh[d*x] - 2*b*e*f*x*Sinh[d*
x] - b*f^2*x^2*Sinh[d*x])*(a + b*Sinh[c + d*x]))/(4*(a^2 + b^2)*d*(b + a*Cs
ch[c + d*x])))
```

Maple [F] time = 2.283, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2}\operatorname{csch}\left(dx+c\right)\left(\operatorname{sech}\left(dx+c\right)\right)^{2}}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.99185, size = 12713, normalized size = 15.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/2*(4*(a^3*b + a*b^3)*d^2*e^2 - 8*(a^3*b + a*b^3)*c*d*e*f + 4*(a^3*b + a*b
^3)*c^2*f^2 - 4*((a^3*b + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x
+ 2*(a^3*b + a*b^3)*c*d*e*f - (a^3*b + a*b^3)*c^2*f^2)*cosh(d*x + c)^2 - 4*
((a^3*b + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x + 2*(a^3*b + a*b
^3)*c*d*e*f - (a^3*b + a*b^3)*c^2*f^2)*sinh(d*x + c)^2 - 4*(b^4*d*f^2*x + b
^4*d*e*f + (b^4*d*f^2*x + b^4*d*e*f)*cosh(d*x + c)^2 + 2*(b^4*d*f^2*x + b^4
*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (b^4*d*f^2*x + b^4*d*e*f)*sinh(d*x +
c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 4*(b^4*
d*f^2*x + b^4*d*e*f + (b^4*d*f^2*x + b^4*d*e*f)*cosh(d*x + c)^2 + 2*(b^4*d*
f^2*x + b^4*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (b^4*d*f^2*x + b^4*d*e*f)*
sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x
+ c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1
) + 2*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2 + (b^4*d^2*e^2 - 2*b^4*c*d
*e*f + b^4*c^2*f^2)*cosh(d*x + c)^2 + 2*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*
c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c
^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*
sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(b^4*d^2*e^2 - 2*b^4*c
*d*e*f + b^4*c^2*f^2 + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*cosh(d*x
 + c)^2 + 2*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*cosh(d*x + c)*sinh(
d*x + c) + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*sinh(d*x + c)^2)*sqr
t((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^
2 + b^2)/b^2) + 2*a) - 2*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f
 - b^4*c^2*f^2 + (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c
^2*f^2)*cosh(d*x + c)^2 + 2*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*
e*f - b^4*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (b^4*d^2*f^2*x^2 + 2*b^4*d
^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b
^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x
 + 2*b^4*c*d*e*f - b^4*c^2*f^2 + (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4
*c*d*e*f - b^4*c^2*f^2)*cosh(d*x + c)^2 + 2*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*
f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (b^4*d^2*f
^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*sinh(d*x + c)^2)*sq
rt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
 c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*(b^4*f^2*cosh(d*x
+ c)^2 + 2*b^4*f^2*cosh(d*x + c)*sinh(d*x + c) + b^4*f^2*sinh(d*x + c)^2 +
b^4*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c
) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 4*(b^4*
f^2*cosh(d*x + c)^2 + 2*b^4*f^2*cosh(d*x + c)*sinh(d*x + c) + b^4*f^2*sinh(
d*x + c)^2 + b^4*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a
*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))
/b) + 4*((a^4 + a^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + (a^4 +
 a^2*b^2)*d^2*e^2)*cosh(d*x + c) + 4*((a^4 + 2*a^2*b^2 + b^4)*d*f^2*x + (a^
4 + 2*a^2*b^2 + b^4)*d*e*f + ((a^4 + 2*a^2*b^2 + b^4)*d*f^2*x + (a^4 + 2*a^
2*b^2 + b^4)*d*e*f)*cosh(d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d*f^2*x +
(a^4 + 2*a^2*b^2 + b^4)*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + ((a^4 + 2*a^2*
b^2 + b^4)*d*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d*e*f)*sinh(d*x + c)^2)*dilog(
cosh(d*x + c) + sinh(d*x + c)) + (-4*I*(a^4 + a^2*b^2)*f^2 + 4*(a^3*b + a*b
^3)*f^2 + (-4*I*(a^4 + a^2*b^2)*f^2 + 4*(a^3*b + a*b^3)*f^2)*cosh(d*x + c)^
2 + (-8*I*(a^4 + a^2*b^2)*f^2 + 8*(a^3*b + a*b^3)*f^2)*cosh(d*x + c)*sinh(d
```

```
*x + c) + (-4*I*(a^4 + a^2*b^2)*f^2 + 4*(a^3*b + a*b^3)*f^2)*sinh(d*x + c)^
2)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + (4*I*(a^4 + a^2*b^2)*f^2 + 4*
(a^3*b + a*b^3)*f^2 + (4*I*(a^4 + a^2*b^2)*f^2 + 4*(a^3*b + a*b^3)*f^2)*cos
h(d*x + c)^2 + (8*I*(a^4 + a^2*b^2)*f^2 + 8*(a^3*b + a*b^3)*f^2)*cosh(d*x +
 c)*sinh(d*x + c) + (4*I*(a^4 + a^2*b^2)*f^2 + 4*(a^3*b + a*b^3)*f^2)*sinh(
d*x + c)^2)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) - 4*((a^4 + 2*a^2*b^2
 + b^4)*d*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d*e*f + ((a^4 + 2*a^2*b^2 + b^4)*
d*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d*e*f)*cosh(d*x + c)^2 + 2*((a^4 + 2*a^2*
b^2 + b^4)*d*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d*e*f)*cosh(d*x + c)*sinh(d*x
+ c) + ((a^4 + 2*a^2*b^2 + b^4)*d*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d*e*f)*si
nh(d*x + c)^2)*dilog(-cosh(d*x + c) - sinh(d*x + c)) - 2*((a^4 + 2*a^2*b^2
+ b^4)*d^2*f^2*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f*x + (a^4 + 2*a^2*b^2
 + b^4)*d^2*e^2 + ((a^4 + 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^4 + 2*a^2*b^2
 + b^4)*d^2*e*f*x + (a^4 + 2*a^2*b^2 + b^4)*d^2*e^2)*cosh(d*x + c)^2 + 2*((
a^4 + 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f*x +
(a^4 + 2*a^2*b^2 + b^4)*d^2*e^2)*cosh(d*x + c)*sinh(d*x + c) + ((a^4 + 2*a^
2*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f*x + (a^4 + 2*a
^2*b^2 + b^4)*d^2*e^2)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) +
 1) + (-4*I*(a^4 + a^2*b^2)*d*e*f + 4*(a^3*b + a*b^3)*d*e*f + 4*I*(a^4 + a^
2*b^2)*c*f^2 - 4*(a^3*b + a*b^3)*c*f^2 + (-4*I*(a^4 + a^2*b^2)*d*e*f + 4*(a
^3*b + a*b^3)*d*e*f + 4*I*(a^4 + a^2*b^2)*c*f^2 - 4*(a^3*b + a*b^3)*c*f^2)*
cosh(d*x + c)^2 + (-8*I*(a^4 + a^2*b^2)*d*e*f + 8*(a^3*b + a*b^3)*d*e*f + 8
*I*(a^4 + a^2*b^2)*c*f^2 - 8*(a^3*b + a*b^3)*c*f^2)*cosh(d*x + c)*sinh(d*x
+ c) + (-4*I*(a^4 + a^2*b^2)*d*e*f + 4*(a^3*b + a*b^3)*d*e*f + 4*I*(a^4 + a
^2*b^2)*c*f^2 - 4*(a^3*b + a*b^3)*c*f^2)*sinh(d*x + c)^2)*log(cosh(d*x + c)
 + sinh(d*x + c) + I) + (4*I*(a^4 + a^2*b^2)*d*e*f + 4*(a^3*b + a*b^3)*d*e*
f - 4*I*(a^4 + a^2*b^2)*c*f^2 - 4*(a^3*b + a*b^3)*c*f^2 + (4*I*(a^4 + a^2*b
^2)*d*e*f + 4*(a^3*b + a*b^3)*d*e*f - 4*I*(a^4 + a^2*b^2)*c*f^2 - 4*(a^3*b
+ a*b^3)*c*f^2)*cosh(d*x + c)^2 + (8*I*(a^4 + a^2*b^2)*d*e*f + 8*(a^3*b + a
*b^3)*d*e*f - 8*I*(a^4 + a^2*b^2)*c*f^2 - 8*(a^3*b + a*b^3)*c*f^2)*cosh(d*x
 + c)*sinh(d*x + c) + (4*I*(a^4 + a^2*b^2)*d*e*f + 4*(a^3*b + a*b^3)*d*e*f
- 4*I*(a^4 + a^2*b^2)*c*f^2 - 4*(a^3*b + a*b^3)*c*f^2)*sinh(d*x + c)^2)*log
(cosh(d*x + c) + sinh(d*x + c) - I) + 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*e^2 -
2*(a^4 + 2*a^2*b^2 + b^4)*c*d*e*f + (a^4 + 2*a^2*b^2 + b^4)*c^2*f^2 + ((a^4
 + 2*a^2*b^2 + b^4)*d^2*e^2 - 2*(a^4 + 2*a^2*b^2 + b^4)*c*d*e*f + (a^4 + 2*
a^2*b^2 + b^4)*c^2*f^2)*cosh(d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*e^
2 - 2*(a^4 + 2*a^2*b^2 + b^4)*c*d*e*f + (a^4 + 2*a^2*b^2 + b^4)*c^2*f^2)*co
sh(d*x + c)*sinh(d*x + c) + ((a^4 + 2*a^2*b^2 + b^4)*d^2*e^2 - 2*(a^4 + 2*a
^2*b^2 + b^4)*c*d*e*f + (a^4 + 2*a^2*b^2 + b^4)*c^2*f^2)*sinh(d*x + c)^2)*l
og(cosh(d*x + c) + sinh(d*x + c) - 1) + (4*I*(a^4 + a^2*b^2)*d*f^2*x + 4*(a
^3*b + a*b^3)*d*f^2*x + 4*I*(a^4 + a^2*b^2)*c*f^2 + 4*(a^3*b + a*b^3)*c*f^2
 + (4*I*(a^4 + a^2*b^2)*d*f^2*x + 4*(a^3*b + a*b^3)*d*f^2*x + 4*I*(a^4 + a^
2*b^2)*c*f^2 + 4*(a^3*b + a*b^3)*c*f^2)*cosh(d*x + c)^2 + (8*I*(a^4 + a^2*b
^2)*d*f^2*x + 8*(a^3*b + a*b^3)*d*f^2*x + 8*I*(a^4 + a^2*b^2)*c*f^2 + 8*(a^
3*b + a*b^3)*c*f^2)*cosh(d*x + c)*sinh(d*x + c) + (4*I*(a^4 + a^2*b^2)*d*f^
2*x + 4*(a^3*b + a*b^3)*d*f^2*x + 4*I*(a^4 + a^2*b^2)*c*f^2 + 4*(a^3*b + a*
b^3)*c*f^2)*sinh(d*x + c)^2)*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) + (
-4*I*(a^4 + a^2*b^2)*d*f^2*x + 4*(a^3*b + a*b^3)*d*f^2*x - 4*I*(a^4 + a^2*b
^2)*c*f^2 + 4*(a^3*b + a*b^3)*c*f^2 + (-4*I*(a^4 + a^2*b^2)*d*f^2*x + 4*(a^
3*b + a*b^3)*d*f^2*x - 4*I*(a^4 + a^2*b^2)*c*f^2 + 4*(a^3*b + a*b^3)*c*f^2)
*cosh(d*x + c)^2 + (-8*I*(a^4 + a^2*b^2)*d*f^2*x + 8*(a^3*b + a*b^3)*d*f^2*
x - 8*I*(a^4 + a^2*b^2)*c*f^2 + 8*(a^3*b + a*b^3)*c*f^2)*cosh(d*x + c)*sinh
(d*x + c) + (-4*I*(a^4 + a^2*b^2)*d*f^2*x + 4*(a^3*b + a*b^3)*d*f^2*x - 4*I
*(a^4 + a^2*b^2)*c*f^2 + 4*(a^3*b + a*b^3)*c*f^2)*sinh(d*x + c)^2)*log(-I*c
osh(d*x + c) - I*sinh(d*x + c) + 1) + 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*f^2*x^
2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*c*d*e*f
 - (a^4 + 2*a^2*b^2 + b^4)*c^2*f^2 + ((a^4 + 2*a^2*b^2 + b^4)*d^2*f^2*x^2 +
 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*c*d*e*f -
(a^4 + 2*a^2*b^2 + b^4)*c^2*f^2)*cosh(d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^
```

```
4)*d^2*f^2*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f*x + 2*(a^4 + 2*a^2*b^2 +
 b^4)*c*d*e*f - (a^4 + 2*a^2*b^2 + b^4)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c
) + ((a^4 + 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*
f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*c*d*e*f - (a^4 + 2*a^2*b^2 + b^4)*c^2*f^2)*
sinh(d*x + c)^2)*log(-cosh(d*x + c) - sinh(d*x + c) + 1) - 4*((a^4 + 2*a^2*
b^2 + b^4)*f^2*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*f^2*cosh(d*x + c
)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*f^2*sinh(d*x + c)^2 + (a^4 + 2*a^
2*b^2 + b^4)*f^2)*polylog(3, cosh(d*x + c) + sinh(d*x + c)) + 4*((a^4 + 2*a
^2*b^2 + b^4)*f^2*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*f^2*cosh(d*x
+ c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*f^2*sinh(d*x + c)^2 + (a^4 + 2
*a^2*b^2 + b^4)*f^2)*polylog(3, -cosh(d*x + c) - sinh(d*x + c)) + 4*((a^4 +
 a^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + (a^4 + a^2*b^2)*d^2*e
^2 - 2*((a^3*b + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x + 2*(a^3*
b + a*b^3)*c*d*e*f - (a^3*b + a*b^3)*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))
/((a^5 + 2*a^3*b^2 + a*b^4)*d^3*cosh(d*x + c)^2 + 2*(a^5 + 2*a^3*b^2 + a*b^
4)*d^3*cosh(d*x + c)*sinh(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4)*d^3*sinh(d*x
 + c)^2 + (a^5 + 2*a^3*b^2 + a*b^4)*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.442 $\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=442

$$-\frac{b^3 f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\left(a^2+b^2\right)^{3/2}}+\frac{b^3 f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2\left(a^2+b^2\right)^{3/2}}-\frac{f\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}+\frac{f\,\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}+\frac{b^2 f\,\text{ta}}{}$$

```
[Out] -((f*ArcTan[Sinh[c + d*x]])/(a*d^2)) + (b^2*f*ArcTan[Sinh[c + d*x]])/(a*(a^
2 + b^2)*d^2) - (2*f*x*ArcTanh[E^(c + d*x)])/(a*d) + (f*x*ArcTanh[Cosh[c +
d*x]])/(a*d) - ((e + f*x)*ArcTanh[Cosh[c + d*x]])/(a*d) - (b^3*(e + f*x)*Lo
g[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^(3/2)*d) + (b^
3*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^
(3/2)*d) + (b*f*Log[Cosh[c + d*x]])/((a^2 + b^2)*d^2) - (f*PolyLog[2, -E^(c
 + d*x)])/(a*d^2) + (f*PolyLog[2, E^(c + d*x)])/(a*d^2) - (b^3*f*PolyLog[2,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^2) + (b^
3*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3
/2)*d^2) + ((e + f*x)*Sech[c + d*x])/(a*d) - (b^2*(e + f*x)*Sech[c + d*x])/
(a*(a^2 + b^2)*d) - (b*(e + f*x)*Tanh[c + d*x])/((a^2 + b^2)*d)
```

Rubi [A] time = 0.805706, antiderivative size = 442, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 19, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.594, Rules used = {5589, 2622, 321, 207, 5462, 6271, 12, 4182, 2279, 2391, 3770, 5573, 3322, 2264, 2190, 6742, 4184, 3475, 5451}

$$-\frac{b^3 f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\left(a^2+b^2\right)^{3/2}}+\frac{b^3 f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2\left(a^2+b^2\right)^{3/2}}-\frac{f\,\text{PolyLog}\left(2,-e^{c+dx}\right)}{ad^2}+\frac{f\,\text{PolyLog}\left(2,e^{c+dx}\right)}{ad^2}+\frac{b^2 f\,\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -((f*ArcTan[Sinh[c + d*x]])/(a*d^2)) + (b^2*f*ArcTan[Sinh[c + d*x]])/(a*(a^
2 + b^2)*d^2) - (2*f*x*ArcTanh[E^(c + d*x)])/(a*d) + (f*x*ArcTanh[Cosh[c +
d*x]])/(a*d) - ((e + f*x)*ArcTanh[Cosh[c + d*x]])/(a*d) - (b^3*(e + f*x)*Lo
g[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^(3/2)*d) + (b^
3*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^
(3/2)*d) + (b*f*Log[Cosh[c + d*x]])/((a^2 + b^2)*d^2) - (f*PolyLog[2, -E^(c
 + d*x)])/(a*d^2) + (f*PolyLog[2, E^(c + d*x)])/(a*d^2) - (b^3*f*PolyLog[2,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^2) + (b^
3*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3
/2)*d^2) + ((e + f*x)*Sech[c + d*x])/(a*d) - (b^2*(e + f*x)*Sech[c + d*x])/
(a*(a^2 + b^2)*d) - (b*(e + f*x)*Tanh[c + d*x])/((a^2 + b^2)*d)
```


Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(e+fx)\operatorname{sech}(c+dx)}{ad} - \frac{b\int (e+fx)\operatorname{sech}^2}{}$$

$$= -\frac{(e+fx)\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(e+fx)\operatorname{sech}(c+dx)}{ad} - \frac{b\int \left(a(e+fx)\operatorname{sec}\right.}{}$$

$$= -\frac{f\tan^{-1}(\sinh(c+dx))}{ad^2} + \frac{fx\tanh^{-1}(\cosh(c+dx))}{ad} - \frac{(e+fx)\tanh^{-1}(\cosh}{ad}$$

$$= -\frac{f\tan^{-1}(\sinh(c+dx))}{ad^2} + \frac{fx\tanh^{-1}(\cosh(c+dx))}{ad} - \frac{(e+fx)\tanh^{-1}(\cosh}{ad}$$

$$= -\frac{f\tan^{-1}(\sinh(c+dx))}{ad^2} + \frac{b^2 f\tan^{-1}(\sinh(c+dx))}{a\left(a^2+b^2\right)d^2} - \frac{2fx\tanh^{-1}\left(e^{c+dx}\right)}{ad} +$$

$$= -\frac{f\tan^{-1}(\sinh(c+dx))}{ad^2} + \frac{b^2 f\tan^{-1}(\sinh(c+dx))}{a\left(a^2+b^2\right)d^2} - \frac{2fx\tanh^{-1}\left(e^{c+dx}\right)}{ad} +$$

$$= -\frac{f\tan^{-1}(\sinh(c+dx))}{ad^2} + \frac{b^2 f\tan^{-1}(\sinh(c+dx))}{a\left(a^2+b^2\right)d^2} - \frac{2fx\tanh^{-1}\left(e^{c+dx}\right)}{ad} +$$

Mathematica [A] time = 7.2462, size = 459, normalized size = 1.04

$$\operatorname{csch}(c+dx)(a+b\sinh(c+dx))\left(\frac{b^3\left(-f\operatorname{PolyLog}\left(2,\frac{b(\sinh(c+dx)+\cosh(c+dx))}{\sqrt{a^2+b^2}-a}\right)+f\operatorname{PolyLog}\left(2,-\frac{b(\sinh(c+dx)+\cosh(c+dx))}{\sqrt{a^2+b^2}+a}\right)+2de\tanh^{-1}\left(\frac{a+b\sinh(c+dx)+b}{\sqrt{a^2+b}}\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (Csch[c + d*x]*(a + b*Sinh[c + d*x])*((-2*a*f*ArcTan[Tanh[(c + d*x)/2]])/(a
^2 + b^2) + (b*f*Log[Cosh[c + d*x]])/(a^2 + b^2) + (d*e*Log[Tanh[(c + d*x)/
2]])/a - (c*f*Log[Tanh[(c + d*x)/2]])/a + (f*((c + d*x)*(Log[1 - E^(-c - d*
x)] - Log[1 + E^(-c - d*x)]) + PolyLog[2, -E^(-c - d*x)] - PolyLog[2, E^(-c
 - d*x)]))/a + (b^3*(2*d*e*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/
Sqrt[a^2 + b^2]] - 2*c*f*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sq
rt[a^2 + b^2]] - f*(c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a
 - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x
]))/(a + Sqrt[a^2 + b^2])] - f*PolyLog[2, (b*(Cosh[c + d*x] + Sinh[c + d*x]
))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*(Cosh[c + d*x] + Sinh[c + d*
x]))/(a + Sqrt[a^2 + b^2]))]))/(a*(a^2 + b^2)^(3/2)) + (d*(e + f*x)*Sech[c
+ d*x]*(a - b*Sinh[c + d*x]))/(a^2 + b^2)))/(d^2*(b + a*Csch[c + d*x]))
```

Maple [B] time = 0.279, size = 1815, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/(a^2+b^2)^(5/2)/d^2*f*b*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2)
)*a^3-2/(a^2+b^2)^(5/2)/d^2*f*b^3*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2
)^(1/2))*a+1/(a^2+b^2)/d*a*e*ln(exp(d*x+c)-1)-1/(a^2+b^2)/d*a*e*ln(exp(d*x+
c)+1)-1/(a^2+b^2)/d^2*a*f*dilog(exp(d*x+c))-1/(a^2+b^2)/d^2*a*f*dilog(exp(d
*x+c)+1)+1/(a^2+b^2)^(5/2)/d^2*a*b^3*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/
(a+(a^2+b^2)^(1/2)))*c-1/(a^2+b^2)^(5/2)/d^2*a*b^3*f*ln((-b*exp(d*x+c)+(a^2
+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/(a^2+b^2)^(5/2)/d*f*b^5/a*ln((b*ex
p(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/(a^2+b^2)^(3/2)/d^2*b^
3*f*c/a*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)^(5/2)
/d^2*f*a^3*b*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2
)^(5/2)/d^2*b^5*f*c/a*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(
a^2+b^2)^(5/2)/d*a*b^3*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)
^(1/2)))*x-1/(a^2+b^2)^(5/2)/d*f*b^5/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)
/(-a+(a^2+b^2)^(1/2)))*x+1/(a^2+b^2)^(5/2)/d*a*b^3*f*ln((b*exp(d*x+c)+(a^2+
b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/(a^2+b^2)^(5/2)/d^2*f*b^5/a*ln((b*ex
p(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/(a^2+b^2)^(5/2)/d^2*f*
b^5/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/(a^2+b
^2)/d^2*b*f*ln(exp(d*x+c))-1/(a^2+b^2)/d*ln(exp(d*x+c)+1)*a*f*x-8/(a^2+b^2)
/d^2*a^3*f/(4*a^2+4*b^2)*arctan(exp(d*x+c))+4/(a^2+b^2)/d^2*f*b^3/(4*a^2+4*
b^2)*ln(1+exp(2*d*x+2*c))-1/(a^2+b^2)/d^2*a*f*c*ln(exp(d*x+c)-1)-1/(a^2+b^2
)/d^2*b^2*f/a*dilog(exp(d*x+c))-1/(a^2+b^2)/d^2*b^2*f/a*dilog(exp(d*x+c)+1)
+1/(a^2+b^2)/d*b^2*e/a*ln(exp(d*x+c)-1)-1/(a^2+b^2)/d*b^2*e/a*ln(exp(d*x+c)
+1)+b/d*e/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
*a+2*(f*x+e)*(a*exp(d*x+c)+b)/d/(a^2+b^2)/(1+exp(2*d*x+2*c))-1/(a^2+b^2)/d^
2*b^2*f*c/a*ln(exp(d*x+c)-1)+4/(a^2+b^2)/d^2*a^2*b*f/(4*a^2+4*b^2)*ln(1+exp
(2*d*x+2*c))-1/(a^2+b^2)^(5/2)/d^2*a*b^3*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(
1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/(a^2+b^2)^(5/2)/d^2*a*b^3*f*dilog((b*exp(d*
x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-8/(a^2+b^2)/d^2*a*b^2*f/(4*a^2
+4*b^2)*arctan(exp(d*x+c))+1/(a^2+b^2)^(3/2)/d^2*f*b^3/a*arctanh(1/2*(2*b*e
xp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)^(5/2)/d^2*f*b^5/a*dilog((-b*exp
(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/(a^2+b^2)^(5/2)/d^2*f*b^
5/a*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/(a^2+b^2)
^(5/2)/d^2*f*b^5/a*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2
+b^2)^(3/2)/d^2*a*f*b*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(
a^2+b^2)^(3/2)/d*b^3*e/a*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-
1/(a^2+b^2)^(5/2)/d*a^3*b*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2
))+1/(a^2+b^2)^(5/2)/d*b^5*e/a*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(
1/2))-1/(a^2+b^2)/d*b^2*f/a*ln(exp(d*x+c)+1)*x-b/d^2*f*c/(a^2+b^2)^(3/2)*ar
ctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.63917, size = 5233, normalized size = 11.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -(2*(a^3*b + a*b^3)*d*f*x*cosh(d*x + c)^2 + 2*(a^3*b + a*b^3)*d*f*x*sinh(d*
x + c)^2 - 2*(a^3*b + a*b^3)*d*e + (b^4*f*cosh(d*x + c)^2 + 2*b^4*f*cosh(d*
x + c)*sinh(d*x + c) + b^4*f*sinh(d*x + c)^2 + b^4*f)*sqrt((a^2 + b^2)/b^2)
*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x +
 c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^4*f*cosh(d*x + c)^2 + 2*b^4*f*c
osh(d*x + c)*sinh(d*x + c) + b^4*f*sinh(d*x + c)^2 + b^4*f)*sqrt((a^2 + b^2
)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^4*d*e - b^4*c*f + (b^4*d*
e - b^4*c*f)*cosh(d*x + c)^2 + 2*(b^4*d*e - b^4*c*f)*cosh(d*x + c)*sinh(d*x
 + c) + (b^4*d*e - b^4*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*
cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^4
*d*e - b^4*c*f + (b^4*d*e - b^4*c*f)*cosh(d*x + c)^2 + 2*(b^4*d*e - b^4*c*f
)*cosh(d*x + c)*sinh(d*x + c) + (b^4*d*e - b^4*c*f)*sinh(d*x + c)^2)*sqrt((
a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 +
 b^2)/b^2) + 2*a) + (b^4*d*f*x + b^4*c*f + (b^4*d*f*x + b^4*c*f)*cosh(d*x +
 c)^2 + 2*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)*sinh(d*x + c) + (b^4*d*f*x +
b^4*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*s
inh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
b)/b) - (b^4*d*f*x + b^4*c*f + (b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^2 + 2*(b
^4*d*f*x + b^4*c*f)*cosh(d*x + c)*sinh(d*x + c) + (b^4*d*f*x + b^4*c*f)*sin
h(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
 - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*((
a^4 + a^2*b^2)*f*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*f*cosh(d*x + c)*sinh(d
*x + c) + (a^4 + a^2*b^2)*f*sinh(d*x + c)^2 + (a^4 + a^2*b^2)*f)*arctan(cos
h(d*x + c) + sinh(d*x + c)) - 2*((a^4 + a^2*b^2)*d*f*x + (a^4 + a^2*b^2)*d*
e)*cosh(d*x + c) - ((a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)^2 + 2*(a^4 + 2*
a^2*b^2 + b^4)*f*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*f*si
nh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*f)*dilog(cosh(d*x + c) + sinh(d*x +
 c)) + ((a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^
4)*f*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*f*sinh(d*x + c)^
2 + (a^4 + 2*a^2*b^2 + b^4)*f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) - ((a^
3*b + a*b^3)*f*cosh(d*x + c)^2 + 2*(a^3*b + a*b^3)*f*cosh(d*x + c)*sinh(d*x
 + c) + (a^3*b + a*b^3)*f*sinh(d*x + c)^2 + (a^3*b + a*b^3)*f)*log(2*cosh(d
*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + ((a^4 + 2*a^2*b^2 + b^4)*d*f*x +
 (a^4 + 2*a^2*b^2 + b^4)*d*e + ((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^
2*b^2 + b^4)*d*e)*cosh(d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4
 + 2*a^2*b^2 + b^4)*d*e)*cosh(d*x + c)*sinh(d*x + c) + ((a^4 + 2*a^2*b^2 +
b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*e)*sinh(d*x + c)^2)*log(cosh(d*x + c
) + sinh(d*x + c) + 1) - ((a^4 + 2*a^2*b^2 + b^4)*d*e - (a^4 + 2*a^2*b^2 +
b^4)*c*f + ((a^4 + 2*a^2*b^2 + b^4)*d*e - (a^4 + 2*a^2*b^2 + b^4)*c*f)*cosh
(d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d*e - (a^4 + 2*a^2*b^2 + b^4)*c*f)
*cosh(d*x + c)*sinh(d*x + c) + ((a^4 + 2*a^2*b^2 + b^4)*d*e - (a^4 + 2*a^2*
b^2 + b^4)*c*f)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) - 1) - (
(a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*c*f + ((a^4 + 2*a^2
*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*c*f)*cosh(d*x + c)^2 + 2*((a^4
+ 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*c*f)*cosh(d*x + c)*sinh(
d*x + c) + ((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*c*f)*si
nh(d*x + c)^2)*log(-cosh(d*x + c) - sinh(d*x + c) + 1) + 2*(2*(a^3*b + a*b^
3)*d*f*x*cosh(d*x + c) - (a^4 + a^2*b^2)*d*f*x - (a^4 + a^2*b^2)*d*e)*sinh(
d*x + c))/((a^5 + 2*a^3*b^2 + a*b^4)*d^2*cosh(d*x + c)^2 + 2*(a^5 + 2*a^3*b
```

```
^2 + a*b^4)*d^2*cosh(d*x + c)*sinh(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4)*d^2
*sinh(d*x + c)^2 + (a^5 + 2*a^3*b^2 + a*b^4)*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.443 $\int \frac{\mathbf{csch}(c+dx)\mathbf{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=113

$$\frac{2b^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{ad\left(a^2+b^2\right)^{3/2}}-\frac{b\mathrm{sech}(c+dx)(a\sinh(c+dx)+b)}{ad\left(a^2+b^2\right)}+\frac{\mathrm{sech}(c+dx)}{ad}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

```
[Out] -(ArcTanh[Cosh[c + d*x]]/(a*d)) + (2*b^3*ArcTanh[(b - a*Tanh[(c + d*x)/2])/
Sqrt[a^2 + b^2]])/(a*(a^2 + b^2)^(3/2)*d) + Sech[c + d*x]/(a*d) - (b*Sech[c
 + d*x]*(b + a*Sinh[c + d*x]))/(a*(a^2 + b^2)*d)
```

Rubi [A] time = 0.259943, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2898, 2622, 321, 207, 2696, 12, 2660, 618, 204}

$$\frac{2b^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{ad\left(a^2+b^2\right)^{3/2}}-\frac{b\mathrm{sech}(c+dx)(a\sinh(c+dx)+b)}{ad\left(a^2+b^2\right)}+\frac{\mathrm{sech}(c+dx)}{ad}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(ArcTanh[Cosh[c + d*x]]/(a*d)) + (2*b^3*ArcTanh[(b - a*Tanh[(c + d*x)/2])/
Sqrt[a^2 + b^2]])/(a*(a^2 + b^2)^(3/2)*d) + Sech[c + d*x]/(a*d) - (b*Sech[c
 + d*x]*(b + a*Sinh[c + d*x]))/(a*(a^2 + b^2)*d)
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = i\int\left(-\frac{i\text{csch}(c+dx)\text{sech}^2(c+dx)}{a}+\frac{ib\text{sech}^2(c+dx)}{a(a+b\sinh(c+dx))}\right)dx$$

$$= \frac{\int \text{csch}(c+dx)\text{sech}^2(c+dx)\,dx}{a} - \frac{b\int \frac{\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{b\text{sech}(c+dx)(b+a\sinh(c+dx))}{a\left(a^2+b^2\right)d} - \frac{b\int \frac{b^2}{a+b\sinh(c+dx)}\,dx}{a\left(a^2+b^2\right)} + \frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\text{se}\right.}{ad}$$

$$= \frac{\text{sech}(c+dx)}{ad} - \frac{b\text{sech}(c+dx)(b+a\sinh(c+dx))}{a\left(a^2+b^2\right)d} - \frac{b^3\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a\left(a^2+b^2\right)} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{\text{sech}(c+dx)}{ad} - \frac{b\text{sech}(c+dx)(b+a\sinh(c+dx))}{a\left(a^2+b^2\right)d} + \frac{\left(2ib^3\right.}{}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{\text{sech}(c+dx)}{ad} - \frac{b\text{sech}(c+dx)(b+a\sinh(c+dx))}{a\left(a^2+b^2\right)d} - \frac{\left(4ib^3\right.}{}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{2b^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^{3/2}d} + \frac{\text{sech}(c+dx)}{ad} - \frac{b\text{sech}(c+}{}$$

Mathematica [A] time = 0.264734, size = 171, normalized size = 1.51

$$-\frac{-ab\sqrt{-a^2-b^2}\tanh(c+dx)+a^2\sqrt{-a^2-b^2}\text{sech}(c+dx)+b^2\sqrt{-a^2-b^2}\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+a^2\sqrt{-a^2-b^2}\log\left(\tan\right.}{ad\left(-a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((-2*b^3*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] + a^2*Sqrt[-a^
2 - b^2]*Log[Tanh[(c + d*x)/2]] + b^2*Sqrt[-a^2 - b^2]*Log[Tanh[(c + d*x)/2
]] + a^2*Sqrt[-a^2 - b^2]*Sech[c + d*x] - a*b*Sqrt[-a^2 - b^2]*Tanh[c + d*x
])/(a*(-a^2 - b^2)^(3/2)*d))
```

Maple [A] time = 0.001, size = 136, normalized size = 1.2

$$\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right) - 2\frac{b^3}{da\left(a^2+b^2\right)^{3/2}}\text{Artanh}\left(1/2\,\frac{2\,a\tanh(1/2\,dx+c/2)-2\,b}{\sqrt{a^2+b^2}}\right) - 2\frac{\tanh(1/2\,dx+c/2)\,b}{d\left(a^2+b^2\right)\left((\tanh(1/2\,dx+c/2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/d/a*ln(tanh(1/2*d*x+1/2*c))-2/d/a*b^3/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*ta
nh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-2/d/(a^2+b^2)/(tanh(1/2*d*x+1/2*c)^
2+1)*tanh(1/2*d*x+1/2*c)*b+2/d/(a^2+b^2)/(tanh(1/2*d*x+1/2*c)^2+1)*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.11662, size = 1455, normalized size = 12.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] (2*a^3*b + 2*a*b^3 + (b^3*cosh(d*x + c)^2 + 2*b^3*cosh(d*x + c)*sinh(d*x +
c) + b^3*sinh(d*x + c)^2 + b^3)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 +
b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x +
 c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x
+ c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(
b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(a^4 + a^2*b^2)*cosh(d*x + c)
- (a^4 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(a^4
 + 2*a^2*b^2 + b^4)*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*s
inh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (a^4 + 2*a^2*b^2 +
 b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*
cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*sinh(d*x + c)^2)*log(
cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(a^4 + a^2*b^2)*sinh(d*x + c))/((a^5
 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c)^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*d*cos
h(d*x + c)*sinh(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4)*d*sinh(d*x + c)^2 + (a
^5 + 2*a^3*b^2 + a*b^4)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21899, size = 209, normalized size = 1.85

$$-\frac{b^3\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^3d+ab^2d\right)\sqrt{a^2+b^2}}+\frac{2\left(ae^{(dx+c)}+b\right)}{\left(a^2d+b^2d\right)\left(e^{(2\,dx+2\,c)}+1\right)}-\frac{\log\left(e^{(dx+c)}+1\right)}{ad}+\frac{\log\left(\left|e^{(dx+c)}-1\right|\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c)
 + 2*a + 2*sqrt(a^2 + b^2)))/((a^3*d + a*b^2*d)*sqrt(a^2 + b^2)) + 2*(a*e^(
d*x + c) + b)/((a^2*d + b^2*d)*(e^(2*d*x + 2*c) + 1)) - log(e^(d*x + c) + 1
)/(a*d) + log(abs(e^(d*x + c) - 1))/(a*d)
```

3.444 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Csch[c + d*x]*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [A] time = 0.0893257, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[c + d*x]*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Csch[c + d*x]*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[c + d*x]*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] $Aborted
```

Maple [A] time = 1.864, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx+c)\left(\operatorname{sech}(dx+c)\right)^2}{\left(fx+e\right)\left(a+b\sinh(dx+c)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-8\,b^3 \int -\frac{e^{(dx+c)}}{4\left(a^3be + ab^3e + \left(a^3bf + ab^3f\right)x - \left(a^3bee^{(2\,c)} + ab^3ee^{(2\,c)} + \left(a^3bfe^{(2\,c)} + ab^3fe^{(2\,c)}\right)x\right)e^{(2\,dx)} - 2\left(a^4ee^c + a^2b^2ee^c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -8*b^3*integrate(-1/4*e^(d*x + c)/(a^3*b*e + a*b^3*e + (a^3*b*f + a*b^3*f)*
x - (a^3*b*e*e^(2*c) + a*b^3*e*e^(2*c) + (a^3*b*f*e^(2*c) + a*b^3*f*e^(2*c)
)*x)*e^(2*d*x) - 2*(a^4*e*e^c + a^2*b^2*e*e^c + (a^4*f*e^c + a^2*b^2*f*e^c)
*x)*e^(d*x)), x) + 2*(a*e^(d*x + c) + b)/(a^2*d*e + b^2*d*e + (a^2*d*f + b^
2*d*f)*x + (a^2*d*e*e^(2*c) + b^2*d*e*e^(2*c) + (a^2*d*f*e^(2*c) + b^2*d*f*
e^(2*c))*x)*e^(2*d*x)) + 8*integrate(1/4*(a*f*e^(d*x + c) + b*f)/(a^2*d*e^2
 + b^2*d*e^2 + (a^2*d*f^2 + b^2*d*f^2)*x^2 + 2*(a^2*d*e*f + b^2*d*e*f)*x +
(a^2*d*e^2*e^(2*c) + b^2*d*e^2*e^(2*c) + (a^2*d*f^2*e^(2*c) + b^2*d*f^2*e^(
2*c))*x^2 + 2*(a^2*d*e*f*e^(2*c) + b^2*d*e*f*e^(2*c))*x)*e^(2*d*x)), x) + 8
*integrate(1/8/(a*f*x + a*e + (a*f*x*e^c + a*e*e^c)*e^(d*x)), x) + 8*integr
ate(-1/8/(a*f*x + a*e - (a*f*x*e^c + a*e*e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{csch}\left(dx+c\right)\operatorname{sech}\left(dx+c\right)^2}{afx+ae+\left(bfx+be\right)\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(csch(d*x + c)*sech(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.445 $\int \frac{(e+fx)^2\mathbf{csch}(c+dx)\mathbf{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1185

result too large to display

```
[Out] (e*f*x)/(a*d) + (f^2*x^2)/(2*a*d) - (2*b^3*(e + f*x)^2*ArcTan[E^(c + d*x)])
/((a^2 + b^2)^2*d) - (b*(e + f*x)^2*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) +
(b*f^2*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d^3) - (2*(e + f*x)^2*ArcTanh[E^
(2*c + 2*d*x)])/(a*d) - (b^4*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2])])/(a*(a^2 + b^2)^2*d) - (b^4*(e + f*x)^2*Log[1 + (b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^2*d) + (b^4*(e + f*x)^2*Log[1 + E^
(2*(c + d*x))])/(a*(a^2 + b^2)^2*d) + (f^2*Log[Cosh[c + d*x]])/(a*d^3) - (b
^2*f^2*Log[Cosh[c + d*x]])/(a*(a^2 + b^2)*d^3) + ((2*I)*b^3*f*(e + f*x)*Pol
yLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) + (I*b*f*(e + f*x)*PolyLog[2
, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) - ((2*I)*b^3*f*(e + f*x)*PolyLog[2,
I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - (I*b*f*(e + f*x)*PolyLog[2, I*E^(c +
d*x)])/((a^2 + b^2)*d^2) - (2*b^4*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^2*d^2) - (2*b^4*f*(e + f*x)*PolyLog
[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^2*d^2) + (b^4
*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(a*(a^2 + b^2)^2*d^2) - (f*(e +
f*x)*PolyLog[2, -E^(2*c + 2*d*x)])/(a*d^2) + (f*(e + f*x)*PolyLog[2, E^(2*c
 + 2*d*x)])/(a*d^2) - ((2*I)*b^3*f^2*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 +
b^2)^2*d^3) - (I*b*f^2*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) + ((
2*I)*b^3*f^2*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)^2*d^3) + (I*b*f^2*Poly
Log[3, I*E^(c + d*x)])/((a^2 + b^2)*d^3) + (2*b^4*f^2*PolyLog[3, -((b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^2*d^3) + (2*b^4*f^2*PolyLog
[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^2*d^3) - (b^4
*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*a*(a^2 + b^2)^2*d^3) + (f^2*PolyLog[3
, -E^(2*c + 2*d*x)])/(2*a*d^3) - (f^2*PolyLog[3, E^(2*c + 2*d*x)])/(2*a*d^3
) - (b*f*(e + f*x)*Sech[c + d*x])/((a^2 + b^2)*d^2) - (b^2*(e + f*x)^2*Sech
[c + d*x]^2)/(2*a*(a^2 + b^2)*d) - (f*(e + f*x)*Tanh[c + d*x])/(a*d^2) + (b
^2*f*(e + f*x)*Tanh[c + d*x])/(a*(a^2 + b^2)*d^2) - (b*(e + f*x)^2*Sech[c +
 d*x]*Tanh[c + d*x])/(2*(a^2 + b^2)*d) - ((e + f*x)^2*Tanh[c + d*x]^2)/(2*a
*d)
```

Rubi [A] time = 2.2249, antiderivative size = 1185, normalized size of antiderivative = 1., number of steps used = 57, number of rules used = 23, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.676, Rules used = {5589, 2620, 14, 5462, 6741, 12, 6742, 2551, 4182, 2531, 2282, 6589, 3720, 3475, 5573, 5561, 2190, 4180, 3718, 4186, 3770, 5451, 4184}

$$-\frac{(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)b^4}{a\left(a^2+b^2\right)^2d}-\frac{(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)b^4}{a\left(a^2+b^2\right)^2d}+\frac{(e+fx)^2\log\left(1+e^{2(c+dx)}\right)b^4}{a\left(a^2+b^2\right)^2d}-\frac{2f(e+fx)\text{PolyLog}}{a\left(a^2+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e*f*x)/(a*d) + (f^2*x^2)/(2*a*d) - (2*b^3*(e + f*x)^2*ArcTan[E^(c + d*x)])
/((a^2 + b^2)^2*d) - (b*(e + f*x)^2*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) +
(b*f^2*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d^3) - (2*(e + f*x)^2*ArcTanh[E^
(2*c + 2*d*x)])/(a*d) - (b^4*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2])])/(a*(a^2 + b^2)^2*d) - (b^4*(e + f*x)^2*Log[1 + (b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^2*d) + (b^4*(e + f*x)^2*Log[1 + E^
(2*(c + d*x))])/(a*(a^2 + b^2)^2*d) + (f^2*Log[Cosh[c + d*x]])/(a*d^3) - (b
```

```
^2*f^2*Log[Cosh[c + d*x]])/(a*(a^2 + b^2)*d^3) + ((2*I)*b^3*f*(e + f*x)*Pol
yLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) + (I*b*f*(e + f*x)*PolyLog[2
, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) - ((2*I)*b^3*f*(e + f*x)*PolyLog[2,
I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - (I*b*f*(e + f*x)*PolyLog[2, I*E^(c +
d*x)])/((a^2 + b^2)*d^2) - (2*b^4*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^2*d^2) - (2*b^4*f*(e + f*x)*PolyLog
[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^2*d^2) + (b^4
*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(a*(a^2 + b^2)^2*d^2) - (f*(e +
f*x)*PolyLog[2, -E^(2*c + 2*d*x)])/(a*d^2) + (f*(e + f*x)*PolyLog[2, E^(2*c
 + 2*d*x)])/(a*d^2) - ((2*I)*b^3*f^2*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 +
b^2)^2*d^3) - (I*b*f^2*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) + ((
2*I)*b^3*f^2*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)^2*d^3) + (I*b*f^2*Poly
Log[3, I*E^(c + d*x)])/((a^2 + b^2)*d^3) + (2*b^4*f^2*PolyLog[3, -((b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^2*d^3) + (2*b^4*f^2*PolyLog
[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^2*d^3) - (b^4
*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*a*(a^2 + b^2)^2*d^3) + (f^2*PolyLog[3
, -E^(2*c + 2*d*x)])/(2*a*d^3) - (f^2*PolyLog[3, E^(2*c + 2*d*x)])/(2*a*d^3
) - (b*f*(e + f*x)*Sech[c + d*x])/((a^2 + b^2)*d^2) - (b^2*(e + f*x)^2*Sech
[c + d*x]^2)/(2*a*(a^2 + b^2)*d) - (f*(e + f*x)*Tanh[c + d*x])/(a*d^2) + (b
^2*f*(e + f*x)*Tanh[c + d*x])/(a*(a^2 + b^2)*d^2) - (b*(e + f*x)^2*Sech[c +
 d*x]*Tanh[c + d*x])/(2*(a^2 + b^2)*d) - ((e + f*x)^2*Tanh[c + d*x]^2)/(2*a
*d)
```

Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
```

```
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\mathrm{csch}(c+dx)\mathrm{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\mathrm{csch}(c+dx)\mathrm{sech}^3(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\mathrm{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= \frac{(e+fx)^2\log(\tanh(c+dx))}{ad} - \frac{(e+fx)^2\tanh^2(c+dx)}{2ad} - \frac{b\int (e+fx)^2\mathrm{sech}^3$$

$$= \frac{(e+fx)^2\log(\tanh(c+dx))}{ad} - \frac{(e+fx)^2\tanh^2(c+dx)}{2ad} - \frac{b^3\int (e+fx)^2\mathrm{sech}$$

$$= \frac{b^4(e+fx)^3}{3a\left(a^2+b^2\right)^2 f} + \frac{(e+fx)^2\log(\tanh(c+dx))}{ad} - \frac{(e+fx)^2\tanh^2(c+dx)}{2ad} -$$

$$= \frac{b^4(e+fx)^3}{3a\left(a^2+b^2\right)^2 f} - \frac{b^4(e+fx)^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^2 d} - \frac{b^4(e+fx)^2\log\left(1+\frac{be^{c}}{a+\sqrt{}}\right.}{a\left(a^2+b^2\right)^2 d}$$

$$= -\frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{bf^2\tan^{-1}(\sinh(c}{\left(a^2+b^2\right)d^3}$$

$$= -\frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + \frac{bf^2\tan^{-1}(\sinh(c}{\left(a^2+b^2\right)d^3}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + bf^2$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + bf^2$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + bf^2$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + bf^2$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} + bf^2$$

Mathematica [B] time = 36.4513, size = 4072, normalized size = 3.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] -(E^(2*c)*((2*(e + f*x)^3)/(E^(2*c)*f) - (3*(1 - E^(-2*c))*(e + f*x)^2*Log[
1 - E^(-c - d*x)])/d - (3*(1 - E^(-2*c))*(e + f*x)^2*Log[1 + E^(-c - d*x)])
/d + (6*(-1 + E^(2*c))*f*(d*(e + f*x)*PolyLog[2, -E^(-c - d*x)] + f*PolyLog
[3, -E^(-c - d*x)]))/(d^3*E^(2*c)) + (6*(-1 + E^(2*c))*f*(d*(e + f*x)*PolyL
og[2, E^(-c - d*x)] + f*PolyLog[3, E^(-c - d*x)]))/(d^3*E^(2*c))))/(3*a*(-1
 + E^(2*c))) - (-12*a^3*d^3*e^2*E^(2*c)*x - 24*a*b^2*d^3*e^2*E^(2*c)*x + 12
*a^3*d*E^(2*c)*f^2*x + 12*a*b^2*d*E^(2*c)*f^2*x - 12*a^3*d^3*e*E^(2*c)*f*x^
2 - 24*a*b^2*d^3*e*E^(2*c)*f*x^2 - 4*a^3*d^3*E^(2*c)*f^2*x^3 - 8*a*b^2*d^3*
E^(2*c)*f^2*x^3 + 6*a^2*b*d^2*e^2*ArcTan[E^(c + d*x)] + 18*b^3*d^2*e^2*ArcT
an[E^(c + d*x)] + 6*a^2*b*d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)] + 18*b^3*d^2*
e^2*E^(2*c)*ArcTan[E^(c + d*x)] - 12*a^2*b*f^2*ArcTan[E^(c + d*x)] - 12*b^3
*f^2*ArcTan[E^(c + d*x)] - 12*a^2*b*E^(2*c)*f^2*ArcTan[E^(c + d*x)] - 12*b^
3*E^(2*c)*f^2*ArcTan[E^(c + d*x)] + (6*I)*a^2*b*d^2*e*f*x*Log[1 - I*E^(c +
d*x)] + (18*I)*b^3*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (6*I)*a^2*b*d^2*e*E^(
2*c)*f*x*Log[1 - I*E^(c + d*x)] + (18*I)*b^3*d^2*e*E^(2*c)*f*x*Log[1 - I*E^
(c + d*x)] + (3*I)*a^2*b*d^2*f^2*x^2*Log[1 - I*E^(c + d*x)] + (9*I)*b^3*d^2
*f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*a^2*b*d^2*E^(2*c)*f^2*x^2*Log[1 - I
*E^(c + d*x)] + (9*I)*b^3*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] - (6*I
)*a^2*b*d^2*e*f*x*Log[1 + I*E^(c + d*x)] - (18*I)*b^3*d^2*e*f*x*Log[1 + I*E
^(c + d*x)] - (6*I)*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (18*I)
*b^3*d^2*e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (3*I)*a^2*b*d^2*f^2*x^2*Log
[1 + I*E^(c + d*x)] - (9*I)*b^3*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - (3*I)*
a^2*b*d^2*E^(2*c)*f^2*x^2*Log[1 + I*E^(c + d*x)] - (9*I)*b^3*d^2*E^(2*c)*f^
2*x^2*Log[1 + I*E^(c + d*x)] + 6*a^3*d^2*e^2*Log[1 + E^(2*(c + d*x))] + 12*
a*b^2*d^2*e^2*Log[1 + E^(2*(c + d*x))] + 6*a^3*d^2*e^2*E^(2*c)*Log[1 + E^(2
*(c + d*x))] + 12*a*b^2*d^2*e^2*E^(2*c)*Log[1 + E^(2*(c + d*x))] - 6*a^3*f^
2*Log[1 + E^(2*(c + d*x))] - 6*a*b^2*f^2*Log[1 + E^(2*(c + d*x))] - 6*a^3*E
^(2*c)*f^2*Log[1 + E^(2*(c + d*x))] - 6*a*b^2*E^(2*c)*f^2*Log[1 + E^(2*(c +
 d*x))] + 12*a^3*d^2*e*f*x*Log[1 + E^(2*(c + d*x))] + 24*a*b^2*d^2*e*f*x*Lo
g[1 + E^(2*(c + d*x))] + 12*a^3*d^2*e*E^(2*c)*f*x*Log[1 + E^(2*(c + d*x))]
+ 24*a*b^2*d^2*e*E^(2*c)*f*x*Log[1 + E^(2*(c + d*x))] + 6*a^3*d^2*f^2*x^2*L
og[1 + E^(2*(c + d*x))] + 12*a*b^2*d^2*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 6
*a^3*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 12*a*b^2*d^2*E^(2*c)*f^
2*x^2*Log[1 + E^(2*(c + d*x))] - (6*I)*b*(a^2 + 3*b^2)*d*(1 + E^(2*c))*f*(e
 + f*x)*PolyLog[2, (-I)*E^(c + d*x)] + (6*I)*b*(a^2 + 3*b^2)*d*(1 + E^(2*c)
)*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)] + 6*a^3*d*e*f*PolyLog[2, -E^(2*(c +
 d*x))] + 12*a*b^2*d*e*f*PolyLog[2, -E^(2*(c + d*x))] + 6*a^3*d*e*E^(2*c)*f
*PolyLog[2, -E^(2*(c + d*x))] + 12*a*b^2*d*e*E^(2*c)*f*PolyLog[2, -E^(2*(c
+ d*x))] + 6*a^3*d*f^2*x*PolyLog[2, -E^(2*(c + d*x))] + 12*a*b^2*d*f^2*x*Po
lyLog[2, -E^(2*(c + d*x))] + 6*a^3*d*E^(2*c)*f^2*x*PolyLog[2, -E^(2*(c + d*
x))] + 12*a*b^2*d*E^(2*c)*f^2*x*PolyLog[2, -E^(2*(c + d*x))] + (6*I)*a^2*b*
f^2*PolyLog[3, (-I)*E^(c + d*x)] + (18*I)*b^3*f^2*PolyLog[3, (-I)*E^(c + d*
x)] + (6*I)*a^2*b*E^(2*c)*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (18*I)*b^3*E^(
2*c)*f^2*PolyLog[3, (-I)*E^(c + d*x)] - (6*I)*a^2*b*f^2*PolyLog[3, I*E^(c +
 d*x)] - (18*I)*b^3*f^2*PolyLog[3, I*E^(c + d*x)] - (6*I)*a^2*b*E^(2*c)*f^2
*PolyLog[3, I*E^(c + d*x)] - (18*I)*b^3*E^(2*c)*f^2*PolyLog[3, I*E^(c + d*x
)] - 3*a^3*f^2*PolyLog[3, -E^(2*(c + d*x))] - 6*a*b^2*f^2*PolyLog[3, -E^(2*
(c + d*x))] - 3*a^3*E^(2*c)*f^2*PolyLog[3, -E^(2*(c + d*x))] - 6*a*b^2*E^(2
*c)*f^2*PolyLog[3, -E^(2*(c + d*x))])/(6*(a^2 + b^2)^2*d^3*(1 + E^(2*c))) +
 (b^4*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[
a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 +
b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x
))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2
*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*
a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b
^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c
```

```
 + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)
))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*
c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 +
b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^
2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^
c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*
E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c +
 d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2
*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log
[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(-1 + E
^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2
)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c +
 d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*f^2*PolyLog[3, -((b*
E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*
PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 -
 (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))
])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 +
 b^2)*E^(2*c)]))])/d^3))/(3*a*(a^2 + b^2)^2*(-1 + E^(2*c))) + (Csch[c]*Sech
[c]*Sech[c + d*x]^2*(-6*a^3*e*f - 6*a*b^2*e*f + 12*a^3*d^2*e^2*x + 24*a*b^2
*d^2*e^2*x - 6*a^3*f^2*x - 6*a*b^2*f^2*x + 12*a^3*d^2*e*f*x^2 + 24*a*b^2*d^
2*e*f*x^2 + 4*a^3*d^2*f^2*x^3 + 8*a*b^2*d^2*f^2*x^3 + 6*a^3*e*f*Cosh[2*c] +
 6*a*b^2*e*f*Cosh[2*c] + 6*a^3*f^2*x*Cosh[2*c] + 6*a*b^2*f^2*x*Cosh[2*c] +
6*a^3*e*f*Cosh[2*d*x] + 6*a*b^2*e*f*Cosh[2*d*x] + 6*a^3*f^2*x*Cosh[2*d*x] +
 6*a*b^2*f^2*x*Cosh[2*d*x] + 3*a^2*b*d*e^2*Cosh[c - d*x] + 3*b^3*d*e^2*Cosh
[c - d*x] + 6*a^2*b*d*e*f*x*Cosh[c - d*x] + 6*b^3*d*e*f*x*Cosh[c - d*x] + 3
*a^2*b*d*f^2*x^2*Cosh[c - d*x] + 3*b^3*d*f^2*x^2*Cosh[c - d*x] - 3*a^2*b*d*
e^2*Cosh[3*c + d*x] - 3*b^3*d*e^2*Cosh[3*c + d*x] - 6*a^2*b*d*e*f*x*Cosh[3*
c + d*x] - 6*b^3*d*e*f*x*Cosh[3*c + d*x] - 3*a^2*b*d*f^2*x^2*Cosh[3*c + d*x
] - 3*b^3*d*f^2*x^2*Cosh[3*c + d*x] - 6*a^3*e*f*Cosh[2*c + 2*d*x] - 6*a*b^2
*e*f*Cosh[2*c + 2*d*x] + 12*a^3*d^2*e^2*x*Cosh[2*c + 2*d*x] + 24*a*b^2*d^2*
e^2*x*Cosh[2*c + 2*d*x] - 6*a^3*f^2*x*Cosh[2*c + 2*d*x] - 6*a*b^2*f^2*x*Cos
h[2*c + 2*d*x] + 12*a^3*d^2*e*f*x^2*Cosh[2*c + 2*d*x] + 24*a*b^2*d^2*e*f*x^
2*Cosh[2*c + 2*d*x] + 4*a^3*d^2*f^2*x^3*Cosh[2*c + 2*d*x] + 8*a*b^2*d^2*f^2
*x^3*Cosh[2*c + 2*d*x] + 6*a^3*d*e^2*Sinh[2*c] + 6*a*b^2*d*e^2*Sinh[2*c] +
12*a^3*d*e*f*x*Sinh[2*c] + 12*a*b^2*d*e*f*x*Sinh[2*c] + 6*a^3*d*f^2*x^2*Sin
h[2*c] + 6*a*b^2*d*f^2*x^2*Sinh[2*c] - 6*a^2*b*e*f*Sinh[c - d*x] - 6*b^3*e*
f*Sinh[c - d*x] - 6*a^2*b*f^2*x*Sinh[c - d*x] - 6*b^3*f^2*x*Sinh[c - d*x] -
 6*a^2*b*e*f*Sinh[3*c + d*x] - 6*b^3*e*f*Sinh[3*c + d*x] - 6*a^2*b*f^2*x*Si
nh[3*c + d*x] - 6*b^3*f^2*x*Sinh[3*c + d*x]))/(24*(a^2 + b^2)^2*d^2)
```

Maple [F] time = 1.372, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 \operatorname{csch}(dx+c)(\operatorname{sech}(dx+c))^3}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -a^2*b*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b
^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2
+ b^4*d^2), x) - 3*b^3*d^2*f^2*integrate(x^2*e^(d*x + c)/(a^4*d^2*e^(2*d*x
+ 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2
+ 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*a^3*d^2*f^2*integrate(x^2/(a^4*d^2*e^(2*
d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*
d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 4*a*b^2*d^2*f^2*integrate(x^2/(a^4*d^2
*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c)
+ a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a^2*b*d^2*e*f*integrate(x*e^(d
*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2*c) + b^4*d^2*
e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 6*b^3*d^2*e*f*in
tegrate(x*e^(d*x + c)/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x + 2
*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 4*
a^3*d^2*e*f*integrate(x/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d*x +
 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) +
8*a*b^2*d^2*e*f*integrate(x/(a^4*d^2*e^(2*d*x + 2*c) + 2*a^2*b^2*d^2*e^(2*d
*x + 2*c) + b^4*d^2*e^(2*d*x + 2*c) + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x
) - a^3*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) - log(e^(2*d*x + 2*c
) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) - a*b^2*f^2*(2*(d*x + c)/((a^4 + 2*a^
2*b^2 + b^4)*d^3) - log(e^(2*d*x + 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3))
 - (b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + 2*a^3*b^2 +
 a*b^4)*d) - (a^2*b + 3*b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*
d) + (a^3 + 2*a*b^2)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d)
+ (b*e^(-d*x - c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2
+ 2*(a^2 + b^2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d) - log(e
^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d))*e^2 + 2*a^2*b*f^2*arc
tan(e^(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3) + 2*b^3*f^2*arctan(e^(d*x +
c))/((a^4 + 2*a^2*b^2 + b^4)*d^3) + (2*a*f^2*x + 2*a*e*f - (b*d*f^2*x^2*e^(
3*c) + 2*b*e*f*e^(3*c) + 2*(d*e*f + f^2)*b*x*e^(3*c))*e^(3*d*x) + 2*(a*d*f^
2*x^2*e^(2*c) + a*e*f*e^(2*c) + (2*d*e*f + f^2)*a*x*e^(2*c))*e^(2*d*x) + (b
*d*f^2*x^2*e^c - 2*b*e*f*e^c + 2*(d*e*f - f^2)*b*x*e^c)*e^(d*x))/(a^2*d^2 +
 b^2*d^2 + (a^2*d^2*e^(4*c) + b^2*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*
c) + b^2*d^2*e^(2*c))*e^(2*d*x)) + 2*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(
d*x + c)))*e*f/(a*d^2) + 2*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))
*e*f/(a*d^2) + (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) -
2*polylog(3, -e^(d*x + c)))*f^2/(a*d^3) + (d^2*x^2*log(-e^(d*x + c) + 1) +
2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*f^2/(a*d^3) - 2/3*(d^
3*f^2*x^3 + 3*d^3*e*f*x^2)/(a*d^3) + integrate(2*(b^5*f^2*x^2 + 2*b^5*e*f*x
 - (a*b^4*f^2*x^2*e^c + 2*a*b^4*e*f*x*e^c)*e^(d*x))/(a^5*b + 2*a^3*b^3 + a*
b^5 - (a^5*b*e^(2*c) + 2*a^3*b^3*e^(2*c) + a*b^5*e^(2*c))*e^(2*d*x) - 2*(a^
6*e^c + 2*a^4*b^2*e^c + a^2*b^4*e^c)*e^(d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.446 $\int \frac{(e+fx)\mathbf{csch}(c+dx)\mathbf{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=746

$$-\frac{b^4 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\left(a^2+b^2\right)^2}-\frac{b^4 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2\left(a^2+b^2\right)^2}+\frac{b^4 f \text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2ad^2\left(a^2+b^2\right)^2}+\frac{ib^3 f \text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)^2}$$

```
[Out] (f*x)/(2*a*d) - (2*b^3*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)^2*d) - (
b*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (2*f*x*ArcTanh[E^(2*c +
2*d*x)])/(a*d) - (b^4*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])])/(a*(a^2 + b^2)^2*d) - (b^4*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])])/(a*(a^2 + b^2)^2*d) + (b^4*(e + f*x)*Log[1 + E^(2*(c + d*x))
])/(a*(a^2 + b^2)^2*d) - (f*x*Log[Tanh[c + d*x]])/(a*d) + ((e + f*x)*Log[Ta
nh[c + d*x]])/(a*d) + (I*b^3*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2
*d^2) + ((I/2)*b*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) - (I*b^3
*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - ((I/2)*b*f*PolyLog[2, I
*E^(c + d*x)])/((a^2 + b^2)*d^2) - (b^4*f*PolyLog[2, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^2*d^2) - (b^4*f*PolyLog[2, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^2*d^2) + (b^4*f*PolyLog[2, -E
^(2*(c + d*x))])/(2*a*(a^2 + b^2)^2*d^2) - (f*PolyLog[2, -E^(2*c + 2*d*x)])
/(2*a*d^2) + (f*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d^2) - (b*f*Sech[c + d*x]
)/(2*(a^2 + b^2)*d^2) - (b^2*(e + f*x)*Sech[c + d*x]^2)/(2*a*(a^2 + b^2)*d)
 - (f*Tanh[c + d*x])/(2*a*d^2) + (b^2*f*Tanh[c + d*x])/(2*a*(a^2 + b^2)*d^2
) - (b*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*(a^2 + b^2)*d) - ((e + f*x
)*Tanh[c + d*x]^2)/(2*a*d)
```

Rubi [A] time = 1.05909, antiderivative size = 746, normalized size of antiderivative = 1., number of steps used = 43, number of rules used = 20, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5589, 2620, 14, 5462, 2548, 12, 4182, 2279, 2391, 3473, 8, 5573, 5561, 2190, 6742, 4180, 3718, 4185, 5451, 3767}

$$-\frac{b^4 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\left(a^2+b^2\right)^2}-\frac{b^4 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2\left(a^2+b^2\right)^2}+\frac{b^4 f \text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2ad^2\left(a^2+b^2\right)^2}+\frac{ib^3 f \text{PolyLog}\left(2,-ie^{c+dx}\right)}{d^2\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (f*x)/(2*a*d) - (2*b^3*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)^2*d) - (
b*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (2*f*x*ArcTanh[E^(2*c +
2*d*x)])/(a*d) - (b^4*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])])/(a*(a^2 + b^2)^2*d) - (b^4*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])])/(a*(a^2 + b^2)^2*d) + (b^4*(e + f*x)*Log[1 + E^(2*(c + d*x))
])/(a*(a^2 + b^2)^2*d) - (f*x*Log[Tanh[c + d*x]])/(a*d) + ((e + f*x)*Log[Ta
nh[c + d*x]])/(a*d) + (I*b^3*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2
*d^2) + ((I/2)*b*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) - (I*b^3
*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - ((I/2)*b*f*PolyLog[2, I
*E^(c + d*x)])/((a^2 + b^2)*d^2) - (b^4*f*PolyLog[2, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^2*d^2) - (b^4*f*PolyLog[2, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^2*d^2) + (b^4*f*PolyLog[2, -E
^(2*(c + d*x))])/(2*a*(a^2 + b^2)^2*d^2) - (f*PolyLog[2, -E^(2*c + 2*d*x)])
/(2*a*d^2) + (f*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d^2) - (b*f*Sech[c + d*x]
)/(2*(a^2 + b^2)*d^2) - (b^2*(e + f*x)*Sech[c + d*x]^2)/(2*a*(a^2 + b^2)*d)
 - (f*Tanh[c + d*x])/(2*a*d^2) + (b^2*f*Tanh[c + d*x])/(2*a*(a^2 + b^2)*d^2
```

```
) - (b*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*(a^2 + b^2)*d) - ((e + f*x
)*Tanh[c + d*x]^2)/(2*a*d)
```

Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{csch}(c+dx)\text{sech}^3(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\text{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= \frac{(e+fx)\log(\tanh(c+dx))}{ad} - \frac{(e+fx)\tanh^2(c+dx)}{2ad} - \frac{b\int (e+fx)\text{sech}^3(c+\ldots}{a(\ldots}$$

$$= \frac{(e+fx)\log(\tanh(c+dx))}{ad} - \frac{(e+fx)\tanh^2(c+dx)}{2ad} - \frac{b^3\int (e+fx)\text{sech}(c+\ldots}{a(\ldots}$$

$$= \frac{b^4(e+fx)^2}{2a\left(a^2+b^2\right)^2 f} - \frac{fx\log(\tanh(c+dx))}{ad} + \frac{(e+fx)\log(\tanh(c+dx))}{ad} - \frac{f\,\text{ta}\ldots}{\ldots}$$

$$= \frac{fx}{2ad} + \frac{b^4(e+fx)^2}{2a\left(a^2+b^2\right)^2 f} - \frac{b^4(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\left(a^2+b^2\right)^2 d} - \frac{b^4(e+fx)\log\left(1+\ldots\right.}{a\left(a^2+b^2\right)^2\ldots}$$

$$= \frac{fx}{2ad} - \frac{2b^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} - \frac{2fx\tanh^{-1}\left(e^{2c}\ldots\right.}{ad}$$

$$= \frac{fx}{2ad} - \frac{2b^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} - \frac{2fx\tanh^{-1}\left(e^{2c}\ldots\right.}{ad}$$

$$= \frac{fx}{2ad} - \frac{2b^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} - \frac{2fx\tanh^{-1}\left(e^{2c}\ldots\right.}{ad}$$

$$= \frac{fx}{2ad} - \frac{2b^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)^2 d} - \frac{b(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{\left(a^2+b^2\right)d} - \frac{2fx\tanh^{-1}\left(e^{2c}\ldots\right.}{ad}$$

Mathematica [A] time = 10.8415, size = 886, normalized size = 1.19

$$-\frac{\left(-\frac{1}{2}f(c+dx)^2 + f\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)(c+dx) + f\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)(c+dx) + de\log(a+b\sinh(c+dx)) - cf\log(a+\ldots\right.}{a\left(a^2+b^2\right)^2 d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (e*Log[Sinh[c + d*x]])/(a*d) - (c*f*Log[Sinh[c + d*x]])/(a*d^2) - (I*f*(I*(
c + d*x)*Log[1 - E^(-2*(c + d*x))] - (I/2)*(-(c + d*x)^2 + PolyLog[2, E^(-2
*(c + d*x))])))/(a*d^2) - (b^4*(-(f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b
*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2])] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[
c + d*x]] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLo
g[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a*(a^2 + b^2)^2*d^2) - (-
2*a^3*d*e*(c + d*x) - 4*a*b^2*d*e*(c + d*x) + 2*a^3*c*f*(c + d*x) + 4*a*b^2
*c*f*(c + d*x) - a^3*f*(c + d*x)^2 - 2*a*b^2*f*(c + d*x)^2 + 2*a^2*b*d*e*Ar
cTan[E^(c + d*x)] + 6*b^3*d*e*ArcTan[E^(c + d*x)] - 2*a^2*b*c*f*ArcTan[E^(c
 + d*x)] - 6*b^3*c*f*ArcTan[E^(c + d*x)] + I*a^2*b*f*(c + d*x)*Log[1 - I*E^
(c + d*x)] + (3*I)*b^3*f*(c + d*x)*Log[1 - I*E^(c + d*x)] - I*a^2*b*f*(c +
d*x)*Log[1 + I*E^(c + d*x)] - (3*I)*b^3*f*(c + d*x)*Log[1 + I*E^(c + d*x)]
+ 2*a^3*d*e*Log[1 + E^(2*(c + d*x))] + 4*a*b^2*d*e*Log[1 + E^(2*(c + d*x))]
 - 2*a^3*c*f*Log[1 + E^(2*(c + d*x))] - 4*a*b^2*c*f*Log[1 + E^(2*(c + d*x))
] + 2*a^3*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] + 4*a*b^2*f*(c + d*x)*Log[1
+ E^(2*(c + d*x))] - I*b*(a^2 + 3*b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] + I*b
*(a^2 + 3*b^2)*f*PolyLog[2, I*E^(c + d*x)] + a^3*f*PolyLog[2, -E^(2*(c + d*
x))] + 2*a*b^2*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^2) + (Sec
h[c + d*x]*(-(b*f) - a*f*Sinh[c + d*x]))/(2*(a^2 + b^2)*d^2) + (Sech[c + d*
x]^2*(a*d*e - a*c*f + a*f*(c + d*x) - b*d*e*Sinh[c + d*x] + b*c*f*Sinh[c +
d*x] - b*f*(c + d*x)*Sinh[c + d*x]))/(2*(a^2 + b^2)*d^2)
```

Maple [B] time = 0.244, size = 2580, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/(a^2+b^2)/d*a*e*ln(exp(d*x+c)-1)+1/(a^2+b^2)/d*a*e*ln(exp(d*x+c)+1)-1/(a^
2+b^2)/d^2*a*f*dilog(exp(d*x+c))+1/(a^2+b^2)/d^2*a*f*dilog(exp(d*x+c)+1)-8/
(a^2+b^2)/d*b^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*a*x-8/(a^2+b^2)/d^2*b^2*
f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*a*c+8/(a^2+b^2)/d^2*b^2*f*c/(4*a^2+4*b^2
)*a*ln(1+exp(2*d*x+2*c))-6*I/(a^2+b^2)/d*b^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x
+c))*x-6*I/(a^2+b^2)/d^2*b^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c+6*I/(a^2+
b^2)/d*b^3*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x+6*I/(a^2+b^2)/d^2*b^3*f/(4*
a^2+4*b^2)*ln(1+I*exp(d*x+c))*c+2*I/(a^2+b^2)/d^2*a^2*f/(4*a^2+4*b^2)*dilog
(1+I*exp(d*x+c))*b-2*I/(a^2+b^2)/d^2*a^2*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+
c))*b-1/2/(a^2+b^2)^(5/2)/d^2*a^2*b^2*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/
(a^2+b^2)^(1/2))+4/(a^2+b^2)/d^2*a^2*f*c/(4*a^2+4*b^2)*b*arctan(exp(d*x+c))
-8/(a^2+b^2)/d*b^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*a*x-8/(a^2+b^2)/d^2*b
^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*a*c+(-b*d*f*x*exp(3*d*x+3*c)+2*a*d*f*
x*exp(2*d*x+2*c)-b*d*e*exp(3*d*x+3*c)+2*a*d*e*exp(2*d*x+2*c)+b*d*f*x*exp(d*
x+c)-b*f*exp(3*d*x+3*c)+a*f*exp(2*d*x+2*c)+b*d*e*exp(d*x+c)-f*b*exp(d*x+c)+
a*f)/d^2/(a^2+b^2)/(1+exp(2*d*x+2*c))^2+1/(a^2+b^2)/d*ln(exp(d*x+c)+1)*a*f*
x-1/(a^2+b^2)/d^2*a*f*c*ln(exp(d*x+c)-1)-1/(a^2+b^2)/d^2*b^2*f/a*dilog(exp(
d*x+c))+1/(a^2+b^2)/d^2*b^2*f/a*dilog(exp(d*x+c)+1)+1/(a^2+b^2)/d*b^2*e/a*l
n(exp(d*x+c)-1)+1/(a^2+b^2)/d*b^2*e/a*ln(exp(d*x+c)+1)-1/(a^2+b^2)/d^2*b^2*
f*c/a*ln(exp(d*x+c)-1)+1/(a^2+b^2)/d*b^2*f/a*ln(exp(d*x+c)+1)*x+2*I/(a^2+b^
2)/d*a^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*b*x+2*I/(a^2+b^2)/d^2*a^2*f/(4*
```

```
a^2+4*b^2)*ln(1+I*exp(d*x+c))*b*c-2*I/(a^2+b^2)/d*a^2*f/(4*a^2+4*b^2)*ln(1-
I*exp(d*x+c))*b*x-2*I/(a^2+b^2)/d^2*a^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*
b*c-1/(a^2+b^2)^2/d^2*b^4*f/a*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(
a^2+b^2)^(1/2)))-1/(a^2+b^2)^2/d^2*b^4*f/a*dilog((b*exp(d*x+c)+(a^2+b^2)^(1
/2)+a)/(a+(a^2+b^2)^(1/2)))-1/2/(a^2+b^2)^(3/2)/d*b^2*e*arctanh(1/2*(2*b*ex
p(d*x+c)+2*a)/(a^2+b^2)^(1/2))-12/(a^2+b^2)/d*b^3*e/(4*a^2+4*b^2)*arctan(ex
p(d*x+c))+1/2/(a^2+b^2)^(5/2)/d*b^4*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2
+b^2)^(1/2))-4/(a^2+b^2)/d^2*a^3*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))-4/(a
^2+b^2)/d^2*a^3*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))-1/(a^2+b^2)^2/d*b^4*e
/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-4/(a^2+b^2)/d*a^3*e/(4*a^2+4*b^2)*
ln(1+exp(2*d*x+2*c))-1/(a^2+b^2)^2/d^2*b^4*f/a*ln((b*exp(d*x+c)+(a^2+b^2)^(
1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+6*I/(a^2+b^2)/d^2*b^3*f/(4*a^2+4*b^2)*dilog(
1+I*exp(d*x+c))-6*I/(a^2+b^2)/d^2*b^3*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))
+1/2/(a^2+b^2)^(3/2)/d^2*b^2*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)
^(1/2))+12/(a^2+b^2)/d^2*b^3*f*c/(4*a^2+4*b^2)*arctan(exp(d*x+c))-1/2/(a^2+
b^2)^(5/2)/d^2*b^4*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-8/
(a^2+b^2)/d^2*b^2*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))*a-8/(a^2+b^2)/d^2*b
^2*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))*a+4/(a^2+b^2)/d^2*a^3*f*c/(4*a^2+4
*b^2)*ln(1+exp(2*d*x+2*c))+1/(a^2+b^2)^2/d^2*b^4*f*c/a*ln(b*exp(2*d*x+2*c)+
2*a*exp(d*x+c)-b)-4/(a^2+b^2)/d*a^2*e/(4*a^2+4*b^2)*b*arctan(exp(d*x+c))-8/
(a^2+b^2)/d*b^2*e/(4*a^2+4*b^2)*a*ln(1+exp(2*d*x+2*c))+1/2/(a^2+b^2)^(5/2)/
d*a^2*b^2*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-4/(a^2+b^2)/d
*a^3*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x-4/(a^2+b^2)/d^2*a^3*f/(4*a^2+4*b^
2)*ln(1+I*exp(d*x+c))*c-4/(a^2+b^2)/d*a^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c)
)*x-4/(a^2+b^2)/d^2*a^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c-1/(a^2+b^2)^2/
d*b^4*f/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/(a
^2+b^2)^2/d^2*b^4*f/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1
/2)))*c-1/(a^2+b^2)^2/d*b^4*f/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2
+b^2)^(1/2)))*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -(b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + 2*a^3*b^2 + a
*b^4)*d) - (a^2*b + 3*b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d)
 + (a^3 + 2*a*b^2)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) +
(b*e^(-d*x - c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 +
2*(a^2 + b^2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d) - log(e^(
-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d))*e - f*(((b*d*x*e^(3*c)
+ b*e^(3*c))*e^(3*d*x) - (2*a*d*x*e^(2*c) + a*e^(2*c))*e^(2*d*x) - (b*d*x*e
^c - b*e^c)*e^(d*x) - a)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) + b^2*d^2*e^
(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*x)) - 16*in
tegrate(-1/8*(a*b^4*x*e^(d*x + c) - b^5*x)/(a^5*b + 2*a^3*b^3 + a*b^5 - (a^
5*b*e^(2*c) + 2*a^3*b^3*e^(2*c) + a*b^5*e^(2*c))*e^(2*d*x) - 2*(a^6*e^c + 2
*a^4*b^2*e^c + a^2*b^4*e^c)*e^(d*x)), x) + 16*integrate(1/16*((a^2*b*e^c +
3*b^3*e^c)*x*e^(d*x) - 2*(a^3 + 2*a*b^2)*x)/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e
^(2*c) + 2*a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x) + 16*integrate(1/1
6*x/(a*e^(d*x + c) + a), x) - 16*integrate(1/16*x/(a*e^(d*x + c) - a), x))
```

Fricas [B] time = 4.2291, size = 17816, normalized size = 23.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -1/2*(2*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e + (a^3*b + a*b^3)*f)*c
osh(d*x + c)^3 + 2*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e + (a^3*b +
a*b^3)*f)*sinh(d*x + c)^3 - 2*(2*(a^4 + a^2*b^2)*d*f*x + 2*(a^4 + a^2*b^2)*
d*e + (a^4 + a^2*b^2)*f)*cosh(d*x + c)^2 - 2*(2*(a^4 + a^2*b^2)*d*f*x + 2*(
a^4 + a^2*b^2)*d*e + (a^4 + a^2*b^2)*f - 3*((a^3*b + a*b^3)*d*f*x + (a^3*b
+ a*b^3)*d*e + (a^3*b + a*b^3)*f)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(a^4 +
 a^2*b^2)*f - 2*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e - (a^3*b + a*b
^3)*f)*cosh(d*x + c) + 2*(b^4*f*cosh(d*x + c)^4 + 4*b^4*f*cosh(d*x + c)*sin
h(d*x + c)^3 + b^4*f*sinh(d*x + c)^4 + 2*b^4*f*cosh(d*x + c)^2 + b^4*f + 2*
(3*b^4*f*cosh(d*x + c)^2 + b^4*f)*sinh(d*x + c)^2 + 4*(b^4*f*cosh(d*x + c)^
3 + b^4*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x
 + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b +
1) + 2*(b^4*f*cosh(d*x + c)^4 + 4*b^4*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^4
*f*sinh(d*x + c)^4 + 2*b^4*f*cosh(d*x + c)^2 + b^4*f + 2*(3*b^4*f*cosh(d*x
+ c)^2 + b^4*f)*sinh(d*x + c)^2 + 4*(b^4*f*cosh(d*x + c)^3 + b^4*f*cosh(d*x
 + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*
x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*((a^4 + 2*a
^2*b^2 + b^4)*f*cosh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)
*sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*f*sinh(d*x + c)^4 + 2*(a^4 + 2*a
^2*b^2 + b^4)*f*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x +
 c)^2 + (a^4 + 2*a^2*b^2 + b^4)*f)*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4
)*f + 4*((a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4
)*f*cosh(d*x + c))*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)) + ((
2*(a^4 + 2*a^2*b^2)*f + I*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^4 + 4*(2*(a^4
+ 2*a^2*b^2)*f + I*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*
(a^4 + 2*a^2*b^2)*f + I*(a^3*b + 3*a*b^3)*f)*sinh(d*x + c)^4 + 2*(2*(a^4 +
2*a^2*b^2)*f + I*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^2 + 2*(3*(2*(a^4 + 2*a^
2*b^2)*f + I*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2)*f +
 I*(a^3*b + 3*a*b^3)*f)*sinh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2)*f + I*(a^3*b
+ 3*a*b^3)*f + 4*((2*(a^4 + 2*a^2*b^2)*f + I*(a^3*b + 3*a*b^3)*f)*cosh(d*x
+ c)^3 + (2*(a^4 + 2*a^2*b^2)*f + I*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c))*sin
h(d*x + c))*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + ((2*(a^4 + 2*a^2*b^2
)*f - I*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^4 + 4*(2*(a^4 + 2*a^2*b^2)*f - I
*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*(a^4 + 2*a^2*b^2)*
f - I*(a^3*b + 3*a*b^3)*f)*sinh(d*x + c)^4 + 2*(2*(a^4 + 2*a^2*b^2)*f - I*(
a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^2 + (6*(2*(a^4 + 2*a^2*b^2)*f - I*(a^3*b
+ 3*a*b^3)*f)*cosh(d*x + c)^2 + 4*(a^4 + 2*a^2*b^2)*f - 2*I*(a^3*b + 3*a*b^
3)*f)*sinh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2)*f - I*(a^3*b + 3*a*b^3)*f + 4*(
(2*(a^4 + 2*a^2*b^2)*f - I*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^3 + (2*(a^4 +
 2*a^2*b^2)*f - I*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(
-I*cosh(d*x + c) - I*sinh(d*x + c)) - 2*((a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x
 + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4
+ 2*a^2*b^2 + b^4)*f*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x
 + c)^2 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2
 + b^4)*f)*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*f + 4*((a^4 + 2*a^2*b^
2 + b^4)*f*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c))*sinh(
d*x + c))*dilog(-cosh(d*x + c) - sinh(d*x + c)) + 2*(b^4*d*e - b^4*c*f + (b
^4*d*e - b^4*c*f)*cosh(d*x + c)^4 + 4*(b^4*d*e - b^4*c*f)*cosh(d*x + c)*sin
h(d*x + c)^3 + (b^4*d*e - b^4*c*f)*sinh(d*x + c)^4 + 2*(b^4*d*e - b^4*c*f)*
cosh(d*x + c)^2 + 2*(b^4*d*e - b^4*c*f + 3*(b^4*d*e - b^4*c*f)*cosh(d*x + c
```

)^2)*sinh(d*x + c)^2 + 4*((b^4*d*e - b^4*c*f)*cosh(d*x + c)^3 + (b^4*d*e - b^4*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(b^4*d*e - b^4*c*f + (b^4*d*e - b^4*c*f)*cosh(d*x + c)^4 + 4*(b^4*d*e - b^4*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*d*e - b^4*c*f)*sinh(d*x + c)^4 + 2*(b^4*d*e - b^4*c*f)*cosh(d*x + c)^2 + 2*(b^4*d*e - b^4*c*f + 3*(b^4*d*e - b^4*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d*e - b^4*c*f)*cosh(d*x + c)^3 + (b^4*d*e - b^4*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(b^4*d*f*x + b^4*c*f + (b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^4 + 4*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*d*f*x + b^4*c*f)*sinh(d*x + c)^4 + 2*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^2 + 2*(b^4*d*f*x + b^4*c*f + 3*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^3 + (b^4*d*f*x + b^4*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(b^4*d*f*x + b^4*c*f + (b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^4 + 4*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*d*f*x + b^4*c*f)*sinh(d*x + c)^4 + 2*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^2 + 2*(b^4*d*f*x + b^4*c*f + 3*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^3 + (b^4*d*f*x + b^4*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*e)*cosh(d*x + c)^4 + 4*((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*e)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*e)*sinh(d*x + c)^4 + (a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*e + 2*((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*e)*cosh(d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*e + 3*((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*e)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*e)*cosh(d*x + c)^3 + ((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*e)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((2*(a^4 + 2*a^2*b^2)*d*e + I*(a^3*b + 3*a*b^3)*d*e - 2*(a^4 + 2*a^2*b^2)*c*f - I*(a^3*b + 3*a*b^3)*c*f)*cosh(d*x + c)^4 + (8*(a^4 + 2*a^2*b^2)*d*e + 4*I*(a^3*b + 3*a*b^3)*d*e - 8*(a^4 + 2*a^2*b^2)*c*f - 4*I*(a^3*b + 3*a*b^3)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*(a^4 + 2*a^2*b^2)*d*e + I*(a^3*b + 3*a*b^3)*d*e - 2*(a^4 + 2*a^2*b^2)*c*f - I*(a^3*b + 3*a*b^3)*c*f)*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2)*d*e + I*(a^3*b + 3*a*b^3)*d*e - 2*(a^4 + 2*a^2*b^2)*c*f - I*(a^3*b + 3*a*b^3)*c*f + (4*(a^4 + 2*a^2*b^2)*d*e + 2*I*(a^3*b + 3*a*b^3)*d*e - 4*(a^4 + 2*a^2*b^2)*c*f - 2*I*(a^3*b + 3*a*b^3)*c*f)*cosh(d*x + c)^2 + (4*(a^4 + 2*a^2*b^2)*d*e + 2*I*(a^3*b + 3*a*b^3)*d*e - 4*(a^4 + 2*a^2*b^2)*c*f - 2*I*(a^3*b + 3*a*b^3)*c*f + (12*(a^4 + 2*a^2*b^2)*d*e + 6*I*(a^3*b + 3*a*b^3)*d*e - 12*(a^4 + 2*a^2*b^2)*c*f - 6*I*(a^3*b + 3*a*b^3)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*(a^4 + 2*a^2*b^2)*d*e + 4*I*(a^3*b + 3*a*b^3)*d*e - 8*(a^4 + 2*a^2*b^2)*c*f - 4*I*(a^3*b + 3*a*b^3)*c*f)*cosh(d*x + c)^3 + (8*(a^4 + 2*a^2*b^2)*d*e + 4*I*(a^3*b + 3*a*b^3)*d*e - 8*(a^4 + 2*a^2*b^2)*c*f - 4*I*(a^3*b + 3*a*b^3)*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + I) + ((2*(a^4 + 2*a^2*b^2)*d*e - I*(a^3*b + 3*a*b^3)*d*e - 2*(a^4 + 2*a^2*b^2)*c*f + I*(a^3*b + 3*a*b^3)*c*f)*cosh(d*x + c)^4 + (8*(a^4 + 2*a^2*b^2)*d*e - 4*I*(a^3*b + 3*a*b^3)*d*e - 8*(a^4 + 2*a^2*b^2)*c*f + 4*I*(a^3*b + 3*a*b^3)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*(a^4 + 2*a^2*b^2)*d*e - I*(a^3*b + 3*a*b^3)*d*e - 2*(a^4 + 2*a^2*b^2)*c*f + I*(a^3*b + 3*a*b^3)*c*f)*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2)*d*e - I*(a^3*b + 3*a*b^3)*d*e - 2*(a^4 + 2*a^2*b^2)*c*f + I*(a^3*b + 3*a*b^3)*c*f + (4*(a^4 + 2*a^2*b^2)*d*e - 2*I*(a^3*b + 3*a*b^3)*d*e - 4*(a^4 + 2*a^2*b^2)*c*f + 2*I*(a^3*b + 3*a*b^3)*c*f)*cosh(d*x + c)^2 + (4*(a^4 + 2*a^2*b^2)*d*e - 2*I*(a^3*b + 3*a*b^3)*d*e - 4*(a^4 + 2*a^2*b^2)*c*f + 2*I*(a^3*b + 3*a*b^3)*c*f + (12*(a^4 + 2*a^2*b^2)*d*e - 6*I*(a^3*b + 3*a*b^3)*d*e - 12*(a^4 + 2*a^2*b^2)*c*f + 6*I*(a^3*b + 3*a*b^3)*c*f)*cosh(d*x + c)

```
^2)*sinh(d*x + c)^2 + ((8*(a^4 + 2*a^2*b^2)*d*e - 4*I*(a^3*b + 3*a*b^3)*d*e
 - 8*(a^4 + 2*a^2*b^2)*c*f + 4*I*(a^3*b + 3*a*b^3)*c*f)*cosh(d*x + c)^3 + (
8*(a^4 + 2*a^2*b^2)*d*e - 4*I*(a^3*b + 3*a*b^3)*d*e - 8*(a^4 + 2*a^2*b^2)*c
*f + 4*I*(a^3*b + 3*a*b^3)*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x
+ c) + sinh(d*x + c) - I) - 2*(((a^4 + 2*a^2*b^2 + b^4)*d*e - (a^4 + 2*a^2*
b^2 + b^4)*c*f)*cosh(d*x + c)^4 + 4*((a^4 + 2*a^2*b^2 + b^4)*d*e - (a^4 + 2
*a^2*b^2 + b^4)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 + 2*a^2*b^2 + b^
4)*d*e - (a^4 + 2*a^2*b^2 + b^4)*c*f)*sinh(d*x + c)^4 + (a^4 + 2*a^2*b^2 +
b^4)*d*e - (a^4 + 2*a^2*b^2 + b^4)*c*f + 2*((a^4 + 2*a^2*b^2 + b^4)*d*e - (
a^4 + 2*a^2*b^2 + b^4)*c*f)*cosh(d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d*
e - (a^4 + 2*a^2*b^2 + b^4)*c*f + 3*((a^4 + 2*a^2*b^2 + b^4)*d*e - (a^4 + 2
*a^2*b^2 + b^4)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^4 + 2*a^2*b^
2 + b^4)*d*e - (a^4 + 2*a^2*b^2 + b^4)*c*f)*cosh(d*x + c)^3 + ((a^4 + 2*a^2
*b^2 + b^4)*d*e - (a^4 + 2*a^2*b^2 + b^4)*c*f)*cosh(d*x + c))*sinh(d*x + c)
)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + ((2*(a^4 + 2*a^2*b^2)*d*f*x - I*
(a^3*b + 3*a*b^3)*d*f*x + 2*(a^4 + 2*a^2*b^2)*c*f - I*(a^3*b + 3*a*b^3)*c*f
)*cosh(d*x + c)^4 + (8*(a^4 + 2*a^2*b^2)*d*f*x - 4*I*(a^3*b + 3*a*b^3)*d*f*
x + 8*(a^4 + 2*a^2*b^2)*c*f - 4*I*(a^3*b + 3*a*b^3)*c*f)*cosh(d*x + c)*sinh
(d*x + c)^3 + (2*(a^4 + 2*a^2*b^2)*d*f*x - I*(a^3*b + 3*a*b^3)*d*f*x + 2*(a
^4 + 2*a^2*b^2)*c*f - I*(a^3*b + 3*a*b^3)*c*f)*sinh(d*x + c)^4 + 2*(a^4 + 2
*a^2*b^2)*d*f*x - I*(a^3*b + 3*a*b^3)*d*f*x + 2*(a^4 + 2*a^2*b^2)*c*f - I*(
a^3*b + 3*a*b^3)*c*f + (4*(a^4 + 2*a^2*b^2)*d*f*x - 2*I*(a^3*b + 3*a*b^3)*d
*f*x + 4*(a^4 + 2*a^2*b^2)*c*f - 2*I*(a^3*b + 3*a*b^3)*c*f)*cosh(d*x + c)^2
 + (4*(a^4 + 2*a^2*b^2)*d*f*x - 2*I*(a^3*b + 3*a*b^3)*d*f*x + 4*(a^4 + 2*a^
2*b^2)*c*f - 2*I*(a^3*b + 3*a*b^3)*c*f + (12*(a^4 + 2*a^2*b^2)*d*f*x - 6*I*
(a^3*b + 3*a*b^3)*d*f*x + 12*(a^4 + 2*a^2*b^2)*c*f - 6*I*(a^3*b + 3*a*b^3)*
c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*(a^4 + 2*a^2*b^2)*d*f*x - 4*I*(
a^3*b + 3*a*b^3)*d*f*x + 8*(a^4 + 2*a^2*b^2)*c*f - 4*I*(a^3*b + 3*a*b^3)*c*
f)*cosh(d*x + c)^3 + (8*(a^4 + 2*a^2*b^2)*d*f*x - 4*I*(a^3*b + 3*a*b^3)*d*f
*x + 8*(a^4 + 2*a^2*b^2)*c*f - 4*I*(a^3*b + 3*a*b^3)*c*f)*cosh(d*x + c))*si
nh(d*x + c))*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) + ((2*(a^4 + 2*a^2*
b^2)*d*f*x + I*(a^3*b + 3*a*b^3)*d*f*x + 2*(a^4 + 2*a^2*b^2)*c*f + I*(a^3*b
 + 3*a*b^3)*c*f)*cosh(d*x + c)^4 + (8*(a^4 + 2*a^2*b^2)*d*f*x + 4*I*(a^3*b
+ 3*a*b^3)*d*f*x + 8*(a^4 + 2*a^2*b^2)*c*f + 4*I*(a^3*b + 3*a*b^3)*c*f)*cos
h(d*x + c)*sinh(d*x + c)^3 + (2*(a^4 + 2*a^2*b^2)*d*f*x + I*(a^3*b + 3*a*b^
3)*d*f*x + 2*(a^4 + 2*a^2*b^2)*c*f + I*(a^3*b + 3*a*b^3)*c*f)*sinh(d*x + c)
^4 + 2*(a^4 + 2*a^2*b^2)*d*f*x + I*(a^3*b + 3*a*b^3)*d*f*x + 2*(a^4 + 2*a^2
*b^2)*c*f + I*(a^3*b + 3*a*b^3)*c*f + (4*(a^4 + 2*a^2*b^2)*d*f*x + 2*I*(a^3
*b + 3*a*b^3)*d*f*x + 4*(a^4 + 2*a^2*b^2)*c*f + 2*I*(a^3*b + 3*a*b^3)*c*f)*
cosh(d*x + c)^2 + (4*(a^4 + 2*a^2*b^2)*d*f*x + 2*I*(a^3*b + 3*a*b^3)*d*f*x
+ 4*(a^4 + 2*a^2*b^2)*c*f + 2*I*(a^3*b + 3*a*b^3)*c*f + (12*(a^4 + 2*a^2*b^
2)*d*f*x + 6*I*(a^3*b + 3*a*b^3)*d*f*x + 12*(a^4 + 2*a^2*b^2)*c*f + 6*I*(a^
3*b + 3*a*b^3)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*(a^4 + 2*a^2*b^2
)*d*f*x + 4*I*(a^3*b + 3*a*b^3)*d*f*x + 8*(a^4 + 2*a^2*b^2)*c*f + 4*I*(a^3*
b + 3*a*b^3)*c*f)*cosh(d*x + c)^3 + (8*(a^4 + 2*a^2*b^2)*d*f*x + 4*I*(a^3*b
 + 3*a*b^3)*d*f*x + 8*(a^4 + 2*a^2*b^2)*c*f + 4*I*(a^3*b + 3*a*b^3)*c*f)*co
sh(d*x + c))*sinh(d*x + c))*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1) - 2
*(((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*c*f)*cosh(d*x +
c)^4 + 4*((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*c*f)*cosh
(d*x + c)*sinh(d*x + c)^3 + ((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b
^2 + b^4)*c*f)*sinh(d*x + c)^4 + (a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a
^2*b^2 + b^4)*c*f + 2*((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b
^4)*c*f)*cosh(d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*
b^2 + b^4)*c*f + 3*((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)
*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^4 + 2*a^2*b^2 + b^4)*d*f*x
+ (a^4 + 2*a^2*b^2 + b^4)*c*f)*cosh(d*x + c)^3 + ((a^4 + 2*a^2*b^2 + b^4)*d
*f*x + (a^4 + 2*a^2*b^2 + b^4)*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-cosh
(d*x + c) - sinh(d*x + c) + 1) - 2*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)
*d*e - 3*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e + (a^3*b + a*b^3)*f)*
```

```
cosh(d*x + c)^2 - (a^3*b + a*b^3)*f + 2*(2*(a^4 + a^2*b^2)*d*f*x + 2*(a^4 +
 a^2*b^2)*d*e + (a^4 + a^2*b^2)*f)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 2*
a^3*b^2 + a*b^4)*d^2*cosh(d*x + c)^4 + 4*(a^5 + 2*a^3*b^2 + a*b^4)*d^2*cosh
(d*x + c)*sinh(d*x + c)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*d^2*sinh(d*x + c)^4 +
 2*(a^5 + 2*a^3*b^2 + a*b^4)*d^2*cosh(d*x + c)^2 + (a^5 + 2*a^3*b^2 + a*b^4
)*d^2 + 2*(3*(a^5 + 2*a^3*b^2 + a*b^4)*d^2*cosh(d*x + c)^2 + (a^5 + 2*a^3*b
^2 + a*b^4)*d^2)*sinh(d*x + c)^2 + 4*((a^5 + 2*a^3*b^2 + a*b^4)*d^2*cosh(d*
x + c)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*d^2*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.447 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=160

$$-\frac{b^4\log(a+b\sinh(c+dx))}{ad\left(a^2+b^2\right)^2}-\frac{b^3\tan^{-1}(\sinh(c+dx))}{d\left(a^2+b^2\right)^2}-\frac{b\tan^{-1}(\sinh(c+dx))}{2d\left(a^2+b^2\right)}-\frac{a\left(a^2+2b^2\right)\log(\cosh(c+dx))}{d\left(a^2+b^2\right)^2}+\text{se}$$

```
[Out] -((b^3*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)^2*d)) - (b*ArcTan[Sinh[c + d*x]]
)/(2*(a^2 + b^2)*d) - (a*(a^2 + 2*b^2)*Log[Cosh[c + d*x]])/((a^2 + b^2)^2*d
) + Log[Sinh[c + d*x]]/(a*d) - (b^4*Log[a + b*Sinh[c + d*x]])/(a*(a^2 + b^2
)^2*d) + (Sech[c + d*x]^2*(a - b*Sinh[c + d*x]))/(2*(a^2 + b^2)*d)
```

Rubi [A] time = 0.270547, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2837, 12, 894, 639, 203, 635, 260}

$$-\frac{b^4\log(a+b\sinh(c+dx))}{ad\left(a^2+b^2\right)^2}-\frac{b^3\tan^{-1}(\sinh(c+dx))}{d\left(a^2+b^2\right)^2}-\frac{b\tan^{-1}(\sinh(c+dx))}{2d\left(a^2+b^2\right)}-\frac{a\left(a^2+2b^2\right)\log(\cosh(c+dx))}{d\left(a^2+b^2\right)^2}+\text{se}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((b^3*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)^2*d)) - (b*ArcTan[Sinh[c + d*x]]
)/(2*(a^2 + b^2)*d) - (a*(a^2 + 2*b^2)*Log[Cosh[c + d*x]])/((a^2 + b^2)^2*d
) + Log[Sinh[c + d*x]]/(a*d) - (b^4*Log[a + b*Sinh[c + d*x]])/(a*(a^2 + b^2
)^2*d) + (Sech[c + d*x]^2*(a - b*Sinh[c + d*x]))/(2*(a^2 + b^2)*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
```

```
Q[p, -1] && NeQ[p, -3/2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b^3\operatorname{Subst}\left(\int \frac{b}{x(a+x)\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= \frac{b^4\operatorname{Subst}\left(\int \frac{1}{x(a+x)\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= \frac{b^4\operatorname{Subst}\left(\int \left(\frac{1}{ab^4x} - \frac{1}{a\left(a^2+b^2\right)^2(a+x)} + \frac{-b^2-ax}{b^2\left(a^2+b^2\right)\left(b^2+x^2\right)^2} + \frac{-b^4-a\left(a^2+2b^2\right)x}{b^4\left(a^2+b^2\right)^2\left(b^2+x^2\right)}\right) dx, x, b\sinh(c$$

$$= \frac{\log(\sinh(c+dx))}{ad} - \frac{b^4\log(a+b\sinh(c+dx))}{a\left(a^2+b^2\right)^2 d} + \frac{\operatorname{Subst}\left(\int \frac{-b^4-a\left(a^2+2b^2\right)x}{b^2+x^2}\,dx, x, b\sinh\right.}{\left(a^2+b^2\right)^2 d}$$

$$= \frac{\log(\sinh(c+dx))}{ad} - \frac{b^4\log(a+b\sinh(c+dx))}{a\left(a^2+b^2\right)^2 d} + \frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2\left(a^2+b^2\right)d}$$

$$= -\frac{b^3\tan^{-1}(\sinh(c+dx))}{\left(a^2+b^2\right)^2 d} - \frac{b\tan^{-1}(\sinh(c+dx))}{2\left(a^2+b^2\right)d} - \frac{a\left(a^2+2b^2\right)\log(\cosh(c+dx))}{\left(a^2+b^2\right)^2 d} +$$

Mathematica [A] time = 0.793089, size = 196, normalized size = 1.22

$$-\frac{-a^2\left(a^2+b^2\right)\operatorname{sech}^2(c+dx) - 2\left(a^2+b^2\right)^2\log(\sinh(c+dx)) + a\left(a^3+2ab^2+\left(-b^2\right)^{3/2}\right)\log\left(\sqrt{-b^2}-b\sinh(c+dx)\right) + a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(a*b*(a^2 + b^2)*ArcTan[Sinh[c + d*x]] - 2*(a^2 + b^2)^2*Log[Sinh[c + d*x]
] + a*(a^3 + 2*a*b^2 + (-b^2)^(3/2))*Log[Sqrt[-b^2] - b*Sinh[c + d*x]] + 2*
b^4*Log[a + b*Sinh[c + d*x]] + a*(a^3 + 2*a*b^2 - (-b^2)^(3/2))*Log[Sqrt[-b
^2] + b*Sinh[c + d*x]] - a^2*(a^2 + b^2)*Sech[c + d*x]^2 + a*b*(a^2 + b^2)*
```

```
Sech[c + d*x]*Tanh[c + d*x])/(2*a*(a^2 + b^2)^2*d)
```

Maple [B] time = 0.003, size = 530, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/d/a*ln(tanh(1/2*d*x+1/2*c))-1/d*b^4/a/(a^4+2*a^2*b^2+b^4)*ln(tanh(1/2*d*x
+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)*b-a)+1/d/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*d*
x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)^3*a^2*b+1/d/(a^4+2*a^2*b^2+b^4)/(tanh(1
/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)^3*b^3-2/d/(a^4+2*a^2*b^2+b^4)/(tan
h(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)^2*a^3-2/d/(a^4+2*a^2*b^2+b^4)/(
tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)^2*a*b^2-1/d/(a^4+2*a^2*b^2+b
^4)/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)*a^2*b-1/d/(a^4+2*a^2*b^
2+b^4)/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)*b^3-1/d/(a^4+2*a^2*b
^2+b^4)*a^3*ln(tanh(1/2*d*x+1/2*c)^2+1)-2/d/(a^4+2*a^2*b^2+b^4)*ln(tanh(1/2
*d*x+1/2*c)^2+1)*a*b^2-1/d/(a^4+2*a^2*b^2+b^4)*arctan(tanh(1/2*d*x+1/2*c))*
a^2*b-3/d/(a^4+2*a^2*b^2+b^4)*arctan(tanh(1/2*d*x+1/2*c))*b^3
```

Maxima [A] time = 1.81423, size = 358, normalized size = 2.24

$$-\frac{b^4\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{\left(a^5+2\,a^3b^2+ab^4\right)d}+\frac{\left(a^2b+3\,b^3\right)\arctan\left(e^{(-dx-c)}\right)}{\left(a^4+2\,a^2b^2+b^4\right)d}-\frac{\left(a^3+2\,ab^2\right)\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\left(a^4+2\,a^2b^2+b^4\right)d}-\frac{}{\left(a^2+b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + 2*a^3*b^2 + a*
b^4)*d) + (a^2*b + 3*b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d)
 - (a^3 + 2*a*b^2)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (
b*e^(-d*x - c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2
*(a^2 + b^2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d) + log(e^(-
d*x - c) + 1)/(a*d) + log(e^(-d*x - c) - 1)/(a*d)
```

Fricas [B] time = 4.2508, size = 3066, normalized size = 19.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -((a^3*b + a*b^3)*cosh(d*x + c)^3 + (a^3*b + a*b^3)*sinh(d*x + c)^3 - 2*(a^
4 + a^2*b^2)*cosh(d*x + c)^2 - (2*a^4 + 2*a^2*b^2 - 3*(a^3*b + a*b^3)*cosh(
d*x + c))*sinh(d*x + c)^2 + ((a^3*b + 3*a*b^3)*cosh(d*x + c)^4 + 4*(a^3*b +
```

```
 3*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b + 3*a*b^3)*sinh(d*x + c)^4
 + a^3*b + 3*a*b^3 + 2*(a^3*b + 3*a*b^3)*cosh(d*x + c)^2 + 2*(a^3*b + 3*a*b
^3 + 3*(a^3*b + 3*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3*b + 3*a
*b^3)*cosh(d*x + c)^3 + (a^3*b + 3*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*arc
tan(cosh(d*x + c) + sinh(d*x + c)) - (a^3*b + a*b^3)*cosh(d*x + c) + (b^4*c
osh(d*x + c)^4 + 4*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + b^4*sinh(d*x + c)^4
+ 2*b^4*cosh(d*x + c)^2 + b^4 + 2*(3*b^4*cosh(d*x + c)^2 + b^4)*sinh(d*x +
c)^2 + 4*(b^4*cosh(d*x + c)^3 + b^4*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*
sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + ((a^4 + 2*a^2*b^2)*co
sh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 +
2*a^2*b^2)*sinh(d*x + c)^4 + a^4 + 2*a^2*b^2 + 2*(a^4 + 2*a^2*b^2)*cosh(d*x
 + c)^2 + 2*(a^4 + 2*a^2*b^2 + 3*(a^4 + 2*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*
x + c)^2 + 4*((a^4 + 2*a^2*b^2)*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2)*cosh(d*
x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)))
 - ((a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*cos
h(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*sinh(d*x + c)^4 + a^4
+ 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(a^4 + 2*
a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 4*((a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*cosh
(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c
))) - (a^3*b + a*b^3 - 3*(a^3*b + a*b^3)*cosh(d*x + c)^2 + 4*(a^4 + a^2*b^2
)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c)^
4 + 4*(a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^5 + 2*
a^3*b^2 + a*b^4)*d*sinh(d*x + c)^4 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x
 + c)^2 + 2*(3*(a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c)^2 + (a^5 + 2*a^3*b
^2 + a*b^4)*d)*sinh(d*x + c)^2 + (a^5 + 2*a^3*b^2 + a*b^4)*d + 4*((a^5 + 2*
a^3*b^2 + a*b^4)*d*cosh(d*x + c)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x +
 c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.76585, size = 485, normalized size = 3.03

$$-\frac{b^5\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2a\right|\right)}{a^5bd+2a^3b^3d+ab^5d}-\frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)\left(a^2b+3b^3\right)}{4\left(a^4d+2a^2b^2d+b^4d\right)}-\frac{\left(a^3+2ab^2\right)\log\left(\left(e^{(dx+c)}\cdots\right.\right.}{2\left(a^4d+2a^2b^2d\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -b^5*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^5*b*d + 2*a^3*b^3*d
+ a*b^5*d) - 1/4*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a
^2*b + 3*b^3)/(a^4*d + 2*a^2*b^2*d + b^4*d) - 1/2*(a^3 + 2*a*b^2)*log((e^(d
*x + c) - e^(-d*x - c))^2 + 4)/(a^4*d + 2*a^2*b^2*d + b^4*d) + 1/2*(a^3*(e^
(d*x + c) - e^(-d*x - c))^2 + 2*a*b^2*(e^(d*x + c) - e^(-d*x - c))^2 - 2*a^
```

```
2*b*(e^(d*x + c) - e^(-d*x - c)) - 2*b^3*(e^(d*x + c) - e^(-d*x - c)) + 8*a
^3 + 12*a*b^2)/((a^4*d + 2*a^2*b^2*d + b^4*d)*((e^(d*x + c) - e^(-d*x - c))
^2 + 4)) + log(abs(e^(d*x + c) - e^(-d*x - c)))/(a*d)
```

3.448 $\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Csch[c + d*x]*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [A] time = 0.0894066, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[c + d*x]*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Csch[c + d*x]*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A] time = 174.592, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[c + d*x]*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] Integrate[(Csch[c + d*x]*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x]))
, x]
```

Maple [A] time = 3.421, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx+c)\left(\operatorname{sech}(dx+c)\right)^3}{\left(fx+e\right)\left(a+b\sinh(dx+c)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -(a*f + (b*d*f*x*e^(3*c) + (d*e - f)*b*e^(3*c))*e^(3*d*x) - (2*a*d*f*x*e^(2
*c) + (2*d*e - f)*a*e^(2*c))*e^(2*d*x) - (b*d*f*x*e^c + (d*e + f)*b*e^c)*e^
(d*x))/(a^2*d^2*e^2 + b^2*d^2*e^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*(a^
2*d^2*e*f + b^2*d^2*e*f)*x + (a^2*d^2*e^2*e^(4*c) + b^2*d^2*e^2*e^(4*c) + (
a^2*d^2*f^2*e^(4*c) + b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^2*d^2*e*f*e^(4*c) + b
^2*d^2*e*f*e^(4*c))*x)*e^(4*d*x) + 2*(a^2*d^2*e^2*e^(2*c) + b^2*d^2*e^2*e^(
2*c) + (a^2*d^2*f^2*e^(2*c) + b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^2*d^2*e*f*e^(
2*c) + b^2*d^2*e*f*e^(2*c))*x)*e^(2*d*x)) + 16*integrate(-1/8*(a*b^4*e^(d*x
 + c) - b^5)/(a^5*b*e + 2*a^3*b^3*e + a*b^5*e + (a^5*b*f + 2*a^3*b^3*f + a*
b^5*f)*x - (a^5*b*e*e^(2*c) + 2*a^3*b^3*e*e^(2*c) + a*b^5*e*e^(2*c) + (a^5*
b*f*e^(2*c) + 2*a^3*b^3*f*e^(2*c) + a*b^5*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^6*
e*e^c + 2*a^4*b^2*e*e^c + a^2*b^4*e*e^c + (a^6*f*e^c + 2*a^4*b^2*f*e^c + a^
2*b^4*f*e^c)*x)*e^(d*x)), x) - 16*integrate(-1/16*(2*(d^2*e^2 - f^2)*a^3 +
2*(2*d^2*e^2 - f^2)*a*b^2 + 2*(a^3*d^2*f^2 + 2*a*b^2*d^2*f^2)*x^2 + 4*(a^3*
d^2*e*f + 2*a*b^2*d^2*e*f)*x - ((d^2*e^2 - 2*f^2)*a^2*b*e^c + (3*d^2*e^2 -
2*f^2)*b^3*e^c + (a^2*b*d^2*f^2*e^c + 3*b^3*d^2*f^2*e^c)*x^2 + 2*(a^2*b*d^2
*e*f*e^c + 3*b^3*d^2*e*f*e^c)*x)*e^(d*x))/(a^4*d^2*e^3 + 2*a^2*b^2*d^2*e^3
+ b^4*d^2*e^3 + (a^4*d^2*f^3 + 2*a^2*b^2*d^2*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^
4*d^2*e*f^2 + 2*a^2*b^2*d^2*e*f^2 + b^4*d^2*e*f^2)*x^2 + 3*(a^4*d^2*e^2*f +
 2*a^2*b^2*d^2*e^2*f + b^4*d^2*e^2*f)*x + (a^4*d^2*e^3*e^(2*c) + 2*a^2*b^2*
d^2*e^3*e^(2*c) + b^4*d^2*e^3*e^(2*c) + (a^4*d^2*f^3*e^(2*c) + 2*a^2*b^2*d^
2*f^3*e^(2*c) + b^4*d^2*f^3*e^(2*c))*x^3 + 3*(a^4*d^2*e*f^2*e^(2*c) + 2*a^2
*b^2*d^2*e*f^2*e^(2*c) + b^4*d^2*e*f^2*e^(2*c))*x^2 + 3*(a^4*d^2*e^2*f*e^(2
*c) + 2*a^2*b^2*d^2*e^2*f*e^(2*c) + b^4*d^2*e^2*f*e^(2*c))*x)*e^(2*d*x)), x
) - 16*integrate(1/16/(a*f*x + a*e + (a*f*x*e^c + a*e*e^c)*e^(d*x)), x) + 1
6*integrate(-1/16/(a*f*x + a*e - (a*f*x*e^c + a*e*e^c)*e^(d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.449 $\int \frac{(e+fx)^3 \coth(c+dx)\mathbf{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=601

$$-\frac{6bf^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3}-\frac{6bf^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^3}+\frac{3bf(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2}$$

```
[Out] (-6*f*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d^2) - ((e + f*x)^3*Csch[c + d*x
])/(a*d) + (b*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
a^2*d) + (b*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^
2*d) - (b*(e + f*x)^3*Log[1 - E^(2*(c + d*x))])/(a^2*d) - (6*f^2*(e + f*x)*
PolyLog[2, -E^(c + d*x)])/(a*d^3) + (6*f^2*(e + f*x)*PolyLog[2, E^(c + d*x)
])/(a*d^3) + (3*b*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2]))])/(a^2*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))])/(a^2*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*
x))])/(2*a^2*d^2) + (6*f^3*PolyLog[3, -E^(c + d*x)])/(a*d^4) - (6*f^3*PolyL
og[3, E^(c + d*x)])/(a*d^4) - (6*b*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^3) + (3*b*f^2*(e + f*x)*PolyLo
g[3, E^(2*(c + d*x))])/(2*a^2*d^3) + (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/(a^2*d^4) + (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/(a^2*d^4) - (3*b*f^3*PolyLog[4, E^(2*(c + d*x))])/
(4*a^2*d^4)
```

Rubi [A] time = 1.00297, antiderivative size = 601, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {5587, 5452, 4182, 2531, 2282, 6589, 5569, 3716, 2190, 6609, 5561}

$$-\frac{6bf^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3}-\frac{6bf^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^3}+\frac{3bf(e+fx)^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (-6*f*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d^2) - ((e + f*x)^3*Csch[c + d*x
])/(a*d) + (b*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
a^2*d) + (b*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^
2*d) - (b*(e + f*x)^3*Log[1 - E^(2*(c + d*x))])/(a^2*d) - (6*f^2*(e + f*x)*
PolyLog[2, -E^(c + d*x)])/(a*d^3) + (6*f^2*(e + f*x)*PolyLog[2, E^(c + d*x)
])/(a*d^3) + (3*b*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2]))])/(a^2*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2]))])/(a^2*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*
x))])/(2*a^2*d^2) + (6*f^3*PolyLog[3, -E^(c + d*x)])/(a*d^4) - (6*f^3*PolyL
og[3, E^(c + d*x)])/(a*d^4) - (6*b*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x
))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^3) + (3*b*f^2*(e + f*x)*PolyLo
g[3, E^(2*(c + d*x))])/(2*a^2*d^3) + (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/(a^2*d^4) + (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/(a^2*d^4) - (3*b*f^3*PolyLog[4, E^(2*(c + d*x))])/
(4*a^2*d^4)
```


Rule 5587

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
```

```
ist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Csch[c + d*x]^(p - 1)*Coth[c + d*x]^n)/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\coth(c+dx)\text{csch}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\coth(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^3\text{csch}(c+dx)}{ad} - \frac{b\int (e+fx)^3\coth(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{(e+fx)^3\cosh\cdots}{a+b\sinh(\cdots}}{a^2}\cdots$$

$$= -\frac{6f(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^3\text{csch}(c+dx)}{ad} + \frac{(2b)\int \frac{e^{2(c+dx)}(e+f\cdots}{1-e^{2(c+dx}\cdots}}{a^2}\cdots$$

$$= -\frac{6f(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^3\text{csch}(c+dx)}{ad} + \frac{b(e+fx)^3\log\left(1\cdots\right.}{a^2d\cdots}$$

$$= -\frac{6f(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^3\text{csch}(c+dx)}{ad} + \frac{b(e+fx)^3\log\left(1\cdots\right.}{a^2d\cdots}$$

$$= -\frac{6f(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^3\text{csch}(c+dx)}{ad} + \frac{b(e+fx)^3\log\left(1\cdots\right.}{a^2d\cdots}$$

$$= -\frac{6f(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^3\text{csch}(c+dx)}{ad} + \frac{b(e+fx)^3\log\left(1\cdots\right.}{a^2d\cdots}$$

$$= -\frac{6f(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^3\text{csch}(c+dx)}{ad} + \frac{b(e+fx)^3\log\left(1\cdots\right.}{a^2d\cdots}$$

Mathematica [C] time = 48.7021, size = 5638, normalized size = 9.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

[Out] Result too large to show

Maple [F] time = 0.909, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \coth(dx+c)\operatorname{csch}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] e^3*(2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) + b*log(-2*a*e^(-d*x - c)
+ b*e^(-2*d*x - 2*c) - b)/(a^2*d) - b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log
(e^(-d*x - c) - 1)/(a^2*d)) - 2*(f^3*x^3*e^c + 3*e*f^2*x^2*e^c + 3*e^2*f*x*
e^c)*e^(d*x)/(a*d*e^(2*d*x + 2*c) - a*d) - 3*e^2*f*log(e^(d*x + c) + 1)/(a*
d^2) + 3*e^2*f*log(e^(d*x + c) - 1)/(a*d^2) - (d^3*x^3*log(e^(d*x + c) + 1)
 + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polyl
og(4, -e^(d*x + c)))*b*f^3/(a^2*d^4) - (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d
^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^
(d*x + c)))*b*f^3/(a^2*d^4) - 3*(b*d*e^2*f + 2*a*e*f^2)*(d*x*log(e^(d*x + c
) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 3*(b*d*e^2*f - 2*a*e*f^2)*(d*x*lo
g(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2*d^3) - 3*(b*d*e*f^2 + a*f^3)
*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -
e^(d*x + c)))/(a^2*d^4) - 3*(b*d*e*f^2 - a*f^3)*(d^2*x^2*log(-e^(d*x + c) +
 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))/(a^2*d^4) + 1/4
*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2 + a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f + 2*a*d*e*
f^2)*d^2*x^2)/(a^2*d^4) + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2 - a*f^3)*d^3*x^
3 + 6*(b*d^2*e^2*f - 2*a*d*e*f^2)*d^2*x^2)/(a^2*d^4) - integrate(-2*(b^2*f^
3*x^3 + 3*b^2*e*f^2*x^2 + 3*b^2*e^2*f*x - (a*b*f^3*x^3*e^c + 3*a*b*e*f^2*x^
2*e^c + 3*a*b*e^2*f*x*e^c)*e^(d*x))/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(d*x +
 c) - a^2*b), x)
```

Fricas [C] time = 3.62349, size = 9829, normalized size = 16.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -(2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*cosh(
d*x + c) + 3*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f - (b*d^2*f^3*x^
2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c)^2 - 2*(b*d^2*f^3*x^2 + 2*b
*d^2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*f^3*x^2 +
2*b*d^2*e*f^2*x + b*d^2*e^2*f)*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*
sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
 b)/b + 1) + 3*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f - (b*d^2*f^3*
x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c)^2 - 2*(b*d^2*f^3*x^2 + 2
*b*d^2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*f^3*x^2
+ 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) +
a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
 - b)/b + 1) - 3*(b*d^2*f^3*x^2 + b*d^2*e^2*f - 2*a*d*e*f^2 - (b*d^2*f^3*x^
2 + b*d^2*e^2*f - 2*a*d*e*f^2 + 2*(b*d^2*e*f^2 - a*d*f^3)*x)*cosh(d*x + c)^
2 - 2*(b*d^2*f^3*x^2 + b*d^2*e^2*f - 2*a*d*e*f^2 + 2*(b*d^2*e*f^2 - a*d*f^3
)*x)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*f^3*x^2 + b*d^2*e^2*f - 2*a*d*e*f
^2 + 2*(b*d^2*e*f^2 - a*d*f^3)*x)*sinh(d*x + c)^2 + 2*(b*d^2*e*f^2 - a*d*f^
3)*x)*dilog(cosh(d*x + c) + sinh(d*x + c)) - 3*(b*d^2*f^3*x^2 + b*d^2*e^2*f
 + 2*a*d*e*f^2 - (b*d^2*f^3*x^2 + b*d^2*e^2*f + 2*a*d*e*f^2 + 2*(b*d^2*e*f^
2 + a*d*f^3)*x)*cosh(d*x + c)^2 - 2*(b*d^2*f^3*x^2 + b*d^2*e^2*f + 2*a*d*e*
f^2 + 2*(b*d^2*e*f^2 + a*d*f^3)*x)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*f^3
*x^2 + b*d^2*e^2*f + 2*a*d*e*f^2 + 2*(b*d^2*e*f^2 + a*d*f^3)*x)*sinh(d*x +
c)^2 + 2*(b*d^2*e*f^2 + a*d*f^3)*x)*dilog(-cosh(d*x + c) - sinh(d*x + c)) +
 (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3 - (b*d^3*e^3 -
3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*cosh(d*x + c)^2 - 2*(b*d^3*e
^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*cosh(d*x + c)*sinh(d*x
+ c) - (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sinh(d*x
 + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b
^2) + 2*a) + (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3 - (
b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*cosh(d*x + c)^2
- 2*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*cosh(d*x +
c)*sinh(d*x + c) - (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f
^3)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((
a^2 + b^2)/b^2) + 2*a) + (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f
*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3 - (b*d^3*f^3*x^3 + 3*b*d
^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*
f^3)*cosh(d*x + c)^2 - 2*(b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f
*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*cosh(d*x + c)*sinh(d*x
+ c) - (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2
*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) +
a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
 - b)/b) + (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2
*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3 - (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 +
 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*cosh(d*x
+ c)^2 - 2*(b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2
*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) - (b*d^3*
f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d
*e*f^2 + b*c^3*f^3)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c
) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b*
d^3*f^3*x^3 + b*d^3*e^3 + 3*a*d^2*e^2*f + 3*(b*d^3*e*f^2 + a*d^2*f^3)*x^2 -
 (b*d^3*f^3*x^3 + b*d^3*e^3 + 3*a*d^2*e^2*f + 3*(b*d^3*e*f^2 + a*d^2*f^3)*x
^2 + 3*(b*d^3*e^2*f + 2*a*d^2*e*f^2)*x)*cosh(d*x + c)^2 - 2*(b*d^3*f^3*x^3
+ b*d^3*e^3 + 3*a*d^2*e^2*f + 3*(b*d^3*e*f^2 + a*d^2*f^3)*x^2 + 3*(b*d^3*e^
2*f + 2*a*d^2*e*f^2)*x)*cosh(d*x + c)*sinh(d*x + c) - (b*d^3*f^3*x^3 + b*d^
3*e^3 + 3*a*d^2*e^2*f + 3*(b*d^3*e*f^2 + a*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f +
2*a*d^2*e*f^2)*x)*sinh(d*x + c)^2 + 3*(b*d^3*e^2*f + 2*a*d^2*e*f^2)*x)*log(
cosh(d*x + c) + sinh(d*x + c) + 1) - (b*d^3*e^3 - 3*(b*c + a)*d^2*e^2*f + 3
*(b*c^2 + 2*a*c)*d*e*f^2 - (b*c^3 + 3*a*c^2)*f^3 - (b*d^3*e^3 - 3*(b*c + a)
*d^2*e^2*f + 3*(b*c^2 + 2*a*c)*d*e*f^2 - (b*c^3 + 3*a*c^2)*f^3)*cosh(d*x +
c)^2 - 2*(b*d^3*e^3 - 3*(b*c + a)*d^2*e^2*f + 3*(b*c^2 + 2*a*c)*d*e*f^2 - (
```

```
b*c^3 + 3*a*c^2)*f^3)*cosh(d*x + c)*sinh(d*x + c) - (b*d^3*e^3 - 3*(b*c + a
)*d^2*e^2*f + 3*(b*c^2 + 2*a*c)*d*e*f^2 - (b*c^3 + 3*a*c^2)*f^3)*sinh(d*x +
 c)^2)*log(cosh(d*x + c) + sinh(d*x + c) - 1) - (b*d^3*f^3*x^3 + 3*b*c*d^2*
e^2*f - 3*(b*c^2 + 2*a*c)*d*e*f^2 + (b*c^3 + 3*a*c^2)*f^3 + 3*(b*d^3*e*f^2
- a*d^2*f^3)*x^2 - (b*d^3*f^3*x^3 + 3*b*c*d^2*e^2*f - 3*(b*c^2 + 2*a*c)*d*e
*f^2 + (b*c^3 + 3*a*c^2)*f^3 + 3*(b*d^3*e*f^2 - a*d^2*f^3)*x^2 + 3*(b*d^3*e
^2*f - 2*a*d^2*e*f^2)*x)*cosh(d*x + c)^2 - 2*(b*d^3*f^3*x^3 + 3*b*c*d^2*e^2
*f - 3*(b*c^2 + 2*a*c)*d*e*f^2 + (b*c^3 + 3*a*c^2)*f^3 + 3*(b*d^3*e*f^2 - a
*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f - 2*a*d^2*e*f^2)*x)*cosh(d*x + c)*sinh(d*x +
 c) - (b*d^3*f^3*x^3 + 3*b*c*d^2*e^2*f - 3*(b*c^2 + 2*a*c)*d*e*f^2 + (b*c^3
 + 3*a*c^2)*f^3 + 3*(b*d^3*e*f^2 - a*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f - 2*a*d^
2*e*f^2)*x)*sinh(d*x + c)^2 + 3*(b*d^3*e^2*f - 2*a*d^2*e*f^2)*x)*log(-cosh(
d*x + c) - sinh(d*x + c) + 1) - 6*(b*f^3*cosh(d*x + c)^2 + 2*b*f^3*cosh(d*x
 + c)*sinh(d*x + c) + b*f^3*sinh(d*x + c)^2 - b*f^3)*polylog(4, (a*cosh(d*x
 + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2))/b) - 6*(b*f^3*cosh(d*x + c)^2 + 2*b*f^3*cosh(d*x + c)*sinh(d*x +
c) + b*f^3*sinh(d*x + c)^2 - b*f^3)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*
x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*
(b*f^3*cosh(d*x + c)^2 + 2*b*f^3*cosh(d*x + c)*sinh(d*x + c) + b*f^3*sinh(d
*x + c)^2 - b*f^3)*polylog(4, cosh(d*x + c) + sinh(d*x + c)) + 6*(b*f^3*cos
h(d*x + c)^2 + 2*b*f^3*cosh(d*x + c)*sinh(d*x + c) + b*f^3*sinh(d*x + c)^2
- b*f^3)*polylog(4, -cosh(d*x + c) - sinh(d*x + c)) - 6*(b*d*f^3*x + b*d*e*
f^2 - (b*d*f^3*x + b*d*e*f^2)*cosh(d*x + c)^2 - 2*(b*d*f^3*x + b*d*e*f^2)*c
osh(d*x + c)*sinh(d*x + c) - (b*d*f^3*x + b*d*e*f^2)*sinh(d*x + c)^2)*polyl
og(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2))/b) - 6*(b*d*f^3*x + b*d*e*f^2 - (b*d*f^3*x + b*d
*e*f^2)*cosh(d*x + c)^2 - 2*(b*d*f^3*x + b*d*e*f^2)*cosh(d*x + c)*sinh(d*x
+ c) - (b*d*f^3*x + b*d*e*f^2)*sinh(d*x + c)^2)*polylog(3, (a*cosh(d*x + c)
 + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2))/b) + 6*(b*d*f^3*x + b*d*e*f^2 - a*f^3 - (b*d*f^3*x + b*d*e*f^2 - a*f^3
)*cosh(d*x + c)^2 - 2*(b*d*f^3*x + b*d*e*f^2 - a*f^3)*cosh(d*x + c)*sinh(d*
x + c) - (b*d*f^3*x + b*d*e*f^2 - a*f^3)*sinh(d*x + c)^2)*polylog(3, cosh(d
*x + c) + sinh(d*x + c)) + 6*(b*d*f^3*x + b*d*e*f^2 + a*f^3 - (b*d*f^3*x +
b*d*e*f^2 + a*f^3)*cosh(d*x + c)^2 - 2*(b*d*f^3*x + b*d*e*f^2 + a*f^3)*cosh
(d*x + c)*sinh(d*x + c) - (b*d*f^3*x + b*d*e*f^2 + a*f^3)*sinh(d*x + c)^2)*
polylog(3, -cosh(d*x + c) - sinh(d*x + c)) + 2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f
^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*sinh(d*x + c))/(a^2*d^4*cosh(d*x + c)
^2 + 2*a^2*d^4*cosh(d*x + c)*sinh(d*x + c) + a^2*d^4*sinh(d*x + c)^2 - a^2*
d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.450 $\int \frac{(e+fx)^2 \coth(c+dx)\mathbf{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=419

$$\frac{2bf(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2}+\frac{2bf(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2}-\frac{2bf^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3}-\frac{2bf^2\text{Poly}}{}$$

```
[Out] (-4*f*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d^2) - ((e + f*x)^2*Csch[c + d*x])
/(a*d) + (b*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^
2*d) + (b*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*
d) - (b*(e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a^2*d) - (2*f^2*PolyLog[2, -
E^(c + d*x)])/(a*d^3) + (2*f^2*PolyLog[2, E^(c + d*x)])/(a*d^3) + (2*b*f*(e
 + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^2) + (
2*b*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*
d^2) - (b*f*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a^2*d^2) - (2*b*f^2*Pol
yLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^3) - (2*b*f^2*Pol
yLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^3) + (b*f^2*PolyL
og[3, E^(2*(c + d*x))])/(2*a^2*d^3)
```

Rubi [A] time = 0.817859, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5587, 5452, 4182, 2279, 2391, 5569, 3716, 2190, 2531, 2282, 6589, 5561}

$$\frac{2bf(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2}+\frac{2bf(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2}-\frac{2bf^2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3}-\frac{2bf^2\text{Poly}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (-4*f*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d^2) - ((e + f*x)^2*Csch[c + d*x])
/(a*d) + (b*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^
2*d) + (b*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*
d) - (b*(e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a^2*d) - (2*f^2*PolyLog[2, -
E^(c + d*x)])/(a*d^3) + (2*f^2*PolyLog[2, E^(c + d*x)])/(a*d^3) + (2*b*f*(e
 + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^2) + (
2*b*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*
d^2) - (b*f*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a^2*d^2) - (2*b*f^2*Pol
yLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^3) - (2*b*f^2*Pol
yLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^3) + (b*f^2*PolyL
og[3, E^(2*(c + d*x))])/(2*a^2*d^3)
```


Rule 5587

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Csch[c + d*x]^(p - 1)*Coth[c + d*x]^n)/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
```

```
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\coth(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^2\text{csch}(c+dx)}{ad} - \frac{b\int (e+fx)^2\coth(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{(e+fx)^2\cosh(c+}{a+b\sinh(c+dx}}{a^2}$$

$$= -\frac{4f(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^2\text{csch}(c+dx)}{ad} + \frac{(2b)\int \frac{e^{2(c+dx)}(e+fx)^2}{1-e^{2(c+dx)}}\,d}{a^2}$$

$$= -\frac{4f(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^2\text{csch}(c+dx)}{ad} + \frac{b(e+fx)^2\log\left(1+\right.}{a^2d}$$

$$= -\frac{4f(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^2\text{csch}(c+dx)}{ad} + \frac{b(e+fx)^2\log\left(1+\right.}{a^2d}$$

$$= -\frac{4f(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^2\text{csch}(c+dx)}{ad} + \frac{b(e+fx)^2\log\left(1+\right.}{a^2d}$$

$$= -\frac{4f(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{(e+fx)^2\text{csch}(c+dx)}{ad} + \frac{b(e+fx)^2\log\left(1+\right.}{a^2d}$$

Mathematica [B] time = 20.2357, size = 1595, normalized size = 3.81

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] -(((e + f*x)^2*Csch[c])/(a*d)) - (b*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 +
2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqr
t[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^
(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) -
 (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]]
)/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a
 + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a
```

```
*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c +
 d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(
a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c
 + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^
(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*L
og[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x
*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E
^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/
d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]
)])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b
^2)*E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c +
d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e +
 f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])
/d^2 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[
(a^2 + b^2)*E^(2*c)]))])/d^3 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c
 + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*
c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3))/(3*a^2*(-1 + E^(2*c)
)) + (b*d^3*(e + f*x)^3*(-1 + Coth[c]) + 3*d*e*f*(b*d*e - 2*a*f)*(d*x - Log
[1 - Cosh[c + d*x] - Sinh[c + d*x]]) - 6*d*f^2*(b*d*e + a*f)*x*Log[1 + Cosh
[c + d*x] - Sinh[c + d*x]] - 3*b*d^2*f^3*x^2*Log[1 + Cosh[c + d*x] - Sinh[c
 + d*x]] - 6*d*f^2*(b*d*e - a*f)*x*Log[1 - Cosh[c + d*x] + Sinh[c + d*x]] -
 3*b*d^2*f^3*x^2*Log[1 - Cosh[c + d*x] + Sinh[c + d*x]] + 3*d*e*f*(b*d*e +
2*a*f)*(d*x - Log[1 + Cosh[c + d*x] + Sinh[c + d*x]]) + 6*f^2*(b*d*e - a*f)
*PolyLog[2, Cosh[c + d*x] - Sinh[c + d*x]] + 6*f^2*(b*d*e + a*f)*PolyLog[2,
 -Cosh[c + d*x] + Sinh[c + d*x]] + 6*b*f^3*(d*x*PolyLog[2, Cosh[c + d*x] -
Sinh[c + d*x]] + PolyLog[3, Cosh[c + d*x] - Sinh[c + d*x]]) + 6*b*f^3*(d*x*
PolyLog[2, -Cosh[c + d*x] + Sinh[c + d*x]] + PolyLog[3, -Cosh[c + d*x] + Si
nh[c + d*x]]))/(3*a^2*d^3*f) + ((e + f*x)^2*Csch[c/2]*Csch[(c + d*x)/2]*Sin
h[(d*x)/2])/(2*a*d) + ((e + f*x)^2*Sech[c/2]*Sech[(c + d*x)/2]*Sinh[(d*x)/2
])/(2*a*d)
```

Maple [F] time = 0.668, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2}\coth\left(dx+c\right)\operatorname{csch}\left(dx+c\right)}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$e^{2}\left(\frac{2\,e^{(-dx-c)}}{\left(ae^{(-2\,dx-2\,c)}-a\right)d}+\frac{b\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{a^{2}d}-\frac{b\log\left(e^{(-dx-c)}+1\right)}{a^{2}d}-\frac{b\log\left(e^{(-dx-c)}-1\right)}{a^{2}d}\right)-\frac{2\left(f^{2}x^{2}e^{c}+\cdots\right.}{ade^{(2d\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] e^2*(2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) + b*log(-2*a*e^(-d*x - c)
+ b*e^(-2*d*x - 2*c) - b)/(a^2*d) - b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log
(e^(-d*x - c) - 1)/(a^2*d)) - 2*(f^2*x^2*e^c + 2*e*f*x*e^c)*e^(d*x)/(a*d*e^
(2*d*x + 2*c) - a*d) - 2*e*f*log(e^(d*x + c) + 1)/(a*d^2) + 2*e*f*log(e^(d*
x + c) - 1)/(a*d^2) - (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x +
 c)) - 2*polylog(3, -e^(d*x + c)))*b*f^2/(a^2*d^3) - (d^2*x^2*log(-e^(d*x +
 c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*b*f^2/(a^2
*d^3) - 2*(b*d*e*f + a*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c))
)/(a^2*d^3) - 2*(b*d*e*f - a*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x
 + c)))/(a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*e*f + a*f^2)*d^2*x^2)/(a^2*
d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*e*f - a*f^2)*d^2*x^2)/(a^2*d^3) - integr
ate(-2*(b^2*f^2*x^2 + 2*b^2*e*f*x - (a*b*f^2*x^2*e^c + 2*a*b*e*f*x*e^c)*e^(
d*x))/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(d*x + c) - a^2*b), x)
```

Fricas [C] time = 2.93885, size = 6053, normalized size = 14.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -(2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*cosh(d*x + c) + 2*(b*d*f^2*
x + b*d*e*f - (b*d*f^2*x + b*d*e*f)*cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*e*
f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f^2*x + b*d*e*f)*sinh(d*x + c)^2)*dil
og((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b*d*f^2*x + b*d*e*f - (b*d*f^2*x +
b*d*e*f)*cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*e*f)*cosh(d*x + c)*sinh(d*x +
 c) - (b*d*f^2*x + b*d*e*f)*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sin
h(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)
/b + 1) - 2*(b*d*f^2*x + b*d*e*f - a*f^2 - (b*d*f^2*x + b*d*e*f - a*f^2)*co
sh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*e*f - a*f^2)*cosh(d*x + c)*sinh(d*x + c)
 - (b*d*f^2*x + b*d*e*f - a*f^2)*sinh(d*x + c)^2)*dilog(cosh(d*x + c) + sin
h(d*x + c)) - 2*(b*d*f^2*x + b*d*e*f + a*f^2 - (b*d*f^2*x + b*d*e*f + a*f^2
)*cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*e*f + a*f^2)*cosh(d*x + c)*sinh(d*x
+ c) - (b*d*f^2*x + b*d*e*f + a*f^2)*sinh(d*x + c)^2)*dilog(-cosh(d*x + c)
- sinh(d*x + c)) + (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2 - (b*d^2*e^2 - 2*b*
c*d*e*f + b*c^2*f^2)*cosh(d*x + c)^2 - 2*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f
^2)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sin
h(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b
^2)/b^2) + 2*a) + (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2 - (b*d^2*e^2 - 2*b*c
*d*e*f + b*c^2*f^2)*cosh(d*x + c)^2 - 2*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^
2)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sinh
(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^
2)/b^2) + 2*a) + (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2 -
 (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*cosh(d*x + c)^2
- 2*(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*cosh(d*x + c)
*sinh(d*x + c) - (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*
sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b*d^2*f^2*x^2 + 2*b*d^
2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2 - (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*
d*e*f - b*c^2*f^2)*cosh(d*x + c)^2 - 2*(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b
*c*d*e*f - b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*f^2*x^2 + 2*b*d^
2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) +
 a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
) - b)/b) - (b*d^2*f^2*x^2 + b*d^2*e^2 + 2*a*d*e*f - (b*d^2*f^2*x^2 + b*d^2
```

```
*e^2 + 2*a*d*e*f + 2*(b*d^2*e*f + a*d*f^2)*x)*cosh(d*x + c)^2 - 2*(b*d^2*f^
2*x^2 + b*d^2*e^2 + 2*a*d*e*f + 2*(b*d^2*e*f + a*d*f^2)*x)*cosh(d*x + c)*si
nh(d*x + c) - (b*d^2*f^2*x^2 + b*d^2*e^2 + 2*a*d*e*f + 2*(b*d^2*e*f + a*d*f
^2)*x)*sinh(d*x + c)^2 + 2*(b*d^2*e*f + a*d*f^2)*x)*log(cosh(d*x + c) + sin
h(d*x + c) + 1) - (b*d^2*e^2 - 2*(b*c + a)*d*e*f + (b*c^2 + 2*a*c)*f^2 - (b
*d^2*e^2 - 2*(b*c + a)*d*e*f + (b*c^2 + 2*a*c)*f^2)*cosh(d*x + c)^2 - 2*(b*
d^2*e^2 - 2*(b*c + a)*d*e*f + (b*c^2 + 2*a*c)*f^2)*cosh(d*x + c)*sinh(d*x +
 c) - (b*d^2*e^2 - 2*(b*c + a)*d*e*f + (b*c^2 + 2*a*c)*f^2)*sinh(d*x + c)^2
)*log(cosh(d*x + c) + sinh(d*x + c) - 1) - (b*d^2*f^2*x^2 + 2*b*c*d*e*f - (
b*c^2 + 2*a*c)*f^2 - (b*d^2*f^2*x^2 + 2*b*c*d*e*f - (b*c^2 + 2*a*c)*f^2 + 2
*(b*d^2*e*f - a*d*f^2)*x)*cosh(d*x + c)^2 - 2*(b*d^2*f^2*x^2 + 2*b*c*d*e*f
- (b*c^2 + 2*a*c)*f^2 + 2*(b*d^2*e*f - a*d*f^2)*x)*cosh(d*x + c)*sinh(d*x +
 c) - (b*d^2*f^2*x^2 + 2*b*c*d*e*f - (b*c^2 + 2*a*c)*f^2 + 2*(b*d^2*e*f - a
*d*f^2)*x)*sinh(d*x + c)^2 + 2*(b*d^2*e*f - a*d*f^2)*x)*log(-cosh(d*x + c)
- sinh(d*x + c) + 1) + 2*(b*f^2*cosh(d*x + c)^2 + 2*b*f^2*cosh(d*x + c)*sin
h(d*x + c) + b*f^2*sinh(d*x + c)^2 - b*f^2)*polylog(3, (a*cosh(d*x + c) + a
*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))
/b) + 2*(b*f^2*cosh(d*x + c)^2 + 2*b*f^2*cosh(d*x + c)*sinh(d*x + c) + b*f^
2*sinh(d*x + c)^2 - b*f^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) -
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*(b*f^2*co
sh(d*x + c)^2 + 2*b*f^2*cosh(d*x + c)*sinh(d*x + c) + b*f^2*sinh(d*x + c)^2
 - b*f^2)*polylog(3, cosh(d*x + c) + sinh(d*x + c)) - 2*(b*f^2*cosh(d*x + c
)^2 + 2*b*f^2*cosh(d*x + c)*sinh(d*x + c) + b*f^2*sinh(d*x + c)^2 - b*f^2)*
polylog(3, -cosh(d*x + c) - sinh(d*x + c)) + 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f
*x + a*d^2*e^2)*sinh(d*x + c))/(a^2*d^3*cosh(d*x + c)^2 + 2*a^2*d^3*cosh(d*
x + c)*sinh(d*x + c) + a^2*d^3*sinh(d*x + c)^2 - a^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.451 $\int \frac{(e+fx)\coth(c+dx)\mathbf{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=243

$$\frac{bf\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2}+\frac{bf\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2}-\frac{bf\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^2d^2}+\frac{b(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^2d}+\frac{b(e}{}$$

```
[Out] -((f*ArcTanh[Cosh[c + d*x]])/(a*d^2)) - ((e + f*x)*Csch[c + d*x])/(a*d) + (
b*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*d) + (b*(e
 + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*d) - (b*(e + f
*x)*Log[1 - E^(2*(c + d*x))])/(a^2*d) + (b*f*PolyLog[2, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/(a^2*d^2) + (b*f*PolyLog[2, -((b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2]))])/(a^2*d^2) - (b*f*PolyLog[2, E^(2*(c + d*x))])/(2*a^2*d^
2)
```

Rubi [A] time = 0.46413, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5587, 5452, 3770, 5569, 3716, 2190, 2279, 2391, 5561}

$$\frac{bf\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2}+\frac{bf\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2}-\frac{bf\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^2d^2}+\frac{b(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^2d}+\frac{b(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((f*ArcTanh[Cosh[c + d*x]])/(a*d^2)) - ((e + f*x)*Csch[c + d*x])/(a*d) + (
b*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*d) + (b*(e
 + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*d) - (b*(e + f
*x)*Log[1 - E^(2*(c + d*x))])/(a^2*d) + (b*f*PolyLog[2, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/(a^2*d^2) + (b*f*PolyLog[2, -((b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2]))])/(a^2*d^2) - (b*f*PolyLog[2, E^(2*(c + d*x))])/(2*a^2*d^
2)
```

Rule 5587

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Csch[c + d*x]^(p - 1)*Coth[c + d*x]^n)/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\coth(c+dx)\operatorname{csch}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)\operatorname{csch}(c+dx)}{ad} - \frac{b\int (e+fx)\coth(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)}\,d}{a^2}$$

$$= -\frac{f\tanh^{-1}(\cosh(c+dx))}{ad^2} - \frac{(e+fx)\operatorname{csch}(c+dx)}{ad} + \frac{(2b)\int \frac{e^{2(c+dx)}(e+fx)}{1-e^{2(c+dx)}}\,dx}{a^2} + \cdots$$

$$= -\frac{f\tanh^{-1}(\cosh(c+dx))}{ad^2} - \frac{(e+fx)\operatorname{csch}(c+dx)}{ad} + \frac{b(e+fx)\log\left(1+\frac{be^{c+d}}{a-\sqrt{a^2+}}\right.}{a^2d}$$

$$= -\frac{f\tanh^{-1}(\cosh(c+dx))}{ad^2} - \frac{(e+fx)\operatorname{csch}(c+dx)}{ad} + \frac{b(e+fx)\log\left(1+\frac{be^{c+d}}{a-\sqrt{a^2+}}\right.}{a^2d}$$

$$= -\frac{f\tanh^{-1}(\cosh(c+dx))}{ad^2} - \frac{(e+fx)\operatorname{csch}(c+dx)}{ad} + \frac{b(e+fx)\log\left(1+\frac{be^{c+d}}{a-\sqrt{a^2+}}\right.}{a^2d}$$

Mathematica [A] time = 1.89411, size = 416, normalized size = 1.71

$$2bf\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+2bf\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+bf\text{PolyLog}\left(2,e^{-2(c+dx)}\right)+2bcf\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)+2bcf\,\text{lo}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*b*c^2*f - 4*b*c*d*f*x - 2*b*d^2*f*x^2 - a*d*e*Coth[(c + d*x)/2] - a*d*f
*x*Coth[(c + d*x)/2] - 2*b*c*f*Log[1 - E^(-2*(c + d*x))] - 2*b*d*f*x*Log[1
- E^(-2*(c + d*x))] + 2*b*c*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
] + 2*b*d*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*b*c*f*Log[
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*b*d*f*x*Log[1 + (b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2])] - 2*b*d*e*Log[Sinh[c + d*x]] + 2*b*c*f*Log[Sinh[
c + d*x]] + 2*b*d*e*Log[a + b*Sinh[c + d*x]] - 2*b*c*f*Log[a + b*Sinh[c + d
*x]] + 2*a*f*Log[Tanh[(c + d*x)/2]] + b*f*PolyLog[2, E^(-2*(c + d*x))] + 2*
b*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*b*f*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + a*d*e*Tanh[(c + d*x)/2] + a*d*f*
x*Tanh[(c + d*x)/2])/(2*a^2*d^2)
```

Maple [B] time = 0.138, size = 528, normalized size = 2.2

$$-2\,\frac{(fx+e)\,e^{dx+c}}{da\left(e^{2\,dx+2\,c}-1\right)}+\frac{bfc\ln\left(e^{dx+c}-1\right)}{a^2d^2}-\frac{bfc\ln\left(be^{2\,dx+2\,c}+2\,ae^{dx+c}-b\right)}{a^2d^2}-\frac{f\ln\left(e^{dx+c}+1\right)}{ad^2}+\frac{f\ln\left(e^{dx+c}-1\right)}{ad^2}+\frac{bf\text{dil}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] -2/d*(f*x+e)/a*exp(d*x+c)/(exp(2*d*x+2*c)-1)+1/a^2/d^2*b*f*c*ln(exp(d*x+c)-
1)-1/a^2/d^2*b*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d^2/a*f*ln(exp(d
*x+c)+1)+1/d^2/a*f*ln(exp(d*x+c)-1)+1/a^2/d^2*b*f*dilog(exp(d*x+c))+1/a^2/d
^2*b*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/a^2/
```

```
d^2*b*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/a^2/d
^2*b*f*dilog(exp(d*x+c)+1)-1/a^2/d*b*e*ln(exp(d*x+c)-1)+1/a^2/d*b*e*ln(b*ex
p(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/a^2/d*b*e*ln(exp(d*x+c)+1)+1/a^2/d*b*f*ln(
(-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/a^2/d^2*b*f*ln(
(-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/a^2/d*b*f*ln((b
*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/a^2/d^2*b*f*ln((b*e
xp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/a^2/d*b*f*ln(exp(d*x+
c)+1)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(2\,bd\int\frac{x}{2\left(a^2de^{(dx+c)}+a^2d\right)}\,dx-2\,bd\int\frac{x}{2\left(a^2de^{(dx+c)}-a^2d\right)}\,dx+a\left(\frac{dx+c}{a^2d^2}-\frac{\log\left(e^{(dx+c)}+1\right)}{a^2d^2}\right)-a\left(\frac{dx+c}{a^2d^2}-\frac{\log\left(e^{(d}\right.}{a^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] (2*b*d*integrate(1/2*x/(a^2*d*e^(d*x + c) + a^2*d), x) - 2*b*d*integrate(1/
2*x/(a^2*d*e^(d*x + c) - a^2*d), x) + a*((d*x + c)/(a^2*d^2) - log(e^(d*x +
 c) + 1)/(a^2*d^2)) - a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^2*d^
2)) - 2*x*e^(d*x + c)/(a*d*e^(2*d*x + 2*c) - a*d) - 2*integrate((a*b*x*e^(d
*x + c) - b^2*x)/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(d*x + c) - a^2*b), x))*f
 + e*(2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) + b*log(-2*a*e^(-d*x - c)
 + b*e^(-2*d*x - 2*c) - b)/(a^2*d) - b*log(e^(-d*x - c) + 1)/(a^2*d) - b*lo
g(e^(-d*x - c) - 1)/(a^2*d))
```

Fricas [B] time = 2.40876, size = 3152, normalized size = 12.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] -(2*(a*d*f*x + a*d*e)*cosh(d*x + c) - (b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*x
 + c)*sinh(d*x + c) + b*f*sinh(d*x + c)^2 - b*f)*dilog((a*cosh(d*x + c) + a
*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b + 1) - (b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c) + b*
f*sinh(d*x + c)^2 - b*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b*f*cosh(
d*x + c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c) + b*f*sinh(d*x + c)^2 - b*f)
*dilog(cosh(d*x + c) + sinh(d*x + c)) + (b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d
*x + c)*sinh(d*x + c) + b*f*sinh(d*x + c)^2 - b*f)*dilog(-cosh(d*x + c) - s
inh(d*x + c)) + (b*d*e - b*c*f - (b*d*e - b*c*f)*cosh(d*x + c)^2 - 2*(b*d*e
 - b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*e - b*c*f)*sinh(d*x + c)^2)*lo
g(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
+ (b*d*e - b*c*f - (b*d*e - b*c*f)*cosh(d*x + c)^2 - 2*(b*d*e - b*c*f)*cosh
(d*x + c)*sinh(d*x + c) - (b*d*e - b*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x
 + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b*d*f*x + b
*c*f - (b*d*f*x + b*c*f)*cosh(d*x + c)^2 - 2*(b*d*f*x + b*c*f)*cosh(d*x + c
)*sinh(d*x + c) - (b*d*f*x + b*c*f)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c)
```

```
+ a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2) - b)/b) + (b*d*f*x + b*c*f - (b*d*f*x + b*c*f)*cosh(d*x + c)^2 - 2*(b*d*
f*x + b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f*x + b*c*f)*sinh(d*x + c)^
2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b*d*f*x + b*d*e - (b*d*f*x + b*d*e +
 a*f)*cosh(d*x + c)^2 - 2*(b*d*f*x + b*d*e + a*f)*cosh(d*x + c)*sinh(d*x +
c) - (b*d*f*x + b*d*e + a*f)*sinh(d*x + c)^2 + a*f)*log(cosh(d*x + c) + sin
h(d*x + c) + 1) - (b*d*e - (b*d*e - (b*c + a)*f)*cosh(d*x + c)^2 - 2*(b*d*e
 - (b*c + a)*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*e - (b*c + a)*f)*sinh(d*
x + c)^2 - (b*c + a)*f)*log(cosh(d*x + c) + sinh(d*x + c) - 1) - (b*d*f*x +
 b*c*f - (b*d*f*x + b*c*f)*cosh(d*x + c)^2 - 2*(b*d*f*x + b*c*f)*cosh(d*x +
 c)*sinh(d*x + c) - (b*d*f*x + b*c*f)*sinh(d*x + c)^2)*log(-cosh(d*x + c) -
 sinh(d*x + c) + 1) + 2*(a*d*f*x + a*d*e)*sinh(d*x + c))/(a^2*d^2*cosh(d*x
+ c)^2 + 2*a^2*d^2*cosh(d*x + c)*sinh(d*x + c) + a^2*d^2*sinh(d*x + c)^2 -
a^2*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\coth(dx+c)\operatorname{csch}(dx+c)}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] integrate((f*x + e)*coth(d*x + c)*csch(d*x + c)/(b*sinh(d*x + c) + a), x)
```

3.452 $\int \frac{\coth(c+dx)\mathbf{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=50

$$-\frac{b\log(\sinh(c+dx))}{a^2d}+\frac{b\log(a+b\sinh(c+dx))}{a^2d}-\frac{\text{csch}(c+dx)}{ad}$$

```
[Out] -(Csch[c + d*x]/(a*d)) - (b*Log[Sinh[c + d*x]])/(a^2*d) + (b*Log[a + b*Sinh
[c + d*x]])/(a^2*d)
```

Rubi [A] time = 0.0749684, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2833, 12, 44}

$$-\frac{b\log(\sinh(c+dx))}{a^2d}+\frac{b\log(a+b\sinh(c+dx))}{a^2d}-\frac{\text{csch}(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(Csch[c + d*x]/(a*d)) - (b*Log[Sinh[c + d*x]])/(a^2*d) + (b*Log[a + b*Sinh
[c + d*x]])/(a^2*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{b^2}{x^2(a+x)}\,dx,x,b\sinh(c+dx)\right)}{bd}\\
&= \frac{b\,\text{Subst}\left(\int \frac{1}{x^2(a+x)}\,dx,x,b\sinh(c+dx)\right)}{d}\\
&= \frac{b\,\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{1}{a^2x}+\frac{1}{a^2(a+x)}\right)dx,x,b\sinh(c+dx)\right)}{d}\\
&= -\frac{\text{csch}(c+dx)}{ad}-\frac{b\log(\sinh(c+dx))}{a^2d}+\frac{b\log(a+b\sinh(c+dx))}{a^2d}
\end{aligned}$$

Mathematica [A] time = 0.0414655, size = 50, normalized size = 1.

$$-\frac{b\log(\sinh(c+dx))}{a^2d}+\frac{b\log(a+b\sinh(c+dx))}{a^2d}-\frac{\operatorname{csch}(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(Csch[c + d*x]/(a*d)) - (b*Log[Sinh[c + d*x]])/(a^2*d) + (b*Log[a + b*Sinh
[c + d*x]])/(a^2*d)
```

Maple [A] time = 0.001, size = 35, normalized size = 0.7

$$-\frac{\operatorname{csch}(dx+c)}{da}+\frac{b\ln(a\operatorname{csch}(dx+c)+b)}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] -csch(d*x+c)/a/d+1/d*b/a^2*ln(a*csch(d*x+c)+b)
```

Maxima [B] time = 1.17884, size = 149, normalized size = 2.98

$$\frac{2\,e^{(-dx-c)}}{\left(ae^{(-2\,dx-2\,c)}-a\right)d}+\frac{b\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{a^2d}-\frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d}-\frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) + b*log(-2*a*e^(-d*x - c) + b*e
^(-2*d*x - 2*c) - b)/(a^2*d) - b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-
d*x - c) - 1)/(a^2*d)
```

Fricas [B] time = 2.1464, size = 554, normalized size = 11.08

$$-\frac{2\,a\cosh(dx+c)-\left(b\cosh(dx+c)^2+2\,b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2-b\right)\log\left(\frac{2\,(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)+}{a^2d\cosh(dx+c)^2+2\,a^2d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*a*cosh(d*x + c) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c)
+ b*sinh(d*x + c)^2 - b)*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(
d*x + c))) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(
d*x + c)^2 - b)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 2*a*
sinh(d*x + c))/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c)*sinh(d*x + c)
```

```
+ a^2*d*sinh(d*x + c)^2 - a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [A] time = 1.33678, size = 132, normalized size = 2.64

$$-\frac{\frac{b\log\left(e^{(dx+c)}+1\right)}{a^2}-\frac{b\log\left(\left|be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}-b\right|\right)}{a^2}+\frac{b\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^2}+\frac{2\,e^{(dx+c)}}{a\left(e^{(dx+c)}+1\right)\left(e^{(dx+c)}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b*log(e^(d*x + c) + 1)/a^2 - b*log(abs(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c
) - b))/a^2 + b*log(abs(e^(d*x + c) - 1))/a^2 + 2*e^(d*x + c)/(a*(e^(d*x +
c) + 1)*(e^(d*x + c) - 1)))/d
```

3.453 $\int \frac{\coth(c+dx)\mathbf{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=34

$$\text{Unintegrable}\left(\frac{\coth(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Coth[c + d*x]*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi [A] time = 0.0663352, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Coth[c + d*x]*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Coth[c + d*x]*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),
 x]
```

Rubi steps

$$\int \frac{\coth(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(c+dx)\text{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Coth[c + d*x]*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x
]
```

```
[Out] $Aborted
```

Maple [A] time = 0.916, size = 0, normalized size = 0.

$$\int \frac{\text{csch}\,(dx+c)\coth\,(dx+c)}{\left(fx+e\right)(a+b\sinh\,(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\,e^{(dx+c)}}{adfx+ade-\left(adfxe^{(2\,c)}+adee^{(2\,c)}\right)e^{(2\,dx)}}-2\int-\frac{bdfx+bde+af}{2\left(a^2dfx^2+2\,a^2defx+a^2de^2-\left(a^2dfx^2e^c+2\,a^2defxe^c+a^2de^2e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="m
axima")
```

```
[Out] 2*e^(d*x + c)/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e*e^(2*c))*e^(2*d*x
)) - 2*integrate(-1/2*(b*d*f*x + b*d*e + a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*
x + a^2*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(
d*x)), x) + 2*integrate(1/2*(b*d*f*x + b*d*e - a*f)/(a^2*d*f^2*x^2 + 2*a^2*
d*e*f*x + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^
c)*e^(d*x)), x) - 2*integrate(-(a*b*e^(d*x + c) - b^2)/(a^2*b*f*x + a^2*b*e
 - (a^2*b*f*x*e^(2*c) + a^2*b*e*e^(2*c))*e^(2*d*x) - 2*(a^3*f*x*e^c + a^3*e
*e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\coth\left(dx+c\right)\operatorname{csch}\left(dx+c\right)}{afx+ae+\left(bfx+be\right)\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] integral(coth(d*x + c)*csch(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
 + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth\left(c+dx\right)\operatorname{csch}\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)*csch(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] Timed out
```

3.454 $\int \frac{(e+fx)^3 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=721

$$-\frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3}+\frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^3}+\frac{3f\sqrt{a^2+b^2}(e+fx)}{}$$

```
[Out] -((e + f*x)^3/(a*d)) + (2*b*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a^2*d) - ((e
 + f*x)^3*Coth[c + d*x])/(a*d) + (Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)^3*L
og[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*d) + (3*f*(e + f*x)^2*L
og[1 - E^(2*(c + d*x))])/(a*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2, -E^(c + d*
x)])/(a^2*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)])/(a^2*d^2) + (3
*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/(a^2*d^2) - (3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^2) + (3*f^2*(e + f*x)*PolyLog[2, E
^(2*(c + d*x))])/(a*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a^
2*d^3) + (6*b*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a^2*d^3) - (6*Sqrt[a^
2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/(a^2*d^3) + (6*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2]))])/(a^2*d^3) - (3*f^3*PolyLog[3, E^(2*(c + d*x))])/(
2*a*d^4) + (6*b*f^3*PolyLog[4, -E^(c + d*x)])/(a^2*d^4) - (6*b*f^3*PolyLog[
4, E^(c + d*x)])/(a^2*d^4) + (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^4) - (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^4)
```

Rubi [A] time = 1.63628, antiderivative size = 721, normalized size of antiderivative = 1., number of steps used = 41, number of rules used = 17, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.607, Rules used = {5569, 3720, 3716, 2190, 2531, 2282, 6589, 32, 5585, 5450, 3296, 2637, 4182, 6609, 5565, 3322, 2264}

$$-\frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3}+\frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^3}+\frac{3f\sqrt{a^2+b^2}(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((e + f*x)^3/(a*d)) + (2*b*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a^2*d) - ((e
 + f*x)^3*Coth[c + d*x])/(a*d) + (Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)^3*L
og[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*d) + (3*f*(e + f*x)^2*L
og[1 - E^(2*(c + d*x))])/(a*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2, -E^(c + d*
x)])/(a^2*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)])/(a^2*d^2) + (3
*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/(a^2*d^2) - (3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c
 + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^2) + (3*f^2*(e + f*x)*PolyLog[2, E
^(2*(c + d*x))])/(a*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a^
2*d^3) + (6*b*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a^2*d^3) - (6*Sqrt[a^
2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/(a^2*d^3) + (6*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2]))])/(a^2*d^3) - (3*f^3*PolyLog[3, E^(2*(c + d*x))])/(
2*a*d^4) + (6*b*f^3*PolyLog[4, -E^(c + d*x)])/(a^2*d^4) - (6*b*f^3*PolyLog[
4, E^(c + d*x)])/(a^2*d^4) + (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^4) - (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4
```

```
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^4)
```

Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
```

```
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\coth^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{\int (e+fx)^3\,dx}{a} - \frac{b\int (e+fx)^3\cosh(c+dx)\coth(c+dx)\,dx}{a^2} +$$

$$= -\frac{(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} - \frac{(e+fx)^3\coth(c+dx)}{ad} - \frac{\int (e+fx)^3\,dx}{a} - \frac{b\int (e+fx)^3\text{csch}(c}{a^2}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{3f(e+fx)^2\log\left(1\right.}{ad^2}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{3f(e+fx)^2\log\left(1\right.}{ad^2}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)^3}{a^2}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)^3}{a^2}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)^3}{a^2}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)^3}{a^2}$$

$$= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^3\coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)^3}{a^2}$$

Mathematica [A] time = 8.68733, size = 1350, normalized size = 1.87

$$\sqrt{a^2+b^2}\left(-2e^3\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)d^3 + f^3x^3\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^3 + 3ef^2x^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)d^3 + 3e^2fx\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (Sqrt[a^2 + b^2]*(-2*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] +
 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 3*d^3*e*f^2
*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + d^3*f^3*x^3*Log[1 + (
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2])] - 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sq
```

```
rt[a^2 + b^2])] - d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
] + 3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
- 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
- 6*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*d*f^3*x*
PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*d*e*f^2*PolyLog[3, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*d*f^3*x*PolyLog[3, -((b*E^(c +
 d*x))/(a + Sqrt[a^2 + b^2]))] + 6*f^3*PolyLog[4, (b*E^(c + d*x))/(-a + Sqr
t[a^2 + b^2])] - 6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
))/(a^2*d^4) - (a*d^3*(e + f*x)^3*(-1 + Coth[c]) - d^2*e^2*(b*d*e - 3*a*f)*
(d*x - Log[1 - Cosh[c + d*x] - Sinh[c + d*x]]) - 3*d^2*e*f*(b*d*e + 2*a*f)*
x*Log[1 + Cosh[c + d*x] - Sinh[c + d*x]] - 3*d^2*f^2*(b*d*e + a*f)*x^2*Log[
1 + Cosh[c + d*x] - Sinh[c + d*x]] - b*d^3*f^3*x^3*Log[1 + Cosh[c + d*x] -
Sinh[c + d*x]] + 3*d^2*e*f*(b*d*e - 2*a*f)*x*Log[1 - Cosh[c + d*x] + Sinh[c
 + d*x]] + 3*d^2*f^2*(b*d*e - a*f)*x^2*Log[1 - Cosh[c + d*x] + Sinh[c + d*x
]] + b*d^3*f^3*x^3*Log[1 - Cosh[c + d*x] + Sinh[c + d*x]] + d^2*e^2*(b*d*e
+ 3*a*f)*(d*x - Log[1 + Cosh[c + d*x] + Sinh[c + d*x]]) - 3*d*e*f*(b*d*e -
2*a*f)*PolyLog[2, Cosh[c + d*x] - Sinh[c + d*x]] + 3*d*e*f*(b*d*e + 2*a*f)*
PolyLog[2, -Cosh[c + d*x] + Sinh[c + d*x]] + 6*f^2*(-(b*d*e) + a*f)*(d*x*Po
lyLog[2, Cosh[c + d*x] - Sinh[c + d*x]] + PolyLog[3, Cosh[c + d*x] - Sinh[c
 + d*x]]) + 6*f^2*(b*d*e + a*f)*(d*x*PolyLog[2, -Cosh[c + d*x] + Sinh[c + d
*x]] + PolyLog[3, -Cosh[c + d*x] + Sinh[c + d*x]]) - 3*b*f^3*(d^2*x^2*PolyL
og[2, Cosh[c + d*x] - Sinh[c + d*x]] + 2*(d*x*PolyLog[3, Cosh[c + d*x] - Si
nh[c + d*x]] + PolyLog[4, Cosh[c + d*x] - Sinh[c + d*x]])) + 3*b*f^3*(d^2*x
^2*PolyLog[2, -Cosh[c + d*x] + Sinh[c + d*x]] + 2*(d*x*PolyLog[3, -Cosh[c +
 d*x] + Sinh[c + d*x]] + PolyLog[4, -Cosh[c + d*x] + Sinh[c + d*x]])))/(a^2
*d^4) + (Sech[c/2]*Sech[c/2 + (d*x)/2]*(-(e^3*Sinh[(d*x)/2]) - 3*e^2*f*x*Si
nh[(d*x)/2] - 3*e*f^2*x^2*Sinh[(d*x)/2] - f^3*x^3*Sinh[(d*x)/2]))/(2*a*d) +
 (Csch[c/2]*Csch[c/2 + (d*x)/2]*(e^3*Sinh[(d*x)/2] + 3*e^2*f*x*Sinh[(d*x)/2
] + 3*e*f^2*x^2*Sinh[(d*x)/2] + f^3*x^3*Sinh[(d*x)/2]))/(2*a*d)
```

Maple [F] time = 0.827, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 (\coth(dx+c))^2}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)

[Out] int((f*x+e)^3*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [C] time = 3.23768, size = 10496, normalized size = 14.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*a*d^3*e^3 - 6*a*c*d^2*e^2*f + 6*a*c^2*d*e*f^2 - 2*a*c^3*f^3 + 2*(a*d^3*
f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + 3*a*c*d^2*e^2*f - 3*a*c^2*d
*e*f^2 + a*c^3*f^3)*cosh(d*x + c)^2 + 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2
+ 3*a*d^3*e^2*f*x + 3*a*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 + a*c^3*f^3)*cosh(d*x
 + c)*sinh(d*x + c) + 2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*
x + 3*a*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 + a*c^3*f^3)*sinh(d*x + c)^2 + 3*(b*d
^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f - (b*d^2*f^3*x^2 + 2*b*d^2*e*f^2
*x + b*d^2*e^2*f)*cosh(d*x + c)^2 - 2*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*
d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x +
 b*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c)
 + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2) - b)/b + 1) - 3*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f - (b*d^2
*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c)^2 - 2*(b*d^2*f^3*x^
2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*f^3
*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2
)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*
b*c^2*d*e*f^2 - b*c^3*f^3 - (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2
- b*c^3*f^3)*cosh(d*x + c)^2 - 2*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e
*f^2 - b*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) - (b*d^3*e^3 - 3*b*c*d^2*e^2*
f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log
(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) +
 (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3 - (b*d^3*e^3 -
3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*cosh(d*x + c)^2 - 2*(b*d^3*e
^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*cosh(d*x + c)*sinh(d*x
+ c) - (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sinh(d*x
 + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) -
2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b
*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3 - (b*d^3*f^3*x
^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^
2 + b*c^3*f^3)*cosh(d*x + c)^2 - 2*(b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b
*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*cosh(d*x + c)
*sinh(d*x + c) - (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b
*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*sinh(d*x + c)^2)*sqrt((a^2 + b^
2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x
^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3 - (b*d
^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^
2*d*e*f^2 + b*c^3*f^3)*cosh(d*x + c)^2 - 2*(b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x
^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*cosh(
d*x + c)*sinh(d*x + c) - (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f
*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*sinh(d*x + c)^2)*sqrt((
a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 6*(b*f^3*cosh(d*x + c)^2
 + 2*b*f^3*cosh(d*x + c)*sinh(d*x + c) + b*f^3*sinh(d*x + c)^2 - b*f^3)*sqr
t((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*(b*f^3*cosh(d*x +
 c)^2 + 2*b*f^3*cosh(d*x + c)*sinh(d*x + c) + b*f^3*sinh(d*x + c)^2 - b*f^3
)*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*(b*d*f^3*x +
 b*d*e*f^2 - (b*d*f^3*x + b*d*e*f^2)*cosh(d*x + c)^2 - 2*(b*d*f^3*x + b*d*e
```

```
*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f^3*x + b*d*e*f^2)*sinh(d*x + c)^2
)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*(b*d*f^3*x +
 b*d*e*f^2 - (b*d*f^3*x + b*d*e*f^2)*cosh(d*x + c)^2 - 2*(b*d*f^3*x + b*d*e
*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f^3*x + b*d*e*f^2)*sinh(d*x + c)^2
)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 3*(b*d^2*f^3*x
^2 + b*d^2*e^2*f - 2*a*d*e*f^2 - (b*d^2*f^3*x^2 + b*d^2*e^2*f - 2*a*d*e*f^2
 + 2*(b*d^2*e*f^2 - a*d*f^3)*x)*cosh(d*x + c)^2 - 2*(b*d^2*f^3*x^2 + b*d^2*
e^2*f - 2*a*d*e*f^2 + 2*(b*d^2*e*f^2 - a*d*f^3)*x)*cosh(d*x + c)*sinh(d*x +
 c) - (b*d^2*f^3*x^2 + b*d^2*e^2*f - 2*a*d*e*f^2 + 2*(b*d^2*e*f^2 - a*d*f^3
)*x)*sinh(d*x + c)^2 + 2*(b*d^2*e*f^2 - a*d*f^3)*x)*dilog(cosh(d*x + c) + s
inh(d*x + c)) + 3*(b*d^2*f^3*x^2 + b*d^2*e^2*f + 2*a*d*e*f^2 - (b*d^2*f^3*x
^2 + b*d^2*e^2*f + 2*a*d*e*f^2 + 2*(b*d^2*e*f^2 + a*d*f^3)*x)*cosh(d*x + c)
^2 - 2*(b*d^2*f^3*x^2 + b*d^2*e^2*f + 2*a*d*e*f^2 + 2*(b*d^2*e*f^2 + a*d*f^
3)*x)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*f^3*x^2 + b*d^2*e^2*f + 2*a*d*e*
f^2 + 2*(b*d^2*e*f^2 + a*d*f^3)*x)*sinh(d*x + c)^2 + 2*(b*d^2*e*f^2 + a*d*f
^3)*x)*dilog(-cosh(d*x + c) - sinh(d*x + c)) + (b*d^3*f^3*x^3 + b*d^3*e^3 +
 3*a*d^2*e^2*f + 3*(b*d^3*e*f^2 + a*d^2*f^3)*x^2 - (b*d^3*f^3*x^3 + b*d^3*e
^3 + 3*a*d^2*e^2*f + 3*(b*d^3*e*f^2 + a*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f + 2*a
*d^2*e*f^2)*x)*cosh(d*x + c)^2 - 2*(b*d^3*f^3*x^3 + b*d^3*e^3 + 3*a*d^2*e^2
*f + 3*(b*d^3*e*f^2 + a*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f + 2*a*d^2*e*f^2)*x)*c
osh(d*x + c)*sinh(d*x + c) - (b*d^3*f^3*x^3 + b*d^3*e^3 + 3*a*d^2*e^2*f + 3
*(b*d^3*e*f^2 + a*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f + 2*a*d^2*e*f^2)*x)*sinh(d*
x + c)^2 + 3*(b*d^3*e^2*f + 2*a*d^2*e*f^2)*x)*log(cosh(d*x + c) + sinh(d*x
+ c) + 1) - (b*d^3*e^3 - 3*(b*c + a)*d^2*e^2*f + 3*(b*c^2 + 2*a*c)*d*e*f^2
- (b*c^3 + 3*a*c^2)*f^3 - (b*d^3*e^3 - 3*(b*c + a)*d^2*e^2*f + 3*(b*c^2 + 2
*a*c)*d*e*f^2 - (b*c^3 + 3*a*c^2)*f^3)*cosh(d*x + c)^2 - 2*(b*d^3*e^3 - 3*(
b*c + a)*d^2*e^2*f + 3*(b*c^2 + 2*a*c)*d*e*f^2 - (b*c^3 + 3*a*c^2)*f^3)*cos
h(d*x + c)*sinh(d*x + c) - (b*d^3*e^3 - 3*(b*c + a)*d^2*e^2*f + 3*(b*c^2 +
2*a*c)*d*e*f^2 - (b*c^3 + 3*a*c^2)*f^3)*sinh(d*x + c)^2)*log(cosh(d*x + c)
+ sinh(d*x + c) - 1) - (b*d^3*f^3*x^3 + 3*b*c*d^2*e^2*f - 3*(b*c^2 + 2*a*c)
*d*e*f^2 + (b*c^3 + 3*a*c^2)*f^3 + 3*(b*d^3*e*f^2 - a*d^2*f^3)*x^2 - (b*d^3
*f^3*x^3 + 3*b*c*d^2*e^2*f - 3*(b*c^2 + 2*a*c)*d*e*f^2 + (b*c^3 + 3*a*c^2)*
f^3 + 3*(b*d^3*e*f^2 - a*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f - 2*a*d^2*e*f^2)*x)*
cosh(d*x + c)^2 - 2*(b*d^3*f^3*x^3 + 3*b*c*d^2*e^2*f - 3*(b*c^2 + 2*a*c)*d*
e*f^2 + (b*c^3 + 3*a*c^2)*f^3 + 3*(b*d^3*e*f^2 - a*d^2*f^3)*x^2 + 3*(b*d^3*
e^2*f - 2*a*d^2*e*f^2)*x)*cosh(d*x + c)*sinh(d*x + c) - (b*d^3*f^3*x^3 + 3*
b*c*d^2*e^2*f - 3*(b*c^2 + 2*a*c)*d*e*f^2 + (b*c^3 + 3*a*c^2)*f^3 + 3*(b*d^
3*e*f^2 - a*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f - 2*a*d^2*e*f^2)*x)*sinh(d*x + c)
^2 + 3*(b*d^3*e^2*f - 2*a*d^2*e*f^2)*x)*log(-cosh(d*x + c) - sinh(d*x + c)
+ 1) + 6*(b*f^3*cosh(d*x + c)^2 + 2*b*f^3*cosh(d*x + c)*sinh(d*x + c) + b*f
^3*sinh(d*x + c)^2 - b*f^3)*polylog(4, cosh(d*x + c) + sinh(d*x + c)) - 6*(
b*f^3*cosh(d*x + c)^2 + 2*b*f^3*cosh(d*x + c)*sinh(d*x + c) + b*f^3*sinh(d*
x + c)^2 - b*f^3)*polylog(4, -cosh(d*x + c) - sinh(d*x + c)) + 6*(b*d*f^3*x
 + b*d*e*f^2 - a*f^3 - (b*d*f^3*x + b*d*e*f^2 - a*f^3)*cosh(d*x + c)^2 - 2*
(b*d*f^3*x + b*d*e*f^2 - a*f^3)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f^3*x +
b*d*e*f^2 - a*f^3)*sinh(d*x + c)^2)*polylog(3, cosh(d*x + c) + sinh(d*x + c
)) - 6*(b*d*f^3*x + b*d*e*f^2 + a*f^3 - (b*d*f^3*x + b*d*e*f^2 + a*f^3)*cos
h(d*x + c)^2 - 2*(b*d*f^3*x + b*d*e*f^2 + a*f^3)*cosh(d*x + c)*sinh(d*x + c
) - (b*d*f^3*x + b*d*e*f^2 + a*f^3)*sinh(d*x + c)^2)*polylog(3, -cosh(d*x +
 c) - sinh(d*x + c)))/(a^2*d^4*cosh(d*x + c)^2 + 2*a^2*d^4*cosh(d*x + c)*si
nh(d*x + c) + a^2*d^4*sinh(d*x + c)^2 - a^2*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.455 $\int \frac{(e+fx)^2 \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=517

$$\frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2}-\frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2}-\frac{2f^2\sqrt{a^2+b^2}\text{PolyLog}\left(\right.}{a^2d^3}$$

```
[Out] -((e + f*x)^2/(a*d)) + (2*b*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d) - ((e
 + f*x)^2*Coth[c + d*x])/(a*d) + (Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)^2*L
og[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*d) + (2*f*(e + f*x)*Log
[1 - E^(2*(c + d*x))])/(a*d^2) + (2*b*f*(e + f*x)*PolyLog[2, -E^(c + d*x)])
/(a^2*d^2) - (2*b*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^2*d^2) + (2*Sqrt[
a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/(a^2*d^2) - (2*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(a^2*d^2) + (f^2*PolyLog[2, E^(2*(c + d*x))])/(a*d^
3) - (2*b*f^2*PolyLog[3, -E^(c + d*x)])/(a^2*d^3) + (2*b*f^2*PolyLog[3, E^(
c + d*x)])/(a^2*d^3) - (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/(a^2*d^3) + (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^3)
```

Rubi [A] time = 1.29593, antiderivative size = 517, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 18, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5569, 3720, 3716, 2190, 2279, 2391, 32, 5585, 5450, 3296, 2638, 4182, 2531, 2282, 6589, 5565, 3322, 2264}

$$\frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2}-\frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2}-\frac{2f^2\sqrt{a^2+b^2}\text{PolyLog}\left(\right.}{a^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -((e + f*x)^2/(a*d)) + (2*b*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d) - ((e
 + f*x)^2*Coth[c + d*x])/(a*d) + (Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)^2*L
og[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*d) + (2*f*(e + f*x)*Log
[1 - E^(2*(c + d*x))])/(a*d^2) + (2*b*f*(e + f*x)*PolyLog[2, -E^(c + d*x)])
/(a^2*d^2) - (2*b*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^2*d^2) + (2*Sqrt[
a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/(a^2*d^2) - (2*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(a^2*d^2) + (f^2*PolyLog[2, E^(2*(c + d*x))])/(a*d^
3) - (2*b*f^2*PolyLog[3, -E^(c + d*x)])/(a^2*d^3) + (2*b*f^2*PolyLog[3, E^(
c + d*x)])/(a^2*d^3) - (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/(a^2*d^3) + (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^3)
```


Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\coth^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{\int (e+fx)^2\,dx}{a} - \frac{b\int (e+fx)^2\cosh(c+dx)\coth(c+dx)\,dx}{a^2} +$$

$$= -\frac{(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} - \frac{(e+fx)^2\coth(c+dx)}{ad} - \frac{\int (e+fx)^2\,dx}{a} - \frac{b\int (e+fx)^2\mathrm{csch}(c}{a^2}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{2f(e+fx)\log\left(1-\right.}{ad^2}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{2f(e+fx)\log\left(1-\right.}{ad^2}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)^2}{a^2}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)^2}{a^2}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)^2}{a^2}$$

$$= -\frac{(e+fx)^2}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)^2\coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)^2}{a^2}$$

Mathematica [A] time = 8.03668, size = 792, normalized size = 1.53

$$\sqrt{a^2+b^2}\left(2df(e+fx)\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)-2df(e+fx)\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)-2f^2\mathrm{PolyLog}\left(3,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+2f^2\mathrm{P}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((((2*a*d^2*(e + f*x)^2)/(-1 + E^(2*c)) + 2*d*f*(b*d*e - a*f)*x*Log[1 - E^(
-c - d*x)] + b*d^2*f^2*x^2*Log[1 - E^(-c - d*x)] - 2*d*f*(b*d*e + a*f)*x*Lo
g[1 + E^(-c - d*x)] - b*d^2*f^2*x^2*Log[1 + E^(-c - d*x)] - d*e*(b*d*e - 2*
a*f)*(d*x - Log[1 - E^(c + d*x)]) + d*e*(b*d*e + 2*a*f)*(d*x - Log[1 + E^(c
 + d*x)]) + 2*f*(b*d*e + a*f)*PolyLog[2, -E^(-c - d*x)] + 2*f*(-(b*d*e) + a
*f)*PolyLog[2, E^(-c - d*x)] + 2*b*f^2*(d*x*PolyLog[2, -E^(-c - d*x)] + Pol
yLog[3, -E^(-c - d*x)]) - 2*b*f^2*(d*x*PolyLog[2, E^(-c - d*x)] + PolyLog[3
, E^(-c - d*x)]))/(a^2*d^3)) + (Sqrt[a^2 + b^2]*(-2*d^2*e^2*ArcTanh[(a + b*
E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])
] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^2*f^2*x^
2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[
2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*f^2*PolyLog[3, (b*E^(c + d*x))/(
-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2]))]))/(a^2*d^3) + (Sech[c/2]*Sech[c/2 + (d*x)/2]*(-(e^2*Sinh[(d*x)/2])
```

```
 - 2*e*f*x*Sinh[(d*x)/2] - f^2*x^2*Sinh[(d*x)/2]))/(2*a*d) + (Csch[c/2]*Csc
h[c/2 + (d*x)/2]*(e^2*Sinh[(d*x)/2] + 2*e*f*x*Sinh[(d*x)/2] + f^2*x^2*Sinh[
(d*x)/2]))/(2*a*d)
```

Maple [F] time = 0.713, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 (\coth (dx+c))^2}{a+b\sinh (dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.81707, size = 6512, normalized size = 12.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*a*d^2*e^2 - 4*a*c*d*e*f + 2*a*c^2*f^2 + 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*
x + 2*a*c*d*e*f - a*c^2*f^2)*cosh(d*x + c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e
*f*x + 2*a*c*d*e*f - a*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + 2*(a*d^2*f^2*
x^2 + 2*a*d^2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*sinh(d*x + c)^2 + 2*(b*d*f^2
*x + b*d*e*f - (b*d*f^2*x + b*d*e*f)*cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*e
*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f^2*x + b*d*e*f)*sinh(d*x + c)^2)*sq
rt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b*d*f^2*x +
b*d*e*f - (b*d*f^2*x + b*d*e*f)*cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*e*f)*c
osh(d*x + c)*sinh(d*x + c) - (b*d*f^2*x + b*d*e*f)*sinh(d*x + c)^2)*sqrt((a
^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b*d^2*e^2 - 2*b*c*d
*e*f + b*c^2*f^2 - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cosh(d*x + c)^2 -
2*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b*d^
2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log
(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) +
 (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2 - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^
2)*cosh(d*x + c)^2 - 2*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cosh(d*x + c)*
```

```
sinh(d*x + c) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sinh(d*x + c)^2)*sqrt
((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2
 + b^2)/b^2) + 2*a) + (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*
f^2 - (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*cosh(d*x +
c)^2 - 2*(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*cosh(d*x
 + c)*sinh(d*x + c) - (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*
f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(
d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b
) - (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2 - (b*d^2*f^2*x
^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*cosh(d*x + c)^2 - 2*(b*d^2*f^
2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c
) - (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*sinh(d*x + c)
^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(b*f^2*cosh(
d*x + c)^2 + 2*b*f^2*cosh(d*x + c)*sinh(d*x + c) + b*f^2*sinh(d*x + c)^2 -
b*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c)
+ (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*(b*f^2*
cosh(d*x + c)^2 + 2*b*f^2*cosh(d*x + c)*sinh(d*x + c) + b*f^2*sinh(d*x + c)
^2 - b*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x
+ c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*(b
*d*f^2*x + b*d*e*f - a*f^2 - (b*d*f^2*x + b*d*e*f - a*f^2)*cosh(d*x + c)^2
- 2*(b*d*f^2*x + b*d*e*f - a*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f^2*x
+ b*d*e*f - a*f^2)*sinh(d*x + c)^2)*dilog(cosh(d*x + c) + sinh(d*x + c)) +
2*(b*d*f^2*x + b*d*e*f + a*f^2 - (b*d*f^2*x + b*d*e*f + a*f^2)*cosh(d*x + c
)^2 - 2*(b*d*f^2*x + b*d*e*f + a*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f^
2*x + b*d*e*f + a*f^2)*sinh(d*x + c)^2)*dilog(-cosh(d*x + c) - sinh(d*x + c
)) + (b*d^2*f^2*x^2 + b*d^2*e^2 + 2*a*d*e*f - (b*d^2*f^2*x^2 + b*d^2*e^2 +
2*a*d*e*f + 2*(b*d^2*e*f + a*d*f^2)*x)*cosh(d*x + c)^2 - 2*(b*d^2*f^2*x^2 +
 b*d^2*e^2 + 2*a*d*e*f + 2*(b*d^2*e*f + a*d*f^2)*x)*cosh(d*x + c)*sinh(d*x
+ c) - (b*d^2*f^2*x^2 + b*d^2*e^2 + 2*a*d*e*f + 2*(b*d^2*e*f + a*d*f^2)*x)*
sinh(d*x + c)^2 + 2*(b*d^2*e*f + a*d*f^2)*x)*log(cosh(d*x + c) + sinh(d*x +
 c) + 1) - (b*d^2*e^2 - 2*(b*c + a)*d*e*f + (b*c^2 + 2*a*c)*f^2 - (b*d^2*e^
2 - 2*(b*c + a)*d*e*f + (b*c^2 + 2*a*c)*f^2)*cosh(d*x + c)^2 - 2*(b*d^2*e^2
 - 2*(b*c + a)*d*e*f + (b*c^2 + 2*a*c)*f^2)*cosh(d*x + c)*sinh(d*x + c) - (
b*d^2*e^2 - 2*(b*c + a)*d*e*f + (b*c^2 + 2*a*c)*f^2)*sinh(d*x + c)^2)*log(c
osh(d*x + c) + sinh(d*x + c) - 1) - (b*d^2*f^2*x^2 + 2*b*c*d*e*f - (b*c^2 +
 2*a*c)*f^2 - (b*d^2*f^2*x^2 + 2*b*c*d*e*f - (b*c^2 + 2*a*c)*f^2 + 2*(b*d^2
*e*f - a*d*f^2)*x)*cosh(d*x + c)^2 - 2*(b*d^2*f^2*x^2 + 2*b*c*d*e*f - (b*c^
2 + 2*a*c)*f^2 + 2*(b*d^2*e*f - a*d*f^2)*x)*cosh(d*x + c)*sinh(d*x + c) - (
b*d^2*f^2*x^2 + 2*b*c*d*e*f - (b*c^2 + 2*a*c)*f^2 + 2*(b*d^2*e*f - a*d*f^2)
*x)*sinh(d*x + c)^2 + 2*(b*d^2*e*f - a*d*f^2)*x)*log(-cosh(d*x + c) - sinh(
d*x + c) + 1) - 2*(b*f^2*cosh(d*x + c)^2 + 2*b*f^2*cosh(d*x + c)*sinh(d*x +
 c) + b*f^2*sinh(d*x + c)^2 - b*f^2)*polylog(3, cosh(d*x + c) + sinh(d*x +
c)) + 2*(b*f^2*cosh(d*x + c)^2 + 2*b*f^2*cosh(d*x + c)*sinh(d*x + c) + b*f^
2*sinh(d*x + c)^2 - b*f^2)*polylog(3, -cosh(d*x + c) - sinh(d*x + c)))/(a^2
*d^3*cosh(d*x + c)^2 + 2*a^2*d^3*cosh(d*x + c)*sinh(d*x + c) + a^2*d^3*sinh
(d*x + c)^2 - a^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e + fx\right)^{2}\coth^{2}\left(c + dx\right)}{a + b\sinh\left(c + dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*coth(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.456 $\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=294

$$\frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2}-\frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2}+\frac{bf\text{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^2}-\frac{bf\text{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^2}$$

```
[Out] (2*b*(e + f*x)*ArcTanh[E^(c + d*x)])/(a^2*d) - ((e + f*x)*Coth[c + d*x])/(a
*d) + (Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])])/(a^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])])/(a^2*d) + (f*Log[Sinh[c + d*x]])/(a*d^2) + (b*f*PolyLog[2,
-E^(c + d*x)])/(a^2*d^2) - (b*f*PolyLog[2, E^(c + d*x)])/(a^2*d^2) + (Sqrt[
a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^2
) - (Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/(a^2*d^2)
```

Rubi [A] time = 0.688467, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 14, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {5569, 3720, 3475, 5585, 5450, 3296, 2637, 4182, 2279, 2391, 5565, 3322, 2264, 2190}

$$\frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2}-\frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2}+\frac{bf\text{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^2}-\frac{bf\text{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (2*b*(e + f*x)*ArcTanh[E^(c + d*x)])/(a^2*d) - ((e + f*x)*Coth[c + d*x])/(a
*d) + (Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])])/(a^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])])/(a^2*d) + (f*Log[Sinh[c + d*x]])/(a*d^2) + (b*f*PolyLog[2,
-E^(c + d*x)])/(a^2*d^2) - (b*f*PolyLog[2, E^(c + d*x)])/(a^2*d^2) + (Sqrt[
a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^2
) - (Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/(a^2*d^2)
```


Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\coth^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cosh(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)\coth(c+dx)}{ad} + \frac{\int (e+fx)\,dx}{a} - \frac{b\int (e+fx)\cosh(c+dx)\coth(c+dx)\,dx}{a^2} + \frac{b^2}{\cdots}$$

$$= \frac{ex}{a} + \frac{fx^2}{2a} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{f\log(\sinh(c+dx))}{ad^2} - \frac{\int (e+fx)\,dx}{a} - \frac{b\int (e+fx)\mathrm{cs}\cdots}{a\cdots}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{f\log(\sinh(c+dx))}{ad^2} + \frac{\left(2\left(a^2+b^2\right)\right)\cdots}{\cdots}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{f\log(\sinh(c+dx))}{ad^2} + \frac{\left(2b\sqrt{a^2+b^2}\cdots\right.}{\cdots}$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d}\cdots$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d}\cdots$$

$$= \frac{2b(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{(e+fx)\coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d}\cdots$$

Mathematica [A] time = 3.66865, size = 364, normalized size = 1.24

$$\frac{2\sqrt{a^2+b^2}\left(f\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) - f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) - 2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) + f(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right) - \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-(a*d*(e + f*x)*Coth[(c + d*x)/2]) + 2*a*f*Log[Sinh[c + d*x]] - 2*b*d*e*Lo
g[Tanh[(c + d*x)/2]] + 2*b*c*f*Log[Tanh[(c + d*x)/2]] + 2*b*f*(-((c + d*x)*
(Log[1 - E^(-c - d*x)] - Log[1 + E^(-c - d*x)])) - PolyLog[2, -E^(-c - d*x)
] + PolyLog[2, E^(-c - d*x)]) + 2*Sqrt[a^2 + b^2]*(-2*d*e*ArcTanh[(a + b*E^
(c + d*x))/Sqrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 +
b^2]] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - f*(c +
 d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*E^(c
 + d*x))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2]))]) - a*d*(e + f*x)*Tanh[(c + d*x)/2])/(2*a^2*d^2)
```

Maple [B] time = 0.213, size = 1017, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] -2/d*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/
d^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)
^(1/2)))-1/d^2*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+
(a^2+b^2)^(1/2)))-2/a^2/d*b^2*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)
+2*a)/(a^2+b^2)^(1/2))+1/a^2/d^2*b*f*c*ln(exp(d*x+c)-1)+1/a^2/d^2*b^2*f/(a^
2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-
1/a^2/d^2*b^2*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(
a^2+b^2)^(1/2)))+1/a^2/d*b*f*ln(exp(d*x+c)+1)*x-1/a^2/d*b^2*f/(a^2+b^2)^(1/
2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/a^2/d^2*b^2
*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))
*c+2/a^2/d^2*b^2*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+
b^2)^(1/2))+1/a^2/d*b^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))*x+1/a^2/d^2*b^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+
c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/d*(f*x+e)/a/(exp(2*d*x+2*c)
-1)-1/a^2/d*b*e*ln(exp(d*x+c)-1)+1/a^2/d*b*e*ln(exp(d*x+c)+1)+1/a^2/d^2*b*f
*dilog(exp(d*x+c))+1/a^2/d^2*b*f*dilog(exp(d*x+c)+1)-2/d^2/a*f*ln(exp(d*x+c
))+1/d^2/a*f*ln(exp(d*x+c)-1)+1/d^2/a*f*ln(exp(d*x+c)+1)+1/d*f/(a^2+b^2)^(1
/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d^2*f/(a
^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-
1/d*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2
)))*x-1/d^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b
^2)^(1/2)))*c+2/d^2*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a
^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.35579, size = 3430, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*a*d*e - 2*a*c*f + 2*(a*d*f*x + a*c*f)*cosh(d*x + c)^2 + 4*(a*d*f*x + a*
c*f)*cosh(d*x + c)*sinh(d*x + c) + 2*(a*d*f*x + a*c*f)*sinh(d*x + c)^2 - (b
*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c) + b*f*sinh(d*x + c)^
2 - b*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b*f
*cosh(d*x + c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c) + b*f*sinh(d*x + c)^2
- b*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b*d*e
 - b*c*f - (b*d*e - b*c*f)*cosh(d*x + c)^2 - 2*(b*d*e - b*c*f)*cosh(d*x + c
)*sinh(d*x + c) - (b*d*e - b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*lo
g(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
+ (b*d*e - b*c*f - (b*d*e - b*c*f)*cosh(d*x + c)^2 - 2*(b*d*e - b*c*f)*cosh
(d*x + c)*sinh(d*x + c) - (b*d*e - b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)
/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2)
 + 2*a) + (b*d*f*x + b*c*f - (b*d*f*x + b*c*f)*cosh(d*x + c)^2 - 2*(b*d*f*x
 + b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f*x + b*c*f)*sinh(d*x + c)^2)*
sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b*d*f*x + b*c*f -
 (b*d*f*x + b*c*f)*cosh(d*x + c)^2 - 2*(b*d*f*x + b*c*f)*cosh(d*x + c)*sinh
(d*x + c) - (b*d*f*x + b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(
a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqr
t((a^2 + b^2)/b^2) - b)/b) + (b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*x + c)*sin
h(d*x + c) + b*f*sinh(d*x + c)^2 - b*f)*dilog(cosh(d*x + c) + sinh(d*x + c)
) - (b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c) + b*f*sinh(d*x
 + c)^2 - b*f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) + (b*d*f*x + b*d*e - (
b*d*f*x + b*d*e + a*f)*cosh(d*x + c)^2 - 2*(b*d*f*x + b*d*e + a*f)*cosh(d*x
 + c)*sinh(d*x + c) - (b*d*f*x + b*d*e + a*f)*sinh(d*x + c)^2 + a*f)*log(co
sh(d*x + c) + sinh(d*x + c) + 1) - (b*d*e - (b*d*e - (b*c + a)*f)*cosh(d*x
+ c)^2 - 2*(b*d*e - (b*c + a)*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*e - (b*
c + a)*f)*sinh(d*x + c)^2 - (b*c + a)*f)*log(cosh(d*x + c) + sinh(d*x + c)
- 1) - (b*d*f*x + b*c*f - (b*d*f*x + b*c*f)*cosh(d*x + c)^2 - 2*(b*d*f*x +
b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f*x + b*c*f)*sinh(d*x + c)^2)*log
(-cosh(d*x + c) - sinh(d*x + c) + 1))/(a^2*d^2*cosh(d*x + c)^2 + 2*a^2*d^2*
cosh(d*x + c)*sinh(d*x + c) + a^2*d^2*sinh(d*x + c)^2 - a^2*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*coth(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

<u>**Giac**</u> **[F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.457 $\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=77

$$-\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d}+\frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}-\frac{\coth(c+dx)}{ad}$$

```
[Out] (b*ArcTanh[Cosh[c + d*x]])/(a^2*d) - (2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh
[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2*d) - Coth[c + d*x]/(a*d)
```

Rubi [A] time = 0.266282, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2723, 3056, 3001, 3770, 2660, 618, 204}

$$-\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d}+\frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}-\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Cosh[c + d*x]])/(a^2*d) - (2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh
[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2*d) - Coth[c + d*x]/(a*d)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\operatorname{csch}^2(c+dx)\left(1+\sinh^2(c+dx)\right)}{a+b\sinh(c+dx)}\,dx$$

$$= -\frac{\coth(c+dx)}{ad} + \frac{i\int \frac{\operatorname{csch}(c+dx)(ib-ia\sinh(c+dx))}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{\coth(c+dx)}{ad} - \frac{b\int \operatorname{csch}(c+dx)\,dx}{a^2} + \frac{\left(a^2+b^2\right)\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a^2}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{\coth(c+dx)}{ad} - \frac{\left(2i\left(a^2+b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(\right.\right.}{a^2d}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{\coth(c+dx)}{ad} + \frac{\left(4i\left(a^2+b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^2+b^2\right)-x^2}\,dx, x, -2ib\right.}{a^2d}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d} - \frac{\coth(c+dx)}{ad}$$

Mathematica [A] time = 0.441296, size = 98, normalized size = 1.27

$$-\frac{4\sqrt{-a^2-b^2}\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)+a\tanh\left(\frac{1}{2}(c+dx)\right)+a\coth\left(\frac{1}{2}(c+dx)\right)+2b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(4*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] + a
*Coth[(c + d*x)/2] + 2*b*Log[Tanh[(c + d*x)/2]] + a*Tanh[(c + d*x)/2])/(2*a
```

```
^2*d)
```

Maple [B] time = 0.003, size = 147, normalized size = 1.9

$$-\frac{1}{2da}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-\frac{b}{da^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\frac{1}{d\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\frac{2a\tanh(1/2dx}{\sqrt{a^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/2/d/a*tanh(1/2*d*x+1/2*c)-1/2/d/a/tanh(1/2*d*x+1/2*c)-1/d/a^2*b*ln(tanh(
1/2*d*x+1/2*c))+2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*
b)/(a^2+b^2)^(1/2))+2/d/a^2*b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d
*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.25519, size = 975, normalized size = 12.66

$$\frac{\sqrt{a^2+b^2}\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2-1\right)\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2ab\cosh(dx+c)}{b\cosh(dx+c)^2+b\sinh}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*
x + c)^2 - 1)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d
*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(
a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*
sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c)
 - b)) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
+ c)^2 - b)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (b*cosh(d*x + c)^2 + 2
*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*log(cosh(d*x + c) +
 sinh(d*x + c) - 1) - 2*a)/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c)*s
inh(d*x + c) + a^2*d*sinh(d*x + c)^2 - a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [B] time = 1.83974, size = 201, normalized size = 2.61

$$\frac{\frac{(a^2e^c+b^2e^c)e^{(-c)}\log\left(\frac{\left|2\,be^{(dx+2\,c)}+2\,ae^c-2\,\sqrt{a^2+b^2}e^c\right|}{\left|2\,be^{(dx+2\,c)}+2\,ae^c+2\,\sqrt{a^2+b^2}e^c\right|}\right)}{\sqrt{a^2+b^2}a^2}+\frac{b\log\left(e^{(dx+c)}+1\right)}{a^2}-\frac{b\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^2}-\frac{2}{a\left(e^{(2\,dx+2\,c)}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] ((a^2*e^c + b^2*e^c)*e^(-c)*log(abs(2*b*e^(d*x + 2*c) + 2*a*e^c - 2*sqrt(a^
2 + b^2)*e^c)/abs(2*b*e^(d*x + 2*c) + 2*a*e^c + 2*sqrt(a^2 + b^2)*e^c))/(sq
rt(a^2 + b^2)*a^2) + b*log(e^(d*x + c) + 1)/a^2 - b*log(abs(e^(d*x + c) - 1
))/a^2 - 2/(a*(e^(2*d*x + 2*c) - 1)))/d
```

3.458 $\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Coth[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0734448, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Coth[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180., size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Coth[c + d*x]^2/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.999, size = 0, normalized size = 0.

$$\int \frac{(\coth(dx+c))^2}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$2\left(a^2e^c + b^2e^c\right)\int -\frac{e^{(dx)}}{a^2bfx + a^2be - \left(a^2bfxe^{(2c)} + a^2bee^{(2c)}\right)e^{(2dx)} - 2\left(a^3fxe^c + a^3ee^c\right)e^{(dx)}}\,dx + \frac{2}{adfx + ade - \left(adfxe^{(2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*(a^2*e^c + b^2*e^c)*integrate(-e^(d*x)/(a^2*b*f*x + a^2*b*e - (a^2*b*f*x*
e^(2*c) + a^2*b*e*e^(2*c))*e^(2*d*x) - 2*(a^3*f*x*e^c + a^3*e*e^c)*e^(d*x))
, x) + 2/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e*e^(2*c))*e^(2*d*x)) -
integrate(-(b*d*f*x + b*d*e + a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e
^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) -
 integrate((b*d*f*x + b*d*e - a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e
^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\coth\left(dx+c\right)^2}{afx+ae+\left(bfx+be\right)\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(coth(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.459 $\int \frac{(e+fx)^3 \cosh(c+dx) \coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=718

$$-\frac{6f^2\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^3}-\frac{6f^2\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2bd^3}+\frac{3f\left(a^2+b^2\right)(e+fx)}{}$$

```
[Out] (b*(e + f*x)^4)/(4*a^2*f) - ((a^2 + b^2)*(e + f*x)^4)/(4*a^2*b*f) - (6*f*(e
 + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d^2) - ((e + f*x)^3*Csch[c + d*x])/(a*d)
 + ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])
/(a^2*b*d) + ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2])])/(a^2*b*d) - (b*(e + f*x)^3*Log[1 - E^(2*(c + d*x))])/(a^2*d) - (
6*f^2*(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^3) + (6*f^2*(e + f*x)*PolyLo
g[2, E^(c + d*x)])/(a*d^3) + (3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*b*d^2) + (3*(a^2 + b^2)*f*(e + f*
x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^2) - (3
*b*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*x))])/(2*a^2*d^2) + (6*f^3*PolyLog[
3, -E^(c + d*x)])/(a*d^4) - (6*f^3*PolyLog[3, E^(c + d*x)])/(a*d^4) - (6*(a
^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))
])/(a^2*b*d^3) - (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^3) + (3*b*f^2*(e + f*x)*PolyLog[3, E^(2*(
c + d*x))])/(2*a^2*d^3) + (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/(a^2*b*d^4) + (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^4) - (3*b*f^3*PolyLog[4, E^(2*
(c + d*x))])/(4*a^2*d^4)
```

Rubi [A] time = 1.64475, antiderivative size = 718, normalized size of antiderivative = 1., number of steps used = 48, number of rules used = 19, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.559, Rules used = {5585, 5450, 3296, 2638, 5452, 4182, 2531, 2282, 6589, 5446, 3311, 32, 2635, 8, 3716, 2190, 6609, 5565, 5561}

$$-\frac{6f^2\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^3}-\frac{6f^2\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2bd^3}+\frac{3f\left(a^2+b^2\right)(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*(e + f*x)^4)/(4*a^2*f) - ((a^2 + b^2)*(e + f*x)^4)/(4*a^2*b*f) - (6*f*(e
 + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d^2) - ((e + f*x)^3*Csch[c + d*x])/(a*d)
 + ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])
/(a^2*b*d) + ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2])])/(a^2*b*d) - (b*(e + f*x)^3*Log[1 - E^(2*(c + d*x))])/(a^2*d) - (
6*f^2*(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^3) + (6*f^2*(e + f*x)*PolyLo
g[2, E^(c + d*x)])/(a*d^3) + (3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*b*d^2) + (3*(a^2 + b^2)*f*(e + f*
x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^2) - (3
*b*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*x))])/(2*a^2*d^2) + (6*f^3*PolyLog[
3, -E^(c + d*x)])/(a*d^4) - (6*f^3*PolyLog[3, E^(c + d*x)])/(a*d^4) - (6*(a
^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))
])/(a^2*b*d^3) - (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^3) + (3*b*f^2*(e + f*x)*PolyLog[3, E^(2*(
c + d*x))])/(2*a^2*d^3) + (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/(a^2*b*d^4) + (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^4) - (3*b*f^3*PolyLog[4, E^(2*
```

```
(c + d*x))])/(4*a^2*d^4)
```

Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\cosh(c+dx)\coth^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cosh^2(c+dx)\coth(c+}{a+b\sinh(c+dx)}}{a}$$

$$= \frac{\int (e+fx)^3\cosh(c+dx)\,dx}{a} + \frac{\int (e+fx)^3\coth(c+dx)\mathbf{csch}(c+dx)\,dx}{a}$$

$$= -\frac{(e+fx)^3\mathbf{csch}(c+dx)}{ad} + \frac{(e+fx)^3\sinh(c+dx)}{ad} - \frac{\int (e+fx)^3\cosh(c}{a}$$

$$= \frac{b(e+fx)^4}{4a^2f} - \frac{(a^2+b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2\tanh^{-1}(e^{c+dx})}{ad^2} - \frac{3f(e+}{}$$

$$= \frac{b(e+fx)^4}{4a^2f} - \frac{(a^2+b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2\tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+f}{}$$

$$= \frac{b(e+fx)^4}{4a^2f} - \frac{(a^2+b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2\tanh^{-1}(e^{c+dx})}{ad^2} - \frac{6f^3\,c}{}$$

$$= \frac{b(e+fx)^4}{4a^2f} - \frac{(a^2+b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2\tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+f}{}$$

$$= \frac{b(e+fx)^4}{4a^2f} - \frac{(a^2+b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2\tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+f}{}$$

$$= \frac{b(e+fx)^4}{4a^2f} - \frac{(a^2+b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2\tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+f}{}$$

Mathematica [C] time = 13.512, size = 10534, normalized size = 14.67

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.986, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3} \cosh \left(dx+c\right) \left(\coth \left(dx+c\right)\right)^{2}}{a+b\sinh \left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] e^3*((d*x + c)/(b*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(
e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + (a^2 + b^2)*l
og(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^2*b*d)) - 3*e^2*f*log(e^(
d*x + c) + 1)/(a*d^2) + 3*e^2*f*log(e^(d*x + c) - 1)/(a*d^2) - 1/4*(a*d*f^3
*x^4 + 4*a*d*e*f^2*x^3 + 6*a*d*e^2*f*x^2 - (a*d*f^3*x^4*e^(2*c) + 4*a*d*e*f
^2*x^3*e^(2*c) + 6*a*d*e^2*f*x^2*e^(2*c))*e^(2*d*x) + 8*(b*f^3*x^3*e^c + 3*
b*e*f^2*x^2*e^c + 3*b*e^2*f*x*e^c)*e^(d*x))/(a*b*d*e^(2*d*x + 2*c) - a*b*d)
 - (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*po
lylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*b*f^3/(a^2*d^4) - (d^3
*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3
, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*b*f^3/(a^2*d^4) - 3*(b*d*e^2*f
+ 2*a*e*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 3
*(b*d*e^2*f - 2*a*e*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(
a^2*d^3) - 3*(b*d*e*f^2 + a*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilo
g(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^2*d^4) - 3*(b*d*e*f^2 - a*
f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(
3, e^(d*x + c)))/(a^2*d^4) + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2 + a*f^3)*d^3
*x^3 + 6*(b*d^2*e^2*f + 2*a*d*e*f^2)*d^2*x^2)/(a^2*d^4) + 1/4*(b*d^4*f^3*x^
4 + 4*(b*d*e*f^2 - a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f - 2*a*d*e*f^2)*d^2*x^2)/
(a^2*d^4) - integrate(-2*((a^2*b*f^3 + b^3*f^3)*x^3 + 3*(a^2*b*e*f^2 + b^3*
e*f^2)*x^2 + 3*(a^2*b*e^2*f + b^3*e^2*f)*x - ((a^3*f^3*e^c + a*b^2*f^3*e^c)
*x^3 + 3*(a^3*e*f^2*e^c + a*b^2*e*f^2*e^c)*x^2 + 3*(a^3*e^2*f*e^c + a*b^2*e
^2*f*e^c)*x)*e^(d*x))/(a^2*b^2*e^(2*d*x + 2*c) + 2*a^3*b*e^(d*x + c) - a^2*
b^2), x)
```

Fricas [C] time = 3.42364, size = 13009, normalized size = 18.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/4*(a^2*d^4*f^3*x^4 + 4*a^2*d^4*e*f^2*x^3 + 6*a^2*d^4*e^2*f*x^2 + 4*a^2*d^
4*e^3*x + 8*a^2*c*d^3*e^3 - 12*a^2*c^2*d^2*e^2*f + 8*a^2*c^3*d*e*f^2 - 2*a^
2*c^4*f^3 - (a^2*d^4*f^3*x^4 + 4*a^2*d^4*e*f^2*x^3 + 6*a^2*d^4*e^2*f*x^2 +
4*a^2*d^4*e^3*x + 8*a^2*c*d^3*e^3 - 12*a^2*c^2*d^2*e^2*f + 8*a^2*c^3*d*e*f^
2 - 2*a^2*c^4*f^3)*cosh(d*x + c)^2 - (a^2*d^4*f^3*x^4 + 4*a^2*d^4*e*f^2*x^3
 + 6*a^2*d^4*e^2*f*x^2 + 4*a^2*d^4*e^3*x + 8*a^2*c*d^3*e^3 - 12*a^2*c^2*d^2
*e^2*f + 8*a^2*c^3*d*e*f^2 - 2*a^2*c^4*f^3)*sinh(d*x + c)^2 - 8*(a*b*d^3*f^
3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*e^2*f*x + a*b*d^3*e^3)*cosh(d*x + c
) - 12*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d
^2*e^2*f - ((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^
2)*d^2*e^2*f)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*
d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) - ((a^2 +
b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*sinh(
d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 12*((a^2 + b^2)*d^2*f^3
*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f - ((a^2 + b^2)*d^2
*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*cosh(d*x + c)
^2 - 2*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d
^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) - ((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 +
 b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*sinh(d*x + c)^2)*dilog((a*cosh(d
*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
 b^2)/b^2) - b)/b + 1) + 12*(b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e*f^
2 - (b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e*f^2 + 2*(b^2*d^2*e*f^2 - a
*b*d*f^3)*x)*cosh(d*x + c)^2 - 2*(b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d
*e*f^2 + 2*(b^2*d^2*e*f^2 - a*b*d*f^3)*x)*cosh(d*x + c)*sinh(d*x + c) - (b^
2*d^2*f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e*f^2 + 2*(b^2*d^2*e*f^2 - a*b*d*f^
3)*x)*sinh(d*x + c)^2 + 2*(b^2*d^2*e*f^2 - a*b*d*f^3)*x)*dilog(cosh(d*x + c
) + sinh(d*x + c)) + 12*(b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f + 2*a*b*d*e*f^2 -
(b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f + 2*a*b*d*e*f^2 + 2*(b^2*d^2*e*f^2 + a*b*d
*f^3)*x)*cosh(d*x + c)^2 - 2*(b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f + 2*a*b*d*e*f
^2 + 2*(b^2*d^2*e*f^2 + a*b*d*f^3)*x)*cosh(d*x + c)*sinh(d*x + c) - (b^2*d^
2*f^3*x^2 + b^2*d^2*e^2*f + 2*a*b*d*e*f^2 + 2*(b^2*d^2*e*f^2 + a*b*d*f^3)*x
)*sinh(d*x + c)^2 + 2*(b^2*d^2*e*f^2 + a*b*d*f^3)*x)*dilog(-cosh(d*x + c) -
 sinh(d*x + c)) - 4*((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a
^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3 - ((a^2 + b^2)*d^3*e^3 - 3*(a^2
 + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*cosh
(d*x + c)^2 - 2*((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 +
 b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) - ((a^
2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 -
(a^2 + b^2)*c^3*f^3)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x
+ c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 4*((a^2 + b^2)*d^3*e^3 - 3*(a^2 +
 b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3 - ((a^2
 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (
a^2 + b^2)*c^3*f^3)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2
)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*cosh(d*x +
 c)*sinh(d*x + c) - ((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a
^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*sinh(d*x + c)^2)*log(2*b*cosh(
d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 4*((a^2 +
 b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x
 + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*
```

```
f^3 - ((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2
)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^
2 + b^2)*c^3*f^3)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b
^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f -
 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*f^3)*cosh(d*x + c)*sinh(d*x +
c) - ((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)
*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2
 + b^2)*c^3*f^3)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) +
 (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 4*((a^
2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*
f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c
^3*f^3 - ((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 +
b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 +
(a^2 + b^2)*c^3*f^3)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2
+ b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*
f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*f^3)*cosh(d*x + c)*sinh(d*x
 + c) - ((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b
^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (
a^2 + b^2)*c^3*f^3)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c
) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*(
b^2*d^3*f^3*x^3 + b^2*d^3*e^3 + 3*a*b*d^2*e^2*f + 3*(b^2*d^3*e*f^2 + a*b*d^
2*f^3)*x^2 - (b^2*d^3*f^3*x^3 + b^2*d^3*e^3 + 3*a*b*d^2*e^2*f + 3*(b^2*d^3*
e*f^2 + a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f + 2*a*b*d^2*e*f^2)*x)*cosh(d*x
+ c)^2 - 2*(b^2*d^3*f^3*x^3 + b^2*d^3*e^3 + 3*a*b*d^2*e^2*f + 3*(b^2*d^3*e*
f^2 + a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f + 2*a*b*d^2*e*f^2)*x)*cosh(d*x +
c)*sinh(d*x + c) - (b^2*d^3*f^3*x^3 + b^2*d^3*e^3 + 3*a*b*d^2*e^2*f + 3*(b^
2*d^3*e*f^2 + a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f + 2*a*b*d^2*e*f^2)*x)*sin
h(d*x + c)^2 + 3*(b^2*d^3*e^2*f + 2*a*b*d^2*e*f^2)*x)*log(cosh(d*x + c) + s
inh(d*x + c) + 1) + 4*(b^2*d^3*e^3 - 3*(b^2*c + a*b)*d^2*e^2*f + 3*(b^2*c^2
 + 2*a*b*c)*d*e*f^2 - (b^2*c^3 + 3*a*b*c^2)*f^3 - (b^2*d^3*e^3 - 3*(b^2*c +
 a*b)*d^2*e^2*f + 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 - (b^2*c^3 + 3*a*b*c^2)*f^3
)*cosh(d*x + c)^2 - 2*(b^2*d^3*e^3 - 3*(b^2*c + a*b)*d^2*e^2*f + 3*(b^2*c^2
 + 2*a*b*c)*d*e*f^2 - (b^2*c^3 + 3*a*b*c^2)*f^3)*cosh(d*x + c)*sinh(d*x + c
) - (b^2*d^3*e^3 - 3*(b^2*c + a*b)*d^2*e^2*f + 3*(b^2*c^2 + 2*a*b*c)*d*e*f^
2 - (b^2*c^3 + 3*a*b*c^2)*f^3)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*
x + c) - 1) + 4*(b^2*d^3*f^3*x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c
)*d*e*f^2 + (b^2*c^3 + 3*a*b*c^2)*f^3 + 3*(b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2
 - (b^2*d^3*f^3*x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 + (
b^2*c^3 + 3*a*b*c^2)*f^3 + 3*(b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(b^2*d^3
*e^2*f - 2*a*b*d^2*e*f^2)*x)*cosh(d*x + c)^2 - 2*(b^2*d^3*f^3*x^3 + 3*b^2*c
*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 + (b^2*c^3 + 3*a*b*c^2)*f^3 + 3*
(b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f - 2*a*b*d^2*e*f^2)*x)*
cosh(d*x + c)*sinh(d*x + c) - (b^2*d^3*f^3*x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2
*c^2 + 2*a*b*c)*d*e*f^2 + (b^2*c^3 + 3*a*b*c^2)*f^3 + 3*(b^2*d^3*e*f^2 - a*
b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f - 2*a*b*d^2*e*f^2)*x)*sinh(d*x + c)^2 + 3
*(b^2*d^3*e^2*f - 2*a*b*d^2*e*f^2)*x)*log(-cosh(d*x + c) - sinh(d*x + c) +
1) + 24*((a^2 + b^2)*f^3*cosh(d*x + c)^2 + 2*(a^2 + b^2)*f^3*cosh(d*x + c)*
sinh(d*x + c) + (a^2 + b^2)*f^3*sinh(d*x + c)^2 - (a^2 + b^2)*f^3)*polylog(
4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2))/b) + 24*((a^2 + b^2)*f^3*cosh(d*x + c)^2 + 2*(a^2 +
 b^2)*f^3*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*f^3*sinh(d*x + c)^2 - (
a^2 + b^2)*f^3)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 24*(b^2*f^3*cosh(d*x +
 c)^2 + 2*b^2*f^3*cosh(d*x + c)*sinh(d*x + c) + b^2*f^3*sinh(d*x + c)^2 - b
^2*f^3)*polylog(4, cosh(d*x + c) + sinh(d*x + c)) - 24*(b^2*f^3*cosh(d*x +
c)^2 + 2*b^2*f^3*cosh(d*x + c)*sinh(d*x + c) + b^2*f^3*sinh(d*x + c)^2 - b^
2*f^3)*polylog(4, -cosh(d*x + c) - sinh(d*x + c)) + 24*((a^2 + b^2)*d*f^3*x
 + (a^2 + b^2)*d*e*f^2 - ((a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*cosh(d
*x + c)^2 - 2*((a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*cosh(d*x + c)*sin
```

```
h(d*x + c) - ((a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*sinh(d*x + c)^2)*p
olylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*((a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d
*e*f^2 - ((a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*cosh(d*x + c)^2 - 2*((
a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) - ((a
^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*sinh(d*x + c)^2)*polylog(3, (a*cos
h(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2))/b) - 24*(b^2*d*f^3*x + b^2*d*e*f^2 - a*b*f^3 - (b^2*d*f^3*x
+ b^2*d*e*f^2 - a*b*f^3)*cosh(d*x + c)^2 - 2*(b^2*d*f^3*x + b^2*d*e*f^2 - a
*b*f^3)*cosh(d*x + c)*sinh(d*x + c) - (b^2*d*f^3*x + b^2*d*e*f^2 - a*b*f^3)
*sinh(d*x + c)^2)*polylog(3, cosh(d*x + c) + sinh(d*x + c)) - 24*(b^2*d*f^3
*x + b^2*d*e*f^2 + a*b*f^3 - (b^2*d*f^3*x + b^2*d*e*f^2 + a*b*f^3)*cosh(d*x
 + c)^2 - 2*(b^2*d*f^3*x + b^2*d*e*f^2 + a*b*f^3)*cosh(d*x + c)*sinh(d*x +
c) - (b^2*d*f^3*x + b^2*d*e*f^2 + a*b*f^3)*sinh(d*x + c)^2)*polylog(3, -cos
h(d*x + c) - sinh(d*x + c)) - 2*(4*a*b*d^3*f^3*x^3 + 12*a*b*d^3*e*f^2*x^2 +
 12*a*b*d^3*e^2*f*x + 4*a*b*d^3*e^3 + (a^2*d^4*f^3*x^4 + 4*a^2*d^4*e*f^2*x^
3 + 6*a^2*d^4*e^2*f*x^2 + 4*a^2*d^4*e^3*x + 8*a^2*c*d^3*e^3 - 12*a^2*c^2*d^
2*e^2*f + 8*a^2*c^3*d*e*f^2 - 2*a^2*c^4*f^3)*cosh(d*x + c))*sinh(d*x + c))/
(a^2*b*d^4*cosh(d*x + c)^2 + 2*a^2*b*d^4*cosh(d*x + c)*sinh(d*x + c) + a^2*
b*d^4*sinh(d*x + c)^2 - a^2*b*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.460 $\int \frac{(e+fx)^2 \cosh(c+dx) \coth^2(c+dx)}{a+b\sinh(c+dx)} dx$

Optimal. Leaf size=518

$$\frac{2f\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^2}+\frac{2f\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2bd^2}-\frac{2f^2\left(a^2+b^2\right)\text{PolyLog}\left(\right.}{a^2bd^3}$$

```
[Out] (b*(e + f*x)^3)/(3*a^2*f) - ((a^2 + b^2)*(e + f*x)^3)/(3*a^2*b*f) - (4*f*(e
 + f*x)*ArcTanh[E^(c + d*x)])/(a*d^2) - ((e + f*x)^2*Csch[c + d*x])/(a*d) +
 ((a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
a^2*b*d) + ((a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(a^2*b*d) - (b*(e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a^2*d) - (2*
f^2*PolyLog[2, -E^(c + d*x)])/(a*d^3) + (2*f^2*PolyLog[2, E^(c + d*x)])/(a*
d^3) + (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/(a^2*b*d^2) + (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^2) - (b*f*(e + f*x)*PolyLog[2, E^
(2*(c + d*x))])/(a^2*d^2) - (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/(a^2*b*d^3) - (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^3) + (b*f^2*PolyLog[3, E^(2*
(c + d*x))])/(2*a^2*d^3)
```

Rubi [A] time = 1.28673, antiderivative size = 518, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 17, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5585, 5450, 3296, 2637, 5452, 4182, 2279, 2391, 5446, 3310, 3716, 2190, 2531, 2282, 6589, 5565, 5561}

$$\frac{2f\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^2}+\frac{2f\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2bd^2}-\frac{2f^2\left(a^2+b^2\right)\text{PolyLog}\left(\right.}{a^2bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (b*(e + f*x)^3)/(3*a^2*f) - ((a^2 + b^2)*(e + f*x)^3)/(3*a^2*b*f) - (4*f*(e
 + f*x)*ArcTanh[E^(c + d*x)])/(a*d^2) - ((e + f*x)^2*Csch[c + d*x])/(a*d) +
 ((a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
a^2*b*d) + ((a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(a^2*b*d) - (b*(e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a^2*d) - (2*
f^2*PolyLog[2, -E^(c + d*x)])/(a*d^3) + (2*f^2*PolyLog[2, E^(c + d*x)])/(a*
d^3) + (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/(a^2*b*d^2) + (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^2) - (b*f*(e + f*x)*PolyLog[2, E^
(2*(c + d*x))])/(a^2*d^2) - (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/(a^2*b*d^3) - (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^3) + (b*f^2*PolyLog[3, E^(2*
(c + d*x))])/(2*a^2*d^3)
```


Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
```

```
0] && IGtQ[p, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
```

]

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\cosh(c+dx)\coth^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cosh^2(c+dx)\coth(c+}{a+b\sinh(c+dx)}}{a}$$

$$= \frac{\int (e+fx)^2\cosh(c+dx)\,dx}{a} + \frac{\int (e+fx)^2\coth(c+dx)\mathbf{csch}(c+dx)\,dx}{a}$$

$$= -\frac{(e+fx)^2\mathbf{csch}(c+dx)}{ad} + \frac{(e+fx)^2\sinh(c+dx)}{ad} - \frac{\int (e+fx)^2\cosh(c}{a}$$

$$= \frac{b(e+fx)^3}{3a^2f} - \frac{(a^2+b^2)(e+fx)^3}{3a^2bf} - \frac{4f(e+fx)\tanh^{-1}(e^{c+dx})}{ad^2} - \frac{2f(e+}{}$$

$$= \frac{b(e+fx)^3}{3a^2f} - \frac{(a^2+b^2)(e+fx)^3}{3a^2bf} - \frac{4f(e+fx)\tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+fx}{}$$

$$= \frac{b(e+fx)^3}{3a^2f} - \frac{(a^2+b^2)(e+fx)^3}{3a^2bf} - \frac{4f(e+fx)\tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+fx}{}$$

$$= \frac{b(e+fx)^3}{3a^2f} - \frac{(a^2+b^2)(e+fx)^3}{3a^2bf} - \frac{4f(e+fx)\tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+fx}{}$$

$$= \frac{b(e+fx)^3}{3a^2f} - \frac{(a^2+b^2)(e+fx)^3}{3a^2bf} - \frac{4f(e+fx)\tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+fx}{}$$

Mathematica [B] time = 14.8317, size = 1720, normalized size = 3.32

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] -((a^2 + b^2)*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6
*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[
-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^
(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2
)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*
d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt
[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 +
 E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(
c + d*x)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^
2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt
[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c -
Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x
))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d
*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*
E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 +
 (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2
*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6
*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a
^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*
E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*f^2*PolyLog[
3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2
*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))
```

```
])/d^3 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^
(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqr
t[(a^2 + b^2)*E^(2*c)]))])/d^3))/(3*a^2*b*(-1 + E^(2*c))) + (b*d^3*(e + f*x
)^3*(-1 + Coth[c]) + 3*d*e*f*(b*d*e - 2*a*f)*(d*x - Log[1 - Cosh[c + d*x] -
 Sinh[c + d*x]]) - 6*d*f^2*(b*d*e + a*f)*x*Log[1 + Cosh[c + d*x] - Sinh[c +
 d*x]] - 3*b*d^2*f^3*x^2*Log[1 + Cosh[c + d*x] - Sinh[c + d*x]] - 6*d*f^2*(
b*d*e - a*f)*x*Log[1 - Cosh[c + d*x] + Sinh[c + d*x]] - 3*b*d^2*f^3*x^2*Log
[1 - Cosh[c + d*x] + Sinh[c + d*x]] + 3*d*e*f*(b*d*e + 2*a*f)*(d*x - Log[1
+ Cosh[c + d*x] + Sinh[c + d*x]]) + 6*f^2*(b*d*e - a*f)*PolyLog[2, Cosh[c +
 d*x] - Sinh[c + d*x]] + 6*f^2*(b*d*e + a*f)*PolyLog[2, -Cosh[c + d*x] + Si
nh[c + d*x]] + 6*b*f^3*(d*x*PolyLog[2, Cosh[c + d*x] - Sinh[c + d*x]] + Pol
yLog[3, Cosh[c + d*x] - Sinh[c + d*x]]) + 6*b*f^3*(d*x*PolyLog[2, -Cosh[c +
 d*x] + Sinh[c + d*x]] + PolyLog[3, -Cosh[c + d*x] + Sinh[c + d*x]]))/(3*a^
2*d^3*f) + ((-3*b*e^2 - 6*b*e*f*x - 3*b*f^2*x^2 + 3*a*d*e^2*x*Cosh[c] + 3*a
*d*e*f*x^2*Cosh[c] + a*d*f^2*x^3*Cosh[c])*Csch[c/2]*Sech[c/2])/(6*a*b*d) +
(Csch[c/2]*Csch[c/2 + (d*x)/2]*(e^2*Sinh[(d*x)/2] + 2*e*f*x*Sinh[(d*x)/2] +
 f^2*x^2*Sinh[(d*x)/2]))/(2*a*d) + (Sech[c/2]*Sech[c/2 + (d*x)/2]*(e^2*Sinh
[(d*x)/2] + 2*e*f*x*Sinh[(d*x)/2] + f^2*x^2*Sinh[(d*x)/2]))/(2*a*d)
```

Maple [F] time = 0.81, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \cosh\left(dx+c\right)\left(\coth\left(dx+c\right)\right)^{2}}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] e^2*((d*x + c)/(b*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(
e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + (a^2 + b^2)*l
og(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^2*b*d)) - 1/3*(a*d*f^2*x^
3 + 3*a*d*e*f*x^2 - (a*d*f^2*x^3*e^(2*c) + 3*a*d*e*f*x^2*e^(2*c))*e^(2*d*x)
 + 6*(b*f^2*x^2*e^c + 2*b*e*f*x*e^c)*e^(d*x))/(a*b*d*e^(2*d*x + 2*c) - a*b*
d) - 2*e*f*log(e^(d*x + c) + 1)/(a*d^2) + 2*e*f*log(e^(d*x + c) - 1)/(a*d^2
) - (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3
, -e^(d*x + c)))*b*f^2/(a^2*d^3) - (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*d
ilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*b*f^2/(a^2*d^3) - 2*(b*d*e*f
 + a*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 2*(b
*d*e*f - a*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2*d^3)
+ 1/3*(b*d^3*f^2*x^3 + 3*(b*d*e*f + a*f^2)*d^2*x^2)/(a^2*d^3) + 1/3*(b*d^3*
f^2*x^3 + 3*(b*d*e*f - a*f^2)*d^2*x^2)/(a^2*d^3) - integrate(-2*((a^2*b*f^2
 + b^3*f^2)*x^2 + 2*(a^2*b*e*f + b^3*e*f)*x - ((a^3*f^2*e^c + a*b^2*f^2*e^c
)*x^2 + 2*(a^3*e*f*e^c + a*b^2*e*f*e^c)*x)*e^(d*x))/(a^2*b^2*e^(2*d*x + 2*c
```

```
) + 2*a^3*b*e^(d*x + c) - a^2*b^2), x)
```

Fricas [C] time = 2.94505, size = 8101, normalized size = 15.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/3*(a^2*d^3*f^2*x^3 + 3*a^2*d^3*e*f*x^2 + 3*a^2*d^3*e^2*x + 6*a^2*c*d^2*e^
2 - 6*a^2*c^2*d*e*f + 2*a^2*c^3*f^2 - (a^2*d^3*f^2*x^3 + 3*a^2*d^3*e*f*x^2
+ 3*a^2*d^3*e^2*x + 6*a^2*c*d^2*e^2 - 6*a^2*c^2*d*e*f + 2*a^2*c^3*f^2)*cosh
(d*x + c)^2 - (a^2*d^3*f^2*x^3 + 3*a^2*d^3*e*f*x^2 + 3*a^2*d^3*e^2*x + 6*a^
2*c*d^2*e^2 - 6*a^2*c^2*d*e*f + 2*a^2*c^3*f^2)*sinh(d*x + c)^2 - 6*(a*b*d^2
*f^2*x^2 + 2*a*b*d^2*e*f*x + a*b*d^2*e^2)*cosh(d*x + c) - 6*((a^2 + b^2)*d*
f^2*x + (a^2 + b^2)*d*e*f - ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(
d*x + c)^2 - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c)*sinh
(d*x + c) - ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*sinh(d*x + c)^2)*dilo
g((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 6*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*
e*f - ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c)^2 - 2*((a^2 +
 b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c)*sinh(d*x + c) - ((a^2 + b^
2)*d*f^2*x + (a^2 + b^2)*d*e*f)*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a
*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b + 1) + 6*(b^2*d*f^2*x + b^2*d*e*f - a*b*f^2 - (b^2*d*f^2*x + b^2*d*e
*f - a*b*f^2)*cosh(d*x + c)^2 - 2*(b^2*d*f^2*x + b^2*d*e*f - a*b*f^2)*cosh(
d*x + c)*sinh(d*x + c) - (b^2*d*f^2*x + b^2*d*e*f - a*b*f^2)*sinh(d*x + c)^
2)*dilog(cosh(d*x + c) + sinh(d*x + c)) + 6*(b^2*d*f^2*x + b^2*d*e*f + a*b*
f^2 - (b^2*d*f^2*x + b^2*d*e*f + a*b*f^2)*cosh(d*x + c)^2 - 2*(b^2*d*f^2*x
+ b^2*d*e*f + a*b*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b^2*d*f^2*x + b^2*d*e
*f + a*b*f^2)*sinh(d*x + c)^2)*dilog(-cosh(d*x + c) - sinh(d*x + c)) - 3*((
a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2 - ((a^2 +
b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)^2
 - 2*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*co
sh(d*x + c)*sinh(d*x + c) - ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f +
(a^2 + b^2)*c^2*f^2)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x
+ c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*((a^2 + b^2)*d^2*e^2 - 2*(a^2 +
 b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2 - ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*
c*d*e*f + (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^2*e^2 - 2
*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) - (
(a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*sinh(d*x
 + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b
^2) + 2*a) - 3*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2
+ b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2 - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 +
b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)
^2 - 2*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c
*d*e*f - (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) - ((a^2 + b^2)*d^
2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c
^2*f^2)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 3*((a^2 + b^2)*
d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)
*c^2*f^2 - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^
2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^2*f^2*
x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2
)*cosh(d*x + c)*sinh(d*x + c) - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^
```

```
2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*sinh(d*x + c)^2)*log
(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2) - b)/b) + 3*(b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + 2*a*b*d*
e*f - (b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + 2*a*b*d*e*f + 2*(b^2*d^2*e*f + a*b*d
*f^2)*x)*cosh(d*x + c)^2 - 2*(b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + 2*a*b*d*e*f +
 2*(b^2*d^2*e*f + a*b*d*f^2)*x)*cosh(d*x + c)*sinh(d*x + c) - (b^2*d^2*f^2*
x^2 + b^2*d^2*e^2 + 2*a*b*d*e*f + 2*(b^2*d^2*e*f + a*b*d*f^2)*x)*sinh(d*x +
 c)^2 + 2*(b^2*d^2*e*f + a*b*d*f^2)*x)*log(cosh(d*x + c) + sinh(d*x + c) +
1) + 3*(b^2*d^2*e^2 - 2*(b^2*c + a*b)*d*e*f + (b^2*c^2 + 2*a*b*c)*f^2 - (b^
2*d^2*e^2 - 2*(b^2*c + a*b)*d*e*f + (b^2*c^2 + 2*a*b*c)*f^2)*cosh(d*x + c)^
2 - 2*(b^2*d^2*e^2 - 2*(b^2*c + a*b)*d*e*f + (b^2*c^2 + 2*a*b*c)*f^2)*cosh(
d*x + c)*sinh(d*x + c) - (b^2*d^2*e^2 - 2*(b^2*c + a*b)*d*e*f + (b^2*c^2 +
2*a*b*c)*f^2)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 3*(
b^2*d^2*f^2*x^2 + 2*b^2*c*d*e*f - (b^2*c^2 + 2*a*b*c)*f^2 - (b^2*d^2*f^2*x^
2 + 2*b^2*c*d*e*f - (b^2*c^2 + 2*a*b*c)*f^2 + 2*(b^2*d^2*e*f - a*b*d*f^2)*x
)*cosh(d*x + c)^2 - 2*(b^2*d^2*f^2*x^2 + 2*b^2*c*d*e*f - (b^2*c^2 + 2*a*b*c
)*f^2 + 2*(b^2*d^2*e*f - a*b*d*f^2)*x)*cosh(d*x + c)*sinh(d*x + c) - (b^2*d
^2*f^2*x^2 + 2*b^2*c*d*e*f - (b^2*c^2 + 2*a*b*c)*f^2 + 2*(b^2*d^2*e*f - a*b
*d*f^2)*x)*sinh(d*x + c)^2 + 2*(b^2*d^2*e*f - a*b*d*f^2)*x)*log(-cosh(d*x +
 c) - sinh(d*x + c) + 1) - 6*((a^2 + b^2)*f^2*cosh(d*x + c)^2 + 2*(a^2 + b^
2)*f^2*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*f^2*sinh(d*x + c)^2 - (a^2
 + b^2)*f^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*((a^2 + b^2)*f^2*cosh(d
*x + c)^2 + 2*(a^2 + b^2)*f^2*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*f^2
*sinh(d*x + c)^2 - (a^2 + b^2)*f^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*
x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*
(b^2*f^2*cosh(d*x + c)^2 + 2*b^2*f^2*cosh(d*x + c)*sinh(d*x + c) + b^2*f^2*
sinh(d*x + c)^2 - b^2*f^2)*polylog(3, cosh(d*x + c) + sinh(d*x + c)) + 6*(b
^2*f^2*cosh(d*x + c)^2 + 2*b^2*f^2*cosh(d*x + c)*sinh(d*x + c) + b^2*f^2*si
nh(d*x + c)^2 - b^2*f^2)*polylog(3, -cosh(d*x + c) - sinh(d*x + c)) - 2*(3*
a*b*d^2*f^2*x^2 + 6*a*b*d^2*e*f*x + 3*a*b*d^2*e^2 + (a^2*d^3*f^2*x^3 + 3*a^
2*d^3*e*f*x^2 + 3*a^2*d^3*e^2*x + 6*a^2*c*d^2*e^2 - 6*a^2*c^2*d*e*f + 2*a^2
*c^3*f^2)*cosh(d*x + c))*sinh(d*x + c))/(a^2*b*d^3*cosh(d*x + c)^2 + 2*a^2*
b*d^3*cosh(d*x + c)*sinh(d*x + c) + a^2*b*d^3*sinh(d*x + c)^2 - a^2*b*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")

[Out] Timed out
```

3.461 $\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=324

$$\frac{f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^2}+\frac{f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2bd^2}-\frac{bf\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^2d^2}+\frac{\left(a^2+b^2\right)(e+$$

```
[Out] (b*(e + f*x)^2)/(2*a^2*f) - ((a^2 + b^2)*(e + f*x)^2)/(2*a^2*b*f) - (f*ArcT
anh[Cosh[c + d*x]])/(a*d^2) - ((e + f*x)*Csch[c + d*x])/(a*d) + ((a^2 + b^2
)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*b*d) + ((a
^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*b*
d) - (b*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a^2*d) + ((a^2 + b^2)*f*PolyLo
g[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*b*d^2) + ((a^2 + b^2)*
f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^2) - (b*f*
PolyLog[2, E^(2*(c + d*x))])/(2*a^2*d^2)
```

Rubi [A] time = 0.742632, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 15, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.469, Rules used = {5585, 5450, 3296, 2638, 5452, 3770, 5446, 2635, 8, 3716, 2190, 2279, 2391, 5565, 5561}

$$\frac{f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^2}+\frac{f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2bd^2}-\frac{bf\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^2d^2}+\frac{\left(a^2+b^2\right)(e+$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (b*(e + f*x)^2)/(2*a^2*f) - ((a^2 + b^2)*(e + f*x)^2)/(2*a^2*b*f) - (f*ArcT
anh[Cosh[c + d*x]])/(a*d^2) - ((e + f*x)*Csch[c + d*x])/(a*d) + ((a^2 + b^2
)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*b*d) + ((a
^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*b*
d) - (b*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a^2*d) + ((a^2 + b^2)*f*PolyLo
g[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*b*d^2) + ((a^2 + b^2)*
f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^2) - (b*f*
PolyLog[2, E^(2*(c + d*x))])/(2*a^2*d^2)
```


Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\cosh(c+dx)\coth^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cosh^2(c+dx)\coth(c+dx)}{a+b\sinh(c+dx)}}{a}$$

$$= \frac{\int (e+fx)\cosh(c+dx)\,dx}{a} + \frac{\int (e+fx)\coth(c+dx)\mathbf{csch}(c+dx)\,dx}{a} - $$

$$= -\frac{(e+fx)\mathbf{csch}(c+dx)}{ad} + \frac{(e+fx)\sinh(c+dx)}{ad} - \frac{\int (e+fx)\cosh(c+dx}{a}$$

$$= \frac{b(e+fx)^2}{2a^2f} - \frac{(a^2+b^2)(e+fx)^2}{2a^2bf} - \frac{f\tanh^{-1}(\cosh(c+dx))}{ad^2} - \frac{f\cosh(c+}{ad^2}$$

$$= \frac{b(e+fx)^2}{2a^2f} - \frac{(a^2+b^2)(e+fx)^2}{2a^2bf} - \frac{f\tanh^{-1}(\cosh(c+dx))}{ad^2} - \frac{(e+fx)\mathrm{cs}}{a}$$

$$= \frac{b(e+fx)^2}{2a^2f} - \frac{(a^2+b^2)(e+fx)^2}{2a^2bf} - \frac{f\tanh^{-1}(\cosh(c+dx))}{ad^2} - \frac{(e+fx)\mathrm{cs}}{a}$$

$$= \frac{b(e+fx)^2}{2a^2f} - \frac{(a^2+b^2)(e+fx)^2}{2a^2bf} - \frac{f\tanh^{-1}(\cosh(c+dx))}{ad^2} - \frac{(e+fx)\mathrm{cs}}{a}$$

Mathematica [A] time = 2.5848, size = 313, normalized size = 0.97

$$\frac{2(a^2+b^2)\left(f\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+f\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+f(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)+f(c+dx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)+de\log(a+b\sinh(c+dx))-cf\log(a+b}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (-(a*d*(e + f*x)*Coth[(c + d*x)/2]) - 2*b*d*e*Log[Sinh[c + d*x]] + 2*b*c*f*
Log[Sinh[c + d*x]] + 2*a*f*Log[Tanh[(c + d*x)/2]] + b*f*(-((c + d*x)*(c + d
*x + 2*Log[1 - E^(-2*(c + d*x))])) + PolyLog[2, E^(-2*(c + d*x))]) + (2*(a^
2 + b^2)*(-(f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
 + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLog[
2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2]))]))/b + a*d*(e + f*x)*Tanh[(c + d*x)/2])/(2*a^2*d^2)
```

Maple [B] time = 0.23, size = 938, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/2*f*x^2/b+1/a^2/d^2*b*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2
+b^2)^(1/2)))+1/a^2/d^2*b*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+
b^2)^(1/2)))+1/a^2/d*b*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d^2/b*f*c^
2+1/d/b*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d/b*e*ln(exp(d*x+c))+1/d^
2/b*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/d^2/b*f
*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/a^2/d*b*f*
ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/a^2/d^2*b*f*ln
((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/a^2/d^2*b*f*c*ln
(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/a^2/d*b*f*ln((-b*exp(d*x+c)+(a^2+b^2)
^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/a^2/d^2*b*f*ln((-b*exp(d*x+c)+(a^2+b^2)
^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/a^2/d^2*b*f*c*ln(exp(d*x+c)-1)-1/a^2/d*
b*f*ln(exp(d*x+c)+1)*x+e*x/b-1/a^2/d*b*e*ln(exp(d*x+c)-1)-1/a^2/d*b*e*ln(ex
p(d*x+c)+1)+1/a^2/d^2*b*f*dilog(exp(d*x+c))-1/a^2/d^2*b*f*dilog(exp(d*x+c)+
1)-2/d/b*f*c*x-1/d^2/b*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d^2/b*f*
c*ln(exp(d*x+c))+1/d/b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)
^(1/2)))*x+1/d^2/b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/
2)))*c+1/d/b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1
/d^2/b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d^2/a
*f*ln(exp(d*x+c)-1)-1/d^2/a*f*ln(exp(d*x+c)+1)-2/d*(f*x+e)/a*exp(d*x+c)/(ex
p(2*d*x+2*c)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(2\,bd\int\frac{x}{a^2de^{(dx+c)}+a^2d}\,dx-2\,bd\int\frac{x}{a^2de^{(dx+c)}-a^2d}\,dx+2\,a\left(\frac{dx+c}{a^2d^2}-\frac{\log\left(e^{(dx+c)}+1\right)}{a^2d^2}\right)-2\,a\left(\frac{dx+c}{a^2d^2}-\frac{\log\left(e^{(dx+c)}-\right.}{a^2d^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] 1/2*(2*b*d*integrate(x/(a^2*d*e^(d*x + c) + a^2*d), x) - 2*b*d*integrate(x/
(a^2*d*e^(d*x + c) - a^2*d), x) + 2*a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c
```

```
) + 1)/(a^2*d^2)) - 2*a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^2*d^
2)) + (a*d*x^2*e^(2*d*x + 2*c) - a*d*x^2 - 4*b*x*e^(d*x + c))/(a*b*d*e^(2*d
*x + 2*c) - a*b*d) - integrate(4*((a^3*e^c + a*b^2*e^c)*x*e^(d*x) - (a^2*b
+ b^3)*x)/(a^2*b^2*e^(2*d*x + 2*c) + 2*a^3*b*e^(d*x + c) - a^2*b^2), x))*f
+ e*((d*x + c)/(b*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(
e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + (a^2 + b^2)*l
og(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^2*b*d))
```

Fricas [B] time = 2.47833, size = 4255, normalized size = 13.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/2*(a^2*d^2*f*x^2 + 2*a^2*d^2*e*x + 4*a^2*c*d*e - 2*a^2*c^2*f - (a^2*d^2*f
*x^2 + 2*a^2*d^2*e*x + 4*a^2*c*d*e - 2*a^2*c^2*f)*cosh(d*x + c)^2 - (a^2*d^
2*f*x^2 + 2*a^2*d^2*e*x + 4*a^2*c*d*e - 2*a^2*c^2*f)*sinh(d*x + c)^2 - 4*(a
*b*d*f*x + a*b*d*e)*cosh(d*x + c) + 2*((a^2 + b^2)*f*cosh(d*x + c)^2 + 2*(a
^2 + b^2)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*f*sinh(d*x + c)^2 - (
a^2 + b^2)*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*((a^2 + b^2)*f*cosh
(d*x + c)^2 + 2*(a^2 + b^2)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*f*s
inh(d*x + c)^2 - (a^2 + b^2)*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) -
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(
b^2*f*cosh(d*x + c)^2 + 2*b^2*f*cosh(d*x + c)*sinh(d*x + c) + b^2*f*sinh(d*
x + c)^2 - b^2*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) - 2*(b^2*f*cosh(d*x
+ c)^2 + 2*b^2*f*cosh(d*x + c)*sinh(d*x + c) + b^2*f*sinh(d*x + c)^2 - b^2*
f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) - 2*((a^2 + b^2)*d*e - (a^2 + b^2)
*c*f - ((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)^2 - 2*((a^2 + b^2)
*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c) - ((a^2 + b^2)*d*e - (a
^2 + b^2)*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) +
 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*((a^2 + b^2)*d*e - (a^2 + b^2)*c*f -
((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*e -
(a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c) - ((a^2 + b^2)*d*e - (a^2 + b^
2)*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sq
rt((a^2 + b^2)/b^2) + 2*a) - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f - ((a^2
 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*f*x + (
a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c) - ((a^2 + b^2)*d*f*x + (a^2 + b
^2)*c*f)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*((a^2 + b^2)
*d*f*x + (a^2 + b^2)*c*f - ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x +
 c)^2 - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)
 - ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*sinh(d*x + c)^2)*log(-(a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2) - b)/b) + 2*(b^2*d*f*x + b^2*d*e + a*b*f - (b^2*d*f*x + b^2*d*e + a
*b*f)*cosh(d*x + c)^2 - 2*(b^2*d*f*x + b^2*d*e + a*b*f)*cosh(d*x + c)*sinh(
d*x + c) - (b^2*d*f*x + b^2*d*e + a*b*f)*sinh(d*x + c)^2)*log(cosh(d*x + c)
 + sinh(d*x + c) + 1) + 2*(b^2*d*e - (b^2*d*e - (b^2*c + a*b)*f)*cosh(d*x +
 c)^2 - 2*(b^2*d*e - (b^2*c + a*b)*f)*cosh(d*x + c)*sinh(d*x + c) - (b^2*d*
e - (b^2*c + a*b)*f)*sinh(d*x + c)^2 - (b^2*c + a*b)*f)*log(cosh(d*x + c) +
 sinh(d*x + c) - 1) + 2*(b^2*d*f*x + b^2*c*f - (b^2*d*f*x + b^2*c*f)*cosh(d
*x + c)^2 - 2*(b^2*d*f*x + b^2*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b^2*d*f*
x + b^2*c*f)*sinh(d*x + c)^2)*log(-cosh(d*x + c) - sinh(d*x + c) + 1) - 2*(
2*a*b*d*f*x + 2*a*b*d*e + (a^2*d^2*f*x^2 + 2*a^2*d^2*e*x + 4*a^2*c*d*e - 2*
```

```
a^2*c^2*f)*cosh(d*x + c))*sinh(d*x + c))/(a^2*b*d^2*cosh(d*x + c)^2 + 2*a^2
*b*d^2*cosh(d*x + c)*sinh(d*x + c) + a^2*b*d^2*sinh(d*x + c)^2 - a^2*b*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)\cosh(c + dx)\coth^2(c + dx)}{a + b\sinh(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*cosh(c + d*x)*coth(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)\cosh(dx + c)\coth(dx + c)^2}{b\sinh(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)*coth(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

3.462 $\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a^2bd}-\frac{b\log(\sinh(c+dx))}{a^2d}-\frac{\operatorname{csch}(c+dx)}{ad}$$

```
[Out] -(Csch[c + d*x]/(a*d)) - (b*Log[Sinh[c + d*x]])/(a^2*d) + ((a^2 + b^2)*Log[
a + b*Sinh[c + d*x]])/(a^2*b*d)
```

Rubi [A] time = 0.122573, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2837, 12, 894}

$$\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a^2bd}-\frac{b\log(\sinh(c+dx))}{a^2d}-\frac{\operatorname{csch}(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(Csch[c + d*x]/(a*d)) - (b*Log[Sinh[c + d*x]])/(a^2*d) + ((a^2 + b^2)*Log[
a + b*Sinh[c + d*x]])/(a^2*b*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{b^2(-b^2-x^2)}{x^2(a+x)}\,dx, x, b\sinh(c+dx)\right)}{b^3d}$$
$$= -\frac{\text{Subst}\left(\int \frac{-b^2-x^2}{x^2(a+x)}\,dx, x, b\sinh(c+dx)\right)}{bd}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{b^2}{ax^2}+\frac{b^2}{a^2x}+\frac{-a^2-b^2}{a^2(a+x)}\right)dx, x, b\sinh(c+dx)\right)}{bd}$$
$$= -\frac{\text{csch}(c+dx)}{ad}-\frac{b\log(\sinh(c+dx))}{a^2d}+\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a^2bd}$$

Mathematica [A] time = 0.0859398, size = 52, normalized size = 0.88

$$\frac{(a^2+b^2)\log(a+b\sinh(c+dx))-ab\text{csch}(c+dx)+b^2(-\log(\sinh(c+dx)))}{a^2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-(a*b*Csch[c + d*x]) - b^2*Log[Sinh[c + d*x]] + (a^2 + b^2)*Log[a + b*Sinh
[c + d*x]])/(a^2*b*d)
```

Maple [B] time = 0.001, size = 172, normalized size = 2.9

$$\frac{1}{2da}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{bd}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{1}{bd}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{1}{2da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-\frac{b}{da^2}\ln\left(\text{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/2/d/a*tanh(1/2*d*x+1/2*c)-1/d/b*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/b*ln(tanh(1
/2*d*x+1/2*c)-1)-1/2/d/a/tanh(1/2*d*x+1/2*c)-1/d/a^2*b*ln(tanh(1/2*d*x+1/2*
c))+1/d/b*ln(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)*b-a)+1/d/a^2*b*l
n(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)*b-a)
```

Maxima [B] time = 1.22645, size = 177, normalized size = 3.

$$\frac{dx+c}{bd}+\frac{2\,e^{(-dx-c)}}{\left(ae^{(-2\,dx-2\,c)}-a\right)d}-\frac{b\log\left(e^{(-dx-c)}+1\right)}{a^2d}-\frac{b\log\left(e^{(-dx-c)}-1\right)}{a^2d}+\frac{(a^2+b^2)\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{a^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] (d*x + c)/(b*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-d
*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) + (a^2 + b^2)*log(-2
```

```
*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^2*b*d)
```

Fricas [B] time = 2.2924, size = 751, normalized size = 12.73

$$-\frac{a^2dx\cosh(dx+c)^2 + a^2dx\sinh(dx+c)^2 - a^2dx + 2ab\cosh(dx+c) - \left(\left(a^2+b^2\right)\cosh(dx+c)^2 + 2\left(a^2+b^2\right)\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -(a^2*d*x*cosh(d*x + c)^2 + a^2*d*x*sinh(d*x + c)^2 - a^2*d*x + 2*a*b*cosh(
d*x + c) - ((a^2 + b^2)*cosh(d*x + c)^2 + 2*(a^2 + b^2)*cosh(d*x + c)*sinh(
d*x + c) + (a^2 + b^2)*sinh(d*x + c)^2 - a^2 - b^2)*log(2*(b*sinh(d*x + c)
+ a)/(cosh(d*x + c) - sinh(d*x + c))) + (b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d
*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - b^2)*log(2*sinh(d*x + c)/(cos
h(d*x + c) - sinh(d*x + c))) + 2*(a^2*d*x*cosh(d*x + c) + a*b)*sinh(d*x + c
))/(a^2*b*d*cosh(d*x + c)^2 + 2*a^2*b*d*cosh(d*x + c)*sinh(d*x + c) + a^2*b
*d*sinh(d*x + c)^2 - a^2*b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(cosh(c + d*x)*coth(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [A] time = 1.38761, size = 153, normalized size = 2.59

$$-\frac{\frac{dx}{b} + \frac{b\log\left(e^{(dx+c)}+1\right)}{a^2} + \frac{b\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^2} - \frac{\left(a^2+b^2\right)\log\left(\left|be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}-b\right|\right)}{a^2b} + \frac{2\,e^{(dx+c)}}{a\left(e^{(dx+c)}+1\right)\left(e^{(dx+c)}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -(d*x/b + b*log(e^(d*x + c) + 1)/a^2 + b*log(abs(e^(d*x + c) - 1))/a^2 - (a
^2 + b^2)*log(abs(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - b))/(a^2*b) + 2*e^(
d*x + c)/(a*(e^(d*x + c) + 1)*(e^(d*x + c) - 1)))/d
```

3.463 $\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Cosh[c + d*x]*Coth[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [A] time = 0.0854025, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cosh[c + d*x]*Coth[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Cosh[c + d*x]*Coth[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.002, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Cosh[c + d*x]*Coth[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.944, size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)(\coth(dx+c))^2}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\,e^{(dx+c)}}{adfx+ade-\left(adfxe^{(2\,c)}+adee^{(2\,c)}\right)e^{(2\,dx)}}+\frac{\log\left(fx+e\right)}{bf}-\frac{1}{2}\int-\frac{2\left(bdfx+bde+af\right)}{a^2df^2x^2+2\,a^2defx+a^2de^2-\left(a^2df^2x^2e^c+2\,a^2de\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] 2*e^(d*x + c)/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e*e^(2*c))*e^(2*d*x
)) + log(f*x + e)/(b*f) - 1/2*integrate(-2*(b*d*f*x + b*d*e + a*f)/(a^2*d*f
^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c
 + a^2*d*e^2*e^c)*e^(d*x)), x) + 1/2*integrate(2*(b*d*f*x + b*d*e - a*f)/(a
^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f
*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) - 1/2*integrate(4*(a^2*b + b^3 - (a^3*
e^c + a*b^2*e^c)*e^(d*x))/(a^2*b^2*f*x + a^2*b^2*e - (a^2*b^2*f*x*e^(2*c) +
 a^2*b^2*e*e^(2*c))*e^(2*d*x) - 2*(a^3*b*f*x*e^c + a^3*b*e*e^c)*e^(d*x)), x
)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh\left(dx+c\right)\coth\left(dx+c\right)^2}{afx+ae+\left(bfx+be\right)\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(cosh(d*x + c)*coth(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh\left(c+dx\right)\coth^2\left(c+dx\right)}{\left(a+b\sinh\left(c+dx\right)\right)\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(cosh(c + d*x)*coth(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)),
x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.464 $\int \frac{(e+fx)^3 \mathbf{csch}^2(c+dx)\mathbf{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1428

result too large to display

```
[Out] (-2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(a*d) + (2*b^2*(e + f*x)^3*ArcTan[E^(c
 + d*x)])/(a*(a^2 + b^2)*d) - (6*f*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d^2
) + (2*b*(e + f*x)^3*ArcTanh[E^(2*c + 2*d*x)])/(a^2*d) - ((e + f*x)^3*Csch[
c + d*x])/(a*d) + (b^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2])])/(a^2*(a^2 + b^2)*d) + (b^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d) - (b^3*(e + f*x)^3*Log[1 + E^(2*(c
+ d*x))])/(a^2*(a^2 + b^2)*d) - (6*f^2*(e + f*x)*PolyLog[2, -E^(c + d*x)])/
(a*d^3) + ((3*I)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) - ((3*
I)*b^2*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) - ((
3*I)*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + ((3*I)*b^2*f*(e + f
*x)^2*PolyLog[2, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) + (6*f^2*(e + f*x)*Pol
yLog[2, E^(c + d*x)])/(a*d^3) + (3*b^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^2) + (3*b^3*f*(e + f*x)^
2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^
2) - (3*b^3*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*a^2*(a^2 + b^2)*
d^2) + (3*b*f*(e + f*x)^2*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a^2*d^2) - (3*b*
f*(e + f*x)^2*PolyLog[2, E^(2*c + 2*d*x)])/(2*a^2*d^2) + (6*f^3*PolyLog[3,
-E^(c + d*x)])/(a*d^4) - ((6*I)*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])
/(a*d^3) + ((6*I)*b^2*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(a*(a^2 +
 b^2)*d^3) + ((6*I)*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a*d^3) - ((6*
I)*b^2*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^3) - (6*f^
3*PolyLog[3, E^(c + d*x)])/(a*d^4) - (6*b^3*f^2*(e + f*x)*PolyLog[3, -((b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^3) - (6*b^3*f^2*(e
+ f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^
2)*d^3) + (3*b^3*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*a^2*(a^2 +
b^2)*d^3) - (3*b*f^2*(e + f*x)*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a^2*d^3) +
(3*b*f^2*(e + f*x)*PolyLog[3, E^(2*c + 2*d*x)])/(2*a^2*d^3) + ((6*I)*f^3*Po
lyLog[4, (-I)*E^(c + d*x)])/(a*d^4) - ((6*I)*b^2*f^3*PolyLog[4, (-I)*E^(c +
 d*x)])/(a*(a^2 + b^2)*d^4) - ((6*I)*f^3*PolyLog[4, I*E^(c + d*x)])/(a*d^4)
 + ((6*I)*b^2*f^3*PolyLog[4, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^4) + (6*b^3*f
^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d
^4) + (6*b^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2
*(a^2 + b^2)*d^4) - (3*b^3*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*a^2*(a^2 +
b^2)*d^4) + (3*b*f^3*PolyLog[4, -E^(2*c + 2*d*x)])/(4*a^2*d^4) - (3*b*f^3*P
olyLog[4, E^(2*c + 2*d*x)])/(4*a^2*d^4)
```

Rubi [A] time = 2.28375, antiderivative size = 1428, normalized size of antiderivative = 1., number of steps used = 64, number of rules used = 20, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {5589, 2621, 321, 207, 5462, 6741, 12, 6742, 5205, 4180, 2531, 6609, 2282, 6589, 4182, 5461, 5573, 5561, 2190, 3718}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(a*d) + (2*b^2*(e + f*x)^3*ArcTan[E^(c
 + d*x)])/(a*(a^2 + b^2)*d) - (6*f*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d^2
) + (2*b*(e + f*x)^3*ArcTanh[E^(2*c + 2*d*x)])/(a^2*d) - ((e + f*x)^3*Csch[
```

```
c + d*x])/(a*d) + (b^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2])])/(a^2*(a^2 + b^2)*d) + (b^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d) - (b^3*(e + f*x)^3*Log[1 + E^(2*(c
+ d*x))])/(a^2*(a^2 + b^2)*d) - (6*f^2*(e + f*x)*PolyLog[2, -E^(c + d*x)])/
(a*d^3) + ((3*I)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) - ((3*
I)*b^2*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) - ((
3*I)*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + ((3*I)*b^2*f*(e + f
*x)^2*PolyLog[2, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) + (6*f^2*(e + f*x)*Pol
yLog[2, E^(c + d*x)])/(a*d^3) + (3*b^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c +
 d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^2) + (3*b^3*f*(e + f*x)^
2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^
2) - (3*b^3*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*a^2*(a^2 + b^2)*
d^2) + (3*b*f*(e + f*x)^2*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a^2*d^2) - (3*b*
f*(e + f*x)^2*PolyLog[2, E^(2*c + 2*d*x)])/(2*a^2*d^2) + (6*f^3*PolyLog[3,
-E^(c + d*x)])/(a*d^4) - ((6*I)*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])
/(a*d^3) + ((6*I)*b^2*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(a*(a^2 +
 b^2)*d^3) + ((6*I)*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a*d^3) - ((6*
I)*b^2*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^3) - (6*f^
3*PolyLog[3, E^(c + d*x)])/(a*d^4) - (6*b^3*f^2*(e + f*x)*PolyLog[3, -((b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^3) - (6*b^3*f^2*(e
+ f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^
2)*d^3) + (3*b^3*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*a^2*(a^2 +
b^2)*d^3) - (3*b*f^2*(e + f*x)*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a^2*d^3) +
(3*b*f^2*(e + f*x)*PolyLog[3, E^(2*c + 2*d*x)])/(2*a^2*d^3) + ((6*I)*f^3*Po
lyLog[4, (-I)*E^(c + d*x)])/(a*d^4) - ((6*I)*b^2*f^3*PolyLog[4, (-I)*E^(c +
 d*x)])/(a*(a^2 + b^2)*d^4) - ((6*I)*f^3*PolyLog[4, I*E^(c + d*x)])/(a*d^4)
 + ((6*I)*b^2*f^3*PolyLog[4, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^4) + (6*b^3*f
^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d
^4) + (6*b^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2
*(a^2 + b^2)*d^4) - (3*b^3*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*a^2*(a^2 +
b^2)*d^4) + (3*b*f^3*PolyLog[4, -E^(2*c + 2*d*x)])/(4*a^2*d^4) - (3*b*f^3*P
olyLog[4, E^(2*c + 2*d*x)])/(4*a^2*d^4)
```

Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
```

```
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,d}{a}$$

$$= -\frac{(e+fx)^3\tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)^3\operatorname{csch}(c+dx)}{ad} - \frac{b\int (e+fx)^3}{}$$

$$= -\frac{(e+fx)^3\tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)^3\operatorname{csch}(c+dx)}{ad} - \frac{(2b)\int (e+f}{}$$

$$= -\frac{b^3(e+fx)^4}{4a^2\left(a^2+b^2\right)f} - \frac{(e+fx)^3\tan^{-1}(\sinh(c+dx))}{ad} + \frac{2b(e+fx)^3\tanh^{-1}}{a^2d}$$

$$= -\frac{b^3(e+fx)^4}{4a^2\left(a^2+b^2\right)f} - \frac{(e+fx)^3\tan^{-1}(\sinh(c+dx))}{ad} + \frac{2b(e+fx)^3\tanh^{-1}}{a^2d}$$

$$= \frac{2b^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} - \frac{(e+fx)^3\tan^{-1}(\sinh(c+dx))}{ad} + \frac{2b(e+fx}{}$$

$$= \frac{2b^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} - \frac{6f(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{2b(e+fx)^3\,\mathrm{t}}{}$$

$$= \frac{2b^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} - \frac{6f(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{2b(e+fx)^3\,\mathrm{t}}{}$$

$$= -\frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} - \frac{6f(e+fx)^2\,\mathrm{ta}}{ad}$$

$$= -\frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} - \frac{6f(e+fx)^2\,\mathrm{ta}}{ad}$$

$$= -\frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} - \frac{6f(e+fx)^2\,\mathrm{ta}}{ad}$$

$$= -\frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} - \frac{6f(e+fx)^2\,\mathrm{ta}}{ad}$$

$$= -\frac{2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} - \frac{6f(e+fx)^2\,\mathrm{ta}}{ad}$$

Mathematica [B] time = 14.0157, size = 7234, normalized size = 5.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 1.48, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 (\operatorname{csch}(dx+c))^2 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)

[Out] int((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")

[Out] (b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + a^2*b^2)*d) +
2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^
2 + b^2)*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-d*x -
 c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d))*e^3 - 2*(f^3*x^3*e^c +
3*e*f^2*x^2*e^c + 3*e^2*f*x*e^c)*e^(d*x)/(a*d*e^(2*d*x + 2*c) - a*d) - 3*e^
2*f*log(e^(d*x + c) + 1)/(a*d^2) + 3*e^2*f*log(e^(d*x + c) - 1)/(a*d^2) - (
d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylo
g(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*b*f^3/(a^2*d^4) - (d^3*x^3
*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^
(d*x + c)) + 6*polylog(4, e^(d*x + c)))*b*f^3/(a^2*d^4) - 3*(b*d*e^2*f + 2*
a*e*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 3*(b*
d*e^2*f - 2*a*e*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2*
d^3) - 3*(b*d*e*f^2 + a*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e
^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^2*d^4) - 3*(b*d*e*f^2 - a*f^3)
*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e
^(d*x + c)))/(a^2*d^4) + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2 + a*f^3)*d^3*x^3
 + 6*(b*d^2*e^2*f + 2*a*d*e*f^2)*d^2*x^2)/(a^2*d^4) + 1/4*(b*d^4*f^3*x^4 +
4*(b*d*e*f^2 - a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f - 2*a*d*e*f^2)*d^2*x^2)/(a^2
*d^4) - integrate(2*(b^4*f^3*x^3 + 3*b^4*e*f^2*x^2 + 3*b^4*e^2*f*x - (a*b^3
*f^3*x^3*e^c + 3*a*b^3*e*f^2*x^2*e^c + 3*a*b^3*e^2*f*x*e^c)*e^(d*x))/(a^4*b
 + a^2*b^3 - (a^4*b*e^(2*c) + a^2*b^3*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c + a^3
*b^2*e^c)*e^(d*x)), x) - integrate(2*(b*f^3*x^3 + 3*b*e*f^2*x^2 + 3*b*e^2*f
*x + (a*f^3*x^3*e^c + 3*a*e*f^2*x^2*e^c + 3*a*e^2*f*x*e^c)*e^(d*x))/(a^2 +
b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [C] time = 5.38672, size = 21464, normalized size = 15.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -(2*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a
*b^2)*d^3*e^2*f*x + (a^3 + a*b^2)*d^3*e^3)*cosh(d*x + c) + 3*(b^3*d^2*f^3*x
^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f - (b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2
*x + b^3*d^2*e^2*f)*cosh(d*x + c)^2 - 2*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*
x + b^3*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d^2*f^3*x^2 + 2*b^3*d
^2*e*f^2*x + b^3*d^2*e^2*f)*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sin
h(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)
/b + 1) + 3*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f - (b^3*d^2
*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*cosh(d*x + c)^2 - 2*(b^3*d^2*
f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) -
(b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*sinh(d*x + c)^2)*dilo
g((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*((a^2*b + b^3)*d^2*f^3*x^2 + (a^2*b +
 b^3)*d^2*e^2*f - 2*(a^3 + a*b^2)*d*e*f^2 - ((a^2*b + b^3)*d^2*f^3*x^2 + (a
^2*b + b^3)*d^2*e^2*f - 2*(a^3 + a*b^2)*d*e*f^2 + 2*((a^2*b + b^3)*d^2*e*f^
2 - (a^3 + a*b^2)*d*f^3)*x)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d^2*f^3*x^2
+ (a^2*b + b^3)*d^2*e^2*f - 2*(a^3 + a*b^2)*d*e*f^2 + 2*((a^2*b + b^3)*d^2*
e*f^2 - (a^3 + a*b^2)*d*f^3)*x)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3
)*d^2*f^3*x^2 + (a^2*b + b^3)*d^2*e^2*f - 2*(a^3 + a*b^2)*d*e*f^2 + 2*((a^2
*b + b^3)*d^2*e*f^2 - (a^3 + a*b^2)*d*f^3)*x)*sinh(d*x + c)^2 + 2*((a^2*b +
 b^3)*d^2*e*f^2 - (a^3 + a*b^2)*d*f^3)*x)*dilog(cosh(d*x + c) + sinh(d*x +
c)) - (3*I*a^3*d^2*f^3*x^2 - 3*a^2*b*d^2*f^3*x^2 + 6*I*a^3*d^2*e*f^2*x - 6*
a^2*b*d^2*e*f^2*x + 3*I*a^3*d^2*e^2*f - 3*a^2*b*d^2*e^2*f + (-3*I*a^3*d^2*f
^3*x^2 + 3*a^2*b*d^2*f^3*x^2 - 6*I*a^3*d^2*e*f^2*x + 6*a^2*b*d^2*e*f^2*x -
3*I*a^3*d^2*e^2*f + 3*a^2*b*d^2*e^2*f)*cosh(d*x + c)^2 + (-6*I*a^3*d^2*f^3*
x^2 + 6*a^2*b*d^2*f^3*x^2 - 12*I*a^3*d^2*e*f^2*x + 12*a^2*b*d^2*e*f^2*x - 6
*I*a^3*d^2*e^2*f + 6*a^2*b*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) + (-3*I*a
^3*d^2*f^3*x^2 + 3*a^2*b*d^2*f^3*x^2 - 6*I*a^3*d^2*e*f^2*x + 6*a^2*b*d^2*e*
f^2*x - 3*I*a^3*d^2*e^2*f + 3*a^2*b*d^2*e^2*f)*sinh(d*x + c)^2)*dilog(I*cos
h(d*x + c) + I*sinh(d*x + c)) - (-3*I*a^3*d^2*f^3*x^2 - 3*a^2*b*d^2*f^3*x^2
 - 6*I*a^3*d^2*e*f^2*x - 6*a^2*b*d^2*e*f^2*x - 3*I*a^3*d^2*e^2*f - 3*a^2*b*
d^2*e^2*f + (3*I*a^3*d^2*f^3*x^2 + 3*a^2*b*d^2*f^3*x^2 + 6*I*a^3*d^2*e*f^2*
x + 6*a^2*b*d^2*e*f^2*x + 3*I*a^3*d^2*e^2*f + 3*a^2*b*d^2*e^2*f)*cosh(d*x +
 c)^2 + (6*I*a^3*d^2*f^3*x^2 + 6*a^2*b*d^2*f^3*x^2 + 12*I*a^3*d^2*e*f^2*x +
 12*a^2*b*d^2*e*f^2*x + 6*I*a^3*d^2*e^2*f + 6*a^2*b*d^2*e^2*f)*cosh(d*x + c
)*sinh(d*x + c) + (3*I*a^3*d^2*f^3*x^2 + 3*a^2*b*d^2*f^3*x^2 + 6*I*a^3*d^2*
e*f^2*x + 6*a^2*b*d^2*e*f^2*x + 3*I*a^3*d^2*e^2*f + 3*a^2*b*d^2*e^2*f)*sinh
(d*x + c)^2)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) - 3*((a^2*b + b^3)*d
^2*f^3*x^2 + (a^2*b + b^3)*d^2*e^2*f + 2*(a^3 + a*b^2)*d*e*f^2 - ((a^2*b +
b^3)*d^2*f^3*x^2 + (a^2*b + b^3)*d^2*e^2*f + 2*(a^3 + a*b^2)*d*e*f^2 + 2*((
a^2*b + b^3)*d^2*e*f^2 + (a^3 + a*b^2)*d*f^3)*x)*cosh(d*x + c)^2 - 2*((a^2*
b + b^3)*d^2*f^3*x^2 + (a^2*b + b^3)*d^2*e^2*f + 2*(a^3 + a*b^2)*d*e*f^2 +
2*((a^2*b + b^3)*d^2*e*f^2 + (a^3 + a*b^2)*d*f^3)*x)*cosh(d*x + c)*sinh(d*x
 + c) - ((a^2*b + b^3)*d^2*f^3*x^2 + (a^2*b + b^3)*d^2*e^2*f + 2*(a^3 + a*b
^2)*d*e*f^2 + 2*((a^2*b + b^3)*d^2*e*f^2 + (a^3 + a*b^2)*d*f^3)*x)*sinh(d*x
 + c)^2 + 2*((a^2*b + b^3)*d^2*e*f^2 + (a^3 + a*b^2)*d*f^3)*x)*dilog(-cosh(
d*x + c) - sinh(d*x + c)) + (b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*
e*f^2 - b^3*c^3*f^3 - (b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2
- b^3*c^3*f^3)*cosh(d*x + c)^2 - 2*(b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3
*c^2*d*e*f^2 - b^3*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d^3*e^3 - 3*
b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*sinh(d*x + c)^2)*log(2*b
*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^
3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3 - (b^3*d^3*
e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*cosh(d*x + c)^2
- 2*(b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*cos
h(d*x + c)*sinh(d*x + c) - (b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e
```

*f^2 - b^3*c^3*f^3)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3 - (b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3)*cosh(d*x + c)^2 - 2*(b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3 - (b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3)*cosh(d*x + c)^2 - 2*(b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - ((a^2*b + b^3)*d^3*f^3*x^3 + (a^2*b + b^3)*d^3*e^3 + 3*(a^3 + a*b^2)*d^2*e^2*f + 3*((a^2*b + b^3)*d^3*e*f^2 + (a^3 + a*b^2)*d^2*f^3)*x^2 - ((a^2*b + b^3)*d^3*f^3*x^3 + (a^2*b + b^3)*d^3*e^3 + 3*(a^3 + a*b^2)*d^2*e^2*f + 3*((a^2*b + b^3)*d^3*e*f^2 + (a^3 + a*b^2)*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f + 2*(a^3 + a*b^2)*d^2*e*f^2)*x)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d^3*f^3*x^3 + (a^2*b + b^3)*d^3*e^3 + 3*(a^3 + a*b^2)*d^2*e^2*f + 3*((a^2*b + b^3)*d^3*e*f^2 + (a^3 + a*b^2)*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f + 2*(a^3 + a*b^2)*d^2*e*f^2)*x)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d^3*f^3*x^3 + (a^2*b + b^3)*d^3*e^3 + 3*(a^3 + a*b^2)*d^2*e^2*f + 3*((a^2*b + b^3)*d^3*e*f^2 + (a^3 + a*b^2)*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f + 2*(a^3 + a*b^2)*d^2*e*f^2)*x)*sinh(d*x + c)^2 + 3*((a^2*b + b^3)*d^3*e^2*f + 2*(a^3 + a*b^2)*d^2*e*f^2)*x)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (I*a^3*d^3*e^3 - a^2*b*d^3*e^3 - 3*I*a^3*c*d^2*e^2*f + 3*a^2*b*c*d^2*e^2*f + 3*I*a^3*c^2*d*e*f^2 - 3*a^2*b*c^2*d*e*f^2 - I*a^3*c^3*f^3 + a^2*b*c^3*f^3 + (-I*a^3*d^3*e^3 + a^2*b*d^3*e^3 + 3*I*a^3*c*d^2*e^2*f - 3*a^2*b*c*d^2*e^2*f - 3*I*a^3*c^2*d*e*f^2 + 3*a^2*b*c^2*d*e*f^2 + I*a^3*c^3*f^3 - a^2*b*c^3*f^3)*cosh(d*x + c)^2 + (-2*I*a^3*d^3*e^3 + 2*a^2*b*d^3*e^3 + 6*I*a^3*c*d^2*e^2*f - 6*a^2*b*c*d^2*e^2*f - 6*I*a^3*c^2*d*e*f^2 + 6*a^2*b*c^2*d*e*f^2 + 2*I*a^3*c^3*f^3 - 2*a^2*b*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) + (-I*a^3*d^3*e^3 + a^2*b*d^3*e^3 + 3*I*a^3*c*d^2*e^2*f - 3*a^2*b*c*d^2*e^2*f - 3*I*a^3*c^2*d*e*f^2 + 3*a^2*b*c^2*d*e*f^2 + I*a^3*c^3*f^3 - a^2*b*c^3*f^3)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) + I) - (-I*a^3*d^3*e^3 - a^2*b*d^3*e^3 + 3*I*a^3*c*d^2*e^2*f + 3*a^2*b*c*d^2*e^2*f - 3*I*a^3*c^2*d*e*f^2 - 3*a^2*b*c^2*d*e*f^2 + I*a^3*c^3*f^3 + a^2*b*c^3*f^3 + (I*a^3*d^3*e^3 + a^2*b*d^3*e^3 - 3*I*a^3*c*d^2*e^2*f - 3*a^2*b*c*d^2*e^2*f + 3*I*a^3*c^2*d*e*f^2 + 3*a^2*b*c^2*d*e*f^2 - I*a^3*c^3*f^3 - a^2*b*c^3*f^3)*cosh(d*x + c)^2 + (2*I*a^3*d^3*e^3 + 2*a^2*b*d^3*e^3 - 6*I*a^3*c*d^2*e^2*f - 6*a^2*b*c*d^2*e^2*f + 6*I*a^3*c^2*d*e*f^2 + 6*a^2*b*c^2*d*e*f^2 - 2*I*a^3*c^3*f^3 - 2*a^2*b*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) + (I*a^3*d^3*e^3 + a^2*b*d^3*e^3 - 3*I*a^3*c*d^2*e^2*f - 3*a^2*b*c*d^2*e^2*f + 3*I*a^3*c^2*d*e*f^2 + 3*a^2*b*c^2*d*e*f^2 - I*a^3*c^3*f^3 - a^2*b*c^3*f^3)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) - I) - ((a^2*b + b^3)*d^3*e^3 - 3*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d^2*e^2*f + 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 - ((a^2*b + b^3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3 - ((a^2*b + b^3)*d^3*e^3 - 3*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d^2*e^2*f + 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 - ((a^2*b + b^3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d^3*e^3 - 3*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d^2*e^2*f + 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 - ((a^2*b + b^3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d^3*e^3 - 3*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d^2*e^2*f + 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 - ((a^2*

```
b + b^3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3)*sinh(d*x + c)^2)*log(cosh(d*x + c)
 + sinh(d*x + c) - 1) - (-I*a^3*d^3*f^3*x^3 - a^2*b*d^3*f^3*x^3 - 3*I*a^3*d
^3*e*f^2*x^2 - 3*a^2*b*d^3*e*f^2*x^2 - 3*I*a^3*d^3*e^2*f*x - 3*a^2*b*d^3*e^
2*f*x - 3*I*a^3*c*d^2*e^2*f - 3*a^2*b*c*d^2*e^2*f + 3*I*a^3*c^2*d*e*f^2 + 3
*a^2*b*c^2*d*e*f^2 - I*a^3*c^3*f^3 - a^2*b*c^3*f^3 + (I*a^3*d^3*f^3*x^3 + a
^2*b*d^3*f^3*x^3 + 3*I*a^3*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e*f^2*x^2 + 3*I*a^3*
d^3*e^2*f*x + 3*a^2*b*d^3*e^2*f*x + 3*I*a^3*c*d^2*e^2*f + 3*a^2*b*c*d^2*e^2
*f - 3*I*a^3*c^2*d*e*f^2 - 3*a^2*b*c^2*d*e*f^2 + I*a^3*c^3*f^3 + a^2*b*c^3*
f^3)*cosh(d*x + c)^2 + (2*I*a^3*d^3*f^3*x^3 + 2*a^2*b*d^3*f^3*x^3 + 6*I*a^3
*d^3*e*f^2*x^2 + 6*a^2*b*d^3*e*f^2*x^2 + 6*I*a^3*d^3*e^2*f*x + 6*a^2*b*d^3*
e^2*f*x + 6*I*a^3*c*d^2*e^2*f + 6*a^2*b*c*d^2*e^2*f - 6*I*a^3*c^2*d*e*f^2 -
 6*a^2*b*c^2*d*e*f^2 + 2*I*a^3*c^3*f^3 + 2*a^2*b*c^3*f^3)*cosh(d*x + c)*sin
h(d*x + c) + (I*a^3*d^3*f^3*x^3 + a^2*b*d^3*f^3*x^3 + 3*I*a^3*d^3*e*f^2*x^2
 + 3*a^2*b*d^3*e*f^2*x^2 + 3*I*a^3*d^3*e^2*f*x + 3*a^2*b*d^3*e^2*f*x + 3*I*
a^3*c*d^2*e^2*f + 3*a^2*b*c*d^2*e^2*f - 3*I*a^3*c^2*d*e*f^2 - 3*a^2*b*c^2*d
*e*f^2 + I*a^3*c^3*f^3 + a^2*b*c^3*f^3)*sinh(d*x + c)^2)*log(I*cosh(d*x + c
) + I*sinh(d*x + c) + 1) - (I*a^3*d^3*f^3*x^3 - a^2*b*d^3*f^3*x^3 + 3*I*a^3
*d^3*e*f^2*x^2 - 3*a^2*b*d^3*e*f^2*x^2 + 3*I*a^3*d^3*e^2*f*x - 3*a^2*b*d^3*
e^2*f*x + 3*I*a^3*c*d^2*e^2*f - 3*a^2*b*c*d^2*e^2*f - 3*I*a^3*c^2*d*e*f^2 +
 3*a^2*b*c^2*d*e*f^2 + I*a^3*c^3*f^3 - a^2*b*c^3*f^3 + (-I*a^3*d^3*f^3*x^3
+ a^2*b*d^3*f^3*x^3 - 3*I*a^3*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e*f^2*x^2 - 3*I*a
^3*d^3*e^2*f*x + 3*a^2*b*d^3*e^2*f*x - 3*I*a^3*c*d^2*e^2*f + 3*a^2*b*c*d^2*
e^2*f + 3*I*a^3*c^2*d*e*f^2 - 3*a^2*b*c^2*d*e*f^2 - I*a^3*c^3*f^3 + a^2*b*c
^3*f^3)*cosh(d*x + c)^2 + (-2*I*a^3*d^3*f^3*x^3 + 2*a^2*b*d^3*f^3*x^3 - 6*I
*a^3*d^3*e*f^2*x^2 + 6*a^2*b*d^3*e*f^2*x^2 - 6*I*a^3*d^3*e^2*f*x + 6*a^2*b*
d^3*e^2*f*x - 6*I*a^3*c*d^2*e^2*f + 6*a^2*b*c*d^2*e^2*f + 6*I*a^3*c^2*d*e*f
^2 - 6*a^2*b*c^2*d*e*f^2 - 2*I*a^3*c^3*f^3 + 2*a^2*b*c^3*f^3)*cosh(d*x + c)
*sinh(d*x + c) + (-I*a^3*d^3*f^3*x^3 + a^2*b*d^3*f^3*x^3 - 3*I*a^3*d^3*e*f^
2*x^2 + 3*a^2*b*d^3*e*f^2*x^2 - 3*I*a^3*d^3*e^2*f*x + 3*a^2*b*d^3*e^2*f*x -
 3*I*a^3*c*d^2*e^2*f + 3*a^2*b*c*d^2*e^2*f + 3*I*a^3*c^2*d*e*f^2 - 3*a^2*b*
c^2*d*e*f^2 - I*a^3*c^3*f^3 + a^2*b*c^3*f^3)*sinh(d*x + c)^2)*log(-I*cosh(d
*x + c) - I*sinh(d*x + c) + 1) - ((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^
3)*c*d^2*e^2*f - 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 + ((a^2*
b + b^3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^3
 + a*b^2)*d^2*f^3)*x^2 - ((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3)*c*d^2
*e^2*f - 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 + ((a^2*b + b^3)
*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^3 + a*b^2
)*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^3 + a*b^2)*d^2*e*f^2)*x)
*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3)*c*d^2*e^2
*f - 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 + ((a^2*b + b^3)*c^3
 + 3*(a^3 + a*b^2)*c^2)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^3 + a*b^2)*d^
2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^3 + a*b^2)*d^2*e*f^2)*x)*cos
h(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3)*c*d
^2*e^2*f - 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 + ((a^2*b + b^
3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^3 + a*b
^2)*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^3 + a*b^2)*d^2*e*f^2)*
x)*sinh(d*x + c)^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^3 + a*b^2)*d^2*e*f^2
)*x)*log(-cosh(d*x + c) - sinh(d*x + c) + 1) - 6*(b^3*f^3*cosh(d*x + c)^2 +
 2*b^3*f^3*cosh(d*x + c)*sinh(d*x + c) + b^3*f^3*sinh(d*x + c)^2 - b^3*f^3)
*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*(b^3*f^3*cosh(d*x + c)^2 + 2*b^3*f^
3*cosh(d*x + c)*sinh(d*x + c) + b^3*f^3*sinh(d*x + c)^2 - b^3*f^3)*polylog(
4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2))/b) + 6*((a^2*b + b^3)*f^3*cosh(d*x + c)^2 + 2*(a^2*
b + b^3)*f^3*cosh(d*x + c)*sinh(d*x + c) + (a^2*b + b^3)*f^3*sinh(d*x + c)^
2 - (a^2*b + b^3)*f^3)*polylog(4, cosh(d*x + c) + sinh(d*x + c)) + 6*(-I*a^
3*f^3 + a^2*b*f^3 + (I*a^3*f^3 - a^2*b*f^3)*cosh(d*x + c)^2 + 2*(I*a^3*f^3
- a^2*b*f^3)*cosh(d*x + c)*sinh(d*x + c) + (I*a^3*f^3 - a^2*b*f^3)*sinh(d*x
 + c)^2)*polylog(4, I*cosh(d*x + c) + I*sinh(d*x + c)) + 6*(I*a^3*f^3 + a^2
```

```
*b*f^3 + (-I*a^3*f^3 - a^2*b*f^3)*cosh(d*x + c)^2 + 2*(-I*a^3*f^3 - a^2*b*f
^3)*cosh(d*x + c)*sinh(d*x + c) + (-I*a^3*f^3 - a^2*b*f^3)*sinh(d*x + c)^2)
*polylog(4, -I*cosh(d*x + c) - I*sinh(d*x + c)) + 6*((a^2*b + b^3)*f^3*cosh
(d*x + c)^2 + 2*(a^2*b + b^3)*f^3*cosh(d*x + c)*sinh(d*x + c) + (a^2*b + b^
3)*f^3*sinh(d*x + c)^2 - (a^2*b + b^3)*f^3)*polylog(4, -cosh(d*x + c) - sin
h(d*x + c)) - 6*(b^3*d*f^3*x + b^3*d*e*f^2 - (b^3*d*f^3*x + b^3*d*e*f^2)*co
sh(d*x + c)^2 - 2*(b^3*d*f^3*x + b^3*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) -
 (b^3*d*f^3*x + b^3*d*e*f^2)*sinh(d*x + c)^2)*polylog(3, (a*cosh(d*x + c) +
 a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2
))/b) - 6*(b^3*d*f^3*x + b^3*d*e*f^2 - (b^3*d*f^3*x + b^3*d*e*f^2)*cosh(d*x
 + c)^2 - 2*(b^3*d*f^3*x + b^3*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b^3*
d*f^3*x + b^3*d*e*f^2)*sinh(d*x + c)^2)*polylog(3, (a*cosh(d*x + c) + a*sin
h(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b)
+ 6*((a^2*b + b^3)*d*f^3*x + (a^2*b + b^3)*d*e*f^2 - (a^3 + a*b^2)*f^3 - ((
a^2*b + b^3)*d*f^3*x + (a^2*b + b^3)*d*e*f^2 - (a^3 + a*b^2)*f^3)*cosh(d*x
+ c)^2 - 2*((a^2*b + b^3)*d*f^3*x + (a^2*b + b^3)*d*e*f^2 - (a^3 + a*b^2)*f
^3)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d*f^3*x + (a^2*b + b^3)*d*
e*f^2 - (a^3 + a*b^2)*f^3)*sinh(d*x + c)^2)*polylog(3, cosh(d*x + c) + sinh
(d*x + c)) - (-6*I*a^3*d*f^3*x + 6*a^2*b*d*f^3*x - 6*I*a^3*d*e*f^2 + 6*a^2*
b*d*e*f^2 + (6*I*a^3*d*f^3*x - 6*a^2*b*d*f^3*x + 6*I*a^3*d*e*f^2 - 6*a^2*b*
d*e*f^2)*cosh(d*x + c)^2 + (12*I*a^3*d*f^3*x - 12*a^2*b*d*f^3*x + 12*I*a^3*
d*e*f^2 - 12*a^2*b*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) + (6*I*a^3*d*f^3*x
- 6*a^2*b*d*f^3*x + 6*I*a^3*d*e*f^2 - 6*a^2*b*d*e*f^2)*sinh(d*x + c)^2)*pol
ylog(3, I*cosh(d*x + c) + I*sinh(d*x + c)) - (6*I*a^3*d*f^3*x + 6*a^2*b*d*f
^3*x + 6*I*a^3*d*e*f^2 + 6*a^2*b*d*e*f^2 + (-6*I*a^3*d*f^3*x - 6*a^2*b*d*f^
3*x - 6*I*a^3*d*e*f^2 - 6*a^2*b*d*e*f^2)*cosh(d*x + c)^2 + (-12*I*a^3*d*f^3
*x - 12*a^2*b*d*f^3*x - 12*I*a^3*d*e*f^2 - 12*a^2*b*d*e*f^2)*cosh(d*x + c)*
sinh(d*x + c) + (-6*I*a^3*d*f^3*x - 6*a^2*b*d*f^3*x - 6*I*a^3*d*e*f^2 - 6*a
^2*b*d*e*f^2)*sinh(d*x + c)^2)*polylog(3, -I*cosh(d*x + c) - I*sinh(d*x + c
)) + 6*((a^2*b + b^3)*d*f^3*x + (a^2*b + b^3)*d*e*f^2 + (a^3 + a*b^2)*f^3 -
 ((a^2*b + b^3)*d*f^3*x + (a^2*b + b^3)*d*e*f^2 + (a^3 + a*b^2)*f^3)*cosh(d
*x + c)^2 - 2*((a^2*b + b^3)*d*f^3*x + (a^2*b + b^3)*d*e*f^2 + (a^3 + a*b^2
)*f^3)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d*f^3*x + (a^2*b + b^3)
*d*e*f^2 + (a^3 + a*b^2)*f^3)*sinh(d*x + c)^2)*polylog(3, -cosh(d*x + c) -
sinh(d*x + c)) + 2*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x
^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + (a^3 + a*b^2)*d^3*e^3)*sinh(d*x + c))/((
a^4 + a^2*b^2)*d^4*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*d^4*cosh(d*x + c)*si
nh(d*x + c) + (a^4 + a^2*b^2)*d^4*sinh(d*x + c)^2 - (a^4 + a^2*b^2)*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)**2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.465 $\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=982

result too large to display

```
[Out] (-2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(a*d) + (2*b^2*(e + f*x)^2*ArcTan[E^(c
 + d*x)])/(a*(a^2 + b^2)*d) - (4*f*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d^2)
+ (2*b*(e + f*x)^2*ArcTanh[E^(2*c + 2*d*x)])/(a^2*d) - ((e + f*x)^2*Csch[c
+ d*x])/(a*d) + (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])])/(a^2*(a^2 + b^2)*d) + (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d) - (b^3*(e + f*x)^2*Log[1 + E^(2*(c +
d*x))])/(a^2*(a^2 + b^2)*d) - (2*f^2*PolyLog[2, -E^(c + d*x)])/(a*d^3) + ((
2*I)*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) - ((2*I)*b^2*f*(e +
f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) - ((2*I)*f*(e + f*x)
*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + ((2*I)*b^2*f*(e + f*x)*PolyLog[2, I*E
^(c + d*x)])/(a*(a^2 + b^2)*d^2) + (2*f^2*PolyLog[2, E^(c + d*x)])/(a*d^3)
+ (2*b^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
(a^2*(a^2 + b^2)*d^2) + (2*b^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^2) - (b^3*f*(e + f*x)*PolyLog[2, -
E^(2*(c + d*x))])/(a^2*(a^2 + b^2)*d^2) + (b*f*(e + f*x)*PolyLog[2, -E^(2*c
 + 2*d*x)])/(a^2*d^2) - (b*f*(e + f*x)*PolyLog[2, E^(2*c + 2*d*x)])/(a^2*d^
2) - ((2*I)*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^3) + ((2*I)*b^2*f^2*Poly
Log[3, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^3) + ((2*I)*f^2*PolyLog[3, I*E^(
c + d*x)])/(a*d^3) - ((2*I)*b^2*f^2*PolyLog[3, I*E^(c + d*x)])/(a*(a^2 + b^
2)*d^3) - (2*b^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
(a^2*(a^2 + b^2)*d^3) - (2*b^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2]))])/(a^2*(a^2 + b^2)*d^3) + (b^3*f^2*PolyLog[3, -E^(2*(c + d*x))]
)/(2*a^2*(a^2 + b^2)*d^3) - (b*f^2*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a^2*d^3
) + (b*f^2*PolyLog[3, E^(2*c + 2*d*x)])/(2*a^2*d^3)
```

Rubi [A] time = 1.66756, antiderivative size = 982, normalized size of antiderivative = 1., number of steps used = 53, number of rules used = 21, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.618, Rules used = {5589, 2621, 321, 207, 5462, 6741, 12, 6742, 5205, 4180, 2531, 2282, 6589, 4182, 2279, 2391, 5461, 5573, 5561, 2190, 3718}

$$\frac{(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)b^3}{a^2\left(a^2+b^2\right)d}+\frac{(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)b^3}{a^2\left(a^2+b^2\right)d}-\frac{(e+fx)^2\log\left(1+e^{2(c+dx)}\right)b^3}{a^2\left(a^2+b^2\right)d}+\frac{2f(e+fx)\text{PolyLog}\left(\right.}{a^2\left(a^2+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(a*d) + (2*b^2*(e + f*x)^2*ArcTan[E^(c
 + d*x)])/(a*(a^2 + b^2)*d) - (4*f*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d^2)
+ (2*b*(e + f*x)^2*ArcTanh[E^(2*c + 2*d*x)])/(a^2*d) - ((e + f*x)^2*Csch[c
+ d*x])/(a*d) + (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])])/(a^2*(a^2 + b^2)*d) + (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d) - (b^3*(e + f*x)^2*Log[1 + E^(2*(c +
d*x))])/(a^2*(a^2 + b^2)*d) - (2*f^2*PolyLog[2, -E^(c + d*x)])/(a*d^3) + ((
2*I)*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) - ((2*I)*b^2*f*(e +
f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) - ((2*I)*f*(e + f*x)
*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + ((2*I)*b^2*f*(e + f*x)*PolyLog[2, I*E
^(c + d*x)])/(a*(a^2 + b^2)*d^2) + (2*f^2*PolyLog[2, E^(c + d*x)])/(a*d^3)
+ (2*b^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
```

```
(a^2*(a^2 + b^2)*d^2) + (2*b^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^2) - (b^3*f*(e + f*x)*PolyLog[2, -
E^(2*(c + d*x))])/(a^2*(a^2 + b^2)*d^2) + (b*f*(e + f*x)*PolyLog[2, -E^(2*c
 + 2*d*x)])/(a^2*d^2) - (b*f*(e + f*x)*PolyLog[2, E^(2*c + 2*d*x)])/(a^2*d^
2) - ((2*I)*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^3) + ((2*I)*b^2*f^2*Poly
Log[3, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^3) + ((2*I)*f^2*PolyLog[3, I*E^(
c + d*x)])/(a*d^3) - ((2*I)*b^2*f^2*PolyLog[3, I*E^(c + d*x)])/(a*(a^2 + b^
2)*d^3) - (2*b^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
(a^2*(a^2 + b^2)*d^3) - (2*b^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2]))])/(a^2*(a^2 + b^2)*d^3) + (b^3*f^2*PolyLog[3, -E^(2*(c + d*x))]
)/(2*a^2*(a^2 + b^2)*d^3) - (b*f^2*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a^2*d^3
) + (b*f^2*PolyLog[3, E^(2*c + 2*d*x)])/(2*a^2*d^3)
```

Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
```

```
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\text{csch}^2(c+dx)\text{sech}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^2\tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)^2\text{csch}(c+dx)}{ad} - \frac{b\int (e+fx)^2\text{csc}$$

$$= -\frac{(e+fx)^2\tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)^2\text{csch}(c+dx)}{ad} - \frac{(2b)\int (e+fx)^2\text{c}$$

$$= -\frac{b^3(e+fx)^3}{3a^2\left(a^2+b^2\right)f} - \frac{(e+fx)^2\tan^{-1}(\sinh(c+dx))}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{2c}\right.}{a^2d}$$

$$= -\frac{b^3(e+fx)^3}{3a^2\left(a^2+b^2\right)f} - \frac{(e+fx)^2\tan^{-1}(\sinh(c+dx))}{ad} + \frac{2b(e+fx)^2\tanh^{-1}\left(e^{2c}\right.}{a^2d}$$

$$= \frac{2b^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} - \frac{(e+fx)^2\tan^{-1}(\sinh(c+dx))}{ad} + \frac{2b(e+fx)^2\text{t}}{a}$$

$$= \frac{2b^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} - \frac{4f(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{2b(e+fx)^2\tanh}{a^2d}$$

$$= \frac{2b^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} - \frac{4f(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^2} + \frac{2b(e+fx)^2\tanh}{a^2d}$$

$$= -\frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} - \frac{4f(e+fx)\tanh^{-}}{ad^2}$$

$$= -\frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} - \frac{4f(e+fx)\tanh^{-}}{ad^2}$$

$$= -\frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} - \frac{4f(e+fx)\tanh^{-}}{ad^2}$$

$$= -\frac{2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} - \frac{4f(e+fx)\tanh^{-}}{ad^2}$$

Mathematica [B] time = 13.7961, size = 2107, normalized size = 2.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] -(((e + f*x)^2*Csch[c])/(a*d)) - (12*b*d^3*e^2*E^(2*c)*x - 12*b*d^3*e^2*(1
+ E^(2*c))*x - 12*b*d^3*e*f*x^2 - 4*b*d^3*f^2*x^3 + 12*a*d^2*e^2*(1 + E^(2*
c))*ArcTan[E^(c + d*x)] + 6*b*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(
c + d*x))]) + (12*I)*a*d*e*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - L
og[1 + I*E^(c + d*x)]) - PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c +
 d*x)]) + 6*b*d*e*(1 + E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) -
 PolyLog[2, -E^(2*(c + d*x))]) + (6*I)*a*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 -
 I*E^(c + d*x)] - d^2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^
```

```
(c + d*x)] + 2*d*x*PolyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x
)] - 2*PolyLog[3, I*E^(c + d*x)]) + b*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x -
 3*Log[1 + E^(2*(c + d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyL
og[3, -E^(2*(c + d*x))]))/(6*(a^2 + b^2)*d^3*(1 + E^(2*c))) - (b^3*(6*e^2*E
^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2
*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (
6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 -
 b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a +
b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 +
 b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 -
 b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d
- (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*
f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*
e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])
])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*
c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2
+ b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^
2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c
+ Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*
E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c
 + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(e +
 f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])
/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c
+ Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))
/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((
b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (6*f^2*PolyLo
g[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^
(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]
))])/d^3))/(3*a^2*(a^2 + b^2)*(-1 + E^(2*c))) + (b*d^3*(e + f*x)^3*(-1 + Co
th[c]) + 3*d*e*f*(b*d*e - 2*a*f)*(d*x - Log[1 - Cosh[c + d*x] - Sinh[c + d*
x]]) - 6*d*f^2*(b*d*e + a*f)*x*Log[1 + Cosh[c + d*x] - Sinh[c + d*x]] - 3*b
*d^2*f^3*x^2*Log[1 + Cosh[c + d*x] - Sinh[c + d*x]] - 6*d*f^2*(b*d*e - a*f)
*x*Log[1 - Cosh[c + d*x] + Sinh[c + d*x]] - 3*b*d^2*f^3*x^2*Log[1 - Cosh[c
+ d*x] + Sinh[c + d*x]] + 3*d*e*f*(b*d*e + 2*a*f)*(d*x - Log[1 + Cosh[c + d
*x] + Sinh[c + d*x]]) + 6*f^2*(b*d*e - a*f)*PolyLog[2, Cosh[c + d*x] - Sinh
[c + d*x]] + 6*f^2*(b*d*e + a*f)*PolyLog[2, -Cosh[c + d*x] + Sinh[c + d*x]]
 + 6*b*f^3*(d*x*PolyLog[2, Cosh[c + d*x] - Sinh[c + d*x]] + PolyLog[3, Cosh
[c + d*x] - Sinh[c + d*x]]) + 6*b*f^3*(d*x*PolyLog[2, -Cosh[c + d*x] + Sinh
[c + d*x]] + PolyLog[3, -Cosh[c + d*x] + Sinh[c + d*x]]))/(3*a^2*d^3*f) - (
b*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Csch[c]*Sech[c])/(3*(a^2 + b^2)) + ((e + f*
x)^2*Csch[c/2]*Csch[(c + d*x)/2]*Sinh[(d*x)/2])/(2*a*d) + ((e + f*x)^2*Sech
[c/2]*Sech[(c + d*x)/2]*Sinh[(d*x)/2])/(2*a*d)
```

Maple [F] time = 1.095, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^2 (\operatorname{csch}(dx+c))^2 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] (b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + a^2*b^2)*d) +
2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^
2 + b^2)*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-d*x -
 c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d))*e^2 - 2*(f^2*x^2*e^c +
2*e*f*x*e^c)*e^(d*x)/(a*d*e^(2*d*x + 2*c) - a*d) - 2*e*f*log(e^(d*x + c) +
1)/(a*d^2) + 2*e*f*log(e^(d*x + c) - 1)/(a*d^2) - (d^2*x^2*log(e^(d*x + c)
+ 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*b*f^2/(a^2*d
^3) - (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog
(3, e^(d*x + c)))*b*f^2/(a^2*d^3) - 2*(b*d*e*f + a*f^2)*(d*x*log(e^(d*x + c
) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 2*(b*d*e*f - a*f^2)*(d*x*log(-e^(
d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d
*e*f + a*f^2)*d^2*x^2)/(a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*e*f - a*f^2)
*d^2*x^2)/(a^2*d^3) - integrate(2*(b^4*f^2*x^2 + 2*b^4*e*f*x - (a*b^3*f^2*x
^2*e^c + 2*a*b^3*e*f*x*e^c)*e^(d*x))/(a^4*b + a^2*b^3 - (a^4*b*e^(2*c) + a^
2*b^3*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c + a^3*b^2*e^c)*e^(d*x)), x) - integra
te(2*(b*f^2*x^2 + 2*b*e*f*x + (a*f^2*x^2*e^c + 2*a*e*f*x*e^c)*e^(d*x))/(a^2
 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [C] time = 4.03983, size = 12925, normalized size = 13.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -(2*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + (a^3 + a*b^2)*
d^2*e^2)*cosh(d*x + c) + 2*(b^3*d*f^2*x + b^3*d*e*f - (b^3*d*f^2*x + b^3*d*
e*f)*cosh(d*x + c)^2 - 2*(b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)*sinh(d*x +
 c) - (b^3*d*f^2*x + b^3*d*e*f)*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a
*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b + 1) + 2*(b^3*d*f^2*x + b^3*d*e*f - (b^3*d*f^2*x + b^3*d*e*f)*cosh(d
*x + c)^2 - 2*(b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*
d*f^2*x + b^3*d*e*f)*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x +
 c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1)
 - 2*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*e*f - (a^3 + a*b^2)*f^2 - ((a
^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*e*f - (a^3 + a*b^2)*f^2)*cosh(d*x + c
)^2 - 2*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*e*f - (a^3 + a*b^2)*f^2)*c
osh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*e*f -
 (a^3 + a*b^2)*f^2)*sinh(d*x + c)^2)*dilog(cosh(d*x + c) + sinh(d*x + c)) -
 (2*I*a^3*d*f^2*x - 2*a^2*b*d*f^2*x + 2*I*a^3*d*e*f - 2*a^2*b*d*e*f + (-2*I
*a^3*d*f^2*x + 2*a^2*b*d*f^2*x - 2*I*a^3*d*e*f + 2*a^2*b*d*e*f)*cosh(d*x +
c)^2 + (-4*I*a^3*d*f^2*x + 4*a^2*b*d*f^2*x - 4*I*a^3*d*e*f + 4*a^2*b*d*e*f)
*cosh(d*x + c)*sinh(d*x + c) + (-2*I*a^3*d*f^2*x + 2*a^2*b*d*f^2*x - 2*I*a^
3*d*e*f + 2*a^2*b*d*e*f)*sinh(d*x + c)^2)*dilog(I*cosh(d*x + c) + I*sinh(d*
x + c)) - (-2*I*a^3*d*f^2*x - 2*a^2*b*d*f^2*x - 2*I*a^3*d*e*f - 2*a^2*b*d*e
*f + (2*I*a^3*d*f^2*x + 2*a^2*b*d*f^2*x + 2*I*a^3*d*e*f + 2*a^2*b*d*e*f)*co
```

```
sh(d*x + c)^2 + (4*I*a^3*d*f^2*x + 4*a^2*b*d*f^2*x + 4*I*a^3*d*e*f + 4*a^2*
b*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (2*I*a^3*d*f^2*x + 2*a^2*b*d*f^2*x +
 2*I*a^3*d*e*f + 2*a^2*b*d*e*f)*sinh(d*x + c)^2)*dilog(-I*cosh(d*x + c) - I
*sinh(d*x + c)) - 2*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*e*f + (a^3 + a
*b^2)*f^2 - ((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*e*f + (a^3 + a*b^2)*f^
2)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*e*f + (a^3
+ a*b^2)*f^2)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d*f^2*x + (a^2*b
 + b^3)*d*e*f + (a^3 + a*b^2)*f^2)*sinh(d*x + c)^2)*dilog(-cosh(d*x + c) -
sinh(d*x + c)) + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2 - (b^3*d^2*e^2
- 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)^2 - 2*(b^3*d^2*e^2 - 2*b^3*c*d
*e*f + b^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d^2*e^2 - 2*b^3*c*d*
e*f + b^3*c^2*f^2)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*
c^2*f^2 - (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)^2 - 2*(
b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b
^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*sinh(d*x + c)^2)*log(2*b*cosh(d*x
 + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*d^2*f^2
*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2 - (b^3*d^2*f^2*x^2 + 2
*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x + c)^2 - 2*(b^3*d^2*
f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x + c)*sinh
(d*x + c) - (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^
2)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
 c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b^3*d^2*f^2*x^2 + 2
*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2 - (b^3*d^2*f^2*x^2 + 2*b^3*d^2
*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x + c)^2 - 2*(b^3*d^2*f^2*x^2
+ 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c
) - (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*sinh(
d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - ((a^2*b + b^3)*d^2*f^2*x^2 +
 (a^2*b + b^3)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f - ((a^2*b + b^3)*d^2*f^2*x^2
 + (a^2*b + b^3)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f
 + (a^3 + a*b^2)*d*f^2)*x)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d^2*f^2*x^2 +
 (a^2*b + b^3)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f +
 (a^3 + a*b^2)*d*f^2)*x)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d^2*f
^2*x^2 + (a^2*b + b^3)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^2*b + b^3)*d
^2*e*f + (a^3 + a*b^2)*d*f^2)*x)*sinh(d*x + c)^2 + 2*((a^2*b + b^3)*d^2*e*f
 + (a^3 + a*b^2)*d*f^2)*x)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (I*a^3*
d^2*e^2 - a^2*b*d^2*e^2 - 2*I*a^3*c*d*e*f + 2*a^2*b*c*d*e*f + I*a^3*c^2*f^2
 - a^2*b*c^2*f^2 + (-I*a^3*d^2*e^2 + a^2*b*d^2*e^2 + 2*I*a^3*c*d*e*f - 2*a^
2*b*c*d*e*f - I*a^3*c^2*f^2 + a^2*b*c^2*f^2)*cosh(d*x + c)^2 + (-2*I*a^3*d^
2*e^2 + 2*a^2*b*d^2*e^2 + 4*I*a^3*c*d*e*f - 4*a^2*b*c*d*e*f - 2*I*a^3*c^2*f
^2 + 2*a^2*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (-I*a^3*d^2*e^2 + a^2*b
*d^2*e^2 + 2*I*a^3*c*d*e*f - 2*a^2*b*c*d*e*f - I*a^3*c^2*f^2 + a^2*b*c^2*f^
2)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) + I) - (-I*a^3*d^2*e^
2 - a^2*b*d^2*e^2 + 2*I*a^3*c*d*e*f + 2*a^2*b*c*d*e*f - I*a^3*c^2*f^2 - a^2
*b*c^2*f^2 + (I*a^3*d^2*e^2 + a^2*b*d^2*e^2 - 2*I*a^3*c*d*e*f - 2*a^2*b*c*d
*e*f + I*a^3*c^2*f^2 + a^2*b*c^2*f^2)*cosh(d*x + c)^2 + (2*I*a^3*d^2*e^2 +
2*a^2*b*d^2*e^2 - 4*I*a^3*c*d*e*f - 4*a^2*b*c*d*e*f + 2*I*a^3*c^2*f^2 + 2*a
^2*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (I*a^3*d^2*e^2 + a^2*b*d^2*e^2
- 2*I*a^3*c*d*e*f - 2*a^2*b*c*d*e*f + I*a^3*c^2*f^2 + a^2*b*c^2*f^2)*sinh(d
*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) - I) - ((a^2*b + b^3)*d^2*e^2
- 2*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d*e*f + ((a^2*b + b^3)*c^2 + 2*(a^3 + a
*b^2)*c)*f^2 - ((a^2*b + b^3)*d^2*e^2 - 2*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d
*e*f + ((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*f^2)*cosh(d*x + c)^2 - 2*((a
^2*b + b^3)*d^2*e^2 - 2*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d*e*f + ((a^2*b + b
^3)*c^2 + 2*(a^3 + a*b^2)*c)*f^2)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b
^3)*d^2*e^2 - 2*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d*e*f + ((a^2*b + b^3)*c^2
+ 2*(a^3 + a*b^2)*c)*f^2)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c
) - 1) - (-I*a^3*d^2*f^2*x^2 - a^2*b*d^2*f^2*x^2 - 2*I*a^3*d^2*e*f*x - 2*a^
```

```
2*b*d^2*e*f*x - 2*I*a^3*c*d*e*f - 2*a^2*b*c*d*e*f + I*a^3*c^2*f^2 + a^2*b*c
^2*f^2 + (I*a^3*d^2*f^2*x^2 + a^2*b*d^2*f^2*x^2 + 2*I*a^3*d^2*e*f*x + 2*a^2
*b*d^2*e*f*x + 2*I*a^3*c*d*e*f + 2*a^2*b*c*d*e*f - I*a^3*c^2*f^2 - a^2*b*c^
2*f^2)*cosh(d*x + c)^2 + (2*I*a^3*d^2*f^2*x^2 + 2*a^2*b*d^2*f^2*x^2 + 4*I*a
^3*d^2*e*f*x + 4*a^2*b*d^2*e*f*x + 4*I*a^3*c*d*e*f + 4*a^2*b*c*d*e*f - 2*I*
a^3*c^2*f^2 - 2*a^2*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (I*a^3*d^2*f^2
*x^2 + a^2*b*d^2*f^2*x^2 + 2*I*a^3*d^2*e*f*x + 2*a^2*b*d^2*e*f*x + 2*I*a^3*
c*d*e*f + 2*a^2*b*c*d*e*f - I*a^3*c^2*f^2 - a^2*b*c^2*f^2)*sinh(d*x + c)^2)
*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) - (I*a^3*d^2*f^2*x^2 - a^2*b*d^
2*f^2*x^2 + 2*I*a^3*d^2*e*f*x - 2*a^2*b*d^2*e*f*x + 2*I*a^3*c*d*e*f - 2*a^2
*b*c*d*e*f - I*a^3*c^2*f^2 + a^2*b*c^2*f^2 + (-I*a^3*d^2*f^2*x^2 + a^2*b*d^
2*f^2*x^2 - 2*I*a^3*d^2*e*f*x + 2*a^2*b*d^2*e*f*x - 2*I*a^3*c*d*e*f + 2*a^2
*b*c*d*e*f + I*a^3*c^2*f^2 - a^2*b*c^2*f^2)*cosh(d*x + c)^2 + (-2*I*a^3*d^2
*f^2*x^2 + 2*a^2*b*d^2*f^2*x^2 - 4*I*a^3*d^2*e*f*x + 4*a^2*b*d^2*e*f*x - 4*
I*a^3*c*d*e*f + 4*a^2*b*c*d*e*f + 2*I*a^3*c^2*f^2 - 2*a^2*b*c^2*f^2)*cosh(d
*x + c)*sinh(d*x + c) + (-I*a^3*d^2*f^2*x^2 + a^2*b*d^2*f^2*x^2 - 2*I*a^3*d
^2*e*f*x + 2*a^2*b*d^2*e*f*x - 2*I*a^3*c*d*e*f + 2*a^2*b*c*d*e*f + I*a^3*c^
2*f^2 - a^2*b*c^2*f^2)*sinh(d*x + c)^2)*log(-I*cosh(d*x + c) - I*sinh(d*x +
 c) + 1) - ((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*c*d*e*f - ((a^2*b +
 b^3)*c^2 + 2*(a^3 + a*b^2)*c)*f^2 - ((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b
+ b^3)*c*d*e*f - ((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*f^2 + 2*((a^2*b +
b^3)*d^2*e*f - (a^3 + a*b^2)*d*f^2)*x)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d
^2*f^2*x^2 + 2*(a^2*b + b^3)*c*d*e*f - ((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)
*c)*f^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^3 + a*b^2)*d*f^2)*x)*cosh(d*x + c)*
sinh(d*x + c) - ((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*c*d*e*f - ((a^
2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*f^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^3 +
 a*b^2)*d*f^2)*x)*sinh(d*x + c)^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^3 + a*b^2
)*d*f^2)*x)*log(-cosh(d*x + c) - sinh(d*x + c) + 1) + 2*(b^3*f^2*cosh(d*x +
 c)^2 + 2*b^3*f^2*cosh(d*x + c)*sinh(d*x + c) + b^3*f^2*sinh(d*x + c)^2 - b
^3*f^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*(b^3*f^2*cosh(d*x + c)^2 + 2
*b^3*f^2*cosh(d*x + c)*sinh(d*x + c) + b^3*f^2*sinh(d*x + c)^2 - b^3*f^2)*p
olylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*((a^2*b + b^3)*f^2*cosh(d*x + c)^2 +
2*(a^2*b + b^3)*f^2*cosh(d*x + c)*sinh(d*x + c) + (a^2*b + b^3)*f^2*sinh(d*
x + c)^2 - (a^2*b + b^3)*f^2)*polylog(3, cosh(d*x + c) + sinh(d*x + c)) + 2
*(I*a^3*f^2 - a^2*b*f^2 + (-I*a^3*f^2 + a^2*b*f^2)*cosh(d*x + c)^2 + 2*(-I*
a^3*f^2 + a^2*b*f^2)*cosh(d*x + c)*sinh(d*x + c) + (-I*a^3*f^2 + a^2*b*f^2)
*sinh(d*x + c)^2)*polylog(3, I*cosh(d*x + c) + I*sinh(d*x + c)) - (2*I*a^3*
f^2 + 2*a^2*b*f^2 - 2*(I*a^3*f^2 + a^2*b*f^2)*cosh(d*x + c)^2 - 4*(I*a^3*f^
2 + a^2*b*f^2)*cosh(d*x + c)*sinh(d*x + c) - 2*(I*a^3*f^2 + a^2*b*f^2)*sinh
(d*x + c)^2)*polylog(3, -I*cosh(d*x + c) - I*sinh(d*x + c)) - 2*((a^2*b + b
^3)*f^2*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*f^2*cosh(d*x + c)*sinh(d*x + c) +
 (a^2*b + b^3)*f^2*sinh(d*x + c)^2 - (a^2*b + b^3)*f^2)*polylog(3, -cosh(d*
x + c) - sinh(d*x + c)) + 2*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^
2*e*f*x + (a^3 + a*b^2)*d^2*e^2)*sinh(d*x + c))/((a^4 + a^2*b^2)*d^3*cosh(d
*x + c)^2 + 2*(a^4 + a^2*b^2)*d^3*cosh(d*x + c)*sinh(d*x + c) + (a^4 + a^2*
b^2)*d^3*sinh(d*x + c)^2 - (a^4 + a^2*b^2)*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)**2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] sage0*x
```

3.466 $\int \frac{(e+fx)\mathbf{csch}^2(c+dx)\mathbf{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=591

$$\frac{b^3f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2\left(a^2+b^2\right)}+\frac{b^3f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2\left(a^2+b^2\right)}-\frac{b^3f\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2a^2d^2\left(a^2+b^2\right)}-\frac{ib^2f\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2\left(a^2+b^2\right)}+\cdots$$

```
[Out] (-2*f*x*ArcTan[E^(c + d*x)])/(a*d) + (2*b^2*(e + f*x)*ArcTan[E^(c + d*x)])/
(a*(a^2 + b^2)*d) + (f*x*ArcTan[Sinh[c + d*x]])/(a*d) - ((e + f*x)*ArcTan[S
inh[c + d*x]])/(a*d) + (2*b*(e + f*x)*ArcTanh[E^(2*c + 2*d*x)])/(a^2*d) - (
f*ArcTanh[Cosh[c + d*x]])/(a*d^2) - ((e + f*x)*Csch[c + d*x])/(a*d) + (b^3*
(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*
d) + (b^3*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*(a
^2 + b^2)*d) - (b^3*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a^2*(a^2 + b^2)*d)
 + (I*f*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) - (I*b^2*f*PolyLog[2, (-I)*E^
(c + d*x)])/(a*(a^2 + b^2)*d^2) - (I*f*PolyLog[2, I*E^(c + d*x)])/(a*d^2) +
 (I*b^2*f*PolyLog[2, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) + (b^3*f*PolyLog[2
, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^2) + (b^3*f
*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^2
) - (b^3*f*PolyLog[2, -E^(2*(c + d*x))])/(2*a^2*(a^2 + b^2)*d^2) + (b*f*Pol
yLog[2, -E^(2*c + 2*d*x)])/(2*a^2*d^2) - (b*f*PolyLog[2, E^(2*c + 2*d*x)])/
(2*a^2*d^2)
```

Rubi [A] time = 0.900204, antiderivative size = 591, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 18, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5589, 2621, 321, 207, 5462, 5203, 12, 4180, 2279, 2391, 3770, 5461, 4182, 5573, 5561, 2190, 6742, 3718}

$$\frac{b^3f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2\left(a^2+b^2\right)}+\frac{b^3f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2\left(a^2+b^2\right)}-\frac{b^3f\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2a^2d^2\left(a^2+b^2\right)}-\frac{ib^2f\text{PolyLog}\left(2,-ie^{c+dx}\right)}{ad^2\left(a^2+b^2\right)}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*f*x*ArcTan[E^(c + d*x)])/(a*d) + (2*b^2*(e + f*x)*ArcTan[E^(c + d*x)])/
(a*(a^2 + b^2)*d) + (f*x*ArcTan[Sinh[c + d*x]])/(a*d) - ((e + f*x)*ArcTan[S
inh[c + d*x]])/(a*d) + (2*b*(e + f*x)*ArcTanh[E^(2*c + 2*d*x)])/(a^2*d) - (
f*ArcTanh[Cosh[c + d*x]])/(a*d^2) - ((e + f*x)*Csch[c + d*x])/(a*d) + (b^3*
(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*
d) + (b^3*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*(a
^2 + b^2)*d) - (b^3*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a^2*(a^2 + b^2)*d)
 + (I*f*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) - (I*b^2*f*PolyLog[2, (-I)*E^
(c + d*x)])/(a*(a^2 + b^2)*d^2) - (I*f*PolyLog[2, I*E^(c + d*x)])/(a*d^2) +
 (I*b^2*f*PolyLog[2, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) + (b^3*f*PolyLog[2
, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^2) + (b^3*f
*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^2
) - (b^3*f*PolyLog[2, -E^(2*(c + d*x))])/(2*a^2*(a^2 + b^2)*d^2) + (b*f*Pol
yLog[2, -E^(2*c + 2*d*x)])/(2*a^2*d^2) - (b*f*PolyLog[2, E^(2*c + 2*d*x)])/
(2*a^2*d^2)
```

Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
```

```
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)\tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)\text{csch}(c+dx)}{ad} - \frac{b\int (e+fx)\text{csch}}{}$$

$$= -\frac{(e+fx)\tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)\text{csch}(c+dx)}{ad} - \frac{(2b)\int (e+fx)\text{c}}{}$$

$$= -\frac{b^3(e+fx)^2}{2a^2\left(a^2+b^2\right)f} + \frac{fx\tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)\tan^{-1}(\sinh(c+dx}{ad}$$

$$= -\frac{b^3(e+fx)^2}{2a^2\left(a^2+b^2\right)f} + \frac{fx\tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)\tan^{-1}(\sinh(c+dx}{ad}$$

$$= -\frac{2fx\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} + \frac{fx\tan^{-1}(\sinh(c+dx)}{ad}$$

$$= -\frac{2fx\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} + \frac{fx\tan^{-1}(\sinh(c+dx)}{ad}$$

$$= -\frac{2fx\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} + \frac{fx\tan^{-1}(\sinh(c+dx)}{ad}$$

$$= -\frac{2fx\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^2(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)d} + \frac{fx\tan^{-1}(\sinh(c+dx)}{ad}$$

Mathematica [A] time = 6.52814, size = 535, normalized size = 0.91

$$\frac{2iafPolyLog(2,-i(\sinh(c+dx)+\cosh(c+dx)))-2iafPolyLog(2,i(\sinh(c+dx)+\cosh(c+dx)))+bfPolyLog(2,-\sinh(2(c+dx))-\cosh(2(c+dx)))-4ade\tan^{-1}(\sin}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (-((d*(e + f*x)*Coth[(c + d*x)/2])/a) - (2*b*d*e*Log[Sinh[c + d*x]])/a^2 +
(2*b*c*f*Log[Sinh[c + d*x]])/a^2 + (2*f*Log[Tanh[(c + d*x)/2]])/a + (b*f*(-
((c + d*x)*(c + d*x + 2*Log[1 - E^(-2*(c + d*x))])) + PolyLog[2, E^(-2*(c +
 d*x))]))/a^2 + (2*b^3*(-(f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]
] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^2*(a^2 + b^2)) + (-2*b*c*d*e +
b*c^2*f - 2*b*d^2*e*x - b*d^2*f*x^2 - 4*a*d*e*ArcTan[Cosh[c + d*x] + Sinh[c
 + d*x]] - 4*a*d*f*x*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] + 2*b*d*e*Log[1
+ Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]] + 2*b*d*f*x*Log[1 + Cosh[2*(c + d*
x)] + Sinh[2*(c + d*x)]] + (2*I)*a*f*PolyLog[2, (-I)*(Cosh[c + d*x] + Sinh[
c + d*x])] - (2*I)*a*f*PolyLog[2, I*(Cosh[c + d*x] + Sinh[c + d*x])] + b*f*
PolyLog[2, -Cosh[2*(c + d*x)] - Sinh[2*(c + d*x)]])/(a^2 + b^2) + (d*(e + f
```

```
*x)*Tanh[(c + d*x)/2])/a)/(2*d^2)
```

Maple [B] time = 0.315, size = 1529, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/a^2/d^2*b*f*c*ln(exp(d*x+c)-1)-1/a^2/d*b*f*ln(exp(d*x+c)+1)*x+1/d*e*b/a/(
a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2
)^(3/2)/d^2*b^3*f*c/a*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d
^2*f*c*b/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2)
)-1/a^2/d*b*e*ln(exp(d*x+c)-1)-1/a^2/d*b*e*ln(exp(d*x+c)+1)+1/a^2/d^2*b*f*d
ilog(exp(d*x+c))-1/a^2/d^2*b*f*dilog(exp(d*x+c)+1)+1/d^2/a*f*ln(exp(d*x+c)-
1)-1/d^2/a*f*ln(exp(d*x+c)+1)-2/d*(f*x+e)/a*exp(d*x+c)/(exp(2*d*x+2*c)-1)-b
/d*e/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a+1/
a^2/d*b^3*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1
/2)))*x+1/a^2/d*b^3*f/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2
+b^2)^(1/2)))*x+1/a^2/d^2*b^3*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))*c+1/a^2/d^2*b^3*f/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2
+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/a^2/d^2*b^3*f*c/(a^2+b^2)*ln(b*exp(
2*d*x+2*c)+2*a*exp(d*x+c)-b)+4*I*a/d*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x-4
*I*a/d*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x+4*I*a/d^2*f/(4*a^2+4*b^2)*ln(1+
I*exp(d*x+c))*c-4*I*a/d^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c+4/d*b*f/(4*a
^2+4*b^2)*ln(1+I*exp(d*x+c))*x+4/d*b*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x+4
/d^2*b*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*c+4/d^2*b*f/(4*a^2+4*b^2)*ln(1-I*
exp(d*x+c))*c-4/d^2*b*f*c/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))+1/a/d^2*b*f/(a
^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/a^2/d^2*b
^3*f/(a^2+b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))
)+1/a^2/d^2*b^3*f/(a^2+b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+
b^2)^(1/2)))+8*a/d^2*f*c/(4*a^2+4*b^2)*arctan(exp(d*x+c))+1/a^2/d*b^3*e/(a^
2+b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-4*I*a/d^2*f/(4*a^2+4*b^2)*dilo
g(1-I*exp(d*x+c))-1/(a^2+b^2)^(3/2)/d^2*f*b^3/a*arctanh(1/2*(2*b*exp(d*x+c)
+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)^(3/2)/d^2*a*f*b*arctanh(1/2*(2*b*exp(d*x
+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)^(3/2)/d*b^3*e/a*arctanh(1/2*(2*b*exp(
d*x+c)+2*a)/(a^2+b^2)^(1/2))+4*I*a/d^2*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c)
)+b/d^2*f*c/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2
))*a+4/d^2*b*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))+4/d^2*b*f/(4*a^2+4*b^2)*
dilog(1-I*exp(d*x+c))+4/d*b*e/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))-8*a/d*e/(4
*a^2+4*b^2)*arctan(exp(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(\frac{b^3\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{\left(a^4+a^2b^2\right)d}+\frac{2\,a\arctan\left(e^{(-dx-c)}\right)}{\left(a^2+b^2\right)d}+\frac{b\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\left(a^2+b^2\right)d}+\frac{2\,e^{(-dx-c)}}{\left(ae^{(-2\,dx-2\,c)}-a\right)d}-\frac{b\log\left(e^{(-dx\cdots}\right.}{a^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] (b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + a^2*b^2)*d) +
2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^
```

```
2 + b^2)*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-d*x -
 c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d))*e + (8*b*d*integrate(1/
8*x/(a^2*d*e^(d*x + c) + a^2*d), x) - 8*b*d*integrate(1/8*x/(a^2*d*e^(d*x +
 c) - a^2*d), x) + a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) + 1)/(a^2*d^2))
 - a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^2*d^2)) - 2*x*e^(d*x +
c)/(a*d*e^(2*d*x + 2*c) - a*d) - 8*integrate(-1/4*(a*b^3*x*e^(d*x + c) - b^
4*x)/(a^4*b + a^2*b^3 - (a^4*b*e^(2*c) + a^2*b^3*e^(2*c))*e^(2*d*x) - 2*(a^
5*e^c + a^3*b^2*e^c)*e^(d*x)), x) - 8*integrate(1/4*(a*x*e^(d*x + c) + b*x)
/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x))*f
```

Fricas [B] time = 3.03856, size = 6273, normalized size = 10.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] -(2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e)*cosh(d*x + c) - (b^3*f*cosh(d
*x + c)^2 + 2*b^3*f*cosh(d*x + c)*sinh(d*x + c) + b^3*f*sinh(d*x + c)^2 - b
^3*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*f*cosh(d*x + c)^2 + 2*b^
3*f*cosh(d*x + c)*sinh(d*x + c) + b^3*f*sinh(d*x + c)^2 - b^3*f)*dilog((a*c
osh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((
a^2 + b^2)/b^2) - b)/b + 1) + ((a^2*b + b^3)*f*cosh(d*x + c)^2 + 2*(a^2*b +
 b^3)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2*b + b^3)*f*sinh(d*x + c)^2 - (a^
2*b + b^3)*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) - (I*a^3*f - a^2*b*f + (
-I*a^3*f + a^2*b*f)*cosh(d*x + c)^2 - 2*(I*a^3*f - a^2*b*f)*cosh(d*x + c)*s
inh(d*x + c) + (-I*a^3*f + a^2*b*f)*sinh(d*x + c)^2)*dilog(I*cosh(d*x + c)
+ I*sinh(d*x + c)) - (-I*a^3*f - a^2*b*f + (I*a^3*f + a^2*b*f)*cosh(d*x + c
)^2 - 2*(-I*a^3*f - a^2*b*f)*cosh(d*x + c)*sinh(d*x + c) + (I*a^3*f + a^2*b
*f)*sinh(d*x + c)^2)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + ((a^2*b +
b^3)*f*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*f*cosh(d*x + c)*sinh(d*x + c) + (a
^2*b + b^3)*f*sinh(d*x + c)^2 - (a^2*b + b^3)*f)*dilog(-cosh(d*x + c) - sin
h(d*x + c)) + (b^3*d*e - b^3*c*f - (b^3*d*e - b^3*c*f)*cosh(d*x + c)^2 - 2*
(b^3*d*e - b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*e - b^3*c*f)*sinh(
d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2
)/b^2) + 2*a) + (b^3*d*e - b^3*c*f - (b^3*d*e - b^3*c*f)*cosh(d*x + c)^2 -
2*(b^3*d*e - b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*e - b^3*c*f)*sin
h(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b
^2)/b^2) + 2*a) + (b^3*d*f*x + b^3*c*f - (b^3*d*f*x + b^3*c*f)*cosh(d*x + c
)^2 - 2*(b^3*d*f*x + b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*f*x + b^
3*c*f)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b^3*d*f*x + b^3
*c*f - (b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^2 - 2*(b^3*d*f*x + b^3*c*f)*cosh
(d*x + c)*sinh(d*x + c) - (b^3*d*f*x + b^3*c*f)*sinh(d*x + c)^2)*log(-(a*co
sh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b) - ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e - ((a^2*b
 + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^3 + a*b^2)*f)*cosh(d*x + c)^2 - 2*((
a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^3 + a*b^2)*f)*cosh(d*x + c)*sin
h(d*x + c) - ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^3 + a*b^2)*f)*si
nh(d*x + c)^2 + (a^3 + a*b^2)*f)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (
I*a^3*d*e - a^2*b*d*e - I*a^3*c*f + a^2*b*c*f + (-I*a^3*d*e + a^2*b*d*e + I
*a^3*c*f - a^2*b*c*f)*cosh(d*x + c)^2 + (-2*I*a^3*d*e + 2*a^2*b*d*e + 2*I*a
^3*c*f - 2*a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (-I*a^3*d*e + a^2*b*d*e
 + I*a^3*c*f - a^2*b*c*f)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c
```

```
) + I) - (-I*a^3*d*e - a^2*b*d*e + I*a^3*c*f + a^2*b*c*f + (I*a^3*d*e + a^2
*b*d*e - I*a^3*c*f - a^2*b*c*f)*cosh(d*x + c)^2 + (2*I*a^3*d*e + 2*a^2*b*d*
e - 2*I*a^3*c*f - 2*a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (I*a^3*d*e + a
^2*b*d*e - I*a^3*c*f - a^2*b*c*f)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh
(d*x + c) - I) - ((a^2*b + b^3)*d*e - ((a^2*b + b^3)*d*e - (a^3 + a*b^2 + (
a^2*b + b^3)*c)*f)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d*e - (a^3 + a*b^2 +
(a^2*b + b^3)*c)*f)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d*e - (a^3
 + a*b^2 + (a^2*b + b^3)*c)*f)*sinh(d*x + c)^2 - (a^3 + a*b^2 + (a^2*b + b^
3)*c)*f)*log(cosh(d*x + c) + sinh(d*x + c) - 1) - (-I*a^3*d*f*x - a^2*b*d*f
*x - I*a^3*c*f - a^2*b*c*f + (I*a^3*d*f*x + a^2*b*d*f*x + I*a^3*c*f + a^2*b
*c*f)*cosh(d*x + c)^2 + (2*I*a^3*d*f*x + 2*a^2*b*d*f*x + 2*I*a^3*c*f + 2*a^
2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (I*a^3*d*f*x + a^2*b*d*f*x + I*a^3*c
*f + a^2*b*c*f)*sinh(d*x + c)^2)*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1)
 - (I*a^3*d*f*x - a^2*b*d*f*x + I*a^3*c*f - a^2*b*c*f + (-I*a^3*d*f*x + a^2
*b*d*f*x - I*a^3*c*f + a^2*b*c*f)*cosh(d*x + c)^2 + (-2*I*a^3*d*f*x + 2*a^2
*b*d*f*x - 2*I*a^3*c*f + 2*a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (-I*a^3
*d*f*x + a^2*b*d*f*x - I*a^3*c*f + a^2*b*c*f)*sinh(d*x + c)^2)*log(-I*cosh(
d*x + c) - I*sinh(d*x + c) + 1) - ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*c*f
- ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*c*f)*cosh(d*x + c)^2 - 2*((a^2*b + b
^3)*d*f*x + (a^2*b + b^3)*c*f)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)
*d*f*x + (a^2*b + b^3)*c*f)*sinh(d*x + c)^2)*log(-cosh(d*x + c) - sinh(d*x
+ c) + 1) + 2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e)*sinh(d*x + c))/((a^
4 + a^2*b^2)*d^2*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*d^2*cosh(d*x + c)*sinh
(d*x + c) + (a^4 + a^2*b^2)*d^2*sinh(d*x + c)^2 - (a^4 + a^2*b^2)*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.467 $\int \frac{\mathbf{csch}^2(c+dx)\mathbf{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=104

$$\frac{b^3\log(a+b\sinh(c+dx))}{a^2d\left(a^2+b^2\right)}-\frac{a\tan^{-1}(\sinh(c+dx))}{d\left(a^2+b^2\right)}+\frac{b\log(\cosh(c+dx))}{d\left(a^2+b^2\right)}-\frac{b\log(\sinh(c+dx))}{a^2d}-\frac{\mathbf{csch}(c+dx)}{ad}$$

```
[Out] -((a*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d)) - Csch[c + d*x]/(a*d) + (b*Log
[Cosh[c + d*x]])/((a^2 + b^2)*d) - (b*Log[Sinh[c + d*x]])/(a^2*d) + (b^3*Lo
g[a + b*Sinh[c + d*x]])/(a^2*(a^2 + b^2)*d)
```

Rubi [A] time = 0.169645, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2837, 12, 894, 635, 203, 260}

$$\frac{b^3\log(a+b\sinh(c+dx))}{a^2d\left(a^2+b^2\right)}-\frac{a\tan^{-1}(\sinh(c+dx))}{d\left(a^2+b^2\right)}+\frac{b\log(\cosh(c+dx))}{d\left(a^2+b^2\right)}-\frac{b\log(\sinh(c+dx))}{a^2d}-\frac{\mathbf{csch}(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d)) - Csch[c + d*x]/(a*d) + (b*Log
[Cosh[c + d*x]])/((a^2 + b^2)*d) - (b*Log[Sinh[c + d*x]])/(a^2*d) + (b^3*Lo
g[a + b*Sinh[c + d*x]])/(a^2*(a^2 + b^2)*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b\,\mathrm{Subst}\left(\int \frac{b^2}{x^2(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= -\frac{b^3\,\mathrm{Subst}\left(\int \frac{1}{x^2(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= -\frac{b^3\,\mathrm{Subst}\left(\int \left(-\frac{1}{ab^2x^2}+\frac{1}{a^2b^2x}-\frac{1}{a^2\left(a^2+b^2\right)(a+x)}+\frac{a-x}{b^2\left(a^2+b^2\right)\left(b^2+x^2\right)}\right) dx, x, b\sinh(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{csch}(c+dx)}{ad}-\frac{b\log(\sinh(c+dx))}{a^2d}+\frac{b^3\log(a+b\sinh(c+dx))}{a^2\left(a^2+b^2\right)d}-\frac{b\,\mathrm{Subst}\left(\int \frac{a-x}{b^2+x}\right.}{\left(a\right.}$$

$$= -\frac{\operatorname{csch}(c+dx)}{ad}-\frac{b\log(\sinh(c+dx))}{a^2d}+\frac{b^3\log(a+b\sinh(c+dx))}{a^2\left(a^2+b^2\right)d}+\frac{b\,\mathrm{Subst}\left(\int \frac{x}{b^2+x}\right.}{\left(a\right.}$$

$$= -\frac{a\tan^{-1}(\sinh(c+dx))}{\left(a^2+b^2\right)d}-\frac{\operatorname{csch}(c+dx)}{ad}+\frac{b\log(\cosh(c+dx))}{\left(a^2+b^2\right)d}-\frac{b\log(\sinh(c+dx))}{a^2d}$$

Mathematica [A] time = 0.593664, size = 160, normalized size = 1.54

$$-\frac{b^3\left(\frac{\log(\sinh(c+dx))}{a^2b^2}-\frac{\left(a\sqrt{-b^2}+b^2\right)\log\left(\sqrt{-b^2}-b\sinh(c+dx)\right)}{2b^4\left(a^2+b^2\right)}-\frac{\log(a+b\sinh(c+dx))}{a^2\left(a^2+b^2\right)}-\frac{\left(\frac{a}{\sqrt{-b^2}}+1\right)\log\left(\sqrt{-b^2}+b\sinh(c+dx)\right)}{2b^2\left(a^2+b^2\right)}+\frac{\operatorname{csch}(c+dx)}{ab^3}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((b^3*(Csch[c + d*x]/(a*b^3) + Log[Sinh[c + d*x]]/(a^2*b^2) - ((b^2 + a*Sq
rt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[c + d*x]])/(2*b^4*(a^2 + b^2)) - Log[a +
b*Sinh[c + d*x]]/(a^2*(a^2 + b^2)) - ((1 + a/Sqrt[-b^2])*Log[Sqrt[-b^2] + b
*Sinh[c + d*x]])/(2*b^2*(a^2 + b^2))))/d)
```

Maple [A] time = 0.003, size = 159, normalized size = 1.5

$$\frac{1}{2\,da}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\,da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-\frac{b}{da^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{b^3}{da^2\left(a^2+b^2\right)}\ln\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 a-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/2/d/a*tanh(1/2*d*x+1/2*c)-1/2/d/a/tanh(1/2*d*x+1/2*c)-1/d/a^2*b*ln(tanh(1
/2*d*x+1/2*c))+1/d*b^3/a^2/(a^2+b^2)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*
d*x+1/2*c)*b-a)+1/d/(a^2+b^2)*b*ln(tanh(1/2*d*x+1/2*c)^2+1)-2/d/(a^2+b^2)*a
*arctan(tanh(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.74286, size = 234, normalized size = 2.25

$$\frac{b^3 \log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{\left(a^4 + a^2b^2\right)d} + \frac{2\,a\arctan\left(e^{(-dx-c)}\right)}{\left(a^2 + b^2\right)d} + \frac{b\log\left(e^{(-2\,dx-2\,c)} + 1\right)}{\left(a^2 + b^2\right)d} + \frac{2\,e^{(-dx-c)}}{\left(ae^{(-2\,dx-2\,c)} - a\right)d} - \frac{b\log\left(e^{(-\right.}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + a^2*b^2)*d) + 2
*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^2
 + b^2)*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-d*x -
c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d)
```

Fricas [B] time = 2.8194, size = 1098, normalized size = 10.56

$$-\frac{2\left(a^3\cosh\left(dx+c\right)^2 + 2\,a^3\cosh\left(dx+c\right)\sinh\left(dx+c\right) + a^3\sinh\left(dx+c\right)^2 - a^3\right)\arctan\left(\cosh\left(dx+c\right) + \sinh\left(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -(2*(a^3*cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*x
 + c)^2 - a^3)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(a^3 + a*b^2)*cosh
(d*x + c) - (b^3*cosh(d*x + c)^2 + 2*b^3*cosh(d*x + c)*sinh(d*x + c) + b^3*
sinh(d*x + c)^2 - b^3)*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*
x + c))) - (a^2*b*cosh(d*x + c)^2 + 2*a^2*b*cosh(d*x + c)*sinh(d*x + c) + a
^2*b*sinh(d*x + c)^2 - a^2*b)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x
 + c))) - (a^2*b + b^3 - (a^2*b + b^3)*cosh(d*x + c)^2 - 2*(a^2*b + b^3)*co
sh(d*x + c)*sinh(d*x + c) - (a^2*b + b^3)*sinh(d*x + c)^2)*log(2*sinh(d*x +
 c)/(cosh(d*x + c) - sinh(d*x + c))) + 2*(a^3 + a*b^2)*sinh(d*x + c))/((a^4
 + a^2*b^2)*d*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x
+ c) + (a^4 + a^2*b^2)*d*sinh(d*x + c)^2 - (a^4 + a^2*b^2)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31908, size = 284, normalized size = 2.73

$$\frac{b^4 \log\left(\left|b\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 2a\right|\right)}{a^4bd + a^2b^3d} - \frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)} - 1\right)e^{(-dx-c)}\right)\right)a}{2\left(a^2d + b^2d\right)} + \frac{b\log\left(\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4\right)}{2\left(a^2d + b^2d\right)} - \frac{b\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^4*b*d + a^2*b^3*d) -
1/2*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*a/(a^2*d + b^2*
d) + 1/2*b*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2*d + b^2*d) - b*log(
abs(e^(d*x + c) - e^(-d*x - c)))/(a^2*d) + (b*(e^(d*x + c) - e^(-d*x - c))
- 2*a)/(a^2*d*(e^(d*x + c) - e^(-d*x - c)))
```

3.468 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Csch[c + d*x]^2*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [A] time = 0.0924191, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[c + d*x]^2*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Csch[c + d*x]^2*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A] time = 76.3949, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[c + d*x]^2*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] Integrate[(Csch[c + d*x]^2*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
, x]
```

Maple [A] time = 1.513, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx+c))^2\operatorname{sech}(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\,e^{(dx+c)}}{adfx+ade-\left(adfxe^{(2\,c)}+adee^{(2\,c)}\right)e^{(2\,dx)}}-8\int -\frac{ab^3e^{(dx}}{4\left(a^4be+a^2b^3e+\left(a^4bf+a^2b^3f\right)x-\left(a^4bee^{(2\,c)}+a^2b^3ee^{(2\,c)}+\left(a^4bfe^{(2}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] 2*e^(d*x + c)/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e*e^(2*c))*e^(2*d*x
)) - 8*integrate(-1/4*(a*b^3*e^(d*x + c) - b^4)/(a^4*b*e + a^2*b^3*e + (a^4
*b*f + a^2*b^3*f)*x - (a^4*b*e*e^(2*c) + a^2*b^3*e*e^(2*c) + (a^4*b*f*e^(2*
c) + a^2*b^3*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^5*e*e^c + a^3*b^2*e*e^c + (a^5*
f*e^c + a^3*b^2*f*e^c)*x)*e^(d*x)), x) - 8*integrate(-1/8*(b*d*f*x + b*d*e
+ a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*
a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) + 8*integrate(1/8*(b*d*f*x +
b*d*e - a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 + (a^2*d*f^2*x^2*e^
c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) - 8*integrate(1/4*(a*e^
(d*x + c) + b)/(a^2*e + b^2*e + (a^2*f + b^2*f)*x + (a^2*e*e^(2*c) + b^2*e*
e^(2*c) + (a^2*f*e^(2*c) + b^2*f*e^(2*c))*x)*e^(2*d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{csch}\left(dx+c\right)^2\operatorname{sech}\left(dx+c\right)}{afx+ae+\left(bfx+be\right)\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(csch(d*x + c)^2*sech(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.469 $\int \frac{(e+fx)^2 \mathbf{csch}^2(c+dx)\mathbf{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=914

result too large to display

```
[Out] (-2*(e + f*x)^2)/(a*d) + (b^2*(e + f*x)^2)/(a*(a^2 + b^2)*d) + (4*b*f*(e +
f*x)*ArcTan[E^(c + d*x)])/(a^2*d^2) - (4*b^3*f*(e + f*x)*ArcTan[E^(c + d*x)
])/(a^2*(a^2 + b^2)*d^2) + (2*b*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d) -
 (2*(e + f*x)^2*Coth[2*c + 2*d*x])/(a*d) + (b^4*(e + f*x)^2*Log[1 + (b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d) - (b^4*(e + f*x)
^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d
) - (2*b^2*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a*(a^2 + b^2)*d^2) + (2*f
*(e + f*x)*Log[1 - E^(4*(c + d*x))])/(a*d^2) + (2*b*f*(e + f*x)*PolyLog[2,
-E^(c + d*x)])/(a^2*d^2) - ((2*I)*b*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*
d^3) + ((2*I)*b^3*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^3) +
 ((2*I)*b*f^2*PolyLog[2, I*E^(c + d*x)])/(a^2*d^3) - ((2*I)*b^3*f^2*PolyLog
[2, I*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^3) - (2*b*f*(e + f*x)*PolyLog[2, E^(
c + d*x)])/(a^2*d^2) + (2*b^4*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2) - (2*b^4*f*(e + f*x)*PolyL
og[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2
) - (b^2*f^2*PolyLog[2, -E^(2*(c + d*x))])/(a*(a^2 + b^2)*d^3) + (f^2*PolyL
og[2, E^(4*(c + d*x))])/(2*a*d^3) - (2*b*f^2*PolyLog[3, -E^(c + d*x)])/(a^2
*d^3) + (2*b*f^2*PolyLog[3, E^(c + d*x)])/(a^2*d^3) - (2*b^4*f^2*PolyLog[3,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^3) + (
2*b^4*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*(a^2 +
 b^2)^(3/2)*d^3) - (b*(e + f*x)^2*Sech[c + d*x])/(a^2*d) + (b^3*(e + f*x)^2
*Sech[c + d*x])/(a^2*(a^2 + b^2)*d) + (b^2*(e + f*x)^2*Tanh[c + d*x])/(a*(a
^2 + b^2)*d)
```

Rubi [A] time = 2.03739, antiderivative size = 914, normalized size of antiderivative = 1., number of steps used = 51, number of rules used = 25, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.694, Rules used = {5589, 5461, 4184, 3716, 2190, 2279, 2391, 2622, 321, 207, 5462, 6741, 12, 6742, 6273, 4182, 2531, 2282, 6589, 4180, 5573, 3322, 2264, 3718, 5451}

$$\frac{(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)b^4}{a^2\left(a^2+b^2\right)^{3/2}d}-\frac{(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)b^4}{a^2\left(a^2+b^2\right)^{3/2}d}+\frac{2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)b^4}{a^2\left(a^2+b^2\right)^{3/2}d^2}-\frac{2f(e+fx)\text{Po}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*(e + f*x)^2)/(a*d) + (b^2*(e + f*x)^2)/(a*(a^2 + b^2)*d) + (4*b*f*(e +
f*x)*ArcTan[E^(c + d*x)])/(a^2*d^2) - (4*b^3*f*(e + f*x)*ArcTan[E^(c + d*x)
])/(a^2*(a^2 + b^2)*d^2) + (2*b*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d) -
 (2*(e + f*x)^2*Coth[2*c + 2*d*x])/(a*d) + (b^4*(e + f*x)^2*Log[1 + (b*E^(c
 + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d) - (b^4*(e + f*x)
^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d
) - (2*b^2*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a*(a^2 + b^2)*d^2) + (2*f
*(e + f*x)*Log[1 - E^(4*(c + d*x))])/(a*d^2) + (2*b*f*(e + f*x)*PolyLog[2,
-E^(c + d*x)])/(a^2*d^2) - ((2*I)*b*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*
d^3) + ((2*I)*b^3*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^3) +
 ((2*I)*b*f^2*PolyLog[2, I*E^(c + d*x)])/(a^2*d^3) - ((2*I)*b^3*f^2*PolyLog
[2, I*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^3) - (2*b*f*(e + f*x)*PolyLog[2, E^(
c + d*x)])/(a^2*d^2) + (2*b^4*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a -
```

```
Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2) - (2*b^4*f*(e + f*x)*PolyL
og[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2
) - (b^2*f^2*PolyLog[2, -E^(2*(c + d*x))])/(a*(a^2 + b^2)*d^3) + (f^2*PolyL
og[2, E^(4*(c + d*x))])/(2*a*d^3) - (2*b*f^2*PolyLog[3, -E^(c + d*x)])/(a^2
*d^3) + (2*b*f^2*PolyLog[3, E^(c + d*x)])/(a^2*d^3) - (2*b^4*f^2*PolyLog[3,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^3) + (
2*b^4*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*(a^2 +
 b^2)^(3/2)*d^3) - (b*(e + f*x)^2*Sech[c + d*x])/(a^2*d) + (b^3*(e + f*x)^2
*Sech[c + d*x])/(a^2*(a^2 + b^2)*d) + (b^2*(e + f*x)^2*Tanh[c + d*x])/(a*(a
^2 + b^2)*d)
```

Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
```

```
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\mathrm{csch}^2(c+dx)\mathrm{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\mathrm{csch}^2(c+dx)\mathrm{sech}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\mathrm{csch}(c+dx)\mathrm{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,d}{a}$$

$$= \frac{4\int (e+fx)^2\mathrm{csch}^2(2c+2dx)\,dx}{a} - \frac{b\int (e+fx)^2\mathrm{csch}(c+dx)\mathrm{sech}^2(c+dx)}{a^2}$$

$$= \frac{b(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{2(e+fx)^2\coth(2c+2dx)}{ad} - \frac{b(e+fx)}{}$$

$$= -\frac{2(e+fx)^2}{ad} + \frac{b(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{2(e+fx)^2\coth(2c+2d}{ad}$$

$$= -\frac{2(e+fx)^2}{ad} + \frac{b(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{2(e+fx)^2\coth(2c+2d}{ad}$$

$$= -\frac{2(e+fx)^2}{ad} + \frac{b(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{2(e+fx)^2\coth(2c+2d}{ad}$$

$$= -\frac{2(e+fx)^2}{ad} + \frac{b^2(e+fx)^2}{a\left(a^2+b^2\right)d} - \frac{4b^3f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d^2} + \frac{b(e+fx)^2\tan}{}$$

$$= -\frac{2(e+fx)^2}{ad} + \frac{b^2(e+fx)^2}{a\left(a^2+b^2\right)d} + \frac{4bf(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{4b^3f(e+fx)\tan}{a^2\left(a^2+\right.}$$

$$= -\frac{2(e+fx)^2}{ad} + \frac{b^2(e+fx)^2}{a\left(a^2+b^2\right)d} + \frac{4bf(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{4b^3f(e+fx)\tan}{a^2\left(a^2+\right.}$$

$$= -\frac{2(e+fx)^2}{ad} + \frac{b^2(e+fx)^2}{a\left(a^2+b^2\right)d} + \frac{4bf(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{4b^3f(e+fx)\tan}{a^2\left(a^2+\right.}$$

$$= -\frac{2(e+fx)^2}{ad} + \frac{b^2(e+fx)^2}{a\left(a^2+b^2\right)d} + \frac{4bf(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{4b^3f(e+fx)\tan}{a^2\left(a^2+\right.}$$

$$= -\frac{2(e+fx)^2}{ad} + \frac{b^2(e+fx)^2}{a\left(a^2+b^2\right)d} + \frac{4bf(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{4b^3f(e+fx)\tan}{a^2\left(a^2+\right.}$$

$$= -\frac{2(e+fx)^2}{ad} + \frac{b^2(e+fx)^2}{a\left(a^2+b^2\right)d} + \frac{4bf(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{4b^3f(e+fx)\tan}{a^2\left(a^2+\right.}$$

Mathematica [B] time = 28.6303, size = 2677, normalized size = 2.93

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x
]),x]
```

```
[Out] 4*(((I/2)*a*(-(d*(e + f*x)*(d*(e + f*x) + (1 + I*E^(2*c))*f*Log[1 - E^(-c -
 d*x)] + (1 + I*E^(2*c))*f*Log[1 + I*E^(-c - d*x)])) + (1 + I*E^(2*c))*f^2*
PolyLog[2, (-I)*E^(-c - d*x)] + (1 + I*E^(2*c))*f^2*PolyLog[2, E^(-c - d*x)
]))/((a^2 + b^2)*d^3*(-I + E^(2*c))) - (b*(4*a*b*d^2*e*E^(2*c)*f*x + 2*a*b*
d^2*E^(2*c)*f^2*x^2 + 2*a^2*d^2*e^2*ArcTanh[E^(c + d*x)] + 2*b^2*d^2*e^2*Ar
cTanh[E^(c + d*x)] - 2*a^2*d^2*e^2*E^(2*c)*ArcTanh[E^(c + d*x)] - 2*b^2*d^2
*e^2*E^(2*c)*ArcTanh[E^(c + d*x)] - 2*a^2*d^2*e*f*x*Log[1 - E^(c + d*x)] -
2*b^2*d^2*e*f*x*Log[1 - E^(c + d*x)] + 2*a^2*d^2*e*E^(2*c)*f*x*Log[1 - E^(c
 + d*x)] + 2*b^2*d^2*e*E^(2*c)*f*x*Log[1 - E^(c + d*x)] - a^2*d^2*f^2*x^2*L
og[1 - E^(c + d*x)] - b^2*d^2*f^2*x^2*Log[1 - E^(c + d*x)] + a^2*d^2*E^(2*c
)*f^2*x^2*Log[1 - E^(c + d*x)] + b^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x
)] + 2*a^2*d^2*e*f*x*Log[1 + E^(c + d*x)] + 2*b^2*d^2*e*f*x*Log[1 + E^(c +
d*x)] - 2*a^2*d^2*e*E^(2*c)*f*x*Log[1 + E^(c + d*x)] - 2*b^2*d^2*e*E^(2*c)*
f*x*Log[1 + E^(c + d*x)] + a^2*d^2*f^2*x^2*Log[1 + E^(c + d*x)] + b^2*d^2*f
^2*x^2*Log[1 + E^(c + d*x)] - a^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)]
- b^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] + 2*a*b*d*e*f*Log[1 - E^(2*(
c + d*x))] - 2*a*b*d*e*E^(2*c)*f*Log[1 - E^(2*(c + d*x))] + 2*a*b*d*f^2*x*L
og[1 - E^(2*(c + d*x))] - 2*a*b*d*E^(2*c)*f^2*x*Log[1 - E^(2*(c + d*x))] -
2*(a^2 + b^2)*d*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -E^(c + d*x)] + 2*(a^
2 + b^2)*d*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, E^(c + d*x)] + a*b*f^2*Pol
yLog[2, E^(2*(c + d*x))] - a*b*E^(2*c)*f^2*PolyLog[2, E^(2*(c + d*x))] - 2*
a^2*f^2*PolyLog[3, -E^(c + d*x)] - 2*b^2*f^2*PolyLog[3, -E^(c + d*x)] + 2*a
^2*E^(2*c)*f^2*PolyLog[3, -E^(c + d*x)] + 2*b^2*E^(2*c)*f^2*PolyLog[3, -E^(
c + d*x)] + 2*a^2*f^2*PolyLog[3, E^(c + d*x)] + 2*b^2*f^2*PolyLog[3, E^(c +
 d*x)] - 2*a^2*E^(2*c)*f^2*PolyLog[3, E^(c + d*x)] - 2*b^2*E^(2*c)*f^2*Poly
Log[3, E^(c + d*x)]))/(4*a^2*(a^2 + b^2)*d^3*(-1 + E^(2*c))) + (b^4*(-2*d^2
*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*x*Log[1 + (b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*
f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*f*(e +
 f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*f^2*PolyLog[
3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[3, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))]))/(4*a^2*(a^2 + b^2)^(3/2)*d^3) + (a*e*f*Sech[
c/2]*(Cosh[c/2]*Log[Cosh[c/2]*Cosh[(d*x)/2] + Sinh[c/2]*Sinh[(d*x)/2]] - (d
*x*Sinh[c/2])/2))/(2*(a^2 + b^2)*d^2*(Cosh[c/2]^2 - Sinh[c/2]^2)) - (a*f^2*
Csch[c/2]*(-(d^2*x^2)/(4*E^ArcTanh[Coth[c/2]]) + (I*Coth[c/2]*(-(d*x*(-Pi +
 (2*I)*ArcTanh[Coth[c/2]]))/2 - Pi*Log[1 + E^(d*x)] - 2*((I/2)*d*x + I*ArcT
anh[Coth[c/2]])*Log[1 - E^((2*I)*((I/2)*d*x + I*ArcTanh[Coth[c/2]]))] + Pi*
Log[Cosh[(d*x)/2]] + (2*I)*ArcTanh[Coth[c/2]]*Log[I*Sinh[(d*x)/2 + ArcTanh[
Coth[c/2]]]] + I*PolyLog[2, E^((2*I)*((I/2)*d*x + I*ArcTanh[Coth[c/2]]))]))
/Sqrt[1 - Coth[c/2]^2])*Sech[c/2])/(2*(a^2 + b^2)*d^3*Sqrt[Csch[c/2]^2*(-Co
sh[c/2]^2 + Sinh[c/2]^2)]) - (e*f*x*Csch[c/2]*Sech[c/2]*(a^2*Cosh[c] - b^2*
Cosh[c] + a^2*Cosh[2*c] - I*a^2*Sinh[c] - I*b^2*Sinh[c]))/(8*a*(a^2 + b^2)*
d*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c] + I*Sinh[c])
) - (f^2*x^2*Csch[c/2]*Sech[c/2]*(a^2*Cosh[c] - b^2*Cosh[c] + a^2*Cosh[2*c]
 - I*a^2*Sinh[c] - I*b^2*Sinh[c]))/(16*a*(a^2 + b^2)*d*(Cosh[c/2] - I*Sinh[
c/2])*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c] + I*Sinh[c])) + (b*e*f*ArcTan[(Sin
h[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]])/((a^2 + b^2)*d^
2*Sqrt[Cosh[c]^2 - Sinh[c]^2]) - ((I/2)*a*f*(Cosh[c] + Sinh[c])*(((e + f*x)
```

```
^2*(Cosh[c] - Sinh[c]))/(2*f) + ((e + f*x)*Log[1 - I*Cosh[c + d*x] + I*Sinh
[c + d*x]]*(I + Cosh[c] - Sinh[c]))/d - (I*f*PolyLog[2, I*(Cosh[c + d*x] -
Sinh[c + d*x])]*(Cosh[c] - Sinh[c])*(-I + Cosh[c] + Sinh[c]))/d^2))/((a^2 +
 b^2)*d*(-I + Cosh[c] + Sinh[c])) + (b*f^2*(((-I)*Csch[c]*(I*(d*x + ArcTanh
[Coth[c]])*(Log[1 - E^(-(d*x) - ArcTanh[Coth[c]])] - Log[1 + E^(-(d*x) - Ar
cTanh[Coth[c]])]) + I*(PolyLog[2, -E^(-(d*x) - ArcTanh[Coth[c]])] - PolyLog
[2, E^(-(d*x) - ArcTanh[Coth[c]])])))/Sqrt[1 - Coth[c]^2] - (2*ArcTan[(Sinh
[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]]*ArcTanh[Coth[c]])
/Sqrt[Cosh[c]^2 - Sinh[c]^2]))/(2*(a^2 + b^2)*d^3) + (Csch[2*c]*Csch[2*c +
2*d*x]*(a*b*e^2*Cosh[c - d*x] + 2*a*b*e*f*x*Cosh[c - d*x] + a*b*f^2*x^2*Cos
h[c - d*x] - a*b*e^2*Cosh[3*c + d*x] - 2*a*b*e*f*x*Cosh[3*c + d*x] - a*b*f^
2*x^2*Cosh[3*c + d*x] - b^2*e^2*Sinh[2*c] - 2*b^2*e*f*x*Sinh[2*c] - b^2*f^2
*x^2*Sinh[2*c] + 2*a^2*e^2*Sinh[2*d*x] + b^2*e^2*Sinh[2*d*x] + 4*a^2*e*f*x*
Sinh[2*d*x] + 2*b^2*e*f*x*Sinh[2*d*x] + 2*a^2*f^2*x^2*Sinh[2*d*x] + b^2*f^2
*x^2*Sinh[2*d*x]))/(4*a*(a^2 + b^2)*d))
```

Maple [F] time = 1.658, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \left(\operatorname{csch}\left(dx+c\right)\right)^{2} \left(\operatorname{sech}\left(dx+c\right)\right)^{2}}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)

[Out] int((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="maxima")

[Out] Exception raised: ValueError
```

Fricas [C] time = 5.02976, size = 23058, normalized size = 25.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="fricas")

[Out] -1/2*(4*(2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*e^2 - 8*(2*a^5 + 3*a^3*b^2 + a*b^4)
*c*d*e*f + 4*(2*a^5 + 3*a^3*b^2 + a*b^4)*c^2*f^2 + 4*((2*a^5 + 3*a^3*b^2 +
a*b^4)*d^2*f^2*x^2 + 2*(2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*e*f*x + 2*(2*a^5 + 3
*a^3*b^2 + a*b^4)*c*d*e*f - (2*a^5 + 3*a^3*b^2 + a*b^4)*c^2*f^2)*cosh(d*x +
```

c)^4 + 4*((2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*e*f*x + 2*(2*a^5 + 3*a^3*b^2 + a*b^4)*c*d*e*f - (2*a^5 + 3*a^3*b^2 + a*b^4)*c^2*f^2)*sinh(d*x + c)^4 + 4*((a^4*b + a^2*b^3)*d^2*f^2*x^2 + 2*(a^4*b + a^2*b^3)*d^2*e*f*x + (a^4*b + a^2*b^3)*d^2*e^2)*cosh(d*x + c)^3 + 4*((a^4*b + a^2*b^3)*d^2*f^2*x^2 + 2*(a^4*b + a^2*b^3)*d^2*e*f*x + (a^4*b + a^2*b^3)*d^2*e^2 + 4*((2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*e*f*x + 2*(2*a^5 + 3*a^3*b^2 + a*b^4)*c*d*e*f - (2*a^5 + 3*a^3*b^2 + a*b^4)*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*((a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(a^3*b^2 + a*b^4)*d^2*e*f*x + (a^3*b^2 + a*b^4)*d^2*e^2)*cosh(d*x + c)^2 + 4*((a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(a^3*b^2 + a*b^4)*d^2*e*f*x + (a^3*b^2 + a*b^4)*d^2*e^2 + 6*((2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*e*f*x + 2*(2*a^5 + 3*a^3*b^2 + a*b^4)*c*d*e*f - (2*a^5 + 3*a^3*b^2 + a*b^4)*c^2*f^2)*cosh(d*x + c)^2 + 3*((a^4*b + a^2*b^3)*d^2*f^2*x^2 + 2*(a^4*b + a^2*b^3)*d^2*e*f*x + (a^4*b + a^2*b^3)*d^2*e^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*(b^5*d*f^2*x + b^5*d*e*f - (b^5*d*f^2*x + b^5*d*e*f)*cosh(d*x + c)^4 - 4*(b^5*d*f^2*x + b^5*d*e*f)*cosh(d*x + c)^3*sinh(d*x + c) - 6*(b^5*d*f^2*x + b^5*d*e*f)*cosh(d*x + c)^2*sinh(d*x + c)^2 - 4*(b^5*d*f^2*x + b^5*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 - (b^5*d*f^2*x + b^5*d*e*f)*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 4*(b^5*d*f^2*x + b^5*d*e*f - (b^5*d*f^2*x + b^5*d*e*f)*cosh(d*x + c)^4 - 4*(b^5*d*f^2*x + b^5*d*e*f)*cosh(d*x + c)^3*sinh(d*x + c) - 6*(b^5*d*f^2*x + b^5*d*e*f)*cosh(d*x + c)^2*sinh(d*x + c)^2 - 4*(b^5*d*f^2*x + b^5*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 - (b^5*d*f^2*x + b^5*d*e*f)*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b^5*d^2*e^2 - 2*b^5*c*d*e*f + b^5*c^2*f^2 - (b^5*d^2*e^2 - 2*b^5*c*d*e*f + b^5*c^2*f^2)*cosh(d*x + c)^4 - 4*(b^5*d^2*e^2 - 2*b^5*c*d*e*f + b^5*c^2*f^2)*cosh(d*x + c)^3*sinh(d*x + c) - 6*(b^5*d^2*e^2 - 2*b^5*c*d*e*f + b^5*c^2*f^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 - 4*(b^5*d^2*e^2 - 2*b^5*c*d*e*f + b^5*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 - (b^5*d^2*e^2 - 2*b^5*c*d*e*f + b^5*c^2*f^2)*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(b^5*d^2*e^2 - 2*b^5*c*d*e*f + b^5*c^2*f^2 - (b^5*d^2*e^2 - 2*b^5*c*d*e*f + b^5*c^2*f^2)*cosh(d*x + c)^4 - 4*(b^5*d^2*e^2 - 2*b^5*c*d*e*f + b^5*c^2*f^2)*cosh(d*x + c)^3*sinh(d*x + c) - 6*(b^5*d^2*e^2 - 2*b^5*c*d*e*f + b^5*c^2*f^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 - 4*(b^5*d^2*e^2 - 2*b^5*c*d*e*f + b^5*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 - (b^5*d^2*e^2 - 2*b^5*c*d*e*f + b^5*c^2*f^2)*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(b^5*d^2*f^2*x^2 + 2*b^5*d^2*e*f*x + 2*b^5*c*d*e*f - b^5*c^2*f^2 - (b^5*d^2*f^2*x^2 + 2*b^5*d^2*e*f*x + 2*b^5*c*d*e*f - b^5*c^2*f^2)*cosh(d*x + c)^4 - 4*(b^5*d^2*f^2*x^2 + 2*b^5*d^2*e*f*x + 2*b^5*c*d*e*f - b^5*c^2*f^2)*cosh(d*x + c)^3*sinh(d*x + c) - 6*(b^5*d^2*f^2*x^2 + 2*b^5*d^2*e*f*x + 2*b^5*c*d*e*f - b^5*c^2*f^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 - 4*(b^5*d^2*f^2*x^2 + 2*b^5*d^2*e*f*x + 2*b^5*c*d*e*f - b^5*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 - (b^5*d^2*f^2*x^2 + 2*b^5*d^2*e*f*x + 2*b^5*c*d*e*f - b^5*c^2*f^2)*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(b^5*d^2*f^2*x^2 + 2*b^5*d^2*e*f*x + 2*b^5*c*d*e*f - b^5*c^2*f^2 - (b^5*d^2*f^2*x^2 + 2*b^5*d^2*e*f*x + 2*b^5*c*d*e*f - b^5*c^2*f^2)*cosh(d*x + c)^4 - 4*(b^5*d^2*f^2*x^2 + 2*b^5*d^2*e*f*x + 2*b^5*c*d*e*f - b^5*c^2*f^2)*cosh(d*x + c)^3*sinh(d*x + c) - 6*(b^5*d^2*f^2*x^2 + 2*b^5*d^2*e*f*x + 2*b^5*c*d*e*f - b^5*c^2*f^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 - 4*(b^5*d^2*f^2*x^2 + 2*b^5*d^2*e*f*x + 2*b^5*c*d*e*f - b^5*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 - (b^5*d^2*f^2*x^2 + 2*b^5*d^2*e*f*x + 2*b^5*c*d*e*f - b^5*c^2*f^2)*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*(b^5*f^2*cosh(d*x + c)^4 + 4*b^5*f^2*cosh(d*x + c)^3*sinh(d*x + c) + 6*b^5*f^2*cosh(d*x +

c)^2*sinh(d*x + c)^2 + 4*b^5*f^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^5*f^2*s
inh(d*x + c)^4 - b^5*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c)
 + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2))/b) - 4*(b^5*f^2*cosh(d*x + c)^4 + 4*b^5*f^2*cosh(d*x + c)^3*sinh(d*x +
 c) + 6*b^5*f^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^5*f^2*cosh(d*x + c)*s
inh(d*x + c)^3 + b^5*f^2*sinh(d*x + c)^4 - b^5*f^2)*sqrt((a^2 + b^2)/b^2)*p
olylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2))/b) - 4*((a^4*b + a^2*b^3)*d^2*f^2*x^2 + 2*(a
^4*b + a^2*b^3)*d^2*e*f*x + (a^4*b + a^2*b^3)*d^2*e^2)*cosh(d*x + c) - 4*((
a^4*b + 2*a^2*b^3 + b^5)*d*f^2*x - ((a^4*b + 2*a^2*b^3 + b^5)*d*f^2*x + (a^
4*b + 2*a^2*b^3 + b^5)*d*e*f - (a^5 + 2*a^3*b^2 + a*b^4)*f^2)*cosh(d*x + c)
^4 - 4*((a^4*b + 2*a^2*b^3 + b^5)*d*f^2*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e*f
 - (a^5 + 2*a^3*b^2 + a*b^4)*f^2)*cosh(d*x + c)^3*sinh(d*x + c) - 6*((a^4*b
 + 2*a^2*b^3 + b^5)*d*f^2*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e*f - (a^5 + 2*a^
3*b^2 + a*b^4)*f^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 - 4*((a^4*b + 2*a^2*b^3
 + b^5)*d*f^2*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e*f - (a^5 + 2*a^3*b^2 + a*b^
4)*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 - ((a^4*b + 2*a^2*b^3 + b^5)*d*f^2*x
+ (a^4*b + 2*a^2*b^3 + b^5)*d*e*f - (a^5 + 2*a^3*b^2 + a*b^4)*f^2)*sinh(d*x
 + c)^4 + (a^4*b + 2*a^2*b^3 + b^5)*d*e*f - (a^5 + 2*a^3*b^2 + a*b^4)*f^2)*
dilog(cosh(d*x + c) + sinh(d*x + c)) - (4*((a^5 + a^3*b^2)*f^2 + I*(a^4*b +
 a^2*b^3)*f^2)*cosh(d*x + c)^4 + 16*((a^5 + a^3*b^2)*f^2 + I*(a^4*b + a^2*b
^3)*f^2)*cosh(d*x + c)^3*sinh(d*x + c) + 24*((a^5 + a^3*b^2)*f^2 + I*(a^4*b
 + a^2*b^3)*f^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 16*((a^5 + a^3*b^2)*f^2
+ I*(a^4*b + a^2*b^3)*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*((a^5 + a^3*b^
2)*f^2 + I*(a^4*b + a^2*b^3)*f^2)*sinh(d*x + c)^4 - 4*(a^5 + a^3*b^2)*f^2 -
 4*I*(a^4*b + a^2*b^3)*f^2)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) - (4*(
(a^5 + a^3*b^2)*f^2 - I*(a^4*b + a^2*b^3)*f^2)*cosh(d*x + c)^4 + 16*((a^5 +
 a^3*b^2)*f^2 - I*(a^4*b + a^2*b^3)*f^2)*cosh(d*x + c)^3*sinh(d*x + c) + 24
*((a^5 + a^3*b^2)*f^2 - I*(a^4*b + a^2*b^3)*f^2)*cosh(d*x + c)^2*sinh(d*x +
 c)^2 + 16*((a^5 + a^3*b^2)*f^2 - I*(a^4*b + a^2*b^3)*f^2)*cosh(d*x + c)*si
nh(d*x + c)^3 + 4*((a^5 + a^3*b^2)*f^2 - I*(a^4*b + a^2*b^3)*f^2)*sinh(d*x
+ c)^4 - 4*(a^5 + a^3*b^2)*f^2 + 4*I*(a^4*b + a^2*b^3)*f^2)*dilog(-I*cosh(d
*x + c) - I*sinh(d*x + c)) + 4*((a^4*b + 2*a^2*b^3 + b^5)*d*f^2*x - ((a^4*b
 + 2*a^2*b^3 + b^5)*d*f^2*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e*f + (a^5 + 2*a^
3*b^2 + a*b^4)*f^2)*cosh(d*x + c)^4 - 4*((a^4*b + 2*a^2*b^3 + b^5)*d*f^2*x
+ (a^4*b + 2*a^2*b^3 + b^5)*d*e*f + (a^5 + 2*a^3*b^2 + a*b^4)*f^2)*cosh(d*x
 + c)^3*sinh(d*x + c) - 6*((a^4*b + 2*a^2*b^3 + b^5)*d*f^2*x + (a^4*b + 2*a
^2*b^3 + b^5)*d*e*f + (a^5 + 2*a^3*b^2 + a*b^4)*f^2)*cosh(d*x + c)^2*sinh(d
x + c)^2 - 4((a^4*b + 2*a^2*b^3 + b^5)*d*f^2*x + (a^4*b + 2*a^2*b^3 + b^5
)*d*e*f + (a^5 + 2*a^3*b^2 + a*b^4)*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 - ((
a^4*b + 2*a^2*b^3 + b^5)*d*f^2*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e*f + (a^5 +
 2*a^3*b^2 + a*b^4)*f^2)*sinh(d*x + c)^4 + (a^4*b + 2*a^2*b^3 + b^5)*d*e*f
+ (a^5 + 2*a^3*b^2 + a*b^4)*f^2)*dilog(-cosh(d*x + c) - sinh(d*x + c)) + 2*
((a^4*b + 2*a^2*b^3 + b^5)*d^2*f^2*x^2 + (a^4*b + 2*a^2*b^3 + b^5)*d^2*e^2
- ((a^4*b + 2*a^2*b^3 + b^5)*d^2*f^2*x^2 + (a^4*b + 2*a^2*b^3 + b^5)*d^2*e^
2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*d*e*f + 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e*
f + (a^5 + 2*a^3*b^2 + a*b^4)*d*f^2)*x)*cosh(d*x + c)^4 - 4*((a^4*b + 2*a^2
*b^3 + b^5)*d^2*f^2*x^2 + (a^4*b + 2*a^2*b^3 + b^5)*d^2*e^2 + 2*(a^5 + 2*a^
3*b^2 + a*b^4)*d*e*f + 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e*f + (a^5 + 2*a^3*
b^2 + a*b^4)*d*f^2)*x)*cosh(d*x + c)^3*sinh(d*x + c) - 6*((a^4*b + 2*a^2*b^
3 + b^5)*d^2*f^2*x^2 + (a^4*b + 2*a^2*b^3 + b^5)*d^2*e^2 + 2*(a^5 + 2*a^3*b
^2 + a*b^4)*d*e*f + 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e*f + (a^5 + 2*a^3*b^2
 + a*b^4)*d*f^2)*x)*cosh(d*x + c)^2*sinh(d*x + c)^2 - 4*((a^4*b + 2*a^2*b^3
 + b^5)*d^2*f^2*x^2 + (a^4*b + 2*a^2*b^3 + b^5)*d^2*e^2 + 2*(a^5 + 2*a^3*b^
2 + a*b^4)*d*e*f + 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e*f + (a^5 + 2*a^3*b^2
+ a*b^4)*d*f^2)*x)*cosh(d*x + c)*sinh(d*x + c)^3 - ((a^4*b + 2*a^2*b^3 + b^
5)*d^2*f^2*x^2 + (a^4*b + 2*a^2*b^3 + b^5)*d^2*e^2 + 2*(a^5 + 2*a^3*b^2 + a
*b^4)*d*e*f + 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e*f + (a^5 + 2*a^3*b^2 + a*b
^4)*d*f^2)*x)*sinh(d*x + c)^4 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*d*e*f + 2*((a^4

*b + 2*a^2*b^3 + b^5)*d^2*e*f + (a^5 + 2*a^3*b^2 + a*b^4)*d*f^2)*x)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - ((4*(a^5 + a^3*b^2)*d*e*f + 4*I*(a^4*b + a^2*b^3)*d*e*f - 4*(a^5 + a^3*b^2)*c*f^2 - 4*I*(a^4*b + a^2*b^3)*c*f^2)*cosh(d*x + c)^4 + (16*(a^5 + a^3*b^2)*d*e*f + 16*I*(a^4*b + a^2*b^3)*d*e*f - 16*(a^5 + a^3*b^2)*c*f^2 - 16*I*(a^4*b + a^2*b^3)*c*f^2)*cosh(d*x + c)^3*sinh(d*x + c) + (24*(a^5 + a^3*b^2)*d*e*f + 24*I*(a^4*b + a^2*b^3)*d*e*f - 24*(a^5 + a^3*b^2)*c*f^2 - 24*I*(a^4*b + a^2*b^3)*c*f^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 + (16*(a^5 + a^3*b^2)*d*e*f + 16*I*(a^4*b + a^2*b^3)*d*e*f - 16*(a^5 + a^3*b^2)*c*f^2 - 16*I*(a^4*b + a^2*b^3)*c*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (4*(a^5 + a^3*b^2)*d*e*f + 4*I*(a^4*b + a^2*b^3)*d*e*f - 4*(a^5 + a^3*b^2)*c*f^2 - 4*I*(a^4*b + a^2*b^3)*c*f^2)*sinh(d*x + c)^4 - 4*(a^5 + a^3*b^2)*d*e*f - 4*I*(a^4*b + a^2*b^3)*d*e*f + 4*(a^5 + a^3*b^2)*c*f^2 + 4*I*(a^4*b + a^2*b^3)*c*f^2)*log(cosh(d*x + c) + sinh(d*x + c) + I) - ((4*(a^5 + a^3*b^2)*d*e*f - 4*I*(a^4*b + a^2*b^3)*d*e*f - 4*(a^5 + a^3*b^2)*c*f^2 + 4*I*(a^4*b + a^2*b^3)*c*f^2)*cosh(d*x + c)^4 + (16*(a^5 + a^3*b^2)*d*e*f - 16*I*(a^4*b + a^2*b^3)*d*e*f - 16*(a^5 + a^3*b^2)*c*f^2 + 16*I*(a^4*b + a^2*b^3)*c*f^2)*cosh(d*x + c)^3*sinh(d*x + c) + (24*(a^5 + a^3*b^2)*d*e*f - 24*I*(a^4*b + a^2*b^3)*d*e*f - 24*(a^5 + a^3*b^2)*c*f^2 + 24*I*(a^4*b + a^2*b^3)*c*f^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 + (16*(a^5 + a^3*b^2)*d*e*f - 16*I*(a^4*b + a^2*b^3)*d*e*f - 16*(a^5 + a^3*b^2)*c*f^2 + 16*I*(a^4*b + a^2*b^3)*c*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (4*(a^5 + a^3*b^2)*d*e*f - 4*I*(a^4*b + a^2*b^3)*d*e*f - 4*(a^5 + a^3*b^2)*c*f^2 + 4*I*(a^4*b + a^2*b^3)*c*f^2)*sinh(d*x + c)^4 - 4*(a^5 + a^3*b^2)*d*e*f + 4*I*(a^4*b + a^2*b^3)*d*e*f + 4*(a^5 + a^3*b^2)*c*f^2 - 4*I*(a^4*b + a^2*b^3)*c*f^2)*log(cosh(d*x + c) + sinh(d*x + c) - I) - 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e^2 - ((a^4*b + 2*a^2*b^3 + b^5)*d^2*e^2 - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*c)*d*e*f + ((a^4*b + 2*a^2*b^3 + b^5)*c^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*c)*f^2)*cosh(d*x + c)^4 - 4*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e^2 - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*c)*d*e*f + ((a^4*b + 2*a^2*b^3 + b^5)*c^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*c)*f^2)*cosh(d*x + c)^3*sinh(d*x + c) - 6*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e^2 - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*c)*d*e*f + ((a^4*b + 2*a^2*b^3 + b^5)*c^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*c)*f^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 - 4*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e^2 - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*c)*d*e*f + ((a^4*b + 2*a^2*b^3 + b^5)*c^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*c)*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 - ((a^4*b + 2*a^2*b^3 + b^5)*d^2*e^2 - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*c)*d*e*f + ((a^4*b + 2*a^2*b^3 + b^5)*c^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*c)*f^2)*sinh(d*x + c)^4 - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*c)*d*e*f + ((a^4*b + 2*a^2*b^3 + b^5)*c^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*c)*f^2)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + (4*(a^5 + a^3*b^2)*d*f^2*x - 4*I*(a^4*b + a^2*b^3)*d*f^2*x - (4*(a^5 + a^3*b^2)*d*f^2*x - 4*I*(a^4*b + a^2*b^3)*d*f^2*x + 4*(a^5 + a^3*b^2)*c*f^2 - 4*I*(a^4*b + a^2*b^3)*c*f^2)*cosh(d*x + c)^4 - (16*(a^5 + a^3*b^2)*d*f^2*x - 16*I*(a^4*b + a^2*b^3)*d*f^2*x + 16*(a^5 + a^3*b^2)*c*f^2 - 16*I*(a^4*b + a^2*b^3)*c*f^2)*cosh(d*x + c)^3*sinh(d*x + c) - (24*(a^5 + a^3*b^2)*d*f^2*x - 24*I*(a^4*b + a^2*b^3)*d*f^2*x + 24*(a^5 + a^3*b^2)*c*f^2 - 24*I*(a^4*b + a^2*b^3)*c*f^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 - (16*(a^5 + a^3*b^2)*d*f^2*x - 16*I*(a^4*b + a^2*b^3)*d*f^2*x + 16*(a^5 + a^3*b^2)*c*f^2 - 16*I*(a^4*b + a^2*b^3)*c*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 - (4*(a^5 + a^3*b^2)*d*f^2*x - 4*I*(a^4*b + a^2*b^3)*d*f^2*x + 4*(a^5 + a^3*b^2)*c*f^2 - 4*I*(a^4*b + a^2*b^3)*c*f^2)*sinh(d*x + c)^4 + 4*(a^5 + a^3*b^2)*c*f^2 - 4*I*(a^4*b + a^2*b^3)*c*f^2)*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) + (4*(a^5 + a^3*b^2)*d*f^2*x + 4*I*(a^4*b + a^2*b^3)*d*f^2*x - (4*(a^5 + a^3*b^2)*d*f^2*x + 4*I*(a^4*b + a^2*b^3)*d*f^2*x + 4*(a^5 + a^3*b^2)*c*f^2 + 4*I*(a^4*b + a^2*b^3)*c*f^2)*cosh(d*x + c)^4 - (16*(a^5 + a^3*b^2)*d*f^2*x + 16*I*(a^4*b + a^2*b^3)*d*f^2*x + 16*(a^5 + a^3*b^2)*c*f^2 + 16*I*(a^4*b + a^2*b^3)*c*f^2)*cosh(d*x + c)^3*sinh(d*x + c) - (24*(a^5 + a^3*b^2)*d*f^2*x + 24*I*(a^4*b + a^2*b^3)*d*f^2*x + 24*(a^5 + a^3*b^2)*c*f^2 + 24*I*(a^4*b + a^2*b^3)*c*f^2)*cosh(d*x + c)^2*sinh(d*x + c)

```
^2 - (16*(a^5 + a^3*b^2)*d*f^2*x + 16*I*(a^4*b + a^2*b^3)*d*f^2*x + 16*(a^5
 + a^3*b^2)*c*f^2 + 16*I*(a^4*b + a^2*b^3)*c*f^2)*cosh(d*x + c)*sinh(d*x +
c)^3 - (4*(a^5 + a^3*b^2)*d*f^2*x + 4*I*(a^4*b + a^2*b^3)*d*f^2*x + 4*(a^5
+ a^3*b^2)*c*f^2 + 4*I*(a^4*b + a^2*b^3)*c*f^2)*sinh(d*x + c)^4 + 4*(a^5 +
a^3*b^2)*c*f^2 + 4*I*(a^4*b + a^2*b^3)*c*f^2)*log(-I*cosh(d*x + c) - I*sinh
(d*x + c) + 1) - 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*f^2*x^2 + 2*(a^4*b + 2*a^
2*b^3 + b^5)*c*d*e*f - ((a^4*b + 2*a^2*b^3 + b^5)*d^2*f^2*x^2 + 2*(a^4*b +
2*a^2*b^3 + b^5)*c*d*e*f - ((a^4*b + 2*a^2*b^3 + b^5)*c^2 + 2*(a^5 + 2*a^3*
b^2 + a*b^4)*c)*f^2 + 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e*f - (a^5 + 2*a^3*b
^2 + a*b^4)*d*f^2)*x)*cosh(d*x + c)^4 - 4*((a^4*b + 2*a^2*b^3 + b^5)*d^2*f^
2*x^2 + 2*(a^4*b + 2*a^2*b^3 + b^5)*c*d*e*f - ((a^4*b + 2*a^2*b^3 + b^5)*c^
2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*c)*f^2 + 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e
*f - (a^5 + 2*a^3*b^2 + a*b^4)*d*f^2)*x)*cosh(d*x + c)^3*sinh(d*x + c) - 6*
((a^4*b + 2*a^2*b^3 + b^5)*d^2*f^2*x^2 + 2*(a^4*b + 2*a^2*b^3 + b^5)*c*d*e*
f - ((a^4*b + 2*a^2*b^3 + b^5)*c^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*c)*f^2 + 2
*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e*f - (a^5 + 2*a^3*b^2 + a*b^4)*d*f^2)*x)*c
osh(d*x + c)^2*sinh(d*x + c)^2 - 4*((a^4*b + 2*a^2*b^3 + b^5)*d^2*f^2*x^2 +
 2*(a^4*b + 2*a^2*b^3 + b^5)*c*d*e*f - ((a^4*b + 2*a^2*b^3 + b^5)*c^2 + 2*(
a^5 + 2*a^3*b^2 + a*b^4)*c)*f^2 + 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e*f - (a
^5 + 2*a^3*b^2 + a*b^4)*d*f^2)*x)*cosh(d*x + c)*sinh(d*x + c)^3 - ((a^4*b +
 2*a^2*b^3 + b^5)*d^2*f^2*x^2 + 2*(a^4*b + 2*a^2*b^3 + b^5)*c*d*e*f - ((a^4
*b + 2*a^2*b^3 + b^5)*c^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*c)*f^2 + 2*((a^4*b
+ 2*a^2*b^3 + b^5)*d^2*e*f - (a^5 + 2*a^3*b^2 + a*b^4)*d*f^2)*x)*sinh(d*x +
 c)^4 - ((a^4*b + 2*a^2*b^3 + b^5)*c^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*c)*f^2
 + 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e*f - (a^5 + 2*a^3*b^2 + a*b^4)*d*f^2)*
x)*log(-cosh(d*x + c) - sinh(d*x + c) + 1) - 4*((a^4*b + 2*a^2*b^3 + b^5)*f
^2*cosh(d*x + c)^4 + 4*(a^4*b + 2*a^2*b^3 + b^5)*f^2*cosh(d*x + c)^3*sinh(d
*x + c) + 6*(a^4*b + 2*a^2*b^3 + b^5)*f^2*cosh(d*x + c)^2*sinh(d*x + c)^2 +
 4*(a^4*b + 2*a^2*b^3 + b^5)*f^2*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + 2
*a^2*b^3 + b^5)*f^2*sinh(d*x + c)^4 - (a^4*b + 2*a^2*b^3 + b^5)*f^2)*polylo
g(3, cosh(d*x + c) + sinh(d*x + c)) + 4*((a^4*b + 2*a^2*b^3 + b^5)*f^2*cosh
(d*x + c)^4 + 4*(a^4*b + 2*a^2*b^3 + b^5)*f^2*cosh(d*x + c)^3*sinh(d*x + c)
 + 6*(a^4*b + 2*a^2*b^3 + b^5)*f^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^4
*b + 2*a^2*b^3 + b^5)*f^2*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + 2*a^2*b^
3 + b^5)*f^2*sinh(d*x + c)^4 - (a^4*b + 2*a^2*b^3 + b^5)*f^2)*polylog(3, -c
osh(d*x + c) - sinh(d*x + c)) - 4*((a^4*b + a^2*b^3)*d^2*f^2*x^2 + 2*(a^4*b
 + a^2*b^3)*d^2*e*f*x + (a^4*b + a^2*b^3)*d^2*e^2 - 4*((2*a^5 + 3*a^3*b^2 +
 a*b^4)*d^2*f^2*x^2 + 2*(2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*e*f*x + 2*(2*a^5 +
3*a^3*b^2 + a*b^4)*c*d*e*f - (2*a^5 + 3*a^3*b^2 + a*b^4)*c^2*f^2)*cosh(d*x
+ c)^3 - 3*((a^4*b + a^2*b^3)*d^2*f^2*x^2 + 2*(a^4*b + a^2*b^3)*d^2*e*f*x +
 (a^4*b + a^2*b^3)*d^2*e^2)*cosh(d*x + c)^2 - 2*((a^3*b^2 + a*b^4)*d^2*f^2*
x^2 + 2*(a^3*b^2 + a*b^4)*d^2*e*f*x + (a^3*b^2 + a*b^4)*d^2*e^2)*cosh(d*x +
 c))*sinh(d*x + c))/((a^6 + 2*a^4*b^2 + a^2*b^4)*d^3*cosh(d*x + c)^4 + 4*(a
^6 + 2*a^4*b^2 + a^2*b^4)*d^3*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^6 + 2*a^
4*b^2 + a^2*b^4)*d^3*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^6 + 2*a^4*b^2 +
 a^2*b^4)*d^3*cosh(d*x + c)*sinh(d*x + c)^3 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d
^3*sinh(d*x + c)^4 - (a^6 + 2*a^4*b^2 + a^2*b^4)*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)**2*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)

[Out] Timed out
```

<u>Giac</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="giac")
```

```
[Out] Timed out
```

3.470 $\int \frac{(e+fx)\mathbf{csch}^2(c+dx)\mathbf{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=499

$$\frac{b^4 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2\left(a^2+b^2\right)^{3/2}}-\frac{b^4 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2\left(a^2+b^2\right)^{3/2}}+\frac{bf\text{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^2}-\frac{bf\text{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^2}-\frac{b^3 f \tan}{a^2 a}$$

```
[Out] (b*f*ArcTan[Sinh[c + d*x]])/(a^2*d^2) - (b^3*f*ArcTan[Sinh[c + d*x]])/(a^2*
(a^2 + b^2)*d^2) + (2*b*f*x*ArcTanh[E^(c + d*x)])/(a^2*d) - (b*f*x*ArcTanh[
Cosh[c + d*x]])/(a^2*d) + (b*(e + f*x)*ArcTanh[Cosh[c + d*x]])/(a^2*d) - (2
*(e + f*x)*Coth[2*c + 2*d*x])/(a*d) + (b^4*(e + f*x)*Log[1 + (b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d) - (b^4*(e + f*x)*Log[1
+ (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d) - (b^2*
f*Log[Cosh[c + d*x]])/(a*(a^2 + b^2)*d^2) + (f*Log[Sinh[2*c + 2*d*x]])/(a*d
^2) + (b*f*PolyLog[2, -E^(c + d*x)])/(a^2*d^2) - (b*f*PolyLog[2, E^(c + d*x
)])/(a^2*d^2) + (b^4*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/(a^2*(a^2 + b^2)^(3/2)*d^2) - (b^4*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2) - (b*(e + f*x)*Sech[c + d*x])
/(a^2*d) + (b^3*(e + f*x)*Sech[c + d*x])/(a^2*(a^2 + b^2)*d) + (b^2*(e + f*
x)*Tanh[c + d*x])/(a*(a^2 + b^2)*d)
```

Rubi [A] time = 0.977582, antiderivative size = 499, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 20, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {5589, 5461, 4184, 3475, 2622, 321, 207, 5462, 6271, 12, 4182, 2279, 2391, 3770, 5573, 3322, 2264, 2190, 6742, 5451}

$$\frac{b^4 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2\left(a^2+b^2\right)^{3/2}}-\frac{b^4 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2\left(a^2+b^2\right)^{3/2}}+\frac{bf\text{PolyLog}\left(2,-e^{c+dx}\right)}{a^2d^2}-\frac{bf\text{PolyLog}\left(2,e^{c+dx}\right)}{a^2d^2}-\frac{b^3 f \tan}{a^2 a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (b*f*ArcTan[Sinh[c + d*x]])/(a^2*d^2) - (b^3*f*ArcTan[Sinh[c + d*x]])/(a^2*
(a^2 + b^2)*d^2) + (2*b*f*x*ArcTanh[E^(c + d*x)])/(a^2*d) - (b*f*x*ArcTanh[
Cosh[c + d*x]])/(a^2*d) + (b*(e + f*x)*ArcTanh[Cosh[c + d*x]])/(a^2*d) - (2
*(e + f*x)*Coth[2*c + 2*d*x])/(a*d) + (b^4*(e + f*x)*Log[1 + (b*E^(c + d*x)
)/(a - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d) - (b^4*(e + f*x)*Log[1
+ (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d) - (b^2*
f*Log[Cosh[c + d*x]])/(a*(a^2 + b^2)*d^2) + (f*Log[Sinh[2*c + 2*d*x]])/(a*d
^2) + (b*f*PolyLog[2, -E^(c + d*x)])/(a^2*d^2) - (b*f*PolyLog[2, E^(c + d*x
)])/(a^2*d^2) + (b^4*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/(a^2*(a^2 + b^2)^(3/2)*d^2) - (b^4*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^(3/2)*d^2) - (b*(e + f*x)*Sech[c + d*x])
/(a^2*d) + (b^3*(e + f*x)*Sech[c + d*x])/(a^2*(a^2 + b^2)*d) + (b^2*(e + f*
x)*Tanh[c + d*x])/(a*(a^2 + b^2)*d)
```


Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
```

```
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,d}{a}$$

$$= \frac{4\int (e+fx)\text{csch}^2(2c+2dx)\,dx}{a} - \frac{b\int (e+fx)\text{csch}(c+dx)\text{sech}^2(c+dx)}{a^2}$$

$$= \frac{b(e+fx)\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{2(e+fx)\coth(2c+2dx)}{ad} - \frac{b(e+fx)}{}$$

$$= \frac{b(e+fx)\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{2(e+fx)\coth(2c+2dx)}{ad} + \frac{f\log(\sin}{}$$

$$= \frac{bf\tan^{-1}(\sinh(c+dx))}{a^2d^2} - \frac{bfx\tanh^{-1}(\cosh(c+dx))}{a^2d} + \frac{b(e+fx)\tanh^{-}}{a^2}$$

$$= \frac{bf\tan^{-1}(\sinh(c+dx))}{a^2d^2} - \frac{bfx\tanh^{-1}(\cosh(c+dx))}{a^2d} + \frac{b(e+fx)\tanh^{-}}{a^2}$$

$$= \frac{bf\tan^{-1}(\sinh(c+dx))}{a^2d^2} - \frac{b^3f\tan^{-1}(\sinh(c+dx))}{a^2\left(a^2+b^2\right)d^2} + \frac{2bfx\tanh^{-1}\left(e^{c+d}\right.}{a^2d}$$

$$= \frac{bf\tan^{-1}(\sinh(c+dx))}{a^2d^2} - \frac{b^3f\tan^{-1}(\sinh(c+dx))}{a^2\left(a^2+b^2\right)d^2} + \frac{2bfx\tanh^{-1}\left(e^{c+d}\right.}{a^2d}$$

$$= \frac{bf\tan^{-1}(\sinh(c+dx))}{a^2d^2} - \frac{b^3f\tan^{-1}(\sinh(c+dx))}{a^2\left(a^2+b^2\right)d^2} + \frac{2bfx\tanh^{-1}\left(e^{c+d}\right.}{a^2d}$$

Mathematica [C] time = 8.95006, size = 1862, normalized size = 3.73

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] 4*(-(f*(c + d*x))/(8*(a + I*b)*d^2) + ((I/8)*((2 - I)*a^3*d*f + (3*I)*a^2*b
*d*f - I*a*b^2*d*f + I*b^3*d*f + a^2*b*c*d*f + I*a*b^2*c*d*f)*(c + d*x))/(a
*(a + I*b)*(a^2 + b^2)*d^3) - ((I/16)*b*f*(c + d*x)^2)/((a^2 + b^2)*d^2) +
((I/4)*f*ArcTan[(a*Cosh[(c + d*x)/2] - b*Cosh[(c + d*x)/2] + a*Sinh[(c + d*
x)/2] + b*Sinh[(c + d*x)/2])/(a*Cosh[(c + d*x)/2] + b*Cosh[(c + d*x)/2] - a
*Sinh[(c + d*x)/2] + b*Sinh[(c + d*x)/2])])/((a + I*b)*d^2) - (a*f*ArcTanh[
1 - (2*I)*Tanh[(c + d*x)/2]])/(2*(a^2 + b^2)*d^2) - (b^2*f*ArcTanh[1 - (2*I
)*Tanh[(c + d*x)/2]])/(2*a*(a^2 + b^2)*d^2) - (b*c*f*ArcTanh[1 - (2*I)*Tanh
```

```
[(c + d*x)/2]])/(2*(a^2 + b^2)*d^2) + ((-(d*e*Cosh[(c + d*x)/2]) + c*f*Cosh
[(c + d*x)/2] - f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(8*a*d^2)
 + (a*f*Log[Cosh[(c + d*x)/2]])/(4*(a^2 + b^2)*d^2) + (b^2*f*Log[Cosh[(c +
d*x)/2]])/(4*a*(a^2 + b^2)*d^2) - (b*c*f*Log[Cosh[(c + d*x)/2]])/(4*(a^2 +
b^2)*d^2) + (f*Log[Cosh[c + d*x]])/(8*(a + I*b)*d^2) + (a*f*((-I)*(c + d*x)
 + 2*ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]] + Log[-1 + Cosh[c + d*x] + I*Sinh
[c + d*x]]))/(4*(a^2 + b^2)*d^2) + ((I/8)*b*f*((-I)*(c + d*x) + 2*ArcTanh[1
 - (2*I)*Tanh[(c + d*x)/2]] + Log[-1 + Cosh[c + d*x] + I*Sinh[c + d*x]]))/(
(a^2 + b^2)*d^2) + (b^2*f*((-I)*(c + d*x) + 2*ArcTanh[1 - (2*I)*Tanh[(c + d
*x)/2]] + Log[-1 + Cosh[c + d*x] + I*Sinh[c + d*x]]))/(8*a*(a^2 + b^2)*d^2)
 + (b*c*f*((-I)*(c + d*x) + 2*ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]] + Log[-1
 + Cosh[c + d*x] + I*Sinh[c + d*x]]))/(8*(a^2 + b^2)*d^2) - (b*e*Log[Tanh[(
c + d*x)/2]])/(4*(a^2 + b^2)*d) - (b^3*e*Log[Tanh[(c + d*x)/2]])/(4*a^2*(a^
2 + b^2)*d) + (b^3*c*f*Log[Tanh[(c + d*x)/2]])/(4*a^2*(a^2 + b^2)*d^2) + ((
I/2)*b*f*((-I/8)*(c + d*x)^2 - (I/2)*(c + d*x)*Log[1 + E^(-c - d*x)] + (I/2
)*PolyLog[2, -E^(-c - d*x)]))/((a^2 + b^2)*d^2) - (b*f*((-I/2)*(c + d*x)^2
+ (I/4)*(3*Pi*(c + d*x) + (1 - I)*(c + d*x)^2 + 2*(Pi - (2*I)*(c + d*x))*Lo
g[1 + I*E^(-c - d*x)] - 4*Pi*Log[1 + E^(c + d*x)] - 2*Pi*Log[-Cos[(Pi + (2*
I)*(c + d*x))/4]] + 4*Pi*Log[Cosh[(c + d*x)/2]] + (4*I)*PolyLog[2, (-I)*E^(
-c - d*x)])))/(4*(a^2 + b^2)*d^2) + ((I/4)*b^3*f*(I*(c + d*x)*(Log[1 - E^(-
c - d*x)] - Log[1 + E^(-c - d*x)]) + I*(PolyLog[2, -E^(-c - d*x)] - PolyLog
[2, E^(-c - d*x)])))/(a^2*(a^2 + b^2)*d^2) - ((I/4)*b*f*((c + d*x)^2/4 + (-
3*Pi*(c + d*x) - (1 - I)*(c + d*x)^2 - 2*(Pi - (2*I)*(c + d*x))*Log[1 + I*E
^(-c - d*x)] + 4*Pi*Log[1 + E^(c + d*x)] + 2*Pi*Log[-Cos[(Pi + (2*I)*(c + d
*x))/4]] - 4*Pi*Log[Cosh[(c + d*x)/2]] - (4*I)*PolyLog[2, (-I)*E^(-c - d*x)
])/4 - (I/2)*(-(c + d*x)^2/2 + 2*(c + d*x)*Log[1 - E^(c + d*x)] + 2*PolyLog
[2, E^(c + d*x)])))/((a^2 + b^2)*d^2) + (b^4*(-2*d*e*ArcTanh[(a + b*Cosh[c
+ d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a + b*Cosh[c +
d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*(Cosh[c +
 d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*(Co
sh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*(Cos
h[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2, -((b*(C
osh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))]))/(4*a^2*(a^2 + b^2)
^(3/2)*d^2) + (Sech[(c + d*x)/2]*(-(d*e*Sinh[(c + d*x)/2]) + c*f*Sinh[(c +
d*x)/2] - f*(c + d*x)*Sinh[(c + d*x)/2]))/(8*a*d^2) + (Sech[c + d*x]*(-(b*d
*e) + b*c*f - b*f*(c + d*x) - a*d*e*Sinh[c + d*x] + a*c*f*Sinh[c + d*x] - a
*f*(c + d*x)*Sinh[c + d*x]))/(4*(a^2 + b^2)*d^2))
```

Maple [B] time = 0.268, size = 1771, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] -2*(f*x+e)*(a*b*exp(3*d*x+3*c)+b^2*exp(2*d*x+2*c)-a*b*exp(d*x+c)+2*a^2+b^2)
/d/(a^2+b^2)/(1+exp(2*d*x+2*c))/a/(exp(2*d*x+2*c)-1)-1/(a^2+b^2)^(5/2)/d^2*
a^2*b^2*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/a^2/(a^2+b^
2)/d^2*b^3*f*c*ln(exp(d*x+c)-1)+1/a^2/(a^2+b^2)/d*b^3*f*ln(exp(d*x+c)+1)*x-
1/a^2/(a^2+b^2)^(5/2)/d*b^6*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1
/2))-1/a^2/(a^2+b^2)^(3/2)/d*b^4*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^
2)^(1/2))+4*a/(a^2+b^2)/d^2*b^2*f/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))+8*a^2/
(a^2+b^2)/d^2*b*f/(4*a^2+4*b^2)*arctan(exp(d*x+c))+1/a^2/(a^2+b^2)^(5/2)/d^
2*b^6*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/a^2
/(a^2+b^2)^(5/2)/d^2*b^6*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b
^2)^(1/2)))+1/(a^2+b^2)^(5/2)/d*b^4*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
```

```
(-a+(a^2+b^2)^(1/2)))*x-1/(a^2+b^2)^(5/2)/d*b^4*f*ln((b*exp(d*x+c)+(a^2+b^2
)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/(a^2+b^2)^(5/2)/d^2*b^4*f*ln((-b*exp(d*
x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/(a^2+b^2)^(5/2)/d^2*b^4*f
*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/a^2/(a^2+b^2)
^(3/2)/d^2*b^4*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/a^2/
(a^2+b^2)^(5/2)/d*b^6*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^
(1/2)))*x-1/a^2/(a^2+b^2)^(5/2)/d*b^6*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)
/(a+(a^2+b^2)^(1/2)))*x+1/a^2/(a^2+b^2)^(5/2)/d^2*b^6*f*ln((-b*exp(d*x+c)+(
a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/a^2/(a^2+b^2)^(5/2)/d^2*b^6*f*l
n((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/a^2/(a^2+b^2)^(
5/2)/d^2*b^6*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-4/d^2/(a
^2+b^2)*a*f*ln(exp(d*x+c))+a/(a^2+b^2)/d^2*f*ln(exp(d*x+c)-1)+a/(a^2+b^2)/d
^2*f*ln(exp(d*x+c)+1)-1/(a^2+b^2)/d*b*e*ln(exp(d*x+c)-1)+1/(a^2+b^2)/d*b*e*
ln(exp(d*x+c)+1)+1/(a^2+b^2)/d^2*b*f*dilog(exp(d*x+c))+1/(a^2+b^2)/d^2*b*f*
dilog(exp(d*x+c)+1)+1/(a^2+b^2)/d*ln(exp(d*x+c)+1)*b*f*x+1/(a^2+b^2)^(5/2)/
d^2*b^4*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/(
a^2+b^2)^(5/2)/d^2*b^4*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2
)^(1/2)))+1/(a^2+b^2)/d^2*b*f*c*ln(exp(d*x+c)-1)+8/(a^2+b^2)/d^2*b^3*f/(4*a
^2+4*b^2)*arctan(exp(d*x+c))-1/a^2/(a^2+b^2)/d*b^3*e*ln(exp(d*x+c)-1)+1/a^2
/(a^2+b^2)/d*b^3*e*ln(exp(d*x+c)+1)+1/a/(a^2+b^2)/d^2*b^2*f*ln(exp(d*x+c)-1
)+1/a/(a^2+b^2)/d^2*b^2*f*ln(exp(d*x+c)+1)-2/a/(a^2+b^2)/d^2*b^2*f*ln(exp(d
*x+c))+4*a^3/(a^2+b^2)/d^2*f/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))+1/a^2/(a^2+
b^2)/d^2*b^3*f*dilog(exp(d*x+c))+1/a^2/(a^2+b^2)/d^2*b^3*f*dilog(exp(d*x+c)
+1)-1/(a^2+b^2)^(3/2)/d*b^2*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1
/2))+1/(a^2+b^2)^(3/2)/d^2*b^2*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^
2)^(1/2))+1/(a^2+b^2)^(5/2)/d*a^2*b^2*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a
^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 3.12856, size = 9453, normalized size = 18.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")

[Out] -(2*((2*a^5 + 3*a^3*b^2 + a*b^4)*d*f*x + (a^5 + 2*a^3*b^2 + a*b^4)*c*f)*cos
h(d*x + c)^4 + 2*((2*a^5 + 3*a^3*b^2 + a*b^4)*d*f*x + (a^5 + 2*a^3*b^2 + a*
b^4)*c*f)*sinh(d*x + c)^4 + 2*((a^4*b + a^2*b^3)*d*f*x + (a^4*b + a^2*b^3)*
d*e)*cosh(d*x + c)^3 + 2*((a^4*b + a^2*b^3)*d*f*x + (a^4*b + a^2*b^3)*d*e +
 4*((2*a^5 + 3*a^3*b^2 + a*b^4)*d*f*x + (a^5 + 2*a^3*b^2 + a*b^4)*c*f)*cosh
(d*x + c))*sinh(d*x + c)^3 + 2*(2*a^5 + 3*a^3*b^2 + a*b^4)*d*e - 2*(a^5 + 2
*a^3*b^2 + a*b^4)*c*f + 2*((a^3*b^2 + a*b^4)*d*f*x + (a^3*b^2 + a*b^4)*d*e)
```

```
*cosh(d*x + c)^2 + 2*((a^3*b^2 + a*b^4)*d*f*x + (a^3*b^2 + a*b^4)*d*e + 6*(
(2*a^5 + 3*a^3*b^2 + a*b^4)*d*f*x + (a^5 + 2*a^3*b^2 + a*b^4)*c*f)*cosh(d*x
 + c)^2 + 3*((a^4*b + a^2*b^3)*d*f*x + (a^4*b + a^2*b^3)*d*e)*cosh(d*x + c)
)*sinh(d*x + c)^2 - (b^5*f*cosh(d*x + c)^4 + 4*b^5*f*cosh(d*x + c)^3*sinh(d
*x + c) + 6*b^5*f*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^5*f*cosh(d*x + c)*s
inh(d*x + c)^3 + b^5*f*sinh(d*x + c)^4 - b^5*f)*sqrt((a^2 + b^2)/b^2)*dilog
((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 + b^2)/b^2) - b)/b + 1) + (b^5*f*cosh(d*x + c)^4 + 4*b^5*f*cosh(d*
x + c)^3*sinh(d*x + c) + 6*b^5*f*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^5*f*
cosh(d*x + c)*sinh(d*x + c)^3 + b^5*f*sinh(d*x + c)^4 - b^5*f)*sqrt((a^2 +
b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*s
inh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^5*d*e - b^5*c*f - (b^5
*d*e - b^5*c*f)*cosh(d*x + c)^4 - 4*(b^5*d*e - b^5*c*f)*cosh(d*x + c)^3*sin
h(d*x + c) - 6*(b^5*d*e - b^5*c*f)*cosh(d*x + c)^2*sinh(d*x + c)^2 - 4*(b^5
*d*e - b^5*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 - (b^5*d*e - b^5*c*f)*sinh(d*
x + c)^4)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) +
 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^5*d*e - b^5*c*f - (b^5*d*e - b^5*c*f
)*cosh(d*x + c)^4 - 4*(b^5*d*e - b^5*c*f)*cosh(d*x + c)^3*sinh(d*x + c) - 6
*(b^5*d*e - b^5*c*f)*cosh(d*x + c)^2*sinh(d*x + c)^2 - 4*(b^5*d*e - b^5*c*f
)*cosh(d*x + c)*sinh(d*x + c)^3 - (b^5*d*e - b^5*c*f)*sinh(d*x + c)^4)*sqrt
((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2
 + b^2)/b^2) + 2*a) + (b^5*d*f*x + b^5*c*f - (b^5*d*f*x + b^5*c*f)*cosh(d*x
 + c)^4 - 4*(b^5*d*f*x + b^5*c*f)*cosh(d*x + c)^3*sinh(d*x + c) - 6*(b^5*d*
f*x + b^5*c*f)*cosh(d*x + c)^2*sinh(d*x + c)^2 - 4*(b^5*d*f*x + b^5*c*f)*co
sh(d*x + c)*sinh(d*x + c)^3 - (b^5*d*f*x + b^5*c*f)*sinh(d*x + c)^4)*sqrt((
a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^5*d*f*x + b^5*c*f - (
b^5*d*f*x + b^5*c*f)*cosh(d*x + c)^4 - 4*(b^5*d*f*x + b^5*c*f)*cosh(d*x + c
)^3*sinh(d*x + c) - 6*(b^5*d*f*x + b^5*c*f)*cosh(d*x + c)^2*sinh(d*x + c)^2
 - 4*(b^5*d*f*x + b^5*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 - (b^5*d*f*x + b^5
*c*f)*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh
(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/
b) - 2*((a^4*b + a^2*b^3)*f*cosh(d*x + c)^4 + 4*(a^4*b + a^2*b^3)*f*cosh(d*
x + c)^3*sinh(d*x + c) + 6*(a^4*b + a^2*b^3)*f*cosh(d*x + c)^2*sinh(d*x + c
)^2 + 4*(a^4*b + a^2*b^3)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + a^2*b^
3)*f*sinh(d*x + c)^4 - (a^4*b + a^2*b^3)*f)*arctan(cosh(d*x + c) + sinh(d*x
 + c)) - 2*((a^4*b + a^2*b^3)*d*f*x + (a^4*b + a^2*b^3)*d*e)*cosh(d*x + c)
+ ((a^4*b + 2*a^2*b^3 + b^5)*f*cosh(d*x + c)^4 + 4*(a^4*b + 2*a^2*b^3 + b^5
)*f*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^4*b + 2*a^2*b^3 + b^5)*f*cosh(d*x
+ c)^2*sinh(d*x + c)^2 + 4*(a^4*b + 2*a^2*b^3 + b^5)*f*cosh(d*x + c)*sinh(d
*x + c)^3 + (a^4*b + 2*a^2*b^3 + b^5)*f*sinh(d*x + c)^4 - (a^4*b + 2*a^2*b^
3 + b^5)*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) - ((a^4*b + 2*a^2*b^3 + b^
5)*f*cosh(d*x + c)^4 + 4*(a^4*b + 2*a^2*b^3 + b^5)*f*cosh(d*x + c)^3*sinh(d
*x + c) + 6*(a^4*b + 2*a^2*b^3 + b^5)*f*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*(a^4*b + 2*a^2*b^3 + b^5)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + 2*a^2
*b^3 + b^5)*f*sinh(d*x + c)^4 - (a^4*b + 2*a^2*b^3 + b^5)*f)*dilog(-cosh(d*
x + c) - sinh(d*x + c)) - ((a^5 + a^3*b^2)*f*cosh(d*x + c)^4 + 4*(a^5 + a^3
*b^2)*f*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^5 + a^3*b^2)*f*cosh(d*x + c)^2
*sinh(d*x + c)^2 + 4*(a^5 + a^3*b^2)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^5
 + a^3*b^2)*f*sinh(d*x + c)^4 - (a^5 + a^3*b^2)*f)*log(2*cosh(d*x + c)/(cos
h(d*x + c) - sinh(d*x + c))) - (((a^4*b + 2*a^2*b^3 + b^5)*d*f*x + (a^4*b +
 2*a^2*b^3 + b^5)*d*e + (a^5 + 2*a^3*b^2 + a*b^4)*f)*cosh(d*x + c)^4 + 4*((
a^4*b + 2*a^2*b^3 + b^5)*d*f*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e + (a^5 + 2*a
^3*b^2 + a*b^4)*f)*cosh(d*x + c)^3*sinh(d*x + c) + 6*((a^4*b + 2*a^2*b^3 +
b^5)*d*f*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e + (a^5 + 2*a^3*b^2 + a*b^4)*f)*c
osh(d*x + c)^2*sinh(d*x + c)^2 + 4*((a^4*b + 2*a^2*b^3 + b^5)*d*f*x + (a^4*
b + 2*a^2*b^3 + b^5)*d*e + (a^5 + 2*a^3*b^2 + a*b^4)*f)*cosh(d*x + c)*sinh(
d*x + c)^3 + ((a^4*b + 2*a^2*b^3 + b^5)*d*f*x + (a^4*b + 2*a^2*b^3 + b^5)*d
*e + (a^5 + 2*a^3*b^2 + a*b^4)*f)*sinh(d*x + c)^4 - (a^4*b + 2*a^2*b^3 + b^
```

```
5)*d*f*x - (a^4*b + 2*a^2*b^3 + b^5)*d*e - (a^5 + 2*a^3*b^2 + a*b^4)*f)*log
(cosh(d*x + c) + sinh(d*x + c) + 1) + (((a^4*b + 2*a^2*b^3 + b^5)*d*e - (a^
5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*c)*f)*cosh(d*x + c)^4 + 4
*((a^4*b + 2*a^2*b^3 + b^5)*d*e - (a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2
*b^3 + b^5)*c)*f)*cosh(d*x + c)^3*sinh(d*x + c) + 6*((a^4*b + 2*a^2*b^3 + b
^5)*d*e - (a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*c)*f)*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*((a^4*b + 2*a^2*b^3 + b^5)*d*e - (a^5 + 2*a^3
*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*c)*f)*cosh(d*x + c)*sinh(d*x + c)^
3 + ((a^4*b + 2*a^2*b^3 + b^5)*d*e - (a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*
a^2*b^3 + b^5)*c)*f)*sinh(d*x + c)^4 - (a^4*b + 2*a^2*b^3 + b^5)*d*e + (a^5
 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*c)*f)*log(cosh(d*x + c) +
sinh(d*x + c) - 1) + (((a^4*b + 2*a^2*b^3 + b^5)*d*f*x + (a^4*b + 2*a^2*b^3
 + b^5)*c*f)*cosh(d*x + c)^4 + 4*((a^4*b + 2*a^2*b^3 + b^5)*d*f*x + (a^4*b
+ 2*a^2*b^3 + b^5)*c*f)*cosh(d*x + c)^3*sinh(d*x + c) + 6*((a^4*b + 2*a^2*b
^3 + b^5)*d*f*x + (a^4*b + 2*a^2*b^3 + b^5)*c*f)*cosh(d*x + c)^2*sinh(d*x +
 c)^2 + 4*((a^4*b + 2*a^2*b^3 + b^5)*d*f*x + (a^4*b + 2*a^2*b^3 + b^5)*c*f)
*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4*b + 2*a^2*b^3 + b^5)*d*f*x + (a^4*b
+ 2*a^2*b^3 + b^5)*c*f)*sinh(d*x + c)^4 - (a^4*b + 2*a^2*b^3 + b^5)*d*f*x -
 (a^4*b + 2*a^2*b^3 + b^5)*c*f)*log(-cosh(d*x + c) - sinh(d*x + c) + 1) - 2
*((a^4*b + a^2*b^3)*d*f*x - 4*((2*a^5 + 3*a^3*b^2 + a*b^4)*d*f*x + (a^5 + 2
*a^3*b^2 + a*b^4)*c*f)*cosh(d*x + c)^3 + (a^4*b + a^2*b^3)*d*e - 3*((a^4*b
+ a^2*b^3)*d*f*x + (a^4*b + a^2*b^3)*d*e)*cosh(d*x + c)^2 - 2*((a^3*b^2 + a
*b^4)*d*f*x + (a^3*b^2 + a*b^4)*d*e)*cosh(d*x + c))*sinh(d*x + c))/((a^6 +
2*a^4*b^2 + a^2*b^4)*d^2*cosh(d*x + c)^4 + 4*(a^6 + 2*a^4*b^2 + a^2*b^4)*d^
2*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^6 + 2*a^4*b^2 + a^2*b^4)*d^2*cosh(d*
x + c)^2*sinh(d*x + c)^2 + 4*(a^6 + 2*a^4*b^2 + a^2*b^4)*d^2*cosh(d*x + c)*
sinh(d*x + c)^3 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d^2*sinh(d*x + c)^4 - (a^6 +
2*a^4*b^2 + a^2*b^4)*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**2*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")

[Out] Timed out
```

3.471 $\int \frac{\mathbf{csch}^2(c+dx)\mathbf{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=144

$$-\frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d\left(a^2+b^2\right)^{3/2}}+\frac{b^2\mathrm{sech}(c+dx)(a\sinh(c+dx)+b)}{a^2d\left(a^2+b^2\right)}-\frac{b\mathrm{sech}(c+dx)}{a^2d}+\frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}-\tanh$$

```
[Out] (b*ArcTanh[Cosh[c + d*x]])/(a^2*d) - (2*b^4*ArcTanh[(b - a*Tanh[(c + d*x)/2
])/Sqrt[a^2 + b^2]])/(a^2*(a^2 + b^2)^(3/2)*d) - Coth[c + d*x]/(a*d) - (b*S
ech[c + d*x])/(a^2*d) + (b^2*Sech[c + d*x]*(b + a*Sinh[c + d*x]))/(a^2*(a^2
 + b^2)*d) - Tanh[c + d*x]/(a*d)
```

Rubi [A] time = 0.310752, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2898, 2622, 321, 207, 2620, 14, 2696, 12, 2660, 618, 204}

$$-\frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d\left(a^2+b^2\right)^{3/2}}+\frac{b^2\mathrm{sech}(c+dx)(a\sinh(c+dx)+b)}{a^2d\left(a^2+b^2\right)}-\frac{b\mathrm{sech}(c+dx)}{a^2d}+\frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}-\tanh$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (b*ArcTanh[Cosh[c + d*x]])/(a^2*d) - (2*b^4*ArcTanh[(b - a*Tanh[(c + d*x)/2
])/Sqrt[a^2 + b^2]])/(a^2*(a^2 + b^2)^(3/2)*d) - Coth[c + d*x]/(a*d) - (b*S
ech[c + d*x])/(a^2*d) + (b^2*Sech[c + d*x]*(b + a*Sinh[c + d*x]))/(a^2*(a^2
 + b^2)*d) - Tanh[c + d*x]/(a*d)
```


Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\int \left(\frac{b\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a^2} - \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a} - \frac{b^2\operatorname{sech}^2(c+}{a^2(a+b\sinh(c}\right.$$

$$= \frac{\int \operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)\,dx}{a} - \frac{b\int \operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{\operatorname{sech}^2}{a+b\sin}}{a^2}$$

$$= \frac{b^2\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a^2\left(a^2+b^2\right)d} + \frac{b^2\int \frac{b^2}{a+b\sinh(c+dx)}\,dx}{a^2\left(a^2+b^2\right)} + \frac{i\operatorname{Subst}\left(\int \frac{1+x^2}{x^2}\,dx,x,\right.}{ad}$$

$$= -\frac{b\operatorname{sech}(c+dx)}{a^2d} + \frac{b^2\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a^2\left(a^2+b^2\right)d} + \frac{b^4\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a^2\left(a^2+b^2\right)} + \frac{i\operatorname{Su}}{}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{\coth(c+dx)}{ad} - \frac{b\operatorname{sech}(c+dx)}{a^2d} + \frac{b^2\operatorname{sech}(c+dx)(b+a\sin}{a^2\left(a^2+b^2\right)d}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{\coth(c+dx)}{ad} - \frac{b\operatorname{sech}(c+dx)}{a^2d} + \frac{b^2\operatorname{sech}(c+dx)(b+a\sin}{a^2\left(a^2+b^2\right)d}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}d} - \frac{\coth(c+dx)}{ad} - \frac{b\operatorname{sech}(c}{a^2d}$$

Mathematica [A] time = 2.64522, size = 135, normalized size = 0.94

$$-\frac{\frac{4b^4\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{a^2\left(-a^2-b^2\right)^{3/2}} + \frac{2\operatorname{sech}(c+dx)(a\sinh(c+dx)+b)}{a^2+b^2} + \frac{2b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a^2} + \frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{a} + \frac{\coth\left(\frac{1}{2}(c+dx)\right)}{a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((4*b^4*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(a^2*(-a^2 - b
^2)^(3/2)) + Coth[(c + d*x)/2]/a + (2*b*Log[Tanh[(c + d*x)/2]])/a^2 + (2*Se
ch[c + d*x]*(b + a*Sinh[c + d*x]))/(a^2 + b^2) + Tanh[(c + d*x)/2]/a)/(2*d)
```

Maple [A] time = 0.003, size = 174, normalized size = 1.2

$$-\frac{1}{2\,da}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{1}{2\,da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1} - \frac{b}{da^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + 2\frac{b^4}{da^2\left(a^2+b^2\right)^{3/2}}\operatorname{Artanh}\left(1/2\,\frac{2\,a\tanh(1}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/2/d/a*tanh(1/2*d*x+1/2*c)-1/2/d/a/tanh(1/2*d*x+1/2*c)-1/d/a^2*b*ln(tanh(
1/2*d*x+1/2*c))+2/d/a^2*b^4/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1
/2*c)-2*b)/(a^2+b^2)^(1/2))-2/d/(a^2+b^2)/(tanh(1/2*d*x+1/2*c)^2+1)*a*tanh(
1/2*d*x+1/2*c)-2/d/(a^2+b^2)/(tanh(1/2*d*x+1/2*c)^2+1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.11544, size = 2483, normalized size = 17.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="frica
s")
```

```
[Out] -(4*a^5 + 6*a^3*b^2 + 2*a*b^4 + 2*(a^4*b + a^2*b^3)*cosh(d*x + c)^3 + 2*(a^
4*b + a^2*b^3)*sinh(d*x + c)^3 + 2*(a^3*b^2 + a*b^4)*cosh(d*x + c)^2 + 2*(a
^3*b^2 + a*b^4 + 3*(a^4*b + a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - (b^4*
cosh(d*x + c)^4 + 4*b^4*cosh(d*x + c)^3*sinh(d*x + c) + 6*b^4*cosh(d*x + c)
^2*sinh(d*x + c)^2 + 4*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + b^4*sinh(d*x + c
)^4 - b^4)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 +
 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x +
 c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*
x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*
sinh(d*x + c) - b)) - 2*(a^4*b + a^2*b^3)*cosh(d*x + c) + (a^4*b + 2*a^2*b^
3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^4 - 4*(a^4*b + 2*a^2*b^3
+ b^5)*cosh(d*x + c)^3*sinh(d*x + c) - 6*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x
 + c)^2*sinh(d*x + c)^2 - 4*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)*sinh(d*
x + c)^3 - (a^4*b + 2*a^2*b^3 + b^5)*sinh(d*x + c)^4)*log(cosh(d*x + c) + s
inh(d*x + c) + 1) - (a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*co
sh(d*x + c)^4 - 4*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^3*sinh(d*x + c) -
 6*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^2*sinh(d*x + c)^2 - 4*(a^4*b + 2
*a^2*b^3 + b^5)*cosh(d*x + c)*sinh(d*x + c)^3 - (a^4*b + 2*a^2*b^3 + b^5)*s
inh(d*x + c)^4)*log(cosh(d*x + c) + sinh(d*x + c) - 1) - 2*(a^4*b + a^2*b^3
 - 3*(a^4*b + a^2*b^3)*cosh(d*x + c)^2 - 2*(a^3*b^2 + a*b^4)*cosh(d*x + c))
*sinh(d*x + c))/((a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^4 + 4*(a^6 + 2
*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^6 + 2*a^4*b^2 +
a^2*b^4)*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^6 + 2*a^4*b^2 + a^2*b^4)*
d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d*sinh(d*x +
c)^4 - (a^6 + 2*a^4*b^2 + a^2*b^4)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.72368, size = 262, normalized size = 1.82

$$-\frac{b^4\log\left(\frac{\left|-2\,be^{(dx+c)}-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|-2\,be^{(dx+c)}-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4d+a^2b^2d\right)\sqrt{a^2+b^2}}-\frac{2\left(abe^{(3\,dx+3\,c)}+b^2e^{(2\,dx+2\,c)}-abe^{(dx+c)}+2\,a^2+b^2\right)}{\left(a^3d+ab^2d\right)\left(e^{(4\,dx+4\,c)}-1\right)}+\frac{b\log\left(e^{(dx+c)}+1\right)}{a^2d}-\frac{b\log\left(\left|e^{(dx+c}\right.\right.}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac"
)
```

```
[Out] -b^4*log(abs(-2*b*e^(d*x + c) - 2*a - 2*sqrt(a^2 + b^2))/abs(-2*b*e^(d*x +
c) - 2*a + 2*sqrt(a^2 + b^2)))/((a^4*d + a^2*b^2*d)*sqrt(a^2 + b^2)) - 2*(a
*b*e^(3*d*x + 3*c) + b^2*e^(2*d*x + 2*c) - a*b*e^(d*x + c) + 2*a^2 + b^2)/(
(a^3*d + a*b^2*d)*(e^(4*d*x + 4*c) - 1)) + b*log(e^(d*x + c) + 1)/(a^2*d) -
 b*log(abs(e^(d*x + c) - 1))/(a^2*d)
```

3.472 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=38

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Csch[c + d*x]^2*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d
*x])), x]
```

Rubi [A] time = 0.129061, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[c + d*x]^2*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Csch[c + d*x]^2*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A] time = 154.862, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]
)),x]
```

```
[Out] Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]
)), x]
```

Maple [A] time = 1.777, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx+c))^2(\operatorname{sech}(dx+c))^2}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$16\,b^4 \int -\frac{e^{(dx+c)}}{8\left(a^4be + a^2b^3e + \left(a^4bf + a^2b^3f\right)x - \left(a^4bee^{(2\,c)} + a^2b^3ee^{(2\,c)} + \left(a^4bfe^{(2\,c)} + a^2b^3fe^{(2\,c)}\right)x\right)e^{(2\,dx)} - 2\left(a^5ee^c + a^3b^2e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] 16*b^4*integrate(-1/8*e^(d*x + c)/(a^4*b*e + a^2*b^3*e + (a^4*b*f + a^2*b^3
*f)*x - (a^4*b*e*e^(2*c) + a^2*b^3*e*e^(2*c) + (a^4*b*f*e^(2*c) + a^2*b^3*f
*e^(2*c))*x)*e^(2*d*x) - 2*(a^5*e*e^c + a^3*b^2*e*e^c + (a^5*f*e^c + a^3*b^
2*f*e^c)*x)*e^(d*x)), x) + 2*(a*b*e^(3*d*x + 3*c) + b^2*e^(2*d*x + 2*c) - a
*b*e^(d*x + c) + 2*a^2 + b^2)/(a^3*d*e + a*b^2*d*e + (a^3*d*f + a*b^2*d*f)*
x - (a^3*d*e*e^(4*c) + a*b^2*d*e*e^(4*c) + (a^3*d*f*e^(4*c) + a*b^2*d*f*e^(
4*c))*x)*e^(4*d*x)) - 16*integrate(-1/16*(b*d*f*x + b*d*e + a*f)/(a^2*d*f^2
*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c +
 a^2*d*e^2*e^c)*e^(d*x)), x) - 16*integrate(1/16*(b*d*f*x + b*d*e - a*f)/(a
^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f
*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) - 16*integrate(1/8*(b*f*e^(d*x + c) -
a*f)/(a^2*d*e^2 + b^2*d*e^2 + (a^2*d*f^2 + b^2*d*f^2)*x^2 + 2*(a^2*d*e*f +
b^2*d*e*f)*x + (a^2*d*e^2*e^(2*c) + b^2*d*e^2*e^(2*c) + (a^2*d*f^2*e^(2*c)
+ b^2*d*f^2*e^(2*c))*x^2 + 2*(a^2*d*e*f*e^(2*c) + b^2*d*e*f*e^(2*c))*x)*e^(
2*d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{csch}\left(dx+c\right)^2\operatorname{sech}\left(dx+c\right)^2}{afx+ae+\left(bfx+be\right)\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] integral(csch(d*x + c)^2*sech(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(
d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*sech(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.473 $\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=978

result too large to display

```
[Out] -(b*f*x)/(2*a^2*d) - (3*f*x*ArcTan[E^(c + d*x)])/(a*d) + (2*b^4*(e + f*x)*A
rcTan[E^(c + d*x)])/(a*(a^2 + b^2)^2*d) + (b^2*(e + f*x)*ArcTan[E^(c + d*x)
])/(a*(a^2 + b^2)*d) + (3*f*x*ArcTan[Sinh[c + d*x]])/(2*a*d) - (3*(e + f*x)
*ArcTan[Sinh[c + d*x]])/(2*a*d) + (2*b*f*x*ArcTanh[E^(2*c + 2*d*x)])/(a^2*d
) - (f*ArcTanh[Cosh[c + d*x]])/(a*d^2) - (3*(e + f*x)*Csch[c + d*x])/(2*a*d
) + (b^5*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*(a^
2 + b^2)^2*d) + (b^5*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)])/(a^2*(a^2 + b^2)^2*d) - (b^5*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a^2*(
a^2 + b^2)^2*d) + (b*f*x*Log[Tanh[c + d*x]])/(a^2*d) - (b*(e + f*x)*Log[Tan
h[c + d*x]])/(a^2*d) + (((3*I)/2)*f*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) -
 (I*b^4*f*PolyLog[2, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)^2*d^2) - ((I/2)*b^2*
f*PolyLog[2, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) - (((3*I)/2)*f*PolyLog[
2, I*E^(c + d*x)])/(a*d^2) + (I*b^4*f*PolyLog[2, I*E^(c + d*x)])/(a*(a^2 +
b^2)^2*d^2) + ((I/2)*b^2*f*PolyLog[2, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) +
 (b^5*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b
^2)^2*d^2) + (b^5*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
a^2*(a^2 + b^2)^2*d^2) - (b^5*f*PolyLog[2, -E^(2*(c + d*x))])/(2*a^2*(a^2 +
 b^2)^2*d^2) + (b*f*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a^2*d^2) - (b*f*PolyLo
g[2, E^(2*c + 2*d*x)])/(2*a^2*d^2) - (f*Sech[c + d*x])/(2*a*d^2) + (b^2*f*S
ech[c + d*x])/(2*a*(a^2 + b^2)*d^2) + (b^3*(e + f*x)*Sech[c + d*x]^2)/(2*a^
2*(a^2 + b^2)*d) + ((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^2)/(2*a*d) + (b*f
*Tanh[c + d*x])/(2*a^2*d^2) - (b^3*f*Tanh[c + d*x])/(2*a^2*(a^2 + b^2)*d^2)
 + (b^2*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*a*(a^2 + b^2)*d) + (b*(e
+ f*x)*Tanh[c + d*x]^2)/(2*a^2*d)
```

Rubi [A] time = 1.411, antiderivative size = 978, normalized size of antiderivative = 1., number of steps used = 57, number of rules used = 27, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.794, Rules used = {5589, 2621, 288, 321, 207, 5462, 5203, 12, 4180, 2279, 2391, 3770, 2622, 2620, 14, 2548, 4182, 3473, 8, 5573, 5561, 2190, 6742, 3718, 4185, 5451, 3767}

$$\frac{(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)b^5}{a^2\left(a^2+b^2\right)^2 d}+\frac{(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)b^5}{a^2\left(a^2+b^2\right)^2 d}-\frac{(e+fx)\log\left(1+e^{2(c+dx)}\right)b^5}{a^2\left(a^2+b^2\right)^2 d}+\frac{f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right.}{a^2\left(a^2+b^2\right)^2 d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] -(b*f*x)/(2*a^2*d) - (3*f*x*ArcTan[E^(c + d*x)])/(a*d) + (2*b^4*(e + f*x)*A
rcTan[E^(c + d*x)])/(a*(a^2 + b^2)^2*d) + (b^2*(e + f*x)*ArcTan[E^(c + d*x)
])/(a*(a^2 + b^2)*d) + (3*f*x*ArcTan[Sinh[c + d*x]])/(2*a*d) - (3*(e + f*x)
*ArcTan[Sinh[c + d*x]])/(2*a*d) + (2*b*f*x*ArcTanh[E^(2*c + 2*d*x)])/(a^2*d
) - (f*ArcTanh[Cosh[c + d*x]])/(a*d^2) - (3*(e + f*x)*Csch[c + d*x])/(2*a*d
) + (b^5*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*(a^
2 + b^2)^2*d) + (b^5*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)])/(a^2*(a^2 + b^2)^2*d) - (b^5*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a^2*(
a^2 + b^2)^2*d) + (b*f*x*Log[Tanh[c + d*x]])/(a^2*d) - (b*(e + f*x)*Log[Tan
h[c + d*x]])/(a^2*d) + (((3*I)/2)*f*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) -
 (I*b^4*f*PolyLog[2, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)^2*d^2) - ((I/2)*b^2*
f*PolyLog[2, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) - (((3*I)/2)*f*PolyLog[
```

2, I*E^(c + d*x)])/(a*d^2) + (I*b^4*f*PolyLog[2, I*E^(c + d*x)])/(a*(a^2 + b^2)^2*d^2) + ((I/2)*b^2*f*PolyLog[2, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) + (b^5*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^2*d^2) + (b^5*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^2*d^2) - (b^5*f*PolyLog[2, -E^(2*(c + d*x))])/(2*a^2*(a^2 + b^2)^2*d^2) + (b*f*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a^2*d^2) - (b*f*PolyLog[2, E^(2*c + 2*d*x)])/(2*a^2*d^2) - (f*Sech[c + d*x])/(2*a*d^2) + (b^2*f*Sech[c + d*x])/(2*a*(a^2 + b^2)*d^2) + (b^3*(e + f*x)*Sech[c + d*x]^2)/(2*a^2*(a^2 + b^2)*d) + ((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^2)/(2*a*d) + (b*f*Tanh[c + d*x])/(2*a^2*d^2) - (b^3*f*Tanh[c + d*x])/(2*a^2*(a^2 + b^2)*d^2) + (b^2*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*a*(a^2 + b^2)*d) + (b*(e + f*x)*Tanh[c + d*x]^2)/(2*a^2*d)

Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{3(e+fx)\tan^{-1}(\sinh(c+dx))}{2ad} - \frac{3(e+fx)\operatorname{csch}(c+dx)}{2ad} + \frac{(e+fx)\operatorname{csch}(c}{}$$

$$= -\frac{3(e+fx)\tan^{-1}(\sinh(c+dx))}{2ad} - \frac{3(e+fx)\operatorname{csch}(c+dx)}{2ad} - \frac{b(e+fx)\log(\mathrm{ta}}{a^2d}$$

$$= \frac{3fx\tan^{-1}(\sinh(c+dx))}{2ad} - \frac{3(e+fx)\tan^{-1}(\sinh(c+dx))}{2ad} - \frac{3f\tanh^{-1}(\cos}{2ad^2}$$

$$= -\frac{b^5(e+fx)^2}{2a^2\left(a^2+b^2\right)^2 f} + \frac{3fx\tan^{-1}(\sinh(c+dx))}{2ad} - \frac{3(e+fx)\tan^{-1}(\sinh(c+d}{2ad}$$

$$= -\frac{bfx}{2a^2d} - \frac{b^5(e+fx)^2}{2a^2\left(a^2+b^2\right)^2 f} - \frac{3fx\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{3fx\tan^{-1}(\sinh(c+dx))}{2ad} -$$

$$= -\frac{bfx}{2a^2d} - \frac{3fx\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^4(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)^2 d} + \frac{b^2(e+fx)\tan^{-1}}{a\left(a^2+b^2\right)}$$

$$= -\frac{bfx}{2a^2d} - \frac{3fx\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^4(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)^2 d} + \frac{b^2(e+fx)\tan^{-1}}{a\left(a^2+b^2\right)}$$

$$= -\frac{bfx}{2a^2d} - \frac{3fx\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^4(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)^2 d} + \frac{b^2(e+fx)\tan^{-1}}{a\left(a^2+b^2\right)}$$

$$= -\frac{bfx}{2a^2d} - \frac{3fx\tan^{-1}\left(e^{c+dx}\right)}{ad} + \frac{2b^4(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a\left(a^2+b^2\right)^2 d} + \frac{b^2(e+fx)\tan^{-1}}{a\left(a^2+b^2\right)}$$

Mathematica [A] time = 10.9538, size = 1337, normalized size = 1.37

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] 8*(((-(d*e*Cosh[(c + d*x)/2]) + c*f*Cosh[(c + d*x)/2] - f*(c + d*x)*Cosh[(c
 + d*x)/2])*Csch[(c + d*x)/2]*Csch[c + d*x]*(a + b*Sinh[c + d*x]))/(16*a*d^
2*(b + a*Csch[c + d*x])) - (b*e*Csch[c + d*x]*Log[Sinh[c + d*x]]*(a + b*Sin
h[c + d*x]))/(8*a^2*d*(b + a*Csch[c + d*x])) + (b*c*f*Csch[c + d*x]*Log[Sin
h[c + d*x]]*(a + b*Sinh[c + d*x]))/(8*a^2*d^2*(b + a*Csch[c + d*x])) + (f*C
sch[c + d*x]*Log[Tanh[(c + d*x)/2]]*(a + b*Sinh[c + d*x]))/(8*a*d^2*(b + a*
Csch[c + d*x])) + ((I/8)*b*f*Csch[c + d*x]*(I*(c + d*x)*Log[1 - E^(-2*(c +
d*x))] - (I/2)*(-(c + d*x)^2 + PolyLog[2, E^(-2*(c + d*x))]))*(a + b*Sinh[c
 + d*x]))/(a^2*d^2*(b + a*Csch[c + d*x])) + (b^5*Csch[c + d*x]*(-(f*(c + d*
x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c
 + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d*e*Log[a + b*Sinh
[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLog[2, (b*E^(c + d*x))/(-
a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
))])*(a + b*Sinh[c + d*x]))/(8*a^2*(a^2 + b^2)^2*d^2*(b + a*Csch[c + d*x]))
 + (Csch[c + d*x]*(-2*a^2*b*d*e*(c + d*x) - 4*b^3*d*e*(c + d*x) + 2*a^2*b*c
*f*(c + d*x) + 4*b^3*c*f*(c + d*x) - a^2*b*f*(c + d*x)^2 - 2*b^3*f*(c + d*x
)^2 - 6*a^3*d*e*ArcTan[E^(c + d*x)] - 10*a*b^2*d*e*ArcTan[E^(c + d*x)] + 6*
a^3*c*f*ArcTan[E^(c + d*x)] + 10*a*b^2*c*f*ArcTan[E^(c + d*x)] - (3*I)*a^3*
f*(c + d*x)*Log[1 - I*E^(c + d*x)] - (5*I)*a*b^2*f*(c + d*x)*Log[1 - I*E^(c
 + d*x)] + (3*I)*a^3*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + (5*I)*a*b^2*f*(c
+ d*x)*Log[1 + I*E^(c + d*x)] + 2*a^2*b*d*e*Log[1 + E^(2*(c + d*x))] + 4*b^
3*d*e*Log[1 + E^(2*(c + d*x))] - 2*a^2*b*c*f*Log[1 + E^(2*(c + d*x))] - 4*b
^3*c*f*Log[1 + E^(2*(c + d*x))] + 2*a^2*b*f*(c + d*x)*Log[1 + E^(2*(c + d*x
))] + 4*b^3*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] + I*a*(3*a^2 + 5*b^2)*f*Po
lyLog[2, (-I)*E^(c + d*x)] - I*a*(3*a^2 + 5*b^2)*f*PolyLog[2, I*E^(c + d*x)
] + a^2*b*f*PolyLog[2, -E^(2*(c + d*x))] + 2*b^3*f*PolyLog[2, -E^(2*(c + d*
x))])*(a + b*Sinh[c + d*x]))/(16*(a^2 + b^2)^2*d^2*(b + a*Csch[c + d*x])) +
 (Csch[c + d*x]*Sech[(c + d*x)/2]*(d*e*Sinh[(c + d*x)/2] - c*f*Sinh[(c + d*
x)/2] + f*(c + d*x)*Sinh[(c + d*x)/2])*(a + b*Sinh[c + d*x]))/(16*a*d^2*(b
+ a*Csch[c + d*x])) + (Csch[c + d*x]*Sech[c + d*x]*(a + b*Sinh[c + d*x])*(-
(a*f) + b*f*Sinh[c + d*x]))/(16*(a^2 + b^2)*d^2*(b + a*Csch[c + d*x])) + (C
sch[c + d*x]*Sech[c + d*x]^2*(a + b*Sinh[c + d*x])*(-(b*d*e) + b*c*f - b*f*
(c + d*x) - a*d*e*Sinh[c + d*x] + a*c*f*Sinh[c + d*x] - a*f*(c + d*x)*Sinh[
c + d*x]))/(16*(a^2 + b^2)*d^2*(b + a*Csch[c + d*x])))
```

Maple [B] time = 0.273, size = 3280, normalized size = 3.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/(a^2+b^2)^(5/2)/d^2*f*b*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2)
)*a^3-2/(a^2+b^2)^(5/2)/d^2*f*b^3*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2
)^(1/2))*a-2/(a^2+b^2)^(3/2)/d^2*b^3*f*c/a*arctanh(1/2*(2*b*exp(d*x+c)+2*a)
/(a^2+b^2)^(1/2))+3/2/(a^2+b^2)^(5/2)/d^2*f*a^3*b*c*arctanh(1/2*(2*b*exp(d*
x+c)+2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)^(5/2)/d^2*b^5*f*c/a*arctanh(1/2*(2*b
*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/a^2/(a^2+b^2)/d^2*b^3*f*c*ln(exp(d*x+c)
-1)-1/a^2/(a^2+b^2)/d*b^3*f*ln(exp(d*x+c)+1)*x+7/2*a/d^2/(a^2+b^2)^(5/2)*b^
3*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+4*a^2/d^2/(a^2+b^2)
*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*b*c+4*a^2/d^2/(a^2+b^2)*f/(4*a^2+4*b^2)
*ln(1-I*exp(d*x+c))*b*c+4*a^2/d/(a^2+b^2)*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c)
)*b*x+4*a^2/d/(a^2+b^2)*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*b*x-4*a^2/d^2/(a
```

```
^2+b^2)*f*c/(4*a^2+4*b^2)*b*ln(1+exp(2*d*x+2*c))-8/d^2/(a^2+b^2)*b^3*f*c/(4
*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))+12*a^3/d^2/(a^2+b^2)*f*c/(4*a^2+4*b^2)*arc
tan(exp(d*x+c))-20*a/d/(a^2+b^2)*b^2*e/(4*a^2+4*b^2)*arctan(exp(d*x+c))+20*
a/d^2/(a^2+b^2)*b^2*f*c/(4*a^2+4*b^2)*arctan(exp(d*x+c))+10*I*a/d^2/(a^2+b^
2)*b^2*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))-10*I*a/d^2/(a^2+b^2)*b^2*f/(4*
a^2+4*b^2)*dilog(1-I*exp(d*x+c))+6*I*a^3/d^2/(a^2+b^2)*f/(4*a^2+4*b^2)*ln(1
+I*exp(d*x+c))*c-6*I*a^3/d^2/(a^2+b^2)*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c
+6*I*a^3/d/(a^2+b^2)*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x-6*I*a^3/d/(a^2+b^
2)*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x+3/2*b/d*e/(a^2+b^2)^(3/2)*arctanh(1
/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a+a/(a^2+b^2)/d^2*f*ln(exp(d*x+c)-
1)-a/(a^2+b^2)/d^2*f*ln(exp(d*x+c)+1)-1/(a^2+b^2)/d*b*e*ln(exp(d*x+c)-1)-1/
(a^2+b^2)/d*b*e*ln(exp(d*x+c)+1)+1/(a^2+b^2)/d^2*b*f*dilog(exp(d*x+c))-1/(a
^2+b^2)/d^2*b*f*dilog(exp(d*x+c)+1)+1/(a^2+b^2)^(3/2)/d^2*f*b^3/a*arctanh(1
/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)^(5/2)/d^2*f*b^5/a*arct
anh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)^(3/2)/d^2*a*f*b*a
rctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)^(3/2)/d*b^3*e/
a*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-3/2/(a^2+b^2)^(5/2)/d*a
^3*b*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/(a^2+b^2)^(5/2)/
d*b^5*e/a*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)/d*l
n(exp(d*x+c)+1)*b*f*x+1/(a^2+b^2)/d^2*b*f*c*ln(exp(d*x+c)-1)-1/a^2/(a^2+b^2
)/d*b^3*e*ln(exp(d*x+c)-1)-1/a^2/(a^2+b^2)/d*b^3*e*ln(exp(d*x+c)+1)+1/a/(a^
2+b^2)/d^2*b^2*f*ln(exp(d*x+c)-1)-1/a/(a^2+b^2)/d^2*b^2*f*ln(exp(d*x+c)+1)+
1/a^2/(a^2+b^2)/d^2*b^3*f*dilog(exp(d*x+c))-1/a^2/(a^2+b^2)/d^2*b^3*f*dilog
(exp(d*x+c)+1)+10*I*a/d/(a^2+b^2)*b^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x-
10*I*a/d/(a^2+b^2)*b^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x+10*I*a/d^2/(a^2
+b^2)*b^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*c-10*I*a/d^2/(a^2+b^2)*b^2*f/(
4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c+1/a^2/d^2/(a^2+b^2)^2*b^5*f*dilog((-b*exp
(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/a^2/d^2/(a^2+b^2)^2*b^5*
f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+8/d/(a^2+b^2)
*b^3*e/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))+8/d^2/(a^2+b^2)*b^3*f/(4*a^2+4*b^
2)*dilog(1+I*exp(d*x+c))+8/d^2/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*dilog(1-I*exp(
d*x+c))-12*a^3/d/(a^2+b^2)*e/(4*a^2+4*b^2)*arctan(exp(d*x+c))+1/a^2/d/(a^2+
b^2)^2*b^5*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-3/2*b/d^2*f*c/(a^2+b^2)^
(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a-(3*a^2*d*f*x*exp(
5*d*x+5*c)+2*b^2*d*f*x*exp(5*d*x+5*c)+3*a^2*d*e*exp(5*d*x+5*c)+2*a*b*d*f*x*
exp(4*d*x+4*c)+2*b^2*d*e*exp(5*d*x+5*c)+2*a^2*d*f*x*exp(3*d*x+3*c)+a^2*f*ex
p(5*d*x+5*c)+2*a*b*d*e*exp(4*d*x+4*c)+4*b^2*d*f*x*exp(3*d*x+3*c)+2*a^2*d*e*
exp(3*d*x+3*c)-2*a*b*d*f*x*exp(2*d*x+2*c)+a*b*f*exp(4*d*x+4*c)+4*b^2*d*e*ex
p(3*d*x+3*c)+3*a^2*d*f*x*exp(d*x+c)-2*a*b*d*e*exp(2*d*x+2*c)+2*b^2*d*f*x*ex
p(d*x+c)+3*a^2*d*e*exp(d*x+c)+2*b^2*d*e*exp(d*x+c)-a^2*f*exp(d*x+c)-a*b*f)/
d^2/(a^2+b^2)/(1+exp(2*d*x+2*c))^2/a/(exp(2*d*x+2*c)-1)-7/2*a/d/(a^2+b^2)^(
5/2)*b^3*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/a^2/d^2/(a^2
+b^2)^2*b^5*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+8/d^2/(a^2+b^2)*b^3*f
/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c+8/d/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*ln(1+
I*exp(d*x+c))*x+8/d/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x+8/d^
2/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*c+4*a^2/d/(a^2+b^2)*e/(4
*a^2+4*b^2)*b*ln(1+exp(2*d*x+2*c))+1/a^2/d^2/(a^2+b^2)^2*b^5*f*ln((b*exp(d*
x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/a^2/d^2/(a^2+b^2)^2*b^5*f*
ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/a^2/d/(a^2+b
^2)^2*b^5*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/a^
2/d/(a^2+b^2)^2*b^5*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1
/2)))*x+4*a^2/d^2/(a^2+b^2)*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))*b+4*a^2/d
^2/(a^2+b^2)*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))*b-6*I*a^3/d^2/(a^2+b^2)*
f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))+6*I*a^3/d^2/(a^2+b^2)*f/(4*a^2+4*b^2)
*dilog(1+I*exp(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] (b^5*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^6 + 2*a^4*b^2 + a^
2*b^4)*d) + (3*a^3 + 5*a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)
*d) + (a^2*b + 2*b^3)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d)
 - (2*a*b*e^(-2*d*x - 2*c) - 2*a*b*e^(-4*d*x - 4*c) + (3*a^2 + 2*b^2)*e^(-d
*x - c) + 2*(a^2 + 2*b^2)*e^(-3*d*x - 3*c) + (3*a^2 + 2*b^2)*e^(-5*d*x - 5*
c))/((a^3 + a*b^2 + (a^3 + a*b^2)*e^(-2*d*x - 2*c) - (a^3 + a*b^2)*e^(-4*d*
x - 4*c) - (a^3 + a*b^2)*e^(-6*d*x - 6*c))*d) - b*log(e^(-d*x - c) + 1)/(a^
2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d))*e + (32*b*d*integrate(1/32*x/(a^2*d
*e^(d*x + c) + a^2*d), x) - 32*b*d*integrate(1/32*x/(a^2*d*e^(d*x + c) - a^
2*d), x) + a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) + 1)/(a^2*d^2)) - a*((d
*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^2*d^2)) - (2*a*b*d*x*e^(2*d*x +
 2*c) - 2*(a^2*d*e^(3*c) + 2*b^2*d*e^(3*c))*x*e^(3*d*x) + a*b - (a^2*e^(5*c
) + (3*a^2*d*e^(5*c) + 2*b^2*d*e^(5*c))*x)*e^(5*d*x) - (2*a*b*d*x*e^(4*c) +
 a*b*e^(4*c))*e^(4*d*x) + (a^2*e^c - (3*a^2*d*e^c + 2*b^2*d*e^c)*x)*e^(d*x)
)/(a^3*d^2 + a*b^2*d^2 - (a^3*d^2*e^(6*c) + a*b^2*d^2*e^(6*c))*e^(6*d*x) -
(a^3*d^2*e^(4*c) + a*b^2*d^2*e^(4*c))*e^(4*d*x) + (a^3*d^2*e^(2*c) + a*b^2*
d^2*e^(2*c))*e^(2*d*x)) - 32*integrate(-1/16*(a*b^5*x*e^(d*x + c) - b^6*x)/
(a^6*b + 2*a^4*b^3 + a^2*b^5 - (a^6*b*e^(2*c) + 2*a^4*b^3*e^(2*c) + a^2*b^5
*e^(2*c))*e^(2*d*x) - 2*(a^7*e^c + 2*a^5*b^2*e^c + a^3*b^4*e^c)*e^(d*x)), x
) - 32*integrate(1/32*((3*a^3*e^c + 5*a*b^2*e^c)*x*e^(d*x) + 2*(a^2*b + 2*b
^3)*x)/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(2*c) + 2*a^2*b^2*e^(2*c) + b^4*e^(2
*c))*e^(2*d*x)), x))*f
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**2*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.474 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=180

$$\frac{b^5\log(a+b\sinh(c+dx))}{a^2d\left(a^2+b^2\right)^2}-\frac{a\left(a^2+2b^2\right)\tan^{-1}(\sinh(c+dx))}{d\left(a^2+b^2\right)^2}-\frac{a\tan^{-1}(\sinh(c+dx))}{2d\left(a^2+b^2\right)}+\frac{b\left(a^2+2b^2\right)\log(\cosh(c+d}{d\left(a^2+b^2\right)^2}$$

```
[Out] -(a*ArcTan[Sinh[c + d*x]])/(2*(a^2 + b^2)*d) - (a*(a^2 + 2*b^2)*ArcTan[Sinh
[c + d*x]])/((a^2 + b^2)^2*d) - Csch[c + d*x]/(a*d) + (b*(a^2 + 2*b^2)*Log[
Cosh[c + d*x]])/((a^2 + b^2)^2*d) - (b*Log[Sinh[c + d*x]])/(a^2*d) + (b^5*L
og[a + b*Sinh[c + d*x]])/(a^2*(a^2 + b^2)^2*d) - (Sech[c + d*x]^2*(b + a*Si
nh[c + d*x]))/(2*(a^2 + b^2)*d)
```

Rubi [A] time = 0.259158, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2837, 12, 894, 639, 203, 635, 260}

$$\frac{b^5\log(a+b\sinh(c+dx))}{a^2d\left(a^2+b^2\right)^2}-\frac{a\left(a^2+2b^2\right)\tan^{-1}(\sinh(c+dx))}{d\left(a^2+b^2\right)^2}-\frac{a\tan^{-1}(\sinh(c+dx))}{2d\left(a^2+b^2\right)}+\frac{b\left(a^2+2b^2\right)\log(\cosh(c+d}{d\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[c + d*x]^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(a*ArcTan[Sinh[c + d*x]])/(2*(a^2 + b^2)*d) - (a*(a^2 + 2*b^2)*ArcTan[Sinh
[c + d*x]])/((a^2 + b^2)^2*d) - Csch[c + d*x]/(a*d) + (b*(a^2 + 2*b^2)*Log[
Cosh[c + d*x]])/((a^2 + b^2)^2*d) - (b*Log[Sinh[c + d*x]])/(a^2*d) + (b^5*L
og[a + b*Sinh[c + d*x]])/(a^2*(a^2 + b^2)^2*d) - (Sech[c + d*x]^2*(b + a*Si
nh[c + d*x]))/(2*(a^2 + b^2)*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
```

```
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b^3 \operatorname{Subst}\left(\int \frac{b^2}{x^2(a+x)\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= \frac{b^5 \operatorname{Subst}\left(\int \frac{1}{x^2(a+x)\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= \frac{b^5 \operatorname{Subst}\left(\int \left(\frac{1}{ab^4x^2} - \frac{1}{a^2b^4x} + \frac{1}{a^2\left(a^2+b^2\right)^2(a+x)} + \frac{-a+x}{b^2\left(a^2+b^2\right)\left(b^2+x^2\right)^2} - \frac{\left(a^2+2b^2\right)(a-x)}{b^4\left(a^2+b^2\right)^2\left(b^2+x^2\right)}\right) dx, x \ldots}{d}$$

$$= -\frac{\operatorname{csch}(c+dx)}{ad} - \frac{b\log(\sinh(c+dx))}{a^2d} + \frac{b^5\log(a+b\sinh(c+dx))}{a^2\left(a^2+b^2\right)^2 d} + \frac{b^3 \operatorname{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{\operatorname{csch}(c+dx)}{ad} - \frac{b\log(\sinh(c+dx))}{a^2d} + \frac{b^5\log(a+b\sinh(c+dx))}{a^2\left(a^2+b^2\right)^2 d} - \frac{\operatorname{sech}^2(c+dx)(\ldots}{2(a \ldots}$$

$$= -\frac{a\tan^{-1}(\sinh(c+dx))}{2\left(a^2+b^2\right)d} - \frac{a\left(a^2+2b^2\right)\tan^{-1}(\sinh(c+dx))}{\left(a^2+b^2\right)^2 d} - \frac{\operatorname{csch}(c+dx)}{ad} + \frac{b\left(a^2 \ldots\right.}{\ldots}$$

Mathematica [C] time = 0.93015, size = 227, normalized size = 1.26

$$-\frac{\operatorname{csch}(c+dx)(a+b\sinh(c+dx))\left(\frac{b\operatorname{sech}^2(c+dx)}{a^2+b^2} - \frac{2b^5\log(a+b\sinh(c+dx))}{a^2\left(a^2+b^2\right)^2} - \frac{(b+ia)\left(a^2+2b^2\right)\log(-\sinh(c+dx)+i)}{\left(a^2+b^2\right)^2} + \frac{(-b+ia)\left(a^2+2b^2\right)\log(\sinh \ldots}{\left(a^2+b^2\right)^2} \ldots\right.}{2d(a\operatorname{csch}(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(Csch[c + d*x]*(a + b*Sinh[c + d*x])*((a*ArcTan[Sinh[c + d*x]])/(a^2 + b^2
) + (2*Csch[c + d*x])/a - ((I*a + b)*(a^2 + 2*b^2)*Log[I - Sinh[c + d*x]])/
(a^2 + b^2)^2 + (2*b*Log[Sinh[c + d*x]])/a^2 + ((I*a - b)*(a^2 + 2*b^2)*Log
```

```
[I + Sinh[c + d*x]])/(a^2 + b^2)^2 - (2*b^5*Log[a + b*Sinh[c + d*x]])/(a^2*
(a^2 + b^2)^2) + (b*Sech[c + d*x]^2)/(a^2 + b^2) + (a*Sech[c + d*x]*Tanh[c
+ d*x])/(a^2 + b^2)))/(2*d*(b + a*Csch[c + d*x]))
```

Maple [B] time = 0.004, size = 478, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/2/d/a*tanh(1/2*d*x+1/2*c)-1/2/d/a/tanh(1/2*d*x+1/2*c)-1/d/a^2*b*ln(tanh(1
/2*d*x+1/2*c))+1/d*b^5/(a^2+b^2)^2/a^2*ln(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/
2*d*x+1/2*c)*b-a)+1/d/(a^2+b^2)^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+
1/2*c)^3*a^3+1/d/(a^2+b^2)^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c
)^3*a*b^2+2/d/(a^2+b^2)^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)^2
*a^2*b+2/d/(a^2+b^2)^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)^2*b^
3-1/d/(a^2+b^2)^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)*a^3-1/d/(
a^2+b^2)^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)*a*b^2+1/d/(a^2+b
^2)^2*ln(tanh(1/2*d*x+1/2*c)^2+1)*a^2*b+2/d/(a^2+b^2)^2*ln(tanh(1/2*d*x+1/2
*c)^2+1)*b^3-3/d/(a^2+b^2)^2*arctan(tanh(1/2*d*x+1/2*c))*a^3-5/d/(a^2+b^2)^
2*arctan(tanh(1/2*d*x+1/2*c))*a*b^2
```

Maxima [A] time = 1.88528, size = 473, normalized size = 2.63

$$\frac{b^5\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{\left(a^6+2\,a^4b^2+a^2b^4\right)d}+\frac{\left(3\,a^3+5\,ab^2\right)\arctan\left(e^{(-dx-c)}\right)}{\left(a^4+2\,a^2b^2+b^4\right)d}+\frac{\left(a^2b+2\,b^3\right)\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\left(a^4+2\,a^2b^2+b^4\right)d}-\frac{2\,abe^{(-2\,c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxim
a")
```

```
[Out] b^5*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^6 + 2*a^4*b^2 + a^2
*b^4)*d) + (3*a^3 + 5*a*b^2)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*
d) + (a^2*b + 2*b^3)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d)
- (2*a*b*e^(-2*d*x - 2*c) - 2*a*b*e^(-4*d*x - 4*c) + (3*a^2 + 2*b^2)*e^(-d*
x - c) + 2*(a^2 + 2*b^2)*e^(-3*d*x - 3*c) + (3*a^2 + 2*b^2)*e^(-5*d*x - 5*c
))/((a^3 + a*b^2 + (a^3 + a*b^2)*e^(-2*d*x - 2*c) - (a^3 + a*b^2)*e^(-4*d*x
 - 4*c) - (a^3 + a*b^2)*e^(-6*d*x - 6*c))*d) - b*log(e^(-d*x - c) + 1)/(a^2
*d) - b*log(e^(-d*x - c) - 1)/(a^2*d)
```

Fricas [B] time = 4.70562, size = 5906, normalized size = 32.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="frica
s")
```

[Out] -((3*a^5 + 5*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^5 + (3*a^5 + 5*a^3*b^2 + 2*a*b^4)*sinh(d*x + c)^5 + 2*(a^4*b + a^2*b^3)*cosh(d*x + c)^4 + (2*a^4*b + 2*a^2*b^3 + 5*(3*a^5 + 5*a^3*b^2 + 2*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + 2*(a^5 + 3*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^3 + 2*(a^5 + 3*a^3*b^2 + 2*a*b^4 + 5*(3*a^5 + 5*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^2 + 4*(a^4*b + a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(a^4*b + a^2*b^3)*cosh(d*x + c)^2 - 2*(a^4*b + a^2*b^3 - 5*(3*a^5 + 5*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^3 - 6*(a^4*b + a^2*b^3)*cosh(d*x + c)^2 - 3*(a^5 + 3*a^3*b^2 + 2*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + ((3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^6 + 6*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^5 + 5*a^3*b^2)*sinh(d*x + c)^6 - 3*a^5 - 5*a^3*b^2 + (3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^4 + (3*a^5 + 5*a^3*b^2 + 15*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^3 + (3*a^5 + 5*a^3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^2 - (3*a^5 + 5*a^3*b^2 - 15*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^4 - 6*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^5 + 2*(3*a^5 + 5*a^3*b^2)*cosh(d*x + c)^3 - (3*a^5 + 5*a^3*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + (3*a^5 + 5*a^3*b^2 + 2*a*b^4)*cosh(d*x + c) - (b^5*cosh(d*x + c)^6 + 6*b^5*cosh(d*x + c)*sinh(d*x + c)^5 + b^5*sinh(d*x + c)^6 + b^5*cosh(d*x + c)^4 - b^5*cosh(d*x + c)^2 - b^5 + (15*b^5*cosh(d*x + c)^2 + b^5)*sinh(d*x + c)^4 + 4*(5*b^5*cosh(d*x + c)^3 + b^5*cosh(d*x + c))*sinh(d*x + c)^3 + (15*b^5*cosh(d*x + c)^4 + 6*b^5*cosh(d*x + c)^2 - b^5)*sinh(d*x + c)^2 + 2*(3*b^5*cosh(d*x + c)^5 + 2*b^5*cosh(d*x + c)^3 - b^5*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) - ((a^4*b + 2*a^2*b^3)*cosh(d*x + c)^6 + 6*(a^4*b + 2*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4*b + 2*a^2*b^3)*sinh(d*x + c)^6 - a^4*b - 2*a^2*b^3 + (a^4*b + 2*a^2*b^3)*cosh(d*x + c)^4 + (a^4*b + 2*a^2*b^3 + 15*(a^4*b + 2*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(a^4*b + 2*a^2*b^3)*cosh(d*x + c)^3 + (a^4*b + 2*a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - (a^4*b + 2*a^2*b^3)*cosh(d*x + c)^2 - (a^4*b + 2*a^2*b^3 - 15*(a^4*b + 2*a^2*b^3)*cosh(d*x + c)^4 - 6*(a^4*b + 2*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(a^4*b + 2*a^2*b^3)*cosh(d*x + c)^5 + 2*(a^4*b + 2*a^2*b^3)*cosh(d*x + c)^3 - (a^4*b + 2*a^2*b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + ((a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^6 + 6*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4*b + 2*a^2*b^3 + b^5)*sinh(d*x + c)^6 - a^4*b - 2*a^2*b^3 - b^5 + (a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^4 + (a^4*b + 2*a^2*b^3 + b^5 + 15*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^3 + (a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - (a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^2 - (a^4*b + 2*a^2*b^3 + b^5 - 15*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^4 - 6*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^3 - (a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + (3*a^5 + 5*a^3*b^2 + 2*a*b^4 + 5*(3*a^5 + 5*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^4 + 8*(a^4*b + a^2*b^3)*cosh(d*x + c)^3 + 6*(a^5 + 3*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^2 - 4*(a^4*b + a^2*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^6 + 6*(a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d*sinh(d*x + c)^6 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^4 + (15*(a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^2 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d)*sinh(d*x + c)^4 - (a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^2 + 4*(5*(a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^3 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^4 + 6*(a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^2 - (a^6 + 2*a^4*b^2 + a^2*b^4)*d)*sinh(d*x + c)^2 - (a^6 + 2*a^4*b^2 + a^2*b^4)*d + 2*(3*(a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^5 + 2*(a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^3 - (a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36711, size = 637, normalized size = 3.54

$$\frac{b^6 \log\left(\left|b\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 2\,a\right|\right)}{a^6bd + 2\,a^4b^3d + a^2b^5d} - \frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)\left(3\,a^3 + 5\,ab^2\right)}{4\left(a^4d + 2\,a^2b^2d + b^4d\right)} + \frac{\left(a^2b + 2\,b^3\right)\log\left(\left(e^{(dx}\right.\right.}{2\left(a^4d + 2\,a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac"
)
```

```
[Out] b^6*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^6*b*d + 2*a^4*b^3*d +
 a^2*b^5*d) - 1/4*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(
3*a^3 + 5*a*b^2)/(a^4*d + 2*a^2*b^2*d + b^4*d) + 1/2*(a^2*b + 2*b^3)*log((e
^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^4*d + 2*a^2*b^2*d + b^4*d) + 1/3*(b^5*
(e^(d*x + c) - e^(-d*x - c))^3 - 9*a^5*(e^(d*x + c) - e^(-d*x - c))^2 - 15*
a^3*b^2*(e^(d*x + c) - e^(-d*x - c))^2 - 6*a*b^4*(e^(d*x + c) - e^(-d*x - c
))^2 - 6*a^4*b*(e^(d*x + c) - e^(-d*x - c)) - 6*a^2*b^3*(e^(d*x + c) - e^(-
d*x - c)) + 4*b^5*(e^(d*x + c) - e^(-d*x - c)) - 24*a^5 - 48*a^3*b^2 - 24*a
*b^4)/((a^6*d + 2*a^4*b^2*d + a^2*b^4*d)*((e^(d*x + c) - e^(-d*x - c))^3 +
4*e^(d*x + c) - 4*e^(-d*x - c))) - b*log(abs(e^(d*x + c) - e^(-d*x - c)))/(
a^2*d)
```

3.475 $\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=38

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Csch[c + d*x]^2*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d
*x])), x]
```

Rubi [A] time = 0.135229, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[c + d*x]^2*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Csch[c + d*x]^2*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.002, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[c + d*x]^2*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x]
)),x]
```

```
[Out] $Aborted
```

Maple [A] time = 3.345, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx+c))^2(\operatorname{sech}(dx+c))^3}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] (a*b*f + (2*b^2*d*e*e^(5*c) + (3*d*e - f)*a^2*e^(5*c) + (3*a^2*d*f*e^(5*c)
+ 2*b^2*d*f*e^(5*c))*x)*e^(5*d*x) + (2*a*b*d*f*x*e^(4*c) + (2*d*e - f)*a*b*
e^(4*c))*e^(4*d*x) + 2*(a^2*d*e*e^(3*c) + 2*b^2*d*e*e^(3*c) + (a^2*d*f*e^(3
*c) + 2*b^2*d*f*e^(3*c))*x)*e^(3*d*x) - 2*(a*b*d*f*x*e^(2*c) + a*b*d*e*e^(2
*c))*e^(2*d*x) + (2*b^2*d*e*e^c + (3*d*e + f)*a^2*e^c + (3*a^2*d*f*e^c + 2*
b^2*d*f*e^c)*x)*e^(d*x))/(a^3*d^2*e^2 + a*b^2*d^2*e^2 + (a^3*d^2*f^2 + a*b^
2*d^2*f^2)*x^2 + 2*(a^3*d^2*e*f + a*b^2*d^2*e*f)*x - (a^3*d^2*e^2*e^(6*c) +
 a*b^2*d^2*e^2*e^(6*c) + (a^3*d^2*f^2*e^(6*c) + a*b^2*d^2*f^2*e^(6*c))*x^2
+ 2*(a^3*d^2*e*f*e^(6*c) + a*b^2*d^2*e*f*e^(6*c))*x)*e^(6*d*x) - (a^3*d^2*e
^2*e^(4*c) + a*b^2*d^2*e^2*e^(4*c) + (a^3*d^2*f^2*e^(4*c) + a*b^2*d^2*f^2*e
^(4*c))*x^2 + 2*(a^3*d^2*e*f*e^(4*c) + a*b^2*d^2*e*f*e^(4*c))*x)*e^(4*d*x)
+ (a^3*d^2*e^2*e^(2*c) + a*b^2*d^2*e^2*e^(2*c) + (a^3*d^2*f^2*e^(2*c) + a*b
^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^3*d^2*e*f*e^(2*c) + a*b^2*d^2*e*f*e^(2*c))*x
)*e^(2*d*x)) - 32*integrate(-1/16*(a*b^5*e^(d*x + c) - b^6)/(a^6*b*e + 2*a^
4*b^3*e + a^2*b^5*e + (a^6*b*f + 2*a^4*b^3*f + a^2*b^5*f)*x - (a^6*b*e*e^(2
*c) + 2*a^4*b^3*e*e^(2*c) + a^2*b^5*e*e^(2*c) + (a^6*b*f*e^(2*c) + 2*a^4*b^
3*f*e^(2*c) + a^2*b^5*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^7*e*e^c + 2*a^5*b^2*e*
e^c + a^3*b^4*e*e^c + (a^7*f*e^c + 2*a^5*b^2*f*e^c + a^3*b^4*f*e^c)*x)*e^(d
*x)), x) - 32*integrate(1/32*(2*(d^2*e^2 - f^2)*a^2*b + 2*(2*d^2*e^2 - f^2)
*b^3 + 2*(a^2*b*d^2*f^2 + 2*b^3*d^2*f^2)*x^2 + 4*(a^2*b*d^2*e*f + 2*b^3*d^2
*e*f)*x + ((3*d^2*e^2 - 2*f^2)*a^3*e^c + (5*d^2*e^2 - 2*f^2)*a*b^2*e^c + (3
*a^3*d^2*f^2*e^c + 5*a*b^2*d^2*f^2*e^c)*x^2 + 2*(3*a^3*d^2*e*f*e^c + 5*a*b^
2*d^2*e*f*e^c)*x)*e^(d*x))/(a^4*d^2*e^3 + 2*a^2*b^2*d^2*e^3 + b^4*d^2*e^3 +
 (a^4*d^2*f^3 + 2*a^2*b^2*d^2*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*e*f^2 + 2
*a^2*b^2*d^2*e*f^2 + b^4*d^2*e*f^2)*x^2 + 3*(a^4*d^2*e^2*f + 2*a^2*b^2*d^2*
e^2*f + b^4*d^2*e^2*f)*x + (a^4*d^2*e^3*e^(2*c) + 2*a^2*b^2*d^2*e^3*e^(2*c)
 + b^4*d^2*e^3*e^(2*c) + (a^4*d^2*f^3*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(2*c) +
 b^4*d^2*f^3*e^(2*c))*x^3 + 3*(a^4*d^2*e*f^2*e^(2*c) + 2*a^2*b^2*d^2*e*f^2*
e^(2*c) + b^4*d^2*e*f^2*e^(2*c))*x^2 + 3*(a^4*d^2*e^2*f*e^(2*c) + 2*a^2*b^2
*d^2*e^2*f*e^(2*c) + b^4*d^2*e^2*f*e^(2*c))*x)*e^(2*d*x)), x) - 32*integrat
e(-1/32*(b*d*f*x + b*d*e + a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2
- (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) + 32
*integrate(1/32*(b*d*f*x + b*d*e - a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^
2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*sech(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.476 $\int \frac{(e+fx)^3 \coth(c+dx)\mathbf{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=752

$$\frac{6b^2f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^3}+\frac{6b^2f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3d^3}-\frac{3b^2f^2(e+fx)\text{PolyLog}\left(3,e^{2(c+dx)}\right)}{2a^3d^3}$$

```
[Out] (-3*f*(e + f*x)^2)/(2*a*d^2) + (6*b*f*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^
2*d^2) - (3*f*(e + f*x)^2*Coth[c + d*x])/(2*a*d^2) + (b*(e + f*x)^3*Csch[c
+ d*x])/(a^2*d) - ((e + f*x)^3*Csch[c + d*x]^2)/(2*a*d) - (b^2*(e + f*x)^3*
Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*d) - (b^2*(e + f*x)^3*
Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*d) + (3*f^2*(e + f*x)*
Log[1 - E^(2*(c + d*x))])/(a*d^3) + (b^2*(e + f*x)^3*Log[1 - E^(2*(c + d*x)
)])/(a^3*d) + (6*b*f^2*(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a^2*d^3) - (6*b
*f^2*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^2*d^3) - (3*b^2*f*(e + f*x)^2*Po
lyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2) - (3*b^2*f*(e
 + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) +
 (3*f^3*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^4) + (3*b^2*f*(e + f*x)^2*PolyL
og[2, E^(2*(c + d*x))])/(2*a^3*d^2) - (6*b*f^3*PolyLog[3, -E^(c + d*x)])/(a
^2*d^4) + (6*b*f^3*PolyLog[3, E^(c + d*x)])/(a^2*d^4) + (6*b^2*f^2*(e + f*x
)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^3) + (6*b^2*
f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^
3) - (3*b^2*f^2*(e + f*x)*PolyLog[3, E^(2*(c + d*x))])/(2*a^3*d^3) - (6*b^2
*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^4) - (6*b
^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^4) + (3
*b^2*f^3*PolyLog[4, E^(2*(c + d*x))])/(4*a^3*d^4)
```

Rubi [A] time = 1.35304, antiderivative size = 752, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 14, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5587, 5452, 4184, 3716, 2190, 2279, 2391, 4182, 2531, 2282, 6589, 5569, 6609, 5561}

$$\frac{6b^2f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^3}+\frac{6b^2f^2(e+fx)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3d^3}-\frac{3b^2f^2(e+fx)\text{PolyLog}\left(3,e^{2(c+dx)}\right)}{2a^3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (-3*f*(e + f*x)^2)/(2*a*d^2) + (6*b*f*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^
2*d^2) - (3*f*(e + f*x)^2*Coth[c + d*x])/(2*a*d^2) + (b*(e + f*x)^3*Csch[c
+ d*x])/(a^2*d) - ((e + f*x)^3*Csch[c + d*x]^2)/(2*a*d) - (b^2*(e + f*x)^3*
Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*d) - (b^2*(e + f*x)^3*
Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*d) + (3*f^2*(e + f*x)*
Log[1 - E^(2*(c + d*x))])/(a*d^3) + (b^2*(e + f*x)^3*Log[1 - E^(2*(c + d*x)
)])/(a^3*d) + (6*b*f^2*(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a^2*d^3) - (6*b
*f^2*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^2*d^3) - (3*b^2*f*(e + f*x)^2*Po
lyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2) - (3*b^2*f*(e
 + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) +
 (3*f^3*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^4) + (3*b^2*f*(e + f*x)^2*PolyL
og[2, E^(2*(c + d*x))])/(2*a^3*d^2) - (6*b*f^3*PolyLog[3, -E^(c + d*x)])/(a
^2*d^4) + (6*b*f^3*PolyLog[3, E^(c + d*x)])/(a^2*d^4) + (6*b^2*f^2*(e + f*x
)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^3) + (6*b^2*
f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^
3) - (3*b^2*f^2*(e + f*x)*PolyLog[3, E^(2*(c + d*x))])/(2*a^3*d^3) - (6*b^2
```

```
*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^4) - (6*b
^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^4) + (3
*b^2*f^3*PolyLog[4, E^(2*(c + d*x))])/(4*a^3*d^4)
```

Rule 5587

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Csch[c + d*x]^(p - 1)*Coth[c + d*x]^n)/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3 \coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3 \coth(c+dx)\text{csch}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3 \coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^3\text{csch}^2(c+dx)}{2ad} - \frac{b\int (e+fx)^3 \coth(c+dx)\text{csch}(c+dx)\,dx}{a^2} + \frac{b^2}{}$$

$$= -\frac{3f(e+fx)^2 \coth(c+dx)}{2ad^2} + \frac{b(e+fx)^3\text{csch}(c+dx)}{a^2 d} - \frac{(e+fx)^3\text{csch}^2(c+}{2ad}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{6bf(e+fx)^2 \tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{3f(e+fx)^2 \coth(c+dx)}{2ad^2} +$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{6bf(e+fx)^2 \tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{3f(e+fx)^2 \coth(c+dx)}{2ad^2} +$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{6bf(e+fx)^2 \tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{3f(e+fx)^2 \coth(c+dx)}{2ad^2} +$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{6bf(e+fx)^2 \tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{3f(e+fx)^2 \coth(c+dx)}{2ad^2} +$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{6bf(e+fx)^2 \tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{3f(e+fx)^2 \coth(c+dx)}{2ad^2} +$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{6bf(e+fx)^2 \tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{3f(e+fx)^2 \coth(c+dx)}{2ad^2} +$$

Mathematica [C] time = 65.6056, size = 6441, normalized size = 8.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.936, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \coth(dx+c)(\text{csch}(dx+c))^2}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -e^3*(2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*
e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + b^2*log(-2*a*e^(-d*x -
c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) - b^2*log(e^(-d*x - c) + 1)/(a^3*d) -
b^2*log(e^(-d*x - c) - 1)/(a^3*d)) + (3*a*f^3*x^2 + 6*a*e*f^2*x + 3*a*e^2*f
 + 2*(b*d*f^3*x^3*e^(3*c) + 3*b*d*e*f^2*x^2*e^(3*c) + 3*b*d*e^2*f*x*e^(3*c)
)*e^(3*d*x) - (2*a*d*f^3*x^3*e^(2*c) + 3*a*e^2*f*e^(2*c) + 3*(2*d*e*f^2 + f
^3)*a*x^2*e^(2*c) + 6*(d*e^2*f + e*f^2)*a*x*e^(2*c))*e^(2*d*x) - 2*(b*d*f^3
*x^3*e^c + 3*b*d*e*f^2*x^2*e^c + 3*b*d*e^2*f*x*e^c)*e^(d*x))/(a^2*d^2*e^(4*
d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) + (d^3*x^3*log(e^(d*x + c
) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6
*polylog(4, -e^(d*x + c)))*b^2*f^3/(a^3*d^4) + (d^3*x^3*log(-e^(d*x + c) +
1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polyl
og(4, e^(d*x + c)))*b^2*f^3/(a^3*d^4) - 3*(b*d*e^2*f + a*e*f^2)*x/(a^2*d^2)
 + 3*(b*d*e^2*f - a*e*f^2)*x/(a^2*d^2) + 3*(b*d*e^2*f + a*e*f^2)*log(e^(d*x
 + c) + 1)/(a^2*d^3) - 3*(b*d*e^2*f - a*e*f^2)*log(e^(d*x + c) - 1)/(a^2*d^
3) + 3*(b^2*d*e*f^2 + a*b*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(
-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^3*d^4) + 3*(b^2*d*e*f^2 - a*
b*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylo
g(3, e^(d*x + c)))/(a^3*d^4) + 3*(b^2*d^2*e^2*f + 2*a*b*d*e*f^2 + a^2*f^3)*
(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^3*d^4) + 3*(b^2*d^2*e^2
*f - 2*a*b*d*e*f^2 + a^2*f^3)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c
)))/(a^3*d^4) - 1/4*(b^2*d^4*f^3*x^4 + 4*(b^2*d*e*f^2 + a*b*f^3)*d^3*x^3 +
6*(b^2*d^2*e^2*f + 2*a*b*d*e*f^2 + a^2*f^3)*d^2*x^2)/(a^3*d^4) - 1/4*(b^2*d
^4*f^3*x^4 + 4*(b^2*d*e*f^2 - a*b*f^3)*d^3*x^3 + 6*(b^2*d^2*e^2*f - 2*a*b*d
*e*f^2 + a^2*f^3)*d^2*x^2)/(a^3*d^4) + integrate(-2*(b^3*f^3*x^3 + 3*b^3*e*
f^2*x^2 + 3*b^3*e^2*f*x - (a*b^2*f^3*x^3*e^c + 3*a*b^2*e*f^2*x^2*e^c + 3*a*
b^2*e^2*f*x*e^c)*e^(d*x))/(a^3*b*e^(2*d*x + 2*c) + 2*a^4*e^(d*x + c) - a^3*
b), x)
```

Fricas [C] time = 4.92579, size = 24935, normalized size = 33.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] (3*a^2*d^2*e^2*f - 6*a^2*c*d*e*f^2 + 3*a^2*c^2*f^3 - 3*(a^2*d^2*f^3*x^2 + 2
*a^2*d^2*e*f^2*x + 2*a^2*c*d*e*f^2 - a^2*c^2*f^3)*cosh(d*x + c)^4 - 3*(a^2*
d^2*f^3*x^2 + 2*a^2*d^2*e*f^2*x + 2*a^2*c*d*e*f^2 - a^2*c^2*f^3)*sinh(d*x +
 c)^4 + 2*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*e^2*f*x + a*b*
d^3*e^3)*cosh(d*x + c)^3 + 2*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b
*d^3*e^2*f*x + a*b*d^3*e^3 - 6*(a^2*d^2*f^3*x^2 + 2*a^2*d^2*e*f^2*x + 2*a^2
*c*d*e*f^2 - a^2*c^2*f^3)*cosh(d*x + c))*sinh(d*x + c)^3 - (2*a^2*d^3*f^3*x
^3 + 2*a^2*d^3*e^3 + 3*a^2*d^2*e^2*f - 12*a^2*c*d*e*f^2 + 6*a^2*c^2*f^3 + 3
*(2*a^2*d^3*e*f^2 - a^2*d^2*f^3)*x^2 + 6*(a^2*d^3*e^2*f - a^2*d^2*e*f^2)*x)
*cosh(d*x + c)^2 - (2*a^2*d^3*f^3*x^3 + 2*a^2*d^3*e^3 + 3*a^2*d^2*e^2*f - 1
2*a^2*c*d*e*f^2 + 6*a^2*c^2*f^3 + 3*(2*a^2*d^3*e*f^2 - a^2*d^2*f^3)*x^2 + 1
8*(a^2*d^2*f^3*x^2 + 2*a^2*d^2*e*f^2*x + 2*a^2*c*d*e*f^2 - a^2*c^2*f^3)*cos
h(d*x + c)^2 + 6*(a^2*d^3*e^2*f - a^2*d^2*e*f^2)*x - 6*(a*b*d^3*f^3*x^3 + 3
*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*e^2*f*x + a*b*d^3*e^3)*cosh(d*x + c))*sinh(d
```

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*x + c)^2 - 2*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*e^2*f*x +
a*b*d^3*e^3)*cosh(d*x + c) - 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d
^2*e^2*f + (b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*cosh(d*x +
 c)^4 + 4*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*cosh(d*x +
c)*sinh(d*x + c)^3 + (b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*
sinh(d*x + c)^4 - 2*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*c
osh(d*x + c)^2 - 2*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f - 3
*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*cosh(d*x + c)^2)*sin
h(d*x + c)^2 + 4*((b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*cos
h(d*x + c)^3 - (b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*cosh(d
*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(b^2*d^2*
f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f + (b^2*d^2*f^3*x^2 + 2*b^2*d^2*
e*f^2*x + b^2*d^2*e^2*f)*cosh(d*x + c)^4 + 4*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e
*f^2*x + b^2*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d^2*f^3*x^2 +
2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*sinh(d*x + c)^4 - 2*(b^2*d^2*f^3*x^2 + 2
*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*cosh(d*x + c)^2 - 2*(b^2*d^2*f^3*x^2 + 2*
b^2*d^2*e*f^2*x + b^2*d^2*e^2*f - 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x +
b^2*d^2*e^2*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d^2*f^3*x^2 + 2*b
^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*cosh(d*x + c)^3 - (b^2*d^2*f^3*x^2 + 2*b^2*
d^2*e*f^2*x + b^2*d^2*e^2*f)*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*
x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b + 1) + 3*(b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e*f^2
+ a^2*f^3 + (b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e*f^2 + a^2*f^3 + 2*
(b^2*d^2*e*f^2 - a*b*d*f^3)*x)*cosh(d*x + c)^4 + 4*(b^2*d^2*f^3*x^2 + b^2*d
^2*e^2*f - 2*a*b*d*e*f^2 + a^2*f^3 + 2*(b^2*d^2*e*f^2 - a*b*d*f^3)*x)*cosh(
d*x + c)*sinh(d*x + c)^3 + (b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e*f^2
 + a^2*f^3 + 2*(b^2*d^2*e*f^2 - a*b*d*f^3)*x)*sinh(d*x + c)^4 - 2*(b^2*d^2*
f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e*f^2 + a^2*f^3 + 2*(b^2*d^2*e*f^2 - a*b*
d*f^3)*x)*cosh(d*x + c)^2 - 2*(b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e*
f^2 + a^2*f^3 - 3*(b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e*f^2 + a^2*f^
3 + 2*(b^2*d^2*e*f^2 - a*b*d*f^3)*x)*cosh(d*x + c)^2 + 2*(b^2*d^2*e*f^2 - a
*b*d*f^3)*x)*sinh(d*x + c)^2 + 2*(b^2*d^2*e*f^2 - a*b*d*f^3)*x + 4*((b^2*d^
2*f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e*f^2 + a^2*f^3 + 2*(b^2*d^2*e*f^2 - a*
b*d*f^3)*x)*cosh(d*x + c)^3 - (b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e*
f^2 + a^2*f^3 + 2*(b^2*d^2*e*f^2 - a*b*d*f^3)*x)*cosh(d*x + c))*sinh(d*x +
c))*dilog(cosh(d*x + c) + sinh(d*x + c)) + 3*(b^2*d^2*f^3*x^2 + b^2*d^2*e^2
*f + 2*a*b*d*e*f^2 + a^2*f^3 + (b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f + 2*a*b*d*e
*f^2 + a^2*f^3 + 2*(b^2*d^2*e*f^2 + a*b*d*f^3)*x)*cosh(d*x + c)^4 + 4*(b^2*
d^2*f^3*x^2 + b^2*d^2*e^2*f + 2*a*b*d*e*f^2 + a^2*f^3 + 2*(b^2*d^2*e*f^2 +
a*b*d*f^3)*x)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d^2*f^3*x^2 + b^2*d^2*e^
2*f + 2*a*b*d*e*f^2 + a^2*f^3 + 2*(b^2*d^2*e*f^2 + a*b*d*f^3)*x)*sinh(d*x +
 c)^4 - 2*(b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f + 2*a*b*d*e*f^2 + a^2*f^3 + 2*(b
^2*d^2*e*f^2 + a*b*d*f^3)*x)*cosh(d*x + c)^2 - 2*(b^2*d^2*f^3*x^2 + b^2*d^2
*e^2*f + 2*a*b*d*e*f^2 + a^2*f^3 - 3*(b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f + 2*a
*b*d*e*f^2 + a^2*f^3 + 2*(b^2*d^2*e*f^2 + a*b*d*f^3)*x)*cosh(d*x + c)^2 + 2
*(b^2*d^2*e*f^2 + a*b*d*f^3)*x)*sinh(d*x + c)^2 + 2*(b^2*d^2*e*f^2 + a*b*d*
f^3)*x + 4*((b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f + 2*a*b*d*e*f^2 + a^2*f^3 + 2*
(b^2*d^2*e*f^2 + a*b*d*f^3)*x)*cosh(d*x + c)^3 - (b^2*d^2*f^3*x^2 + b^2*d^2
*e^2*f + 2*a*b*d*e*f^2 + a^2*f^3 + 2*(b^2*d^2*e*f^2 + a*b*d*f^3)*x)*cosh(d*
x + c))*sinh(d*x + c))*dilog(-cosh(d*x + c) - sinh(d*x + c)) - (b^2*d^3*e^3
 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3 + (b^2*d^3*e^3 - 3*b
^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*cosh(d*x + c)^4 + 4*(b^2*
d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*cosh(d*x + c
)*sinh(d*x + c)^3 + (b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 -
b^2*c^3*f^3)*sinh(d*x + c)^4 - 2*(b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c
^2*d*e*f^2 - b^2*c^3*f^3)*cosh(d*x + c)^2 - 2*(b^2*d^3*e^3 - 3*b^2*c*d^2*e^
2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3 - 3*(b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f
+ 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b
```

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^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*cosh(d*x
+ c)^3 - (b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3
)*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) +
 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^
2*c^2*d*e*f^2 - b^2*c^3*f^3 + (b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*
d*e*f^2 - b^2*c^3*f^3)*cosh(d*x + c)^4 + 4*(b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f
 + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d^
3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*sinh(d*x + c)^
4 - 2*(b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*c
osh(d*x + c)^2 - 2*(b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b
^2*c^3*f^3 - 3*(b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c
^3*f^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d^3*e^3 - 3*b^2*c*d^2*e^
2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*cosh(d*x + c)^3 - (b^2*d^3*e^3 - 3*b
^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*cosh(d*x + c))*sinh(d*x +
 c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2)
+ 2*a) - (b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2
*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3 + (b^2*d^3*f^3*x^3 + 3*b^2*d
^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 +
b^2*c^3*f^3)*cosh(d*x + c)^4 + 4*(b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3
*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*cos
h(d*x + c)*sinh(d*x + c)^3 + (b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2
*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*sinh(d*
x + c)^4 - 2*(b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3
*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*cosh(d*x + c)^2 - 2*(b^
2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f
 - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3 - 3*(b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x
^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2
*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*
f^3)*cosh(d*x + c)^3 - (b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e
^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*cosh(d*x + c)
)*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^2*d^3*f^3*x^3 + 3*b^
2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2
 + b^2*c^3*f^3 + (b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x
 + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*cosh(d*x + c)^4 + 4
*(b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e
^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^
2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f
 - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*sinh(d*x + c)^4 - 2*(b^2*d^3*f^3*x^3 +
3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e
*f^2 + b^2*c^3*f^3)*cosh(d*x + c)^2 - 2*(b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*
x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f
^3 - 3*(b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c
*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 + 4*((b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2
*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*cosh(d*x + c)^3 - (b^2*d^3*
f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b
^2*c^2*d*e*f^2 + b^2*c^3*f^3)*cosh(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*
x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b) + (b^2*d^3*f^3*x^3 + b^2*d^3*e^3 + 3*a*b*d^2*e^2*f + 3*a^
2*d*e*f^2 + (b^2*d^3*f^3*x^3 + b^2*d^3*e^3 + 3*a*b*d^2*e^2*f + 3*a^2*d*e*f^
2 + 3*(b^2*d^3*e*f^2 + a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f + 2*a*b*d^2*e*f^
2 + a^2*d*f^3)*x)*cosh(d*x + c)^4 + 4*(b^2*d^3*f^3*x^3 + b^2*d^3*e^3 + 3*a*
b*d^2*e^2*f + 3*a^2*d*e*f^2 + 3*(b^2*d^3*e*f^2 + a*b*d^2*f^3)*x^2 + 3*(b^2*
d^3*e^2*f + 2*a*b*d^2*e*f^2 + a^2*d*f^3)*x)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (b^2*d^3*f^3*x^3 + b^2*d^3*e^3 + 3*a*b*d^2*e^2*f + 3*a^2*d*e*f^2 + 3*(b^2*
d^3*e*f^2 + a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f + 2*a*b*d^2*e*f^2 + a^2*d*f
^3)*x)*sinh(d*x + c)^4 + 3*(b^2*d^3*e*f^2 + a*b*d^2*f^3)*x^2 - 2*(b^2*d^3*f
```

^3*x^3 + b^2*d^3*e^3 + 3*a*b*d^2*e^2*f + 3*a^2*d*e*f^2 + 3*(b^2*d^3*e*f^2 + a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f + 2*a*b*d^2*e*f^2 + a^2*d*f^3)*x)*cosh (d*x + c)^2 - 2*(b^2*d^3*f^3*x^3 + b^2*d^3*e^3 + 3*a*b*d^2*e^2*f + 3*a^2*d* e*f^2 + 3*(b^2*d^3*e*f^2 + a*b*d^2*f^3)*x^2 - 3*(b^2*d^3*f^3*x^3 + b^2*d^3* e^3 + 3*a*b*d^2*e^2*f + 3*a^2*d*e*f^2 + 3*(b^2*d^3*e*f^2 + a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f + 2*a*b*d^2*e*f^2 + a^2*d*f^3)*x)*cosh(d*x + c)^2 + 3*(b^2*d^3*e^2*f + 2*a*b*d^2*e*f^2 + a^2*d*f^3)*x)*sinh(d*x + c)^2 + 3*(b^2*d^ 3*e^2*f + 2*a*b*d^2*e*f^2 + a^2*d*f^3)*x + 4*((b^2*d^3*f^3*x^3 + b^2*d^3*e^ 3 + 3*a*b*d^2*e^2*f + 3*a^2*d*e*f^2 + 3*(b^2*d^3*e*f^2 + a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f + 2*a*b*d^2*e*f^2 + a^2*d*f^3)*x)*cosh(d*x + c)^3 - (b^2* d^3*f^3*x^3 + b^2*d^3*e^3 + 3*a*b*d^2*e^2*f + 3*a^2*d*e*f^2 + 3*(b^2*d^3*e* f^2 + a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f + 2*a*b*d^2*e*f^2 + a^2*d*f^3)*x) *cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (b^ 2*d^3*e^3 - 3*(b^2*c + a*b)*d^2*e^2*f + 3*(b^2*c^2 + 2*a*b*c + a^2)*d*e*f^2 + (b^2*d^3*e^3 - 3*(b^2*c + a*b)*d^2*e^2*f + 3*(b^2*c^2 + 2*a*b*c + a^2)*d *e*f^2 - (b^2*c^3 + 3*a*b*c^2 + 3*a^2*c)*f^3)*cosh(d*x + c)^4 + 4*(b^2*d^3* e^3 - 3*(b^2*c + a*b)*d^2*e^2*f + 3*(b^2*c^2 + 2*a*b*c + a^2)*d*e*f^2 - (b^ 2*c^3 + 3*a*b*c^2 + 3*a^2*c)*f^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d^3* e^3 - 3*(b^2*c + a*b)*d^2*e^2*f + 3*(b^2*c^2 + 2*a*b*c + a^2)*d*e*f^2 - (b^ 2*c^3 + 3*a*b*c^2 + 3*a^2*c)*f^3)*sinh(d*x + c)^4 - (b^2*c^3 + 3*a*b*c^2 + 3*a^2*c)*f^3 - 2*(b^2*d^3*e^3 - 3*(b^2*c + a*b)*d^2*e^2*f + 3*(b^2*c^2 + 2* a*b*c + a^2)*d*e*f^2 - (b^2*c^3 + 3*a*b*c^2 + 3*a^2*c)*f^3)*cosh(d*x + c)^2 - 2*(b^2*d^3*e^3 - 3*(b^2*c + a*b)*d^2*e^2*f + 3*(b^2*c^2 + 2*a*b*c + a^2) *d*e*f^2 - (b^2*c^3 + 3*a*b*c^2 + 3*a^2*c)*f^3 - 3*(b^2*d^3*e^3 - 3*(b^2*c + a*b)*d^2*e^2*f + 3*(b^2*c^2 + 2*a*b*c + a^2)*d*e*f^2 - (b^2*c^3 + 3*a*b*c ^2 + 3*a^2*c)*f^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d^3*e^3 - 3*(b^2*c + a*b)*d^2*e^2*f + 3*(b^2*c^2 + 2*a*b*c + a^2)*d*e*f^2 - (b^2*c^3 + 3 *a*b*c^2 + 3*a^2*c)*f^3)*cosh(d*x + c)^3 - (b^2*d^3*e^3 - 3*(b^2*c + a*b)*d ^2*e^2*f + 3*(b^2*c^2 + 2*a*b*c + a^2)*d*e*f^2 - (b^2*c^3 + 3*a*b*c^2 + 3*a ^2*c)*f^3)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + (b^2*d^3*f^3*x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 + (b^2*d^3*f^3*x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 + (b^2*c^3 + 3*a*b*c^2 + 3*a^2*c)*f^3 + 3*(b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f - 2*a*b*d^2*e*f^2 + a^2*d*f^3)*x)*cosh(d*x + c)^4 + 4*(b^2 *d^3*f^3*x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 + (b^2*c^3 + 3*a*b*c^2 + 3*a^2*c)*f^3 + 3*(b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(b^2* d^3*e^2*f - 2*a*b*d^2*e*f^2 + a^2*d*f^3)*x)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d^3*f^3*x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 + (b^ 2*c^3 + 3*a*b*c^2 + 3*a^2*c)*f^3 + 3*(b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3* (b^2*d^3*e^2*f - 2*a*b*d^2*e*f^2 + a^2*d*f^3)*x)*sinh(d*x + c)^4 + (b^2*c^3 + 3*a*b*c^2 + 3*a^2*c)*f^3 + 3*(b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2 - 2*(b^2* d^3*f^3*x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 + (b^2*c^3 + 3*a*b*c^2 + 3*a^2*c)*f^3 + 3*(b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(b^2*d ^3*e^2*f - 2*a*b*d^2*e*f^2 + a^2*d*f^3)*x)*cosh(d*x + c)^2 - 2*(b^2*d^3*f^3 *x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 + (b^2*c^3 + 3*a*b *c^2 + 3*a^2*c)*f^3 + 3*(b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2 - 3*(b^2*d^3*f^3* x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 + (b^2*c^3 + 3*a*b* c^2 + 3*a^2*c)*f^3 + 3*(b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f - 2*a*b*d^2*e*f^2 + a^2*d*f^3)*x)*cosh(d*x + c)^2 + 3*(b^2*d^3*e^2*f - 2*a *b*d^2*e*f^2 + a^2*d*f^3)*x)*sinh(d*x + c)^2 + 3*(b^2*d^3*e^2*f - 2*a*b*d^2 *e*f^2 + a^2*d*f^3)*x + 4*((b^2*d^3*f^3*x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^ 2 + 2*a*b*c)*d*e*f^2 + (b^2*c^3 + 3*a*b*c^2 + 3*a^2*c)*f^3 + 3*(b^2*d^3*e*f ^2 - a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f - 2*a*b*d^2*e*f^2 + a^2*d*f^3)*x)* cosh(d*x + c)^3 - (b^2*d^3*f^3*x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b *c)*d*e*f^2 + (b^2*c^3 + 3*a*b*c^2 + 3*a^2*c)*f^3 + 3*(b^2*d^3*e*f^2 - a*b* d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f - 2*a*b*d^2*e*f^2 + a^2*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c))*log(-cosh(d*x + c) - sinh(d*x + c) + 1) - 6*(b^2*f^3*c osh(d*x + c)^4 + 4*b^2*f^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*f^3*sinh(d*x + c)^4 - 2*b^2*f^3*cosh(d*x + c)^2 + b^2*f^3 + 2*(3*b^2*f^3*cosh(d*x + c)^

```
2 - b^2*f^3)*sinh(d*x + c)^2 + 4*(b^2*f^3*cosh(d*x + c)^3 - b^2*f^3*cosh(d*
x + c))*sinh(d*x + c))*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*(b^2*f^3*cosh
(d*x + c)^4 + 4*b^2*f^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*f^3*sinh(d*x +
c)^4 - 2*b^2*f^3*cosh(d*x + c)^2 + b^2*f^3 + 2*(3*b^2*f^3*cosh(d*x + c)^2 -
 b^2*f^3)*sinh(d*x + c)^2 + 4*(b^2*f^3*cosh(d*x + c)^3 - b^2*f^3*cosh(d*x +
 c))*sinh(d*x + c))*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*(b^2*f^3*cosh(d*
x + c)^4 + 4*b^2*f^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*f^3*sinh(d*x + c)^
4 - 2*b^2*f^3*cosh(d*x + c)^2 + b^2*f^3 + 2*(3*b^2*f^3*cosh(d*x + c)^2 - b^
2*f^3)*sinh(d*x + c)^2 + 4*(b^2*f^3*cosh(d*x + c)^3 - b^2*f^3*cosh(d*x + c)
)*sinh(d*x + c))*polylog(4, cosh(d*x + c) + sinh(d*x + c)) + 6*(b^2*f^3*cos
h(d*x + c)^4 + 4*b^2*f^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*f^3*sinh(d*x +
 c)^4 - 2*b^2*f^3*cosh(d*x + c)^2 + b^2*f^3 + 2*(3*b^2*f^3*cosh(d*x + c)^2
- b^2*f^3)*sinh(d*x + c)^2 + 4*(b^2*f^3*cosh(d*x + c)^3 - b^2*f^3*cosh(d*x
+ c))*sinh(d*x + c))*polylog(4, -cosh(d*x + c) - sinh(d*x + c)) + 6*(b^2*d*
f^3*x + b^2*d*e*f^2 + (b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c)^4 + 4*(b^2*
d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*f^3*x + b^2*d
*e*f^2)*sinh(d*x + c)^4 - 2*(b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c)^2 - 2
*(b^2*d*f^3*x + b^2*d*e*f^2 - 3*(b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 4*((b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c)^3 - (b^2*d
*f^3*x + b^2*d*e*f^2)*cosh(d*x + c))*sinh(d*x + c))*polylog(3, (a*cosh(d*x
+ c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2))/b) + 6*(b^2*d*f^3*x + b^2*d*e*f^2 + (b^2*d*f^3*x + b^2*d*e*f^2)*co
sh(d*x + c)^4 + 4*(b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c)^3
 + (b^2*d*f^3*x + b^2*d*e*f^2)*sinh(d*x + c)^4 - 2*(b^2*d*f^3*x + b^2*d*e*f
^2)*cosh(d*x + c)^2 - 2*(b^2*d*f^3*x + b^2*d*e*f^2 - 3*(b^2*d*f^3*x + b^2*d
*e*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d*f^3*x + b^2*d*e*f^2)*c
osh(d*x + c)^3 - (b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c))*sinh(d*x + c))*
polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*(b^2*d*f^3*x + b^2*d*e*f^2 - a*b*f^3
 + (b^2*d*f^3*x + b^2*d*e*f^2 - a*b*f^3)*cosh(d*x + c)^4 + 4*(b^2*d*f^3*x +
 b^2*d*e*f^2 - a*b*f^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*f^3*x + b^2*
d*e*f^2 - a*b*f^3)*sinh(d*x + c)^4 - 2*(b^2*d*f^3*x + b^2*d*e*f^2 - a*b*f^3
)*cosh(d*x + c)^2 - 2*(b^2*d*f^3*x + b^2*d*e*f^2 - a*b*f^3 - 3*(b^2*d*f^3*x
 + b^2*d*e*f^2 - a*b*f^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d*f^3*
x + b^2*d*e*f^2 - a*b*f^3)*cosh(d*x + c)^3 - (b^2*d*f^3*x + b^2*d*e*f^2 - a
*b*f^3)*cosh(d*x + c))*sinh(d*x + c))*polylog(3, cosh(d*x + c) + sinh(d*x +
 c)) - 6*(b^2*d*f^3*x + b^2*d*e*f^2 + a*b*f^3 + (b^2*d*f^3*x + b^2*d*e*f^2
+ a*b*f^3)*cosh(d*x + c)^4 + 4*(b^2*d*f^3*x + b^2*d*e*f^2 + a*b*f^3)*cosh(d
*x + c)*sinh(d*x + c)^3 + (b^2*d*f^3*x + b^2*d*e*f^2 + a*b*f^3)*sinh(d*x +
c)^4 - 2*(b^2*d*f^3*x + b^2*d*e*f^2 + a*b*f^3)*cosh(d*x + c)^2 - 2*(b^2*d*f
^3*x + b^2*d*e*f^2 + a*b*f^3 - 3*(b^2*d*f^3*x + b^2*d*e*f^2 + a*b*f^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d*f^3*x + b^2*d*e*f^2 + a*b*f^3)*cos
h(d*x + c)^3 - (b^2*d*f^3*x + b^2*d*e*f^2 + a*b*f^3)*cosh(d*x + c))*sinh(d*
x + c))*polylog(3, -cosh(d*x + c) - sinh(d*x + c)) - 2*(a*b*d^3*f^3*x^3 + 3
*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*e^2*f*x + a*b*d^3*e^3 + 6*(a^2*d^2*f^3*x^2 +
 2*a^2*d^2*e*f^2*x + 2*a^2*c*d*e*f^2 - a^2*c^2*f^3)*cosh(d*x + c)^3 - 3*(a*
b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*e^2*f*x + a*b*d^3*e^3)*cosh
(d*x + c)^2 + (2*a^2*d^3*f^3*x^3 + 2*a^2*d^3*e^3 + 3*a^2*d^2*e^2*f - 12*a^2
*c*d*e*f^2 + 6*a^2*c^2*f^3 + 3*(2*a^2*d^3*e*f^2 - a^2*d^2*f^3)*x^2 + 6*(a^2
*d^3*e^2*f - a^2*d^2*e*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d^4*cosh(
d*x + c)^4 + 4*a^3*d^4*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d^4*sinh(d*x + c
)^4 - 2*a^3*d^4*cosh(d*x + c)^2 + a^3*d^4 + 2*(3*a^3*d^4*cosh(d*x + c)^2 -
a^3*d^4)*sinh(d*x + c)^2 + 4*(a^3*d^4*cosh(d*x + c)^3 - a^3*d^4*cosh(d*x +
c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*coth(d*x+c)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")

[Out] Timed out
```

3.477 $\int \frac{(e+fx)^2 \coth(c+dx)\mathbf{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=502

$$-\frac{2b^2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}-\frac{2b^2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3d^2}+\frac{b^2f(e+fx)\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{a^3d^2}+$$

```
[Out] (4*b*f*(e + f*x)*ArcTanh[E^(c + d*x)])/(a^2*d^2) - (f*(e + f*x)*Coth[c + d*
x])/(a*d^2) + (b*(e + f*x)^2*Csch[c + d*x])/(a^2*d) - ((e + f*x)^2*Csch[c +
 d*x]^2)/(2*a*d) - (b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/(a^3*d) - (b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(a^3*d) + (b^2*(e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a^3*d) + (f^
2*Log[Sinh[c + d*x]])/(a*d^3) + (2*b*f^2*PolyLog[2, -E^(c + d*x)])/(a^2*d^3
) - (2*b*f^2*PolyLog[2, E^(c + d*x)])/(a^2*d^3) - (2*b^2*f*(e + f*x)*PolyLo
g[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2) - (2*b^2*f*(e + f
*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) + (b^2*
f*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a^3*d^2) + (2*b^2*f^2*PolyLog[3,
-((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^3) + (2*b^2*f^2*PolyLog[3
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^3) - (b^2*f^2*PolyLog[3
, E^(2*(c + d*x))])/(2*a^3*d^3)
```

Rubi [A] time = 1.00402, antiderivative size = 502, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 14, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5587, 5452, 4184, 3475, 4182, 2279, 2391, 5569, 3716, 2190, 2531, 2282, 6589, 5561}

$$-\frac{2b^2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}-\frac{2b^2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3d^2}+\frac{b^2f(e+fx)\text{PolyLog}\left(2,e^{2(c+dx)}\right)}{a^3d^2}+$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (4*b*f*(e + f*x)*ArcTanh[E^(c + d*x)])/(a^2*d^2) - (f*(e + f*x)*Coth[c + d*
x])/(a*d^2) + (b*(e + f*x)^2*Csch[c + d*x])/(a^2*d) - ((e + f*x)^2*Csch[c +
 d*x]^2)/(2*a*d) - (b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/(a^3*d) - (b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
 b^2])])/(a^3*d) + (b^2*(e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a^3*d) + (f^
2*Log[Sinh[c + d*x]])/(a*d^3) + (2*b*f^2*PolyLog[2, -E^(c + d*x)])/(a^2*d^3
) - (2*b*f^2*PolyLog[2, E^(c + d*x)])/(a^2*d^3) - (2*b^2*f*(e + f*x)*PolyLo
g[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2) - (2*b^2*f*(e + f
*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) + (b^2*
f*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a^3*d^2) + (2*b^2*f^2*PolyLog[3,
-((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^3) + (2*b^2*f^2*PolyLog[3
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^3) - (b^2*f^2*PolyLog[3
, E^(2*(c + d*x))])/(2*a^3*d^3)
```


Rule 5587

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Csch[c + d*x]^(p - 1)*Coth[c + d*x]^n)/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
```

```
IGtQ[p, 0]
```

Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\coth(c+dx)\text{csch}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}dx}{a}$$

$$= -\frac{(e+fx)^2\text{csch}^2(c+dx)}{2ad} - \frac{b\int (e+fx)^2\coth(c+dx)\text{csch}(c+dx)\,dx}{a^2} + \frac{b^2}{}$$

$$= -\frac{f(e+fx)\coth(c+dx)}{ad^2} + \frac{b(e+fx)^2\text{csch}(c+dx)}{a^2d} - \frac{(e+fx)^2\text{csch}^2(c+d}{2ad}$$

$$= \frac{4bf(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{f(e+fx)\coth(c+dx)}{ad^2} + \frac{b(e+fx)^2\text{csch}(c+}{a^2d}$$

$$= \frac{4bf(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{f(e+fx)\coth(c+dx)}{ad^2} + \frac{b(e+fx)^2\text{csch}(c+}{a^2d}$$

$$= \frac{4bf(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{f(e+fx)\coth(c+dx)}{ad^2} + \frac{b(e+fx)^2\text{csch}(c+}{a^2d}$$

$$= \frac{4bf(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{f(e+fx)\coth(c+dx)}{ad^2} + \frac{b(e+fx)^2\text{csch}(c+}{a^2d}$$

$$= \frac{4bf(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - \frac{f(e+fx)\coth(c+dx)}{ad^2} + \frac{b(e+fx)^2\text{csch}(c+}{a^2d}$$

Mathematica [B] time = 30.4058, size = 1816, normalized size = 3.62

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (b*(e + f*x)^2*Csch[c])/(a^2*d) + ((-e^2 - 2*e*f*x - f^2*x^2)*Csch[c/2 + (d
*x)/2]^2)/(8*a*d) - (12*d*E^(2*c)*(b^2*d^2*e^2 + a^2*f^2)*x - 12*d*(-1 + E^
(2*c))*(b^2*d^2*e^2 + a^2*f^2)*x + 12*b^2*d^3*e*f*x^2 + 4*b^2*d^3*f^2*x^3 -
 24*a*b*d*e*(-1 + E^(2*c))*f*ArcTanh[E^(c + d*x)] + 6*b^2*d^2*e^2*(-1 + E^(
2*c))*(2*d*x - Log[1 - E^(2*(c + d*x))]) + 6*a^2*(-1 + E^(2*c))*f^2*(2*d*x
- Log[1 - E^(2*(c + d*x))]) + 12*a*b*(-1 + E^(2*c))*f^2*(d*x*(Log[1 - E^(c
+ d*x)] - Log[1 + E^(c + d*x)]) - PolyLog[2, -E^(c + d*x)] + PolyLog[2, E^(
c + d*x)]) + 6*b^2*d*e*(-1 + E^(2*c))*f*(2*d*x*(d*x - Log[1 - E^(2*(c + d*x
))]) - PolyLog[2, E^(2*(c + d*x))]) + b^2*(-1 + E^(2*c))*f^2*(2*d^2*x^2*(2*
d*x - 3*Log[1 - E^(2*(c + d*x))]) - 6*d*x*PolyLog[2, E^(2*(c + d*x))] + 3*P
olyLog[3, E^(2*(c + d*x))]))/(6*a^3*d^3*(-1 + E^(2*c))) + (b^2*(6*e^2*E^(2*
c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*Arc
Tan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*
Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2
]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^
(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2
)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2
)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d - (3
*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e*f*x*
Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^
(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d
 + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])
])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^
2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 +
```

```
b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sq
rt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c
+ Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d
*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(e + f*x
)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2
 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sq
rt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*
E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^
(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (6*f^2*PolyLog[3,
 -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c
)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])
/d^3))/(3*a^3*(-1 + E^(2*c))) + ((e^2 + 2*e*f*x + f^2*x^2)*Sech[c/2 + (d*x)
/2]^2)/(8*a*d) + (Sech[c/2]*Sech[c/2 + (d*x)/2]*(-(b*d*e^2*Sinh[(d*x)/2]) -
 a*e*f*Sinh[(d*x)/2] - 2*b*d*e*f*x*Sinh[(d*x)/2] - a*f^2*x*Sinh[(d*x)/2] -
b*d*f^2*x^2*Sinh[(d*x)/2]))/(2*a^2*d^2) + (Csch[c/2]*Csch[c/2 + (d*x)/2]*(-
(b*d*e^2*Sinh[(d*x)/2]) + a*e*f*Sinh[(d*x)/2] - 2*b*d*e*f*x*Sinh[(d*x)/2] +
 a*f^2*x*Sinh[(d*x)/2] - b*d*f^2*x^2*Sinh[(d*x)/2]))/(2*a^2*d^2)
```

Maple [F] time = 0.615, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{2} \coth\left(dx+c\right) \left(\operatorname{csch}\left(dx+c\right)\right)^{2}}{a+b\sinh\left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -e^2*(2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*
e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + b^2*log(-2*a*e^(-d*x -
c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) - b^2*log(e^(-d*x - c) + 1)/(a^3*d) -
b^2*log(e^(-d*x - c) - 1)/(a^3*d)) + 2*(a*f^2*x + a*e*f + (b*d*f^2*x^2*e^(3
*c) + 2*b*d*e*f*x*e^(3*c))*e^(3*d*x) - (a*d*f^2*x^2*e^(2*c) + a*e*f*e^(2*c)
 + (2*d*e*f + f^2)*a*x*e^(2*c))*e^(2*d*x) - (b*d*f^2*x^2*e^c + 2*b*d*e*f*x*
e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^
2) + (d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(
3, -e^(d*x + c)))*b^2*f^2/(a^3*d^3) + (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*
x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*b^2*f^2/(a^3*d^3) - (2*b*
d*e*f + a*f^2)*x/(a^2*d^2) + (2*b*d*e*f - a*f^2)*x/(a^2*d^2) + (2*b*d*e*f +
 a*f^2)*log(e^(d*x + c) + 1)/(a^2*d^3) - (2*b*d*e*f - a*f^2)*log(e^(d*x + c
) - 1)/(a^2*d^3) + 2*(b^2*d*e*f + a*b*f^2)*(d*x*log(e^(d*x + c) + 1) + dilo
g(-e^(d*x + c)))/(a^3*d^3) + 2*(b^2*d*e*f - a*b*f^2)*(d*x*log(-e^(d*x + c)
+ 1) + dilog(e^(d*x + c)))/(a^3*d^3) - 1/3*(b^2*d^3*f^2*x^3 + 3*(b^2*d*e*f
+ a*b*f^2)*d^2*x^2)/(a^3*d^3) - 1/3*(b^2*d^3*f^2*x^3 + 3*(b^2*d*e*f - a*b*f
```

```
^2)*d^2*x^2)/(a^3*d^3) + integrate(-2*(b^3*f^2*x^2 + 2*b^3*e*f*x - (a*b^2*f
^2*x^2*e^c + 2*a*b^2*e*f*x*e^c)*e^(d*x))/(a^3*b*e^(2*d*x + 2*c) + 2*a^4*e^(
d*x + c) - a^3*b), x)
```

Fricas [C] time = 3.81377, size = 14549, normalized size = 28.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] (2*a^2*d*e*f - 2*a^2*c*f^2 - 2*(a^2*d*f^2*x + a^2*c*f^2)*cosh(d*x + c)^4 -
2*(a^2*d*f^2*x + a^2*c*f^2)*sinh(d*x + c)^4 + 2*(a*b*d^2*f^2*x^2 + 2*a*b*d^
2*e*f*x + a*b*d^2*e^2)*cosh(d*x + c)^3 + 2*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f
*x + a*b*d^2*e^2 - 4*(a^2*d*f^2*x + a^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c)
^3 - 2*(a^2*d^2*f^2*x^2 + a^2*d^2*e^2 + a^2*d*e*f - 2*a^2*c*f^2 + (2*a^2*d^
2*e*f - a^2*d*f^2)*x)*cosh(d*x + c)^2 - 2*(a^2*d^2*f^2*x^2 + a^2*d^2*e^2 +
a^2*d*e*f - 2*a^2*c*f^2 + 6*(a^2*d*f^2*x + a^2*c*f^2)*cosh(d*x + c)^2 + (2*
a^2*d^2*e*f - a^2*d*f^2)*x - 3*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + a*b*d^2
*e^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x
 + a*b*d^2*e^2)*cosh(d*x + c) - 2*(b^2*d*f^2*x + b^2*d*e*f + (b^2*d*f^2*x +
 b^2*d*e*f)*cosh(d*x + c)^4 + 4*(b^2*d*f^2*x + b^2*d*e*f)*cosh(d*x + c)*sin
h(d*x + c)^3 + (b^2*d*f^2*x + b^2*d*e*f)*sinh(d*x + c)^4 - 2*(b^2*d*f^2*x +
 b^2*d*e*f)*cosh(d*x + c)^2 - 2*(b^2*d*f^2*x + b^2*d*e*f - 3*(b^2*d*f^2*x +
 b^2*d*e*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d*f^2*x + b^2*d*e*f)
*cosh(d*x + c)^3 - (b^2*d*f^2*x + b^2*d*e*f)*cosh(d*x + c))*sinh(d*x + c))*
dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b^2*d*f^2*x + b^2*d*e*f + (b^2*d
*f^2*x + b^2*d*e*f)*cosh(d*x + c)^4 + 4*(b^2*d*f^2*x + b^2*d*e*f)*cosh(d*x
+ c)*sinh(d*x + c)^3 + (b^2*d*f^2*x + b^2*d*e*f)*sinh(d*x + c)^4 - 2*(b^2*d
*f^2*x + b^2*d*e*f)*cosh(d*x + c)^2 - 2*(b^2*d*f^2*x + b^2*d*e*f - 3*(b^2*d
*f^2*x + b^2*d*e*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d*f^2*x + b^
2*d*e*f)*cosh(d*x + c)^3 - (b^2*d*f^2*x + b^2*d*e*f)*cosh(d*x + c))*sinh(d*
x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^2*d*f^2*x + b^2*d*e*f
+ (b^2*d*f^2*x + b^2*d*e*f - a*b*f^2)*cosh(d*x + c)^4 + 4*(b^2*d*f^2*x + b^
2*d*e*f - a*b*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*f^2*x + b^2*d*e*f
 - a*b*f^2)*sinh(d*x + c)^4 - a*b*f^2 - 2*(b^2*d*f^2*x + b^2*d*e*f - a*b*f^
2)*cosh(d*x + c)^2 - 2*(b^2*d*f^2*x + b^2*d*e*f - a*b*f^2 - 3*(b^2*d*f^2*x
+ b^2*d*e*f - a*b*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d*f^2*x +
 b^2*d*e*f - a*b*f^2)*cosh(d*x + c)^3 - (b^2*d*f^2*x + b^2*d*e*f - a*b*f^2)
*cosh(d*x + c))*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)) + 2*(b^
2*d*f^2*x + b^2*d*e*f + (b^2*d*f^2*x + b^2*d*e*f + a*b*f^2)*cosh(d*x + c)^4
 + 4*(b^2*d*f^2*x + b^2*d*e*f + a*b*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (b
^2*d*f^2*x + b^2*d*e*f + a*b*f^2)*sinh(d*x + c)^4 + a*b*f^2 - 2*(b^2*d*f^2*
x + b^2*d*e*f + a*b*f^2)*cosh(d*x + c)^2 - 2*(b^2*d*f^2*x + b^2*d*e*f + a*b
*f^2 - 3*(b^2*d*f^2*x + b^2*d*e*f + a*b*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)
^2 + 4*((b^2*d*f^2*x + b^2*d*e*f + a*b*f^2)*cosh(d*x + c)^3 - (b^2*d*f^2*x
+ b^2*d*e*f + a*b*f^2)*cosh(d*x + c))*sinh(d*x + c))*dilog(-cosh(d*x + c) -
 sinh(d*x + c)) - (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2 + (b^2*d^2*e^2
 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*cosh(d*x + c)^4 + 4*(b^2*d^2*e^2 - 2*b^2*c*
d*e*f + b^2*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d^2*e^2 - 2*b^2*c
*d*e*f + b^2*c^2*f^2)*sinh(d*x + c)^4 - 2*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^
2*c^2*f^2)*cosh(d*x + c)^2 - 2*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2 -
 3*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x +
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c)^2 + 4*((b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*cosh(d*x + c)^3 - (b^
2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(
2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) -
(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2 + (b^2*d^2*e^2 - 2*b^2*c*d*e*f +
 b^2*c^2*f^2)*cosh(d*x + c)^4 + 4*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^
2)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f
^2)*sinh(d*x + c)^4 - 2*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*cosh(d*
x + c)^2 - 2*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2 - 3*(b^2*d^2*e^2 -
2*b^2*c*d*e*f + b^2*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d^2
*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*cosh(d*x + c)^3 - (b^2*d^2*e^2 - 2*b^2*
c*d*e*f + b^2*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c)
+ 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*d^2*f^2*x^2 +
 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2 + (b^2*d^2*f^2*x^2 + 2*b^2*d
^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*cosh(d*x + c)^4 + 4*(b^2*d^2*f^2*x^
2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*cosh(d*x + c)*sinh(d*x +
 c)^3 + (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*s
inh(d*x + c)^4 - 2*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2
*c^2*f^2)*cosh(d*x + c)^2 - 2*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*
d*e*f - b^2*c^2*f^2 - 3*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f
- b^2*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d^2*f^2*x^2 + 2*b
^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*cosh(d*x + c)^3 - (b^2*d^2*f^2*
x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*cosh(d*x + c))*sinh(d*
x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f
*x + 2*b^2*c*d*e*f - b^2*c^2*f^2 + (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b
^2*c*d*e*f - b^2*c^2*f^2)*cosh(d*x + c)^4 + 4*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*
e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d
^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*sinh(d*x + c)^4
 - 2*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*cosh
(d*x + c)^2 - 2*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^
2*f^2 - 3*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x +
 2*b^2*c*d*e*f - b^2*c^2*f^2)*cosh(d*x + c)^3 - (b^2*d^2*f^2*x^2 + 2*b^2*d^
2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(-(
a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqr
t((a^2 + b^2)/b^2) - b)/b) + (b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + 2*a*b*d*e*f +
 (b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + 2*a*b*d*e*f + a^2*f^2 + 2*(b^2*d^2*e*f +
a*b*d*f^2)*x)*cosh(d*x + c)^4 + 4*(b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + 2*a*b*d*
e*f + a^2*f^2 + 2*(b^2*d^2*e*f + a*b*d*f^2)*x)*cosh(d*x + c)*sinh(d*x + c)^
3 + (b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + 2*a*b*d*e*f + a^2*f^2 + 2*(b^2*d^2*e*f
 + a*b*d*f^2)*x)*sinh(d*x + c)^4 + a^2*f^2 - 2*(b^2*d^2*f^2*x^2 + b^2*d^2*e
^2 + 2*a*b*d*e*f + a^2*f^2 + 2*(b^2*d^2*e*f + a*b*d*f^2)*x)*cosh(d*x + c)^2
 - 2*(b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + 2*a*b*d*e*f + a^2*f^2 - 3*(b^2*d^2*f^
2*x^2 + b^2*d^2*e^2 + 2*a*b*d*e*f + a^2*f^2 + 2*(b^2*d^2*e*f + a*b*d*f^2)*x
)*cosh(d*x + c)^2 + 2*(b^2*d^2*e*f + a*b*d*f^2)*x)*sinh(d*x + c)^2 + 2*(b^2
*d^2*e*f + a*b*d*f^2)*x + 4*((b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + 2*a*b*d*e*f +
 a^2*f^2 + 2*(b^2*d^2*e*f + a*b*d*f^2)*x)*cosh(d*x + c)^3 - (b^2*d^2*f^2*x^
2 + b^2*d^2*e^2 + 2*a*b*d*e*f + a^2*f^2 + 2*(b^2*d^2*e*f + a*b*d*f^2)*x)*co
sh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (b^2*d
^2*e^2 + (b^2*d^2*e^2 - 2*(b^2*c + a*b)*d*e*f + (b^2*c^2 + 2*a*b*c + a^2)*f
^2)*cosh(d*x + c)^4 + 4*(b^2*d^2*e^2 - 2*(b^2*c + a*b)*d*e*f + (b^2*c^2 + 2
*a*b*c + a^2)*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d^2*e^2 - 2*(b^2*c
+ a*b)*d*e*f + (b^2*c^2 + 2*a*b*c + a^2)*f^2)*sinh(d*x + c)^4 - 2*(b^2*c +
a*b)*d*e*f + (b^2*c^2 + 2*a*b*c + a^2)*f^2 - 2*(b^2*d^2*e^2 - 2*(b^2*c + a*
b)*d*e*f + (b^2*c^2 + 2*a*b*c + a^2)*f^2)*cosh(d*x + c)^2 - 2*(b^2*d^2*e^2
- 2*(b^2*c + a*b)*d*e*f + (b^2*c^2 + 2*a*b*c + a^2)*f^2 - 3*(b^2*d^2*e^2 -
2*(b^2*c + a*b)*d*e*f + (b^2*c^2 + 2*a*b*c + a^2)*f^2)*cosh(d*x + c)^2)*sin
h(d*x + c)^2 + 4*((b^2*d^2*e^2 - 2*(b^2*c + a*b)*d*e*f + (b^2*c^2 + 2*a*b*c
 + a^2)*f^2)*cosh(d*x + c)^3 - (b^2*d^2*e^2 - 2*(b^2*c + a*b)*d*e*f + (b^2*

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c^2 + 2*a*b*c + a^2)*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) +
 sinh(d*x + c) - 1) + (b^2*d^2*f^2*x^2 + 2*b^2*c*d*e*f + (b^2*d^2*f^2*x^2 +
 2*b^2*c*d*e*f - (b^2*c^2 + 2*a*b*c)*f^2 + 2*(b^2*d^2*e*f - a*b*d*f^2)*x)*c
osh(d*x + c)^4 + 4*(b^2*d^2*f^2*x^2 + 2*b^2*c*d*e*f - (b^2*c^2 + 2*a*b*c)*f
^2 + 2*(b^2*d^2*e*f - a*b*d*f^2)*x)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d^
2*f^2*x^2 + 2*b^2*c*d*e*f - (b^2*c^2 + 2*a*b*c)*f^2 + 2*(b^2*d^2*e*f - a*b*
d*f^2)*x)*sinh(d*x + c)^4 - (b^2*c^2 + 2*a*b*c)*f^2 - 2*(b^2*d^2*f^2*x^2 +
2*b^2*c*d*e*f - (b^2*c^2 + 2*a*b*c)*f^2 + 2*(b^2*d^2*e*f - a*b*d*f^2)*x)*co
sh(d*x + c)^2 - 2*(b^2*d^2*f^2*x^2 + 2*b^2*c*d*e*f - (b^2*c^2 + 2*a*b*c)*f^
2 - 3*(b^2*d^2*f^2*x^2 + 2*b^2*c*d*e*f - (b^2*c^2 + 2*a*b*c)*f^2 + 2*(b^2*d
^2*e*f - a*b*d*f^2)*x)*cosh(d*x + c)^2 + 2*(b^2*d^2*e*f - a*b*d*f^2)*x)*sin
h(d*x + c)^2 + 2*(b^2*d^2*e*f - a*b*d*f^2)*x + 4*((b^2*d^2*f^2*x^2 + 2*b^2*
c*d*e*f - (b^2*c^2 + 2*a*b*c)*f^2 + 2*(b^2*d^2*e*f - a*b*d*f^2)*x)*cosh(d*x
 + c)^3 - (b^2*d^2*f^2*x^2 + 2*b^2*c*d*e*f - (b^2*c^2 + 2*a*b*c)*f^2 + 2*(b
^2*d^2*e*f - a*b*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))*log(-cosh(d*x + c)
 - sinh(d*x + c) + 1) + 2*(b^2*f^2*cosh(d*x + c)^4 + 4*b^2*f^2*cosh(d*x + c
)*sinh(d*x + c)^3 + b^2*f^2*sinh(d*x + c)^4 - 2*b^2*f^2*cosh(d*x + c)^2 + b
^2*f^2 + 2*(3*b^2*f^2*cosh(d*x + c)^2 - b^2*f^2)*sinh(d*x + c)^2 + 4*(b^2*f
^2*cosh(d*x + c)^3 - b^2*f^2*cosh(d*x + c))*sinh(d*x + c))*polylog(3, (a*co
sh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2))/b) + 2*(b^2*f^2*cosh(d*x + c)^4 + 4*b^2*f^2*cosh(d*x + c)*s
inh(d*x + c)^3 + b^2*f^2*sinh(d*x + c)^4 - 2*b^2*f^2*cosh(d*x + c)^2 + b^2*
f^2 + 2*(3*b^2*f^2*cosh(d*x + c)^2 - b^2*f^2)*sinh(d*x + c)^2 + 4*(b^2*f^2*
cosh(d*x + c)^3 - b^2*f^2*cosh(d*x + c))*sinh(d*x + c))*polylog(3, (a*cosh(
d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2))/b) - 2*(b^2*f^2*cosh(d*x + c)^4 + 4*b^2*f^2*cosh(d*x + c)*sinh
(d*x + c)^3 + b^2*f^2*sinh(d*x + c)^4 - 2*b^2*f^2*cosh(d*x + c)^2 + b^2*f^2
 + 2*(3*b^2*f^2*cosh(d*x + c)^2 - b^2*f^2)*sinh(d*x + c)^2 + 4*(b^2*f^2*cos
h(d*x + c)^3 - b^2*f^2*cosh(d*x + c))*sinh(d*x + c))*polylog(3, cosh(d*x +
c) + sinh(d*x + c)) - 2*(b^2*f^2*cosh(d*x + c)^4 + 4*b^2*f^2*cosh(d*x + c)*
sinh(d*x + c)^3 + b^2*f^2*sinh(d*x + c)^4 - 2*b^2*f^2*cosh(d*x + c)^2 + b^2
*f^2 + 2*(3*b^2*f^2*cosh(d*x + c)^2 - b^2*f^2)*sinh(d*x + c)^2 + 4*(b^2*f^2
*cosh(d*x + c)^3 - b^2*f^2*cosh(d*x + c))*sinh(d*x + c))*polylog(3, -cosh(d
*x + c) - sinh(d*x + c)) - 2*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + a*b*d^2*e
^2 + 4*(a^2*d*f^2*x + a^2*c*f^2)*cosh(d*x + c)^3 - 3*(a*b*d^2*f^2*x^2 + 2*a
*b*d^2*e*f*x + a*b*d^2*e^2)*cosh(d*x + c)^2 + 2*(a^2*d^2*f^2*x^2 + a^2*d^2*
e^2 + a^2*d*e*f - 2*a^2*c*f^2 + (2*a^2*d^2*e*f - a^2*d*f^2)*x)*cosh(d*x + c
))*sinh(d*x + c))/(a^3*d^3*cosh(d*x + c)^4 + 4*a^3*d^3*cosh(d*x + c)*sinh(d
*x + c)^3 + a^3*d^3*sinh(d*x + c)^4 - 2*a^3*d^3*cosh(d*x + c)^2 + a^3*d^3 +
 2*(3*a^3*d^3*cosh(d*x + c)^2 - a^3*d^3)*sinh(d*x + c)^2 + 4*(a^3*d^3*cosh(
d*x + c)^3 - a^3*d^3*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*coth(d*x+c)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.478 $\int \frac{(e+fx)\coth(c+dx)\mathbf{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=298

$$-\frac{b^2 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}-\frac{b^2 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3d^2}+\frac{b^2 f \text{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^3d^2}-\frac{b^2(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^3d}$$

```
[Out] (b*f*ArcTanh[Cosh[c + d*x]])/(a^2*d^2) - (f*Coth[c + d*x])/(2*a*d^2) + (b*(
e + f*x)*Csch[c + d*x])/(a^2*d) - ((e + f*x)*Csch[c + d*x]^2)/(2*a*d) - (b^
2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*d) - (b^2*
(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*d) + (b^2*(e
 + f*x)*Log[1 - E^(2*(c + d*x))])/(a^3*d) - (b^2*f*PolyLog[2, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2) - (b^2*f*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) + (b^2*f*PolyLog[2, E^(2*(c + d*x))])
/(2*a^3*d^2)
```

Rubi [A] time = 0.570451, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {5587, 5452, 3767, 8, 3770, 5569, 3716, 2190, 2279, 2391, 5561}

$$-\frac{b^2 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}-\frac{b^2 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3d^2}+\frac{b^2 f \text{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^3d^2}-\frac{b^2(e+fx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*f*ArcTanh[Cosh[c + d*x]])/(a^2*d^2) - (f*Coth[c + d*x])/(2*a*d^2) + (b*(
e + f*x)*Csch[c + d*x])/(a^2*d) - ((e + f*x)*Csch[c + d*x]^2)/(2*a*d) - (b^
2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*d) - (b^2*
(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*d) + (b^2*(e
 + f*x)*Log[1 - E^(2*(c + d*x))])/(a^3*d) - (b^2*f*PolyLog[2, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2) - (b^2*f*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) + (b^2*f*PolyLog[2, E^(2*(c + d*x))])
/(2*a^3*d^2)
```

Rule 5587

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Csch[c + d*x]^(p - 1)*Coth[c + d*x]^n)/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)\coth(c+dx)\text{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\coth(c+dx)\text{csch}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\coth(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)\text{csch}^2(c+dx)}{2ad} - \frac{b\int (e+fx)\coth(c+dx)\text{csch}(c+dx)\,dx}{a^2} + \frac{b^2\int (\ldots}{\ldots}$$

$$= \frac{b(e+fx)\text{csch}(c+dx)}{a^2d} - \frac{(e+fx)\text{csch}^2(c+dx)}{2ad} + \frac{b^2\int (e+fx)\coth(c+dx)\ldots}{a^3}$$

$$= \frac{bf\tanh^{-1}(\cosh(c+dx))}{a^2d^2} - \frac{f\coth(c+dx)}{2ad^2} + \frac{b(e+fx)\text{csch}(c+dx)}{a^2d} - (e+\ldots$$

$$= \frac{bf\tanh^{-1}(\cosh(c+dx))}{a^2d^2} - \frac{f\coth(c+dx)}{2ad^2} + \frac{b(e+fx)\text{csch}(c+dx)}{a^2d} - (e+\ldots$$

$$= \frac{bf\tanh^{-1}(\cosh(c+dx))}{a^2d^2} - \frac{f\coth(c+dx)}{2ad^2} + \frac{b(e+fx)\text{csch}(c+dx)}{a^2d} - (e+\ldots$$

$$= \frac{bf\tanh^{-1}(\cosh(c+dx))}{a^2d^2} - \frac{f\coth(c+dx)}{2ad^2} + \frac{b(e+fx)\text{csch}(c+dx)}{a^2d} - (e+\ldots$$

Mathematica [A] time = 6.8722, size = 376, normalized size = 1.26

$$\frac{-8b^2\left(f\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+f(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)+f(c+dx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (2*a*(-(a*f) + 2*b*d*(e + f*x))*Coth[(c + d*x)/2] - a^2*d*(e + f*x)*Csch[(c
 + d*x)/2]^2 + 8*b^2*d*e*Log[Sinh[c + d*x]] - 8*b^2*c*f*Log[Sinh[c + d*x]]
- 8*a*b*f*Log[Tanh[(c + d*x)/2]] + 4*b^2*f*((c + d*x)*(c + d*x + 2*Log[1 -
E^(-2*(c + d*x))]) - PolyLog[2, E^(-2*(c + d*x))]) - 8*b^2*(-(f*(c + d*x)^2
)/2 + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d
*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d*e*Log[a + b*Sinh[c +
 d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLog[2, (b*E^(c + d*x))/(-a +
Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])
 + a^2*d*(e + f*x)*Sech[(c + d*x)/2]^2 - 2*a*(a*f + 2*b*d*(e + f*x))*Tanh[(
c + d*x)/2])/(8*a^3*d^2)
```

Maple [B] time = 0.143, size = 649, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] -(-2*b*d*f*x*exp(3*d*x+3*c)+2*a*d*f*x*exp(2*d*x+2*c)-2*b*d*e*exp(3*d*x+3*c)
+2*a*d*e*exp(2*d*x+2*c)+2*b*d*f*x*exp(d*x+c)+a*f*exp(2*d*x+2*c)+2*b*d*e*exp
(d*x+c)-a*f)/d^2/a^2/(exp(2*d*x+2*c)-1)^2-1/a^3/d^2*b^2*f*c*ln(exp(d*x+c)-1
```

```
)+1/a^3*b^2/d^2*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/a^2/d^2*b*f*ln(
exp(d*x+c)+1)-1/a^2/d^2*b*f*ln(exp(d*x+c)-1)-1/a^3/d^2*b^2*f*dilog(exp(d*x+
c))-1/a^3*b^2/d^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(
1/2)))-1/a^3*b^2/d^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^
(1/2)))+1/a^3/d^2*b^2*f*dilog(exp(d*x+c)+1)+1/a^3/d*b^2*e*ln(exp(d*x+c)-1)-
1/a^3*b^2/d*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/a^3/d*b^2*e*ln(exp(d*
x+c)+1)-1/a^3*b^2/d*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1
/2)))*x-1/a^3*b^2/d^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^
(1/2)))*c-1/a^3*b^2/d*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1
/2)))*x-1/a^3*b^2/d^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1
/2)))*c+1/a^3/d*b^2*f*ln(exp(d*x+c)+1)*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\left(4\,b^2d\int\frac{x}{4\left(a^3de^{(dx+c)}+a^3d\right)}\,dx-4\,b^2d\int\frac{x}{4\left(a^3de^{(dx+c)}-a^3d\right)}\,dx+ab\left(\frac{dx+c}{a^3d^2}-\frac{\log\left(e^{(dx+c)}+1\right)}{a^3d^2}\right)-ab\left(\frac{dx+c}{a^3d^2}-\frac{\mathrm{lo}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -(4*b^2*d*integrate(1/4*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 4*b^2*d*integra
te(1/4*x/(a^3*d*e^(d*x + c) - a^3*d), x) + a*b*((d*x + c)/(a^3*d^2) - log(e
^(d*x + c) + 1)/(a^3*d^2)) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) - 1
)/(a^3*d^2)) - (2*b*d*x*e^(3*d*x + 3*c) - 2*b*d*x*e^(d*x + c) - (2*a*d*x*e^
(2*c) + a*e^(2*c))*e^(2*d*x) + a)/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2
*d*x + 2*c) + a^2*d^2) - 4*integrate(1/2*(a*b^2*x*e^(d*x + c) - b^3*x)/(a^3
*b*e^(2*d*x + 2*c) + 2*a^4*e^(d*x + c) - a^3*b), x))*f - e*(2*(b*e^(-d*x -
c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*e^(-2*d*x - 2*c) - a^
2*e^(-4*d*x - 4*c) - a^2)*d) + b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*
c) - b)/(a^3*d) - b^2*log(e^(-d*x - c) + 1)/(a^3*d) - b^2*log(e^(-d*x - c)
- 1)/(a^3*d))
```

Fricas [B] time = 2.60422, size = 7004, normalized size = 23.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] (2*(a*b*d*f*x + a*b*d*e)*cosh(d*x + c)^3 + 2*(a*b*d*f*x + a*b*d*e)*sinh(d*x
 + c)^3 + a^2*f - (2*a^2*d*f*x + 2*a^2*d*e + a^2*f)*cosh(d*x + c)^2 - (2*a^
2*d*f*x + 2*a^2*d*e + a^2*f - 6*(a*b*d*f*x + a*b*d*e)*cosh(d*x + c))*sinh(d
*x + c)^2 - 2*(a*b*d*f*x + a*b*d*e)*cosh(d*x + c) - (b^2*f*cosh(d*x + c)^4
+ 4*b^2*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*f*sinh(d*x + c)^4 - 2*b^2*f*c
osh(d*x + c)^2 + b^2*f + 2*(3*b^2*f*cosh(d*x + c)^2 - b^2*f)*sinh(d*x + c)^
2 + 4*(b^2*f*cosh(d*x + c)^3 - b^2*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a
*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b + 1) - (b^2*f*cosh(d*x + c)^4 + 4*b^2*f*cosh(d*x +
 c)*sinh(d*x + c)^3 + b^2*f*sinh(d*x + c)^4 - 2*b^2*f*cosh(d*x + c)^2 + b^2
*f + 2*(3*b^2*f*cosh(d*x + c)^2 - b^2*f)*sinh(d*x + c)^2 + 4*(b^2*f*cosh(d*
```

x + c)^3 - b^2*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*s
inh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
b)/b + 1) + (b^2*f*cosh(d*x + c)^4 + 4*b^2*f*cosh(d*x + c)*sinh(d*x + c)^3
+ b^2*f*sinh(d*x + c)^4 - 2*b^2*f*cosh(d*x + c)^2 + b^2*f + 2*(3*b^2*f*cosh
(d*x + c)^2 - b^2*f)*sinh(d*x + c)^2 + 4*(b^2*f*cosh(d*x + c)^3 - b^2*f*cos
h(d*x + c))*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)) + (b^2*f*co
sh(d*x + c)^4 + 4*b^2*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*f*sinh(d*x + c)
^4 - 2*b^2*f*cosh(d*x + c)^2 + b^2*f + 2*(3*b^2*f*cosh(d*x + c)^2 - b^2*f)*
sinh(d*x + c)^2 + 4*(b^2*f*cosh(d*x + c)^3 - b^2*f*cosh(d*x + c))*sinh(d*x
+ c))*dilog(-cosh(d*x + c) - sinh(d*x + c)) - ((b^2*d*e - b^2*c*f)*cosh(d*x
 + c)^4 + 4*(b^2*d*e - b^2*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*e -
b^2*c*f)*sinh(d*x + c)^4 + b^2*d*e - b^2*c*f - 2*(b^2*d*e - b^2*c*f)*cosh(d
x + c)^2 - 2(b^2*d*e - b^2*c*f - 3*(b^2*d*e - b^2*c*f)*cosh(d*x + c)^2)*s
inh(d*x + c)^2 + 4*((b^2*d*e - b^2*c*f)*cosh(d*x + c)^3 - (b^2*d*e - b^2*c*
f)*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c)
+ 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - ((b^2*d*e - b^2*c*f)*cosh(d*x + c)^4 +
 4*(b^2*d*e - b^2*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*e - b^2*c*f)*
sinh(d*x + c)^4 + b^2*d*e - b^2*c*f - 2*(b^2*d*e - b^2*c*f)*cosh(d*x + c)^2
 - 2*(b^2*d*e - b^2*c*f - 3*(b^2*d*e - b^2*c*f)*cosh(d*x + c)^2)*sinh(d*x +
 c)^2 + 4*((b^2*d*e - b^2*c*f)*cosh(d*x + c)^3 - (b^2*d*e - b^2*c*f)*cosh(d
*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqr
t((a^2 + b^2)/b^2) + 2*a) - (b^2*d*f*x + (b^2*d*f*x + b^2*c*f)*cosh(d*x + c
)^4 + 4*(b^2*d*f*x + b^2*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*f*x +
b^2*c*f)*sinh(d*x + c)^4 + b^2*c*f - 2*(b^2*d*f*x + b^2*c*f)*cosh(d*x + c)^
2 - 2*(b^2*d*f*x + b^2*c*f - 3*(b^2*d*f*x + b^2*c*f)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 + 4*((b^2*d*f*x + b^2*c*f)*cosh(d*x + c)^3 - (b^2*d*f*x + b^2*c*
f)*cosh(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) +
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^2*d*
f*x + (b^2*d*f*x + b^2*c*f)*cosh(d*x + c)^4 + 4*(b^2*d*f*x + b^2*c*f)*cosh(
d*x + c)*sinh(d*x + c)^3 + (b^2*d*f*x + b^2*c*f)*sinh(d*x + c)^4 + b^2*c*f
- 2*(b^2*d*f*x + b^2*c*f)*cosh(d*x + c)^2 - 2*(b^2*d*f*x + b^2*c*f - 3*(b^2
*d*f*x + b^2*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d*f*x + b^2*c*
f)*cosh(d*x + c)^3 - (b^2*d*f*x + b^2*c*f)*cosh(d*x + c))*sinh(d*x + c))*lo
g(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))
*sqrt((a^2 + b^2)/b^2) - b)/b) + (b^2*d*f*x + (b^2*d*f*x + b^2*d*e + a*b*f)
*cosh(d*x + c)^4 + 4*(b^2*d*f*x + b^2*d*e + a*b*f)*cosh(d*x + c)*sinh(d*x +
 c)^3 + (b^2*d*f*x + b^2*d*e + a*b*f)*sinh(d*x + c)^4 + b^2*d*e + a*b*f - 2
*(b^2*d*f*x + b^2*d*e + a*b*f)*cosh(d*x + c)^2 - 2*(b^2*d*f*x + b^2*d*e + a
*b*f - 3*(b^2*d*f*x + b^2*d*e + a*b*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4
*((b^2*d*f*x + b^2*d*e + a*b*f)*cosh(d*x + c)^3 - (b^2*d*f*x + b^2*d*e + a*
b*f)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) +
 ((b^2*d*e - (b^2*c + a*b)*f)*cosh(d*x + c)^4 + 4*(b^2*d*e - (b^2*c + a*b)*
f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*e - (b^2*c + a*b)*f)*sinh(d*x + c
)^4 + b^2*d*e - 2*(b^2*d*e - (b^2*c + a*b)*f)*cosh(d*x + c)^2 - 2*(b^2*d*e
- 3*(b^2*d*e - (b^2*c + a*b)*f)*cosh(d*x + c)^2 - (b^2*c + a*b)*f)*sinh(d*x
 + c)^2 - (b^2*c + a*b)*f + 4*((b^2*d*e - (b^2*c + a*b)*f)*cosh(d*x + c)^3
- (b^2*d*e - (b^2*c + a*b)*f)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x +
c) + sinh(d*x + c) - 1) + (b^2*d*f*x + (b^2*d*f*x + b^2*c*f)*cosh(d*x + c)^
4 + 4*(b^2*d*f*x + b^2*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*f*x + b^
2*c*f)*sinh(d*x + c)^4 + b^2*c*f - 2*(b^2*d*f*x + b^2*c*f)*cosh(d*x + c)^2
- 2*(b^2*d*f*x + b^2*c*f - 3*(b^2*d*f*x + b^2*c*f)*cosh(d*x + c)^2)*sinh(d*
x + c)^2 + 4*((b^2*d*f*x + b^2*c*f)*cosh(d*x + c)^3 - (b^2*d*f*x + b^2*c*f)
*cosh(d*x + c))*sinh(d*x + c))*log(-cosh(d*x + c) - sinh(d*x + c) + 1) - 2*
(a*b*d*f*x + a*b*d*e - 3*(a*b*d*f*x + a*b*d*e)*cosh(d*x + c)^2 + (2*a^2*d*f
*x + 2*a^2*d*e + a^2*f)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d^2*cosh(d*x + c
)^4 + 4*a^3*d^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d^2*sinh(d*x + c)^4 - 2
*a^3*d^2*cosh(d*x + c)^2 + a^3*d^2 + 2*(3*a^3*d^2*cosh(d*x + c)^2 - a^3*d^2
)*sinh(d*x + c)^2 + 4*(a^3*d^2*cosh(d*x + c)^3 - a^3*d^2*cosh(d*x + c))*sin
h(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)\coth(dx+c)\operatorname{csch}(dx+c)^2}{b\sinh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate((f*x + e)*coth(d*x + c)*csch(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

3.479 $\int \frac{\coth(c+dx)\mathbf{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{b^2\log(\sinh(c+dx))}{a^3d}-\frac{b^2\log(a+b\sinh(c+dx))}{a^3d}+\frac{b\operatorname{csch}(c+dx)}{a^2d}-\frac{\operatorname{csch}^2(c+dx)}{2ad}$$

```
[Out] (b*Csch[c + d*x])/(a^2*d) - Csch[c + d*x]^2/(2*a*d) + (b^2*Log[Sinh[c + d*x
]])/(a^3*d) - (b^2*Log[a + b*Sinh[c + d*x]])/(a^3*d)
```

Rubi [A] time = 0.110946, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2833, 12, 44}

$$\frac{b^2\log(\sinh(c+dx))}{a^3d}-\frac{b^2\log(a+b\sinh(c+dx))}{a^3d}+\frac{b\operatorname{csch}(c+dx)}{a^2d}-\frac{\operatorname{csch}^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*Csch[c + d*x])/(a^2*d) - Csch[c + d*x]^2/(2*a*d) + (b^2*Log[Sinh[c + d*x
]])/(a^3*d) - (b^2*Log[a + b*Sinh[c + d*x]])/(a^3*d)
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{b^3}{x^3(a+x)}\,dx,x,b\sinh(c+dx)\right)}{bd}\\
&= \frac{b^2\operatorname{Subst}\left(\int \frac{1}{x^3(a+x)}\,dx,x,b\sinh(c+dx)\right)}{d}\\
&= \frac{b^2\operatorname{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{1}{a^2x^2}+\frac{1}{a^3x}-\frac{1}{a^3(a+x)}\right)dx,x,b\sinh(c+dx)\right)}{d}\\
&= \frac{b\operatorname{csch}(c+dx)}{a^2d}-\frac{\operatorname{csch}^2(c+dx)}{2ad}+\frac{b^2\log(\sinh(c+dx))}{a^3d}-\frac{b^2\log(a+b\sinh(c+dx))}{a^3d}
\end{aligned}$$

Mathematica [A] time = 0.100926, size = 60, normalized size = 0.83

$$\frac{-a^2\text{csch}^2(c+dx)+2b^2(\log(\sinh(c+dx))-\log(a+b\sinh(c+dx)))+2ab\text{csch}(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a*b*Csch[c + d*x] - a^2*Csch[c + d*x]^2 + 2*b^2*(Log[Sinh[c + d*x]] - Lo
g[a + b*Sinh[c + d*x]]))/(2*a^3*d)
```

Maple [A] time = 0.001, size = 73, normalized size = 1.

$$-\frac{1}{2\,da\left(\sinh\left(dx+c\right)\right)^{2}}+\frac{b^{2}\ln\left(\sinh\left(dx+c\right)\right)}{a^{3}d}+\frac{b}{da^{2}\sinh\left(dx+c\right)}-\frac{b^{2}\ln\left(a+b\sinh\left(dx+c\right)\right)}{a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/2/d/a/sinh(d*x+c)^2+b^2*ln(sinh(d*x+c))/a^3/d+1/d/a^2*b/sinh(d*x+c)-b^2*
ln(a+b*sinh(d*x+c))/a^3/d
```

Maxima [B] time = 1.26917, size = 217, normalized size = 3.01

$$-\frac{2\left(be^{(-dx-c)}-ae^{(-2\,dx-2\,c)}-be^{(-3\,dx-3\,c)}\right)}{\left(2\,a^{2}e^{(-2\,dx-2\,c)}-a^{2}e^{(-4\,dx-4\,c)}-a^{2}\right)d}-\frac{b^{2}\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{a^{3}d}+\frac{b^{2}\log\left(e^{(-dx-c)}+1\right)}{a^{3}d}+\frac{b^{2}\log\left(e^{(-dx}\right.}{a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*e^(-2
*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - b^2*log(-2*a*e^(-d*x - c) +
b*e^(-2*d*x - 2*c) - b)/(a^3*d) + b^2*log(e^(-d*x - c) + 1)/(a^3*d) + b^2*l
og(e^(-d*x - c) - 1)/(a^3*d)
```

Fricas [B] time = 2.19279, size = 1370, normalized size = 19.03

$$\frac{2\,ab\cosh\left(dx+c\right)^{3}+2\,ab\sinh\left(dx+c\right)^{3}-2\,a^{2}\cosh\left(dx+c\right)^{2}-2\,ab\cosh\left(dx+c\right)+2\left(3\,ab\cosh\left(dx+c\right)-a^{2}\right)\sinh\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] (2*a*b*cosh(d*x + c)^3 + 2*a*b*sinh(d*x + c)^3 - 2*a^2*cosh(d*x + c)^2 - 2*
a*b*cosh(d*x + c) + 2*(3*a*b*cosh(d*x + c) - a^2)*sinh(d*x + c)^2 - (b^2*co
```

```
sh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 -
 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^2 +
b^2 + 4*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*s
inh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + (b^2*cosh(d*x + c)^4 +
 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 - 2*b^2*cosh(d*x
 + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*co
sh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh
(d*x + c) - sinh(d*x + c))) + 2*(3*a*b*cosh(d*x + c)^2 - 2*a^2*cosh(d*x + c
) - a*b)*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)*sinh
(d*x + c)^3 + a^3*d*sinh(d*x + c)^4 - 2*a^3*d*cosh(d*x + c)^2 + a^3*d + 2*(
3*a^3*d*cosh(d*x + c)^2 - a^3*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3
 - a^3*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)*csch(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

Giac [A] time = 1.43603, size = 181, normalized size = 2.51

$$\frac{\frac{b^2\log\left(e^{(dx+c)}+1\right)}{a^3}-\frac{b^2\log\left(\left|be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}-b\right|\right)}{a^3}+\frac{b^2\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^3}+\frac{2\left(abe^{(3\,dx+3\,c)}-a^2e^{(2\,dx+2\,c)}-abe^{(dx+c)}\right)}{a^3\left(e^{(dx+c)}+1\right)^2\left(e^{(dx+c)}-1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (b^2*log(e^(d*x + c) + 1)/a^3 - b^2*log(abs(b*e^(2*d*x + 2*c) + 2*a*e^(d*x
+ c) - b))/a^3 + b^2*log(abs(e^(d*x + c) - 1))/a^3 + 2*(a*b*e^(3*d*x + 3*c)
 - a^2*e^(2*d*x + 2*c) - a*b*e^(d*x + c))/(a^3*(e^(d*x + c) + 1)^2*(e^(d*x
+ c) - 1)^2))/d
```

3.480 $\int \frac{\coth(c+dx)\mathbf{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{\coth(c+dx)\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Coth[c + d*x]*Csch[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [A] time = 0.0920873, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Coth[c + d*x]*Csch[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Coth[c + d*x]*Csch[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth(c+dx)\text{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Coth[c + d*x]*Csch[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] $Aborted
```

Maple [A] time = 1.971, size = 0, normalized size = 0.

$$\int \frac{\coth(dx+c)\,(\text{csch}\,(dx+c))^2}{\left(fx+e\right)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{af-2\left(bdfxe^{(3c)}+bdee^{(3c)}\right)e^{(3dx)}+\left(2adfxe^{(2c)}+\left(2de-f\right)ae^{(2c)}\right)e^{(2dx)}+2\left(bdfxe^{c}+bdee^{c}\right)e^{(dx)}}{a^2d^2f^2x^2+2a^2d^2efx+a^2d^2e^2+\left(a^2d^2f^2x^2e^{(4c)}+2a^2d^2efxe^{(4c)}+a^2d^2e^2e^{(4c)}\right)e^{(4dx)}-2\left(a^2d^2f^2x^2e^{(2c)}+2a^2d^2efxe^{(2c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -(a*f - 2*(b*d*f*x*e^(3*c) + b*d*e*e^(3*c))*e^(3*d*x) + (2*a*d*f*x*e^(2*c)
+ (2*d*e - f)*a*e^(2*c))*e^(2*d*x) + 2*(b*d*f*x*e^c + b*d*e*e^c)*e^(d*x))/(
a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + a^2*d^2*e^2 + (a^2*d^2*f^2*x^2*e^(4*c)
+ 2*a^2*d^2*e*f*x*e^(4*c) + a^2*d^2*e^2*e^(4*c))*e^(4*d*x) - 2*(a^2*d^2*f^2
*x^2*e^(2*c) + 2*a^2*d^2*e*f*x*e^(2*c) + a^2*d^2*e^2*e^(2*c))*e^(2*d*x)) +
4*integrate(-1/4*(b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + a*b*d*e*f + a^2*f^2 + (2*
b^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*
d^2*e^2*f*x + a^3*d^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c
+ 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) - 4*integrate(1/4*(
b^2*d^2*f^2*x^2 + b^2*d^2*e^2 - a*b*d*e*f + a^2*f^2 + (2*b^2*d^2*e*f - a*b*
d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*
d^2*e^3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*
x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) + 4*integrate(-1/2*(a*b^2*e^(d*x + c)
 - b^3)/(a^3*b*f*x + a^3*b*e - (a^3*b*f*x*e^(2*c) + a^3*b*e*e^(2*c))*e^(2*d
*x) - 2*(a^4*f*x*e^c + a^4*e*e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\coth\left(dx+c\right)\operatorname{csch}\left(dx+c\right)^{2}}{afx+ae+\left(bfx+be\right)\sinh\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(coth(d*x + c)*csch(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.481 $\int \frac{(e+fx)^3 \coth^2(c+dx)\mathbf{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1038

result too large to display

```
[Out] (b*(e + f*x)^3)/(a^2*d) - (6*f^2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d^3) -
((e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (2*b^2*(e + f*x)^3*ArcTanh[E^(c
+ d*x)])/(a^3*d) + (b*(e + f*x)^3*Coth[c + d*x])/(a^2*d) - (3*f*(e + f*x)^2
*Csch[c + d*x])/(2*a*d^2) - ((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x])/(2*a*
d) - (b*Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/(a^3*d) + (b*Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/(a^3*d) - (3*b*f*(e + f*x)^2*Log[1 - E^(2*(c + d*x))
])/(a^2*d^2) - (3*f^3*PolyLog[2, -E^(c + d*x)])/(a*d^4) - (3*f*(e + f*x)^2*
PolyLog[2, -E^(c + d*x)])/(2*a*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -E^(c
 + d*x)])/(a^3*d^2) + (3*f^3*PolyLog[2, E^(c + d*x)])/(a*d^4) + (3*f*(e + f
*x)^2*PolyLog[2, E^(c + d*x)])/(2*a*d^2) + (3*b^2*f*(e + f*x)^2*PolyLog[2,
E^(c + d*x)])/(a^3*d^2) - (3*b*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2) + (3*b*Sqrt[a^2 + b^2]*f*
(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2)
 - (3*b*f^2*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a^2*d^3) + (3*f^2*(e +
f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) + (6*b^2*f^2*(e + f*x)*PolyLog[3, -E
^(c + d*x)])/(a^3*d^3) - (3*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a*d^3)
- (6*b^2*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a^3*d^3) + (6*b*Sqrt[a^2 +
 b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(
a^3*d^3) - (6*b*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/(a^3*d^3) + (3*b*f^3*PolyLog[3, E^(2*(c + d*x))])/
(2*a^2*d^4) - (3*f^3*PolyLog[4, -E^(c + d*x)])/(a*d^4) - (6*b^2*f^3*PolyLog
[4, -E^(c + d*x)])/(a^3*d^4) + (3*f^3*PolyLog[4, E^(c + d*x)])/(a*d^4) + (6
*b^2*f^3*PolyLog[4, E^(c + d*x)])/(a^3*d^4) - (6*b*Sqrt[a^2 + b^2]*f^3*Poly
Log[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^4) + (6*b*Sqrt[a^2
 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^4)
```

Rubi [A] time = 2.24077, antiderivative size = 1038, normalized size of antiderivative = 1., number of steps used = 67, number of rules used = 22, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {5587, 5457, 4182, 2531, 6609, 2282, 6589, 4186, 2279, 2391, 5569, 3720, 3716, 2190, 32, 5585, 5450, 3296, 2637, 5565, 3322, 2264}

$$-\frac{3\text{PolyLog}\left(2,-e^{c+dx}\right)f^3}{ad^4}+\frac{3\text{PolyLog}\left(2,e^{c+dx}\right)f^3}{ad^4}+\frac{3b\text{PolyLog}\left(3,e^{2(c+dx)}\right)f^3}{2a^2d^4}-\frac{6b^2\text{PolyLog}\left(4,-e^{c+dx}\right)f^3}{a^3d^4}-3\text{Poly}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*(e + f*x)^3)/(a^2*d) - (6*f^2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d^3) -
((e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (2*b^2*(e + f*x)^3*ArcTanh[E^(c
+ d*x)])/(a^3*d) + (b*(e + f*x)^3*Coth[c + d*x])/(a^2*d) - (3*f*(e + f*x)^2
*Csch[c + d*x])/(2*a*d^2) - ((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x])/(2*a*
d) - (b*Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2])])/(a^3*d) + (b*Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])])/(a^3*d) - (3*b*f*(e + f*x)^2*Log[1 - E^(2*(c + d*x))
])/(a^2*d^2) - (3*f^3*PolyLog[2, -E^(c + d*x)])/(a*d^4) - (3*f*(e + f*x)^2*
PolyLog[2, -E^(c + d*x)])/(2*a*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -E^(c
 + d*x)])/(a^3*d^2) + (3*f^3*PolyLog[2, E^(c + d*x)])/(a*d^4) + (3*f*(e + f
```

```
*x)^2*PolyLog[2, E^(c + d*x)])/(2*a*d^2) + (3*b^2*f*(e + f*x)^2*PolyLog[2,
E^(c + d*x)])/(a^3*d^2) - (3*b*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2) + (3*b*Sqrt[a^2 + b^2]*f*
(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2)
 - (3*b*f^2*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a^2*d^3) + (3*f^2*(e +
f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) + (6*b^2*f^2*(e + f*x)*PolyLog[3, -E
^(c + d*x)])/(a^3*d^3) - (3*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a*d^3)
- (6*b^2*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a^3*d^3) + (6*b*Sqrt[a^2 +
 b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(
a^3*d^3) - (6*b*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/(a^3*d^3) + (3*b*f^3*PolyLog[3, E^(2*(c + d*x))])/
(2*a^2*d^4) - (3*f^3*PolyLog[4, -E^(c + d*x)])/(a*d^4) - (6*b^2*f^3*PolyLog
[4, -E^(c + d*x)])/(a^3*d^4) + (3*f^3*PolyLog[4, E^(c + d*x)])/(a*d^4) + (6
*b^2*f^3*PolyLog[4, E^(c + d*x)])/(a^3*d^4) - (6*b*Sqrt[a^2 + b^2]*f^3*Poly
Log[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^4) + (6*b*Sqrt[a^2
 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^4)
```

<u>Rule 5587</u>

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Csch[c + d*x]^(p - 1)*Coth[c + d*x]^n)/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

<u>Rule 5457</u>

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

<u>Rule 4182</u>

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

<u>Rule 2531</u>

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

<u>Rule 6609</u>

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

<u>Rule 2282</u>

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
```

```
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\coth^2(c+dx)\operatorname{csch}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)^3\operatorname{csch}(c+dx)\,dx}{a} + \frac{\int (e+fx)^3\operatorname{csch}^3(c+dx)\,dx}{a} - \frac{b\int (e+fx)^3\,\mathrm{c}}{a}$$

$$= -\frac{2(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{b(e+fx)^3\coth(c+dx)}{a^2d} - \frac{3f(e+fx)^2\operatorname{csch}(c}{2ad^2}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{b(e+fx)^4}{4a^2f} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} - \frac{(e+fx)^3\tanh^{-1}\left(\right.}{ad}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} - \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(}{}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} - \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(}{}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} - \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(}{}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} - \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(}{}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} - \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(}{}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} - \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(}{}$$

$$= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad^3} - \frac{(e+fx)^3\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(}{}$$

Mathematica [C] time = 43.6309, size = 2384, normalized size = 2.3

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (e^3*Log[Tanh[(c + d*x)/2]])/(2*a*d) + (b^2*e^3*Log[Tanh[(c + d*x)/2]])/(a^
3*d) + (3*e*f^2*Log[Tanh[(c + d*x)/2]])/(a*d^3) + (3*e^2*f*(-(c*Log[Tanh[(c
 + d*x)/2]]) - I*((I*c + I*d*x)*(Log[1 - E^(I*(I*c + I*d*x))] - Log[1 + E^(
I*(I*c + I*d*x))]) + I*(PolyLog[2, -E^(I*(I*c + I*d*x))] - PolyLog[2, E^(I*
(I*c + I*d*x))]))))/(2*a*d^2) + (3*b^2*e^2*f*(-(c*Log[Tanh[(c + d*x)/2]]) -
 I*((I*c + I*d*x)*(Log[1 - E^(I*(I*c + I*d*x))] - Log[1 + E^(I*(I*c + I*d*x
))]) + I*(PolyLog[2, -E^(I*(I*c + I*d*x))] - PolyLog[2, E^(I*(I*c + I*d*x))
]))))/(a^3*d^2) + (3*f^3*(-(c*Log[Tanh[(c + d*x)/2]]) - I*((I*c + I*d*x)*(L
og[1 - E^(I*(I*c + I*d*x))] - Log[1 + E^(I*(I*c + I*d*x))]) + I*(PolyLog[2,
 -E^(I*(I*c + I*d*x))] - PolyLog[2, E^(I*(I*c + I*d*x))]))))/(a*d^4) + (b*E
```

```
^c*f^3*Csch[c]*((2*d^3*x^3)/E^(2*c) - 3*d^2*(1 - E^(-2*c))*x^2*Log[1 - E^(-
c - d*x)] - 3*d^2*(1 - E^(-2*c))*x^2*Log[1 + E^(-c - d*x)] + 6*(1 - E^(-2*c
))*(d*x*PolyLog[2, -E^(-c - d*x)] + PolyLog[3, -E^(-c - d*x)]) + 6*(1 - E^(
-2*c))*(d*x*PolyLog[2, E^(-c - d*x)] + PolyLog[3, E^(-c - d*x)])))/(2*a^2*d
^4) - (3*e*f^2*(d^2*x^2*ArcTanh[Cosh[c + d*x] + Sinh[c + d*x]] + d*x*PolyLo
g[2, -Cosh[c + d*x] - Sinh[c + d*x]] - d*x*PolyLog[2, Cosh[c + d*x] + Sinh[
c + d*x]] - PolyLog[3, -Cosh[c + d*x] - Sinh[c + d*x]] + PolyLog[3, Cosh[c
+ d*x] + Sinh[c + d*x]]))/(a*d^3) - (6*b^2*e*f^2*(d^2*x^2*ArcTanh[Cosh[c +
d*x] + Sinh[c + d*x]] + d*x*PolyLog[2, -Cosh[c + d*x] - Sinh[c + d*x]] - d*
x*PolyLog[2, Cosh[c + d*x] + Sinh[c + d*x]] - PolyLog[3, -Cosh[c + d*x] - S
inh[c + d*x]] + PolyLog[3, Cosh[c + d*x] + Sinh[c + d*x]]))/(a^3*d^3) + (b*
Sqrt[a^2 + b^2]*(2*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 3
*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 3*d^3*e*f^2*x
^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^3*f^3*x^3*Log[1 + (b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])] + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
- 3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] +
3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] +
6*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*d*f^3*x*Po
lyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*d*e*f^2*PolyLog[3, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*f^3*x*PolyLog[3, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))] - 6*f^3*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[
a^2 + b^2])] + 6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))
/(a^3*d^4) + (f^3*(-2*d^3*x^3*ArcTanh[Cosh[c + d*x] + Sinh[c + d*x]] - 3*d^
2*x^2*PolyLog[2, -Cosh[c + d*x] - Sinh[c + d*x]] + 3*d^2*x^2*PolyLog[2, Cos
h[c + d*x] + Sinh[c + d*x]] + 6*d*x*PolyLog[3, -Cosh[c + d*x] - Sinh[c + d*
x]] - 6*d*x*PolyLog[3, Cosh[c + d*x] + Sinh[c + d*x]] - 6*PolyLog[4, -Cosh[
c + d*x] - Sinh[c + d*x]] + 6*PolyLog[4, Cosh[c + d*x] + Sinh[c + d*x]]))/(
2*a*d^4) + (b^2*f^3*(-2*d^3*x^3*ArcTanh[Cosh[c + d*x] + Sinh[c + d*x]] - 3*
d^2*x^2*PolyLog[2, -Cosh[c + d*x] - Sinh[c + d*x]] + 3*d^2*x^2*PolyLog[2, C
osh[c + d*x] + Sinh[c + d*x]] + 6*d*x*PolyLog[3, -Cosh[c + d*x] - Sinh[c +
d*x]] - 6*d*x*PolyLog[3, Cosh[c + d*x] + Sinh[c + d*x]] - 6*PolyLog[4, -Cos
h[c + d*x] - Sinh[c + d*x]] + 6*PolyLog[4, Cosh[c + d*x] + Sinh[c + d*x]]))
/(a^3*d^4) + (3*b*e^2*f*Csch[c]*(-(d*x*Cosh[c]) + Log[Cosh[d*x]*Sinh[c] + C
osh[c]*Sinh[d*x]]*Sinh[c]))/(a^2*d^2*(-Cosh[c]^2 + Sinh[c]^2)) + (Csch[c]*C
sch[c + d*x]^2*(2*b*d*e^3*Cosh[c] + 6*b*d*e^2*f*x*Cosh[c] + 6*b*d*e*f^2*x^2
*Cosh[c] + 2*b*d*f^3*x^3*Cosh[c] + 3*a*e^2*f*Cosh[d*x] + 6*a*e*f^2*x*Cosh[d
*x] + 3*a*f^3*x^2*Cosh[d*x] - 3*a*e^2*f*Cosh[2*c + d*x] - 6*a*e*f^2*x*Cosh[
2*c + d*x] - 3*a*f^3*x^2*Cosh[2*c + d*x] - 2*b*d*e^3*Cosh[c + 2*d*x] - 6*b*
d*e^2*f*x*Cosh[c + 2*d*x] - 6*b*d*e*f^2*x^2*Cosh[c + 2*d*x] - 2*b*d*f^3*x^3
*Cosh[c + 2*d*x] + a*d*e^3*Sinh[d*x] + 3*a*d*e^2*f*x*Sinh[d*x] + 3*a*d*e*f^
2*x^2*Sinh[d*x] + a*d*f^3*x^3*Sinh[d*x] - a*d*e^3*Sinh[2*c + d*x] - 3*a*d*e
^2*f*x*Sinh[2*c + d*x] - 3*a*d*e*f^2*x^2*Sinh[2*c + d*x] - a*d*f^3*x^3*Sinh
[2*c + d*x]))/(4*a^2*d^2) - (3*b*e*f^2*Csch[c]*Sech[c]*(-((d^2*x^2)/E^ArcTa
nh[Tanh[c]]) + (I*(-(d*x*(-Pi + (2*I)*ArcTanh[Tanh[c]])) - Pi*Log[1 + E^(2*
d*x)] - 2*(I*d*x + I*ArcTanh[Tanh[c]])*Log[1 - E^((2*I)*(I*d*x + I*ArcTanh[
Tanh[c]]))] + Pi*Log[Cosh[d*x]] + (2*I)*ArcTanh[Tanh[c]]*Log[I*Sinh[d*x + A
rcTanh[Tanh[c]]]] + I*PolyLog[2, E^((2*I)*(I*d*x + I*ArcTanh[Tanh[c]]))])*T
anh[c])/Sqrt[1 - Tanh[c]^2]))/(a^2*d^3*Sqrt[Sech[c]^2*(Cosh[c]^2 - Sinh[c]^
2)])
```

Maple [F] time = 0.889, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 (\coth(dx+c))^2 \operatorname{csch}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 4.75378, size = 29336, normalized size = 28.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/2*(4*a*b*d^3*e^3 - 12*a*b*c*d^2*e^2*f + 12*a*b*c^2*d*e*f^2 - 4*a*b*c^3*f
^3 - 4*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*e^2*f*x + 3*a*b*c
*d^2*e^2*f - 3*a*b*c^2*d*e*f^2 + a*b*c^3*f^3)*cosh(d*x + c)^4 - 4*(a*b*d^3*
f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*e^2*f*x + 3*a*b*c*d^2*e^2*f - 3*a
*b*c^2*d*e*f^2 + a*b*c^3*f^3)*sinh(d*x + c)^4 + 2*(a^2*d^3*f^3*x^3 + a^2*d^
3*e^3 + 3*a^2*d^2*e^2*f + 3*(a^2*d^3*e*f^2 + a^2*d^2*f^3)*x^2 + 3*(a^2*d^3*
e^2*f + 2*a^2*d^2*e*f^2)*x)*cosh(d*x + c)^3 + 2*(a^2*d^3*f^3*x^3 + a^2*d^3*
e^3 + 3*a^2*d^2*e^2*f + 3*(a^2*d^3*e*f^2 + a^2*d^2*f^3)*x^2 + 3*(a^2*d^3*e^
2*f + 2*a^2*d^2*e*f^2)*x - 8*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b
*d^3*e^2*f*x + 3*a*b*c*d^2*e^2*f - 3*a*b*c^2*d*e*f^2 + a*b*c^3*f^3)*cosh(d*
x + c))*sinh(d*x + c)^3 + 4*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b*
d^3*e^2*f*x - a*b*d^3*e^3 + 6*a*b*c*d^2*e^2*f - 6*a*b*c^2*d*e*f^2 + 2*a*b*c
^3*f^3)*cosh(d*x + c)^2 + 2*(2*a*b*d^3*f^3*x^3 + 6*a*b*d^3*e*f^2*x^2 + 6*a*
b*d^3*e^2*f*x - 2*a*b*d^3*e^3 + 12*a*b*c*d^2*e^2*f - 12*a*b*c^2*d*e*f^2 + 4
*a*b*c^3*f^3 - 12*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*e^2*f*
x + 3*a*b*c*d^2*e^2*f - 3*a*b*c^2*d*e*f^2 + a*b*c^3*f^3)*cosh(d*x + c)^2 +
3*(a^2*d^3*f^3*x^3 + a^2*d^3*e^3 + 3*a^2*d^2*e^2*f + 3*(a^2*d^3*e*f^2 + a^2
*d^2*f^3)*x^2 + 3*(a^2*d^3*e^2*f + 2*a^2*d^2*e*f^2)*x)*cosh(d*x + c))*sinh(
d*x + c)^2 + 6*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f + (b^2*
d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*cosh(d*x + c)^4 + 4*(b^2*d
^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c)
^3 + (b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*sinh(d*x + c)^4
- 2*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*cosh(d*x + c)^2 -
 2*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f - 3*(b^2*d^2*f^3*x^
2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4
*((b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*cosh(d*x + c)^3 - (
b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*cosh(d*x + c))*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 6*(b^
2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f + (b^2*d^2*f^3*x^2 + 2*b^
```

2*d^2*e*f^2*x + b^2*d^2*e^2*f)*cosh(d*x + c)^4 + 4*(b^2*d^2*f^3*x^2 + 2*b^2
*d^2*e*f^2*x + b^2*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d^2*f^3*
x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*sinh(d*x + c)^4 - 2*(b^2*d^2*f^3*x
^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*cosh(d*x + c)^2 - 2*(b^2*d^2*f^3*x^
2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f - 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^
2*x + b^2*d^2*e^2*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d^2*f^3*x^2
 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*cosh(d*x + c)^3 - (b^2*d^2*f^3*x^2 +
2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b^2*d^3*e^3 - 3*b^2*
c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3 + (b^2*d^3*e^3 - 3*b^2*c*d^2*
e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*cosh(d*x + c)^4 + 4*(b^2*d^3*e^3 -
 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*cosh(d*x + c)*sinh(d*
x + c)^3 + (b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f
^3)*sinh(d*x + c)^4 - 2*(b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^
2 - b^2*c^3*f^3)*cosh(d*x + c)^2 - 2*(b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b
^2*c^2*d*e*f^2 - b^2*c^3*f^3 - 3*(b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c
^2*d*e*f^2 - b^2*c^3*f^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d^3*e^
3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*cosh(d*x + c)^3 -
(b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*cosh(d*
x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*si
nh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(b^2*d^3*e^3 - 3*b^2*c*d
^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3 + (b^2*d^3*e^3 - 3*b^2*c*d^2*e^2
*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*cosh(d*x + c)^4 + 4*(b^2*d^3*e^3 - 3*
b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*cosh(d*x + c)*sinh(d*x +
 c)^3 + (b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)
*sinh(d*x + c)^4 - 2*(b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 -
 b^2*c^3*f^3)*cosh(d*x + c)^2 - 2*(b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*
c^2*d*e*f^2 - b^2*c^3*f^3 - 3*(b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*
d*e*f^2 - b^2*c^3*f^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d^3*e^3 -
 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*cosh(d*x + c)^3 - (b^
2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*cosh(d*x +
 c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(
d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(b^2*d^3*f^3*x^3 + 3*b^2*d^
3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b
^2*c^3*f^3 + (b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3
*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*cosh(d*x + c)^4 + 4*(b^
2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f
 - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d^
3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3
*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*sinh(d*x + c)^4 - 2*(b^2*d^3*f^3*x^3 + 3*b^
2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2
 + b^2*c^3*f^3)*cosh(d*x + c)^2 - 2*(b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2
+ 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3 -
 3*(b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2
*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 4*((b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d
^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*cosh(d*x + c)^3 - (b^2*d^3*f^3*
x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c
^2*d*e*f^2 + b^2*c^3*f^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*
x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f
^3 + (b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d
^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*cosh(d*x + c)^4 + 4*(b^2*d^3*f^
3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2
*c^2*d*e*f^2 + b^2*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d^3*f^3*x^
3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2
*d*e*f^2 + b^2*c^3*f^3)*sinh(d*x + c)^4 - 2*(b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*

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f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c
^3*f^3)*cosh(d*x + c)^2 - 2*(b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*
d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3 - 3*(b^2*
d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f -
 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^
2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f
 - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*cosh(d*x + c)^3 - (b^2*d^3*f^3*x^3 + 3*
b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f
^2 + b^2*c^3*f^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-
(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sq
rt((a^2 + b^2)/b^2) - b)/b) + 12*(b^2*f^3*cosh(d*x + c)^4 + 4*b^2*f^3*cosh(
d*x + c)*sinh(d*x + c)^3 + b^2*f^3*sinh(d*x + c)^4 - 2*b^2*f^3*cosh(d*x + c
)^2 + b^2*f^3 + 2*(3*b^2*f^3*cosh(d*x + c)^2 - b^2*f^3)*sinh(d*x + c)^2 + 4
*(b^2*f^3*cosh(d*x + c)^3 - b^2*f^3*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 12*(b^2*f^3*cosh(d*x + c)
^4 + 4*b^2*f^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*f^3*sinh(d*x + c)^4 - 2*
b^2*f^3*cosh(d*x + c)^2 + b^2*f^3 + 2*(3*b^2*f^3*cosh(d*x + c)^2 - b^2*f^3)
*sinh(d*x + c)^2 + 4*(b^2*f^3*cosh(d*x + c)^3 - b^2*f^3*cosh(d*x + c))*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x +
 c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 12*(b
^2*d*f^3*x + b^2*d*e*f^2 + (b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c)^4 + 4*
(b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*f^3*x +
b^2*d*e*f^2)*sinh(d*x + c)^4 - 2*(b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c)^
2 - 2*(b^2*d*f^3*x + b^2*d*e*f^2 - 3*(b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x +
 c)^2)*sinh(d*x + c)^2 + 4*((b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c)^3 - (
b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 12*(b^2*d*f^3*x + b^2*d*e*f^2 + (b
^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c)^4 + 4*(b^2*d*f^3*x + b^2*d*e*f^2)*c
osh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*f^3*x + b^2*d*e*f^2)*sinh(d*x + c)^4
- 2*(b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c)^2 - 2*(b^2*d*f^3*x + b^2*d*e*
f^2 - 3*(b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((
b^2*d*f^3*x + b^2*d*e*f^2)*cosh(d*x + c)^3 - (b^2*d*f^3*x + b^2*d*e*f^2)*co
sh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x +
c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2))/b) + 2*(a^2*d^3*f^3*x^3 + a^2*d^3*e^3 - 3*a^2*d^2*e^2*f + 3*(a^2*d^3
*e*f^2 - a^2*d^2*f^3)*x^2 + 3*(a^2*d^3*e^2*f - 2*a^2*d^2*e*f^2)*x)*cosh(d*x
 + c) - 3*((a^2 + 2*b^2)*d^2*f^3*x^2 + (a^2 + 2*b^2)*d^2*e^2*f - 4*a*b*d*e*
f^2 + 2*a^2*f^3 + ((a^2 + 2*b^2)*d^2*f^3*x^2 + (a^2 + 2*b^2)*d^2*e^2*f - 4*
a*b*d*e*f^2 + 2*a^2*f^3 + 2*((a^2 + 2*b^2)*d^2*e*f^2 - 2*a*b*d*f^3)*x)*cosh
(d*x + c)^4 + 4*((a^2 + 2*b^2)*d^2*f^3*x^2 + (a^2 + 2*b^2)*d^2*e^2*f - 4*a*
b*d*e*f^2 + 2*a^2*f^3 + 2*((a^2 + 2*b^2)*d^2*e*f^2 - 2*a*b*d*f^3)*x)*cosh(d
*x + c)*sinh(d*x + c)^3 + ((a^2 + 2*b^2)*d^2*f^3*x^2 + (a^2 + 2*b^2)*d^2*e^
2*f - 4*a*b*d*e*f^2 + 2*a^2*f^3 + 2*((a^2 + 2*b^2)*d^2*e*f^2 - 2*a*b*d*f^3)
*x)*sinh(d*x + c)^4 - 2*((a^2 + 2*b^2)*d^2*f^3*x^2 + (a^2 + 2*b^2)*d^2*e^2*
f - 4*a*b*d*e*f^2 + 2*a^2*f^3 + 2*((a^2 + 2*b^2)*d^2*e*f^2 - 2*a*b*d*f^3)*x
)*cosh(d*x + c)^2 - 2*((a^2 + 2*b^2)*d^2*f^3*x^2 + (a^2 + 2*b^2)*d^2*e^2*f
- 4*a*b*d*e*f^2 + 2*a^2*f^3 - 3*((a^2 + 2*b^2)*d^2*f^3*x^2 + (a^2 + 2*b^2)*
d^2*e^2*f - 4*a*b*d*e*f^2 + 2*a^2*f^3 + 2*((a^2 + 2*b^2)*d^2*e*f^2 - 2*a*b*
d*f^3)*x)*cosh(d*x + c)^2 + 2*((a^2 + 2*b^2)*d^2*e*f^2 - 2*a*b*d*f^3)*x)*si
nh(d*x + c)^2 + 2*((a^2 + 2*b^2)*d^2*e*f^2 - 2*a*b*d*f^3)*x + 4*(((a^2 + 2*
b^2)*d^2*f^3*x^2 + (a^2 + 2*b^2)*d^2*e^2*f - 4*a*b*d*e*f^2 + 2*a^2*f^3 + 2*
((a^2 + 2*b^2)*d^2*e*f^2 - 2*a*b*d*f^3)*x)*cosh(d*x + c)^3 - ((a^2 + 2*b^2)
*d^2*f^3*x^2 + (a^2 + 2*b^2)*d^2*e^2*f - 4*a*b*d*e*f^2 + 2*a^2*f^3 + 2*((a^
2 + 2*b^2)*d^2*e*f^2 - 2*a*b*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c))*dilog(
cosh(d*x + c) + sinh(d*x + c)) + 3*((a^2 + 2*b^2)*d^2*f^3*x^2 + (a^2 + 2*b^
2)*d^2*e^2*f + 4*a*b*d*e*f^2 + 2*a^2*f^3 + ((a^2 + 2*b^2)*d^2*f^3*x^2 + (a^
2 + 2*b^2)*d^2*e^2*f + 4*a*b*d*e*f^2 + 2*a^2*f^3 + 2*((a^2 + 2*b^2)*d^2*e*f
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^2 + 2*a*b*d*f^3)*x)*cosh(d*x + c)^4 + 4*((a^2 + 2*b^2)*d^2*f^3*x^2 + (a^2
+ 2*b^2)*d^2*e^2*f + 4*a*b*d*e*f^2 + 2*a^2*f^3 + 2*((a^2 + 2*b^2)*d^2*e*f^2
 + 2*a*b*d*f^3)*x)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 + 2*b^2)*d^2*f^3*x
^2 + (a^2 + 2*b^2)*d^2*e^2*f + 4*a*b*d*e*f^2 + 2*a^2*f^3 + 2*((a^2 + 2*b^2)
*d^2*e*f^2 + 2*a*b*d*f^3)*x)*sinh(d*x + c)^4 - 2*((a^2 + 2*b^2)*d^2*f^3*x^2
 + (a^2 + 2*b^2)*d^2*e^2*f + 4*a*b*d*e*f^2 + 2*a^2*f^3 + 2*((a^2 + 2*b^2)*d
^2*e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*x + c)^2 - 2*((a^2 + 2*b^2)*d^2*f^3*x^2 +
 (a^2 + 2*b^2)*d^2*e^2*f + 4*a*b*d*e*f^2 + 2*a^2*f^3 - 3*((a^2 + 2*b^2)*d^2
*f^3*x^2 + (a^2 + 2*b^2)*d^2*e^2*f + 4*a*b*d*e*f^2 + 2*a^2*f^3 + 2*((a^2 +
2*b^2)*d^2*e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*x + c)^2 + 2*((a^2 + 2*b^2)*d^2*e
*f^2 + 2*a*b*d*f^3)*x)*sinh(d*x + c)^2 + 2*((a^2 + 2*b^2)*d^2*e*f^2 + 2*a*b
*d*f^3)*x + 4*(((a^2 + 2*b^2)*d^2*f^3*x^2 + (a^2 + 2*b^2)*d^2*e^2*f + 4*a*b
*d*e*f^2 + 2*a^2*f^3 + 2*((a^2 + 2*b^2)*d^2*e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*
x + c)^3 - ((a^2 + 2*b^2)*d^2*f^3*x^2 + (a^2 + 2*b^2)*d^2*e^2*f + 4*a*b*d*e
*f^2 + 2*a^2*f^3 + 2*((a^2 + 2*b^2)*d^2*e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*x +
c))*sinh(d*x + c))*dilog(-cosh(d*x + c) - sinh(d*x + c)) + ((a^2 + 2*b^2)*d
^3*f^3*x^3 + (a^2 + 2*b^2)*d^3*e^3 + 6*a*b*d^2*e^2*f + 6*a^2*d*e*f^2 + ((a^
2 + 2*b^2)*d^3*f^3*x^3 + (a^2 + 2*b^2)*d^3*e^3 + 6*a*b*d^2*e^2*f + 6*a^2*d*
e*f^2 + 3*((a^2 + 2*b^2)*d^3*e*f^2 + 2*a*b*d^2*f^3)*x^2 + 3*((a^2 + 2*b^2)*
d^3*e^2*f + 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x)*cosh(d*x + c)^4 + 4*((a^2 + 2
*b^2)*d^3*f^3*x^3 + (a^2 + 2*b^2)*d^3*e^3 + 6*a*b*d^2*e^2*f + 6*a^2*d*e*f^2
 + 3*((a^2 + 2*b^2)*d^3*e*f^2 + 2*a*b*d^2*f^3)*x^2 + 3*((a^2 + 2*b^2)*d^3*e
^2*f + 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x)*cosh(d*x + c)*sinh(d*x + c)^3 + ((
a^2 + 2*b^2)*d^3*f^3*x^3 + (a^2 + 2*b^2)*d^3*e^3 + 6*a*b*d^2*e^2*f + 6*a^2*
d*e*f^2 + 3*((a^2 + 2*b^2)*d^3*e*f^2 + 2*a*b*d^2*f^3)*x^2 + 3*((a^2 + 2*b^2
)*d^3*e^2*f + 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x)*sinh(d*x + c)^4 + 3*((a^2 +
 2*b^2)*d^3*e*f^2 + 2*a*b*d^2*f^3)*x^2 - 2*((a^2 + 2*b^2)*d^3*f^3*x^3 + (a^
2 + 2*b^2)*d^3*e^3 + 6*a*b*d^2*e^2*f + 6*a^2*d*e*f^2 + 3*((a^2 + 2*b^2)*d^3
*e*f^2 + 2*a*b*d^2*f^3)*x^2 + 3*((a^2 + 2*b^2)*d^3*e^2*f + 4*a*b*d^2*e*f^2
+ 2*a^2*d*f^3)*x)*cosh(d*x + c)^2 - 2*((a^2 + 2*b^2)*d^3*f^3*x^3 + (a^2 + 2
*b^2)*d^3*e^3 + 6*a*b*d^2*e^2*f + 6*a^2*d*e*f^2 + 3*((a^2 + 2*b^2)*d^3*e*f^
2 + 2*a*b*d^2*f^3)*x^2 - 3*((a^2 + 2*b^2)*d^3*f^3*x^3 + (a^2 + 2*b^2)*d^3*e
^3 + 6*a*b*d^2*e^2*f + 6*a^2*d*e*f^2 + 3*((a^2 + 2*b^2)*d^3*e*f^2 + 2*a*b*d
^2*f^3)*x^2 + 3*((a^2 + 2*b^2)*d^3*e^2*f + 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x
)*cosh(d*x + c)^2 + 3*((a^2 + 2*b^2)*d^3*e^2*f + 4*a*b*d^2*e*f^2 + 2*a^2*d*
f^3)*x)*sinh(d*x + c)^2 + 3*((a^2 + 2*b^2)*d^3*e^2*f + 4*a*b*d^2*e*f^2 + 2*
a^2*d*f^3)*x + 4*(((a^2 + 2*b^2)*d^3*f^3*x^3 + (a^2 + 2*b^2)*d^3*e^3 + 6*a*
b*d^2*e^2*f + 6*a^2*d*e*f^2 + 3*((a^2 + 2*b^2)*d^3*e*f^2 + 2*a*b*d^2*f^3)*x
^2 + 3*((a^2 + 2*b^2)*d^3*e^2*f + 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x)*cosh(d*
x + c)^3 - ((a^2 + 2*b^2)*d^3*f^3*x^3 + (a^2 + 2*b^2)*d^3*e^3 + 6*a*b*d^2*e
^2*f + 6*a^2*d*e*f^2 + 3*((a^2 + 2*b^2)*d^3*e*f^2 + 2*a*b*d^2*f^3)*x^2 + 3*
((a^2 + 2*b^2)*d^3*e^2*f + 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x)*cosh(d*x + c))
*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - ((a^2 + 2*b^2)*d^3
*e^3 - 3*(2*a*b + (a^2 + 2*b^2)*c)*d^2*e^2*f + 3*(4*a*b*c + (a^2 + 2*b^2)*c
^2 + 2*a^2)*d*e*f^2 + ((a^2 + 2*b^2)*d^3*e^3 - 3*(2*a*b + (a^2 + 2*b^2)*c)*
d^2*e^2*f + 3*(4*a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*d*e*f^2 - (6*a*b*c^2 +
(a^2 + 2*b^2)*c^3 + 6*a^2*c)*f^3)*cosh(d*x + c)^4 + 4*((a^2 + 2*b^2)*d^3*e^
3 - 3*(2*a*b + (a^2 + 2*b^2)*c)*d^2*e^2*f + 3*(4*a*b*c + (a^2 + 2*b^2)*c^2
+ 2*a^2)*d*e*f^2 - (6*a*b*c^2 + (a^2 + 2*b^2)*c^3 + 6*a^2*c)*f^3)*cosh(d*x
+ c)*sinh(d*x + c)^3 + ((a^2 + 2*b^2)*d^3*e^3 - 3*(2*a*b + (a^2 + 2*b^2)*c)
*d^2*e^2*f + 3*(4*a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*d*e*f^2 - (6*a*b*c^2 +
 (a^2 + 2*b^2)*c^3 + 6*a^2*c)*f^3)*sinh(d*x + c)^4 - (6*a*b*c^2 + (a^2 + 2*
b^2)*c^3 + 6*a^2*c)*f^3 - 2*((a^2 + 2*b^2)*d^3*e^3 - 3*(2*a*b + (a^2 + 2*b^
2)*c)*d^2*e^2*f + 3*(4*a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*d*e*f^2 - (6*a*b*
c^2 + (a^2 + 2*b^2)*c^3 + 6*a^2*c)*f^3)*cosh(d*x + c)^2 - 2*((a^2 + 2*b^2)*
d^3*e^3 - 3*(2*a*b + (a^2 + 2*b^2)*c)*d^2*e^2*f + 3*(4*a*b*c + (a^2 + 2*b^2
)*c^2 + 2*a^2)*d*e*f^2 - (6*a*b*c^2 + (a^2 + 2*b^2)*c^3 + 6*a^2*c)*f^3 - 3*
((a^2 + 2*b^2)*d^3*e^3 - 3*(2*a*b + (a^2 + 2*b^2)*c)*d^2*e^2*f + 3*(4*a*b*c
 + (a^2 + 2*b^2)*c^2 + 2*a^2)*d*e*f^2 - (6*a*b*c^2 + (a^2 + 2*b^2)*c^3 + 6*
```

```
a^2*c)*f^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^2 + 2*b^2)*d^3*e^3 -
3*(2*a*b + (a^2 + 2*b^2)*c)*d^2*e^2*f + 3*(4*a*b*c + (a^2 + 2*b^2)*c^2 + 2*
a^2)*d*e*f^2 - (6*a*b*c^2 + (a^2 + 2*b^2)*c^3 + 6*a^2*c)*f^3)*cosh(d*x + c)
^3 - ((a^2 + 2*b^2)*d^3*e^3 - 3*(2*a*b + (a^2 + 2*b^2)*c)*d^2*e^2*f + 3*(4*
a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*d*e*f^2 - (6*a*b*c^2 + (a^2 + 2*b^2)*c^3
 + 6*a^2*c)*f^3)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x
 + c) - 1) - ((a^2 + 2*b^2)*d^3*f^3*x^3 + 3*(a^2 + 2*b^2)*c*d^2*e^2*f - 3*(
4*a*b*c + (a^2 + 2*b^2)*c^2)*d*e*f^2 + ((a^2 + 2*b^2)*d^3*f^3*x^3 + 3*(a^2
+ 2*b^2)*c*d^2*e^2*f - 3*(4*a*b*c + (a^2 + 2*b^2)*c^2)*d*e*f^2 + (6*a*b*c^2
 + (a^2 + 2*b^2)*c^3 + 6*a^2*c)*f^3 + 3*((a^2 + 2*b^2)*d^3*e*f^2 - 2*a*b*d^
2*f^3)*x^2 + 3*((a^2 + 2*b^2)*d^3*e^2*f - 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x)
*cosh(d*x + c)^4 + 4*((a^2 + 2*b^2)*d^3*f^3*x^3 + 3*(a^2 + 2*b^2)*c*d^2*e^2
*f - 3*(4*a*b*c + (a^2 + 2*b^2)*c^2)*d*e*f^2 + (6*a*b*c^2 + (a^2 + 2*b^2)*c
^3 + 6*a^2*c)*f^3 + 3*((a^2 + 2*b^2)*d^3*e*f^2 - 2*a*b*d^2*f^3)*x^2 + 3*((a
^2 + 2*b^2)*d^3*e^2*f - 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x)*cosh(d*x + c)*sin
h(d*x + c)^3 + ((a^2 + 2*b^2)*d^3*f^3*x^3 + 3*(a^2 + 2*b^2)*c*d^2*e^2*f - 3
*(4*a*b*c + (a^2 + 2*b^2)*c^2)*d*e*f^2 + (6*a*b*c^2 + (a^2 + 2*b^2)*c^3 + 6
*a^2*c)*f^3 + 3*((a^2 + 2*b^2)*d^3*e*f^2 - 2*a*b*d^2*f^3)*x^2 + 3*((a^2 + 2
*b^2)*d^3*e^2*f - 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x)*sinh(d*x + c)^4 + (6*a*
b*c^2 + (a^2 + 2*b^2)*c^3 + 6*a^2*c)*f^3 + 3*((a^2 + 2*b^2)*d^3*e*f^2 - 2*a
*b*d^2*f^3)*x^2 - 2*((a^2 + 2*b^2)*d^3*f^3*x^3 + 3*(a^2 + 2*b^2)*c*d^2*e^2*
f - 3*(4*a*b*c + (a^2 + 2*b^2)*c^2)*d*e*f^2 + (6*a*b*c^2 + (a^2 + 2*b^2)*c^
3 + 6*a^2*c)*f^3 + 3*((a^2 + 2*b^2)*d^3*e*f^2 - 2*a*b*d^2*f^3)*x^2 + 3*((a^
2 + 2*b^2)*d^3*e^2*f - 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x)*cosh(d*x + c)^2 -
2*((a^2 + 2*b^2)*d^3*f^3*x^3 + 3*(a^2 + 2*b^2)*c*d^2*e^2*f - 3*(4*a*b*c + (
a^2 + 2*b^2)*c^2)*d*e*f^2 + (6*a*b*c^2 + (a^2 + 2*b^2)*c^3 + 6*a^2*c)*f^3 +
 3*((a^2 + 2*b^2)*d^3*e*f^2 - 2*a*b*d^2*f^3)*x^2 - 3*((a^2 + 2*b^2)*d^3*f^3
*x^3 + 3*(a^2 + 2*b^2)*c*d^2*e^2*f - 3*(4*a*b*c + (a^2 + 2*b^2)*c^2)*d*e*f^
2 + (6*a*b*c^2 + (a^2 + 2*b^2)*c^3 + 6*a^2*c)*f^3 + 3*((a^2 + 2*b^2)*d^3*e*
f^2 - 2*a*b*d^2*f^3)*x^2 + 3*((a^2 + 2*b^2)*d^3*e^2*f - 4*a*b*d^2*e*f^2 + 2
*a^2*d*f^3)*x)*cosh(d*x + c)^2 + 3*((a^2 + 2*b^2)*d^3*e^2*f - 4*a*b*d^2*e*f
^2 + 2*a^2*d*f^3)*x)*sinh(d*x + c)^2 + 3*((a^2 + 2*b^2)*d^3*e^2*f - 4*a*b*d
^2*e*f^2 + 2*a^2*d*f^3)*x + 4*(((a^2 + 2*b^2)*d^3*f^3*x^3 + 3*(a^2 + 2*b^2)
*c*d^2*e^2*f - 3*(4*a*b*c + (a^2 + 2*b^2)*c^2)*d*e*f^2 + (6*a*b*c^2 + (a^2
+ 2*b^2)*c^3 + 6*a^2*c)*f^3 + 3*((a^2 + 2*b^2)*d^3*e*f^2 - 2*a*b*d^2*f^3)*x
^2 + 3*((a^2 + 2*b^2)*d^3*e^2*f - 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x)*cosh(d*
x + c)^3 - ((a^2 + 2*b^2)*d^3*f^3*x^3 + 3*(a^2 + 2*b^2)*c*d^2*e^2*f - 3*(4*
a*b*c + (a^2 + 2*b^2)*c^2)*d*e*f^2 + (6*a*b*c^2 + (a^2 + 2*b^2)*c^3 + 6*a^2
*c)*f^3 + 3*((a^2 + 2*b^2)*d^3*e*f^2 - 2*a*b*d^2*f^3)*x^2 + 3*((a^2 + 2*b^2
)*d^3*e^2*f - 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c
))*log(-cosh(d*x + c) - sinh(d*x + c) + 1) - 6*((a^2 + 2*b^2)*f^3*cosh(d*x
+ c)^4 + 4*(a^2 + 2*b^2)*f^3*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*b^2)*
f^3*sinh(d*x + c)^4 - 2*(a^2 + 2*b^2)*f^3*cosh(d*x + c)^2 + (a^2 + 2*b^2)*f
^3 + 2*(3*(a^2 + 2*b^2)*f^3*cosh(d*x + c)^2 - (a^2 + 2*b^2)*f^3)*sinh(d*x +
 c)^2 + 4*((a^2 + 2*b^2)*f^3*cosh(d*x + c)^3 - (a^2 + 2*b^2)*f^3*cosh(d*x +
 c))*sinh(d*x + c))*polylog(4, cosh(d*x + c) + sinh(d*x + c)) + 6*((a^2 + 2
*b^2)*f^3*cosh(d*x + c)^4 + 4*(a^2 + 2*b^2)*f^3*cosh(d*x + c)*sinh(d*x + c)
^3 + (a^2 + 2*b^2)*f^3*sinh(d*x + c)^4 - 2*(a^2 + 2*b^2)*f^3*cosh(d*x + c)^
2 + (a^2 + 2*b^2)*f^3 + 2*(3*(a^2 + 2*b^2)*f^3*cosh(d*x + c)^2 - (a^2 + 2*b
^2)*f^3)*sinh(d*x + c)^2 + 4*((a^2 + 2*b^2)*f^3*cosh(d*x + c)^3 - (a^2 + 2*
b^2)*f^3*cosh(d*x + c))*sinh(d*x + c))*polylog(4, -cosh(d*x + c) - sinh(d*x
 + c)) + 6*((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2 - 2*a*b*f^3 + ((a
^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2 - 2*a*b*f^3)*cosh(d*x + c)^4 +
4*((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2 - 2*a*b*f^3)*cosh(d*x + c)
*sinh(d*x + c)^3 + ((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2 - 2*a*b*f
^3)*sinh(d*x + c)^4 - 2*((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2 - 2*
a*b*f^3)*cosh(d*x + c)^2 - 2*((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2
 - 2*a*b*f^3 - 3*((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2 - 2*a*b*f^3
)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b
```

```
^2)*d*e*f^2 - 2*a*b*f^3)*cosh(d*x + c)^3 - ((a^2 + 2*b^2)*d*f^3*x + (a^2 +
2*b^2)*d*e*f^2 - 2*a*b*f^3)*cosh(d*x + c))*sinh(d*x + c))*polylog(3, cosh(d
*x + c) + sinh(d*x + c)) - 6*((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2
 + 2*a*b*f^3 + ((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2 + 2*a*b*f^3)*
cosh(d*x + c)^4 + 4*((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2 + 2*a*b*
f^3)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)
*d*e*f^2 + 2*a*b*f^3)*sinh(d*x + c)^4 - 2*((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2
*b^2)*d*e*f^2 + 2*a*b*f^3)*cosh(d*x + c)^2 - 2*((a^2 + 2*b^2)*d*f^3*x + (a^
2 + 2*b^2)*d*e*f^2 + 2*a*b*f^3 - 3*((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d
*e*f^2 + 2*a*b*f^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^2 + 2*b^2)*d*
f^3*x + (a^2 + 2*b^2)*d*e*f^2 + 2*a*b*f^3)*cosh(d*x + c)^3 - ((a^2 + 2*b^2)
*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2 + 2*a*b*f^3)*cosh(d*x + c))*sinh(d*x + c))
*polylog(3, -cosh(d*x + c) - sinh(d*x + c)) + 2*(a^2*d^3*f^3*x^3 + a^2*d^3*
e^3 - 3*a^2*d^2*e^2*f - 8*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^
3*e^2*f*x + 3*a*b*c*d^2*e^2*f - 3*a*b*c^2*d*e*f^2 + a*b*c^3*f^3)*cosh(d*x +
 c)^3 + 3*(a^2*d^3*e*f^2 - a^2*d^2*f^3)*x^2 + 3*(a^2*d^3*f^3*x^3 + a^2*d^3*
e^3 + 3*a^2*d^2*e^2*f + 3*(a^2*d^3*e*f^2 + a^2*d^2*f^3)*x^2 + 3*(a^2*d^3*e^
2*f + 2*a^2*d^2*e*f^2)*x)*cosh(d*x + c)^2 + 3*(a^2*d^3*e^2*f - 2*a^2*d^2*e*
f^2)*x + 4*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*e^2*f*x - a*b
*d^3*e^3 + 6*a*b*c*d^2*e^2*f - 6*a*b*c^2*d*e*f^2 + 2*a*b*c^3*f^3)*cosh(d*x
+ c))*sinh(d*x + c))/(a^3*d^4*cosh(d*x + c)^4 + 4*a^3*d^4*cosh(d*x + c)*sin
h(d*x + c)^3 + a^3*d^4*sinh(d*x + c)^4 - 2*a^3*d^4*cosh(d*x + c)^2 + a^3*d^
4 + 2*(3*a^3*d^4*cosh(d*x + c)^2 - a^3*d^4)*sinh(d*x + c)^2 + 4*(a^3*d^4*co
sh(d*x + c)^3 - a^3*d^4*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*coth(d*x+c)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.482 $\int \frac{(e+fx)^2 \coth^2(c+dx)\mathbf{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=714

$$-\frac{2b^2f(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{a^3d^2}+\frac{2b^2f(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)}{a^3d^2}-\frac{2bf\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}+$$

```
[Out] (b*(e + f*x)^2)/(a^2*d) - ((e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (2*b^2
*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^3*d) - (f^2*ArcTanh[Cosh[c + d*x]])/(
a*d^3) + (b*(e + f*x)^2*Coth[c + d*x])/(a^2*d) - (f*(e + f*x)*Csch[c + d*x]
)/(a*d^2) - ((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x])/(2*a*d) - (b*Sqrt[a^2
 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*d)
 + (b*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])])/(a^3*d) - (2*b*f*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a^2*d^2) - (f*
(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2,
 -E^(c + d*x)])/(a^3*d^2) + (f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) +
 (2*b^2*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^3*d^2) - (2*b*Sqrt[a^2 + b^
2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d
^2) + (2*b*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/(a^3*d^2) - (b*f^2*PolyLog[2, E^(2*(c + d*x))])/(a^2*d^3)
 + (f^2*PolyLog[3, -E^(c + d*x)])/(a*d^3) + (2*b^2*f^2*PolyLog[3, -E^(c + d
*x)])/(a^3*d^3) - (f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) - (2*b^2*f^2*PolyLo
g[3, E^(c + d*x)])/(a^3*d^3) + (2*b*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^3) - (2*b*Sqrt[a^2 + b^2]*f^2*Pol
yLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^3)
```

Rubi [A] time = 1.728, antiderivative size = 714, normalized size of antiderivative = 1., number of steps used = 52, number of rules used = 22, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {5587, 5457, 4182, 2531, 2282, 6589, 4186, 3770, 5569, 3720, 3716, 2190, 2279, 2391, 32, 5585, 5450, 3296, 2638, 5565, 3322, 2264}

$$-\frac{2b^2f(e+fx)\text{PolyLog}\left(2,-e^{c+dx}\right)}{a^3d^2}+\frac{2b^2f(e+fx)\text{PolyLog}\left(2,e^{c+dx}\right)}{a^3d^2}-\frac{2bf\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}+$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (b*(e + f*x)^2)/(a^2*d) - ((e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (2*b^2
*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^3*d) - (f^2*ArcTanh[Cosh[c + d*x]])/(
a*d^3) + (b*(e + f*x)^2*Coth[c + d*x])/(a^2*d) - (f*(e + f*x)*Csch[c + d*x]
)/(a*d^2) - ((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x])/(2*a*d) - (b*Sqrt[a^2
 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*d)
 + (b*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])])/(a^3*d) - (2*b*f*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a^2*d^2) - (f*
(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2,
 -E^(c + d*x)])/(a^3*d^2) + (f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) +
 (2*b^2*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^3*d^2) - (2*b*Sqrt[a^2 + b^
2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d
^2) + (2*b*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/(a^3*d^2) - (b*f^2*PolyLog[2, E^(2*(c + d*x))])/(a^2*d^3)
 + (f^2*PolyLog[3, -E^(c + d*x)])/(a*d^3) + (2*b^2*f^2*PolyLog[3, -E^(c + d
*x)])/(a^3*d^3) - (f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) - (2*b^2*f^2*PolyLo
g[3, E^(c + d*x)])/(a^3*d^3) + (2*b*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^3) - (2*b*Sqrt[a^2 + b^2]*f^2*Pol
```

```
yLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^3)
```

Rule 5587

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Csch[c + d*x]^(p - 1)*Coth[c + d*x]^n)/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 5457

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
```

```
 0] && IGtQ[p, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\coth^2(c+dx)\text{csch}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)^2\text{csch}(c+dx)\,dx}{a} + \frac{\int (e+fx)^2\text{csch}^3(c+dx)\,dx}{a} - \frac{b\int (e+fx)^2\,\text{c}}{a}$$

$$= -\frac{2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{b(e+fx)^2\coth(c+dx)}{a^2d} - \frac{f(e+fx)\text{csch}(c+}{ad^2}$$

$$= \frac{b(e+fx)^2}{a^2d} - \frac{b(e+fx)^3}{3a^2f} - \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{f^2\tanh^{-1}(\cosh(c+}{ad^3}$$

$$= \frac{b(e+fx)^2}{a^2d} - \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} - \frac{f^2\,\text{t}}{}$$

$$= \frac{b(e+fx)^2}{a^2d} - \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} - \frac{f^2\,\text{t}}{}$$

$$= \frac{b(e+fx)^2}{a^2d} - \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} - \frac{f^2\,\text{t}}{}$$

$$= \frac{b(e+fx)^2}{a^2d} - \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} - \frac{f^2\,\text{t}}{}$$

$$= \frac{b(e+fx)^2}{a^2d} - \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} - \frac{f^2\,\text{t}}{}$$

$$= \frac{b(e+fx)^2}{a^2d} - \frac{(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} - \frac{f^2\,\text{t}}{}$$

Mathematica [B] time = 26.4172, size = 1529, normalized size = 2.14

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (8*a*b*d^2*e*E^(2*c)*f*x + 4*a*b*d^2*E^(2*c)*f^2*x^2 + 2*a^2*d^2*e^2*ArcTan
h[E^(c + d*x)] + 4*b^2*d^2*e^2*ArcTanh[E^(c + d*x)] - 2*a^2*d^2*e^2*E^(2*c)
*ArcTanh[E^(c + d*x)] - 4*b^2*d^2*e^2*E^(2*c)*ArcTanh[E^(c + d*x)] + 4*a^2*
f^2*ArcTanh[E^(c + d*x)] - 4*a^2*E^(2*c)*f^2*ArcTanh[E^(c + d*x)] - 2*a^2*d
^2*e*f*x*Log[1 - E^(c + d*x)] - 4*b^2*d^2*e*f*x*Log[1 - E^(c + d*x)] + 2*a^
2*d^2*e*E^(2*c)*f*x*Log[1 - E^(c + d*x)] + 4*b^2*d^2*e*E^(2*c)*f*x*Log[1 -
E^(c + d*x)] - a^2*d^2*f^2*x^2*Log[1 - E^(c + d*x)] - 2*b^2*d^2*f^2*x^2*Log
[1 - E^(c + d*x)] + a^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + 2*b^2*d^
2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + 2*a^2*d^2*e*f*x*Log[1 + E^(c + d*x
)] + 4*b^2*d^2*e*f*x*Log[1 + E^(c + d*x)] - 2*a^2*d^2*e*E^(2*c)*f*x*Log[1 +
 E^(c + d*x)] - 4*b^2*d^2*e*E^(2*c)*f*x*Log[1 + E^(c + d*x)] + a^2*d^2*f^2*
x^2*Log[1 + E^(c + d*x)] + 2*b^2*d^2*f^2*x^2*Log[1 + E^(c + d*x)] - a^2*d^2
*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] - 2*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E
^(c + d*x)] + 4*a*b*d*e*f*Log[1 - E^(2*(c + d*x))] - 4*a*b*d*e*E^(2*c)*f*Lo
g[1 - E^(2*(c + d*x))] + 4*a*b*d*f^2*x*Log[1 - E^(2*(c + d*x))] - 4*a*b*d*E
^(2*c)*f^2*x*Log[1 - E^(2*(c + d*x))] - 2*(a^2 + 2*b^2)*d*(-1 + E^(2*c))*f*
(e + f*x)*PolyLog[2, -E^(c + d*x)] + 2*(a^2 + 2*b^2)*d*(-1 + E^(2*c))*f*(e
+ f*x)*PolyLog[2, E^(c + d*x)] + 2*a*b*f^2*PolyLog[2, E^(2*(c + d*x))] - 2*
```

```
a*b*E^(2*c)*f^2*PolyLog[2, E^(2*(c + d*x))] - 2*a^2*f^2*PolyLog[3, -E^(c +
d*x)] - 4*b^2*f^2*PolyLog[3, -E^(c + d*x)] + 2*a^2*E^(2*c)*f^2*PolyLog[3, -
E^(c + d*x)] + 4*b^2*E^(2*c)*f^2*PolyLog[3, -E^(c + d*x)] + 2*a^2*f^2*PolyL
og[3, E^(c + d*x)] + 4*b^2*f^2*PolyLog[3, E^(c + d*x)] - 2*a^2*E^(2*c)*f^2*
PolyLog[3, E^(c + d*x)] - 4*b^2*E^(2*c)*f^2*PolyLog[3, E^(c + d*x)])/(2*a^3
*d^3*(-1 + E^(2*c))) + (b*Sqrt[a^2 + b^2]*(2*d^2*e^2*ArcTanh[(a + b*E^(c +
d*x))/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*d
^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1
 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, (b*E
^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))] + 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sq
rt[a^2 + b^2])] - 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
]))/(a^3*d^3) + (Csch[c]*Csch[c + d*x]^2*(2*b*d*e^2*Cosh[c] + 4*b*d*e*f*x*C
osh[c] + 2*b*d*f^2*x^2*Cosh[c] + 2*a*e*f*Cosh[d*x] + 2*a*f^2*x*Cosh[d*x] -
2*a*e*f*Cosh[2*c + d*x] - 2*a*f^2*x*Cosh[2*c + d*x] - 2*b*d*e^2*Cosh[c + 2*
d*x] - 4*b*d*e*f*x*Cosh[c + 2*d*x] - 2*b*d*f^2*x^2*Cosh[c + 2*d*x] + a*d*e^
2*Sinh[d*x] + 2*a*d*e*f*x*Sinh[d*x] + a*d*f^2*x^2*Sinh[d*x] - a*d*e^2*Sinh[
2*c + d*x] - 2*a*d*e*f*x*Sinh[2*c + d*x] - a*d*f^2*x^2*Sinh[2*c + d*x]))/(4
*a^2*d^2)
```

Maple [F] time = 0.684, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 (\coth(dx+c))^2 \operatorname{csch}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)

[Out] int((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")

[Out] Exception raised: ValueError
```

Fricas [C] time = 3.62596, size = 17416, normalized size = 24.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -1/2*(4*a*b*d^2*e^2 - 8*a*b*c*d*e*f + 4*a*b*c^2*f^2 - 4*(a*b*d^2*f^2*x^2 +
2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*f^2)*cosh(d*x + c)^4 - 4*(a*b*d^2
*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*f^2)*sinh(d*x + c)^4 +
 2*(a^2*d^2*f^2*x^2 + a^2*d^2*e^2 + 2*a^2*d*e*f + 2*(a^2*d^2*e*f + a^2*d*f^
2)*x)*cosh(d*x + c)^3 + 2*(a^2*d^2*f^2*x^2 + a^2*d^2*e^2 + 2*a^2*d*e*f + 2*
(a^2*d^2*e*f + a^2*d*f^2)*x - 8*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*
c*d*e*f - a*b*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a*b*d^2*f^2*x^2
+ 2*a*b*d^2*e*f*x - a*b*d^2*e^2 + 4*a*b*c*d*e*f - 2*a*b*c^2*f^2)*cosh(d*x +
 c)^2 + 2*(2*a*b*d^2*f^2*x^2 + 4*a*b*d^2*e*f*x - 2*a*b*d^2*e^2 + 8*a*b*c*d*
e*f - 4*a*b*c^2*f^2 - 12*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f
 - a*b*c^2*f^2)*cosh(d*x + c)^2 + 3*(a^2*d^2*f^2*x^2 + a^2*d^2*e^2 + 2*a^2*
d*e*f + 2*(a^2*d^2*e*f + a^2*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*(
b^2*d*f^2*x + b^2*d*e*f + (b^2*d*f^2*x + b^2*d*e*f)*cosh(d*x + c)^4 + 4*(b^
2*d*f^2*x + b^2*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*f^2*x + b^2*d
*e*f)*sinh(d*x + c)^4 - 2*(b^2*d*f^2*x + b^2*d*e*f)*cosh(d*x + c)^2 - 2*(b^
2*d*f^2*x + b^2*d*e*f - 3*(b^2*d*f^2*x + b^2*d*e*f)*cosh(d*x + c)^2)*sinh(d
*x + c)^2 + 4*((b^2*d*f^2*x + b^2*d*e*f)*cosh(d*x + c)^3 - (b^2*d*f^2*x + b
^2*d*e*f)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh
(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
 + b^2)/b^2) - b)/b + 1) - 4*(b^2*d*f^2*x + b^2*d*e*f + (b^2*d*f^2*x + b^2*
d*e*f)*cosh(d*x + c)^4 + 4*(b^2*d*f^2*x + b^2*d*e*f)*cosh(d*x + c)*sinh(d*x
 + c)^3 + (b^2*d*f^2*x + b^2*d*e*f)*sinh(d*x + c)^4 - 2*(b^2*d*f^2*x + b^2*
d*e*f)*cosh(d*x + c)^2 - 2*(b^2*d*f^2*x + b^2*d*e*f - 3*(b^2*d*f^2*x + b^2*
d*e*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d*f^2*x + b^2*d*e*f)*cosh
(d*x + c)^3 - (b^2*d*f^2*x + b^2*d*e*f)*cosh(d*x + c))*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b^2*d^2*e^2 - 2
*b^2*c*d*e*f + b^2*c^2*f^2 + (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*co
sh(d*x + c)^4 + 4*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*sinh(d*x + c
)^4 - 2*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*cosh(d*x + c)^2 - 2*(b^
2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2 - 3*(b^2*d^2*e^2 - 2*b^2*c*d*e*f +
b^2*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d^2*e^2 - 2*b^2*c*d
*e*f + b^2*c^2*f^2)*cosh(d*x + c)^3 - (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^
2*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x
 + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(b^2*d^2*e
^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2 + (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f
^2)*cosh(d*x + c)^4 + 4*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*cosh(d*
x + c)*sinh(d*x + c)^3 + (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*sinh(d
*x + c)^4 - 2*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*cosh(d*x + c)^2 -
 2*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2 - 3*(b^2*d^2*e^2 - 2*b^2*c*d*
e*f + b^2*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d^2*e^2 - 2*b
^2*c*d*e*f + b^2*c^2*f^2)*cosh(d*x + c)^3 - (b^2*d^2*e^2 - 2*b^2*c*d*e*f +
b^2*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*co
sh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(b^2
*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2 + (b^2*d^2*f^2
*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*cosh(d*x + c)^4 + 4*(
b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f -
b^2*c^2*f^2)*sinh(d*x + c)^4 - 2*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2
*c*d*e*f - b^2*c^2*f^2)*cosh(d*x + c)^2 - 2*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*
f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2 - 3*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x +
2*b^2*c*d*e*f - b^2*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d^2
*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*cosh(d*x + c)^3 -
 (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*cosh(d*x
 + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(
d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b
) - 2*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2 + (b
^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*cosh(d*x +
```

```
c)^4 + 4*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*
cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*
c*d*e*f - b^2*c^2*f^2)*sinh(d*x + c)^4 - 2*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f
*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*cosh(d*x + c)^2 - 2*(b^2*d^2*f^2*x^2 + 2*
b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2 - 3*(b^2*d^2*f^2*x^2 + 2*b^2*d^
2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4
*((b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*cosh(d*
x + c)^3 - (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2
)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c)
 + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2) - b)/b) - 4*(b^2*f^2*cosh(d*x + c)^4 + 4*b^2*f^2*cosh(d*x + c)*sinh(d*x
 + c)^3 + b^2*f^2*sinh(d*x + c)^4 - 2*b^2*f^2*cosh(d*x + c)^2 + b^2*f^2 + 2
*(3*b^2*f^2*cosh(d*x + c)^2 - b^2*f^2)*sinh(d*x + c)^2 + 4*(b^2*f^2*cosh(d*
x + c)^3 - b^2*f^2*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*poly
log(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x +
 c))*sqrt((a^2 + b^2)/b^2))/b) + 4*(b^2*f^2*cosh(d*x + c)^4 + 4*b^2*f^2*cos
h(d*x + c)*sinh(d*x + c)^3 + b^2*f^2*sinh(d*x + c)^4 - 2*b^2*f^2*cosh(d*x +
 c)^2 + b^2*f^2 + 2*(3*b^2*f^2*cosh(d*x + c)^2 - b^2*f^2)*sinh(d*x + c)^2 +
 4*(b^2*f^2*cosh(d*x + c)^3 - b^2*f^2*cosh(d*x + c))*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*(a^2*d^2*f^2*x^2 + a^
2*d^2*e^2 - 2*a^2*d*e*f + 2*(a^2*d^2*e*f - a^2*d*f^2)*x)*cosh(d*x + c) - 2*
((a^2 + 2*b^2)*d*f^2*x + ((a^2 + 2*b^2)*d*f^2*x + (a^2 + 2*b^2)*d*e*f - 2*a
*b*f^2)*cosh(d*x + c)^4 + 4*((a^2 + 2*b^2)*d*f^2*x + (a^2 + 2*b^2)*d*e*f -
2*a*b*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 + 2*b^2)*d*f^2*x + (a^2 +
2*b^2)*d*e*f - 2*a*b*f^2)*sinh(d*x + c)^4 + (a^2 + 2*b^2)*d*e*f - 2*a*b*f^2
 - 2*((a^2 + 2*b^2)*d*f^2*x + (a^2 + 2*b^2)*d*e*f - 2*a*b*f^2)*cosh(d*x + c
)^2 - 2*((a^2 + 2*b^2)*d*f^2*x + (a^2 + 2*b^2)*d*e*f - 2*a*b*f^2 - 3*((a^2
+ 2*b^2)*d*f^2*x + (a^2 + 2*b^2)*d*e*f - 2*a*b*f^2)*cosh(d*x + c)^2)*sinh(d
*x + c)^2 + 4*(((a^2 + 2*b^2)*d*f^2*x + (a^2 + 2*b^2)*d*e*f - 2*a*b*f^2)*co
sh(d*x + c)^3 - ((a^2 + 2*b^2)*d*f^2*x + (a^2 + 2*b^2)*d*e*f - 2*a*b*f^2)*c
osh(d*x + c))*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)) + 2*((a^2
 + 2*b^2)*d*f^2*x + ((a^2 + 2*b^2)*d*f^2*x + (a^2 + 2*b^2)*d*e*f + 2*a*b*f^
2)*cosh(d*x + c)^4 + 4*((a^2 + 2*b^2)*d*f^2*x + (a^2 + 2*b^2)*d*e*f + 2*a*b
*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 + 2*b^2)*d*f^2*x + (a^2 + 2*b^2
)*d*e*f + 2*a*b*f^2)*sinh(d*x + c)^4 + (a^2 + 2*b^2)*d*e*f + 2*a*b*f^2 - 2*
((a^2 + 2*b^2)*d*f^2*x + (a^2 + 2*b^2)*d*e*f + 2*a*b*f^2)*cosh(d*x + c)^2 -
 2*((a^2 + 2*b^2)*d*f^2*x + (a^2 + 2*b^2)*d*e*f + 2*a*b*f^2 - 3*((a^2 + 2*b
^2)*d*f^2*x + (a^2 + 2*b^2)*d*e*f + 2*a*b*f^2)*cosh(d*x + c)^2)*sinh(d*x +
c)^2 + 4*(((a^2 + 2*b^2)*d*f^2*x + (a^2 + 2*b^2)*d*e*f + 2*a*b*f^2)*cosh(d*
x + c)^3 - ((a^2 + 2*b^2)*d*f^2*x + (a^2 + 2*b^2)*d*e*f + 2*a*b*f^2)*cosh(d
*x + c))*sinh(d*x + c))*dilog(-cosh(d*x + c) - sinh(d*x + c)) + ((a^2 + 2*b
^2)*d^2*f^2*x^2 + (a^2 + 2*b^2)*d^2*e^2 + 4*a*b*d*e*f + ((a^2 + 2*b^2)*d^2*
f^2*x^2 + (a^2 + 2*b^2)*d^2*e^2 + 4*a*b*d*e*f + 2*a^2*f^2 + 2*((a^2 + 2*b^2
)*d^2*e*f + 2*a*b*d*f^2)*x)*cosh(d*x + c)^4 + 4*((a^2 + 2*b^2)*d^2*f^2*x^2
+ (a^2 + 2*b^2)*d^2*e^2 + 4*a*b*d*e*f + 2*a^2*f^2 + 2*((a^2 + 2*b^2)*d^2*e*
f + 2*a*b*d*f^2)*x)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 + 2*b^2)*d^2*f^2*
x^2 + (a^2 + 2*b^2)*d^2*e^2 + 4*a*b*d*e*f + 2*a^2*f^2 + 2*((a^2 + 2*b^2)*d^
2*e*f + 2*a*b*d*f^2)*x)*sinh(d*x + c)^4 + 2*a^2*f^2 - 2*((a^2 + 2*b^2)*d^2*
f^2*x^2 + (a^2 + 2*b^2)*d^2*e^2 + 4*a*b*d*e*f + 2*a^2*f^2 + 2*((a^2 + 2*b^2
)*d^2*e*f + 2*a*b*d*f^2)*x)*cosh(d*x + c)^2 - 2*((a^2 + 2*b^2)*d^2*f^2*x^2
+ (a^2 + 2*b^2)*d^2*e^2 + 4*a*b*d*e*f + 2*a^2*f^2 - 3*((a^2 + 2*b^2)*d^2*f^
2*x^2 + (a^2 + 2*b^2)*d^2*e^2 + 4*a*b*d*e*f + 2*a^2*f^2 + 2*((a^2 + 2*b^2)*
d^2*e*f + 2*a*b*d*f^2)*x)*cosh(d*x + c)^2 + 2*((a^2 + 2*b^2)*d^2*e*f + 2*a*
b*d*f^2)*x)*sinh(d*x + c)^2 + 2*((a^2 + 2*b^2)*d^2*e*f + 2*a*b*d*f^2)*x + 4
*(((a^2 + 2*b^2)*d^2*f^2*x^2 + (a^2 + 2*b^2)*d^2*e^2 + 4*a*b*d*e*f + 2*a^2*
f^2 + 2*((a^2 + 2*b^2)*d^2*e*f + 2*a*b*d*f^2)*x)*cosh(d*x + c)^3 - ((a^2 +
2*b^2)*d^2*f^2*x^2 + (a^2 + 2*b^2)*d^2*e^2 + 4*a*b*d*e*f + 2*a^2*f^2 + 2*((
a^2 + 2*b^2)*d^2*e*f + 2*a*b*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))*log(co
```

```
sh(d*x + c) + sinh(d*x + c) + 1) - ((a^2 + 2*b^2)*d^2*e^2 + ((a^2 + 2*b^2)*
d^2*e^2 - 2*(2*a*b + (a^2 + 2*b^2)*c)*d*e*f + (4*a*b*c + (a^2 + 2*b^2)*c^2
+ 2*a^2)*f^2)*cosh(d*x + c)^4 + 4*((a^2 + 2*b^2)*d^2*e^2 - 2*(2*a*b + (a^2
+ 2*b^2)*c)*d*e*f + (4*a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*f^2)*cosh(d*x + c
)*sinh(d*x + c)^3 + ((a^2 + 2*b^2)*d^2*e^2 - 2*(2*a*b + (a^2 + 2*b^2)*c)*d*
e*f + (4*a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*f^2)*sinh(d*x + c)^4 - 2*(2*a*b
 + (a^2 + 2*b^2)*c)*d*e*f + (4*a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*f^2 - 2*(
(a^2 + 2*b^2)*d^2*e^2 - 2*(2*a*b + (a^2 + 2*b^2)*c)*d*e*f + (4*a*b*c + (a^2
 + 2*b^2)*c^2 + 2*a^2)*f^2)*cosh(d*x + c)^2 - 2*((a^2 + 2*b^2)*d^2*e^2 - 2*
(2*a*b + (a^2 + 2*b^2)*c)*d*e*f + (4*a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*f^2
 - 3*((a^2 + 2*b^2)*d^2*e^2 - 2*(2*a*b + (a^2 + 2*b^2)*c)*d*e*f + (4*a*b*c
+ (a^2 + 2*b^2)*c^2 + 2*a^2)*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a
^2 + 2*b^2)*d^2*e^2 - 2*(2*a*b + (a^2 + 2*b^2)*c)*d*e*f + (4*a*b*c + (a^2 +
 2*b^2)*c^2 + 2*a^2)*f^2)*cosh(d*x + c)^3 - ((a^2 + 2*b^2)*d^2*e^2 - 2*(2*a
*b + (a^2 + 2*b^2)*c)*d*e*f + (4*a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*f^2)*co
sh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) - ((a^2
+ 2*b^2)*d^2*f^2*x^2 + 2*(a^2 + 2*b^2)*c*d*e*f + ((a^2 + 2*b^2)*d^2*f^2*x^2
 + 2*(a^2 + 2*b^2)*c*d*e*f - (4*a*b*c + (a^2 + 2*b^2)*c^2)*f^2 + 2*((a^2 +
2*b^2)*d^2*e*f - 2*a*b*d*f^2)*x)*cosh(d*x + c)^4 + 4*((a^2 + 2*b^2)*d^2*f^2
*x^2 + 2*(a^2 + 2*b^2)*c*d*e*f - (4*a*b*c + (a^2 + 2*b^2)*c^2)*f^2 + 2*((a^
2 + 2*b^2)*d^2*e*f - 2*a*b*d*f^2)*x)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2
+ 2*b^2)*d^2*f^2*x^2 + 2*(a^2 + 2*b^2)*c*d*e*f - (4*a*b*c + (a^2 + 2*b^2)*c
^2)*f^2 + 2*((a^2 + 2*b^2)*d^2*e*f - 2*a*b*d*f^2)*x)*sinh(d*x + c)^4 - (4*a
*b*c + (a^2 + 2*b^2)*c^2)*f^2 - 2*((a^2 + 2*b^2)*d^2*f^2*x^2 + 2*(a^2 + 2*b
^2)*c*d*e*f - (4*a*b*c + (a^2 + 2*b^2)*c^2)*f^2 + 2*((a^2 + 2*b^2)*d^2*e*f
- 2*a*b*d*f^2)*x)*cosh(d*x + c)^2 - 2*((a^2 + 2*b^2)*d^2*f^2*x^2 + 2*(a^2 +
 2*b^2)*c*d*e*f - (4*a*b*c + (a^2 + 2*b^2)*c^2)*f^2 - 3*((a^2 + 2*b^2)*d^2*
f^2*x^2 + 2*(a^2 + 2*b^2)*c*d*e*f - (4*a*b*c + (a^2 + 2*b^2)*c^2)*f^2 + 2*(
(a^2 + 2*b^2)*d^2*e*f - 2*a*b*d*f^2)*x)*cosh(d*x + c)^2 + 2*((a^2 + 2*b^2)*
d^2*e*f - 2*a*b*d*f^2)*x)*sinh(d*x + c)^2 + 2*((a^2 + 2*b^2)*d^2*e*f - 2*a*
b*d*f^2)*x + 4*(((a^2 + 2*b^2)*d^2*f^2*x^2 + 2*(a^2 + 2*b^2)*c*d*e*f - (4*a
*b*c + (a^2 + 2*b^2)*c^2)*f^2 + 2*((a^2 + 2*b^2)*d^2*e*f - 2*a*b*d*f^2)*x)*
cosh(d*x + c)^3 - ((a^2 + 2*b^2)*d^2*f^2*x^2 + 2*(a^2 + 2*b^2)*c*d*e*f - (4
*a*b*c + (a^2 + 2*b^2)*c^2)*f^2 + 2*((a^2 + 2*b^2)*d^2*e*f - 2*a*b*d*f^2)*x
)*cosh(d*x + c))*sinh(d*x + c))*log(-cosh(d*x + c) - sinh(d*x + c) + 1) + 2
*((a^2 + 2*b^2)*f^2*cosh(d*x + c)^4 + 4*(a^2 + 2*b^2)*f^2*cosh(d*x + c)*sin
h(d*x + c)^3 + (a^2 + 2*b^2)*f^2*sinh(d*x + c)^4 - 2*(a^2 + 2*b^2)*f^2*cosh
(d*x + c)^2 + (a^2 + 2*b^2)*f^2 + 2*(3*(a^2 + 2*b^2)*f^2*cosh(d*x + c)^2 -
(a^2 + 2*b^2)*f^2)*sinh(d*x + c)^2 + 4*((a^2 + 2*b^2)*f^2*cosh(d*x + c)^3 -
 (a^2 + 2*b^2)*f^2*cosh(d*x + c))*sinh(d*x + c))*polylog(3, cosh(d*x + c) +
 sinh(d*x + c)) - 2*((a^2 + 2*b^2)*f^2*cosh(d*x + c)^4 + 4*(a^2 + 2*b^2)*f^
2*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*b^2)*f^2*sinh(d*x + c)^4 - 2*(a^
2 + 2*b^2)*f^2*cosh(d*x + c)^2 + (a^2 + 2*b^2)*f^2 + 2*(3*(a^2 + 2*b^2)*f^2
*cosh(d*x + c)^2 - (a^2 + 2*b^2)*f^2)*sinh(d*x + c)^2 + 4*((a^2 + 2*b^2)*f^
2*cosh(d*x + c)^3 - (a^2 + 2*b^2)*f^2*cosh(d*x + c))*sinh(d*x + c))*polylog
(3, -cosh(d*x + c) - sinh(d*x + c)) + 2*(a^2*d^2*f^2*x^2 + a^2*d^2*e^2 - 2*
a^2*d*e*f - 8*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*
f^2)*cosh(d*x + c)^3 + 3*(a^2*d^2*f^2*x^2 + a^2*d^2*e^2 + 2*a^2*d*e*f + 2*(
a^2*d^2*e*f + a^2*d*f^2)*x)*cosh(d*x + c)^2 + 2*(a^2*d^2*e*f - a^2*d*f^2)*x
 + 4*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x - a*b*d^2*e^2 + 4*a*b*c*d*e*f - 2*a
*b*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d^3*cosh(d*x + c)^4 + 4*a^3*
d^3*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d^3*sinh(d*x + c)^4 - 2*a^3*d^3*cos
h(d*x + c)^2 + a^3*d^3 + 2*(3*a^3*d^3*cosh(d*x + c)^2 - a^3*d^3)*sinh(d*x +
 c)^2 + 4*(a^3*d^3*cosh(d*x + c)^3 - a^3*d^3*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*coth(d*x+c)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.483 $\int \frac{(e+fx)\coth^2(c+dx)\mathbf{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=413

$$-\frac{b^2 f \mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{a^3d^2}+\frac{b^2 f \mathrm{PolyLog}\left(2,e^{c+dx}\right)}{a^3d^2}-\frac{bf\sqrt{a^2+b^2}\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}+\frac{bf\sqrt{a^2+b^2}\mathrm{PolyLog}\left(2,-\right.}{a^3d^2}$$

```
[Out] -(((e + f*x)*ArcTanh[E^(c + d*x)])/(a*d)) - (2*b^2*(e + f*x)*ArcTanh[E^(c +
 d*x)])/(a^3*d) + (b*(e + f*x)*Coth[c + d*x])/(a^2*d) - (f*Csch[c + d*x])/(
2*a*d^2) - ((e + f*x)*Coth[c + d*x]*Csch[c + d*x])/(2*a*d) - (b*Sqrt[a^2 +
b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*d) + (b
*Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/
(a^3*d) - (b*f*Log[Sinh[c + d*x]])/(a^2*d^2) - (f*PolyLog[2, -E^(c + d*x)])
/(2*a*d^2) - (b^2*f*PolyLog[2, -E^(c + d*x)])/(a^3*d^2) + (f*PolyLog[2, E^(
c + d*x)])/(2*a*d^2) + (b^2*f*PolyLog[2, E^(c + d*x)])/(a^3*d^2) - (b*Sqrt[
a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2
) + (b*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)])/(a^3*d^2)
```

Rubi [A] time = 0.928764, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 17, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.531, Rules used = {5587, 5457, 4182, 2279, 2391, 4185, 5569, 3720, 3475, 5585, 5450, 3296, 2637, 5565, 3322, 2264, 2190}

$$-\frac{b^2 f \mathrm{PolyLog}\left(2,-e^{c+dx}\right)}{a^3d^2}+\frac{b^2 f \mathrm{PolyLog}\left(2,e^{c+dx}\right)}{a^3d^2}-\frac{bf\sqrt{a^2+b^2}\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}+\frac{bf\sqrt{a^2+b^2}\mathrm{PolyLog}\left(2,-\right.}{a^3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(((e + f*x)*ArcTanh[E^(c + d*x)])/(a*d)) - (2*b^2*(e + f*x)*ArcTanh[E^(c +
 d*x)])/(a^3*d) + (b*(e + f*x)*Coth[c + d*x])/(a^2*d) - (f*Csch[c + d*x])/(
2*a*d^2) - ((e + f*x)*Coth[c + d*x]*Csch[c + d*x])/(2*a*d) - (b*Sqrt[a^2 +
b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*d) + (b
*Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/
(a^3*d) - (b*f*Log[Sinh[c + d*x]])/(a^2*d^2) - (f*PolyLog[2, -E^(c + d*x)])
/(2*a*d^2) - (b^2*f*PolyLog[2, -E^(c + d*x)])/(a^3*d^2) + (f*PolyLog[2, E^(
c + d*x)])/(2*a*d^2) + (b^2*f*PolyLog[2, E^(c + d*x)])/(a^3*d^2) - (b*Sqrt[
a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2
) + (b*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)])/(a^3*d^2)
```

Rule 5587

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Csch[c + d*x]^(p - 1)*Coth[c + d*x]^n)/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 5457

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
```

```
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
```

```
 0] && IGtQ[p, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\coth^2(c+dx)\operatorname{csch}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= \frac{\int (e+fx)\operatorname{csch}(c+dx)\,dx}{a} + \frac{\int (e+fx)\operatorname{csch}^3(c+dx)\,dx}{a} - \frac{b\int (e+fx)\operatorname{co}}{a}$$

$$= -\frac{2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{b(e+fx)\coth(c+dx)}{a^2d} - \frac{f\operatorname{csch}(c+dx)}{2ad^2} - ($$

$$= -\frac{bex}{a^2} - \frac{bfx^2}{2a^2} - \frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} + \frac{b(e+fx)\coth(c+dx)}{a^2d} - \frac{f\operatorname{csc}}{}$$

$$= -\frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} + \frac{b(e+fx)\coth(}{a^2d}$$

$$= -\frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} + \frac{b(e+fx)\coth(}{a^2d}$$

$$= -\frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} + \frac{b(e+fx)\coth(}{a^2d}$$

$$= -\frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} + \frac{b(e+fx)\coth(}{a^2d}$$

$$= -\frac{(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{2b^2(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^3d} + \frac{b(e+fx)\coth(}{a^2d}$$

Mathematica [C] time = 8.17093, size = 734, normalized size = 1.78

$$\frac{b\sqrt{a^2+b^2}\left(-f\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+f\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-f(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+\right.\right.}{a^3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] ((2*b*d*e*Cosh[(c + d*x)/2] - a*f*Cosh[(c + d*x)/2] - 2*b*c*f*Cosh[(c + d*x
)/2] + 2*b*f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(4*a^2*d^2) +
((-(d*e) + c*f - f*(c + d*x))*Csch[(c + d*x)/2]^2)/(8*a*d^2) - (b*f*Log[Sin
h[c + d*x]])/(a^2*d^2) + (e*Log[Tanh[(c + d*x)/2]])/(2*a*d) + (b^2*e*Log[Ta
nh[(c + d*x)/2]])/(a^3*d) - (c*f*Log[Tanh[(c + d*x)/2]])/(2*a*d^2) - (b^2*c
*f*Log[Tanh[(c + d*x)/2]])/(a^3*d^2) - ((I/2)*f*(I*(c + d*x)*(Log[1 - E^(-c
 - d*x)] - Log[1 + E^(-c - d*x)]) + I*(PolyLog[2, -E^(-c - d*x)] - PolyLog[
2, E^(-c - d*x)])))/(a*d^2) - (I*b^2*f*(I*(c + d*x)*(Log[1 - E^(-c - d*x)]
- Log[1 + E^(-c - d*x)]) + I*(PolyLog[2, -E^(-c - d*x)] - PolyLog[2, E^(-c
- d*x)])))/(a^3*d^2) + (b*Sqrt[a^2 + b^2]*(2*d*e*ArcTanh[(a + b*E^(c + d*x)
)/Sqrt[a^2 + b^2]] - 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - f
*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log
[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - f*PolyLog[2, (b*E^(c + d*x))/
(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2]))]))/(a^3*d^2) + ((-(d*e) + c*f - f*(c + d*x))*Sech[(c + d*x)/2]^2)/(8*a
*d^2) + (Sech[(c + d*x)/2]*(2*b*d*e*Sinh[(c + d*x)/2] + a*f*Sinh[(c + d*x)/
2] - 2*b*c*f*Sinh[(c + d*x)/2] + 2*b*f*(c + d*x)*Sinh[(c + d*x)/2]))/(4*a^2
*d^2)
```

Maple [B] time = 0.233, size = 1284, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/a^3/d^2*b^2*f*c*ln(exp(d*x+c)-1)-1/a^3/d^2*b^3*f/(a^2+b^2)^(1/2)*dilog((
-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/a^3/d^2*b^3*f/(a^2
+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/a
^3/d*b^2*f*ln(exp(d*x+c)+1)*x+2/a^3/d*b^3*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*
b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/a^3/d*b^3*f/(a^2+b^2)^(1/2)*ln((b*exp(
d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/a^3/d^2*b^3*f/(a^2+b^2)^
(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/a^3/d^
2*b^3*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1
/2)))*c-2/a^3/d^2*b^3*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/
(a^2+b^2)^(1/2))-1/a^3/d*b^3*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^
(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+2/d*e*b/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b
*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d^2*f*b/a/(a^2+b^2)^(1/2)*dilog((-b*exp
(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/d^2*f*b/a/(a^2+b^2)^(1/2
)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-2/d^2*f*c*b/a
/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d*f*b/
a/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))
)*x-1/d^2*f*b/a/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a
^2+b^2)^(1/2)))*c+1/d*f*b/a/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2
)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2*f*b/a/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a
^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/a^3/d^2*b^2*f*dilog(exp(d*x+c))-1
/a^3/d^2*b^2*f*dilog(exp(d*x+c)+1)+2/a^2/d^2*b*f*ln(exp(d*x+c))-1/a^2/d^2*b
*f*ln(exp(d*x+c)-1)-1/a^2/d^2*b*f*ln(exp(d*x+c)+1)+1/a^3/d*b^2*e*ln(exp(d*x
+c)-1)-1/a^3/d*b^2*e*ln(exp(d*x+c)+1)-1/2/d^2*f/a*dilog(exp(d*x+c))-1/2/d^2
*f/a*dilog(exp(d*x+c)+1)+1/2/d/a*e*ln(exp(d*x+c)-1)-1/2/d/a*e*ln(exp(d*x+c)
+1)-1/2/d/a*ln(exp(d*x+c)+1)*f*x-1/2/d^2/a*f*c*ln(exp(d*x+c)-1)-(a*d*f*x*ex
p(3*d*x+3*c)+a*d*e*exp(3*d*x+3*c)-2*b*d*f*x*exp(2*d*x+2*c)+a*d*f*x*exp(d*x+
c)+a*f*exp(3*d*x+3*c)-2*b*d*e*exp(2*d*x+2*c)+a*d*e*exp(d*x+c)+2*b*d*f*x-a*f
*exp(d*x+c)+2*b*d*e)/d^2/a^2/(exp(2*d*x+2*c)-1)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.71947, size = 8606, normalized size = 20.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")

[Out] 1/2*(4*(a*b*d*f*x + a*b*c*f)*cosh(d*x + c)^4 + 4*(a*b*d*f*x + a*b*c*f)*sinh
(d*x + c)^4 - 4*a*b*d*e + 4*a*b*c*f - 2*(a^2*d*f*x + a^2*d*e + a^2*f)*cosh(
d*x + c)^3 - 2*(a^2*d*f*x + a^2*d*e + a^2*f - 8*(a*b*d*f*x + a*b*c*f)*cosh(
d*x + c))*sinh(d*x + c)^3 - 4*(a*b*d*f*x - a*b*d*e + 2*a*b*c*f)*cosh(d*x +
c)^2 - 2*(2*a*b*d*f*x - 2*a*b*d*e + 4*a*b*c*f - 12*(a*b*d*f*x + a*b*c*f)*co
sh(d*x + c)^2 + 3*(a^2*d*f*x + a^2*d*e + a^2*f)*cosh(d*x + c))*sinh(d*x + c
)^2 - 2*(b^2*f*cosh(d*x + c)^4 + 4*b^2*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^
2*f*sinh(d*x + c)^4 - 2*b^2*f*cosh(d*x + c)^2 + b^2*f + 2*(3*b^2*f*cosh(d*x
 + c)^2 - b^2*f)*sinh(d*x + c)^2 + 4*(b^2*f*cosh(d*x + c)^3 - b^2*f*cosh(d*
x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sin
h(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)
/b + 1) + 2*(b^2*f*cosh(d*x + c)^4 + 4*b^2*f*cosh(d*x + c)*sinh(d*x + c)^3
+ b^2*f*sinh(d*x + c)^4 - 2*b^2*f*cosh(d*x + c)^2 + b^2*f + 2*(3*b^2*f*cosh
(d*x + c)^2 - b^2*f)*sinh(d*x + c)^2 + 4*(b^2*f*cosh(d*x + c)^3 - b^2*f*cos
h(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a
*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b + 1) + 2*((b^2*d*e - b^2*c*f)*cosh(d*x + c)^4 + 4*(b^2*d*e - b^2*c*f
)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*e - b^2*c*f)*sinh(d*x + c)^4 + b^2
*d*e - b^2*c*f - 2*(b^2*d*e - b^2*c*f)*cosh(d*x + c)^2 - 2*(b^2*d*e - b^2*c
f - 3(b^2*d*e - b^2*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d*e -
 b^2*c*f)*cosh(d*x + c)^3 - (b^2*d*e - b^2*c*f)*cosh(d*x + c))*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sq
rt((a^2 + b^2)/b^2) + 2*a) - 2*((b^2*d*e - b^2*c*f)*cosh(d*x + c)^4 + 4*(b^
2*d*e - b^2*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*e - b^2*c*f)*sinh(d
*x + c)^4 + b^2*d*e - b^2*c*f - 2*(b^2*d*e - b^2*c*f)*cosh(d*x + c)^2 - 2*(
b^2*d*e - b^2*c*f - 3*(b^2*d*e - b^2*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2
 + 4*((b^2*d*e - b^2*c*f)*cosh(d*x + c)^3 - (b^2*d*e - b^2*c*f)*cosh(d*x + c
))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*
x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(b^2*d*f*x + (b^2*d*f*x + b^2
*c*f)*cosh(d*x + c)^4 + 4*(b^2*d*f*x + b^2*c*f)*cosh(d*x + c)*sinh(d*x + c)
^3 + (b^2*d*f*x + b^2*c*f)*sinh(d*x + c)^4 + b^2*c*f - 2*(b^2*d*f*x + b^2*c
*f)*cosh(d*x + c)^2 - 2*(b^2*d*f*x + b^2*c*f - 3*(b^2*d*f*x + b^2*c*f)*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^2*d*f*x + b^2*c*f)*cosh(d*x + c)^3 - (
b^2*d*f*x + b^2*c*f)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*lo
g(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))
sqrt((a^2 + b^2)/b^2) - b)/b) + 2(b^2*d*f*x + (b^2*d*f*x + b^2*c*f)*cosh(
d*x + c)^4 + 4*(b^2*d*f*x + b^2*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d
*f*x + b^2*c*f)*sinh(d*x + c)^4 + b^2*c*f - 2*(b^2*d*f*x + b^2*c*f)*cosh(d*
x + c)^2 - 2*(b^2*d*f*x + b^2*c*f - 3*(b^2*d*f*x + b^2*c*f)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 4*((b^2*d*f*x + b^2*c*f)*cosh(d*x + c)^3 - (b^2*d*f*x +
 b^2*c*f)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(
d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2) - b)/b) - 2*(a^2*d*f*x + a^2*d*e - a^2*f)*cosh(d*x + c) + ((a^2
 + 2*b^2)*f*cosh(d*x + c)^4 + 4*(a^2 + 2*b^2)*f*cosh(d*x + c)*sinh(d*x + c)
^3 + (a^2 + 2*b^2)*f*sinh(d*x + c)^4 - 2*(a^2 + 2*b^2)*f*cosh(d*x + c)^2 +
2*(3*(a^2 + 2*b^2)*f*cosh(d*x + c)^2 - (a^2 + 2*b^2)*f)*sinh(d*x + c)^2 + (
a^2 + 2*b^2)*f + 4*((a^2 + 2*b^2)*f*cosh(d*x + c)^3 - (a^2 + 2*b^2)*f*cosh(
d*x + c))*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)) - ((a^2 + 2*b
^2)*f*cosh(d*x + c)^4 + 4*(a^2 + 2*b^2)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (
a^2 + 2*b^2)*f*sinh(d*x + c)^4 - 2*(a^2 + 2*b^2)*f*cosh(d*x + c)^2 + 2*(3*(
a^2 + 2*b^2)*f*cosh(d*x + c)^2 - (a^2 + 2*b^2)*f)*sinh(d*x + c)^2 + (a^2 +
2*b^2)*f + 4*((a^2 + 2*b^2)*f*cosh(d*x + c)^3 - (a^2 + 2*b^2)*f*cosh(d*x +
c))*sinh(d*x + c))*dilog(-cosh(d*x + c) - sinh(d*x + c)) - (((a^2 + 2*b^2)*
d*f*x + (a^2 + 2*b^2)*d*e + 2*a*b*f)*cosh(d*x + c)^4 + 4*((a^2 + 2*b^2)*d*f
*x + (a^2 + 2*b^2)*d*e + 2*a*b*f)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 + 2
*b^2)*d*f*x + (a^2 + 2*b^2)*d*e + 2*a*b*f)*sinh(d*x + c)^4 + (a^2 + 2*b^2)*

```
d*f*x + (a^2 + 2*b^2)*d*e + 2*a*b*f - 2*((a^2 + 2*b^2)*d*f*x + (a^2 + 2*b^2
)*d*e + 2*a*b*f)*cosh(d*x + c)^2 - 2*((a^2 + 2*b^2)*d*f*x + (a^2 + 2*b^2)*d
*e + 2*a*b*f - 3*((a^2 + 2*b^2)*d*f*x + (a^2 + 2*b^2)*d*e + 2*a*b*f)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^2 + 2*b^2)*d*f*x + (a^2 + 2*b^2)*d*e +
2*a*b*f)*cosh(d*x + c)^3 - ((a^2 + 2*b^2)*d*f*x + (a^2 + 2*b^2)*d*e + 2*a*b
*f)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) +
(((a^2 + 2*b^2)*d*e - (2*a*b + (a^2 + 2*b^2)*c)*f)*cosh(d*x + c)^4 + 4*((a^
2 + 2*b^2)*d*e - (2*a*b + (a^2 + 2*b^2)*c)*f)*cosh(d*x + c)*sinh(d*x + c)^3
 + ((a^2 + 2*b^2)*d*e - (2*a*b + (a^2 + 2*b^2)*c)*f)*sinh(d*x + c)^4 + (a^2
 + 2*b^2)*d*e - 2*((a^2 + 2*b^2)*d*e - (2*a*b + (a^2 + 2*b^2)*c)*f)*cosh(d*
x + c)^2 - 2*((a^2 + 2*b^2)*d*e - 3*((a^2 + 2*b^2)*d*e - (2*a*b + (a^2 + 2*
b^2)*c)*f)*cosh(d*x + c)^2 - (2*a*b + (a^2 + 2*b^2)*c)*f)*sinh(d*x + c)^2 -
 (2*a*b + (a^2 + 2*b^2)*c)*f + 4*(((a^2 + 2*b^2)*d*e - (2*a*b + (a^2 + 2*b^
2)*c)*f)*cosh(d*x + c)^3 - ((a^2 + 2*b^2)*d*e - (2*a*b + (a^2 + 2*b^2)*c)*f
)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + ((
(a^2 + 2*b^2)*d*f*x + (a^2 + 2*b^2)*c*f)*cosh(d*x + c)^4 + 4*((a^2 + 2*b^2)
*d*f*x + (a^2 + 2*b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 + 2*b^2)*
d*f*x + (a^2 + 2*b^2)*c*f)*sinh(d*x + c)^4 + (a^2 + 2*b^2)*d*f*x + (a^2 + 2
*b^2)*c*f - 2*((a^2 + 2*b^2)*d*f*x + (a^2 + 2*b^2)*c*f)*cosh(d*x + c)^2 - 2
*((a^2 + 2*b^2)*d*f*x + (a^2 + 2*b^2)*c*f - 3*((a^2 + 2*b^2)*d*f*x + (a^2 +
 2*b^2)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^2 + 2*b^2)*d*f*x + (
a^2 + 2*b^2)*c*f)*cosh(d*x + c)^3 - ((a^2 + 2*b^2)*d*f*x + (a^2 + 2*b^2)*c*
f)*cosh(d*x + c))*sinh(d*x + c))*log(-cosh(d*x + c) - sinh(d*x + c) + 1) -
2*(a^2*d*f*x + a^2*d*e - 8*(a*b*d*f*x + a*b*c*f)*cosh(d*x + c)^3 - a^2*f +
3*(a^2*d*f*x + a^2*d*e + a^2*f)*cosh(d*x + c)^2 + 4*(a*b*d*f*x - a*b*d*e +
2*a*b*c*f)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d^2*cosh(d*x + c)^4 + 4*a^3*d
^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d^2*sinh(d*x + c)^4 - 2*a^3*d^2*cosh
(d*x + c)^2 + a^3*d^2 + 2*(3*a^3*d^2*cosh(d*x + c)^2 - a^3*d^2)*sinh(d*x +
c)^2 + 4*(a^3*d^2*cosh(d*x + c)^3 - a^3*d^2*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")

[Out] Timed out
```

3.484 $\int \frac{\coth^2(c+dx)\mathrm{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=111

$$\frac{2b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3d}-\frac{\left(a^2+2b^2\right)\tanh^{-1}(\cosh(c+dx))}{2a^3d}+\frac{b\coth(c+dx)}{a^2d}-\frac{\coth(c+dx)\mathrm{csch}(c+a}{2ad}$$

```
[Out] -((a^2 + 2*b^2)*ArcTanh[Cosh[c + d*x]])/(2*a^3*d) + (2*b*Sqrt[a^2 + b^2]*Ar
cTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^3*d) + (b*Coth[c + d*x
])/(a^2*d) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.569431, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2889, 3056, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3d}-\frac{\left(a^2+2b^2\right)\tanh^{-1}(\cosh(c+dx))}{2a^3d}+\frac{b\coth(c+dx)}{a^2d}-\frac{\coth(c+dx)\mathrm{csch}(c+a}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a^2 + 2*b^2)*ArcTanh[Cosh[c + d*x]])/(2*a^3*d) + (2*b*Sqrt[a^2 + b^2]*Ar
cTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^3*d) + (b*Coth[c + d*x
])/(a^2*d) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d)
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\coth^2(c+dx)\text{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx = \int \frac{\text{csch}^3(c+dx)\left(1+\sinh^2(c+dx)\right)}{a+b\sinh(c+dx)}\,dx$$

$$= -\frac{\coth(c+dx)\text{csch}(c+dx)}{2ad} + \frac{i\int \frac{\text{csch}^2(c+dx)\left(2ib-ia\sinh(c+dx)+ib\sinh^2(c+dx)\right)}{a+b\sinh(c+dx)}\,dx}{2a}$$

$$= \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\text{csch}(c+dx)}{2ad} - \frac{\int \frac{\text{csch}(c+dx)\left(-a^2-2b^2+ab\sinh(c+dx)\right)}{a+b\sinh(c+dx)}\,dx}{2a^2}$$

$$= \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\text{csch}(c+dx)}{2ad} - \frac{\left(b\left(a^2+b^2\right)\right)\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a^3} + \ldots$$

$$= -\frac{\left(a^2+2b^2\right)\tanh^{-1}(\cosh(c+dx))}{2a^3d} + \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\text{csch}(c+dx)}{2ad}$$

$$= -\frac{\left(a^2+2b^2\right)\tanh^{-1}(\cosh(c+dx))}{2a^3d} + \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\text{csch}(c+dx)}{2ad}$$

$$= -\frac{\left(a^2+2b^2\right)\tanh^{-1}(\cosh(c+dx))}{2a^3d} + \frac{2b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3d} + \frac{b\,\mathrm{c}\ldots}{}$$

Mathematica [A] time = 1.28068, size = 145, normalized size = 1.31

$$\frac{4\left(a^2+2b^2\right)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+16b\sqrt{-a^2-b^2}\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)-a^2\text{csch}^2\left(\frac{1}{2}(c+dx)\right)-a^2\text{sech}^2\left(\frac{1}{2}(c+\ldots\right.}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (16*b*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] +
 4*a*b*Coth[(c + d*x)/2] - a^2*Csch[(c + d*x)/2]^2 + 4*(a^2 + 2*b^2)*Log[Ta
nh[(c + d*x)/2]] - a^2*Sech[(c + d*x)/2]^2 + 4*a*b*Tanh[(c + d*x)/2])/(8*a^
3*d)
```

Maple [A] time = 0.003, size = 162, normalized size = 1.5

$$\frac{1}{8da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+\frac{b}{2da^2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{8da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-2}+\frac{1}{2da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{b^2}{da^3}\ln\left(\tan\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/8/d/a*tanh(1/2*d*x+1/2*c)^2+1/2/d/a^2*tanh(1/2*d*x+1/2*c)*b-1/8/d/a/tanh(
1/2*d*x+1/2*c)^2+1/2/d/a*ln(tanh(1/2*d*x+1/2*c))+1/d/a^3*ln(tanh(1/2*d*x+1/
2*c))*b^2+1/2/d*b/a^2/tanh(1/2*d*x+1/2*c)-2/d*b*(a^2+b^2)^(1/2)/a^3*arctanh
(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.46252, size = 2272, normalized size = 20.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/2*(2*a^2*cosh(d*x + c)^3 + 2*a^2*sinh(d*x + c)^3 - 4*a*b*cosh(d*x + c)^2
 + 2*a^2*cosh(d*x + c) + 2*(3*a^2*cosh(d*x + c) - 2*a*b)*sinh(d*x + c)^2 -
2*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^
4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 4*(
b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(a^2 + b^2)*log
((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 +
 b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cos
h(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 +
 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 4*a*b +
((a^2 + 2*b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*b^2)*cosh(d*x + c)*sinh(d*x + c
)^3 + (a^2 + 2*b^2)*sinh(d*x + c)^4 - 2*(a^2 + 2*b^2)*cosh(d*x + c)^2 + 2*(
3*(a^2 + 2*b^2)*cosh(d*x + c)^2 - a^2 - 2*b^2)*sinh(d*x + c)^2 + a^2 + 2*b^
2 + 4*((a^2 + 2*b^2)*cosh(d*x + c)^3 - (a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*
x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - ((a^2 + 2*b^2)*cosh(d*x +
c)^4 + 4*(a^2 + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*b^2)*sinh(d
*x + c)^4 - 2*(a^2 + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*b^2)*cosh(d*x +
 c)^2 - a^2 - 2*b^2)*sinh(d*x + c)^2 + a^2 + 2*b^2 + 4*((a^2 + 2*b^2)*cosh(
d*x + c)^3 - (a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c)
+ sinh(d*x + c) - 1) + 2*(3*a^2*cosh(d*x + c)^2 - 4*a*b*cosh(d*x + c) + a^2
)*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)*sinh(d*x +
c)^3 + a^3*d*sinh(d*x + c)^4 - 2*a^3*d*cosh(d*x + c)^2 + a^3*d + 2*(3*a^3*d
*cosh(d*x + c)^2 - a^3*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3 - a^3*
d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)**2*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

Giac [B] time = 1.99255, size = 298, normalized size = 2.68

$$-\frac{\frac{\left(a^2e^c+2\,b^2e^c\right)e^{(-c)}\log\left(e^{(dx+c)}+1\right)}{a^3}-\frac{\left(a^2e^c+2\,b^2e^c\right)e^{(-c)}\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^3}+\frac{2\left(a^2be^c+b^3e^c\right)e^{(-c)}\log\left(\frac{\left|2\,be^{(dx+2\,c)}+2\,ae^c-2\,\sqrt{a^2+b^2}e^c\right|}{\left|2\,be^{(dx+2\,c)}+2\,ae^c+2\,\sqrt{a^2+b^2}e^c\right|}\right)}{\sqrt{a^2+b^2}a^3}+\frac{2\left(ae^{(3\,dx+3\,c)}-2\,be^{(2\,d\cdots}\right.}{a^2\left(e^{(2\,dx+\cdots}\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*((a^2*e^c + 2*b^2*e^c)*e^(-c)*log(e^(d*x + c) + 1)/a^3 - (a^2*e^c + 2*
b^2*e^c)*e^(-c)*log(abs(e^(d*x + c) - 1))/a^3 + 2*(a^2*b*e^c + b^3*e^c)*e^(
-c)*log(abs(2*b*e^(d*x + 2*c) + 2*a*e^c - 2*sqrt(a^2 + b^2)*e^c)/abs(2*b*e^
(d*x + 2*c) + 2*a*e^c + 2*sqrt(a^2 + b^2)*e^c))/(sqrt(a^2 + b^2)*a^3) + 2*(
a*e^(3*d*x + 3*c) - 2*b*e^(2*d*x + 2*c) + a*e^(d*x + c) + 2*b)/(a^2*(e^(2*d
*x + 2*c) - 1)^2))/d
```

3.485 $\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Coth[c + d*x]^2*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [A] time = 0.0877947, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Coth[c + d*x]^2*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Coth[c + d*x]^2*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.008, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Coth[c + d*x]^2*Csch[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] $Aborted
```

Maple [A] time = 2.181, size = 0, normalized size = 0.

$$\int \frac{(\coth(dx+c))^2\operatorname{csch}(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -2*(a^2*b*e^c + b^3*e^c)*integrate(-e^(d*x)/(a^3*b*f*x + a^3*b*e - (a^3*b*f
*x*e^(2*c) + a^3*b*e*e^(2*c))*e^(2*d*x) - 2*(a^4*f*x*e^c + a^4*e*e^c)*e^(d*
x)), x) - (2*b*d*f*x + 2*b*d*e + (a*d*f*x*e^(3*c) + (d*e - f)*a*e^(3*c))*e^
(3*d*x) - 2*(b*d*f*x*e^(2*c) + b*d*e*e^(2*c))*e^(2*d*x) + (a*d*f*x*e^c + (d
*e + f)*a*e^c)*e^(d*x))/(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + a^2*d^2*e^2 +
(a^2*d^2*f^2*x^2*e^(4*c) + 2*a^2*d^2*e*f*x*e^(4*c) + a^2*d^2*e^2*e^(4*c))*e
^(4*d*x) - 2*(a^2*d^2*f^2*x^2*e^(2*c) + 2*a^2*d^2*e*f*x*e^(2*c) + a^2*d^2*e
^2*e^(2*c))*e^(2*d*x)) + 2*integrate(-1/4*(2*b^2*d^2*e^2 + 2*a*b*d*e*f + (d
^2*e^2 + 2*f^2)*a^2 + (a^2*d^2*f^2 + 2*b^2*d^2*f^2)*x^2 + 2*(a^2*d^2*e*f +
2*b^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^
3*d^2*e^2*f*x + a^3*d^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^
c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) + 2*integrate(1/4
*(2*b^2*d^2*e^2 - 2*a*b*d*e*f + (d^2*e^2 + 2*f^2)*a^2 + (a^2*d^2*f^2 + 2*b^
2*d^2*f^2)*x^2 + 2*(a^2*d^2*e*f + 2*b^2*d^2*e*f - a*b*d*f^2)*x)/(a^3*d^2*f^
3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 + (a^3*d^2*f^
3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e
^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\coth(dx+c)^2\operatorname{csch}(dx+c)}{afx+ae+\left(bfx+be\right)\sinh(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(coth(d*x + c)^2*csch(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.486 $\int \frac{(e+fx)^3 \coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=972

result too large to display

```
[Out] (-3*f*(e + f*x)^2)/(2*a*d^2) + (e + f*x)^3/(2*a*d) - (e + f*x)^4/(4*a*f) -
(b^2*(e + f*x)^4)/(4*a^3*f) + ((a^2 + b^2)*(e + f*x)^4)/(4*a^3*f) + (6*b*f*
(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d^2) - (3*f*(e + f*x)^2*Coth[c + d*x
])/(2*a*d^2) - ((e + f*x)^3*Coth[c + d*x]^2)/(2*a*d) + (b*(e + f*x)^3*Csch[
c + d*x])/(a^2*d) - ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])])/(a^3*d) - ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2])])/(a^3*d) + (3*f^2*(e + f*x)*Log[1 - E^(2*(c + d*x)
)])/(a*d^3) + ((e + f*x)^3*Log[1 - E^(2*(c + d*x))])/(a*d) + (b^2*(e + f*x)
^3*Log[1 - E^(2*(c + d*x))])/(a^3*d) + (6*b*f^2*(e + f*x)*PolyLog[2, -E^(c
+ d*x)])/(a^2*d^3) - (6*b*f^2*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^2*d^3)
- (3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/(a^3*d^2) - (3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) + (3*f^3*PolyLog[2, E^(2*(c + d*x))
])/(2*a*d^4) + (3*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^2) + (3
*b^2*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*x))])/(2*a^3*d^2) - (6*b*f^3*Poly
Log[3, -E^(c + d*x)])/(a^2*d^4) + (6*b*f^3*PolyLog[3, E^(c + d*x)])/(a^2*d^
4) + (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/(a^3*d^3) + (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^3) - (3*f^2*(e + f*x)*PolyLog[3, E^
(2*(c + d*x))])/(2*a*d^3) - (3*b^2*f^2*(e + f*x)*PolyLog[3, E^(2*(c + d*x))
])/(2*a^3*d^3) - (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2]))])/(a^3*d^4) - (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/(a^3*d^4) + (3*f^3*PolyLog[4, E^(2*(c + d*x))])/(4
*a*d^4) + (3*b^2*f^3*PolyLog[4, E^(2*(c + d*x))])/(4*a^3*d^4)
```

Rubi [A] time = 2.20277, antiderivative size = 972, normalized size of antiderivative = 1., number of steps used = 62, number of rules used = 23, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.821, Rules used = {5569, 3720, 3716, 2190, 2279, 2391, 32, 2531, 6609, 2282, 6589, 5585, 5450, 3296, 2638, 5452, 4182, 5446, 3311, 2635, 8, 5565, 5561}

$$-\frac{b^2(e+fx)^4}{4a^3f}+\frac{\left(a^2+b^2\right)(e+fx)^4}{4a^3f}-\frac{(e+fx)^4}{4af}-\frac{\coth^2(c+dx)(e+fx)^3}{2ad}+\frac{b\,\mathrm{csch}(c+dx)(e+fx)^3}{a^2d}-\frac{\left(a^2+b^2\right)\log(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (-3*f*(e + f*x)^2)/(2*a*d^2) + (e + f*x)^3/(2*a*d) - (e + f*x)^4/(4*a*f) -
(b^2*(e + f*x)^4)/(4*a^3*f) + ((a^2 + b^2)*(e + f*x)^4)/(4*a^3*f) + (6*b*f*
(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d^2) - (3*f*(e + f*x)^2*Coth[c + d*x
])/(2*a*d^2) - ((e + f*x)^3*Coth[c + d*x]^2)/(2*a*d) + (b*(e + f*x)^3*Csch[
c + d*x])/(a^2*d) - ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])])/(a^3*d) - ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2])])/(a^3*d) + (3*f^2*(e + f*x)*Log[1 - E^(2*(c + d*x)
)])/(a*d^3) + ((e + f*x)^3*Log[1 - E^(2*(c + d*x))])/(a*d) + (b^2*(e + f*x)
^3*Log[1 - E^(2*(c + d*x))])/(a^3*d) + (6*b*f^2*(e + f*x)*PolyLog[2, -E^(c
+ d*x)])/(a^2*d^3) - (6*b*f^2*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^2*d^3)
- (3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
 b^2]))])/(a^3*d^2) - (3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d
```

```
*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) + (3*f^3*PolyLog[2, E^(2*(c + d*x))
])/(2*a*d^4) + (3*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^2) + (3
*b^2*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*x))])/(2*a^3*d^2) - (6*b*f^3*Poly
Log[3, -E^(c + d*x)])/(a^2*d^4) + (6*b*f^3*PolyLog[3, E^(c + d*x)])/(a^2*d^
4) + (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/(a^3*d^3) + (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^3) - (3*f^2*(e + f*x)*PolyLog[3, E^
(2*(c + d*x))])/(2*a*d^3) - (3*b^2*f^2*(e + f*x)*PolyLog[3, E^(2*(c + d*x))
])/(2*a^3*d^3) - (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2]))])/(a^3*d^4) - (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/(a^3*d^4) + (3*f^3*PolyLog[4, E^(2*(c + d*x))])/(4
*a*d^4) + (3*b^2*f^3*PolyLog[4, E^(2*(c + d*x))])/(4*a^3*d^4)
```

Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
```

```
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\coth^3(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^3\coth^2(c+dx)}{2ad} + \frac{\int (e+fx)^3\coth(c+dx)\,dx}{a} - \frac{b\int (e+fx)^3\cosh(c+dx)}{a^2}$$

$$= -\frac{(e+fx)^4}{4af} - \frac{3f(e+fx)^2\coth(c+dx)}{2ad^2} - \frac{(e+fx)^3\coth^2(c+dx)}{2ad} - \frac{2\int \frac{e^{2(c+dx)}(e+fx)^3}{1-e^{2(c+dx)}}}{a}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{3f(e+fx)^2\coth(c+dx)}{2ad^2} - \frac{(e+fx)^3\coth^2}{2ad}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{b^2(e+fx)^4}{4a^3f} + \frac{\left(a^2+b^2\right)(e+fx)^4}{4a^3f} + \frac{6bf(e}{}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{b^2(e+fx)^4}{4a^3f} + \frac{\left(a^2+b^2\right)(e+fx)^4}{4a^3f} + \frac{6bf(e}{}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{b^2(e+fx)^4}{4a^3f} + \frac{\left(a^2+b^2\right)(e+fx)^4}{4a^3f} + \frac{6bf(e}{}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{b^2(e+fx)^4}{4a^3f} + \frac{\left(a^2+b^2\right)(e+fx)^4}{4a^3f} + \frac{6bf(e}{}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{b^2(e+fx)^4}{4a^3f} + \frac{\left(a^2+b^2\right)(e+fx)^4}{4a^3f} + \frac{6bf(e}{}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{b^2(e+fx)^4}{4a^3f} + \frac{\left(a^2+b^2\right)(e+fx)^4}{4a^3f} + \frac{6bf(e}{}$$

Mathematica [C] time = 73.7676, size = 11848, normalized size = 12.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.872, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3(\coth(dx+c))^3}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -e^3*(2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*
e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 + b^2)*log(-2*a*e^
(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x - c)
 + 1)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x - c) - 1)/(a^3*d)) + (3*a*f^3*x^2 +
 6*a*e*f^2*x + 3*a*e^2*f + 2*(b*d*f^3*x^3*e^(3*c) + 3*b*d*e*f^2*x^2*e^(3*c)
 + 3*b*d*e^2*f*x*e^(3*c))*e^(3*d*x) - (2*a*d*f^3*x^3*e^(2*c) + 3*a*e^2*f*e^
(2*c) + 3*(2*d*e*f^2 + f^3)*a*x^2*e^(2*c) + 6*(d*e^2*f + e*f^2)*a*x*e^(2*c)
)*e^(2*d*x) - 2*(b*d*f^3*x^3*e^c + 3*b*d*e*f^2*x^2*e^c + 3*b*d*e^2*f*x*e^c)
*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) -
 3*(b*d*e^2*f + a*e*f^2)*x/(a^2*d^2) + 3*(b*d*e^2*f - a*e*f^2)*x/(a^2*d^2)
+ 3*(b*d*e^2*f + a*e*f^2)*log(e^(d*x + c) + 1)/(a^2*d^3) - 3*(b*d*e^2*f - a
*e*f^2)*log(e^(d*x + c) - 1)/(a^2*d^3) + (d^3*x^3*log(e^(d*x + c) + 1) + 3*
d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4,
 -e^(d*x + c)))*(a^2*f^3 + b^2*f^3)/(a^3*d^4) + (d^3*x^3*log(-e^(d*x + c) +
 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*poly
log(4, e^(d*x + c)))*(a^2*f^3 + b^2*f^3)/(a^3*d^4) + 3*(a^2*d*e*f^2 + b^2*d
*e*f^2 + a*b*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c))
 - 2*polylog(3, -e^(d*x + c)))/(a^3*d^4) + 3*(a^2*d*e*f^2 + b^2*d*e*f^2 - a
*b*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polyl
og(3, e^(d*x + c)))/(a^3*d^4) + 3*(b^2*d^2*e^2*f + 2*a*b*d*e*f^2 + (d^2*e^2
*f + f^3)*a^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^3*d^4) +
 3*(b^2*d^2*e^2*f - 2*a*b*d*e*f^2 + (d^2*e^2*f + f^3)*a^2)*(d*x*log(-e^(d*x
 + c) + 1) + dilog(e^(d*x + c)))/(a^3*d^4) - 1/4*((a^2*f^3 + b^2*f^3)*d^4*x
^4 + 4*(a^2*d*e*f^2 + b^2*d*e*f^2 + a*b*f^3)*d^3*x^3 + 6*(b^2*d^2*e^2*f + 2
*a*b*d*e*f^2 + (d^2*e^2*f + f^3)*a^2)*d^2*x^2)/(a^3*d^4) - 1/4*((a^2*f^3 +
b^2*f^3)*d^4*x^4 + 4*(a^2*d*e*f^2 + b^2*d*e*f^2 - a*b*f^3)*d^3*x^3 + 6*(b^2
*d^2*e^2*f - 2*a*b*d*e*f^2 + (d^2*e^2*f + f^3)*a^2)*d^2*x^2)/(a^3*d^4) + in
tegrate(-2*((a^2*b*f^3 + b^3*f^3)*x^3 + 3*(a^2*b*e*f^2 + b^3*e*f^2)*x^2 + 3
*(a^2*b*e^2*f + b^3*e^2*f)*x - ((a^3*f^3*e^c + a*b^2*f^3*e^c)*x^3 + 3*(a^3*
e*f^2*e^c + a*b^2*e*f^2*e^c)*x^2 + 3*(a^3*e^2*f*e^c + a*b^2*e^2*f*e^c)*x)*e
^(d*x))/(a^3*b*e^(2*d*x + 2*c) + 2*a^4*e^(d*x + c) - a^3*b), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*coth(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.487 $\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=689

$$-\frac{2f\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}-\frac{2f\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3d^2}+\frac{b^2f(e+fx)\text{PolyLog}(}{a^3d^2}$$

```
[Out] (e*f*x)/(a*d) + (f^2*x^2)/(2*a*d) - (e + f*x)^3/(3*a*f) - (b^2*(e + f*x)^3)
/(3*a^3*f) + ((a^2 + b^2)*(e + f*x)^3)/(3*a^3*f) + (4*b*f*(e + f*x)*ArcTanh
[E^(c + d*x)])/(a^2*d^2) - (f*(e + f*x)*Coth[c + d*x])/(a*d^2) - ((e + f*x)
^2*Coth[c + d*x]^2)/(2*a*d) + (b*(e + f*x)^2*Csch[c + d*x])/(a^2*d) - ((a^2
 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*d)
 - ((a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])
/(a^3*d) + ((e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a*d) + (b^2*(e + f*x)^2*
Log[1 - E^(2*(c + d*x))])/(a^3*d) + (f^2*Log[Sinh[c + d*x]])/(a*d^3) + (2*b
*f^2*PolyLog[2, -E^(c + d*x)])/(a^2*d^3) - (2*b*f^2*PolyLog[2, E^(c + d*x)]
)/(a^2*d^3) - (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2]))])/(a^3*d^2) - (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) + (f*(e + f*x)*PolyLog[2, E
^(2*(c + d*x))])/(a*d^2) + (b^2*f*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a
^3*d^2) + (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/(a^3*d^3) + (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/(a^3*d^3) - (f^2*PolyLog[3, E^(2*(c + d*x))])/(2*a*d^3) -
 (b^2*f^2*PolyLog[3, E^(2*(c + d*x))])/(2*a^3*d^3)
```

Rubi [A] time = 1.7085, antiderivative size = 689, normalized size of antiderivative = 1., number of steps used = 47, number of rules used = 20, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5569, 3720, 3475, 3716, 2190, 2531, 2282, 6589, 5585, 5450, 3296, 2637, 5452, 4182, 2279, 2391, 5446, 3310, 5565, 5561}

$$-\frac{2f\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}-\frac{2f\left(a^2+b^2\right)(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3d^2}+\frac{b^2f(e+fx)\text{PolyLog}(}{a^3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (e*f*x)/(a*d) + (f^2*x^2)/(2*a*d) - (e + f*x)^3/(3*a*f) - (b^2*(e + f*x)^3)
/(3*a^3*f) + ((a^2 + b^2)*(e + f*x)^3)/(3*a^3*f) + (4*b*f*(e + f*x)*ArcTanh
[E^(c + d*x)])/(a^2*d^2) - (f*(e + f*x)*Coth[c + d*x])/(a*d^2) - ((e + f*x)
^2*Coth[c + d*x]^2)/(2*a*d) + (b*(e + f*x)^2*Csch[c + d*x])/(a^2*d) - ((a^2
 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*d)
 - ((a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])
/(a^3*d) + ((e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a*d) + (b^2*(e + f*x)^2*
Log[1 - E^(2*(c + d*x))])/(a^3*d) + (f^2*Log[Sinh[c + d*x]])/(a*d^3) + (2*b
*f^2*PolyLog[2, -E^(c + d*x)])/(a^2*d^3) - (2*b*f^2*PolyLog[2, E^(c + d*x)]
)/(a^2*d^3) - (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2]))])/(a^3*d^2) - (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) + (f*(e + f*x)*PolyLog[2, E
^(2*(c + d*x))])/(a*d^2) + (b^2*f*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a
^3*d^2) + (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/(a^3*d^3) + (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/(a^3*d^3) - (f^2*PolyLog[3, E^(2*(c + d*x))])/(2*a*d^3) -
 (b^2*f^2*PolyLog[3, E^(2*(c + d*x))])/(2*a^3*d^3)
```

Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\coth^3(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^2\coth^2(c+dx)}{2ad} + \frac{\int (e+fx)^2\coth(c+dx)\,dx}{a} - \frac{b\int (e+fx)^2\cosh(c+dx)}{a^2}$$

$$= -\frac{(e+fx)^3}{3af} - \frac{f(e+fx)\coth(c+dx)}{ad^2} - \frac{(e+fx)^2\coth^2(c+dx)}{2ad} - \frac{2\int \frac{e^{2(c+dx)}(e+fx)^2}{1-e^{2(c+dx)}}\,dx}{a}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{(e+fx)^3}{3af} - \frac{f(e+fx)\coth(c+dx)}{ad^2} - \frac{(e+fx)^2\coth^2(c+dx)}{2ad} + b(e+$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{(e+fx)^3}{3af} - \frac{b^2(e+fx)^3}{3a^3f} + \frac{\left(a^2+b^2\right)(e+fx)^3}{3a^3f} + \frac{4bf(e+fx)\tanh^{-1}(}{a^2d^2}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{(e+fx)^3}{3af} - \frac{b^2(e+fx)^3}{3a^3f} + \frac{\left(a^2+b^2\right)(e+fx)^3}{3a^3f} + \frac{4bf(e+fx)\tanh^{-1}(}{a^2d^2}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{(e+fx)^3}{3af} - \frac{b^2(e+fx)^3}{3a^3f} + \frac{\left(a^2+b^2\right)(e+fx)^3}{3a^3f} + \frac{4bf(e+fx)\tanh^{-1}(}{a^2d^2}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{(e+fx)^3}{3af} - \frac{b^2(e+fx)^3}{3a^3f} + \frac{\left(a^2+b^2\right)(e+fx)^3}{3a^3f} + \frac{4bf(e+fx)\tanh^{-1}(}{a^2d^2}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{(e+fx)^3}{3af} - \frac{b^2(e+fx)^3}{3a^3f} + \frac{\left(a^2+b^2\right)(e+fx)^3}{3a^3f} + \frac{4bf(e+fx)\tanh^{-1}(}{a^2d^2}$$

Mathematica [B] time = 42.2086, size = 2277, normalized size = 3.3

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*(e + f*x)^2*Csch[c])/(a^2*d) - ((e + f*x)^2*Csch[(c + d*x)/2]^2)/(8*a*d)
 - (12*a^2*d^3*e^2*E^(2*c)*x + 12*b^2*d^3*e^2*E^(2*c)*x + 12*a^2*d*E^(2*c)*
f^2*x + 12*a^2*d^3*e*E^(2*c)*f*x^2 + 12*b^2*d^3*e*E^(2*c)*f*x^2 + 4*a^2*d^3
*E^(2*c)*f^2*x^3 + 4*b^2*d^3*E^(2*c)*f^2*x^3 + 24*a*b*d*e*f*ArcTanh[E^(c +
d*x)] - 24*a*b*d*e*E^(2*c)*f*ArcTanh[E^(c + d*x)] - 12*a*b*d*f^2*x*Log[1 -
E^(c + d*x)] + 12*a*b*d*E^(2*c)*f^2*x*Log[1 - E^(c + d*x)] + 12*a*b*d*f^2*x
*Log[1 + E^(c + d*x)] - 12*a*b*d*E^(2*c)*f^2*x*Log[1 + E^(c + d*x)] + 6*a^2
*d^2*e^2*Log[1 - E^(2*(c + d*x))] + 6*b^2*d^2*e^2*Log[1 - E^(2*(c + d*x))]
- 6*a^2*d^2*e^2*E^(2*c)*Log[1 - E^(2*(c + d*x))] - 6*b^2*d^2*e^2*E^(2*c)*Lo
g[1 - E^(2*(c + d*x))] + 6*a^2*f^2*Log[1 - E^(2*(c + d*x))] - 6*a^2*E^(2*c)
*f^2*Log[1 - E^(2*(c + d*x))] + 12*a^2*d^2*e*f*x*Log[1 - E^(2*(c + d*x))] +
 12*b^2*d^2*e*f*x*Log[1 - E^(2*(c + d*x))] - 12*a^2*d^2*e*E^(2*c)*f*x*Log[1
 - E^(2*(c + d*x))] - 12*b^2*d^2*e*E^(2*c)*f*x*Log[1 - E^(2*(c + d*x))] + 6
*a^2*d^2*f^2*x^2*Log[1 - E^(2*(c + d*x))] + 6*b^2*d^2*f^2*x^2*Log[1 - E^(2*
(c + d*x))] - 6*a^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(2*(c + d*x))] - 6*b^2*d^
2*E^(2*c)*f^2*x^2*Log[1 - E^(2*(c + d*x))] - 12*a*b*(-1 + E^(2*c))*f^2*Poly
Log[2, -E^(c + d*x)] + 12*a*b*(-1 + E^(2*c))*f^2*PolyLog[2, E^(c + d*x)] +
6*a^2*d*e*f*PolyLog[2, E^(2*(c + d*x))] + 6*b^2*d*e*f*PolyLog[2, E^(2*(c +
d*x))] - 6*a^2*d*e*E^(2*c)*f*PolyLog[2, E^(2*(c + d*x))] - 6*b^2*d*e*E^(2*c
)*f*PolyLog[2, E^(2*(c + d*x))] + 6*a^2*d*f^2*x*PolyLog[2, E^(2*(c + d*x))]
 + 6*b^2*d*f^2*x*PolyLog[2, E^(2*(c + d*x))] - 6*a^2*d*E^(2*c)*f^2*x*PolyLo
g[2, E^(2*(c + d*x))] - 6*b^2*d*E^(2*c)*f^2*x*PolyLog[2, E^(2*(c + d*x))] -
 3*a^2*f^2*PolyLog[3, E^(2*(c + d*x))] - 3*b^2*f^2*PolyLog[3, E^(2*(c + d*x
))] + 3*a^2*E^(2*c)*f^2*PolyLog[3, E^(2*(c + d*x))] + 3*b^2*E^(2*c)*f^2*Pol
yLog[3, E^(2*(c + d*x))])/(6*a^3*d^3*(-1 + E^(2*c))) + ((a^2 + b^2)*(6*e^2*
E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^
2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) +
(6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2
- b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a +
 b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2
+ b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2
- b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d
 - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))])/d + (6*e
*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6
*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]
)])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2
 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a
^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c
 + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a
*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*
c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(e
+ f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))]
)/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c
 + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x)
)/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -(
(b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 - (6*f^2*PolyL
og[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E
^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)
]))])/d^3))/(3*a^3*(-1 + E^(2*c))) + ((e + f*x)^2*Sech[(c + d*x)/2]^2)/(8*a
*d) - ((e + f*x)*(-(a*f) + b*d*(e + f*x))*Csch[c/2]*Csch[(c + d*x)/2]*Sinh[
(d*x)/2])/(2*a^2*d^2) - ((e + f*x)*(a*f + b*d*(e + f*x))*Sech[c/2]*Sech[(c
+ d*x)/2]*Sinh[(d*x)/2])/(2*a^2*d^2)
```

Maple [F] time = 0.671, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 (\coth (dx+c))^3}{a+b\sinh (dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -e^2*(2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*
e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 + b^2)*log(-2*a*e^
(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x - c)
 + 1)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x - c) - 1)/(a^3*d)) + 2*(a*f^2*x + a
*e*f + (b*d*f^2*x^2*e^(3*c) + 2*b*d*e*f*x*e^(3*c))*e^(3*d*x) - (a*d*f^2*x^2
*e^(2*c) + a*e*f*e^(2*c) + (2*d*e*f + f^2)*a*x*e^(2*c))*e^(2*d*x) - (b*d*f^
2*x^2*e^c + 2*b*d*e*f*x*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*
e^(2*d*x + 2*c) + a^2*d^2) - (2*b*d*e*f + a*f^2)*x/(a^2*d^2) + (2*b*d*e*f -
 a*f^2)*x/(a^2*d^2) + (2*b*d*e*f + a*f^2)*log(e^(d*x + c) + 1)/(a^2*d^3) -
(2*b*d*e*f - a*f^2)*log(e^(d*x + c) - 1)/(a^2*d^3) + (d^2*x^2*log(e^(d*x +
c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*(a^2*f^2
+ b^2*f^2)/(a^3*d^3) + (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x
+ c)) - 2*polylog(3, e^(d*x + c)))*(a^2*f^2 + b^2*f^2)/(a^3*d^3) + 2*(a^2*d
*e*f + b^2*d*e*f + a*b*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c))
)/(a^3*d^3) + 2*(a^2*d*e*f + b^2*d*e*f - a*b*f^2)*(d*x*log(-e^(d*x + c) + 1
) + dilog(e^(d*x + c)))/(a^3*d^3) - 1/3*((a^2*f^2 + b^2*f^2)*d^3*x^3 + 3*(a
^2*d*e*f + b^2*d*e*f + a*b*f^2)*d^2*x^2)/(a^3*d^3) - 1/3*((a^2*f^2 + b^2*f^
2)*d^3*x^3 + 3*(a^2*d*e*f + b^2*d*e*f - a*b*f^2)*d^2*x^2)/(a^3*d^3) + integ
rate(-2*((a^2*b*f^2 + b^3*f^2)*x^2 + 2*(a^2*b*e*f + b^3*e*f)*x - ((a^3*f^2*
e^c + a*b^2*f^2*e^c)*x^2 + 2*(a^3*e*f*e^c + a*b^2*e*f*e^c)*x)*e^(d*x))/(a^3
*b*e^(2*d*x + 2*c) + 2*a^4*e^(d*x + c) - a^3*b), x)
```

Fricas [C] time = 2.99641, size = 18048, normalized size = 26.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*a^2*d*e*f - 2*a^2*c*f^2 - 2*(a^2*d*f^2*x + a^2*c*f^2)*cosh(d*x + c)^4 -
2*(a^2*d*f^2*x + a^2*c*f^2)*sinh(d*x + c)^4 + 2*(a*b*d^2*f^2*x^2 + 2*a*b*d^
2*e*f*x + a*b*d^2*e^2)*cosh(d*x + c)^3 + 2*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f
```

```
*x + a*b*d^2*e^2 - 4*(a^2*d*f^2*x + a^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c)
^3 - 2*(a^2*d^2*f^2*x^2 + a^2*d^2*e^2 + a^2*d*e*f - 2*a^2*c*f^2 + (2*a^2*d^
2*e*f - a^2*d*f^2)*x)*cosh(d*x + c)^2 - 2*(a^2*d^2*f^2*x^2 + a^2*d^2*e^2 +
a^2*d*e*f - 2*a^2*c*f^2 + 6*(a^2*d*f^2*x + a^2*c*f^2)*cosh(d*x + c)^2 + (2*
a^2*d^2*e*f - a^2*d*f^2)*x - 3*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + a*b*d^2
*e^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x
 + a*b*d^2*e^2)*cosh(d*x + c) - 2*((a^2 + b^2)*d*f^2*x + ((a^2 + b^2)*d*f^2
*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c)^4 + 4*((a^2 + b^2)*d*f^2*x + (a^2 + b
^2)*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 + b^2)*d*f^2*x + (a^2 + b^
2)*d*e*f)*sinh(d*x + c)^4 + (a^2 + b^2)*d*e*f - 2*((a^2 + b^2)*d*f^2*x + (a
^2 + b^2)*d*e*f)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e
*f - 3*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c)^2)*sinh(d*x
+ c)^2 + 4*(((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c)^3 - ((a
^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(
(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sq
rt((a^2 + b^2)/b^2) - b)/b + 1) - 2*((a^2 + b^2)*d*f^2*x + ((a^2 + b^2)*d*f
^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c)^4 + 4*((a^2 + b^2)*d*f^2*x + (a^2 +
 b^2)*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 + b^2)*d*f^2*x + (a^2 +
b^2)*d*e*f)*sinh(d*x + c)^4 + (a^2 + b^2)*d*e*f - 2*((a^2 + b^2)*d*f^2*x +
(a^2 + b^2)*d*e*f)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d
*e*f - 3*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c)^2)*sinh(d*
x + c)^2 + 4*(((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c)^3 - (
(a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c))*sinh(d*x + c))*dilo
g((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*((a^2 + b^2)*d*f^2*x + ((a^2 + b^2)*d
*f^2*x + (a^2 + b^2)*d*e*f - a*b*f^2)*cosh(d*x + c)^4 + 4*((a^2 + b^2)*d*f^
2*x + (a^2 + b^2)*d*e*f - a*b*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 +
b^2)*d*f^2*x + (a^2 + b^2)*d*e*f - a*b*f^2)*sinh(d*x + c)^4 + (a^2 + b^2)*d
*e*f - a*b*f^2 - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f - a*b*f^2)*cosh
(d*x + c)^2 - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f - a*b*f^2 - 3*((a^
2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f - a*b*f^2)*cosh(d*x + c)^2)*sinh(d*x +
 c)^2 + 4*(((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f - a*b*f^2)*cosh(d*x + c
)^3 - ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f - a*b*f^2)*cosh(d*x + c))*si
nh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)) + 2*((a^2 + b^2)*d*f^2*x
+ ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f + a*b*f^2)*cosh(d*x + c)^4 + 4*(
(a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f + a*b*f^2)*cosh(d*x + c)*sinh(d*x +
 c)^3 + ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f + a*b*f^2)*sinh(d*x + c)^4
 + (a^2 + b^2)*d*e*f + a*b*f^2 - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f
 + a*b*f^2)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f +
a*b*f^2 - 3*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f + a*b*f^2)*cosh(d*x +
c)^2)*sinh(d*x + c)^2 + 4*(((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f + a*b*f
^2)*cosh(d*x + c)^3 - ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f + a*b*f^2)*c
osh(d*x + c))*sinh(d*x + c))*dilog(-cosh(d*x + c) - sinh(d*x + c)) - ((a^2
+ b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2 + ((a^2 + b^2)
*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)^4 + 4
*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*cosh(d
*x + c)*sinh(d*x + c)^3 + ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a
^2 + b^2)*c^2*f^2)*sinh(d*x + c)^4 - 2*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)
*c*d*e*f + (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^2*e^2 -
2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2 - 3*((a^2 + b^2)*d^2*e^2 - 2*(a
^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
 4*(((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*cos
h(d*x + c)^3 - ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c
^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x
+ c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b
^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2 + ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*
d*e*f + (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)^4 + 4*((a^2 + b^2)*d^2*e^2 - 2*(
a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (
(a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*sinh(d*x
```

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 + c)^4 - 2*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*
f^2)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^
2 + b^2)*c^2*f^2 - 3*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 +
b^2)*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^2 + b^2)*d^2*e^2 -
2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)^3 - ((a^2 + b^2)
*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*cosh(d*x + c))*sinh
(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)
/b^2) + 2*a) - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2
+ b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2 + ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 +
b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)
^4 + 4*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c
*d*e*f - (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 + b^2)*
d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)
*c^2*f^2)*sinh(d*x + c)^4 - 2*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*
e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)^2 - 2*((
a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f -
(a^2 + b^2)*c^2*f^2 - 3*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x
+ 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x +
c)^2 + 4*(((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2
)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)^3 - ((a^2 + b^2)*d^2*f^2*x^2
 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*c
osh(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - ((a^2 + b^2
)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^
2)*c^2*f^2 + ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 +
b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)^4 + 4*((a^2 + b^2)*d^2*f^
2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f
^2)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2
)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*sinh(d*x + c)^4
- 2*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*
e*f - (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^2*f^2*x^2 + 2
*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2 - 3*((
a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f -
(a^2 + b^2)*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^2 + b^2)*d^2
*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^
2*f^2)*cosh(d*x + c)^3 - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x
 + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c
))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b) + ((a^2 + b^2)*d^2*f^2*x^2 + (a^2 + b^2
)*d^2*e^2 + 2*a*b*d*e*f + ((a^2 + b^2)*d^2*f^2*x^2 + (a^2 + b^2)*d^2*e^2 +
2*a*b*d*e*f + a^2*f^2 + 2*((a^2 + b^2)*d^2*e*f + a*b*d*f^2)*x)*cosh(d*x + c
)^4 + 4*((a^2 + b^2)*d^2*f^2*x^2 + (a^2 + b^2)*d^2*e^2 + 2*a*b*d*e*f + a^2*
f^2 + 2*((a^2 + b^2)*d^2*e*f + a*b*d*f^2)*x)*cosh(d*x + c)*sinh(d*x + c)^3
+ ((a^2 + b^2)*d^2*f^2*x^2 + (a^2 + b^2)*d^2*e^2 + 2*a*b*d*e*f + a^2*f^2 +
2*((a^2 + b^2)*d^2*e*f + a*b*d*f^2)*x)*sinh(d*x + c)^4 + a^2*f^2 - 2*((a^2
+ b^2)*d^2*f^2*x^2 + (a^2 + b^2)*d^2*e^2 + 2*a*b*d*e*f + a^2*f^2 + 2*((a^2
+ b^2)*d^2*e*f + a*b*d*f^2)*x)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^2*f^2*x^2
 + (a^2 + b^2)*d^2*e^2 + 2*a*b*d*e*f + a^2*f^2 - 3*((a^2 + b^2)*d^2*f^2*x^2
 + (a^2 + b^2)*d^2*e^2 + 2*a*b*d*e*f + a^2*f^2 + 2*((a^2 + b^2)*d^2*e*f + a
*b*d*f^2)*x)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*d^2*e*f + a*b*d*f^2)*x)*sinh(
d*x + c)^2 + 2*((a^2 + b^2)*d^2*e*f + a*b*d*f^2)*x + 4*(((a^2 + b^2)*d^2*f^
2*x^2 + (a^2 + b^2)*d^2*e^2 + 2*a*b*d*e*f + a^2*f^2 + 2*((a^2 + b^2)*d^2*e*
f + a*b*d*f^2)*x)*cosh(d*x + c)^3 - ((a^2 + b^2)*d^2*f^2*x^2 + (a^2 + b^2)*
d^2*e^2 + 2*a*b*d*e*f + a^2*f^2 + 2*((a^2 + b^2)*d^2*e*f + a*b*d*f^2)*x)*co
sh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a^2
+ b^2)*d^2*e^2 + ((a^2 + b^2)*d^2*e^2 - 2*(a*b + (a^2 + b^2)*c)*d*e*f + (2*
a*b*c + (a^2 + b^2)*c^2 + a^2)*f^2)*cosh(d*x + c)^4 + 4*((a^2 + b^2)*d^2*e^
2 - 2*(a*b + (a^2 + b^2)*c)*d*e*f + (2*a*b*c + (a^2 + b^2)*c^2 + a^2)*f^2)*
cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 + b^2)*d^2*e^2 - 2*(a*b + (a^2 + b^2)
```

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*c)*d*e*f + (2*a*b*c + (a^2 + b^2)*c^2 + a^2)*f^2)*sinh(d*x + c)^4 - 2*(a*b
 + (a^2 + b^2)*c)*d*e*f + (2*a*b*c + (a^2 + b^2)*c^2 + a^2)*f^2 - 2*((a^2 +
 b^2)*d^2*e^2 - 2*(a*b + (a^2 + b^2)*c)*d*e*f + (2*a*b*c + (a^2 + b^2)*c^2
+ a^2)*f^2)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^2*e^2 - 2*(a*b + (a^2 + b^2)
*c)*d*e*f + (2*a*b*c + (a^2 + b^2)*c^2 + a^2)*f^2 - 3*((a^2 + b^2)*d^2*e^2
- 2*(a*b + (a^2 + b^2)*c)*d*e*f + (2*a*b*c + (a^2 + b^2)*c^2 + a^2)*f^2)*co
sh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^2 + b^2)*d^2*e^2 - 2*(a*b + (a^2 +
b^2)*c)*d*e*f + (2*a*b*c + (a^2 + b^2)*c^2 + a^2)*f^2)*cosh(d*x + c)^3 - ((
a^2 + b^2)*d^2*e^2 - 2*(a*b + (a^2 + b^2)*c)*d*e*f + (2*a*b*c + (a^2 + b^2)
*c^2 + a^2)*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x
 + c) - 1) + ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*c*d*e*f + ((a^2 + b^2
)*d^2*f^2*x^2 + 2*(a^2 + b^2)*c*d*e*f - (2*a*b*c + (a^2 + b^2)*c^2)*f^2 + 2
*((a^2 + b^2)*d^2*e*f - a*b*d*f^2)*x)*cosh(d*x + c)^4 + 4*((a^2 + b^2)*d^2*
f^2*x^2 + 2*(a^2 + b^2)*c*d*e*f - (2*a*b*c + (a^2 + b^2)*c^2)*f^2 + 2*((a^2
 + b^2)*d^2*e*f - a*b*d*f^2)*x)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 + b^2
)*d^2*f^2*x^2 + 2*(a^2 + b^2)*c*d*e*f - (2*a*b*c + (a^2 + b^2)*c^2)*f^2 + 2
*((a^2 + b^2)*d^2*e*f - a*b*d*f^2)*x)*sinh(d*x + c)^4 - (2*a*b*c + (a^2 + b
^2)*c^2)*f^2 - 2*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*c*d*e*f - (2*a*b*
c + (a^2 + b^2)*c^2)*f^2 + 2*((a^2 + b^2)*d^2*e*f - a*b*d*f^2)*x)*cosh(d*x
+ c)^2 - 2*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*c*d*e*f - (2*a*b*c + (a
^2 + b^2)*c^2)*f^2 - 3*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*c*d*e*f - (
2*a*b*c + (a^2 + b^2)*c^2)*f^2 + 2*((a^2 + b^2)*d^2*e*f - a*b*d*f^2)*x)*cos
h(d*x + c)^2 + 2*((a^2 + b^2)*d^2*e*f - a*b*d*f^2)*x)*sinh(d*x + c)^2 + 2*(
(a^2 + b^2)*d^2*e*f - a*b*d*f^2)*x + 4*(((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 +
 b^2)*c*d*e*f - (2*a*b*c + (a^2 + b^2)*c^2)*f^2 + 2*((a^2 + b^2)*d^2*e*f -
a*b*d*f^2)*x)*cosh(d*x + c)^3 - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*c*
d*e*f - (2*a*b*c + (a^2 + b^2)*c^2)*f^2 + 2*((a^2 + b^2)*d^2*e*f - a*b*d*f^
2)*x)*cosh(d*x + c))*sinh(d*x + c))*log(-cosh(d*x + c) - sinh(d*x + c) + 1)
 + 2*((a^2 + b^2)*f^2*cosh(d*x + c)^4 + 4*(a^2 + b^2)*f^2*cosh(d*x + c)*sin
h(d*x + c)^3 + (a^2 + b^2)*f^2*sinh(d*x + c)^4 - 2*(a^2 + b^2)*f^2*cosh(d*x
 + c)^2 + (a^2 + b^2)*f^2 + 2*(3*(a^2 + b^2)*f^2*cosh(d*x + c)^2 - (a^2 + b
^2)*f^2)*sinh(d*x + c)^2 + 4*((a^2 + b^2)*f^2*cosh(d*x + c)^3 - (a^2 + b^2)
*f^2*cosh(d*x + c))*sinh(d*x + c))*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x
 + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*(
(a^2 + b^2)*f^2*cosh(d*x + c)^4 + 4*(a^2 + b^2)*f^2*cosh(d*x + c)*sinh(d*x
+ c)^3 + (a^2 + b^2)*f^2*sinh(d*x + c)^4 - 2*(a^2 + b^2)*f^2*cosh(d*x + c)^
2 + (a^2 + b^2)*f^2 + 2*(3*(a^2 + b^2)*f^2*cosh(d*x + c)^2 - (a^2 + b^2)*f^
2)*sinh(d*x + c)^2 + 4*((a^2 + b^2)*f^2*cosh(d*x + c)^3 - (a^2 + b^2)*f^2*c
osh(d*x + c))*sinh(d*x + c))*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c)
- (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*((a^2 +
 b^2)*f^2*cosh(d*x + c)^4 + 4*(a^2 + b^2)*f^2*cosh(d*x + c)*sinh(d*x + c)^3
 + (a^2 + b^2)*f^2*sinh(d*x + c)^4 - 2*(a^2 + b^2)*f^2*cosh(d*x + c)^2 + (a
^2 + b^2)*f^2 + 2*(3*(a^2 + b^2)*f^2*cosh(d*x + c)^2 - (a^2 + b^2)*f^2)*sin
h(d*x + c)^2 + 4*((a^2 + b^2)*f^2*cosh(d*x + c)^3 - (a^2 + b^2)*f^2*cosh(d*
x + c))*sinh(d*x + c))*polylog(3, cosh(d*x + c) + sinh(d*x + c)) - 2*((a^2
+ b^2)*f^2*cosh(d*x + c)^4 + 4*(a^2 + b^2)*f^2*cosh(d*x + c)*sinh(d*x + c)^
3 + (a^2 + b^2)*f^2*sinh(d*x + c)^4 - 2*(a^2 + b^2)*f^2*cosh(d*x + c)^2 + (
a^2 + b^2)*f^2 + 2*(3*(a^2 + b^2)*f^2*cosh(d*x + c)^2 - (a^2 + b^2)*f^2)*si
nh(d*x + c)^2 + 4*((a^2 + b^2)*f^2*cosh(d*x + c)^3 - (a^2 + b^2)*f^2*cosh(d
*x + c))*sinh(d*x + c))*polylog(3, -cosh(d*x + c) - sinh(d*x + c)) - 2*(a*b
*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + a*b*d^2*e^2 + 4*(a^2*d*f^2*x + a^2*c*f^2)*
cosh(d*x + c)^3 - 3*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + a*b*d^2*e^2)*cosh(
d*x + c)^2 + 2*(a^2*d^2*f^2*x^2 + a^2*d^2*e^2 + a^2*d*e*f - 2*a^2*c*f^2 + (
2*a^2*d^2*e*f - a^2*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d^3*cosh(d
*x + c)^4 + 4*a^3*d^3*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d^3*sinh(d*x + c)
^4 - 2*a^3*d^3*cosh(d*x + c)^2 + a^3*d^3 + 2*(3*a^3*d^3*cosh(d*x + c)^2 - a
^3*d^3)*sinh(d*x + c)^2 + 4*(a^3*d^3*cosh(d*x + c)^3 - a^3*d^3*cosh(d*x + c
))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*coth(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.488 $\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=435

$$\frac{b^2 f \text{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^3d^2}-\frac{f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}-\frac{f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3d^2}+\frac{f\text{PolyLog}\left(2,e\right.}{2ad^2}$$

```
[Out] (f*x)/(2*a*d) - (e + f*x)^2/(2*a*f) - (b^2*(e + f*x)^2)/(2*a^3*f) + ((a^2 +
 b^2)*(e + f*x)^2)/(2*a^3*f) + (b*f*ArcTanh[Cosh[c + d*x]])/(a^2*d^2) - (f*
Coth[c + d*x])/(2*a*d^2) - ((e + f*x)*Coth[c + d*x]^2)/(2*a*d) + (b*(e + f*
x)*Csch[c + d*x])/(a^2*d) - ((a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])])/(a^3*d) - ((a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*d) + ((e + f*x)*Log[1 - E^(2*(c + d*x))]
)/(a*d) + (b^2*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a^3*d) - ((a^2 + b^2)*f
*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2) - ((a^2 +
b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) + (f
*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^2) + (b^2*f*PolyLog[2, E^(2*(c + d*x))
])/(2*a^3*d^2)
```

Rubi [A] time = 0.976158, antiderivative size = 435, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 18, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {5569, 3720, 3473, 8, 3716, 2190, 2279, 2391, 5585, 5450, 3296, 2638, 5452, 3770, 5446, 2635, 5565, 5561}

$$\frac{b^2 f \text{PolyLog}\left(2,e^{2(c+dx)}\right)}{2a^3d^2}-\frac{f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2}-\frac{f\left(a^2+b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3d^2}+\frac{f\text{PolyLog}\left(2,e\right.}{2ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (f*x)/(2*a*d) - (e + f*x)^2/(2*a*f) - (b^2*(e + f*x)^2)/(2*a^3*f) + ((a^2 +
 b^2)*(e + f*x)^2)/(2*a^3*f) + (b*f*ArcTanh[Cosh[c + d*x]])/(a^2*d^2) - (f*
Coth[c + d*x])/(2*a*d^2) - ((e + f*x)*Coth[c + d*x]^2)/(2*a*d) + (b*(e + f*
x)*Csch[c + d*x])/(a^2*d) - ((a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])])/(a^3*d) - ((a^2 + b^2)*(e + f*x)*Log[1 + (b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*d) + ((e + f*x)*Log[1 - E^(2*(c + d*x))]
)/(a*d) + (b^2*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a^3*d) - ((a^2 + b^2)*f
*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2) - ((a^2 +
b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) + (f
*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^2) + (b^2*f*PolyLog[2, E^(2*(c + d*x))
])/(2*a^3*d^2)
```


Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
 + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
```

```
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
 (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
 0] && IGtQ[p, 0]
```

Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{(e+fx)\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\coth^3(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)\coth^2(c+dx)}{2ad} + \frac{\int (e+fx)\coth(c+dx)\,dx}{a} - \frac{b\int (e+fx)\cosh(c+dx)\,\mathrm{cotl}}{a^2}$$

$$= -\frac{(e+fx)^2}{2af} - \frac{f\coth(c+dx)}{2ad^2} - \frac{(e+fx)\coth^2(c+dx)}{2ad} - \frac{2\int \frac{e^{2(c+dx)}(e+fx)}{1-e^{2(c+dx)}}\,dx}{a} - \frac{b\int (e+}{}$$

$$= \frac{fx}{2ad} - \frac{(e+fx)^2}{2af} - \frac{f\coth(c+dx)}{2ad^2} - \frac{(e+fx)\coth^2(c+dx)}{2ad} + \frac{b(e+fx)\mathrm{csch}(c+dx)}{a^2d}$$

$$= \frac{fx}{2ad} - \frac{(e+fx)^2}{2af} - \frac{b^2(e+fx)^2}{2a^3f} + \frac{(a^2+b^2)(e+fx)^2}{2a^3f} + \frac{bf\tanh^{-1}(\cosh(c+dx))}{a^2d^2} + \frac{bj}{}$$

$$= \frac{fx}{2ad} - \frac{(e+fx)^2}{2af} - \frac{b^2(e+fx)^2}{2a^3f} + \frac{(a^2+b^2)(e+fx)^2}{2a^3f} + \frac{bf\tanh^{-1}(\cosh(c+dx))}{a^2d^2} - \frac{f}{}$$

$$= \frac{fx}{2ad} - \frac{(e+fx)^2}{2af} - \frac{b^2(e+fx)^2}{2a^3f} + \frac{(a^2+b^2)(e+fx)^2}{2a^3f} + \frac{bf\tanh^{-1}(\cosh(c+dx))}{a^2d^2} - \frac{f}{}$$

$$= \frac{fx}{2ad} - \frac{(e+fx)^2}{2af} - \frac{b^2(e+fx)^2}{2a^3f} + \frac{(a^2+b^2)(e+fx)^2}{2a^3f} + \frac{bf\tanh^{-1}(\cosh(c+dx))}{a^2d^2} - \frac{f}{}$$

Mathematica [A] time = 4.72672, size = 455, normalized size = 1.05

$$\frac{-8(a^2+b^2)\left(f\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+f\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+f(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)+f(c+dx)\log\left(\frac{b}{\sqrt{a^2}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a*(-(a*f) + 2*b*d*(e + f*x))*Coth[(c + d*x)/2] - a^2*d*(e + f*x)*Csch[(c
 + d*x)/2]^2 + 8*a^2*d*e*Log[Sinh[c + d*x]] + 8*b^2*d*e*Log[Sinh[c + d*x]]
- 8*a^2*c*f*Log[Sinh[c + d*x]] - 8*b^2*c*f*Log[Sinh[c + d*x]] - 8*a*b*f*Log
[Tanh[(c + d*x)/2]] + 4*a^2*f*((c + d*x)*(c + d*x + 2*Log[1 - E^(-2*(c + d*
x))]) - PolyLog[2, E^(-2*(c + d*x))]) + 4*b^2*f*((c + d*x)*(c + d*x + 2*Log
[1 - E^(-2*(c + d*x))]) - PolyLog[2, E^(-2*(c + d*x))]) - 8*(a^2 + b^2)*(-(
f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
)] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d*e*Log[a
 + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLog[2, (b*E^(c +
 d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2]))]) + a^2*d*(e + f*x)*Sech[(c + d*x)/2]^2 - 2*a*(a*f + 2*b*d*(e +
 f*x))*Tanh[(c + d*x)/2])/(8*a^3*d^2)
```

Maple [B] time = 0.171, size = 1098, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/a^3*b^2/d^2*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/a^3*b^2/d*f*ln((-
b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/a^3*b^2/d^2*f*ln(
(-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/a^3*b^2/d*f*ln(
(b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/a^3*b^2/d^2*f*ln(
(b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/a^3/d^2*b^2*f*c*l
n(exp(d*x+c)-1)+1/a^3/d*b^2*f*ln(exp(d*x+c)+1)*x-1/d*f/a*ln((-b*exp(d*x+c)+
(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/d^2*f/a*ln((-b*exp(d*x+c)+(a^2
+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/d*f/a*ln((b*exp(d*x+c)+(a^2+b^2)^(
1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/d^2*f/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a
)/(a+(a^2+b^2)^(1/2)))*c+1/d^2*f*c/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-
1/a^3*b^2/d^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)
))-1/a^3*b^2/d^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2
)))-1/a^3*b^2/d*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d^2*f/a*dilog((-b
*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2*f/a*dilog((b*exp
(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/a^3/d^2*b^2*f*dilog(exp(d
*x+c))+1/a^3/d^2*b^2*f*dilog(exp(d*x+c)+1)-1/a^2/d^2*b*f*ln(exp(d*x+c)-1)+1
/a^2/d^2*b*f*ln(exp(d*x+c)+1)+1/a^3/d*b^2*e*ln(exp(d*x+c)-1)+1/a^3/d*b^2*e*
ln(exp(d*x+c)+1)-1/d^2*f/a*dilog(exp(d*x+c))+1/d^2*f/a*dilog(exp(d*x+c)+1)+
1/d/a*e*ln(exp(d*x+c)-1)+1/d/a*e*ln(exp(d*x+c)+1)+1/d/a*ln(exp(d*x+c)+1)*f*
x-1/d^2/a*f*c*ln(exp(d*x+c)-1)-(-2*b*d*f*x*exp(3*d*x+3*c)+2*a*d*f*x*exp(2*d
*x+2*c)-2*b*d*e*exp(3*d*x+3*c)+2*a*d*e*exp(2*d*x+2*c)+2*b*d*f*x*exp(d*x+c)+
a*f*exp(2*d*x+2*c)+2*b*d*e*exp(d*x+c)-a*f)/d^2/a^2/(exp(2*d*x+2*c)-1)^2-1/d
*e/a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\left(a^2d\int\frac{x}{a^3de^{(dx+c)}+a^3d}\,dx+b^2d\int\frac{x}{a^3de^{(dx+c)}+a^3d}\,dx-a^2d\int\frac{x}{a^3de^{(dx+c)}-a^3d}\,dx-b^2d\int\frac{x}{a^3de^{(dx+c)}-a^3d}\,dx+ab\Big(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a^2*d*integrate(x/(a^3*d*e^(d*x + c) + a^3*d), x) + b^2*d*integrate(x/(a^
3*d*e^(d*x + c) + a^3*d), x) - a^2*d*integrate(x/(a^3*d*e^(d*x + c) - a^3*d
), x) - b^2*d*integrate(x/(a^3*d*e^(d*x + c) - a^3*d), x) + a*b*((d*x + c)/
(a^3*d^2) - log(e^(d*x + c) + 1)/(a^3*d^2)) - a*b*((d*x + c)/(a^3*d^2) - lo
g(e^(d*x + c) - 1)/(a^3*d^2)) - (2*b*d*x*e^(3*d*x + 3*c) - 2*b*d*x*e^(d*x +
 c) - (2*a*d*x*e^(2*c) + a*e^(2*c))*e^(2*d*x) + a)/(a^2*d^2*e^(4*d*x + 4*c)
 - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) - integrate(2*((a^3*e^c + a*b^2*e^c
)*x*e^(d*x) - (a^2*b + b^3)*x)/(a^3*b*e^(2*d*x + 2*c) + 2*a^4*e^(d*x + c) -
 a^3*b), x))*f - e*(2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x -
3*c))/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 + b^
2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) - (a^2 + b^2)*lo
g(e^(-d*x - c) + 1)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x - c) - 1)/(a^3*d))
```

Fricas [B] time = 2.38878, size = 8753, normalized size = 20.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*(a*b*d*f*x + a*b*d*e)*cosh(d*x + c)^3 + 2*(a*b*d*f*x + a*b*d*e)*sinh(d*x
 + c)^3 + a^2*f - (2*a^2*d*f*x + 2*a^2*d*e + a^2*f)*cosh(d*x + c)^2 - (2*a^
2*d*f*x + 2*a^2*d*e + a^2*f - 6*(a*b*d*f*x + a*b*d*e)*cosh(d*x + c))*sinh(d
*x + c)^2 - 2*(a*b*d*f*x + a*b*d*e)*cosh(d*x + c) - ((a^2 + b^2)*f*cosh(d*x
 + c)^4 + 4*(a^2 + b^2)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + b^2)*f*sin
h(d*x + c)^4 - 2*(a^2 + b^2)*f*cosh(d*x + c)^2 + 2*(3*(a^2 + b^2)*f*cosh(d*
x + c)^2 - (a^2 + b^2)*f)*sinh(d*x + c)^2 + (a^2 + b^2)*f + 4*((a^2 + b^2)*
f*cosh(d*x + c)^3 - (a^2 + b^2)*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*co
sh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a
^2 + b^2)/b^2) - b)/b + 1) - ((a^2 + b^2)*f*cosh(d*x + c)^4 + 4*(a^2 + b^2)
*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + b^2)*f*sinh(d*x + c)^4 - 2*(a^2 +
 b^2)*f*cosh(d*x + c)^2 + 2*(3*(a^2 + b^2)*f*cosh(d*x + c)^2 - (a^2 + b^2)*
f)*sinh(d*x + c)^2 + (a^2 + b^2)*f + 4*((a^2 + b^2)*f*cosh(d*x + c)^3 - (a^
2 + b^2)*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*
x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b +
 1) + ((a^2 + b^2)*f*cosh(d*x + c)^4 + 4*(a^2 + b^2)*f*cosh(d*x + c)*sinh(d
*x + c)^3 + (a^2 + b^2)*f*sinh(d*x + c)^4 - 2*(a^2 + b^2)*f*cosh(d*x + c)^2
 + 2*(3*(a^2 + b^2)*f*cosh(d*x + c)^2 - (a^2 + b^2)*f)*sinh(d*x + c)^2 + (a
^2 + b^2)*f + 4*((a^2 + b^2)*f*cosh(d*x + c)^3 - (a^2 + b^2)*f*cosh(d*x + c
))*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)) + ((a^2 + b^2)*f*cos
h(d*x + c)^4 + 4*(a^2 + b^2)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + b^2)*
f*sinh(d*x + c)^4 - 2*(a^2 + b^2)*f*cosh(d*x + c)^2 + 2*(3*(a^2 + b^2)*f*co
sh(d*x + c)^2 - (a^2 + b^2)*f)*sinh(d*x + c)^2 + (a^2 + b^2)*f + 4*((a^2 +
b^2)*f*cosh(d*x + c)^3 - (a^2 + b^2)*f*cosh(d*x + c))*sinh(d*x + c))*dilog(
-cosh(d*x + c) - sinh(d*x + c)) - (((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh
(d*x + c)^4 + 4*((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x
+ c)^3 + ((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*sinh(d*x + c)^4 + (a^2 + b^2)*
d*e - (a^2 + b^2)*c*f - 2*((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)
^2 - 2*((a^2 + b^2)*d*e - (a^2 + b^2)*c*f - 3*((a^2 + b^2)*d*e - (a^2 + b^2
)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^2 + b^2)*d*e - (a^2 + b^2)
*c*f)*cosh(d*x + c)^3 - ((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c))*
sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 +
b^2)/b^2) + 2*a) - (((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)^4 + 4
*((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2
+ b^2)*d*e - (a^2 + b^2)*c*f)*sinh(d*x + c)^4 + (a^2 + b^2)*d*e - (a^2 + b^
2)*c*f - 2*((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)^2 - 2*((a^2 +
b^2)*d*e - (a^2 + b^2)*c*f - 3*((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 + 4*(((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x
+ c)^3 - ((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c))*sinh(d*x + c))*
log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a
) - (((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)^4 + 4*((a^2 + b^2)
*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 + b^2)*d*f*
x + (a^2 + b^2)*c*f)*sinh(d*x + c)^4 + (a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f
- 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*
d*f*x + (a^2 + b^2)*c*f - 3*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 + 4*(((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x
 + c)^3 - ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c))*sinh(d*x + c
))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f
)*cosh(d*x + c)^4 + 4*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)*s
inh(d*x + c)^3 + ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*sinh(d*x + c)^4 + (a
^2 + b^2)*d*f*x + (a^2 + b^2)*c*f - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)
*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f - 3*((a^2 + b^2)*
d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^2 + b^2)
*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)^3 - ((a^2 + b^2)*d*f*x + (a^2 + b^2
)*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c
) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (((
a^2 + b^2)*d*f*x + (a^2 + b^2)*d*e + a*b*f)*cosh(d*x + c)^4 + 4*((a^2 + b^2
)*d*f*x + (a^2 + b^2)*d*e + a*b*f)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 +
```

```
b^2)*d*f*x + (a^2 + b^2)*d*e + a*b*f)*sinh(d*x + c)^4 + (a^2 + b^2)*d*f*x +
 (a^2 + b^2)*d*e + a*b*f - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*d*e + a*b*f)*
cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*d*e + a*b*f - 3*((a^2
+ b^2)*d*f*x + (a^2 + b^2)*d*e + a*b*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
4*(((a^2 + b^2)*d*f*x + (a^2 + b^2)*d*e + a*b*f)*cosh(d*x + c)^3 - ((a^2 +
b^2)*d*f*x + (a^2 + b^2)*d*e + a*b*f)*cosh(d*x + c))*sinh(d*x + c))*log(cos
h(d*x + c) + sinh(d*x + c) + 1) + (((a^2 + b^2)*d*e - (a*b + (a^2 + b^2)*c)
*f)*cosh(d*x + c)^4 + 4*((a^2 + b^2)*d*e - (a*b + (a^2 + b^2)*c)*f)*cosh(d*
x + c)*sinh(d*x + c)^3 + ((a^2 + b^2)*d*e - (a*b + (a^2 + b^2)*c)*f)*sinh(d
*x + c)^4 + (a^2 + b^2)*d*e - 2*((a^2 + b^2)*d*e - (a*b + (a^2 + b^2)*c)*f)
*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*e - 3*((a^2 + b^2)*d*e - (a*b + (a^2 +
b^2)*c)*f)*cosh(d*x + c)^2 - (a*b + (a^2 + b^2)*c)*f)*sinh(d*x + c)^2 - (a*
b + (a^2 + b^2)*c)*f + 4*(((a^2 + b^2)*d*e - (a*b + (a^2 + b^2)*c)*f)*cosh(
d*x + c)^3 - ((a^2 + b^2)*d*e - (a*b + (a^2 + b^2)*c)*f)*cosh(d*x + c))*sin
h(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + (((a^2 + b^2)*d*f*x +
(a^2 + b^2)*c*f)*cosh(d*x + c)^4 + 4*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*
cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*sinh(
d*x + c)^4 + (a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f - 2*((a^2 + b^2)*d*f*x + (
a^2 + b^2)*c*f)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f -
3*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
4*(((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)^3 - ((a^2 + b^2)*d*f
*x + (a^2 + b^2)*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-cosh(d*x + c) - si
nh(d*x + c) + 1) - 2*(a*b*d*f*x + a*b*d*e - 3*(a*b*d*f*x + a*b*d*e)*cosh(d*
x + c)^2 + (2*a^2*d*f*x + 2*a^2*d*e + a^2*f)*cosh(d*x + c))*sinh(d*x + c))/
(a^3*d^2*cosh(d*x + c)^4 + 4*a^3*d^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d^
2*sinh(d*x + c)^4 - 2*a^3*d^2*cosh(d*x + c)^2 + a^3*d^2 + 2*(3*a^3*d^2*cosh
(d*x + c)^2 - a^3*d^2)*sinh(d*x + c)^2 + 4*(a^3*d^2*cosh(d*x + c)^3 - a^3*d
^2*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e + fx\right)\coth^3\left(c + dx\right)}{a + b\sinh\left(c + dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*coth(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx + e\right)\coth\left(dx + c\right)^3}{b\sinh\left(dx + c\right) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*coth(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

3.489 $\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{(a^2+b^2)\log(\sinh(c+dx))}{a^3d}-\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a^3d}+\frac{b\operatorname{csch}(c+dx)}{a^2d}-\frac{\operatorname{csch}^2(c+dx)}{2ad}$$

```
[Out] (b*Csch[c + d*x])/(a^2*d) - Csch[c + d*x]^2/(2*a*d) + ((a^2 + b^2)*Log[Sinh
[c + d*x]])/(a^3*d) - ((a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/(a^3*d)
```

Rubi [A] time = 0.105165, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$\frac{(a^2+b^2)\log(\sinh(c+dx))}{a^3d}-\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a^3d}+\frac{b\operatorname{csch}(c+dx)}{a^2d}-\frac{\operatorname{csch}^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*Csch[c + d*x])/(a^2*d) - Csch[c + d*x]^2/(2*a*d) + ((a^2 + b^2)*Log[Sinh
[c + d*x]])/(a^3*d) - ((a^2 + b^2)*Log[a + b*Sinh[c + d*x]])/(a^3*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{-b^2-x^2}{x^3(a+x)}\,dx,x,b\sinh(c+dx)\right)}{d}\\ &= -\frac{\operatorname{Subst}\left(\int \left(-\frac{b^2}{ax^3}+\frac{b^2}{a^2x^2}+\frac{-a^2-b^2}{a^3x}+\frac{a^2+b^2}{a^3(a+x)}\right)dx,x,b\sinh(c+dx)\right)}{d}\\ &= \frac{b\operatorname{csch}(c+dx)}{a^2d}-\frac{\operatorname{csch}^2(c+dx)}{2ad}+\frac{(a^2+b^2)\log(\sinh(c+dx))}{a^3d}-\frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a^3d}\end{aligned}$$

Mathematica [A] time = 0.11834, size = 64, normalized size = 0.8

$$\frac{2(a^2+b^2)(\log(\sinh(c+dx))-\log(a+b\sinh(c+dx)))-a^2\operatorname{csch}^2(c+dx)+2ab\operatorname{csch}(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a*b*Csch[c + d*x] - a^2*Csch[c + d*x]^2 + 2*(a^2 + b^2)*(Log[Sinh[c + d*
x]] - Log[a + b*Sinh[c + d*x]]))/(2*a^3*d)
```

Maple [B] time = 0.001, size = 194, normalized size = 2.4

$$-\frac{1}{8da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{b}{2da^2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{8da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-2}+\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{b^2}{da^3}\ln\left(\tanh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/8/d/a*tanh(1/2*d*x+1/2*c)^2-1/2/d/a^2*tanh(1/2*d*x+1/2*c)*b-1/8/d/a/tanh
(1/2*d*x+1/2*c)^2+1/d/a*ln(tanh(1/2*d*x+1/2*c))+1/d/a^3*ln(tanh(1/2*d*x+1/2
*c))*b^2+1/2/d*b/a^2/tanh(1/2*d*x+1/2*c)-1/d/a*ln(tanh(1/2*d*x+1/2*c)^2*a-2
*tanh(1/2*d*x+1/2*c)*b-a)-1/d/a^3*ln(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x
+1/2*c)*b-a)*b^2
```

Maxima [B] time = 1.29418, size = 234, normalized size = 2.92

$$-\frac{2\left(be^{(-dx-c)}-ae^{(-2\,dx-2\,c)}-be^{(-3\,dx-3\,c)}\right)}{\left(2\,a^2e^{(-2\,dx-2\,c)}-a^2e^{(-4\,dx-4\,c)}-a^2\right)d}-\frac{\left(a^2+b^2\right)\log\left(-2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}-b\right)}{a^3d}+\frac{\left(a^2+b^2\right)\log\left(e^{(-dx-c)}+1\right)}{a^3d}+\frac{\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*e^(-2
*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - (a^2 + b^2)*log(-2*a*e^(-d*x
 - c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) + (a^2 + b^2)*log(e^(-d*x - c) + 1)
/(a^3*d) + (a^2 + b^2)*log(e^(-d*x - c) - 1)/(a^3*d)
```

Fricas [B] time = 1.84773, size = 1554, normalized size = 19.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*a*b*cosh(d*x + c)^3 + 2*a*b*sinh(d*x + c)^3 - 2*a^2*cosh(d*x + c)^2 - 2*
a*b*cosh(d*x + c) + 2*(3*a*b*cosh(d*x + c) - a^2)*sinh(d*x + c)^2 - ((a^2 +
 b^2)*cosh(d*x + c)^4 + 4*(a^2 + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2
+ b^2)*sinh(d*x + c)^4 - 2*(a^2 + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + b^2)*c
osh(d*x + c)^2 - a^2 - b^2)*sinh(d*x + c)^2 + a^2 + b^2 + 4*((a^2 + b^2)*co
sh(d*x + c)^3 - (a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x
 + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + ((a^2 + b^2)*cosh(d*x + c)^4
```

```
+ 4*(a^2 + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + b^2)*sinh(d*x + c)^4
 - 2*(a^2 + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + b^2)*cosh(d*x + c)^2 - a^2 -
 b^2)*sinh(d*x + c)^2 + a^2 + b^2 + 4*((a^2 + b^2)*cosh(d*x + c)^3 - (a^2 +
 b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - si
nh(d*x + c))) + 2*(3*a*b*cosh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - a*b)*sinh(
d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)*sinh(d*x + c)^3 +
a^3*d*sinh(d*x + c)^4 - 2*a^3*d*cosh(d*x + c)^2 + a^3*d + 2*(3*a^3*d*cosh(d
*x + c)^2 - a^3*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3 - a^3*d*cosh(
d*x + c))*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

Giac [B] time = 1.41714, size = 224, normalized size = 2.8

$$\frac{\frac{(a^2e^c+b^2e^c)e^{(-c)}\log\left(e^{(dx+c)}+1\right)}{a^3}+\frac{(a^2e^c+b^2e^c)e^{(-c)}\log\left(\left|e^{(dx+c)}-1\right|\right)}{a^3}-\frac{(a^2+b^2)\log\left(\left|be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}-b\right|\right)}{a^3}+\frac{2\left(abe^{(3\,dx+3\,c)}-a^2e^{(2\,dx+2\,c)}-abe^{(dx+c)}\right)}{a^3\left(e^{(dx+c)}+1\right)^2\left(e^{(dx+c)}-1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] ((a^2*e^c + b^2*e^c)*e^(-c)*log(e^(d*x + c) + 1)/a^3 + (a^2*e^c + b^2*e^c)*
e^(-c)*log(abs(e^(d*x + c) - 1))/a^3 - (a^2 + b^2)*log(abs(b*e^(2*d*x + 2*c
) + 2*a*e^(d*x + c) - b))/a^3 + 2*(a*b*e^(3*d*x + 3*c) - a^2*e^(2*d*x + 2*c
) - a*b*e^(d*x + c))/(a^3*(e^(d*x + c) + 1)^2*(e^(d*x + c) - 1)^2))/d
```

3.490 $\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[Coth[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi [A] time = 0.0761779, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][Coth[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Coth[c + d*x]^3/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.499, size = 0, normalized size = 0.

$$\int \frac{(\coth(dx+c))^3}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a*f - 2*(b*d*f*x*e^(3*c) + b*d*e*e^(3*c))*e^(3*d*x) + (2*a*d*f*x*e^(2*c)
+ (2*d*e - f)*a*e^(2*c))*e^(2*d*x) + 2*(b*d*f*x*e^c + b*d*e*e^c)*e^(d*x))/(
a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + a^2*d^2*e^2 + (a^2*d^2*f^2*x^2*e^(4*c)
+ 2*a^2*d^2*e*f*x*e^(4*c) + a^2*d^2*e^2*e^(4*c))*e^(4*d*x) - 2*(a^2*d^2*f^2
*x^2*e^(2*c) + 2*a^2*d^2*e*f*x*e^(2*c) + a^2*d^2*e^2*e^(2*c))*e^(2*d*x)) +
integrate(-(b^2*d^2*e^2 + a*b*d*e*f + (d^2*e^2 + f^2)*a^2 + (a^2*d^2*f^2 +
b^2*d^2*f^2)*x^2 + (2*a^2*d^2*e*f + 2*b^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*d^2*
f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 - (a^3*d^2*
f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3
*e^c)*e^(d*x)), x) - integrate((b^2*d^2*e^2 - a*b*d*e*f + (d^2*e^2 + f^2)*a
^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + (2*a^2*d^2*e*f + 2*b^2*d^2*e*f - a*b
*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3
*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f
*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) + integrate(2*(a^2*b + b^3 - (a^3*e^
c + a*b^2*e^c)*e^(d*x))/(a^3*b*f*x + a^3*b*e - (a^3*b*f*x*e^(2*c) + a^3*b*e
*e^(2*c))*e^(2*d*x) - 2*(a^4*f*x*e^c + a^4*e*e^c)*e^(d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\coth(dx+c)^3}{afx+ae+\left(bfx+be\right)\sinh(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(coth(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(c+dx)}{(a+b\sinh(c+dx))\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)**3/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.491 $\int \frac{(e+fx)^3\mathbf{csch}^3(c+dx)\mathbf{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1795

result too large to display

```
[Out] (-3*f*(e + f*x)^2)/(2*a*d^2) + (e + f*x)^3/(2*a*d) + (2*b*(e + f*x)^3*ArcTa
n[E^(c + d*x)])/(a^2*d) - (2*b^3*(e + f*x)^3*ArcTan[E^(c + d*x)])/(a^2*(a^2
 + b^2)*d) + (6*b*f*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d^2) + (2*(e + f
*x)^3*ArcTanh[E^(2*c + 2*d*x)])/(a*d) - (2*b^2*(e + f*x)^3*ArcTanh[E^(2*c +
 2*d*x)])/(a^3*d) - (3*f*(e + f*x)^2*Coth[c + d*x])/(2*a*d^2) - ((e + f*x)^
3*Coth[c + d*x]^2)/(2*a*d) + (b*(e + f*x)^3*Csch[c + d*x])/(a^2*d) - (b^4*(
e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)
*d) - (b^4*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3
*(a^2 + b^2)*d) + (3*f^2*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a*d^3) + (b^4
*(e + f*x)^3*Log[1 + E^(2*(c + d*x))])/(a^3*(a^2 + b^2)*d) + (6*b*f^2*(e +
f*x)*PolyLog[2, -E^(c + d*x)])/(a^2*d^3) - ((3*I)*b*f*(e + f*x)^2*PolyLog[2
, (-I)*E^(c + d*x)])/(a^2*d^2) + ((3*I)*b^3*f*(e + f*x)^2*PolyLog[2, (-I)*E
^(c + d*x)])/(a^2*(a^2 + b^2)*d^2) + ((3*I)*b*f*(e + f*x)^2*PolyLog[2, I*E^
(c + d*x)])/(a^2*d^2) - ((3*I)*b^3*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])
/(a^2*(a^2 + b^2)*d^2) - (6*b*f^2*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^2*d
^3) - (3*b^4*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/(a^3*(a^2 + b^2)*d^2) - (3*b^4*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^2) + (3*b^4*f*(e + f*x)^2*
PolyLog[2, -E^(2*(c + d*x))])/(2*a^3*(a^2 + b^2)*d^2) + (3*f^3*PolyLog[2, E
^(2*(c + d*x))])/(2*a*d^4) + (3*f*(e + f*x)^2*PolyLog[2, -E^(2*c + 2*d*x)])
/(2*a*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a^3*d^2)
 - (3*f*(e + f*x)^2*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d^2) + (3*b^2*f*(e +
f*x)^2*PolyLog[2, E^(2*c + 2*d*x)])/(2*a^3*d^2) - (6*b*f^3*PolyLog[3, -E^(c
 + d*x)])/(a^2*d^4) + ((6*I)*b*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/
(a^2*d^3) - ((6*I)*b^3*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(a^2*(a^
2 + b^2)*d^3) - ((6*I)*b*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a^2*d^3)
 + ((6*I)*b^3*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^3
) + (6*b*f^3*PolyLog[3, E^(c + d*x)])/(a^2*d^4) + (6*b^4*f^2*(e + f*x)*Poly
Log[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^3) + (
6*b^4*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
a^3*(a^2 + b^2)*d^3) - (3*b^4*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(
2*a^3*(a^2 + b^2)*d^3) - (3*f^2*(e + f*x)*PolyLog[3, -E^(2*c + 2*d*x)])/(2*
a*d^3) + (3*b^2*f^2*(e + f*x)*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a^3*d^3) + (
3*f^2*(e + f*x)*PolyLog[3, E^(2*c + 2*d*x)])/(2*a*d^3) - (3*b^2*f^2*(e + f*
x)*PolyLog[3, E^(2*c + 2*d*x)])/(2*a^3*d^3) - ((6*I)*b*f^3*PolyLog[4, (-I)*
E^(c + d*x)])/(a^2*d^4) + ((6*I)*b^3*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(a^2
*(a^2 + b^2)*d^4) + ((6*I)*b*f^3*PolyLog[4, I*E^(c + d*x)])/(a^2*d^4) - ((6
*I)*b^3*f^3*PolyLog[4, I*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^4) - (6*b^4*f^3*P
olyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^4)
- (6*b^4*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^
2 + b^2)*d^4) + (3*b^4*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*a^3*(a^2 + b^2)
*d^4) + (3*f^3*PolyLog[4, -E^(2*c + 2*d*x)])/(4*a*d^4) - (3*b^2*f^3*PolyLog
[4, -E^(2*c + 2*d*x)])/(4*a^3*d^4) - (3*f^3*PolyLog[4, E^(2*c + 2*d*x)])/(4
*a*d^4) + (3*b^2*f^3*PolyLog[4, E^(2*c + 2*d*x)])/(4*a^3*d^4)
```

Rubi [A] time = 3.27209, antiderivative size = 1795, normalized size of antiderivative = 1., number of steps used = 87, number of rules used = 28, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.824, Rules used = {5589, 2620, 14, 5462, 6741, 12, 6742, 3720, 3716, 2190, 2279, 2391, 32, 2551, 4182, 2531, 6609, 2282, 6589, 2621, 321, 207, 5205, 4180, 5461, 5573, 5561, 3718}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-3*f*(e + f*x)^2)/(2*a*d^2) + (e + f*x)^3/(2*a*d) + (2*b*(e + f*x)^3*ArcTa
n[E^(c + d*x)])/(a^2*d) - (2*b^3*(e + f*x)^3*ArcTan[E^(c + d*x)])/(a^2*(a^2
 + b^2)*d) + (6*b*f*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d^2) + (2*(e + f
*x)^3*ArcTanh[E^(2*c + 2*d*x)])/(a*d) - (2*b^2*(e + f*x)^3*ArcTanh[E^(2*c +
 2*d*x)])/(a^3*d) - (3*f*(e + f*x)^2*Coth[c + d*x])/(2*a*d^2) - ((e + f*x)^
3*Coth[c + d*x]^2)/(2*a*d) + (b*(e + f*x)^3*Csch[c + d*x])/(a^2*d) - (b^4*(
e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)
*d) - (b^4*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3
*(a^2 + b^2)*d) + (3*f^2*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a*d^3) + (b^4
*(e + f*x)^3*Log[1 + E^(2*(c + d*x))])/(a^3*(a^2 + b^2)*d) + (6*b*f^2*(e +
f*x)*PolyLog[2, -E^(c + d*x)])/(a^2*d^3) - ((3*I)*b*f*(e + f*x)^2*PolyLog[2
, (-I)*E^(c + d*x)])/(a^2*d^2) + ((3*I)*b^3*f*(e + f*x)^2*PolyLog[2, (-I)*E
^(c + d*x)])/(a^2*(a^2 + b^2)*d^2) + ((3*I)*b*f*(e + f*x)^2*PolyLog[2, I*E^
(c + d*x)])/(a^2*d^2) - ((3*I)*b^3*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])
/(a^2*(a^2 + b^2)*d^2) - (6*b*f^2*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^2*d
^3) - (3*b^4*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]))])/(a^3*(a^2 + b^2)*d^2) - (3*b^4*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^2) + (3*b^4*f*(e + f*x)^2*
PolyLog[2, -E^(2*(c + d*x))])/(2*a^3*(a^2 + b^2)*d^2) + (3*f^3*PolyLog[2, E
^(2*(c + d*x))])/(2*a*d^4) + (3*f*(e + f*x)^2*PolyLog[2, -E^(2*c + 2*d*x)])
/(2*a*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a^3*d^2)
 - (3*f*(e + f*x)^2*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d^2) + (3*b^2*f*(e +
f*x)^2*PolyLog[2, E^(2*c + 2*d*x)])/(2*a^3*d^2) - (6*b*f^3*PolyLog[3, -E^(c
 + d*x)])/(a^2*d^4) + ((6*I)*b*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/
(a^2*d^3) - ((6*I)*b^3*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(a^2*(a^
2 + b^2)*d^3) - ((6*I)*b*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a^2*d^3)
 + ((6*I)*b^3*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^3
) + (6*b*f^3*PolyLog[3, E^(c + d*x)])/(a^2*d^4) + (6*b^4*f^2*(e + f*x)*Poly
Log[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^3) + (
6*b^4*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
a^3*(a^2 + b^2)*d^3) - (3*b^4*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(
2*a^3*(a^2 + b^2)*d^3) - (3*f^2*(e + f*x)*PolyLog[3, -E^(2*c + 2*d*x)])/(2*
a*d^3) + (3*b^2*f^2*(e + f*x)*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a^3*d^3) + (
3*f^2*(e + f*x)*PolyLog[3, E^(2*c + 2*d*x)])/(2*a*d^3) - (3*b^2*f^2*(e + f*
x)*PolyLog[3, E^(2*c + 2*d*x)])/(2*a^3*d^3) - ((6*I)*b*f^3*PolyLog[4, (-I)*
E^(c + d*x)])/(a^2*d^4) + ((6*I)*b^3*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(a^2
*(a^2 + b^2)*d^4) + ((6*I)*b*f^3*PolyLog[4, I*E^(c + d*x)])/(a^2*d^4) - ((6
*I)*b^3*f^3*PolyLog[4, I*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^4) - (6*b^4*f^3*P
olyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^4)
- (6*b^4*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^
2 + b^2)*d^4) + (3*b^4*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*a^3*(a^2 + b^2)
*d^4) + (3*f^3*PolyLog[4, -E^(2*c + 2*d*x)])/(4*a*d^4) - (3*b^2*f^3*PolyLog
[4, -E^(2*c + 2*d*x)])/(4*a^3*d^4) - (3*f^3*PolyLog[4, E^(2*c + 2*d*x)])/(4
*a*d^4) + (3*b^2*f^3*PolyLog[4, E^(2*c + 2*d*x)])/(4*a^3*d^4)
```

Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
```

```
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^3\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^3\text{csch}^3(c+dx)\text{sech}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^3\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^3\coth^2(c+dx)}{2ad} - \frac{(e+fx)^3\log(\tanh(c+dx))}{ad} - \frac{b\int (e+fx)^3\text{csc}$$

$$= \frac{b(e+fx)^3\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{(e+fx)^3\coth^2(c+dx)}{2ad} + \frac{b(e+fx)^3\text{csc}}{a^2d}$$

$$= \frac{b(e+fx)^3\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{(e+fx)^3\coth^2(c+dx)}{2ad} + \frac{b(e+fx)^3\text{csc}}{a^2d}$$

$$= \frac{b^4(e+fx)^4}{4a^3\left(a^2+b^2\right)f} + \frac{b(e+fx)^3\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{2b^2(e+fx)^3\tanh^{-1}\left(e^{2}\right.}{a^3d}$$

$$= \frac{b^4(e+fx)^4}{4a^3\left(a^2+b^2\right)f} + \frac{b(e+fx)^3\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{2b^2(e+fx)^3\tanh^{-1}\left(e^{2}\right.}{a^3d}$$

$$= -\frac{2b^3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d} + \frac{b(e+fx)^3\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{2b^2(e+fx}{}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{2b^3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d} + \frac{6bf(e+fx)^2\tan}{a^2d^2}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{2b^3(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d} + \frac{6bf(e+fx)^2\tan}{a^2d^2}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} + \frac{2b(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^3(e+fx)^3\tan^{-}}{a^2\left(a^2+b^2\right)}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} + \frac{2b(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^3(e+fx)^3\tan^{-}}{a^2\left(a^2+b^2\right)}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} + \frac{2b(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^3(e+fx)^3\tan^{-}}{a^2\left(a^2+b^2\right)}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} + \frac{2b(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^3(e+fx)^3\tan^{-}}{a^2\left(a^2+b^2\right)}$$

$$= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} + \frac{2b(e+fx)^3\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^3(e+fx)^3\tan^{-}}{a^2\left(a^2+b^2\right)}$$

Mathematica [B] time = 87.9895, size = 9045, normalized size = 5.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 2.194, size = 0, normalized size = 0.

$$\int \frac{\left(fx+e\right)^{3} \left(\operatorname{csch}\left(dx+c\right)\right)^{3} \operatorname{sech}\left(dx+c\right)}{a+b\sinh\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -(b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + a^3*b^2)*d) +
 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a
^2 + b^2)*d) + 2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))
/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 - b^2)*lo
g(e^(-d*x - c) + 1)/(a^3*d) + (a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d))*e^
3 + (3*a*f^3*x^2 + 6*a*e*f^2*x + 3*a*e^2*f + 2*(b*d*f^3*x^3*e^(3*c) + 3*b*d
*e*f^2*x^2*e^(3*c) + 3*b*d*e^2*f*x*e^(3*c))*e^(3*d*x) - (2*a*d*f^3*x^3*e^(2
*c) + 3*a*e^2*f*e^(2*c) + 3*(2*d*e*f^2 + f^3)*a*x^2*e^(2*c) + 6*(d*e^2*f +
e*f^2)*a*x*e^(2*c))*e^(2*d*x) - 2*(b*d*f^3*x^3*e^c + 3*b*d*e*f^2*x^2*e^c +
3*b*d*e^2*f*x*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x +
 2*c) + a^2*d^2) - 3*(b*d*e^2*f + a*e*f^2)*x/(a^2*d^2) + 3*(b*d*e^2*f - a*e
*f^2)*x/(a^2*d^2) + 3*(b*d*e^2*f + a*e*f^2)*log(e^(d*x + c) + 1)/(a^2*d^3)
- 3*(b*d*e^2*f - a*e*f^2)*log(e^(d*x + c) - 1)/(a^2*d^3) - (d^3*x^3*log(e^(
d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x +
c)) + 6*polylog(4, -e^(d*x + c)))*(a^2*f^3 - b^2*f^3)/(a^3*d^4) - (d^3*x^3*
log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(
d*x + c)) + 6*polylog(4, e^(d*x + c)))*(a^2*f^3 - b^2*f^3)/(a^3*d^4) - 3*(a
^2*d*e*f^2 - b^2*d*e*f^2 - a*b*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*d
ilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^3*d^4) - 3*(a^2*d*e*f^2
 - b^2*d*e*f^2 + a*b*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d
*x + c)) - 2*polylog(3, e^(d*x + c)))/(a^3*d^4) + 3*(b^2*d^2*e^2*f + 2*a*b*
d*e*f^2 - (d^2*e^2*f - f^3)*a^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x
```

```
+ c)))/(a^3*d^4) + 3*(b^2*d^2*e^2*f - 2*a*b*d*e*f^2 - (d^2*e^2*f - f^3)*a^2
)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^3*d^4) + 1/4*((a^2*f^
3 - b^2*f^3)*d^4*x^4 + 4*(a^2*d*e*f^2 - b^2*d*e*f^2 + a*b*f^3)*d^3*x^3 - 6*
(b^2*d^2*e^2*f - 2*a*b*d*e*f^2 - (d^2*e^2*f - f^3)*a^2)*d^2*x^2)/(a^3*d^4)
+ 1/4*((a^2*f^3 - b^2*f^3)*d^4*x^4 + 4*(a^2*d*e*f^2 - b^2*d*e*f^2 - a*b*f^3
)*d^3*x^3 - 6*(b^2*d^2*e^2*f + 2*a*b*d*e*f^2 - (d^2*e^2*f - f^3)*a^2)*d^2*x
^2)/(a^3*d^4) + integrate(2*(b^5*f^3*x^3 + 3*b^5*e*f^2*x^2 + 3*b^5*e^2*f*x
- (a*b^4*f^3*x^3*e^c + 3*a*b^4*e*f^2*x^2*e^c + 3*a*b^4*e^2*f*x*e^c)*e^(d*x)
)/(a^5*b + a^3*b^3 - (a^5*b*e^(2*c) + a^3*b^3*e^(2*c))*e^(2*d*x) - 2*(a^6*e
^c + a^4*b^2*e^c)*e^(d*x)), x) + integrate(-2*(a*f^3*x^3 + 3*a*e*f^2*x^2 +
3*a*e^2*f*x - (b*f^3*x^3*e^c + 3*b*e*f^2*x^2*e^c + 3*b*e^2*f*x*e^c)*e^(d*x)
)/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)**3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.492 $\int \frac{(e+fx)^2\mathbf{csch}^3(c+dx)\mathbf{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1219

result too large to display

```
[Out] (e*f*x)/(a*d) + (f^2*x^2)/(2*a*d) + (2*b*(e + f*x)^2*ArcTan[E^(c + d*x)])/(
a^2*d) - (2*b^3*(e + f*x)^2*ArcTan[E^(c + d*x)])/(a^2*(a^2 + b^2)*d) + (4*b
*f*(e + f*x)*ArcTanh[E^(c + d*x)])/(a^2*d^2) + (2*(e + f*x)^2*ArcTanh[E^(2*
c + 2*d*x)])/(a*d) - (2*b^2*(e + f*x)^2*ArcTanh[E^(2*c + 2*d*x)])/(a^3*d) -
 (f*(e + f*x)*Coth[c + d*x])/(a*d^2) - ((e + f*x)^2*Coth[c + d*x]^2)/(2*a*d
) + (b*(e + f*x)^2*Csch[c + d*x])/(a^2*d) - (b^4*(e + f*x)^2*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)*d) - (b^4*(e + f*x)^2*Lo
g[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)*d) + (b^4*(e
 + f*x)^2*Log[1 + E^(2*(c + d*x))])/(a^3*(a^2 + b^2)*d) + (f^2*Log[Sinh[c +
 d*x]])/(a*d^3) + (2*b*f^2*PolyLog[2, -E^(c + d*x)])/(a^2*d^3) - ((2*I)*b*f
*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*d^2) + ((2*I)*b^3*f*(e + f*x)
*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^2) + ((2*I)*b*f*(e + f*x)
*PolyLog[2, I*E^(c + d*x)])/(a^2*d^2) - ((2*I)*b^3*f*(e + f*x)*PolyLog[2, I
*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^2) - (2*b*f^2*PolyLog[2, E^(c + d*x)])/(a
^2*d^3) - (2*b^4*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/(a^3*(a^2 + b^2)*d^2) - (2*b^4*f*(e + f*x)*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^2) + (b^4*f*(e + f*x)*Poly
Log[2, -E^(2*(c + d*x))])/(a^3*(a^2 + b^2)*d^2) + (f*(e + f*x)*PolyLog[2, -
E^(2*c + 2*d*x)])/(a*d^2) - (b^2*f*(e + f*x)*PolyLog[2, -E^(2*c + 2*d*x)])/
(a^3*d^2) - (f*(e + f*x)*PolyLog[2, E^(2*c + 2*d*x)])/(a*d^2) + (b^2*f*(e +
 f*x)*PolyLog[2, E^(2*c + 2*d*x)])/(a^3*d^2) + ((2*I)*b*f^2*PolyLog[3, (-I)
*E^(c + d*x)])/(a^2*d^3) - ((2*I)*b^3*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(a^
2*(a^2 + b^2)*d^3) - ((2*I)*b*f^2*PolyLog[3, I*E^(c + d*x)])/(a^2*d^3) + ((
2*I)*b^3*f^2*PolyLog[3, I*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^3) + (2*b^4*f^2*
PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^3)
 + (2*b^4*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a
^2 + b^2)*d^3) - (b^4*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*a^3*(a^2 + b^2)*
d^3) - (f^2*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a*d^3) + (b^2*f^2*PolyLog[3, -
E^(2*c + 2*d*x)])/(2*a^3*d^3) + (f^2*PolyLog[3, E^(2*c + 2*d*x)])/(2*a*d^3)
 - (b^2*f^2*PolyLog[3, E^(2*c + 2*d*x)])/(2*a^3*d^3)
```

Rubi [A] time = 2.22243, antiderivative size = 1219, normalized size of antiderivative = 1., number of steps used = 71, number of rules used = 26, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.765, Rules used = {5589, 2620, 14, 5462, 6741, 12, 6742, 3720, 3475, 2551, 4182, 2531, 2282, 6589, 2621, 321, 207, 5205, 4180, 2279, 2391, 5461, 5573, 5561, 2190, 3718}

$$-\frac{(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)b^4}{a^3\left(a^2+b^2\right)d}-\frac{(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)b^4}{a^3\left(a^2+b^2\right)d}+\frac{(e+fx)^2\log\left(1+e^{2(c+dx)}\right)b^4}{a^3\left(a^2+b^2\right)d}-\frac{2f(e+fx)\text{PolyL}}{a^3\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (e*f*x)/(a*d) + (f^2*x^2)/(2*a*d) + (2*b*(e + f*x)^2*ArcTan[E^(c + d*x)])/(
a^2*d) - (2*b^3*(e + f*x)^2*ArcTan[E^(c + d*x)])/(a^2*(a^2 + b^2)*d) + (4*b
*f*(e + f*x)*ArcTanh[E^(c + d*x)])/(a^2*d^2) + (2*(e + f*x)^2*ArcTanh[E^(2*
c + 2*d*x)])/(a*d) - (2*b^2*(e + f*x)^2*ArcTanh[E^(2*c + 2*d*x)])/(a^3*d) -
 (f*(e + f*x)*Coth[c + d*x])/(a*d^2) - ((e + f*x)^2*Coth[c + d*x]^2)/(2*a*d
) + (b*(e + f*x)^2*Csch[c + d*x])/(a^2*d) - (b^4*(e + f*x)^2*Log[1 + (b*E^(
```

```
c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)*d) - (b^4*(e + f*x)^2*Lo
g[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)*d) + (b^4*(e
 + f*x)^2*Log[1 + E^(2*(c + d*x))])/(a^3*(a^2 + b^2)*d) + (f^2*Log[Sinh[c +
 d*x]])/(a*d^3) + (2*b*f^2*PolyLog[2, -E^(c + d*x)])/(a^2*d^3) - ((2*I)*b*f
*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*d^2) + ((2*I)*b^3*f*(e + f*x)
*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^2) + ((2*I)*b*f*(e + f*x)
*PolyLog[2, I*E^(c + d*x)])/(a^2*d^2) - ((2*I)*b^3*f*(e + f*x)*PolyLog[2, I
*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^2) - (2*b*f^2*PolyLog[2, E^(c + d*x)])/(a
^2*d^3) - (2*b^4*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/(a^3*(a^2 + b^2)*d^2) - (2*b^4*f*(e + f*x)*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^2) + (b^4*f*(e + f*x)*Poly
Log[2, -E^(2*(c + d*x))])/(a^3*(a^2 + b^2)*d^2) + (f*(e + f*x)*PolyLog[2, -
E^(2*c + 2*d*x)])/(a*d^2) - (b^2*f*(e + f*x)*PolyLog[2, -E^(2*c + 2*d*x)])/
(a^3*d^2) - (f*(e + f*x)*PolyLog[2, E^(2*c + 2*d*x)])/(a*d^2) + (b^2*f*(e +
 f*x)*PolyLog[2, E^(2*c + 2*d*x)])/(a^3*d^2) + ((2*I)*b*f^2*PolyLog[3, (-I)
*E^(c + d*x)])/(a^2*d^3) - ((2*I)*b^3*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(a^
2*(a^2 + b^2)*d^3) - ((2*I)*b*f^2*PolyLog[3, I*E^(c + d*x)])/(a^2*d^3) + ((
2*I)*b^3*f^2*PolyLog[3, I*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^3) + (2*b^4*f^2*
PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^3)
 + (2*b^4*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a
^2 + b^2)*d^3) - (b^4*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*a^3*(a^2 + b^2)*
d^3) - (f^2*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a*d^3) + (b^2*f^2*PolyLog[3, -
E^(2*c + 2*d*x)])/(2*a^3*d^3) + (f^2*PolyLog[3, E^(2*c + 2*d*x)])/(2*a*d^3)
 - (b^2*f^2*PolyLog[3, E^(2*c + 2*d*x)])/(2*a^3*d^3)
```

Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
```

```
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)^2\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)^2\coth^2(c+dx)}{2ad} - \frac{(e+fx)^2\log(\tanh(c+dx))}{ad} - \frac{b\int (e+fx)^2\operatorname{csc}$$

$$= \frac{b(e+fx)^2\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{(e+fx)^2\coth^2(c+dx)}{2ad} + \frac{b(e+fx)^2\operatorname{csc}}{a^2d}$$

$$= \frac{b(e+fx)^2\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{(e+fx)^2\coth^2(c+dx)}{2ad} + \frac{b(e+fx)^2\operatorname{csc}}{a^2d}$$

$$= \frac{b^4(e+fx)^3}{3a^3\left(a^2+b^2\right)f} + \frac{b(e+fx)^2\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{2}\right.}{a^3d}$$

$$= \frac{b^4(e+fx)^3}{3a^3\left(a^2+b^2\right)f} + \frac{b(e+fx)^2\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{2b^2(e+fx)^2\tanh^{-1}\left(e^{2}\right.}{a^3d}$$

$$= -\frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d} + \frac{b(e+fx)^2\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{2b^2(e+fx}{}$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d} + \frac{4bf(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} - 2$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d} + \frac{4bf(e+fx)\tanh^{-1}\left(e^{c+dx}\right)}{a^2d^2} + 2$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} + \frac{2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d} + 4b$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} + \frac{2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d} + 4b$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} + \frac{2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d} + 4b$$

$$= \frac{efx}{ad} + \frac{f^2x^2}{2ad} + \frac{2b(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^3(e+fx)^2\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d} + 4b$$

Mathematica [B] time = 42.4488, size = 2784, normalized size = 2.28

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (b*(e + f*x)^2*Csch[c])/(a^2*d) - ((e + f*x)^2*Csch[(c + d*x)/2]^2)/(8*a*d)
 + (-12*a*d^3*e^2*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*d^3*e*f*x
^2 + 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] - 6*a
*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*b*d*e*(1
 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - Poly
```

Log[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*e*(1 + E^(2*c
))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]
) + (6*I)*b*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log
[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2,
 I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*
x)]) - a*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))])
- 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))]))/(6*
(a^2 + b^2)*d^3*(1 + E^(2*c))) - (-12*a^2*d^3*e^2*E^(2*c)*x + 12*b^2*d^3*e^
2*E^(2*c)*x + 12*a^2*d*E^(2*c)*f^2*x - 12*a^2*d^3*e*E^(2*c)*f*x^2 + 12*b^2*
d^3*e*E^(2*c)*f*x^2 - 4*a^2*d^3*E^(2*c)*f^2*x^3 + 4*b^2*d^3*E^(2*c)*f^2*x^3
 + 24*a*b*d*e*f*ArcTanh[E^(c + d*x)] - 24*a*b*d*e*E^(2*c)*f*ArcTanh[E^(c +
d*x)] - 12*a*b*d*f^2*x*Log[1 - E^(c + d*x)] + 12*a*b*d*E^(2*c)*f^2*x*Log[1
- E^(c + d*x)] + 12*a*b*d*f^2*x*Log[1 + E^(c + d*x)] - 12*a*b*d*E^(2*c)*f^2
*x*Log[1 + E^(c + d*x)] - 6*a^2*d^2*e^2*Log[1 - E^(2*(c + d*x))] + 6*b^2*d^
2*e^2*Log[1 - E^(2*(c + d*x))] + 6*a^2*d^2*e^2*E^(2*c)*Log[1 - E^(2*(c + d*
x))] - 6*b^2*d^2*e^2*E^(2*c)*Log[1 - E^(2*(c + d*x))] + 6*a^2*f^2*Log[1 - E
^(2*(c + d*x))] - 6*a^2*E^(2*c)*f^2*Log[1 - E^(2*(c + d*x))] - 12*a^2*d^2*e
*f*x*Log[1 - E^(2*(c + d*x))] + 12*b^2*d^2*e*f*x*Log[1 - E^(2*(c + d*x))] +
 12*a^2*d^2*e*E^(2*c)*f*x*Log[1 - E^(2*(c + d*x))] - 12*b^2*d^2*e*E^(2*c)*f
*x*Log[1 - E^(2*(c + d*x))] - 6*a^2*d^2*f^2*x^2*Log[1 - E^(2*(c + d*x))] +
6*b^2*d^2*f^2*x^2*Log[1 - E^(2*(c + d*x))] + 6*a^2*d^2*E^(2*c)*f^2*x^2*Log[
1 - E^(2*(c + d*x))] - 6*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(2*(c + d*x))] -
 12*a*b*(-1 + E^(2*c))*f^2*PolyLog[2, -E^(c + d*x)] + 12*a*b*(-1 + E^(2*c))
*f^2*PolyLog[2, E^(c + d*x)] - 6*a^2*d*e*f*PolyLog[2, E^(2*(c + d*x))] + 6*
b^2*d*e*f*PolyLog[2, E^(2*(c + d*x))] + 6*a^2*d*e*E^(2*c)*f*PolyLog[2, E^(2
*(c + d*x))] - 6*b^2*d*e*E^(2*c)*f*PolyLog[2, E^(2*(c + d*x))] - 6*a^2*d*f^
2*x*PolyLog[2, E^(2*(c + d*x))] + 6*b^2*d*f^2*x*PolyLog[2, E^(2*(c + d*x))]
 + 6*a^2*d*E^(2*c)*f^2*x*PolyLog[2, E^(2*(c + d*x))] - 6*b^2*d*E^(2*c)*f^2*
x*PolyLog[2, E^(2*(c + d*x))] + 3*a^2*f^2*PolyLog[3, E^(2*(c + d*x))] - 3*b
^2*f^2*PolyLog[3, E^(2*(c + d*x))] - 3*a^2*E^(2*c)*f^2*PolyLog[3, E^(2*(c +
 d*x))] + 3*b^2*E^(2*c)*f^2*PolyLog[3, E^(2*(c + d*x))])/(6*a^3*d^3*(-1 + E
^(2*c))) + (b^4*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 +
(6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqr
t[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*
E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b
^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2
)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sq
rt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1
 + E^(2*(c + d*x)))])/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2
*(c + d*x)))])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 +
b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sq
rt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c
- Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d
*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*Log[1 + (b*E^(2*c +
 d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(2*c)*f*x*Log[1 + (
b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1
 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c)*f
^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d -
(6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[
(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((
b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*f^2*PolyLo
g[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^
(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]
))])/d^3 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*
E^(2*c)]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + S
qrt[(a^2 + b^2)*E^(2*c)]))])/d^3))/(3*a^3*(a^2 + b^2)*(-1 + E^(2*c))) - (a*
x*(3*e^2 + 3*e*f*x + f^2*x^2)*Csch[c]*Sech[c])/(3*(a^2 + b^2)) + ((e + f*x)
^2*Sech[(c + d*x)/2]^2)/(8*a*d) - ((e + f*x)*(-(a*f) + b*d*(e + f*x))*Csch[
c/2]*Csch[(c + d*x)/2]*Sinh[(d*x)/2])/(2*a^2*d^2) - ((e + f*x)*(a*f + b*d*(

```
e + f*x))*Sech[c/2]*Sech[(c + d*x)/2]*Sinh[(d*x)/2])/(2*a^2*d^2)
```

Maple [F] time = 1.398, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 (\operatorname{csch}(dx+c))^3 \operatorname{sech}(dx+c)}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -(b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + a^3*b^2)*d) +
 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a
^2 + b^2)*d) + 2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))
/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 - b^2)*lo
g(e^(-d*x - c) + 1)/(a^3*d) + (a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d))*e^
2 + 2*(a*f^2*x + a*e*f + (b*d*f^2*x^2*e^(3*c) + 2*b*d*e*f*x*e^(3*c))*e^(3*d
*x) - (a*d*f^2*x^2*e^(2*c) + a*e*f*e^(2*c) + (2*d*e*f + f^2)*a*x*e^(2*c))*e
^(2*d*x) - (b*d*f^2*x^2*e^c + 2*b*d*e*f*x*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x +
 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) - (2*b*d*e*f + a*f^2)*x/(a^2*d
^2) + (2*b*d*e*f - a*f^2)*x/(a^2*d^2) + (2*b*d*e*f + a*f^2)*log(e^(d*x + c)
 + 1)/(a^2*d^3) - (2*b*d*e*f - a*f^2)*log(e^(d*x + c) - 1)/(a^2*d^3) - (d^2
*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*
x + c)))*(a^2*f^2 - b^2*f^2)/(a^3*d^3) - (d^2*x^2*log(-e^(d*x + c) + 1) + 2
*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))*(a^2*f^2 - b^2*f^2)/(a
^3*d^3) - 2*(a^2*d*e*f - b^2*d*e*f - a*b*f^2)*(d*x*log(e^(d*x + c) + 1) + d
ilog(-e^(d*x + c)))/(a^3*d^3) - 2*(a^2*d*e*f - b^2*d*e*f + a*b*f^2)*(d*x*lo
g(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^3*d^3) + 1/3*((a^2*f^2 - b^2*f
^2)*d^3*x^3 + 3*(a^2*d*e*f - b^2*d*e*f + a*b*f^2)*d^2*x^2)/(a^3*d^3) + 1/3*
((a^2*f^2 - b^2*f^2)*d^3*x^3 + 3*(a^2*d*e*f - b^2*d*e*f - a*b*f^2)*d^2*x^2)
/(a^3*d^3) + integrate(2*(b^5*f^2*x^2 + 2*b^5*e*f*x - (a*b^4*f^2*x^2*e^c +
2*a*b^4*e*f*x*e^c)*e^(d*x))/(a^5*b + a^3*b^3 - (a^5*b*e^(2*c) + a^3*b^3*e^(
2*c))*e^(2*d*x) - 2*(a^6*e^c + a^4*b^2*e^c)*e^(d*x)), x) + integrate(-2*(a*
f^2*x^2 + 2*a*e*f*x - (b*f^2*x^2*e^c + 2*b*e*f*x*e^c)*e^(d*x))/(a^2 + b^2 +
 (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [C] time = 6.1085, size = 29429, normalized size = 24.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] -(2*((a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*c*f^2)*cosh(d*x + c)^4 + 2*(
(a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*c*f^2)*sinh(d*x + c)^4 - 2*(a^4 +
 a^2*b^2)*d*e*f + 2*(a^4 + a^2*b^2)*c*f^2 - 2*((a^3*b + a*b^3)*d^2*f^2*x^2
+ 2*(a^3*b + a*b^3)*d^2*e*f*x + (a^3*b + a*b^3)*d^2*e^2)*cosh(d*x + c)^3 -
2*((a^3*b + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x + (a^3*b + a*b
^3)*d^2*e^2 - 4*((a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*c*f^2)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 2*((a^4 + a^2*b^2)*d^2*f^2*x^2 + (a^4 + a^2*b^2)*d^
2*e^2 + (a^4 + a^2*b^2)*d*e*f - 2*(a^4 + a^2*b^2)*c*f^2 + (2*(a^4 + a^2*b^2
)*d^2*e*f - (a^4 + a^2*b^2)*d*f^2)*x)*cosh(d*x + c)^2 + 2*((a^4 + a^2*b^2)*
d^2*f^2*x^2 + (a^4 + a^2*b^2)*d^2*e^2 + (a^4 + a^2*b^2)*d*e*f - 2*(a^4 + a^
2*b^2)*c*f^2 + 6*((a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*c*f^2)*cosh(d*x
 + c)^2 + (2*(a^4 + a^2*b^2)*d^2*e*f - (a^4 + a^2*b^2)*d*f^2)*x - 3*((a^3*b
 + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x + (a^3*b + a*b^3)*d^2*e
^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*((a^3*b + a*b^3)*d^2*f^2*x^2 + 2*(a^
3*b + a*b^3)*d^2*e*f*x + (a^3*b + a*b^3)*d^2*e^2)*cosh(d*x + c) + 2*(b^4*d*
f^2*x + b^4*d*e*f + (b^4*d*f^2*x + b^4*d*e*f)*cosh(d*x + c)^4 + 4*(b^4*d*f^
2*x + b^4*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*d*f^2*x + b^4*d*e*f)*
sinh(d*x + c)^4 - 2*(b^4*d*f^2*x + b^4*d*e*f)*cosh(d*x + c)^2 - 2*(b^4*d*f^
2*x + b^4*d*e*f - 3*(b^4*d*f^2*x + b^4*d*e*f)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 + 4*((b^4*d*f^2*x + b^4*d*e*f)*cosh(d*x + c)^3 - (b^4*d*f^2*x + b^4*d*e
*f)*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c)
+ (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2
*(b^4*d*f^2*x + b^4*d*e*f + (b^4*d*f^2*x + b^4*d*e*f)*cosh(d*x + c)^4 + 4*(
b^4*d*f^2*x + b^4*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*d*f^2*x + b^4
*d*e*f)*sinh(d*x + c)^4 - 2*(b^4*d*f^2*x + b^4*d*e*f)*cosh(d*x + c)^2 - 2*(
b^4*d*f^2*x + b^4*d*e*f - 3*(b^4*d*f^2*x + b^4*d*e*f)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + 4*((b^4*d*f^2*x + b^4*d*e*f)*cosh(d*x + c)^3 - (b^4*d*f^2*x +
 b^4*d*e*f)*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d
*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b
+ 1) + 2*((a^4 - b^4)*d*f^2*x + ((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f +
(a^3*b + a*b^3)*f^2)*cosh(d*x + c)^4 + 4*((a^4 - b^4)*d*f^2*x + (a^4 - b^4)
*d*e*f + (a^3*b + a*b^3)*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 - b^4)*
d*f^2*x + (a^4 - b^4)*d*e*f + (a^3*b + a*b^3)*f^2)*sinh(d*x + c)^4 + (a^4 -
 b^4)*d*e*f + (a^3*b + a*b^3)*f^2 - 2*((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*
e*f + (a^3*b + a*b^3)*f^2)*cosh(d*x + c)^2 - 2*((a^4 - b^4)*d*f^2*x + (a^4
- b^4)*d*e*f + (a^3*b + a*b^3)*f^2 - 3*((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d
*e*f + (a^3*b + a*b^3)*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^4 - b
^4)*d*f^2*x + (a^4 - b^4)*d*e*f + (a^3*b + a*b^3)*f^2)*cosh(d*x + c)^3 - ((
a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f + (a^3*b + a*b^3)*f^2)*cosh(d*x + c)
)*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)) - (2*a^4*d*f^2*x + 2*
I*a^3*b*d*f^2*x + 2*a^4*d*e*f + 2*I*a^3*b*d*e*f + (2*a^4*d*f^2*x + 2*I*a^3*
b*d*f^2*x + 2*a^4*d*e*f + 2*I*a^3*b*d*e*f)*cosh(d*x + c)^4 + (8*a^4*d*f^2*x
 + 8*I*a^3*b*d*f^2*x + 8*a^4*d*e*f + 8*I*a^3*b*d*e*f)*cosh(d*x + c)*sinh(d*
x + c)^3 + (2*a^4*d*f^2*x + 2*I*a^3*b*d*f^2*x + 2*a^4*d*e*f + 2*I*a^3*b*d*e
*f)*sinh(d*x + c)^4 - (4*a^4*d*f^2*x + 4*I*a^3*b*d*f^2*x + 4*a^4*d*e*f + 4*
I*a^3*b*d*e*f)*cosh(d*x + c)^2 - (4*a^4*d*f^2*x + 4*I*a^3*b*d*f^2*x + 4*a^4
*d*e*f + 4*I*a^3*b*d*e*f - (12*a^4*d*f^2*x + 12*I*a^3*b*d*f^2*x + 12*a^4*d*
e*f + 12*I*a^3*b*d*e*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a^4*d*f^2*x
+ 8*I*a^3*b*d*f^2*x + 8*a^4*d*e*f + 8*I*a^3*b*d*e*f)*cosh(d*x + c)^3 - (8*a
^4*d*f^2*x + 8*I*a^3*b*d*f^2*x + 8*a^4*d*e*f + 8*I*a^3*b*d*e*f)*cosh(d*x +
c))*sinh(d*x + c))*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) - (2*a^4*d*f^2*
x - 2*I*a^3*b*d*f^2*x + 2*a^4*d*e*f - 2*I*a^3*b*d*e*f + (2*a^4*d*f^2*x - 2*
I*a^3*b*d*f^2*x + 2*a^4*d*e*f - 2*I*a^3*b*d*e*f)*cosh(d*x + c)^4 + (8*a^4*d
*f^2*x - 8*I*a^3*b*d*f^2*x + 8*a^4*d*e*f - 8*I*a^3*b*d*e*f)*cosh(d*x + c)*s
inh(d*x + c)^3 + (2*a^4*d*f^2*x - 2*I*a^3*b*d*f^2*x + 2*a^4*d*e*f - 2*I*a^3
*b*d*e*f)*sinh(d*x + c)^4 - (4*a^4*d*f^2*x - 4*I*a^3*b*d*f^2*x + 4*a^4*d*e*
```

f - 4*I*a^3*b*d*e*f)*cosh(d*x + c)^2 - (4*a^4*d*f^2*x - 4*I*a^3*b*d*f^2*x + 4*a^4*d*e*f - 4*I*a^3*b*d*e*f - (12*a^4*d*f^2*x - 12*I*a^3*b*d*f^2*x + 12*a^4*d*e*f - 12*I*a^3*b*d*e*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a^4*d*f^2*x - 8*I*a^3*b*d*f^2*x + 8*a^4*d*e*f - 8*I*a^3*b*d*e*f)*cosh(d*x + c)^3 - (8*a^4*d*f^2*x - 8*I*a^3*b*d*f^2*x + 8*a^4*d*e*f - 8*I*a^3*b*d*e*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + 2*((a^4 - b^4)*d*f^2*x + ((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f - (a^3*b + a*b^3)*f^2)*cosh(d*x + c)^4 + 4*((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f - (a^3*b + a*b^3)*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f - (a^3*b + a*b^3)*f^2)*sinh(d*x + c)^4 + (a^4 - b^4)*d*e*f - (a^3*b + a*b^3)*f^2 - 2*((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f - (a^3*b + a*b^3)*f^2)*cosh(d*x + c)^2 - 2*((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f - (a^3*b + a*b^3)*f^2 - 3*((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f - (a^3*b + a*b^3)*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f - (a^3*b + a*b^3)*f^2)*cosh(d*x + c)^3 - ((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f - (a^3*b + a*b^3)*f^2)*cosh(d*x + c))*sinh(d*x + c))*dilog(-cosh(d*x + c) - sinh(d*x + c)) + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2 + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*cosh(d*x + c)^4 + 4*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*sinh(d*x + c)^4 - 2*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*cosh(d*x + c)^2 - 2*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2 - 3*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*cosh(d*x + c)^3 - (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2 + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*cosh(d*x + c)^4 + 4*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*sinh(d*x + c)^4 - 2*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*cosh(d*x + c)^2 - 2*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2 - 3*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*cosh(d*x + c)^3 - (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2 + (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh(d*x + c)^4 + 4*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*sinh(d*x + c)^4 - 2*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh(d*x + c)^2 - 2*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2 - 3*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh(d*x + c)^3 - (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2 + (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh(d*x + c)^4 + 4*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*sinh(d*x + c)^4 - 2*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh(d*x + c)^2 - 2*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2 - 3*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh(d*x + c)^3 - (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + ((a^4 - b^4)*d^2*f^2*x^2 + (a^4 - b^4)*d^2*e^2 + ((a^4 - b^4)*d^2*f^2*x^2 + (a^4 - b^4)*d^2*e^2

- 2*(a^3*b + a*b^3)*d*e*f - (a^4 + a^2*b^2)*f^2 + 2*((a^4 - b^4)*d^2*e*f - (a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c)^4 + 4*((a^4 - b^4)*d^2*f^2*x^2 + (a^4 - b^4)*d^2*e^2 - 2*(a^3*b + a*b^3)*d*e*f - (a^4 + a^2*b^2)*f^2 + 2*((a^ 4 - b^4)*d^2*e*f - (a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 - b^4)*d^2*f^2*x^2 + (a^4 - b^4)*d^2*e^2 - 2*(a^3*b + a*b^3)*d*e*f - (a^4 + a^2*b^2)*f^2 + 2*((a^4 - b^4)*d^2*e*f - (a^3*b + a*b^3)*d*f^2)*x)* sinh(d*x + c)^4 - 2*(a^3*b + a*b^3)*d*e*f - (a^4 + a^2*b^2)*f^2 - 2*((a^4 - b^4)*d^2*f^2*x^2 + (a^4 - b^4)*d^2*e^2 - 2*(a^3*b + a*b^3)*d*e*f - (a^4 + a^2*b^2)*f^2 + 2*((a^4 - b^4)*d^2*e*f - (a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c)^2 - 2*((a^4 - b^4)*d^2*f^2*x^2 + (a^4 - b^4)*d^2*e^2 - 2*(a^3*b + a*b^ 3)*d*e*f - (a^4 + a^2*b^2)*f^2 - 3*((a^4 - b^4)*d^2*f^2*x^2 + (a^4 - b^4)*d ^2*e^2 - 2*(a^3*b + a*b^3)*d*e*f - (a^4 + a^2*b^2)*f^2 + 2*((a^4 - b^4)*d^2 *e*f - (a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c)^2 + 2*((a^4 - b^4)*d^2*e*f - (a^3*b + a*b^3)*d*f^2)*x)*sinh(d*x + c)^2 + 2*((a^4 - b^4)*d^2*e*f - (a^3* b + a*b^3)*d*f^2)*x + 4*(((a^4 - b^4)*d^2*f^2*x^2 + (a^4 - b^4)*d^2*e^2 - 2 *(a^3*b + a*b^3)*d*e*f - (a^4 + a^2*b^2)*f^2 + 2*((a^4 - b^4)*d^2*e*f - (a^ 3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c)^3 - ((a^4 - b^4)*d^2*f^2*x^2 + (a^4 - b^4)*d^2*e^2 - 2*(a^3*b + a*b^3)*d*e*f - (a^4 + a^2*b^2)*f^2 + 2*((a^4 - b^ 4)*d^2*e*f - (a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))*log(co sh(d*x + c) + sinh(d*x + c) + 1) - (a^4*d^2*e^2 + I*a^3*b*d^2*e^2 - 2*a^4*c *d*e*f - 2*I*a^3*b*c*d*e*f + a^4*c^2*f^2 + I*a^3*b*c^2*f^2 + (a^4*d^2*e^2 + I*a^3*b*d^2*e^2 - 2*a^4*c*d*e*f - 2*I*a^3*b*c*d*e*f + a^4*c^2*f^2 + I*a^3* b*c^2*f^2)*cosh(d*x + c)^4 + (4*a^4*d^2*e^2 + 4*I*a^3*b*d^2*e^2 - 8*a^4*c*d *e*f - 8*I*a^3*b*c*d*e*f + 4*a^4*c^2*f^2 + 4*I*a^3*b*c^2*f^2)*cosh(d*x + c) *sinh(d*x + c)^3 + (a^4*d^2*e^2 + I*a^3*b*d^2*e^2 - 2*a^4*c*d*e*f - 2*I*a^3 *b*c*d*e*f + a^4*c^2*f^2 + I*a^3*b*c^2*f^2)*sinh(d*x + c)^4 - (2*a^4*d^2*e^ 2 + 2*I*a^3*b*d^2*e^2 - 4*a^4*c*d*e*f - 4*I*a^3*b*c*d*e*f + 2*a^4*c^2*f^2 + 2*I*a^3*b*c^2*f^2)*cosh(d*x + c)^2 - (2*a^4*d^2*e^2 + 2*I*a^3*b*d^2*e^2 - 4*a^4*c*d*e*f - 4*I*a^3*b*c*d*e*f + 2*a^4*c^2*f^2 + 2*I*a^3*b*c^2*f^2 - (6* a^4*d^2*e^2 + 6*I*a^3*b*d^2*e^2 - 12*a^4*c*d*e*f - 12*I*a^3*b*c*d*e*f + 6*a ^4*c^2*f^2 + 6*I*a^3*b*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((4*a^4* d^2*e^2 + 4*I*a^3*b*d^2*e^2 - 8*a^4*c*d*e*f - 8*I*a^3*b*c*d*e*f + 4*a^4*c^2 *f^2 + 4*I*a^3*b*c^2*f^2)*cosh(d*x + c)^3 - (4*a^4*d^2*e^2 + 4*I*a^3*b*d^2* e^2 - 8*a^4*c*d*e*f - 8*I*a^3*b*c*d*e*f + 4*a^4*c^2*f^2 + 4*I*a^3*b*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + I) - (a ^4*d^2*e^2 - I*a^3*b*d^2*e^2 - 2*a^4*c*d*e*f + 2*I*a^3*b*c*d*e*f + a^4*c^2* f^2 - I*a^3*b*c^2*f^2 + (a^4*d^2*e^2 - I*a^3*b*d^2*e^2 - 2*a^4*c*d*e*f + 2* I*a^3*b*c*d*e*f + a^4*c^2*f^2 - I*a^3*b*c^2*f^2)*cosh(d*x + c)^4 + (4*a^4*d ^2*e^2 - 4*I*a^3*b*d^2*e^2 - 8*a^4*c*d*e*f + 8*I*a^3*b*c*d*e*f + 4*a^4*c^2* f^2 - 4*I*a^3*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*d^2*e^2 - I*a ^3*b*d^2*e^2 - 2*a^4*c*d*e*f + 2*I*a^3*b*c*d*e*f + a^4*c^2*f^2 - I*a^3*b*c^ 2*f^2)*sinh(d*x + c)^4 - (2*a^4*d^2*e^2 - 2*I*a^3*b*d^2*e^2 - 4*a^4*c*d*e*f + 4*I*a^3*b*c*d*e*f + 2*a^4*c^2*f^2 - 2*I*a^3*b*c^2*f^2)*cosh(d*x + c)^2 - (2*a^4*d^2*e^2 - 2*I*a^3*b*d^2*e^2 - 4*a^4*c*d*e*f + 4*I*a^3*b*c*d*e*f + 2 *a^4*c^2*f^2 - 2*I*a^3*b*c^2*f^2 - (6*a^4*d^2*e^2 - 6*I*a^3*b*d^2*e^2 - 12* a^4*c*d*e*f + 12*I*a^3*b*c*d*e*f + 6*a^4*c^2*f^2 - 6*I*a^3*b*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((4*a^4*d^2*e^2 - 4*I*a^3*b*d^2*e^2 - 8*a^4*c *d*e*f + 8*I*a^3*b*c*d*e*f + 4*a^4*c^2*f^2 - 4*I*a^3*b*c^2*f^2)*cosh(d*x + c)^3 - (4*a^4*d^2*e^2 - 4*I*a^3*b*d^2*e^2 - 8*a^4*c*d*e*f + 8*I*a^3*b*c*d*e *f + 4*a^4*c^2*f^2 - 4*I*a^3*b*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(c osh(d*x + c) + sinh(d*x + c) - I) + ((a^4 - b^4)*d^2*e^2 + ((a^4 - b^4)*d^2 *e^2 + 2*(a^3*b + a*b^3 - (a^4 - b^4)*c)*d*e*f - (a^4 + a^2*b^2 - (a^4 - b^ 4)*c^2 + 2*(a^3*b + a*b^3)*c)*f^2)*cosh(d*x + c)^4 + 4*((a^4 - b^4)*d^2*e^2 + 2*(a^3*b + a*b^3 - (a^4 - b^4)*c)*d*e*f - (a^4 + a^2*b^2 - (a^4 - b^4)*c ^2 + 2*(a^3*b + a*b^3)*c)*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 - b^4) *d^2*e^2 + 2*(a^3*b + a*b^3 - (a^4 - b^4)*c)*d*e*f - (a^4 + a^2*b^2 - (a^4 - b^4)*c^2 + 2*(a^3*b + a*b^3)*c)*f^2)*sinh(d*x + c)^4 + 2*(a^3*b + a*b^3 - (a^4 - b^4)*c)*d*e*f - (a^4 + a^2*b^2 - (a^4 - b^4)*c^2 + 2*(a^3*b + a*b^3)*c)*f^2 - 2*((a^4 - b^4)*d^2*e^2 + 2*(a^3*b + a*b^3 - (a^4 - b^4)*c)*d*e*f

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 - (a^4 + a^2*b^2 - (a^4 - b^4)*c^2 + 2*(a^3*b + a*b^3)*c)*f^2)*cosh(d*x +
c)^2 - 2*((a^4 - b^4)*d^2*e^2 + 2*(a^3*b + a*b^3 - (a^4 - b^4)*c)*d*e*f - (
a^4 + a^2*b^2 - (a^4 - b^4)*c^2 + 2*(a^3*b + a*b^3)*c)*f^2 - 3*((a^4 - b^4)
*d^2*e^2 + 2*(a^3*b + a*b^3 - (a^4 - b^4)*c)*d*e*f - (a^4 + a^2*b^2 - (a^4
- b^4)*c^2 + 2*(a^3*b + a*b^3)*c)*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4
*(((a^4 - b^4)*d^2*e^2 + 2*(a^3*b + a*b^3 - (a^4 - b^4)*c)*d*e*f - (a^4 + a
^2*b^2 - (a^4 - b^4)*c^2 + 2*(a^3*b + a*b^3)*c)*f^2)*cosh(d*x + c)^3 - ((a^
4 - b^4)*d^2*e^2 + 2*(a^3*b + a*b^3 - (a^4 - b^4)*c)*d*e*f - (a^4 + a^2*b^2
 - (a^4 - b^4)*c^2 + 2*(a^3*b + a*b^3)*c)*f^2)*cosh(d*x + c))*sinh(d*x + c)
)*log(cosh(d*x + c) + sinh(d*x + c) - 1) - (a^4*d^2*f^2*x^2 - I*a^3*b*d^2*f
^2*x^2 + 2*a^4*d^2*e*f*x - 2*I*a^3*b*d^2*e*f*x + 2*a^4*c*d*e*f - 2*I*a^3*b*
c*d*e*f - a^4*c^2*f^2 + I*a^3*b*c^2*f^2 + (a^4*d^2*f^2*x^2 - I*a^3*b*d^2*f^
2*x^2 + 2*a^4*d^2*e*f*x - 2*I*a^3*b*d^2*e*f*x + 2*a^4*c*d*e*f - 2*I*a^3*b*c
*d*e*f - a^4*c^2*f^2 + I*a^3*b*c^2*f^2)*cosh(d*x + c)^4 + (4*a^4*d^2*f^2*x^
2 - 4*I*a^3*b*d^2*f^2*x^2 + 8*a^4*d^2*e*f*x - 8*I*a^3*b*d^2*e*f*x + 8*a^4*c
*d*e*f - 8*I*a^3*b*c*d*e*f - 4*a^4*c^2*f^2 + 4*I*a^3*b*c^2*f^2)*cosh(d*x +
c)*sinh(d*x + c)^3 + (a^4*d^2*f^2*x^2 - I*a^3*b*d^2*f^2*x^2 + 2*a^4*d^2*e*f
*x - 2*I*a^3*b*d^2*e*f*x + 2*a^4*c*d*e*f - 2*I*a^3*b*c*d*e*f - a^4*c^2*f^2
+ I*a^3*b*c^2*f^2)*sinh(d*x + c)^4 - (2*a^4*d^2*f^2*x^2 - 2*I*a^3*b*d^2*f^2
*x^2 + 4*a^4*d^2*e*f*x - 4*I*a^3*b*d^2*e*f*x + 4*a^4*c*d*e*f - 4*I*a^3*b*c*
d*e*f - 2*a^4*c^2*f^2 + 2*I*a^3*b*c^2*f^2)*cosh(d*x + c)^2 - (2*a^4*d^2*f^2
*x^2 - 2*I*a^3*b*d^2*f^2*x^2 + 4*a^4*d^2*e*f*x - 4*I*a^3*b*d^2*e*f*x + 4*a^
4*c*d*e*f - 4*I*a^3*b*c*d*e*f - 2*a^4*c^2*f^2 + 2*I*a^3*b*c^2*f^2 - (6*a^4*
d^2*f^2*x^2 - 6*I*a^3*b*d^2*f^2*x^2 + 12*a^4*d^2*e*f*x - 12*I*a^3*b*d^2*e*f
*x + 12*a^4*c*d*e*f - 12*I*a^3*b*c*d*e*f - 6*a^4*c^2*f^2 + 6*I*a^3*b*c^2*f^
2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((4*a^4*d^2*f^2*x^2 - 4*I*a^3*b*d^2*f
^2*x^2 + 8*a^4*d^2*e*f*x - 8*I*a^3*b*d^2*e*f*x + 8*a^4*c*d*e*f - 8*I*a^3*b*
c*d*e*f - 4*a^4*c^2*f^2 + 4*I*a^3*b*c^2*f^2)*cosh(d*x + c)^3 - (4*a^4*d^2*f
^2*x^2 - 4*I*a^3*b*d^2*f^2*x^2 + 8*a^4*d^2*e*f*x - 8*I*a^3*b*d^2*e*f*x + 8*
a^4*c*d*e*f - 8*I*a^3*b*c*d*e*f - 4*a^4*c^2*f^2 + 4*I*a^3*b*c^2*f^2)*cosh(d
*x + c))*sinh(d*x + c))*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) - (a^4*d
^2*f^2*x^2 + I*a^3*b*d^2*f^2*x^2 + 2*a^4*d^2*e*f*x + 2*I*a^3*b*d^2*e*f*x +
2*a^4*c*d*e*f + 2*I*a^3*b*c*d*e*f - a^4*c^2*f^2 - I*a^3*b*c^2*f^2 + (a^4*d^
2*f^2*x^2 + I*a^3*b*d^2*f^2*x^2 + 2*a^4*d^2*e*f*x + 2*I*a^3*b*d^2*e*f*x + 2
*a^4*c*d*e*f + 2*I*a^3*b*c*d*e*f - a^4*c^2*f^2 - I*a^3*b*c^2*f^2)*cosh(d*x
+ c)^4 + (4*a^4*d^2*f^2*x^2 + 4*I*a^3*b*d^2*f^2*x^2 + 8*a^4*d^2*e*f*x + 8*I
*a^3*b*d^2*e*f*x + 8*a^4*c*d*e*f + 8*I*a^3*b*c*d*e*f - 4*a^4*c^2*f^2 - 4*I*
a^3*b*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*d^2*f^2*x^2 + I*a^3*b*d
^2*f^2*x^2 + 2*a^4*d^2*e*f*x + 2*I*a^3*b*d^2*e*f*x + 2*a^4*c*d*e*f + 2*I*a^
3*b*c*d*e*f - a^4*c^2*f^2 - I*a^3*b*c^2*f^2)*sinh(d*x + c)^4 - (2*a^4*d^2*f
^2*x^2 + 2*I*a^3*b*d^2*f^2*x^2 + 4*a^4*d^2*e*f*x + 4*I*a^3*b*d^2*e*f*x + 4*
a^4*c*d*e*f + 4*I*a^3*b*c*d*e*f - 2*a^4*c^2*f^2 - 2*I*a^3*b*c^2*f^2)*cosh(d
*x + c)^2 - (2*a^4*d^2*f^2*x^2 + 2*I*a^3*b*d^2*f^2*x^2 + 4*a^4*d^2*e*f*x +
4*I*a^3*b*d^2*e*f*x + 4*a^4*c*d*e*f + 4*I*a^3*b*c*d*e*f - 2*a^4*c^2*f^2 - 2
*I*a^3*b*c^2*f^2 - (6*a^4*d^2*f^2*x^2 + 6*I*a^3*b*d^2*f^2*x^2 + 12*a^4*d^2*
e*f*x + 12*I*a^3*b*d^2*e*f*x + 12*a^4*c*d*e*f + 12*I*a^3*b*c*d*e*f - 6*a^4*
c^2*f^2 - 6*I*a^3*b*c^2*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((4*a^4*d^2
*f^2*x^2 + 4*I*a^3*b*d^2*f^2*x^2 + 8*a^4*d^2*e*f*x + 8*I*a^3*b*d^2*e*f*x +
8*a^4*c*d*e*f + 8*I*a^3*b*c*d*e*f - 4*a^4*c^2*f^2 - 4*I*a^3*b*c^2*f^2)*cosh
(d*x + c)^3 - (4*a^4*d^2*f^2*x^2 + 4*I*a^3*b*d^2*f^2*x^2 + 8*a^4*d^2*e*f*x
+ 8*I*a^3*b*d^2*e*f*x + 8*a^4*c*d*e*f + 8*I*a^3*b*c*d*e*f - 4*a^4*c^2*f^2 -
 4*I*a^3*b*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c))*log(-I*cosh(d*x + c) - I*
sinh(d*x + c) + 1) + ((a^4 - b^4)*d^2*f^2*x^2 + 2*(a^4 - b^4)*c*d*e*f + ((a
^4 - b^4)*d^2*f^2*x^2 + 2*(a^4 - b^4)*c*d*e*f - ((a^4 - b^4)*c^2 - 2*(a^3*b
 + a*b^3)*c)*f^2 + 2*((a^4 - b^4)*d^2*e*f + (a^3*b + a*b^3)*d*f^2)*x)*cosh(
d*x + c)^4 + 4*((a^4 - b^4)*d^2*f^2*x^2 + 2*(a^4 - b^4)*c*d*e*f - ((a^4 - b
^4)*c^2 - 2*(a^3*b + a*b^3)*c)*f^2 + 2*((a^4 - b^4)*d^2*e*f + (a^3*b + a*b^
3)*d*f^2)*x)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 - b^4)*d^2*f^2*x^2 + 2*(
a^4 - b^4)*c*d*e*f - ((a^4 - b^4)*c^2 - 2*(a^3*b + a*b^3)*c)*f^2 + 2*((a^4
```

```
- b^4)*d^2*e*f + (a^3*b + a*b^3)*d*f^2)*x)*sinh(d*x + c)^4 - ((a^4 - b^4)*c
^2 - 2*(a^3*b + a*b^3)*c)*f^2 - 2*((a^4 - b^4)*d^2*f^2*x^2 + 2*(a^4 - b^4)*
c*d*e*f - ((a^4 - b^4)*c^2 - 2*(a^3*b + a*b^3)*c)*f^2 + 2*((a^4 - b^4)*d^2*
e*f + (a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c)^2 - 2*((a^4 - b^4)*d^2*f^2*x^
2 + 2*(a^4 - b^4)*c*d*e*f - ((a^4 - b^4)*c^2 - 2*(a^3*b + a*b^3)*c)*f^2 - 3
*((a^4 - b^4)*d^2*f^2*x^2 + 2*(a^4 - b^4)*c*d*e*f - ((a^4 - b^4)*c^2 - 2*(a
^3*b + a*b^3)*c)*f^2 + 2*((a^4 - b^4)*d^2*e*f + (a^3*b + a*b^3)*d*f^2)*x)*c
osh(d*x + c)^2 + 2*((a^4 - b^4)*d^2*e*f + (a^3*b + a*b^3)*d*f^2)*x)*sinh(d*
x + c)^2 + 2*((a^4 - b^4)*d^2*e*f + (a^3*b + a*b^3)*d*f^2)*x + 4*(((a^4 - b
^4)*d^2*f^2*x^2 + 2*(a^4 - b^4)*c*d*e*f - ((a^4 - b^4)*c^2 - 2*(a^3*b + a*b
^3)*c)*f^2 + 2*((a^4 - b^4)*d^2*e*f + (a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x +
c)^3 - ((a^4 - b^4)*d^2*f^2*x^2 + 2*(a^4 - b^4)*c*d*e*f - ((a^4 - b^4)*c^2
- 2*(a^3*b + a*b^3)*c)*f^2 + 2*((a^4 - b^4)*d^2*e*f + (a^3*b + a*b^3)*d*f^2
)*x)*cosh(d*x + c))*sinh(d*x + c))*log(-cosh(d*x + c) - sinh(d*x + c) + 1)
- 2*(b^4*f^2*cosh(d*x + c)^4 + 4*b^4*f^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^
4*f^2*sinh(d*x + c)^4 - 2*b^4*f^2*cosh(d*x + c)^2 + b^4*f^2 + 2*(3*b^4*f^2*
cosh(d*x + c)^2 - b^4*f^2)*sinh(d*x + c)^2 + 4*(b^4*f^2*cosh(d*x + c)^3 - b
^4*f^2*cosh(d*x + c))*sinh(d*x + c))*polylog(3, (a*cosh(d*x + c) + a*sinh(d
*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2
*(b^4*f^2*cosh(d*x + c)^4 + 4*b^4*f^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^4*f
^2*sinh(d*x + c)^4 - 2*b^4*f^2*cosh(d*x + c)^2 + b^4*f^2 + 2*(3*b^4*f^2*cos
h(d*x + c)^2 - b^4*f^2)*sinh(d*x + c)^2 + 4*(b^4*f^2*cosh(d*x + c)^3 - b^4*
f^2*cosh(d*x + c))*sinh(d*x + c))*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x
+ c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*((
a^4 - b^4)*f^2*cosh(d*x + c)^4 + 4*(a^4 - b^4)*f^2*cosh(d*x + c)*sinh(d*x +
 c)^3 + (a^4 - b^4)*f^2*sinh(d*x + c)^4 - 2*(a^4 - b^4)*f^2*cosh(d*x + c)^2
 + (a^4 - b^4)*f^2 + 2*(3*(a^4 - b^4)*f^2*cosh(d*x + c)^2 - (a^4 - b^4)*f^2
)*sinh(d*x + c)^2 + 4*((a^4 - b^4)*f^2*cosh(d*x + c)^3 - (a^4 - b^4)*f^2*co
sh(d*x + c))*sinh(d*x + c))*polylog(3, cosh(d*x + c) + sinh(d*x + c)) + (2*
a^4*f^2 + 2*I*a^3*b*f^2 + (2*a^4*f^2 + 2*I*a^3*b*f^2)*cosh(d*x + c)^4 + (8*
a^4*f^2 + 8*I*a^3*b*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^4*f^2 + 2*I*a
^3*b*f^2)*sinh(d*x + c)^4 - (4*a^4*f^2 + 4*I*a^3*b*f^2)*cosh(d*x + c)^2 - (
4*a^4*f^2 + 4*I*a^3*b*f^2 - (12*a^4*f^2 + 12*I*a^3*b*f^2)*cosh(d*x + c)^2)*
sinh(d*x + c)^2 + ((8*a^4*f^2 + 8*I*a^3*b*f^2)*cosh(d*x + c)^3 - (8*a^4*f^2
 + 8*I*a^3*b*f^2)*cosh(d*x + c))*sinh(d*x + c))*polylog(3, I*cosh(d*x + c)
+ I*sinh(d*x + c)) + (2*a^4*f^2 - 2*I*a^3*b*f^2 + (2*a^4*f^2 - 2*I*a^3*b*f^
2)*cosh(d*x + c)^4 + (8*a^4*f^2 - 8*I*a^3*b*f^2)*cosh(d*x + c)*sinh(d*x + c
)^3 + (2*a^4*f^2 - 2*I*a^3*b*f^2)*sinh(d*x + c)^4 - (4*a^4*f^2 - 4*I*a^3*b*
f^2)*cosh(d*x + c)^2 - (4*a^4*f^2 - 4*I*a^3*b*f^2 - (12*a^4*f^2 - 12*I*a^3*
b*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a^4*f^2 - 8*I*a^3*b*f^2)*cosh
(d*x + c)^3 - (8*a^4*f^2 - 8*I*a^3*b*f^2)*cosh(d*x + c))*sinh(d*x + c))*pol
ylog(3, -I*cosh(d*x + c) - I*sinh(d*x + c)) - 2*((a^4 - b^4)*f^2*cosh(d*x +
 c)^4 + 4*(a^4 - b^4)*f^2*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 - b^4)*f^2*s
inh(d*x + c)^4 - 2*(a^4 - b^4)*f^2*cosh(d*x + c)^2 + (a^4 - b^4)*f^2 + 2*(3
*(a^4 - b^4)*f^2*cosh(d*x + c)^2 - (a^4 - b^4)*f^2)*sinh(d*x + c)^2 + 4*((a
^4 - b^4)*f^2*cosh(d*x + c)^3 - (a^4 - b^4)*f^2*cosh(d*x + c))*sinh(d*x + c
))*polylog(3, -cosh(d*x + c) - sinh(d*x + c)) + 2*((a^3*b + a*b^3)*d^2*f^2*
x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x + (a^3*b + a*b^3)*d^2*e^2 + 4*((a^4 + a^2
*b^2)*d*f^2*x + (a^4 + a^2*b^2)*c*f^2)*cosh(d*x + c)^3 - 3*((a^3*b + a*b^3)
*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x + (a^3*b + a*b^3)*d^2*e^2)*cosh(
d*x + c)^2 + 2*((a^4 + a^2*b^2)*d^2*f^2*x^2 + (a^4 + a^2*b^2)*d^2*e^2 + (a^
4 + a^2*b^2)*d*e*f - 2*(a^4 + a^2*b^2)*c*f^2 + (2*(a^4 + a^2*b^2)*d^2*e*f -
 (a^4 + a^2*b^2)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + a^3*b^2)*d
^3*cosh(d*x + c)^4 + 4*(a^5 + a^3*b^2)*d^3*cosh(d*x + c)*sinh(d*x + c)^3 +
(a^5 + a^3*b^2)*d^3*sinh(d*x + c)^4 - 2*(a^5 + a^3*b^2)*d^3*cosh(d*x + c)^2
 + (a^5 + a^3*b^2)*d^3 + 2*(3*(a^5 + a^3*b^2)*d^3*cosh(d*x + c)^2 - (a^5 +
a^3*b^2)*d^3)*sinh(d*x + c)^2 + 4*((a^5 + a^3*b^2)*d^3*cosh(d*x + c)^3 - (a
^5 + a^3*b^2)*d^3*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)**3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.493 $\int \frac{(e+fx)\mathbf{csch}^3(c+dx)\mathbf{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=762

$$-\frac{b^4 f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2\left(a^2+b^2\right)}-\frac{b^4 f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3d^2\left(a^2+b^2\right)}+\frac{b^4 f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2a^3d^2\left(a^2+b^2\right)}+\frac{ib^3 f\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{a^2d^2\left(a^2+b^2\right)}$$

```
[Out] (f*x)/(2*a*d) + (2*b*f*x*ArcTan[E^(c + d*x)])/(a^2*d) - (2*b^3*(e + f*x)*Ar
cTan[E^(c + d*x)])/(a^2*(a^2 + b^2)*d) - (b*f*x*ArcTan[Sinh[c + d*x]])/(a^2
*d) + (b*(e + f*x)*ArcTan[Sinh[c + d*x]])/(a^2*d) + (2*f*x*ArcTanh[E^(2*c +
 2*d*x)])/(a*d) - (2*b^2*(e + f*x)*ArcTanh[E^(2*c + 2*d*x)])/(a^3*d) + (b*f
*ArcTanh[Cosh[c + d*x]])/(a^2*d^2) - (f*Coth[c + d*x])/(2*a*d^2) - ((e + f*
x)*Coth[c + d*x]^2)/(2*a*d) + (b*(e + f*x)*Csch[c + d*x])/(a^2*d) - (b^4*(e
 + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)*d)
 - (b^4*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*(a^2
 + b^2)*d) + (b^4*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a^3*(a^2 + b^2)*d) +
 (f*x*Log[Tanh[c + d*x]])/(a*d) - ((e + f*x)*Log[Tanh[c + d*x]])/(a*d) - (I
*b*f*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*d^2) + (I*b^3*f*PolyLog[2, (-I)*E^(
c + d*x)])/(a^2*(a^2 + b^2)*d^2) + (I*b*f*PolyLog[2, I*E^(c + d*x)])/(a^2*d
^2) - (I*b^3*f*PolyLog[2, I*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^2) - (b^4*f*Po
lyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^2) -
 (b^4*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b
^2)*d^2) + (b^4*f*PolyLog[2, -E^(2*(c + d*x))])/(2*a^3*(a^2 + b^2)*d^2) + (
f*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a*d^2) - (b^2*f*PolyLog[2, -E^(2*c + 2*d
*x)])/(2*a^3*d^2) - (f*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d^2) + (b^2*f*Poly
Log[2, E^(2*c + 2*d*x)])/(2*a^3*d^2)
```

Rubi [A] time = 1.13785, antiderivative size = 762, normalized size of antiderivative = 1., number of steps used = 49, number of rules used = 23, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.719, Rules used = {5589, 2620, 14, 5462, 3473, 8, 2548, 12, 4182, 2279, 2391, 2621, 321, 207, 5203, 4180, 3770, 5461, 5573, 5561, 2190, 6742, 3718}

$$-\frac{b^4 f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2\left(a^2+b^2\right)}-\frac{b^4 f\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3d^2\left(a^2+b^2\right)}+\frac{b^4 f\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)}{2a^3d^2\left(a^2+b^2\right)}+\frac{ib^3 f\,\text{PolyLog}\left(2,-ie^{c+dx}\right)}{a^2d^2\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (f*x)/(2*a*d) + (2*b*f*x*ArcTan[E^(c + d*x)])/(a^2*d) - (2*b^3*(e + f*x)*Ar
cTan[E^(c + d*x)])/(a^2*(a^2 + b^2)*d) - (b*f*x*ArcTan[Sinh[c + d*x]])/(a^2
*d) + (b*(e + f*x)*ArcTan[Sinh[c + d*x]])/(a^2*d) + (2*f*x*ArcTanh[E^(2*c +
 2*d*x)])/(a*d) - (2*b^2*(e + f*x)*ArcTanh[E^(2*c + 2*d*x)])/(a^3*d) + (b*f
*ArcTanh[Cosh[c + d*x]])/(a^2*d^2) - (f*Coth[c + d*x])/(2*a*d^2) - ((e + f*
x)*Coth[c + d*x]^2)/(2*a*d) + (b*(e + f*x)*Csch[c + d*x])/(a^2*d) - (b^4*(e
 + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)*d)
 - (b^4*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*(a^2
 + b^2)*d) + (b^4*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a^3*(a^2 + b^2)*d) +
 (f*x*Log[Tanh[c + d*x]])/(a*d) - ((e + f*x)*Log[Tanh[c + d*x]])/(a*d) - (I
*b*f*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*d^2) + (I*b^3*f*PolyLog[2, (-I)*E^(
c + d*x)])/(a^2*(a^2 + b^2)*d^2) + (I*b*f*PolyLog[2, I*E^(c + d*x)])/(a^2*d
^2) - (I*b^3*f*PolyLog[2, I*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^2) - (b^4*f*Po
lyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^2) -
 (b^4*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b
```

```
^2)*d^2) + (b^4*f*PolyLog[2, -E^(2*(c + d*x))])/(2*a^3*(a^2 + b^2)*d^2) + (
f*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a*d^2) - (b^2*f*PolyLog[2, -E^(2*c + 2*d
*x)])/(2*a^3*d^2) - (f*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d^2) + (b^2*f*Poly
Log[2, E^(2*c + 2*d*x)])/(2*a^3*d^2)
```

Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
```

```
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
```

```
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{csch}^3(c+dx)\text{sech}(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= -\frac{(e+fx)\coth^2(c+dx)}{2ad} - \frac{(e+fx)\log(\tanh(c+dx))}{ad} - \frac{b\int (e+fx)\text{csch}$$

$$= \frac{b(e+fx)\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{(e+fx)\coth^2(c+dx)}{2ad} + \frac{b(e+fx)\text{csch}}{a^2d}$$

$$= \frac{b(e+fx)\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{f\coth(c+dx)}{2ad^2} - \frac{(e+fx)\coth^2(c+dx)}{2ad}$$

$$= \frac{fx}{2ad} + \frac{b^4(e+fx)^2}{2a^3\left(a^2+b^2\right)f} - \frac{bfx\tan^{-1}(\sinh(c+dx))}{a^2d} + \frac{b(e+fx)\tan^{-1}(\sinh}{a^2d}$$

$$= \frac{fx}{2ad} + \frac{b^4(e+fx)^2}{2a^3\left(a^2+b^2\right)f} - \frac{bfx\tan^{-1}(\sinh(c+dx))}{a^2d} + \frac{b(e+fx)\tan^{-1}(\sinh}{a^2d}$$

$$= \frac{fx}{2ad} + \frac{2bfx\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d} - \frac{bfx\tan^{-1}(\sinh(}{a^2d}$$

$$= \frac{fx}{2ad} + \frac{2bfx\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d} - \frac{bfx\tan^{-1}(\sinh(}{a^2d}$$

$$= \frac{fx}{2ad} + \frac{2bfx\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d} - \frac{bfx\tan^{-1}(\sinh(}{a^2d}$$

$$= \frac{fx}{2ad} + \frac{2bfx\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^3(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)d} - \frac{bfx\tan^{-1}(\sinh(}{a^2d}$$

Mathematica [A] time = 7.78415, size = 913, normalized size = 1.2

$$-\frac{\left(-\frac{1}{2}f(c+dx)^2 + f\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)(c+dx) + f\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)(c+dx) + de\log(a+b\sinh(c+dx)) - cf\log(a}{a^3\left(a^2+b^2\right)d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] ((2*b*d*e*Cosh[(c + d*x)/2] - a*f*Cosh[(c + d*x)/2] - 2*b*c*f*Cosh[(c + d*x
)/2] + 2*b*f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(4*a^2*d^2) +
((-(d*e) + c*f - f*(c + d*x))*Csch[(c + d*x)/2]^2)/(8*a*d^2) - (e*Log[Sinh[
c + d*x]])/(a*d) + (b^2*e*Log[Sinh[c + d*x]])/(a^3*d) + (c*f*Log[Sinh[c + d
*x]])/(a*d^2) - (b^2*c*f*Log[Sinh[c + d*x]])/(a^3*d^2) - (b*f*Log[Tanh[(c +
 d*x)/2]])/(a^2*d^2) + (I*f*(I*(c + d*x)*Log[1 - E^(-2*(c + d*x))] - (I/2)*
(-(c + d*x)^2 + PolyLog[2, E^(-2*(c + d*x))])))/(a*d^2) - (I*b^2*f*(I*(c +
d*x)*Log[1 - E^(-2*(c + d*x))] - (I/2)*(-(c + d*x)^2 + PolyLog[2, E^(-2*(c
+ d*x))])))/(a^3*d^2) - (b^4*(-(f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a
 + Sqrt[a^2 + b^2])] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c
+ d*x]] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[
2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^3*(a^2 + b^2)*d^2) + (-(a
```

```
*d*e*(c + d*x)) + a*c*f*(c + d*x) - (a*f*(c + d*x)^2)/2 + 2*b*d*e*ArcTan[Co
sh[c + d*x] + Sinh[c + d*x]] - 2*b*c*f*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]
] + 2*b*f*(c + d*x)*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] + a*d*e*Log[1 + C
osh[2*(c + d*x)] + Sinh[2*(c + d*x)]] - a*c*f*Log[1 + Cosh[2*(c + d*x)] + S
inh[2*(c + d*x)]] + a*f*(c + d*x)*Log[1 + Cosh[2*(c + d*x)] + Sinh[2*(c + d
*x)]] - I*b*f*PolyLog[2, (-I)*(Cosh[c + d*x] + Sinh[c + d*x])] + I*b*f*Poly
Log[2, I*(Cosh[c + d*x] + Sinh[c + d*x])] + (a*f*PolyLog[2, -Cosh[2*(c + d*
x)] - Sinh[2*(c + d*x)]])/2)/((a^2 + b^2)*d^2) + ((d*e - c*f + f*(c + d*x))
*Sech[(c + d*x)/2]^2)/(8*a*d^2) + (Sech[(c + d*x)/2]*(-2*b*d*e*Sinh[(c + d*
x)/2] - a*f*Sinh[(c + d*x)/2] + 2*b*c*f*Sinh[(c + d*x)/2] - 2*b*f*(c + d*x)
*Sinh[(c + d*x)/2]))/(4*a^2*d^2)
```

Maple [B] time = 0.204, size = 1478, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/a^2/d^2*f*b^4/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^
(1/2))-1/a^2/d^2*f*b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^
2+b^2)^(1/2))-1/a^3/d*b^4*e/(a^2+b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)
-1/a^3/d^2*b^2*f*c*ln(exp(d*x+c)-1)+1/a^3/d*b^2*f*ln(exp(d*x+c)+1)*x-1/a^3/
d^2*f*b^4/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2
)))*c-1/a^3/d^2*f*b^4/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2
+b^2)^(1/2)))*c-1/a^3/d*f*b^4/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a
)/(-a+(a^2+b^2)^(1/2)))*x-1/a^3/d*f*b^4/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b^2
)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/a^3/d^2*f*b^4*c/(a^2+b^2)*ln(b*exp(2*d*
x+2*c)+2*a*exp(d*x+c)-b)-4*I/d^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*b*c+4/d
^2*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))*a+4/d^2*f/(4*a^2+4*b^2)*dilog(1-I*
exp(d*x+c))*a+8/d*e/(4*a^2+4*b^2)*b*arctan(exp(d*x+c))+4/d*e/(4*a^2+4*b^2)*
a*ln(1+exp(2*d*x+2*c))-1/a^3/d^2*b^2*f*dilog(exp(d*x+c))+1/a^3/d^2*b^2*f*di
log(exp(d*x+c)+1)-1/a^2/d^2*b*f*ln(exp(d*x+c)-1)+1/a^2/d^2*b*f*ln(exp(d*x+c
)+1)+1/a^3/d*b^2*e*ln(exp(d*x+c)-1)+1/a^3/d*b^2*e*ln(exp(d*x+c)+1)+1/d^2*f/
a*dilog(exp(d*x+c))-1/d^2*f/a*dilog(exp(d*x+c)+1)-8/d^2*f*c/(4*a^2+4*b^2)*b
*arctan(exp(d*x+c))+4/d*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*a*x+4/d^2*f/(4*a
^2+4*b^2)*ln(1+I*exp(d*x+c))*a*c+4/d*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*a*x
+4/d^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*a*c-4/d^2*f*c/(4*a^2+4*b^2)*a*ln(
1+exp(2*d*x+2*c))-1/d/a*e*ln(exp(d*x+c)-1)-1/d/a*e*ln(exp(d*x+c)+1)-1/a^3/d
^2*f*b^4/(a^2+b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2
)))-1/a^3/d^2*f*b^4/(a^2+b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(
a^2+b^2)^(1/2)))-4*I/d^2*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))*b+4*I/d^2*f/
(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))*b+1/d^2*b^2*f/(a^2+b^2)^(3/2)*arctanh(1
/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d/a*ln(exp(d*x+c)+1)*f*x+1/d^2/a
*f*c*ln(exp(d*x+c)-1)-(-2*b*d*f*x*exp(3*d*x+3*c)+2*a*d*f*x*exp(2*d*x+2*c)-2
*b*d*e*exp(3*d*x+3*c)+2*a*d*e*exp(2*d*x+2*c)+2*b*d*f*x*exp(d*x+c)+a*f*exp(2
*d*x+2*c)+2*b*d*e*exp(d*x+c)-a*f)/d^2/a^2/(exp(2*d*x+2*c)-1)^2+4*I/d^2*f/(4
*a^2+4*b^2)*ln(1-I*exp(d*x+c))*b*c-4*I/d*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))
*b*x+4*I/d*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*b*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -(b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + a^3*b^2)*d) +
 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a
^2 + b^2)*d) + 2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))
/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 - b^2)*lo
g(e^(-d*x - c) + 1)/(a^3*d) + (a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d))*e
+ (16*a^2*d*integrate(1/16*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 16*b^2*d*int
egrate(1/16*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 16*a^2*d*integrate(1/16*x/(
a^3*d*e^(d*x + c) - a^3*d), x) + 16*b^2*d*integrate(1/16*x/(a^3*d*e^(d*x +
c) - a^3*d), x) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) + 1)/(a^3*d^2)
) + a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) - 1)/(a^3*d^2)) + (2*b*d*x*e
^(3*d*x + 3*c) - 2*b*d*x*e^(d*x + c) - (2*a*d*x*e^(2*c) + a*e^(2*c))*e^(2*d
*x) + a)/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) +
16*integrate(-1/8*(a*b^4*x*e^(d*x + c) - b^5*x)/(a^5*b + a^3*b^3 - (a^5*b*e
^(2*c) + a^3*b^3*e^(2*c))*e^(2*d*x) - 2*(a^6*e^c + a^4*b^2*e^c)*e^(d*x)), x
) + 16*integrate(1/8*(b*x*e^(d*x + c) - a*x)/(a^2 + b^2 + (a^2*e^(2*c) + b^
2*e^(2*c))*e^(2*d*x)), x))*f
```

Fricas [B] time = 3.73601, size = 13535, normalized size = 17.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] (2*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e)*cosh(d*x + c)^3 + 2*((a^3*
b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e)*sinh(d*x + c)^3 - (2*(a^4 + a^2*b^2
)*d*f*x + 2*(a^4 + a^2*b^2)*d*e + (a^4 + a^2*b^2)*f)*cosh(d*x + c)^2 - (2*(
a^4 + a^2*b^2)*d*f*x + 2*(a^4 + a^2*b^2)*d*e + (a^4 + a^2*b^2)*f - 6*((a^3*
b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e)*cosh(d*x + c))*sinh(d*x + c)^2 + (a
^4 + a^2*b^2)*f - 2*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e)*cosh(d*x
+ c) - (b^4*f*cosh(d*x + c)^4 + 4*b^4*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^4
*f*sinh(d*x + c)^4 - 2*b^4*f*cosh(d*x + c)^2 + b^4*f + 2*(3*b^4*f*cosh(d*x
+ c)^2 - b^4*f)*sinh(d*x + c)^2 + 4*(b^4*f*cosh(d*x + c)^3 - b^4*f*cosh(d*x
 + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*
x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^4*f*cosh(d
*x + c)^4 + 4*b^4*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^4*f*sinh(d*x + c)^4 -
 2*b^4*f*cosh(d*x + c)^2 + b^4*f + 2*(3*b^4*f*cosh(d*x + c)^2 - b^4*f)*sinh
(d*x + c)^2 + 4*(b^4*f*cosh(d*x + c)^3 - b^4*f*cosh(d*x + c))*sinh(d*x + c)
)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - ((a^4 - b^4)*f*cosh(d*x + c)^4 +
4*(a^4 - b^4)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 - b^4)*f*sinh(d*x + c)
^4 - 2*(a^4 - b^4)*f*cosh(d*x + c)^2 + 2*(3*(a^4 - b^4)*f*cosh(d*x + c)^2 -
 (a^4 - b^4)*f)*sinh(d*x + c)^2 + (a^4 - b^4)*f + 4*((a^4 - b^4)*f*cosh(d*x
 + c)^3 - (a^4 - b^4)*f*cosh(d*x + c))*sinh(d*x + c))*dilog(cosh(d*x + c) +
 sinh(d*x + c)) + (a^4*f + I*a^3*b*f + (a^4*f + I*a^3*b*f)*cosh(d*x + c)^4
+ (4*a^4*f + 4*I*a^3*b*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*f + I*a^3*b*
f)*sinh(d*x + c)^4 - (2*a^4*f + 2*I*a^3*b*f)*cosh(d*x + c)^2 - (2*a^4*f + 2
*I*a^3*b*f - (6*a^4*f + 6*I*a^3*b*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((4
*a^4*f + 4*I*a^3*b*f)*cosh(d*x + c)^3 - (4*a^4*f + 4*I*a^3*b*f)*cosh(d*x +
c))*sinh(d*x + c))*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + (a^4*f - I*a^
3*b*f + (a^4*f - I*a^3*b*f)*cosh(d*x + c)^4 + (4*a^4*f - 4*I*a^3*b*f)*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^4*f - I*a^3*b*f)*sinh(d*x + c)^4 - (2*a^4*f -
```

```
 2*I*a^3*b*f)*cosh(d*x + c)^2 - (2*a^4*f - 2*I*a^3*b*f - (6*a^4*f - 6*I*a^3
*b*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((4*a^4*f - 4*I*a^3*b*f)*cosh(d*x
+ c)^3 - (4*a^4*f - 4*I*a^3*b*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(-I*cos
h(d*x + c) - I*sinh(d*x + c)) - ((a^4 - b^4)*f*cosh(d*x + c)^4 + 4*(a^4 - b
^4)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 - b^4)*f*sinh(d*x + c)^4 - 2*(a^
4 - b^4)*f*cosh(d*x + c)^2 + 2*(3*(a^4 - b^4)*f*cosh(d*x + c)^2 - (a^4 - b^
4)*f)*sinh(d*x + c)^2 + (a^4 - b^4)*f + 4*((a^4 - b^4)*f*cosh(d*x + c)^3 -
(a^4 - b^4)*f*cosh(d*x + c))*sinh(d*x + c))*dilog(-cosh(d*x + c) - sinh(d*x
 + c)) - (b^4*d*e - b^4*c*f + (b^4*d*e - b^4*c*f)*cosh(d*x + c)^4 + 4*(b^4*
d*e - b^4*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*d*e - b^4*c*f)*sinh(d*x
 + c)^4 - 2*(b^4*d*e - b^4*c*f)*cosh(d*x + c)^2 - 2*(b^4*d*e - b^4*c*f - 3*
(b^4*d*e - b^4*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d*e - b^4*c*
f)*cosh(d*x + c)^3 - (b^4*d*e - b^4*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(
2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) -
(b^4*d*e - b^4*c*f + (b^4*d*e - b^4*c*f)*cosh(d*x + c)^4 + 4*(b^4*d*e - b^4
*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*d*e - b^4*c*f)*sinh(d*x + c)^4 -
 2*(b^4*d*e - b^4*c*f)*cosh(d*x + c)^2 - 2*(b^4*d*e - b^4*c*f - 3*(b^4*d*e
- b^4*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d*e - b^4*c*f)*cosh(d
*x + c)^3 - (b^4*d*e - b^4*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(
d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^4*d*f*
x + b^4*c*f + (b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^4 + 4*(b^4*d*f*x + b^4*c*
f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^4*d*f*x + b^4*c*f)*sinh(d*x + c)^4 -
2*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^2 - 2*(b^4*d*f*x + b^4*c*f - 3*(b^4*d
*f*x + b^4*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((b^4*d*f*x + b^4*c*f)
*cosh(d*x + c)^3 - (b^4*d*f*x + b^4*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(
-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 + b^2)/b^2) - b)/b) - (b^4*d*f*x + b^4*c*f + (b^4*d*f*x + b^4*c*f)
*cosh(d*x + c)^4 + 4*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 +
(b^4*d*f*x + b^4*c*f)*sinh(d*x + c)^4 - 2*(b^4*d*f*x + b^4*c*f)*cosh(d*x +
c)^2 - 2*(b^4*d*f*x + b^4*c*f - 3*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^2)*si
nh(d*x + c)^2 + 4*((b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^3 - (b^4*d*f*x + b^4
*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
 - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (((a
^4 - b^4)*d*f*x + (a^4 - b^4)*d*e - (a^3*b + a*b^3)*f)*cosh(d*x + c)^4 + 4*
((a^4 - b^4)*d*f*x + (a^4 - b^4)*d*e - (a^3*b + a*b^3)*f)*cosh(d*x + c)*sin
h(d*x + c)^3 + ((a^4 - b^4)*d*f*x + (a^4 - b^4)*d*e - (a^3*b + a*b^3)*f)*si
nh(d*x + c)^4 + (a^4 - b^4)*d*f*x + (a^4 - b^4)*d*e - 2*((a^4 - b^4)*d*f*x
+ (a^4 - b^4)*d*e - (a^3*b + a*b^3)*f)*cosh(d*x + c)^2 - 2*((a^4 - b^4)*d*f
*x + (a^4 - b^4)*d*e - 3*((a^4 - b^4)*d*f*x + (a^4 - b^4)*d*e - (a^3*b + a*
b^3)*f)*cosh(d*x + c)^2 - (a^3*b + a*b^3)*f)*sinh(d*x + c)^2 - (a^3*b + a*b
^3)*f + 4*(((a^4 - b^4)*d*f*x + (a^4 - b^4)*d*e - (a^3*b + a*b^3)*f)*cosh(d
*x + c)^3 - ((a^4 - b^4)*d*f*x + (a^4 - b^4)*d*e - (a^3*b + a*b^3)*f)*cosh(
d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (a^4*d*e
+ I*a^3*b*d*e - a^4*c*f - I*a^3*b*c*f + (a^4*d*e + I*a^3*b*d*e - a^4*c*f -
I*a^3*b*c*f)*cosh(d*x + c)^4 + (4*a^4*d*e + 4*I*a^3*b*d*e - 4*a^4*c*f - 4*I
*a^3*b*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*d*e + I*a^3*b*d*e - a^4*c*
f - I*a^3*b*c*f)*sinh(d*x + c)^4 - (2*a^4*d*e + 2*I*a^3*b*d*e - 2*a^4*c*f -
 2*I*a^3*b*c*f)*cosh(d*x + c)^2 - (2*a^4*d*e + 2*I*a^3*b*d*e - 2*a^4*c*f -
2*I*a^3*b*c*f - (6*a^4*d*e + 6*I*a^3*b*d*e - 6*a^4*c*f - 6*I*a^3*b*c*f)*cos
h(d*x + c)^2)*sinh(d*x + c)^2 + ((4*a^4*d*e + 4*I*a^3*b*d*e - 4*a^4*c*f - 4
*I*a^3*b*c*f)*cosh(d*x + c)^3 - (4*a^4*d*e + 4*I*a^3*b*d*e - 4*a^4*c*f - 4*
I*a^3*b*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c
) + I) + (a^4*d*e - I*a^3*b*d*e - a^4*c*f + I*a^3*b*c*f + (a^4*d*e - I*a^3*
b*d*e - a^4*c*f + I*a^3*b*c*f)*cosh(d*x + c)^4 + (4*a^4*d*e - 4*I*a^3*b*d*e
 - 4*a^4*c*f + 4*I*a^3*b*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*d*e - I*
a^3*b*d*e - a^4*c*f + I*a^3*b*c*f)*sinh(d*x + c)^4 - (2*a^4*d*e - 2*I*a^3*b
*d*e - 2*a^4*c*f + 2*I*a^3*b*c*f)*cosh(d*x + c)^2 - (2*a^4*d*e - 2*I*a^3*b*
d*e - 2*a^4*c*f + 2*I*a^3*b*c*f - (6*a^4*d*e - 6*I*a^3*b*d*e - 6*a^4*c*f +
6*I*a^3*b*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((4*a^4*d*e - 4*I*a^3*b*d
```

```
*e - 4*a^4*c*f + 4*I*a^3*b*c*f)*cosh(d*x + c)^3 - (4*a^4*d*e - 4*I*a^3*b*d*
e - 4*a^4*c*f + 4*I*a^3*b*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x +
 c) + sinh(d*x + c) - I) - (((a^4 - b^4)*d*e + (a^3*b + a*b^3 - (a^4 - b^4)
*c)*f)*cosh(d*x + c)^4 + 4*((a^4 - b^4)*d*e + (a^3*b + a*b^3 - (a^4 - b^4)*
c)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^4 - b^4)*d*e + (a^3*b + a*b^3 - (
a^4 - b^4)*c)*f)*sinh(d*x + c)^4 + (a^4 - b^4)*d*e - 2*((a^4 - b^4)*d*e + (
a^3*b + a*b^3 - (a^4 - b^4)*c)*f)*cosh(d*x + c)^2 - 2*((a^4 - b^4)*d*e - 3*
((a^4 - b^4)*d*e + (a^3*b + a*b^3 - (a^4 - b^4)*c)*f)*cosh(d*x + c)^2 + (a^
3*b + a*b^3 - (a^4 - b^4)*c)*f)*sinh(d*x + c)^2 + (a^3*b + a*b^3 - (a^4 - b
^4)*c)*f + 4*(((a^4 - b^4)*d*e + (a^3*b + a*b^3 - (a^4 - b^4)*c)*f)*cosh(d*
x + c)^3 - ((a^4 - b^4)*d*e + (a^3*b + a*b^3 - (a^4 - b^4)*c)*f)*cosh(d*x +
 c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + (a^4*d*f*x - I
*a^3*b*d*f*x + a^4*c*f - I*a^3*b*c*f + (a^4*d*f*x - I*a^3*b*d*f*x + a^4*c*f
 - I*a^3*b*c*f)*cosh(d*x + c)^4 + (4*a^4*d*f*x - 4*I*a^3*b*d*f*x + 4*a^4*c*
f - 4*I*a^3*b*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*d*f*x - I*a^3*b*d*f
*x + a^4*c*f - I*a^3*b*c*f)*sinh(d*x + c)^4 - (2*a^4*d*f*x - 2*I*a^3*b*d*f*
x + 2*a^4*c*f - 2*I*a^3*b*c*f)*cosh(d*x + c)^2 - (2*a^4*d*f*x - 2*I*a^3*b*d
*f*x + 2*a^4*c*f - 2*I*a^3*b*c*f - (6*a^4*d*f*x - 6*I*a^3*b*d*f*x + 6*a^4*c
*f - 6*I*a^3*b*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((4*a^4*d*f*x - 4*I*
a^3*b*d*f*x + 4*a^4*c*f - 4*I*a^3*b*c*f)*cosh(d*x + c)^3 - (4*a^4*d*f*x - 4
*I*a^3*b*d*f*x + 4*a^4*c*f - 4*I*a^3*b*c*f)*cosh(d*x + c))*sinh(d*x + c))*l
og(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) + (a^4*d*f*x + I*a^3*b*d*f*x + a^
4*c*f + I*a^3*b*c*f + (a^4*d*f*x + I*a^3*b*d*f*x + a^4*c*f + I*a^3*b*c*f)*c
osh(d*x + c)^4 + (4*a^4*d*f*x + 4*I*a^3*b*d*f*x + 4*a^4*c*f + 4*I*a^3*b*c*f
)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*d*f*x + I*a^3*b*d*f*x + a^4*c*f + I*
a^3*b*c*f)*sinh(d*x + c)^4 - (2*a^4*d*f*x + 2*I*a^3*b*d*f*x + 2*a^4*c*f + 2
*I*a^3*b*c*f)*cosh(d*x + c)^2 - (2*a^4*d*f*x + 2*I*a^3*b*d*f*x + 2*a^4*c*f
+ 2*I*a^3*b*c*f - (6*a^4*d*f*x + 6*I*a^3*b*d*f*x + 6*a^4*c*f + 6*I*a^3*b*c*
f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((4*a^4*d*f*x + 4*I*a^3*b*d*f*x + 4*a
^4*c*f + 4*I*a^3*b*c*f)*cosh(d*x + c)^3 - (4*a^4*d*f*x + 4*I*a^3*b*d*f*x +
4*a^4*c*f + 4*I*a^3*b*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-I*cosh(d*x +
c) - I*sinh(d*x + c) + 1) - (((a^4 - b^4)*d*f*x + (a^4 - b^4)*c*f)*cosh(d*x
 + c)^4 + 4*((a^4 - b^4)*d*f*x + (a^4 - b^4)*c*f)*cosh(d*x + c)*sinh(d*x +
c)^3 + ((a^4 - b^4)*d*f*x + (a^4 - b^4)*c*f)*sinh(d*x + c)^4 + (a^4 - b^4)*
d*f*x + (a^4 - b^4)*c*f - 2*((a^4 - b^4)*d*f*x + (a^4 - b^4)*c*f)*cosh(d*x
+ c)^2 - 2*((a^4 - b^4)*d*f*x + (a^4 - b^4)*c*f - 3*((a^4 - b^4)*d*f*x + (a
^4 - b^4)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^4 - b^4)*d*f*x + (
a^4 - b^4)*c*f)*cosh(d*x + c)^3 - ((a^4 - b^4)*d*f*x + (a^4 - b^4)*c*f)*cos
h(d*x + c))*sinh(d*x + c))*log(-cosh(d*x + c) - sinh(d*x + c) + 1) - 2*((a^
3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e - 3*((a^3*b + a*b^3)*d*f*x + (a^3*
b + a*b^3)*d*e)*cosh(d*x + c)^2 + (2*(a^4 + a^2*b^2)*d*f*x + 2*(a^4 + a^2*b
^2)*d*e + (a^4 + a^2*b^2)*f)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + a^3*b^2)
*d^2*cosh(d*x + c)^4 + 4*(a^5 + a^3*b^2)*d^2*cosh(d*x + c)*sinh(d*x + c)^3
+ (a^5 + a^3*b^2)*d^2*sinh(d*x + c)^4 - 2*(a^5 + a^3*b^2)*d^2*cosh(d*x + c)
^2 + (a^5 + a^3*b^2)*d^2 + 2*(3*(a^5 + a^3*b^2)*d^2*cosh(d*x + c)^2 - (a^5
+ a^3*b^2)*d^2)*sinh(d*x + c)^2 + 4*((a^5 + a^3*b^2)*d^2*cosh(d*x + c)^3 -
(a^5 + a^3*b^2)*d^2*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.494 $\int \frac{\text{csch}^3(c+dx)\text{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=130

$$-\frac{b^4\log(a+b\sinh(c+dx))}{a^3d\left(a^2+b^2\right)}-\frac{\left(a^2-b^2\right)\log(\sinh(c+dx))}{a^3d}+\frac{b\tan^{-1}(\sinh(c+dx))}{d\left(a^2+b^2\right)}+\frac{a\log(\cosh(c+dx))}{d\left(a^2+b^2\right)}+\frac{b\text{csch}(c\cdot}{a^2d}$$

```
[Out] (b*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d) + (b*Csch[c + d*x])/(a^2*d) - Csc
h[c + d*x]^2/(2*a*d) + (a*Log[Cosh[c + d*x]])/((a^2 + b^2)*d) - ((a^2 - b^2
)*Log[Sinh[c + d*x]])/(a^3*d) - (b^4*Log[a + b*Sinh[c + d*x]])/(a^3*(a^2 +
b^2)*d)
```

Rubi [A] time = 0.235471, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2837, 12, 894, 635, 203, 260}

$$-\frac{b^4\log(a+b\sinh(c+dx))}{a^3d\left(a^2+b^2\right)}-\frac{\left(a^2-b^2\right)\log(\sinh(c+dx))}{a^3d}+\frac{b\tan^{-1}(\sinh(c+dx))}{d\left(a^2+b^2\right)}+\frac{a\log(\cosh(c+dx))}{d\left(a^2+b^2\right)}+\frac{b\text{csch}(c\cdot}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d) + (b*Csch[c + d*x])/(a^2*d) - Csc
h[c + d*x]^2/(2*a*d) + (a*Log[Cosh[c + d*x]])/((a^2 + b^2)*d) - ((a^2 - b^2
)*Log[Sinh[c + d*x]])/(a^3*d) - (b^4*Log[a + b*Sinh[c + d*x]])/(a^3*(a^2 +
b^2)*d)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)}\,dx = -\frac{b\,\mathrm{Subst}\left(\int \frac{b^3}{x^3(a+x)(-b^2-x^2)}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= -\frac{b^4\,\mathrm{Subst}\left(\int \frac{1}{x^3(a+x)(-b^2-x^2)}\,dx, x, b\sinh(c+dx)\right)}{d}$$

$$= -\frac{b^4\,\mathrm{Subst}\left(\int \left(-\frac{1}{ab^2x^3}+\frac{1}{a^2b^2x^2}+\frac{a^2-b^2}{a^3b^4x}+\frac{1}{a^3(a^2+b^2)(a+x)}+\frac{-b^2-ax}{b^4(a^2+b^2)(b^2+x^2)}\right) dx, x, b\sinh(\right.}{d}$$

$$= \frac{b\operatorname{csch}(c+dx)}{a^2d} - \frac{\operatorname{csch}^2(c+dx)}{2ad} - \frac{(a^2-b^2)\log(\sinh(c+dx))}{a^3d} - \frac{b^4\log(a+b\sinh(c}{a^3(a^2+b^2)d}$$

$$= \frac{b\operatorname{csch}(c+dx)}{a^2d} - \frac{\operatorname{csch}^2(c+dx)}{2ad} - \frac{(a^2-b^2)\log(\sinh(c+dx))}{a^3d} - \frac{b^4\log(a+b\sinh(c}{a^3(a^2+b^2)d}$$

$$= \frac{b\tan^{-1}(\sinh(c+dx))}{(a^2+b^2)d} + \frac{b\operatorname{csch}(c+dx)}{a^2d} - \frac{\operatorname{csch}^2(c+dx)}{2ad} + \frac{a\log(\cosh(c+dx))}{(a^2+b^2)d} - \frac{(a}{}$$

Mathematica [A] time = 0.36427, size = 164, normalized size = 1.26

$$\frac{-\frac{2b^4\log(a+b\sinh(c+dx))}{a^3(a^2+b^2)}+\frac{\left(a-\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\sinh(c+dx)\right)}{a^2+b^2}+\frac{\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\sinh(c+dx)\right)}{a^2+b^2}+\frac{2b\operatorname{csch}(c+dx)}{a^2}-\frac{2(a-b)(a+b)\log(\sinh(c+dx))}{a^3}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[c + d*x]^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((2*b*Csch[c + d*x])/a^2 - Csch[c + d*x]^2/a - (2*(a - b)*(a + b)*Log[Sinh[
c + d*x]])/a^3 + ((a - Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[c + d*x]])/(a^2
+ b^2) - (2*b^4*Log[a + b*Sinh[c + d*x]])/(a^3*(a^2 + b^2)) + ((a + Sqrt[-b
^2])*Log[Sqrt[-b^2] + b*Sinh[c + d*x]])/(a^2 + b^2))/(2*d)
```

Maple [A] time = 0.003, size = 219, normalized size = 1.7

$$-\frac{1}{8da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{b}{2da^2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{8da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-2}-\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{b^2}{da^3}\ln\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)

[Out] -1/8/d/a*tanh(1/2*d*x+1/2*c)^2-1/2/d/a^2*tanh(1/2*d*x+1/2*c)*b-1/8/d/a/tanh(1/2*d*x+1/2*c)^2-1/d/a*ln(tanh(1/2*d*x+1/2*c))+1/d/a^3*ln(tanh(1/2*d*x+1/2*c))*b^2+1/2/d*b/a^2/tanh(1/2*d*x+1/2*c)-1/d*b^4/a^3/(a^2+b^2)*ln(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)*b-a)+1/d/(a^2+b^2)*a*ln(tanh(1/2*d*x+1/2*c)^2+1)+2/d/(a^2+b^2)*b*arctan(tanh(1/2*d*x+1/2*c))

Maxima [A] time = 1.69105, size = 319, normalized size = 2.45

$$-\frac{b^4 \log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{\left(a^5 + a^3b^2\right)d} - \frac{2\,b\arctan\left(e^{(-dx-c)}\right)}{\left(a^2 + b^2\right)d} + \frac{a\log\left(e^{(-2\,dx-2\,c)} + 1\right)}{\left(a^2 + b^2\right)d} - \frac{2\left(be^{(-dx-c)} - ae^{(-2\,dx-2\,c)} - be\right.}{\left(2\,a^2e^{(-2\,dx-2\,c)} - a^2e^{(-4\,dx-4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")

[Out] -b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + a^3*b^2)*d) - 2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + a*log(e^(-2*d*x - 2*c) + 1)/((a^2 + b^2)*d) - 2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) - (a^2 - b^2)*log(e^(-d*x - c) + 1)/(a^3*d) - (a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d)

Fricas [B] time = 3.37535, size = 2503, normalized size = 19.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")

[Out] (2*(a^3*b + a*b^3)*cosh(d*x + c)^3 + 2*(a^3*b + a*b^3)*sinh(d*x + c)^3 - 2*(a^4 + a^2*b^2)*cosh(d*x + c)^2 - 2*(a^4 + a^2*b^2 - 3*(a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(a^3*b*cosh(d*x + c)^4 + 4*a^3*b*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*b*sinh(d*x + c)^4 - 2*a^3*b*cosh(d*x + c)^2 + a^3*b + 2*(3*a^3*b*cosh(d*x + c)^2 - a^3*b)*sinh(d*x + c)^2 + 4*(a^3*b*cosh(d*x + c)^3 - a^3*b*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*(a^3*b + a*b^3)*cosh(d*x + c) - (b^4*cosh(d*x + c)^4 + 4*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + b^4*sinh(d*x + c)^4 - 2*b^4*cosh(d*x + c)^2 + b^4 + 2*(3*b^4*cosh(d*x + c)^2 - b^4)*sinh(d*x + c)^2 + 4*(b^4*cosh(d*x + c)^3 - b^4*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + (a^4*cosh(d*x + c)^4 + 4*a^4*cosh(d*x + c)*sinh(d*x + c)^3 + a^4*sinh(d*x + c)^4 - 2*a^4*cosh(d*x + c)^2 + a^4 + 2*(3*a^4*cosh(d*x + c)^2 - a^4)*sinh(d*x + c)^2 + 4*(a^4*cosh(d*x + c)^3 - a^4*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - ((a^4 - b^4)*cosh(d*x + c)^4 + 4*(a^4 - b^4)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 - b^4)*sinh(d*x + c)^4 + a^4 - b^4 - 2*(a^4 - b^4)*cosh(d*x + c)^2 - 2*(a^4 - b^4 - 3*(a^4 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^4 - b^4)*cosh(d*x + c)^3 - (a^4 - b^4)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - 2*(a^3*b + a*b^3 - 3*(a^3*b + a*b^3)*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^5 + a^3*b^2)*d*cosh(d*x + c)

```
*sinh(d*x + c)^3 + (a^5 + a^3*b^2)*d*sinh(d*x + c)^4 - 2*(a^5 + a^3*b^2)*d*
cosh(d*x + c)^2 + 2*(3*(a^5 + a^3*b^2)*d*cosh(d*x + c)^2 - (a^5 + a^3*b^2)*
d)*sinh(d*x + c)^2 + (a^5 + a^3*b^2)*d + 4*((a^5 + a^3*b^2)*d*cosh(d*x + c)
^3 - (a^5 + a^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.346, size = 370, normalized size = 2.85

$$-\frac{b^5 \log\left(\left|b\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 2\,a\right|\right)}{a^5bd + a^3b^3d} + \frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)b}{2\left(a^2d + b^2d\right)} + \frac{a\log\left(\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4\right)}{2\left(a^2d + b^2d\right)} - \frac{(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -b^5*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^5*b*d + a^3*b^3*d) +
 1/2*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*b/(a^2*d + b^2
*d) + 1/2*a*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^2*d + b^2*d) - (a^2
- b^2)*log(abs(e^(d*x + c) - e^(-d*x - c)))/(a^3*d) + 1/2*(3*a^2*(e^(d*x +
c) - e^(-d*x - c))^2 - 3*b^2*(e^(d*x + c) - e^(-d*x - c))^2 + 4*a*b*(e^(d*x
 + c) - e^(-d*x - c)) - 4*a^2)/(a^3*d*(e^(d*x + c) - e^(-d*x - c))^2)
```

3.495 $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Csch[c + d*x]^3*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi [A] time = 0.09124, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[c + d*x]^3*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Csch[c + d*x]^3*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x])
), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx = \int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [A] time = 158.407, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[c + d*x]^3*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
,x]
```

```
[Out] Integrate[(Csch[c + d*x]^3*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
, x]
```

Maple [A] time = 3.241, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx+c))^3\operatorname{sech}(dx+c)}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -(a*f - 2*(b*d*f*x*e^(3*c) + b*d*e*e^(3*c))*e^(3*d*x) + (2*a*d*f*x*e^(2*c)
+ (2*d*e - f)*a*e^(2*c))*e^(2*d*x) + 2*(b*d*f*x*e^c + b*d*e*e^c)*e^(d*x))/(
a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + a^2*d^2*e^2 + (a^2*d^2*f^2*x^2*e^(4*c)
+ 2*a^2*d^2*e*f*x*e^(4*c) + a^2*d^2*e^2*e^(4*c))*e^(4*d*x) - 2*(a^2*d^2*f^2
*x^2*e^(2*c) + 2*a^2*d^2*e*f*x*e^(2*c) + a^2*d^2*e^2*e^(2*c))*e^(2*d*x)) -
16*integrate(1/16*(b^2*d^2*e^2 + a*b*d*e*f - (d^2*e^2 - f^2)*a^2 - (a^2*d^2
*f^2 - b^2*d^2*f^2)*x^2 - (2*a^2*d^2*e*f - 2*b^2*d^2*e*f - a*b*d*f^2)*x)/(a
^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 - (a
^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*
d^2*e^3*e^c)*e^(d*x)), x) + 16*integrate(-1/16*(b^2*d^2*e^2 - a*b*d*e*f - (
d^2*e^2 - f^2)*a^2 - (a^2*d^2*f^2 - b^2*d^2*f^2)*x^2 - (2*a^2*d^2*e*f - 2*b
^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d
^2*e^2*f*x + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c +
 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) + 16*integrate(-1/8*
(a*b^4*e^(d*x + c) - b^5)/(a^5*b*e + a^3*b^3*e + (a^5*b*f + a^3*b^3*f)*x -
(a^5*b*e*e^(2*c) + a^3*b^3*e*e^(2*c) + (a^5*b*f*e^(2*c) + a^3*b^3*f*e^(2*c)
)*x)*e^(2*d*x) - 2*(a^6*e*e^c + a^4*b^2*e*e^c + (a^6*f*e^c + a^4*b^2*f*e^c)
*x)*e^(d*x)), x) + 16*integrate(1/8*(b*e^(d*x + c) - a)/(a^2*e + b^2*e + (a
^2*f + b^2*f)*x + (a^2*e*e^(2*c) + b^2*e*e^(2*c) + (a^2*f*e^(2*c) + b^2*f*e
^(2*c))*x)*e^(2*d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] Timed out
```

3.496 $\int \frac{(e+fx)^2\mathbf{csch}^3(c+dx)\mathbf{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1245

result too large to display

```
[Out] (2*b*(e + f*x)^2)/(a^2*d) - (b^3*(e + f*x)^2)/(a^2*(a^2 + b^2)*d) + (4*f^2*
x*ArcTan[E^(c + d*x)])/(a*d^2) - (4*b^2*f*(e + f*x)*ArcTan[E^(c + d*x)])/(a
^3*d^2) + (4*b^4*f*(e + f*x)*ArcTan[E^(c + d*x)])/(a^3*(a^2 + b^2)*d^2) + (
2*e*f*ArcTan[Sinh[c + d*x]])/(a*d^2) + (3*(e + f*x)^2*ArcTanh[E^(c + d*x)])
/(a*d) - (2*b^2*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^3*d) - (f^2*ArcTanh[Co
sh[c + d*x]])/(a*d^3) + (2*b*(e + f*x)^2*Coth[2*c + 2*d*x])/(a^2*d) - (e*f*
Csch[c + d*x])/(a*d^2) - (f^2*x*Csch[c + d*x])/(a*d^2) - (b^5*(e + f*x)^2*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)^(3/2)*d) +
(b^5*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*(a^2
+ b^2)^(3/2)*d) + (2*b^3*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a^2*(a^2 +
b^2)*d^2) - (2*b*f*(e + f*x)*Log[1 - E^(4*(c + d*x))])/(a^2*d^2) + (3*f*(e
+ f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2, -E
^(c + d*x)])/(a^3*d^2) - ((2*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^3) +
 ((2*I)*b^2*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a^3*d^3) - ((2*I)*b^4*f^2*Po
lyLog[2, (-I)*E^(c + d*x)])/(a^3*(a^2 + b^2)*d^3) + ((2*I)*f^2*PolyLog[2, I
*E^(c + d*x)])/(a*d^3) - ((2*I)*b^2*f^2*PolyLog[2, I*E^(c + d*x)])/(a^3*d^3
) + ((2*I)*b^4*f^2*PolyLog[2, I*E^(c + d*x)])/(a^3*(a^2 + b^2)*d^3) - (3*f*
(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) + (2*b^2*f*(e + f*x)*PolyLog[2,
E^(c + d*x)])/(a^3*d^2) - (2*b^5*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)^(3/2)*d^2) + (2*b^5*f*(e + f*x)*Po
lyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)^(3/2)*
d^2) + (b^3*f^2*PolyLog[2, -E^(2*(c + d*x))])/(a^2*(a^2 + b^2)*d^3) - (b*f^
2*PolyLog[2, E^(4*(c + d*x))])/(2*a^2*d^3) - (3*f^2*PolyLog[3, -E^(c + d*x)
])/(a*d^3) + (2*b^2*f^2*PolyLog[3, -E^(c + d*x)])/(a^3*d^3) + (3*f^2*PolyLo
g[3, E^(c + d*x)])/(a*d^3) - (2*b^2*f^2*PolyLog[3, E^(c + d*x)])/(a^3*d^3)
+ (2*b^5*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^
2 + b^2)^(3/2)*d^3) - (2*b^5*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2]))])/(a^3*(a^2 + b^2)^(3/2)*d^3) - (3*(e + f*x)^2*Sech[c + d*x])/(2*
a*d) + (b^2*(e + f*x)^2*Sech[c + d*x])/(a^3*d) - (b^4*(e + f*x)^2*Sech[c +
d*x])/(a^3*(a^2 + b^2)*d) - ((e + f*x)^2*Csch[c + d*x]^2*Sech[c + d*x])/(2*
a*d) - (b^3*(e + f*x)^2*Tanh[c + d*x])/(a^2*(a^2 + b^2)*d)
```

Rubi [A] time = 3.43733, antiderivative size = 1245, normalized size of antiderivative = 1., number of steps used = 88, number of rules used = 33, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {5589, 2622, 288, 321, 207, 5462, 6688, 12, 6742, 6273, 4182, 2531, 2282, 6589, 4133, 453, 203, 4180, 2279, 2391, 2621, 5203, 3770, 5461, 4184, 3716, 2190, 6741, 5573, 3322, 2264, 3718, 5451}

$$-\frac{(e+fx)^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)b^5}{a^3\left(a^2+b^2\right)^{3/2}d}+\frac{(e+fx)^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)b^5}{a^3\left(a^2+b^2\right)^{3/2}d}-\frac{2f(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)b^5}{a^3\left(a^2+b^2\right)^{3/2}d^2}+\frac{2f(e+fx)\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*b*(e + f*x)^2)/(a^2*d) - (b^3*(e + f*x)^2)/(a^2*(a^2 + b^2)*d) + (4*f^2*
x*ArcTan[E^(c + d*x)])/(a*d^2) - (4*b^2*f*(e + f*x)*ArcTan[E^(c + d*x)])/(a
^3*d^2) + (4*b^4*f*(e + f*x)*ArcTan[E^(c + d*x)])/(a^3*(a^2 + b^2)*d^2) + (
2*e*f*ArcTan[Sinh[c + d*x]])/(a*d^2) + (3*(e + f*x)^2*ArcTanh[E^(c + d*x)])
```

```
/(a*d) - (2*b^2*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^3*d) - (f^2*ArcTanh[Co
sh[c + d*x]])/(a*d^3) + (2*b*(e + f*x)^2*Coth[2*c + 2*d*x])/(a^2*d) - (e*f*
Csch[c + d*x])/(a*d^2) - (f^2*x*Csch[c + d*x])/(a*d^2) - (b^5*(e + f*x)^2*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)^(3/2)*d) +
(b^5*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*(a^2
+ b^2)^(3/2)*d) + (2*b^3*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a^2*(a^2 +
b^2)*d^2) - (2*b*f*(e + f*x)*Log[1 - E^(4*(c + d*x))])/(a^2*d^2) + (3*f*(e
+ f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2, -E
^(c + d*x)])/(a^3*d^2) - ((2*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^3) +
 ((2*I)*b^2*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a^3*d^3) - ((2*I)*b^4*f^2*Po
lyLog[2, (-I)*E^(c + d*x)])/(a^3*(a^2 + b^2)*d^3) + ((2*I)*f^2*PolyLog[2, I
*E^(c + d*x)])/(a*d^3) - ((2*I)*b^2*f^2*PolyLog[2, I*E^(c + d*x)])/(a^3*d^3
) + ((2*I)*b^4*f^2*PolyLog[2, I*E^(c + d*x)])/(a^3*(a^2 + b^2)*d^3) - (3*f*
(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) + (2*b^2*f*(e + f*x)*PolyLog[2,
E^(c + d*x)])/(a^3*d^2) - (2*b^5*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)^(3/2)*d^2) + (2*b^5*f*(e + f*x)*Po
lyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)^(3/2)*
d^2) + (b^3*f^2*PolyLog[2, -E^(2*(c + d*x))])/(a^2*(a^2 + b^2)*d^3) - (b*f^
2*PolyLog[2, E^(4*(c + d*x))])/(2*a^2*d^3) - (3*f^2*PolyLog[3, -E^(c + d*x)
])/(a*d^3) + (2*b^2*f^2*PolyLog[3, -E^(c + d*x)])/(a^3*d^3) + (3*f^2*PolyLo
g[3, E^(c + d*x)])/(a*d^3) - (2*b^2*f^2*PolyLog[3, E^(c + d*x)])/(a^3*d^3)
+ (2*b^5*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^
2 + b^2)^(3/2)*d^3) - (2*b^5*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2
 + b^2]))])/(a^3*(a^2 + b^2)^(3/2)*d^3) - (3*(e + f*x)^2*Sech[c + d*x])/(2*
a*d) + (b^2*(e + f*x)^2*Sech[c + d*x])/(a^3*d) - (b^4*(e + f*x)^2*Sech[c +
d*x])/(a^3*(a^2 + b^2)*d) - ((e + f*x)^2*Csch[c + d*x]^2*Sech[c + d*x])/(2*
a*d) - (b^3*(e + f*x)^2*Tanh[c + d*x])/(a^2*(a^2 + b^2)*d)
```

Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
```

```
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)^2\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int(e+fx)^2\text{csch}^3(c+dx)\text{sech}^2(c+dx)\,dx}{a} - \frac{b\int\frac{(e+fx)^2\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}}{a}$$

$$= \frac{3(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{3(e+fx)^2\text{sech}(c+dx)}{2ad} - (e+fx)^2\text{csc}$$

$$= \frac{3(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{3(e+fx)^2\text{sech}(c+dx)}{2ad} - (e+fx)^2\text{csc}$$

$$= \frac{3(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{b^2(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{a^3d} + 2b$$

$$= \frac{2b(e+fx)^2}{a^2d} + \frac{3(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{b^2(e+fx)^2\tanh^{-1}(\cosh}{a^3d}$$

$$= \frac{2b(e+fx)^2}{a^2d} + \frac{3(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{b^2(e+fx)^2\tanh^{-1}(\cosh}{a^3d}$$

$$= \frac{2b(e+fx)^2}{a^2d} - \frac{b^2(e+fx)^2\tanh^{-1}(\cosh(c+dx))}{a^3d} + \frac{2b(e+fx)^2\coth(2c+}{a^2d}$$

$$= \frac{2b(e+fx)^2}{a^2d} - \frac{b^3(e+fx)^2}{a^2\left(a^2+b^2\right)d} + \frac{4b^4f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^3\left(a^2+b^2\right)d^2} - b^2(e+fx)^2\text{ta}$$

$$= \frac{2b(e+fx)^2}{a^2d} - \frac{b^3(e+fx)^2}{a^2\left(a^2+b^2\right)d} - \frac{4b^2f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^3d^2} + \frac{4b^4f(e+fx)}{a^3\left(a^2+\right.}$$

$$= \frac{2b(e+fx)^2}{a^2d} - \frac{b^3(e+fx)^2}{a^2\left(a^2+b^2\right)d} - \frac{4b^2f(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^3d^2} + \frac{4b^4f(e+fx)}{a^3\left(a^2+\right.}$$

$$= \frac{2b(e+fx)^2}{a^2d} - \frac{b^3(e+fx)^2}{a^2\left(a^2+b^2\right)d} + \frac{6f^2x\tan^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{4b^2f(e+fx)\tan^{-1}\left(e\right.}{a^3d^2}$$

$$= \frac{2b(e+fx)^2}{a^2d} - \frac{b^3(e+fx)^2}{a^2\left(a^2+b^2\right)d} + \frac{6f^2x\tan^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{4b^2f(e+fx)\tan^{-1}\left(e\right.}{a^3d^2}$$

$$= \frac{2b(e+fx)^2}{a^2d} - \frac{b^3(e+fx)^2}{a^2\left(a^2+b^2\right)d} + \frac{6f^2x\tan^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{4b^2f(e+fx)\tan^{-1}\left(e\right.}{a^3d^2}$$

$$= \frac{2b(e+fx)^2}{a^2d} - \frac{b^3(e+fx)^2}{a^2\left(a^2+b^2\right)d} + \frac{6f^2x\tan^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{4b^2f(e+fx)\tan^{-1}\left(e\right.}{a^3d^2}$$

$$= \frac{2b(e+fx)^2}{a^2d} - \frac{b^3(e+fx)^2}{a^2\left(a^2+b^2\right)d} + \frac{4f^2x\tan^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{4b^2f(e+fx)\tan^{-1}\left(e\right.}{a^3d^2}$$

$$= \frac{2b(e+fx)^2}{a^2d} - \frac{b^3(e+fx)^2}{a^2\left(a^2+b^2\right)d} + \frac{4f^2x\tan^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{4b^2f(e+fx)\tan^{-1}\left(e\right.}{a^3d^2}$$

$$= \frac{2b(e+fx)^2}{a^2d} - \frac{b^3(e+fx)^2}{a^2\left(a^2+b^2\right)d} + \frac{4f^2x\tan^{-1}\left(e^{c+dx}\right)}{ad^2} - \frac{4b^2f(e+fx)\tan^{-1}\left(e\right.}{a^3d^2}$$

Mathematica [B] time = 27.5585, size = 2574, normalized size = 2.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x
]),x]
```

```
[Out] (8*a*b*d^2*e*E^(2*c)*f*x + 4*a*b*d^2*E^(2*c)*f^2*x^2 - 6*a^2*d^2*e^2*ArcTan
h[E^(c + d*x)] + 4*b^2*d^2*e^2*ArcTanh[E^(c + d*x)] + 6*a^2*d^2*e^2*E^(2*c)
*ArcTanh[E^(c + d*x)] - 4*b^2*d^2*e^2*E^(2*c)*ArcTanh[E^(c + d*x)] + 4*a^2*
f^2*ArcTanh[E^(c + d*x)] - 4*a^2*E^(2*c)*f^2*ArcTanh[E^(c + d*x)] + 6*a^2*d
^2*e*f*x*Log[1 - E^(c + d*x)] - 4*b^2*d^2*e*f*x*Log[1 - E^(c + d*x)] - 6*a^
2*d^2*e*E^(2*c)*f*x*Log[1 - E^(c + d*x)] + 4*b^2*d^2*e*E^(2*c)*f*x*Log[1 -
E^(c + d*x)] + 3*a^2*d^2*f^2*x^2*Log[1 - E^(c + d*x)] - 2*b^2*d^2*f^2*x^2*L
og[1 - E^(c + d*x)] - 3*a^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + 2*b^
2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] - 6*a^2*d^2*e*f*x*Log[1 + E^(c +
 d*x)] + 4*b^2*d^2*e*f*x*Log[1 + E^(c + d*x)] + 6*a^2*d^2*e*E^(2*c)*f*x*Log
[1 + E^(c + d*x)] - 4*b^2*d^2*e*E^(2*c)*f*x*Log[1 + E^(c + d*x)] - 3*a^2*d^
2*f^2*x^2*Log[1 + E^(c + d*x)] + 2*b^2*d^2*f^2*x^2*Log[1 + E^(c + d*x)] + 3
*a^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] - 2*b^2*d^2*E^(2*c)*f^2*x^2*L
og[1 + E^(c + d*x)] + 4*a*b*d*e*f*Log[1 - E^(2*(c + d*x))] - 4*a*b*d*e*E^(2
*c)*f*Log[1 - E^(2*(c + d*x))] + 4*a*b*d*f^2*x*Log[1 - E^(2*(c + d*x))] - 4
*a*b*d*E^(2*c)*f^2*x*Log[1 - E^(2*(c + d*x))] + 2*(3*a^2 - 2*b^2)*d*(-1 + E
^(2*c))*f*(e + f*x)*PolyLog[2, -E^(c + d*x)] - 2*(3*a^2 - 2*b^2)*d*(-1 + E^
(2*c))*f*(e + f*x)*PolyLog[2, E^(c + d*x)] + 2*a*b*f^2*PolyLog[2, E^(2*(c +
 d*x))] - 2*a*b*E^(2*c)*f^2*PolyLog[2, E^(2*(c + d*x))] + 6*a^2*f^2*PolyLog
[3, -E^(c + d*x)] - 4*b^2*f^2*PolyLog[3, -E^(c + d*x)] - 6*a^2*E^(2*c)*f^2*
PolyLog[3, -E^(c + d*x)] + 4*b^2*E^(2*c)*f^2*PolyLog[3, -E^(c + d*x)] - 6*a
^2*f^2*PolyLog[3, E^(c + d*x)] + 4*b^2*f^2*PolyLog[3, E^(c + d*x)] + 6*a^2*
E^(2*c)*f^2*PolyLog[3, E^(c + d*x)] - 4*b^2*E^(2*c)*f^2*PolyLog[3, E^(c + d
*x)])/(2*a^3*d^3*(-1 + E^(2*c))) + (b^5*(2*d^2*e^2*ArcTanh[(a + b*E^(c + d*
x))/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*d^2
*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 +
 (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, (b*E^(
c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))] + 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt
[a^2 + b^2])] - 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])
)/(a^3*(a^2 + b^2)^(3/2)*d^3) - (2*b*e*f*Sech[c]*(Cosh[c]*Log[Cosh[c]*Cosh[
d*x] + Sinh[c]*Sinh[d*x]] - d*x*Sinh[c]))/((a^2 + b^2)*d^2*(Cosh[c]^2 - Sin
h[c]^2)) + (4*a*e*f*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2
 - Sinh[c]^2]])/((a^2 + b^2)*d^2*Sqrt[Cosh[c]^2 - Sinh[c]^2]) + (b*f^2*Csch
[c]*(-((d^2*x^2)/E^ArcTanh[Coth[c]]) + (I*Coth[c]*(-(d*x*(-Pi + (2*I)*ArcTa
nh[Coth[c]])) - Pi*Log[1 + E^(2*d*x)] - 2*(I*d*x + I*ArcTanh[Coth[c]])*Log[
1 - E^((2*I)*(I*d*x + I*ArcTanh[Coth[c]]))] + Pi*Log[Cosh[d*x]] + (2*I)*Arc
Tanh[Coth[c]]*Log[I*Sinh[d*x + ArcTanh[Coth[c]]]] + I*PolyLog[2, E^((2*I)*(
I*d*x + I*ArcTanh[Coth[c]]))]))/Sqrt[1 - Coth[c]^2])*Sech[c])/((a^2 + b^2)*
d^3*Sqrt[Csch[c]^2*(-Cosh[c]^2 + Sinh[c]^2)]) + (2*a*f^2*(((-I)*Csch[c]*(I*
(d*x + ArcTanh[Coth[c]])*(Log[1 - E^(-(d*x) - ArcTanh[Coth[c]])] - Log[1 +
E^(-(d*x) - ArcTanh[Coth[c]])]) + I*(PolyLog[2, -E^(-(d*x) - ArcTanh[Coth[c
]])] - PolyLog[2, E^(-(d*x) - ArcTanh[Coth[c]])])))/Sqrt[1 - Coth[c]^2] - (
2*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]]*Arc
Tanh[Coth[c]])/Sqrt[Cosh[c]^2 - Sinh[c]^2]))/((a^2 + b^2)*d^3) + (Csch[c]*C
sch[c + d*x]^2*Sech[c]*Sech[c + d*x]*(2*a^3*e*f*Cosh[2*d*x] + 2*a*b^2*e*f*C
osh[2*d*x] + 2*a^3*f^2*x*Cosh[2*d*x] + 2*a*b^2*f^2*x*Cosh[2*d*x] + 4*a^2*b*
d*e^2*Cosh[c - d*x] + 8*a^2*b*d*e*f*x*Cosh[c - d*x] + 4*a^2*b*d*f^2*x^2*Cos
h[c - d*x] + 2*b^3*d*e^2*Cosh[c + d*x] + 4*b^3*d*e*f*x*Cosh[c + d*x] + 2*b^
```

```
3*d*f^2*x^2*Cosh[c + d*x] + 2*b^3*d*e^2*Cosh[3*c + d*x] + 4*b^3*d*e*f*x*Cos
h[3*c + d*x] + 2*b^3*d*f^2*x^2*Cosh[3*c + d*x] - 2*a^3*e*f*Cosh[4*c + 2*d*x
] - 2*a*b^2*e*f*Cosh[4*c + 2*d*x] - 2*a^3*f^2*x*Cosh[4*c + 2*d*x] - 2*a*b^2
*f^2*x*Cosh[4*c + 2*d*x] - 4*a^2*b*d*e^2*Cosh[c + 3*d*x] - 2*b^3*d*e^2*Cosh
[c + 3*d*x] - 8*a^2*b*d*e*f*x*Cosh[c + 3*d*x] - 4*b^3*d*e*f*x*Cosh[c + 3*d*
x] - 4*a^2*b*d*f^2*x^2*Cosh[c + 3*d*x] - 2*b^3*d*f^2*x^2*Cosh[c + 3*d*x] -
2*b^3*d*e^2*Cosh[3*c + 3*d*x] - 4*b^3*d*e*f*x*Cosh[3*c + 3*d*x] - 2*b^3*d*f
^2*x^2*Cosh[3*c + 3*d*x] + 2*a^3*d*e^2*Sinh[2*c] - 2*a*b^2*d*e^2*Sinh[2*c]
+ 4*a^3*d*e*f*x*Sinh[2*c] - 4*a*b^2*d*e*f*x*Sinh[2*c] + 2*a^3*d*f^2*x^2*Sin
h[2*c] - 2*a*b^2*d*f^2*x^2*Sinh[2*c] + 3*a^3*d*e^2*Sinh[2*d*x] + a*b^2*d*e^
2*Sinh[2*d*x] + 6*a^3*d*e*f*x*Sinh[2*d*x] + 2*a*b^2*d*e*f*x*Sinh[2*d*x] + 3
*a^3*d*f^2*x^2*Sinh[2*d*x] + a*b^2*d*f^2*x^2*Sinh[2*d*x] - 3*a^3*d*e^2*Sinh
[4*c + 2*d*x] - a*b^2*d*e^2*Sinh[4*c + 2*d*x] - 6*a^3*d*e*f*x*Sinh[4*c + 2*
d*x] - 2*a*b^2*d*e*f*x*Sinh[4*c + 2*d*x] - 3*a^3*d*f^2*x^2*Sinh[4*c + 2*d*x
] - a*b^2*d*f^2*x^2*Sinh[4*c + 2*d*x]))/(16*a^2*(a^2 + b^2)*d^2)
```

Maple [F] time = 1.557, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 (\operatorname{csch}(dx+c))^3 (\operatorname{sech}(dx+c))^2}{a+b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)**3*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algori
thm="giac")
```

```
[Out] Timed out
```

3.497 $\int \frac{(e+fx)\mathbf{csch}^3(c+dx)\mathbf{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=699

$$-\frac{b^5 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2\left(a^2+b^2\right)^{3/2}}+\frac{b^5 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3d^2\left(a^2+b^2\right)^{3/2}}-\frac{b^2 f \text{PolyLog}\left(2,-e^{c+dx}\right)}{a^3d^2}+\frac{b^2 f \text{PolyLog}\left(2,e^{c+dx}\right)}{a^3d^2}+3f\text{P}$$

```
[Out] (f*ArcTan[Sinh[c + d*x]])/(a*d^2) - (b^2*f*ArcTan[Sinh[c + d*x]])/(a^3*d^2)
 + (b^4*f*ArcTan[Sinh[c + d*x]])/(a^3*(a^2 + b^2)*d^2) + (3*f*x*ArcTanh[E^(
c + d*x)])/(a*d) - (2*b^2*f*x*ArcTanh[E^(c + d*x)])/(a^3*d) - (3*f*x*ArcTan
h[Cosh[c + d*x]])/(2*a*d) + (b^2*f*x*ArcTanh[Cosh[c + d*x]])/(a^3*d) + (3*(
e + f*x)*ArcTanh[Cosh[c + d*x]])/(2*a*d) - (b^2*(e + f*x)*ArcTanh[Cosh[c +
d*x]])/(a^3*d) + (2*b*(e + f*x)*Coth[2*c + 2*d*x])/(a^2*d) - (f*Csch[c + d*
x])/(2*a*d^2) - (b^5*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
)])/(a^3*(a^2 + b^2)^(3/2)*d) + (b^5*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)^(3/2)*d) + (b^3*f*Log[Cosh[c + d*x]])/
(a^2*(a^2 + b^2)*d^2) - (b*f*Log[Sinh[2*c + 2*d*x]])/(a^2*d^2) + (3*f*PolyL
og[2, -E^(c + d*x)])/(2*a*d^2) - (b^2*f*PolyLog[2, -E^(c + d*x)])/(a^3*d^2)
 - (3*f*PolyLog[2, E^(c + d*x)])/(2*a*d^2) + (b^2*f*PolyLog[2, E^(c + d*x)]
)/(a^3*d^2) - (b^5*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
(a^3*(a^2 + b^2)^(3/2)*d^2) + (b^5*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2]))])/(a^3*(a^2 + b^2)^(3/2)*d^2) - (3*(e + f*x)*Sech[c + d*x])/(
2*a*d) + (b^2*(e + f*x)*Sech[c + d*x])/(a^3*d) - (b^4*(e + f*x)*Sech[c + d*
x])/(a^3*(a^2 + b^2)*d) - ((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x])/(2*a*d)
 - (b^3*(e + f*x)*Tanh[c + d*x])/(a^2*(a^2 + b^2)*d)
```

Rubi [A] time = 1.35635, antiderivative size = 699, normalized size of antiderivative = 1., number of steps used = 44, number of rules used = 22, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {5589, 2622, 288, 321, 207, 5462, 6271, 12, 4182, 2279, 2391, 3770, 2621, 5461, 4184, 3475, 5573, 3322, 2264, 2190, 6742, 5451}

$$-\frac{b^5 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2\left(a^2+b^2\right)^{3/2}}+\frac{b^5 f \text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3d^2\left(a^2+b^2\right)^{3/2}}-\frac{b^2 f \text{PolyLog}\left(2,-e^{c+dx}\right)}{a^3d^2}+\frac{b^2 f \text{PolyLog}\left(2,e^{c+dx}\right)}{a^3d^2}+3f\text{P}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (f*ArcTan[Sinh[c + d*x]])/(a*d^2) - (b^2*f*ArcTan[Sinh[c + d*x]])/(a^3*d^2)
 + (b^4*f*ArcTan[Sinh[c + d*x]])/(a^3*(a^2 + b^2)*d^2) + (3*f*x*ArcTanh[E^(
c + d*x)])/(a*d) - (2*b^2*f*x*ArcTanh[E^(c + d*x)])/(a^3*d) - (3*f*x*ArcTan
h[Cosh[c + d*x]])/(2*a*d) + (b^2*f*x*ArcTanh[Cosh[c + d*x]])/(a^3*d) + (3*(
e + f*x)*ArcTanh[Cosh[c + d*x]])/(2*a*d) - (b^2*(e + f*x)*ArcTanh[Cosh[c +
d*x]])/(a^3*d) + (2*b*(e + f*x)*Coth[2*c + 2*d*x])/(a^2*d) - (f*Csch[c + d*
x])/(2*a*d^2) - (b^5*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
)])/(a^3*(a^2 + b^2)^(3/2)*d) + (b^5*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a +
 Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)^(3/2)*d) + (b^3*f*Log[Cosh[c + d*x]])/
(a^2*(a^2 + b^2)*d^2) - (b*f*Log[Sinh[2*c + 2*d*x]])/(a^2*d^2) + (3*f*PolyL
og[2, -E^(c + d*x)])/(2*a*d^2) - (b^2*f*PolyLog[2, -E^(c + d*x)])/(a^3*d^2)
 - (3*f*PolyLog[2, E^(c + d*x)])/(2*a*d^2) + (b^2*f*PolyLog[2, E^(c + d*x)]
)/(a^3*d^2) - (b^5*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/
(a^3*(a^2 + b^2)^(3/2)*d^2) + (b^5*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2]))])/(a^3*(a^2 + b^2)^(3/2)*d^2) - (3*(e + f*x)*Sech[c + d*x])/(
2*a*d) + (b^2*(e + f*x)*Sech[c + d*x])/(a^3*d) - (b^4*(e + f*x)*Sech[c + d*
```

```
x])/(a^3*(a^2 + b^2)*d) - ((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x])/(2*a*d)
 - (b^3*(e + f*x)*Tanh[c + d*x])/(a^2*(a^2 + b^2)*d)
```

Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
```

```
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\text{csch}^3(c+dx)\text{sech}^2(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\text{csch}^2(c+dx)\text{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= \frac{3(e+fx)\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{3(e+fx)\text{sech}(c+dx)}{2ad} - \frac{(e+fx)\text{csch}^2(}{}$$

$$= \frac{3(e+fx)\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{3(e+fx)\text{sech}(c+dx)}{2ad} - \frac{(e+fx)\text{csch}^2(}{}$$

$$= \frac{3f\tan^{-1}(\sinh(c+dx))}{2ad^2} - \frac{3fx\tanh^{-1}(\cosh(c+dx))}{2ad} + \frac{3(e+fx)\tanh^{-1}(c}{2ad}$$

$$= \frac{3f\tan^{-1}(\sinh(c+dx))}{2ad^2} - \frac{3fx\tanh^{-1}(\cosh(c+dx))}{2ad} + \frac{3(e+fx)\tanh^{-1}(c}{2ad}$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{ad^2} - \frac{b^2f\tan^{-1}(\sinh(c+dx))}{a^3d^2} + \frac{3fx\tanh^{-1}\left(e^{c+dx}\right)}{ad} - 3$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{ad^2} - \frac{b^2f\tan^{-1}(\sinh(c+dx))}{a^3d^2} + \frac{3fx\tanh^{-1}\left(e^{c+dx}\right)}{ad} - 3$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{ad^2} - \frac{b^2f\tan^{-1}(\sinh(c+dx))}{a^3d^2} + \frac{b^4f\tan^{-1}(\sinh(c+dx}{a^3\left(a^2+b^2\right)d^2}$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{ad^2} - \frac{b^2f\tan^{-1}(\sinh(c+dx))}{a^3d^2} + \frac{b^4f\tan^{-1}(\sinh(c+dx}{a^3\left(a^2+b^2\right)d^2}$$

$$= \frac{f\tan^{-1}(\sinh(c+dx))}{ad^2} - \frac{b^2f\tan^{-1}(\sinh(c+dx))}{a^3d^2} + \frac{b^4f\tan^{-1}(\sinh(c+dx}{a^3\left(a^2+b^2\right)d^2}$$

Mathematica [C] time = 8.4167, size = 863, normalized size = 1.23

$$\frac{\left(2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) - 2cf\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) - f(c+dx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right) + f(c+dx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right) - f\text{PolyLog}}{a^3\left(a^2+b^2\right)^{3/2}d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] (2*a*f*ArcTan[Tanh[(c + d*x)/2]])/(d*(a^2*d + b^2*d)) + ((2*b*d*e*Cosh[(c +
 d*x)/2] - a*f*Cosh[(c + d*x)/2] - 2*b*c*f*Cosh[(c + d*x)/2] + 2*b*f*(c + d
*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(4*a^2*d^2) + ((-(d*e) + c*f - f*
(c + d*x))*Csch[(c + d*x)/2]^2)/(8*a*d^2) - (b*f*Log[Cosh[c + d*x]])/((a^2
+ b^2)*d^2) - (b*f*Log[Sinh[c + d*x]])/(a^2*d^2) - (3*e*Log[Tanh[(c + d*x)/
2]])/(2*a*d) + (b^2*e*Log[Tanh[(c + d*x)/2]])/(a^3*d) + (3*c*f*Log[Tanh[(c
+ d*x)/2]])/(2*a*d^2) - (b^2*c*f*Log[Tanh[(c + d*x)/2]])/(a^3*d^2) + (((3*I
)/2)*f*(I*(c + d*x)*(Log[1 - E^(-c - d*x)] - Log[1 + E^(-c - d*x)]) + I*(Po
lyLog[2, -E^(-c - d*x)] - PolyLog[2, E^(-c - d*x)])))/(a*d^2) - (I*b^2*f*(I
*(c + d*x)*(Log[1 - E^(-c - d*x)] - Log[1 + E^(-c - d*x)]) + I*(PolyLog[2,
-E^(-c - d*x)] - PolyLog[2, E^(-c - d*x)])))/(a^3*d^2) + (b^5*(2*d*e*ArcTan
```

```
h[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*c*f*ArcTanh[(a + b*E^(c + d*x))/
Sqrt[a^2 + b^2]] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
)] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - f*PolyLog
[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))]))/(a^3*(a^2 + b^2)^(3/2)*d^2) + ((-(d*e) + c*f -
f*(c + d*x))*Sech[(c + d*x)/2]^2)/(8*a*d^2) + (Sech[(c + d*x)/2]*(2*b*d*e*S
inh[(c + d*x)/2] + a*f*Sinh[(c + d*x)/2] - 2*b*c*f*Sinh[(c + d*x)/2] + 2*b*
f*(c + d*x)*Sinh[(c + d*x)/2]))/(4*a^2*d^2) + (Sech[c + d*x]*(-(a*d*e) + a*
c*f - a*f*(c + d*x) + b*d*e*Sinh[c + d*x] - b*c*f*Sinh[c + d*x] + b*f*(c +
d*x)*Sinh[c + d*x]))/((a^2 + b^2)*d^2)
```

Maple [B] time = 0.276, size = 2767, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/(a^2+b^2)^(5/2)/d^2*f*b*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
*a^3+2/(a^2+b^2)^(5/2)/d^2*f*b^3*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)
^(1/2))*a-3/2/(a^2+b^2)/d*a*e*ln(exp(d*x+c)-1)+3/2/(a^2+b^2)/d*a*e*ln(exp(d
*x+c)+1)+3/2/(a^2+b^2)/d^2*a*f*dilog(exp(d*x+c))+3/2/(a^2+b^2)/d^2*a*f*dilo
g(exp(d*x+c)+1)+1/(a^2+b^2)^(5/2)/d*f*b^5/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2
)+a)/(a+(a^2+b^2)^(1/2)))*x+1/2/(a^2+b^2)^(3/2)/d^2*b^3*f*c/a*arctanh(1/2*(
2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-3/2/(a^2+b^2)^(5/2)/d^2*f*a^3*b*c*arct
anh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-3/2/(a^2+b^2)^(5/2)/d^2*b^5*f
*c/a*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)^(5/2)/d*
f*b^5/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/(a^2
+b^2)^(5/2)/d^2*f*b^5/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1
/2)))*c-1/(a^2+b^2)^(5/2)/d^2*f*b^5/a*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
(-a+(a^2+b^2)^(1/2)))*c-1/a^3/(a^2+b^2)^(5/2)/d^2*b^7*f*dilog((-b*exp(d*x+c
)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/a^3/(a^2+b^2)^(5/2)/d^2*b^7*f*
dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/a^3/(a^2+b^2)
/d^2*b^4*f*c*ln(exp(d*x+c)-1)+1/a^3/(a^2+b^2)^(3/2)/d*b^5*e*arctanh(1/2*(2*
b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/a^3/(a^2+b^2)^(5/2)/d*b^7*e*arctanh(1/
2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/a^3/(a^2+b^2)/d*b^4*f*ln(exp(d*x+
c)+1)*x-1/a^3/(a^2+b^2)/d*b^4*e*ln(exp(d*x+c)+1)+1/a^3/(a^2+b^2)/d*b^4*e*ln
(exp(d*x+c)-1)-1/a^3/(a^2+b^2)/d^2*b^4*f*dilog(exp(d*x+c))-1/a^3/(a^2+b^2)/
d^2*b^4*f*dilog(exp(d*x+c)+1)-2*a/d^2/(a^2+b^2)^(5/2)*b^3*f*c*arctanh(1/2*(
2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+4/(a^2+b^2)/d^2*b*f*ln(exp(d*x+c))+3/2
/(a^2+b^2)/d*ln(exp(d*x+c)+1)*a*f*x+8/(a^2+b^2)/d^2*a^3*f/(4*a^2+4*b^2)*arc
tan(exp(d*x+c))-4/(a^2+b^2)/d^2*f*b^3/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))+3/
2/(a^2+b^2)/d^2*a*f*c*ln(exp(d*x+c)-1)+1/2/(a^2+b^2)/d^2*b^2*f/a*dilog(exp(
d*x+c))+1/2/(a^2+b^2)/d^2*b^2*f/a*dilog(exp(d*x+c)+1)-1/2/(a^2+b^2)/d*b^2*e
/a*ln(exp(d*x+c)-1)+1/2/(a^2+b^2)/d*b^2*e/a*ln(exp(d*x+c)+1)-3/2*b/d*e/(a^2
+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a+1/a^3/(a^2+
b^2)^(5/2)/d^2*b^7*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)
))*c-1/a^3/(a^2+b^2)^(5/2)/d^2*b^7*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^
2+b^2)^(1/2))-1/a^3/(a^2+b^2)^(5/2)/d*b^7*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/
2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/a^3/(a^2+b^2)^(5/2)/d*b^7*f*ln((b*exp(d*x+c
)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/a^3/(a^2+b^2)^(3/2)/d^2*b^5*f
*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/a^3/(a^2+b^2)^(5/2)/
d^2*b^7*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/2/
(a^2+b^2)/d^2*b^2*f*c/a*ln(exp(d*x+c)-1)-4/(a^2+b^2)/d^2*a^2*b*f/(4*a^2+4*b
^2)*ln(1+exp(2*d*x+2*c))+8/(a^2+b^2)/d^2*a*b^2*f/(4*a^2+4*b^2)*arctan(exp(d
*x+c))-1/(a^2+b^2)^(3/2)/d^2*f*b^3/a*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+
```

```
b^2)^(1/2))-1/(a^2+b^2)^(5/2)/d^2*f*b^5/a*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1
/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/(a^2+b^2)^(5/2)/d^2*f*b^5/a*dilog((b*exp(d*x
+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/(a^2+b^2)^(5/2)/d^2*f*b^5/a*a
rctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)^(3/2)/d^2*a*f*
b*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/2/(a^2+b^2)^(3/2)/d*b
^3*e/a*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+3/2/(a^2+b^2)^(5/2
)/d*a^3*b*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+3/2/(a^2+b^2)
^(5/2)/d*b^5*e/a*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/2/(a^2
+b^2)/d*b^2*f/a*ln(exp(d*x+c)+1)*x-(3*a^3*d*f*x*exp(5*d*x+5*c)+a*b^2*d*f*x*
exp(5*d*x+5*c)+3*a^3*d*e*exp(5*d*x+5*c)+a*b^2*d*e*exp(5*d*x+5*c)-2*b^3*d*f*
x*exp(4*d*x+4*c)-2*a^3*d*f*x*exp(3*d*x+3*c)+a^3*f*exp(5*d*x+5*c)+2*a*b^2*d*
f*x*exp(3*d*x+3*c)+a*b^2*f*exp(5*d*x+5*c)-2*b^3*d*e*exp(4*d*x+4*c)-2*a^3*d*
e*exp(3*d*x+3*c)-4*a^2*b*d*f*x*exp(2*d*x+2*c)+2*a*b^2*d*e*exp(3*d*x+3*c)+3*
a^3*d*f*x*exp(d*x+c)-4*a^2*b*d*e*exp(2*d*x+2*c)+a*b^2*d*f*x*exp(d*x+c)+3*a^
3*d*e*exp(d*x+c)+4*a^2*b*d*f*x+a*b^2*d*e*exp(d*x+c)+2*b^3*d*f*x-a^3*f*exp(d
*x+c)+4*a^2*b*d*e-a*b^2*f*exp(d*x+c)+2*b^3*d*e)/d^2/a^2/(exp(2*d*x+2*c)-1)^
2/(a^2+b^2)/(1+exp(2*d*x+2*c))+3/2*b/d^2*f*c/(a^2+b^2)^(3/2)*arctanh(1/2*(2
*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a+2/a^2/(a^2+b^2)/d^2*b^3*f*ln(exp(d*x+
c))-1/a^2/(a^2+b^2)/d^2*b^3*f*ln(exp(d*x+c)-1)-1/a^2/(a^2+b^2)/d^2*b^3*f*ln
(exp(d*x+c)+1)+2*a/d/(a^2+b^2)^(5/2)*b^3*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)
/(a^2+b^2)^(1/2))-1/(a^2+b^2)/d^2*b*f*ln(exp(d*x+c)-1)-1/(a^2+b^2)/d^2*b*f*
ln(exp(d*x+c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.71445, size = 24195, normalized size = 34.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/2*(4*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)*c
*f)*cosh(d*x + c)^6 + 4*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d*f*x + (a^5*b + 2*a
^3*b^3 + a*b^5)*c*f)*sinh(d*x + c)^6 - 2*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*f
*x + (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*e + (a^6 + 2*a^4*b^2 + a^2*b^4)*f)*cos
h(d*x + c)^5 - 2*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*f*x + (3*a^6 + 4*a^4*b^2
+ a^2*b^4)*d*e + (a^6 + 2*a^4*b^2 + a^2*b^4)*f - 12*((2*a^5*b + 3*a^3*b^3 +
 a*b^5)*d*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)*c*f)*cosh(d*x + c))*sinh(d*x +
c)^5 - 4*(2*(a^5*b + a^3*b^3)*d*f*x - (a^3*b^3 + a*b^5)*d*e + (a^5*b + 2*a^
3*b^3 + a*b^5)*c*f)*cosh(d*x + c)^4 - 2*(4*(a^5*b + a^3*b^3)*d*f*x - 2*(a^3
*b^3 + a*b^5)*d*e + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*c*f - 30*((2*a^5*b + 3*a^
3*b^3 + a*b^5)*d*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)*c*f)*cosh(d*x + c)^2 + 5
*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*f*x + (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*e +
```

(a^6 + 2*a^4*b^2 + a^2*b^4)*f)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*((a^6 - a^2*b^4)*d*f*x + (a^6 - a^2*b^4)*d*e)*cosh(d*x + c)^3 + 4*((a^6 - a^2*b^4)*d*f*x + 20*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)*c*f)*cosh(d*x + c)^3 + (a^6 - a^2*b^4)*d*e - 5*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*f*x + (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*e + (a^6 + 2*a^4*b^2 + a^2*b^4)*f)*cosh(d*x + c)^2 - 4*(2*(a^5*b + a^3*b^3)*d*f*x - (a^3*b^3 + a*b^5)*d*e + (a^5*b + 2*a^3*b^3 + a*b^5)*c*f)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(2*a^5*b + 3*a^3*b^3 + a*b^5)*d*e + 4*(a^5*b + 2*a^3*b^3 + a*b^5)*c*f - 4*((a^3*b^3 + a*b^5)*d*f*x - 2*(a^5*b + a^3*b^3)*d*e + (a^5*b + 2*a^3*b^3 + a*b^5)*c*f)*cosh(d*x + c)^2 + 4*(15*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)*c*f)*cosh(d*x + c)^4 - (a^3*b^3 + a*b^5)*d*f*x - 5*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*f*x + (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*e + (a^6 + 2*a^4*b^2 + a^2*b^4)*f)*cosh(d*x + c)^3 + 2*(a^5*b + a^3*b^3)*d*e - (a^5*b + 2*a^3*b^3 + a*b^5)*c*f - 6*(2*(a^5*b + a^3*b^3)*d*f*x - (a^3*b^3 + a*b^5)*d*e + (a^5*b + 2*a^3*b^3 + a*b^5)*c*f)*cosh(d*x + c)^2 + 3*((a^6 - a^2*b^4)*d*f*x + (a^6 - a^2*b^4)*d*e)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(b^6*f*cosh(d*x + c)^6 + 6*b^6*f*cosh(d*x + c)*sinh(d*x + c)^5 + b^6*f*sinh(d*x + c)^6 - b^6*f*cosh(d*x + c)^4 - b^6*f*cosh(d*x + c)^2 + b^6*f + (15*b^6*f*cosh(d*x + c)^2 - b^6*f)*sinh(d*x + c)^4 + 4*(5*b^6*f*cosh(d*x + c)^3 - b^6*f*cosh(d*x + c))*sinh(d*x + c)^3 + (15*b^6*f*cosh(d*x + c)^4 - 6*b^6*f*cosh(d*x + c)^2 - b^6*f)*sinh(d*x + c)^2 + 2*(3*b^6*f*cosh(d*x + c)^5 - 2*b^6*f*cosh(d*x + c)^3 - b^6*f*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^6*f*cosh(d*x + c)^6 + 6*b^6*f*cosh(d*x + c)*sinh(d*x + c)^5 + b^6*f*sinh(d*x + c)^6 - b^6*f*cosh(d*x + c)^4 - b^6*f*cosh(d*x + c)^2 + b^6*f + (15*b^6*f*cosh(d*x + c)^2 - b^6*f)*sinh(d*x + c)^4 + 4*(5*b^6*f*cosh(d*x + c)^3 - b^6*f*cosh(d*x + c))*sinh(d*x + c)^3 + (15*b^6*f*cosh(d*x + c)^4 - 6*b^6*f*cosh(d*x + c)^2 - b^6*f)*sinh(d*x + c)^2 + 2*(3*b^6*f*cosh(d*x + c)^5 - 2*b^6*f*cosh(d*x + c)^3 - b^6*f*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^6*d*e - b^6*c*f + (b^6*d*e - b^6*c*f)*cosh(d*x + c)^6 + 6*(b^6*d*e - b^6*c*f)*cosh(d*x + c)*sinh(d*x + c)^5 + (b^6*d*e - b^6*c*f)*sinh(d*x + c)^6 - (b^6*d*e - b^6*c*f)*cosh(d*x + c)^4 - (b^6*d*e - b^6*c*f - 15*(b^6*d*e - b^6*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(b^6*d*e - b^6*c*f)*cosh(d*x + c)^3 - (b^6*d*e - b^6*c*f)*cosh(d*x + c))*sinh(d*x + c)^3 - (b^6*d*e - b^6*c*f)*cosh(d*x + c)^2 - (b^6*d*e - b^6*c*f - 15*(b^6*d*e - b^6*c*f)*cosh(d*x + c)^4 + 6*(b^6*d*e - b^6*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(b^6*d*e - b^6*c*f)*cosh(d*x + c)^5 - 2*(b^6*d*e - b^6*c*f)*cosh(d*x + c)^3 - (b^6*d*e - b^6*c*f)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(b^6*d*e - b^6*c*f + (b^6*d*e - b^6*c*f)*cosh(d*x + c)^6 + 6*(b^6*d*e - b^6*c*f)*cosh(d*x + c)*sinh(d*x + c)^5 + (b^6*d*e - b^6*c*f)*sinh(d*x + c)^6 - (b^6*d*e - b^6*c*f)*cosh(d*x + c)^4 - (b^6*d*e - b^6*c*f - 15*(b^6*d*e - b^6*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(b^6*d*e - b^6*c*f)*cosh(d*x + c)^3 - (b^6*d*e - b^6*c*f)*cosh(d*x + c))*sinh(d*x + c)^3 - (b^6*d*e - b^6*c*f)*cosh(d*x + c)^2 - (b^6*d*e - b^6*c*f - 15*(b^6*d*e - b^6*c*f)*cosh(d*x + c)^4 + 6*(b^6*d*e - b^6*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(b^6*d*e - b^6*c*f)*cosh(d*x + c)^5 - 2*(b^6*d*e - b^6*c*f)*cosh(d*x + c)^3 - (b^6*d*e - b^6*c*f)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(b^6*d*f*x + b^6*c*f + (b^6*d*f*x + b^6*c*f)*cosh(d*x + c)^6 + 6*(b^6*d*f*x + b^6*c*f)*cosh(d*x + c)*sinh(d*x + c)^5 + (b^6*d*f*x + b^6*c*f)*sinh(d*x + c)^6 - (b^6*d*f*x + b^6*c*f)*cosh(d*x + c)^4 - (b^6*d*f*x + b^6*c*f - 15*(b^6*d*f*x + b^6*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(b^6*d*f*x + b^6*c*f)*cosh(d*x + c)^3 - (b^6*d*f*x + b^6*c*f)*cosh(d*x + c))*sinh(d*x + c)^3 - (b^6*d*f*x + b^6*c*f)*cosh(d*x + c)^2 - (b^6*d*f*x + b^6*c*f - 15*(b^6*d*f*x + b^6*c*f)*cosh(d*x + c)^4 + 6*(b^6*d*f*x + b^6*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(b^6*d*f*x +

b^6*c*f)*cosh(d*x + c)^5 - 2*(b^6*d*f*x + b^6*c*f)*cosh(d*x + c)^3 - (b^6*
d*f*x + b^6*c*f)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(
a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqr
t((a^2 + b^2)/b^2) - b)/b) + 2*(b^6*d*f*x + b^6*c*f + (b^6*d*f*x + b^6*c*f)
*cosh(d*x + c)^6 + 6*(b^6*d*f*x + b^6*c*f)*cosh(d*x + c)*sinh(d*x + c)^5 +
(b^6*d*f*x + b^6*c*f)*sinh(d*x + c)^6 - (b^6*d*f*x + b^6*c*f)*cosh(d*x + c)
^4 - (b^6*d*f*x + b^6*c*f - 15*(b^6*d*f*x + b^6*c*f)*cosh(d*x + c)^2)*sinh(
d*x + c)^4 + 4*(5*(b^6*d*f*x + b^6*c*f)*cosh(d*x + c)^3 - (b^6*d*f*x + b^6*
c*f)*cosh(d*x + c))*sinh(d*x + c)^3 - (b^6*d*f*x + b^6*c*f)*cosh(d*x + c)^2
 - (b^6*d*f*x + b^6*c*f - 15*(b^6*d*f*x + b^6*c*f)*cosh(d*x + c)^4 + 6*(b^6
*d*f*x + b^6*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(b^6*d*f*x + b^6*
c*f)*cosh(d*x + c)^5 - 2*(b^6*d*f*x + b^6*c*f)*cosh(d*x + c)^3 - (b^6*d*f*x
 + b^6*c*f)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cos
h(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b) + 4*((a^6 + a^4*b^2)*f*cosh(d*x + c)^6 + 6*(a^6 + a^4
*b^2)*f*cosh(d*x + c)*sinh(d*x + c)^5 + (a^6 + a^4*b^2)*f*sinh(d*x + c)^6 -
 (a^6 + a^4*b^2)*f*cosh(d*x + c)^4 + (15*(a^6 + a^4*b^2)*f*cosh(d*x + c)^2
 - (a^6 + a^4*b^2)*f)*sinh(d*x + c)^4 - (a^6 + a^4*b^2)*f*cosh(d*x + c)^2 +
4*(5*(a^6 + a^4*b^2)*f*cosh(d*x + c)^3 - (a^6 + a^4*b^2)*f*cosh(d*x + c))*s
inh(d*x + c)^3 + (15*(a^6 + a^4*b^2)*f*cosh(d*x + c)^4 - 6*(a^6 + a^4*b^2)*
f*cosh(d*x + c)^2 - (a^6 + a^4*b^2)*f)*sinh(d*x + c)^2 + (a^6 + a^4*b^2)*f
 + 2*(3*(a^6 + a^4*b^2)*f*cosh(d*x + c)^5 - 2*(a^6 + a^4*b^2)*f*cosh(d*x + c
)^3 - (a^6 + a^4*b^2)*f*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c)
 + sinh(d*x + c)) - 2*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*f*x + (3*a^6 + 4*a^4*
b^2 + a^2*b^4)*d*e - (a^6 + 2*a^4*b^2 + a^2*b^4)*f)*cosh(d*x + c) - ((3*a^6
 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*f*cosh(d*x + c)^6 + 6*(3*a^6 + 4*a^4*b^2 -
a^2*b^4 - 2*b^6)*f*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^6 + 4*a^4*b^2 - a^2
*b^4 - 2*b^6)*f*sinh(d*x + c)^6 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*f*c
osh(d*x + c)^4 + (15*(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*f*cosh(d*x + c)^
2 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*f)*sinh(d*x + c)^4 - (3*a^6 + 4*a
^4*b^2 - a^2*b^4 - 2*b^6)*f*cosh(d*x + c)^2 + 4*(5*(3*a^6 + 4*a^4*b^2 - a^2
*b^4 - 2*b^6)*f*cosh(d*x + c)^3 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*f*c
osh(d*x + c))*sinh(d*x + c)^3 + (15*(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*f
*cosh(d*x + c)^4 - 6*(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*f*cosh(d*x + c)^
2 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*f)*sinh(d*x + c)^2 + (3*a^6 + 4*a
^4*b^2 - a^2*b^4 - 2*b^6)*f + 2*(3*(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*f*
cosh(d*x + c)^5 - 2*(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*f*cosh(d*x + c)^3
 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*f*cosh(d*x + c))*sinh(d*x + c))*di
log(cosh(d*x + c) + sinh(d*x + c)) + ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)
*f*cosh(d*x + c)^6 + 6*(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*f*cosh(d*x + c
)*sinh(d*x + c)^5 + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*f*sinh(d*x + c)^6
 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*f*cosh(d*x + c)^4 + (15*(3*a^6 + 4
*a^4*b^2 - a^2*b^4 - 2*b^6)*f*cosh(d*x + c)^2 - (3*a^6 + 4*a^4*b^2 - a^2*b^
4 - 2*b^6)*f)*sinh(d*x + c)^4 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*f*cos
h(d*x + c)^2 + 4*(5*(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*f*cosh(d*x + c)^3
 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*f*cosh(d*x + c))*sinh(d*x + c)^3 +
 (15*(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*f*cosh(d*x + c)^4 - 6*(3*a^6 + 4
*a^4*b^2 - a^2*b^4 - 2*b^6)*f*cosh(d*x + c)^2 - (3*a^6 + 4*a^4*b^2 - a^2*b^
4 - 2*b^6)*f)*sinh(d*x + c)^2 + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*f + 2
(3(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*f*cosh(d*x + c)^5 - 2*(3*a^6 + 4*
a^4*b^2 - a^2*b^4 - 2*b^6)*f*cosh(d*x + c)^3 - (3*a^6 + 4*a^4*b^2 - a^2*b^4
 - 2*b^6)*f*cosh(d*x + c))*sinh(d*x + c))*dilog(-cosh(d*x + c) - sinh(d*x +
 c)) - 2*((a^5*b + a^3*b^3)*f*cosh(d*x + c)^6 + 6*(a^5*b + a^3*b^3)*f*cosh(
d*x + c)*sinh(d*x + c)^5 + (a^5*b + a^3*b^3)*f*sinh(d*x + c)^6 - (a^5*b + a
^3*b^3)*f*cosh(d*x + c)^4 + (15*(a^5*b + a^3*b^3)*f*cosh(d*x + c)^2 - (a^5*
b + a^3*b^3)*f)*sinh(d*x + c)^4 - (a^5*b + a^3*b^3)*f*cosh(d*x + c)^2 + 4*(
5*(a^5*b + a^3*b^3)*f*cosh(d*x + c)^3 - (a^5*b + a^3*b^3)*f*cosh(d*x + c))*
sinh(d*x + c)^3 + (15*(a^5*b + a^3*b^3)*f*cosh(d*x + c)^4 - 6*(a^5*b + a^3*
b^3)*f*cosh(d*x + c)^2 - (a^5*b + a^3*b^3)*f)*sinh(d*x + c)^2 + (a^5*b + a^

3*b^3)*f + 2*(3*(a^5*b + a^3*b^3)*f*cosh(d*x + c)^5 - 2*(a^5*b + a^3*b^3)*f
*cosh(d*x + c)^3 - (a^5*b + a^3*b^3)*f*cosh(d*x + c))*sinh(d*x + c))*log(2*
cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + (((3*a^6 + 4*a^4*b^2 - a^2
*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 2*(a^5*b
+ 2*a^3*b^3 + a*b^5)*f)*cosh(d*x + c)^6 + 6*((3*a^6 + 4*a^4*b^2 - a^2*b^4 -
2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 2*(a^5*b + 2*a^
3*b^3 + a*b^5)*f)*cosh(d*x + c)*sinh(d*x + c)^5 + ((3*a^6 + 4*a^4*b^2 - a^2
*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 2*(a^5*b
+ 2*a^3*b^3 + a*b^5)*f)*sinh(d*x + c)^6 - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2
*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 2*(a^5*b + 2*a^3*
b^3 + a*b^5)*f)*cosh(d*x + c)^4 - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*
f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 15*((3*a^6 + 4*a^4*b^2 -
a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 2*(a^5
*b + 2*a^3*b^3 + a*b^5)*f)*cosh(d*x + c)^2 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*
f)*sinh(d*x + c)^4 + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + 4*(5*((3
*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 -
2*b^6)*d*e - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*f)*cosh(d*x + c)^3 - ((3*a^6 + 4
*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d
e - 2(a^5*b + 2*a^3*b^3 + a*b^5)*f)*cosh(d*x + c))*sinh(d*x + c)^3 + (3*a
^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b
^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 2*(a^5*b + 2*a^3*b^
3 + a*b^5)*f)*cosh(d*x + c)^2 + (15*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*
d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 2*(a^5*b + 2*a^3*b^3 +
a*b^5)*f)*cosh(d*x + c)^4 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x - (
3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 6*((3*a^6 + 4*a^4*b^2 - a^2*b^4
- 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 2*(a^5*b + 2*a
^3*b^3 + a*b^5)*f)*cosh(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*f)*sinh(
d*x + c)^2 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*f + 2*(3*((3*a^6 + 4*a^4*b^2 - a
^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 2*(a^5*
b + 2*a^3*b^3 + a*b^5)*f)*cosh(d*x + c)^5 - 2*((3*a^6 + 4*a^4*b^2 - a^2*b^4
- 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 2*(a^5*b + 2*
a^3*b^3 + a*b^5)*f)*cosh(d*x + c)^3 - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6
)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 2*(a^5*b + 2*a^3*b^3
+ a*b^5)*f)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c)
+ 1) - (((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e + (2*a^5*b + 4*a^3*b^3
+ 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*cosh(d*x + c)^6 + 6
*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*a*b^
5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*cosh(d*x + c)*sinh(d*x + c)
^5 + ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*
a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*sinh(d*x + c)^6 - ((3*a
^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3
*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*cosh(d*x + c)^4 - ((3*a^6 + 4*a^4
*b^2 - a^2*b^4 - 2*b^6)*d*e - 15*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e
+ (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c
)*f)*cosh(d*x + c)^2 + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2
- a^2*b^4 - 2*b^6)*c)*f)*sinh(d*x + c)^4 + 4*(5*((3*a^6 + 4*a^4*b^2 - a^2*b
^4 - 2*b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2
*b^4 - 2*b^6)*c)*f)*cosh(d*x + c)^3 - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6
)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b
^6)*c)*f)*cosh(d*x + c))*sinh(d*x + c)^3 + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2
*b^6)*d*e - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e + (2*a^5*b + 4*a^3*b
^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*cosh(d*x + c)^2
+ (15*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2
*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*cosh(d*x + c)^4 - (3*a
^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 6*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2
*b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 -
2*b^6)*c)*f)*cosh(d*x + c)^2 - (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4
*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*sinh(d*x + c)^2 + (2*a^5*b + 4*a^3*b^3 +
2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f + 2*(3*((3*a^6 + 4*a^4

```
*b^2 - a^2*b^4 - 2*b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a
^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*cosh(d*x + c)^5 - 2*((3*a^6 + 4*a^4*b^2 - a
^2*b^4 - 2*b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 -
 a^2*b^4 - 2*b^6)*c)*f)*cosh(d*x + c)^3 - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2
*b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 -
 2*b^6)*c)*f)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x +
c) - 1) - (((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^
2 - a^2*b^4 - 2*b^6)*c*f)*cosh(d*x + c)^6 + 6*((3*a^6 + 4*a^4*b^2 - a^2*b^4
 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f)*cosh(d*x + c)*
sinh(d*x + c)^5 + ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4
*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f)*sinh(d*x + c)^6 - ((3*a^6 + 4*a^4*b^2 - a^
2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f)*cosh(d*x
+ c)^4 - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2
- a^2*b^4 - 2*b^6)*c*f - 15*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x +
(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^4
 + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + 4*(5*((3*a^6 + 4*a^4*b^2 -
 a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f)*cosh(d
*x + c)^3 - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b
^2 - a^2*b^4 - 2*b^6)*c*f)*cosh(d*x + c))*sinh(d*x + c)^3 + (3*a^6 + 4*a^4*
b^2 - a^2*b^4 - 2*b^6)*c*f - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x +
 (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f)*cosh(d*x + c)^2 + (15*((3*a^6 +
 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)
*c*f)*cosh(d*x + c)^4 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x - (3*a^
6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f - 6*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*
b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f)*cosh(d*x + c)^2)*si
nh(d*x + c)^2 + 2*(3*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6
+ 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f)*cosh(d*x + c)^5 - 2*((3*a^6 + 4*a^4*b^2
 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f)*cosh
(d*x + c)^3 - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4
*b^2 - a^2*b^4 - 2*b^6)*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-cosh(d*x +
c) - sinh(d*x + c) + 1) + 2*(12*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d*f*x + (a^5
*b + 2*a^3*b^3 + a*b^5)*c*f)*cosh(d*x + c)^5 - 5*((3*a^6 + 4*a^4*b^2 + a^2*
b^4)*d*f*x + (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*e + (a^6 + 2*a^4*b^2 + a^2*b^4
)*f)*cosh(d*x + c)^4 - (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*f*x - 8*(2*(a^5*b +
a^3*b^3)*d*f*x - (a^3*b^3 + a*b^5)*d*e + (a^5*b + 2*a^3*b^3 + a*b^5)*c*f)*c
osh(d*x + c)^3 - (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*e + 6*((a^6 - a^2*b^4)*d*f
*x + (a^6 - a^2*b^4)*d*e)*cosh(d*x + c)^2 + (a^6 + 2*a^4*b^2 + a^2*b^4)*f -
 4*((a^3*b^3 + a*b^5)*d*f*x - 2*(a^5*b + a^3*b^3)*d*e + (a^5*b + 2*a^3*b^3
+ a*b^5)*c*f)*cosh(d*x + c))*sinh(d*x + c))/((a^7 + 2*a^5*b^2 + a^3*b^4)*d^
2*cosh(d*x + c)^6 + 6*(a^7 + 2*a^5*b^2 + a^3*b^4)*d^2*cosh(d*x + c)*sinh(d*
x + c)^5 + (a^7 + 2*a^5*b^2 + a^3*b^4)*d^2*sinh(d*x + c)^6 - (a^7 + 2*a^5*b
^2 + a^3*b^4)*d^2*cosh(d*x + c)^4 - (a^7 + 2*a^5*b^2 + a^3*b^4)*d^2*cosh(d*
x + c)^2 + (15*(a^7 + 2*a^5*b^2 + a^3*b^4)*d^2*cosh(d*x + c)^2 - (a^7 + 2*a
^5*b^2 + a^3*b^4)*d^2)*sinh(d*x + c)^4 + 4*(5*(a^7 + 2*a^5*b^2 + a^3*b^4)*d
^2*cosh(d*x + c)^3 - (a^7 + 2*a^5*b^2 + a^3*b^4)*d^2*cosh(d*x + c))*sinh(d*
x + c)^3 + (a^7 + 2*a^5*b^2 + a^3*b^4)*d^2 + (15*(a^7 + 2*a^5*b^2 + a^3*b^4
)*d^2*cosh(d*x + c)^4 - 6*(a^7 + 2*a^5*b^2 + a^3*b^4)*d^2*cosh(d*x + c)^2 -
 (a^7 + 2*a^5*b^2 + a^3*b^4)*d^2)*sinh(d*x + c)^2 + 2*(3*(a^7 + 2*a^5*b^2 +
 a^3*b^4)*d^2*cosh(d*x + c)^5 - 2*(a^7 + 2*a^5*b^2 + a^3*b^4)*d^2*cosh(d*x
+ c)^3 - (a^7 + 2*a^5*b^2 + a^3*b^4)*d^2*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**3*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.498 $\int \frac{\mathbf{csch}^3(c+dx)\mathbf{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=206

$$\frac{2b^5\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3d\left(a^2+b^2\right)^{3/2}}+\frac{b^2\mathrm{sech}(c+dx)}{a^3d}-\frac{b^2\tanh^{-1}(\cosh(c+dx))}{a^3d}-\frac{b^3\mathrm{sech}(c+dx)(a\sinh(c+dx)+b)}{a^3d\left(a^2+b^2\right)}+\frac{b\,\mathrm{ta}}{}$$

```
[Out] (3*ArcTanh[Cosh[c + d*x]])/(2*a*d) - (b^2*ArcTanh[Cosh[c + d*x]])/(a^3*d) +
 (2*b^5*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^3*(a^2 + b^2
)^(3/2)*d) + (b*Coth[c + d*x])/(a^2*d) - (3*Sech[c + d*x])/(2*a*d) + (b^2*S
ech[c + d*x])/(a^3*d) - (Csch[c + d*x]^2*Sech[c + d*x])/(2*a*d) - (b^3*Sech
[c + d*x]*(b + a*Sinh[c + d*x]))/(a^3*(a^2 + b^2)*d) + (b*Tanh[c + d*x])/(a
^2*d)
```

Rubi [A] time = 0.420027, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {2898, 2622, 321, 207, 2620, 14, 288, 2696, 12, 2660, 618, 204}

$$\frac{2b^5\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3d\left(a^2+b^2\right)^{3/2}}+\frac{b^2\mathrm{sech}(c+dx)}{a^3d}-\frac{b^2\tanh^{-1}(\cosh(c+dx))}{a^3d}-\frac{b^3\mathrm{sech}(c+dx)(a\sinh(c+dx)+b)}{a^3d\left(a^2+b^2\right)}+\frac{b\,\mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Cosh[c + d*x]])/(2*a*d) - (b^2*ArcTanh[Cosh[c + d*x]])/(a^3*d) +
 (2*b^5*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^3*(a^2 + b^2
)^(3/2)*d) + (b*Coth[c + d*x])/(a^2*d) - (3*Sech[c + d*x])/(2*a*d) + (b^2*S
ech[c + d*x])/(a^3*d) - (Csch[c + d*x]^2*Sech[c + d*x])/(2*a*d) - (b^3*Sech
[c + d*x]*(b + a*Sinh[c + d*x]))/(a^3*(a^2 + b^2)*d) + (b*Tanh[c + d*x])/(a
^2*d)
```

Rule 2898

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)}\,dx = -\left(i\int\left(\frac{ib^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a^3} - \frac{ib\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a^2} + \frac{i\operatorname{csch}^3(c\right.\right.$$

$$= \frac{\int \operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)\,dx}{a} - \frac{b\int \operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)\,dx}{a^2} + \frac{b^2\int \operatorname{csch}}{}$$

$$= -\frac{b^3\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a^3\left(a^2+b^2\right)d} - \frac{b^3\int \frac{b^2}{a+b\sinh(c+dx)}\,dx}{a^3\left(a^2+b^2\right)} - \frac{\operatorname{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx\right.}{ad}$$

$$= \frac{b^2\operatorname{sech}(c+dx)}{a^3d} - \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{2ad} - \frac{b^3\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a^3\left(a^2+b^2\right)d}$$

$$= -\frac{b^2\tanh^{-1}(\cosh(c+dx))}{a^3d} + \frac{b\coth(c+dx)}{a^2d} - \frac{3\operatorname{sech}(c+dx)}{2ad} + \frac{b^2\operatorname{sech}(c+dx)}{a^3d} - $$

$$= \frac{3\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{b^2\tanh^{-1}(\cosh(c+dx))}{a^3d} + \frac{b\coth(c+dx)}{a^2d} - \frac{3\operatorname{sech}(c+}{2ad}$$

$$= \frac{3\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{b^2\tanh^{-1}(\cosh(c+dx))}{a^3d} + \frac{2b^5\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)^{3/2}d}$$

Mathematica [A] time = 2.47148, size = 185, normalized size = 0.9

$$\frac{-\frac{4\left(3a^2-2b^2\right)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} + \frac{16b^5\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{a^3\left(-a^2-b^2\right)^{3/2}} + \frac{8\operatorname{sech}(c+dx)(b\sinh(c+dx)-a)}{a^2+b^2} + \frac{4b\tanh\left(\frac{1}{2}(c+dx)\right)}{a^2} + \frac{4b\coth\left(\frac{1}{2}(c+dx)\right)}{a^2} - \operatorname{csch}^2}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((16*b^5*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(a^3*(-a^2 - b
^2)^(3/2)) + (4*b*Coth[(c + d*x)/2])/a^2 - Csch[(c + d*x)/2]^2/a - (4*(3*a^
2 - 2*b^2)*Log[Tanh[(c + d*x)/2]])/a^3 - Sech[(c + d*x)/2]^2/a + (8*Sech[c
+ d*x]*(-a + b*Sinh[c + d*x]))/(a^2 + b^2) + (4*b*Tanh[(c + d*x)/2])/a^2)/(
8*d)
```

Maple [A] time = 0.002, size = 233, normalized size = 1.1

$$\frac{1}{8da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 + \frac{b}{2da^2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{1}{8da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-2} - \frac{3}{2da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + \frac{b^2}{da^3}\ln\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/8/d/a*tanh(1/2*d*x+1/2*c)^2+1/2/d/a^2*tanh(1/2*d*x+1/2*c)*b-1/8/d/a/tanh(
1/2*d*x+1/2*c)^2-3/2/d/a*ln(tanh(1/2*d*x+1/2*c))+1/d/a^3*ln(tanh(1/2*d*x+1/
2*c))*b^2+1/2/d*b/a^2/tanh(1/2*d*x+1/2*c)-2/d/a^3*b^5/(a^2+b^2)^(3/2)*arcta
nh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))+2/d/(a^2+b^2)/(tanh(1
/2*d*x+1/2*c)^2+1)*tanh(1/2*d*x+1/2*c)*b-2/d/(a^2+b^2)/(tanh(1/2*d*x+1/2*c)
^2+1)*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.7177, size = 5921, normalized size = 28.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="frica
s")
```

```
[Out] -1/2*(8*a^5*b + 12*a^3*b^3 + 4*a*b^5 + 2*(3*a^6 + 4*a^4*b^2 + a^2*b^4)*cosh
(d*x + c)^5 + 2*(3*a^6 + 4*a^4*b^2 + a^2*b^4)*sinh(d*x + c)^5 - 4*(a^3*b^3
+ a*b^5)*cosh(d*x + c)^4 - 2*(2*a^3*b^3 + 2*a*b^5 - 5*(3*a^6 + 4*a^4*b^2 +
a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(a^6 - a^2*b^4)*cosh(d*x + c)^3
 - 4*(a^6 - a^2*b^4 - 5*(3*a^6 + 4*a^4*b^2 + a^2*b^4)*cosh(d*x + c)^2 + 4*(
a^3*b^3 + a*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*(a^5*b + a^3*b^3)*cosh(
d*x + c)^2 - 4*(2*a^5*b + 2*a^3*b^3 - 5*(3*a^6 + 4*a^4*b^2 + a^2*b^4)*cosh(
d*x + c)^3 + 6*(a^3*b^3 + a*b^5)*cosh(d*x + c)^2 + 3*(a^6 - a^2*b^4)*cosh(d
*x + c))*sinh(d*x + c)^2 - 2*(b^5*cosh(d*x + c)^6 + 6*b^5*cosh(d*x + c)*sin
h(d*x + c)^5 + b^5*sinh(d*x + c)^6 - b^5*cosh(d*x + c)^4 - b^5*cosh(d*x + c
)^2 + b^5 + (15*b^5*cosh(d*x + c)^2 - b^5)*sinh(d*x + c)^4 + 4*(5*b^5*cosh(
d*x + c)^3 - b^5*cosh(d*x + c))*sinh(d*x + c)^3 + (15*b^5*cosh(d*x + c)^4 -
 6*b^5*cosh(d*x + c)^2 - b^5)*sinh(d*x + c)^2 + 2*(3*b^5*cosh(d*x + c)^5 -
2*b^5*cosh(d*x + c)^3 - b^5*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*l
og((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2
 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*c
osh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(3*a
^6 + 4*a^4*b^2 + a^2*b^4)*cosh(d*x + c) - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2
*b^6)*cosh(d*x + c)^6 + 6*(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(d*x +
c)*sinh(d*x + c)^5 + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*sinh(d*x + c)^6
+ 3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^
6)*cosh(d*x + c)^4 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6 - 15*(3*a^6 + 4*a
^4*b^2 - a^2*b^4 - 2*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(3*a^6 +
4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(d*x + c)^3 - (3*a^6 + 4*a^4*b^2 - a^2*b^4
```

```
 - 2*b^6)*cosh(d*x + c))*sinh(d*x + c)^3 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2
*b^6)*cosh(d*x + c)^2 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6 - 15*(3*a^6 +
4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(d*x + c)^4 + 6*(3*a^6 + 4*a^4*b^2 - a^2*b
^4 - 2*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(3*a^6 + 4*a^4*b^2 - a^
2*b^4 - 2*b^6)*cosh(d*x + c)^5 - 2*(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*co
sh(d*x + c)^3 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(d*x + c))*sinh(d
*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((3*a^6 + 4*a^4*b^2 - a^2
*b^4 - 2*b^6)*cosh(d*x + c)^6 + 6*(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*cos
h(d*x + c)*sinh(d*x + c)^5 + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*sinh(d*x
 + c)^6 + 3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6 - (3*a^6 + 4*a^4*b^2 - a^2*b^
4 - 2*b^6)*cosh(d*x + c)^4 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6 - 15*(3*a
^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(
3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(d*x + c)^3 - (3*a^6 + 4*a^4*b^2 -
 a^2*b^4 - 2*b^6)*cosh(d*x + c))*sinh(d*x + c)^3 - (3*a^6 + 4*a^4*b^2 - a^2
*b^4 - 2*b^6)*cosh(d*x + c)^2 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6 - 15*(
3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(d*x + c)^4 + 6*(3*a^6 + 4*a^4*b^2
 - a^2*b^4 - 2*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(3*a^6 + 4*a^4*
b^2 - a^2*b^4 - 2*b^6)*cosh(d*x + c)^5 - 2*(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2
*b^6)*cosh(d*x + c)^3 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(d*x + c)
)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(3*a^6 + 4*a^4*
b^2 + a^2*b^4 + 5*(3*a^6 + 4*a^4*b^2 + a^2*b^4)*cosh(d*x + c)^4 - 8*(a^3*b^
3 + a*b^5)*cosh(d*x + c)^3 - 6*(a^6 - a^2*b^4)*cosh(d*x + c)^2 - 8*(a^5*b +
 a^3*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh
(d*x + c)^6 + 6*(a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5
 + (a^7 + 2*a^5*b^2 + a^3*b^4)*d*sinh(d*x + c)^6 - (a^7 + 2*a^5*b^2 + a^3*b
^4)*d*cosh(d*x + c)^4 + (15*(a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^2 -
 (a^7 + 2*a^5*b^2 + a^3*b^4)*d)*sinh(d*x + c)^4 - (a^7 + 2*a^5*b^2 + a^3*b^
4)*d*cosh(d*x + c)^2 + 4*(5*(a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^3 -
 (a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^7 +
2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^4 - 6*(a^7 + 2*a^5*b^2 + a^3*b^4)*d*co
sh(d*x + c)^2 - (a^7 + 2*a^5*b^2 + a^3*b^4)*d)*sinh(d*x + c)^2 + (a^7 + 2*a
^5*b^2 + a^3*b^4)*d + 2*(3*(a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^5 -
2*(a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^3 - (a^7 + 2*a^5*b^2 + a^3*b^
4)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.77048, size = 317, normalized size = 1.54

$$\frac{b^5 \log\left(\frac{\left|-2\,be^{(dx+c)}-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|-2\,be^{(dx+c)}-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^5d + a^3b^2d\right)\sqrt{a^2+b^2}} - \frac{2\left(ae^{(dx+c)} + b\right)}{\left(a^2d + b^2d\right)\left(e^{(2\,dx+2\,c)} + 1\right)} + \frac{\left(3\,a^2 - 2\,b^2\right)\log\left(e^{(dx+c)} + 1\right)}{2\,a^3d} - \frac{\left(3\,a^2 - 2\,b^2\right)\log\left(\left|e^{(dx+c)} - \right.\right.}{2\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac"
)
```

```
[Out] b^5*log(abs(-2*b*e^(d*x + c) - 2*a - 2*sqrt(a^2 + b^2))/abs(-2*b*e^(d*x + c
) - 2*a + 2*sqrt(a^2 + b^2)))/((a^5*d + a^3*b^2*d)*sqrt(a^2 + b^2)) - 2*(a*
e^(d*x + c) + b)/((a^2*d + b^2*d)*(e^(2*d*x + 2*c) + 1)) + 1/2*(3*a^2 - 2*b
^2)*log(e^(d*x + c) + 1)/(a^3*d) - 1/2*(3*a^2 - 2*b^2)*log(abs(e^(d*x + c)
- 1))/(a^3*d) - (a*e^(3*d*x + 3*c) - 2*b*e^(2*d*x + 2*c) + a*e^(d*x + c) +
2*b)/(a^2*d*(e^(2*d*x + 2*c) - 1)^2)
```

3.499 $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=38

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Csch[c + d*x]^3*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d
*x])), x]
```

Rubi [A] time = 0.134132, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[c + d*x]^3*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Csch[c + d*x]^3*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[c + d*x]^3*Sech[c + d*x]^2)/((e + f*x)*(a + b*Sinh[c + d*x]
)),x]
```

```
[Out] $Aborted
```

Maple [A] time = 3.536, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx+c))^3(\operatorname{sech}(dx+c))^2}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -32*b^5*integrate(-1/16*e^(d*x + c)/(a^5*b*e + a^3*b^3*e + (a^5*b*f + a^3*b
^3*f)*x - (a^5*b*e*e^(2*c) + a^3*b^3*e*e^(2*c) + (a^5*b*f*e^(2*c) + a^3*b^3
*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^6*e*e^c + a^4*b^2*e*e^c + (a^6*f*e^c + a^4*
b^2*f*e^c)*x)*e^(d*x)), x) - (4*a^2*b*d*e + 2*b^3*d*e + 2*(2*a^2*b*d*f + b^
3*d*f)*x + ((3*d*e - f)*a^3*e^(5*c) + (d*e - f)*a*b^2*e^(5*c) + (3*a^3*d*f*
e^(5*c) + a*b^2*d*f*e^(5*c))*x)*e^(5*d*x) - 2*(b^3*d*f*x*e^(4*c) + b^3*d*e*
e^(4*c))*e^(4*d*x) - 2*(a^3*d*e*e^(3*c) - a*b^2*d*e*e^(3*c) + (a^3*d*f*e^(3
*c) - a*b^2*d*f*e^(3*c))*x)*e^(3*d*x) - 4*(a^2*b*d*f*x*e^(2*c) + a^2*b*d*e*
e^(2*c))*e^(2*d*x) + ((3*d*e + f)*a^3*e^c + (d*e + f)*a*b^2*e^c + (3*a^3*d*
f*e^c + a*b^2*d*f*e^c)*x)*e^(d*x))/(a^4*d^2*e^2 + a^2*b^2*d^2*e^2 + (a^4*d^
2*f^2 + a^2*b^2*d^2*f^2)*x^2 + 2*(a^4*d^2*e*f + a^2*b^2*d^2*e*f)*x + (a^4*d
^2*e^2*e^(6*c) + a^2*b^2*d^2*e^2*e^(6*c) + (a^4*d^2*f^2*e^(6*c) + a^2*b^2*d
^2*f^2*e^(6*c))*x^2 + 2*(a^4*d^2*e*f*e^(6*c) + a^2*b^2*d^2*e*f*e^(6*c))*x)*
e^(6*d*x) - (a^4*d^2*e^2*e^(4*c) + a^2*b^2*d^2*e^2*e^(4*c) + (a^4*d^2*f^2*e
^(4*c) + a^2*b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^4*d^2*e*f*e^(4*c) + a^2*b^2*d^
2*e*f*e^(4*c))*x)*e^(4*d*x) - (a^4*d^2*e^2*e^(2*c) + a^2*b^2*d^2*e^2*e^(2*c
) + (a^4*d^2*f^2*e^(2*c) + a^2*b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^4*d^2*e*f*e^
(2*c) + a^2*b^2*d^2*e*f*e^(2*c))*x)*e^(2*d*x)) - 32*integrate(1/64*(2*b^2*d
^2*e^2 + 2*a*b*d*e*f - (3*d^2*e^2 - 2*f^2)*a^2 - (3*a^2*d^2*f^2 - 2*b^2*d^2
*f^2)*x^2 - 2*(3*a^2*d^2*e*f - 2*b^2*d^2*e*f - a*b*d*f^2)*x)/(a^3*d^2*f^3*x
^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 - (a^3*d^2*f^3*x
^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)
*e^(d*x)), x) - 32*integrate(-1/64*(2*b^2*d^2*e^2 - 2*a*b*d*e*f - (3*d^2*e^
2 - 2*f^2)*a^2 - (3*a^2*d^2*f^2 - 2*b^2*d^2*f^2)*x^2 - 2*(3*a^2*d^2*e*f - 2
*b^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3
*d^2*e^2*f*x + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c
 + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) - 32*integrate(1/1
6*(a*f*e^(d*x + c) + b*f)/(a^2*d*e^2 + b^2*d*e^2 + (a^2*d*f^2 + b^2*d*f^2)*
x^2 + 2*(a^2*d*e*f + b^2*d*e*f)*x + (a^2*d*e^2*e^(2*c) + b^2*d*e^2*e^(2*c)
+ (a^2*d*f^2*e^(2*c) + b^2*d*f^2*e^(2*c))*x^2 + 2*(a^2*d*e*f*e^(2*c) + b^2*
d*e*f*e^(2*c))*x)*e^(2*d*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{csch}(dx+c)^3\operatorname{sech}(dx+c)^2}{afx+ae+\left(bfx+be\right)\sinh(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] integral(csch(d*x + c)^3*sech(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(
d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*sech(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.500 $\int \frac{(e+fx)\textbf{csch}^3(c+dx)\textbf{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=1122

result too large to display

```
[Out] (b^2*f*x)/(2*a^3*d) + (3*b*f*x*ArcTan[E^(c + d*x)])/(a^2*d) - (2*b^5*(e + f
*x)*ArcTan[E^(c + d*x)])/(a^2*(a^2 + b^2)^2*d) - (b^3*(e + f*x)*ArcTan[E^(c
 + d*x)])/(a^2*(a^2 + b^2)*d) - (3*b*f*x*ArcTan[Sinh[c + d*x]])/(2*a^2*d) +
 (3*b*(e + f*x)*ArcTan[Sinh[c + d*x]])/(2*a^2*d) - (2*b^2*f*x*ArcTanh[E^(2*
c + 2*d*x)])/(a^3*d) + (4*(e + f*x)*ArcTanh[E^(2*c + 2*d*x)])/(a*d) + (b*f*
ArcTanh[Cosh[c + d*x]])/(a^2*d^2) + (3*b*(e + f*x)*Csch[c + d*x])/(2*a^2*d)
 - (f*Csch[2*c + 2*d*x])/(a*d^2) - (2*(e + f*x)*Coth[2*c + 2*d*x]*Csch[2*c
+ 2*d*x])/(a*d) - (b^6*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])])/(a^3*(a^2 + b^2)^2*d) - (b^6*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])])/(a^3*(a^2 + b^2)^2*d) + (b^6*(e + f*x)*Log[1 + E^(2*(c +
d*x))])/(a^3*(a^2 + b^2)^2*d) - (b^2*f*x*Log[Tanh[c + d*x]])/(a^3*d) + (b^2
*(e + f*x)*Log[Tanh[c + d*x]])/(a^3*d) - (((3*I)/2)*b*f*PolyLog[2, (-I)*E^(
c + d*x)])/(a^2*d^2) + (I*b^5*f*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*(a^2 + b
^2)^2*d^2) + ((I/2)*b^3*f*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^
2) + (((3*I)/2)*b*f*PolyLog[2, I*E^(c + d*x)])/(a^2*d^2) - (I*b^5*f*PolyLog
[2, I*E^(c + d*x)])/(a^2*(a^2 + b^2)^2*d^2) - ((I/2)*b^3*f*PolyLog[2, I*E^(
c + d*x)])/(a^2*(a^2 + b^2)*d^2) - (b^6*f*PolyLog[2, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)^2*d^2) - (b^6*f*PolyLog[2, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)^2*d^2) + (b^6*f*PolyLog[2
, -E^(2*(c + d*x))])/(2*a^3*(a^2 + b^2)^2*d^2) + (f*PolyLog[2, -E^(2*c + 2*
d*x)])/(a*d^2) - (b^2*f*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a^3*d^2) - (f*Poly
Log[2, E^(2*c + 2*d*x)])/(a*d^2) + (b^2*f*PolyLog[2, E^(2*c + 2*d*x)])/(2*a
^3*d^2) + (b*f*Sech[c + d*x])/(2*a^2*d^2) - (b^3*f*Sech[c + d*x])/(2*a^2*(a
^2 + b^2)*d^2) - (b^4*(e + f*x)*Sech[c + d*x]^2)/(2*a^3*(a^2 + b^2)*d) - (b
*(e + f*x)*Csch[c + d*x]*Sech[c + d*x]^2)/(2*a^2*d) - (b^2*f*Tanh[c + d*x])
/(2*a^3*d^2) + (b^4*f*Tanh[c + d*x])/(2*a^3*(a^2 + b^2)*d^2) - (b^3*(e + f*
x)*Sech[c + d*x]*Tanh[c + d*x])/(2*a^2*(a^2 + b^2)*d) - (b^2*(e + f*x)*Tanh
[c + d*x]^2)/(2*a^3*d)
```

Rubi [A] time = 1.80698, antiderivative size = 1122, normalized size of antiderivative = 1., number of steps used = 65, number of rules used = 28, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.824, Rules used = {5589, 5461, 4185, 4182, 2279, 2391, 2621, 288, 321, 207, 5462, 5203, 12, 4180, 3770, 2622, 2620, 14, 2548, 3473, 8, 5573, 5561, 2190, 6742, 3718, 5451, 3767}

$$-\frac{(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)b^6}{a^3\left(a^2+b^2\right)^2 d}-\frac{(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)b^6}{a^3\left(a^2+b^2\right)^2 d}+\frac{(e+fx)\log\left(1+e^{2(c+dx)}\right)b^6}{a^3\left(a^2+b^2\right)^2 d}-\frac{f\text{PolyLog}\left(2,-\frac{be^{c}}{a-\sqrt{}}\right.}{a^3\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```


```
[Out] (b^2*f*x)/(2*a^3*d) + (3*b*f*x*ArcTan[E^(c + d*x)])/(a^2*d) - (2*b^5*(e + f
*x)*ArcTan[E^(c + d*x)])/(a^2*(a^2 + b^2)^2*d) - (b^3*(e + f*x)*ArcTan[E^(c
 + d*x)])/(a^2*(a^2 + b^2)*d) - (3*b*f*x*ArcTan[Sinh[c + d*x]])/(2*a^2*d) +
 (3*b*(e + f*x)*ArcTan[Sinh[c + d*x]])/(2*a^2*d) - (2*b^2*f*x*ArcTanh[E^(2*
c + 2*d*x)])/(a^3*d) + (4*(e + f*x)*ArcTanh[E^(2*c + 2*d*x)])/(a*d) + (b*f*
ArcTanh[Cosh[c + d*x]])/(a^2*d^2) + (3*b*(e + f*x)*Csch[c + d*x])/(2*a^2*d)
 - (f*Csch[2*c + 2*d*x])/(a*d^2) - (2*(e + f*x)*Coth[2*c + 2*d*x]*Csch[2*c
+ 2*d*x])/(a*d) - (b^6*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
```

```
2])])/(a^3*(a^2 + b^2)^2*d) - (b^6*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])])/(a^3*(a^2 + b^2)^2*d) + (b^6*(e + f*x)*Log[1 + E^(2*(c +
d*x))])/(a^3*(a^2 + b^2)^2*d) - (b^2*f*x*Log[Tanh[c + d*x]])/(a^3*d) + (b^2
*(e + f*x)*Log[Tanh[c + d*x]])/(a^3*d) - (((3*I)/2)*b*f*PolyLog[2, (-I)*E^(
c + d*x)])/(a^2*d^2) + (I*b^5*f*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*(a^2 + b
^2)^2*d^2) + ((I/2)*b^3*f*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^
2) + (((3*I)/2)*b*f*PolyLog[2, I*E^(c + d*x)])/(a^2*d^2) - (I*b^5*f*PolyLog
[2, I*E^(c + d*x)])/(a^2*(a^2 + b^2)^2*d^2) - ((I/2)*b^3*f*PolyLog[2, I*E^(
c + d*x)])/(a^2*(a^2 + b^2)*d^2) - (b^6*f*PolyLog[2, -((b*E^(c + d*x))/(a -
 Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)^2*d^2) - (b^6*f*PolyLog[2, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)^2*d^2) + (b^6*f*PolyLog[2
, -E^(2*(c + d*x))])/(2*a^3*(a^2 + b^2)^2*d^2) + (f*PolyLog[2, -E^(2*c + 2*
d*x)])/(a*d^2) - (b^2*f*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a^3*d^2) - (f*Poly
Log[2, E^(2*c + 2*d*x)])/(a*d^2) + (b^2*f*PolyLog[2, E^(2*c + 2*d*x)])/(2*a
^3*d^2) + (b*f*Sech[c + d*x])/(2*a^2*d^2) - (b^3*f*Sech[c + d*x])/(2*a^2*(a
^2 + b^2)*d^2) - (b^4*(e + f*x)*Sech[c + d*x]^2)/(2*a^3*(a^2 + b^2)*d) - (b
*(e + f*x)*Csch[c + d*x]*Sech[c + d*x]^2)/(2*a^2*d) - (b^2*f*Tanh[c + d*x])
/(2*a^3*d^2) + (b^4*f*Tanh[c + d*x])/(2*a^3*(a^2 + b^2)*d^2) - (b^3*(e + f*
x)*Sech[c + d*x]*Tanh[c + d*x])/(2*a^2*(a^2 + b^2)*d) - (b^2*(e + f*x)*Tanh
[c + d*x]^2)/(2*a^3*d)
```

Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
 IGtQ[p, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
 b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{\int (e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)\,dx}{a} - \frac{b\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx}{a}$$

$$= \frac{8\int (e+fx)\operatorname{csch}^3(2c+2dx)\,dx}{a} - \frac{b\int (e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)\,dx}{a^2}$$

$$= \frac{3b(e+fx)\tan^{-1}(\sinh(c+dx))}{2a^2d} + \frac{3b(e+fx)\operatorname{csch}(c+dx)}{2a^2d} - \frac{f\operatorname{csch}(2c+2d}{ad^2}$$

$$= \frac{3b(e+fx)\tan^{-1}(\sinh(c+dx))}{2a^2d} + \frac{4(e+fx)\tanh^{-1}\left(e^{2c+2dx}\right)}{ad} + \frac{3b(e+fx)c}{2a}$$

$$= -\frac{3bfx\tan^{-1}(\sinh(c+dx))}{2a^2d} + \frac{3b(e+fx)\tan^{-1}(\sinh(c+dx))}{2a^2d} + \frac{4(e+fx)\,t}{}$$

$$= \frac{b^6(e+fx)^2}{2a^3\left(a^2+b^2\right)^2 f} - \frac{3bfx\tan^{-1}(\sinh(c+dx))}{2a^2d} + \frac{3b(e+fx)\tan^{-1}(\sinh(c+}{2a^2d}$$

$$= \frac{b^2fx}{2a^3d} + \frac{b^6(e+fx)^2}{2a^3\left(a^2+b^2\right)^2 f} + \frac{3bfx\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{3bfx\tan^{-1}(\sinh(c+dx))}{2a^2d}$$

$$= \frac{b^2fx}{2a^3d} + \frac{3bfx\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^5(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)^2 d} - \frac{b^3(e+fx)\tan^{-1}}{a^2\left(a^2+b^2\right)}$$

$$= \frac{b^2fx}{2a^3d} + \frac{3bfx\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^5(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)^2 d} - \frac{b^3(e+fx)\tan^{-1}}{a^2\left(a^2+b^2\right)}$$

$$= \frac{b^2fx}{2a^3d} + \frac{3bfx\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^5(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)^2 d} - \frac{b^3(e+fx)\tan^{-1}}{a^2\left(a^2+b^2\right)}$$

$$= \frac{b^2fx}{2a^3d} + \frac{3bfx\tan^{-1}\left(e^{c+dx}\right)}{a^2d} - \frac{2b^5(e+fx)\tan^{-1}\left(e^{c+dx}\right)}{a^2\left(a^2+b^2\right)^2 d} - \frac{b^3(e+fx)\tan^{-1}}{a^2\left(a^2+b^2\right)}$$

Mathematica [B] time = 10.1996, size = 2868, normalized size = 2.56

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] 8*(((I/16)*(2*a^6 + 3*a^4*b^2 + b^6)*(d*e - c*f)*(c + d*x))/(a^3*(a^2 + b^2
)^2*d^2) + ((I/32)*(2*a^6 + 3*a^4*b^2 + b^6)*f*(c + d*x)^2)/(a^3*(a^2 + b^2
)^2*d^2) + (a^3*e*ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]])/(2*(a^2 + b^2)^2*d)
 + (3*a*b^2*e*ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]])/(4*(a^2 + b^2)^2*d) - (
b^6*e*ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]])/(4*a^3*(a^2 + b^2)^2*d) - (a^3*
c*f*ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]])/(2*(a^2 + b^2)^2*d^2) - (3*a*b^2*
c*f*ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]])/(4*(a^2 + b^2)^2*d^2) + (b^6*c*f*
ArcTanh[1 - (2*I)*Tanh[(c + d*x)/2]])/(4*a^3*(a^2 + b^2)^2*d^2) - (e*Log[Co
sh[(c + d*x)/2]])/(4*a*d) + (b^2*e*Log[Cosh[(c + d*x)/2]])/(8*a^3*d) + (c*f
*Log[Cosh[(c + d*x)/2]])/(4*a*d^2) - (b^2*c*f*Log[Cosh[(c + d*x)/2]])/(8*a^
```

```
3*d^2) + (a^3*e*((-I/2)*(c + d*x) + Log[Cosh[(c + d*x)/2] + I*Sinh[(c + d*x
)/2]]))/(4*(a^2 + b^2)^2*d) + (3*a*b^2*e*((-I/2)*(c + d*x) + Log[Cosh[(c +
d*x)/2] + I*Sinh[(c + d*x)/2]]))/(8*(a^2 + b^2)^2*d) - (a^3*c*f*((-I/2)*(c
+ d*x) + Log[Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]]))/(4*(a^2 + b^2)^2*d^
2) - (3*a*b^2*c*f*((-I/2)*(c + d*x) + Log[Cosh[(c + d*x)/2] + I*Sinh[(c + d
*x)/2]]))/(8*(a^2 + b^2)^2*d^2) + (b^6*e*((-I)*(c + d*x) + 2*ArcTanh[1 - (2
*I)*Tanh[(c + d*x)/2]] + Log[-1 + Cosh[c + d*x] + I*Sinh[c + d*x]]))/(16*a^
3*(a^2 + b^2)^2*d) - (b^6*c*f*((-I)*(c + d*x) + 2*ArcTanh[1 - (2*I)*Tanh[(c
 + d*x)/2]] + Log[-1 + Cosh[c + d*x] + I*Sinh[c + d*x]]))/(16*a^3*(a^2 + b^
2)^2*d^2) - (b*f*Log[Tanh[(c + d*x)/2]])/(8*a^2*d^2) - ((I/2)*f*((-I/8)*(c
+ d*x)^2 - (I/2)*(c + d*x)*Log[1 + E^(-c - d*x)] + (I/2)*PolyLog[2, -E^(-c
- d*x)]))/(a*d^2) + ((I/4)*b^2*f*((-I/8)*(c + d*x)^2 - (I/2)*(c + d*x)*Log[
1 + E^(-c - d*x)] + (I/2)*PolyLog[2, -E^(-c - d*x)]))/(a^3*d^2) + (b^6*f*((
-I/2)*(c + d*x)^2 + (I/4)*(3*Pi*(c + d*x) + (1 - I)*(c + d*x)^2 + 2*(Pi - (
2*I)*(c + d*x))*Log[1 + I*E^(-c - d*x)] - 4*Pi*Log[1 + E^(c + d*x)] - 2*Pi*
Log[-Cos[(Pi + (2*I)*(c + d*x))/4]] + 4*Pi*Log[Cosh[(c + d*x)/2]] + (4*I)*P
olyLog[2, (-I)*E^(-c - d*x)])))/(8*a^3*(a^2 + b^2)^2*d^2) - ((I/4)*a^3*f*((
c + d*x)^2/4 + (-3*Pi*(c + d*x) - (1 - I)*(c + d*x)^2 - 2*(Pi - (2*I)*(c +
d*x))*Log[1 + I*E^(-c - d*x)] + 4*Pi*Log[1 + E^(c + d*x)] + 2*Pi*Log[-Cos[(
Pi + (2*I)*(c + d*x))/4]] - 4*Pi*Log[Cosh[(c + d*x)/2]] - (4*I)*PolyLog[2,
(-I)*E^(-c - d*x)])/4 - (I/2)*(-(c + d*x)^2/2 + 2*(c + d*x)*Log[1 - E^(c +
d*x)] + 2*PolyLog[2, E^(c + d*x)])))/((a^2 + b^2)^2*d^2) - (((3*I)/8)*a*b^2
*f*((c + d*x)^2/4 + (-3*Pi*(c + d*x) - (1 - I)*(c + d*x)^2 - 2*(Pi - (2*I)*
(c + d*x))*Log[1 + I*E^(-c - d*x)] + 4*Pi*Log[1 + E^(c + d*x)] + 2*Pi*Log[-
Cos[(Pi + (2*I)*(c + d*x))/4]] - 4*Pi*Log[Cosh[(c + d*x)/2]] - (4*I)*PolyLo
g[2, (-I)*E^(-c - d*x)])/4 - (I/2)*(-(c + d*x)^2/2 + 2*(c + d*x)*Log[1 - E^
(c + d*x)] + 2*PolyLog[2, E^(c + d*x)])))/((a^2 + b^2)^2*d^2) + ((I/8)*b^6*
f*((c + d*x)^2/4 + (-3*Pi*(c + d*x) - (1 - I)*(c + d*x)^2 - 2*(Pi - (2*I)*(
c + d*x))*Log[1 + I*E^(-c - d*x)] + 4*Pi*Log[1 + E^(c + d*x)] + 2*Pi*Log[-C
os[(Pi + (2*I)*(c + d*x))/4]] - 4*Pi*Log[Cosh[(c + d*x)/2]] - (4*I)*PolyLog
[2, (-I)*E^(-c - d*x)])/4 - (I/2)*(-(c + d*x)^2/2 + 2*(c + d*x)*Log[1 - E^(
c + d*x)] + 2*PolyLog[2, E^(c + d*x)])))/(a^3*(a^2 + b^2)^2*d^2) - (b^6*(-(
f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
)] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d*e*Log[a
 + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLog[2, (b*E^(c +
 d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2]))]))/(8*a^3*(a^2 + b^2)^2*d^2) + ((I/2)*a^3*f*(-(E^((I/4)*Pi)*(c
+ d*x)^2)/4 + ((Pi*(c + d*x))/4 - Pi*Log[1 + E^(c + d*x)] - 2*(Pi/4 + (I/2)
*(c + d*x))*Log[1 - E^((2*I)*(Pi/4 + (I/2)*(c + d*x)))] + Pi*Log[Cosh[(c +
d*x)/2]] + (Pi*Log[Sin[Pi/4 + (I/2)*(c + d*x)]])/2 + I*PolyLog[2, E^((2*I)*
(Pi/4 + (I/2)*(c + d*x)))])/Sqrt[2]))/(Sqrt[2]*(a^2 + b^2)^2*d^2) + (((3*I)
/4)*a*b^2*f*(-(E^((I/4)*Pi)*(c + d*x)^2)/4 + ((Pi*(c + d*x))/4 - Pi*Log[1 +
 E^(c + d*x)] - 2*(Pi/4 + (I/2)*(c + d*x))*Log[1 - E^((2*I)*(Pi/4 + (I/2)*(
c + d*x)))] + Pi*Log[Cosh[(c + d*x)/2]] + (Pi*Log[Sin[Pi/4 + (I/2)*(c + d*x
)]])/2 + I*PolyLog[2, E^((2*I)*(Pi/4 + (I/2)*(c + d*x)))])/Sqrt[2]))/(Sqrt[
2]*(a^2 + b^2)^2*d^2) + (b*(3*a^2 + 5*b^2)*(2*(d*e - c*f + f*(c + d*x))*Arc
Tan[Cosh[c + d*x] + Sinh[c + d*x]] - I*f*PolyLog[2, (-I)*(Cosh[c + d*x] + S
inh[c + d*x])] + I*f*PolyLog[2, I*(Cosh[c + d*x] + Sinh[c + d*x])]))/(16*(a
^2 + b^2)^2*d^2) + (Csch[c + d*x]^2*Sech[c + d*x]^2*(-4*a*b^2*d*e + 4*a*b^2
*c*f - 4*a*b^2*f*(c + d*x) - 2*a^2*b*f*Cosh[c + d*x] - 8*a^3*d*e*Cosh[2*(c
+ d*x)] - 4*a*b^2*d*e*Cosh[2*(c + d*x)] + 8*a^3*c*f*Cosh[2*(c + d*x)] + 4*a
*b^2*c*f*Cosh[2*(c + d*x)] - 8*a^3*f*(c + d*x)*Cosh[2*(c + d*x)] - 4*a*b^2*
f*(c + d*x)*Cosh[2*(c + d*x)] + 2*a^2*b*f*Cosh[3*(c + d*x)] - 2*a^2*b*d*e*S
inh[c + d*x] + 4*b^3*d*e*Sinh[c + d*x] + 2*a^2*b*c*f*Sinh[c + d*x] - 4*b^3*
c*f*Sinh[c + d*x] - 2*a^2*b*f*(c + d*x)*Sinh[c + d*x] + 4*b^3*f*(c + d*x)*S
inh[c + d*x] - 4*a^3*f*Sinh[2*(c + d*x)] - 2*a*b^2*f*Sinh[2*(c + d*x)] + 6*
a^2*b*d*e*Sinh[3*(c + d*x)] + 4*b^3*d*e*Sinh[3*(c + d*x)] - 6*a^2*b*c*f*Sin
h[3*(c + d*x)] - 4*b^3*c*f*Sinh[3*(c + d*x)] + 6*a^2*b*f*(c + d*x)*Sinh[3*(
c + d*x)] + 4*b^3*f*(c + d*x)*Sinh[3*(c + d*x)] - a*b^2*f*Sinh[4*(c + d*x)]
```

```
))/(128*a^2*(a^2 + b^2)*d^2))
```

Maple [B] time = 0.289, size = 3563, normalized size = 3.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/a^2/d^2*f*b^4/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)
^(1/2))+1/a^2/d^2/(a^2+b^2)^(5/2)*b^6*f*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a
^2+b^2)^(1/2))-2/(a^2+b^2)/d*a*e*ln(exp(d*x+c)-1)-2/(a^2+b^2)/d*a*e*ln(exp(
d*x+c)+1)+2/(a^2+b^2)/d^2*a*f*dilog(exp(d*x+c))-2/(a^2+b^2)/d^2*a*f*dilog(e
xp(d*x+c)+1)+1/d^2/(a^2+b^2)^(5/2)*a^2*b^2*f*arctanh(1/2*(2*b*exp(d*x+c)+2*
a)/(a^2+b^2)^(1/2))-1/a^3/d/(a^2+b^2)^2*b^6*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d
*x+c)-b)-1/a^3/(a^2+b^2)/d^2*b^4*f*c*ln(exp(d*x+c)-1)+1/a^3/(a^2+b^2)/d*b^4
*f*ln(exp(d*x+c)+1)*x+1/a^3/(a^2+b^2)/d*b^4*e*ln(exp(d*x+c)+1)+1/a^3/(a^2+b
^2)/d*b^4*e*ln(exp(d*x+c)-1)-1/a^3/(a^2+b^2)/d^2*b^4*f*dilog(exp(d*x+c))+1/
a^3/(a^2+b^2)/d^2*b^4*f*dilog(exp(d*x+c)+1)+12/(a^2+b^2)/d*b^2*f/(4*a^2+4*b
^2)*ln(1-I*exp(d*x+c))*a*x+12/(a^2+b^2)/d^2*b^2*f/(4*a^2+4*b^2)*ln(1-I*exp(
d*x+c))*a*c-12/(a^2+b^2)/d^2*b^2*f*c/(4*a^2+4*b^2)*a*ln(1+exp(2*d*x+2*c))+1
/2/(a^2+b^2)^(5/2)/d^2*a^2*b^2*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^
2)^(1/2))-12/(a^2+b^2)/d^2*a^2*f*c/(4*a^2+4*b^2)*b*arctan(exp(d*x+c))+12/(a
^2+b^2)/d*b^2*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*a*x+12/(a^2+b^2)/d^2*b^2*f
/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*a*c+6*I*a^2/d^2/(a^2+b^2)*f/(4*a^2+4*b^2)
*dilog(1-I*exp(d*x+c))*b-6*I*a^2/d^2/(a^2+b^2)*f/(4*a^2+4*b^2)*dilog(1+I*ex
p(d*x+c))*b+10*I/d^2/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c-10*
I/d^2/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*c+10*I/d/(a^2+b^2)*b
^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x-10*I/d/(a^2+b^2)*b^3*f/(4*a^2+4*b^2
)*ln(1+I*exp(d*x+c))*x-2/(a^2+b^2)/d*ln(exp(d*x+c)+1)*a*f*x+2/(a^2+b^2)/d^2
*a*f*c*ln(exp(d*x+c)-1)+1/(a^2+b^2)/d^2*b^2*f/a*dilog(exp(d*x+c))-1/(a^2+b^
2)/d^2*b^2*f/a*dilog(exp(d*x+c)+1)-1/(a^2+b^2)/d*b^2*e/a*ln(exp(d*x+c)-1)-1
/(a^2+b^2)/d*b^2*e/a*ln(exp(d*x+c)+1)-1/d^2*b^2*f/(a^2+b^2)^(3/2)*arctanh(1
/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d^2/(a^2+b^2)^(5/2)*b^4*f*arctan
h(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/a^3/d^2/(a^2+b^2)^2*b^6*f*dil
og((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/a^3/d^2/(a^2+b
^2)^2*b^6*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/(
a^2+b^2)/d^2*b^2*f*c/a*ln(exp(d*x+c)-1)-1/(a^2+b^2)/d*b^2*f/a*ln(exp(d*x+c)
+1)*x-(2*a*b^2*d*e*exp(6*d*x+6*c)-2*b^3*d*f*x*exp(5*d*x+5*c)+a^2*b*d*e*exp(
5*d*x+5*c)+4*a*b^2*d*e*exp(4*d*x+4*c)+3*a^2*b*d*f*x*exp(d*x+c)-a^2*b*d*f*x*
exp(3*d*x+3*c)-2*b^3*d*f*x*exp(7*d*x+7*c)+4*a^3*d*f*x*exp(6*d*x+6*c)-3*a^2*
b*d*e*exp(7*d*x+7*c)+4*a^3*d*f*x*exp(2*d*x+2*c)-a^2*b*d*e*exp(3*d*x+3*c)+2*
a*b^2*d*e*exp(2*d*x+2*c)+2*b^3*d*e*exp(d*x+c)-a^2*b*f*exp(d*x+c)+2*b^3*d*e*
exp(3*d*x+3*c)+4*a^3*d*e*exp(2*d*x+2*c)+a^2*b*f*exp(3*d*x+3*c)-a*b^2*f*exp(
2*d*x+2*c)+2*a^3*f*exp(6*d*x+6*c)-a*b^2*f-2*a^3*f*exp(2*d*x+2*c)+2*a*b^2*d*
f*x*exp(2*d*x+2*c)+2*b^3*d*f*x*exp(d*x+c)+3*a^2*b*d*e*exp(d*x+c)+2*b^3*d*f*
x*exp(3*d*x+3*c)+a^2*b*f*exp(5*d*x+5*c)+a*b^2*f*exp(4*d*x+4*c)-2*b^3*d*e*ex
p(7*d*x+7*c)+4*a^3*d*e*exp(6*d*x+6*c)-a^2*b*f*exp(7*d*x+7*c)+a*b^2*f*exp(6*
d*x+6*c)-2*b^3*d*e*exp(5*d*x+5*c)-3*a^2*b*d*f*x*exp(7*d*x+7*c)+2*a*b^2*d*f*
x*exp(6*d*x+6*c)+a^2*b*d*f*x*exp(5*d*x+5*c)+4*a*b^2*d*f*x*exp(4*d*x+4*c))/d
^2/(a^2+b^2)/(1+exp(2*d*x+2*c))^2/a^2/(exp(2*d*x+2*c)-1)^2-6*I*a^2/d^2/(a^2
+b^2)*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*b*c+6*I*a^2/d^2/(a^2+b^2)*f/(4*a^2
+4*b^2)*ln(1-I*exp(d*x+c))*b*c-6*I*a^2/d/(a^2+b^2)*f/(4*a^2+4*b^2)*ln(1+I*e
xp(d*x+c))*b*x+6*I*a^2/d/(a^2+b^2)*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*b*x-1
/a^2/(a^2+b^2)/d^2*b^3*f*ln(exp(d*x+c)-1)+1/a^2/(a^2+b^2)/d^2*b^3*f*ln(exp(
d*x+c)+1)+1/2/(a^2+b^2)^(3/2)/d*b^2*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2
```

```
+b^2)^(1/2))+20/(a^2+b^2)/d*b^3*e/(4*a^2+4*b^2)*arctan(exp(d*x+c))-1/2/(a^2
+b^2)^(5/2)/d*b^4*e*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+8/(a^
2+b^2)/d^2*a^3*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))+8/(a^2+b^2)/d^2*a^3*f/
(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))+8/(a^2+b^2)/d*a^3*e/(4*a^2+4*b^2)*ln(1+
exp(2*d*x+2*c))+1/a^3/d^2/(a^2+b^2)^2*b^6*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d
*x+c)-b)-1/a^3/d/(a^2+b^2)^2*b^6*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a
+(a^2+b^2)^(1/2)))*x-1/a^3/d/(a^2+b^2)^2*b^6*f*ln((b*exp(d*x+c)+(a^2+b^2)^(
1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/a^3/d^2/(a^2+b^2)^2*b^6*f*ln((-b*exp(d*x+c
)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/a^3/d^2/(a^2+b^2)^2*b^6*f*ln
((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+10*I/d^2/(a^2+b^2)
*b^3*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))-10*I/d^2/(a^2+b^2)*b^3*f/(4*a^2+
4*b^2)*dilog(1+I*exp(d*x+c))-1/2/(a^2+b^2)^(3/2)/d^2*b^2*f*c*arctanh(1/2*(2
*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-20/(a^2+b^2)/d^2*b^3*f*c/(4*a^2+4*b^2)*
arctan(exp(d*x+c))+1/2/(a^2+b^2)^(5/2)/d^2*b^4*f*c*arctanh(1/2*(2*b*exp(d*x
+c)+2*a)/(a^2+b^2)^(1/2))+12/(a^2+b^2)/d^2*b^2*f/(4*a^2+4*b^2)*dilog(1+I*ex
p(d*x+c))*a+12/(a^2+b^2)/d^2*b^2*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))*a-8/
(a^2+b^2)/d^2*a^3*f*c/(4*a^2+4*b^2)*ln(1+exp(2*d*x+2*c))+12/(a^2+b^2)/d*a^2
*e/(4*a^2+4*b^2)*b*arctan(exp(d*x+c))+12/(a^2+b^2)/d*b^2*e/(4*a^2+4*b^2)*a*
ln(1+exp(2*d*x+2*c))-1/2/(a^2+b^2)^(5/2)/d*a^2*b^2*e*arctanh(1/2*(2*b*exp(d
*x+c)+2*a)/(a^2+b^2)^(1/2))+8/(a^2+b^2)/d*a^3*f/(4*a^2+4*b^2)*ln(1+I*exp(d*
x+c))*x+8/(a^2+b^2)/d^2*a^3*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*c+8/(a^2+b^2
)/d*a^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x+8/(a^2+b^2)/d^2*a^3*f/(4*a^2+4
*b^2)*ln(1-I*exp(d*x+c))*c-1/(a^2+b^2)/d^2*b*f*ln(exp(d*x+c)-1)+1/(a^2+b^2)
/d^2*b*f*ln(exp(d*x+c)+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -(b^6*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^7 + 2*a^5*b^2 + a
^3*b^4)*d) + (3*a^2*b + 5*b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4
)*d) - (2*a^3 + 3*a*b^2)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)
*d) + (4*a*b^2*e^(-4*d*x - 4*c) - (3*a^2*b + 2*b^3)*e^(-d*x - c) + 2*(2*a^3
 + a*b^2)*e^(-2*d*x - 2*c) + (a^2*b - 2*b^3)*e^(-3*d*x - 3*c) - (a^2*b - 2*
b^3)*e^(-5*d*x - 5*c) + 2*(2*a^3 + a*b^2)*e^(-6*d*x - 6*c) + (3*a^2*b + 2*b
^3)*e^(-7*d*x - 7*c))/((a^4 + a^2*b^2 - 2*(a^4 + a^2*b^2)*e^(-4*d*x - 4*c)
+ (a^4 + a^2*b^2)*e^(-8*d*x - 8*c))*d) + (2*a^2 - b^2)*log(e^(-d*x - c) + 1
)/(a^3*d) + (2*a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d))*e + (128*a^2*d*int
egrate(1/64*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 64*b^2*d*integrate(1/64*x/(
a^3*d*e^(d*x + c) + a^3*d), x) - 128*a^2*d*integrate(1/64*x/(a^3*d*e^(d*x +
 c) - a^3*d), x) + 64*b^2*d*integrate(1/64*x/(a^3*d*e^(d*x + c) - a^3*d), x
) - a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) + 1)/(a^3*d^2)) + a*b*((d*x
+ c)/(a^3*d^2) - log(e^(d*x + c) - 1)/(a^3*d^2)) + (a*b^2 + (a^2*b*e^(7*c)
+ (3*a^2*b*d*e^(7*c) + 2*b^3*d*e^(7*c))*x)*e^(7*d*x) - (2*a^3*e^(6*c) + a*b
^2*e^(6*c) + 2*(2*a^3*d*e^(6*c) + a*b^2*d*e^(6*c))*x)*e^(6*d*x) - (a^2*b*e^
(5*c) + (a^2*b*d*e^(5*c) - 2*b^3*d*e^(5*c))*x)*e^(5*d*x) - (4*a*b^2*d*x*e^(
4*c) + a*b^2*e^(4*c))*e^(4*d*x) - (a^2*b*e^(3*c) - (a^2*b*d*e^(3*c) - 2*b^3
*d*e^(3*c))*x)*e^(3*d*x) + (2*a^3*e^(2*c) + a*b^2*e^(2*c) - 2*(2*a^3*d*e^(2
*c) + a*b^2*d*e^(2*c))*x)*e^(2*d*x) + (a^2*b*e^c - (3*a^2*b*d*e^c + 2*b^3*d
*e^c)*x)*e^(d*x))/(a^4*d^2 + a^2*b^2*d^2 + (a^4*d^2*e^(8*c) + a^2*b^2*d^2*e
^(8*c))*e^(8*d*x) - 2*(a^4*d^2*e^(4*c) + a^2*b^2*d^2*e^(4*c))*e^(4*d*x)) +
64*integrate(-1/32*(a*b^6*x*e^(d*x + c) - b^7*x)/(a^7*b + 2*a^5*b^3 + a^3*b
```

```
^5 - (a^7*b*e^(2*c) + 2*a^5*b^3*e^(2*c) + a^3*b^5*e^(2*c))*e^(2*d*x) - 2*(a
^8*e^c + 2*a^6*b^2*e^c + a^4*b^4*e^c)*e^(d*x)), x) + 64*integrate(1/64*((3*
a^2*b*e^c + 5*b^3*e^c)*x*e^(d*x) - 2*(2*a^3 + 3*a*b^2)*x)/(a^4 + 2*a^2*b^2
+ b^4 + (a^4*e^(2*c) + 2*a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x))*f
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**3*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.501 $\int \frac{\mathbf{csch}^3(c+dx)\mathbf{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=211

$$-\frac{b^6\log(a+b\sinh(c+dx))}{a^3d\left(a^2+b^2\right)^2}-\frac{\left(2a^2-b^2\right)\log(\sinh(c+dx))}{a^3d}+\frac{b\left(a^2+2b^2\right)\tan^{-1}(\sinh(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{b\tan^{-1}(\sinh(c+dx}{2d\left(a^2+b^2\right)}$$

```
[Out] (b*ArcTan[Sinh[c + d*x]])/(2*(a^2 + b^2)*d) + (b*(a^2 + 2*b^2)*ArcTan[Sinh[
c + d*x]])/((a^2 + b^2)^2*d) + (b*Csch[c + d*x])/(a^2*d) - Csch[c + d*x]^2/
(2*a*d) + (a*(2*a^2 + 3*b^2)*Log[Cosh[c + d*x]])/((a^2 + b^2)^2*d) - ((2*a^
2 - b^2)*Log[Sinh[c + d*x]])/(a^3*d) - (b^6*Log[a + b*Sinh[c + d*x]])/(a^3*
(a^2 + b^2)^2*d) - (Sech[c + d*x]^2*(a - b*Sinh[c + d*x]))/(2*(a^2 + b^2)*d
)
```

Rubi [A] time = 0.366715, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2837, 12, 894, 639, 203, 635, 260}

$$-\frac{b^6\log(a+b\sinh(c+dx))}{a^3d\left(a^2+b^2\right)^2}-\frac{\left(2a^2-b^2\right)\log(\sinh(c+dx))}{a^3d}+\frac{b\left(a^2+2b^2\right)\tan^{-1}(\sinh(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{b\tan^{-1}(\sinh(c+dx}{2d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[c + d*x]^3*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/(2*(a^2 + b^2)*d) + (b*(a^2 + 2*b^2)*ArcTan[Sinh[
c + d*x]])/((a^2 + b^2)^2*d) + (b*Csch[c + d*x])/(a^2*d) - Csch[c + d*x]^2/
(2*a*d) + (a*(2*a^2 + 3*b^2)*Log[Cosh[c + d*x]])/((a^2 + b^2)^2*d) - ((2*a^
2 - b^2)*Log[Sinh[c + d*x]])/(a^3*d) - (b^6*Log[a + b*Sinh[c + d*x]])/(a^3*
(a^2 + b^2)^2*d) - (Sech[c + d*x]^2*(a - b*Sinh[c + d*x]))/(2*(a^2 + b^2)*d
)
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)}\,dx = \frac{b^3\operatorname{Subst}\left(\int\frac{b^3}{x^3(a+x)\left(-b^2-x^2\right)^2}\,dx,x,b\sinh(c+dx)\right)}{d}$$

$$= \frac{b^6\operatorname{Subst}\left(\int\frac{1}{x^3(a+x)\left(-b^2-x^2\right)^2}\,dx,x,b\sinh(c+dx)\right)}{d}$$

$$= \frac{b^6\operatorname{Subst}\left(\int\left(\frac{1}{ab^4x^3}-\frac{1}{a^2b^4x^2}+\frac{-2a^2+b^2}{a^3b^6x}-\frac{1}{a^3\left(a^2+b^2\right)^2(a+x)}+\frac{b^2+ax}{b^4\left(a^2+b^2\right)\left(b^2+x^2\right)^2}+\frac{b^2\left(a^2+2b^2\right)+\cdots}{b^6\left(a^2+b^2\right)\cdots}\right.\right.}{d}$$

$$= \frac{b\operatorname{csch}(c+dx)}{a^2d}-\frac{\operatorname{csch}^2(c+dx)}{2ad}-\frac{\left(2a^2-b^2\right)\log(\sinh(c+dx))}{a^3d}-\frac{b^6\log(a+b\sinh(\cdots}{a^3\left(a^2+b^2\right)^{\cdots}}$$

$$= \frac{b\operatorname{csch}(c+dx)}{a^2d}-\frac{\operatorname{csch}^2(c+dx)}{2ad}-\frac{\left(2a^2-b^2\right)\log(\sinh(c+dx))}{a^3d}-\frac{b^6\log(a+b\sinh(\cdots}{a^3\left(a^2+b^2\right)^{\cdots}}$$

$$= \frac{b\tan^{-1}(\sinh(c+dx))}{2\left(a^2+b^2\right)d}+\frac{b\left(a^2+2b^2\right)\tan^{-1}(\sinh(c+dx))}{\left(a^2+b^2\right)^2d}+\frac{b\operatorname{csch}(c+dx)}{a^2d}-\frac{\operatorname{csch}^{2}\cdots}{2\cdots}$$

Mathematica [C] time = 0.807566, size = 237, normalized size = 1.12

$$\frac{-\frac{a\operatorname{sech}^2(c+dx)}{a^2+b^2}-\frac{2b^6\log(a+b\sinh(c+dx))}{a^3\left(a^2+b^2\right)^2}+\frac{(a-ib)\left(2a^2+iab+2b^2\right)\log(-\sinh(c+dx)+i)}{\left(a^2+b^2\right)^2}-\frac{2\left(2a^2-b^2\right)\log(\sinh(c+dx))}{a^3}+\frac{(a+ib)\left(2a^2-iab+2b^2\right)\log(\sinh(c+\cdots}{\left(a^2+b^2\right)^2}\cdots}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csch[c + d*x]^3*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((b*ArcTan[Sinh[c + d*x]])/(a^2 + b^2) + (2*b*Csch[c + d*x])/a^2 - Csch[c +
 d*x]^2/a + ((a - I*b)*(2*a^2 + I*a*b + 2*b^2)*Log[I - Sinh[c + d*x]])/(a^2
 + b^2)^2 - (2*(2*a^2 - b^2)*Log[Sinh[c + d*x]])/a^3 + ((a + I*b)*(2*a^2 -
I*a*b + 2*b^2)*Log[I + Sinh[c + d*x]])/(a^2 + b^2)^2 - (2*b^6*Log[a + b*Sin
h[c + d*x]])/(a^3*(a^2 + b^2)^2) - (a*Sech[c + d*x]^2)/(a^2 + b^2) + (b*Sec
h[c + d*x]*Tanh[c + d*x])/(a^2 + b^2))/(2*d)
```

Maple [B] time = 0.003, size = 539, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/8/d/a*tanh(1/2*d*x+1/2*c)^2-1/2/d/a^2*tanh(1/2*d*x+1/2*c)*b-1/8/d/a/tanh
(1/2*d*x+1/2*c)^2-2/d/a*ln(tanh(1/2*d*x+1/2*c))+1/d/a^3*ln(tanh(1/2*d*x+1/2
*c))*b^2+1/2/d*b/a^2/tanh(1/2*d*x+1/2*c)-1/d*b^6/(a^2+b^2)^2/a^3*ln(tanh(1/
2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)*b-a)-1/d/(a^2+b^2)^2/(tanh(1/2*d*x+1
/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)^3*a^2*b-1/d/(a^2+b^2)^2/(tanh(1/2*d*x+1/2*
c)^2+1)^2*tanh(1/2*d*x+1/2*c)^3*b^3+2/d/(a^2+b^2)^2/(tanh(1/2*d*x+1/2*c)^2+
1)^2*tanh(1/2*d*x+1/2*c)^2*a^3+2/d/(a^2+b^2)^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*
tanh(1/2*d*x+1/2*c)^2*a*b^2+1/d/(a^2+b^2)^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tan
h(1/2*d*x+1/2*c)*a^2*b+1/d/(a^2+b^2)^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2
*d*x+1/2*c)*b^3+2/d/(a^2+b^2)^2*ln(tanh(1/2*d*x+1/2*c)^2+1)*a^3+3/d/(a^2+b^
2)^2*ln(tanh(1/2*d*x+1/2*c)^2+1)*a*b^2+3/d/(a^2+b^2)^2*arctan(tanh(1/2*d*x+
1/2*c))*a^2*b+5/d/(a^2+b^2)^2*arctan(tanh(1/2*d*x+1/2*c))*b^3
```

Maxima [B] time = 1.7642, size = 564, normalized size = 2.67

$$-\frac{b^6 \log\left(-2\,ae^{(-dx-c)} + be^{(-2\,dx-2\,c)} - b\right)}{\left(a^7 + 2\,a^5b^2 + a^3b^4\right)d} - \frac{\left(3\,a^2b + 5\,b^3\right)\arctan\left(e^{(-dx-c)}\right)}{\left(a^4 + 2\,a^2b^2 + b^4\right)d} + \frac{\left(2\,a^3 + 3\,ab^2\right)\log\left(e^{(-2\,dx-2\,c)} + 1\right)}{\left(a^4 + 2\,a^2b^2 + b^4\right)d} - \frac{4\,ab^2 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxim
a")
```

```
[Out] -b^6*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^7 + 2*a^5*b^2 + a^
3*b^4)*d) - (3*a^2*b + 5*b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)
*d) + (2*a^3 + 3*a*b^2)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*
d) - (4*a*b^2*e^(-4*d*x - 4*c) - (3*a^2*b + 2*b^3)*e^(-d*x - c) + 2*(2*a^3
+ a*b^2)*e^(-2*d*x - 2*c) + (a^2*b - 2*b^3)*e^(-3*d*x - 3*c) - (a^2*b - 2*b
^3)*e^(-5*d*x - 5*c) + 2*(2*a^3 + a*b^2)*e^(-6*d*x - 6*c) + (3*a^2*b + 2*b^
3)*e^(-7*d*x - 7*c))/((a^4 + a^2*b^2 - 2*(a^4 + a^2*b^2)*e^(-4*d*x - 4*c) +
 (a^4 + a^2*b^2)*e^(-8*d*x - 8*c))*d) - (2*a^2 - b^2)*log(e^(-d*x - c) + 1)
/(a^3*d) - (2*a^2 - b^2)*log(e^(-d*x - c) - 1)/(a^3*d)
```

Fricas [B] time = 7.08528, size = 7170, normalized size = 33.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="frica
s")

[Out] ((3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*cosh(d*x + c)^7 + (3*a^5*b + 5*a^3*b^3 + 2
*a*b^5)*sinh(d*x + c)^7 - 2*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*cosh(d*x + c)^6 -
 (4*a^6 + 6*a^4*b^2 + 2*a^2*b^4 - 7*(3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*cosh(d*
x + c))*sinh(d*x + c)^6 - (a^5*b - a^3*b^3 - 2*a*b^5)*cosh(d*x + c)^5 - (a^
5*b - a^3*b^3 - 2*a*b^5 - 21*(3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*cosh(d*x + c)^
2 + 12*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*(a^
4*b^2 + a^2*b^4)*cosh(d*x + c)^4 - (4*a^4*b^2 + 4*a^2*b^4 - 35*(3*a^5*b + 5
*a^3*b^3 + 2*a*b^5)*cosh(d*x + c)^3 + 30*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*cosh
(d*x + c)^2 + 5*(a^5*b - a^3*b^3 - 2*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^4
 + (a^5*b - a^3*b^3 - 2*a*b^5)*cosh(d*x + c)^3 + (a^5*b - a^3*b^3 - 2*a*b^5
 + 35*(3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*cosh(d*x + c)^4 - 40*(2*a^6 + 3*a^4*b^
2 + a^2*b^4)*cosh(d*x + c)^3 - 10*(a^5*b - a^3*b^3 - 2*a*b^5)*cosh(d*x + c)
^2 - 16*(a^4*b^2 + a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(2*a^6 + 3*a
^4*b^2 + a^2*b^4)*cosh(d*x + c)^2 - (4*a^6 + 6*a^4*b^2 + 2*a^2*b^4 - 21*(3*
a^5*b + 5*a^3*b^3 + 2*a*b^5)*cosh(d*x + c)^5 + 30*(2*a^6 + 3*a^4*b^2 + a^2*
b^4)*cosh(d*x + c)^4 + 10*(a^5*b - a^3*b^3 - 2*a*b^5)*cosh(d*x + c)^3 + 24*
(a^4*b^2 + a^2*b^4)*cosh(d*x + c)^2 - 3*(a^5*b - a^3*b^3 - 2*a*b^5)*cosh(d*
x + c))*sinh(d*x + c)^2 + ((3*a^5*b + 5*a^3*b^3)*cosh(d*x + c)^8 + 56*(3*a^
5*b + 5*a^3*b^3)*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*(3*a^5*b + 5*a^3*b^3)
*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*(3*a^5*b + 5*a^3*b^3)*cosh(d*x + c)*si
nh(d*x + c)^7 + (3*a^5*b + 5*a^3*b^3)*sinh(d*x + c)^8 + 3*a^5*b + 5*a^3*b^3
 - 2*(3*a^5*b + 5*a^3*b^3)*cosh(d*x + c)^4 - 2*(3*a^5*b + 5*a^3*b^3 - 35*(3
*a^5*b + 5*a^3*b^3)*cosh(d*x + c)^4)*sinh(d*x + c)^4 + 8*(7*(3*a^5*b + 5*a^
3*b^3)*cosh(d*x + c)^5 - (3*a^5*b + 5*a^3*b^3)*cosh(d*x + c))*sinh(d*x + c)
^3 + 4*(7*(3*a^5*b + 5*a^3*b^3)*cosh(d*x + c)^6 - 3*(3*a^5*b + 5*a^3*b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^5*b + 5*a^3*b^3)*cosh(d*x + c)^7
 - (3*a^5*b + 5*a^3*b^3)*cosh(d*x + c)^3)*sinh(d*x + c))*arctan(cosh(d*x + c
) + sinh(d*x + c)) - (3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*cosh(d*x + c) - (b^6*c
osh(d*x + c)^8 + 56*b^6*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*b^6*cosh(d*x +
 c)^2*sinh(d*x + c)^6 + 8*b^6*cosh(d*x + c)*sinh(d*x + c)^7 + b^6*sinh(d*x
 + c)^8 - 2*b^6*cosh(d*x + c)^4 + b^6 + 2*(35*b^6*cosh(d*x + c)^4 - b^6)*sin
h(d*x + c)^4 + 8*(7*b^6*cosh(d*x + c)^5 - b^6*cosh(d*x + c))*sinh(d*x + c)^
3 + 4*(7*b^6*cosh(d*x + c)^6 - 3*b^6*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(
b^6*cosh(d*x + c)^7 - b^6*cosh(d*x + c)^3)*sinh(d*x + c))*log(2*(b*sinh(d*x
 + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + ((2*a^6 + 3*a^4*b^2)*cosh(d*x
 + c)^8 + 56*(2*a^6 + 3*a^4*b^2)*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*(2*a^
6 + 3*a^4*b^2)*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*(2*a^6 + 3*a^4*b^2)*cosh
(d*x + c)*sinh(d*x + c)^7 + (2*a^6 + 3*a^4*b^2)*sinh(d*x + c)^8 + 2*a^6 + 3
*a^4*b^2 - 2*(2*a^6 + 3*a^4*b^2)*cosh(d*x + c)^4 - 2*(2*a^6 + 3*a^4*b^2 - 3
5*(2*a^6 + 3*a^4*b^2)*cosh(d*x + c)^4)*sinh(d*x + c)^4 + 8*(7*(2*a^6 + 3*a^
4*b^2)*cosh(d*x + c)^5 - (2*a^6 + 3*a^4*b^2)*cosh(d*x + c))*sinh(d*x + c)^3
 + 4*(7*(2*a^6 + 3*a^4*b^2)*cosh(d*x + c)^6 - 3*(2*a^6 + 3*a^4*b^2)*cosh(d*
x + c)^2)*sinh(d*x + c)^2 + 8*((2*a^6 + 3*a^4*b^2)*cosh(d*x + c)^7 - (2*a^6
 + 3*a^4*b^2)*cosh(d*x + c)^3)*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x
 + c) - sinh(d*x + c))) - ((2*a^6 + 3*a^4*b^2 - b^6)*cosh(d*x + c)^8 + 56*(
2*a^6 + 3*a^4*b^2 - b^6)*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*(2*a^6 + 3*a^
4*b^2 - b^6)*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*(2*a^6 + 3*a^4*b^2 - b^6)*
cosh(d*x + c)*sinh(d*x + c)^7 + (2*a^6 + 3*a^4*b^2 - b^6)*sinh(d*x + c)^8 +
 2*a^6 + 3*a^4*b^2 - b^6 - 2*(2*a^6 + 3*a^4*b^2 - b^6)*cosh(d*x + c)^4 - 2*
(2*a^6 + 3*a^4*b^2 - b^6 - 35*(2*a^6 + 3*a^4*b^2 - b^6)*cosh(d*x + c)^4)*si
nh(d*x + c)^4 + 8*(7*(2*a^6 + 3*a^4*b^2 - b^6)*cosh(d*x + c)^5 - (2*a^6 + 3
*a^4*b^2 - b^6)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(2*a^6 + 3*a^4*b^2 -
b^6)*cosh(d*x + c)^6 - 3*(2*a^6 + 3*a^4*b^2 - b^6)*cosh(d*x + c)^2)*sinh(d*
x + c)^2 + 8*((2*a^6 + 3*a^4*b^2 - b^6)*cosh(d*x + c)^7 - (2*a^6 + 3*a^4*b^

```
2 - b^6)*cosh(d*x + c)^3)*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c)
 - sinh(d*x + c))) + (7*(3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*cosh(d*x + c)^6 - 3
*a^5*b - 5*a^3*b^3 - 2*a*b^5 - 12*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*cosh(d*x +
c)^5 - 5*(a^5*b - a^3*b^3 - 2*a*b^5)*cosh(d*x + c)^4 - 16*(a^4*b^2 + a^2*b^
4)*cosh(d*x + c)^3 + 3*(a^5*b - a^3*b^3 - 2*a*b^5)*cosh(d*x + c)^2 - 4*(2*a
^6 + 3*a^4*b^2 + a^2*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^7 + 2*a^5*b^2 +
 a^3*b^4)*d*cosh(d*x + c)^8 + 56*(a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c
)^3*sinh(d*x + c)^5 + 28*(a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^2*sinh
(d*x + c)^6 + 8*(a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7
 + (a^7 + 2*a^5*b^2 + a^3*b^4)*d*sinh(d*x + c)^8 - 2*(a^7 + 2*a^5*b^2 + a^3
*b^4)*d*cosh(d*x + c)^4 + 2*(35*(a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)
^4 - (a^7 + 2*a^5*b^2 + a^3*b^4)*d)*sinh(d*x + c)^4 + 8*(7*(a^7 + 2*a^5*b^2
 + a^3*b^4)*d*cosh(d*x + c)^5 - (a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)
)*sinh(d*x + c)^3 + 4*(7*(a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^6 - 3*
(a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + (a^7 + 2*a
^5*b^2 + a^3*b^4)*d + 8*((a^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^7 - (a
^7 + 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^3)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.77478, size = 652, normalized size = 3.09

$$-\frac{b^7\log\left(\left|b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,a\right|\right)}{a^7bd+2\,a^5b^3d+a^3b^5d}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(3\,a^2b+5\,b^3\right)}{4\left(a^4d+2\,a^2b^2d+b^4d\right)}+\frac{\left(2\,a^3+3\,ab^2\right)\log\left(\left(\right.\right.}{2\left(a^4d+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac"
)
```

```
[Out] -b^7*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^7*b*d + 2*a^5*b^3*d
+ a^3*b^5*d) + 1/4*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*
(3*a^2*b + 5*b^3)/(a^4*d + 2*a^2*b^2*d + b^4*d) + 1/2*(2*a^3 + 3*a*b^2)*log
((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^4*d + 2*a^2*b^2*d + b^4*d) - 1/2*(2
*a^3*(e^(d*x + c) - e^(-d*x - c))^2 + 3*a*b^2*(e^(d*x + c) - e^(-d*x - c))^
2 - 2*a^2*b*(e^(d*x + c) - e^(-d*x - c)) - 2*b^3*(e^(d*x + c) - e^(-d*x - c
)) + 12*a^3 + 16*a*b^2)/((a^4*d + 2*a^2*b^2*d + b^4*d)*((e^(d*x + c) - e^(-
d*x - c))^2 + 4)) - (2*a^2 - b^2)*log(abs(e^(d*x + c) - e^(-d*x - c)))/(a^3
*d) + 1/2*(6*a^2*(e^(d*x + c) - e^(-d*x - c))^2 - 3*b^2*(e^(d*x + c) - e^(-
d*x - c))^2 + 4*a*b*(e^(d*x + c) - e^(-d*x - c)) - 4*a^2)/(a^3*d*(e^(d*x +
c) - e^(-d*x - c))^2)
```

3.502 $\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$

Optimal. Leaf size=38

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))},x\right)$$

```
[Out] Unintegrable[(Csch[c + d*x]^3*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d
*x])), x]
```

Rubi [A] time = 0.133629, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csch[c + d*x]^3*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x])),x]
```

```
[Out] Defer[Int][(Csch[c + d*x]^3*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x
])), x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx=\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}\,dx$$

Mathematica [F] time = 180.001, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(Csch[c + d*x]^3*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x]
)),x]
```

```
[Out] $Aborted
```

Maple [A] time = 3.319, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx+c))^3(\operatorname{sech}(dx+c))^3}{(fx+e)(a+b\sinh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="maxima")
```

```
[Out] -(a*b^2*f - (2*b^3*d*e*e^(7*c) + (3*d*e - f)*a^2*b*e^(7*c) + (3*a^2*b*d*f*e
^(7*c) + 2*b^3*d*f*e^(7*c))*x)*e^(7*d*x) + (2*(2*d*e - f)*a^3*e^(6*c) + (2*
d*e - f)*a*b^2*e^(6*c) + 2*(2*a^3*d*f*e^(6*c) + a*b^2*d*f*e^(6*c))*x)*e^(6*
d*x) - (2*b^3*d*e*e^(5*c) - (d*e - f)*a^2*b*e^(5*c) - (a^2*b*d*f*e^(5*c) -
2*b^3*d*f*e^(5*c))*x)*e^(5*d*x) + (4*a*b^2*d*f*x*e^(4*c) + (4*d*e - f)*a*b^
2*e^(4*c))*e^(4*d*x) + (2*b^3*d*e*e^(3*c) - (d*e + f)*a^2*b*e^(3*c) - (a^2*
b*d*f*e^(3*c) - 2*b^3*d*f*e^(3*c))*x)*e^(3*d*x) + (2*(2*d*e + f)*a^3*e^(2*c
) + (2*d*e + f)*a*b^2*e^(2*c) + 2*(2*a^3*d*f*e^(2*c) + a*b^2*d*f*e^(2*c))*x
)*e^(2*d*x) + (2*b^3*d*e*e^c + (3*d*e + f)*a^2*b*e^c + (3*a^2*b*d*f*e^c + 2
*b^3*d*f*e^c)*x)*e^(d*x))/(a^4*d^2*e^2 + a^2*b^2*d^2*e^2 + (a^4*d^2*f^2 + a
^2*b^2*d^2*f^2)*x^2 + 2*(a^4*d^2*e*f + a^2*b^2*d^2*e*f)*x + (a^4*d^2*e^2*e^
(8*c) + a^2*b^2*d^2*e^2*e^(8*c) + (a^4*d^2*f^2*e^(8*c) + a^2*b^2*d^2*f^2*e^
(8*c))*x^2 + 2*(a^4*d^2*e*f*e^(8*c) + a^2*b^2*d^2*e*f*e^(8*c))*x)*e^(8*d*x)
 - 2*(a^4*d^2*e^2*e^(4*c) + a^2*b^2*d^2*e^2*e^(4*c) + (a^4*d^2*f^2*e^(4*c)
+ a^2*b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^4*d^2*e*f*e^(4*c) + a^2*b^2*d^2*e*f*e
^(4*c))*x)*e^(4*d*x)) + 64*integrate(-1/32*(a*b^6*e^(d*x + c) - b^7)/(a^7*b
*e + 2*a^5*b^3*e + a^3*b^5*e + (a^7*b*f + 2*a^5*b^3*f + a^3*b^5*f)*x - (a^7
*b*e*e^(2*c) + 2*a^5*b^3*e*e^(2*c) + a^3*b^5*e*e^(2*c) + (a^7*b*f*e^(2*c) +
 2*a^5*b^3*f*e^(2*c) + a^3*b^5*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^8*e*e^c + 2*a
^6*b^2*e*e^c + a^4*b^4*e*e^c + (a^8*f*e^c + 2*a^6*b^2*f*e^c + a^4*b^4*f*e^c
)*x)*e^(d*x)), x) - 64*integrate(1/64*(b^2*d^2*e^2 + a*b*d*e*f - (2*d^2*e^2
 - f^2)*a^2 - (2*a^2*d^2*f^2 - b^2*d^2*f^2)*x^2 - (4*a^2*d^2*e*f - 2*b^2*d^
2*e*f - a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^
2*f*x + a^3*d^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^
3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) + 64*integrate(-1/64*(b^2
*d^2*e^2 - a*b*d*e*f - (2*d^2*e^2 - f^2)*a^2 - (2*a^2*d^2*f^2 - b^2*d^2*f^2
)*x^2 - (4*a^2*d^2*e*f - 2*b^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3
*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c
 + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*
x)), x) + 64*integrate(-1/64*(2*(2*d^2*e^2 - f^2)*a^3 + 2*(3*d^2*e^2 - f^2)
*a*b^2 + 2*(2*a^3*d^2*f^2 + 3*a*b^2*d^2*f^2)*x^2 + 4*(2*a^3*d^2*e*f + 3*a*b
^2*d^2*e*f)*x - ((3*d^2*e^2 - 2*f^2)*a^2*b*e^c + (5*d^2*e^2 - 2*f^2)*b^3*e^
c + (3*a^2*b*d^2*f^2*e^c + 5*b^3*d^2*f^2*e^c)*x^2 + 2*(3*a^2*b*d^2*e*f*e^c
+ 5*b^3*d^2*e*f*e^c)*x)*e^(d*x))/(a^4*d^2*e^3 + 2*a^2*b^2*d^2*e^3 + b^4*d^2
*e^3 + (a^4*d^2*f^3 + 2*a^2*b^2*d^2*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*e*f
^2 + 2*a^2*b^2*d^2*e*f^2 + b^4*d^2*e*f^2)*x^2 + 3*(a^4*d^2*e^2*f + 2*a^2*b^
2*d^2*e^2*f + b^4*d^2*e^2*f)*x + (a^4*d^2*e^3*e^(2*c) + 2*a^2*b^2*d^2*e^3*e
^(2*c) + b^4*d^2*e^3*e^(2*c) + (a^4*d^2*f^3*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(
2*c) + b^4*d^2*f^3*e^(2*c))*x^3 + 3*(a^4*d^2*e*f^2*e^(2*c) + 2*a^2*b^2*d^2*
e*f^2*e^(2*c) + b^4*d^2*e*f^2*e^(2*c))*x^2 + 3*(a^4*d^2*e^2*f*e^(2*c) + 2*a
^2*b^2*d^2*e^2*f*e^(2*c) + b^4*d^2*e^2*f*e^(2*c))*x)*e^(2*d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*sech(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```

```
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal, debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list( expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list( expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list( expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```